Title: Graduate Course on Standard Model & Quantum Field Theory - 10B

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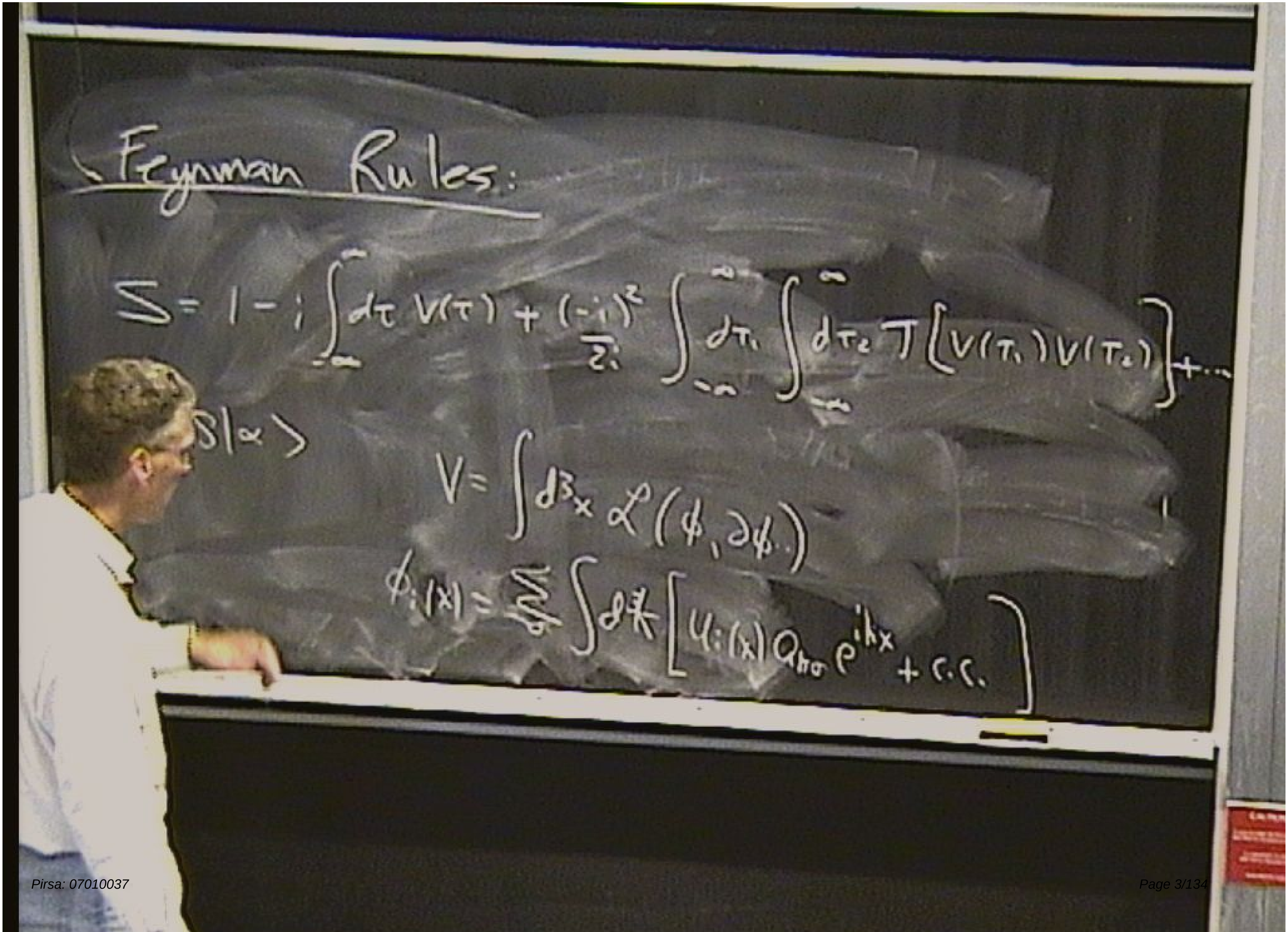
URL: http://pirsa.org/07010037

Abstract: Graduate Course on Standard Model & Quantum Field Theory

Feynman Rules:

$$S = 1 - i\int_{-\infty}^{\infty} d\tau \, V(\tau) + \frac{(-i)^2}{2!}\int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \, T\left[V(\tau_1)V(\tau_2)\right] + \cdots$$


Feynman Rules:
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$S|\alpha\rangle$
$V = \int d^3x\, \mathcal{L}(\phi, \partial\phi)$
$\phi_i(x) = \sum_\sigma \int d^3k \left[u_i(x)\, a_{k\sigma}\, e^{ikx} + c.c. \right]$

Feynman Rules:

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$$\langle \beta | S | \alpha \rangle$$

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Feynman Rules:

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$$V = \int d^3x\, \mathcal{L}(\phi, \partial\phi ..)$$

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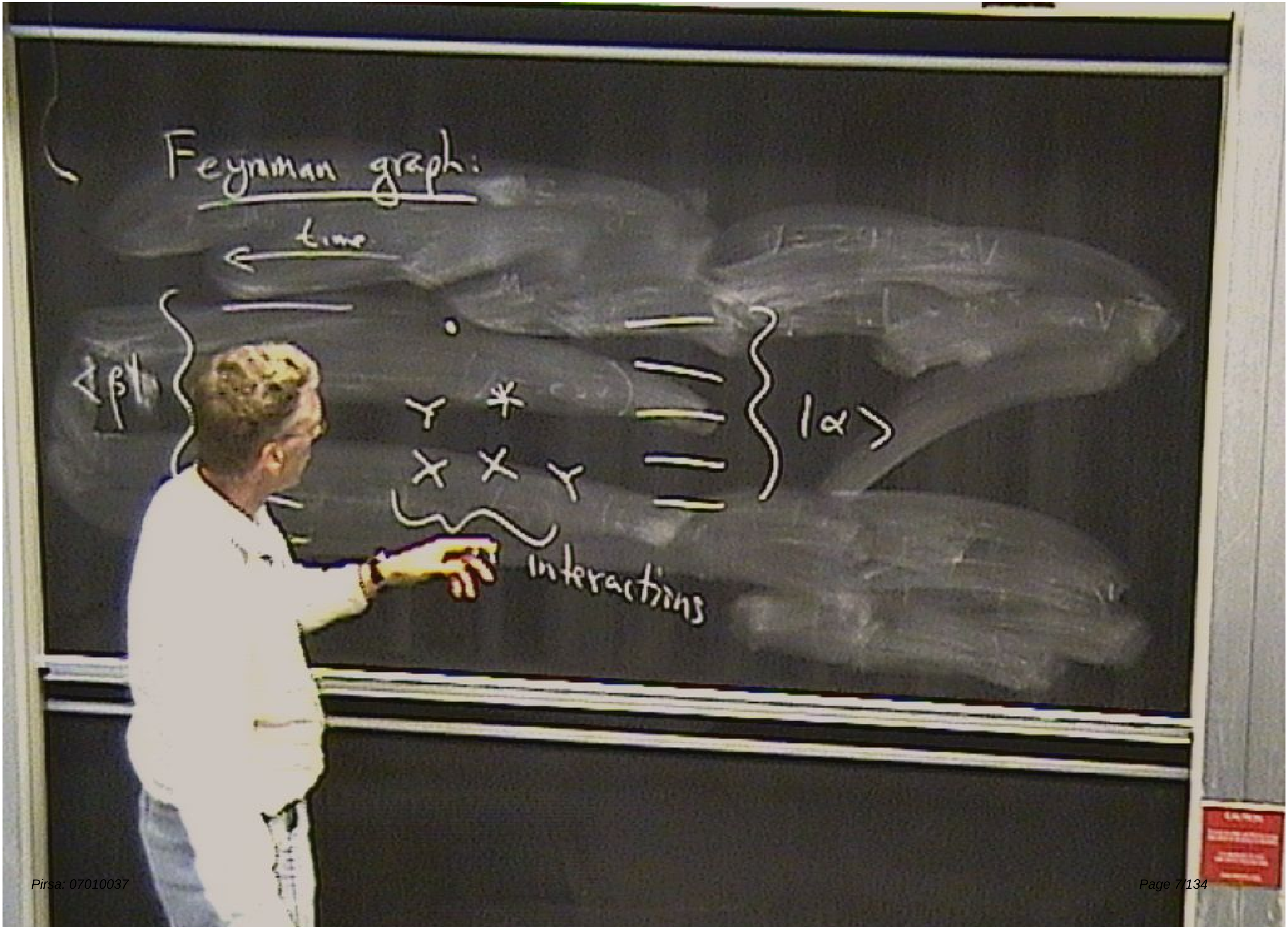
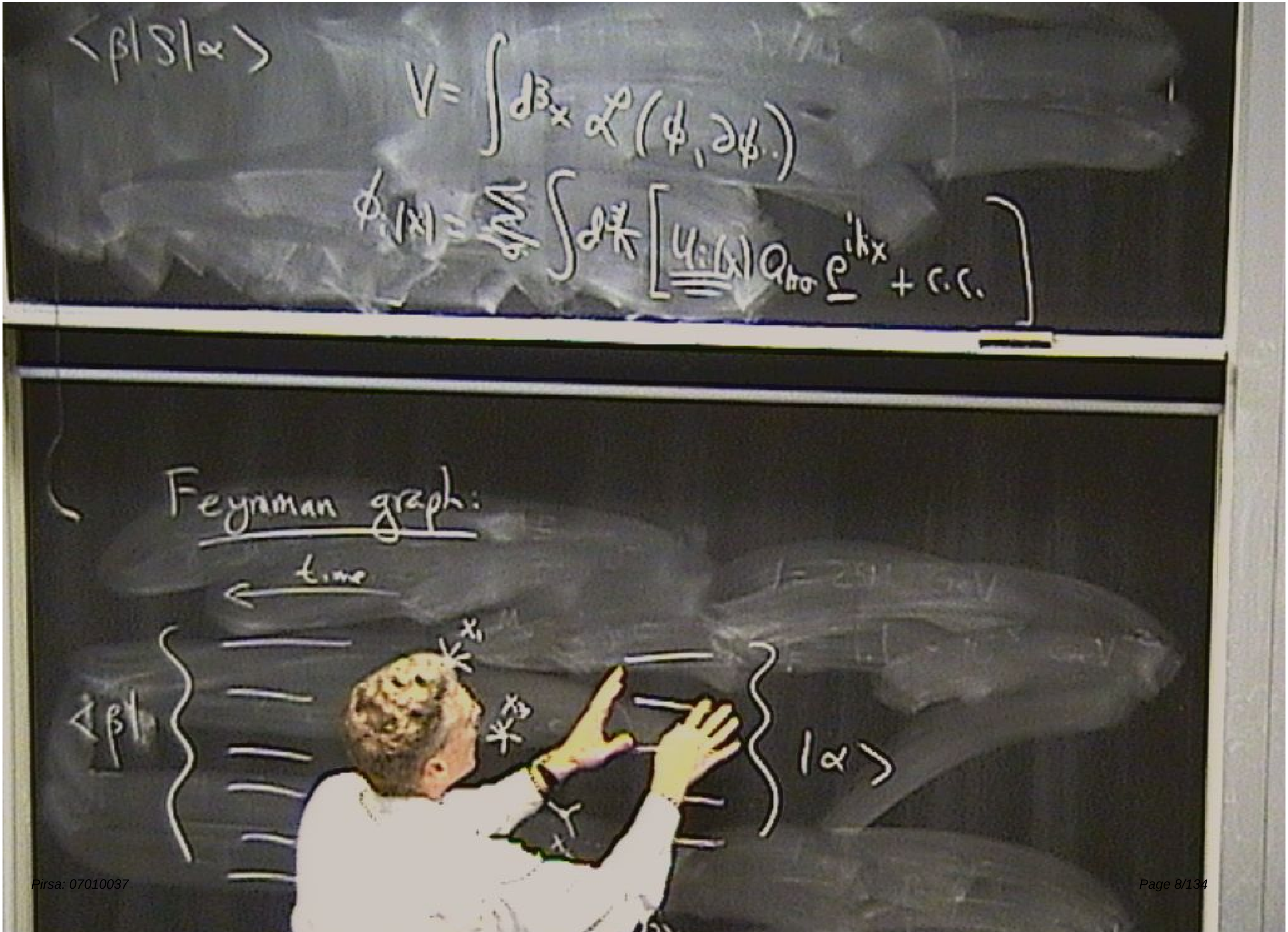
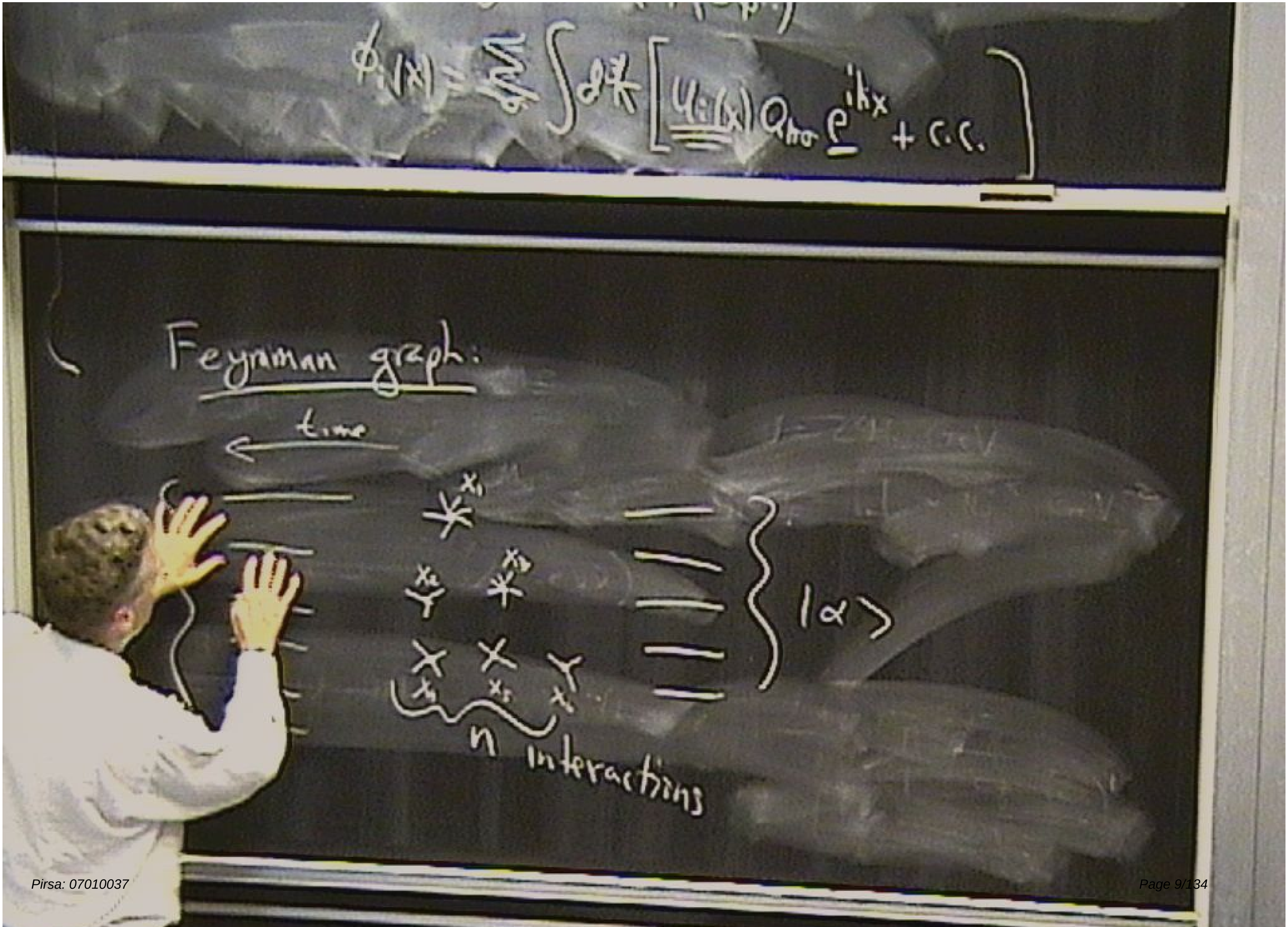
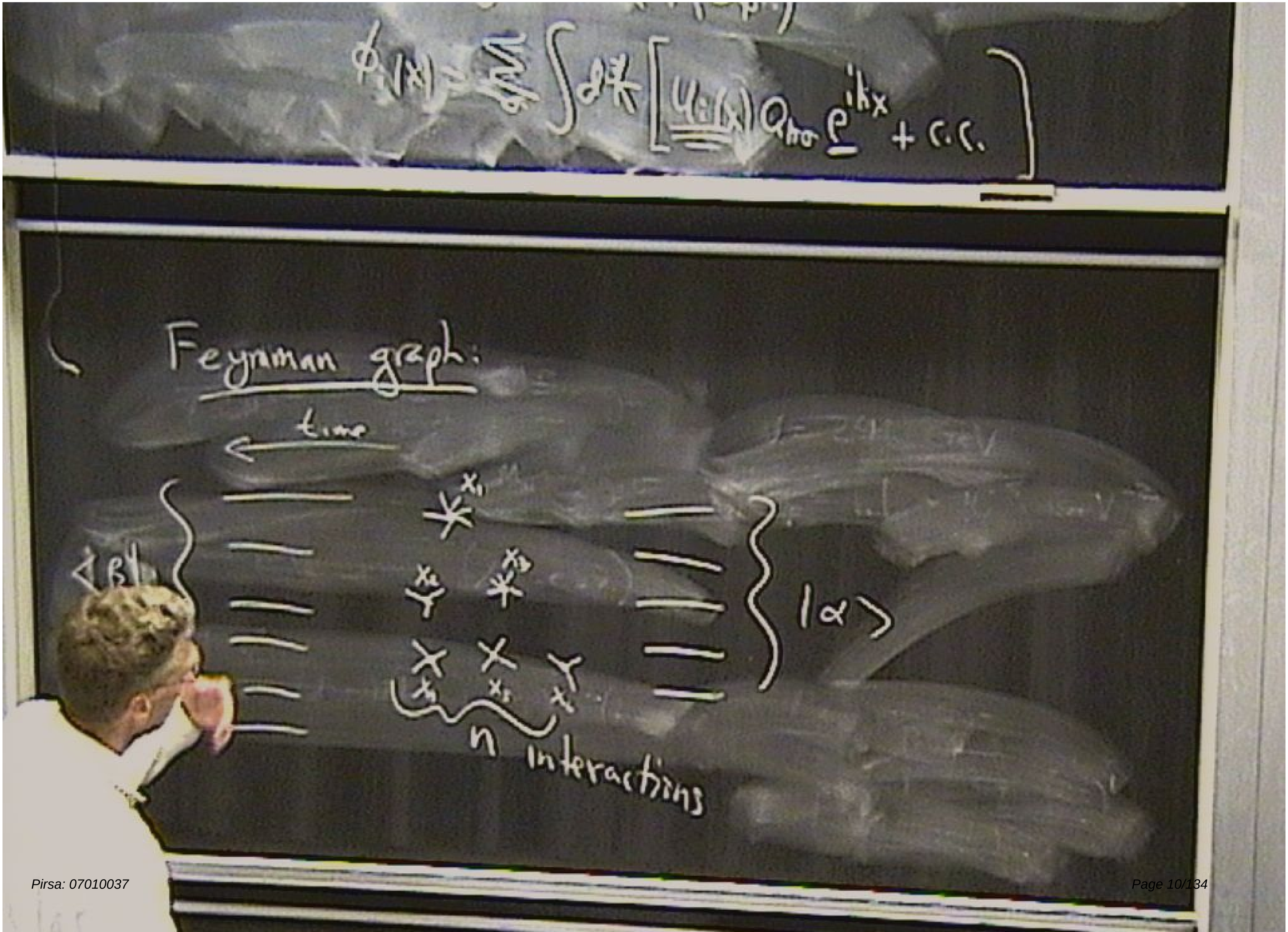
Feynman graph:


Feynman graph:
time
⟨β|
|α⟩
interactions

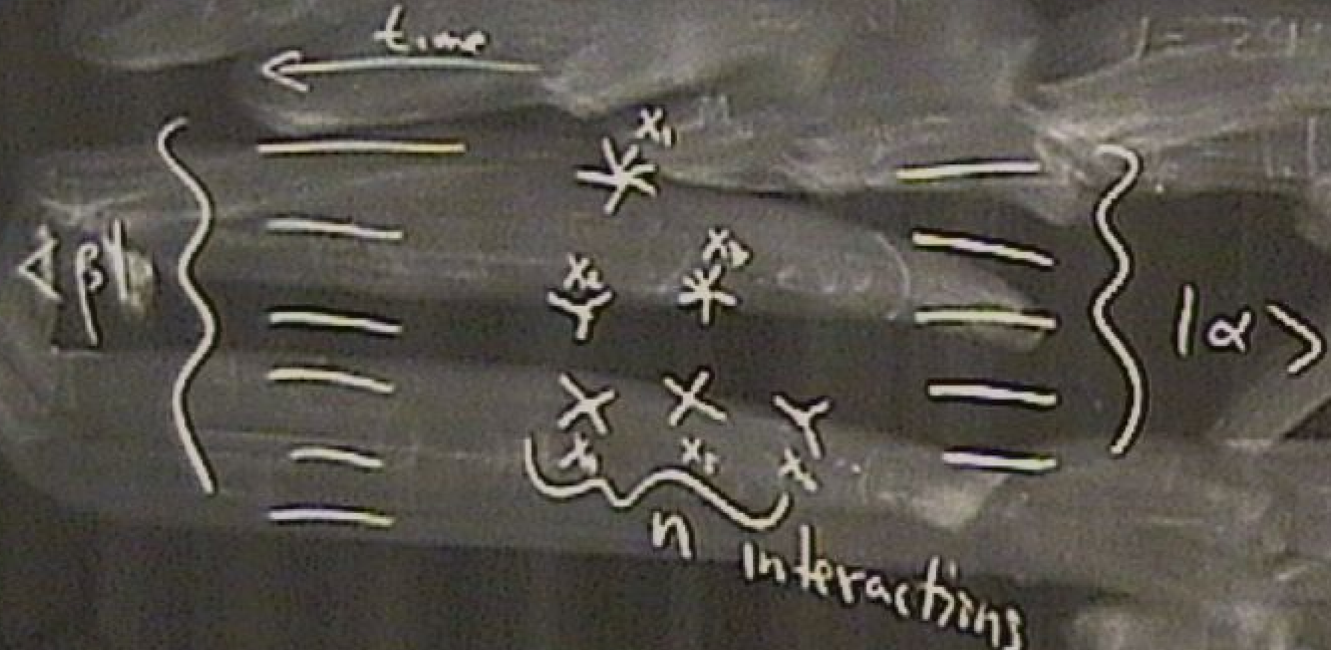

⟨β|S|α⟩
V = ∫d³x ℒ(φ, ∂φ..)
+ c.c.
Feynman graph:
time
x₁
x₃
⟨β|
|α⟩


Feynman graph:
time
$|\alpha\rangle$
n interactions


+ c.c.
Feynman graph:
time
⟨β|
|α⟩
n interactions

$$\phi(x) = \sum_{n} \int d^3k \left[u_{n\underline{k}}(x)\, a_{n\underline{k}}\, e^{i k x} + c.c. \right]$$

Feynman graph:



$$S = 1 - i\int_{-\infty}^{\infty} d\tau\, V(\tau) + \frac{(-i)^2}{2!}\int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2\, T\left[V(\tau_1)V(\tau_2)\right] + \cdots$$

$$\langle\beta|S|\alpha\rangle$$

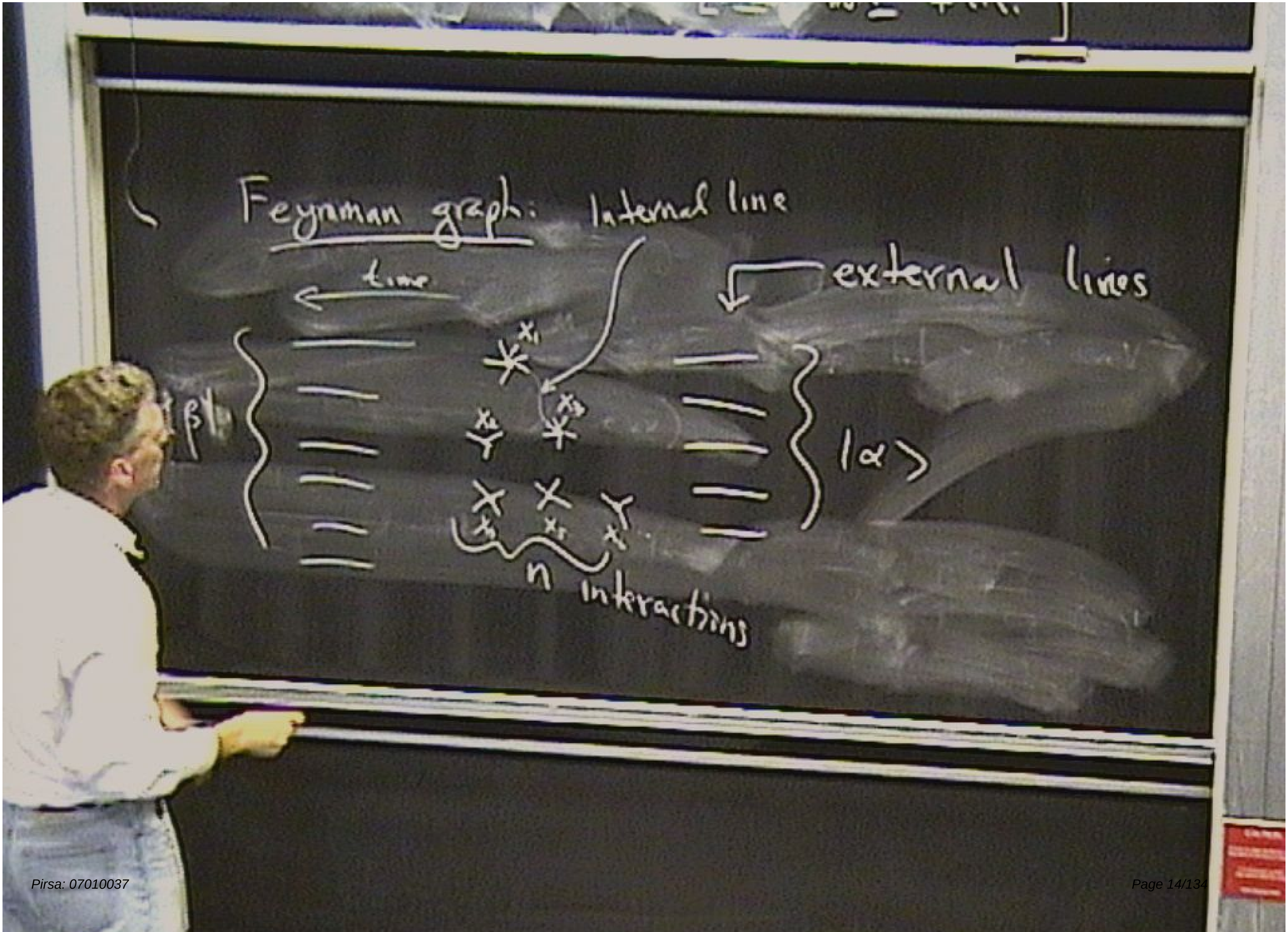
$$V = \int d^3x\, \mathcal{L}(\phi, \partial\phi \ldots)$$

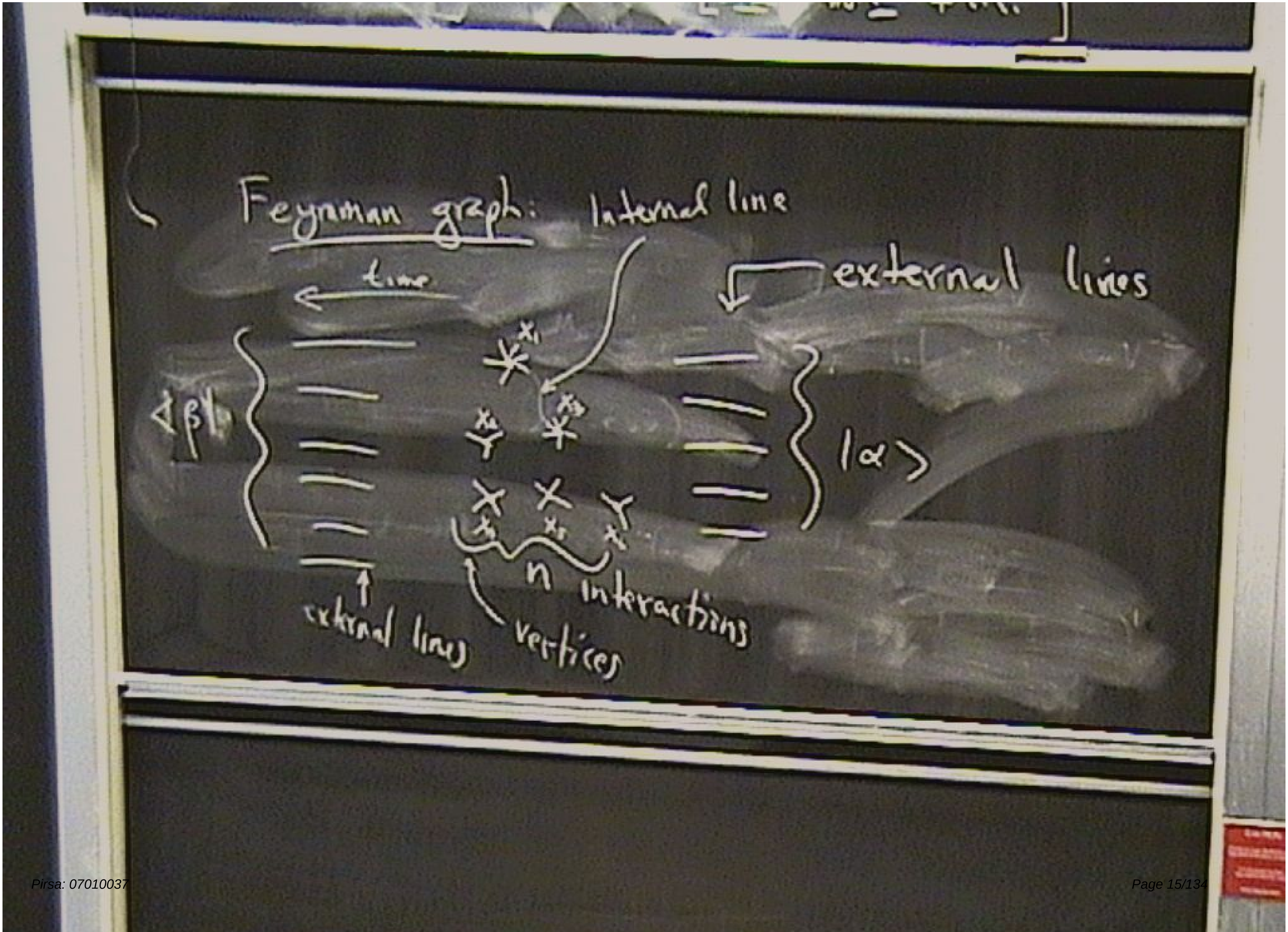
$$\phi(x) = \sum_{\sigma}\int d^3k \left[u_\sigma(k)\, a_{k\sigma}\, e^{ikx} + c.c. \right]$$

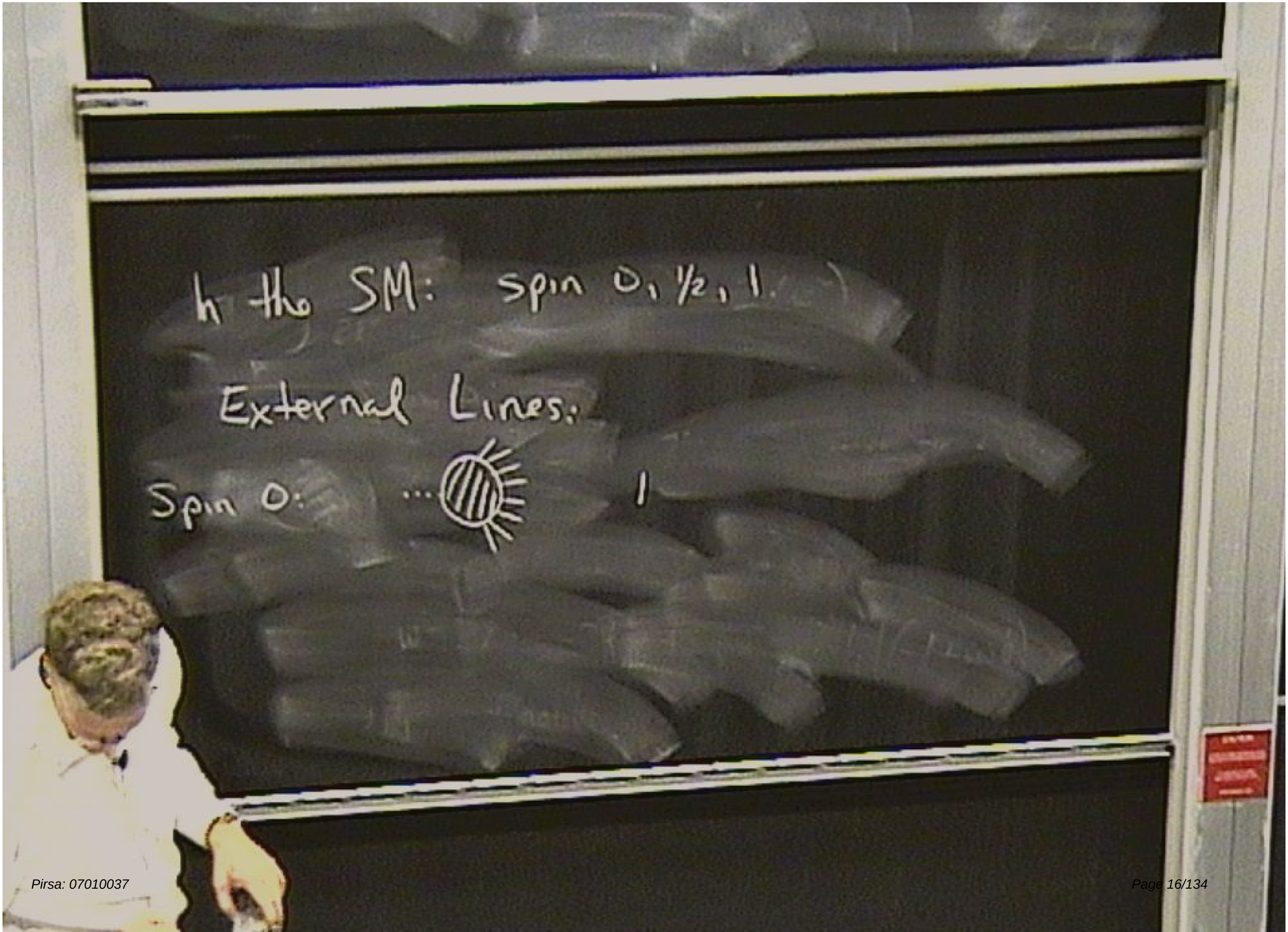
Feynman graph:

In the SM: spin 0, $\frac{1}{2}$, 1.

$\xrightarrow{p}$ spin 0


Feynman graph:
Internal line
time
external lines
$\langle\beta|$
$|\alpha\rangle$
n interactions

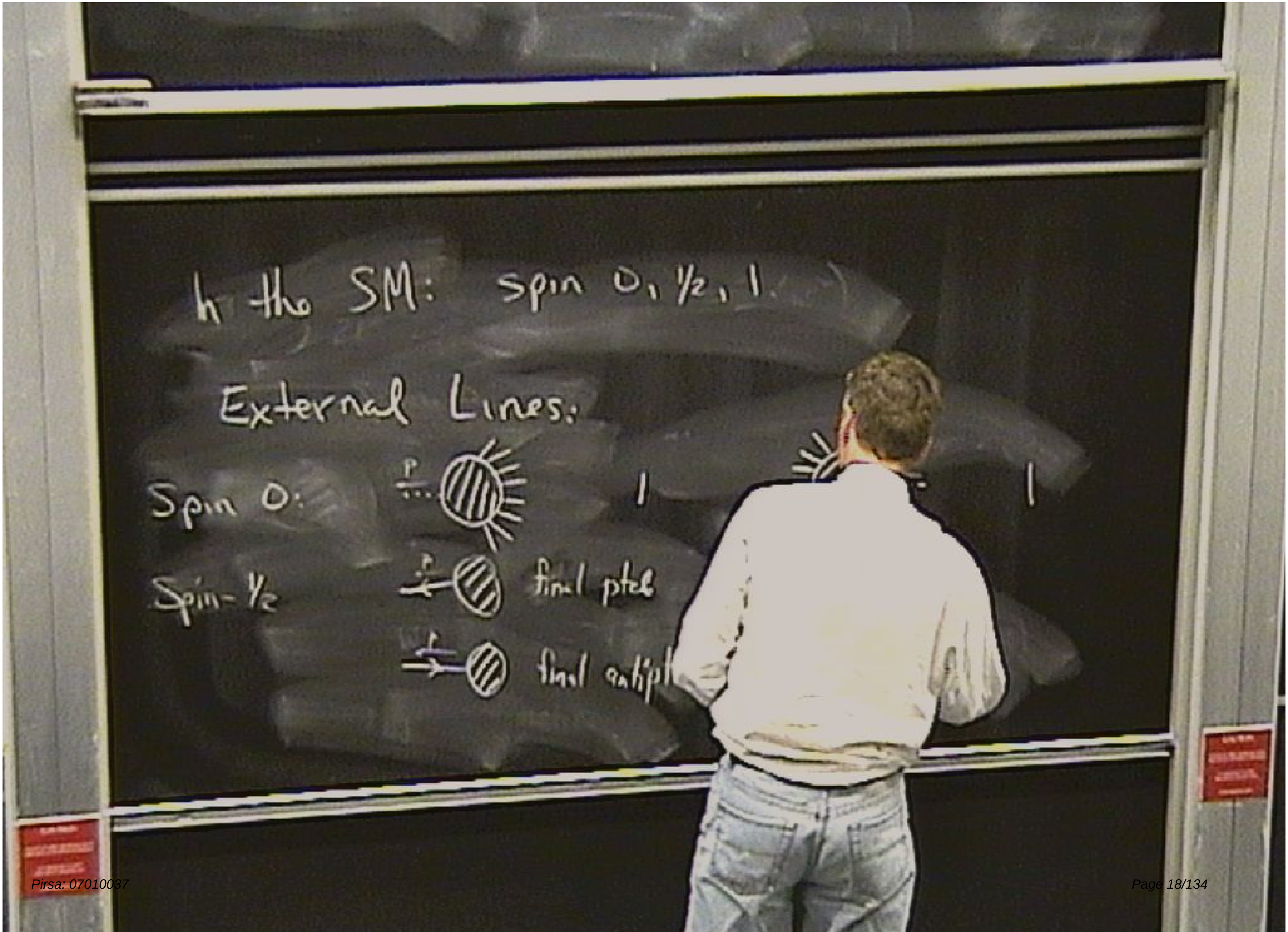

Feynman graph:
Internal line
time
external lines
⟨β|
|α⟩
n interactions
vertices
external lines


In the SM: spin 0, ½, 1.
External Lines:
Spin 0: ... 1

In the SM: spin 0, $\frac{1}{2}$, 1.

External Lines:

Spin 0:  1  1


In the SM: spin 0, 1/2, 1.
External Lines:
Spin 0:
p
1
1
Spin-1/2
p
final ptcl
final

In the SM: spin 0, ½, 1.

External Lines:

Spin 0: $\overset{p}{\cdots}$ 1 $\overset{p}{\cdots}$ 1

Spin-½ $\overset{p}{\rightarrow}$ final ptcl | initial ptcl $\overset{p}{\leftarrow}$

$\overset{p}{\leftarrow}$ final antiptcl | anti- $\overset{p}{\rightarrow}$

Feynman

$$S = 1 - i\int_{-\infty}^{\infty} d\tau\, V(\tau) + \frac{(-i)^2}{2!}\int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2\, T\left[V(\tau_1)V(\tau_2)\right] + \cdots$$

$\langle \beta | S | \alpha \rangle$

$$V = \int d^3x\, \mathcal{L}(\phi, \partial\phi \ldots)$$

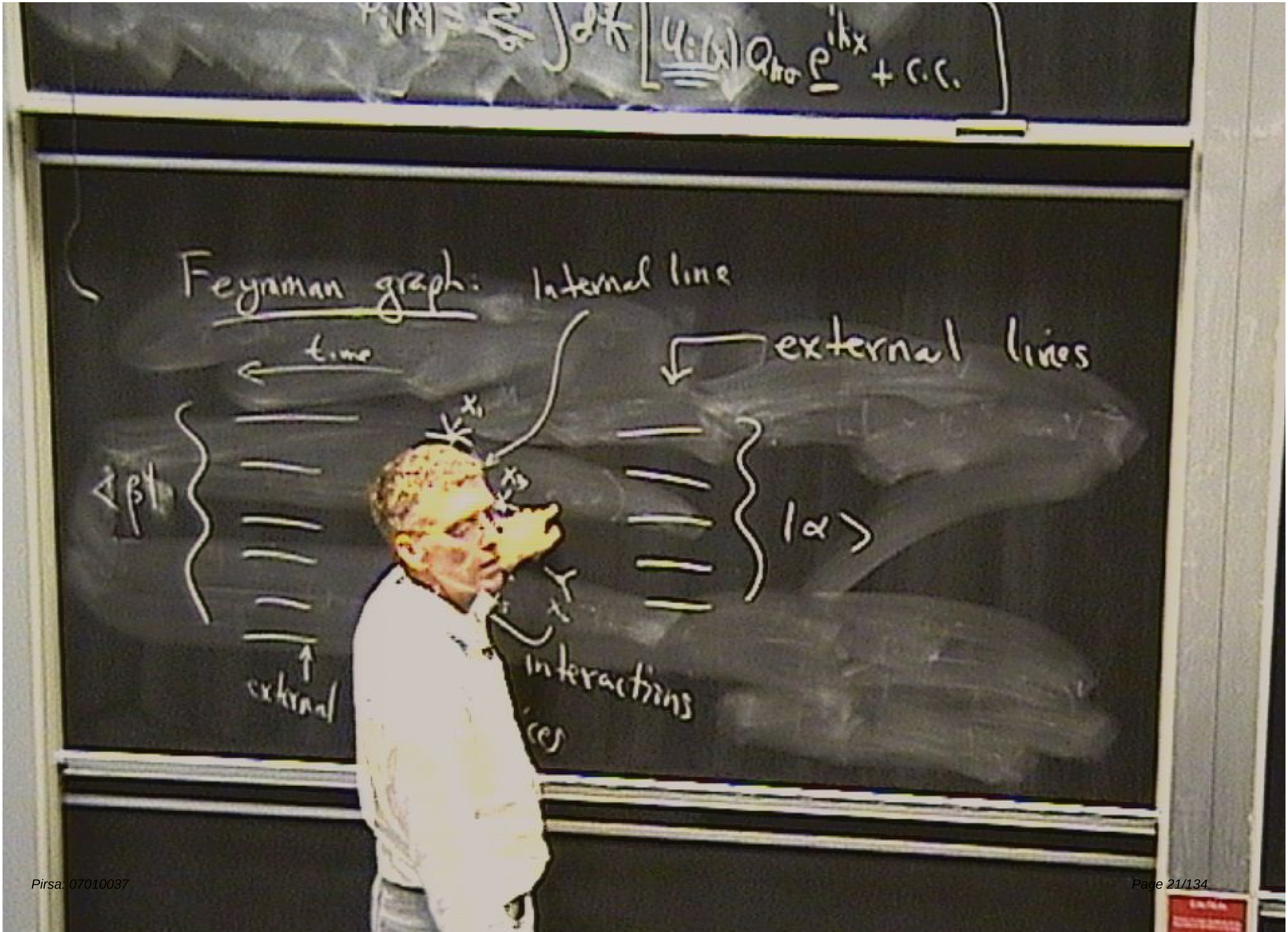
$$\phi_\ell(x) = \sum_\sigma \int d^3k \left[u_\ell(k)\, a_{k\sigma}\, e^{ikx} + \text{c.c.} \right]$$

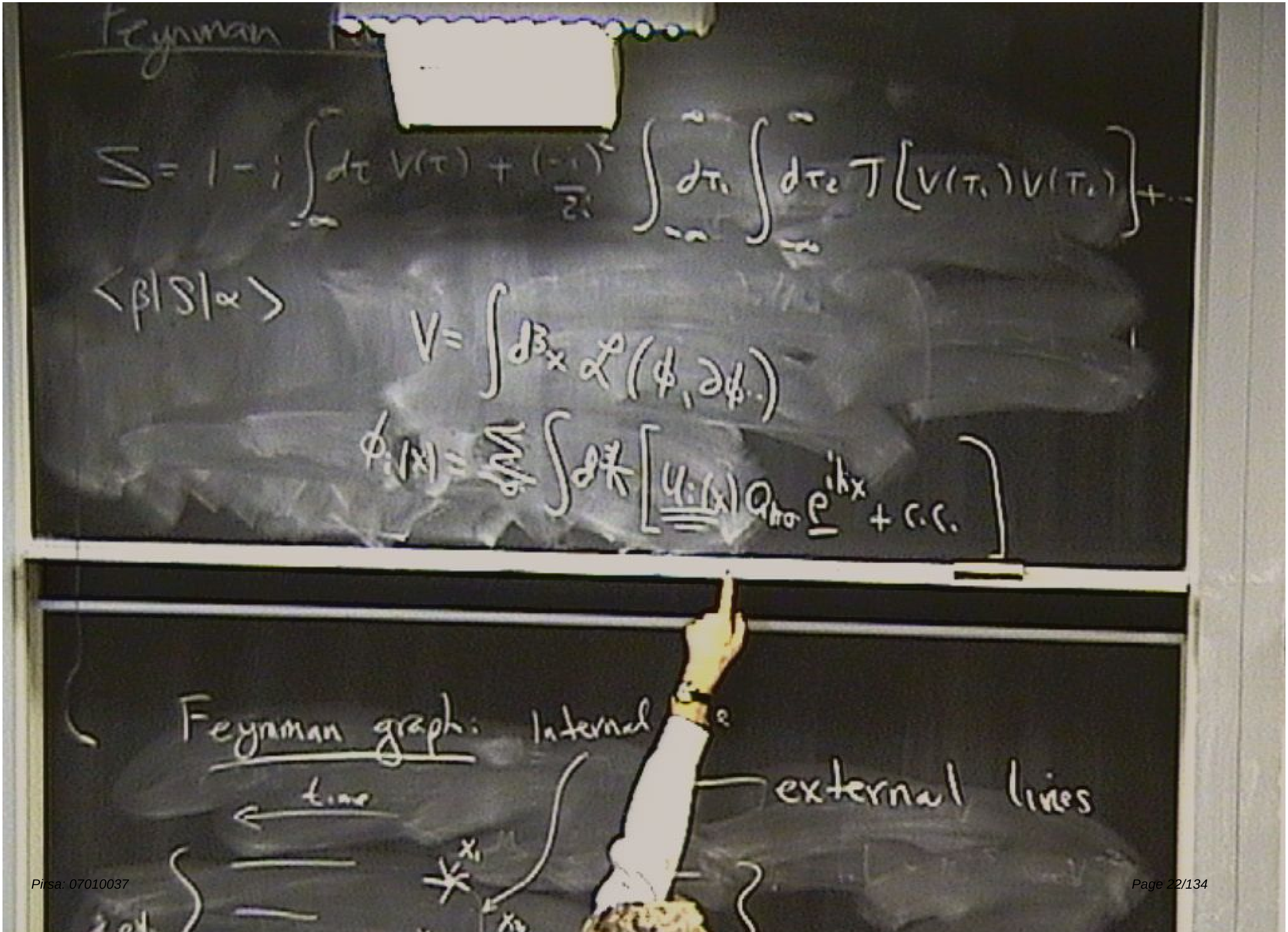
Feynman graph: internal line

time ←

external lines

x_1


+ c.c.
Feynman graph:
internal line
external lines
time
x_1
x_2
$\langle\beta|$
$|\alpha\rangle$
external
interactions


Feynman
S = 1 − i ∫ dτ V(τ) + (−i)²/2! ∫ dτ₁ ∫ dτ₂ T[V(τ₁)V(τ₂)] + ...
⟨β|S|α⟩
V = ∫ d³x ℒ(φ, ∂φ..)
φᵢ(x) = Σ ∫ d³k [uᵢ(k) a e^{ikx} + c.c.]
Feynman graph: Internal
external lines
time

In the SM: spin 0, ½, 1.

