Title: Graduate Course on Standard Model & Quantum Field Theory - 11B

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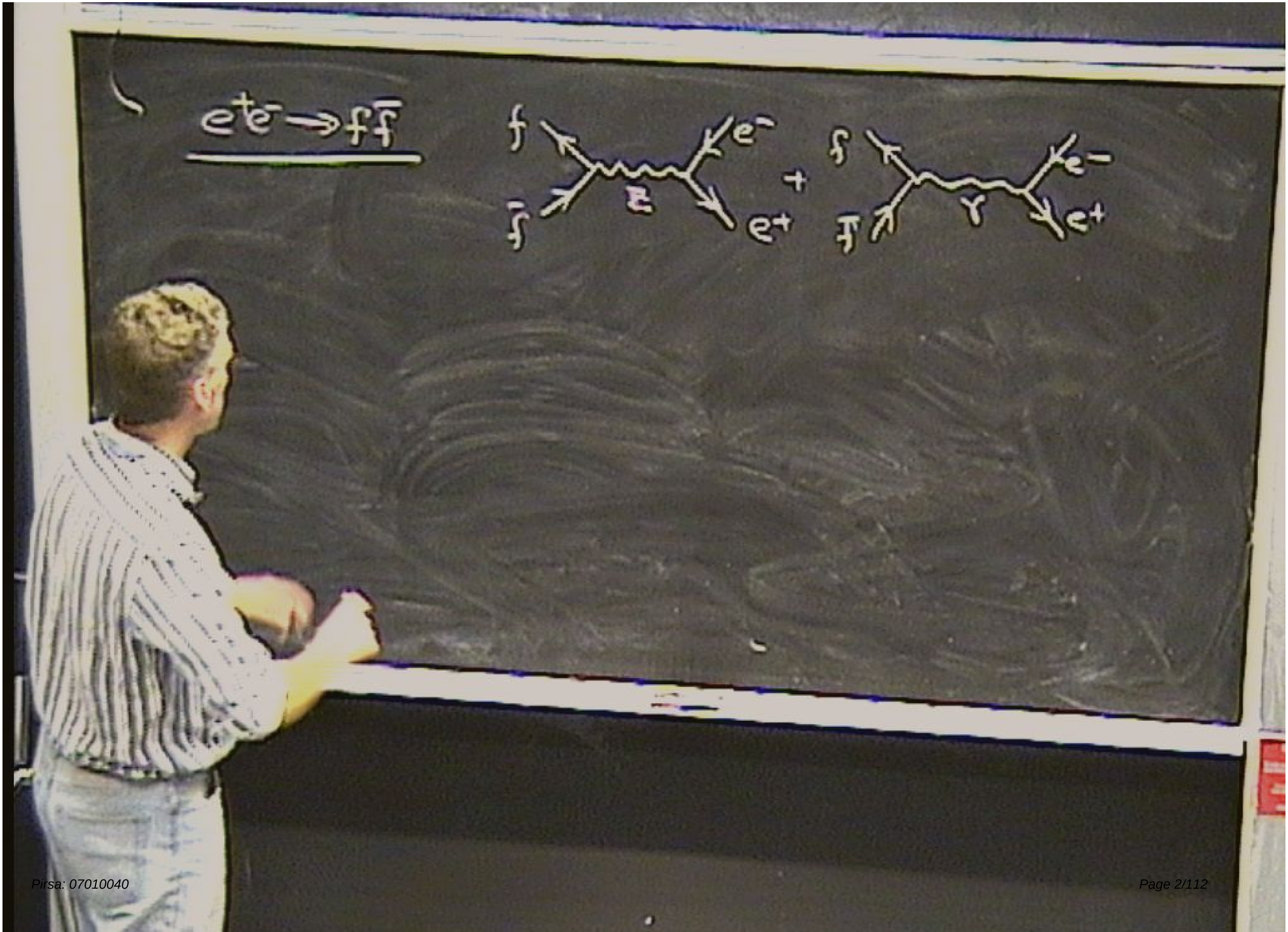
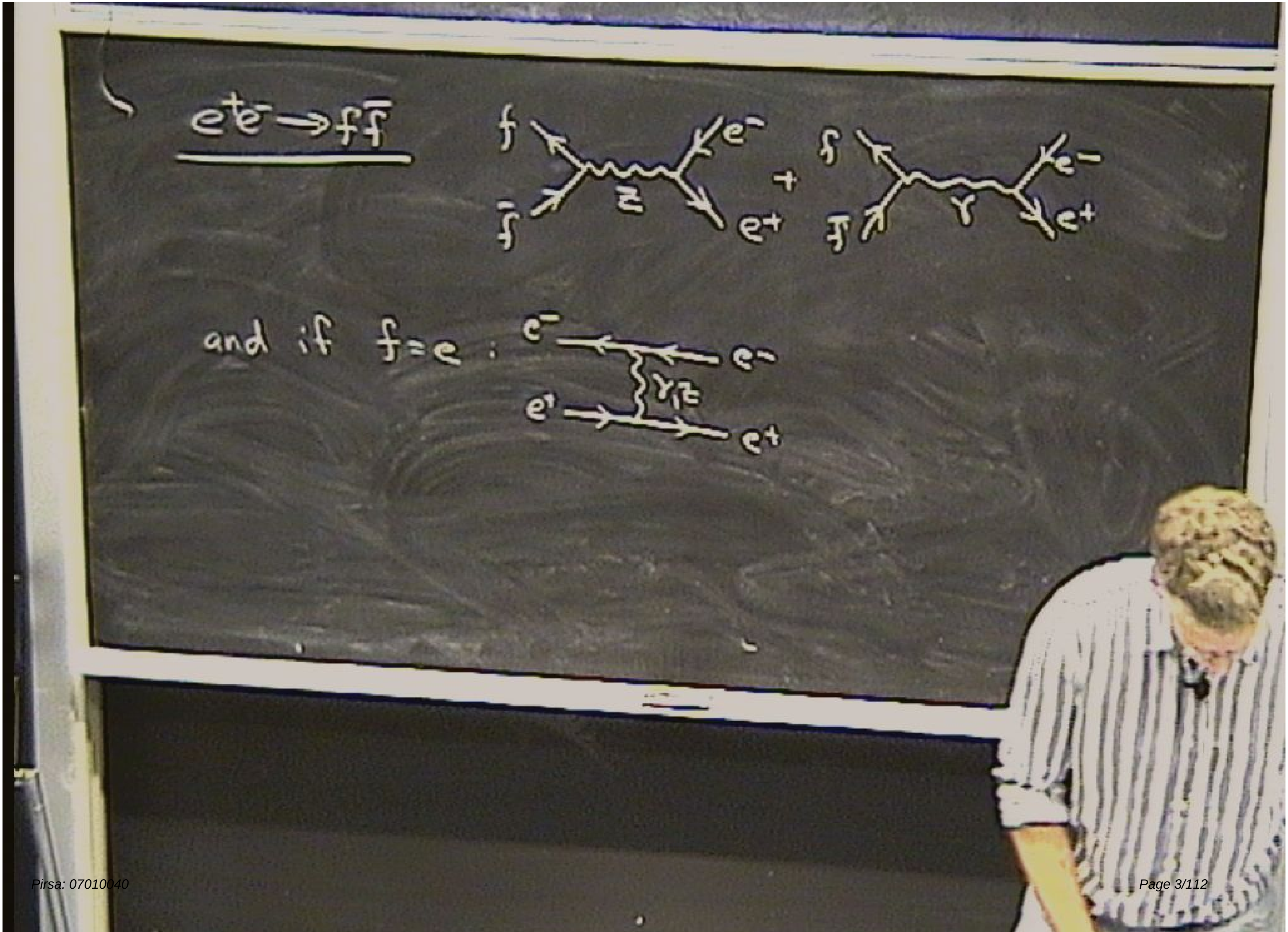
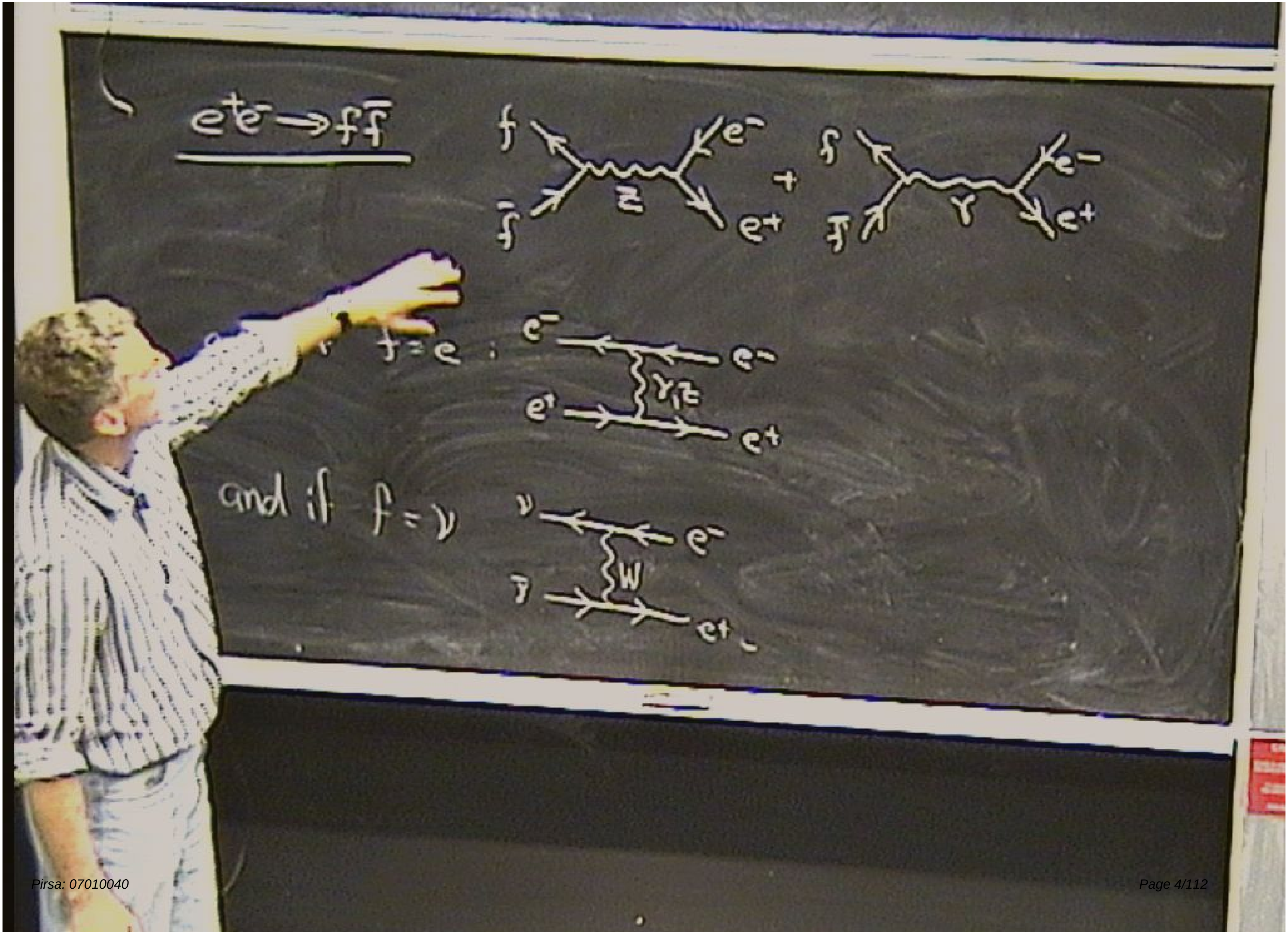
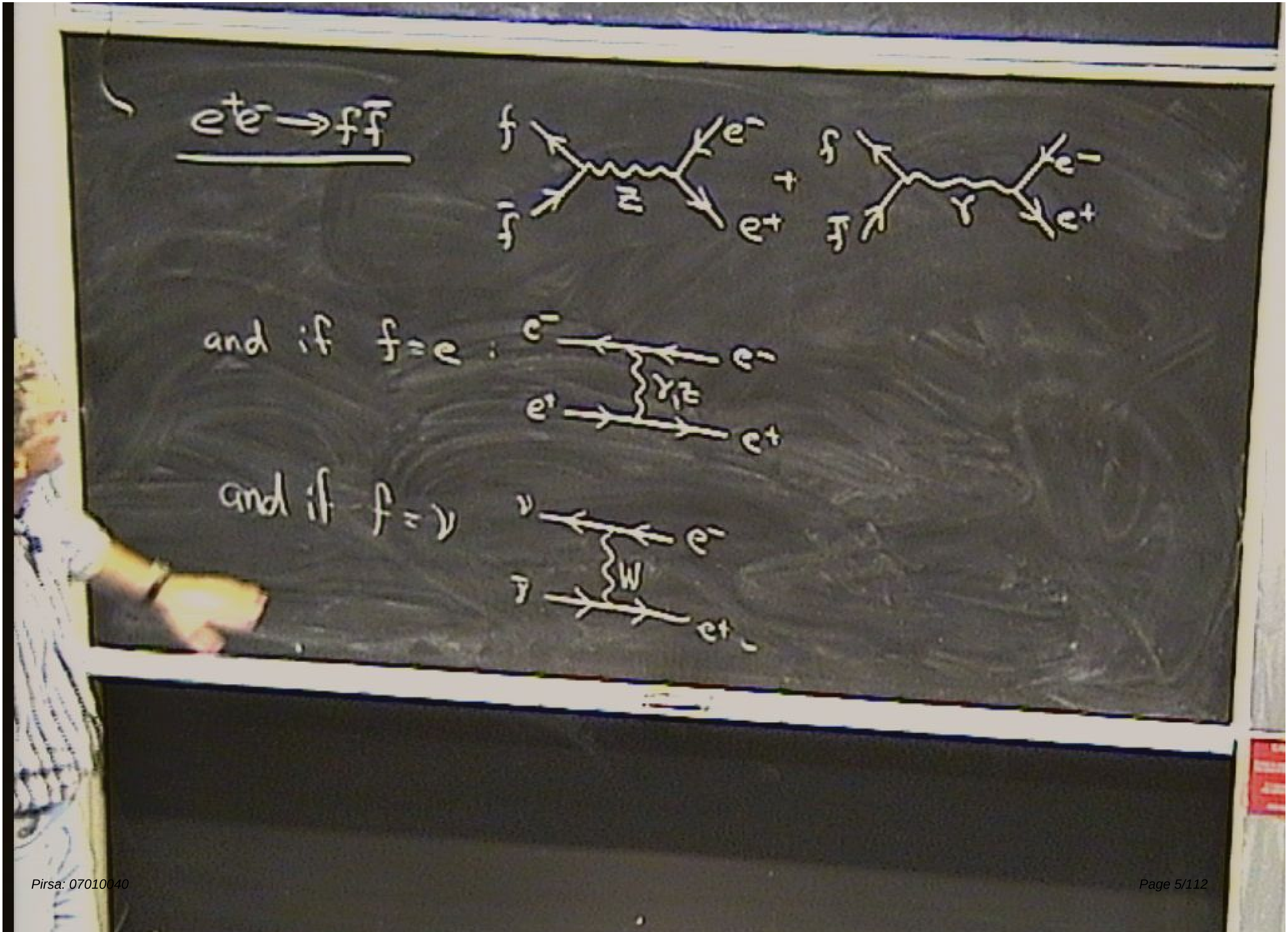
Abstract: Graduate Course on Standard Model & Quantum Field Theory

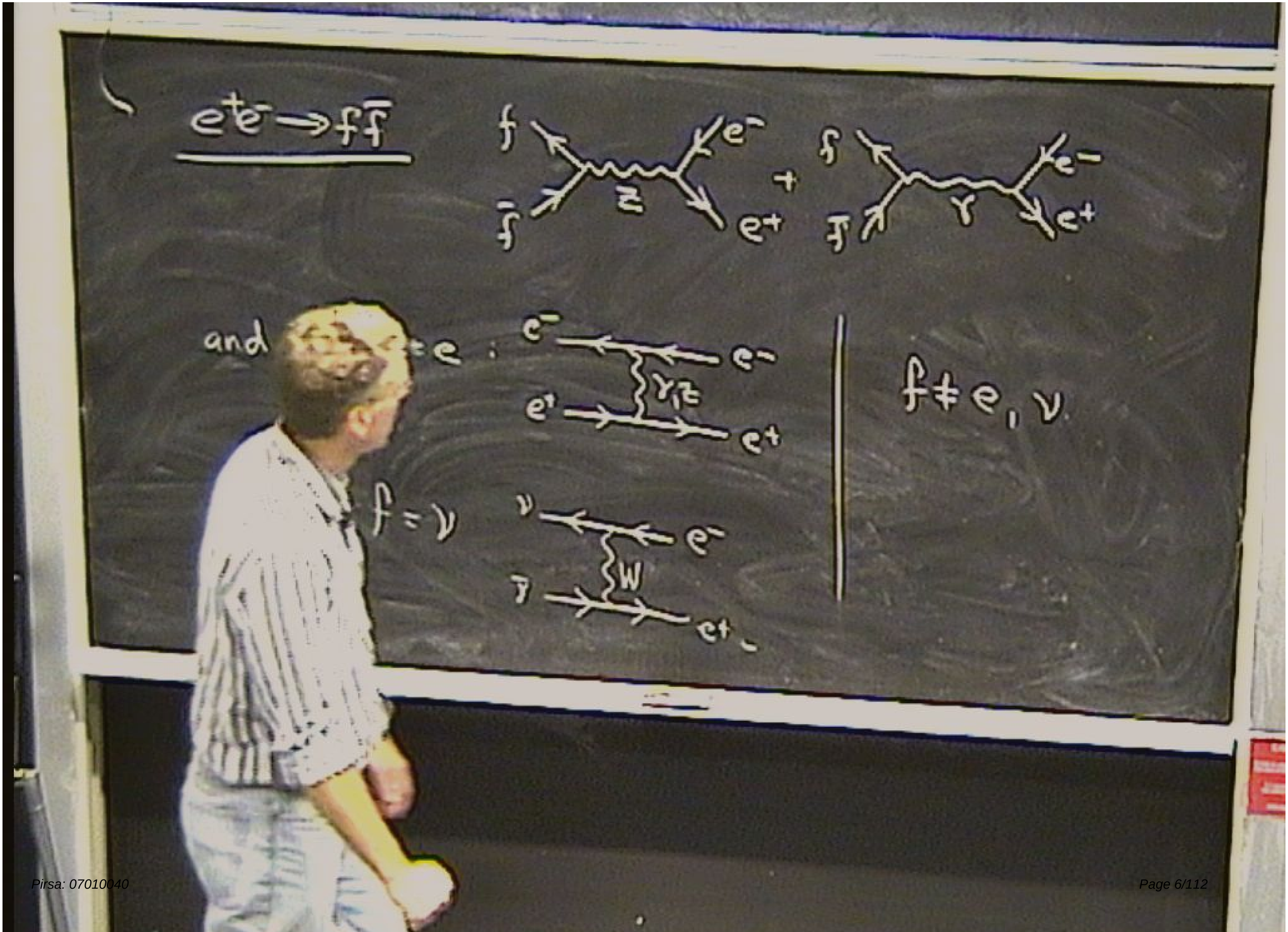

e+e- → f f̄
f
f̄
Z
e-
e+
+
f
f̄
γ
e-
e+


e⁺e⁻ → f f̄
f
f̄
Z
e⁻
e⁺
+
f
f̄
γ
e⁻
e⁺
and if f=e :
e⁻
e⁺
γ,Z
e⁻
e⁺

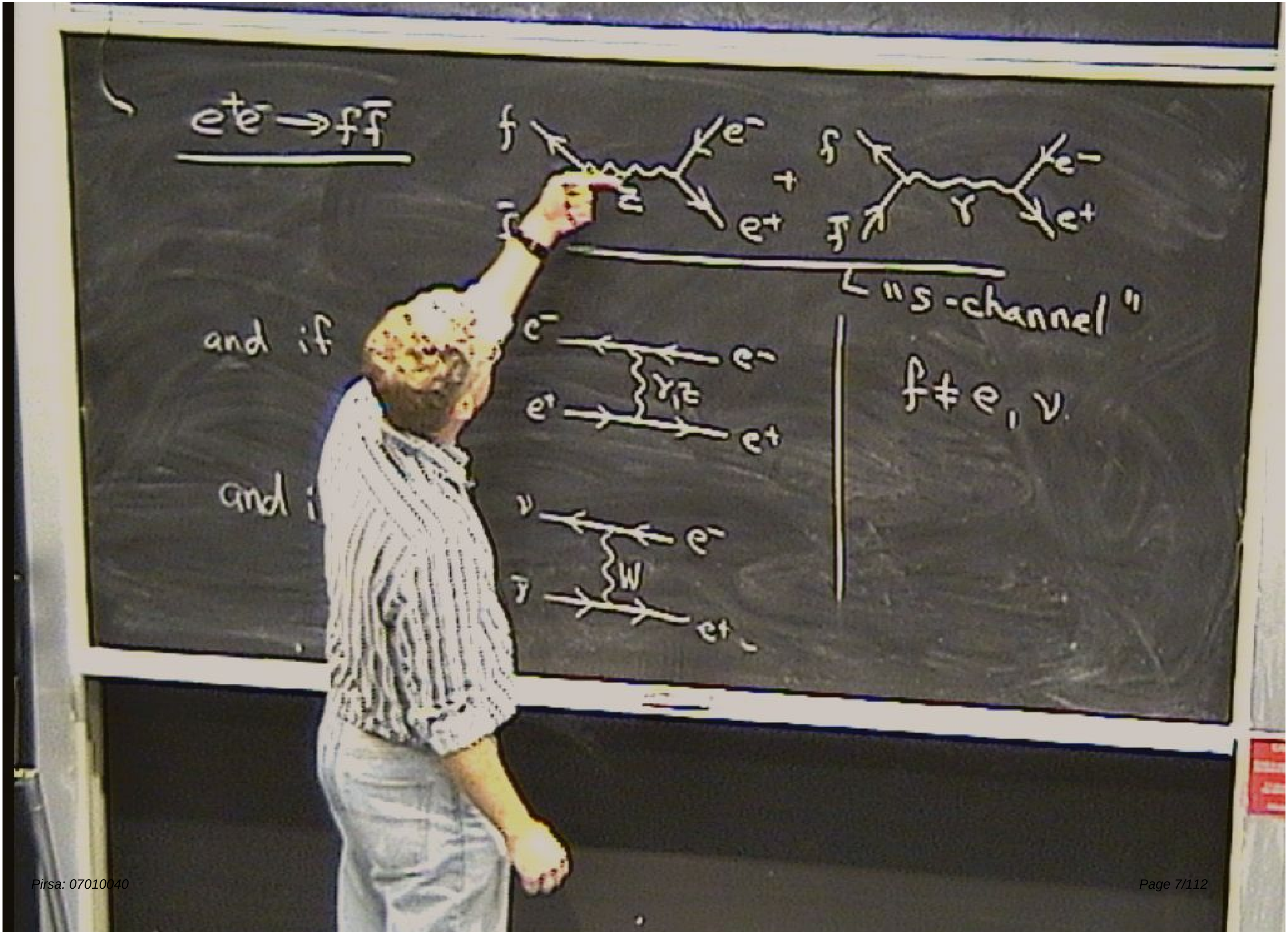


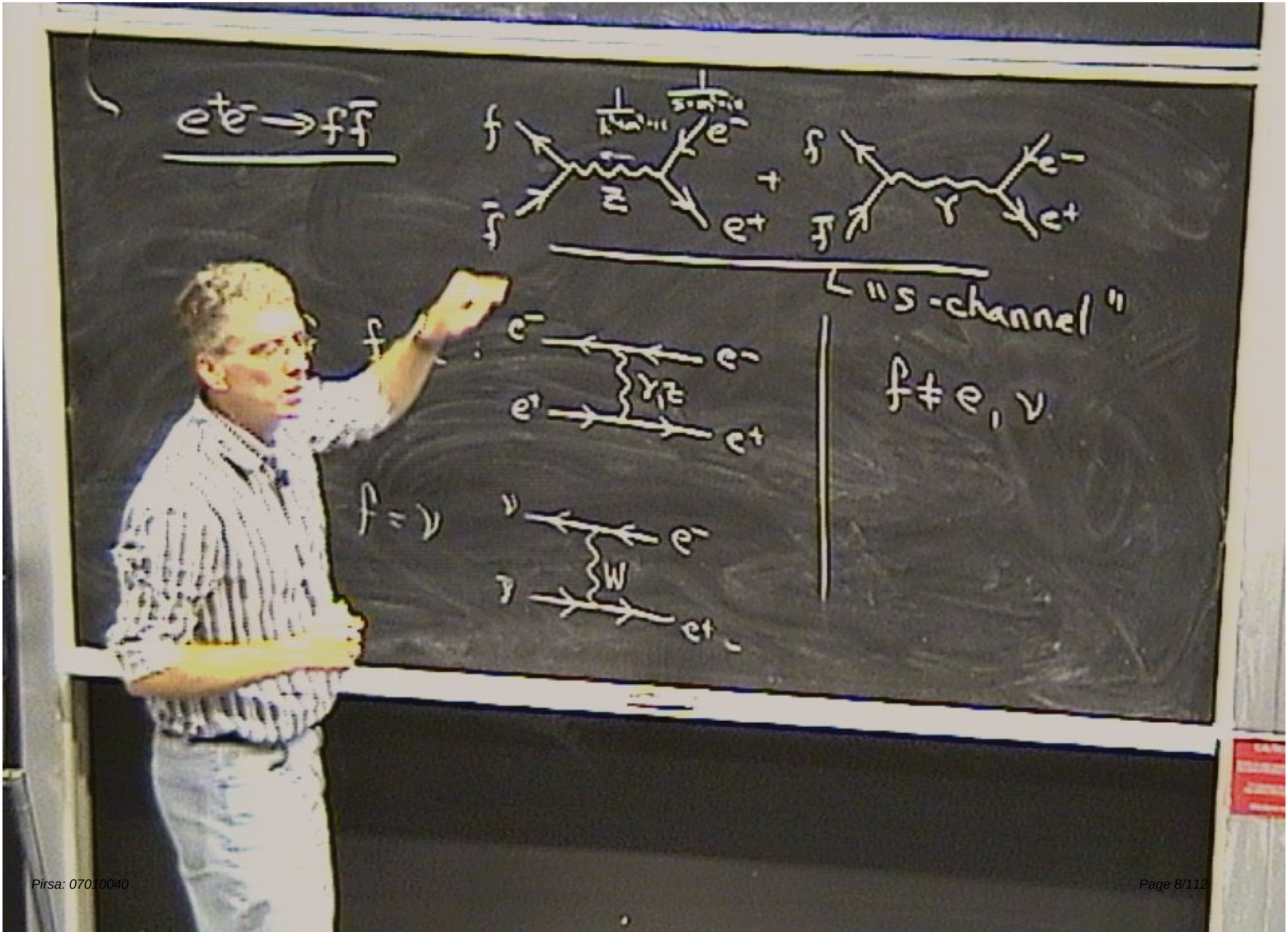
$e^+e^- \to f\bar{f}$
f
e⁻
Z
+
γ
e⁺
f = e :
γ, Z
and if f = ν
W


$e^+e^- \to f\bar{f}$
f
$\bar{f}$
Z
e^-
e^+
+
f
$\bar{f}$
γ
e^-
e^+
and if f = e :
e^-
e^-
e^+
e^+
γ, Z
and if f = ν
ν
$\bar{\nu}$
W
e^-
e^+

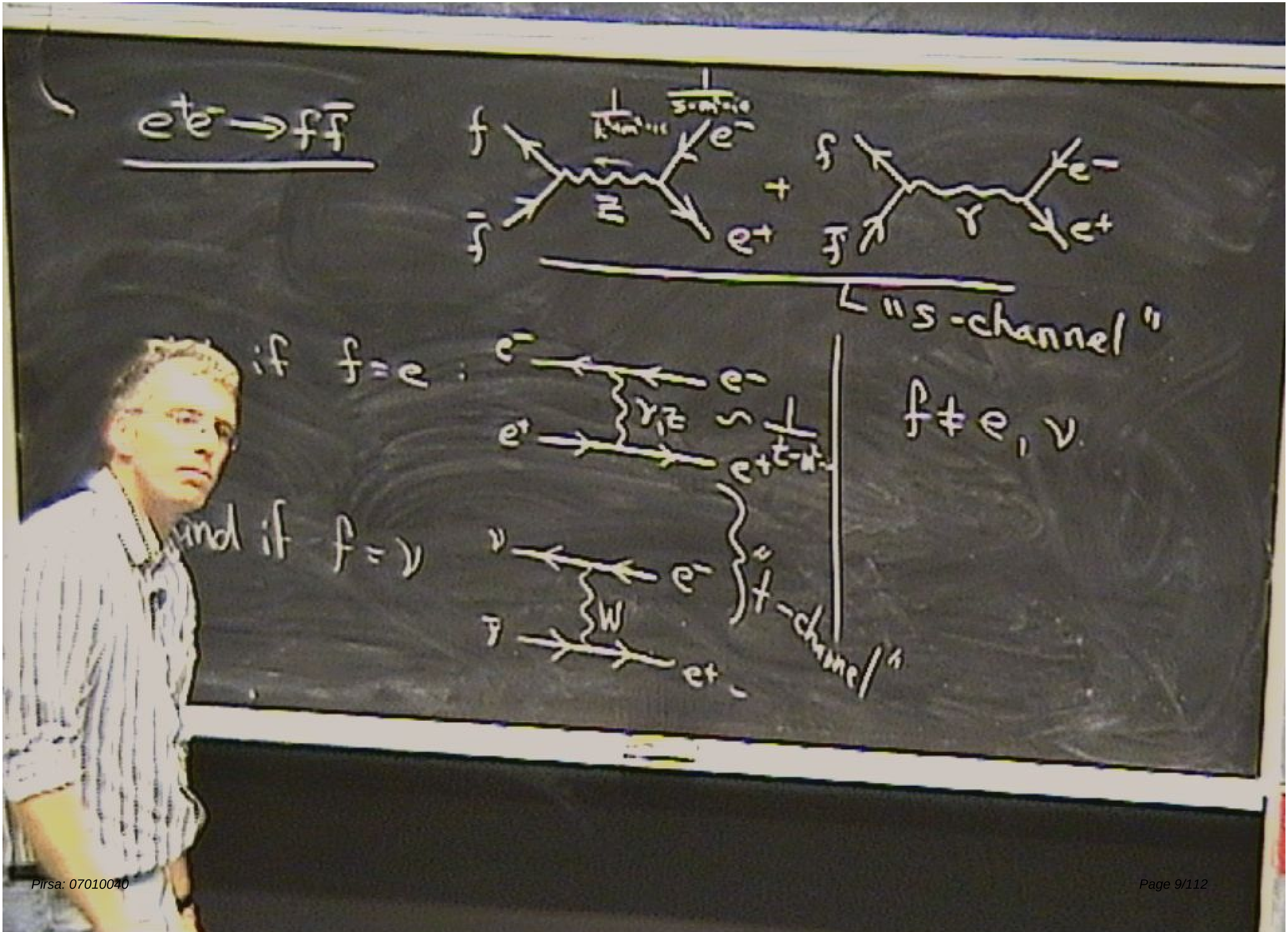


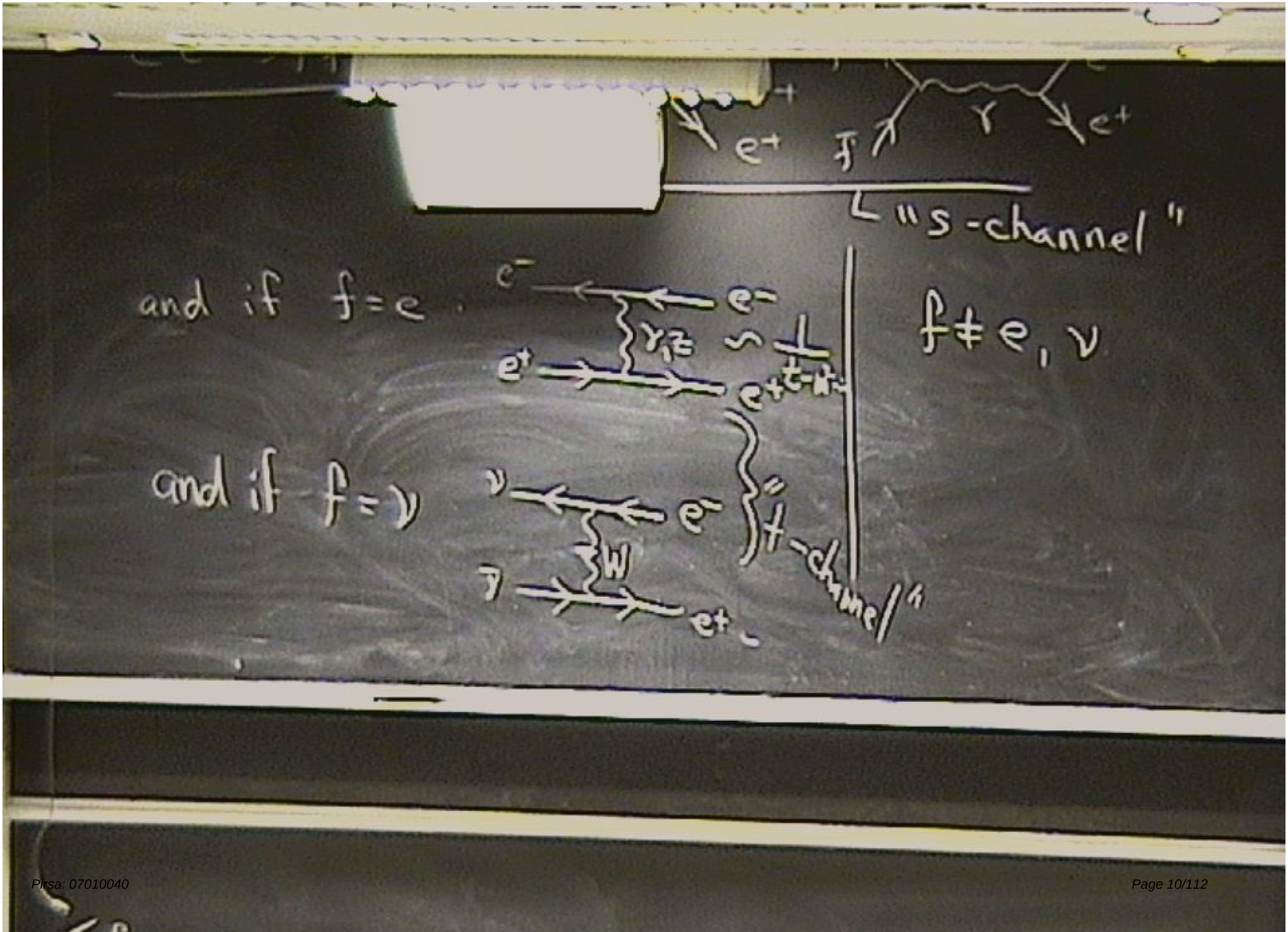
$e^+e^- \to f\bar{f}$
f
e^-
$\bar{f}$
Z
e^+
+
f
e^-
$\bar{f}$
γ
e^+
and
=e :
e^-
e^-
γ,Z
e^+
e^+
f = ν
ν
e^-
W
$\bar{\nu}$
e^+
f ≠ e, ν

