

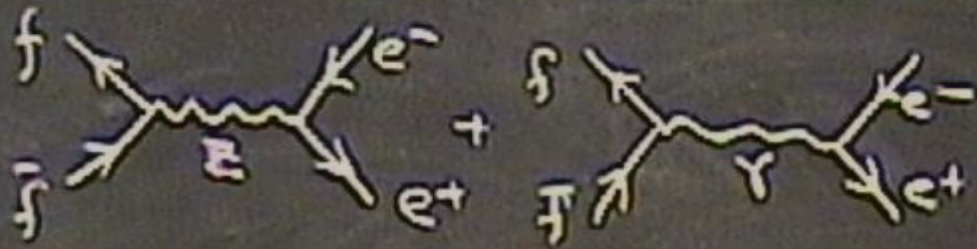
Title: Graduate Course on Standard Model & Quantum Field Theory - 11B

Date: Jan 31, 2007 03:00 PM

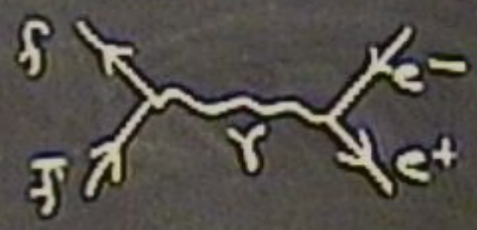
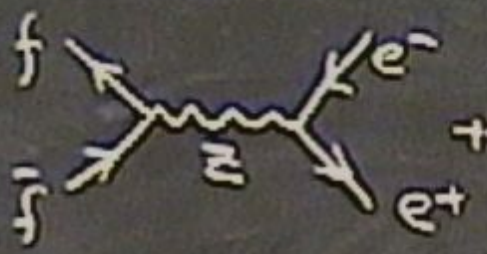
URL: <http://pirsa.org/07010040>

Abstract: Graduate Course on Standard Model & Quantum Field Theory

$$\underline{e^+e^- \rightarrow f\bar{f}}$$



$$\underline{e^+e^- \rightarrow f\bar{f}}$$

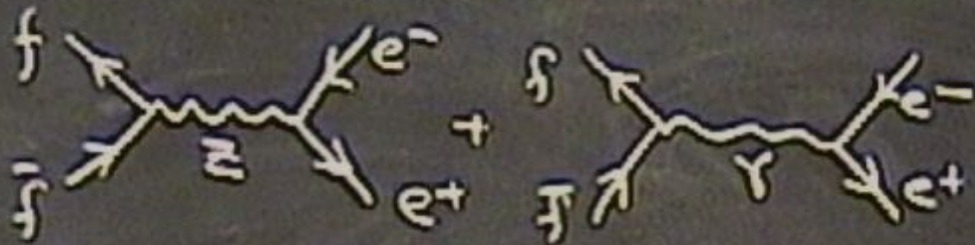


and if $f=e$:

A Feynman diagram showing the annihilation of an electron (e^-) and a positron (e^+) into an electron (e^-) and a positron (e^+) via a photon (γ). The incoming particles are on the left, and the outgoing particles are on the right. A wavy line representing the photon connects the two vertices.



$$\underline{e^+e^- \rightarrow f\bar{f}}$$



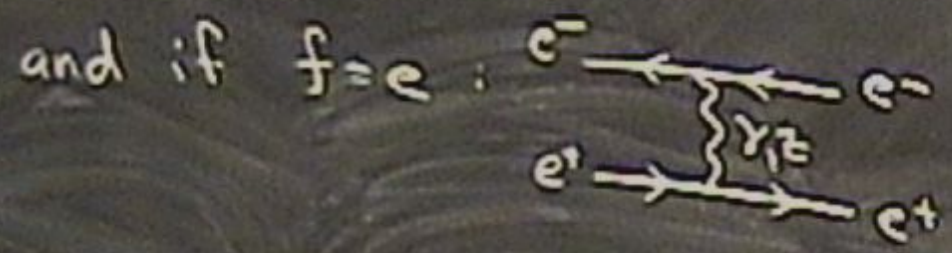
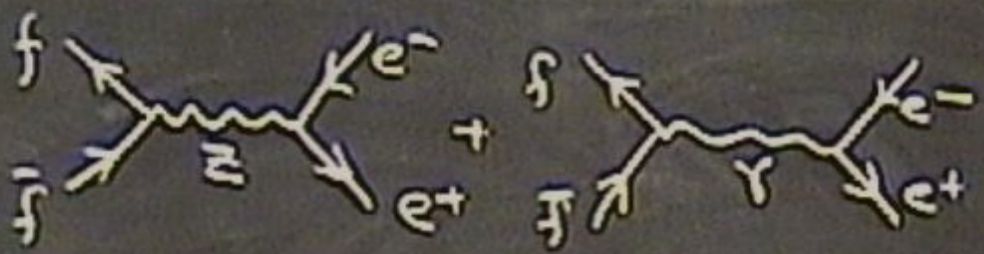
$f = e$



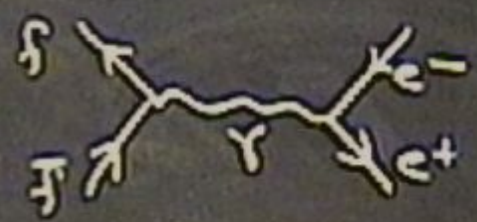
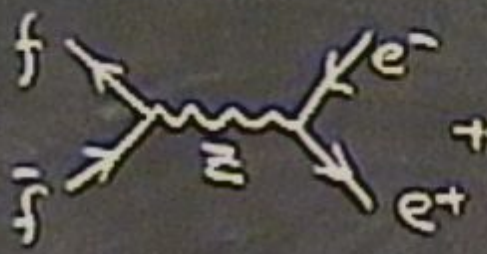
and if $f = \nu$



$$\underline{e^+e^- \rightarrow f\bar{f}}$$



$$\underline{e^+e^- \rightarrow f\bar{f}}$$



and

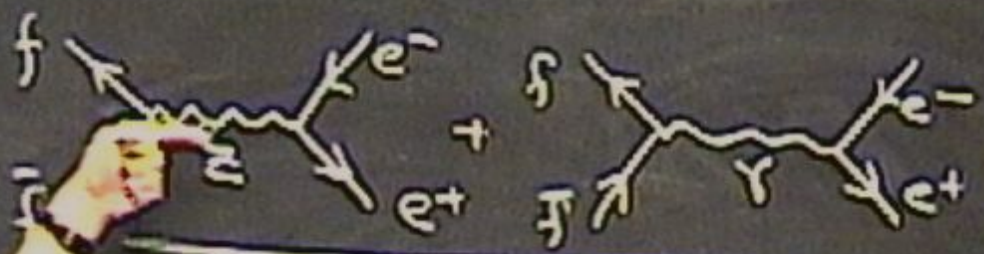


$f \neq e, \nu$

$f = \nu$



$$\underline{e^+e^- \rightarrow f\bar{f}}$$



↳ "s-channel"

and if

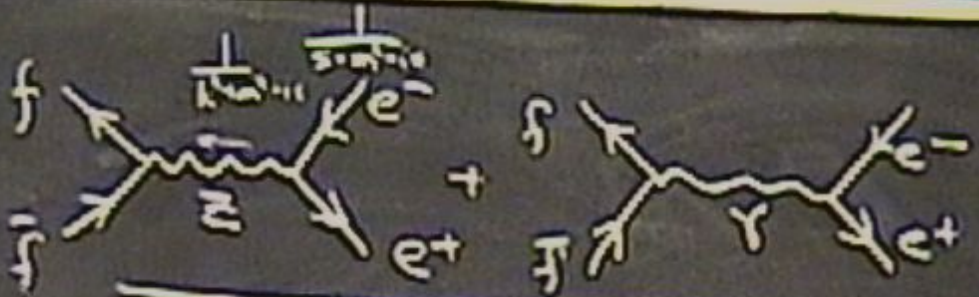


$$f \neq e, \nu$$

and if

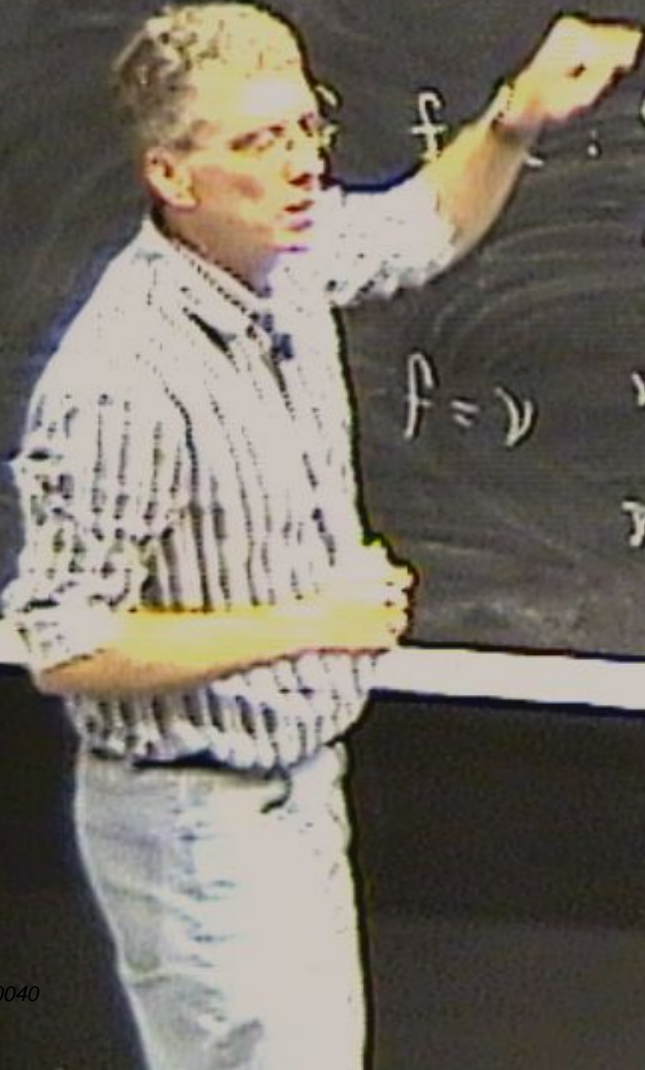
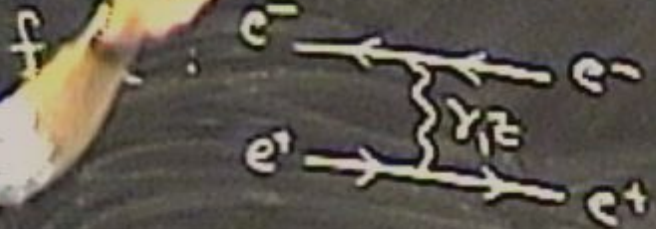