External Lines:

Spin 0: p 1 p 1

Spin-½ p final ptcl | initial ptcl p u

p final antiptcl | anti... p

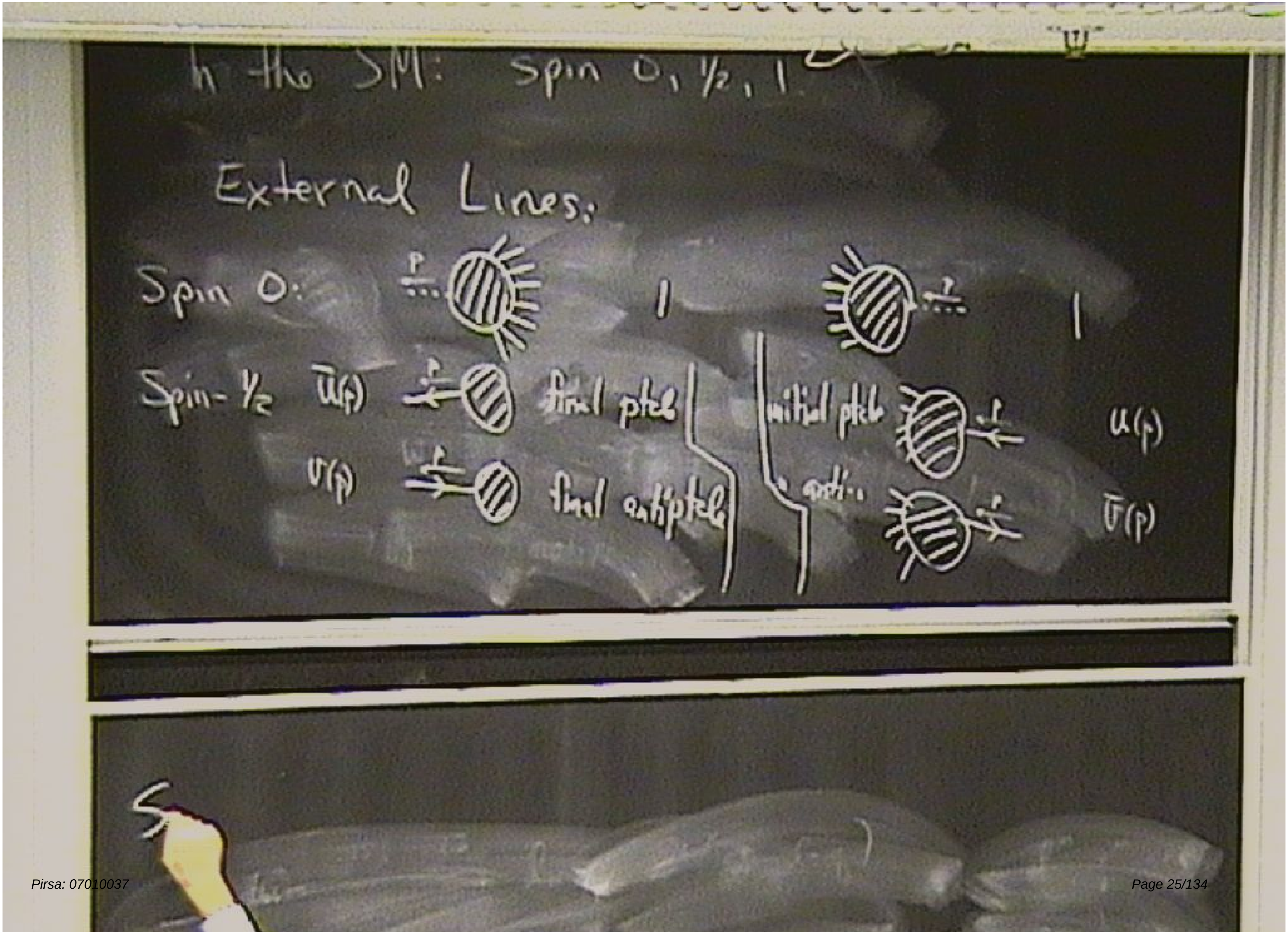
In the SM: spin 0, ½, 1.

External Lines:

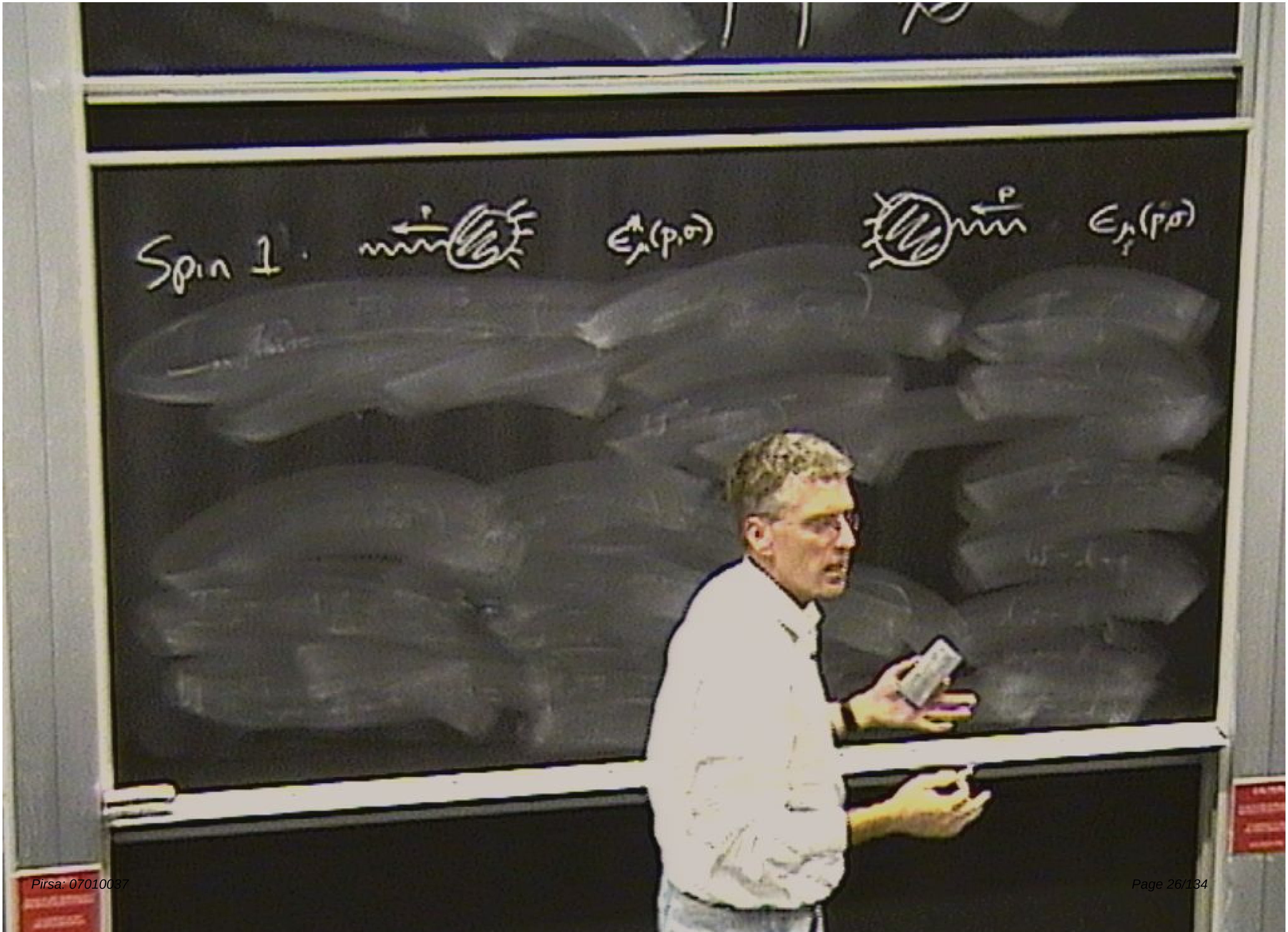
Spin 0: 1 1

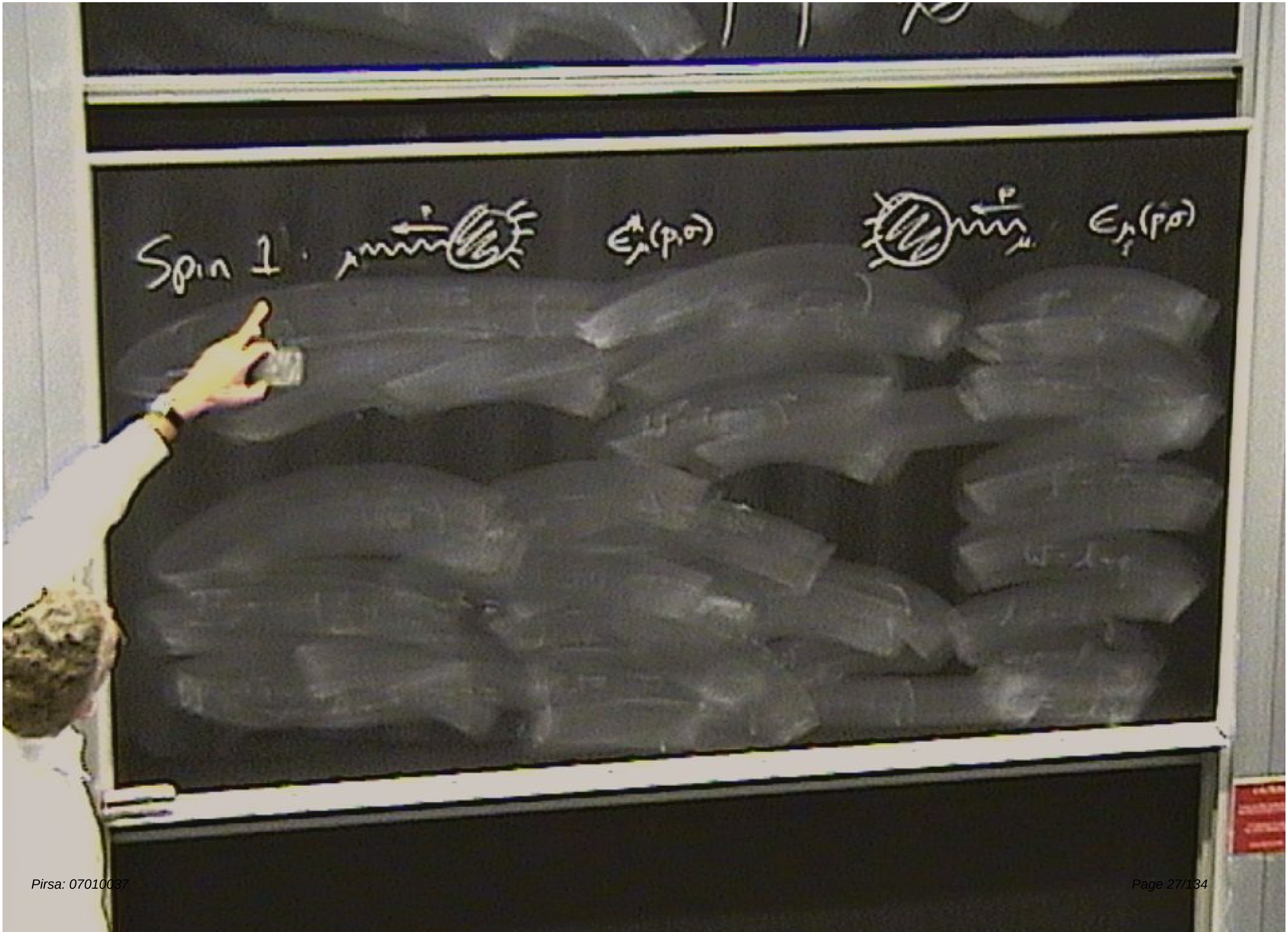
Spin-½ $\bar{u}(p)$ final ptcle | initial ptcle $u(p)$

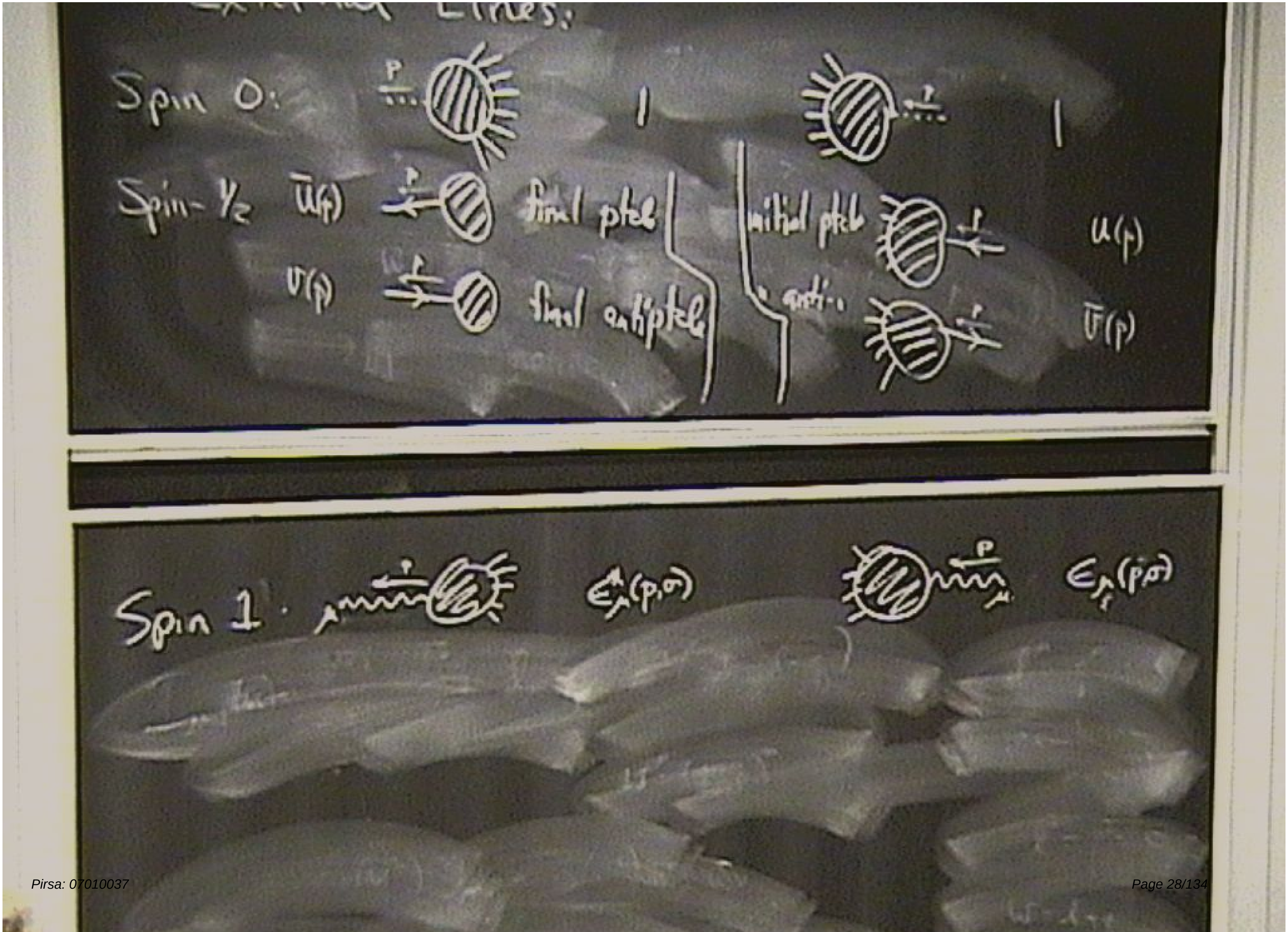
final antiptcle | anti..



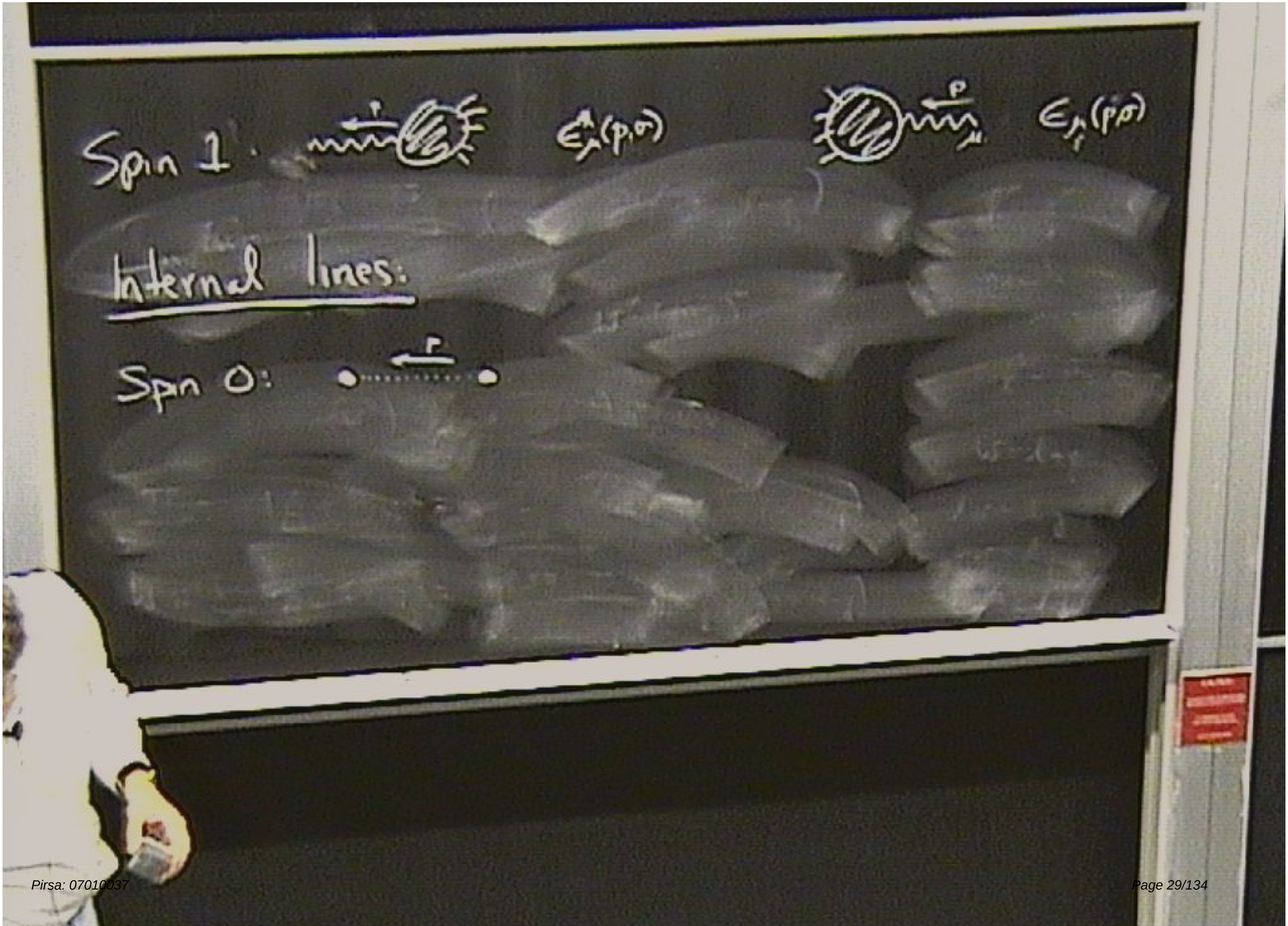
In the SM: Spin 0, 1/2, 1
External Lines:
Spin 0:
1
1
Spin-1/2
ū(p)
final ptcle
initial ptcle
u(p)
v(p)
final antiptcle
v̄(p)

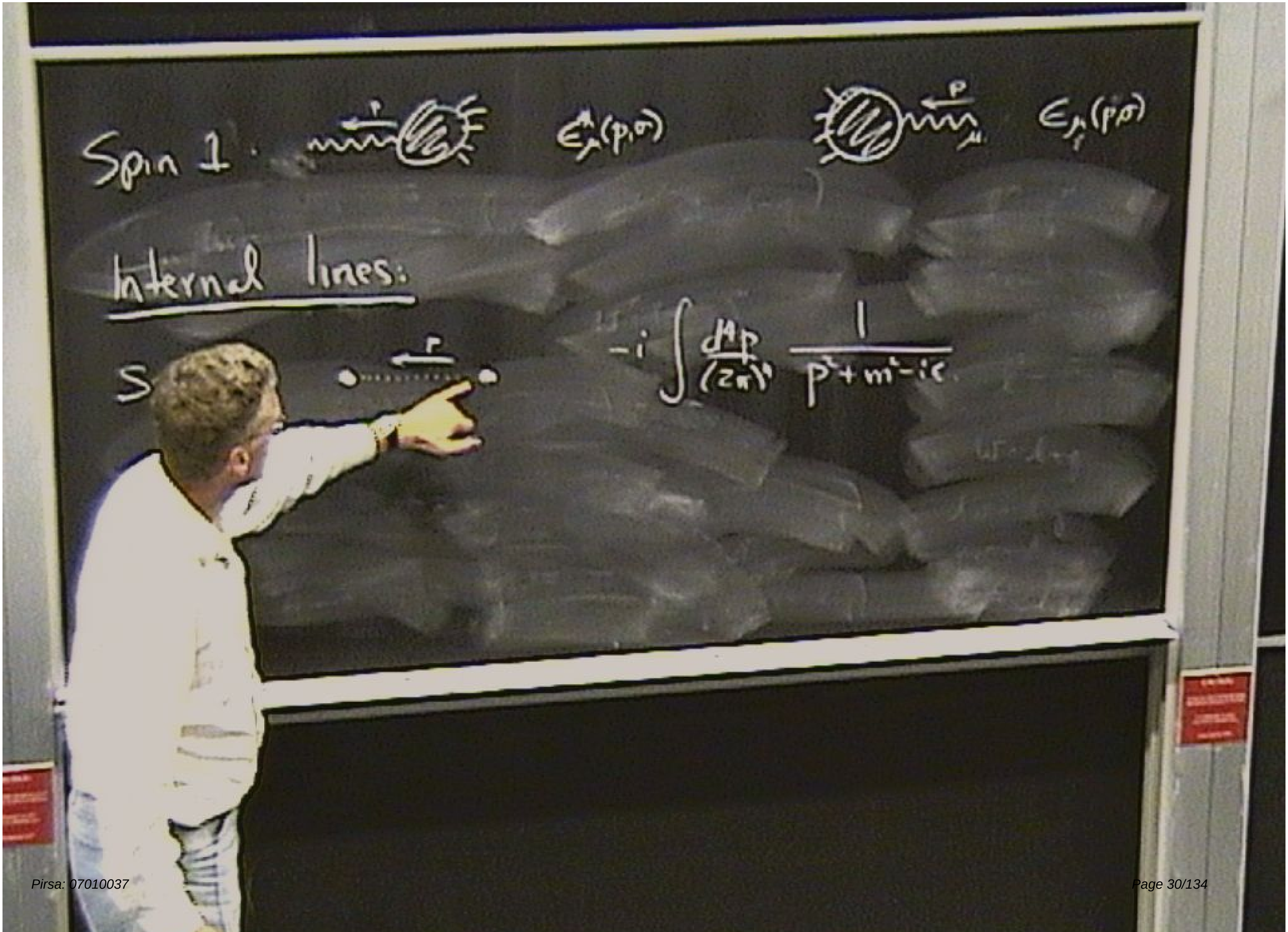

Spin 1 :
p
$\epsilon^*_\mu(p,\sigma)$
p
$\epsilon_\mu(p,\sigma)$


Spin 1
$\epsilon^*_\mu(p,\sigma)$
$\epsilon_\mu(p,\sigma)$



Lines:
Spin 0:
p
1
Spin-1/2
ū(p)
final ptcl
initial ptcl
u(p)
v(p)
final antiptcl
anti-
v̄(p)
Spin 1
ε*μ(p,σ)
εμ(p,σ)


Spin 1:
$\epsilon^*_\mu(p,\sigma)$
$\epsilon_\mu(p,\sigma)$
Internal lines:
Spin 0:

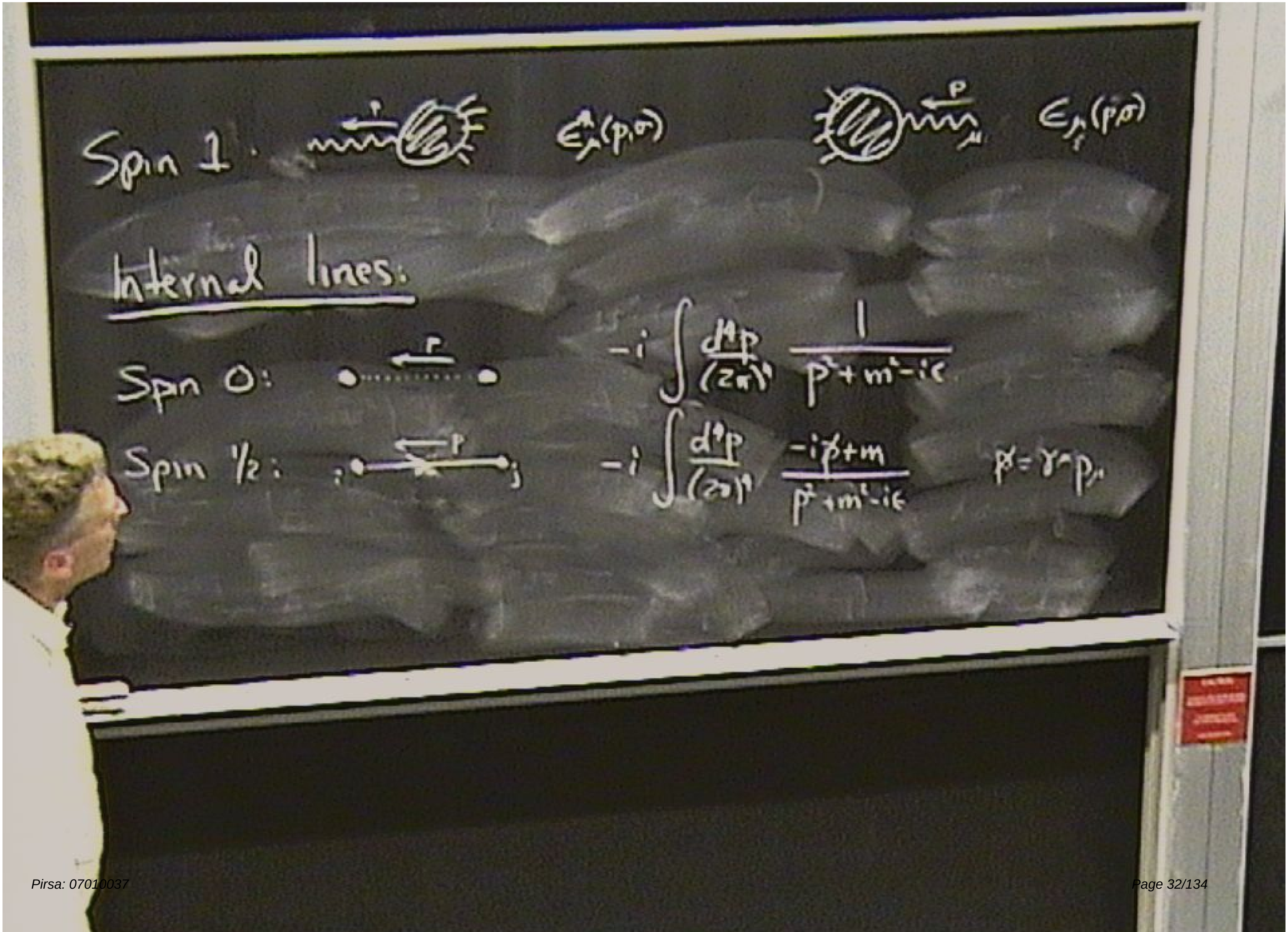

Spin 1
$\epsilon_\mu(p,\sigma)$
$\epsilon^*_\mu(p,\sigma)$
Internal lines:
S
$-i\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2-i\epsilon}$

Spin 1:  $\epsilon_\mu(p,\sigma)$  $\epsilon^*_\nu(p,\sigma)$

Internal lines:

Spin 0:  $-i\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2-i\epsilon}$

Spin 1/2: 



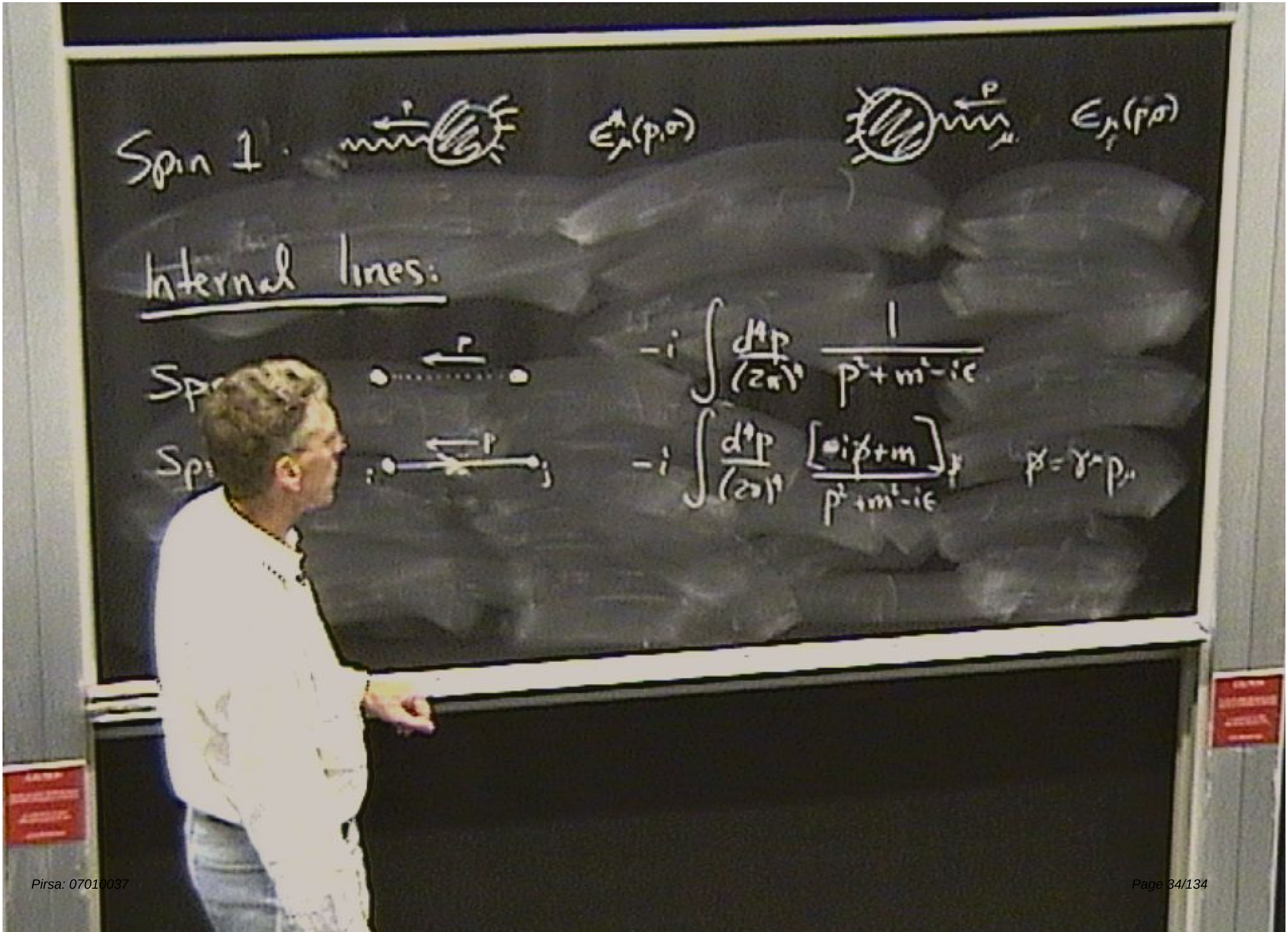
Spin 1: p $\epsilon_\mu(p,\sigma)$ p μ $\epsilon^*_\mu(p,\sigma)$
Internal lines:
Spin 0: p $-i\int \frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+m^2-i\epsilon}$
Spin 1/2: i p j $-i\int \frac{d^4p}{(2\pi)^4}\,\frac{-i\not{p}+m}{p^2+m^2-i\epsilon}$ $\not{p}=\gamma^\mu p_\mu$

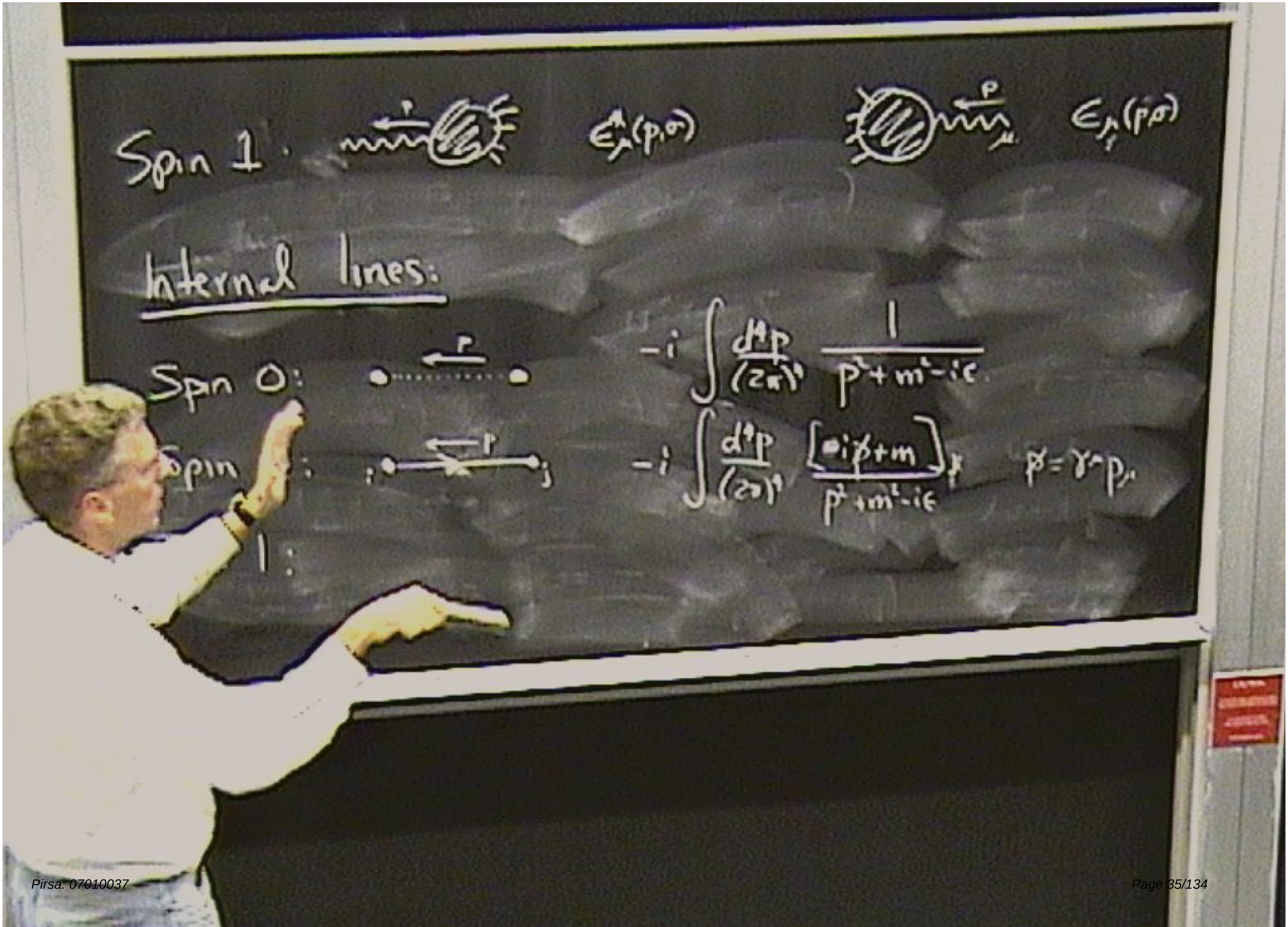
Spin 1: $\epsilon^*_\mu(p,\sigma)$ $\epsilon_\mu(p,\sigma)$

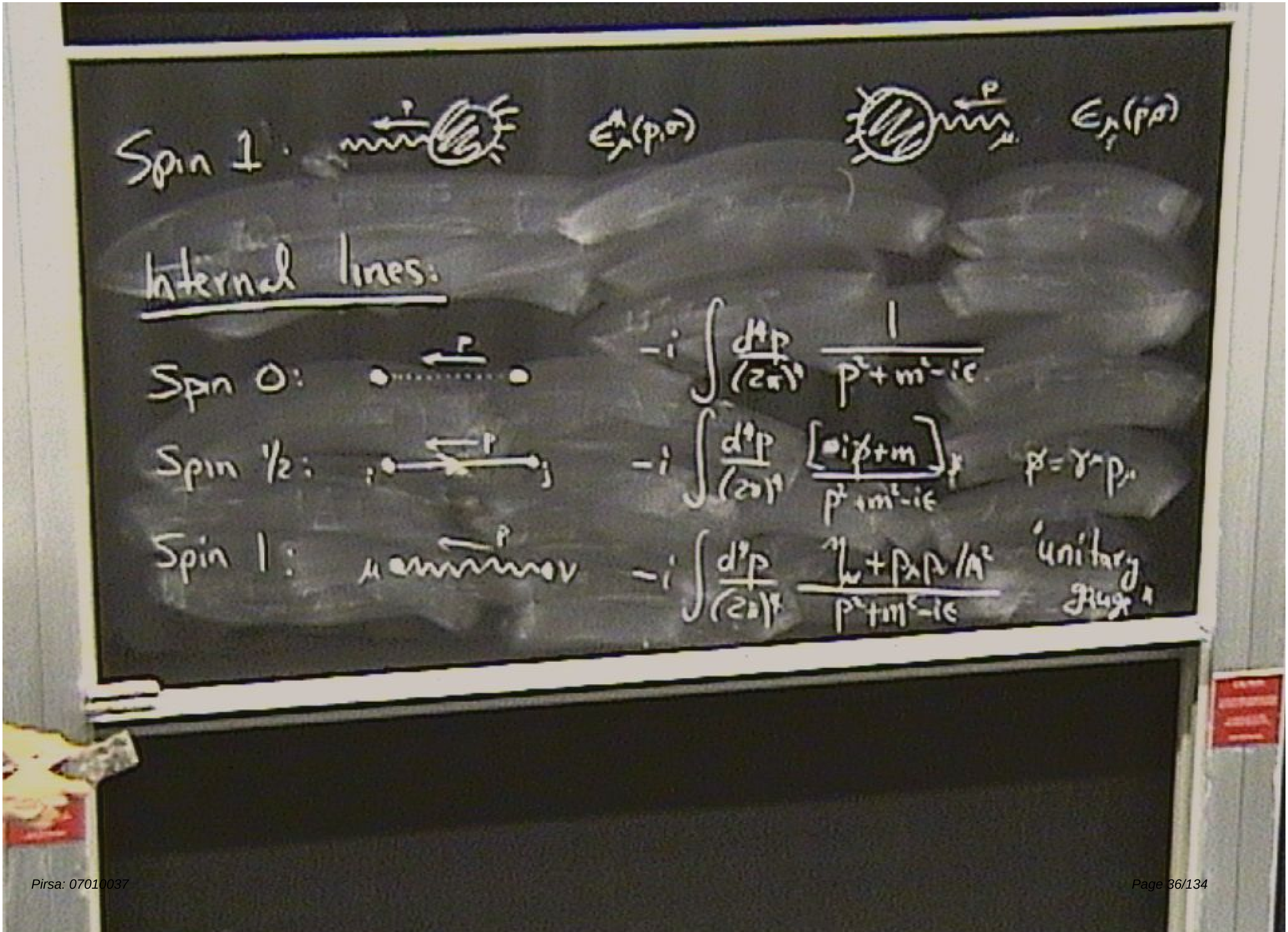
Internal lines:

Spin 0: $-i\int \frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+m^2-i\epsilon}$

Spin 1/2: $-i\int \frac{d^4p}{(2\pi)^4}\,\frac{-i\not{p}+m}{p^2+m^2-i\epsilon}$ $\not{p}=\gamma^\mu p_\mu$


Spin 1 · $\epsilon_\mu(p,\sigma)$
$\epsilon^*_\nu(p,\sigma)$
Internal lines:
Sp
p
$-i\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2-i\epsilon}$
Sp
i j
$-i\int \frac{d^4p}{(2\pi)^4}\frac{[-i\not p+m]_{ij}}{p^2+m^2-i\epsilon}$
$\not p=\gamma^\mu p_\mu$


Spin 1:
Internal lines:
Spin 0:
-i ∫ d⁴p/(2π)⁴ 1/(p² + m² − iε)
Spin
-i ∫ d⁴p/(2π)⁴ [−ip̸ + m]ᵢⱼ/(p² + m² − iε)
p̸ = γ^μ p_μ
1:


Spin 1: p $\epsilon^*_\mu(p,\sigma)$
p μ $\epsilon_\mu(p,\sigma)$
Internal lines:
Spin 0: p $-i\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m^2-i\epsilon}$
Spin 1/2: i p j $-i\int \frac{d^4p}{(2\pi)^4}\frac{[-i\not p+m]_{ij}}{p^2+m^2-i\epsilon}$
$\not p=\gamma^\mu p_\mu$
Spin 1: μ p ν $-i\int \frac{d^4p}{(2\pi)^4}\frac{\eta_{\mu\nu}+p_\mu p_\nu/m^2}{p^2+m^2-i\epsilon}$
"unitary gauge"