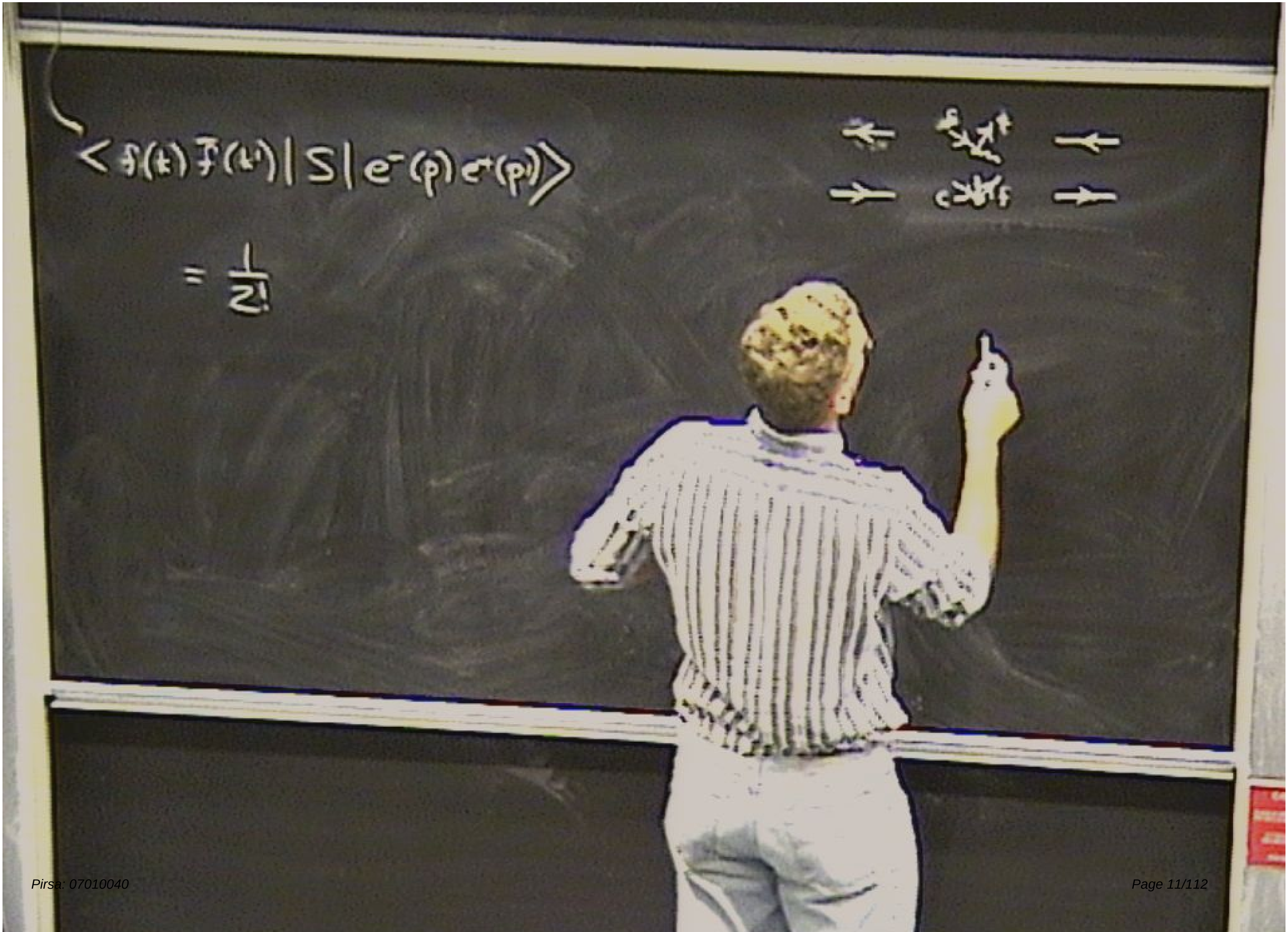

$e^+e^- \to f\bar{f}$
f
e⁻
+
f
e⁻
Z
$\bar{f}$
e⁺
$\bar{f}$
γ
e⁺
"s-channel"
and if
e⁻
e⁻
γ,Z
e⁺
e⁺
$f \neq e, \nu$
and i
ν
e⁻
W
$\bar{\nu}$
e⁺

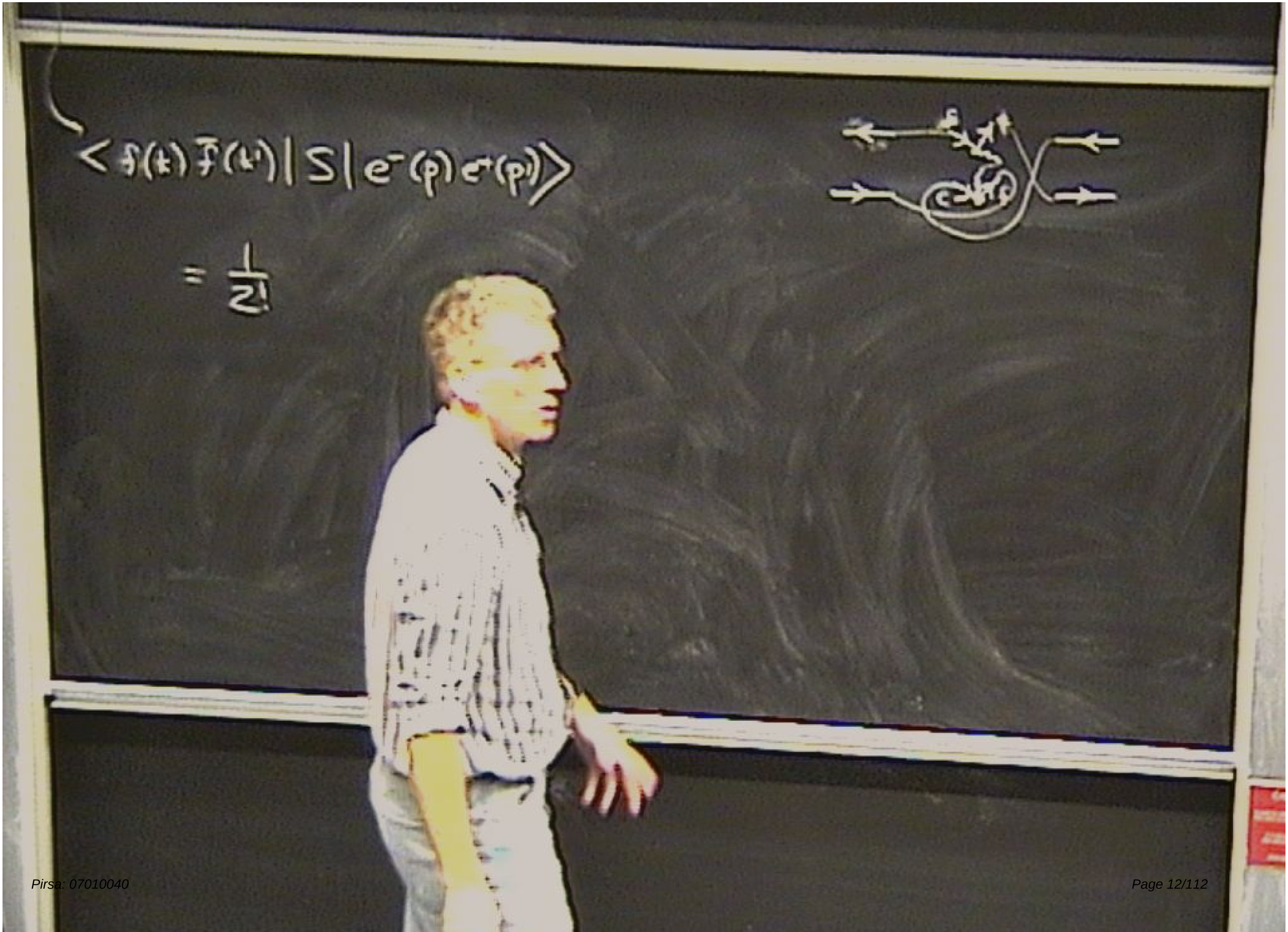


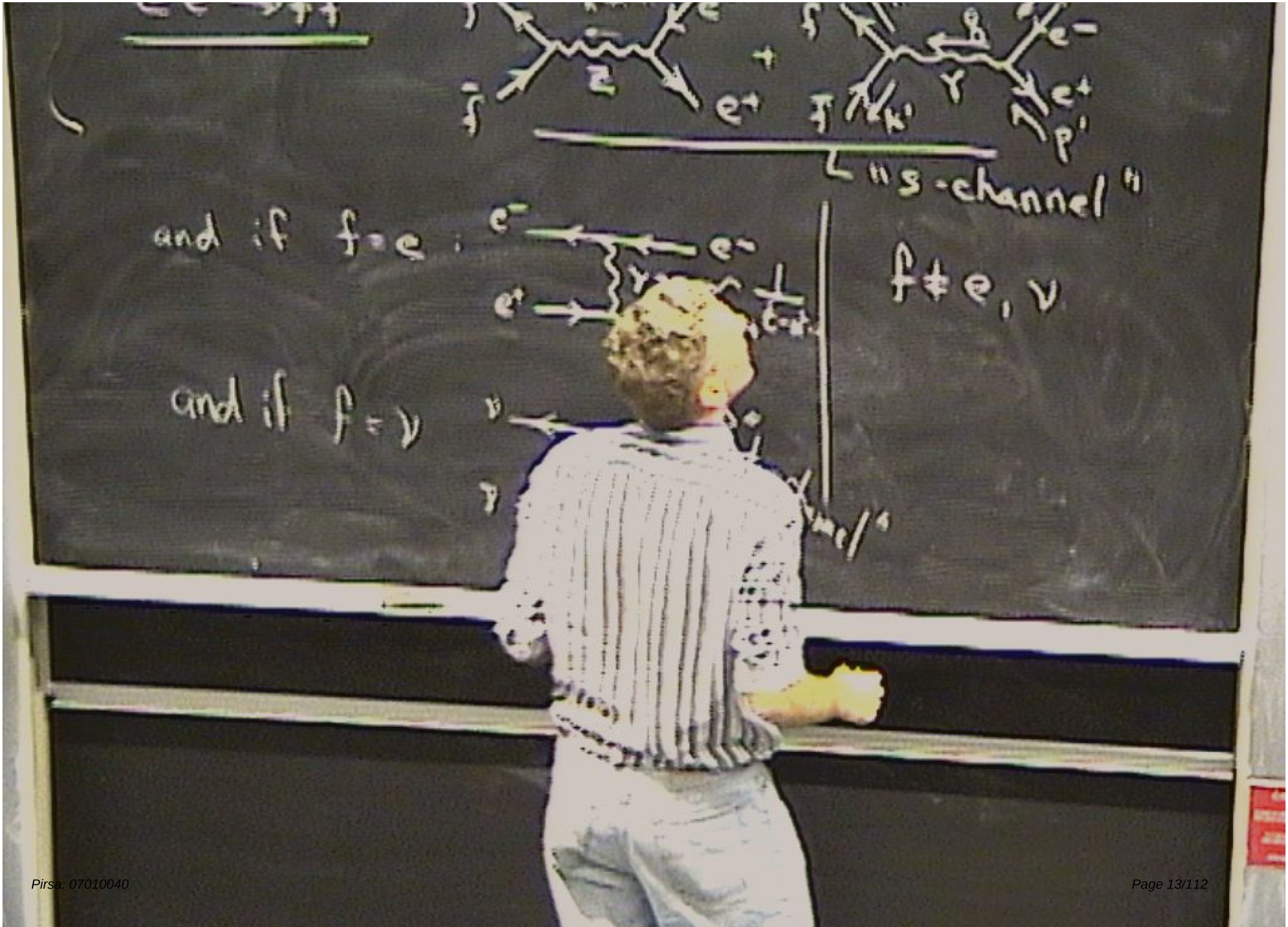
e+e- → f f̄
f
f̄
Z
e-
e+
+
γ
"s-channel"
f ≠ e, ν
γ, Z
f = ν
ν
W

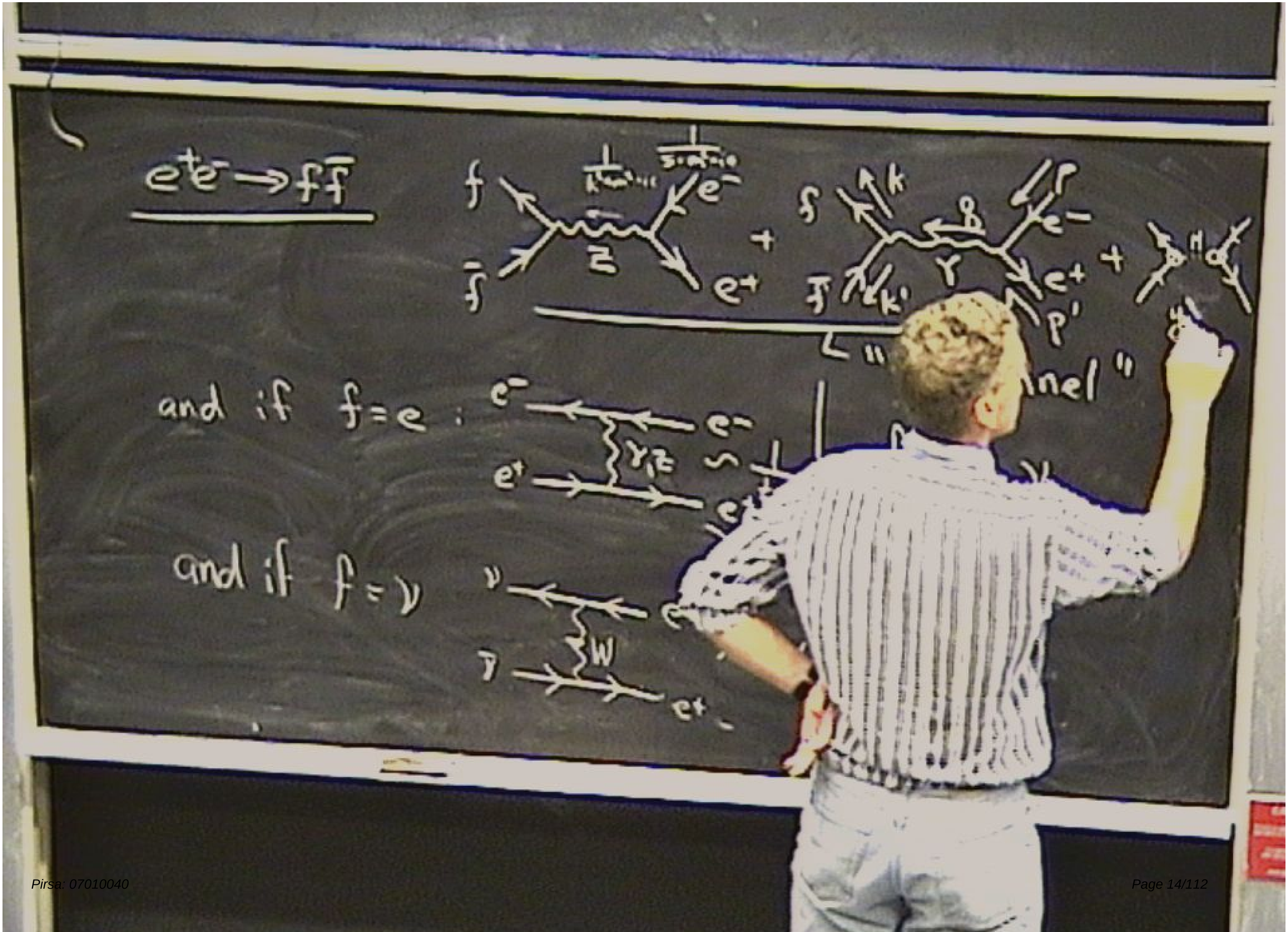

e⁺e⁻ → f f̄
f
f̄
e⁻
e⁺
Z
+
γ
"s-channel"
if f = e :
γ, Z
and if f = ν
W
"t-channel"
f ≠ e, ν.

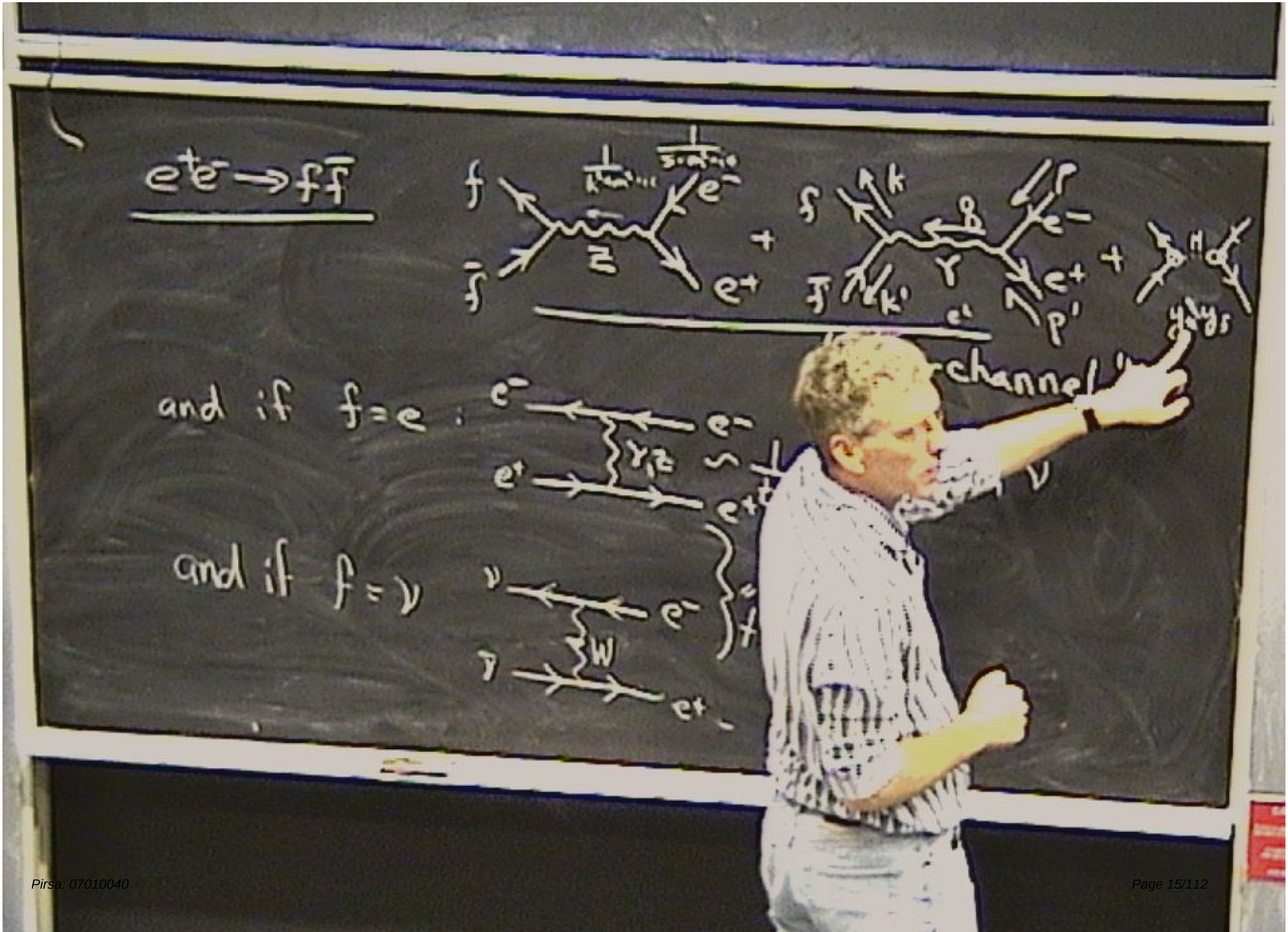

e+
f̄
γ
e+
"s-channel"
and if f=e
e−
e−
γ, Z
∼ 1/(t−M²)
e+
e+
f ≠ e, ν
and if f=ν
ν
e−
W
ν̄
e+
"t-channel"


$\langle f(k)\, \bar{f}(k') \,|\, S \,|\, e^-(p)\, e^+(p') \rangle$
$= \frac{1}{2!}$

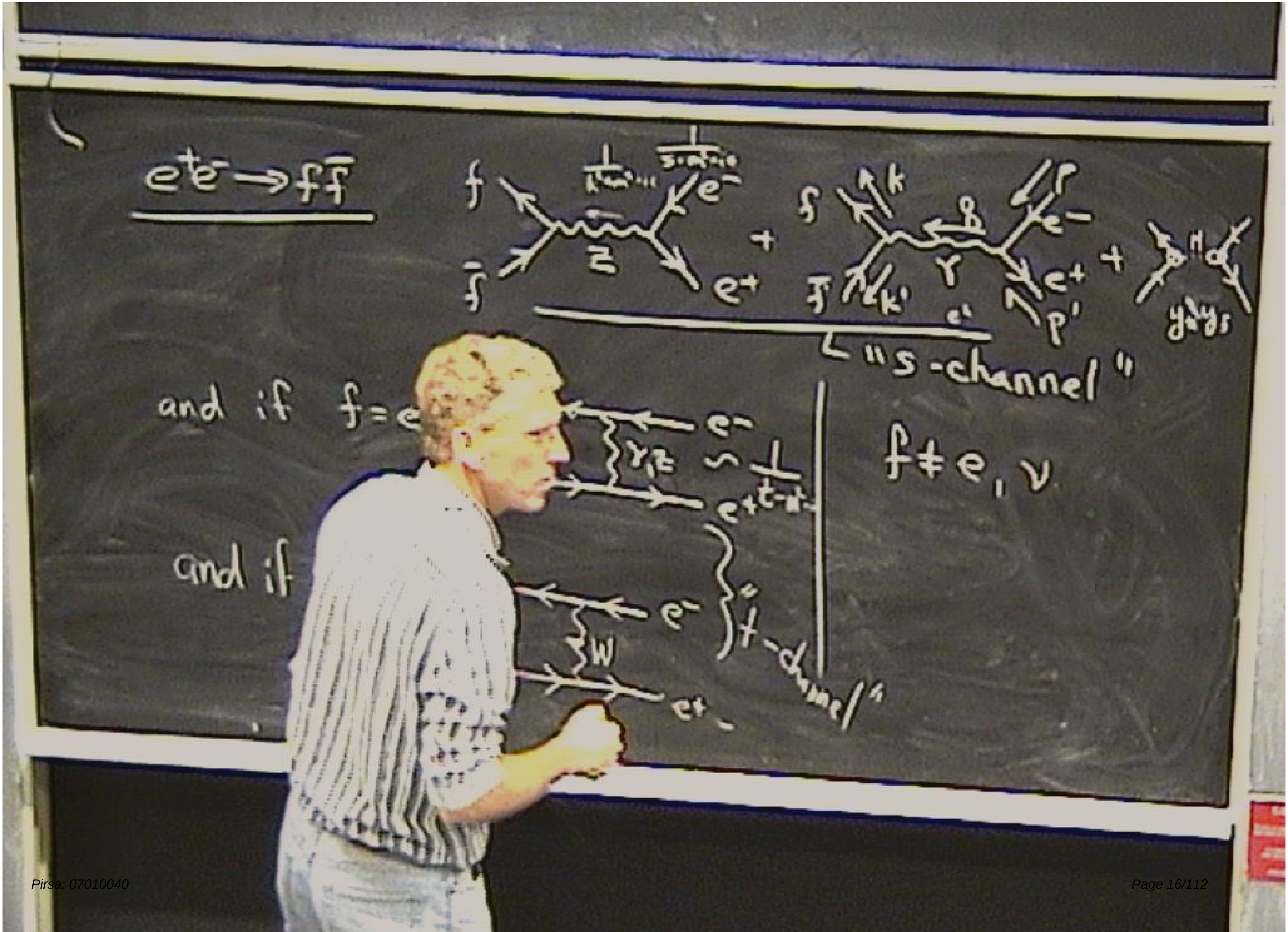

⟨γ(k) γ(k′) | S | e⁻(p) e⁺(p′)⟩
= 1/2!


"s-channel"
and if f=e
and if f=ν
f≠e, ν

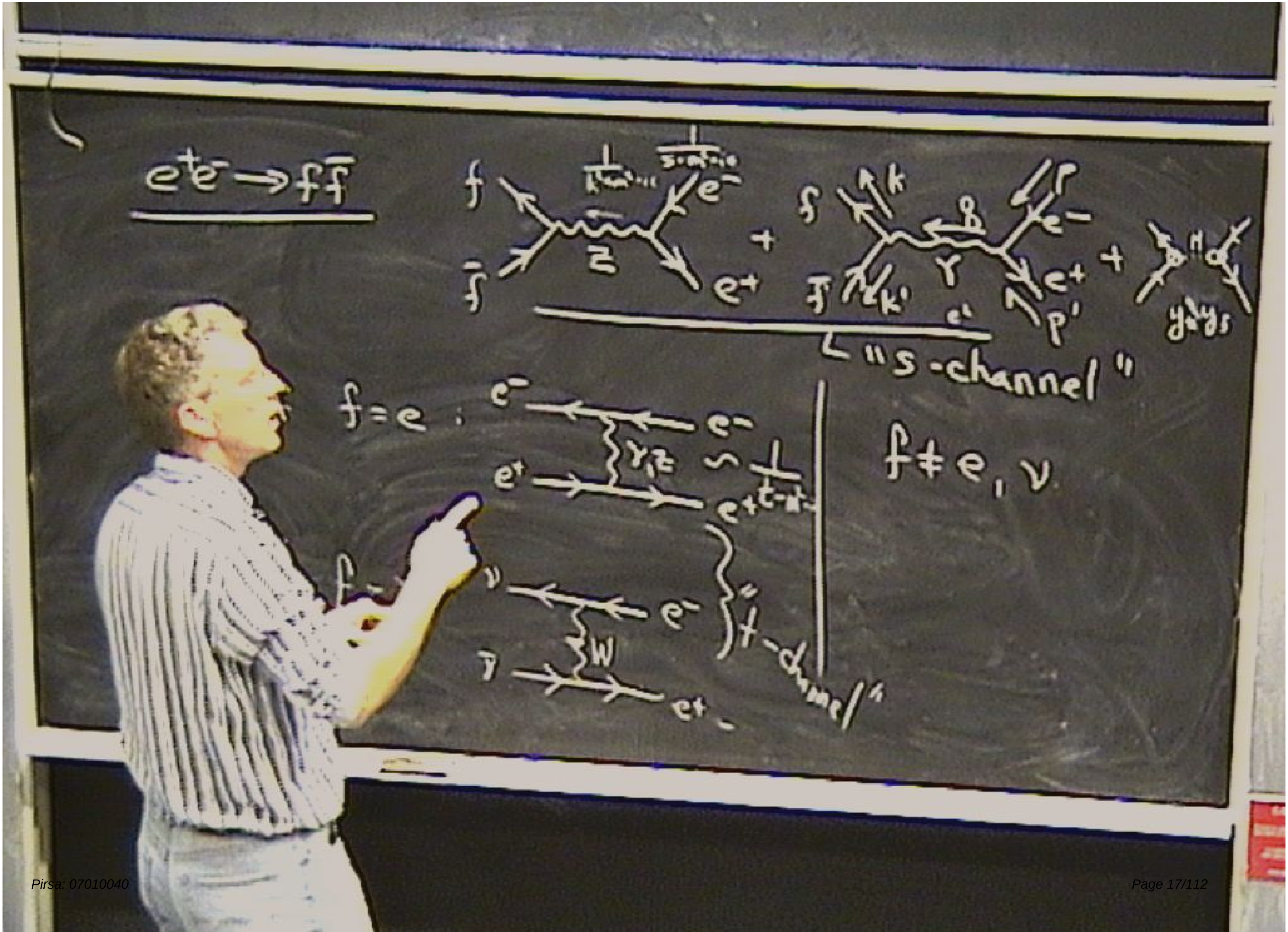

e+e- → f f̄
and if f=e :
and if f=ν
Z
γ
γ, Z
W
"channel"