$$\underline{e^- e^- \rightarrow f \bar{f}}$$

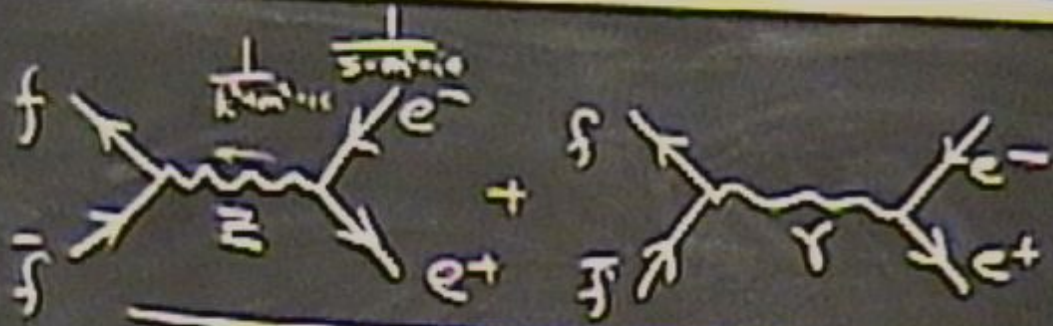


“s-channel”

$$f \neq e, \nu$$

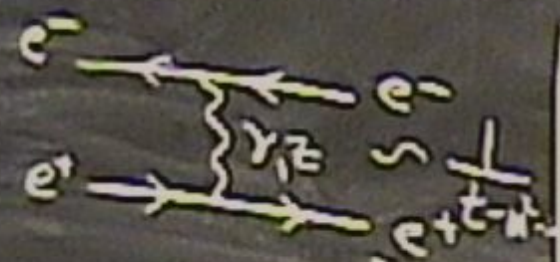


$e^-e^- \rightarrow f\bar{f}$



↳ "s-channel"

if $f = e$:

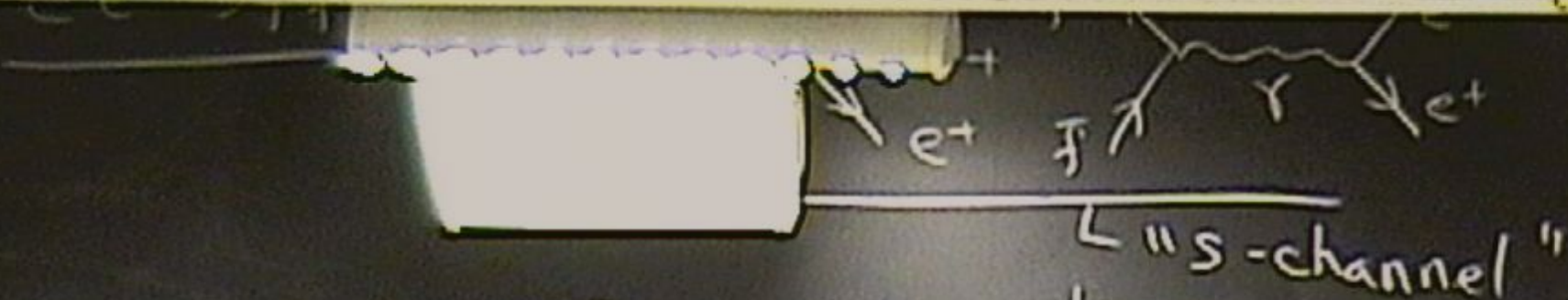


$f \neq e, \nu$

and if $f = \nu$



↳ "t-channel"



and if $f = e$



$f \neq e, \nu$

and if $f = \nu$



$$\langle \bar{\psi}(x) \bar{\psi}(x') | S | e^{-}(p) e^{+}(p) \rangle$$

$$= \frac{1}{2i}$$



$$\langle f(k) \bar{f}(k') | S | e^-(p) e^+(p) \rangle$$

$$= \frac{1}{2i}$$





"s-channel"

and if $f = e$

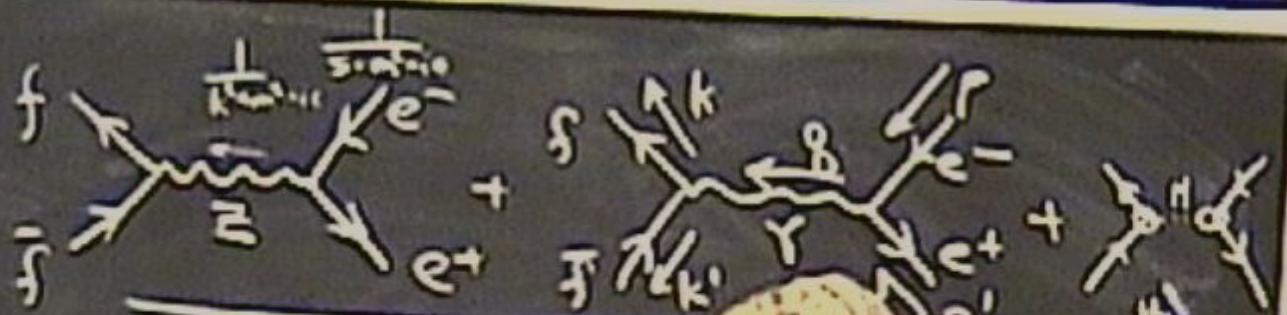


$f \neq e, \nu$

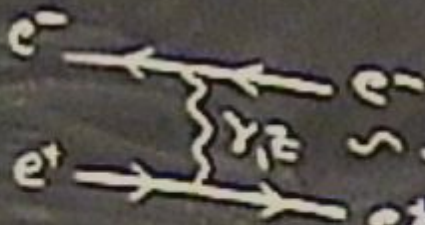
and if $f = \nu$



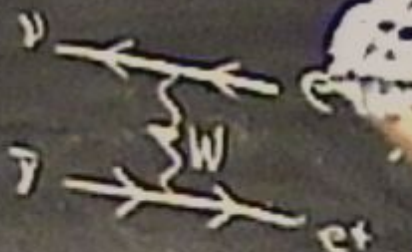
$$\underline{e^+e^- \rightarrow f\bar{f}}$$



and if $f=e$:

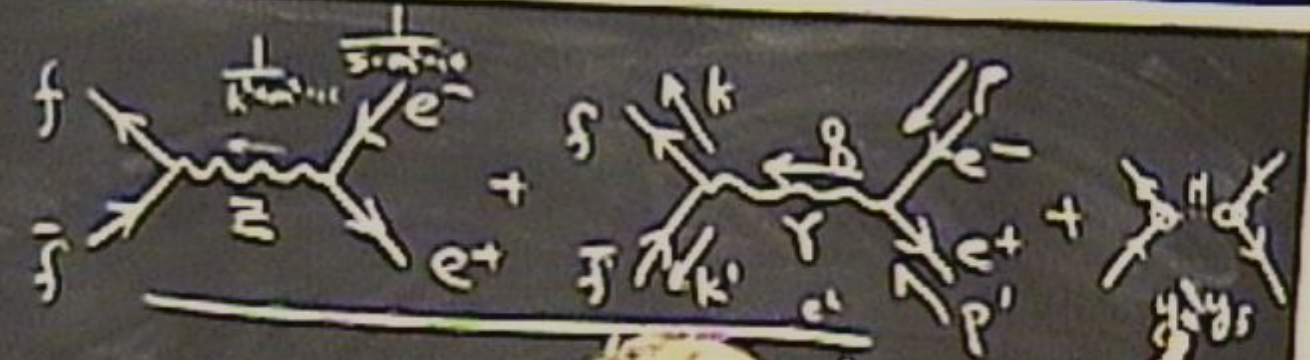


and if $f=\nu$



"inel"

$e^+e^- \rightarrow f\bar{f}$

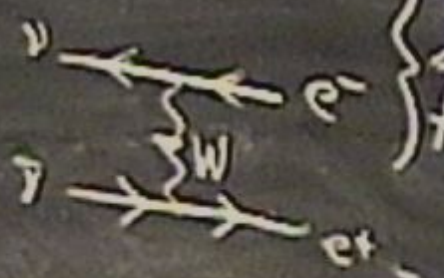


channel

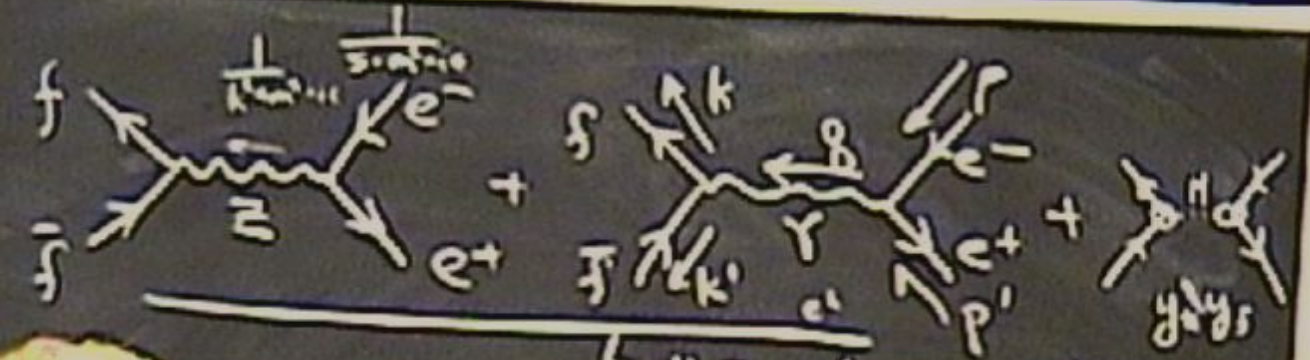
and if $f=e$:



and if $f=\nu$:

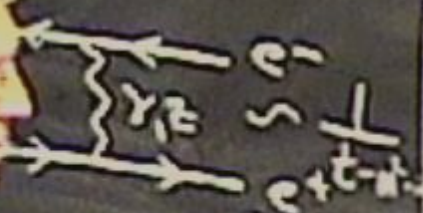


$$\underline{e^+e^- \rightarrow f\bar{f}}$$



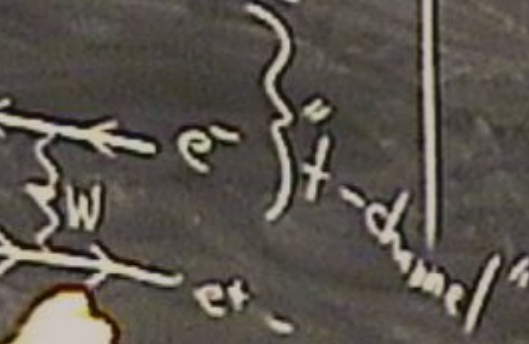
"s-channel"

and if $f = e$



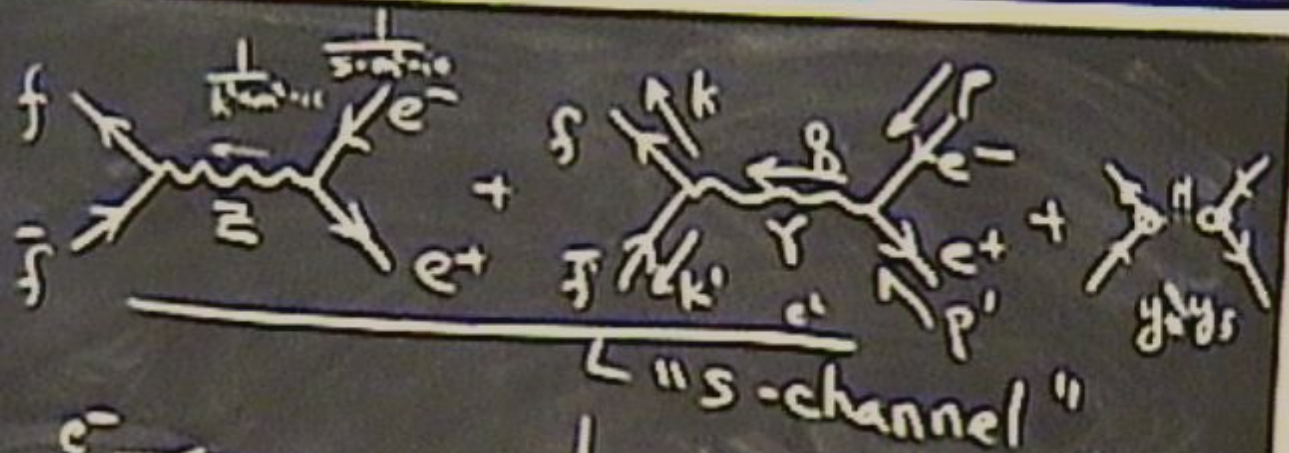
$f \neq e, \nu$

and if

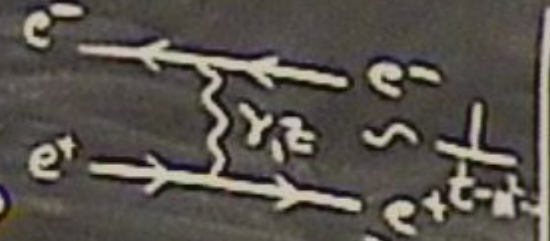


"t-channel"

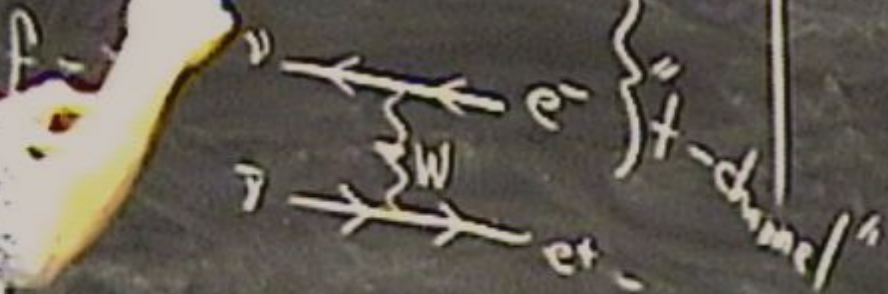
$$\underline{e^+e^- \rightarrow f\bar{f}}$$



$$f=e$$



$$f \neq e, \nu$$

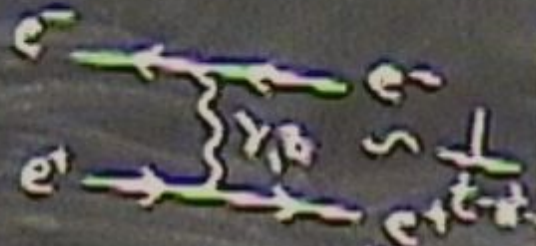


$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | e^-(p) e^+(p') \rangle$$



$$= \frac{1}{2!} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4 q \delta^4(p+p'-q) \delta^4(q-k-k')$$

and if $f = e$



and if $f = \nu$



"s-channel"

$$f \neq e, \nu$$

$$\mathcal{L}_{nc} = \frac{ie}{s_W c_W} Z_\mu \bar{\psi} \gamma^\mu (g_V + g_A \gamma_5) \psi$$

$$\bar{\psi} \gamma^\mu (g_V + g_A \gamma_5) \psi$$

and if $f = \nu$



$$\mathcal{L}_{nc} = \frac{i e}{2 m_W} \bar{\psi} \gamma^\mu (g_V \gamma_\mu + g_A \gamma_5) \psi A_\mu$$



$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | e^-(p) e^+(p') \rangle$$



$$= \frac{1}{2!} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\bar{U}(p) \gamma^\mu M_\nu$$

$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | e^-(p) e^+(p') \rangle$$



$$= \frac{e^2}{2!} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4 q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \lambda} \left[\bar{u}(p) \gamma^\mu M_\nu u(p') \right] \left[\bar{u}(k) \gamma^\nu M_\lambda \right]$$

$$M_\nu^\mu = \begin{cases} \gamma^\mu & \text{if } \nu = \gamma \\ \gamma^5 \gamma^\mu & \text{if } \nu = z \end{cases}$$

$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | e^-(p) e^+(p') \rangle$$



$$= \frac{e^2}{Z!} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4 q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \gamma} \left[\bar{u}(p) \gamma^\mu M_\nu^\mu u(p) \right] \left[\bar{u}(k) \gamma^\nu M_\nu^\nu u(k') \right] \frac{\gamma_\mu + \not{q} \gamma_\nu / M_\nu^2}{q^2 + M_\nu^2 - i\epsilon}$$

$$M_\nu^\mu = \begin{cases} \gamma^\mu & \text{if } \nu = \gamma \\ \frac{g^{\mu\nu} \not{q}}{2M_\nu} & \text{if } \nu = Z \end{cases}$$

$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | e^{-}(p) e^{+}(p') \rangle$$



$$= \frac{e^2}{2!} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4 q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \mu} \left[\bar{u}(p) \gamma^\mu M_\nu^\mu u(p) \right] \left[\bar{u}(k) \gamma^\nu M_\nu^\nu u(k') \right] \frac{\eta_{\mu\nu} + g_\mu g_\nu / M_\nu^2}{g^2 + M_\nu^2 - i\epsilon}$$

$$M_\nu^\mu = \begin{cases} \gamma_\nu & \text{if } \nu = \mu \\ \frac{g_\mu g_\nu}{M_\nu^2} & \text{if } \nu \neq \mu \end{cases}$$

$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | e^{-i(p)} e^{i(p')} \rangle$$



$$= \frac{e^2}{2i} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \mu} \left[\bar{u}(p) \gamma^\nu M_\nu u(p) \right] \left[\bar{u}(k) \gamma^\mu M_\mu u(k') \right] \frac{\gamma_\mu + \not{q} \gamma_\nu / M_\nu}{q^2 + M_\nu^2 - i\epsilon}$$

$$M_\nu = \begin{cases} \not{V} & \text{if } V=\gamma \\ \frac{g_{\mu\nu} \not{V} \not{V}}{2M_\nu} & \text{if } V=Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

$$\overline{M^2} = \frac{1}{N_c} \sum_{\mathbf{k} \in \text{BZ}} |M|^2, \text{ where } S = -2\pi i \delta^4(p+p'-k-k') M.$$

$$= N_c$$

$$N_c = \begin{cases} 1, & \text{if } f \in \text{A.T.} \\ 3, & \text{if } f = g \end{cases}$$

$$\langle f(k) \bar{f}(k') | S | e^-(p) e^+(p') \rangle$$



$$= \frac{e^2}{Z_1} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \gamma} \left[\bar{u}(p) \gamma^\mu M_\nu^\mu u(p) \right] \left[\bar{u}(k) \gamma^\nu M_\nu^\nu u(k') \right] \frac{\gamma_\mu + \not{q} \gamma_\nu / M_\nu^2}{q^2 + M_\nu^2 - i\epsilon}$$

$$M_\nu^\mu = \begin{cases} g_{\mu\nu} & \text{if } \nu = \gamma \\ \frac{g_{\mu\nu} + \not{v} \gamma_\mu \not{v}}{2v \cdot v} & \text{if } \nu = Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

$$\overline{M^2} = \frac{1}{4} \sum_{\gamma, \gamma'} |M|^2, \text{ where } S = -2\pi^2 i \delta^4(p_1 p_2 - k - k') M.$$

$$= N_c e^2 \left\{ \sum_{\gamma, \gamma'} g_i^2 g_f^2 \right.$$

$$N_c = \begin{cases} 1, & \text{if } f = \text{quark} \\ 3, & \text{if } f = \text{gluon} \end{cases}$$

$$\overline{M^2} = \frac{1}{4} \sum_{\alpha\beta\gamma\delta} |M|^2, \text{ where } S = -2\pi^2 i \delta^4(p+p'-k-k') M.$$

$$= N_c e^2 \left\{ \left| \sum_{\gamma\delta} \frac{g_i^{\alpha} g_i^{\beta}}{s - M_V^2 - i\epsilon} \right|^2 \right.$$

$$N_c = \begin{cases} 1, & \text{if } f = \text{quark} \\ 3, & \text{if } f = \text{gluon} \end{cases}$$

$$\overline{M^2} = \frac{1}{4} \sum_{\lambda, \lambda'} |M|^2, \text{ where } S = -2\pi i \delta^4(p+p'-k-k') M.$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{s - M_V^2 - i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1, & \text{if } f=q, \bar{q} \\ 3, & \text{if } f=g \end{cases}$$

$$+ \left| \sum_{V=\gamma, Z} \right.$$

$$\overline{M^2} = \frac{1}{4} \sum_{\alpha\beta\gamma\delta} |M|^2, \text{ where } S = -2\pi^2 i S^4 (p_1 p_1' - k - k')$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{S - M_V^2 - i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1, & \text{if } f=u, r \\ 3, & \text{if } f=s, b \end{cases}$$

