Title: Quantum cosmology and the conditions at the birth of the universe

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Abstract: Cosmology ultimately aims to explain the initial conditions at the beginning of time and the entire subsequent evolution of the universe. The "beginning of time" can be understood in the Wheeler-DeWitt approach to quantum gravity, where homogeneous universes are described by a Schroedinger equation with a potential barrier. Quantum tunneling through the barrier is interpreted as a spontaneous creation of a small (Planck-size) closed universe, which then enters the regime of cosmological inflation and reaches an extremely large size. After sufficient growth, the universe can be adequately described as a classical spacetime with quantum matter. The initial quantum state of matter in the created universe can be determined by solving the Schroedinger equation with appropriate boundary conditions. I show that the most likely initial state is close to the vacuum state. This is the initial condition for inflation favored both by observational data and theoretical considerations.

A Possible History of the Universe

time	
?	dark energy era: empty? bubble nucleation? landscape?
$13.7 \cdot 10^9$yr	present time
10^9yr	gravitational instability, galaxy formation
380,000 yr	recombination, CMB last scattering
1s	Nucleosynthesis (D, He, Li)
10^{-6}s	Phase transitions: leptogenesis, baryosynthesis...
10^{-35}s	Inflation: many models; reheating
0? 10^{-43}s?	Birth: Singularity? Quantum gravity? Eternal inflation?

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The Fate of Quantum Fluctuations

- Inflation = Accelerated expansion of space

[Guth 1981, Linde 1982, ...]

- Quantum fluctuations are stretched beyond causal distances

→ become classical inhogeneities

[Linde 1982, Starobinsky 1982, ... See Winitzki 2006 for review]

- Classical inhomogeneities collapse and form galaxies

[Peebles 1982]

- Fluctuations of temperature → CMB

[See Straumann 2006 for a complete calculation]

The Longest Calculation in Cosmology

Ann. Phys. (Leipzig) 15, No. 10–11, 701–845 (2006) / DOI 10.1002/andp.200610212

From primordial quantum fluctuations to the anisotropies of the cosmic microwave background radiation*

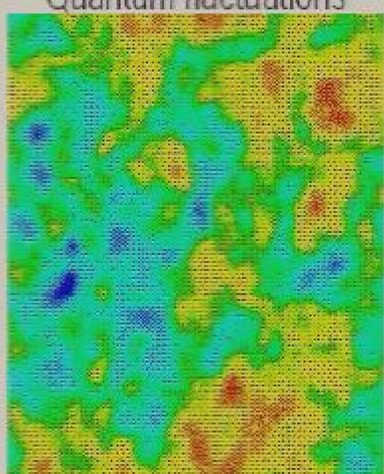
Norbert Straumann**

Institute for Theoretical Physics, University of Zurich, 8057 Zurich, Switzerland

Received 25 May 2005, accepted 17 January 2006 by A. Wipf
Published online 22 May 2006

[Straumann 2006]