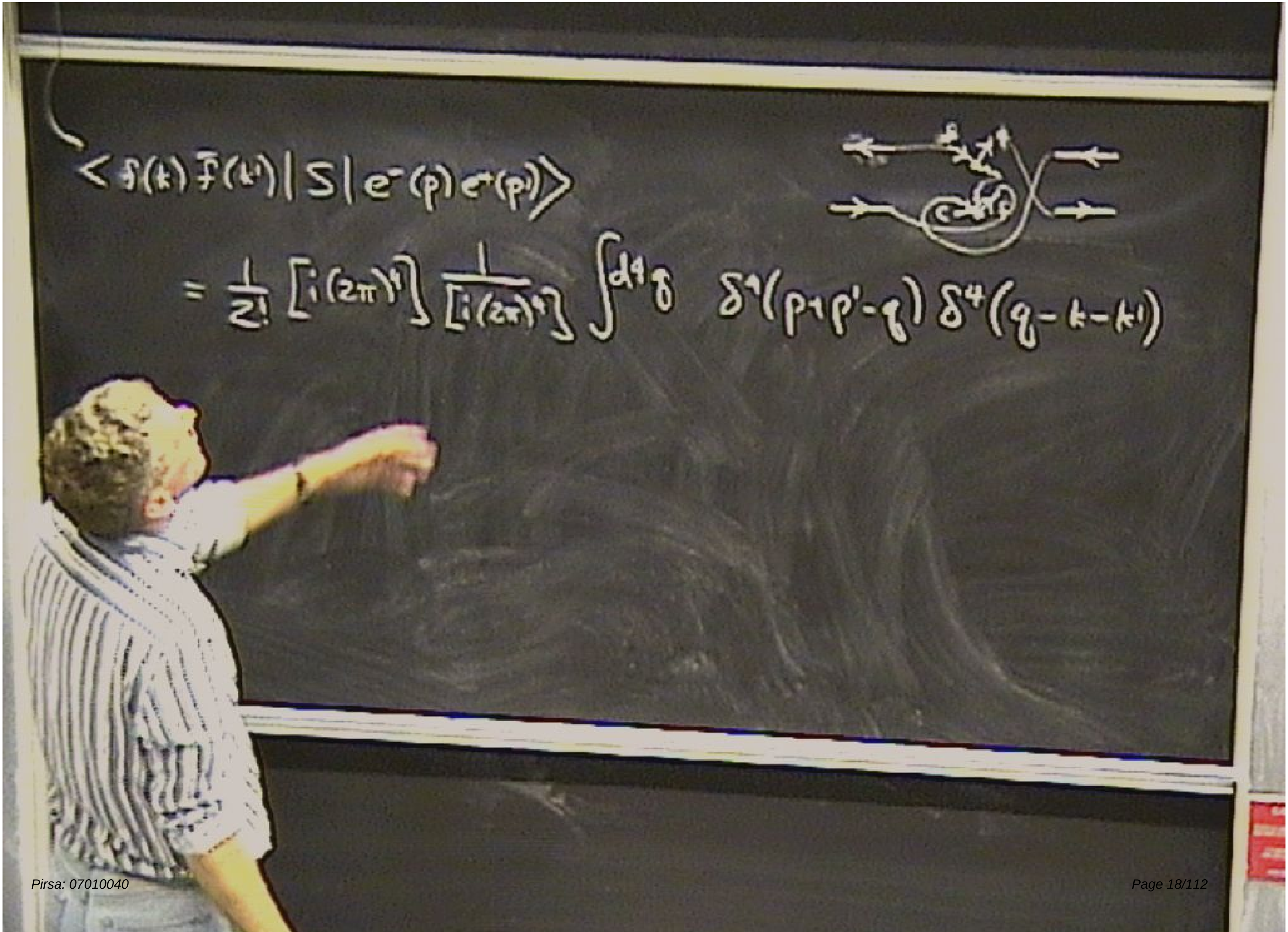


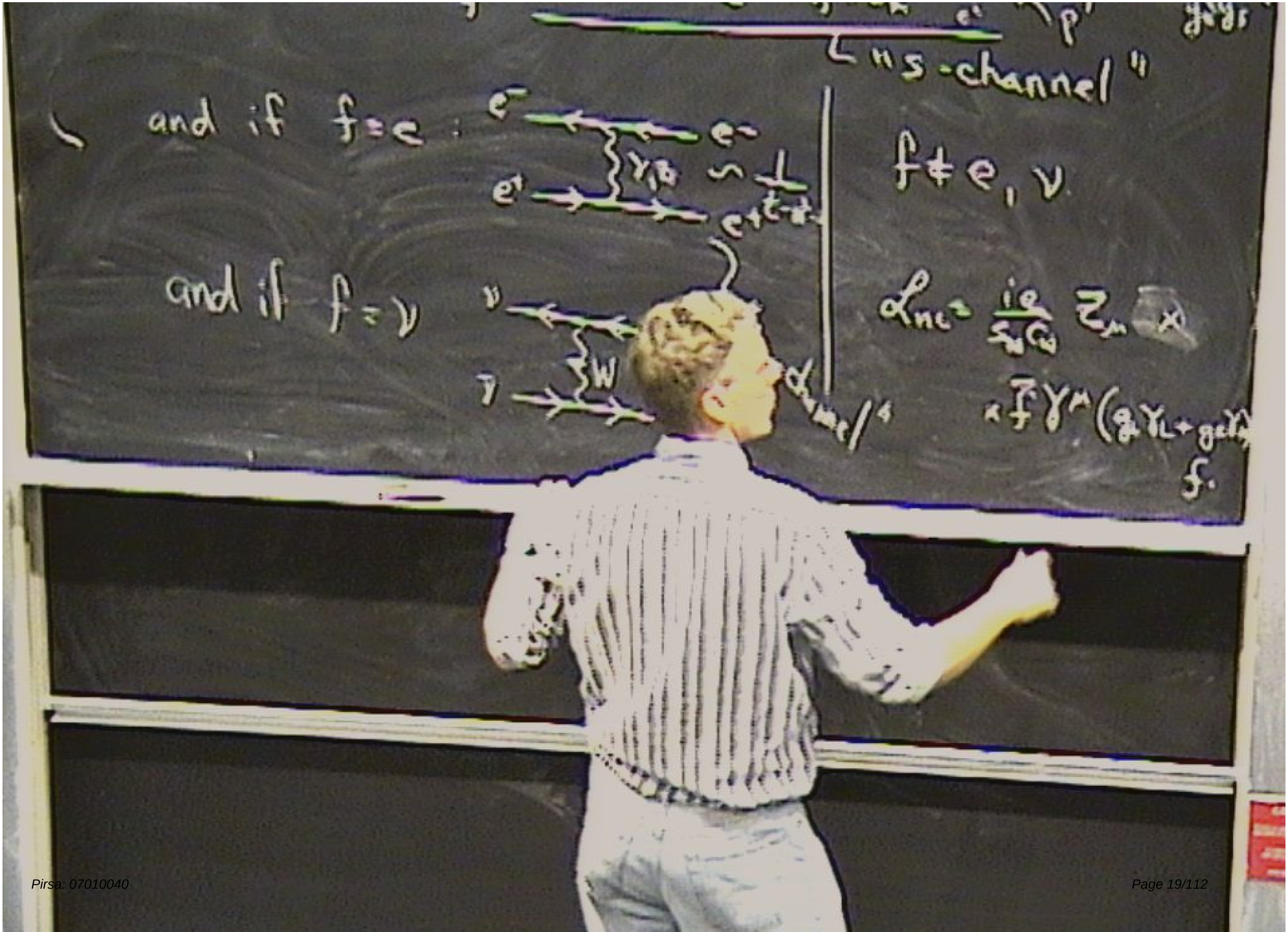
e+e- → f f̄
f
f̄
Z
e-
e+
k
k'
p
p'
γ
channel
and if f=e :
γ, Z
and if f = ν
W

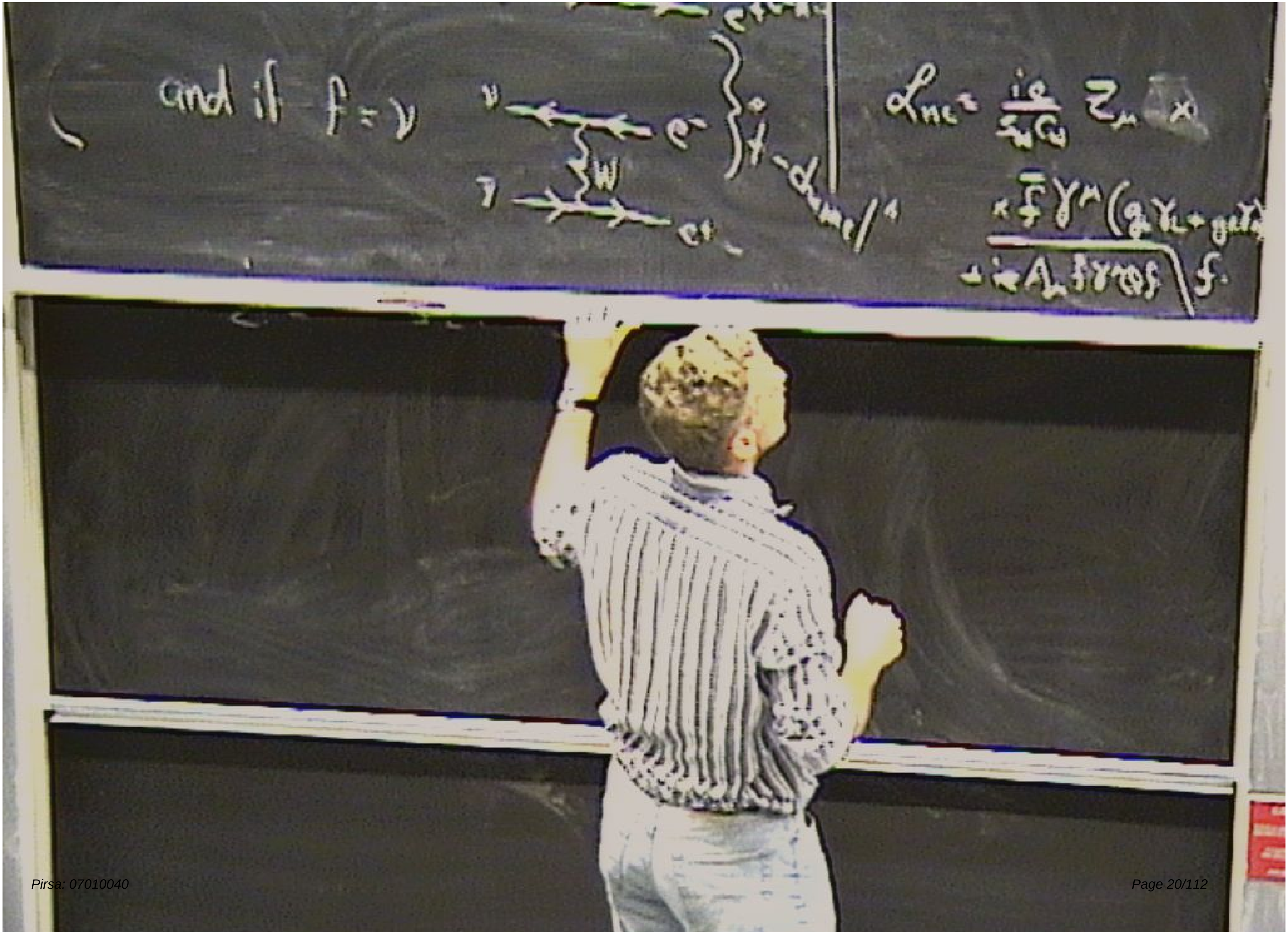


e+e- → f f̄
f
f̄
Z
e-
e+
+
k
k'
p
p'
γ
"s-channel"
and if f=e
γ, Z
e-
e+
f≠e, ν
and if
W
"t-channel"


e⁺e⁻ → f f̄
"s-channel"
f = e
f ≠ e, ν
"t-channel"


$\langle f(k)\,\bar{f}(k')|\,S\,|e^-(p)\,e^+(p')\rangle$
$= \frac{1}{2!}\left[i(2\pi)^4\right]\frac{1}{\left[i(2\pi)^4\right]}\int d^4q\;\delta^4(p+p'-q)\,\delta^4(q-k-k')$


"s-channel"
and if f=e :
e⁻
e⁺
γ, Z
and if f = ν
ν
W
f ≠ e, ν
f.

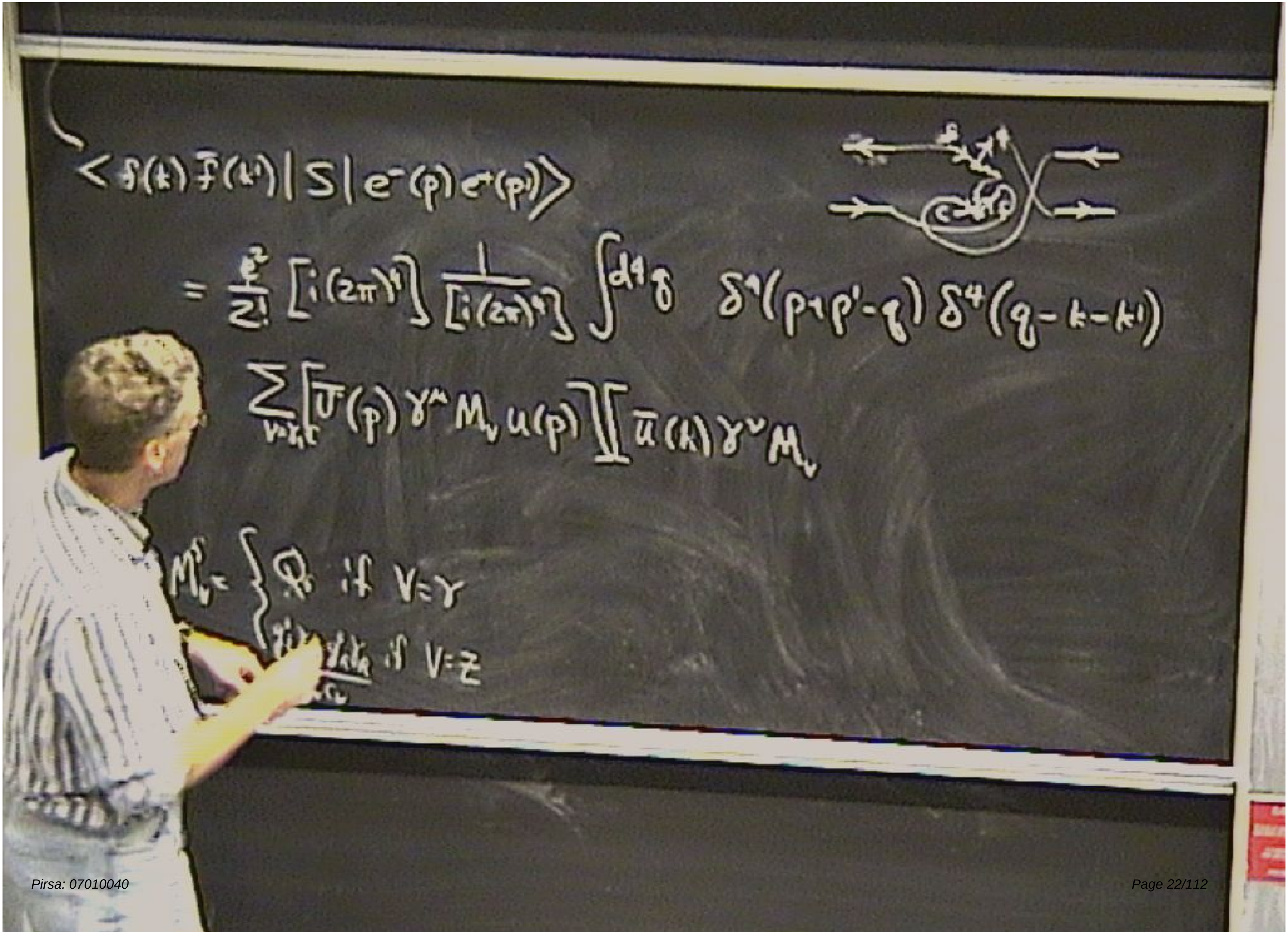

and if f = ν

$$\langle \gamma(k)\,\gamma(k')|\,S\,|\,e^-(p)\,e^+(p')\rangle$$

$$= \frac{1}{2!}\left[i(2\pi)^4\right]\frac{1}{\left[i(2\pi)^4\right]}\int d^4q \quad \delta^4(p+p'-q)\,\delta^4(q-k-k')$$

$$\bar{v}(p)\,\gamma^\mu M_\nu$$

M



< f(k) f̄(k') | S | e⁻(p) e⁺(p') >
= e²/2! [i(2π)⁴] 1/[i(2π)⁴] ∫d⁴q δ⁴(p+p'−q) δ⁴(q−k−k')
Σ V=γ,Z [v̄(p') γ^μ M_V u(p)] [ū(k) γ^ν M_V
M_V = { Q_f if V=γ
if V=Z

$$\langle f(k)\,\bar f(k')\,|\,S\,|\,e^-(p)\,e^+(p')\rangle$$

$$= \frac{e^2}{2!}\left[i(2\pi)^4\right]\frac{1}{\left[i(2\pi)^4\right]}\int d^4q \;\; \delta^4(p+p'-q)\,\delta^4(q-k-k')$$

$$\sum_{V=\gamma,Z}\left[\bar v(p)\,\gamma^\mu M^e_V\,u(p)\right]\left[\bar u(k)\,\gamma^\nu M^f_V\,v(k')\right]\frac{\eta_{\mu\nu}+q_\mu q_\nu/M_V^2}{q^2+M_V^2-i\epsilon}$$

$$M^f_V = \begin{cases} Q_f & \text{if } V=\gamma \\ \dfrac{[illegible]}{s_W c_W} & \text{if } V=Z \end{cases}$$

$$\langle f(k)\,\bar f(k')\,|\,S\,|\,e^-(p)\,e^+(p')\rangle$$

$$= \frac{e^2}{2!}\left[i(2\pi)^4\right]\frac{1}{\left[i(2\pi)^4\right]}\int d^4q \quad \delta^4(p+p'-q)\,\delta^4(q-k-k')$$

$$\sum_{V=\gamma,Z}\left[\bar v(p')\gamma^\mu M^e_V u(p)\right]\left[\bar u(k)\gamma^\nu M^f_V v(k')\right]\frac{\eta_{\mu\nu}+q_\mu q_\nu/M_V^2}{q^2+M_V^2-i\epsilon}$$

$$M^f_V = \begin{cases} Q_f & \text{if } V=\gamma \\ \dfrac{g^f_L P_L + g^f_R P_R}{s_W c_W} & \text{if } V=Z \end{cases}$$

$$\langle f(k)\bar{f}(k')|S|e^-(p)e^+(p')\rangle$$

$$= \frac{e^2}{2!}\left[i(2\pi)^4\right]\frac{1}{\left[i(2\pi)^4\right]}\int d^4q \; \delta^4(p+p'-q)\,\delta^4(q-k-k')$$

$$\sum_{V=\gamma,Z}\left[\bar{v}(p')\gamma^\mu M_V^e u(p)\right]\left[\bar{u}(k)\gamma^\nu M_V^f v(k')\right]\frac{\eta_{\mu\nu}+q_\mu q_\nu/M_V^2}{q^2+M_V^2-i\epsilon}$$

$$M_V^f = \begin{cases} Q_f & \text{if } V=\gamma \\ \dfrac{g_L^f\gamma_L + g_R^f\gamma_R}{s_W c_W} & \text{if } V=Z \end{cases}$$

neglect $\dfrac{m_f^2}{M_Z^2}, \dfrac{m_e^2}{M_Z^2}$

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{\text{all spins}} |\mathcal{M}|^2 \quad \text{, where } S = -(2\pi)^4 i\, \delta^4(p+p'-k-k')\,\mathcal{M}$$

$$= N_c$$

$$N_c = \begin{cases} 1 & \text{if } f = \mu, \tau \\ 3 & \text{if } f = q \end{cases}$$

$$\langle f(k)\bar{f}(k')|S|e^-(p)e^+(p')\rangle$$

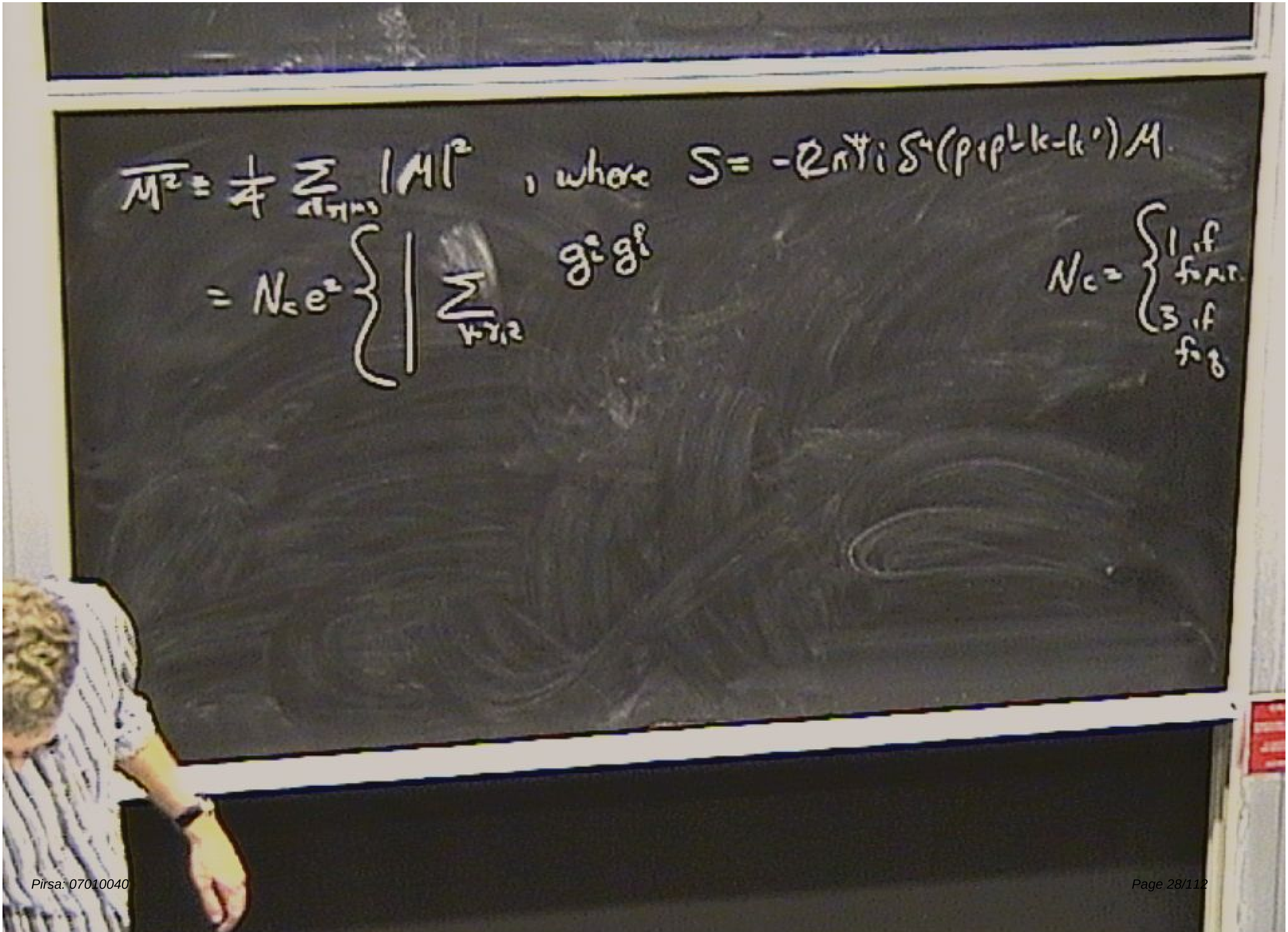
$$= \frac{e^2}{2!}\left[i(2\pi)^4\right]\frac{1}{\left[i(2\pi)^4\right]}\int d^4q \;\; \delta^4(p+p'-q)\,\delta^4(q-k-k')$$

$$\sum_{V=\gamma,Z}\left[\bar{v}(p')\gamma^\mu M_V^e u(p)\right]\left[\bar{u}(k)\gamma^\nu M_V^f v(k')\right]\frac{\eta_{\mu\nu}+q_\mu q_\nu/M_V^2}{q^2+M_V^2-i\epsilon}$$

$g_V^f - Q_f - g_A^f$

$$M_V^f = \begin{cases} Q_f & \text{if } V=\gamma \\ \dfrac{g_V^f - g_A^f\gamma_5}{s_W c_W} & \text{if } V=Z \end{cases}$$

neglect $\dfrac{m_f^2}{M_Z^2}, \dfrac{m_e^2}{M_Z^2}$


$\overline{|M|^2} = \frac{1}{4}\sum |M|^2$, where $S = -(2\pi)^4 i\,\delta^4(p+p'-k-k')M$
$= N_c e^2 \left\{ \left| \sum_{k=\gamma, Z} \right. \right.$ $g_i^e g_i^f$
$N_c = \begin{cases} 1 \text{ if } f = \mu, \tau \\ 3 \text{ if } f = q \end{cases}$

$$\overline{\mathcal{M}^2} = \frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 \quad , \text{ where } S = -(2\pi)^4 i\, \delta^4(p+p'-k-k')\,\mathcal{M}$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma,Z} \frac{g^e g^f}{s - M_V^2 - i\epsilon} \right|^2 \right.$$

$$N_c = \begin{cases} 1 \text{ if } f = \mu, \tau \\ 3 \text{ if } f = q \end{cases}$$

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 \quad , \text{ where } S = -(2\pi)^4 i\, \delta^4(p+p'-k-k')\,\mathcal{M}$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma,Z} \frac{g^e g^f}{s - M_V^2 - i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1 & \text{if } f = \mu, \tau \\ 3 & \text{if } f = q \end{cases}$$

$$+ \left| \sum_{V=\gamma,Z} \right.$$

$$\overline{\mathcal{M}^2} = \frac{1}{4}\sum_{\text{all spins}} |\mathcal{M}|^2 \quad \text{, where } S = -(2\pi)^4 i\, \delta^4(p+p'-k-k')\,\mathcal{M}$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma,Z} \frac{g_L^e g_L^f}{s - M_V^2 - i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1 & \text{if } f=\mu,\tau \\ 3 & \text{if } f=q \end{cases}$$

$$+ \left| \sum_{V=\gamma,Z} \frac{g_R^e g_R^f}{s - M_V^2 - i\epsilon} \right|^2 u^2 + \left| \sum_{V=\gamma,Z} \frac{g_L^e g_R^f}{s - M_V^2 - i\epsilon} \right|^2 t^2$$

$$+ \left| \sum_{V=\gamma,Z} \frac{g_R^e g_L^f}{s - M_V^2 - i\epsilon} \right|^2 t^2$$

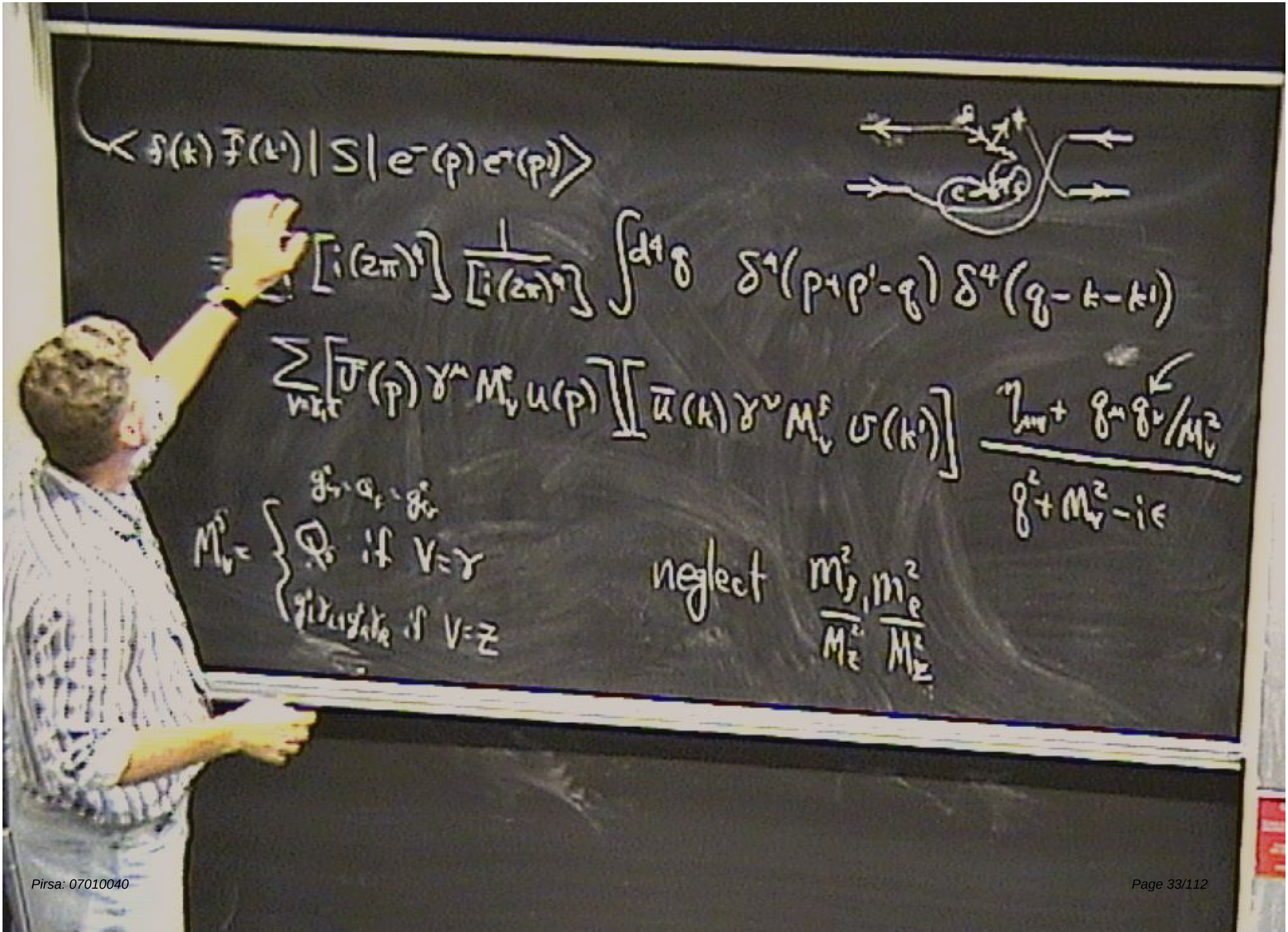
$$\overline{\mathcal{M}^2} = \frac{1}{4}\sum_{\text{all spins}} |\mathcal{M}|^2 \quad , \text{ where } S = -(2\pi)^4 i\,\delta^4(p+p'-k-k')\,\mathcal{M}$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma,Z} \frac{g_L^e g_L^f}{s-M_V^2-i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1 \text{ if } f=\mu,\tau \\ 3 \text{ if } f=q \end{cases}$$

$$+ \left| \sum_{V=\gamma,Z} \frac{g_R^e g_R^f}{s-M_V^2-i\epsilon} \right|^2 u^2 + \left| \sum_{V=\gamma,Z} \frac{g_L^e g_R^f}{s-M_V^2-i\epsilon} \right|^2 t^2$$

$$\left. + \left| \sum_{V=\gamma,Z} \frac{g_R^e g_L^f}{s-M_V^2-i\epsilon} \right|^2 t^2 \right\}$$


⟨ f(k) f̄(k') | S | e⁻(p) e⁺(p') ⟩
[i(2π)⁴] 1/[i(2π)⁴] ∫d⁴q δ⁴(p+p'−q) δ⁴(q−k−k')
Σ_{V=Z,γ} [v̄(p') γ^μ M^e_V u(p)] [ū(k) γ^ν M^f_V v(k')] (η_{μν} + q_μ q_ν/M_V²)/(q² + M_V² − iε)
M^f_V = { Q_f if V=γ ; g_L γ_L + g_R γ_R if V=Z
g_V, a_f, g_A
neglect m_f²/M_Z², m_e²/M_Z²

$$\langle f(k)\,\bar{f}(k')\,|\,S\,|\,e^-(p)\,e^+(p')\rangle$$

$$= \frac{e^2}{2!}\left[i(2\pi)^4\right]\frac{1}{\left[i(2\pi)^4\right]}\int d^4q \quad \delta^4(p+p'-q)\,\delta^4(q-k-k')$$

$$\sum_{V=\gamma,Z}\left[\bar{v}(p')\,\gamma^\mu M_V^e\,u(p)\right]\left[\bar{u}(k)\,\gamma^\nu M_V^f\,v(k')\right]\frac{\eta_{\mu\nu}+q_\mu q_\nu/M_V^2}{q^2+M_V^2-i\epsilon}$$

$$M_V^f = \begin{cases} \overset{g_V \cdot Q_f \cdot g_V^f}{Q_f} & \text{if } V=\gamma \\ \ldots\gamma_L + g_R^f\gamma_R & \text{if } V=Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

$$\langle f(k)\bar{f}(k')|S|e^-(p)e^+(p')\rangle$$

$$= \frac{e^2}{2!}\left[i(2\pi)^4\right]\frac{1}{\left[i(2\pi)^4\right]}\int d^4q \;\; \delta^4(p+p'-q)\,\delta^4(q-k-k')$$

$$\sum_{V=\gamma,Z}\left[\bar{v}(p')\gamma^\mu M_V^e u(p)\right]\left[\bar{u}(k)\gamma^\nu M_V^f v(k')\right]\frac{\eta_{\mu\nu}+q_\mu q_\nu/M_V^2}{q^2+M_V^2-i\epsilon}$$

$g_V^f,\ a_f,\ g_A^f$

$$M_V^f = \begin{cases} Q_f & \text{if } V=\gamma \\ g_L^f\gamma_L + g_R^f\gamma_R & \text{if } V=Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

$$\langle f(k)\,\bar f(k')\,|\,S\,|\,e^-(p)\,e^+(p')\rangle$$

$$= \frac{e^2}{2!}\left[i(2\pi)^4\right]^2 \frac{1}{\left[i(2\pi)^4\right]} \int d^4q \;\; \delta^4(p+p'-q)\,\delta^4(q-k-k')$$

$$\sum_{V=\gamma,Z}\left[\bar v(p')\,\gamma^\mu M_V^e\, u(p)\right]\left[\bar u(k)\,\gamma^\nu M_V^f\, v(k')\right] \frac{\eta_{\mu\nu} + q_\mu q_\nu / M_V^2}{q^2 + M_V^2 - i\epsilon}$$

g_L, a_L, g_R

$$M_V^f = \begin{cases} Q_f & \text{if } V=\gamma \\ g_L^f\gamma_L + g_R^f\gamma_R & \text{if } V=Z \end{cases}$$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$

