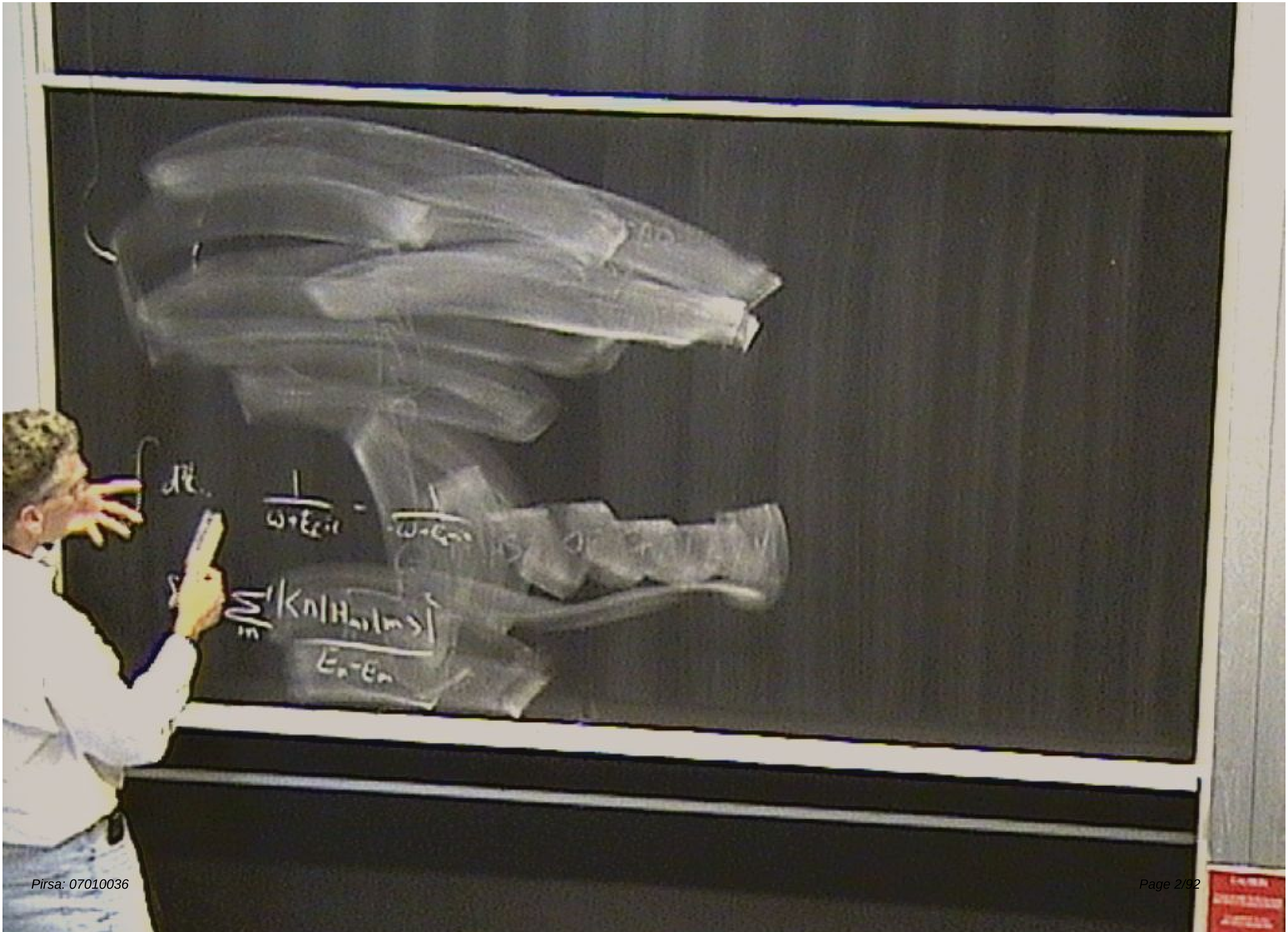
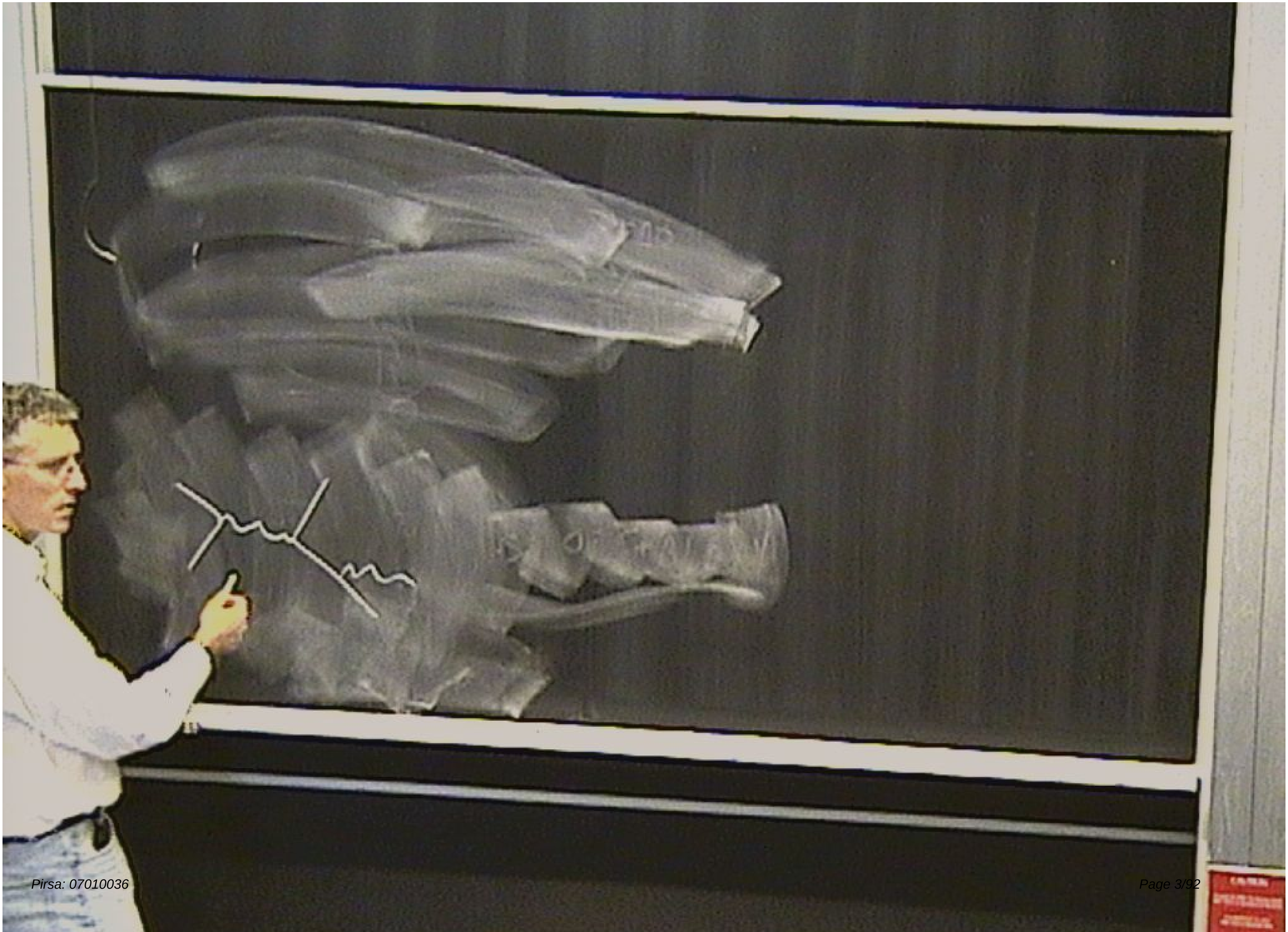
Title: Graduate Course on Standard Model & Quantum Field Theory - 10A