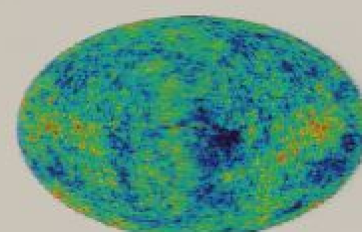
Quantum fluctuations



$> 10^3$ equations

linear map

CMB as seen today



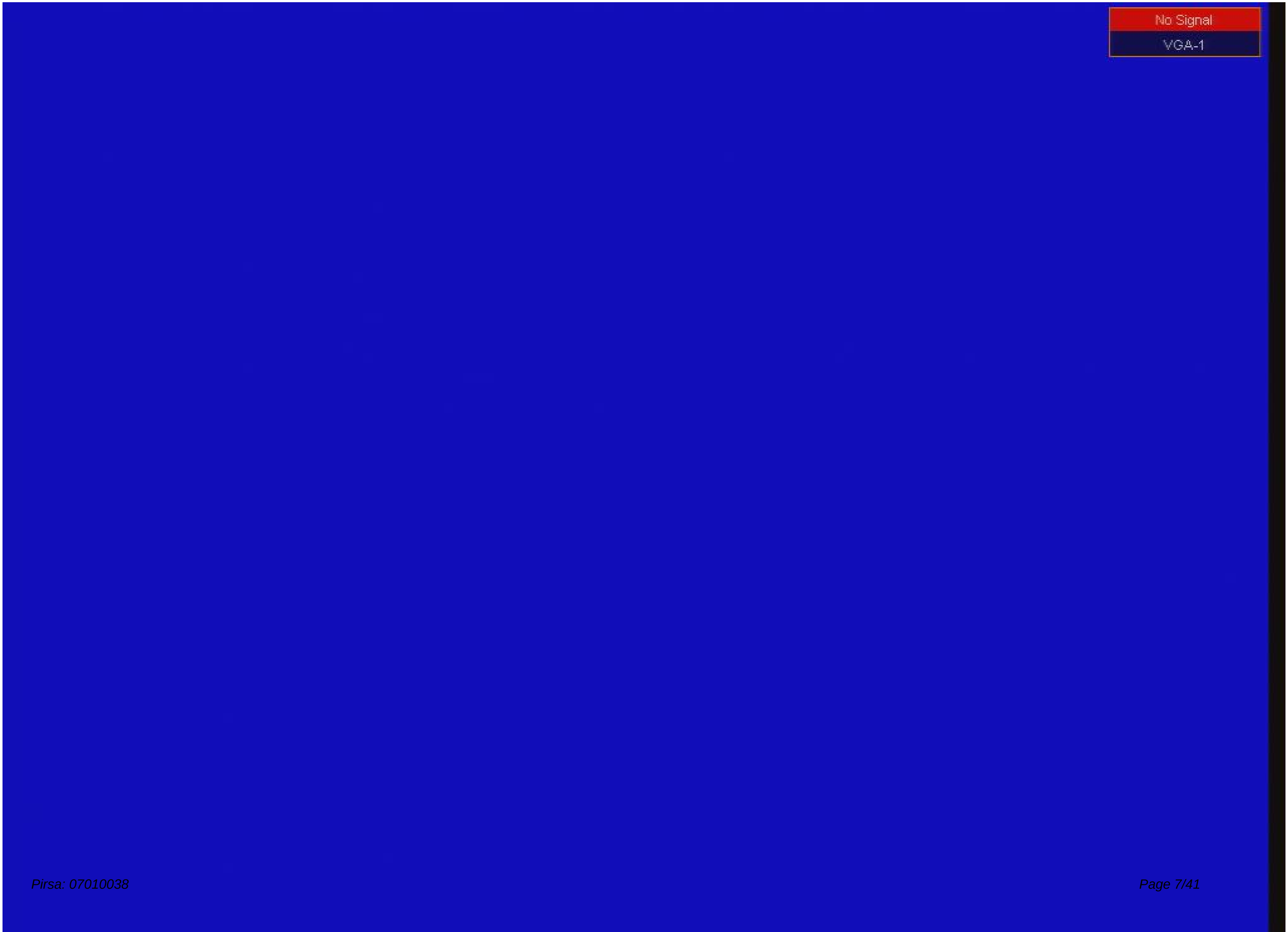
The Enigma of the Initial State

What we know:

- Universe begins inflating
- Quantum fields are in vacuum state

What is missing:

- Explanation of the "birth" of the universe
- Explanation of the initial quantum state of fields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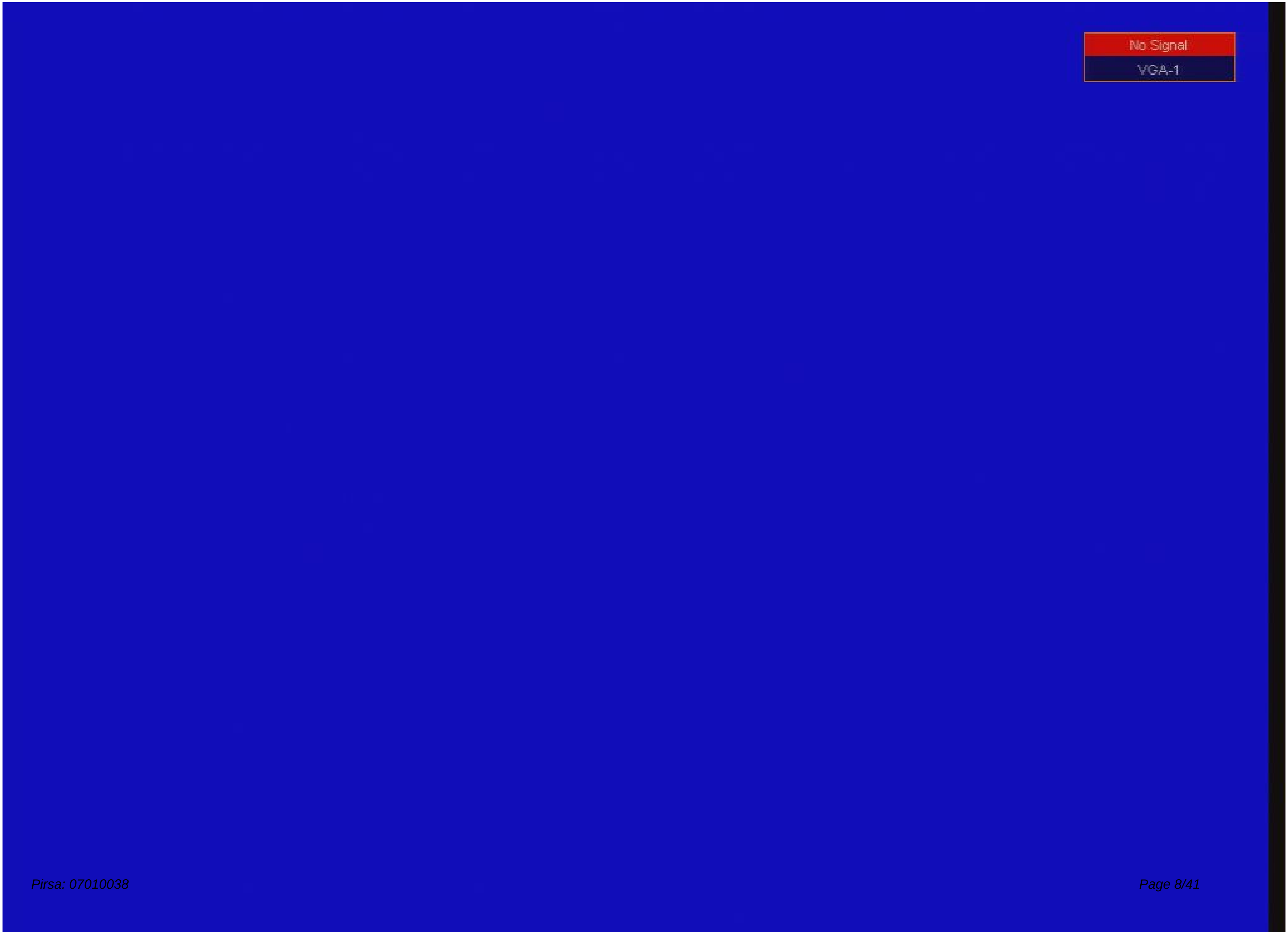
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Outline

- Goal: Describe the creation of the universe and derive the initial quantum state of fields

- Introduction to quantum cosmology: "tunneling from nothing" and "perturbative minisuperspace"

- Interpretation of the wave function of the universe by means of QFT in curved spacetime

- Claims of highly non-vacuum initial conditions (Rubakov *et al.* 1984, 2002)

- Claims of vacuum initial conditions (Vilenkin, Vachaspati, Garriga 1988, 1997)

Resolution: Select a consistent, quasi-classical quantum state of gravity $\Rightarrow$ **initial conditions close to vacuum**

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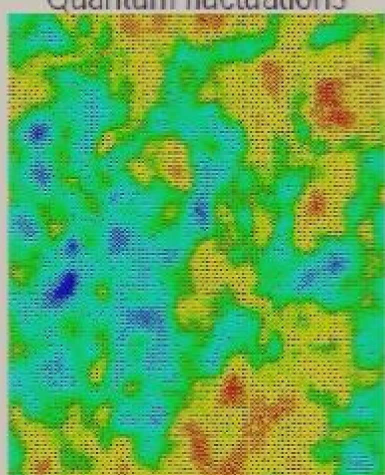