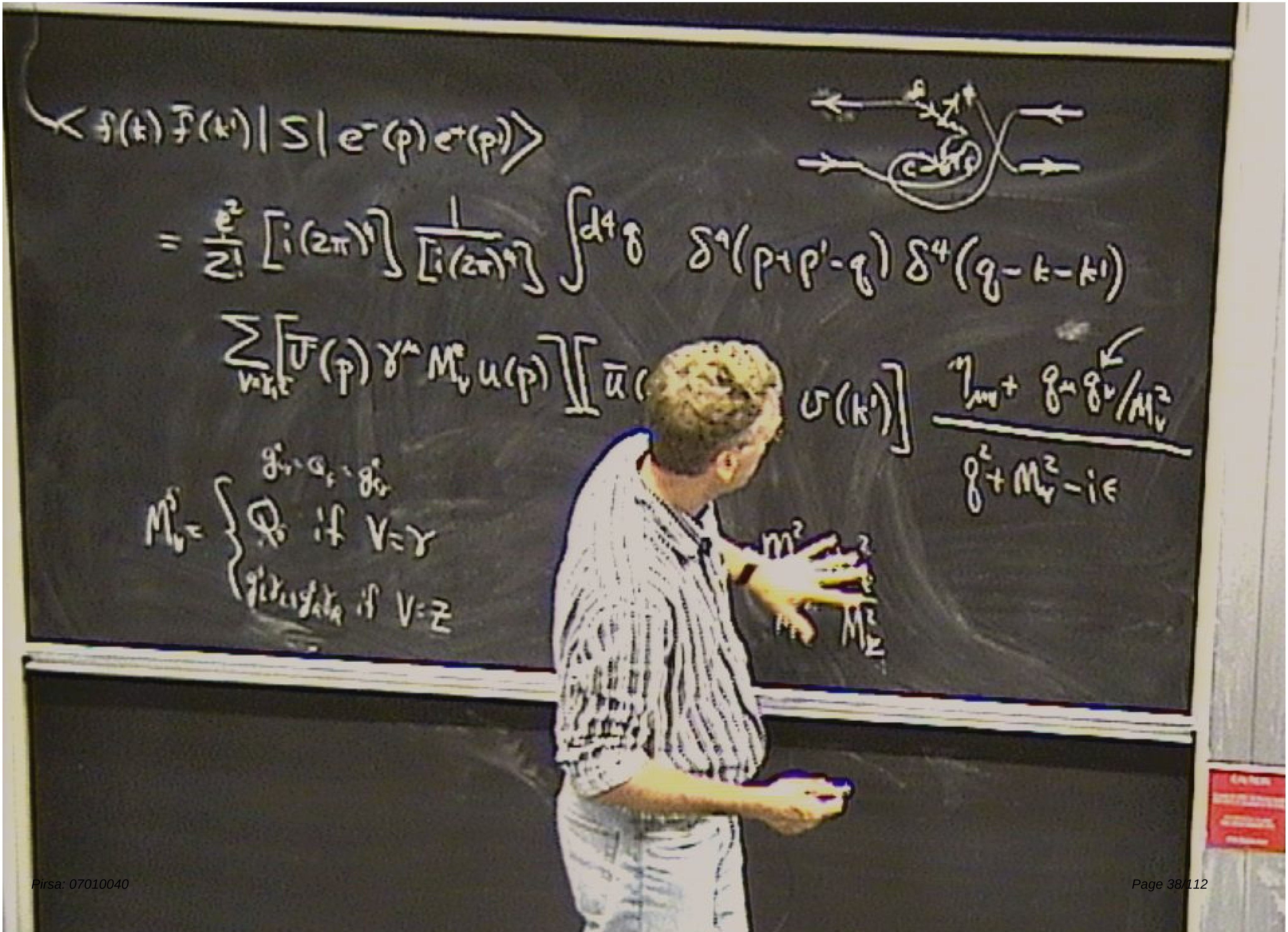
$$\overline{\mathcal{M}^2} = \frac{1}{4}\sum_{\text{all spins}} |\mathcal{M}|^2 \quad , \text{ where } S = -(2\pi)^4 i\, \delta^4(p+p'-k-k')\,\mathcal{M}$$

$$= N_c e^2 \left\{ \left| \sum_{V=\gamma,Z} \frac{g_L^e g_L^f}{s-M_V^2-i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1 & \text{if } f=\mu,\tau \\ 3 & \text{if } f=q \end{cases}$$

$$+ \left| \sum_{V=\gamma,Z} \frac{g_R^e g_R^f}{s-M_V^2-i\epsilon} \right|^2 u^2 + \left| \sum_{V=\gamma,Z} \frac{g_L^e g_R^f}{s-M_V^2-i\epsilon} \right|^2 t^2$$

$$\left. + \left| \sum_{V=\gamma,Z} \frac{g_R^e g_L^f}{s-M_V^2-i\epsilon} \right|^2 t^2 \right\}$$



if V=γ
if V=Z

$$\overline{\mathcal{M}^2} = \frac{1}{4}\sum_{\text{all spins}} |\mathcal{M}|^2 \quad \text{, where } S = -(2\pi)^4 i\, \delta^4(p+p'-k-k')\,\mathcal{M}$$

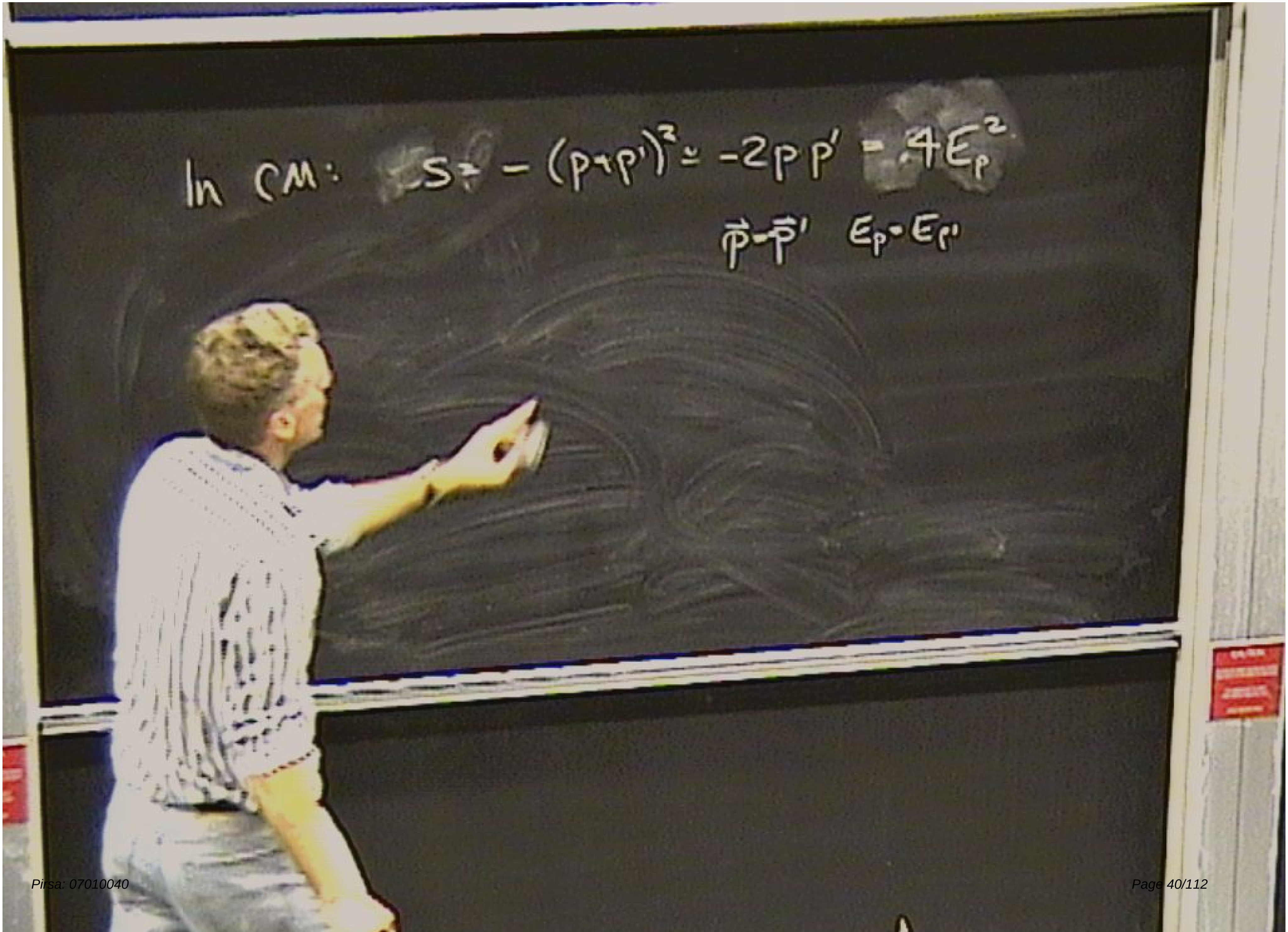
$$= N_c e^2 \left\{ \left| \sum_{V=\gamma,Z} \frac{g_L^e g_L^f}{s - M_V^2 - i\epsilon} \right|^2 u^2 \right.$$

$$N_c = \begin{cases} 1 & \text{if } f = \mu, \tau \\ 3 & \text{if } f = q \end{cases}$$

$$+ \left| \sum_{V=\gamma,Z} \frac{g_R^e g_R^f}{s - M_V^2 - i\epsilon} \right|^2 u^2 + \left| \sum_{V=\gamma,Z} \frac{g_L^e g_R^f}{s - M_V^2 - i\epsilon} \right|^2 t^2$$

$$\left. + \left| \sum_{V=\gamma,Z} \frac{g_R^e g_L^f}{s - M_V^2 - i\epsilon} \right|^2 t^2 \right\}$$

In CM:


In CM: $s = -(p+p')^2 = -2pp' = 4E_p^2$
$\vec{p} = -\vec{p}'$ $E_p = E_{p'}$

$p_c \quad p_d \qquad\qquad p_a \quad p_b$

$\langle f(k)\,\bar f(k')\,|\,S\,|\,e^-(p)\,e^+(p')\rangle$

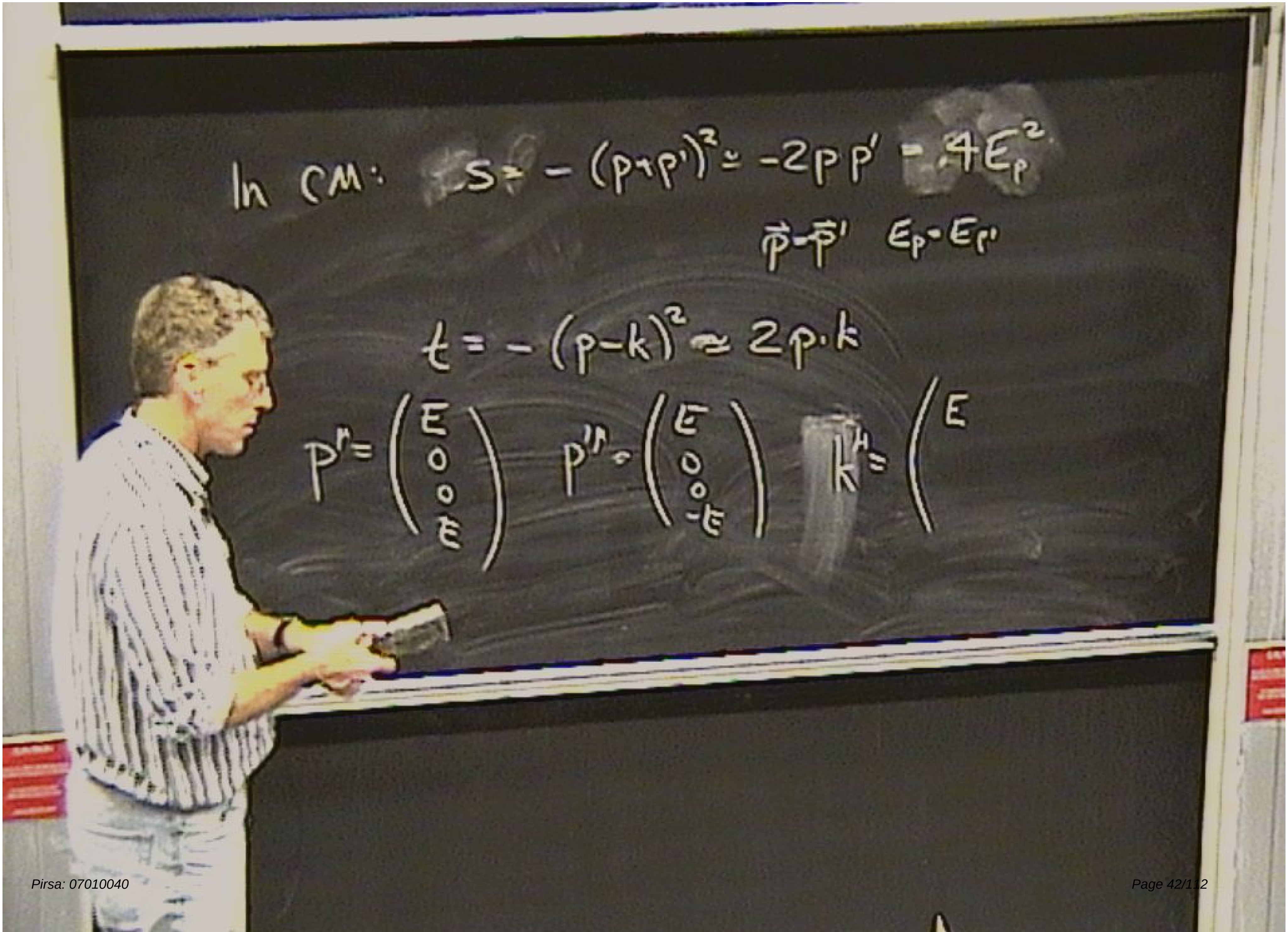
$= \frac{e^2}{2!}\,[i(2\pi)^4]^2\,\frac{1}{[i(2\pi)^4]}\int d^4q\;\delta^4(p+p'-q)\,\delta^4(q-k-k')$

$\sum_{V=\gamma,Z}[\bar v(p')\gamma^\mu M_V^e\,u(p)][\bar u(k)\gamma^\nu M_V^f\,v(k')]\;\frac{\eta_{\mu\nu}+q_\mu q_\nu/M_V^2}{q^2+M_V^2-i\epsilon}$ $\quad Z$ only

$g_V^f = a_f - g_A^f$

$M_V^f = \begin{cases} Q_f & \text{if } V=\gamma \\ g_V^f\,\gamma_5\,g_A^f & \text{if } V=Z \end{cases}$

neglect $\frac{m_f^2}{M_Z^2}, \frac{m_e^2}{M_Z^2}$


In CM: $s = -(p+p')^2 \simeq -2p\,p' = 4E_p^2$
$\vec{p} = -\vec{p}'$ $E_p = E_{p'}$
$t = -(p-k)^2 \simeq 2p\cdot k$
$p^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix}$ $p'^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$ $k^\mu = (E$

In CM: $s = -(p+p')^2 \simeq -2p\,p' = 4E_p^2$

$\vec{p} = -\vec{p}'$ $\quad E_p = E_{p'}$

$t = -(p-k)^2 \simeq 2p\cdot k = +2E^2[-1$

$$p^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \qquad p'^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix} \qquad k^\mu = \begin{pmatrix} E \\ E\sin\theta \\ 0 \\ E\cos\theta \end{pmatrix}$$

$$k'^\mu = \begin{pmatrix} E \\ -E\sin\theta \\ 0 \\ -E\cos\theta \end{pmatrix}$$

In CM: $s = -(p+p')^2 \simeq -2p\,p' = 4E_p^2$

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$t = -(p-k)^2 \simeq 2p\cdot k = +2E^2\left[-1+\cos\theta\right]$

$$p^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \quad p'^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix} \quad k^\mu = \begin{pmatrix} E \\ E\sin\theta \\ 0 \\ E\cos\theta \end{pmatrix}$$

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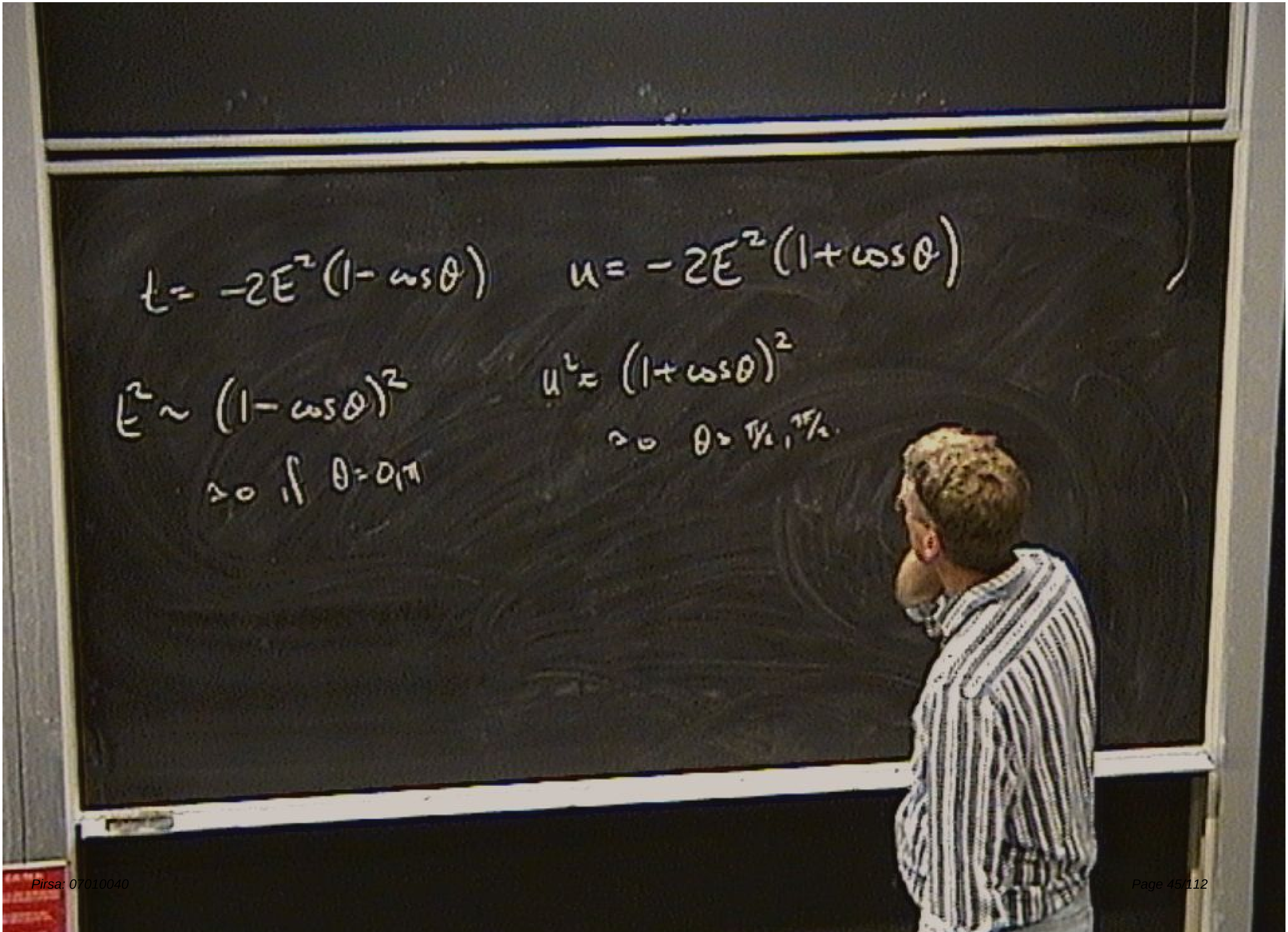

$t = -2E^2(1-\cos\theta)$
$u = -2E^2(1+\cos\theta)$
$t^2 \sim (1-\cos\theta)^2$
$\to 0$ if $\theta = 0, \pi$
$u^2 \simeq (1+\cos\theta)^2$
$\to 0$ $\theta = \pi/2, 3\pi/2$

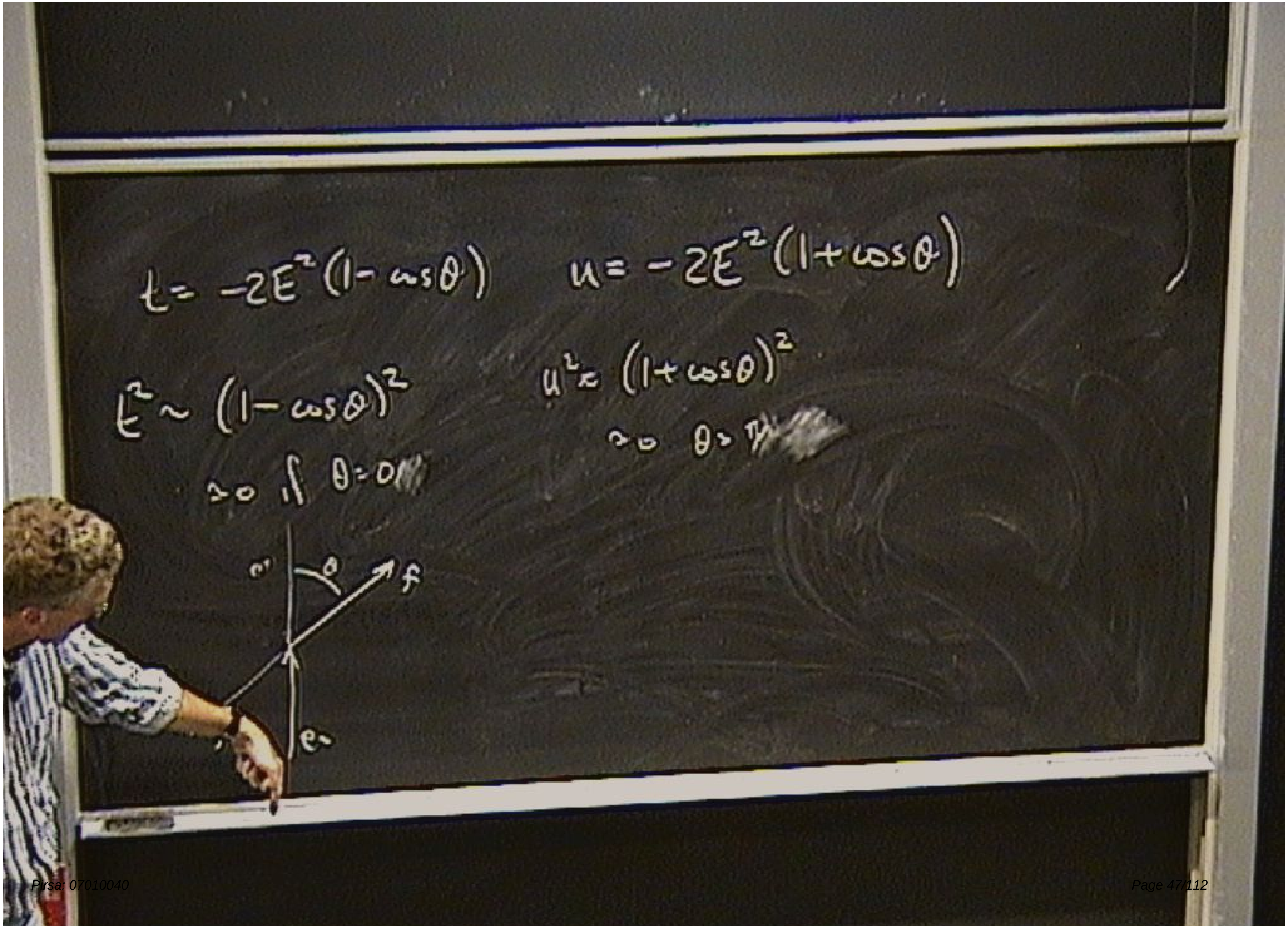
$$t = -2E^2(1-\cos\theta) \qquad u = -2E^2(1+\cos\theta)$$

$$t^2 \sim (1-\cos\theta)^2 \qquad u^2 \sim (1+\cos\theta)^2$$

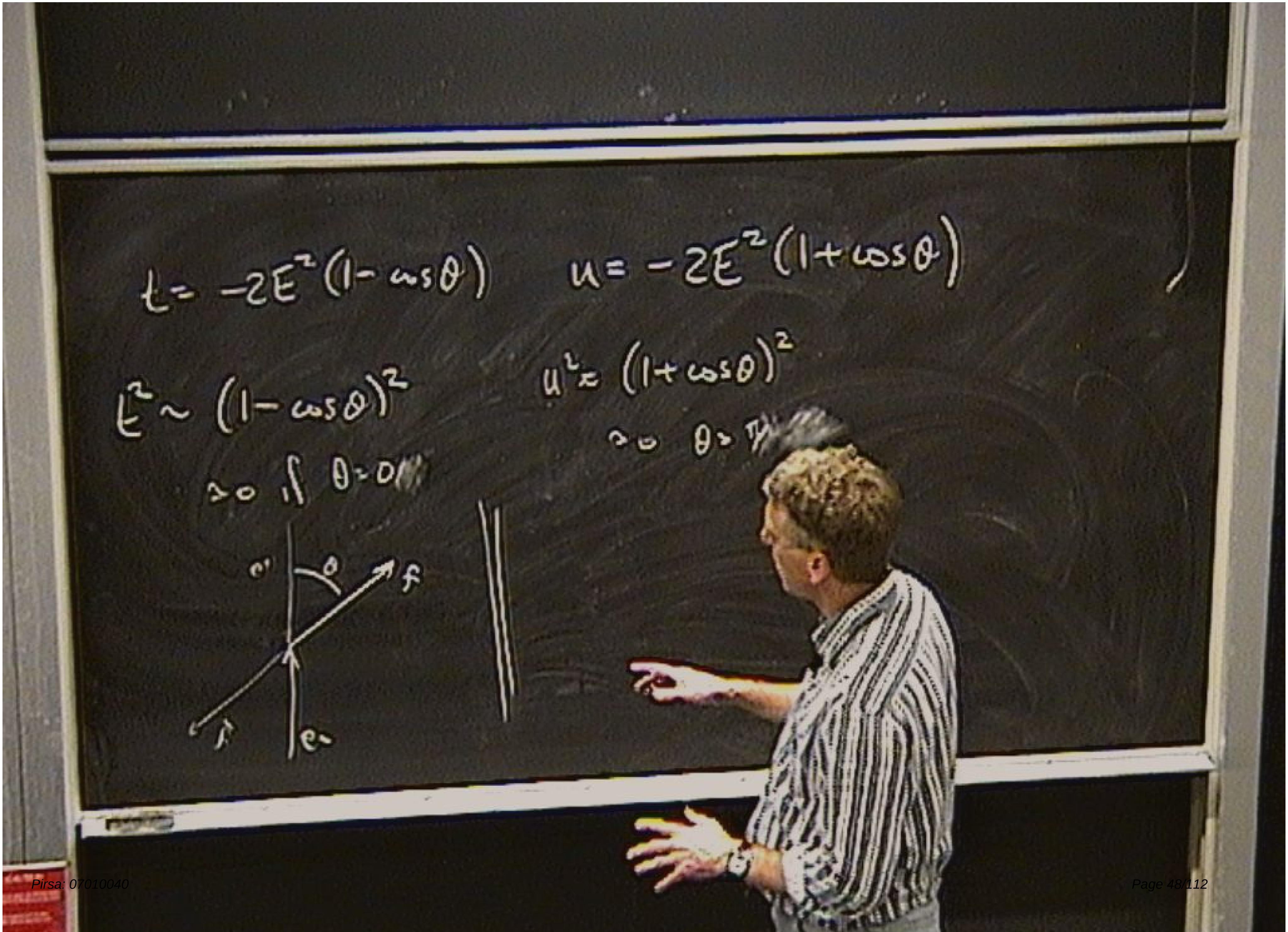
$$\to 0 \text{ if } \theta = 0 \qquad \to 0 \quad \theta = \pi$$