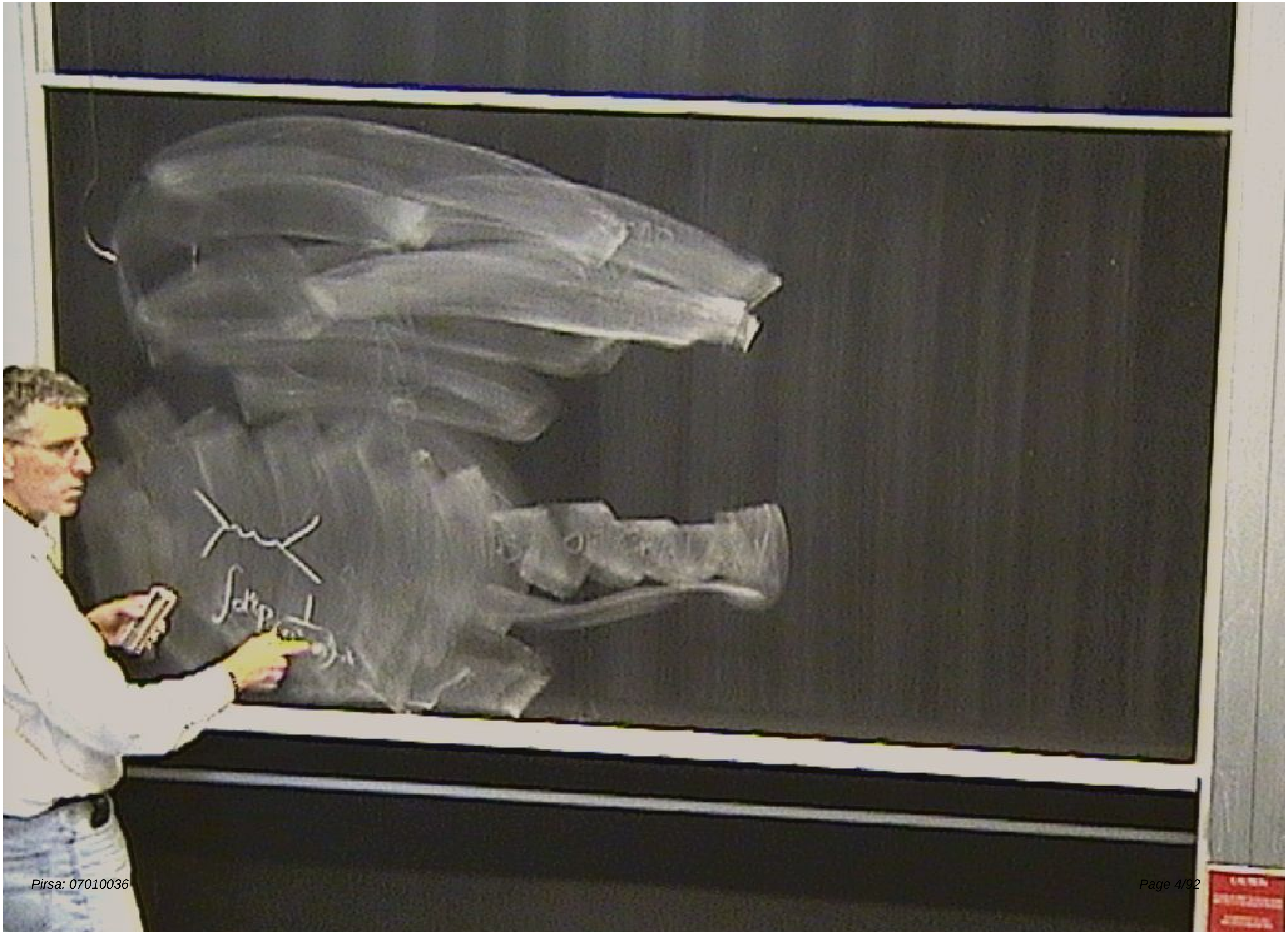
Date: Jan 24, 2007 11:00 AM

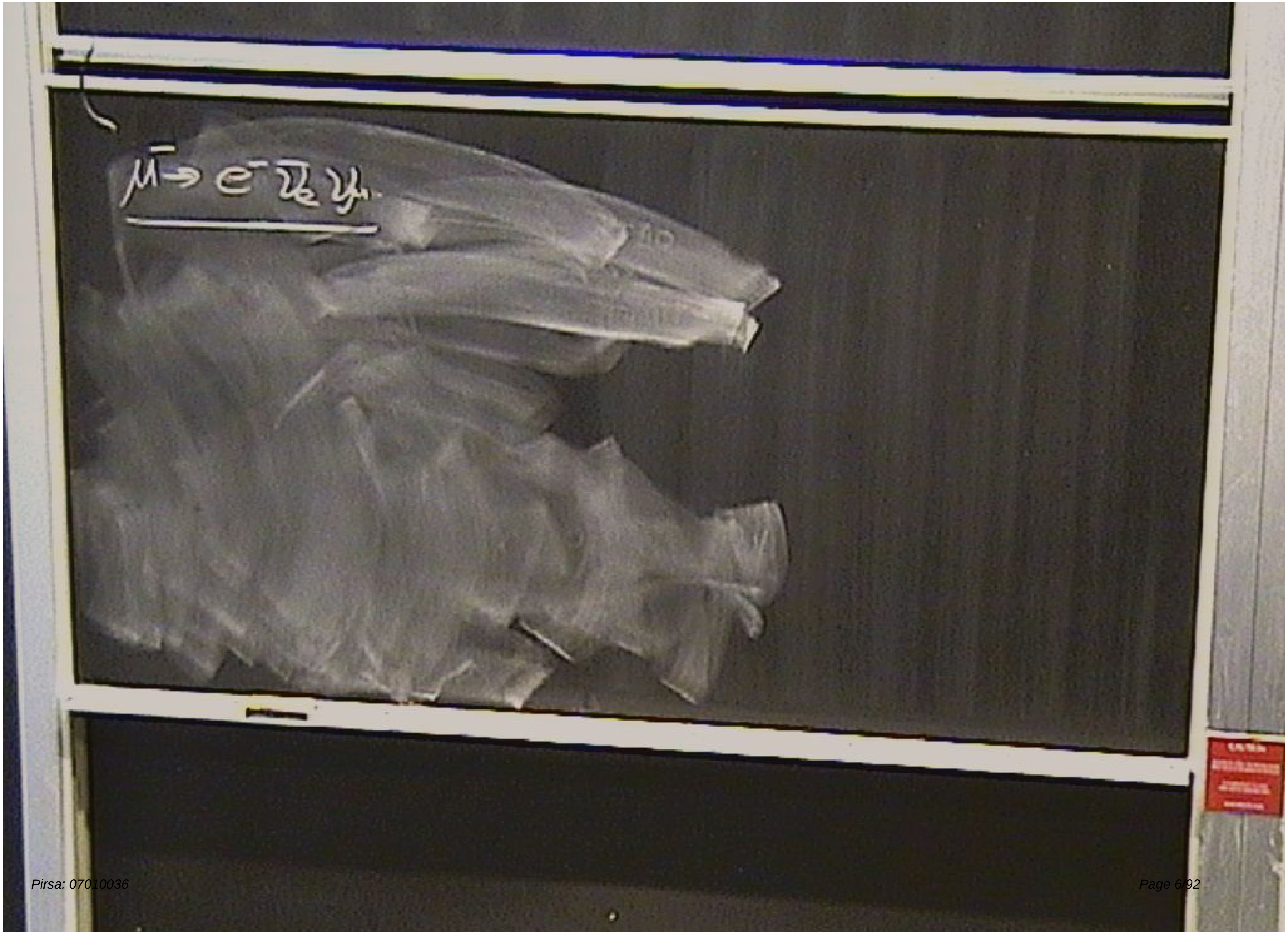
URL: http://pirsa.org/07010036

Abstract: Graduate Course on