$$+ \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{S - M_V^2 - i\epsilon} \right|^2 u^2 + \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{S - M_V^2 - i\epsilon} \right|^2 t^2$$

$$+ \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{S - M_V^2 - i\epsilon} \right|^2 t^2$$

$$\overline{M^2} = \frac{1}{4} \sum_{\alpha, \beta, \gamma, \delta} |M|^2, \text{ where } S = -2\pi i \delta^4(p_1 p_2 - k - k') M.$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{S - M_V^2 - i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1, & \text{if } f = \bar{u}, r \\ 3, & \text{if } f = g \end{cases}$$

$$+ \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{S - M_V^2 - i\epsilon} \right|^2 u^2 + \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{S - M_V^2 - i\epsilon} \right|^2 t^2$$

$$+ \left. \left| \sum_{V=\gamma, Z} \frac{g_V^i g_V^f}{S - M_V^2 - i\epsilon} \right|^2 t^2 \right\}$$

$$\langle f(k) \bar{f}(k') | S | e^-(p) e^+(p') \rangle$$



$$= \frac{1}{[i(2\pi)^4]} \int d^4 q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \gamma} \left[\bar{u}(p) \gamma^\mu M_\nu^\mu u(p) \right] \left[\bar{u}(k) \gamma^\nu M_\nu^\nu u(k) \right] \frac{\eta_{\mu\nu} + g_\mu g_\nu / M_V^2}{q^2 + M_V^2 - i\epsilon}$$

$$M_\nu^\mu = \begin{cases} g_\mu g_\nu & \text{if } V=Z \\ g_\mu g_\nu & \text{if } V=W \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | \psi(p) \psi(p') \rangle$$



$$= \frac{e^2}{z!} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4 q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \nu'} \left[\bar{\psi}(p) \gamma^\mu M_\nu^\mu u(p) \right] \left[\bar{u}(k) \gamma^\nu M_{\nu'}^\nu \psi(k') \right] \frac{\eta_{\mu\nu} + g_\mu g_\nu / M_V^2}{q^2 + M_V^2 - i\epsilon}$$

$$M_V^\mu = \begin{cases} g_\mu & \text{if } V=\gamma \\ g_\mu - g_\nu g_\nu / M_V^2 & \text{if } V=Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

$$\langle f(k) \bar{f}(k') | S | e^-(p) e^+(p') \rangle$$



$$= \frac{e^2}{2i} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \lambda} \left[\bar{u}(p) \gamma^\nu M_\nu^\lambda u(p') \right] \left[\bar{u}(k) \gamma^\lambda M_\lambda^\nu u(k') \right] \frac{\eta_{\mu\nu} + \not{q} \not{q} / M_V^2}{q^2 + M_V^2 - i\epsilon}$$

$$M_\nu^\lambda = \begin{cases} \delta_\nu^\lambda & \text{if } V = \gamma \\ g_\nu^\lambda & \text{if } V = Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | e^-(p) e^-(p') \rangle$$



$$= \frac{e^2}{2i} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \nu'} \left[\bar{u}(p) \gamma^\mu M_\nu^\mu u(p) \right] \left[\bar{u}(k) \gamma^\nu M_{\nu'}^\nu u(k) \right] \frac{\eta_{\mu\nu} + g_\mu g_\nu / M_V^2}{q^2 + M_V^2 - i\epsilon}$$

$$M_\nu^\mu = \begin{cases} g_\mu^\nu & \text{if } V=\gamma \\ g_\mu^\nu - g_\mu g_\nu / M_V^2 & \text{if } V=Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

$$\overline{M^2} = \frac{1}{4} \sum_{\text{all spins}} |M|^2, \text{ where } S = -2\pi i \delta^4(p+p'-k-k') M$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma, Z} \frac{g_V^e g_V^f}{S - M_V^2 - i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1, & \text{if } f=q, \bar{q} \\ 3, & \text{if } f=g \end{cases}$$

$$+ \left| \sum_{V=\gamma, Z} \frac{g_V^e g_V^f}{S - M_V^2 - i\epsilon} \right|^2 u^2 + \left| \sum_{V=\gamma, Z} \frac{g_V^e g_V^f}{S - M_V^2 - i\epsilon} \right|^2 t^2$$

$$+ \left. \left| \sum_{V=\gamma, Z} \frac{g_V^e g_V^f}{S - M_V^2 - i\epsilon} \right|^2 t^2 \right\}$$

$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | e^-(p) e^+(p') \rangle$$



$$= \frac{e^2}{2i} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{\nu, \lambda} [\bar{u}(p) \gamma^\nu M_\nu u(p)] [\bar{u}(k') \gamma^\lambda u(k')] \frac{\eta_{\mu\nu} + g_\mu g_\nu / M_V^2}{q^2 + M_V^2 - i\epsilon}$$

$$M_\nu^\mu = \begin{cases} g_\nu^\mu & \text{if } V = \gamma \\ g_\nu^\mu - g_\nu^\mu & \text{if } V = Z \end{cases}$$



$$\overline{M^2} = \frac{1}{4} \sum_{\text{all spins}} |M|^2, \text{ where } S = -2\pi i \delta^4(p+p'-k-k') M$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma, Z} \frac{g_V^e g_V^f}{S - M_V^2 - i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1, & \text{if } f=q, \bar{q} \\ 3, & \text{if } f=g \end{cases}$$

$$+ \left| \sum_{V=\gamma, Z} \frac{g_V^e g_V^f}{S - M_V^2 - i\epsilon} \right|^2 u^2 + \left| \sum_{V=\gamma, Z} \frac{g_V^e g_V^f}{S - M_V^2 - i\epsilon} \right|^2 t^2$$

$$+ \left. \left| \sum_{V=\gamma, Z} \frac{g_V^e g_V^f}{S - M_V^2 - i\epsilon} \right|^2 t^2 \right\}$$

In CM: $S = -(\vec{p} + \vec{p}')^2 = -2\vec{p} \cdot \vec{p}' = -4E_p^2$
 $\vec{p} = -\vec{p}' \quad E_p = E_{p'}$

$$\langle \bar{\psi}(k) \bar{\psi}(k') | S | \psi(p) \psi(p') \rangle$$



$$= \frac{e^2}{Z_1} [i(2\pi)^4] \frac{1}{[i(2\pi)^4]} \int d^4 q \delta^4(p+p'-q) \delta^4(q-k-k')$$

$$\sum_{V=V, Z} [\bar{\psi}(p) \gamma^\mu M_V^\mu u(p)] [\bar{u}(k) \gamma^\nu M_V^\nu \psi(k')] \frac{\gamma_\mu + \not{q} \gamma_\nu / M_V^2}{q^2 + M_V^2 - i\epsilon}$$

$$M_V^\mu = \begin{cases} \gamma^\mu & \text{if } V=\gamma \\ \gamma^\mu \gamma_5 & \text{if } V=Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_f^2}{M_W^2}$

In CM: $S = -(\vec{p} + \vec{p}')^2 \approx -2\vec{p} \cdot \vec{p}' = -4E_p^2$
 $\vec{p} = -\vec{p}' \quad E_p = E_{p'}$

$$t = -(\vec{p} - \vec{k})^2 \approx 2\vec{p} \cdot \vec{k}$$

$$p^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \quad p'^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix} \quad k^\mu = \begin{pmatrix} E \\ \dots \\ \dots \\ \dots \end{pmatrix}$$

In CM: $S = -(\vec{p} + \vec{p}')^2 = -2\vec{p} \cdot \vec{p}' = -4E_p^2$

$\vec{p} = -\vec{p}' \quad E_p = E_{p'}$

$t = -(\vec{p} - \vec{k})^2 \approx 2\vec{p} \cdot \vec{k} = +2E^2[-1]$

$P^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix}$

$P'^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$

$K^\mu = \begin{pmatrix} E \\ E \sin \theta \\ 0 \\ E \cos \theta \end{pmatrix}$

$K'^\mu = \begin{pmatrix} E \\ -E \sin \theta \\ 0 \\ -E \cos \theta \end{pmatrix}$

$$\text{In CM: } s = -(p+p')^2 = -2pp' = 4E_p^2$$

$$\vec{p} = \vec{p}' \quad E_p = E_{p'}$$

$$t = -(p-k)^2 = 2p \cdot k = +2E^2[-1 + \cos\theta]$$

$$p^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix}$$

$$p'^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$$

$$k^\mu = \begin{pmatrix} E \\ E \sin\theta \\ 0 \\ E \cos\theta \end{pmatrix}$$

$$k'^\mu = \begin{pmatrix} E \\ -E \sin\theta \\ 0 \\ E \cos\theta \end{pmatrix}$$

$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$t^2 \sim (1 - \cos\theta)^2$$

$$\approx 0 \text{ if } \theta = 0, \pi$$

$$u^2 \sim (1 + \cos\theta)^2$$

$$\approx 0 \text{ if } \theta = \pi/2, 3\pi/2$$

$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$t^2 \sim (1 - \cos\theta)^2$$

≈ 0 if $\theta = 0$

$$u^2 \sim (1 + \cos\theta)^2$$

≈ 0 if $\theta = \pi$



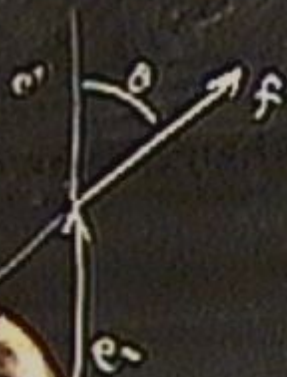
$$L = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$L^2 \sim (1 - \cos\theta)^2$$

≈ 0 if $\theta = 0$

$$u^2 \sim (1 + \cos\theta)^2$$

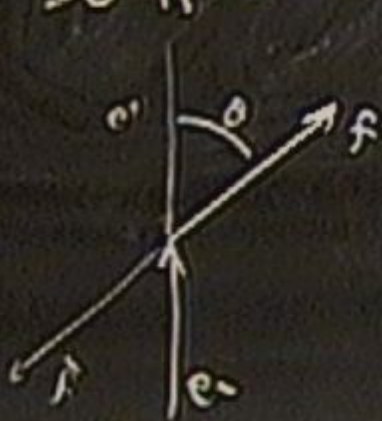
≈ 0 if $\theta = \pi$



$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$t^2 \sim (1 - \cos\theta)^2$$