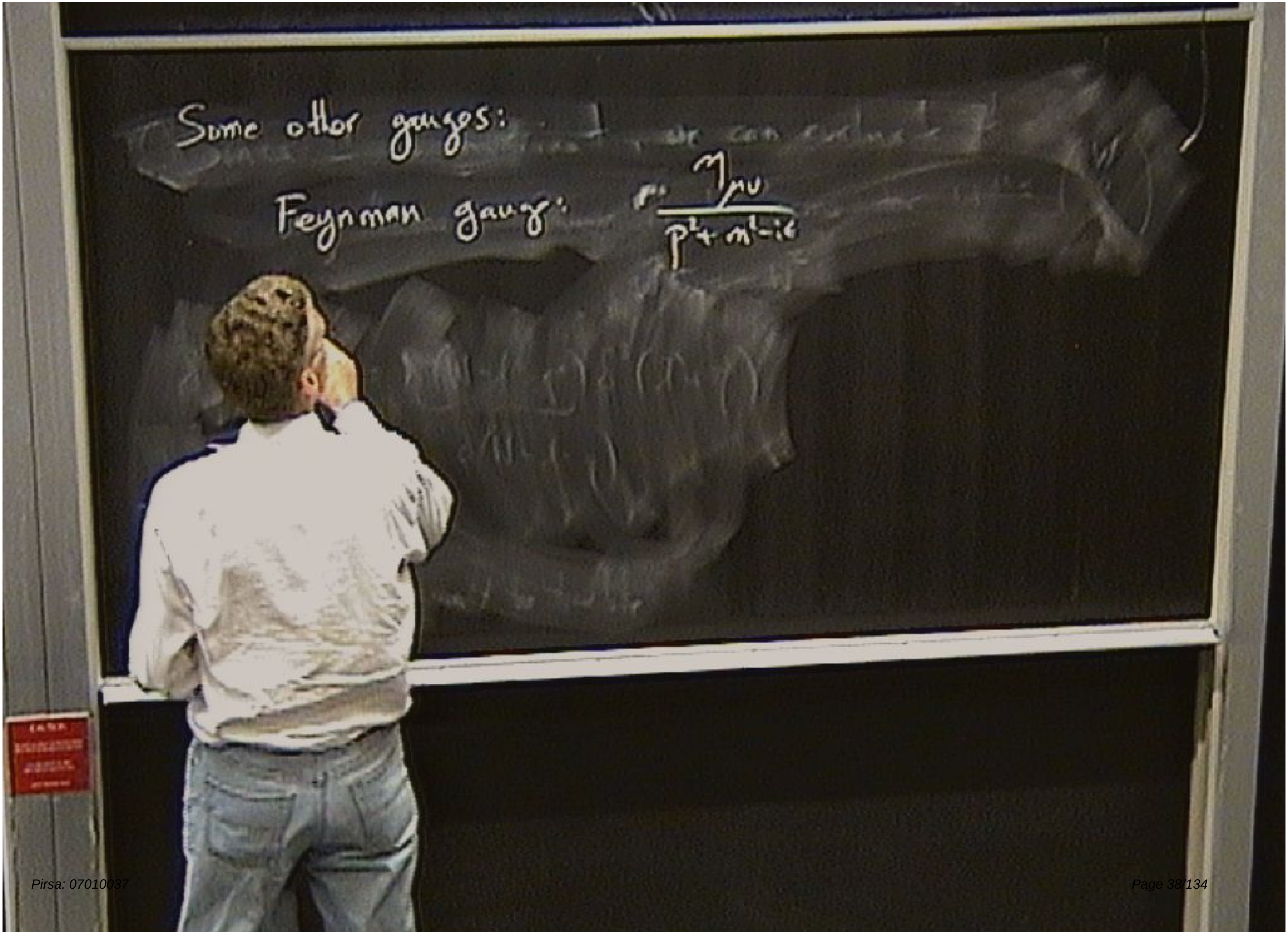
Spin 1: $\epsilon_\mu(p,\sigma)$ $\epsilon^*_\mu(p,\sigma)$

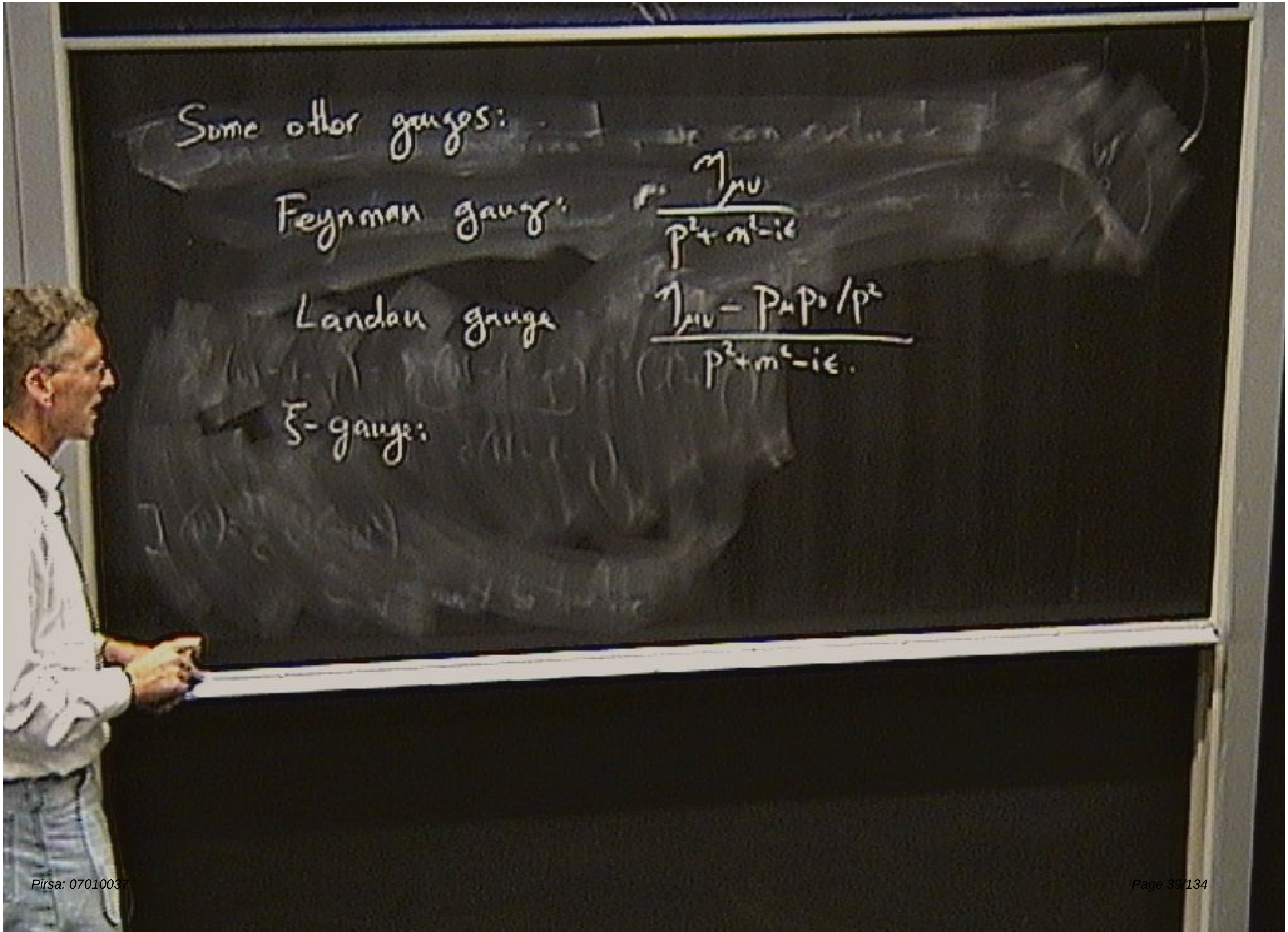
<u>Internal lines:</u>

Spin 0: $-i\int \frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+m^2-i\epsilon}$

Spin ½: $i \quad j$ $-i\int \frac{d^4p}{(2\pi)^4}\,\frac{\left[-i\not{p}+m\right]_{ij}}{p^2+m^2-i\epsilon}$ $\not{p}=\gamma^\mu p_\mu$

Spin 1: $\mu \quad \nu$ $-i\int \frac{d^4p}{(2\pi)^4}\,\frac{\eta_{\mu\nu}+p_\mu p_\nu/m^2}{p^2+m^2-i\epsilon}$ "unitary gauge"


Some other gauges:
Feynman gauge: $\frac{\eta_{\mu\nu}}{p^2+m^2-i\epsilon}$

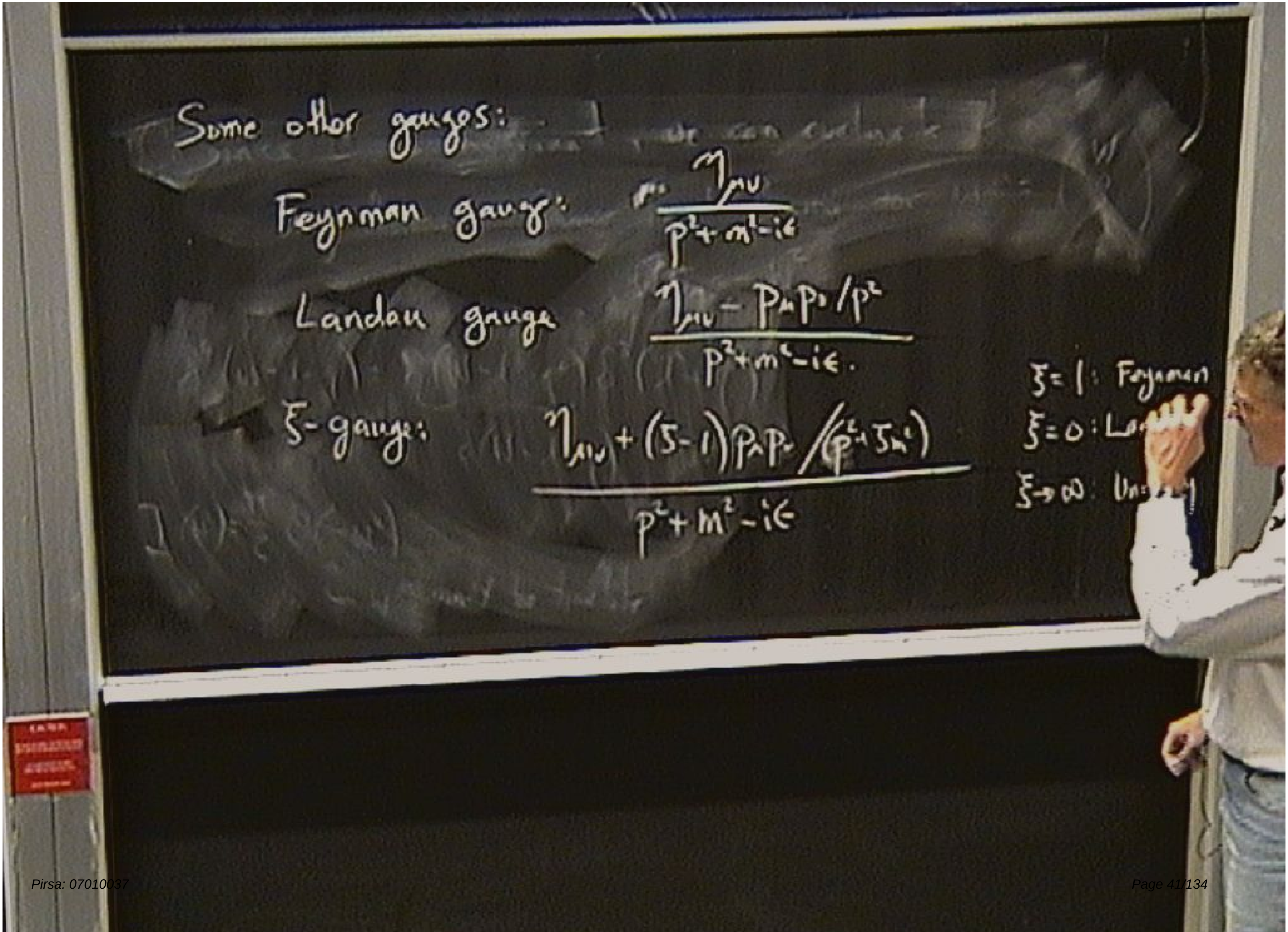

Some other gauges:
Feynman gauge: $\frac{\eta_{\mu\nu}}{p^2+m^2-i\epsilon}$
Landau gauge $\frac{\eta_{\mu\nu}-p_\mu p_\nu/p^2}{p^2+m^2-i\epsilon}$.
ξ-gauge:

Some other gauges:

Feynman gauge: $$\frac{\eta_{\mu\nu}}{p^2+m^2-i\epsilon}$$

Landau gauge $$\frac{\eta_{\mu\nu}-p_\mu p_\nu/p^2}{p^2+m^2-i\epsilon}.$$

ξ-gauge: $$\eta_{\mu\nu}+(\xi-1)p_\mu p_\nu /$$



Some other gauges:
Feynman gauge: $\frac{\eta_{\mu\nu}}{p^2+m^2-i\epsilon}$
Landau gauge $\frac{\eta_{\mu\nu}-p_\mu p_\nu/p^2}{p^2+m^2-i\epsilon}$.
ξ-gauge: $\frac{\eta_{\mu\nu}+(\xi-1)p_\mu p_\nu/(p^2+\xi m^2)}{p^2+m^2-i\epsilon}$
$\xi=1$: Feynman
$\xi=0$: La
$\xi\to\infty$: Un

Some other gauges:

Feynman gauge: $\dfrac{\eta_{\mu\nu}}{p^2+m^2-i\epsilon}$

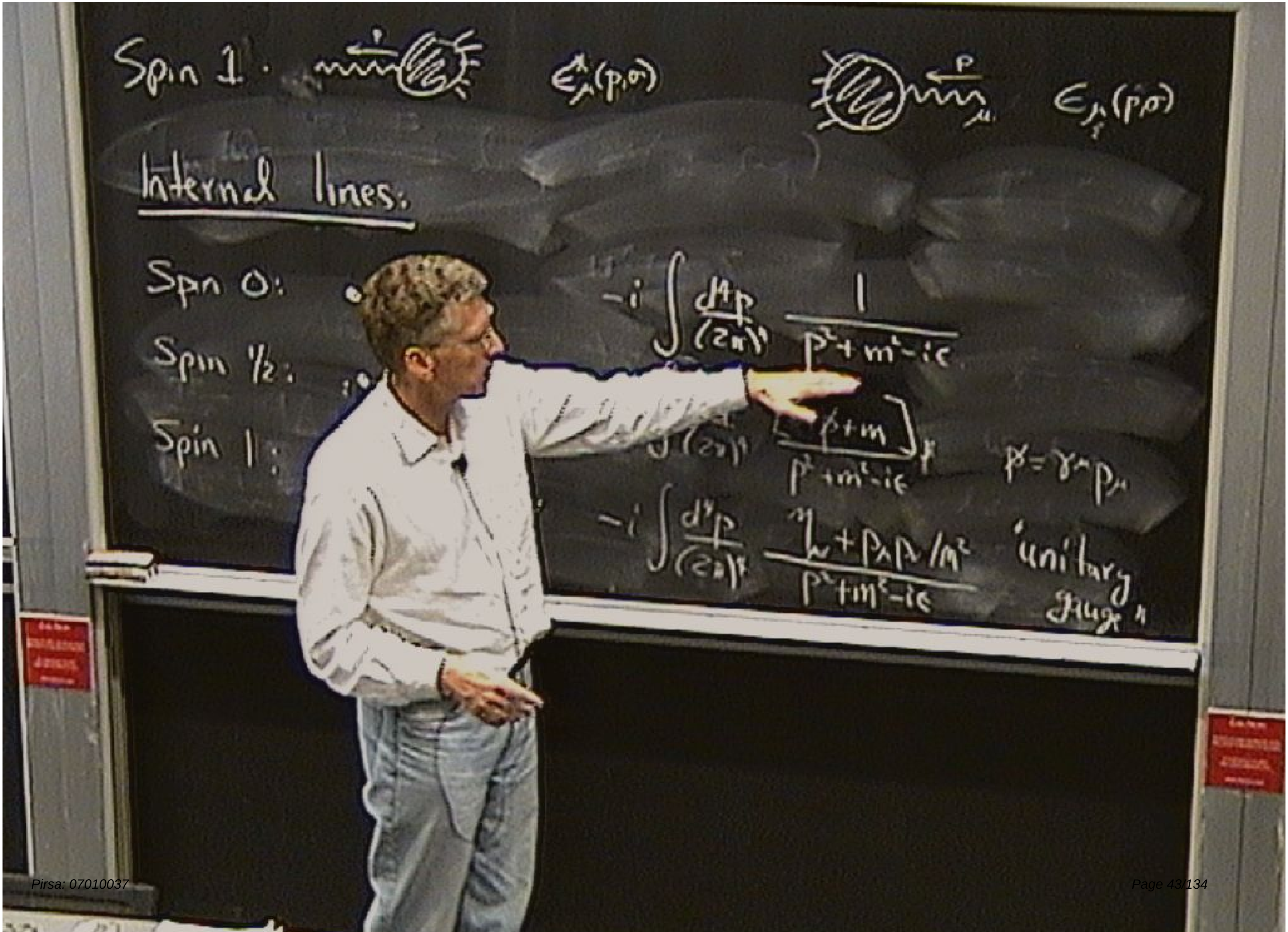
Landau gauge $\dfrac{\eta_{\mu\nu}-p_\mu p_\nu/p^2}{p^2+m^2-i\epsilon}$.

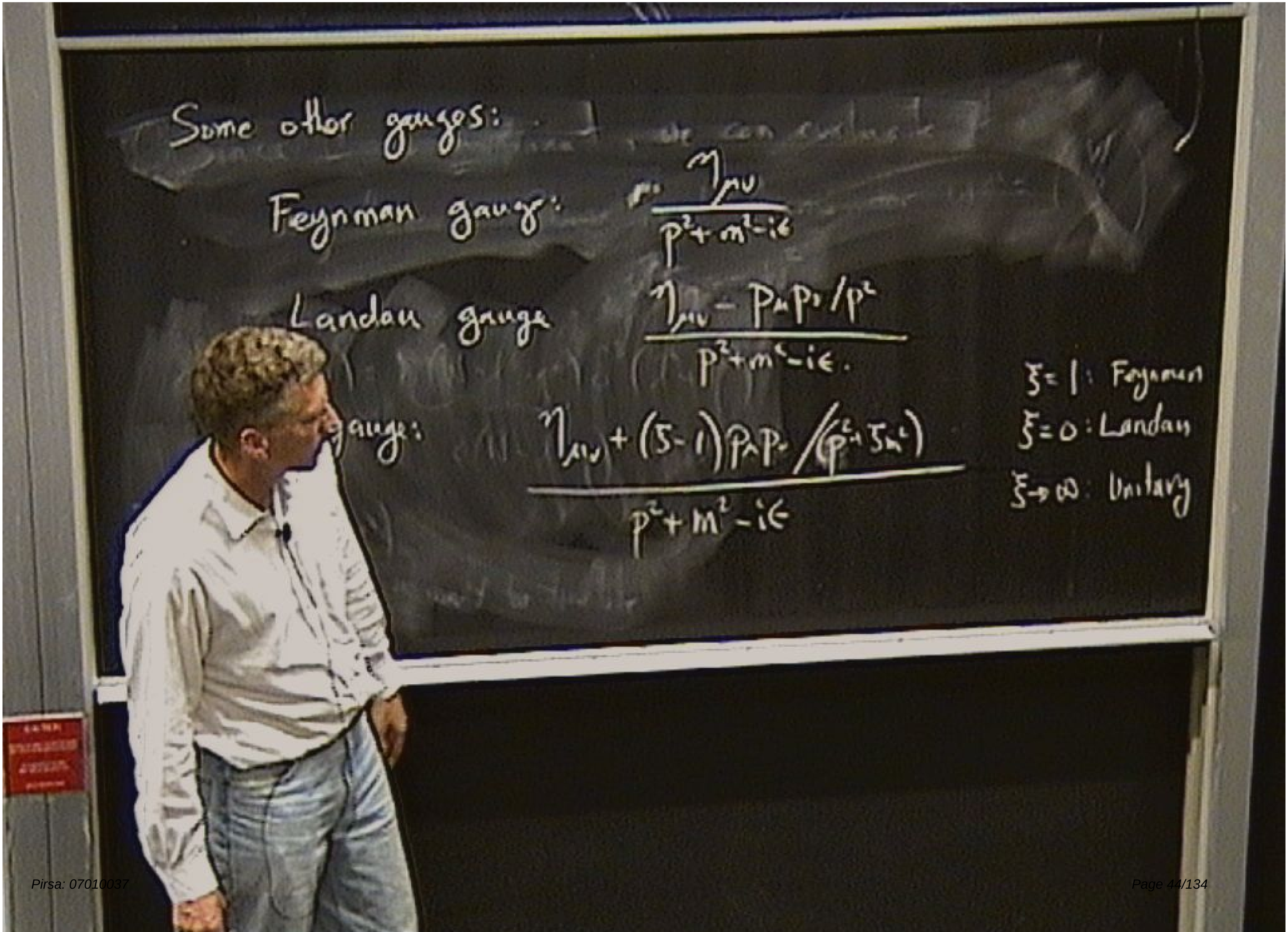
ξ-gauge: $\dfrac{\eta_{\mu\nu}+(\xi-1)p_\mu p_\nu/(p^2+\xi m^2)}{p^2+m^2-i\epsilon}$

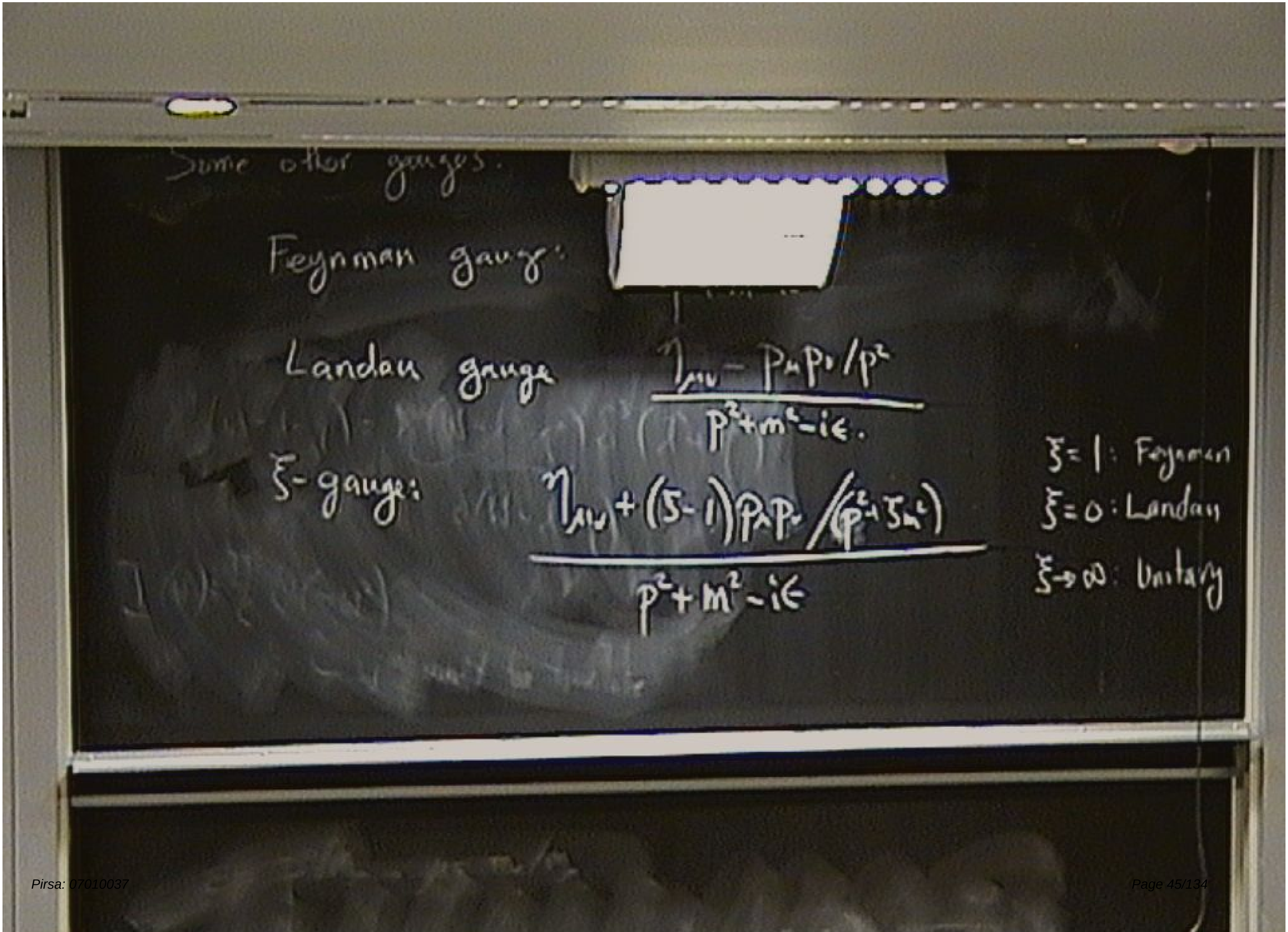
$\xi=1$: Feynman

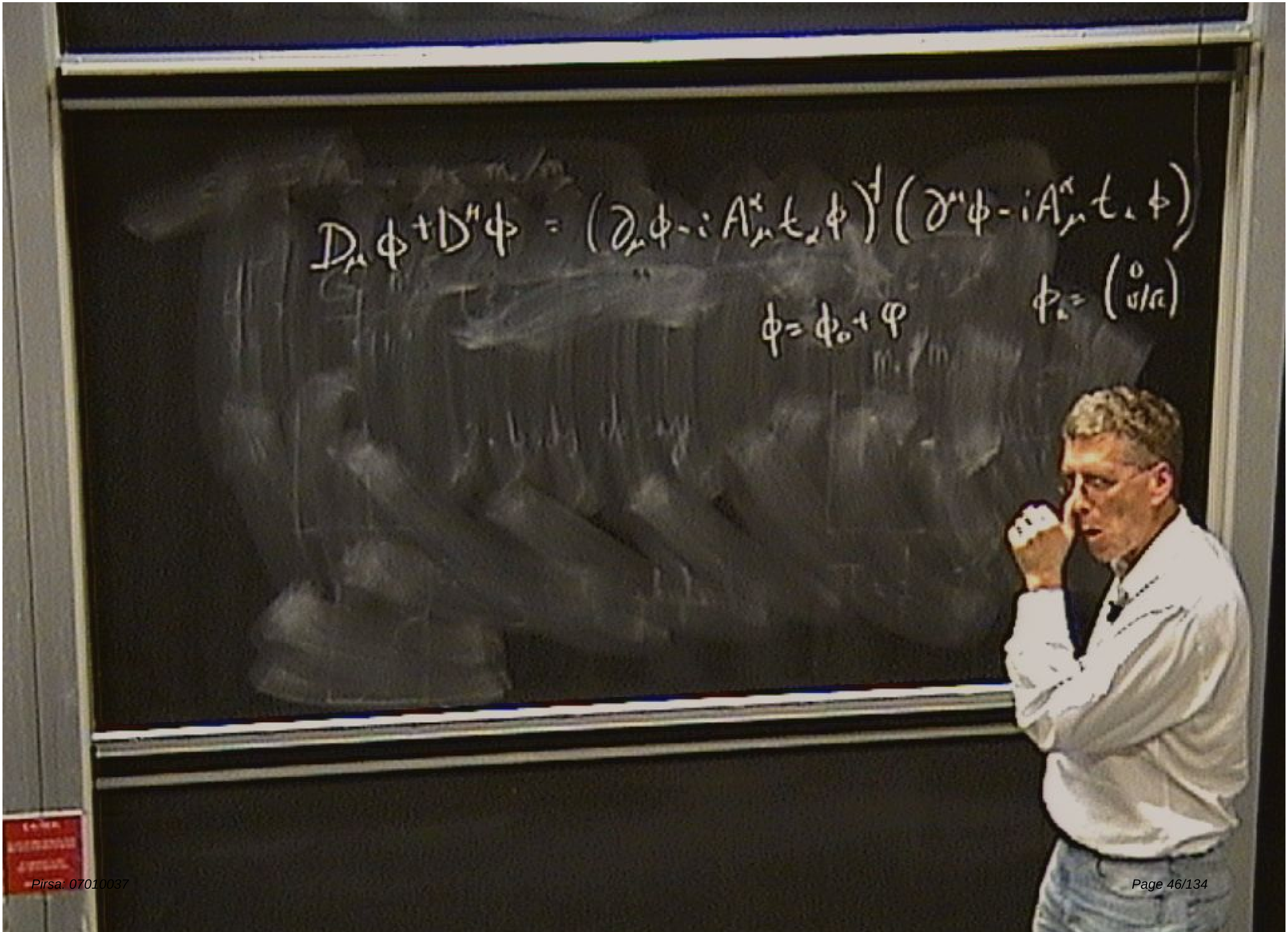
$\xi=0$: Landau

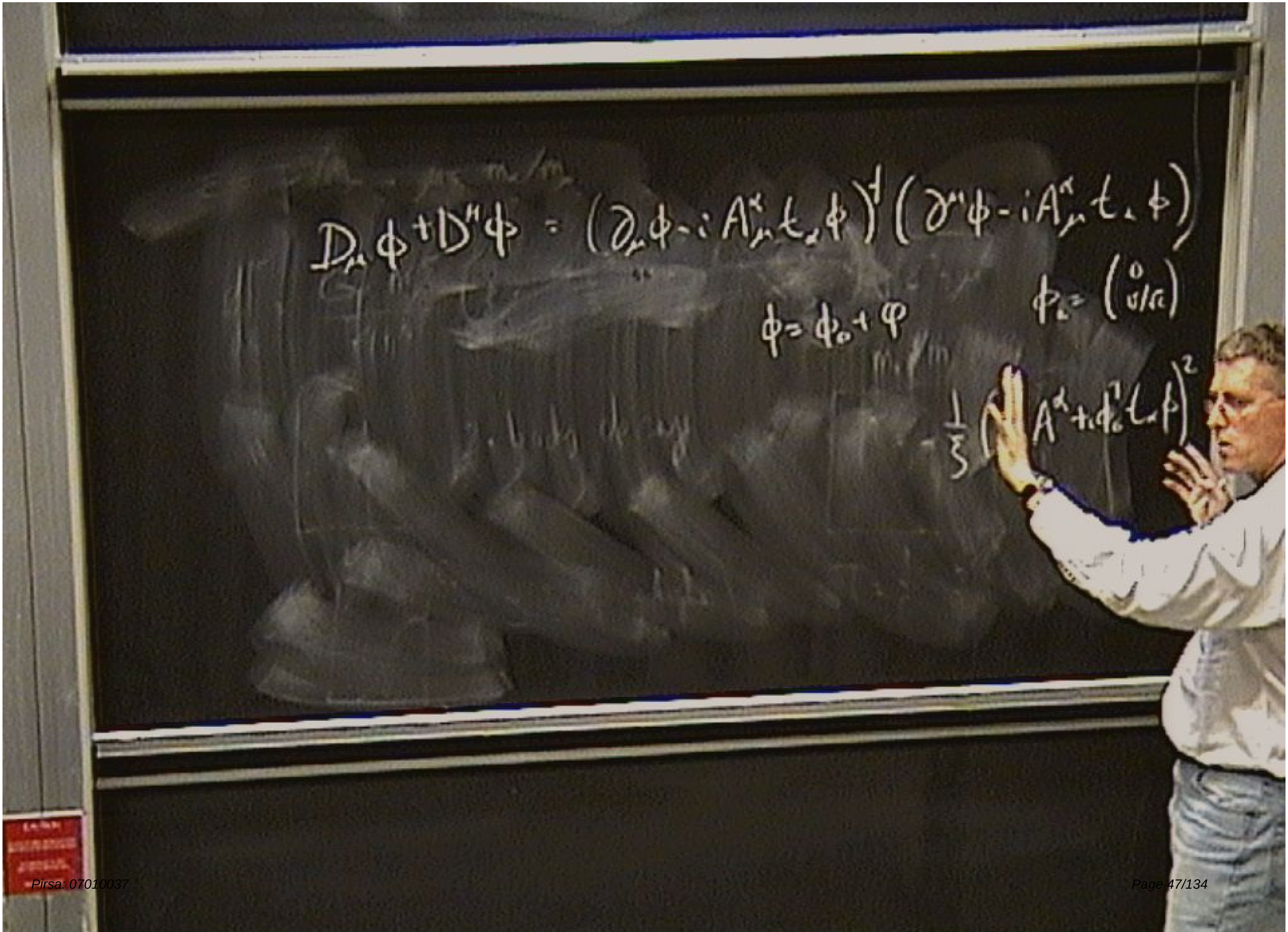
$\xi\to\infty$: Unitary


Spin 1:
Internal lines:
Spin 0:
Spin 1/2:
Spin 1:
'unitary gauge'


Some other gauges:
Feynman gauge: $\frac{\eta_{\mu\nu}}{p^2+m^2-i\epsilon}$
Landau gauge $\frac{\eta_{\mu\nu} - p_\mu p_\nu / p^2}{p^2+m^2-i\epsilon}$.
gauge: $\frac{\eta_{\mu\nu} + (\xi-1) p_\mu p_\nu / (p^2+\xi m^2)}{p^2+m^2-i\epsilon}$
$\xi=1$: Feynman
$\xi=0$: Landau
$\xi\to\infty$: Unitary


Some other gauges.
Feynman gauge:
Landau gauge $\frac{\eta_{\mu\nu} - p_\mu p_\nu / p^2}{p^2 + m^2 - i\epsilon}$.
ξ-gauge: $\frac{\eta_{\mu\nu} + (\xi - 1) p_\mu p_\nu / (p^2 + \xi m^2)}{p^2 + m^2 - i\epsilon}$
$\xi = 1$: Feynman
$\xi = 0$: Landau
$\xi \to \infty$: Unitary


$D_\mu \phi^\dagger D^\mu \phi = (\partial_\mu \phi - i A^\alpha_\mu t_\alpha \phi)^\dagger (\partial^\mu \phi - i A^\alpha_\mu t_\alpha \phi)$
$\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$
$\phi = \phi_0 + \varphi$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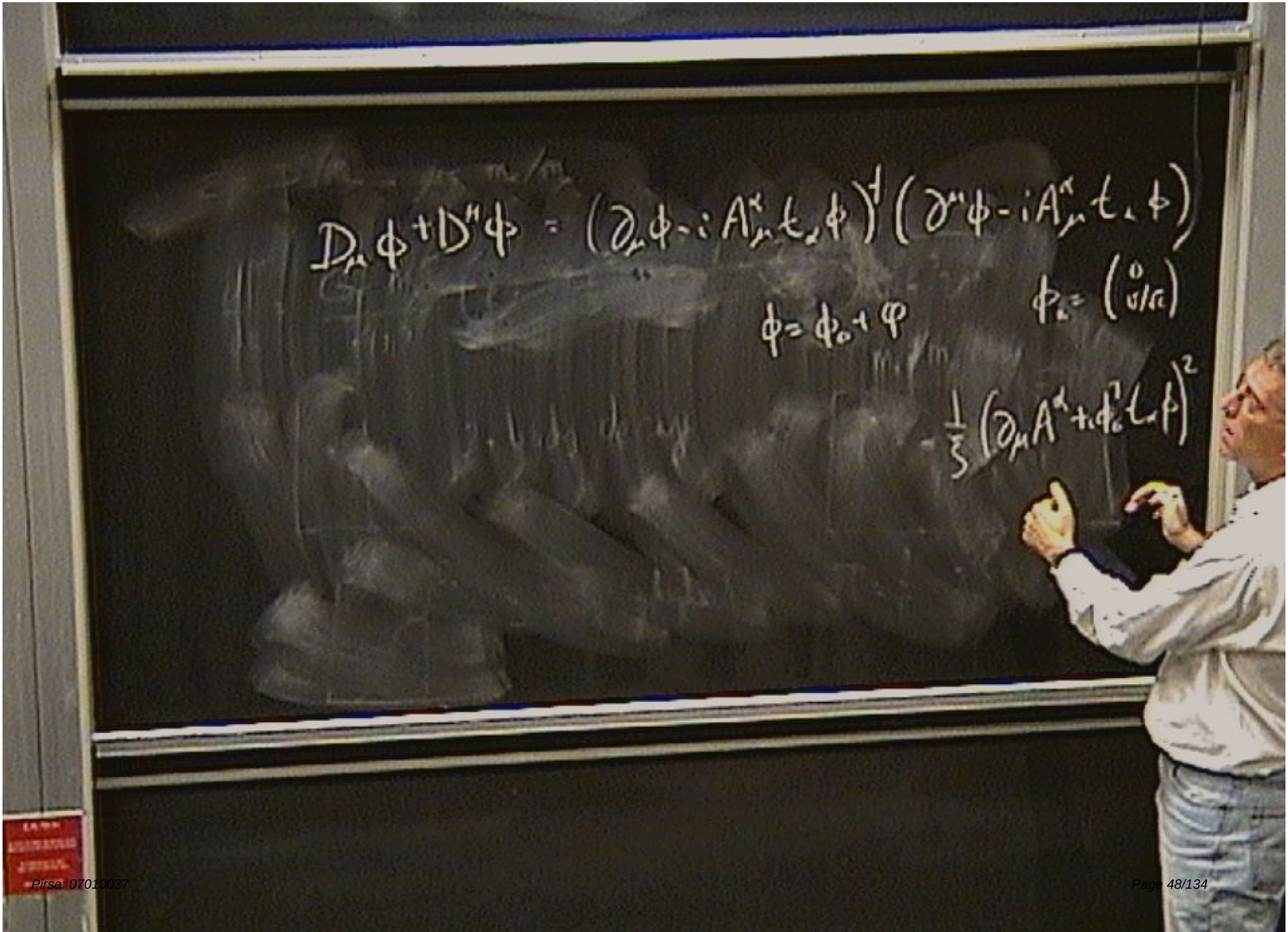
$\phi_0 = \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$
$\phi = \phi_0 + \varphi$
$-\frac{1}{\xi}(A^\alpha + i\phi_0^\dagger t_\alpha\phi)^2$



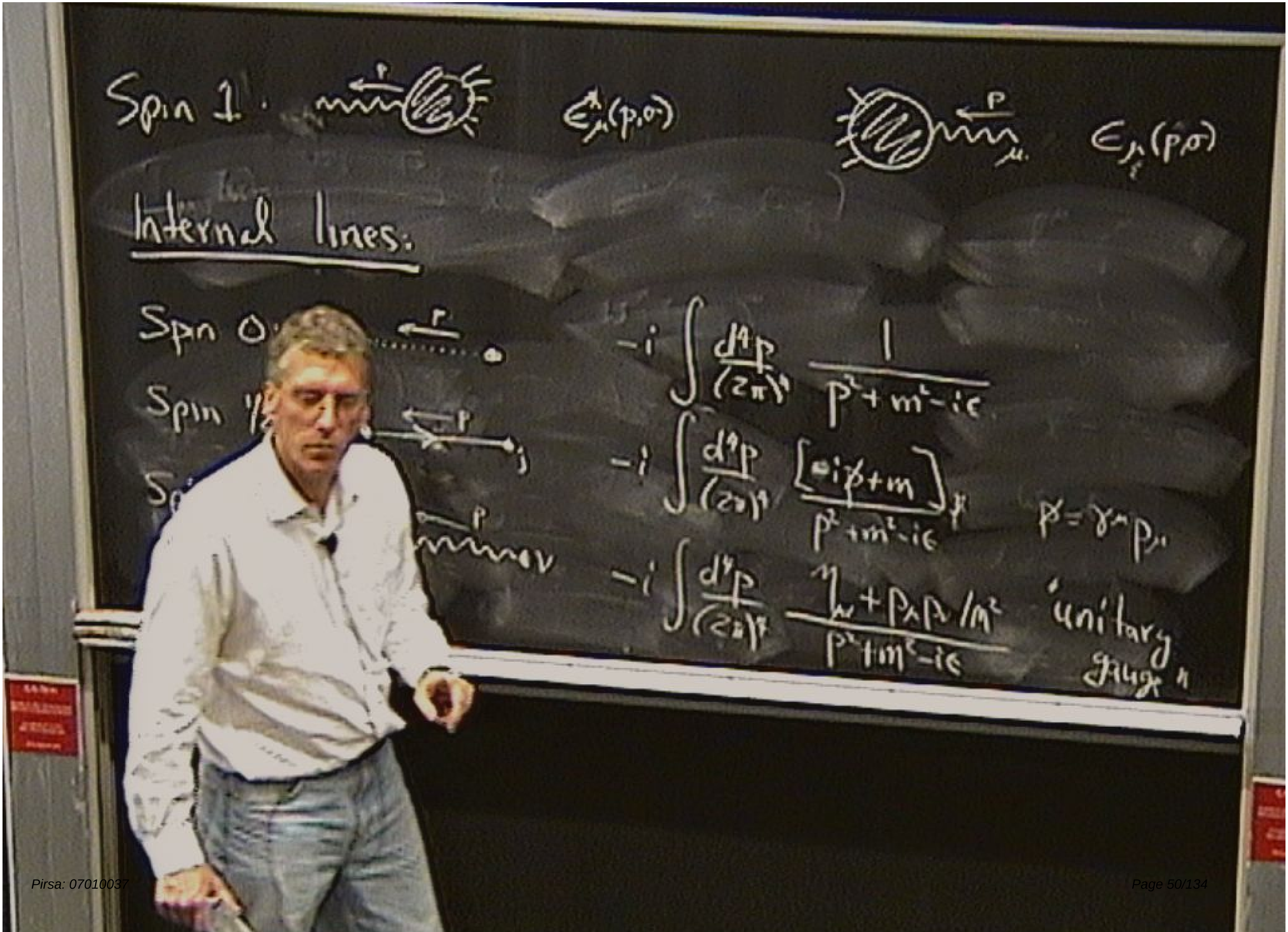
$D_\mu \phi^\dagger D^\mu \phi = (\partial_\mu \phi - i A^\alpha_\mu t_\alpha \phi)^\dagger (\partial^\mu \phi - i A^\alpha_\mu t_\alpha \phi)$
$\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$
$\phi = \phi_0 + \varphi$
$-\frac{1}{\xi}(\partial_\mu A^\alpha + i \xi \phi_0^\dagger t_\alpha \varphi)^2$

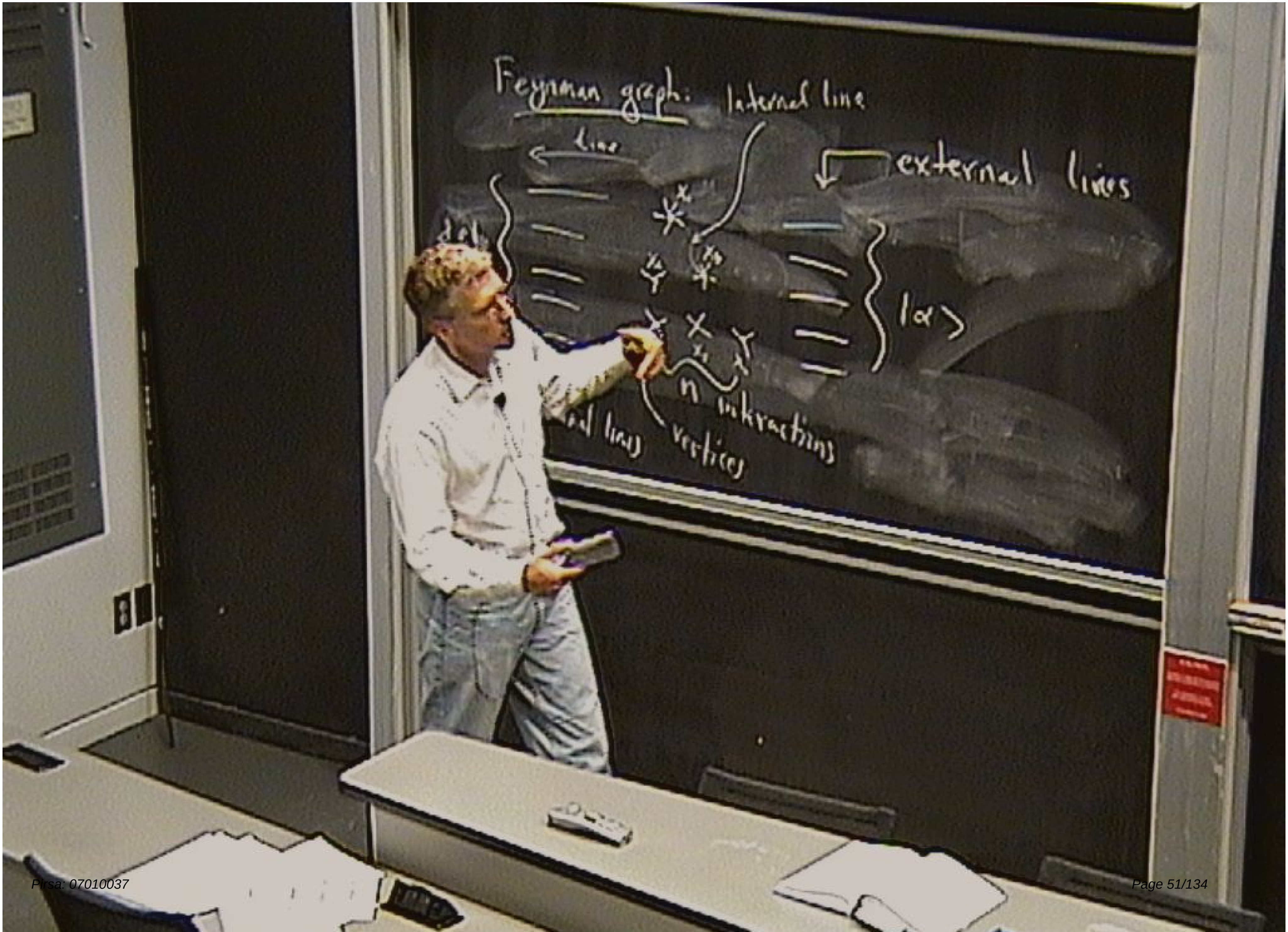
$$D_\mu\phi^\dagger D^\mu\phi = (\partial_\mu\phi - iA^\alpha_\mu t_\alpha\phi)^\dagger(\partial^\mu\phi - iA^\alpha_\mu t_\alpha\phi)$$