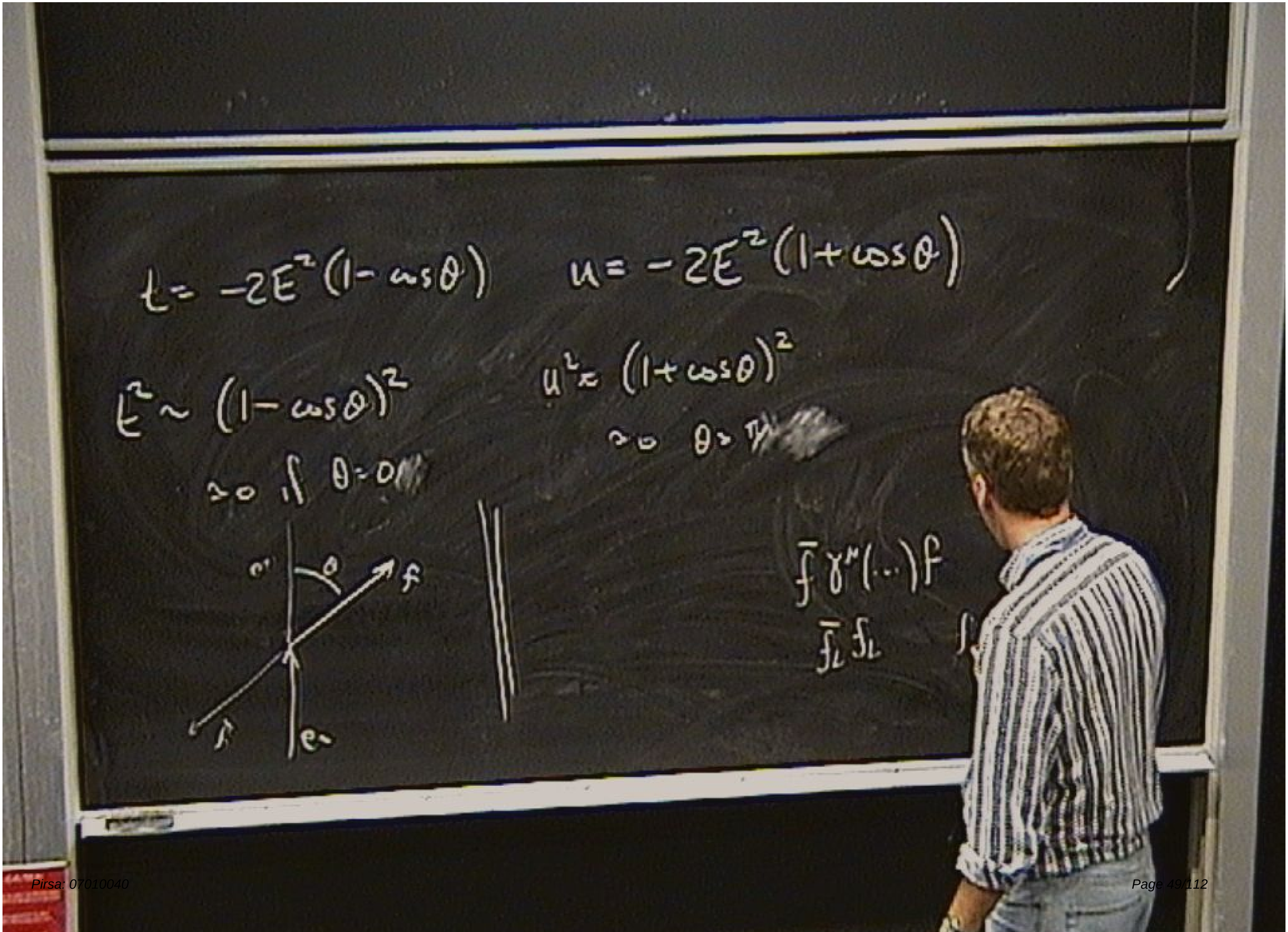
e^+

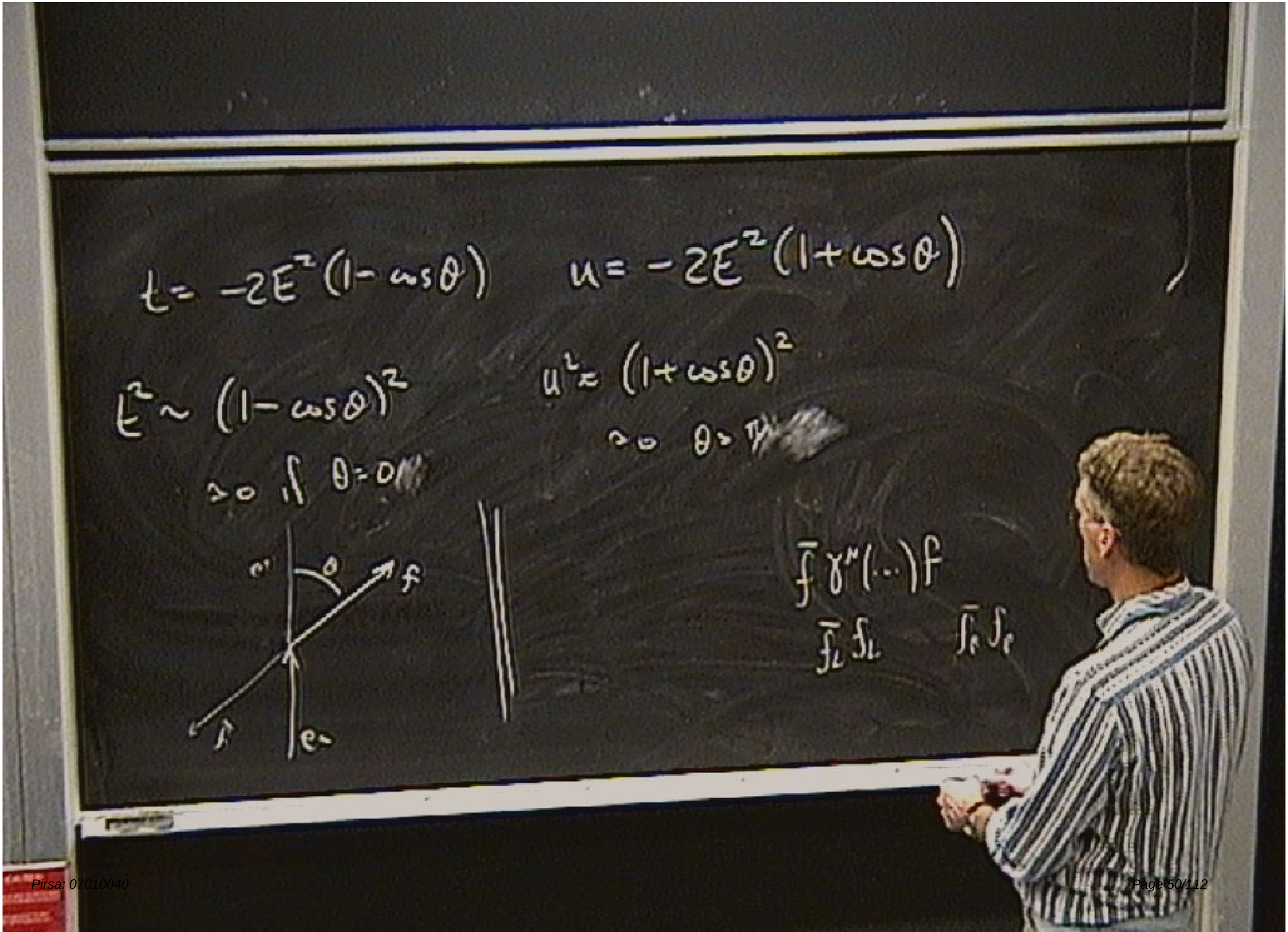
e^-



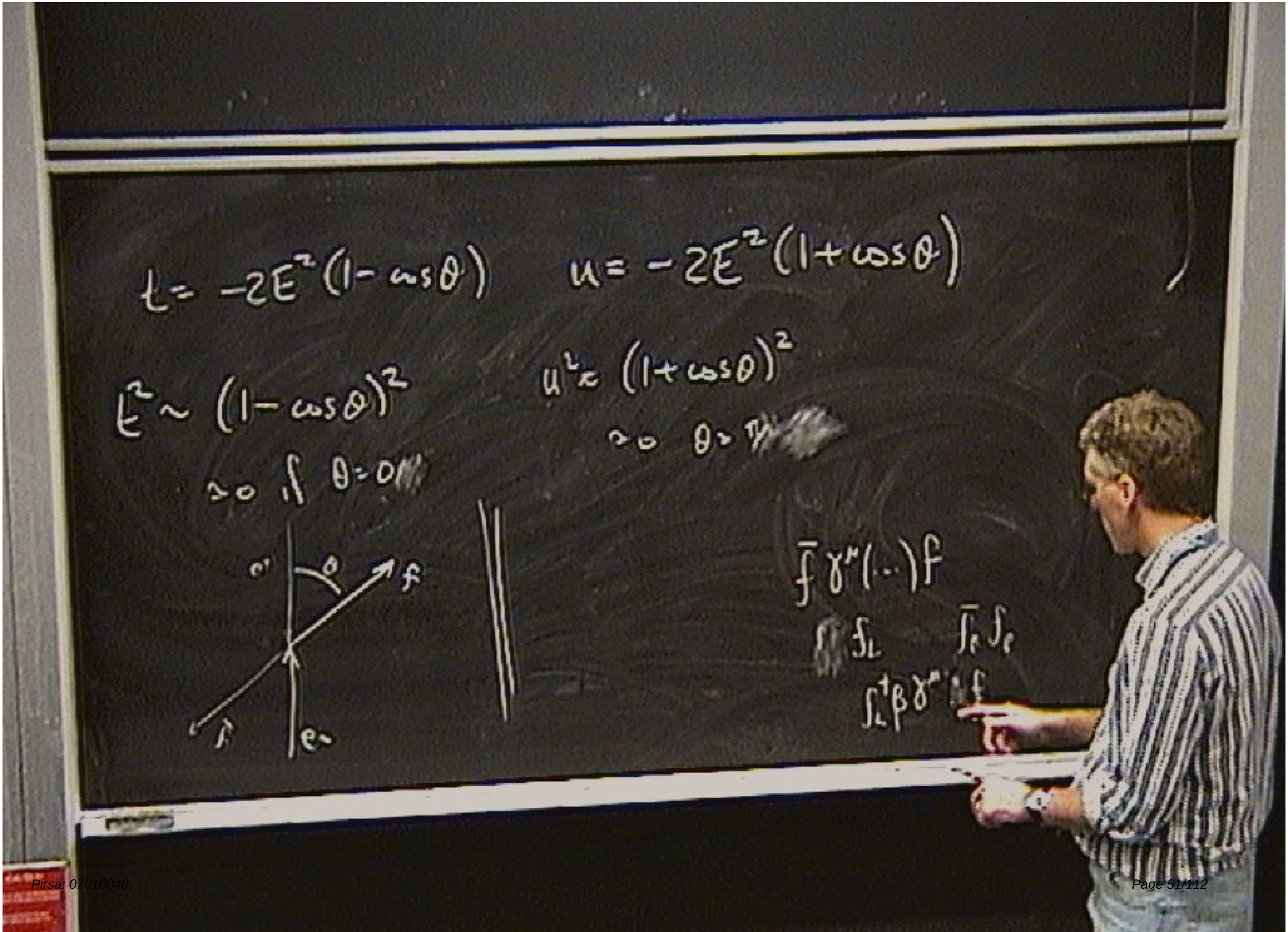
$t = -2E^2(1-\cos\theta)$
$u = -2E^2(1+\cos\theta)$
$t^2 \sim (1-\cos\theta)^2$
$\to 0$ if $\theta = 0$
$u^2 \approx (1+\cos\theta)^2$
$\to 0$ $\theta = \pi$
e^+
θ
f
e^-


$t = -2E^2(1-\cos\theta)$
$u = -2E^2(1+\cos\theta)$
$t^2 \sim (1-\cos\theta)^2$
$\to 0$ if $\theta = 0$
$u^2 \approx (1+\cos\theta)^2$
$\to 0$ $\theta = \pi$
e^-
θ
f
e^-

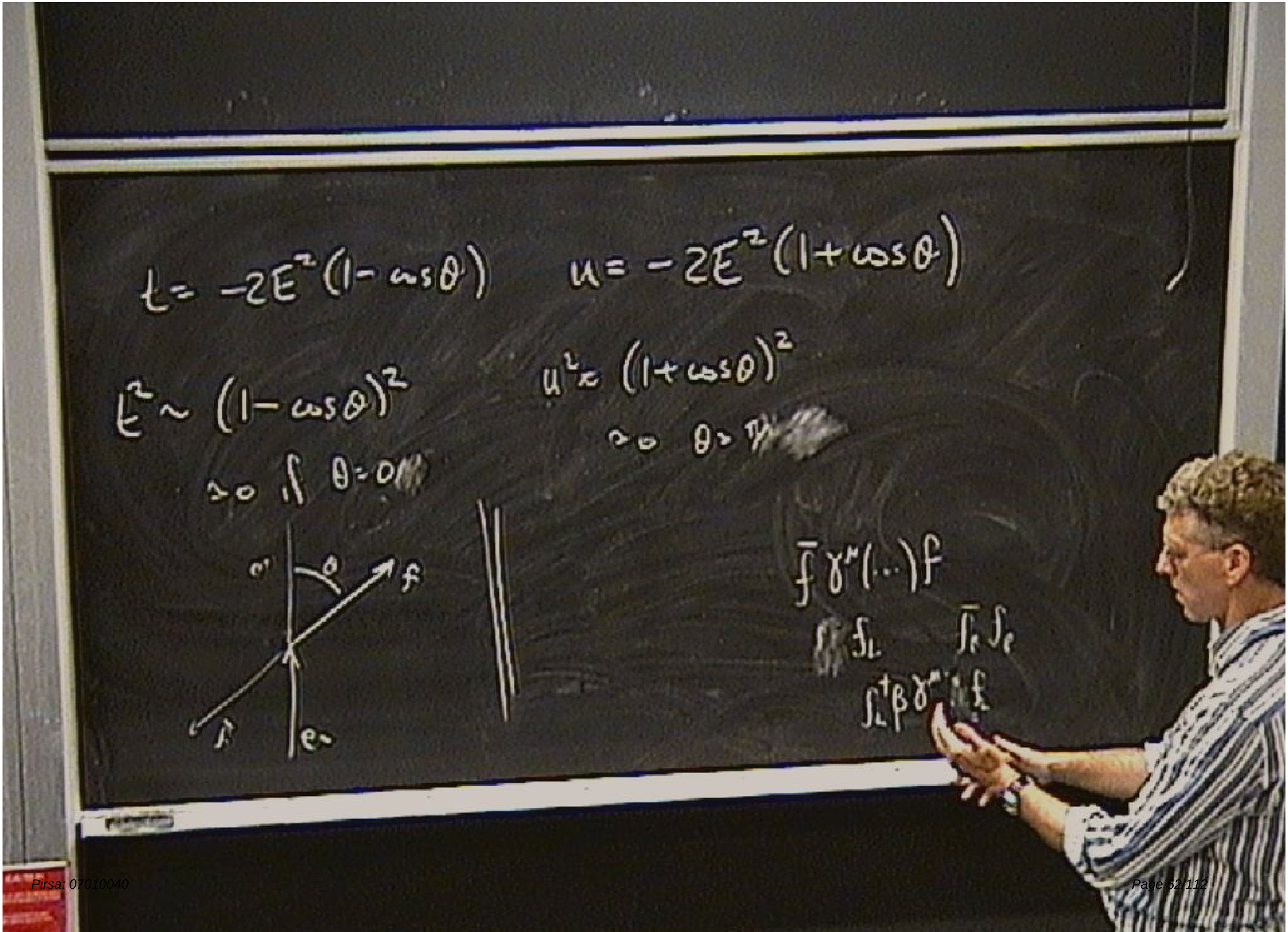

$t = -2E^2(1-\cos\theta)$
$u = -2E^2(1+\cos\theta)$
$t^2 \sim (1-\cos\theta)^2$
$\to 0$ if $\theta = 0$
$u^2 \approx (1+\cos\theta)^2$
$\to 0 \quad \theta \to \pi$
e^+
θ
f
$\bar{f}$
e^-
$\bar{f}\gamma^\mu(\ldots)f$
$\bar{f}_L f_L$



$t = -2E^2(1-\cos\theta)$
$u = -2E^2(1+\cos\theta)$
$t^2 \sim (1-\cos\theta)^2$
$\to 0$ if $\theta = 0$
$u^2 \sim (1+\cos\theta)^2$
$\to 0$ $\theta \to \pi$
e^+
θ
f
$\bar{f}$
e^-
$\bar{f}\gamma^\mu(\dots)f$
$\bar{f}_L f_L$
$\bar{f}_R f_R$



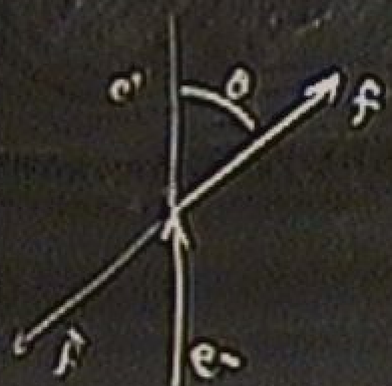
$t = -2E^2(1-\cos\theta)$
$u = -2E^2(1+\cos\theta)$
$t^2 \sim (1-\cos\theta)^2$
$\to 0$ if $\theta = 0$
$u^2 \propto (1+\cos\theta)^2$
$\to 0$ $\theta \to \pi$
e^-
θ
f
$\bar{f}$
e^-
$\bar{f}\gamma^\mu(\ldots)f$
$\bar{f}_L f_L$
$\bar{f}_R f_R$
$f_L^\dagger \beta \gamma^\mu f$


t = -2E^2 (1 - cos θ)
u = -2E^2 (1 + cos θ)
t^2 ~ (1 - cos θ)^2
u^2 ~ (1 + cos θ)^2
= 0 if θ = 0
= 0 θ = π
θ
f
e-
f̄ γ^μ (...) f
f_L
f̄_R f_R
f_L† β γ^μ f_L

$$t = -2E^2(1-\cos\theta) \qquad u = -2E^2(1+\cos\theta)$$

$$t^2 \sim (1-\cos\theta)^2 \qquad u^2 \sim (1+\cos\theta)^2$$

$$\to 0 \text{ if } \theta = 0 \qquad \to 0 \quad \theta = \pi$$



$$\bar{f}\gamma^\mu(\ldots)f$$

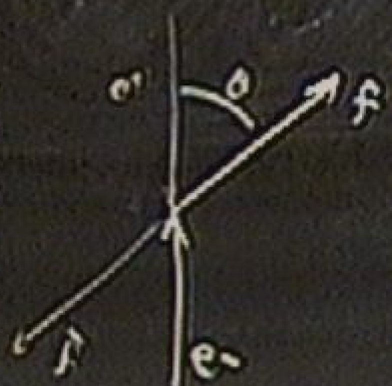
$$f_L \qquad \bar{f}_R f_R$$

$$f_L^\dagger \beta \gamma^\mu f_L$$

$$t = -2E^2(1-\cos\theta) \qquad u = -2E^2(1+\cos\theta)$$

$$t^2 \sim (1-\cos\theta)^2 \qquad u^2 \sim (1+\cos\theta)^2$$

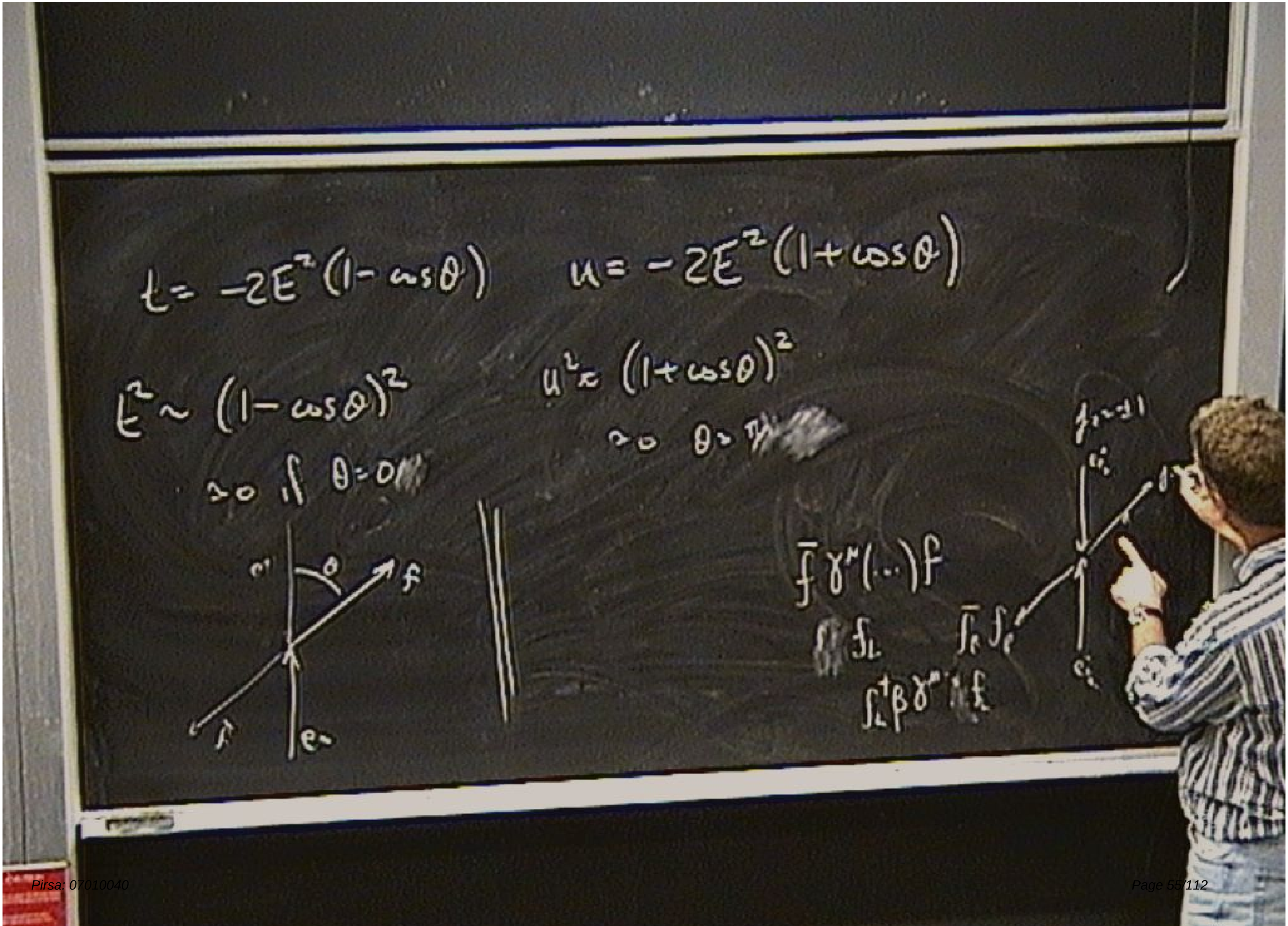
$\to 0$ if $\theta = 0$ $\qquad \to 0 \quad \theta = \pi$



$$\bar f \gamma^\mu (\ldots) f$$

$$f_L \qquad \bar f_R f_R$$

$$f_L^\dagger \beta \gamma^\mu f_L$$


$t = -2E^2(1-\cos\theta)$
$u = -2E^2(1+\cos\theta)$
$t^2 \sim (1-\cos\theta)^2$
$\to 0$ if $\theta = 0$
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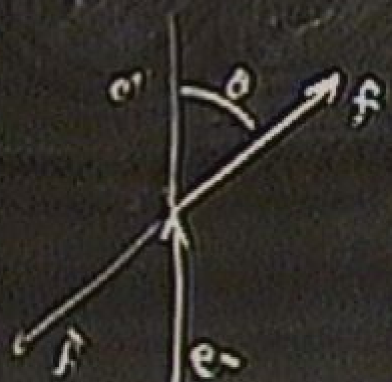
$\to 0$ $\theta = \pi$
$\bar{f}\gamma^\mu(\ldots)f$

$$t = -2E^2(1-\cos\theta) \qquad u = -2E^2(1+\cos\theta)$$

$$t^2 \sim (1-\cos\theta)^2 \qquad u^2 \sim (1+\cos\theta)^2$$

$\rightarrow 0$ if $\theta = 0$ $\qquad$ $\rightarrow 0 \quad \theta = \pi$



$\bar{f}\gamma^\mu(\ldots)f$

$f_L \qquad \bar{f}_R f_R$

$f_L^\dagger \beta \gamma^\mu f_L$

$t = -2E^2(1-\cos\theta)$ $\qquad$ $u = -2E^2(1+\cos\theta)$

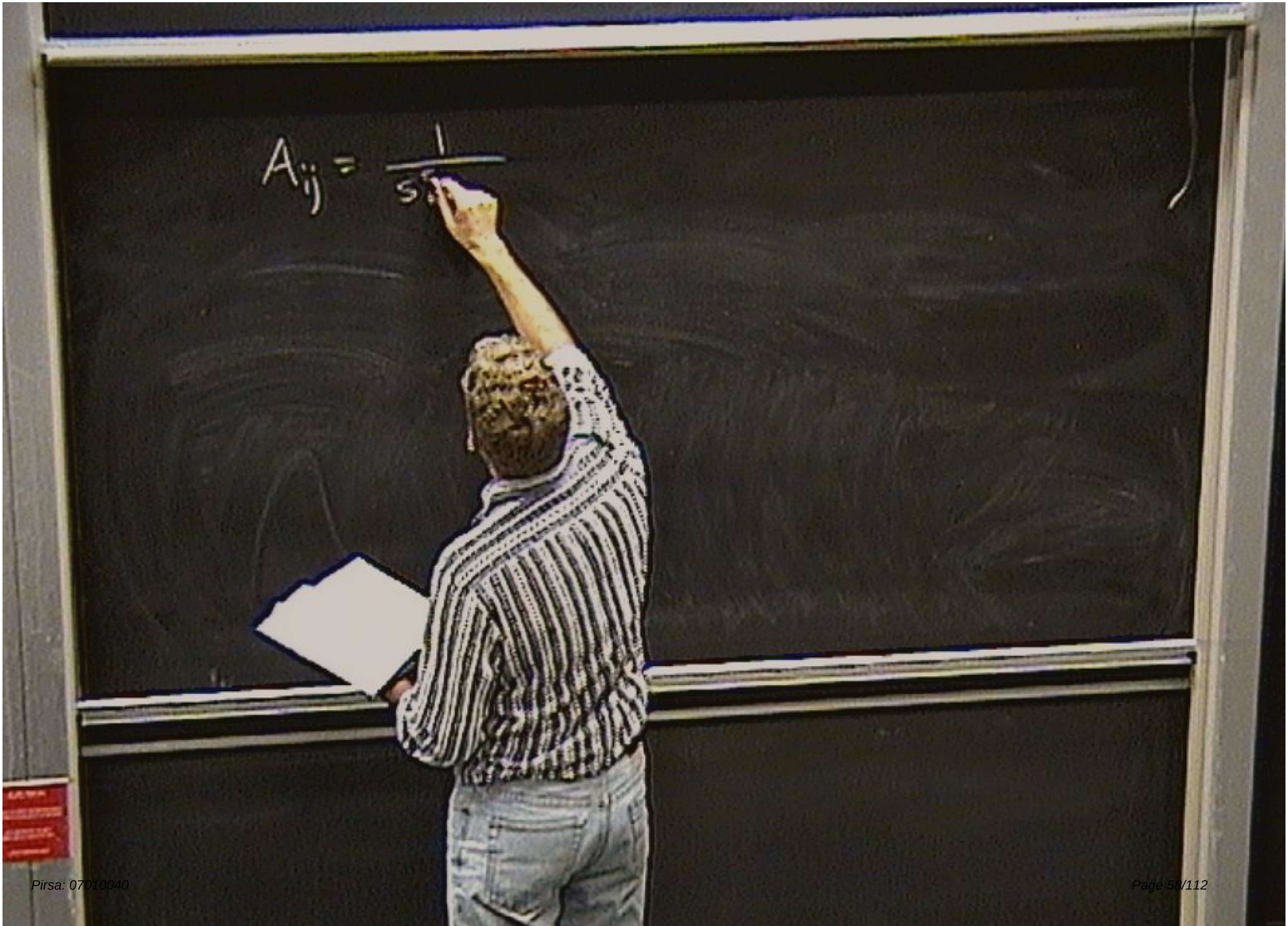
$t^2 \sim (1-\cos\theta)^2$ $\qquad$ $u^2 \sim (1+\cos\theta)^2$

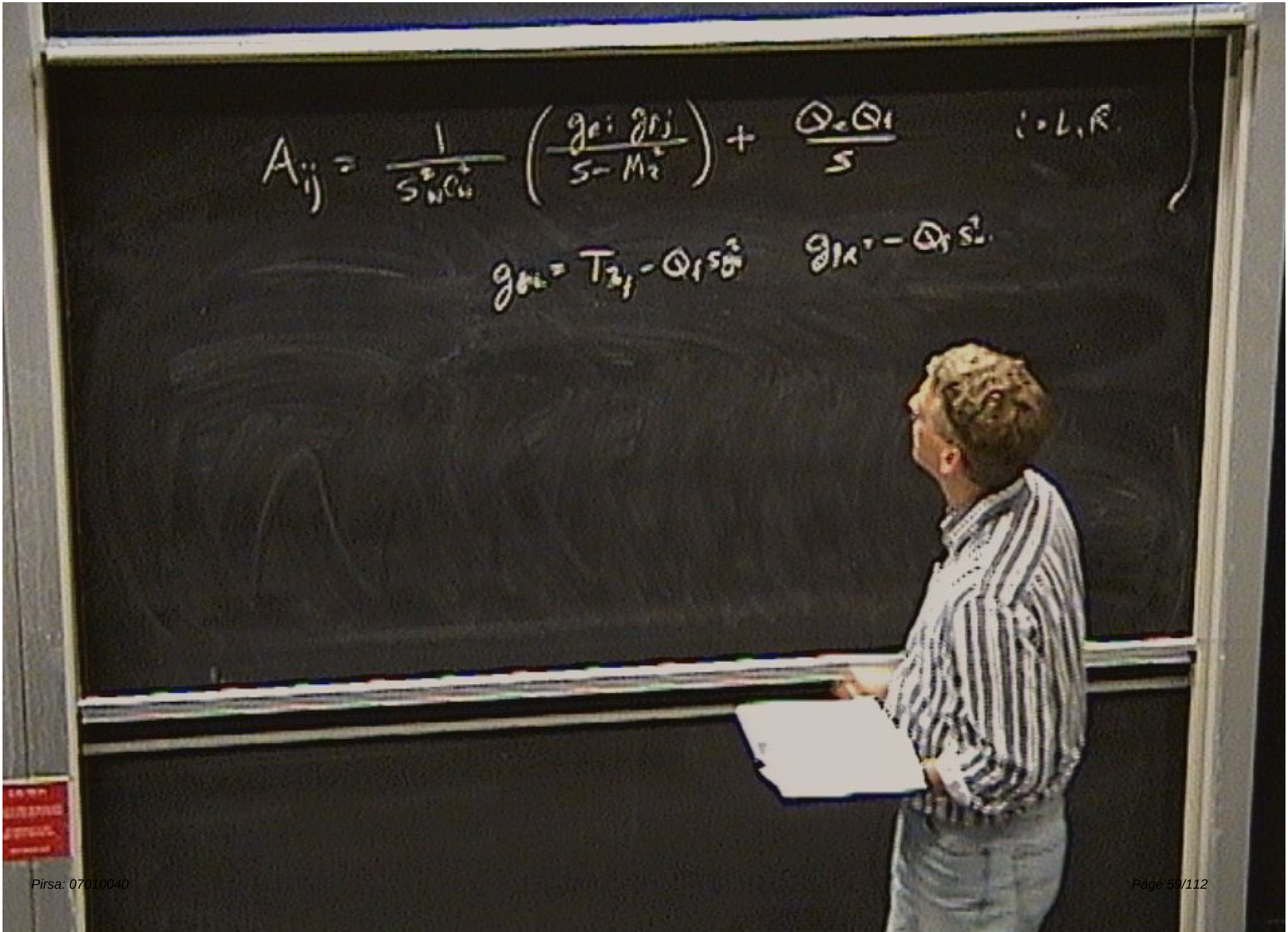
$\simeq 0$ if $\theta = 0$ $\qquad$ $\simeq 0$ $\theta = \pi$

$\bar{f}\gamma^\mu(\ldots)f$

f_L $\qquad$ $\bar{f}_R f_L$

$f_L^\dagger \beta \gamma^\mu f_L$




$A_{ij} = \frac{1}{s_W^2 c_W^2}\left(\frac{g_{ei}\, g_{fj}}{s - M_Z^2}\right) + \frac{Q_e Q_f}{s}$ $i = L, R$
$g_{fL} = T_{3_f} - Q_f s_W^2$ $g_{fR} = -Q_f s_W^2$

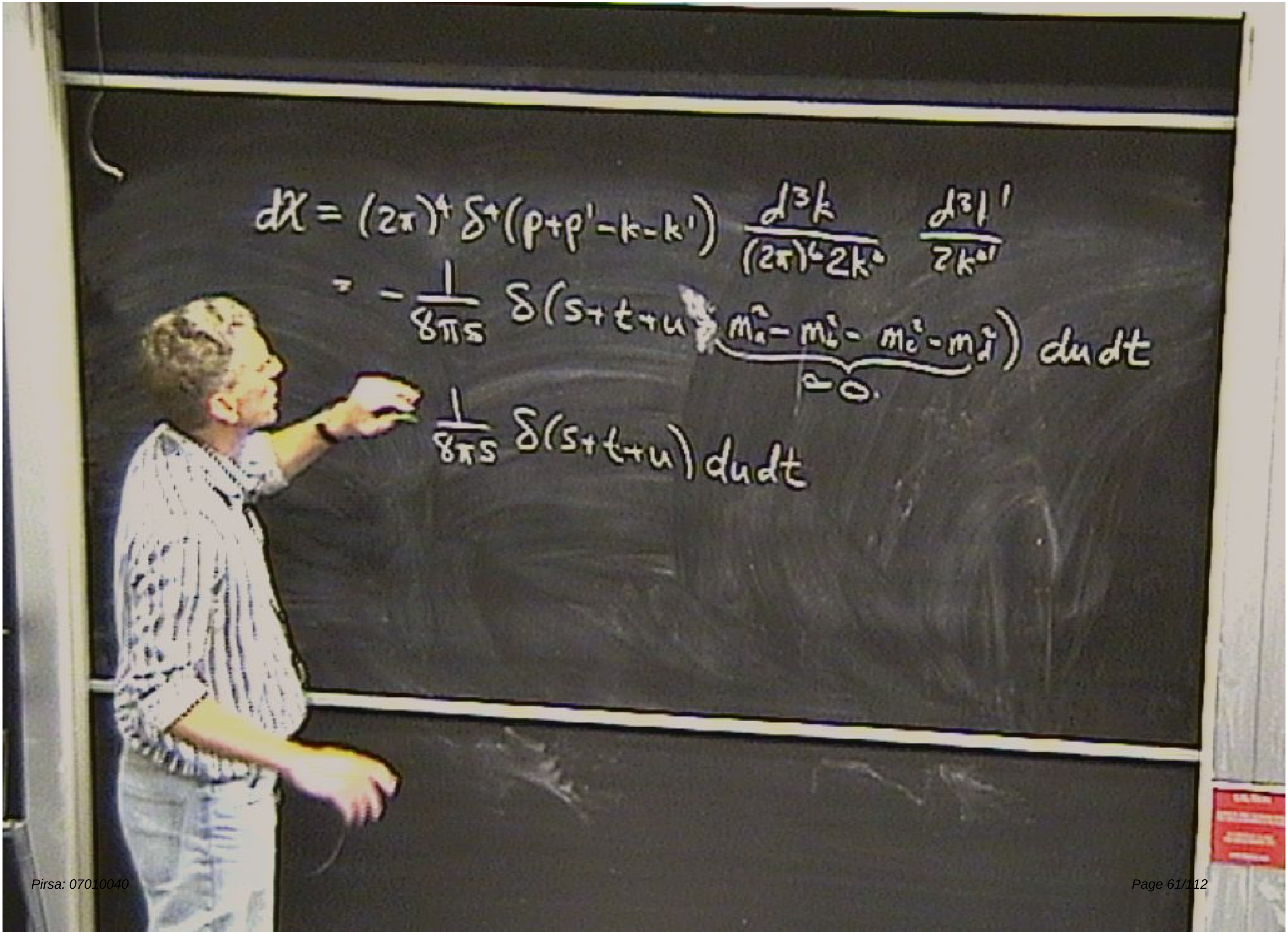
$$A_{ij} = \frac{1}{s_W^2 c_W^2}\left(\frac{g_{ei}\, g_{fj}}{s - M_Z^2}\right) + \frac{Q_e Q_f}{s} \qquad i = L, R.$$

$$g_{fL} = T_{3_f} - Q_f s_W^2 \qquad g_{fR} = -Q_f s_W^2.$$

$$d\sigma(e^+e^- \to f\bar{f}) = \frac{8\pi^2\alpha^2 N_c}{s}\Big[|A_{LL}(s)|^2 + |A_{RR}(s)|^2$$

$(f \neq e^-, \nu_e)$

$$+\,|A_{LR}(s)|^2 + |A_{RL}(s)|^2\Big]\, d\chi$$


$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6 2k^0} \frac{d^3k'}{2k^{0\prime}}$
$= -\frac{1}{8\pi s} \delta(s+t+u \underbrace{- m_a^2 - m_b^2 - m_c^2 - m_d^2}_{\simeq 0.}) \, du\, dt$
$\frac{1}{8\pi s} \delta(s+t+u) \, du\, dt$

$$d\chi = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6 2k^0} \frac{d^3k'}{2k^{0'}}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u \underbrace{- m_a^2 - m_b^2 - m_c^2 - m_d^2}_{\simeq 0}) \, du\, dt$$

$$\simeq -\frac{1}{8\pi s} \delta(s+t+u) \, du\, dt$$

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6 2k^0} \frac{d^3k'}{2k^{0\prime}}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u \underbrace{- m_1^2 - m_2^2 - m_3^2 - m_4^2}_{\simeq 0}) \, du \, dt$$

$$\simeq -\frac{1}{8\pi s} \delta(s+t+u) \, du \, dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} \left[\dots \right]$$

$$d\chi = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6\, 2k^0} \frac{d^3k'}{2k^{0\prime}}$$

$$= -\frac{1}{8\pi s}\,\delta(s+t+u \underbrace{- m_a^2 - m_b^2 - m_c^2 - m_d^2}_{\simeq 0})\, du\, dt$$

$$\simeq -\frac{1}{8\pi s}\,\delta(s+t+u)\, du\, dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2}\left[\ \cdots\ \right]$$

$$t = -2E^2(1-\cos\theta)$$

$$dt = +2E^2\, d\cos\theta$$
$$= -2E^2 \sin\theta\, d\theta$$

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6\, 2k^0} \frac{d^3k'}{2k^{0'}}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u \underbrace{- m_1^2 - m_2^2 - m_3^2 - m_4^2}_{\simeq 0}) \, du\, dt$$

$$\simeq -\frac{1}{8\pi s} \delta(s+t+u)\, du\, dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} \left[\cdots \right]$$

$$\alpha = e^2/4\pi$$

$$t = -2E^2(1-\cos\theta)$$

$$dt = +2E^2\, d\cos\theta$$

$$= -2E^2 \sin\theta\, d\theta$$

$$dX = (2\pi)^4\,\delta^4(p+p'-k-k')\,\frac{d^3k}{(2\pi)^6\,2k^0}\,\frac{d^3k'}{2k^{0'}}$$

$$= -\frac{1}{8\pi s}\,\delta(s+t+u-\underbrace{m_1^2-m_2^2-m_3^2-m_4^2}_{\simeq 0})\,du\,dt$$

$$\simeq -\frac{1}{8\pi s}\,\delta(s+t+u)\,du\,dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2}\left[\;\cdots\;\right]$$