Standard Model & Quantum Field Theory







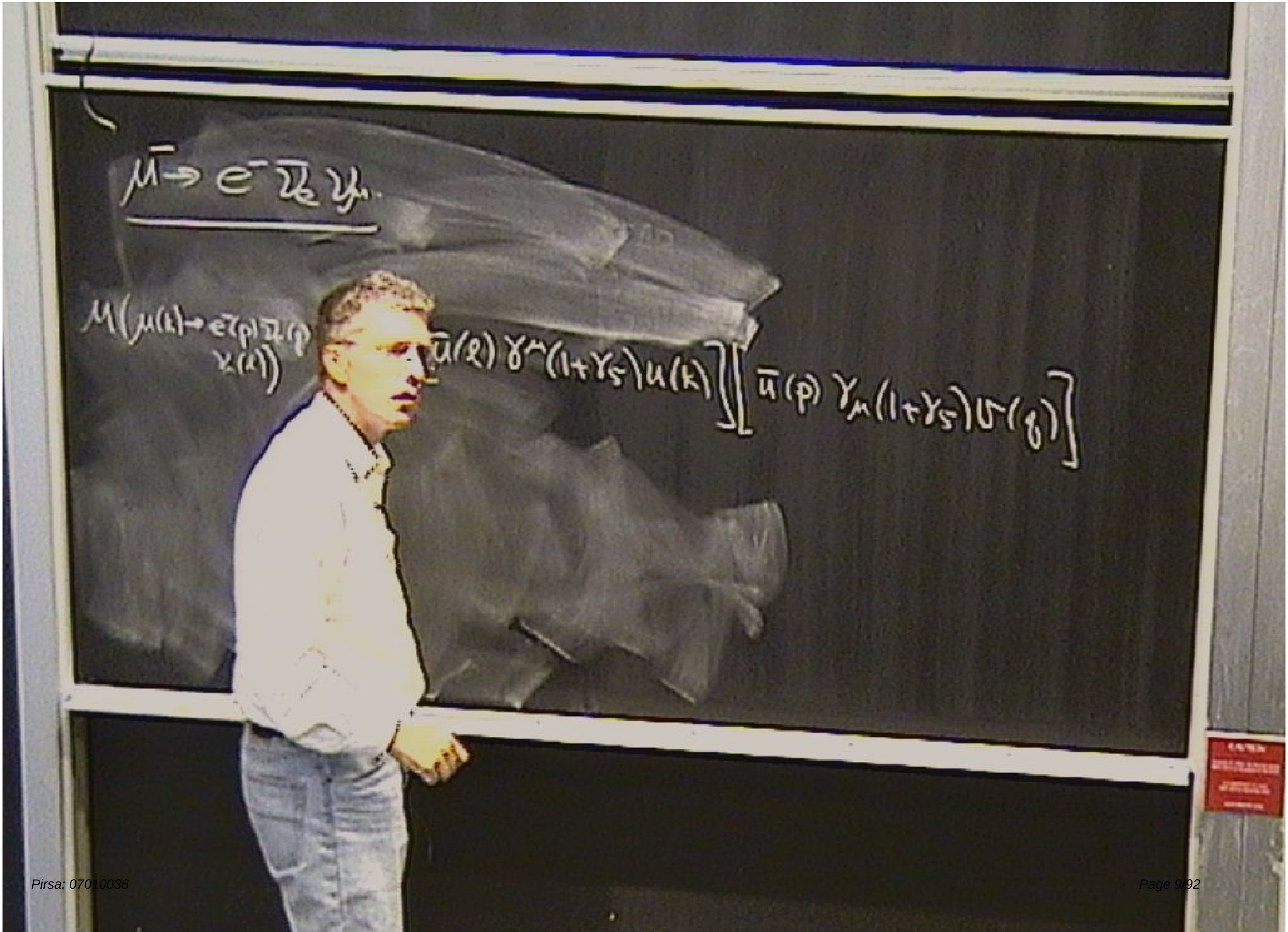

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

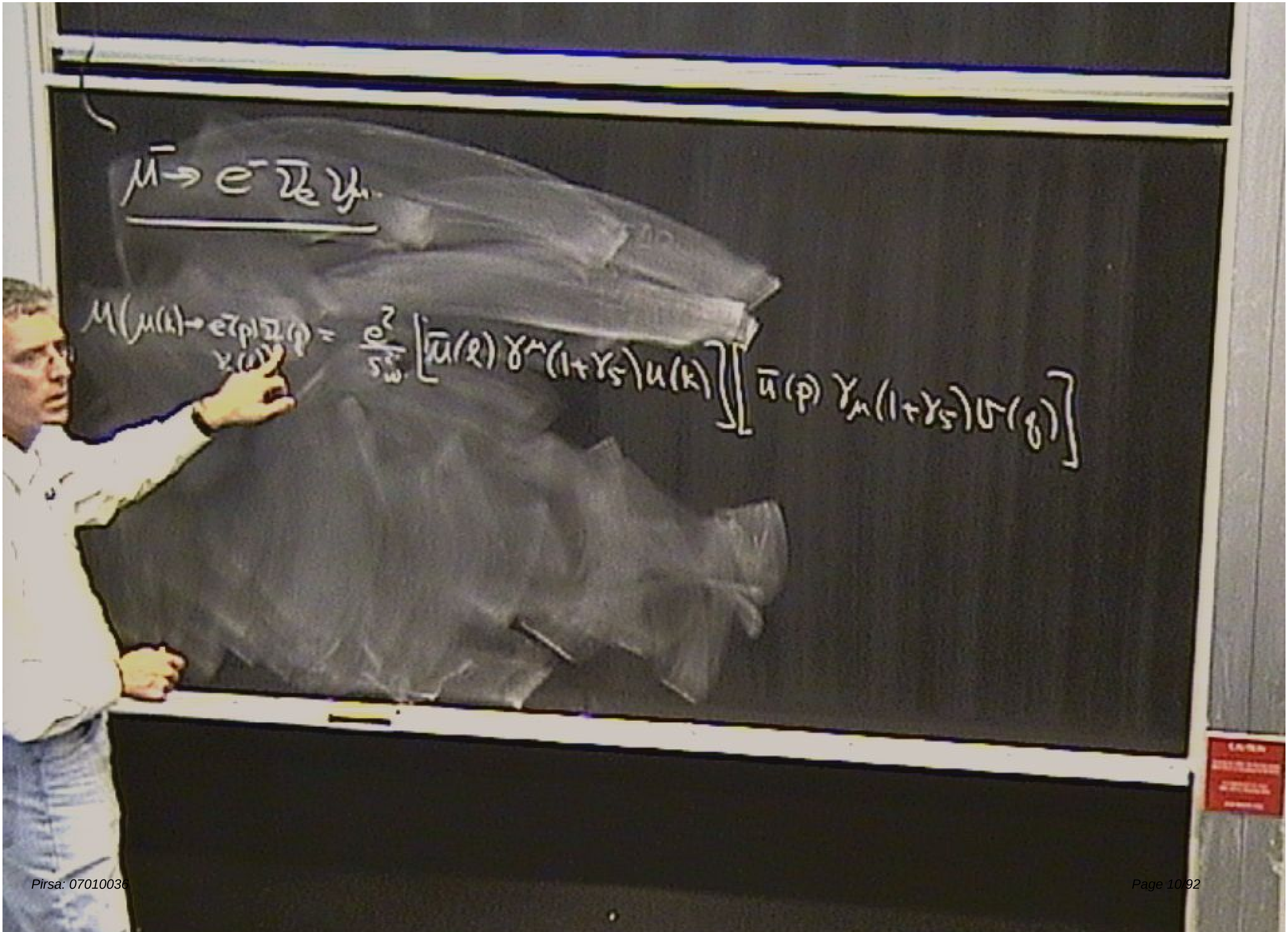