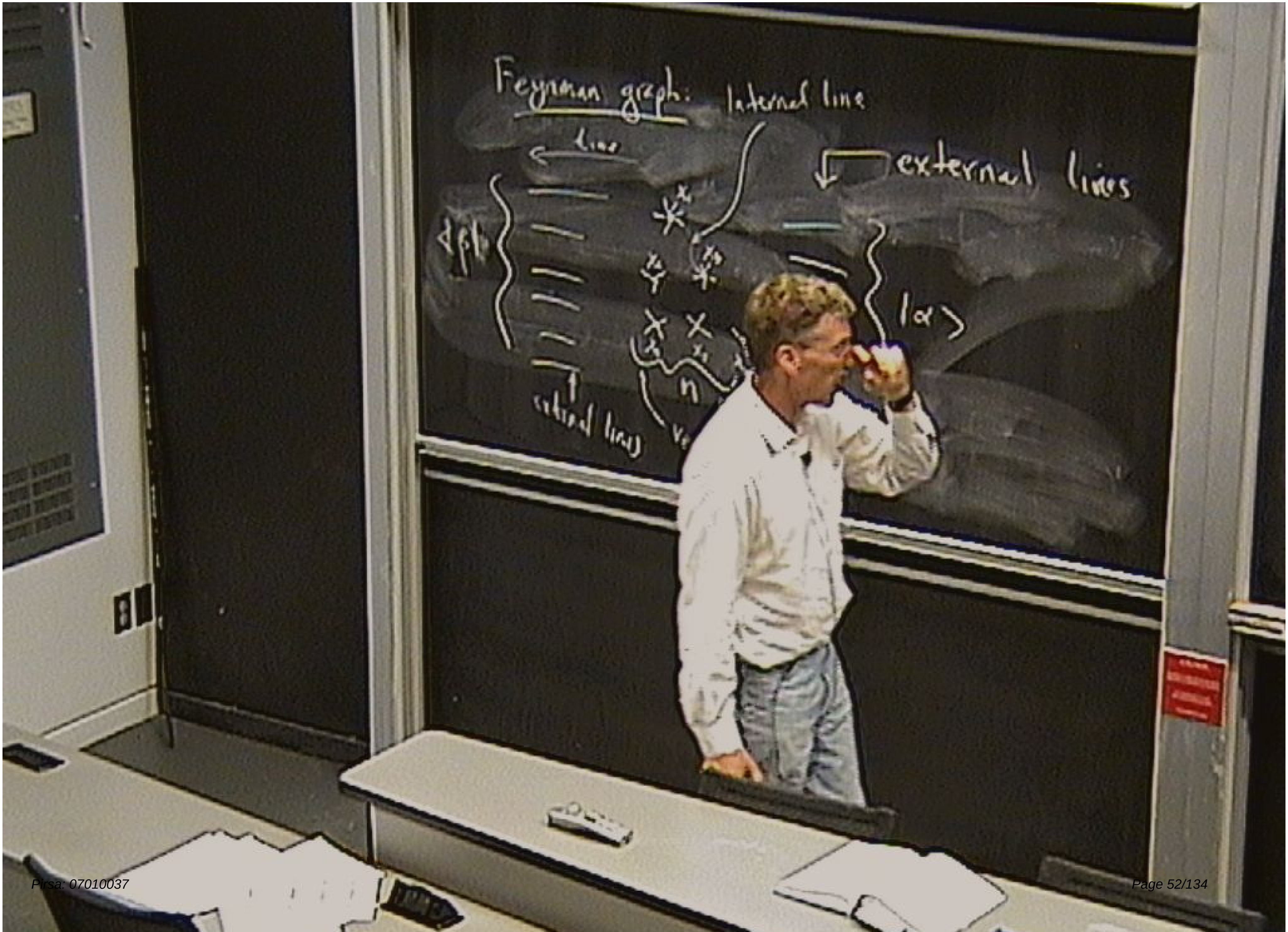
$$\phi_0 = \begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$$

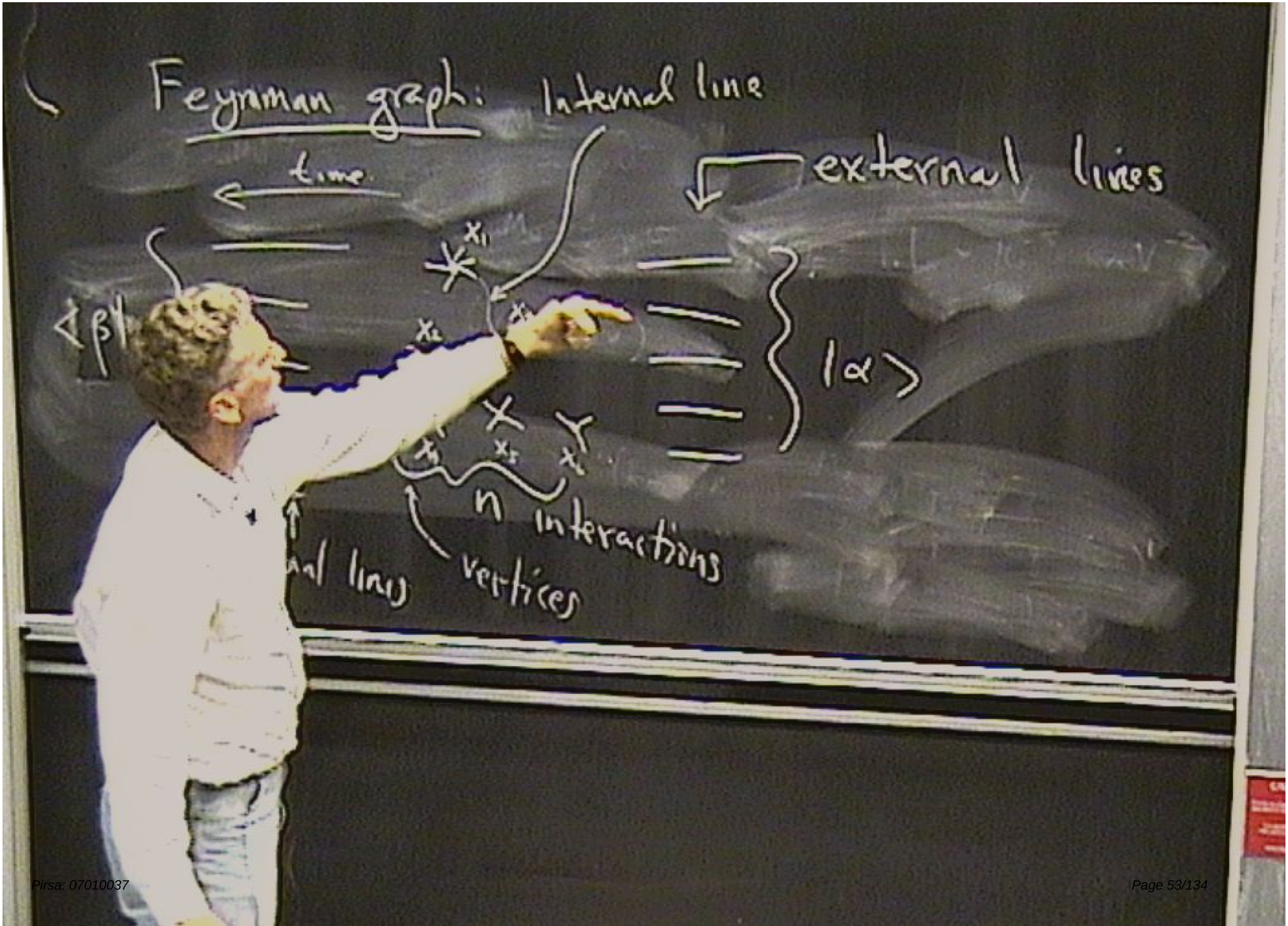
$$\phi = \phi_0 + \varphi$$

$$-\frac{1}{\xi}(\partial_\mu A^\alpha + i\phi_0^\dagger t_\alpha\varphi)^2$$


Spin 1:
Internal lines:
Spin 0
Spin 1/2
unitary gauge

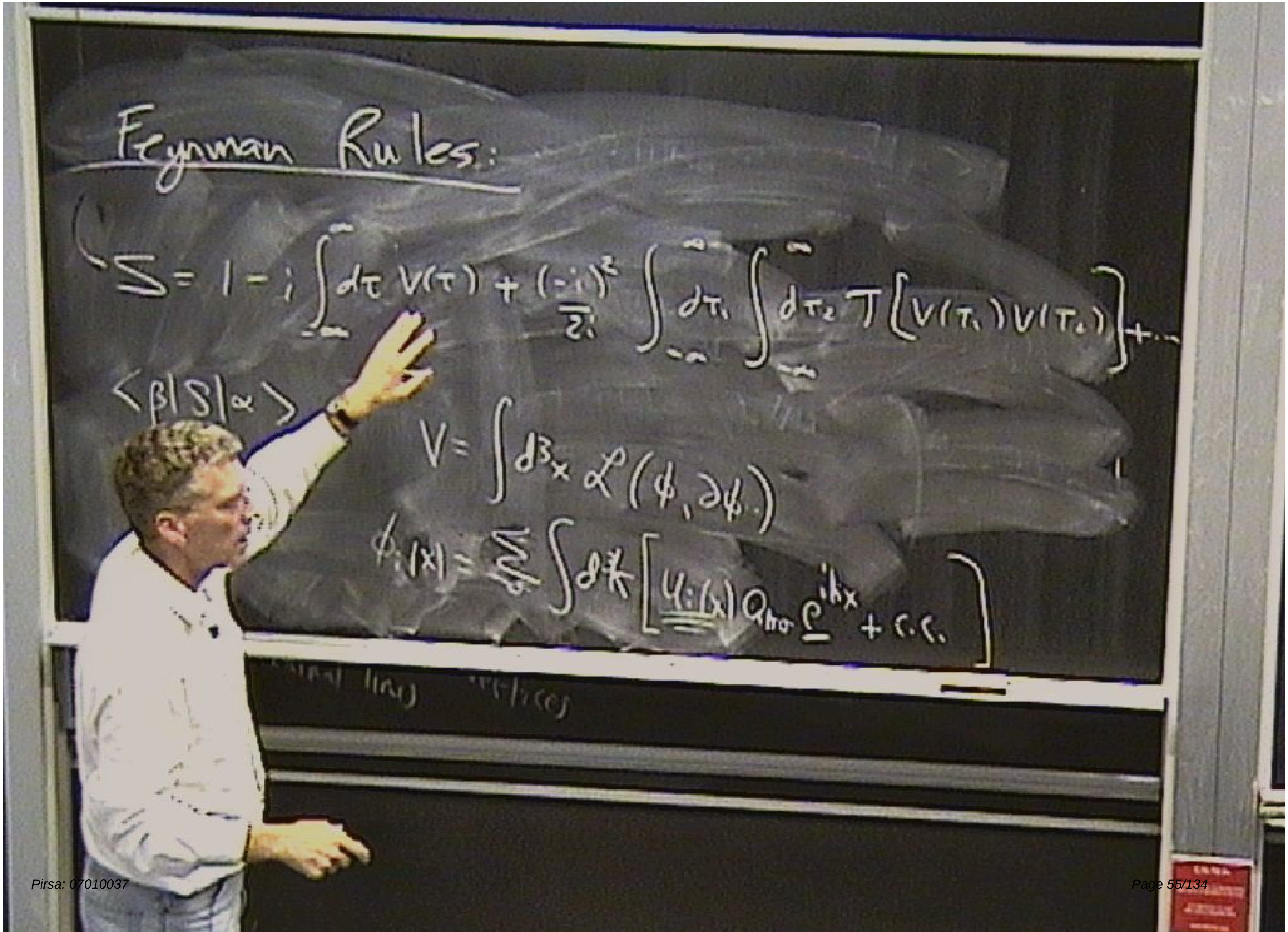

Feynman graph:
internal line
line
external lines
$|\alpha\rangle$
n interactions
vertices

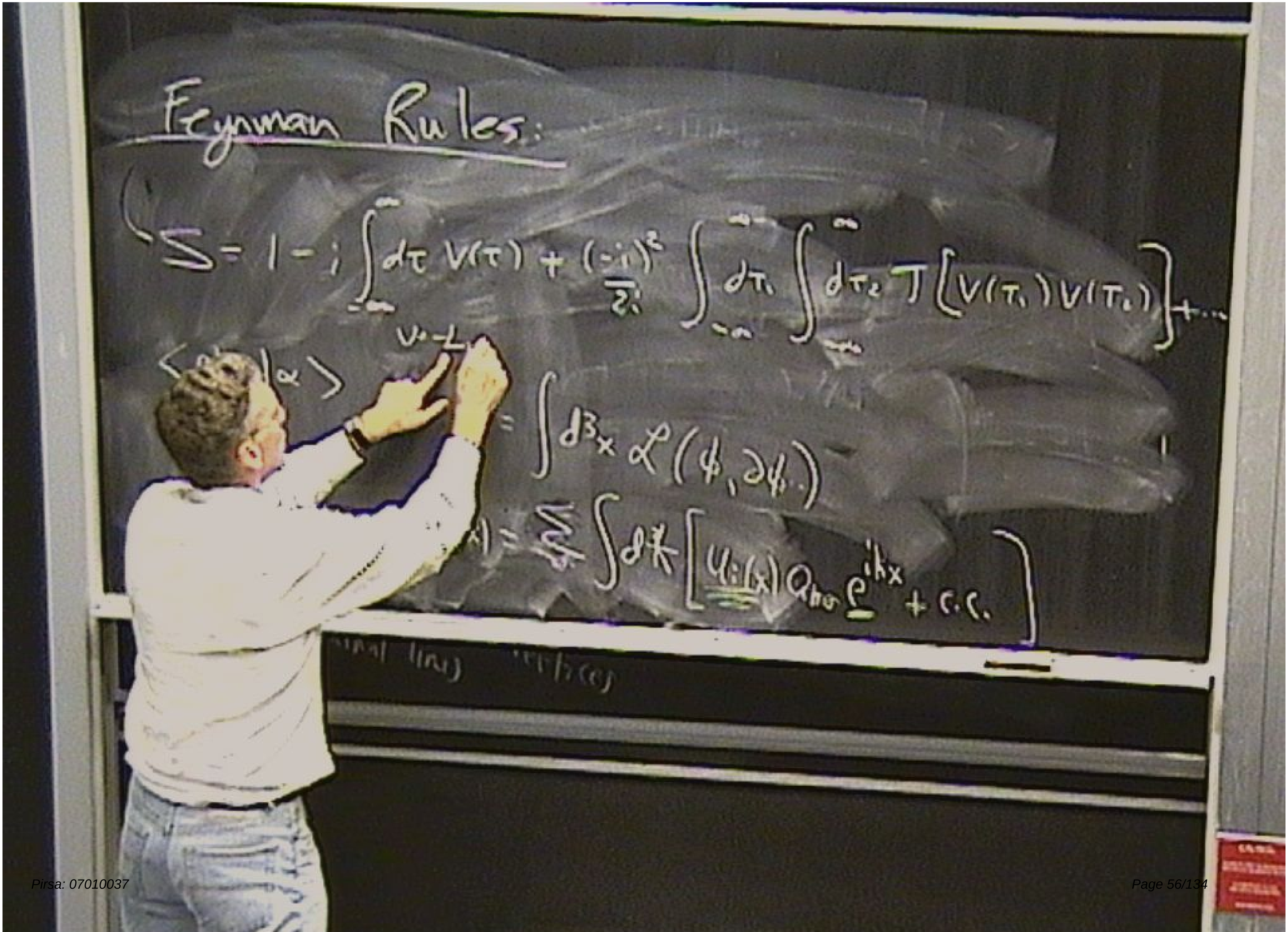

Feynman graph:
internal line
line
external lines
|α⟩
external lines
n

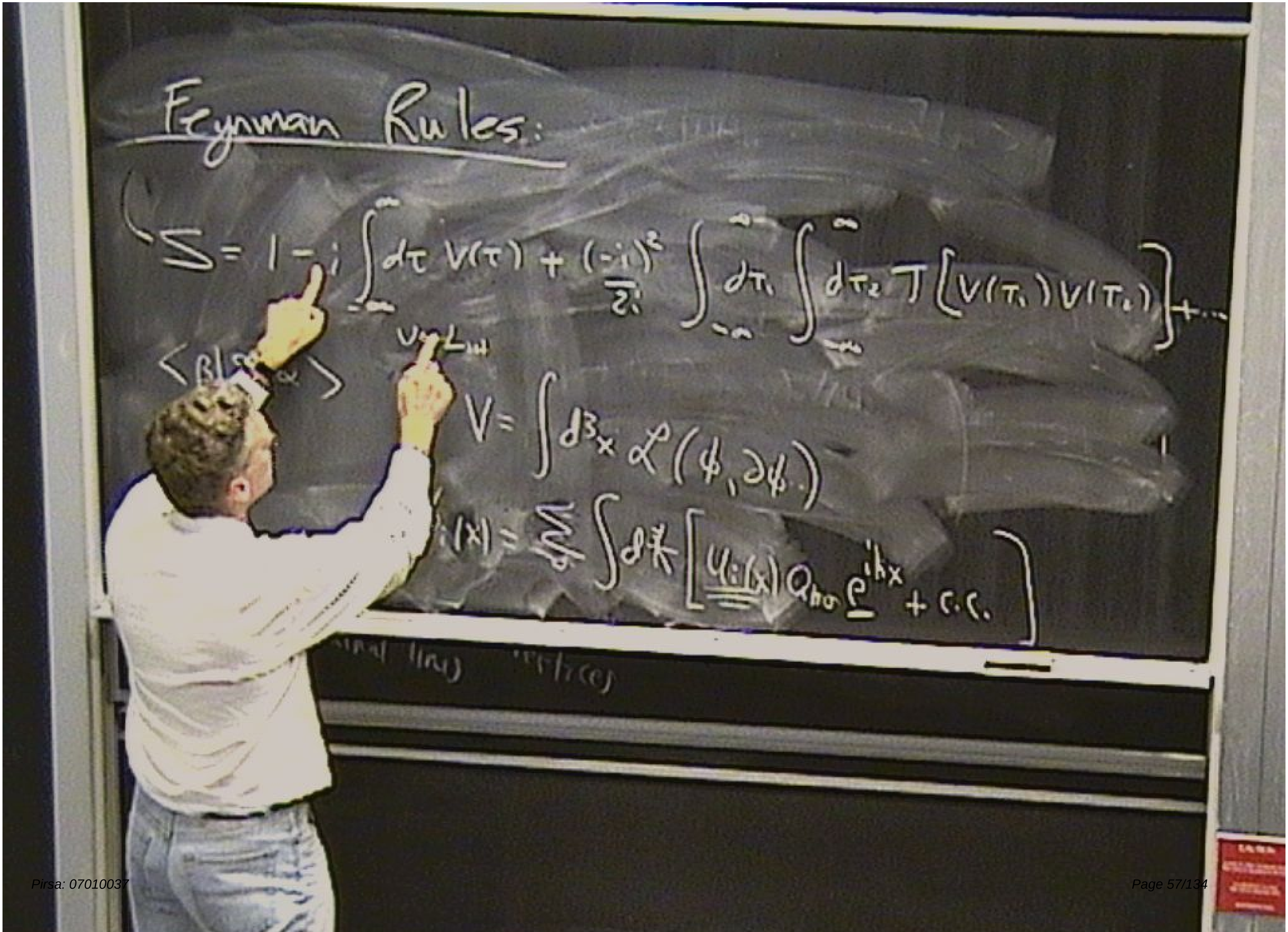

Feynman graph:
Internal line
time
external lines
x_1
x_2
$\langle\beta|$
$|\alpha\rangle$
n interactions
vertices

Vertices : λH^3

 : λ


Feynman Rules:
$S = 1 - i\int_{-\infty}^{\infty} d\tau\, V(\tau) + \frac{(-i)^2}{2!}\int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2\, T\left[V(\tau_1)V(\tau_2)\right] + \dots$
$\langle\beta|S|\alpha\rangle$
$V = \int d^3x\, \mathcal{L}(\phi, \partial\phi ..)$
$\phi_i(x) = \sum_{\sigma}\int d^3k \left[u_i(x)\, a_{k\sigma}\, e^{ikx} + c.c.\right]$


Feynman Rules:
S = 1 − i ∫ dτ V(τ) + (−i)²/2! ∫ dτ₁ ∫ dτ₂ T[V(τ₁)V(τ₂)] + ...
= ∫ d³x ℒ(φ, ∂φ)
+ c.c.


Feynman Rules:
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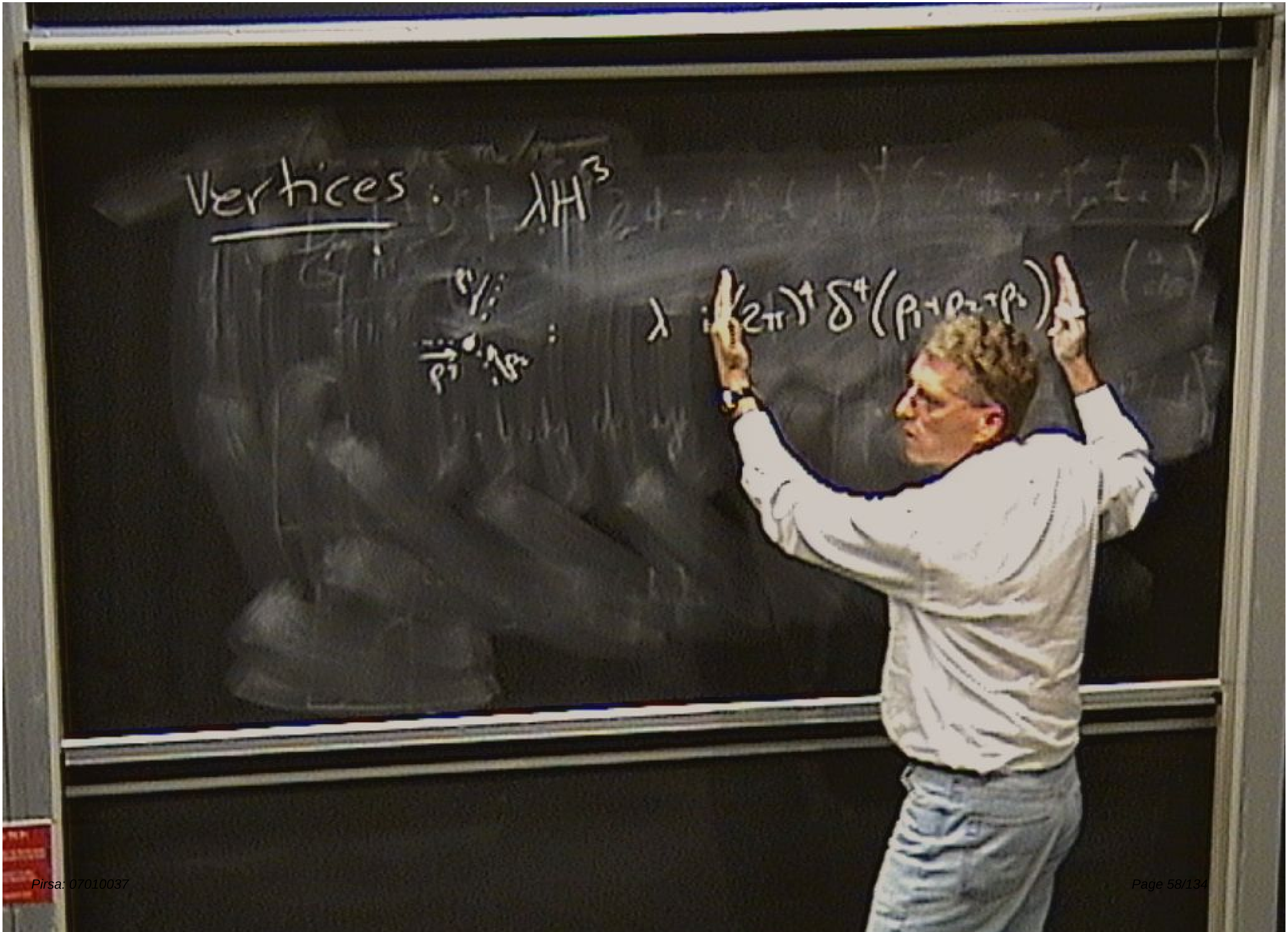
$\langle\beta|S|\alpha\rangle$
$V = \int d^3x\, \mathcal{L}(\phi, \partial\phi)$


Vertices : λH^3
$\lambda \; i(2\pi)^4 \delta^4(p_1+p_2+p_3)$

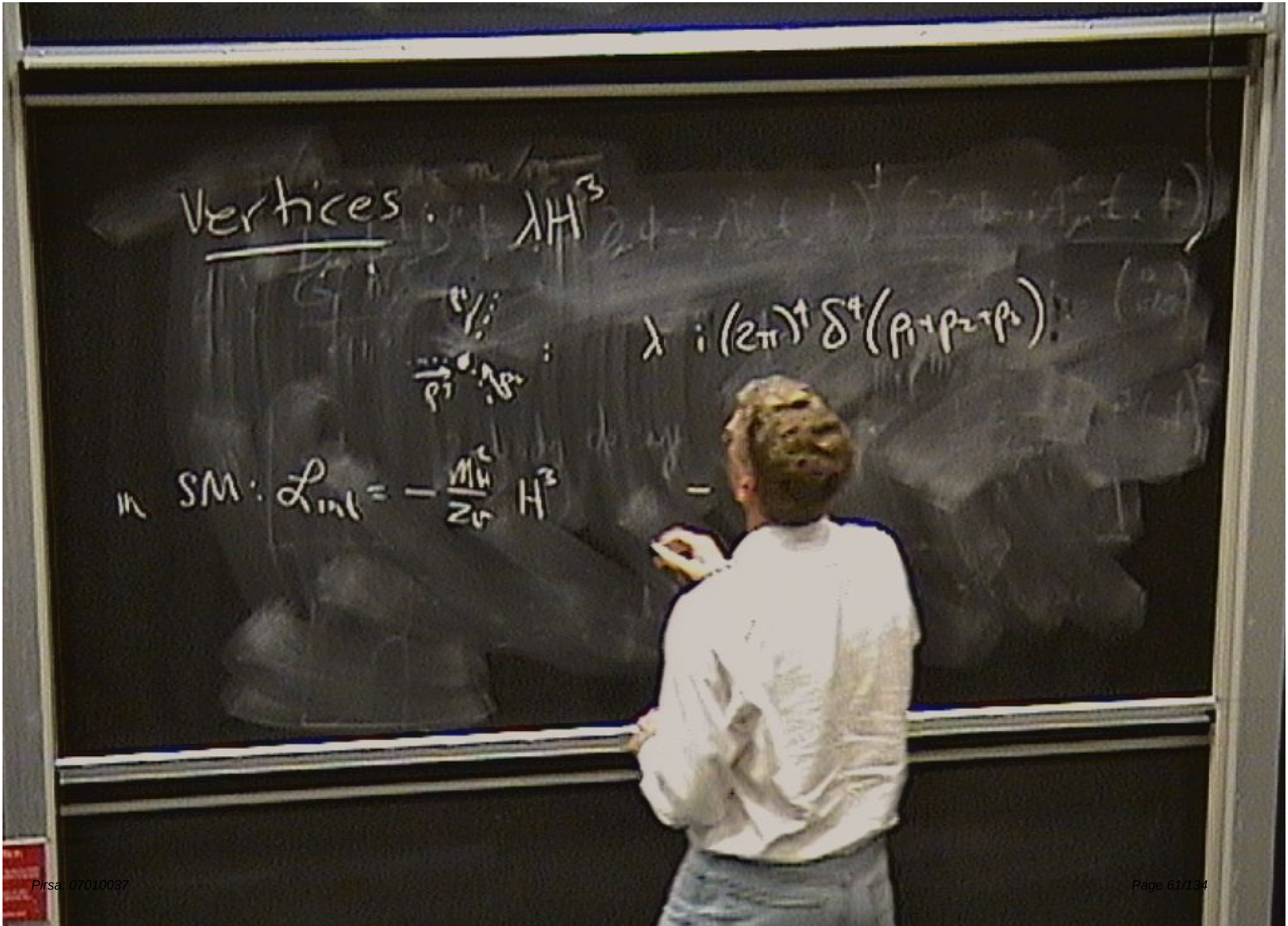
Vertices: λH^3

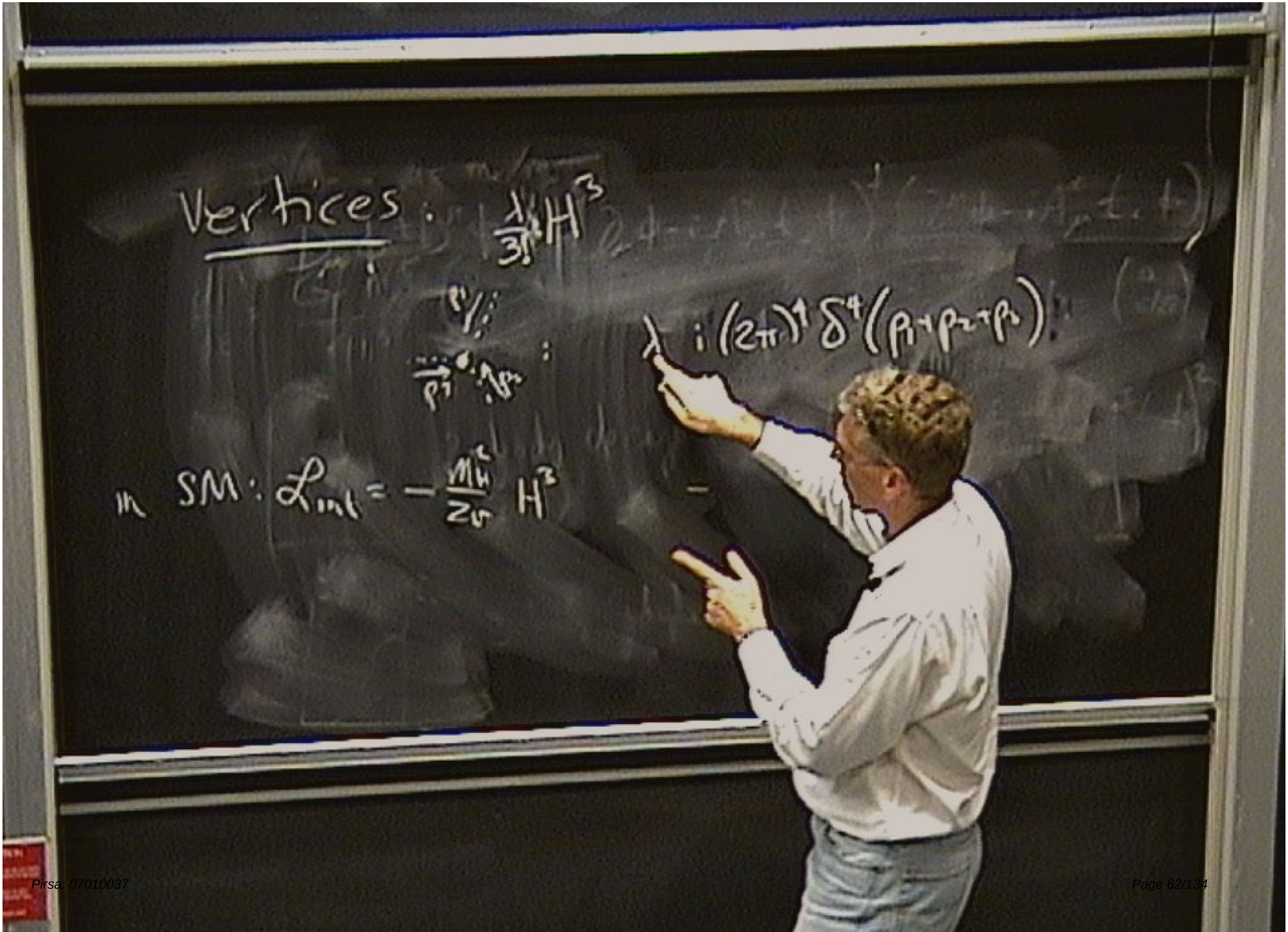
$: \quad \lambda \; i(2\pi)^4 \delta^4(p_1+p_2+p_3)$

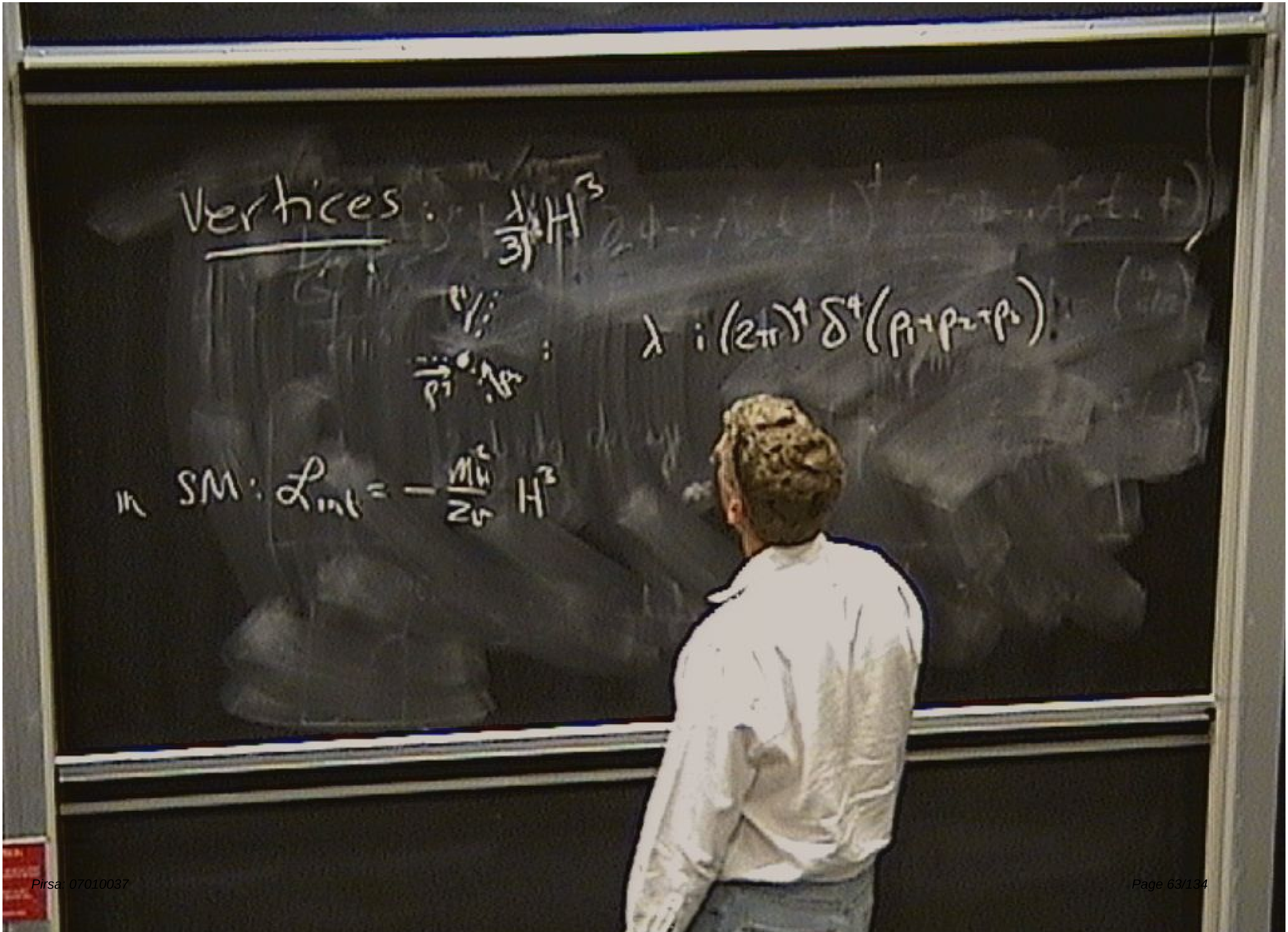
Vertices: λH^3

$: \quad \lambda \; i (2\pi)^4 \delta^4(p_1+p_2+p_3)$

in SM: $\mathcal{L}_{int} = -$


Vertices: λH³
λ i (2π)⁴ δ⁴(p₁+p₂+p₃)
in SM: L_int = − m_H²/2v H³


Vertices: $\frac{\lambda}{3!} H^3$
$\lambda \; i (2\pi)^4 \delta^4(p_1+p_2+p_3)$
in SM: $\mathcal{L}_{int} = -\frac{m_H^2}{2v} H^3$


Vertices: $\frac{\lambda}{3!} H^3$
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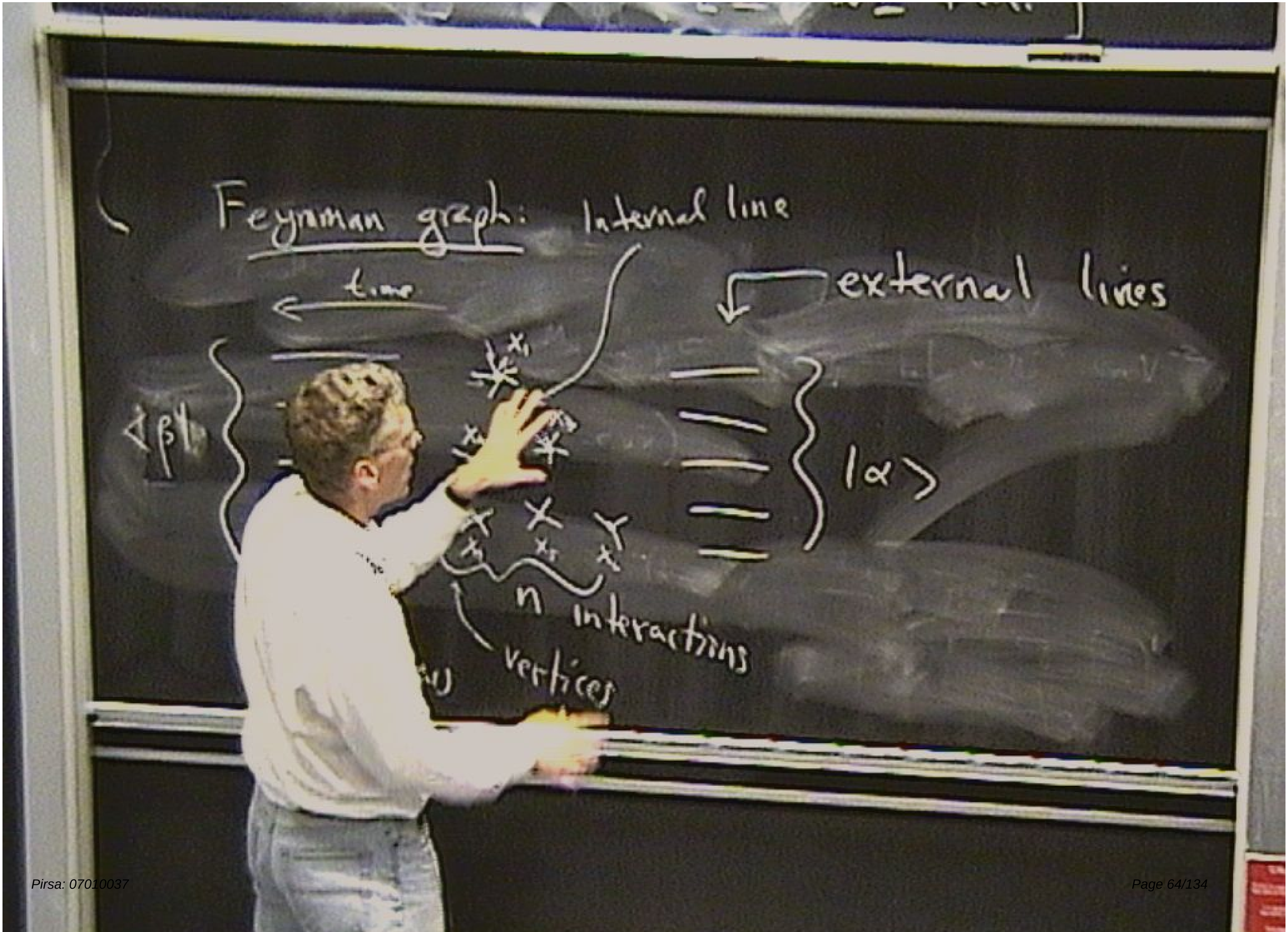
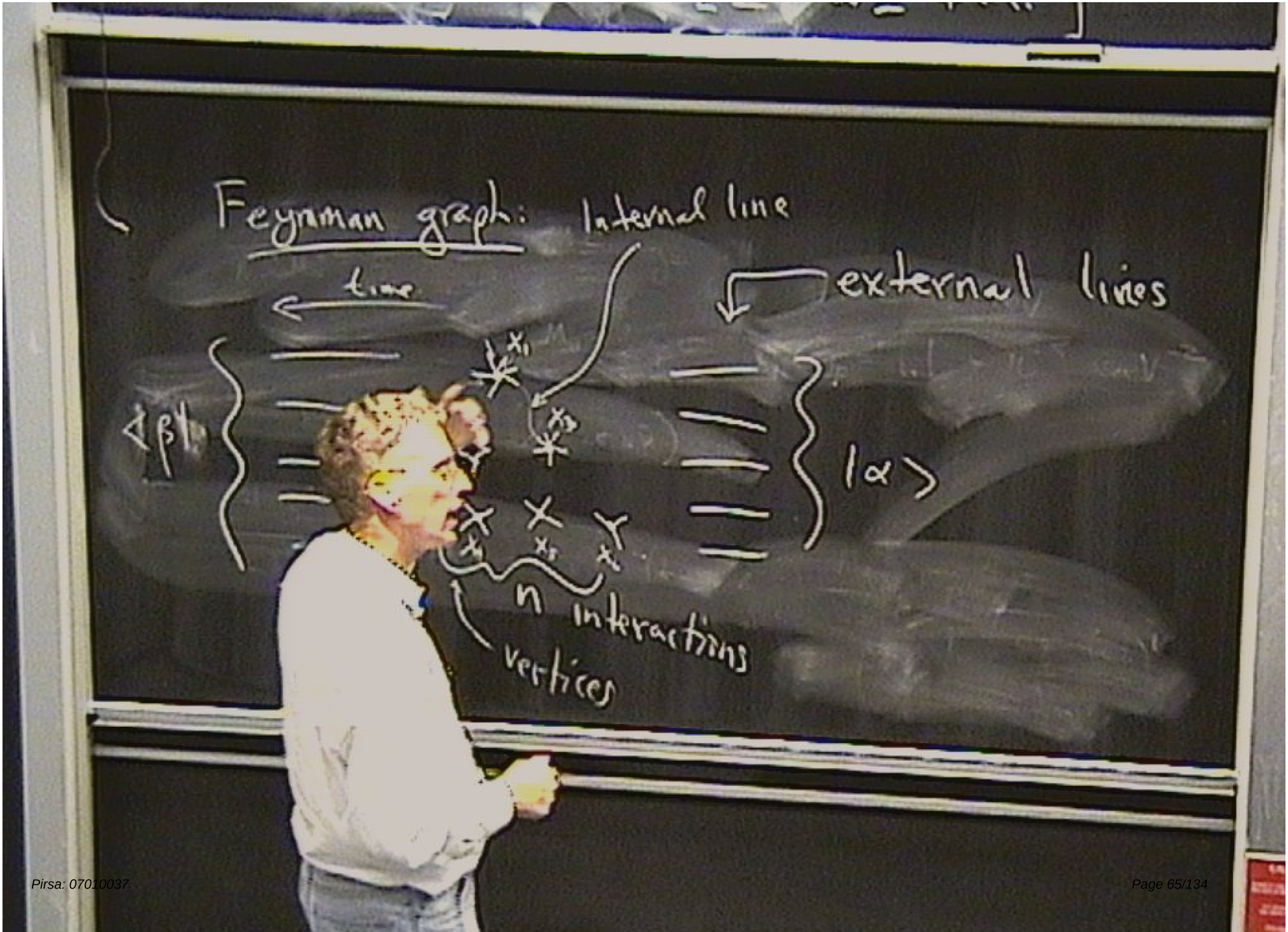