$\approx 0 \quad \text{if } \theta = 0$



$$u^2 \sim (1 + \cos\theta)^2$$

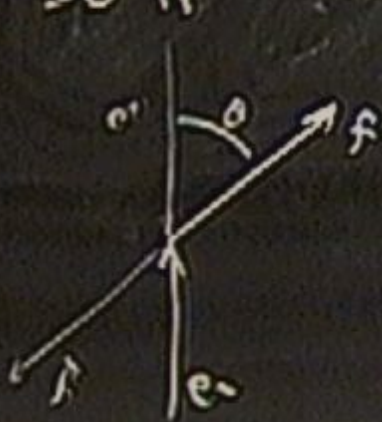
$\approx 0 \quad \theta = \pi$



$$L = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$L^2 \sim (1 - \cos\theta)^2$$

$\approx 0 \quad \text{if } \theta = 0$



$$u^2 \sim (1 + \cos\theta)^2$$

$\approx 0 \quad \theta = \pi$



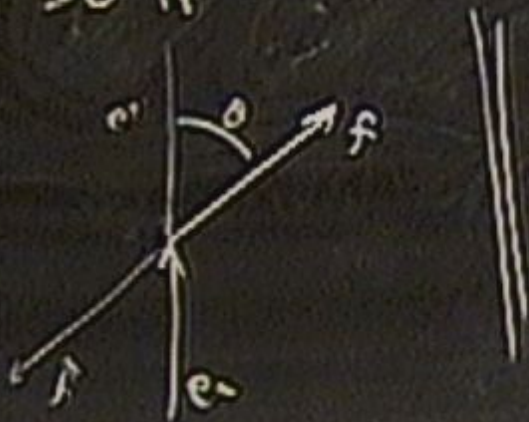
$$\bar{f} \gamma^\mu (\dots) f$$

$$\bar{f}_L \gamma_L$$

$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$E^2 \sim (1 - \cos\theta)^2$$

≈ 0 if $\theta = 0$



$$u^2 \sim (1 + \cos\theta)^2$$

≈ 0 if $\theta = \pi$

$$\bar{f} \gamma^\mu (\dots) f$$

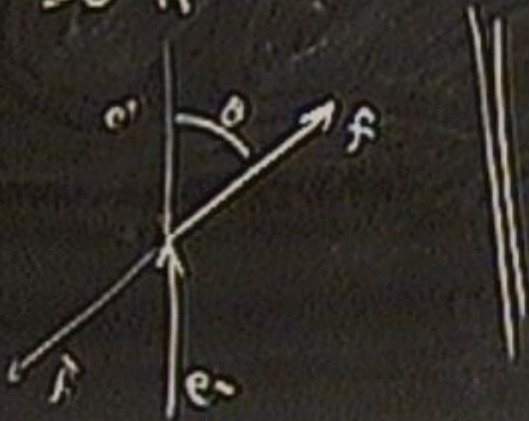
$$\bar{f}_L \gamma_L$$

$$\bar{f}_R \gamma_R$$

$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$t^2 \sim (1 - \cos\theta)^2$$

≈ 0 if $\theta = 0$



$$u^2 \sim (1 + \cos\theta)^2$$

≈ 0 if $\theta = \pi$

$$\bar{f} \gamma^\mu (\dots) f$$

$$\int dL$$

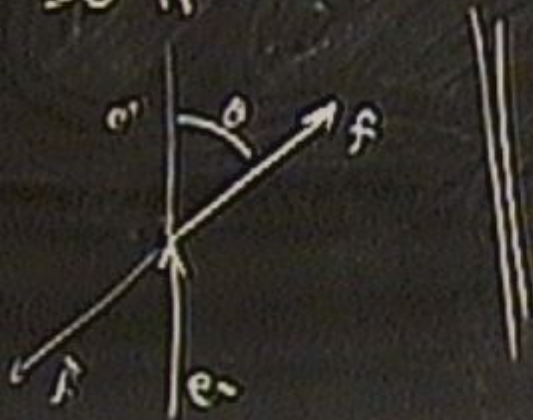
$$\bar{f}_e \delta_e$$

$$\int d^4x \beta \delta^4(x) f$$

$$L = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$L^2 \sim (1 - \cos\theta)^2$$

≈ 0 if $\theta = 0$



$$u^2 \sim (1 + \cos\theta)^2$$

≈ 0 if $\theta = \pi$

$$\bar{f} \gamma^\mu (\dots) f$$

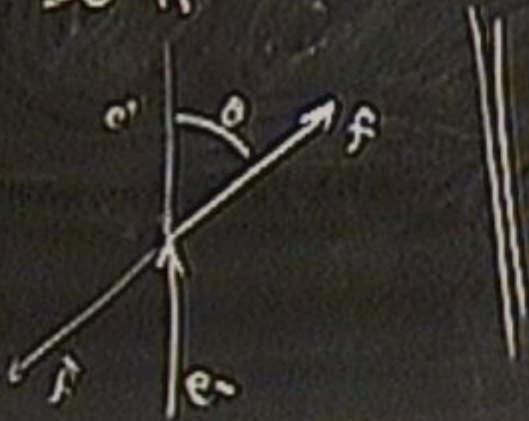
$$\int_{\Omega_L} \int_{\Omega_R}$$

$$\int_{\Omega_L} \beta \gamma^\mu \dots$$

$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$t^2 \sim (1 - \cos\theta)^2$$

≈ 0 if $\theta = 0$



$$u^2 \sim (1 + \cos\theta)^2$$

≈ 0 if $\theta = \pi$

$$\bar{f} \gamma^\mu (\dots) f$$

$$\int_L \int_R$$

$$\int_L \int_R$$

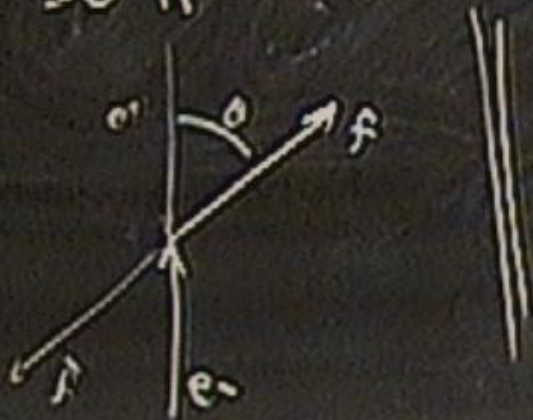
$$\int_L \beta \gamma^\mu \dots$$



$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$t^2 \sim (1 - \cos\theta)^2$$

$\rightarrow 0$ if $\theta = 0$



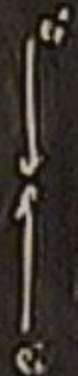
$$u^2 \sim (1 + \cos\theta)^2$$

$\rightarrow 0$ if $\theta = \pi$

$$\bar{f} \gamma^\mu (\dots) f$$

$$\int_{\mathcal{L}_L} \int_{\mathcal{L}_R}$$

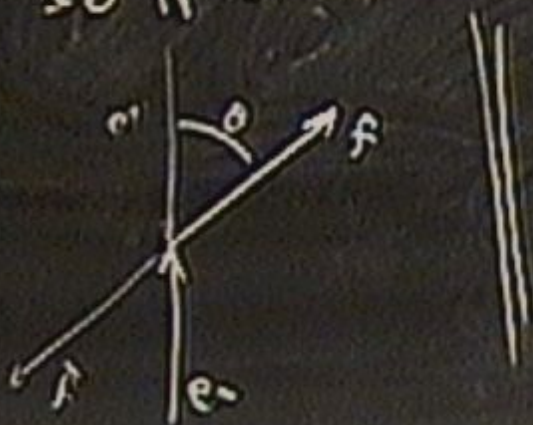
$$\int_{\mathcal{L}_L} \beta \gamma^\mu \dots$$



$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$t^2 \sim (1 - \cos\theta)^2$$

≈ 0 if $\theta = 0$



$$u^2 \sim (1 + \cos\theta)^2$$

≈ 0 if $\theta = \pi$

$$\bar{f} \gamma^\mu (\dots) f$$

$$\bar{u} \gamma_\mu$$

$$\bar{u} \gamma_\mu$$

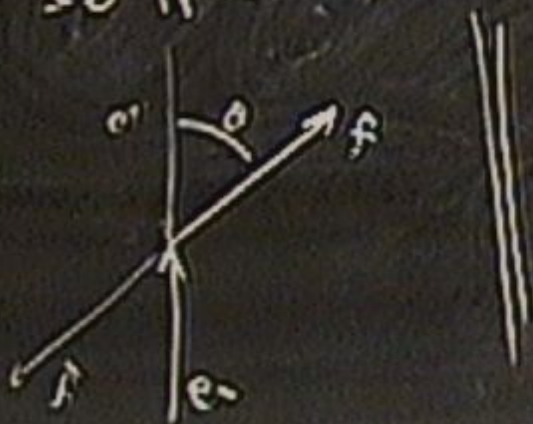
$$\int d^4x \delta^4(x) \dots$$



$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$t^2 \sim (1 - \cos\theta)^2$$

≈ 0 if $\theta = 0$



$$u^2 \sim (1 + \cos\theta)^2$$

≈ 0 if $\theta = \pi$

$$\bar{f} \gamma^\mu (\dots) f$$

$$\gamma^\mu \gamma^\nu$$

$$\gamma^\mu \gamma^\nu$$

$$\int d^4x \beta \delta^4(x) \dots$$

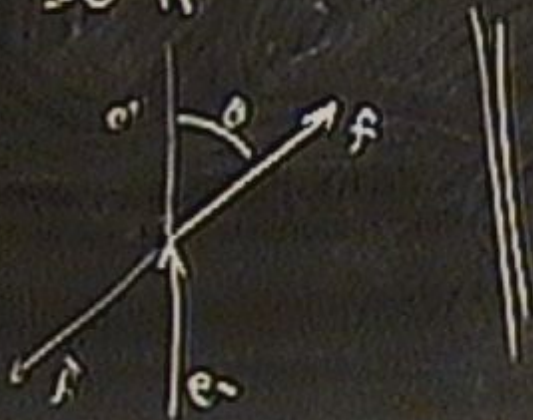


$$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$$

$$E^2 \sim (1 - \cos\theta)^2 \quad u^2 \sim (1 + \cos\theta)^2$$

≈ 0 if $\theta = 0$

≈ 0 if $\theta = \pi$



$$\bar{f} \gamma^\mu (\dots) f$$

$$\int dL$$

$$\int dR$$

$$\int d^4x \delta^4(x)$$



$$A_{ij} = \frac{1}{s_{ij}}$$

$$A_{ij} = \frac{1}{s^2 \omega^2} \left(\frac{g_{0i} g_{0j}}{s - M_2} \right) + \frac{Q_0 Q_1}{s} \quad i, j = L, R$$