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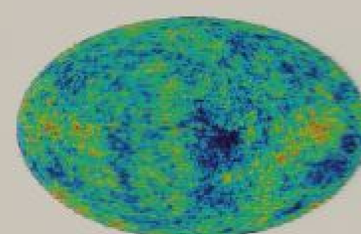

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Birth of FRW universe I: pure gravity

Quantum mechanics of closed FRW universe (minisuperspace):

$$ds^2 = N^2(t)dt^2 - a^2(t)d\Omega^2$$

Einstein-Hilbert action (pure gravity):

$$S_G = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \Lambda\right) \equiv \int \mathcal{L}_G dt$$

$$\mathcal{L}_G = -\frac{a\dot{a}^2}{2N} + \frac{Na}{2} - \frac{Na^3H_0^2}{2}, \quad H_0 \equiv \frac{4G_N}{3}\sqrt{\Lambda}$$

Wheeler-DeWitt equation

Canonical quantization:

$$p_a = -\frac{a\dot{a}}{N} \rightarrow i\frac{\partial}{\partial a}$$
$$\mathcal{H}_G = \frac{N}{2}\left(\frac{p_a^2}{a} - a + H_0^2 a^3\right) \rightarrow \hat{\mathcal{H}}_G$$

$\mathcal{H}_G = N \cdot (...)$, so N is a Lagrange multiplier $\Rightarrow \mathcal{H}_G = 0$

Hamiltonian constraint, a.k.a. Wheeler-DeWitt equation:

$$\hat{\mathcal{H}}_G |\Psi\rangle = 0$$

Wave function of the universe: $|\Psi\rangle = \Psi(a,t)$

$$i\frac{\partial |\Psi\rangle}{\partial t} = \hat{\mathcal{H}}_G |\Psi\rangle = 0 \quad \Rightarrow \quad |\Psi\rangle = \Psi(a)$$

Issues of interpretation ("problem of time")

Interpretation of the wave function

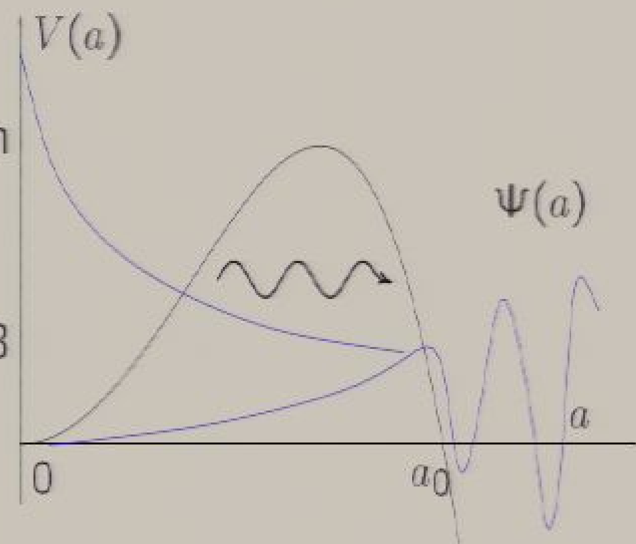
Wheeler-DeWitt equation:

$$\left[\frac{\partial^2}{\partial a^2} - a^2 + H_0^2 a^4\right]\Psi(a) \equiv \left[\frac{\partial^2}{\partial a^2} - V(a)\right]\Psi(a) = 0$$

Stationary Schrödinger equation ("particle in a potential")

Semiclassical regime at $a \gtrsim a_0$

Solutions can be found through WKB approximation



Interpretation: Expanding (de Sitter) universe nucleated at $a = a_0$

"Tunneling from nothing"

Birth of FRW universe II: gravity + radiation

Add homogeneous radiation (energy density E):