Feynman graph:
Internal line
time
external lines
⟨β|
|α⟩
n interactions
vertices


Feynman graph:
Internal line
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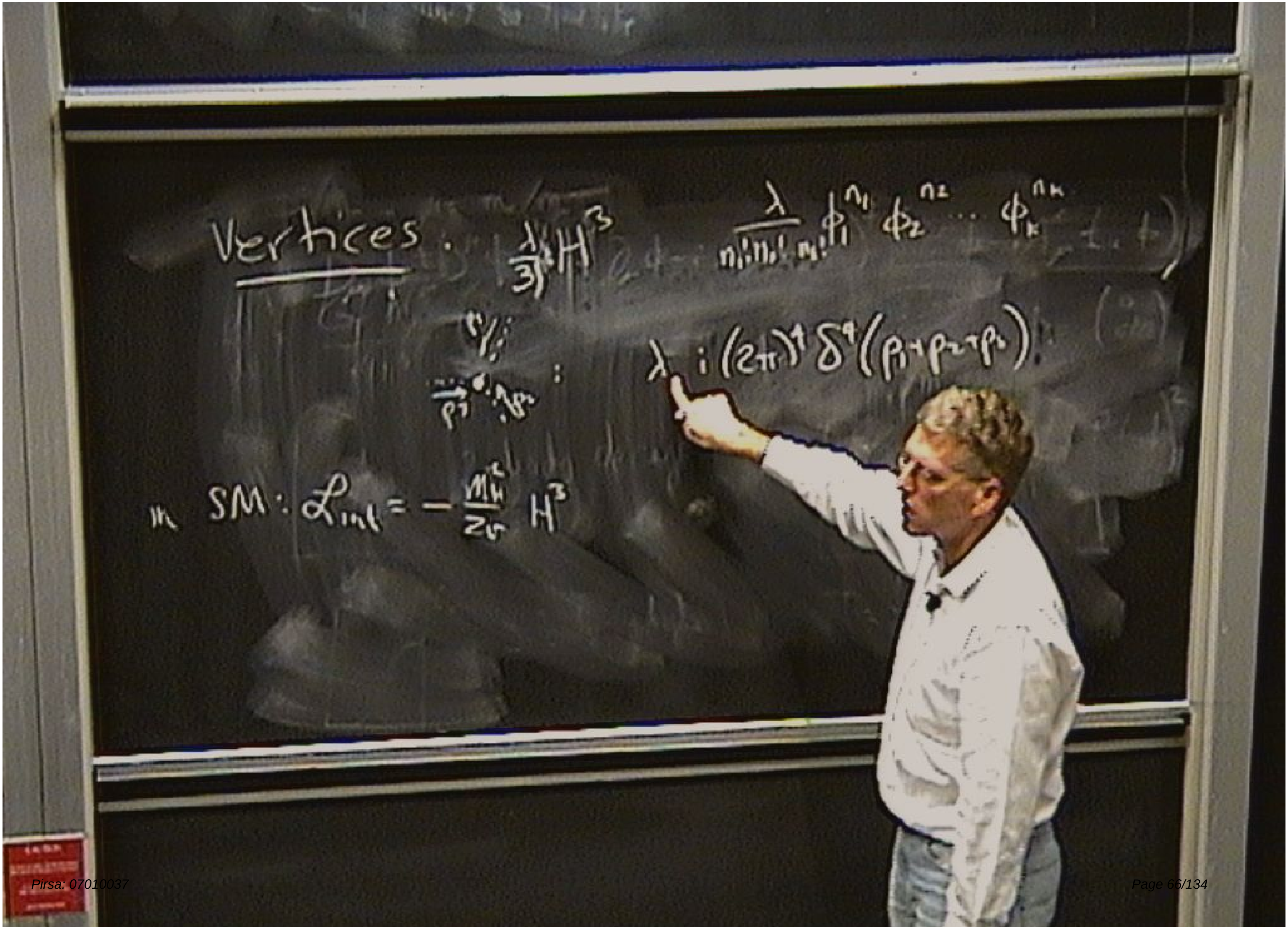
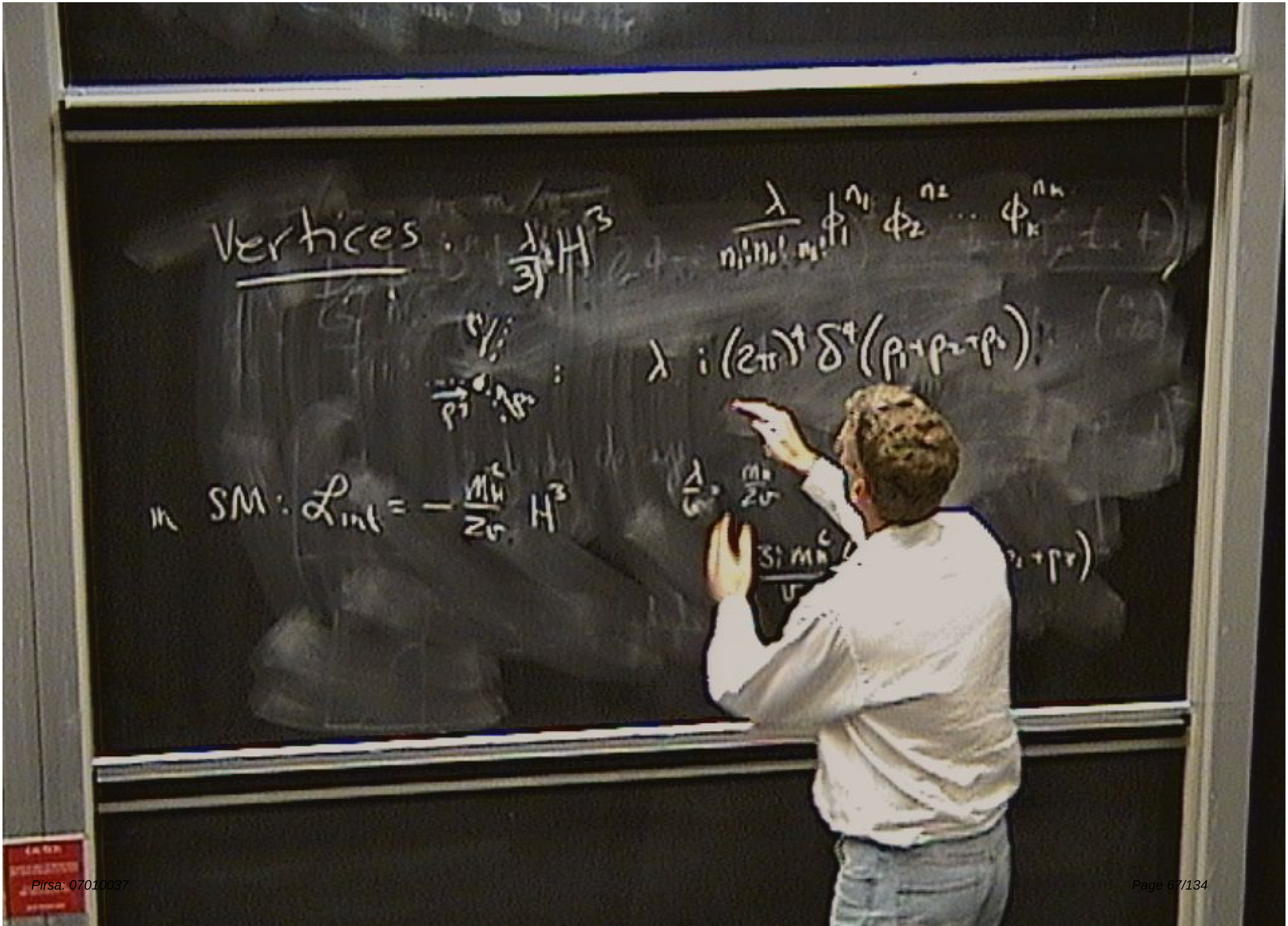
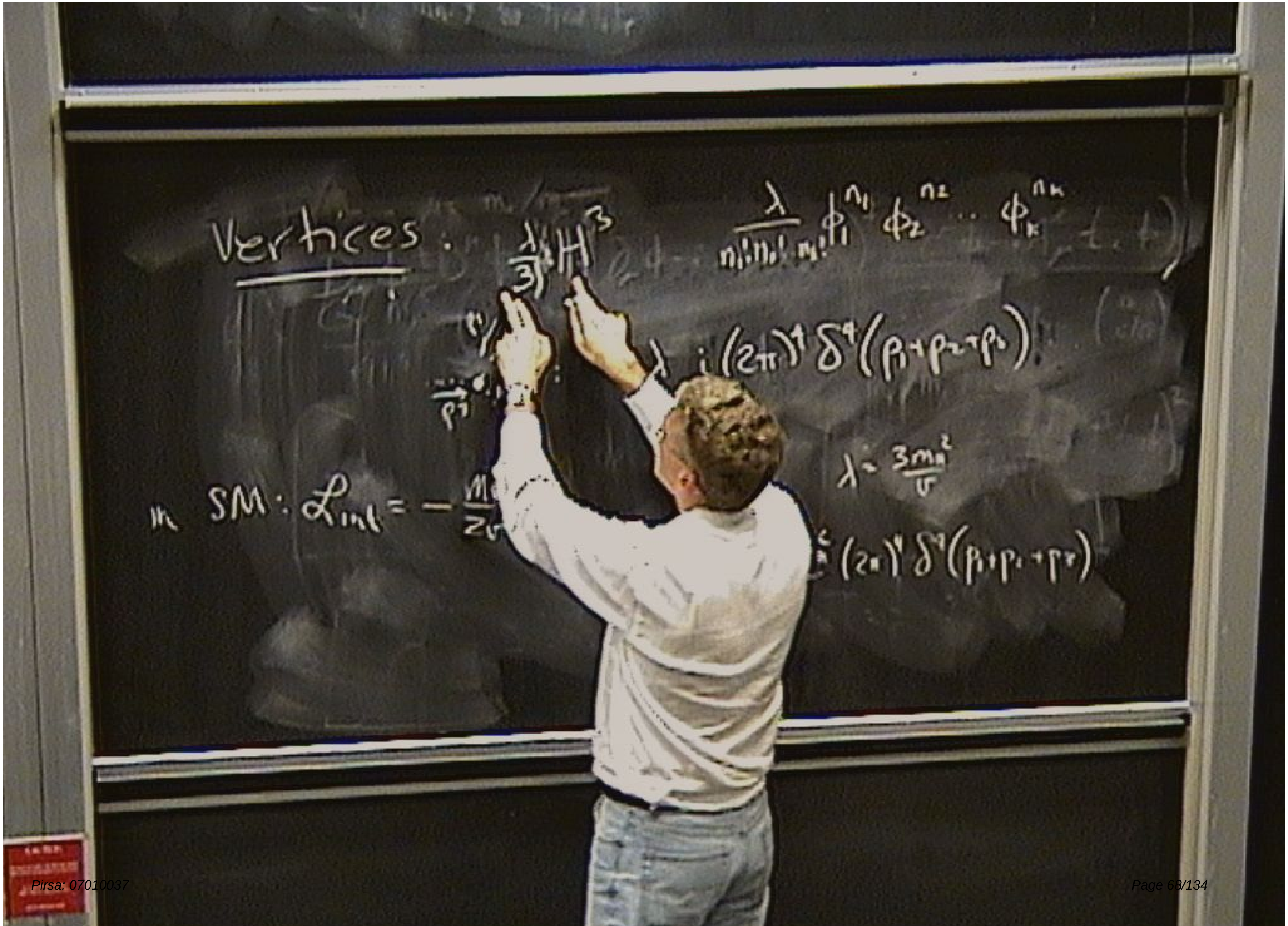
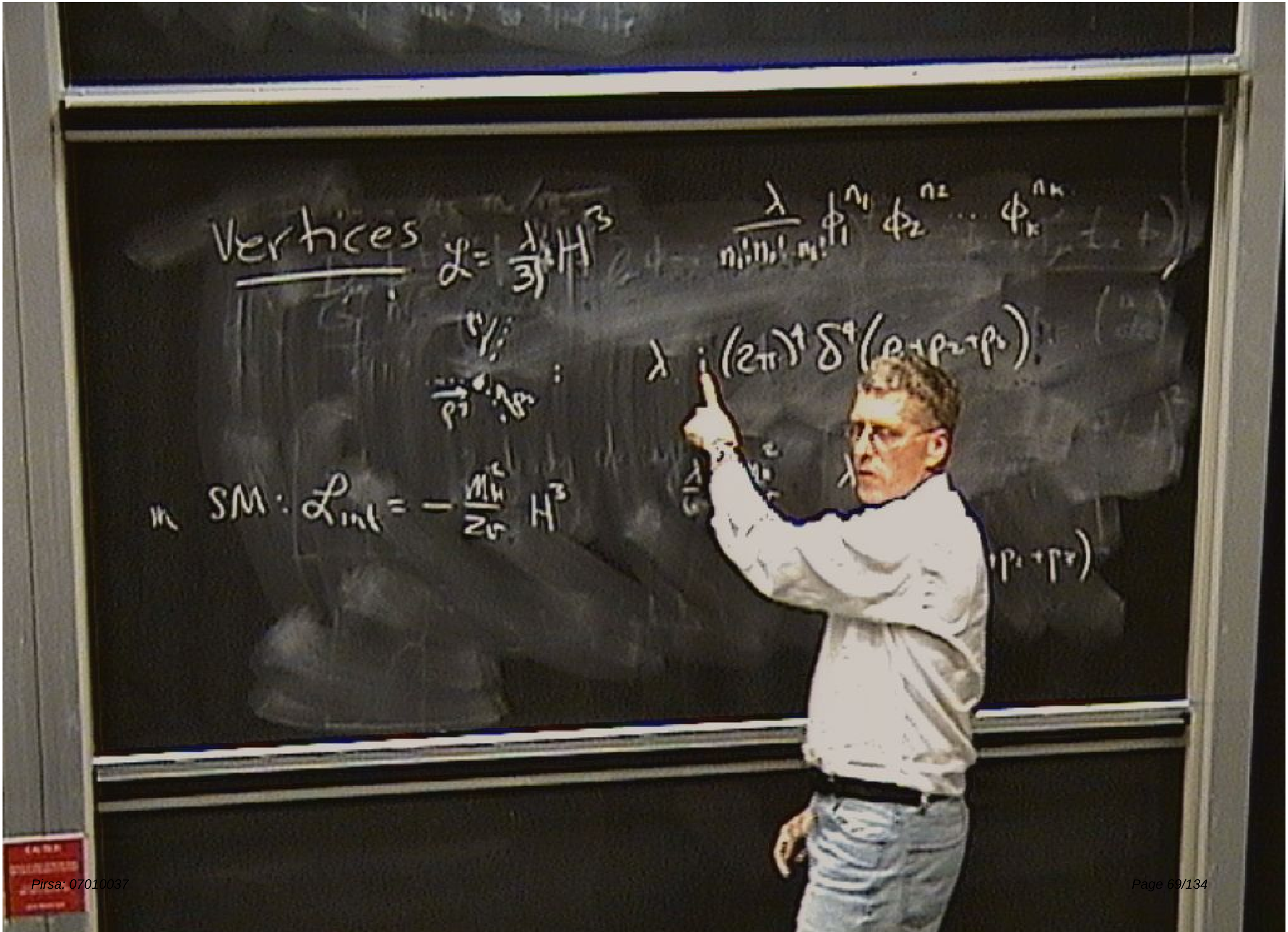

Vertices
$\frac{\lambda}{3!}H^3$
$\frac{\lambda}{n_1! n_2! \cdots n_k!}\phi_1^{n_1}\phi_2^{n_2}\cdots\phi_k^{n_k}$
$\lambda \, i(2\pi)^4\delta^4(p_1+p_2+p_3)$
SM: $\mathcal{L}_{int} = -\frac{m_H^2}{2v} H^3$


Vertices : λ/3! H^3
λ/(n1! n2! ... nk!) φ1^n1 φ2^n2 ... φk^nk
λ i (2π)^4 δ^4(p1+p2+p3)
in SM: L_int = − mH²/2v H^3


Vertices:
λ/(n1! n2! ... nk!) φ1^n1 φ2^n2 ... φk^nk
i(2π)^4 δ^4(p1+p2+p3)
λ = 3mh^2/v
in SM: Lint = −
(2π)^4 δ^4(p1+p2+p3)

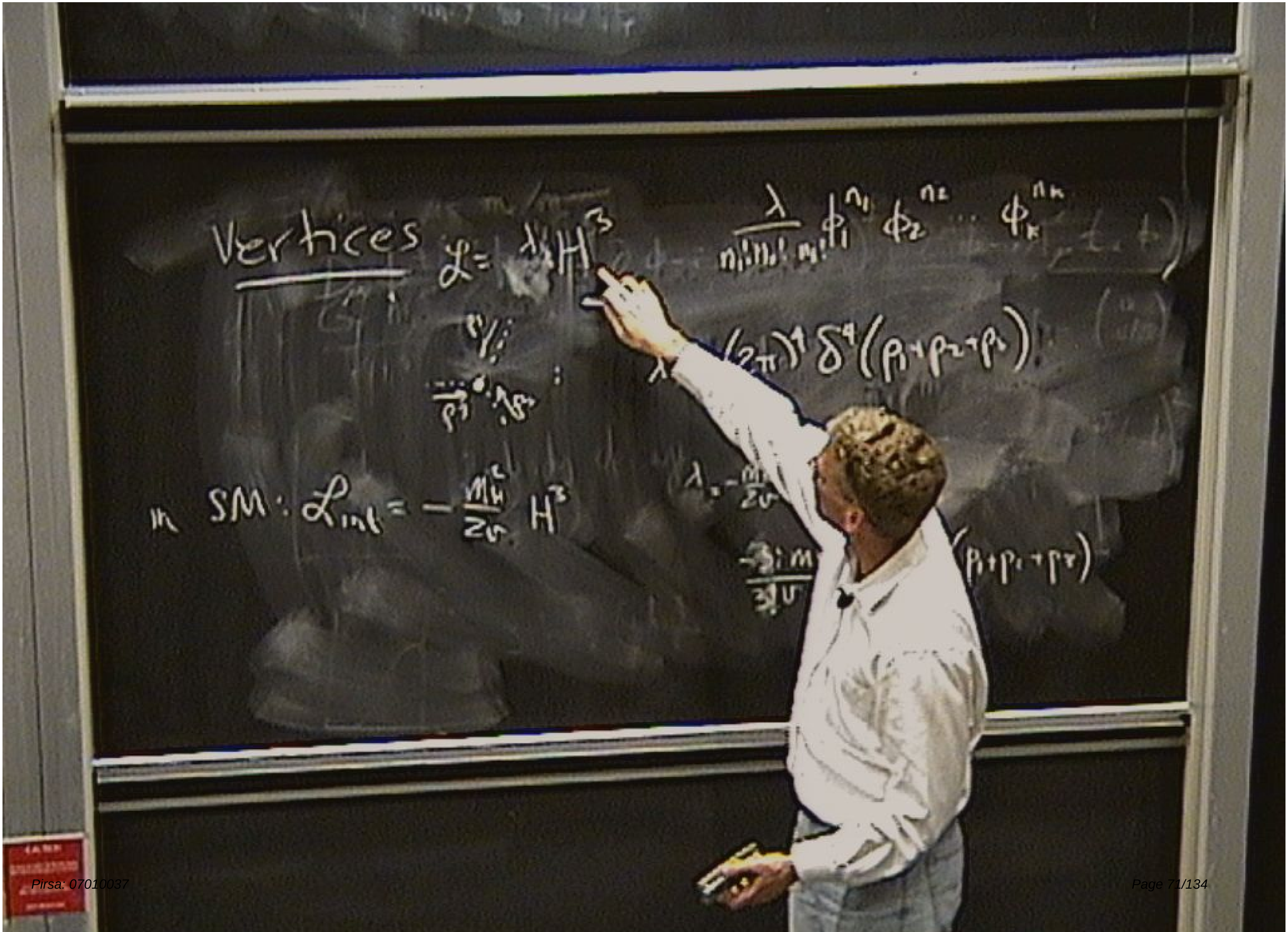

Vertices

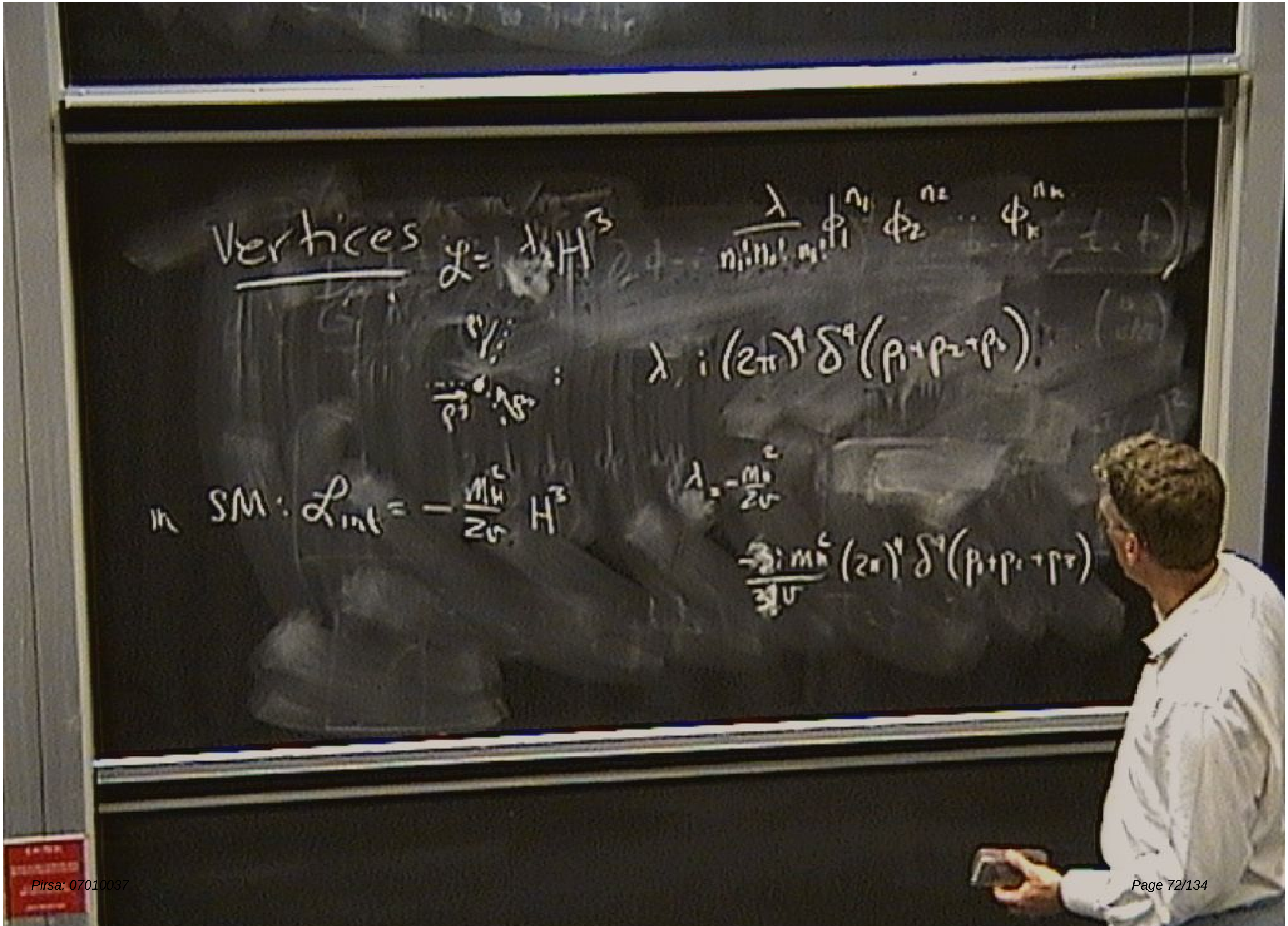
Vertices $\mathcal{L} = \frac{\lambda}{3!} H^3$ $\qquad \frac{\lambda}{n_1!\, n_2! \cdots n_k!} \phi_1^{n_1} \phi_2^{n_2} \cdots \phi_k^{n_k}$

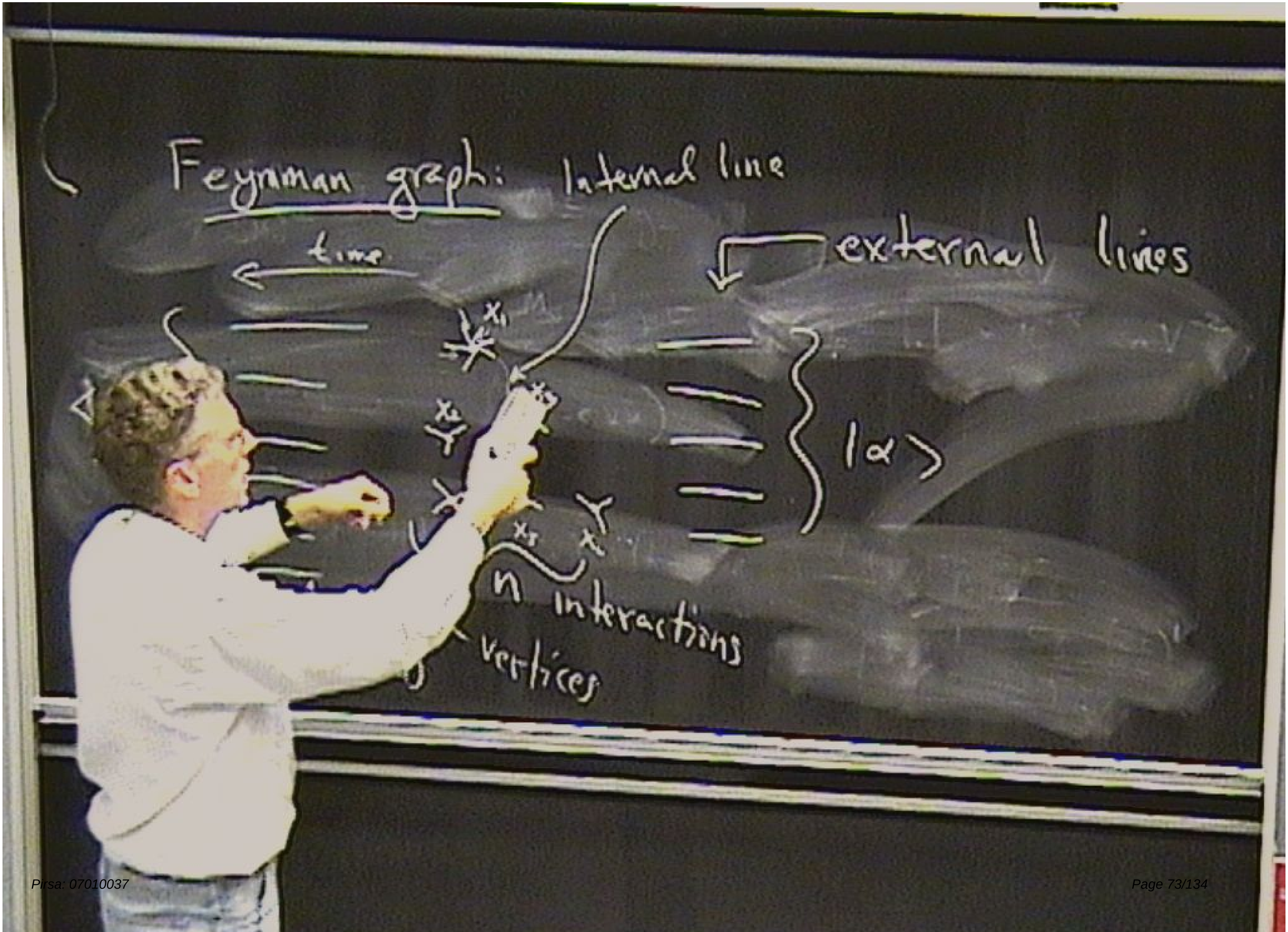
: $\lambda \; i (2\pi)^4 \delta^4(p_1+p_2+p_3)$

in SM: $\mathcal{L}_{int} = -\frac{m_H^2}{2v} H^3$ $\qquad \frac{\lambda}{6} = -\frac{m_H^2}{2v}$ $\qquad \lambda = \frac{3m_H^2}{v}$

$$-\frac{3i\, m_H^2}{v} (2\pi)^4 \delta^4(p_1+p_2+p_3)$$


Vertices
$\mathcal{L} = \lambda_3 H^3$
$\frac{\lambda}{n_1! n_2! \dots n_k!} \phi_1^{n_1} \phi_2^{n_2} \dots \phi_k^{n_k}$
$(2\pi)^4 \delta^4(p_1+p_2+p_3)$
in SM: $\mathcal{L}_{int} = -\frac{m_H^2}{2v} H^3$


Vertices
$\mathcal{L} = \lambda_3 H^3$
$\frac{\lambda}{n_1! n_2! \dots n_k!} \phi_1^{n_1} \phi_2^{n_2} \dots \phi_k^{n_k}$
$\lambda\, i (2\pi)^4 \delta^4(p_1+p_2+p_3)$
in SM: $\mathcal{L}_{int} = -\frac{m_H^2}{2v} H^3$
$\lambda_3 = -\frac{m_H^2}{2v}$
$-\frac{3!\, i m_H^2}{3!\, v} (2\pi)^4 \delta^4(p_1+p_2+p_3)$


Feynman graph: Internal line
time
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x_1
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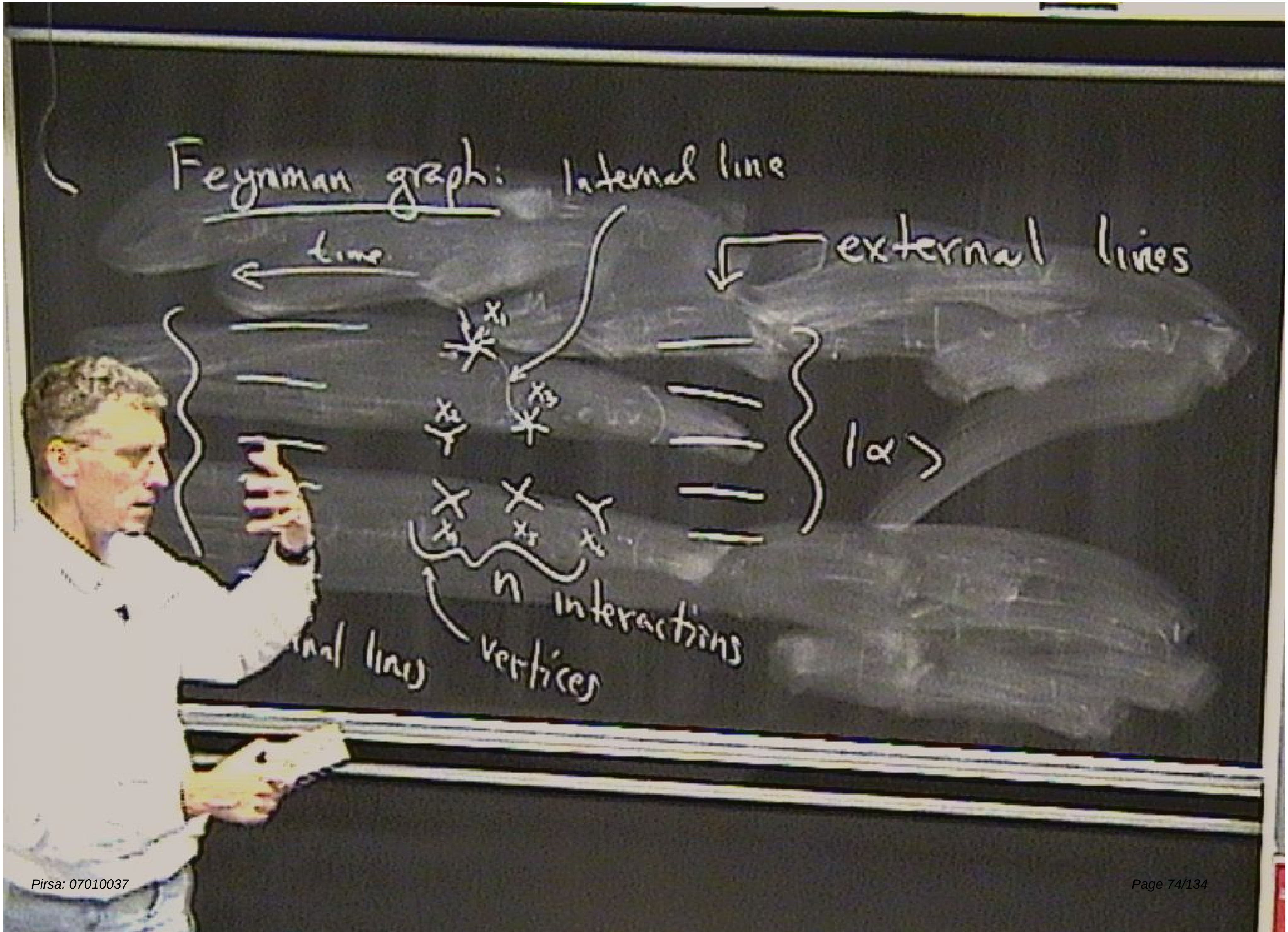
Feynman graph:
internal line
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|α>
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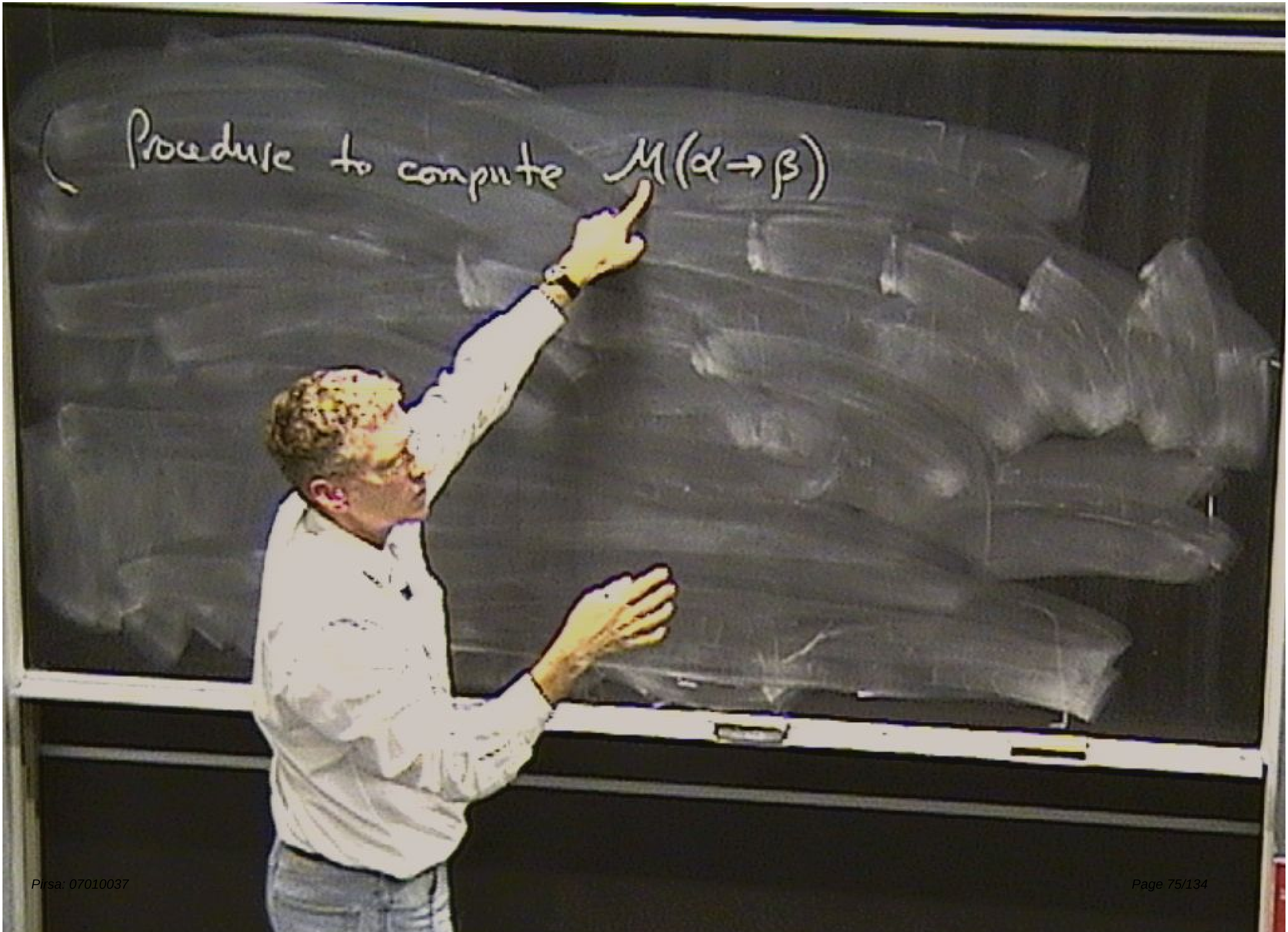
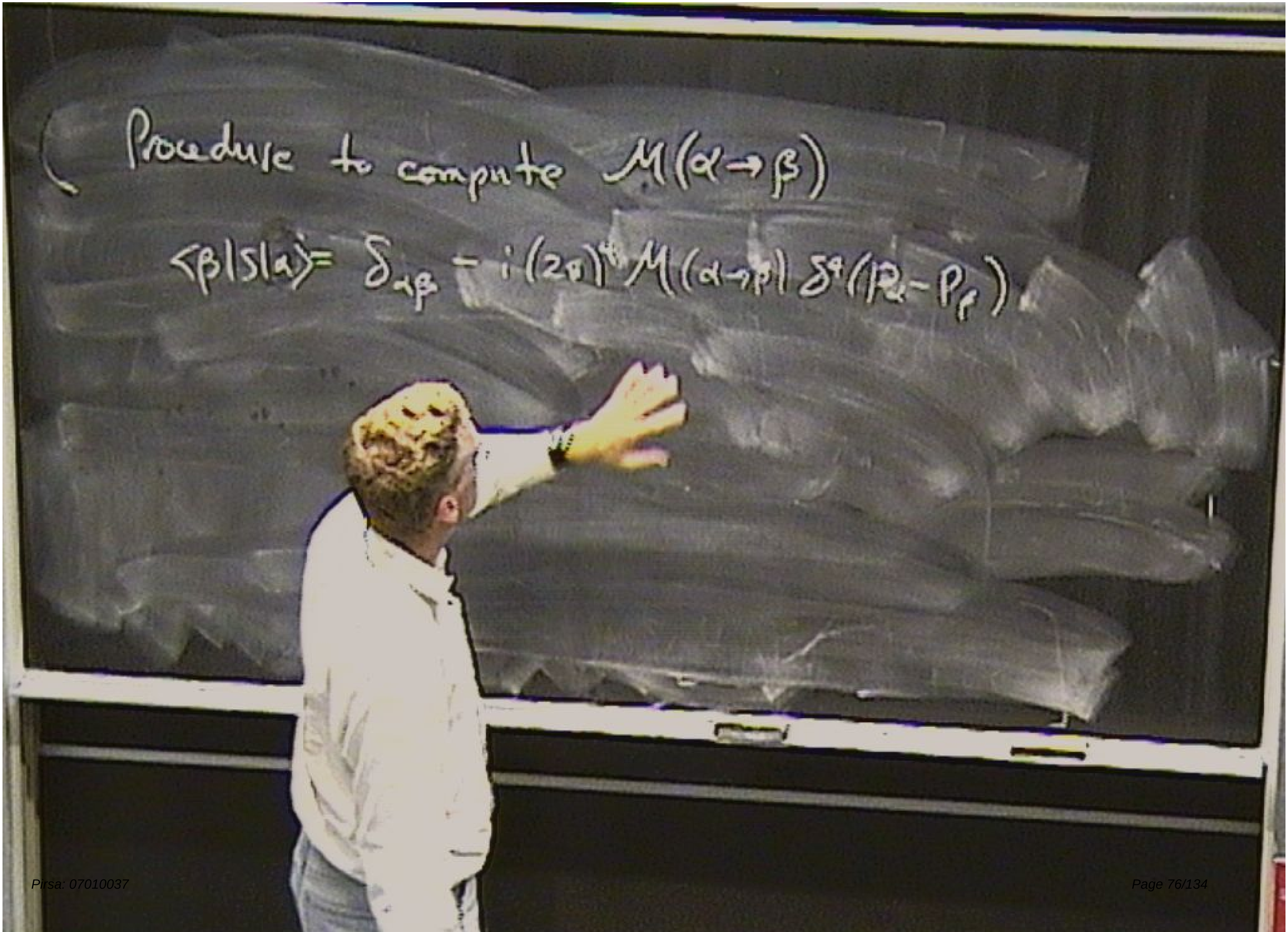

Procedure to compute M(α → β)


Procedure to compute $\mathcal{M}(\alpha \to \beta)$
$\langle \beta | S | \alpha \rangle = \delta_{\alpha\beta} - i(2\pi)^4 \mathcal{M}(\alpha \to \beta)\, \delta^4(P_\alpha - P_f)$

Procedure to compute $S(\alpha \to \beta)$

$$\langle \beta | S | \alpha \rangle = \delta_{\alpha\beta} - i(2\pi)^4 \mathcal{M}(\alpha \to \beta)\, \delta^4(P_i - P_f)$$

1) Connect initial particles + vertices in all possible ways.

Procedure to compute $\mathcal{S}(\alpha \to \beta)$

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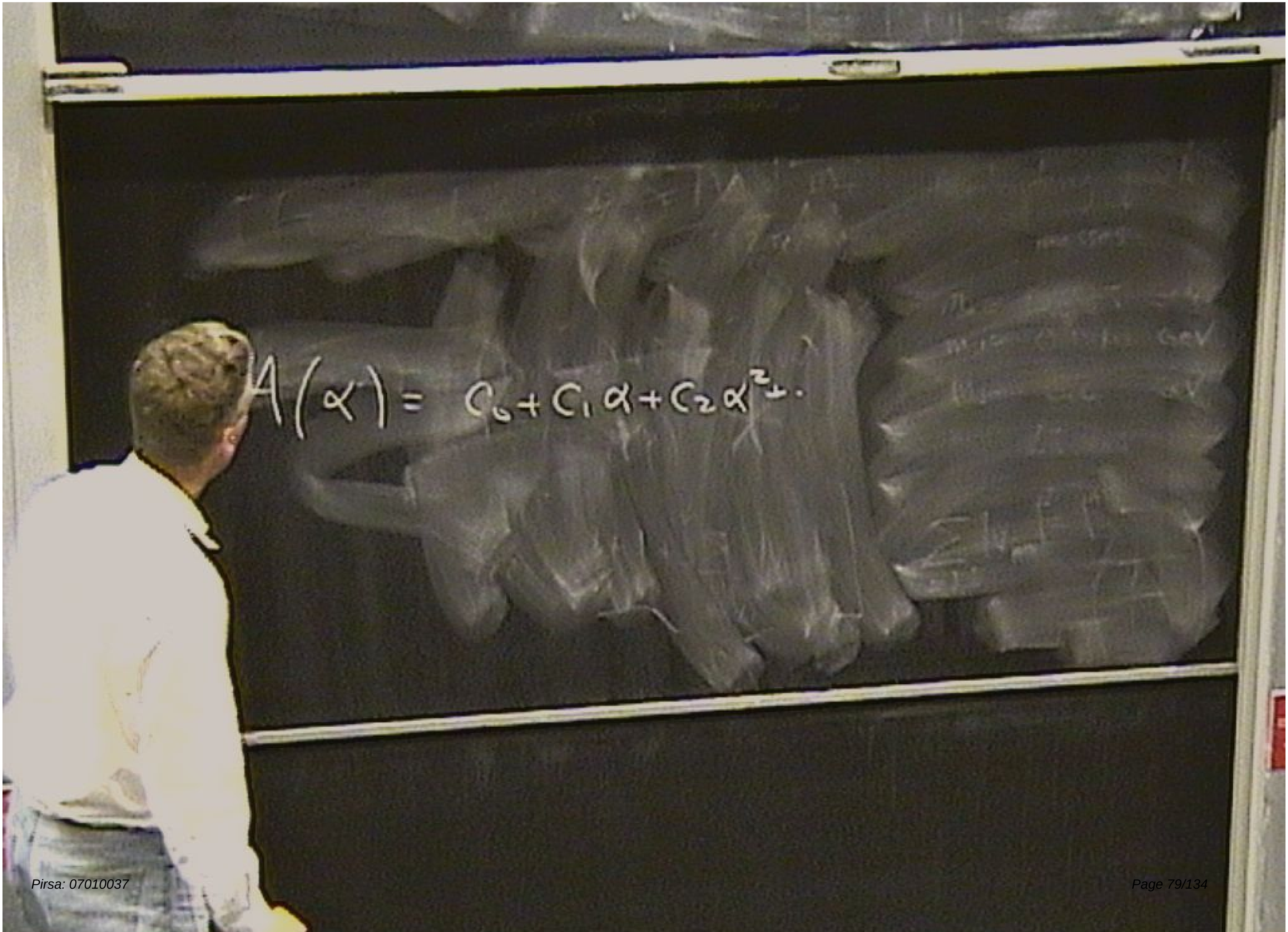
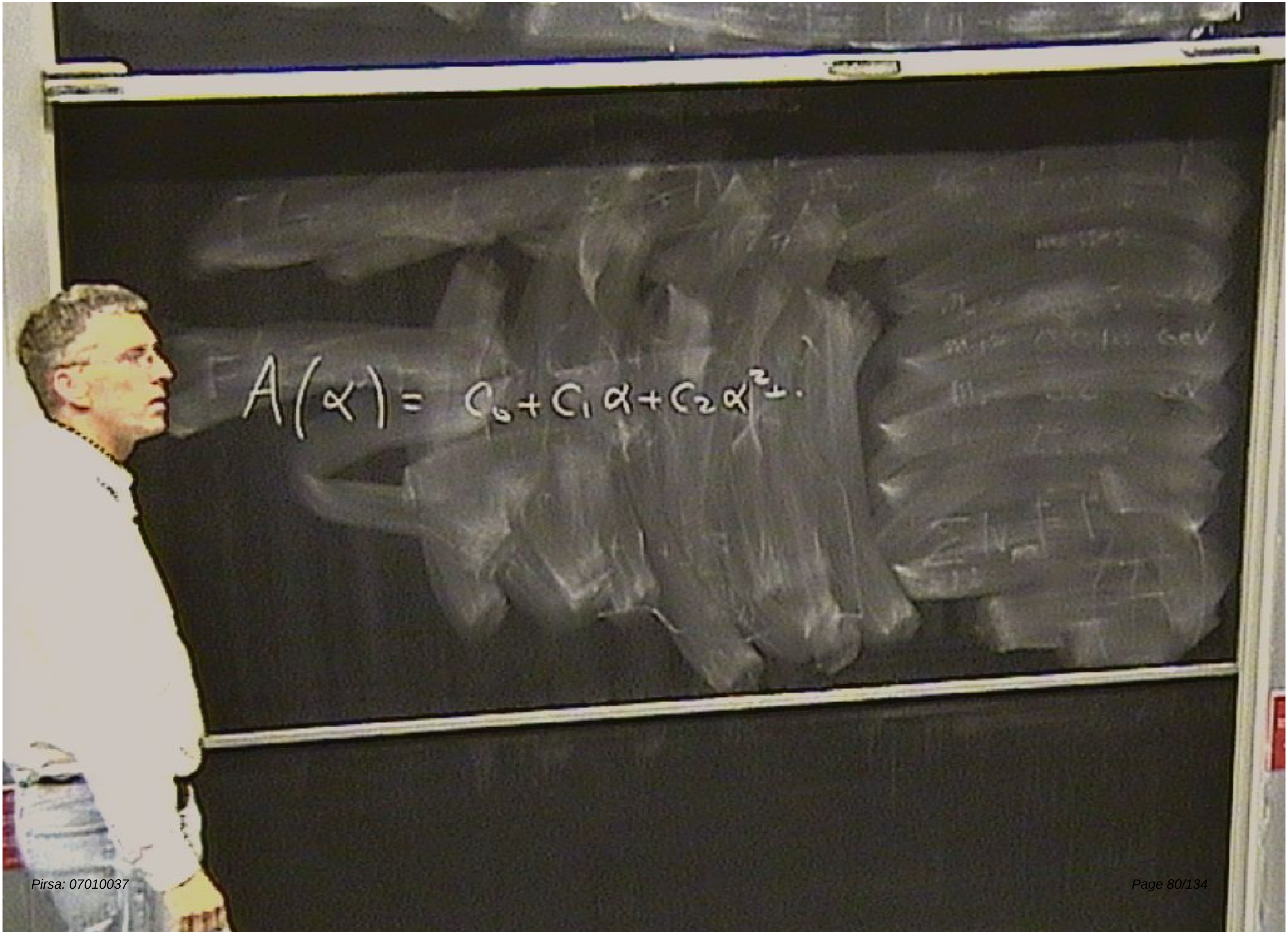
1) Connect initial particles + vertices in all possible ways.