$$g_{0L} = T_{31} - Q_1 s^2 \quad g_{0R} = -Q_1 s^2$$

$$A_{ij} = \frac{1}{s N \Omega_i} \left(\frac{g_{\alpha i} g_{\beta j}}{s - M_i} \right) + \frac{Q_i Q_j}{s} \quad i = L, R.$$

$$g_{\beta L} = T s_f - Q_f s_f^2 \quad g_{\beta R} = -Q_f s_f^2$$

$$d\sigma(e^+e^- \rightarrow f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s} \left[|A_{LL}(s)|^2 + |A_{RR}(s)|^2 \right. \\ \left. + |A_{LR}(s)|^2 + |A_{RL}(s)|^2 \right] dX \\ (f \neq e, \nu_e)$$

$$\begin{aligned}
 dX &= (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0} \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt \\
 &= \frac{1}{8\pi s} \delta(s+t+u) du dt
 \end{aligned}$$

$$\begin{aligned}
 dX &= (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0} \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) du dt
 \end{aligned}$$

$$\begin{aligned}
 d\mathcal{X} &= (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0} \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) du dt
 \end{aligned}$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} [\dots]$$

$$\begin{aligned}
 dX &= (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0} \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) du dt
 \end{aligned}$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} [\dots]$$

$$\begin{aligned}
 t &= -2E^2(1 - \cos\theta) \\
 dt &= +2E^2 d\cos\theta \\
 &= -2E^2 \sin\theta d\theta
 \end{aligned}$$

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt$$

$$\rightarrow \frac{1}{8\pi s} \delta(s+t+u) du dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} [\dots]$$

$$\alpha = \frac{e^2}{4}$$

$$t = -2E^2(1 - \cos\theta)$$

$$dt = +2E^2 d\cos\theta$$

$$= -2E^2 \sin\theta d\theta$$

$$\begin{aligned}
 dX &= (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0} \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt \\
 &\approx -\frac{1}{8\pi s} \delta(s+t+u) du dt
 \end{aligned}$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} [\dots]$$

$\alpha = e^2/4$

$$\begin{aligned}
 t &= -2E^2(1 - \cos\theta) \\
 dt &= +2E^2 d\cos\theta \\
 &= -2E^2 \sin\theta d\theta
 \end{aligned}$$

Different energy regimes:

Different energy regimes:

$$\underline{\underline{S \ll Mz^2}}$$

Different energy regimes:

$S \ll M^2$

$A_{ij}(s) \approx$

Different energy regimes:

$$\underline{\underline{s \ll M_z^2}}$$

$$A_{ij}(s) \sim \frac{Q_i Q_j}{s}$$

Different energy regimes:

$$\underline{\underline{s \ll M_z^2}}$$

$$A_{ij}(s) \sim \frac{Q_i Q_j}{s}$$

independent of
 $i, j = L, R.$

$$\begin{aligned}
 dX &= (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0} \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) du dt
 \end{aligned}$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} [\dots]$$

$\alpha = e^2/4$

$$\begin{aligned}
 t &= -2E^2(1 - \cos\theta) \\
 dt &= +2E^2 d\cos\theta \\
 &= -2E^2 \sin\theta d\theta
 \end{aligned}$$

$$\begin{aligned}
 dX &= (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0} \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt \\
 &= -\frac{1}{8\pi s} \delta(s+t+u) du dt
 \end{aligned}$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} [\dots]$$

$$\alpha = e^2/4$$

$$t = -2E^2(1 - \cos\theta)$$

$$\begin{aligned}
 dt &= +2E^2 d\cos\theta \\
 &= -2E^2 \sin\theta d\theta
 \end{aligned}$$

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt$$

$$\sigma \sim \frac{1}{M^4}$$

$$L \sim M^4$$

$$= -\frac{1}{8\pi s} \delta(s+t+u) du dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} [\dots]$$

$$\alpha = e^2/4$$

$$t = -2E^2(1 - \cos\theta)$$

$$dt = +2E^2 d\cos\theta$$

$$= -2E^2 \sin\theta d\theta$$

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt$$

$$\sigma \sim \frac{1}{M^2}$$

$$L \sim M^2$$

$$\approx -\frac{1}{8\pi s} \delta(s+t+u) du dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} [\dots]$$

$$\alpha = e^2/4\pi$$

$$t = -2E^2(1 - \cos\theta)$$

$$dt = +2E^2 d\cos\theta$$

$$= -2E^2 \sin\theta d\theta$$

$$A_{ij} = \frac{1}{s^2 N_c^2} \left(\frac{g_{ei} g_{rj}}{s - M_i^2} \right) + \frac{Q_e Q_f}{s} \quad i = L, R$$

$$g_{eL} = T_{3f} - Q_f s_0^2 \quad g_{fR} = -Q_f s_0^2$$

$$d\sigma(e^+e^- \rightarrow f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s} \left[|A_{eL}|^2 + |A_{eR}(s)|^2 + |A_{fL}(s)|^2 + |A_{fR}(s)|^2 \right]$$

(f ≠ e, ν_e)

$$A_{ij} = \frac{1}{s^2 u_{ij}^2} \left(\frac{g_{ij} g_{ji}}{s - M_i^2} \right) + \frac{Q_i Q_j}{s} \quad i = L, R.$$

$$g_{LL} = T_{3f} - Q_f s^2 \quad g_{RR} = -Q_f s^2.$$

$$d\sigma(e^+e^- \rightarrow f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s} \left[|A_{LL}(s)|^2 + |A_{RR}(s)|^2 \right. \\ \left. + |A_{LR}(s)|^2 + |A_{RL}(s)|^2 \right] d\chi \\ (f \neq e, \nu_e)$$

$$A_{ij} = \frac{1}{s \omega_0} \left(\frac{g_{ij} \partial_i}{s - M_i} \right) + \frac{Q_i Q_j}{s} \quad i, j = L, R$$

$$g_{0L} = T_{21} - Q_1 s^2 \quad g_{1R} = -Q_1 s^2$$

$$d\sigma(e^+e^- \rightarrow f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s} \left[(|A_{LL}(s)|^2 + |A_{RR}(s)|^2) u^2 + (|A_{LR}(s)|^2 + |A_{RL}(s)|^2) t^2 \right] dx$$

Different energy regimes:

$S \ll M_z^2$

$A_{ij}(s) \approx \frac{Q_i Q_j}{s}$

independent of $j=L,R$.

$\frac{d\sigma}{dE} \approx$



Different energy regimes:

$s \ll M_z^2$

$A_{ij}(s) \approx \frac{Q_i Q_j}{s}$

independent of $i, j = L, R$.

$\frac{d\sigma}{d\Omega} \approx$

Different energy regimes:

$s \ll M_e^2$

$A_{ij}(s) \approx \frac{Q_i Q_j}{s}$

independent of $(ij) = L, R$.

$\frac{d\sigma}{dE} \approx \frac{Q_0^2 Q_1^2 / 16\pi\alpha^2 N_c}{s^4} \left(u^2 + t^2 \right)$

Different energy regimes:

$$\underline{s \ll M_z^2}$$

$$A_{ij}(s) \approx \frac{Q_i Q_j}{s}$$

independent of
 $i, j = L, R.$

$$\frac{d\sigma}{dE} \approx \frac{Q_0^2 Q_f^2}{s^2} \frac{\pi \alpha^2 N_c}{s} \left(u^2 + t^2 \right)$$

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^3 2k^0} \frac{d^3k'}{2k'^0}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u) \underbrace{(m_a^2 - m_b^2 - m_c^2 - m_d^2)}_{=0} du dt$$

$$\sigma \sim \frac{1}{M^2}$$

$$L \sim M^3$$

$$\approx -\frac{1}{8\pi s} \delta(s+t+u) du dt$$

$$\frac{d\sigma}{dt} = -\frac{\pi \alpha^2 N_c}{s^2} [\dots]$$

$$\alpha = e^2/4\pi$$

$$t = -2E^2(1 - \cos\theta)$$

$$dt = +2E^2 d\cos\theta$$

$$= -2E^2 \sin\theta d\theta$$

Different energy regimes:

$$\underline{s \ll M_e^2}$$

$$A_{ij}(s) \approx \frac{Q_e Q_j}{s}$$

independent of
 $ij = L, R.$

$$\frac{d\sigma}{dE} \approx \frac{-Q_e^2 Q_j^2 \pi \alpha^2 N_c}{s^2} \left(u^2 + t^2 \right)$$

$$0 < \theta < \pi \rightarrow 0 > t \gg -4E^2 = -s$$

$$\sigma = \int_{-\infty}^0 \frac{d\sigma}{dt} dt \rightarrow$$



$$\sigma = \int_{-\infty}^0 \frac{d\sigma}{dt} dt \rightarrow \frac{4\pi\alpha^2}{3} Q_e^2 Q_f^2 N_c$$



$$\sigma = \int_{-\infty}^0 \frac{d\sigma}{dt} dt \rightarrow \frac{4\pi\alpha^2}{3s} Q_e^2 Q_f^2 N_c$$

Different energy regimes:

$s \ll M_z^2$

$A_{ij}(s) \approx \frac{Q_i Q_j}{s}$

independent of $i, j = L, R$.