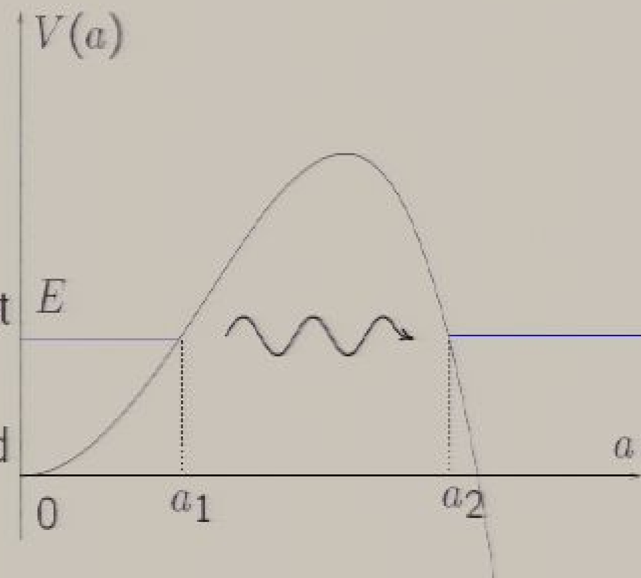
$$\rho = H_0^2 + \frac{E}{a^4}$$

Wheeler-DeWitt equation:

$$\left[\frac{\partial^2}{\partial a^2} - V(a) + E\right]\Psi(a) = 0$$

Stationary Schrödinger equation at energy E

Semiclassical regimes: $a \lesssim a_1$ and $a \gtrsim a_2$



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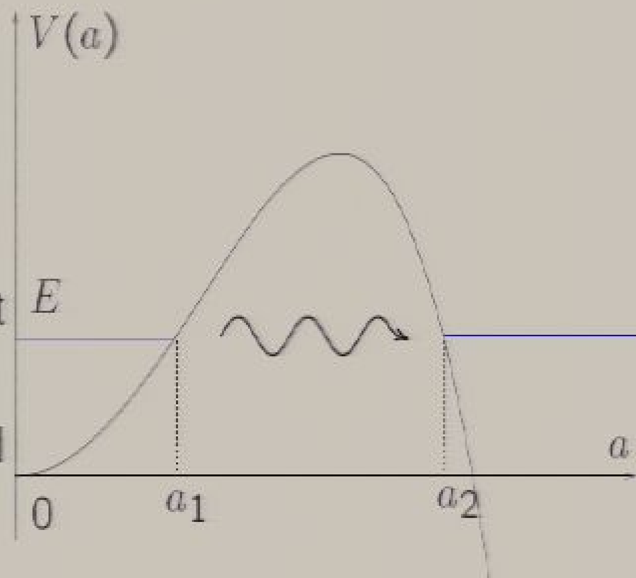

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"Tunneling from something"

Quantum cosmology with inhomogeneous matter

Conformally coupled scalar field ϕ:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{12}R\phi^2\right]$$

Expand $\phi(x,t)$ in spherical harmonics $Q_n(x)$, where $x \in S^3$,

$$\phi(x,t) = \frac{1}{a(t)}\sum_n f_n(t)Q_n(x)$$

Wheeler-DeWitt equation for $\Psi(a,\{f_n\})$:

$$\left[\frac{\partial^2}{\partial a^2} - V(a) + E + \sum_n\left(-\frac{\partial^2}{\partial f_n^2} + \left(n^2 + m^2a^2\right) f_n^2\right)\right]\Psi = 0$$

Goal: to describe the quantum state of fields in the nucleated universe in terms of QFT in (semiclassical) curved spacetime

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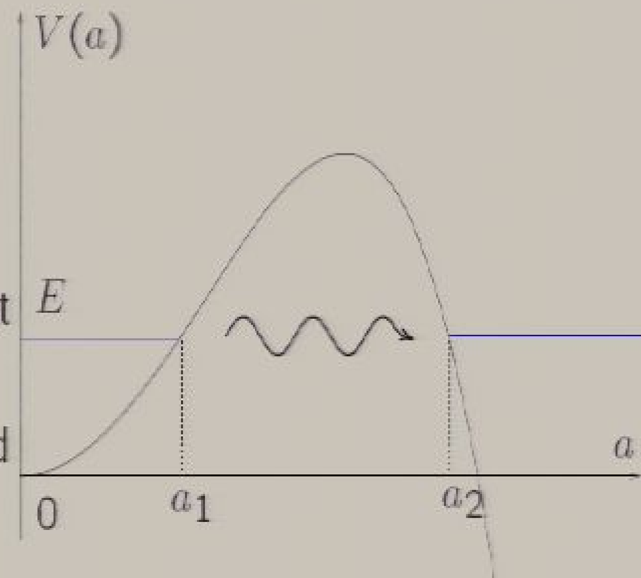