$$\mu^- \to e^- \bar{\nu}_e \nu_\mu$$

$$\mathcal{M}(\mu \to e \nu \bar{\nu}) = \frac{e^2}{s_w^2} \bar{u}$$

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

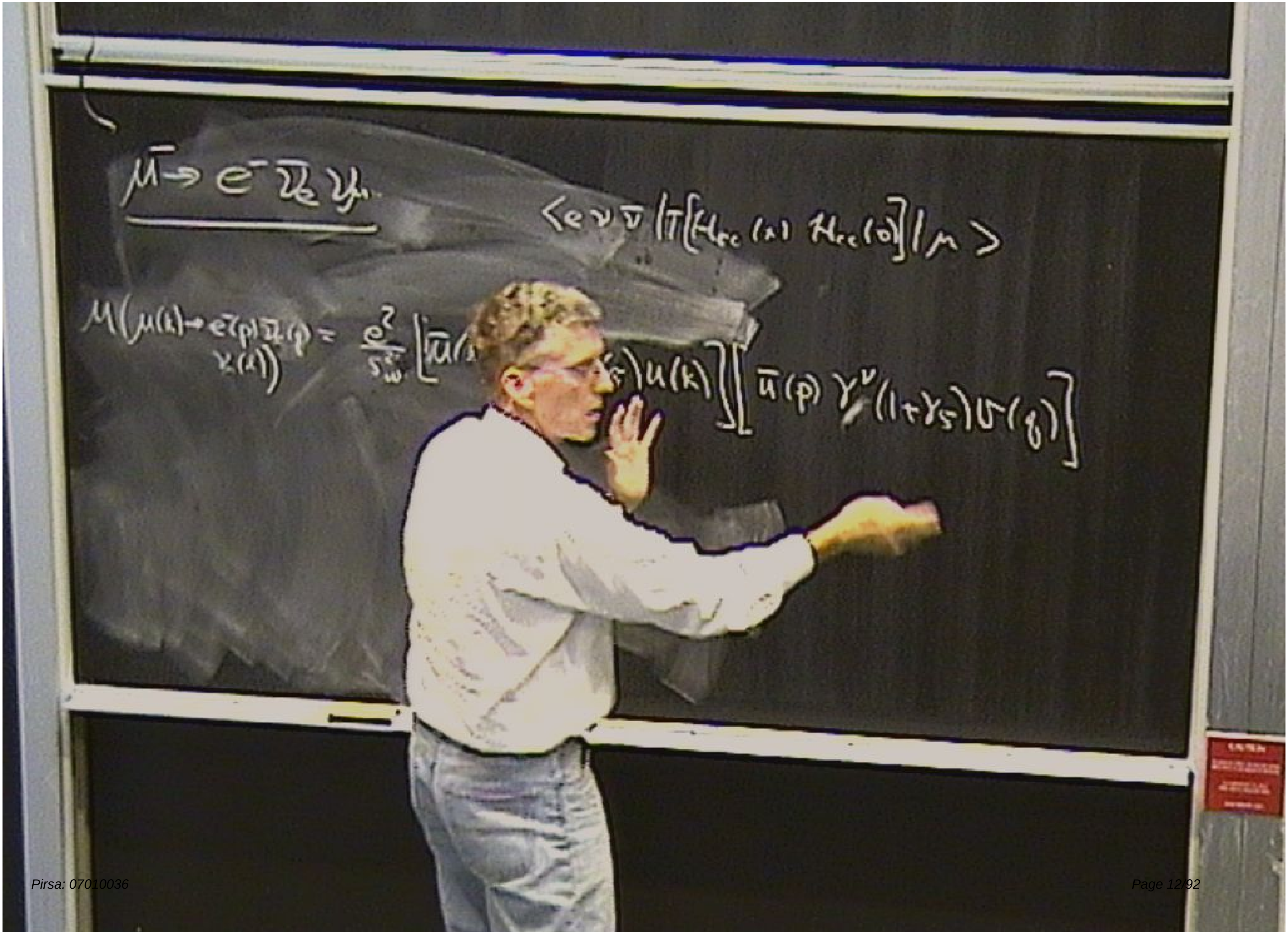
$$\mathcal{M}(\mu \to e \nu \bar{\nu}) = \frac{e^2}{s_W^2} \left[\bar{u}(\ell) \gamma^\mu (1+\gamma_5) u(k) \right] \left[\right.$$