2) Collect all distinct graphs:



$A(\alpha) = c_0 + c_1\alpha + c_2\alpha^2 + \cdots$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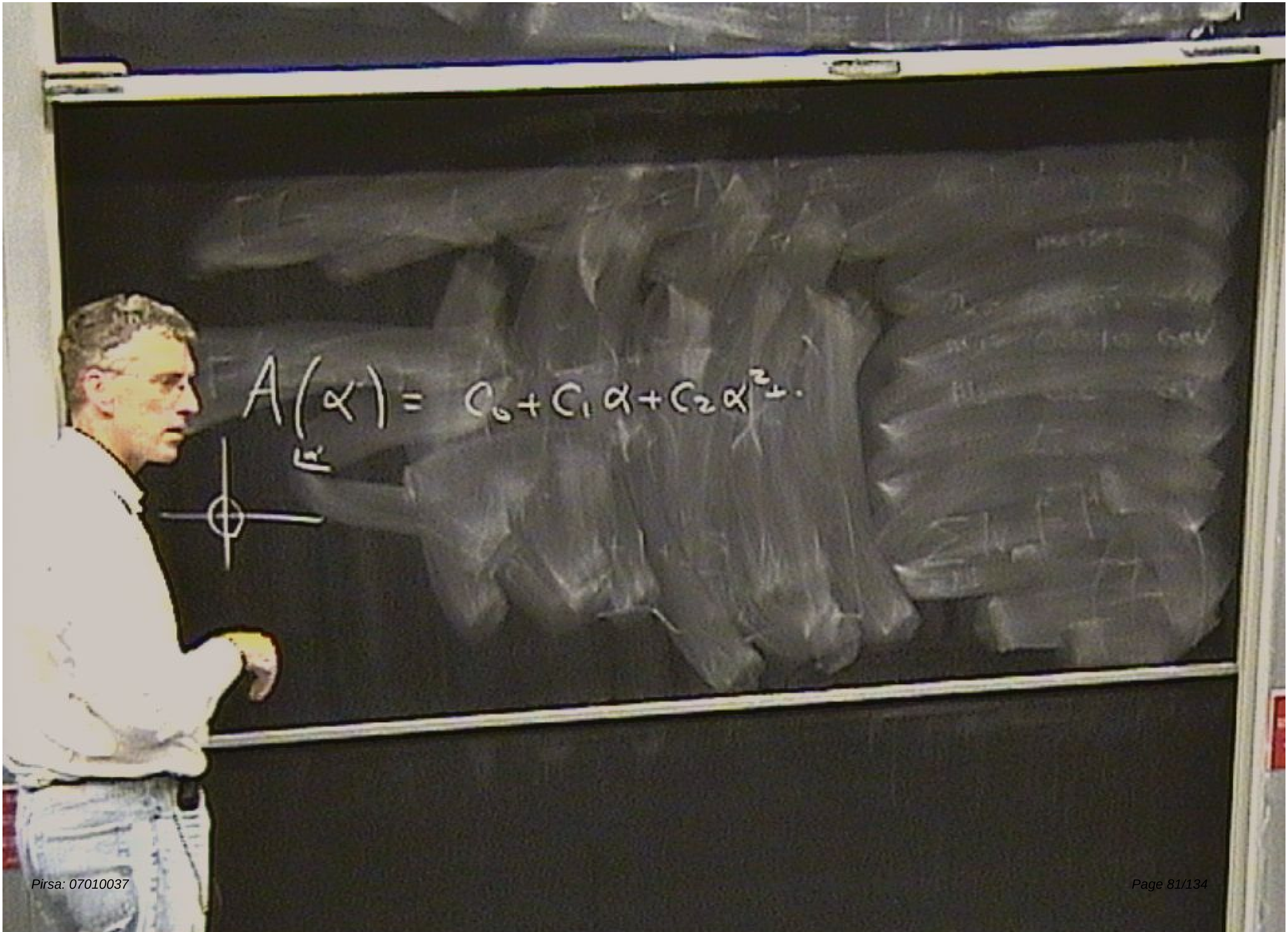
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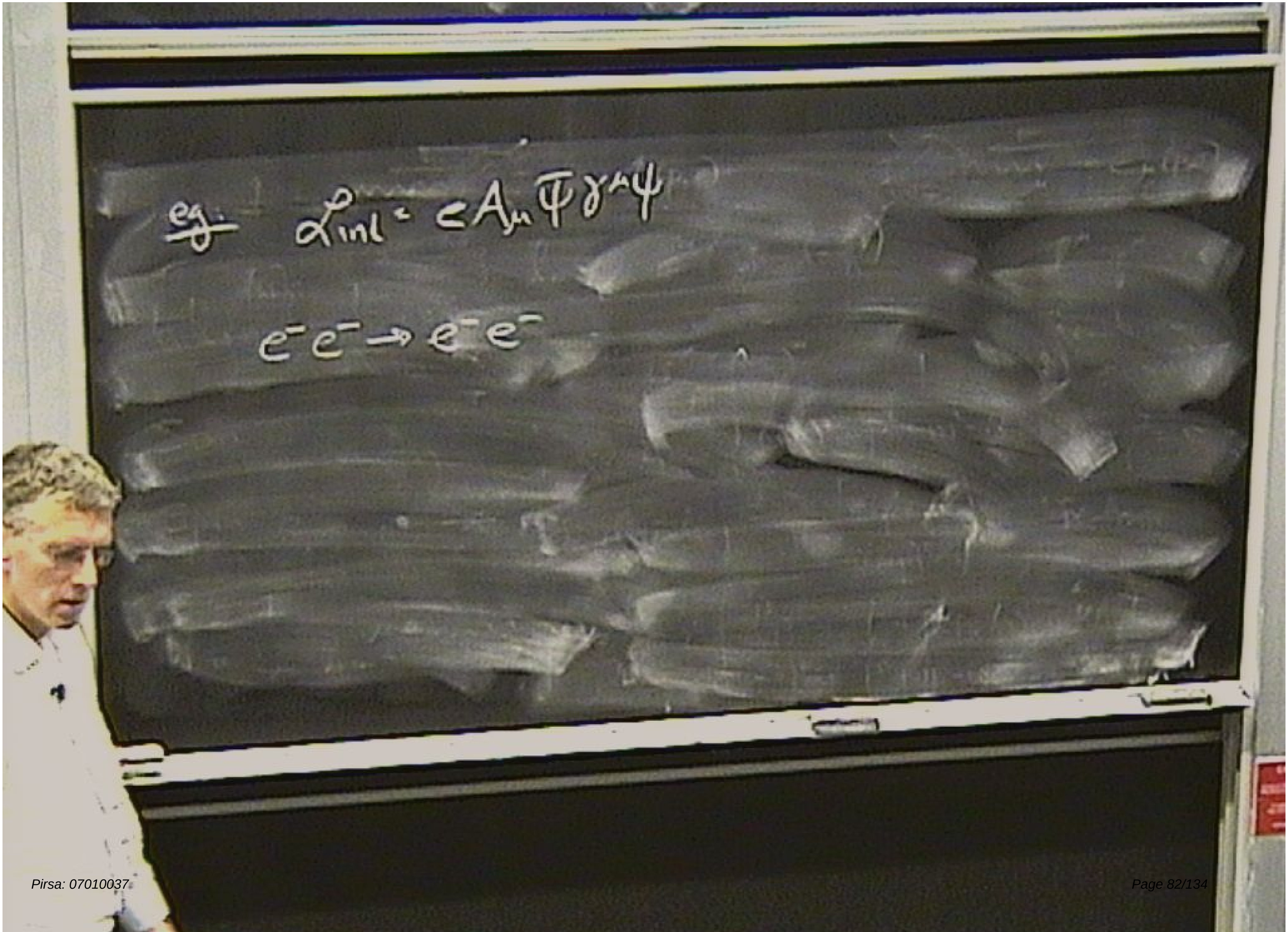
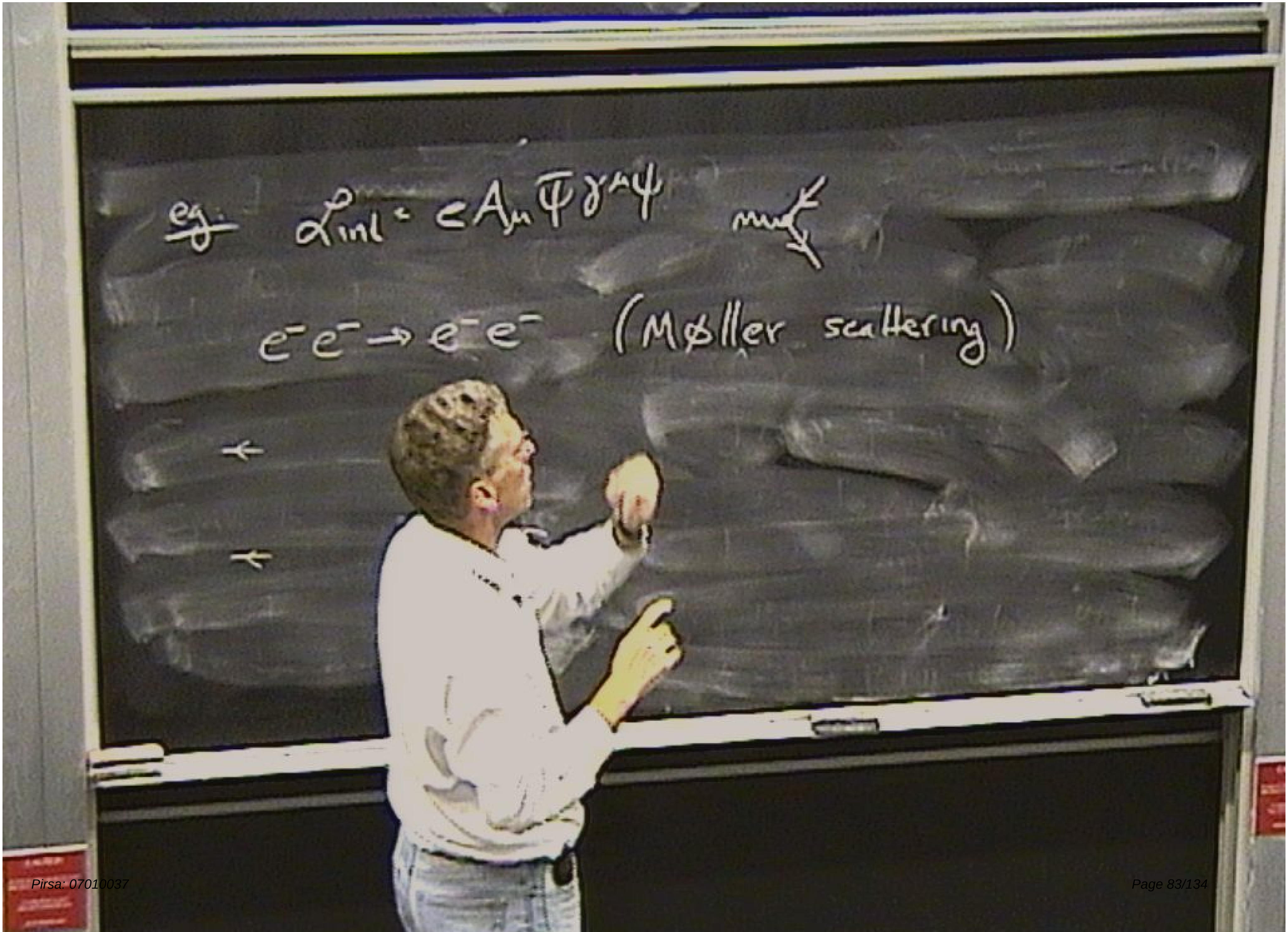
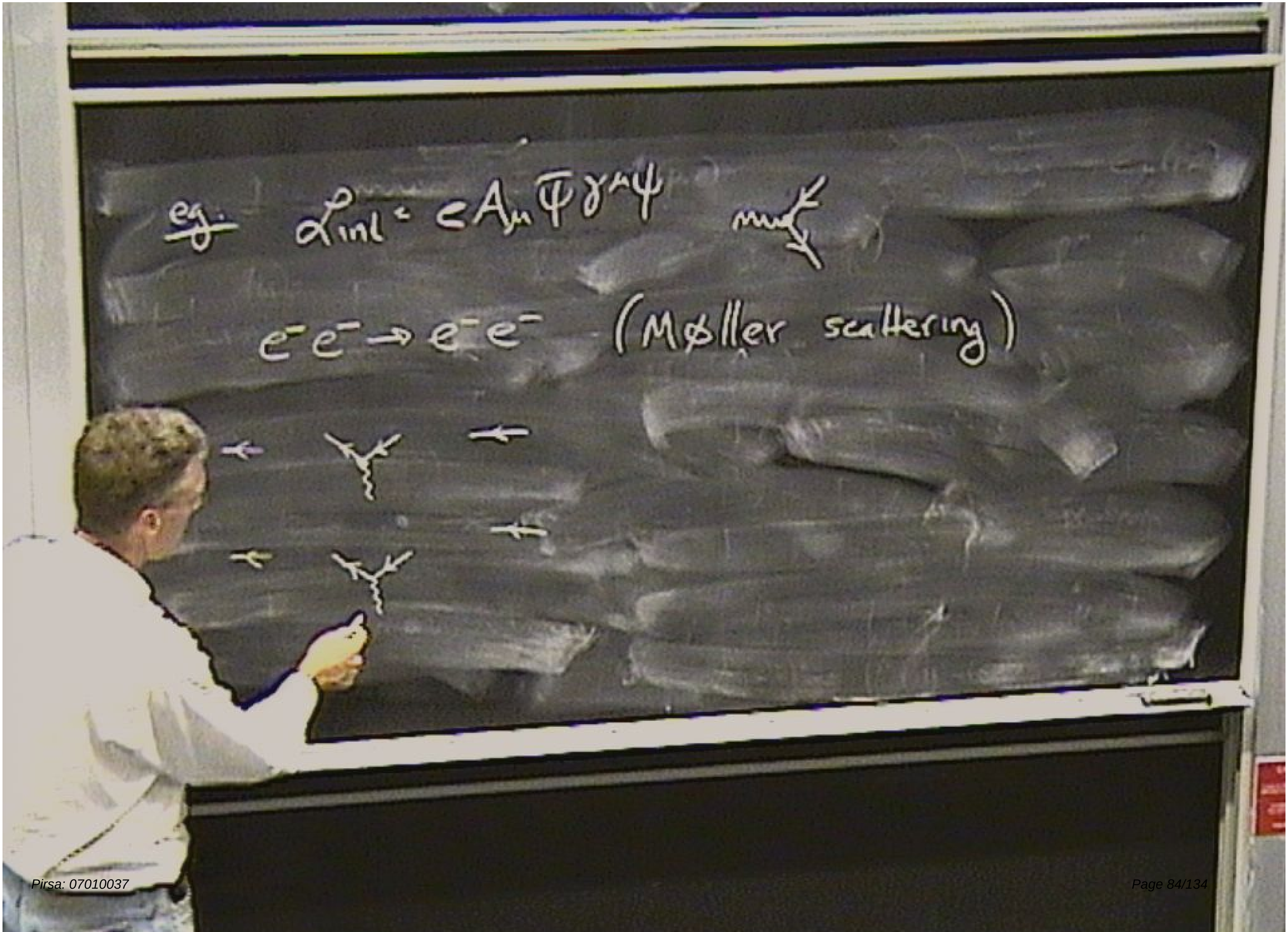

e.g. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \psi$
$e^- e^- \to e^- e^-$


e.g. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \psi$
$e^- e^- \rightarrow e^- e^-$ (Møller scattering)



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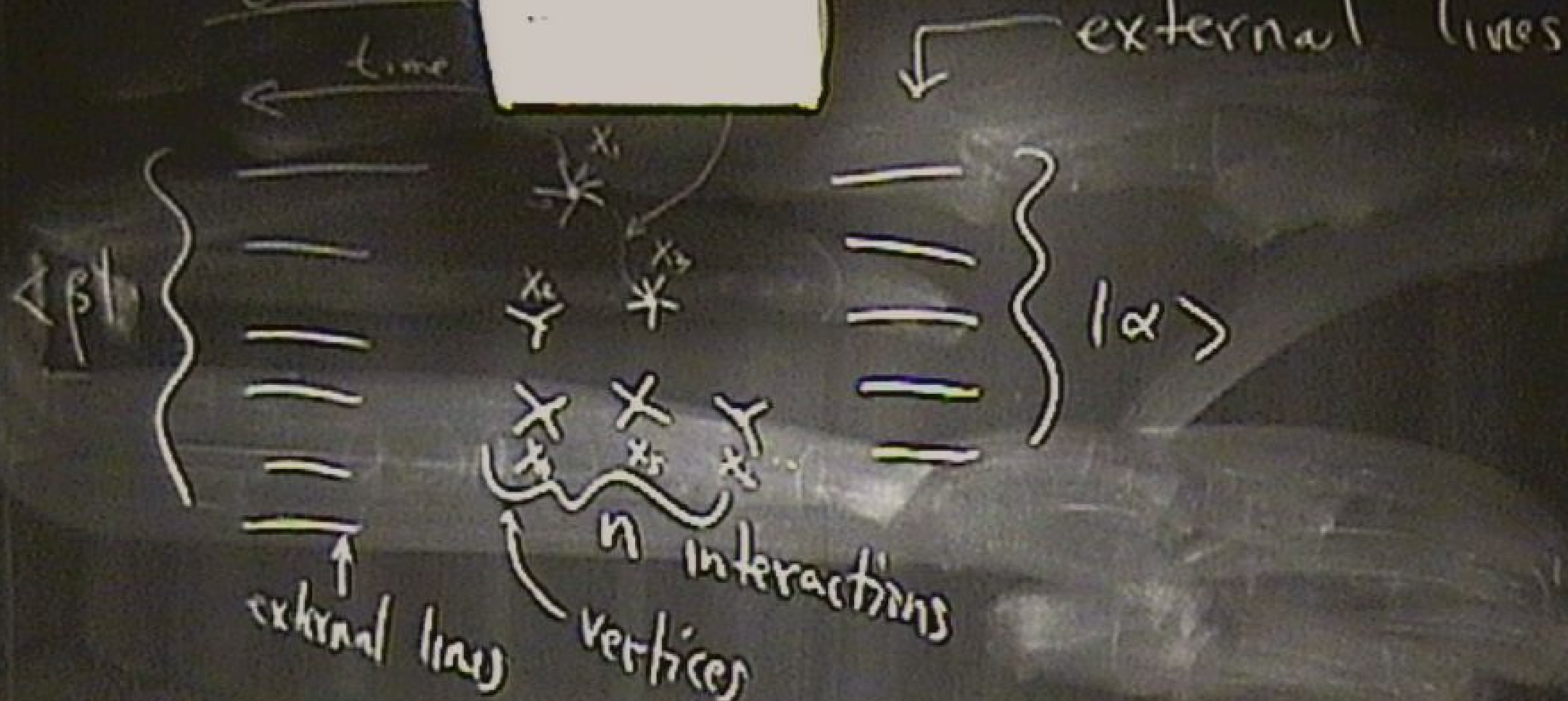
Feynman



Procedure to compute $S(\alpha\to\beta)$

$\langle\beta|S|\alpha\rangle = S_{\beta\alpha} = i(2\pi)^4 M(\alpha\to\beta)\,\delta^4(\ldots)$

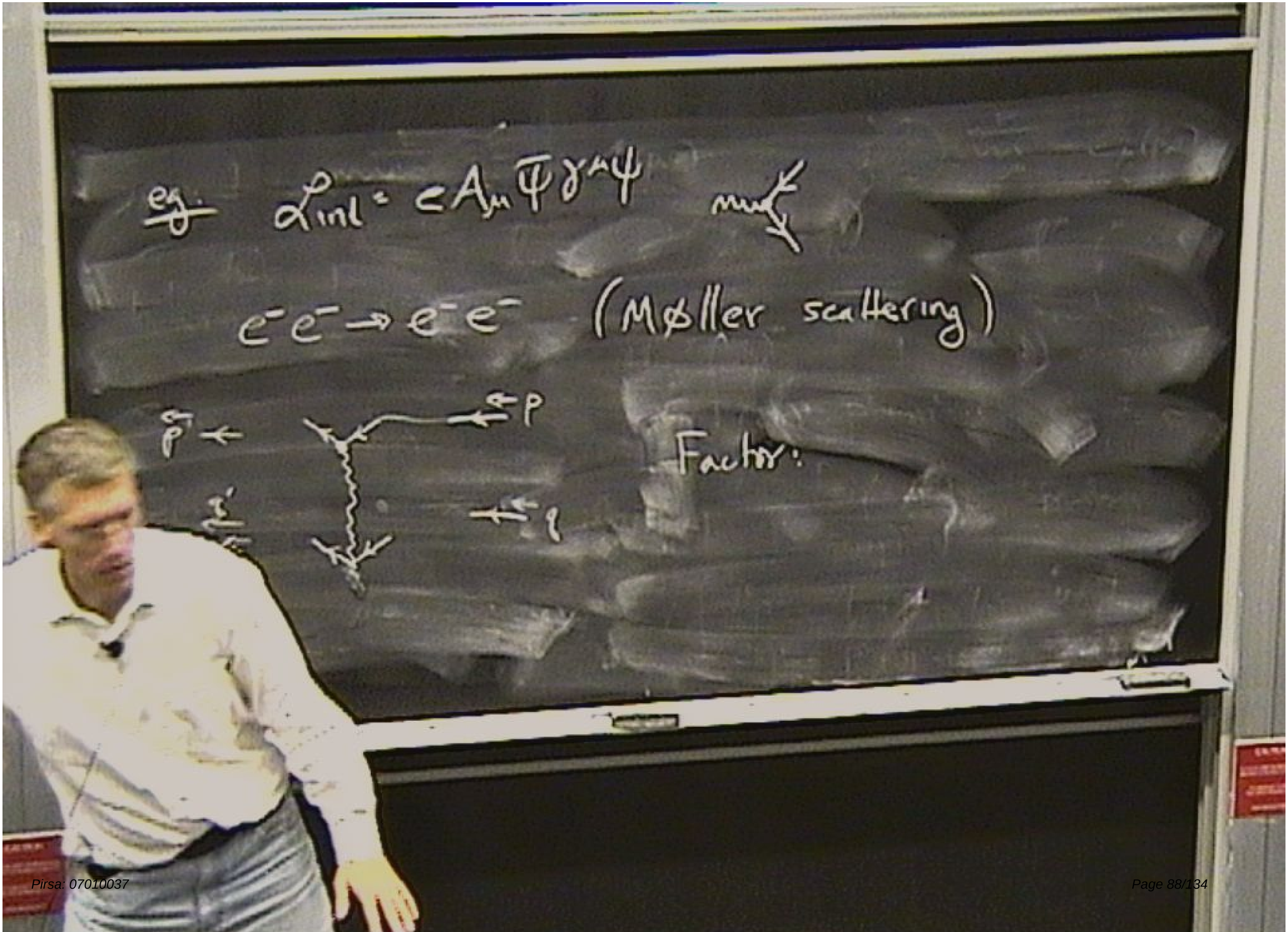
Procedure to compute $S(\alpha \to \beta)$

$$\langle \beta | S | \alpha \rangle = \delta_{\alpha\beta} - i (2\pi)^4 \mathcal{M}(\alpha \to \beta)\, \delta^4(P_i - P_f)$$

1) Connect initial particles + vertices in all possible ways.

2) Collect all distinct graphs.

3) multiply by the appropriate statistical factor $1/n!$ $n = \#$ vertices



e.g. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \psi$
$e^- e^- \to e^- e^-$ (Møller scattering)
p'
p
q
Factor:

e.g. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \psi$ 

$e^- e^- \to e^- e^-$ (Møller scattering)

Factor: $\frac{1}{2!} \times 2$

Procedure to compute $S(\alpha \to \beta)$

$$\langle\beta|S|\alpha\rangle = \delta_{\alpha\beta} - i(2\pi)^4 \mathcal{M}(\alpha\to\beta)\, \delta^4(P_\alpha - P_f)$$

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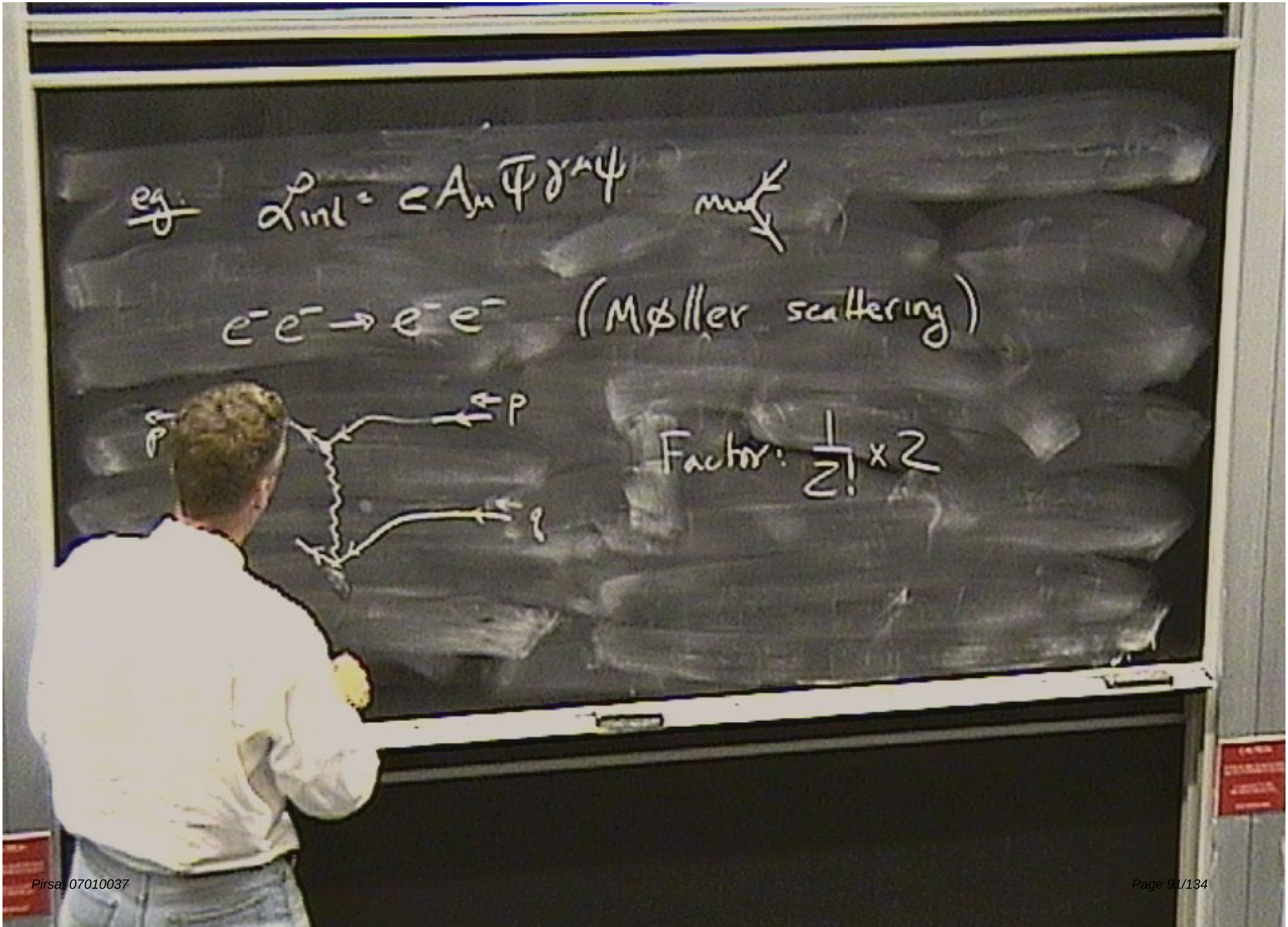

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p
p
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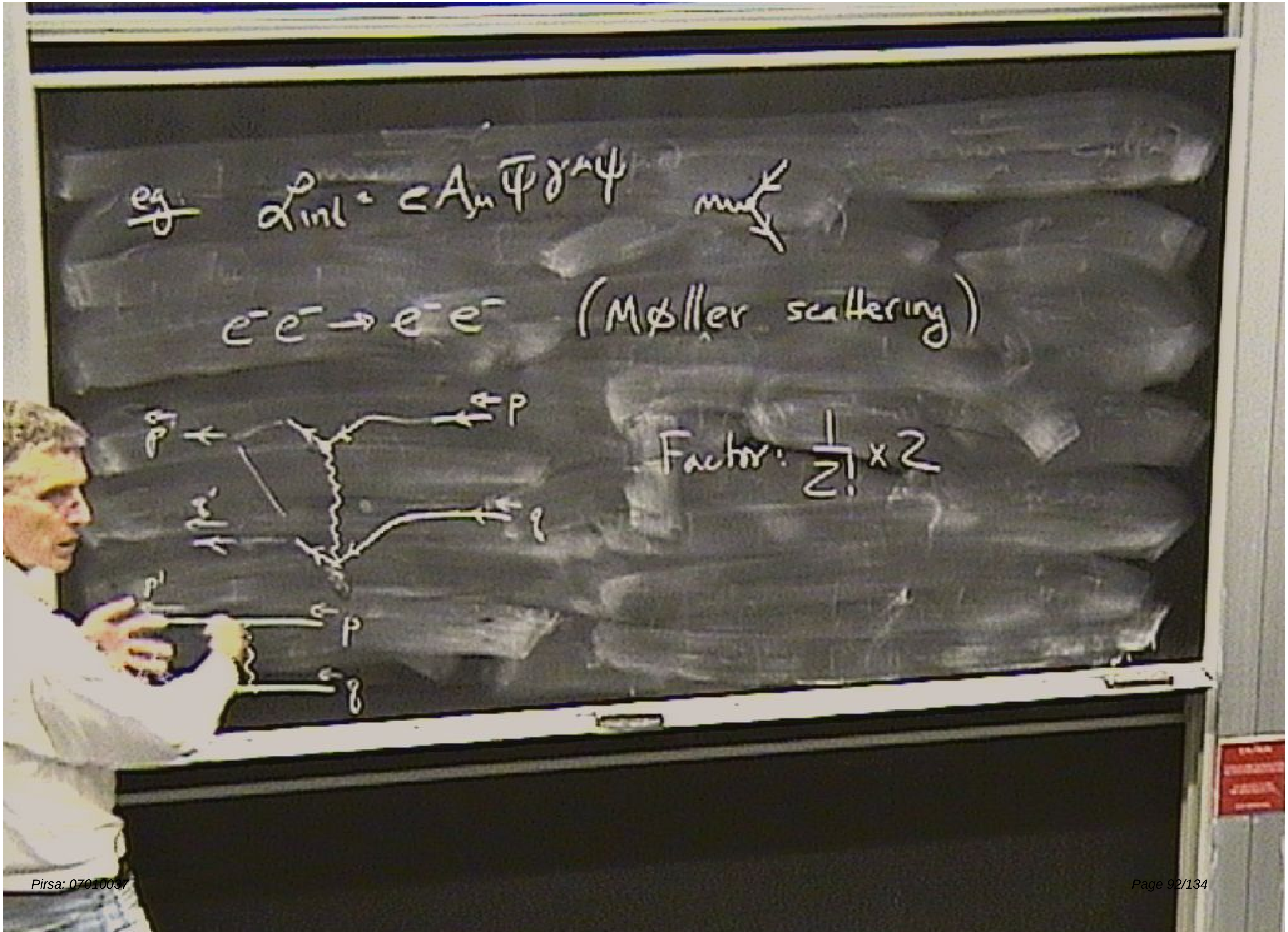

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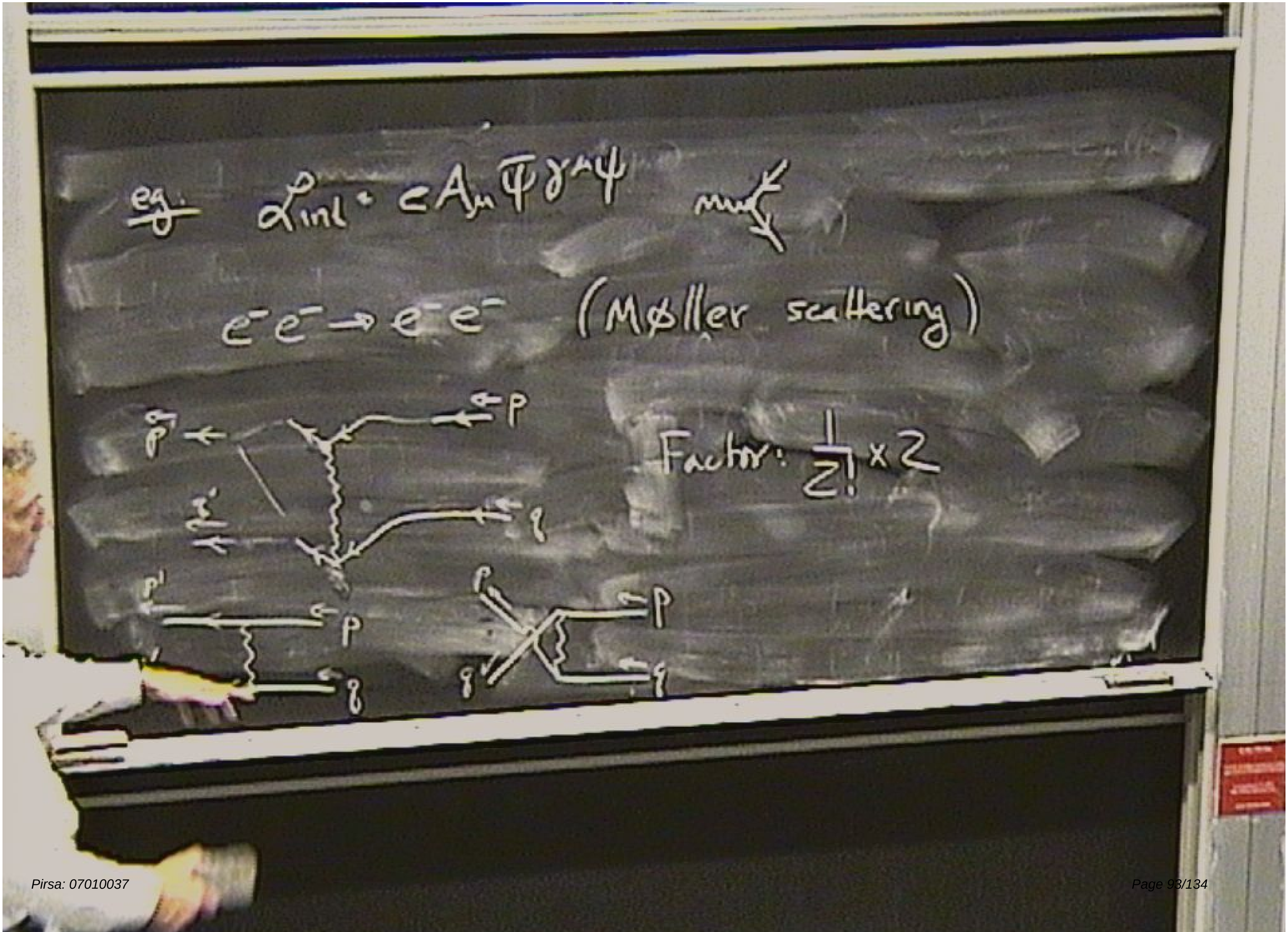
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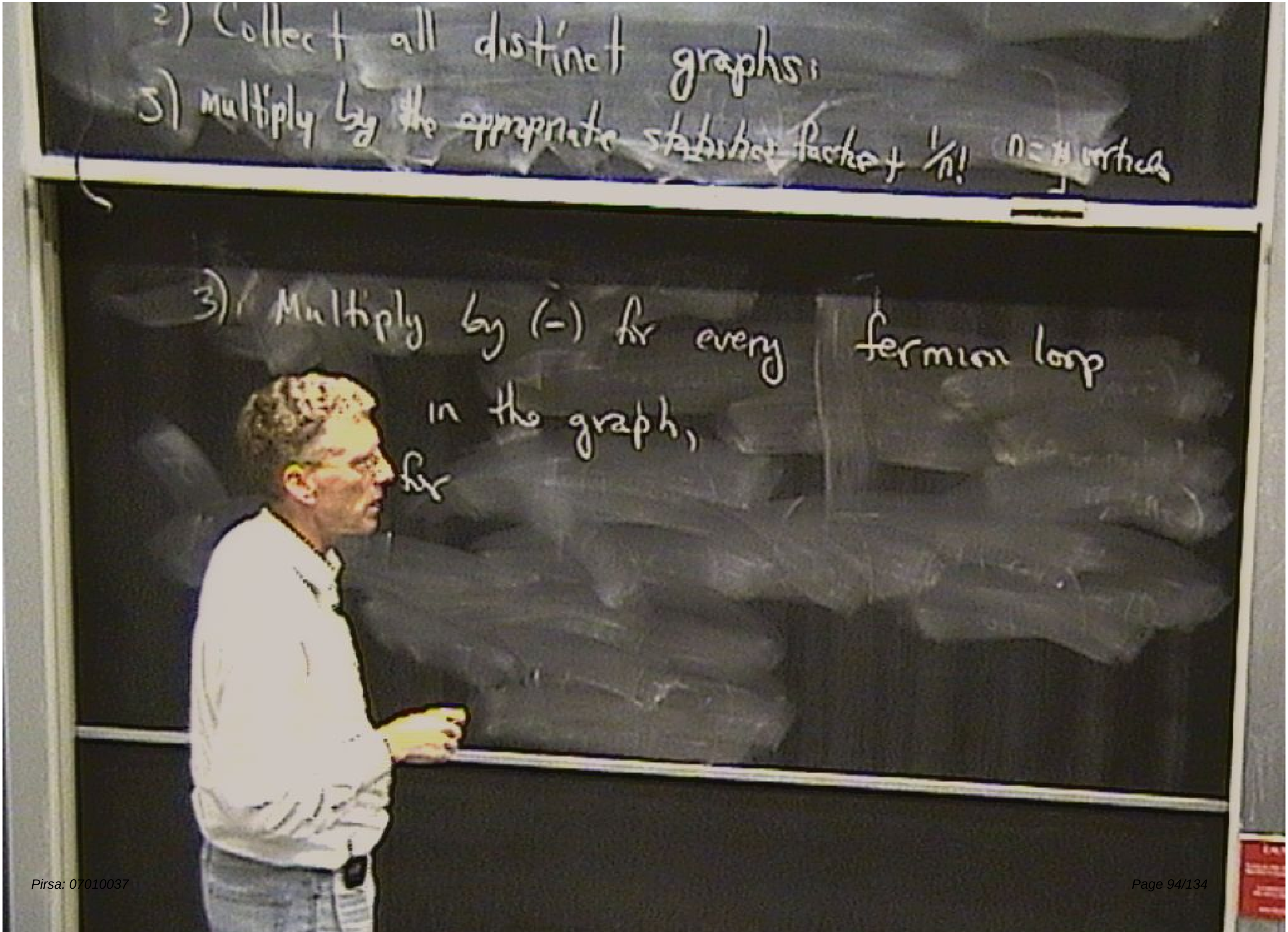
2) Collect all distinct graphs;
3) multiply by the appropriate statistical factor 1/n! n = # vertices
3) Multiply by (-) for every fermion loop
in the graph,
for

2) Collect all distinct graphs;

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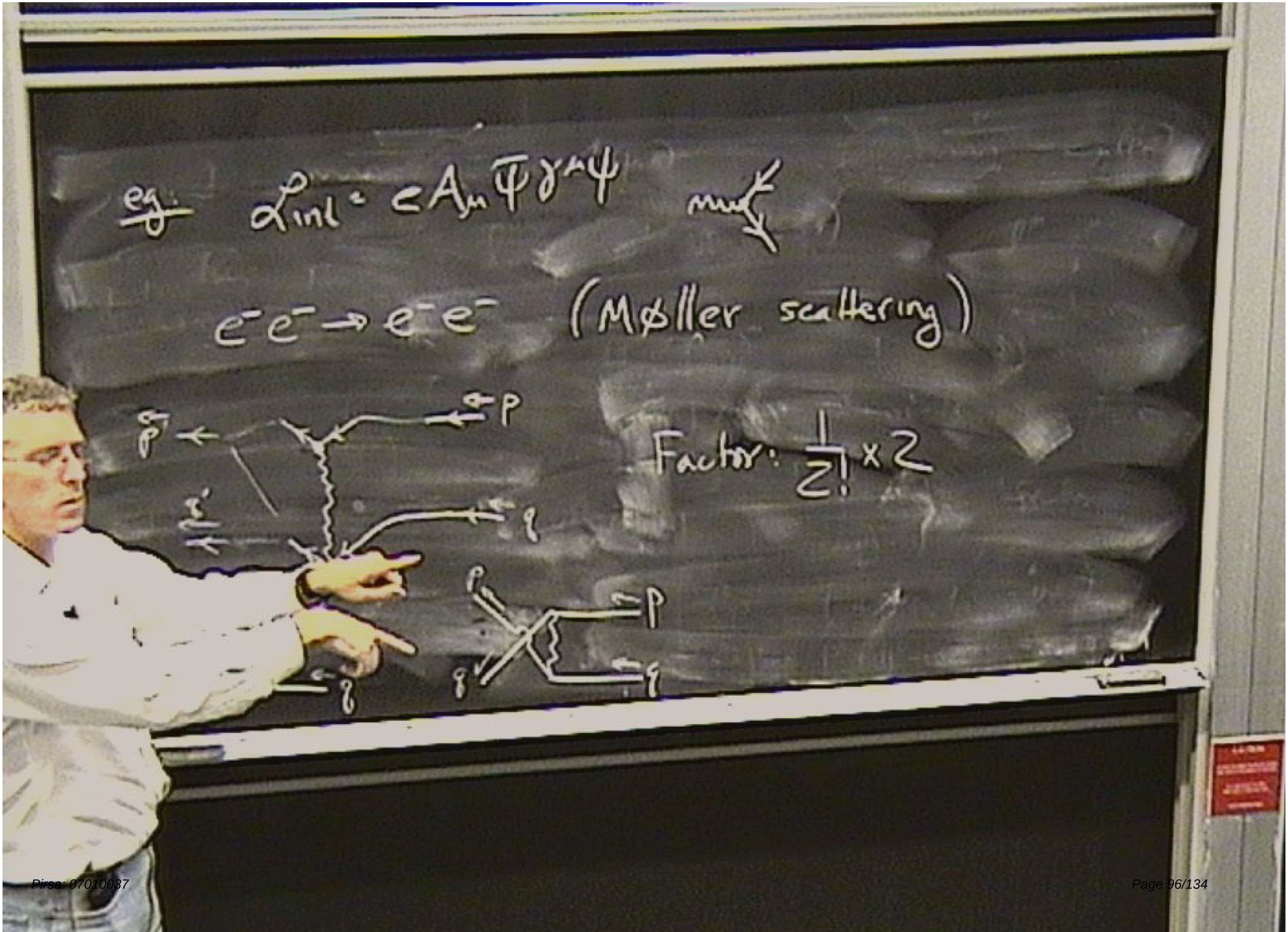
3) Multiply by $(-)$ for every fermion loop in the graph,

and for every time a pair of external fermions are interchanged.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p
p'
q
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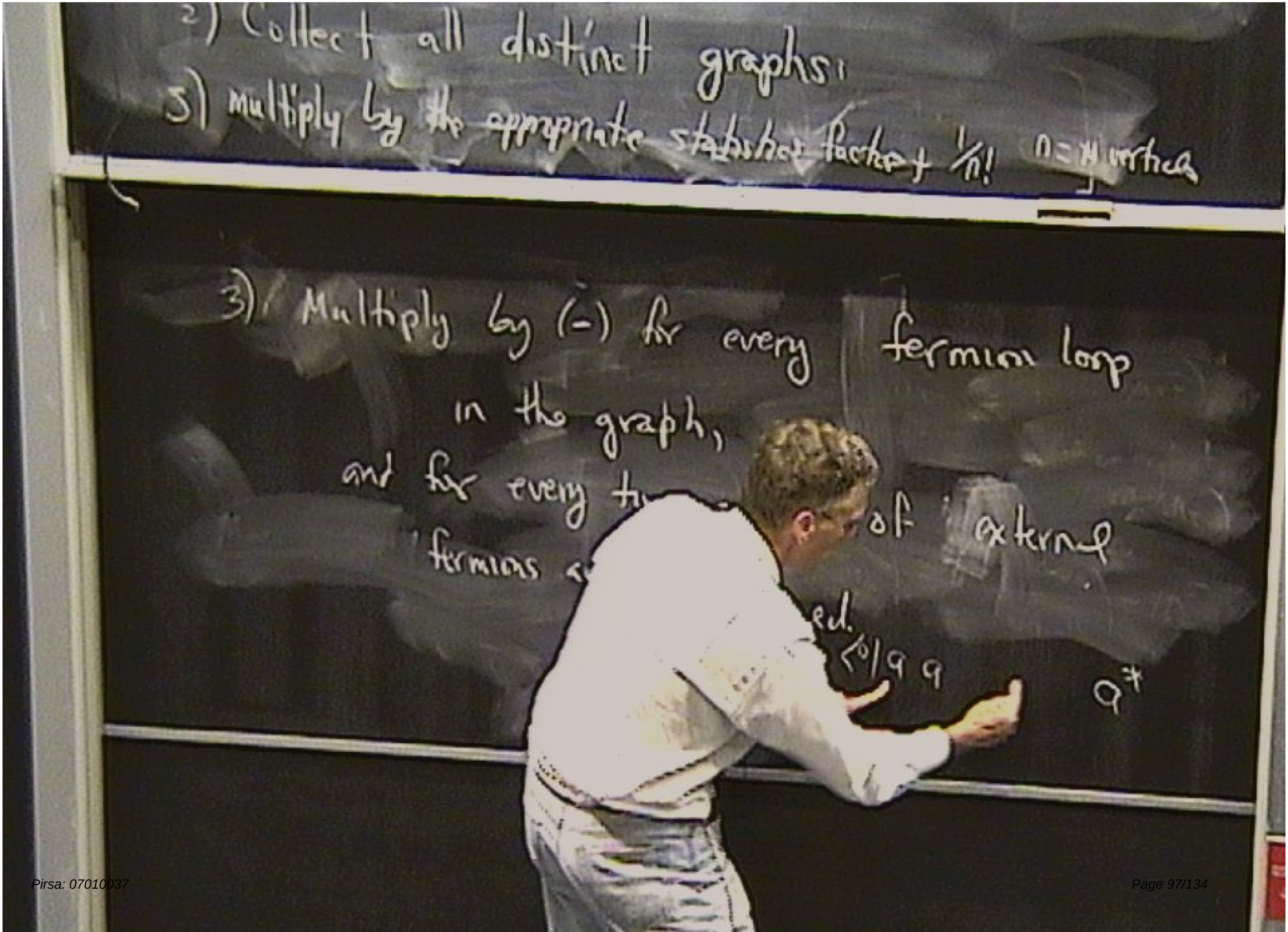
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fermions