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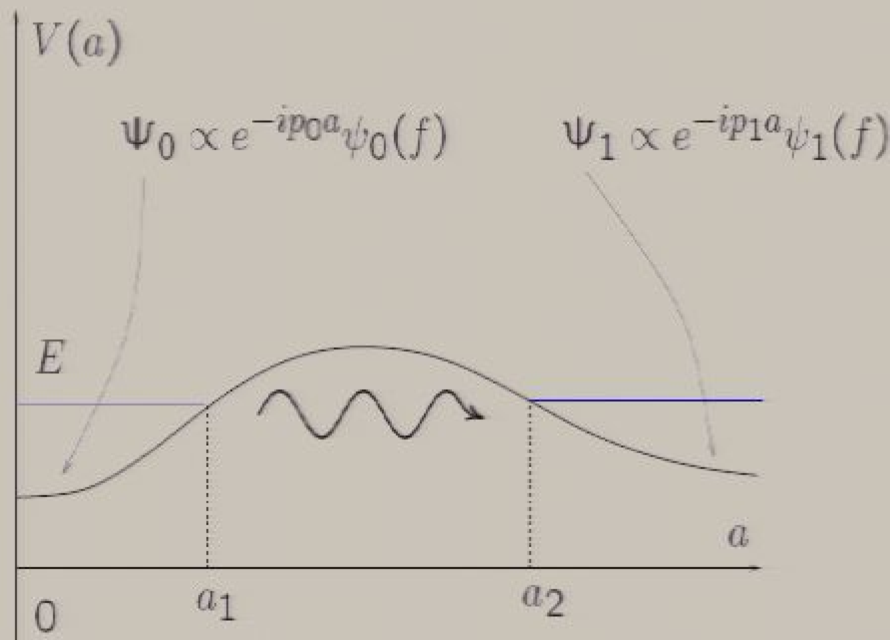
Analogy with quantum mechanics

A harmonic oscillator $f(t)$ at fixed energy E tunnels in direction $a(t)$:



$$\left[-\frac{\partial^2}{\partial a^2} + V(a) - E - \frac{\partial^2}{\partial f^2} + \omega^2(a) f^2\right]\Psi(a, f) = 0$$

How to determine the final state $\psi_1(f)$, given the initial state $\psi_0(f)$?

- Answer: $\psi_1(f)$ is almost equal to the ground state of the oscillator

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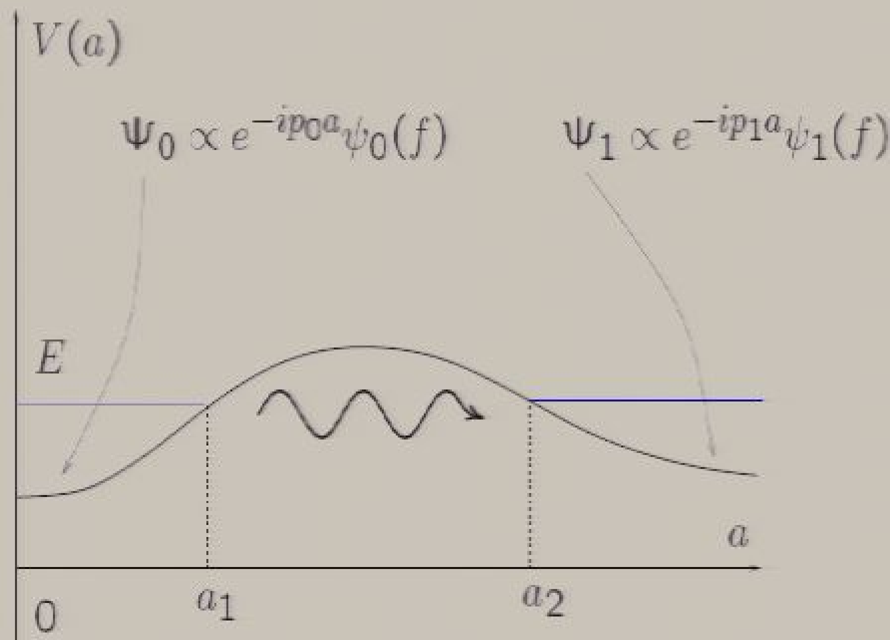