$\frac{d\sigma}{dE} \approx \frac{Q_0^2 Q_1^2 2\pi \alpha^2 N_c}{s^4} \left(u^2 + t^2 \right) \Big|_{t = -s - u}$

$0 < u < \pi \rightarrow 0 \geq t \geq -4E^2 = -s$

$$\sigma = \int_{-\infty}^0 \frac{d\sigma}{dt} dt \rightarrow \frac{4\pi\alpha^2}{3s} Q_e^2 Q_f^2 N_c \leftarrow s \ll M_Z^2$$



$$\sigma = \int_{-\infty}^0 \frac{d\sigma}{dt} dt \rightarrow \underbrace{\frac{4\pi\alpha^2}{3s}}_{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} Q_e^2 Q_f^2 N_c \leftarrow s \ll M_Z^2$$



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$$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$$

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_f Q_f^2$$



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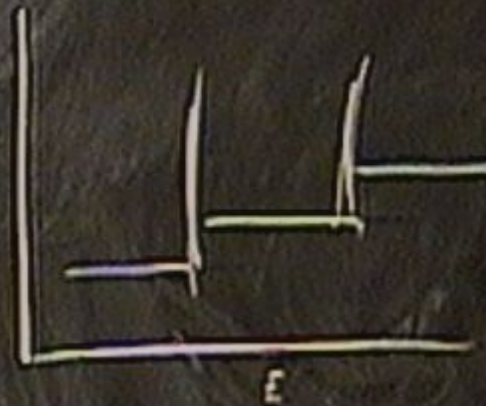
$$\sigma = \int_{-1}^0 \frac{d\sigma}{dt} dt \rightarrow \frac{4\pi\alpha^2}{3s} Q_e^2 Q_f^2 N_c \leftarrow s \ll M_Z^2$$

$$\sigma = (\sigma(e^+e^- \rightarrow \mu^+\mu^-))$$

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum_q Q_q^2$$

$$Q_u = 2/3$$

$$Q_d = -1/3$$



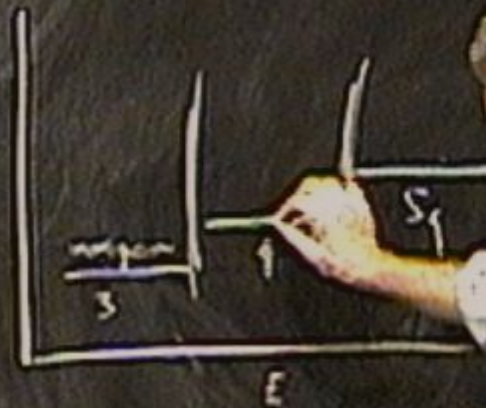
$$\sigma = \int_{-\sqrt{s}}^{\sqrt{s}} \frac{d\sigma}{dt} dt \rightarrow \underbrace{\frac{4\pi\alpha^2}{3s}}_{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} Q_e^2 Q_f^2 N_c \quad \leftarrow s \ll M_Z^2$$

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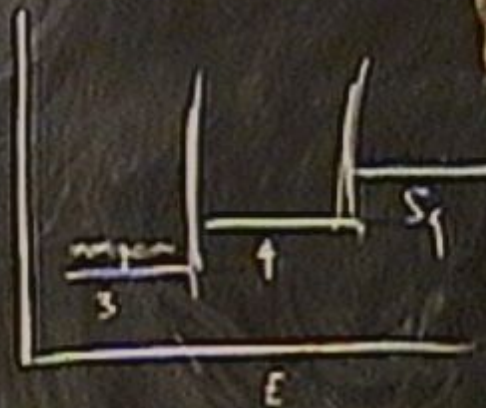
$$\sigma = \int_{-s}^0 \frac{d\sigma}{dt} dt \rightarrow \underbrace{\frac{4\pi\alpha^2}{3s}}_{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} Q_e^2 Q_f^2 N_c \leftarrow s \ll M_Z^2$$

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Intermediate energies:

$$S \approx Mc^2$$



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$$S \ll Mc^2$$

$$A_{ij} \approx \frac{Q_e Q_f}{S} =$$

$N_c = \begin{cases} 1 \text{ if } f_1 = f_2 \\ 3 \text{ if } f_1 \neq f_2 \end{cases}$

$\frac{2}{t^2}$

$g_{\mu\nu} = T_{31} = 0, \delta_{ij} \quad g_{1\mu} = -\delta_{i\mu}$

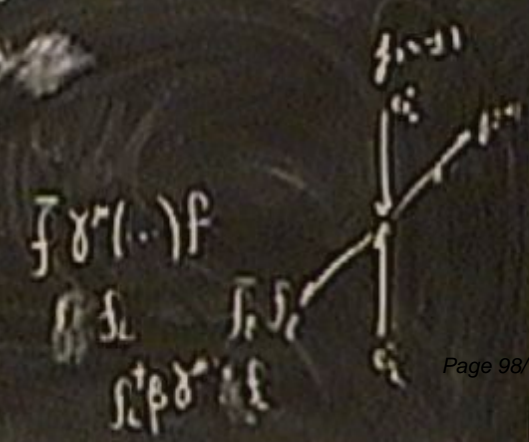
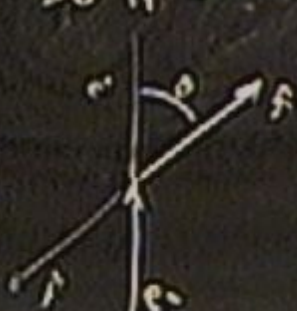
$$d\sigma(e^+e^- \rightarrow f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s} \left[(|A_{\mu\mu}(s)|^2 + |A_{\mu\nu}(s)|^2) u^2 + (|A_{\nu\mu}(s)|^2 + |A_{\nu\nu}(s)|^2) t^2 \right] dX$$

(p^+e^-, ν_e)

$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$

$t^2 \sim (1 - \cos\theta)^2 \quad u^2 \sim (1 + \cos\theta)^2$
 $\geq 0 \text{ if } 0 < \theta < \pi$

dependent of $(ij) = L, R$.



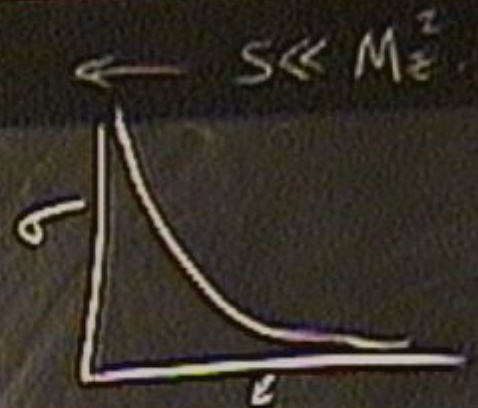
Intermediate energies:

$$S \lesssim M_E^2$$

$$A_{ij} \approx \frac{Q_e Q_f}{S} - \frac{g_E^i g_E^j}{S \hbar c M_E^2} + O\left(\frac{S}{M_E^2}\right)^{L-1}$$

$$\sigma \Rightarrow \int \frac{d\sigma}{dt} dt \Rightarrow \frac{4\pi\alpha'^2}{3s} Q_e^2 Q_f^2 N_c$$

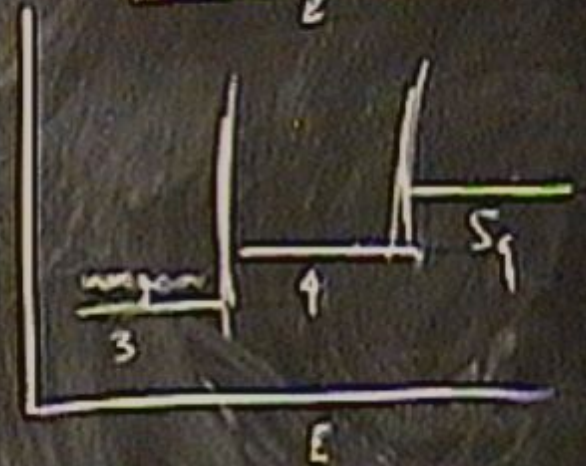
$$\sigma = (\text{e}^+\text{e}^- \rightarrow \mu^+\mu^-)$$



$$R = \frac{\sigma(\text{e}^+\text{e}^- \rightarrow \text{hadrons})}{\sigma(\text{e}^+\text{e}^- \rightarrow \mu^+\mu^-)} = N_c \sum_q Q_q^2$$

$$Q_{u,c} = \frac{2}{3}$$

$$Q_{d,s} = -\frac{1}{3}$$



Intermediate energies:

$$S \ll M_E^2$$

$$A_{ij} \approx \frac{Q_e Q_f}{S} - \frac{g_{Ei} g_{Ej}}{S_{HCW} M_E^2} + O\left(\frac{S}{M_E^4}\right)$$

Intermediate energies:

$$S \lesssim M_e^2$$

$$A_{ij} \approx \frac{Q_e Q_f}{S} - \frac{g_{Ei} g_{Ej}}{S \hbar c M_e^2} + O\left(\frac{S}{M_e^2}\right)^{L+1}$$

$$\underline{\underline{\sigma(\bar{e}_L) - \sigma(\bar{e}_R)}}$$

Intermediate energies:

$$S \ll M_e^2$$

$$A_{ij} \approx \frac{Q_e Q_f}{S} - \frac{g_{ei} g_{fj}}{S \omega M_e^2} + O\left(\frac{S}{M_e^2}\right)^{L-L'}$$

$$A_{LR}(S) = \frac{\sigma(\bar{e}_L) - \sigma(\bar{e}_R)}{\sigma(\bar{e}_L) + \sigma(\bar{e}_R)}$$

$\left. \begin{array}{l} 1. f \\ 2. f \\ 3. f \\ 4. f \end{array} \right\}$

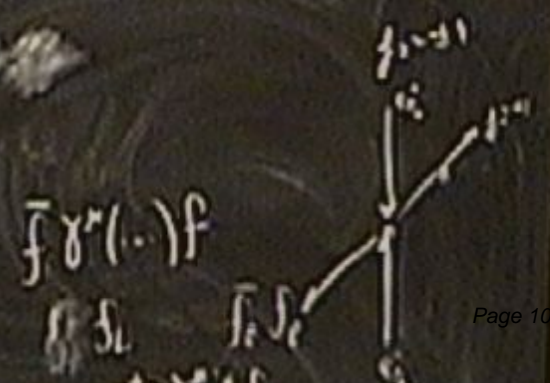
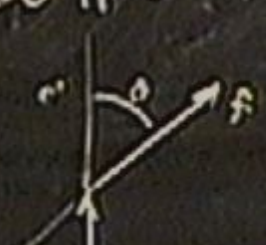
$g_{00} = T_{31} = 0, \quad g_{11} = -0, \quad g_{22} = -0, \quad g_{33} = -0$

$$d\sigma(e^+e^- \rightarrow f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s} \left[(|A_{uL}(s)|^2 + |A_{uR}(s)|^2) u^2 + (|A_{dL}(s)|^2 + |A_{dR}(s)|^2) t^2 \right] dX$$

$t = -2E^2(1 - \cos\theta) \quad u = -2E^2(1 + \cos\theta)$

$t^2 \sim (1 - \cos\theta)^2$
 $\Rightarrow \int_0^{2\pi} \int_0^\pi \dots$

$u^2 \sim (1 + \cos\theta)^2$
 $\Rightarrow \dots$



part of
 L.R.