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$\{a, a\}$ a a^*

e.g. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \psi$ 

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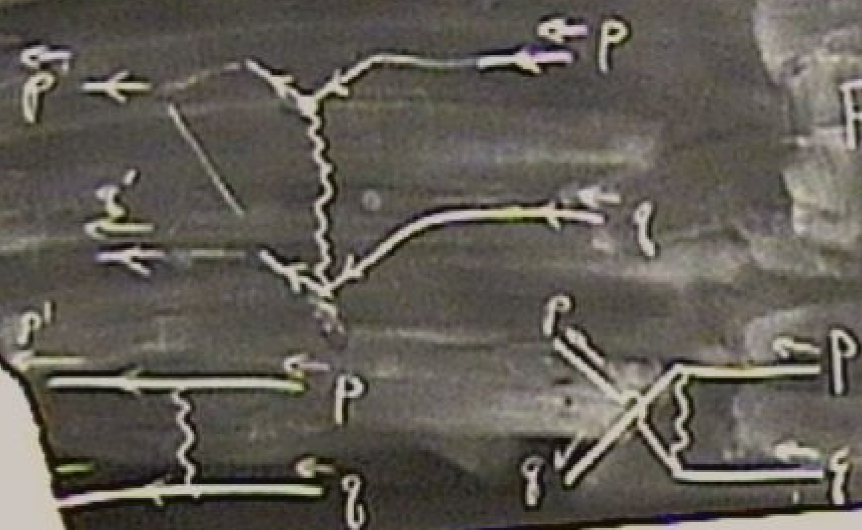

Factor: $\frac{1}{2!} \times 2$

e.g. $\mathcal{L}_{int} = e A_\mu \bar{\psi} \gamma^\mu \psi$

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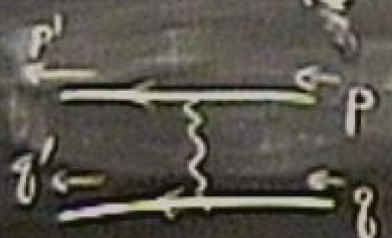

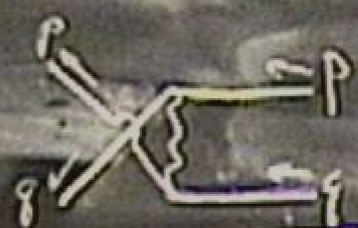

Factor: $\frac{1}{2!} \times 2$

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3) Multiply by $(-)$ for every fermion loop in the graph,

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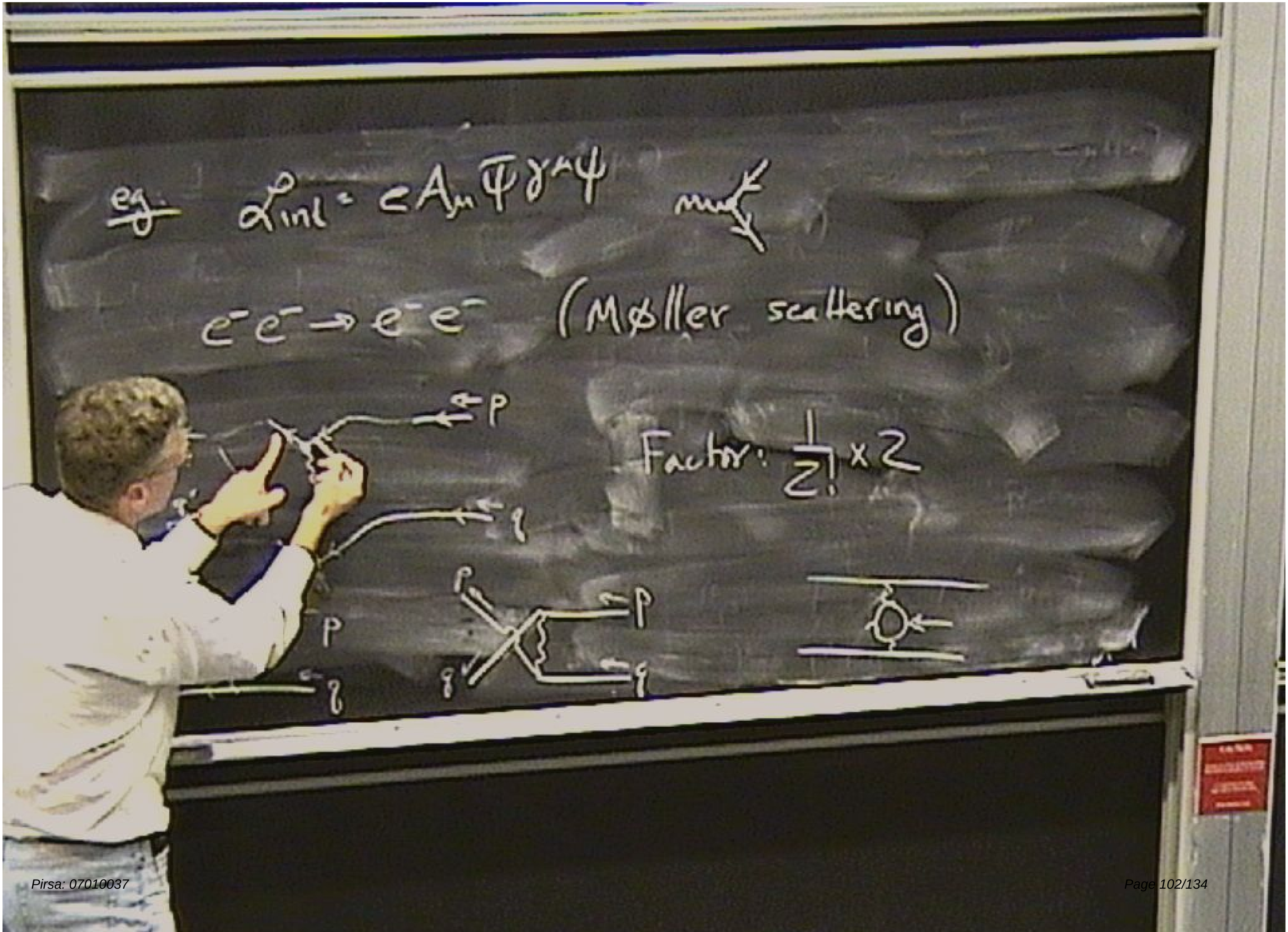
$\langle 0 | q \, q \quad a \quad a^{*}$

$\psi \bar{\psi} \psi \quad \bar{\psi} \psi$


e.g. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \psi$
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Factor: $\frac{1}{2!} \times 2$
p
q
p
q
p
q
p
q

$\mathcal{L}_{int} = \lambda\phi^4$

$\phi\phi \to \phi\phi$

$\xi = 1$: Feynman

$\xi = 0$: Landau

$\xi \to \infty$: Unitary

$\mathcal{L}_{int} = \lambda\phi^4$

$\phi\phi \to \phi\phi$

$\xi = 1$: Feynman

$\xi = 0$: Landau

$\xi \to \infty$: Unitary

$\mathcal{L}_{int} = \lambda\phi^4$

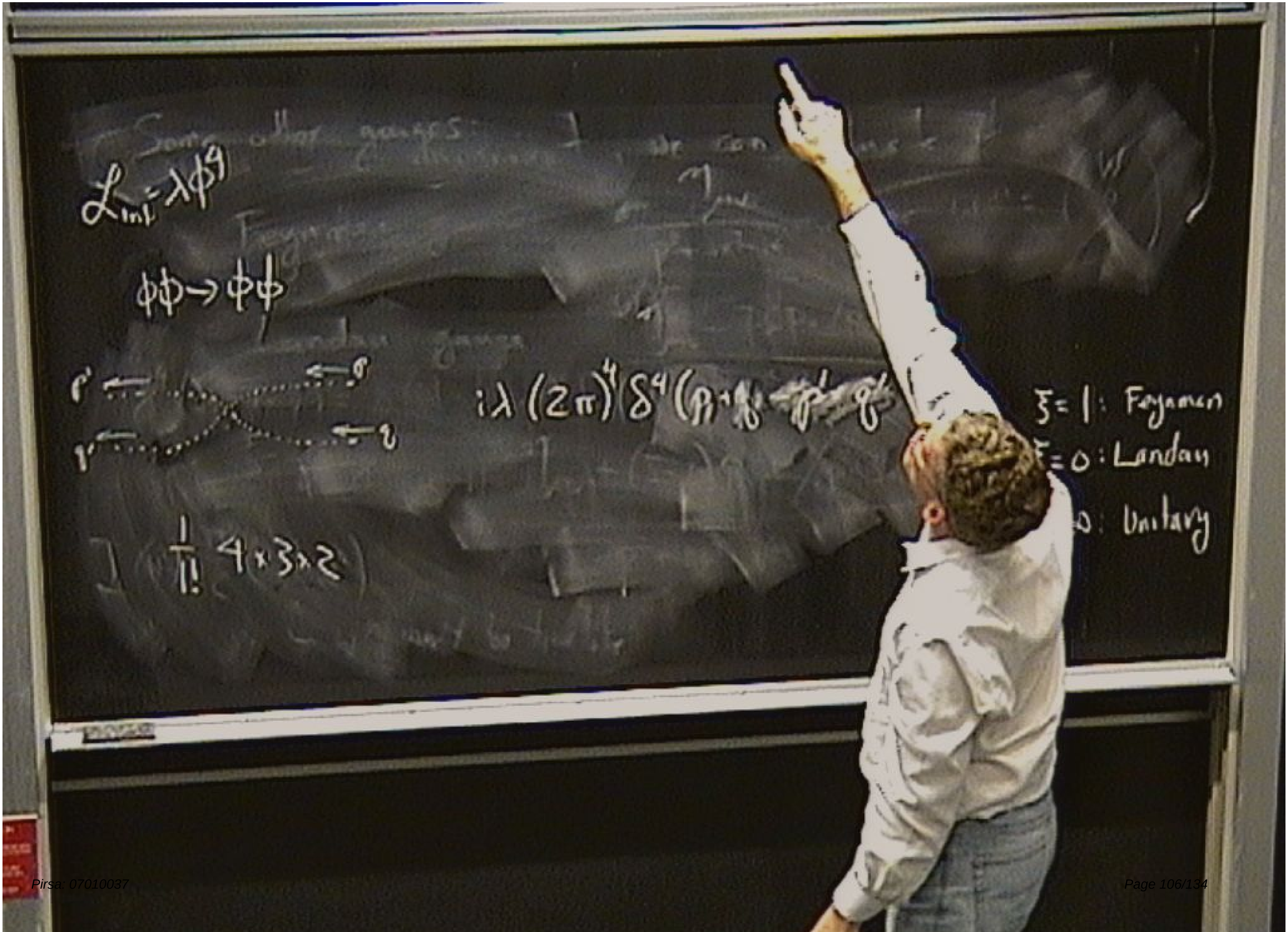
$\phi\phi \to \phi\phi$

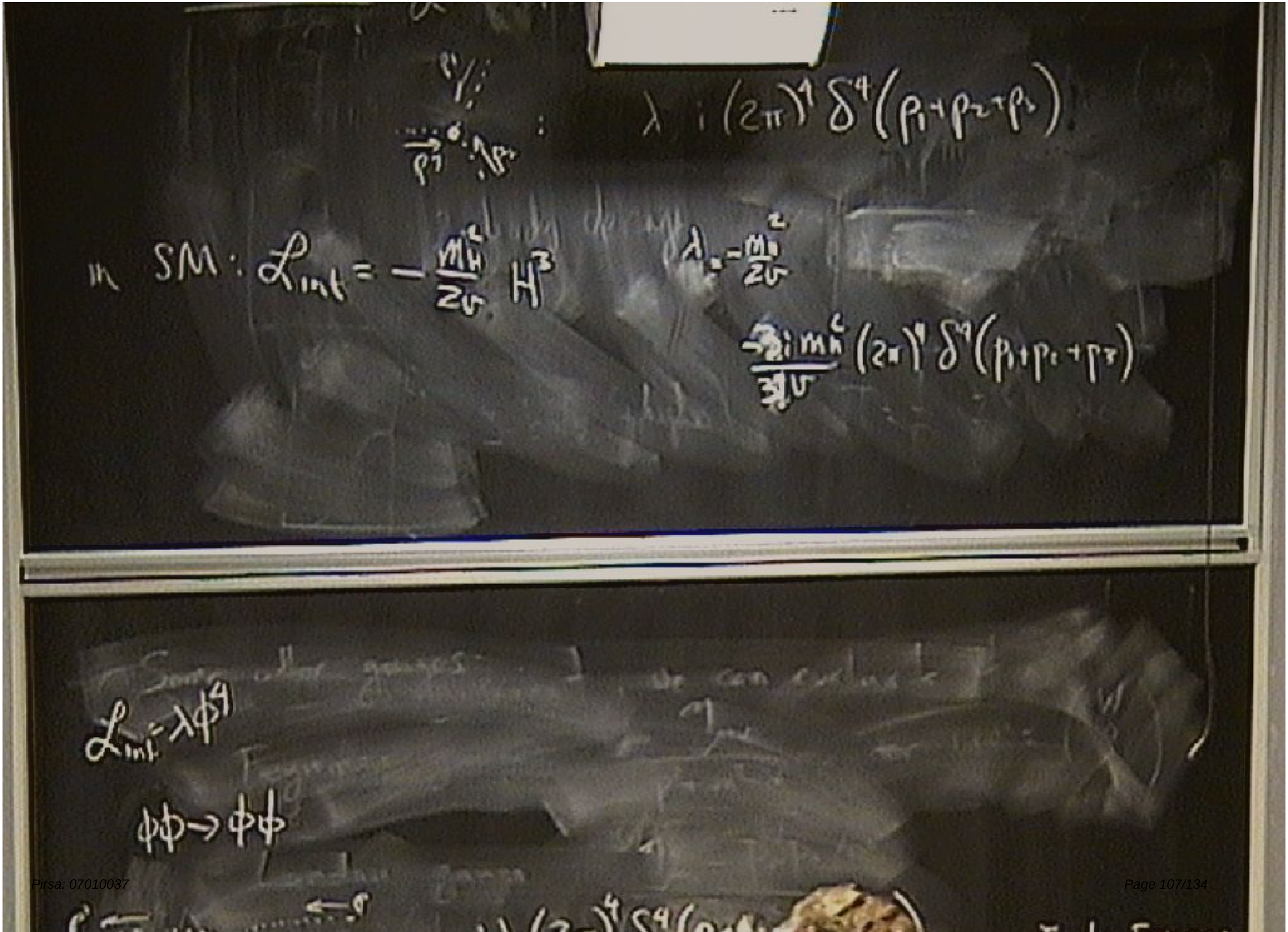
$\frac{1}{4!}\, 4\times 3\times 2$

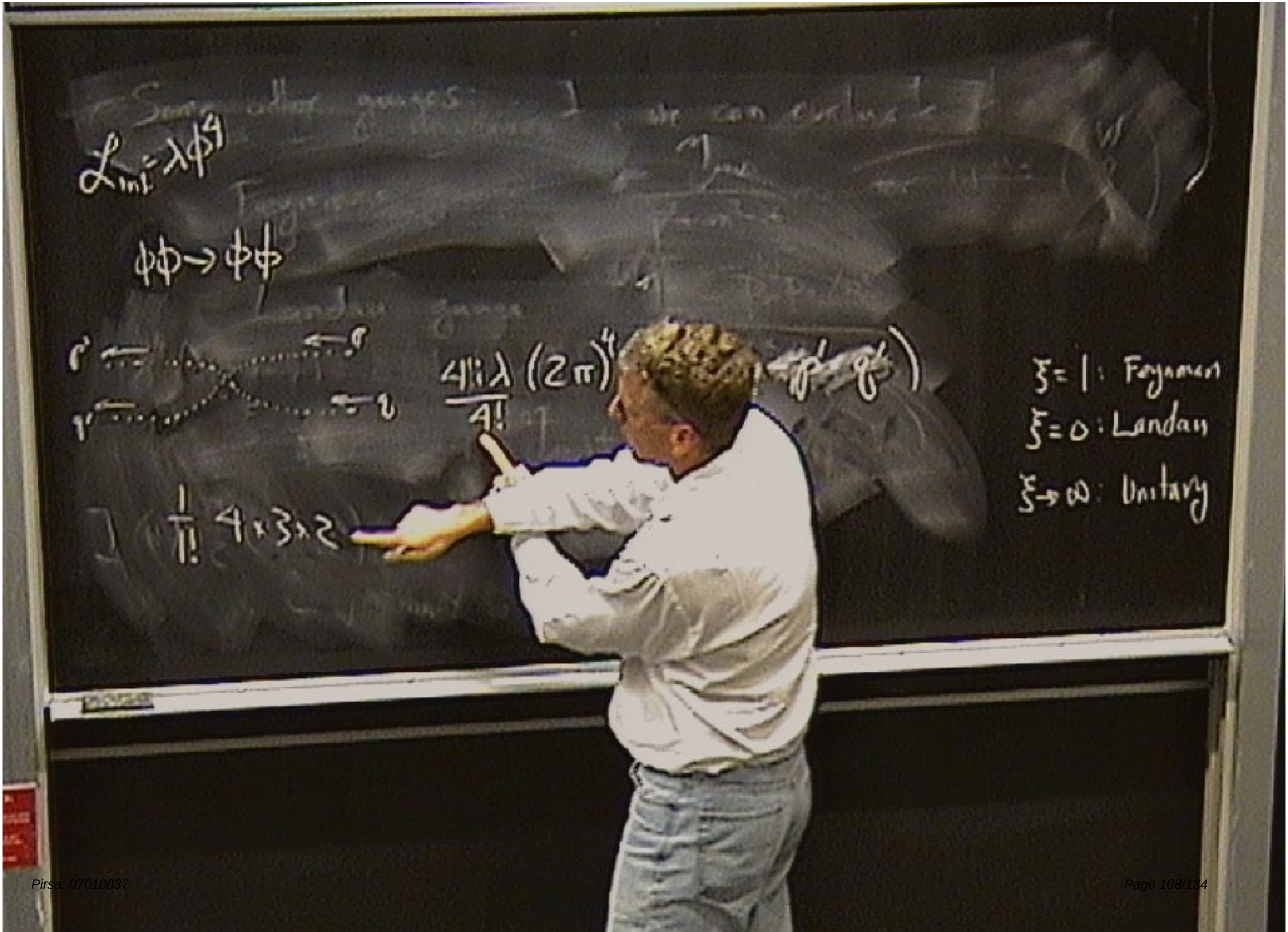
$\xi = 1$: Feynman

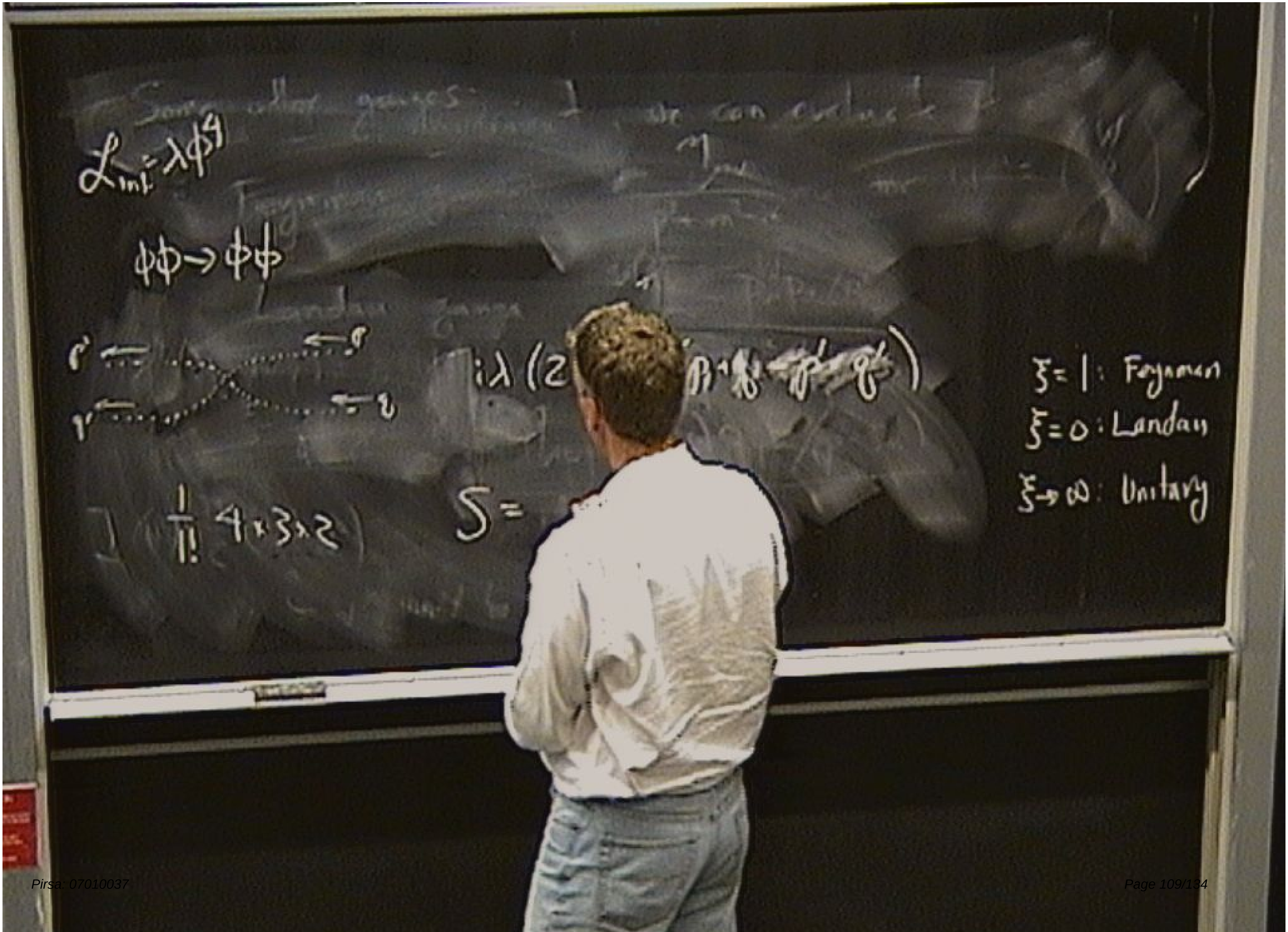
$\xi = 0$: Landau

$\xi \to \infty$: Unitary

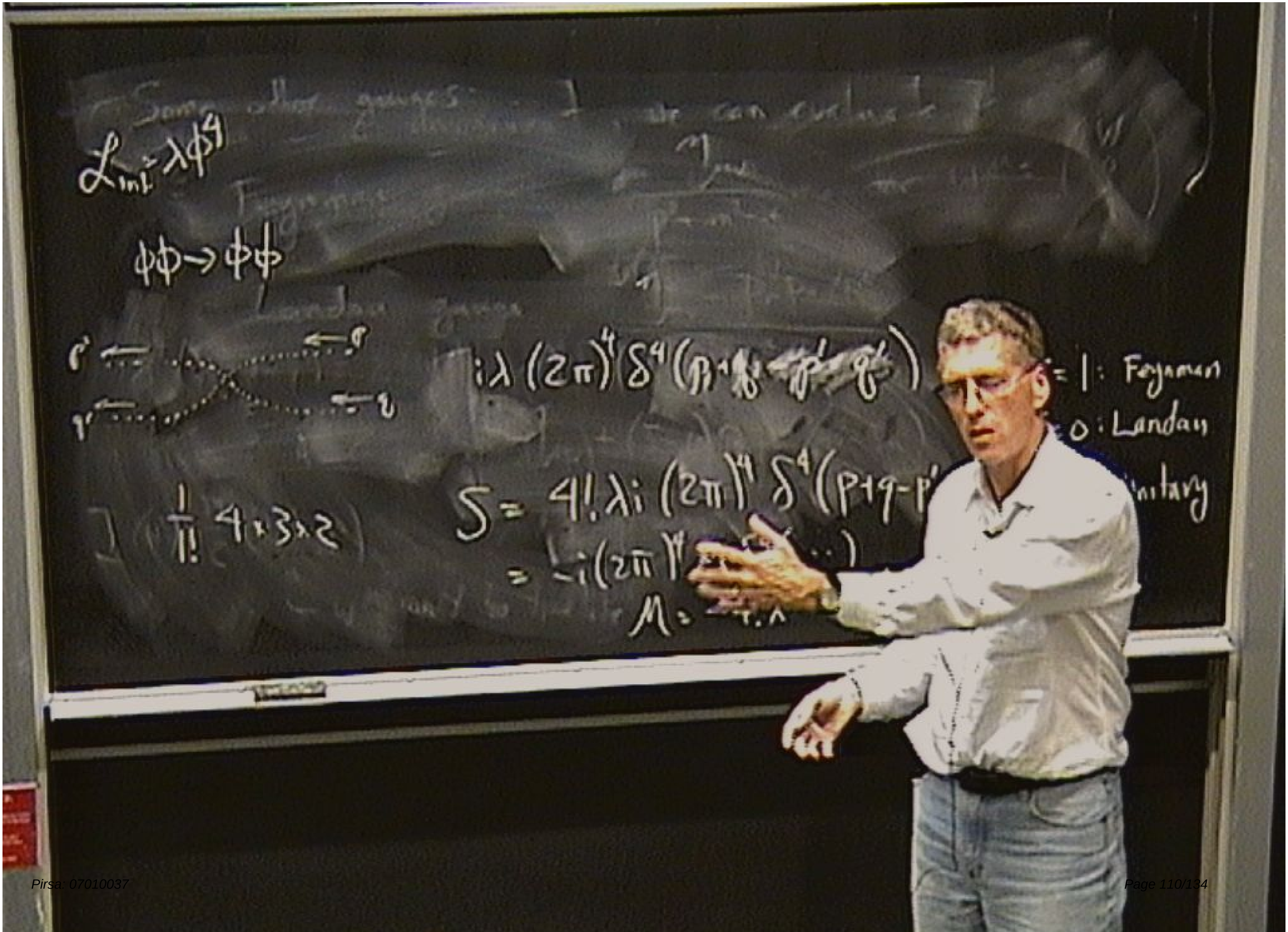

L_int = λϕ^4
ϕϕ → ϕϕ
p' p
q' q
iλ (2π)^4 δ^4 (p + q − p' − q')
1/4! 4×3×2
ξ = 1 : Feynman
ξ = 0 : Landau
Unitary


$\lambda\, i(2\pi)^4\delta^4(p_1+p_2+p_3)$
in SM: $\mathcal{L}_{int} = -\frac{m_H^2}{2v} H^3$
$\lambda = -\frac{m_H^2}{2v}$
$-\frac{3 i m_H^2}{v}(2\pi)^4\delta^4(p_1+p_2+p_3)$
$\mathcal{L}_{int} = \lambda\phi^4$
$\phi\phi \to \phi\phi$


$\mathcal{L}_{int} = \lambda\phi^4$
$\phi\phi \to \phi\phi$
$\frac{4!\, i\lambda}{4!}\,(2\pi)^4$
$\frac{1}{1!}\, 4\times3\times2$
$\xi = 1$: Feynman
$\xi = 0$: Landau
$\xi \to \infty$: Unitary



L_int = λφ^4
φφ → φφ
ξ = 1 : Feynman
ξ = 0 : Landau
ξ → ∞ : Unitary
S =
1/1! 4×3×2



$\mathcal{L}_{int} = \lambda\phi^4$
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$i\lambda\,(2\pi)^4\,\delta^4(p+q-p'-q')$
= 1: Feynman
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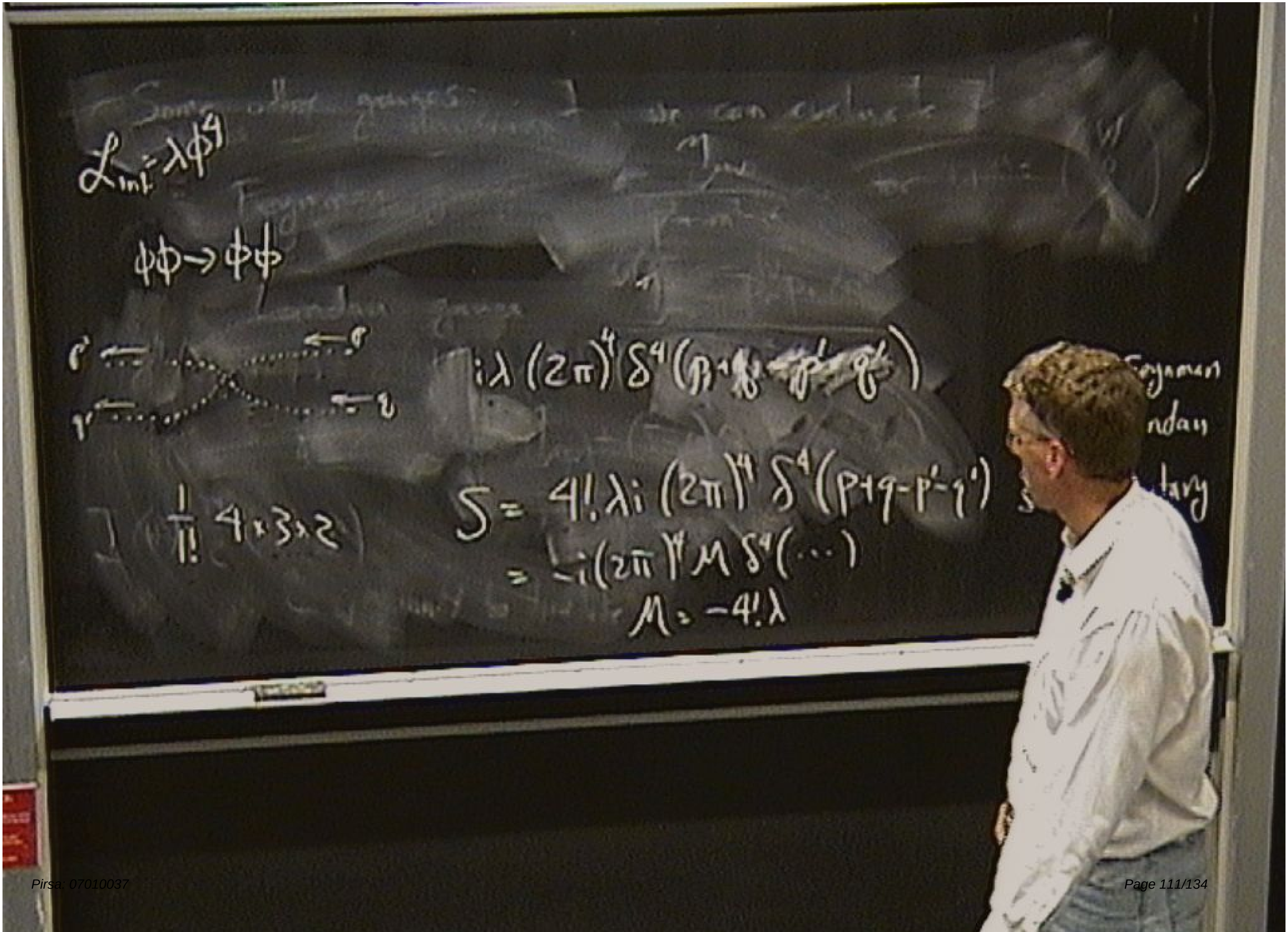

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$S = 4!\, \lambda i\, (2\pi)^4 \delta^4(p+q-p'-q')$
$= i(2\pi)^4 M \delta^4(\cdots)$
$M = -4!\lambda$

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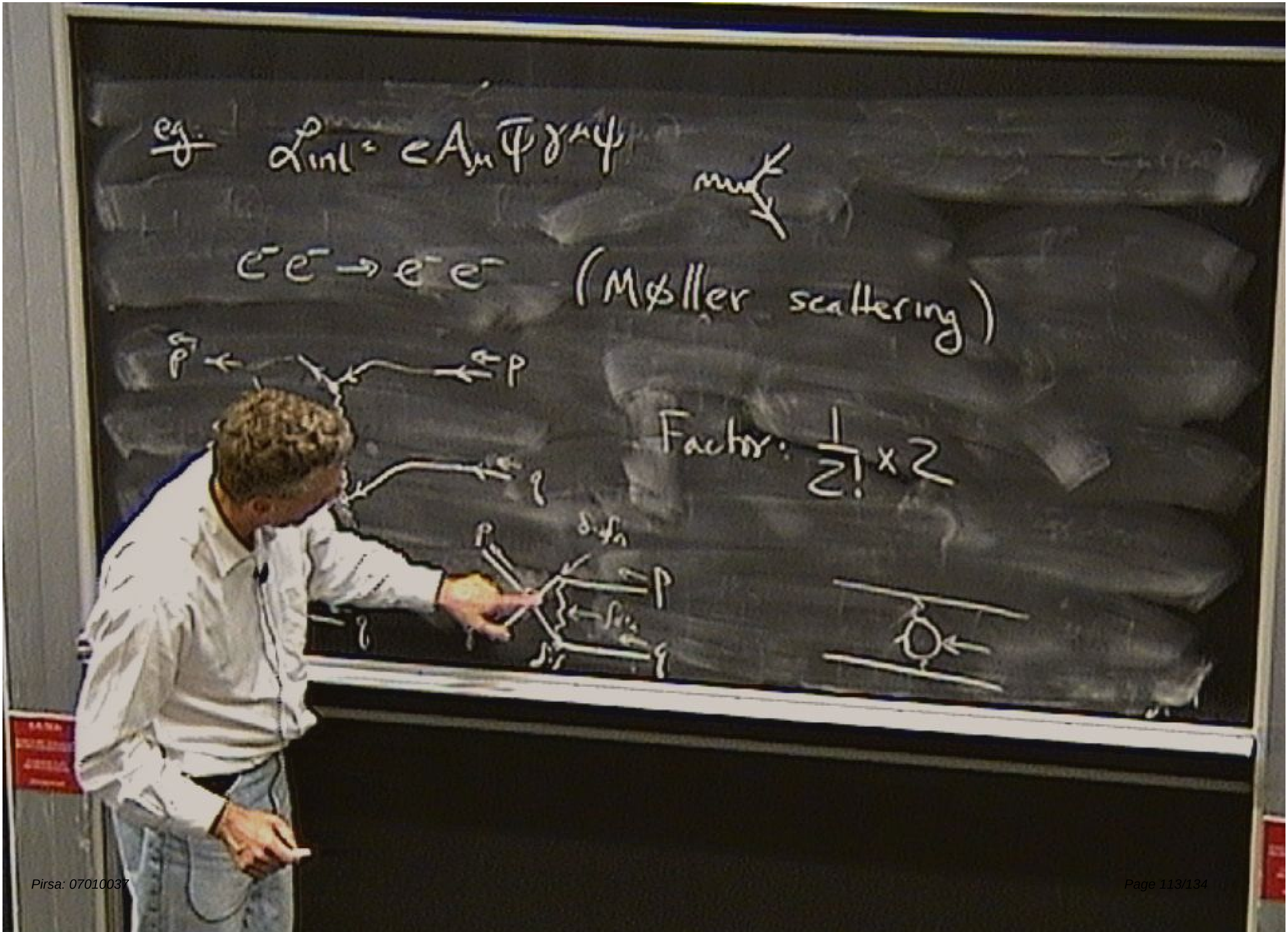
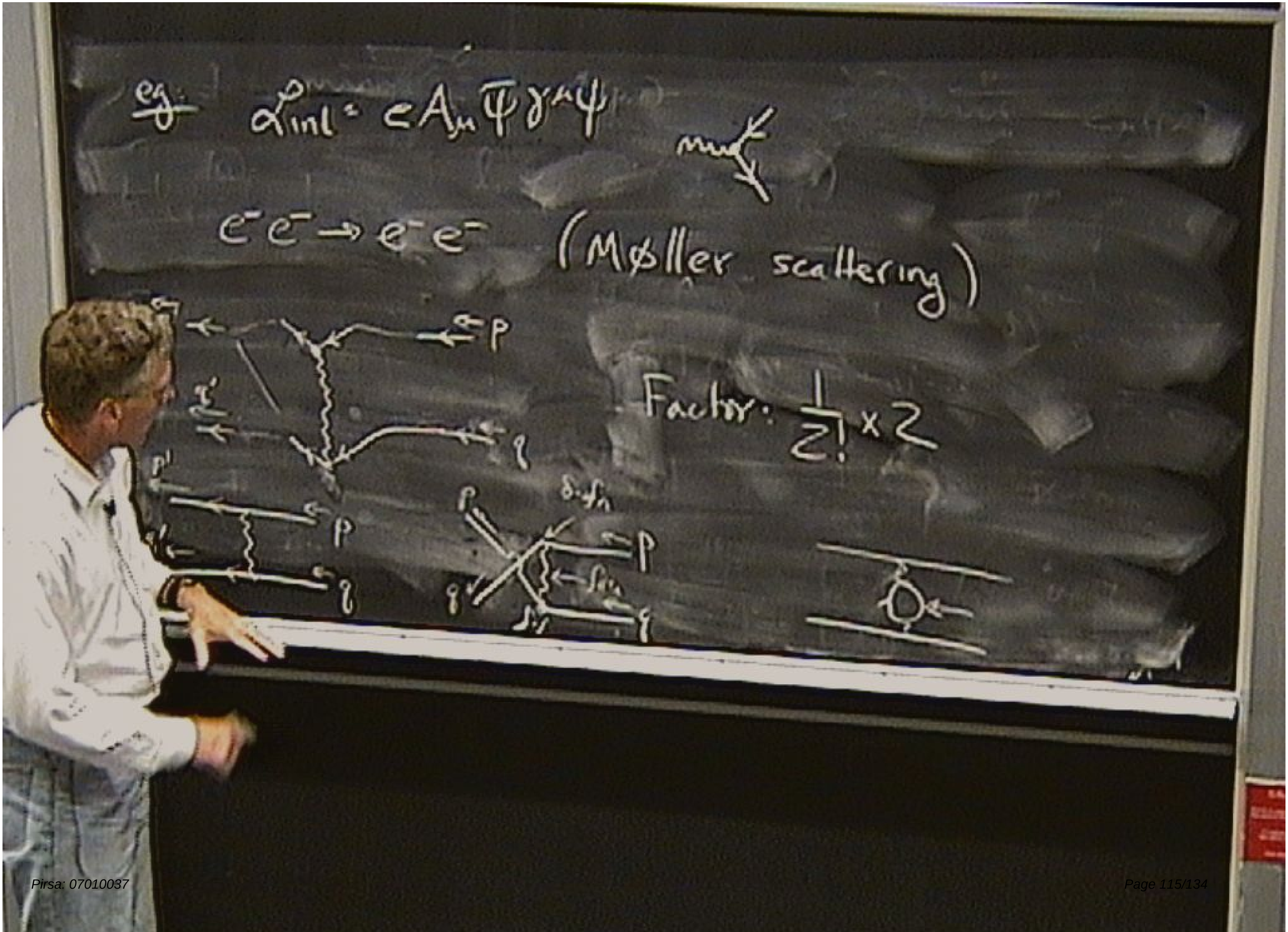

e.g. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \psi$
$e^- e^- \to e^- e^-$ (Møller scattering)
Factor: $\frac{1}{2!} \times 2$

How many momentum integrals survive after using all the δ-fns?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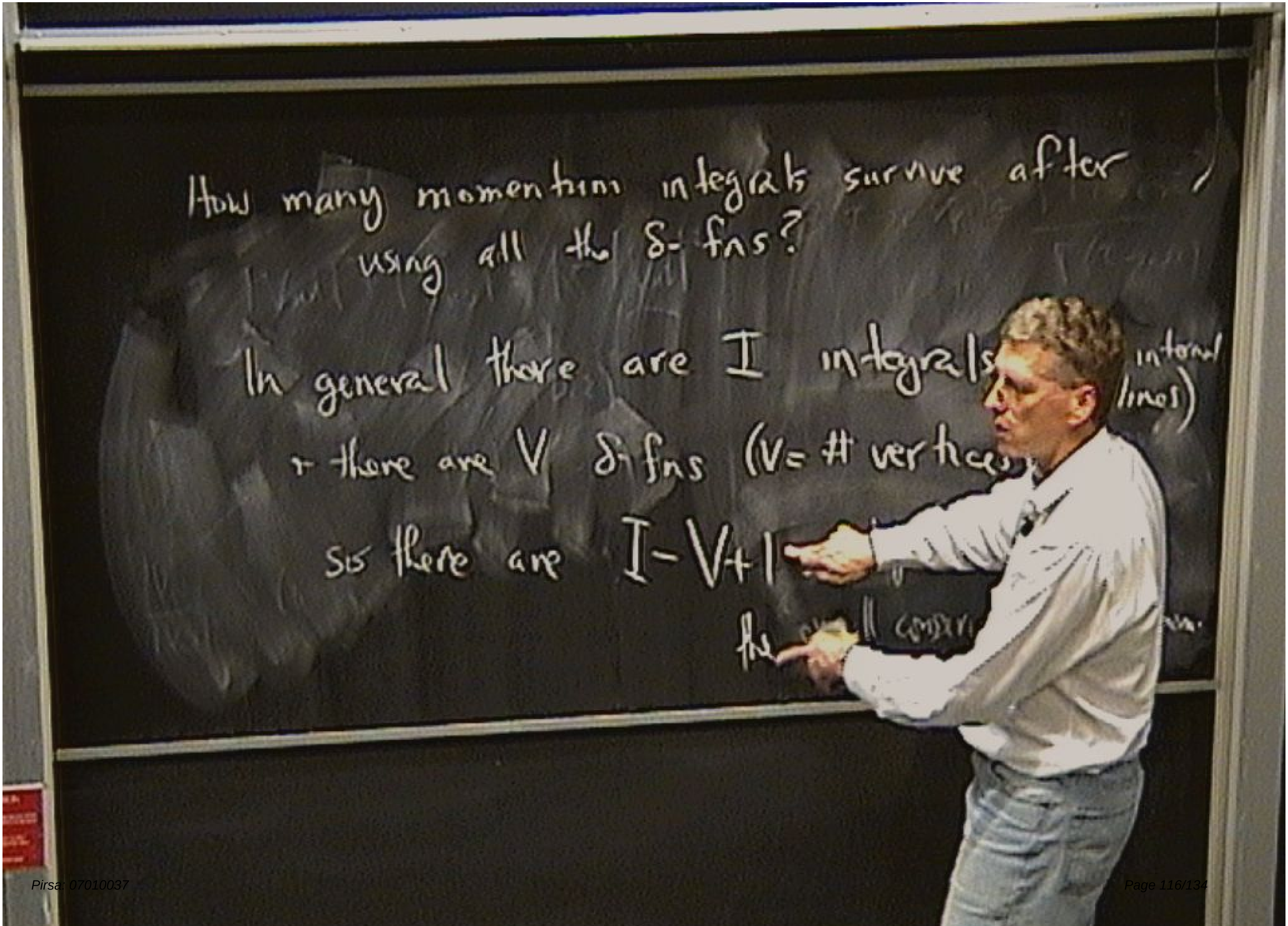
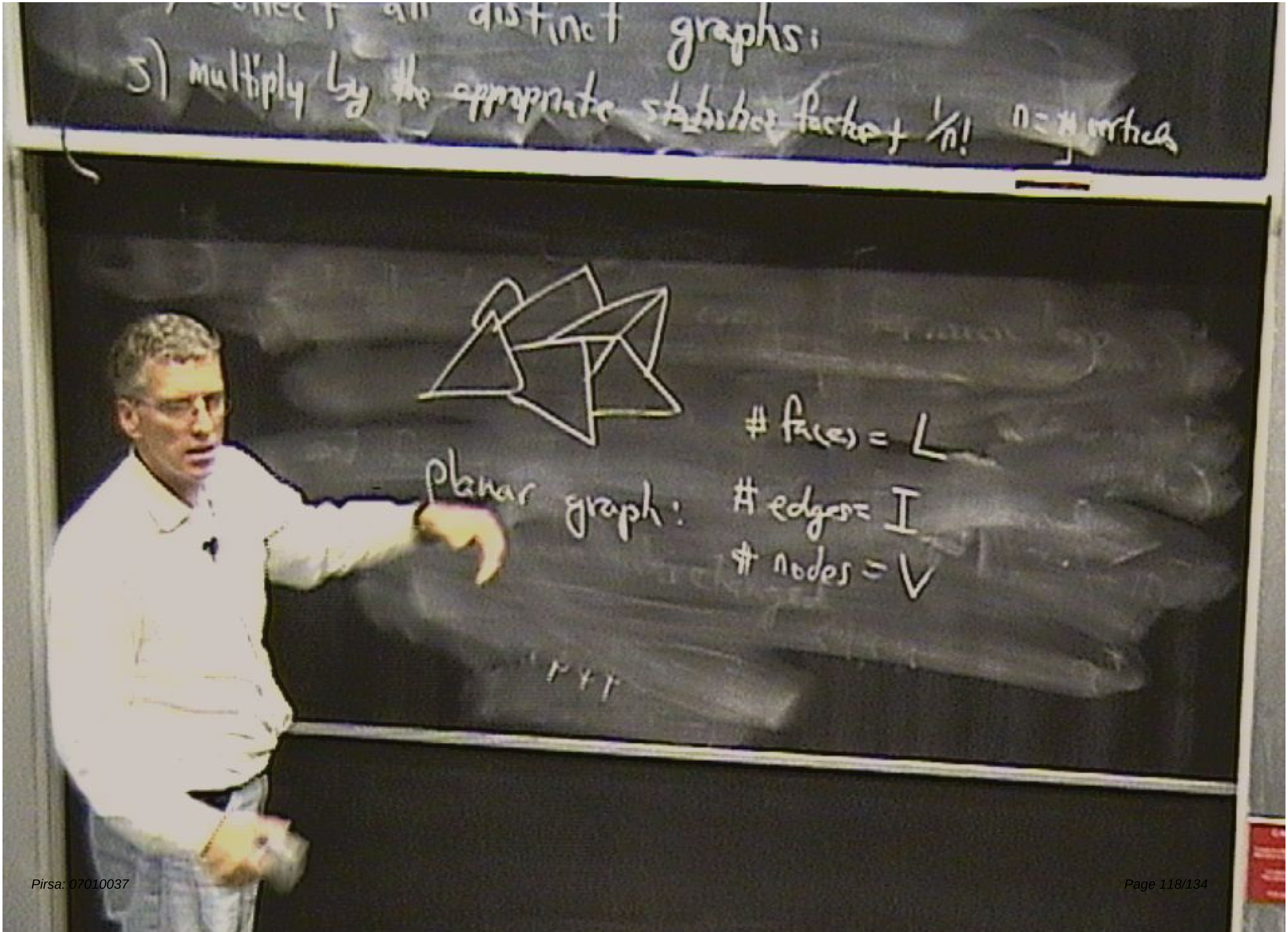
p
q
p'
q'
Factor: $\frac{1}{2!} \times 2$

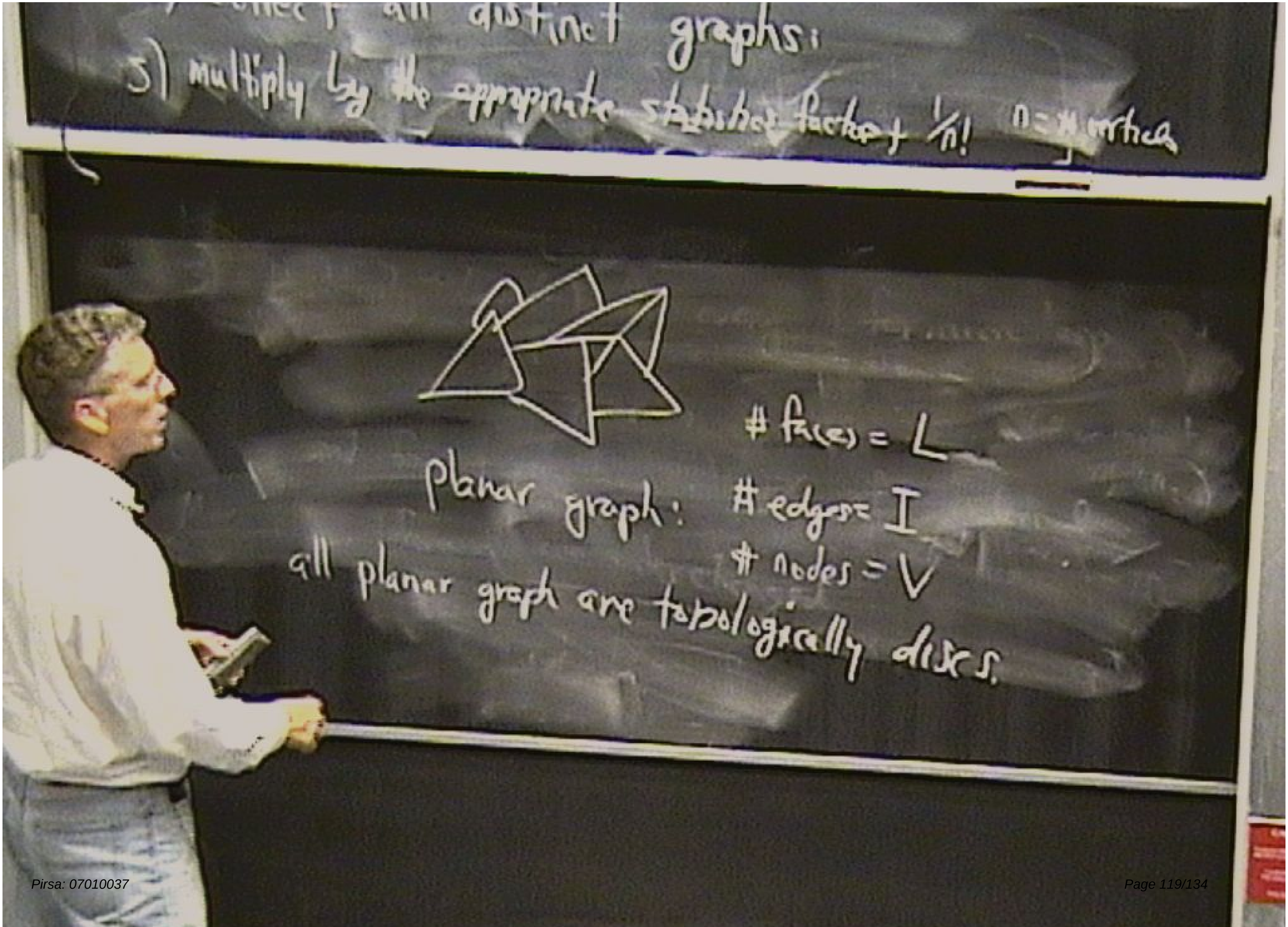

How many momentum integrals survive after
using all the δ-fns?
In general there are I integrals
internal lines)
+ there are V δ-fns (V = # vertices
So there are I−V+1

How many momentum integrals survive after using all the δ-fns?