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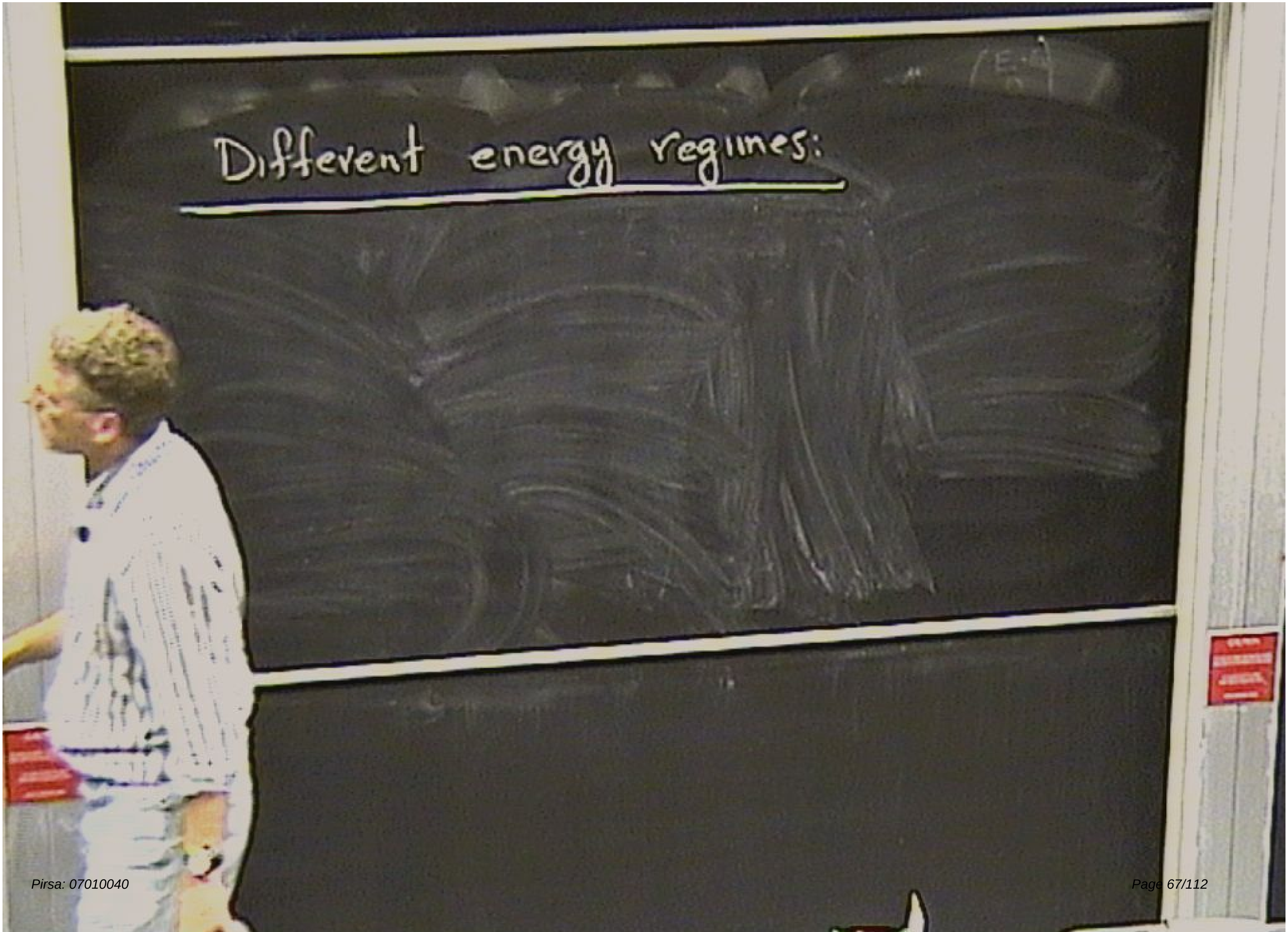
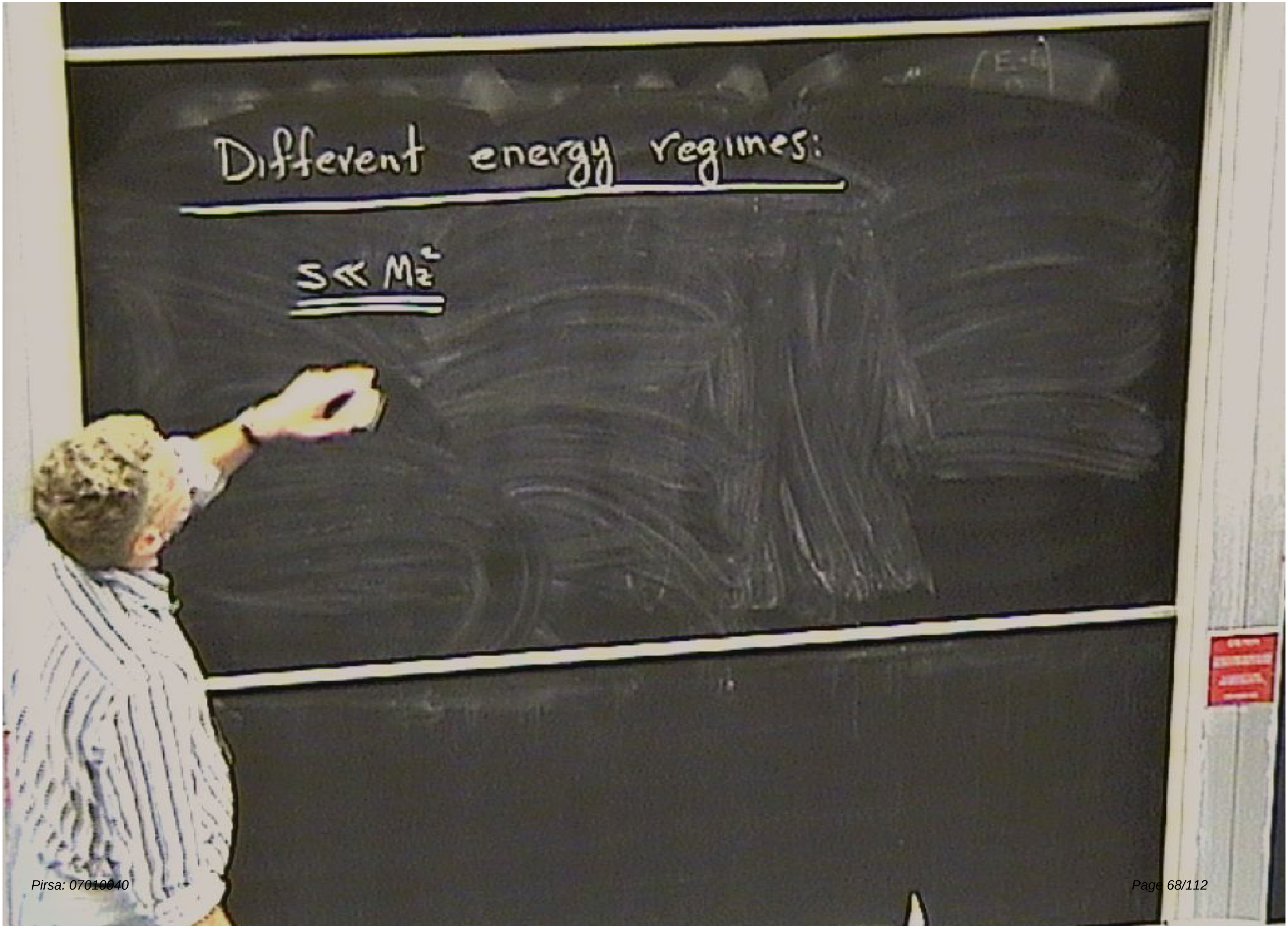
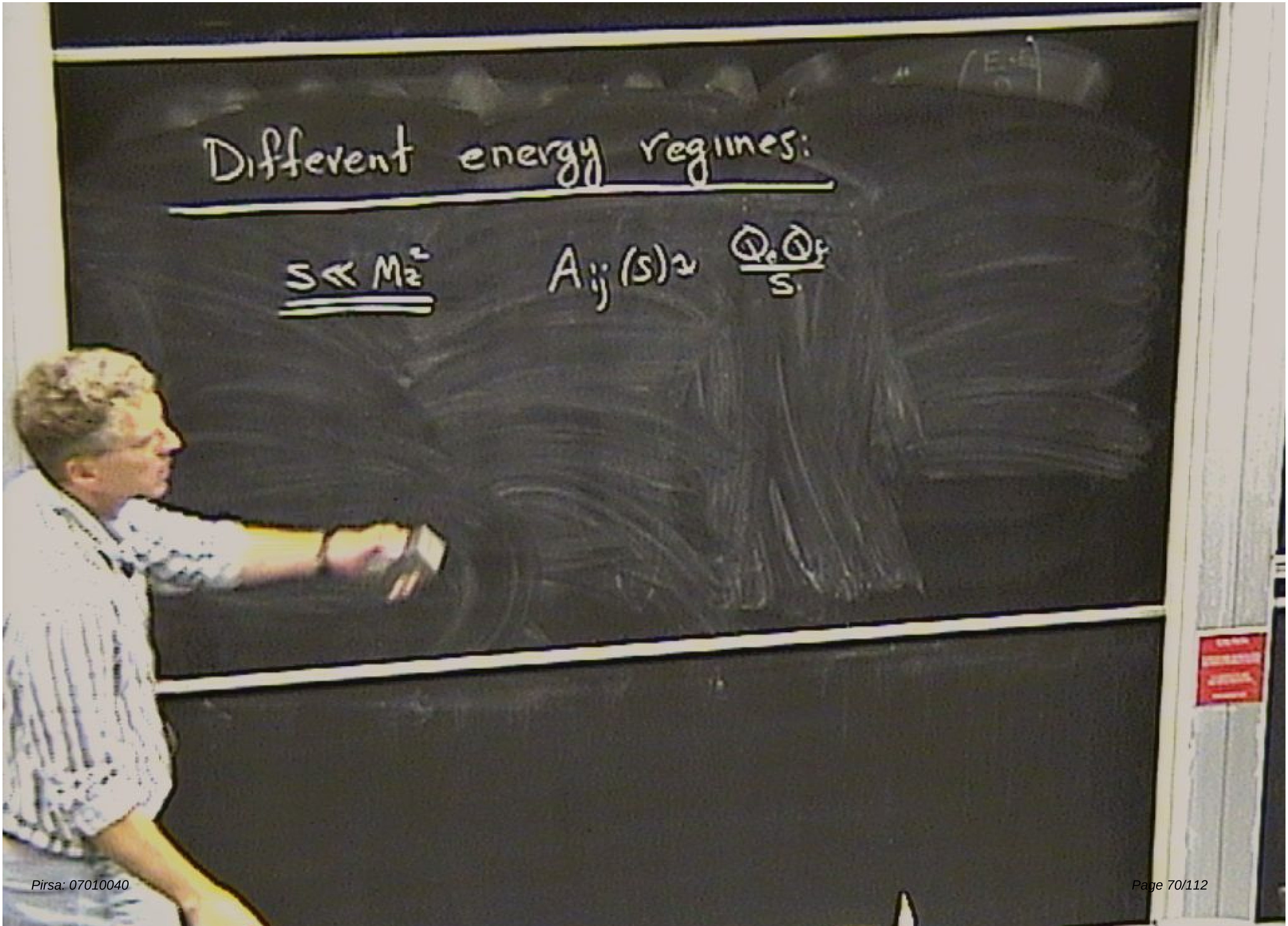

Different energy regimes:


Different energy regimes:
$s \ll M_Z^2$

Different energy regimes:

$s \ll M_Z^2$ $\quad A_{ij}(s) \sim$


Different energy regimes:
$s \ll M_Z^2$
$A_{ij}(s) \approx \frac{Q_e Q_f}{s}$

Different energy regimes:

$\underline{\underline{s \ll M_Z^2}}$ $\qquad A_{ij}(s) \approx \dfrac{Q_e Q_f}{s}$ $\qquad$ independent of $i,j = L,R$.

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6 2k^0} \frac{d^3k'}{2k^{0'}}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u \underbrace{- m_a^2 - m_b^2 - m_c^2 - m_d^2}_{\simeq 0}) \, du\, dt$$

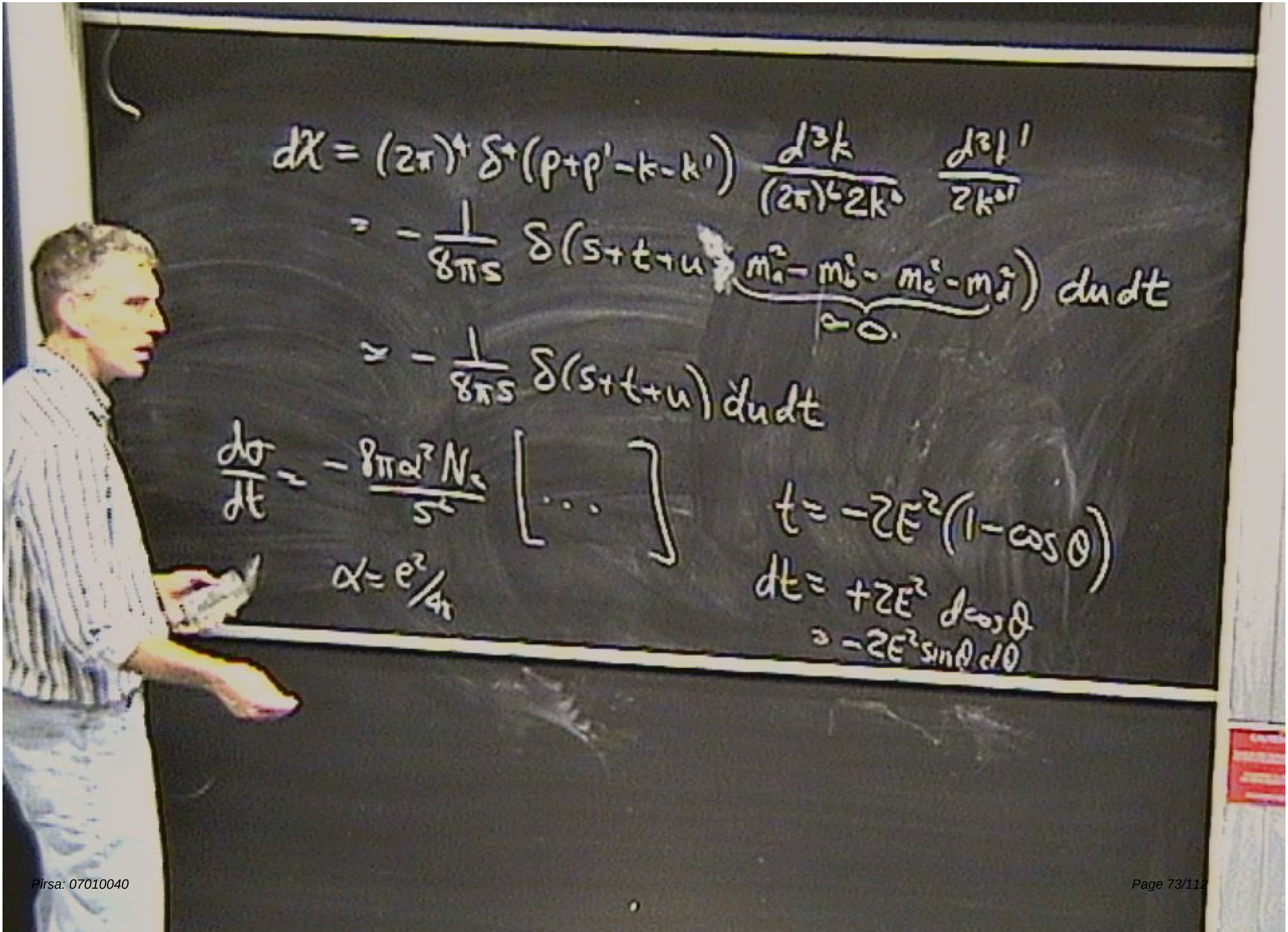
$$\simeq -\frac{1}{8\pi s} \delta(s+t+u) \, du\, dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} \left[\cdots \right]$$

$$\alpha = e^2/4\pi$$

$$t = -2E^2(1-\cos\theta)$$

$$dt = +2E^2 \, d\cos\theta$$
$$= -2E^2 \sin\theta \, d\theta$$


$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6 2k^0} \frac{d^3k'}{2k^{0'}}$
$= -\frac{1}{8\pi s} \delta(s+t+u \underbrace{- m_a^2 - m_b^2 - m_c^2 - m_d^2}_{\simeq 0}) \, du \, dt$
$\simeq -\frac{1}{8\pi s} \delta(s+t+u) \, du \, dt$
$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} \left[\cdots \right]$
$\alpha = e^2/4\pi$
$t = -2E^2(1-\cos\theta)$
$dt = +2E^2 \, d\cos\theta$
$= -2E^2 \sin\theta \, d\theta$

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6 2k^0} \frac{d^3k'}{2k^{0\prime}}$$

$$= -\frac{1}{8\pi s}\,\delta(s+t+u - \underbrace{m_a^2 - m_b^2 - m_c^2 - m_d^2}_{\approx 0})\, du\, dt$$

$\sigma \sim \frac{1}{M^2}$

$t \sim M^2$

$$\simeq -\frac{1}{8\pi s}\,\delta(s+t+u)\, du\, dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2}\left[\ \cdots\ \right]$$

$$\alpha = e^2/4\pi$$

$$t = -2E^2(1-\cos\theta)$$

$$dt = +2E^2\, d\cos\theta$$

$$= -2E^2 \sin\theta\, d\theta$$

$$d\chi = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6 2k^0} \frac{d^3k'}{2k^{0\prime}}$$

$$= -\frac{1}{8\pi s} \delta(s+t+u \underbrace{-m_a^2-m_b^2-m_c^2-m_d^2}_{\simeq 0}) \, du\, dt$$

$\sigma \sim \frac{1}{M^2}$

$t \sim M^2$

$$\simeq -\frac{1}{8\pi s} \delta(s+t+u) \, du\, dt$$

$$\frac{d\sigma}{dt} = -\frac{8\pi\alpha^2 N_c}{s^2} \left[\ \cdots \ \right]$$

$\alpha = e^2/4\pi$

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$$dt = +2E^2 \, d\cos\theta$$

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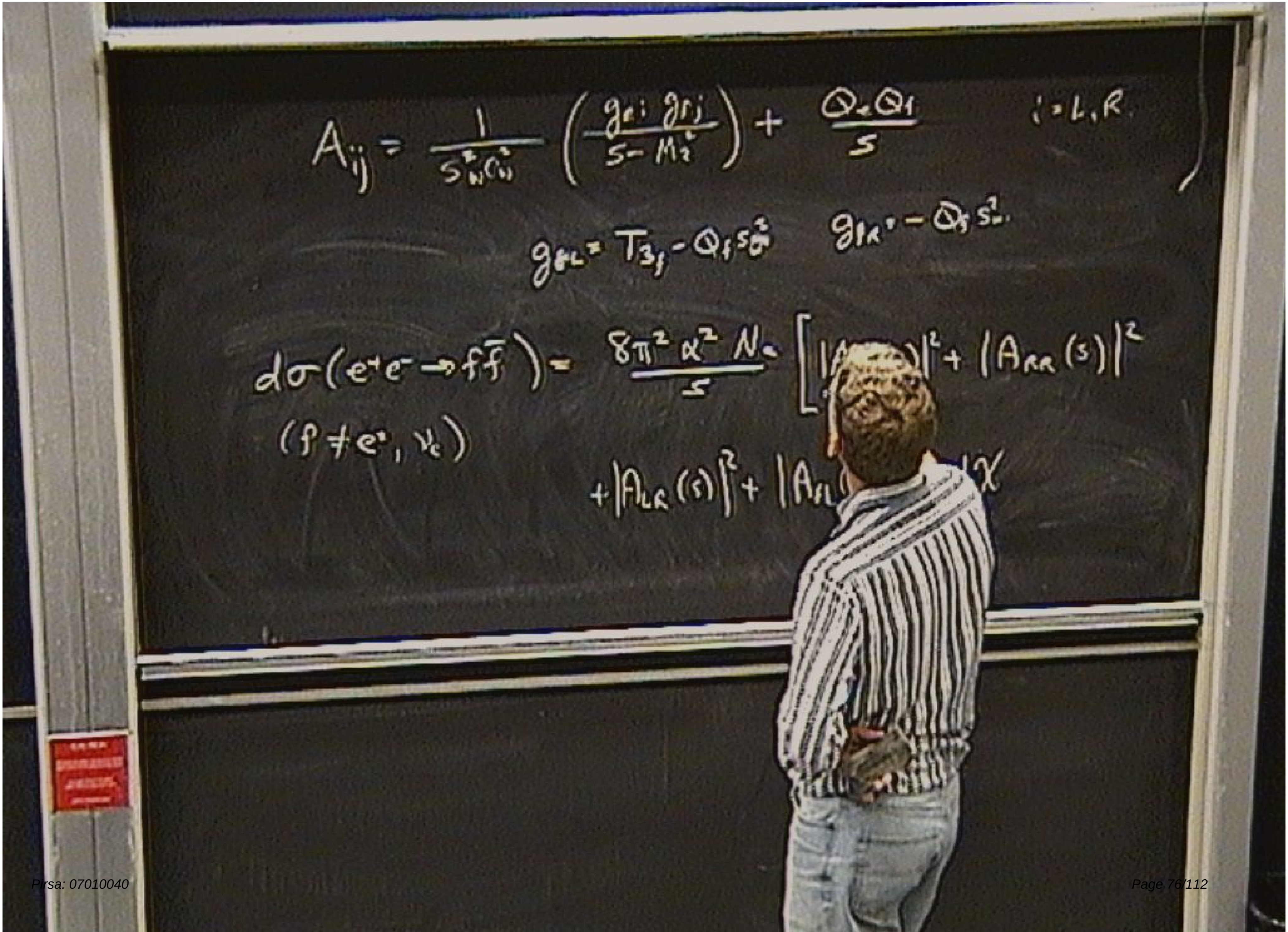

$A_{ij} = \frac{1}{s_W^2 c_W^2} \left(\frac{g_{ei}\, g_{fj}}{s - M_Z^2} \right) + \frac{Q_e Q_f}{s}$
$i = L, R$
$g_{fL} = T_{3f} - Q_f s_W^2$
$g_{fR} = -Q_f s_W^2$
$d\sigma(e^+e^- \to f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s}$
$|A_{RR}(s)|^2$
$(f \neq e^-, \nu_e)$
$+ |A_{LR}(s)|^2 +$

$$A_{ij} = \frac{1}{s_W^2 c_W^2}\left(\frac{g_{ei}\, g_{fj}}{s - M_Z^2}\right) + \frac{Q_e Q_f}{s} \qquad i = L, R$$

$$g_{fL} = T_{3_f} - Q_f s_W^2 \qquad g_{fR} = -Q_f s_W^2$$

$$d\sigma(e^+e^- \to f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s}\Big[|A_{LL}(s)|^2 + |A_{RR}(s)|^2 + |A_{LR}(s)|^2 + |A_{RL}(s)|^2\Big]\, d\chi$$

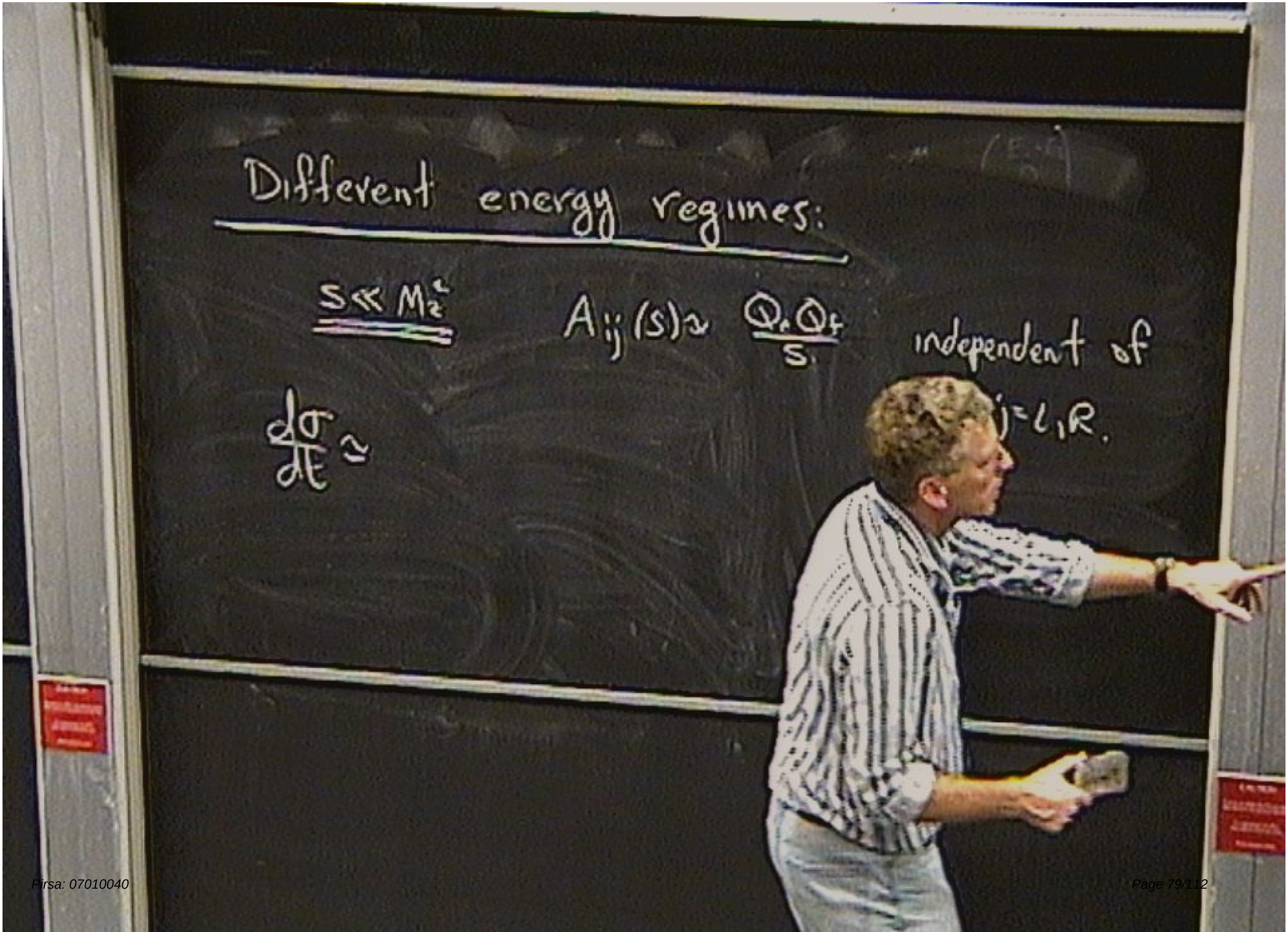
$(f \neq e^-, \nu_e)$

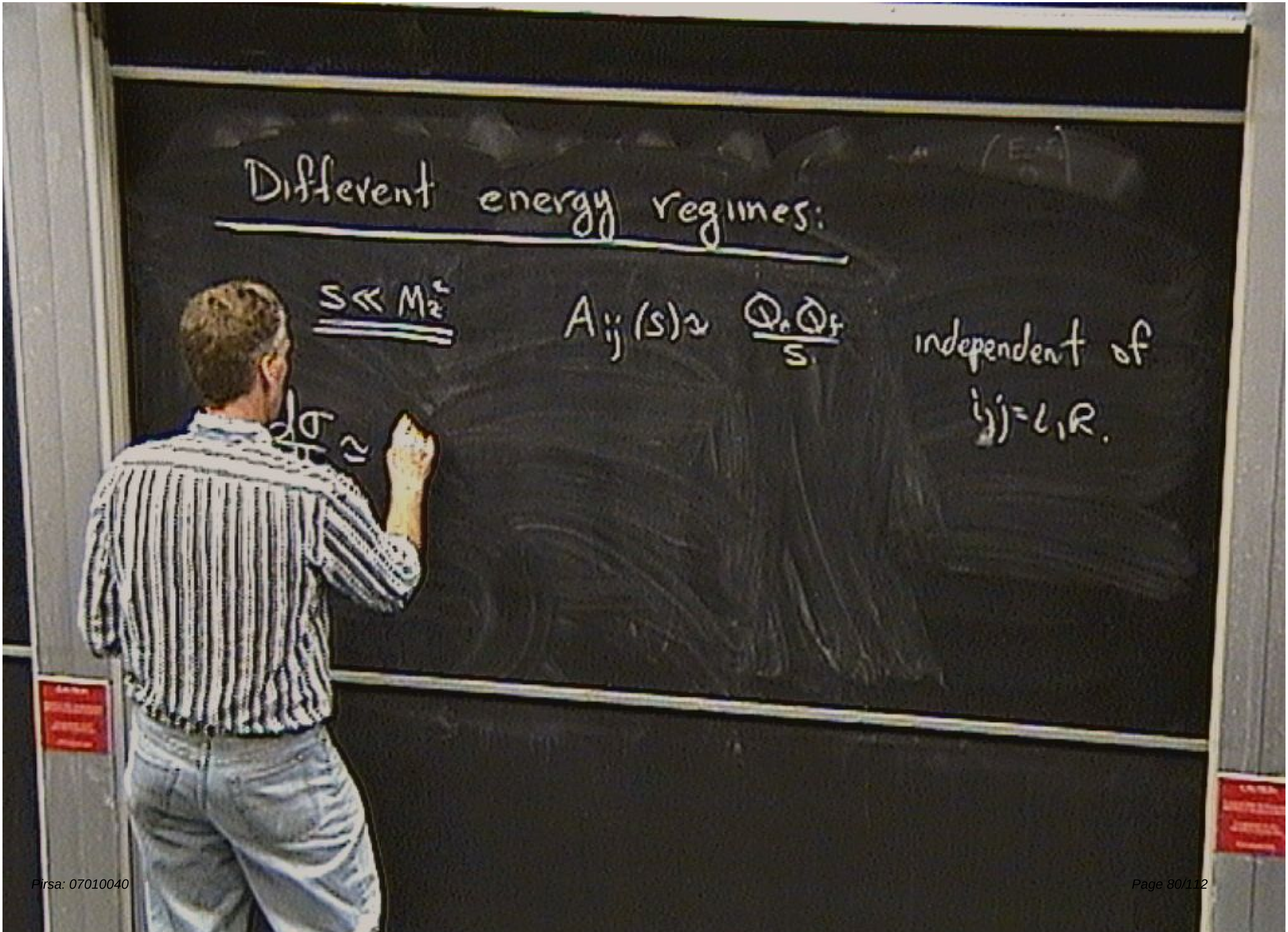
$$A_{ij} = \frac{1}{s_w^2 c_w^2}\left(\frac{g_{ei}\, g_{fj}}{s - M_Z^2}\right) + \frac{Q_e Q_f}{s} \qquad i = L, R$$

$$g_{fL} = T_{3f} - Q_f s_w^2 \qquad g_{fR} = -Q_f s_w^2$$

$$d\sigma(e^+e^- \to f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s}\Big[\left(|A_{LL}(s)|^2 + |A_{RR}(s)|^2\right)u^2 + \left(|A_{LR}(s)|^2 + |A_{RL}(s)|^2\right)t^2\Big]\, dX$$

$(f \neq e^-, \nu_e)$


Different energy regimes:
$s \ll M_Z^2$
$A_{ij}(s) \approx \frac{Q_e Q_f}{s}$
independent of
$i,j = L,R.$
$\frac{d\sigma}{dt} \approx$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Different energy regimes:

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$$\frac{d\sigma}{dt} \simeq \frac{-Q_e^2 Q_f^2 \, \pi \alpha^2 N_c}{s^4} \left(u^2 + t^2 \right)$$

$$dX = (2\pi)^4 \delta^4(p+p'-k-k') \frac{d^3k}{(2\pi)^6 2k^0} \frac{d^3k'}{2k^{0'}}$$

$$= -\frac{1}{8\pi s}\,\delta(s+t+u-\underbrace{m_a^2-m_b^2-m_c^2-m_d^2}_{\simeq 0})\,du\,dt$$

$\sigma \sim \frac{1}{M^2}$

$t \sim M^2$

$$\simeq -\frac{1}{8\pi s}\,\delta(s+t+u)\,du\,dt$$

$$\frac{d\sigma}{dt} = -\frac{\pi\alpha^2 N_c}{s^2}\left[\ \cdots\ \right]$$

$$\alpha = e^2/4\pi$$

$$t = -2E^2(1-\cos\theta)$$

$$dt = +2E^2\, d\cos\theta$$

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Different energy regimes:

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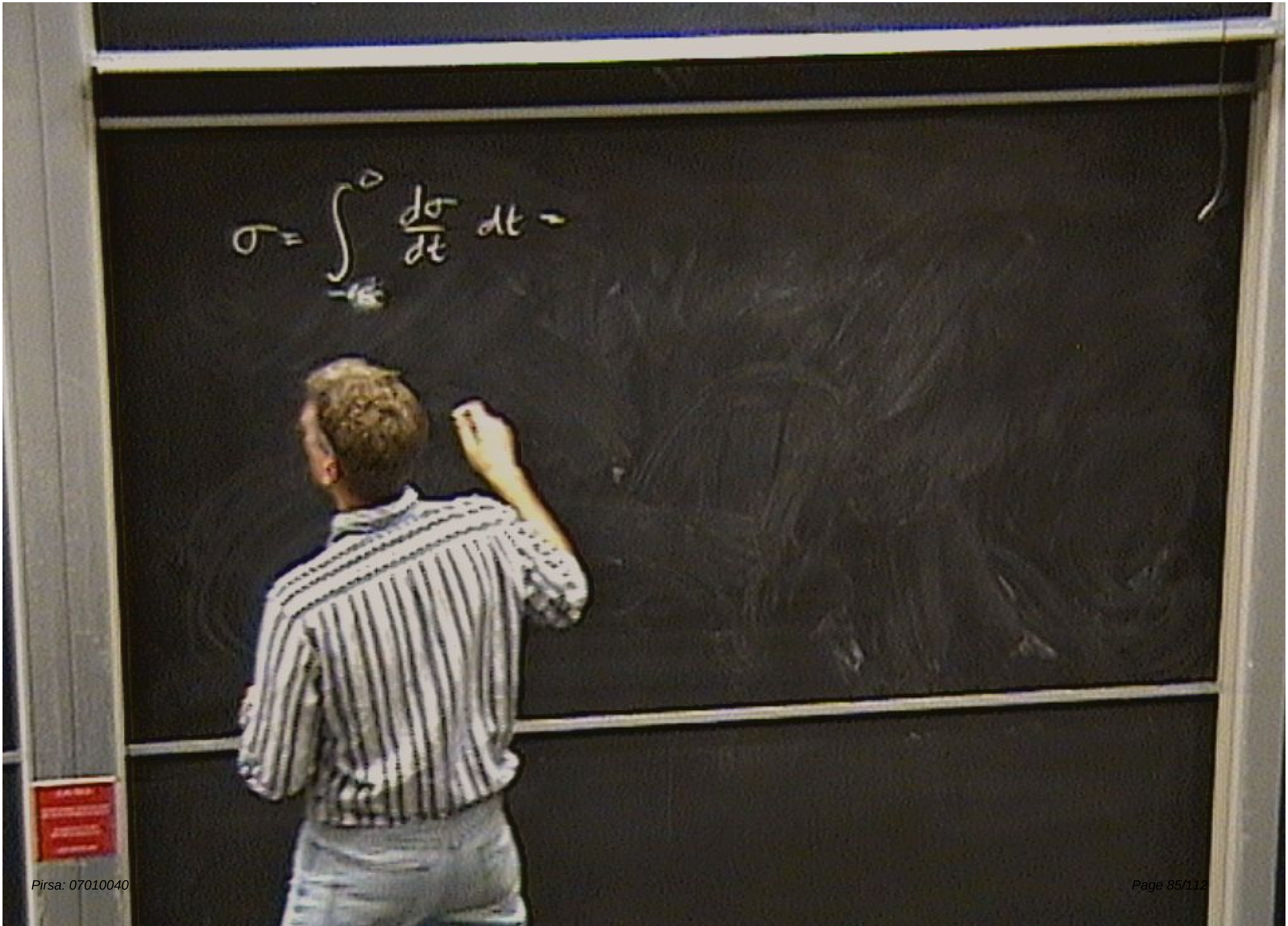
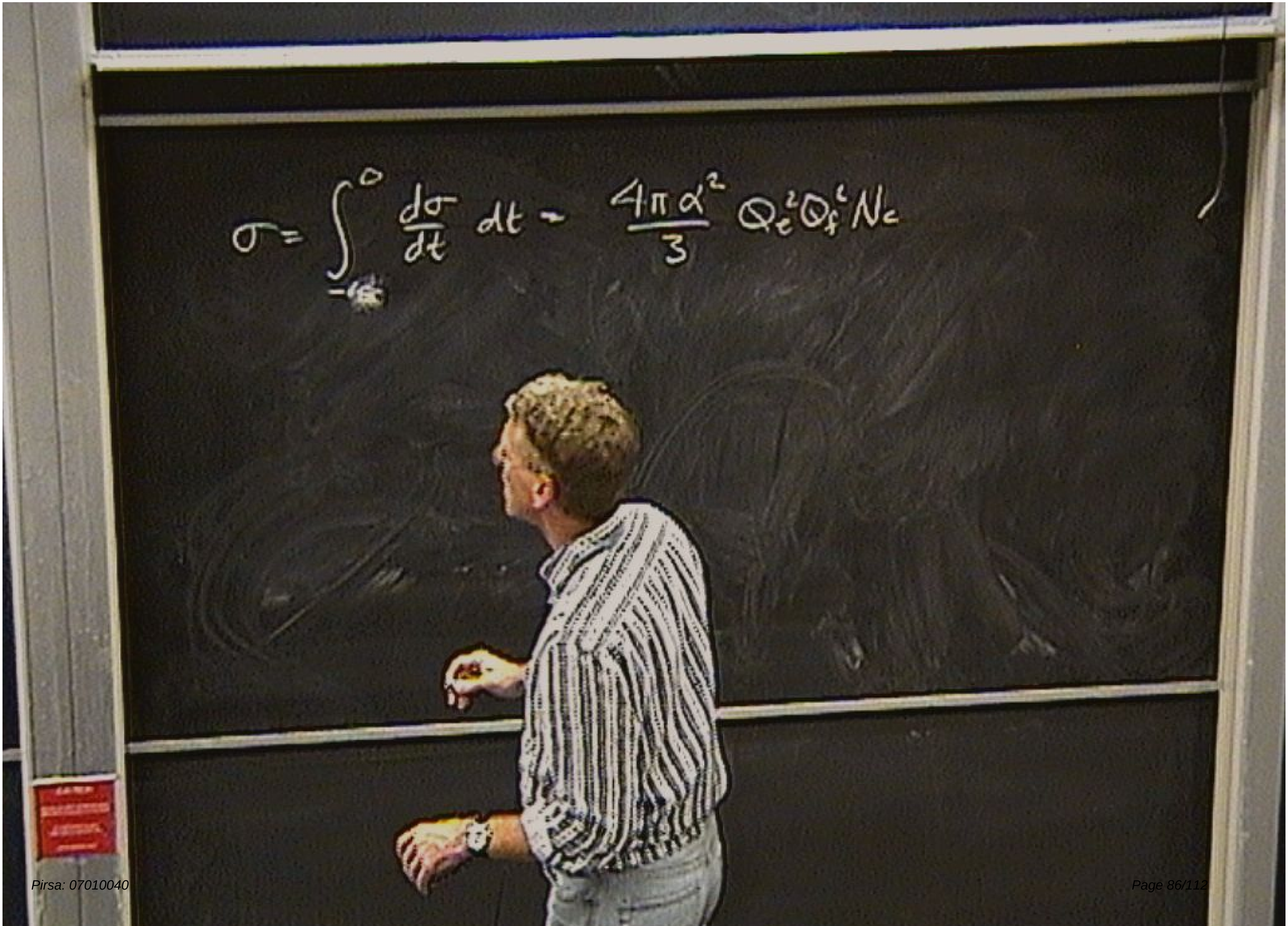
$0 < \theta < \pi \longrightarrow 0 > t \geq -4E^2 = -s$


σ = ∫ dσ/dt dt


$\sigma = \int^{0} \frac{d\sigma}{dt}\, dt = \frac{4\pi\alpha^2}{3} Q_e^2 Q_f^2 N_c$

$$\sigma = \int_{-s}^{0} \frac{d\sigma}{dt}\, dt = \frac{4\pi\alpha^2}{3s} Q_e^2 Q_f^2 N_c$$

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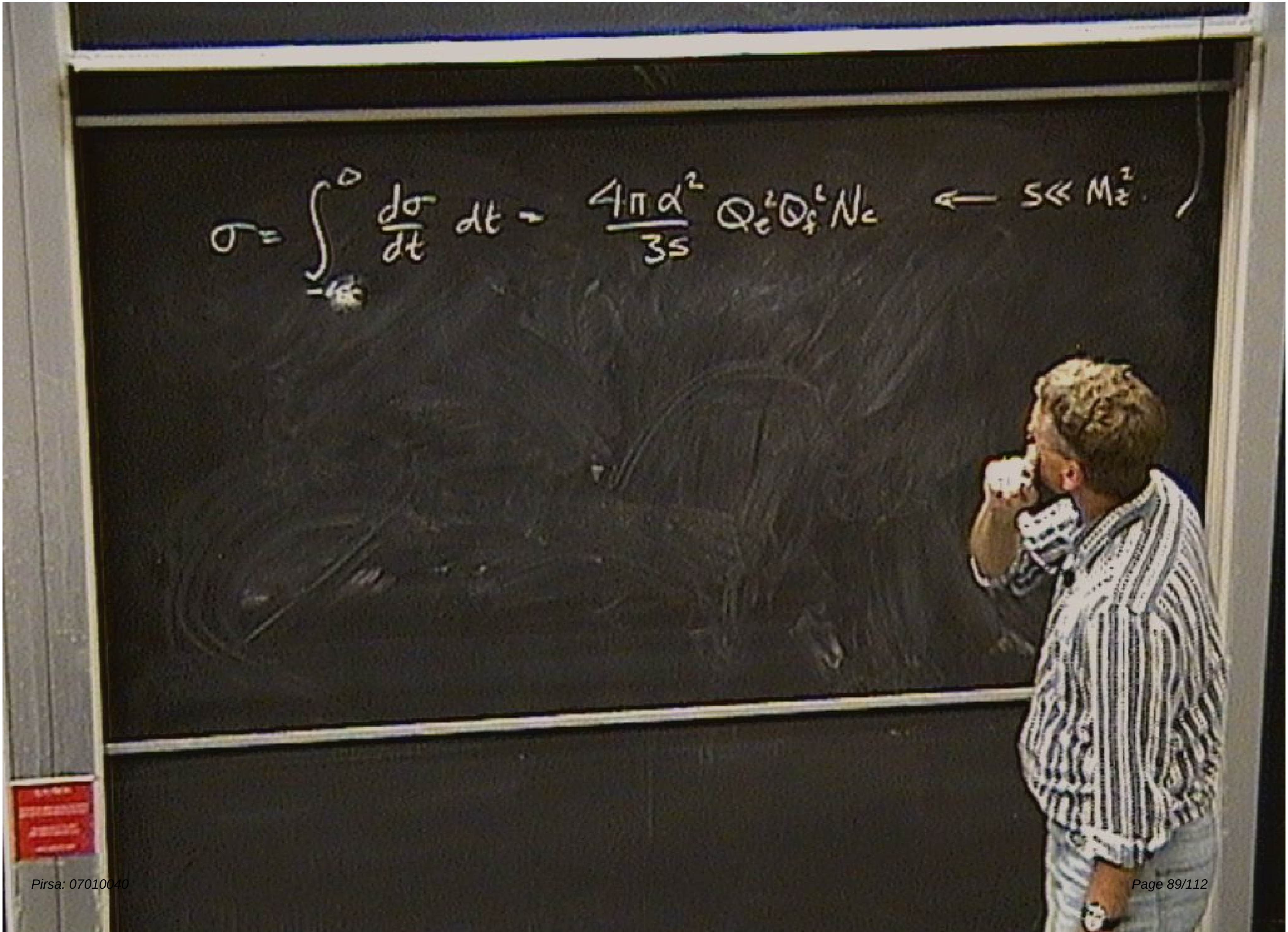
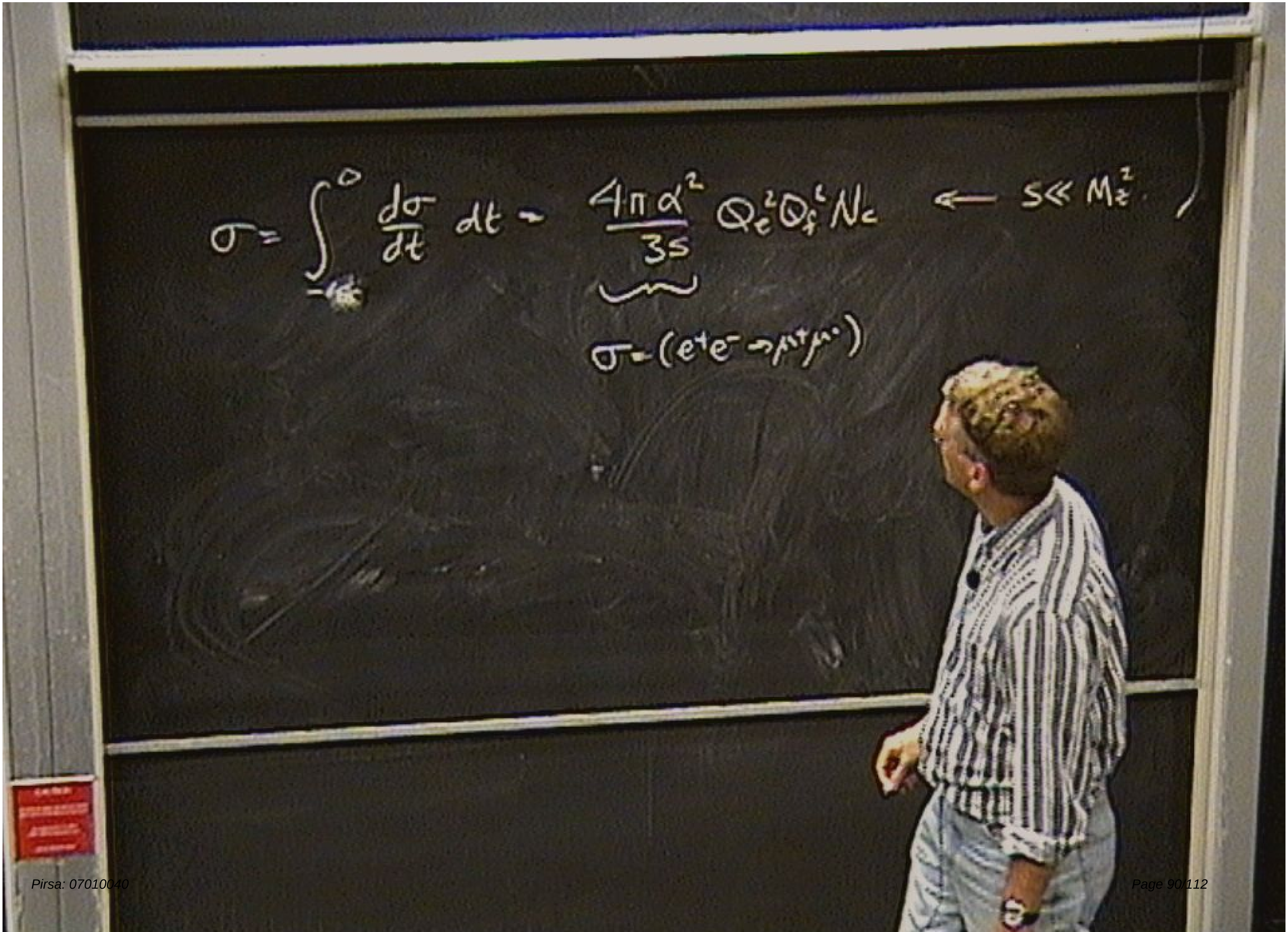
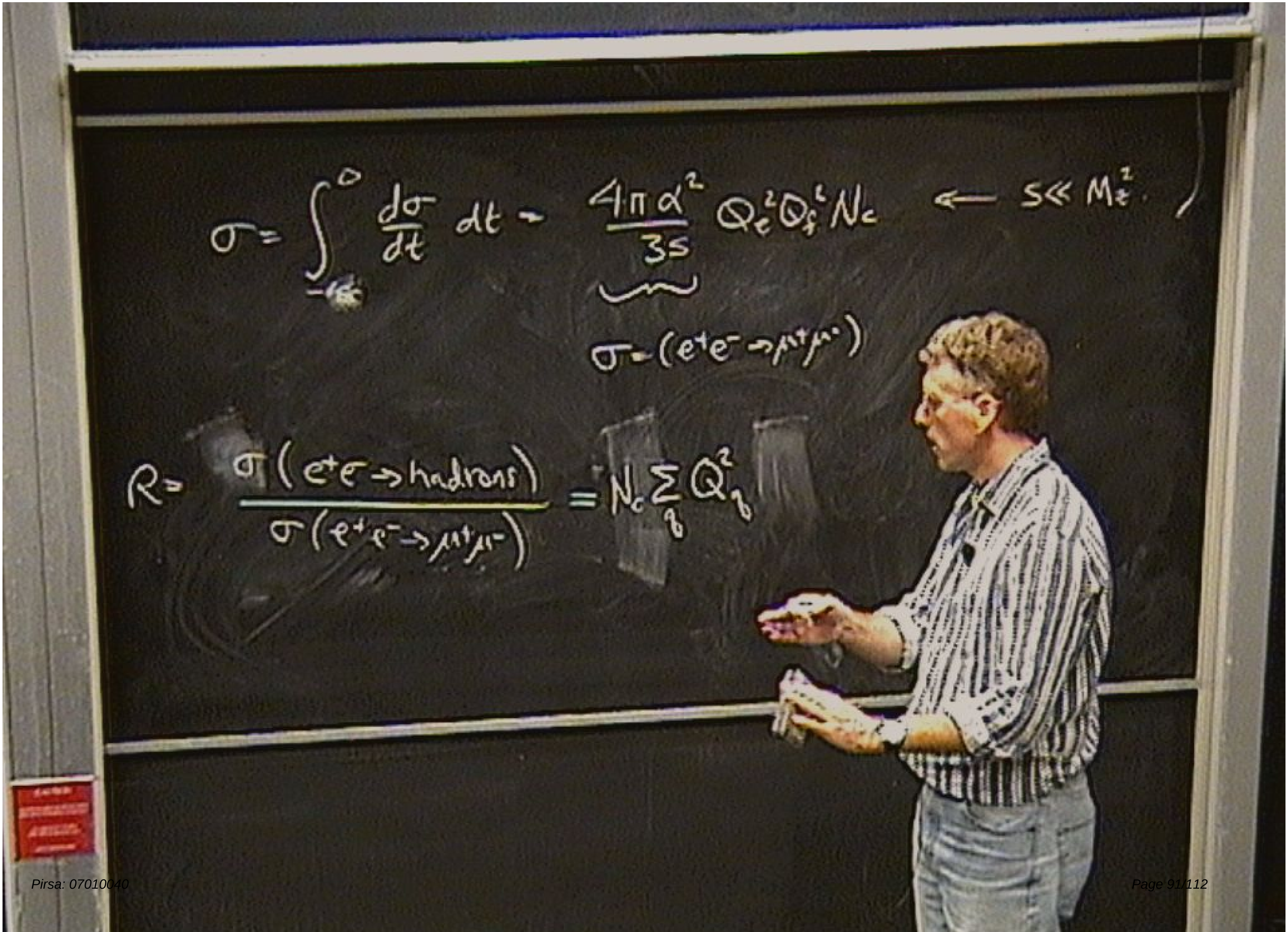
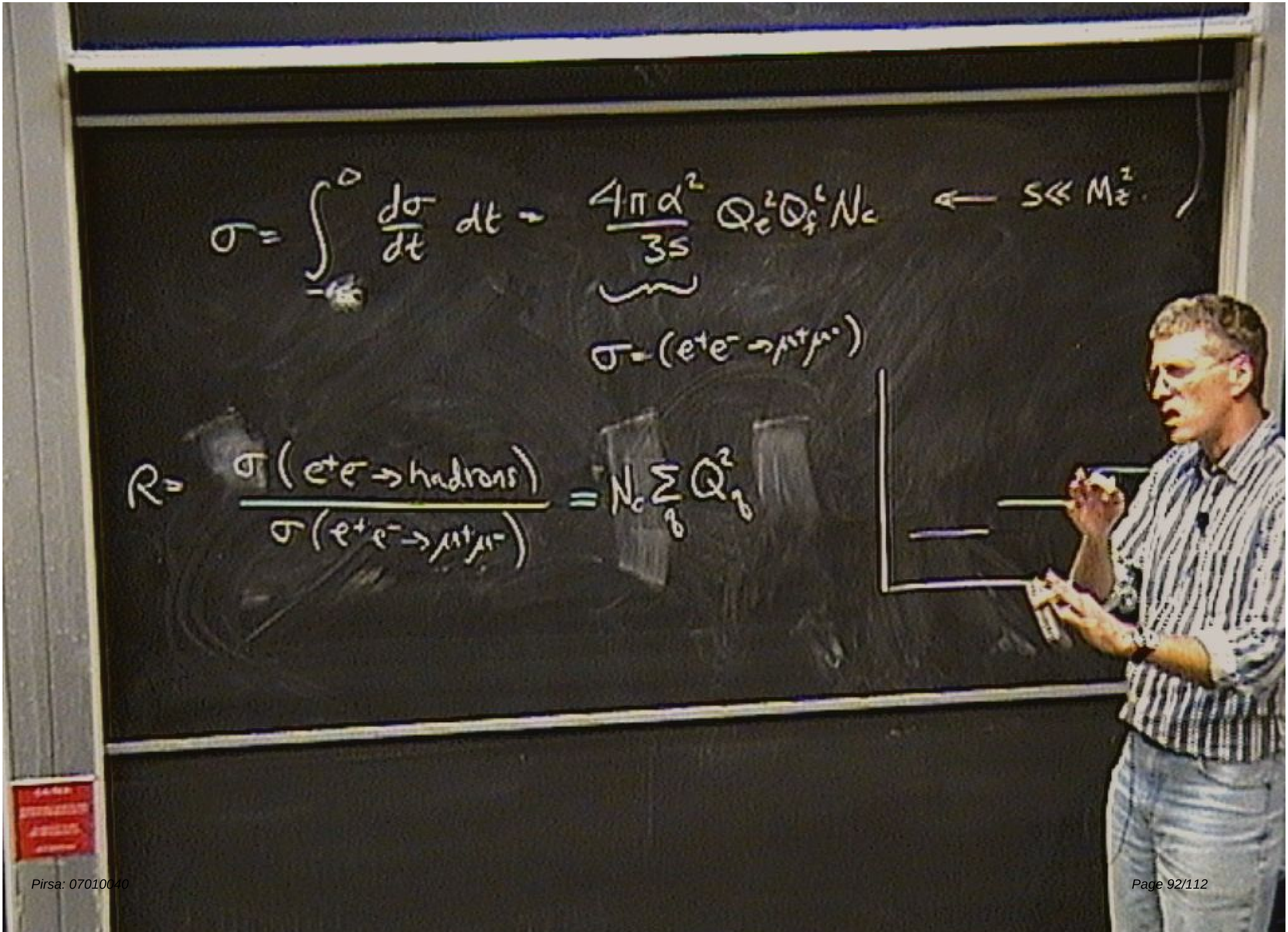
$$\frac{d\sigma}{dt} \simeq \frac{Q_e^2 Q_f^2\, 2\pi\alpha^2 N_c}{s^4}\left(u^2+t^2\right)\Big|_{u=-s-t}$$

$0 < \theta < \pi \longrightarrow 0 \geq t \geq -4E^2 = -s$


$\sigma = \int^{0} \frac{d\sigma}{dt}\, dt = \frac{4\pi\alpha^2}{3s} Q_e^2 Q_f^2 N_c \longleftarrow s \ll M_Z^2$


σ = ∫^0 dσ/dt dt = 4πα²/3s Q_e² Q_f² N_c ⟵ s ≪ M_z²
σ = (e⁺e⁻ → μ⁺μ⁻)


$\sigma = \int^{0} \frac{d\sigma}{dt}\, dt = \underbrace{\frac{4\pi\alpha^2}{3s}}_{\sigma = (e^+e^- \to \mu^+\mu^-)} Q_e^2 Q_f^2 N_c \leftarrow s \ll M_Z^2$
$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q Q_q^2$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$$Q_u = 2/3$$

$$Q_{d,s} = -1/3$$

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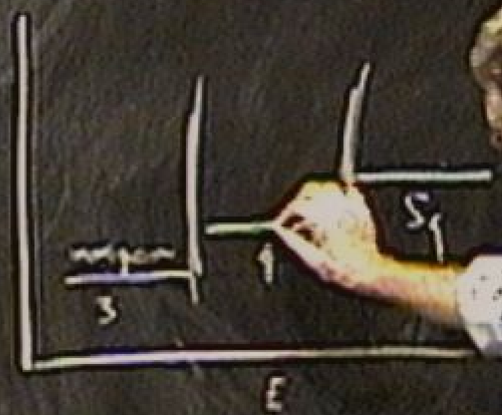

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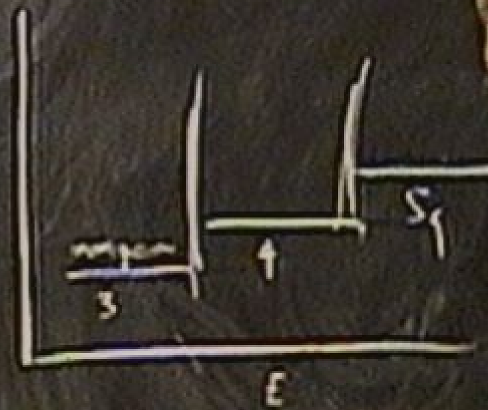

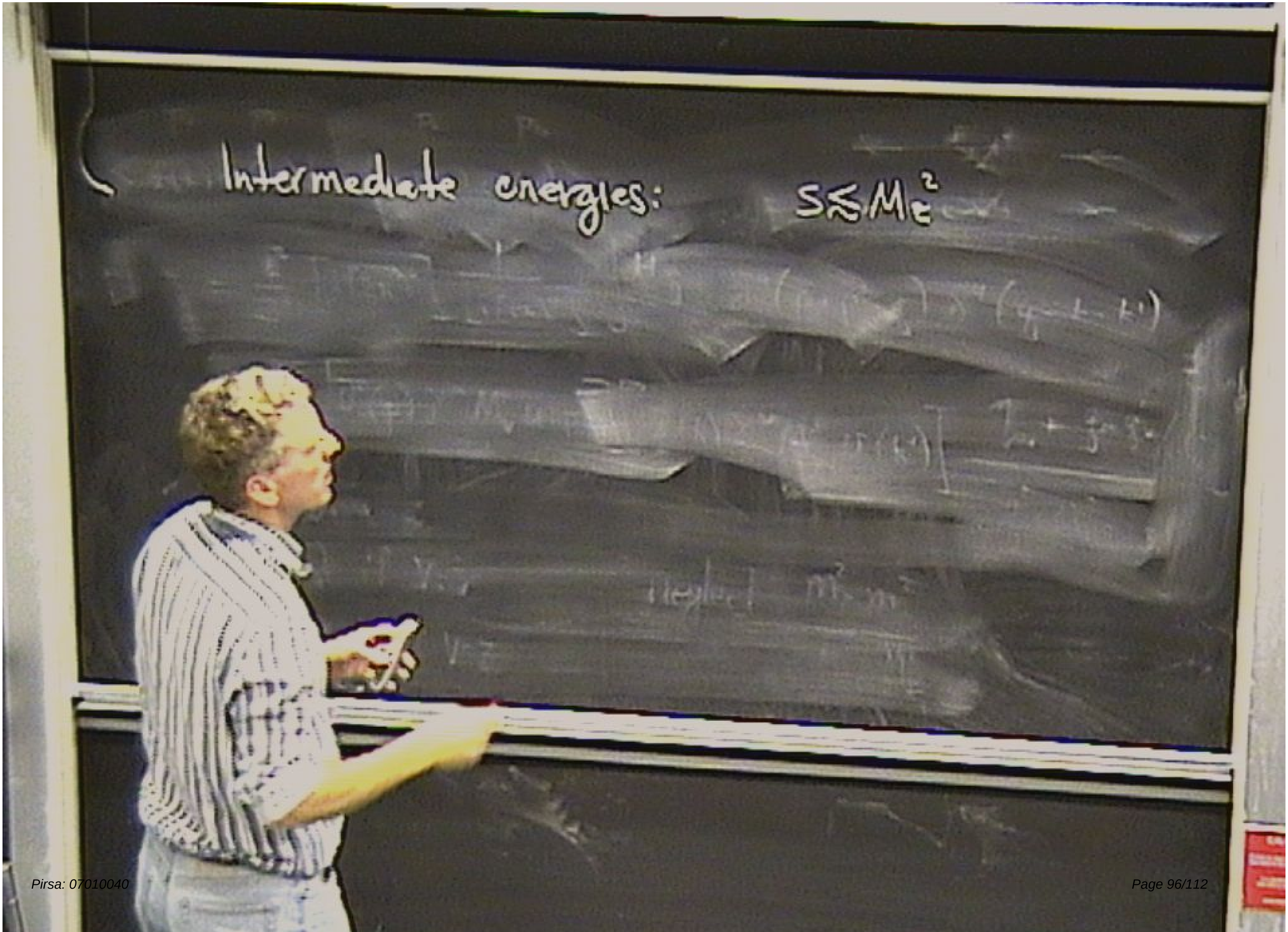
Intermediate energies: $s \lesssim M_Z^2$

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$$A_{ij} \simeq \frac{Q_e Q_f}{s} -$$

$N_c = \begin{cases} 1, & \text{if } f \neq q \\ 3, & \text{if } f = q \end{cases}$

t^2

$g_{fL} = T_{3f} - Q_f s_W^2 \qquad g_{fR} = -Q_f s_W^2$

$$d\sigma(e^+e^- \to f\bar{f}) = \frac{8\pi^2 \alpha^2 N_c}{s}\left[\left(|A_{LL}(s)|^2 + |A_{RR}(s)|^2\right)u^2 + \left(|A_{LR}(s)|^2 + |A_{RL}(s)|^2\right)t^2\right] dX$$

$(f \neq e^-, \nu_e)$

$t = -2E^2(1-\cos\theta) \qquad u = -2E^2(1+\cos\theta)$

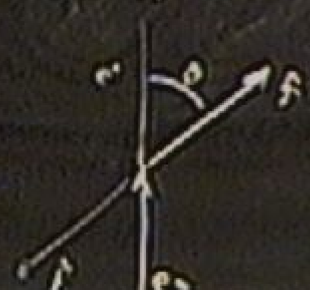
$t^2 \sim (1-\cos\theta)^2 \qquad u^2 \sim (1+\cos\theta)^2$