$\rightarrow i e A_{\mu} \bar{\psi} \gamma^{\mu} \psi$

Intermediate energies: $S \lesssim M_Z^2$

$$A_{ij} \approx \frac{Q_e Q_f}{S} - \frac{g_{e_i} g_{f_j}}{S \sin^2 \theta} \frac{1}{M_Z^2} + O\left(\frac{S}{M_Z^2}\right)$$

$$A_{LR}^{(S)} \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R^-)} \approx - \left(\frac{S}{M_Z^2}\right) \left[\frac{(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 + g_{fR}^2)}{2Q_e Q_f s_W^2 c_W^2} \right]$$

$\alpha = e^2 / 4\pi$

$$dt = +2E^2 d\cos\theta \\ = -2E^2 \sin\theta d\theta$$

$\rightarrow i e A_{ij} \gamma^0 \gamma^j / s$

Intermediate energies: $s \lesssim M_Z^2$

$$A_{ij} \approx \frac{Q_e Q_f}{s} - \frac{g_{e_i} g_{f_j}}{s_W c_W M_Z^2} + \mathcal{O}\left(\frac{s}{M_Z^2}\right)$$

$$A_{LR}^{(s)} \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R^-)} \approx - \left(\frac{s}{M_Z^2}\right) \left[\frac{(g_{eL}^2 - g_{eR}^2)(g_{\nu e}^2 + g_{\nu R}^2)}{2Q_e Q_f s_W^2 c_W^2} \right]$$



$$\alpha = e^2 / 4\pi$$

$$dt = +2E^2 d\cos\theta \\ = -2E^2 \sin\theta d\theta$$

$+ieA_\mu \bar{\psi} \gamma^\mu \psi$

Intermediate energies: $s \lesssim M_Z^2$

$$A_{ij} \approx \frac{Q_e Q_f}{s} - \frac{g_{e_i} g_{f_j}}{s_W c_W M_Z^2} + O\left(\frac{s}{M_Z^2}\right)$$

$$A_{LR}^{(s)} \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e^-)} \approx - \left(\frac{s}{M_Z^2}\right) \left[\frac{(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 + g_{fR}^2)}{2Q_e Q_f s_W^2 c_W^2} \right]$$

$$dt = +ZE^2 d\cos\theta$$

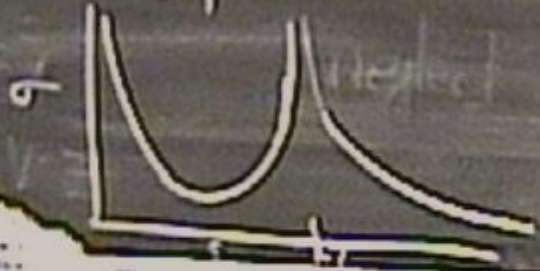
$$= -ZE^2 \sin\theta d\theta$$

$\pm i e A_\mu \gamma^\mu \not{\partial} / f$

Intermediate energies: $S \lesssim M_Z^2$

$$A_{ij} \approx \frac{Q_e Q_f}{S} - \frac{g_{e_i} g_{f_j}}{S_W C_W M_Z^2} + O\left(\frac{S}{M_Z^2}\right)$$

$$A_{LR}^{(S)} \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R^-)} \approx - \left(\frac{S}{M_Z^2}\right) \left[\frac{(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 + g_{fR}^2)}{2Q_e Q_f S_W^2 C_W^2} \right]$$



$$dt = +ZE^2 d\cos\theta$$

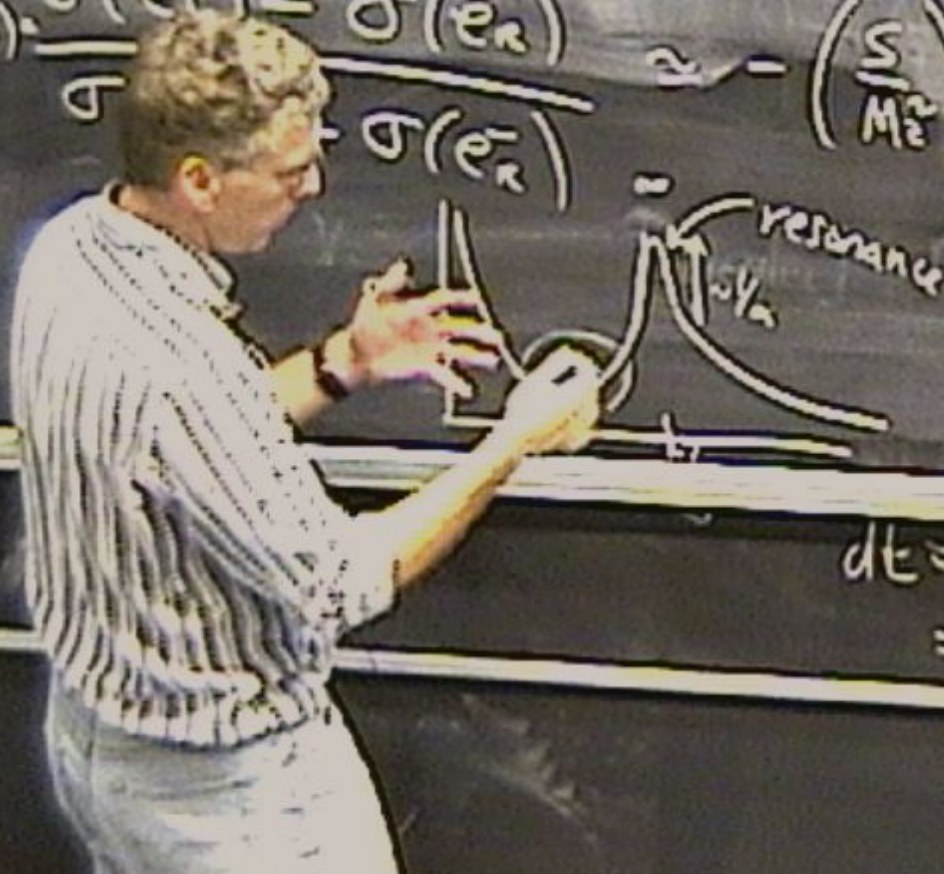
$$= -ZE^2 \sin\theta d\theta$$

$+ieA_{\mu} \gamma^{\mu} \psi / \hbar$

Intermediate energies: $S \lesssim M_e^2$

$$A_{ij} \approx \frac{Q_e Q_f}{S} - \frac{g_{e_i} g_{f_j}}{S \omega c_w M_Z^2} + O\left(\frac{S}{M_Z^2}\right)$$

$$A_{LR}^{(S)} = \frac{\sigma(e^-) - \sigma(e_R^-)}{\sigma + \sigma(e_R^-)} \approx - \left(\frac{S}{M_Z^2}\right) \left[\frac{(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 + g_{fR}^2)}{2Q_e Q_f s_w^2 c_w^2} \right]$$



$$dt = +ZE^2 d\cos\theta$$

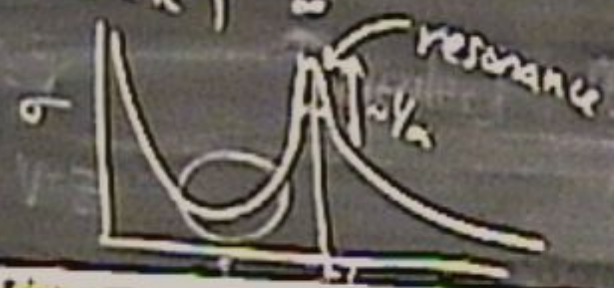
$$= -ZE^2 \sin\theta d\theta$$

$\pm i e A_{\mu} \gamma^{\mu} \not{\partial} / s$

Intermediate energies: $s \lesssim M_Z^2$

$$A_{ij} \approx \frac{Q_e Q_f}{s} - \frac{g_e^i g_f^j}{s_W c_W M_Z^2} + O\left(\frac{s}{M_Z^2}\right)$$

$$\frac{A_{LR}(s) \cdot \sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R^-)} \approx - \left(\frac{s}{M_Z^2}\right) \left[\frac{(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 + g_{fR}^2)}{2Q_e Q_f s_W^2 c_W^2} \right]$$



$$dt = +ZE^2 d\cos\theta$$

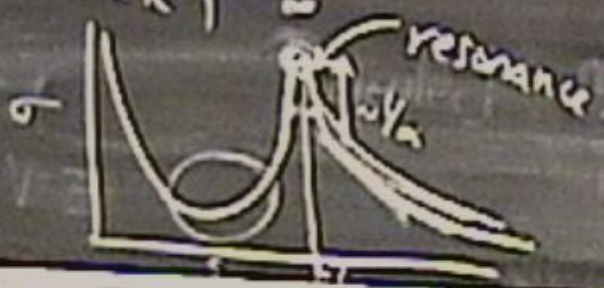
$$= -ZE^2 \sin\theta d\theta$$

$\pm i e A_{\mu} \gamma^{\mu} \not{p} / s$

Intermediate energies: $s \lesssim M_e^2$

$$A_{ij} \approx \frac{Q_e Q_f}{s} - \frac{g_{eL}^i g_{fL}^j}{s_{\mu} c_{\mu}} \frac{1}{M_e^2} + O\left(\frac{s}{M_e^2}\right)$$

$$A_{LR}^{(s)} \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R^-)} \approx - \left(\frac{s}{M_e^2}\right) \left[\frac{(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 + g_{fR}^2)}{2Q_e Q_f s_{\mu}^2 c_{\mu}^2} \right]$$



$$t = \frac{e^2}{4s}$$

$$dt = +2E^2 d\cos\theta$$

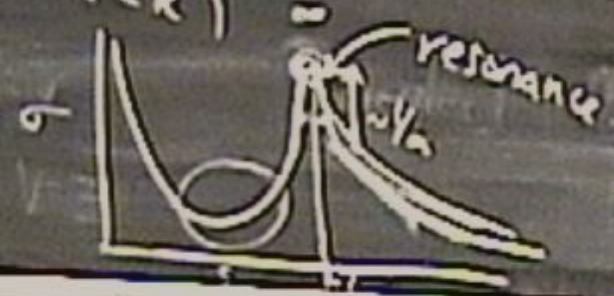
$$= -2E^2 \sin\theta d\theta$$

$\pm i e A_{\mu} \gamma^{\mu} \not{\partial} / s$

Intermediate energies: $S \lesssim M_e^2$

$$A_{ij} \approx \frac{Q_e Q_f}{s} - \frac{g_{eL}^i g_{fL}^j}{s_{Wc} M_Z^2} + O\left(\frac{s}{M_e^2}\right)$$

$$A_{LR}^{(S)} \cdot \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R^-)} \approx - \left(\frac{s}{M_Z^2}\right) \left[\frac{(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 + g_{fR}^2)}{2Q_e Q_f s_W^2 c_W^2} \right]$$



$$\alpha = e^2 / 4\pi$$

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