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Method: Squeezed-state/WKB approximation

Idea: Squeezed vacuum state w.r.t. f, semiclassical motion w.r.t. a

Ansatz: [Banks, Bender, Wu 1973]

$$\Psi(a,f) = \exp\left[-S_0(a) - \frac{1}{2}S_1(a)f^2\right]$$

- Neglect f^4, keep f^2, use WKB for a-dependence in $S_0(a)$, $S_1(a)$

$$\frac{dS_0^{\pm}}{da} = \pm\sqrt{V(a)-E}; \quad \frac{dS_1^{\pm}}{da} = \pm\frac{\omega^2(a) - S_1^2}{\sqrt{V(a)-E}}$$

- Solve for $S_0(a)$, $S_1(a)$ with appropriate boundary conditions

Obtain wave function:

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Interpretation: The state ψ_1 after tunneling is a squeezed vacuum state

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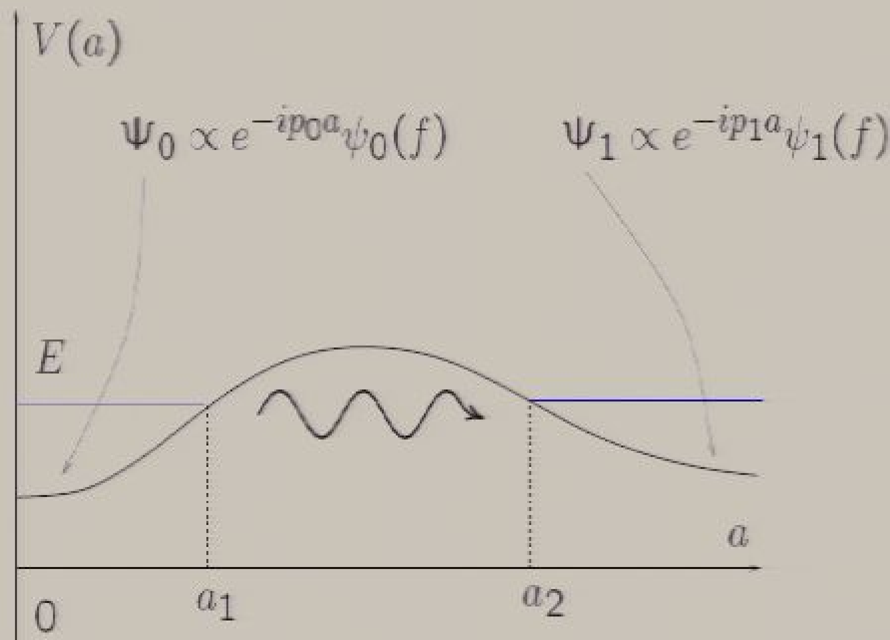

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- QFT in flat spacetime:

$$\hat{\phi}_n(t) = \hat{a}_n e^{i\omega_n t} + \hat{a}_n^\dagger e^{-i\omega_n t}$$

- QFT in curved spacetime: (see book by Mukhanov & S.W., 2007)

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Schrödinger picture: Gaussian wave function $\Psi(t, \{f_n\})$ = squeezed vacuum state

"Adiabatic vacuum state" can be defined with a certain precision [Winitzki 2005]

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Determining the initial quantum state

- Obtain $\Psi(a, \{f_n\})$ in the squeezed-state/WKB approximation:

$\Psi \propto \exp\left(-S_0(a) - \frac{1}{2}\sum_n S_n(a) f_n^2\right)$

- $\Rightarrow$ A single WKB branch of Ψ yields a certain squeezed state of fields

- Compare with adiabatic vacuum:

$$u_n^{(ad)} = \frac{1}{\sqrt{\omega_n(t)}} \exp\left[-i \int^t \omega_n(t) dt\right], \quad \omega_n^2 \equiv n^2 + m^2 a^2 \quad \Rightarrow S_n^{(ad)}$$