$\to 0$ if $\theta = 0$ $\qquad \to 0 \quad \theta = \pi$

...dependent of

$i,j = L,R.$

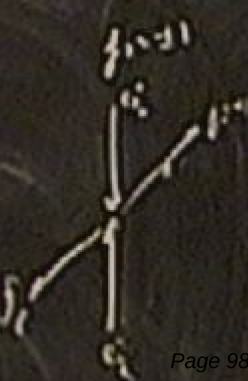
$= s - t$

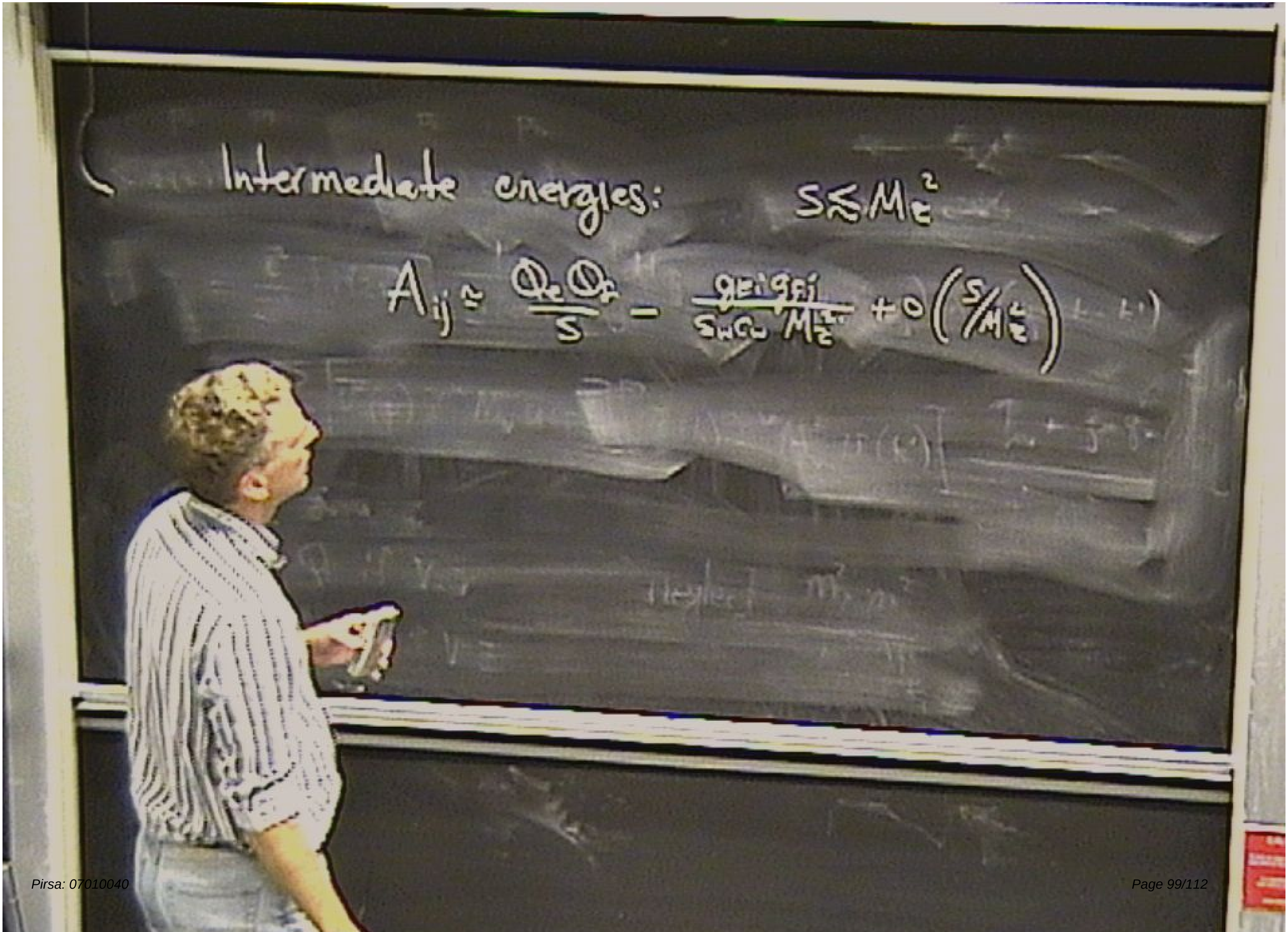


$\bar{f}\gamma^\mu(\ldots)f$

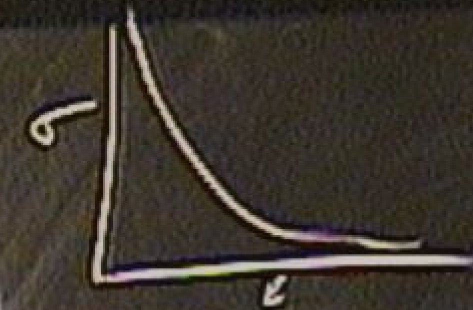
$\bar{f}_L f_L \qquad \bar{f}_L f_L$

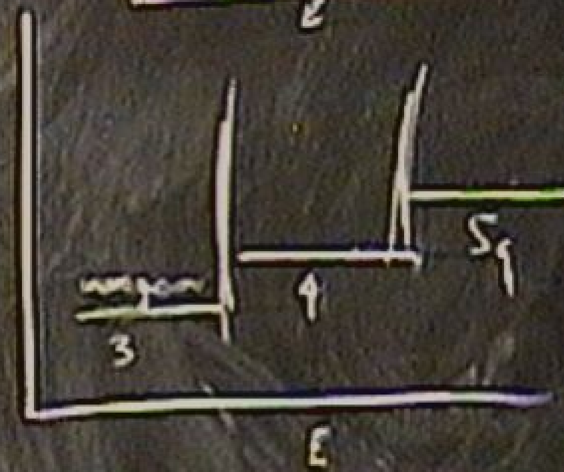
$f_L^\dagger \beta\gamma^\mu f_L$




Intermediate energies: $s \lesssim M_Z^2$
$A_{ij} \simeq \frac{Q_e Q_f}{s} - \frac{g_{ei} g_{fj}}{s_W c_W M_Z^2} + O\left(s/M_Z^2\right)$

$$\sigma = \int_{-s} \frac{d\sigma}{dt}\, dt = \underbrace{\frac{4\pi\alpha^2}{3s}}_{\sigma = (e^+e^- \to \mu^+\mu^-)} Q_e^2 Q_f^2 N_c \quad \leftarrow \quad s \ll M_Z^2$$





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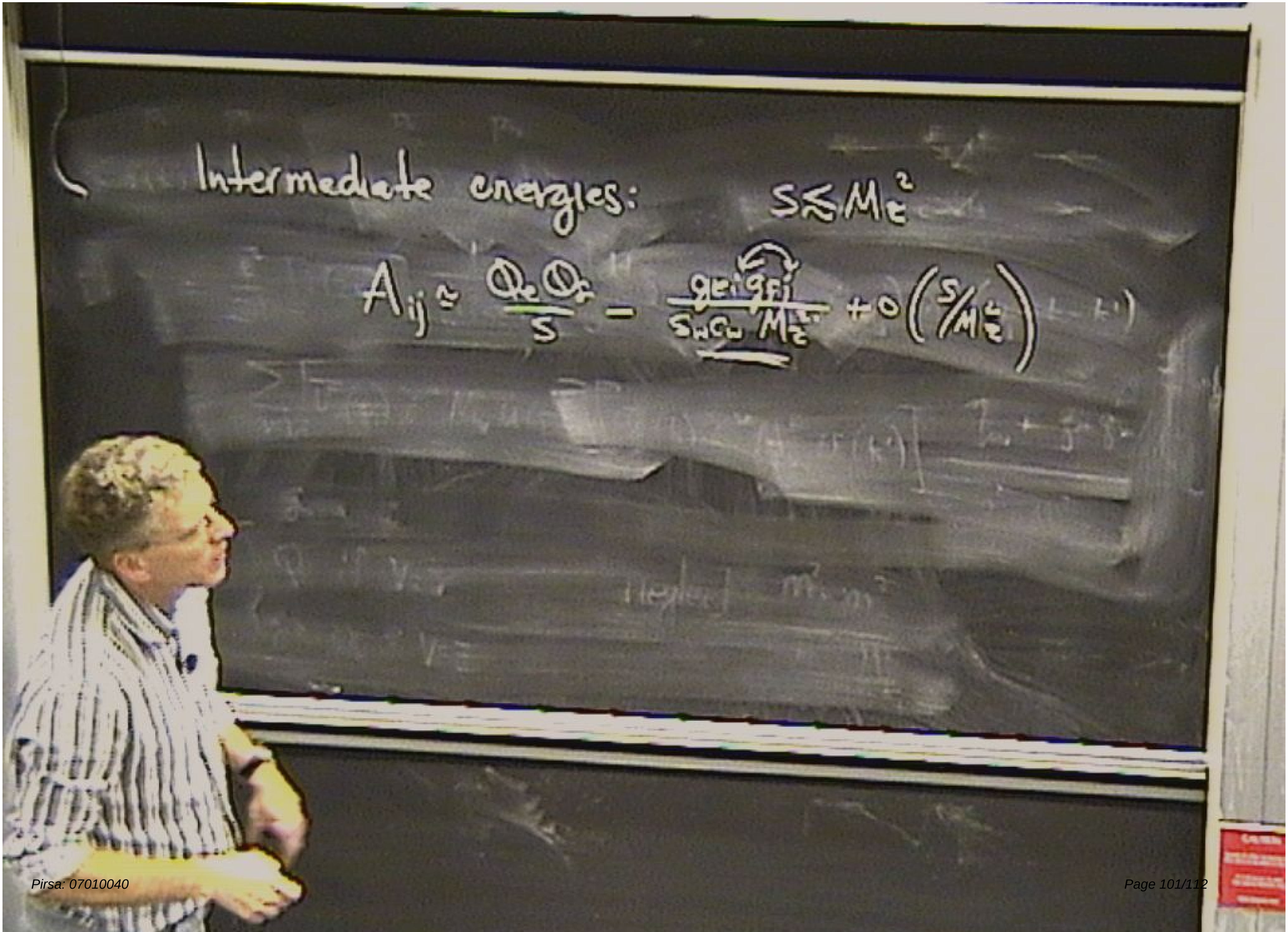
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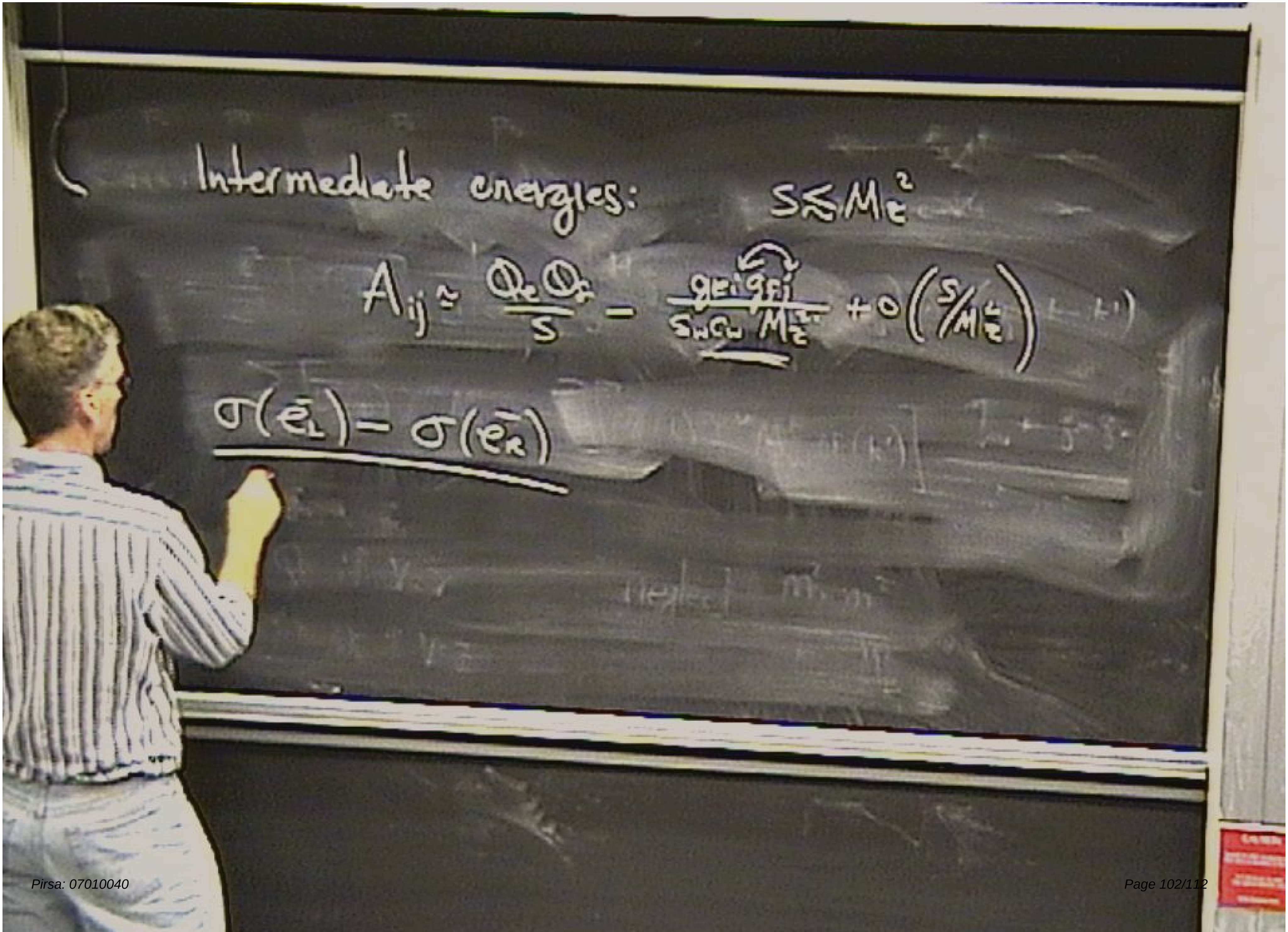
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$(f \neq e^-, \nu_e)$

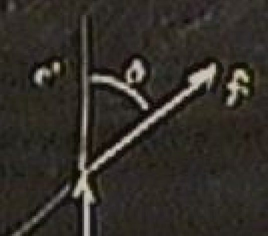
...ent of

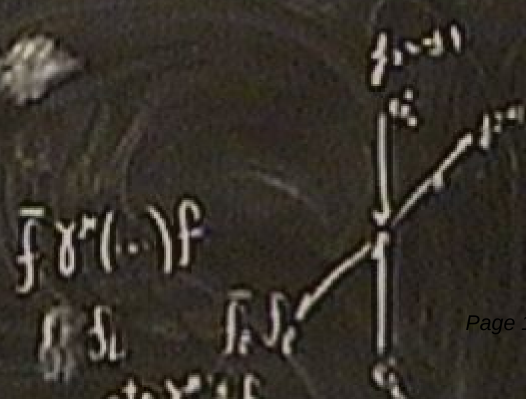
$= L, R.$

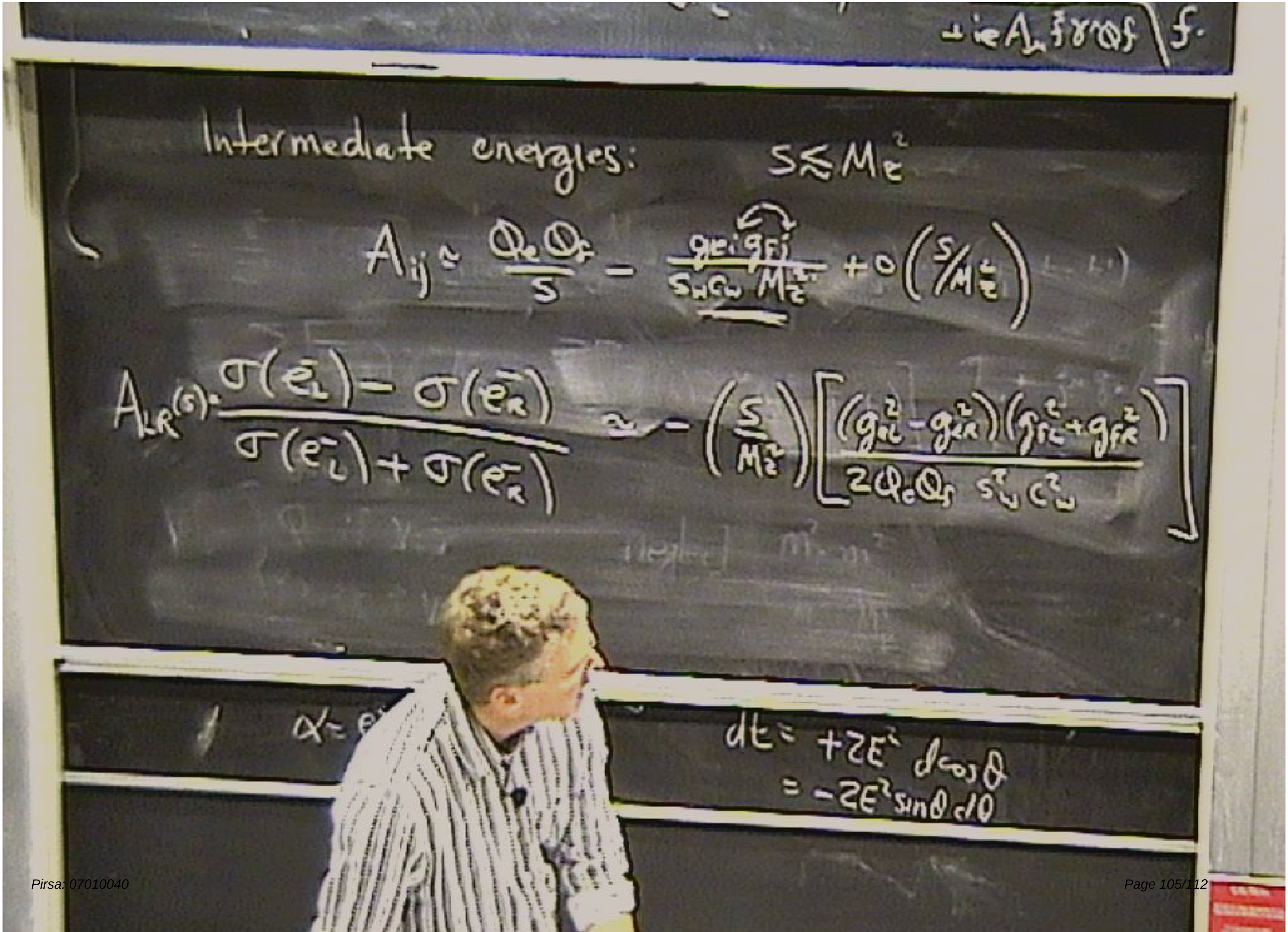
$t = -2E^2(1-\cos\theta) \qquad u = -2E^2(1+\cos\theta)$

$t^2 \sim (1-\cos\theta)^2 \qquad u^2 \sim (1+\cos\theta)^2$

$\to 0$ if $\theta = 0 \qquad \to 0 \;\; \theta = \pi$



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$dt = +2E^2 \, d\cos\theta$
$= -2E^2 \sin\theta \, d\theta$

$$+ ie A_\mu \bar{f}\gamma^\mu Q_f \Big) f.$$

Intermediate energies: $s \lesssim M_Z^2$

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$$\alpha = e^2/4\pi$$

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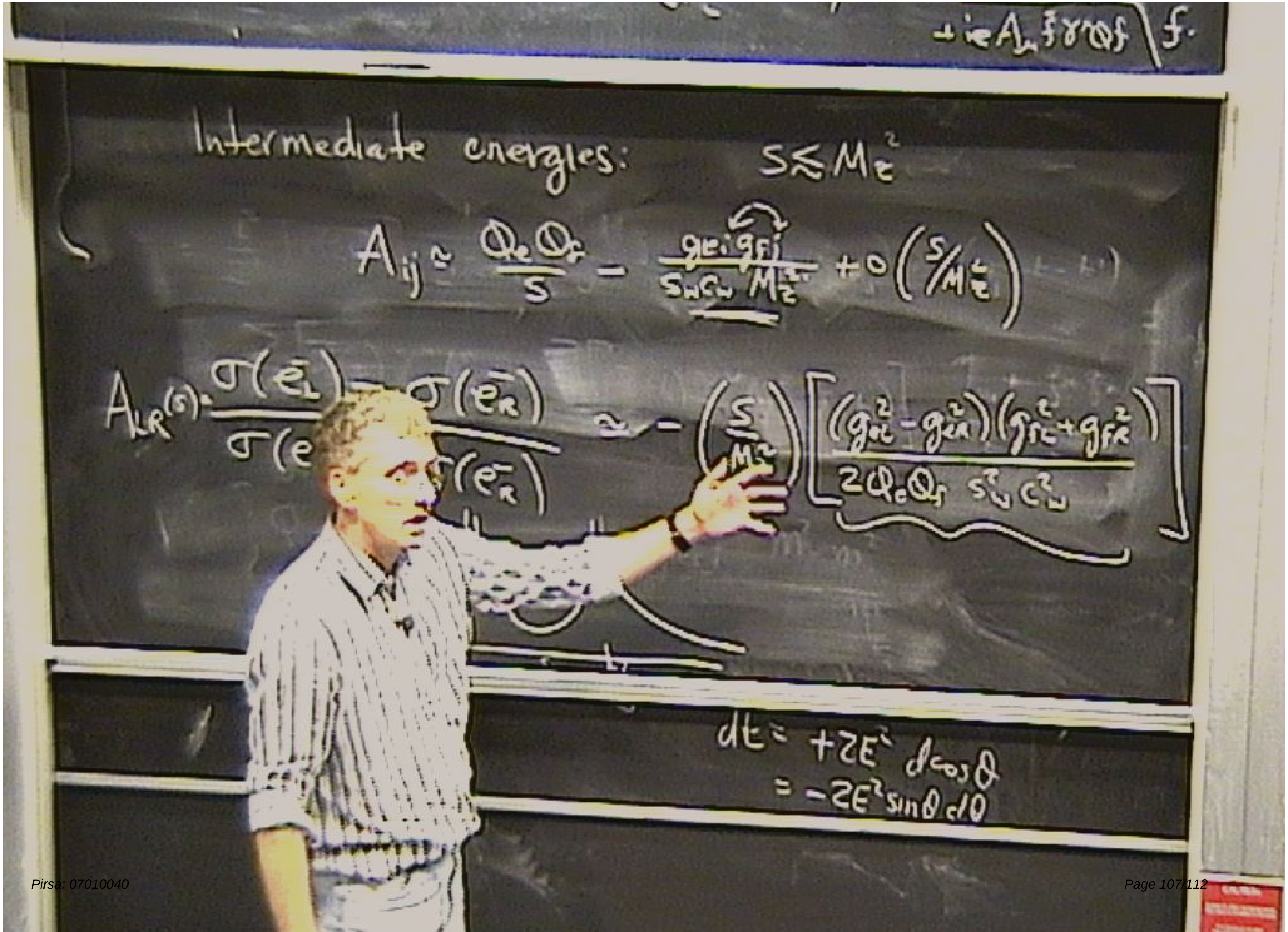

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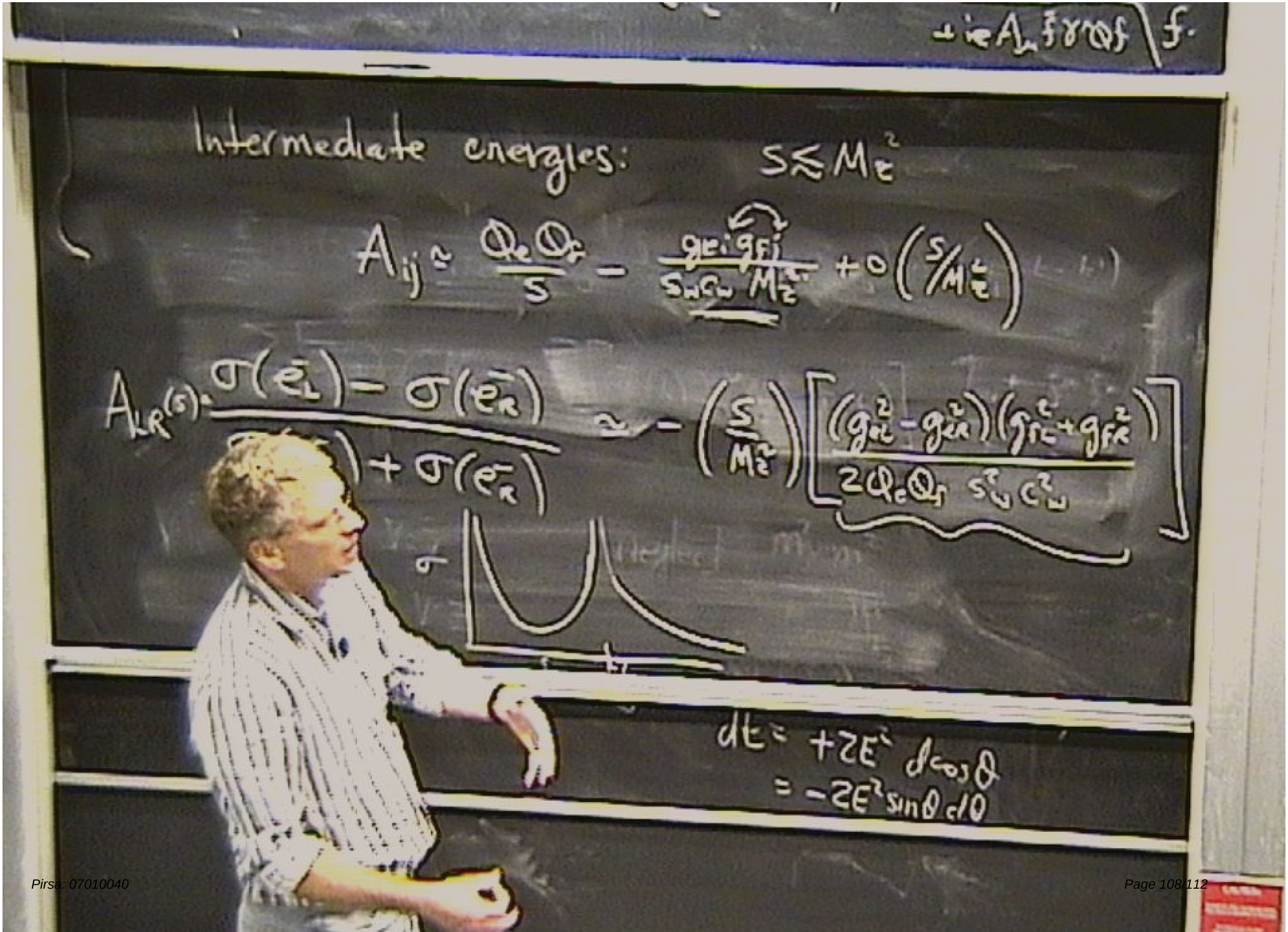
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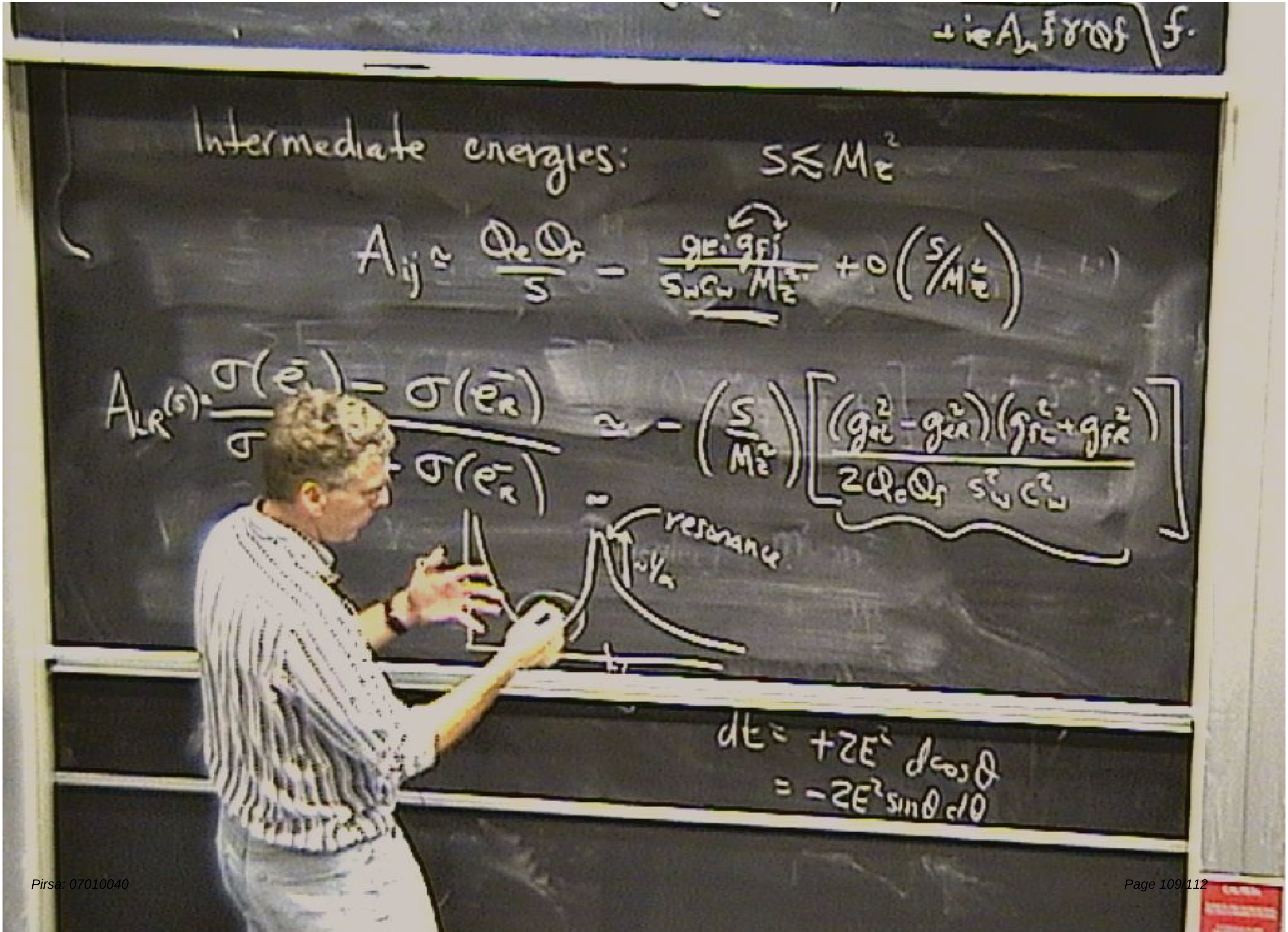

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$dt = +2E^2 \, d\cos\theta$
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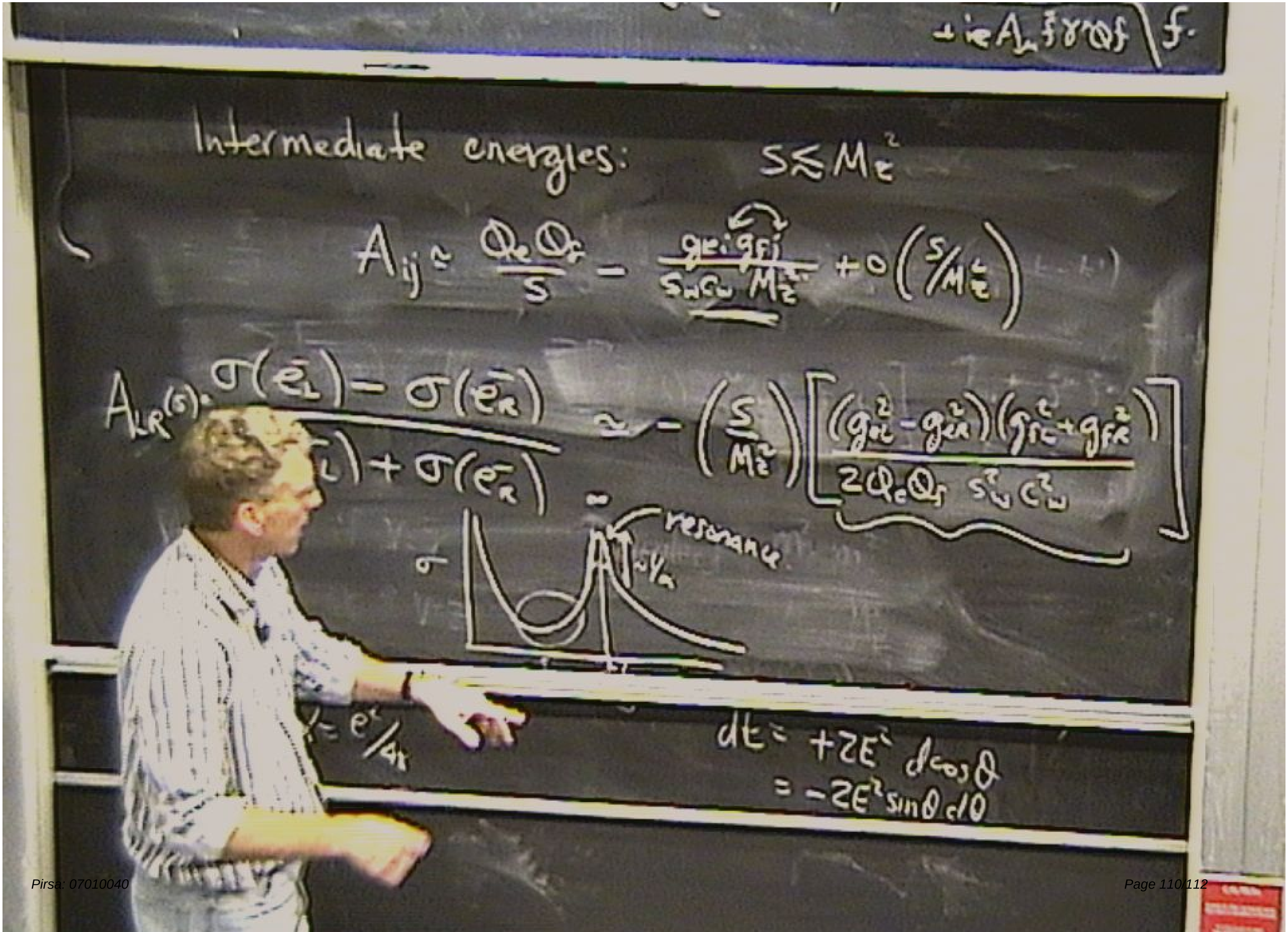
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resonance
σ
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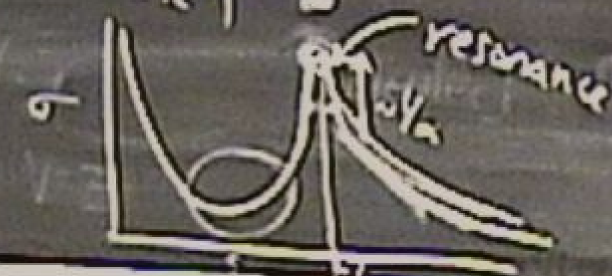

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Intermediate energies: $s \lesssim M_Z^2$

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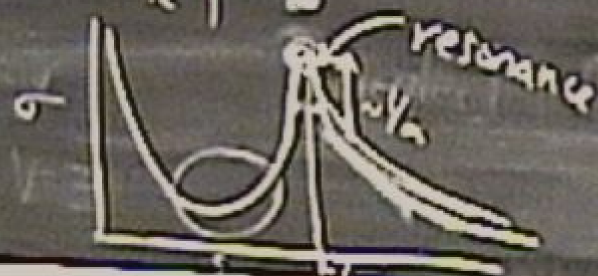

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