In general there are I integrals (I = # internal lines)
+ there are V δ-fns (V = # vertices)

so there are $I - V + 1$ integrals left once the overall conservation of 4-mom. is extracted


all distinct graphs;
5) multiply by the appropriate statistics factor 1/n! n = # vertices
faces = L
planar graph: # edges = I
nodes = V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all planar graph are topologically discs.

...ollect all distinct graphs;

3) multiply by the appropriate statistical factor $1/n!$ $n = \#$ vertices

$\chi =$ faces − edges + nodes $= L - I + V$

planar graph: # faces $= L$

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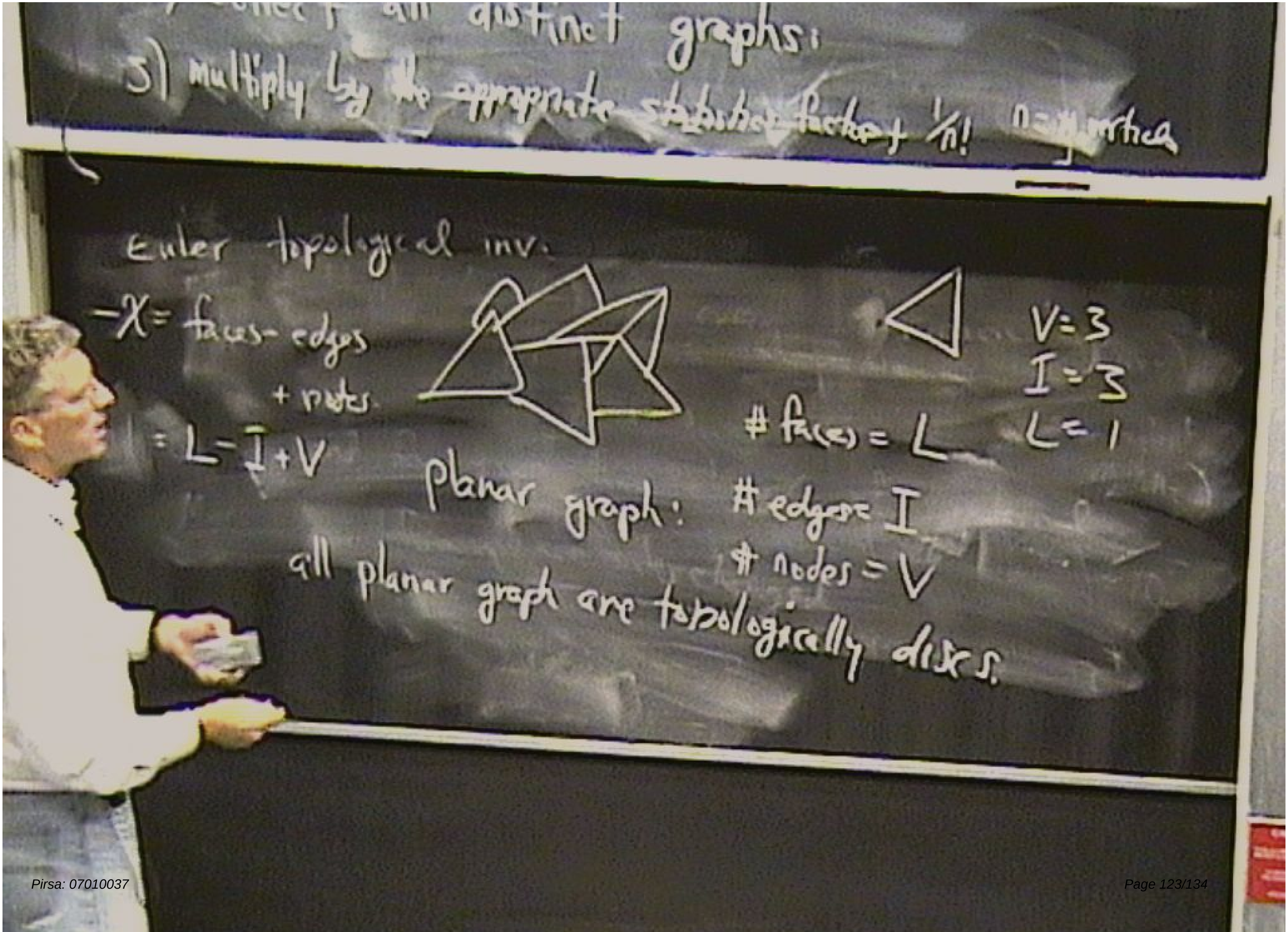
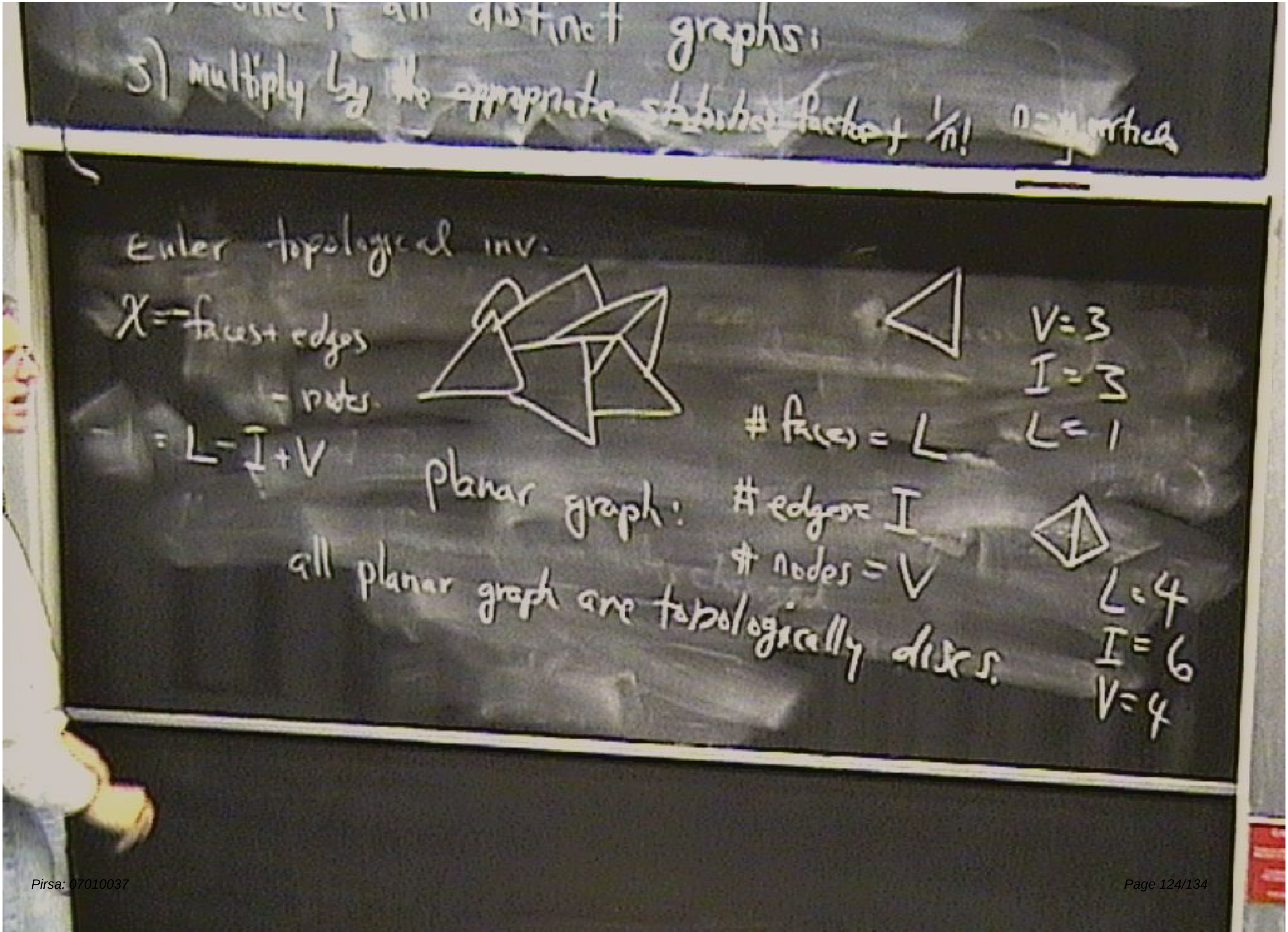
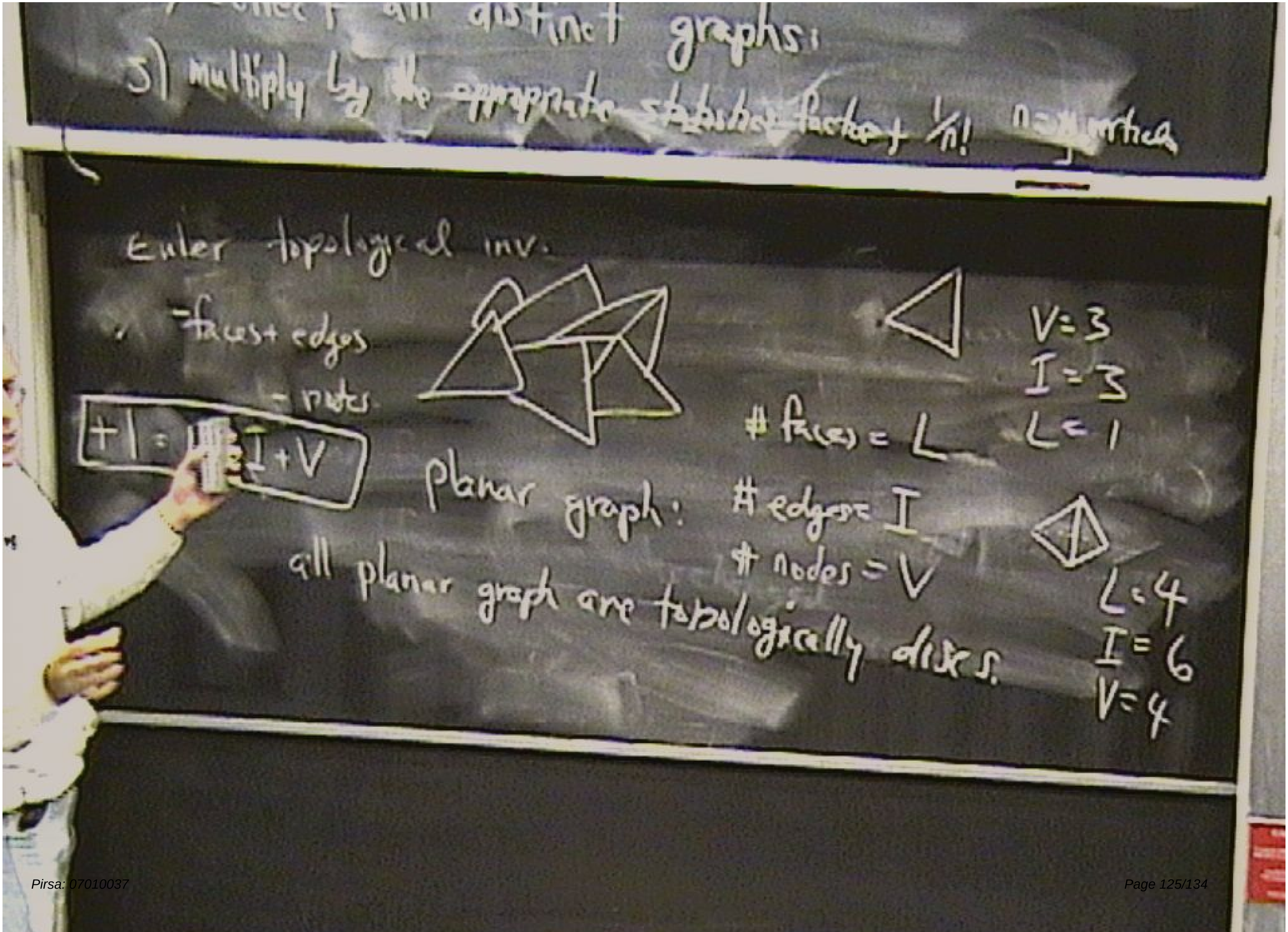

graphs:
5) multiply by the appropriate statistical factor 1/n!
Euler topological inv.
−χ = faces − edges + nodes
= L − I + V
V = 3
I = 3
L = 1
# faces = L
planar graph: # edges = I
nodes = V
all planar graph are topologically discs.


collect all distinct graphs;
5) multiply by the appropriate stabiliser factor 1/n! n = # vertices
Euler topological inv.
χ =− faces + edges
− nodes.
= L − I + V
V = 3
I = 3
L = 1
# faces = L
planar graph: # edges = I
nodes = V
all planar graph are topologically discs.
L = 4
I = 6
V = 4


distinct graphs;
5) multiply by the appropriate statistical factor 1/n! n = # vertices
Euler topological inv.
- faces + edges
- nodes
+1 = I + V
planar graph: # faces = L
edges = I
nodes = V
all planar graph are topologically discs.
V = 3
I = 3
L = 1
L = 4
I = 6
V = 4

... all distinct graphs;

3) multiply by the appropriate stabilizer factor $1/n!$ $n = \#$ vertices

Euler topological inv.

− faces + edges

− nodes.

$\boxed{\chi = L - I + V}$

$V = 3$

$I = 3$

$L = 1$

planar graph: $\#$ faces $= L$

$\#$ edges $= I$

$\#$ nodes $= V$

$L = 4$

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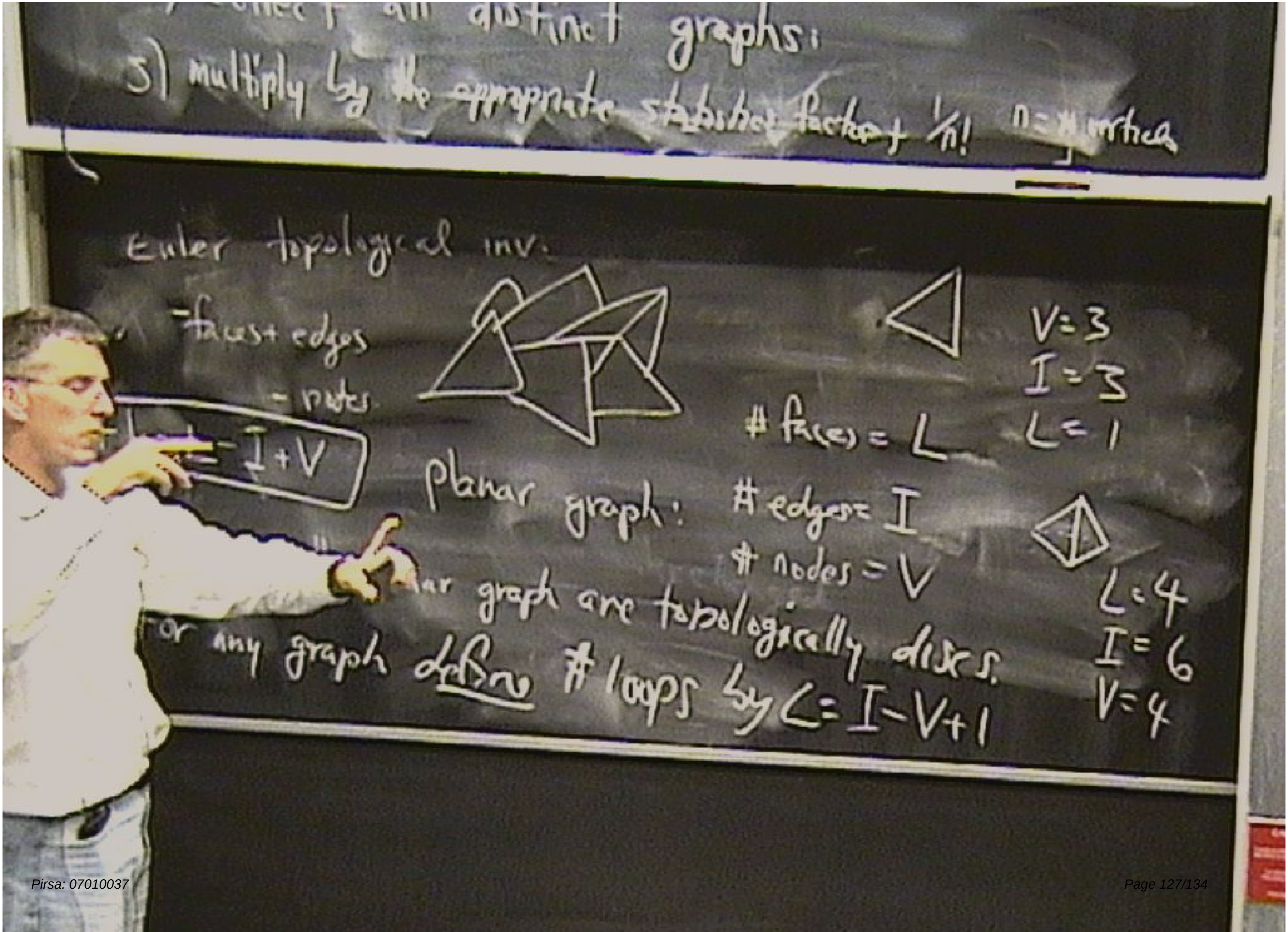
$V = 4$

all planar graph are topologically discs.



distinct graphs;
5) multiply by the appropriate stabilizer factor 1/n! n = # vertices
Euler topological inv.
-faces + edges
- nodes
I + V
planar graph: # edges = I
nodes = V
faces = L
graph are topologically discs.
For any graph define # loops by L = I - V + 1
V = 3
I = 3
L = 1
L = 4
I = 6
V = 4

... all distinct graphs;

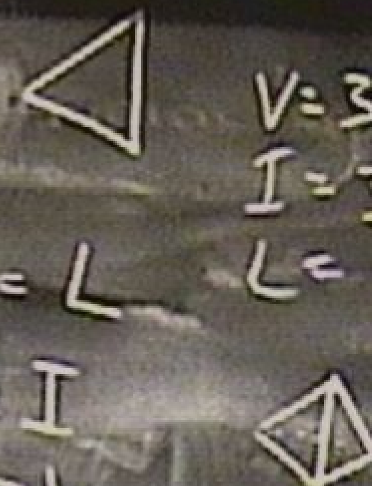
5) multiply by the appropriate stabilizer factor $1/n!$ $n = \#$ vertices

Euler topological inv.

- faces + edges

- nodes

$\boxed{+1 = L - I + V}$



$V = 3$
$I = 3$
$L = 1$

\# face(s) = L

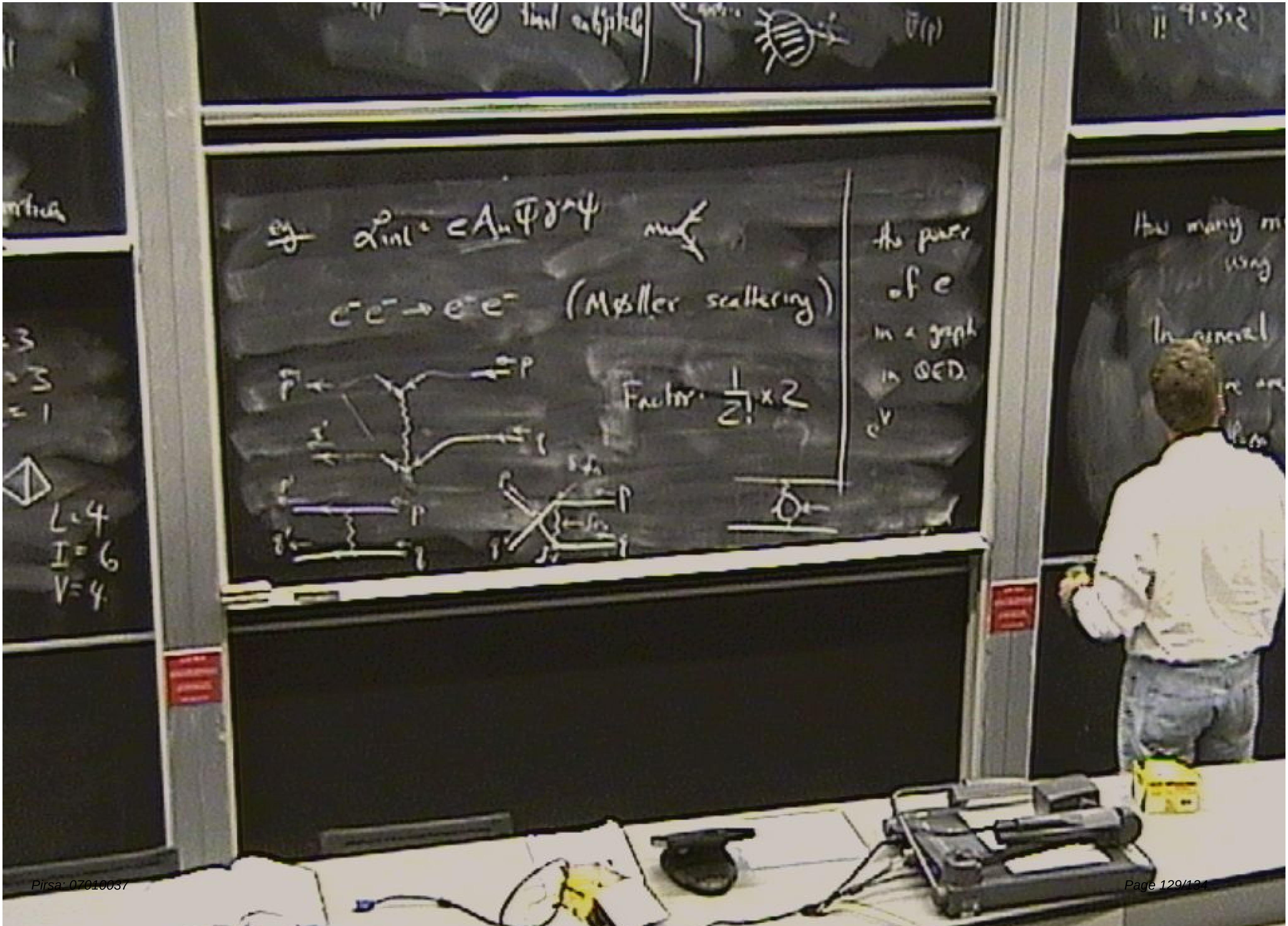
planar graph: \# edges = I

\# nodes = V

$L = 4$
$I = 6$
$V = 4$

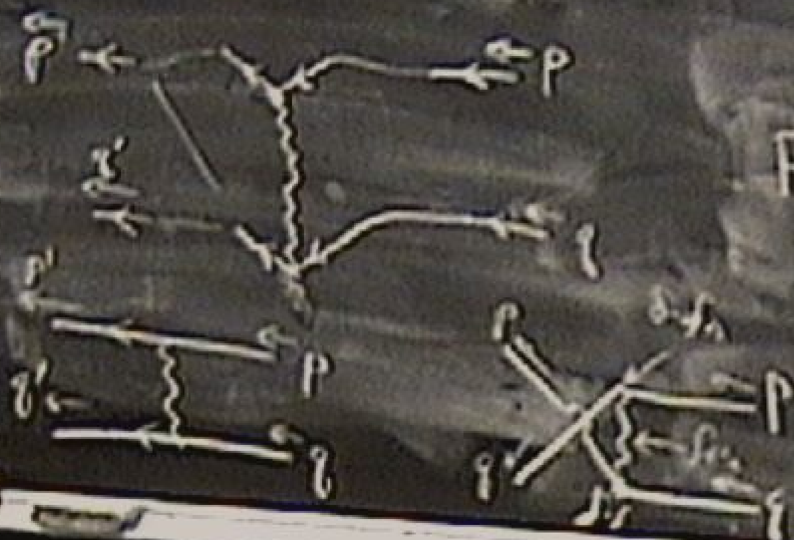
all planar graph are topologically discs.

For any graph define \# loops by $L = I - V + 1$


e.g. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \Psi$
$e^- e^- \to e^- e^-$ (Møller scattering)
Factor: $\frac{1}{2!} \times 2$
the power of e in a graph in QED
How many
In general
L=4
I=6
V=4

e.g. $\mathcal{L}_{int} = e A_\mu \bar{\psi} \gamma^\mu \psi$

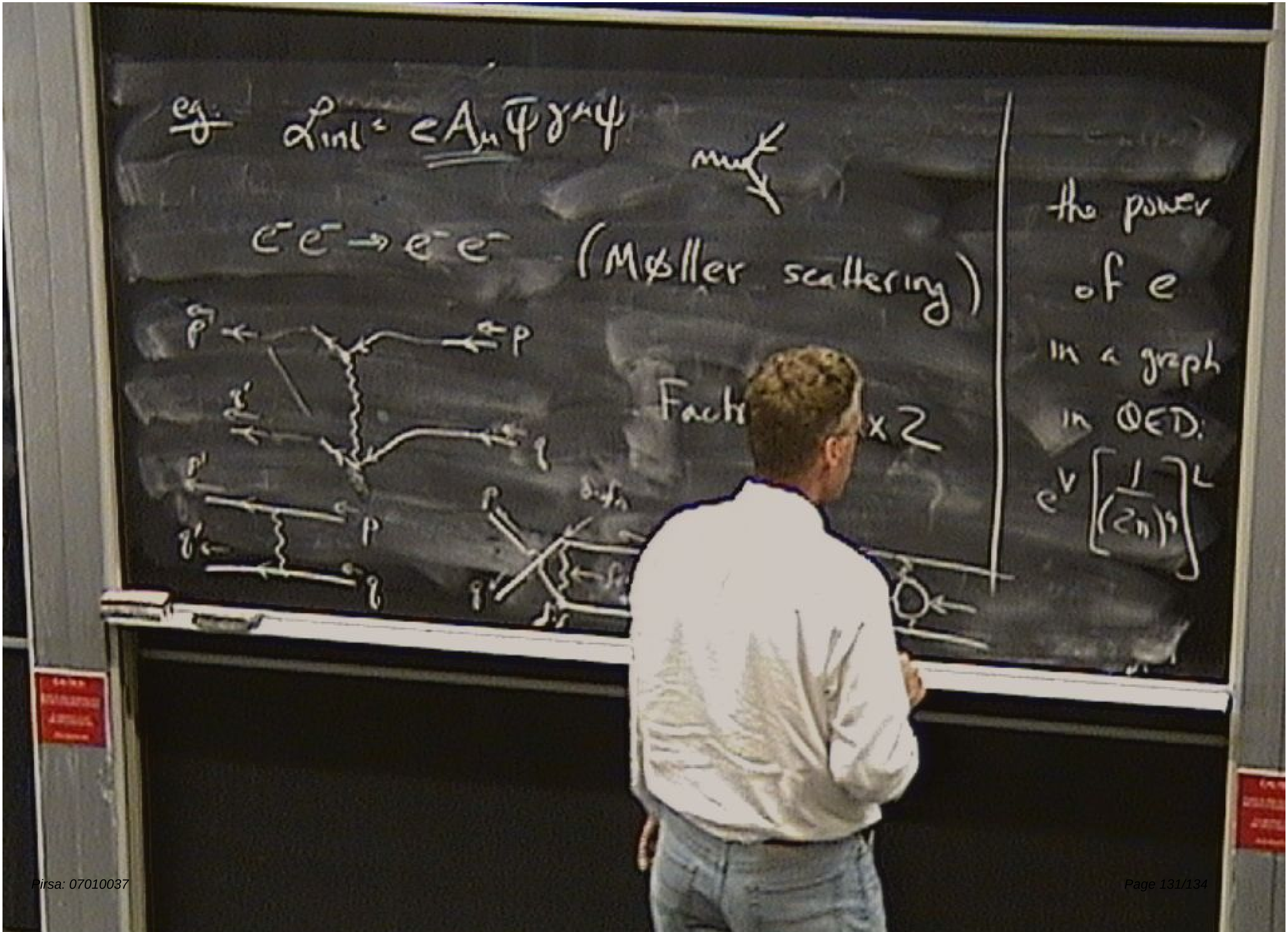
$e^- e^- \to e^- e^-$ (Møller scattering)

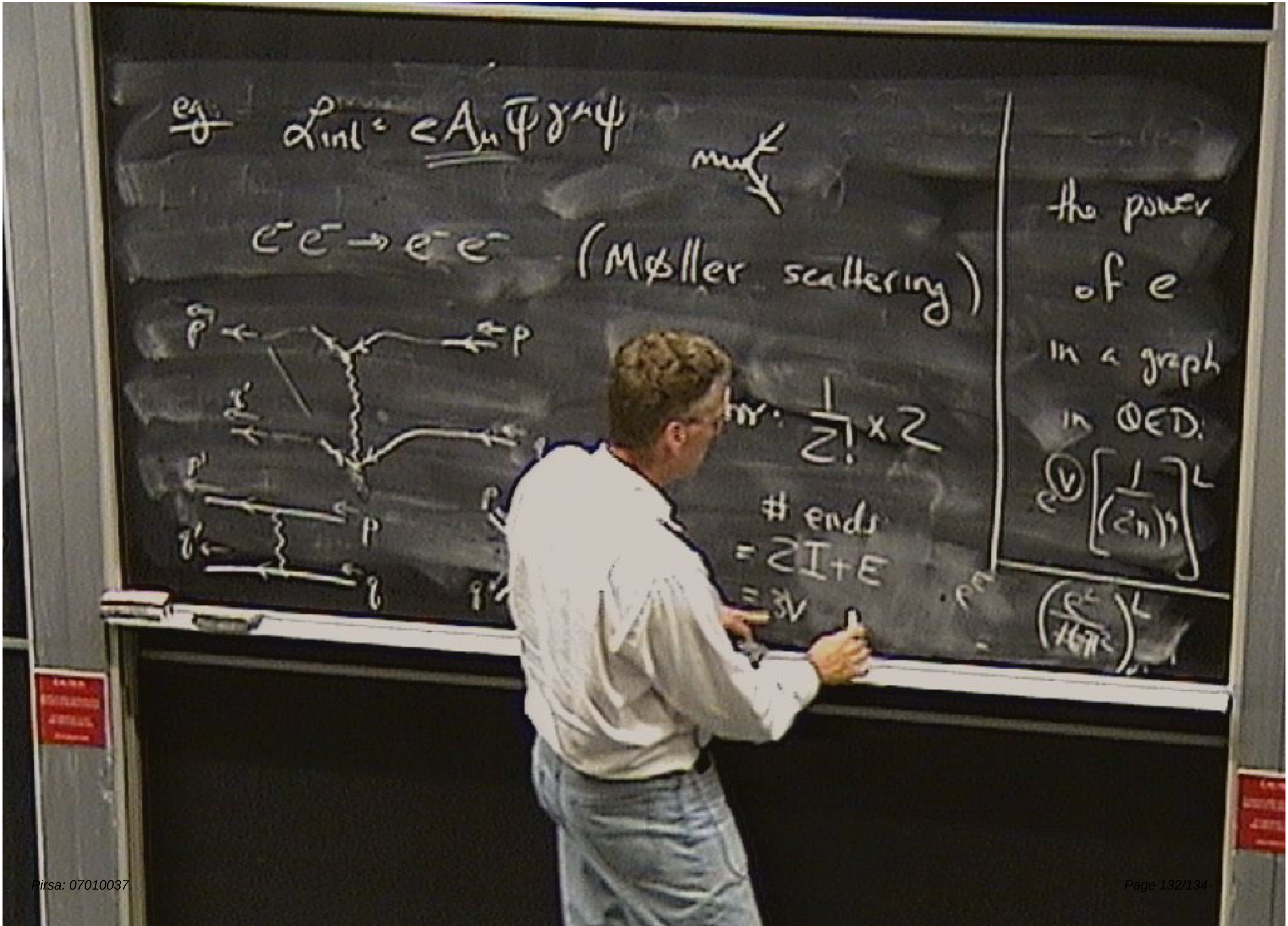


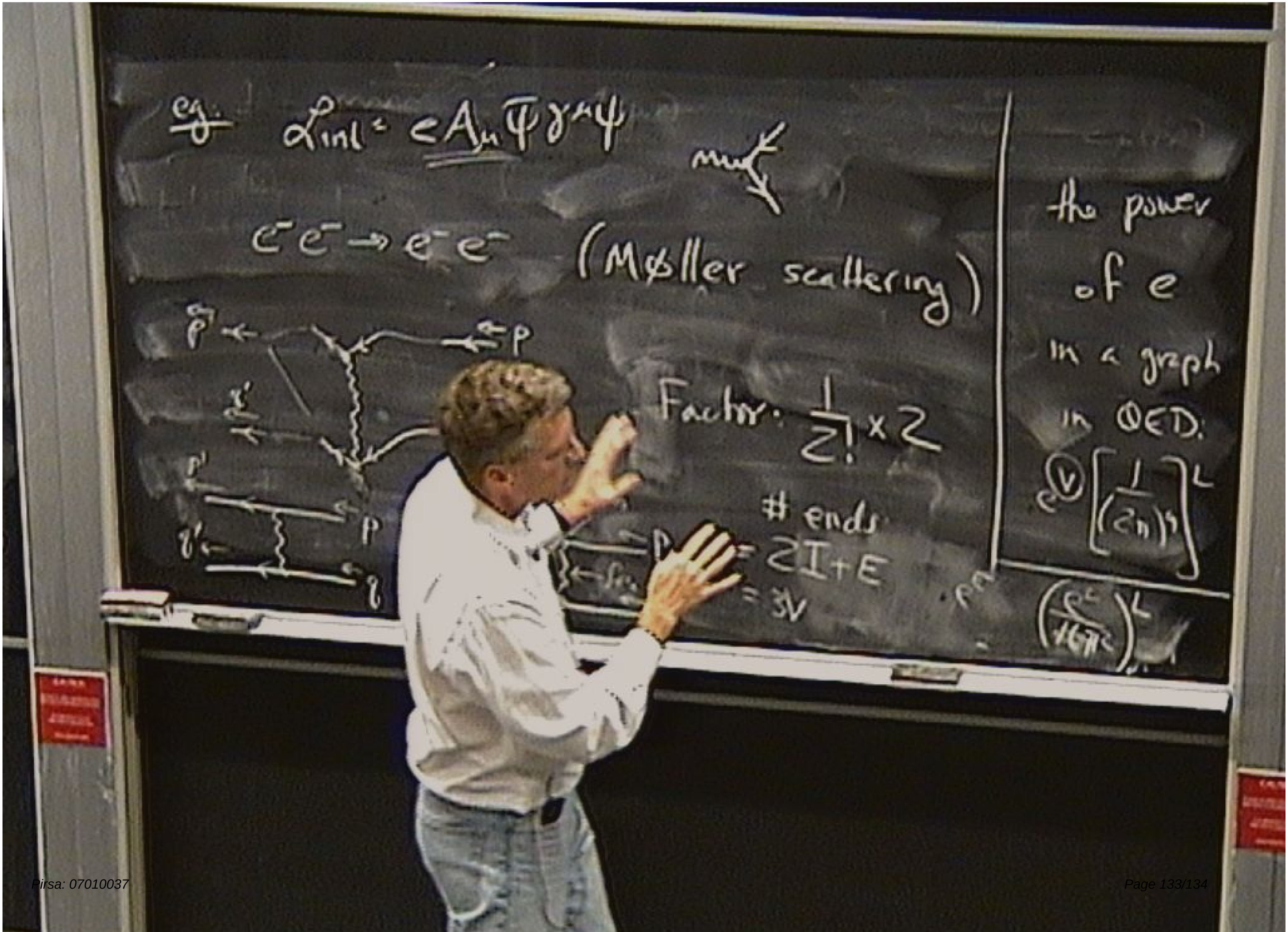
Factor: $\frac{1}{2!} \times 2$

the power of e in a graph in QED.

e^V


e.g. $\mathcal{L}_{int} = eA_\mu \bar{\Psi} \gamma^\mu \psi$
$e^- e^- \to e^- e^-$ (Møller scattering)
p'
p
q'
q
x 2
the power of e in a graph in QED:
$e^V \left[\frac{1}{(2\pi)^4}\right]^L$


eg. $\mathcal{L}_{int} = e A_\mu \bar{\Psi} \gamma^\mu \psi$
$e^- e^- \to e^- e^-$ (Møller scattering)
$\frac{1}{2!} \times 2$
ends
$= 2I + E$
$= 3V$
the power of e in a graph in QED.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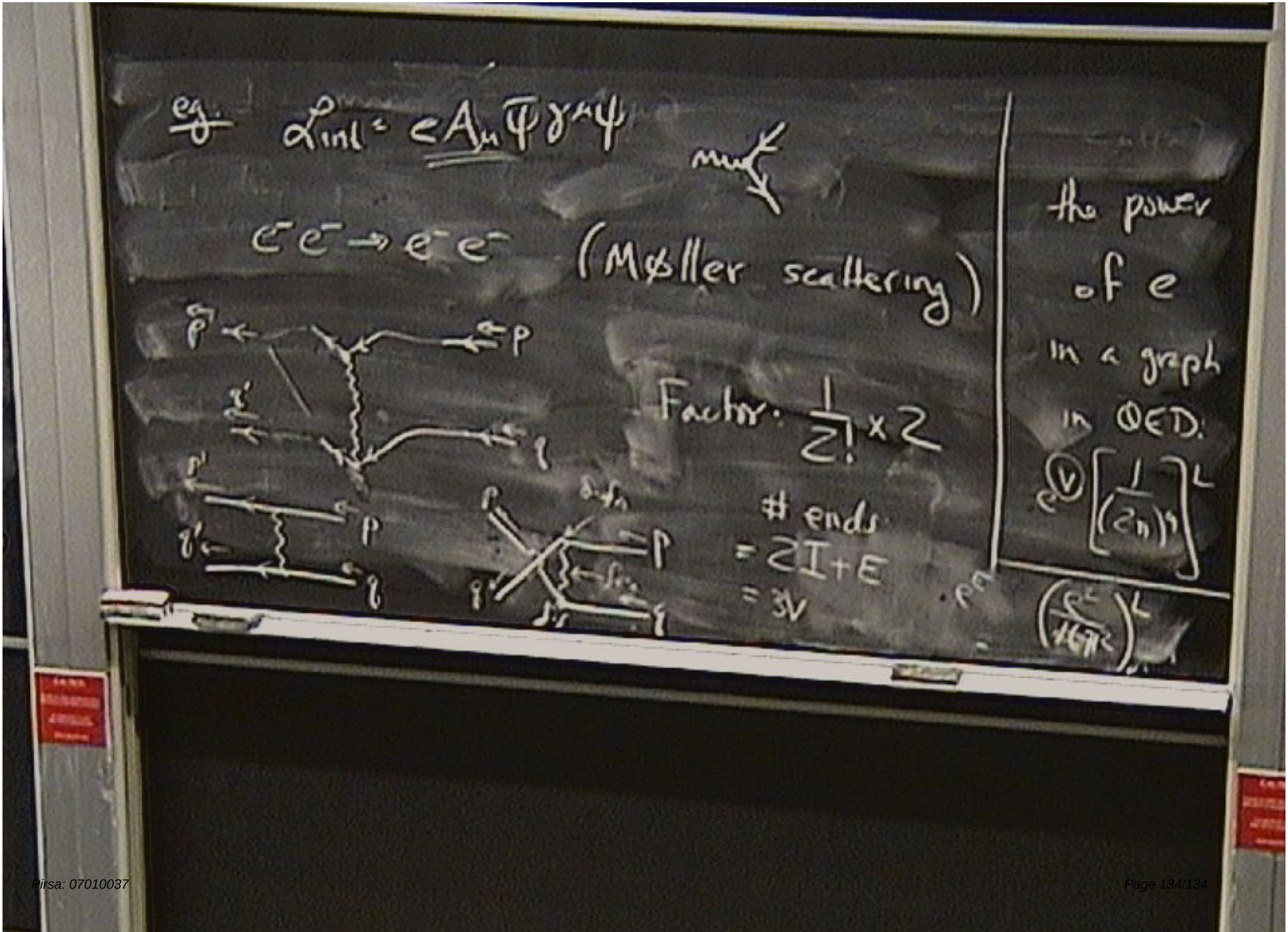

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$e^- e^- \to e^- e^-$ (Møller scattering)
Factor: $\frac{1}{2!} \times 2$
ends $= 2I + E = 3V$
the power of e in a graph in QED:
$e^V \left[\frac{1}{(2\pi)^4}\right]^L$
$\left(\frac{e^2}{16\pi^2}\right)^L$