$\Rightarrow$ Can compute mean particle number w.r.t. adiabatic vacuum:

$$|\beta_n|^2 = \frac{\left|S_n - S_n^{(ad)}\right|^2}{4\,(\mathrm{Re}\, S_n)\left(\mathrm{Re}\, S_n^{(ad)}\right)}$$

Previous results

"Tunneling from nothing": Perturbations of metric and minimally coupled scalar field start out in vacuum state [Halliwell & Hawking 1985]

Further results:

- massless scalar field [Vachaspati & Vilenkin 1988]
- massive scalar field [Garriga & Vilenkin 1997]
- conformally coupled scalar field [Bouhmadi-López et al. 2002]

"Tunneling from something:" claims of infinite particle production during tunneling [Rubakov 1984; Levkov, Rebbi, Rubakov 2002]

- Contradiction or controversy?

Resolution of the controversy

[Hong, Vilenkin, Winitzki 2003]

Technical reason for "infinite" particle production: solutions $S_n(a)$ reach Re $S_n(a) < 0$ for sufficiently wide barriers

- Explanation: Breakdown of the squeezed-state/WKB approximation

Interpretation of Ψ is based on choosing a *semiclassical* state of gravity

Choice of $S_0(a)$, $S_n(a)$ is due to boundary conditions at $a = 0$, $a = \infty$

A non-semiclassical state: superposition of different $S_n(a)$ — looks like a state with infinitely many particles even in case $m = 0$ where particle production is absent! [explicit calculations]

There exists a consistent choice of semiclassical state of gravity such that all approximations are valid, Re $S_n > 0$, and fields are in near-vacuum state [explicit calculations]

Conclusions

Quantum cosmology promises to explain the birth of the universe and to provide initial conditions for inflation

Well-defined procedure leads from a wave function of the universe to interpretation through QFT in curved *semiclassical* spacetime

A consistent semiclassical state of gravity describes an expanding universe created by tunneling with quantum fields in near-vacuum state

Toy models give intuitively sensible answers; more detailed picture to be constructed (need more detailed theory of quantum gravity)

- Use loop quantum gravity?

- Quantize more degrees of freedom in GR (but still not all)?

Quantum cosmology with inhomogeneous matter

Conformally coupled scalar field ϕ:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{12}R\phi^2\right]$$

Expand $\phi(x,t)$ in spherical harmonics $Q_n(x)$, where $x \in S^3$,

$$\phi(x,t) = \frac{1}{a(t)}\sum_n f_n(t)Q_n(x)$$

Wheeler-DeWitt equation for $\Psi(a,\{f_n\})$:

$$\left[\frac{\partial^2}{\partial a^2} - V(a) + E + \sum_n\left(-\frac{\partial^2}{\partial f_n^2} + \left(n^2 + m^2a^2\right)f_n^2\right)\right]\Psi = 0$$

Goal: to describe the quantum state of fields in the nucleated universe in terms of QFT in (semiclassical) curved spacetime

Previous results

"Tunneling from nothing": Perturbations of metric and minimally coupled scalar field start out in vacuum state [Halliwell & Hawking 1985]

Further results:

- massless scalar field [Vachaspati & Vilenkin 1988]
- massive scalar field [Garriga & Vilenkin 1997]
- conformally coupled scalar field [Bouhmadi-López et al. 2002]

"Tunneling from something:" claims of infinite particle production during tunneling [Rubakov 1984; Levkov, Rebbi, Rubakov 2002]

- Contradiction or controversy?

Conclusions

Quantum cosmology promises to explain the birth of the universe and to provide initial conditions for inflation

Well-defined procedure leads from a wave function of the universe to interpretation through QFT in curved *semiclassical* spacetime

A consistent semiclassical state of gravity describes an expanding universe created by tunneling with quantum fields in near-vacuum state

Toy models give intuitively sensible answers; more detailed picture to be constructed (need more detailed theory of quantum gravity)

- Use loop quantum gravity?

- Quantize more degrees of freedom in GR (but still not all)?