$\mu^- \to e^- \bar{\nu}_e \nu_\mu$
$\mathcal{M}(\mu(k) \to e^-(p)\bar{\nu}_e(q)\nu_\mu(\ell))$
$\bar{u}(\ell)\gamma^\mu(1+\gamma_5)u(k)\,]\,[\,\bar{u}(p)\gamma_\mu(1+\gamma_5)v(q)\,]$


$\mu^- \to e^- \bar{\nu}_e \nu_\mu$
$[\bar{u}(\ell)\gamma^\mu(1+\gamma_5)u(k)][\bar{u}(p)\gamma_\mu(1+\gamma_5)v(q)]$

$$\mu^- \to e^- \bar{\nu}_e \nu_\mu$$

$$\mathcal{M}\big(\mu(k) \to e^-(p)\,\bar{\nu}_e(q)\,\nu_\mu(l)\big) = \frac{e^2}{s_W^2}\left[\bar{u}(l)\,\gamma^\mu(1+\gamma_5)\,u(k)\right]\left[\bar{u}(p)\,\gamma_\mu(1+\gamma_5)\,v(q)\right]$$

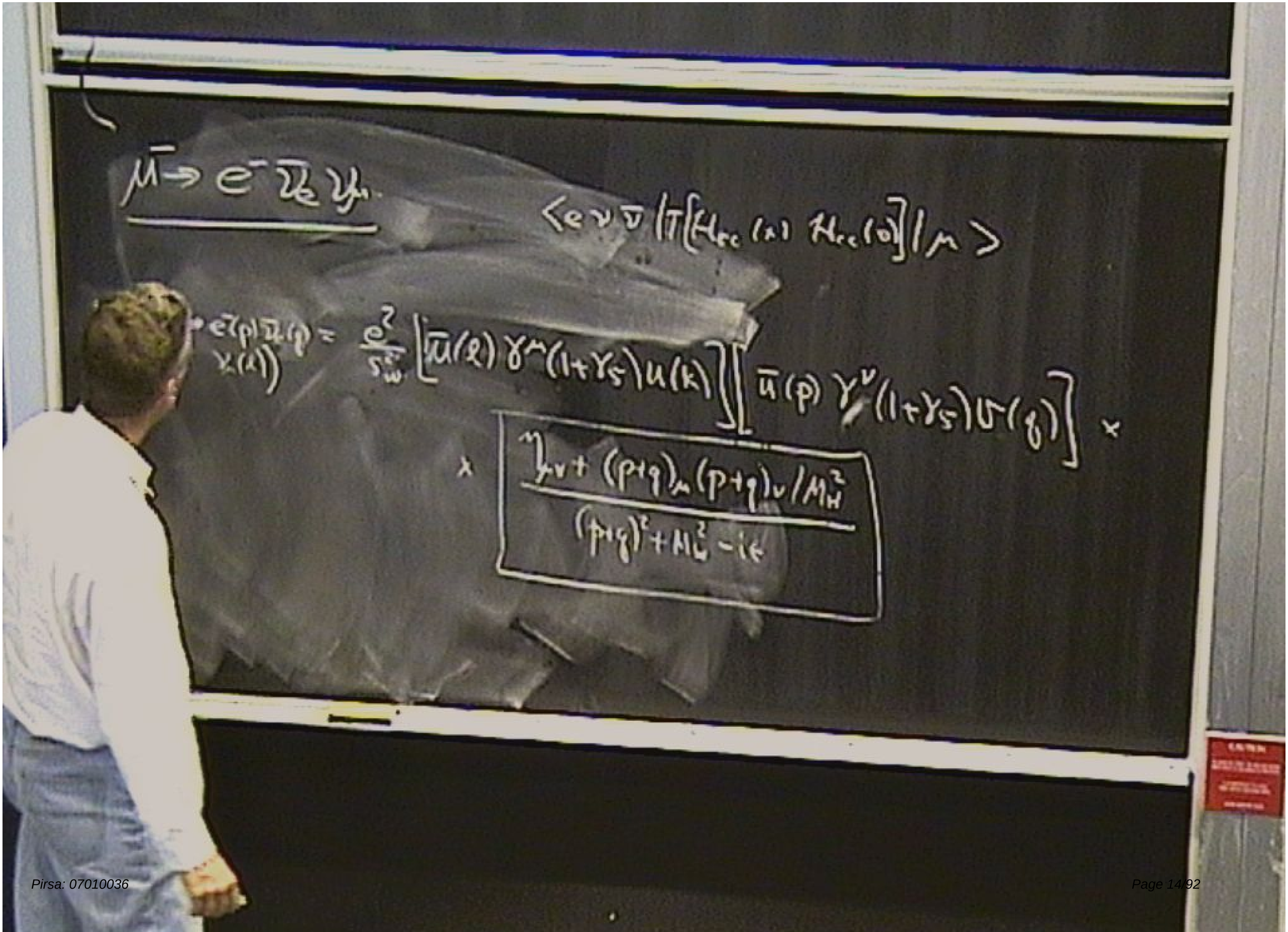


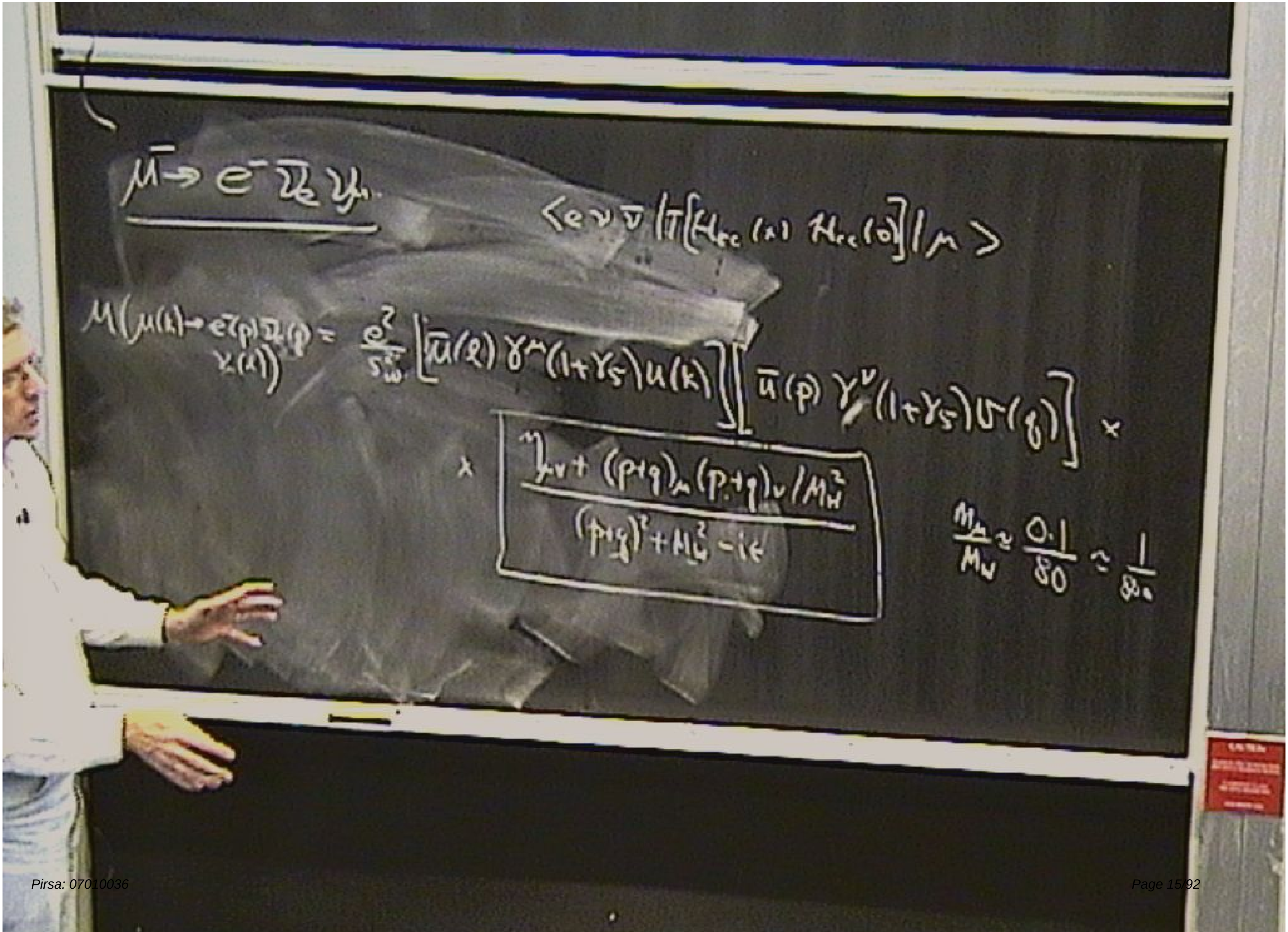
μ⁻ → e⁻ ν̄ₑ ν_μ
<e ν ν̄ | T[H_cc(x) H_cc(0)] | μ>

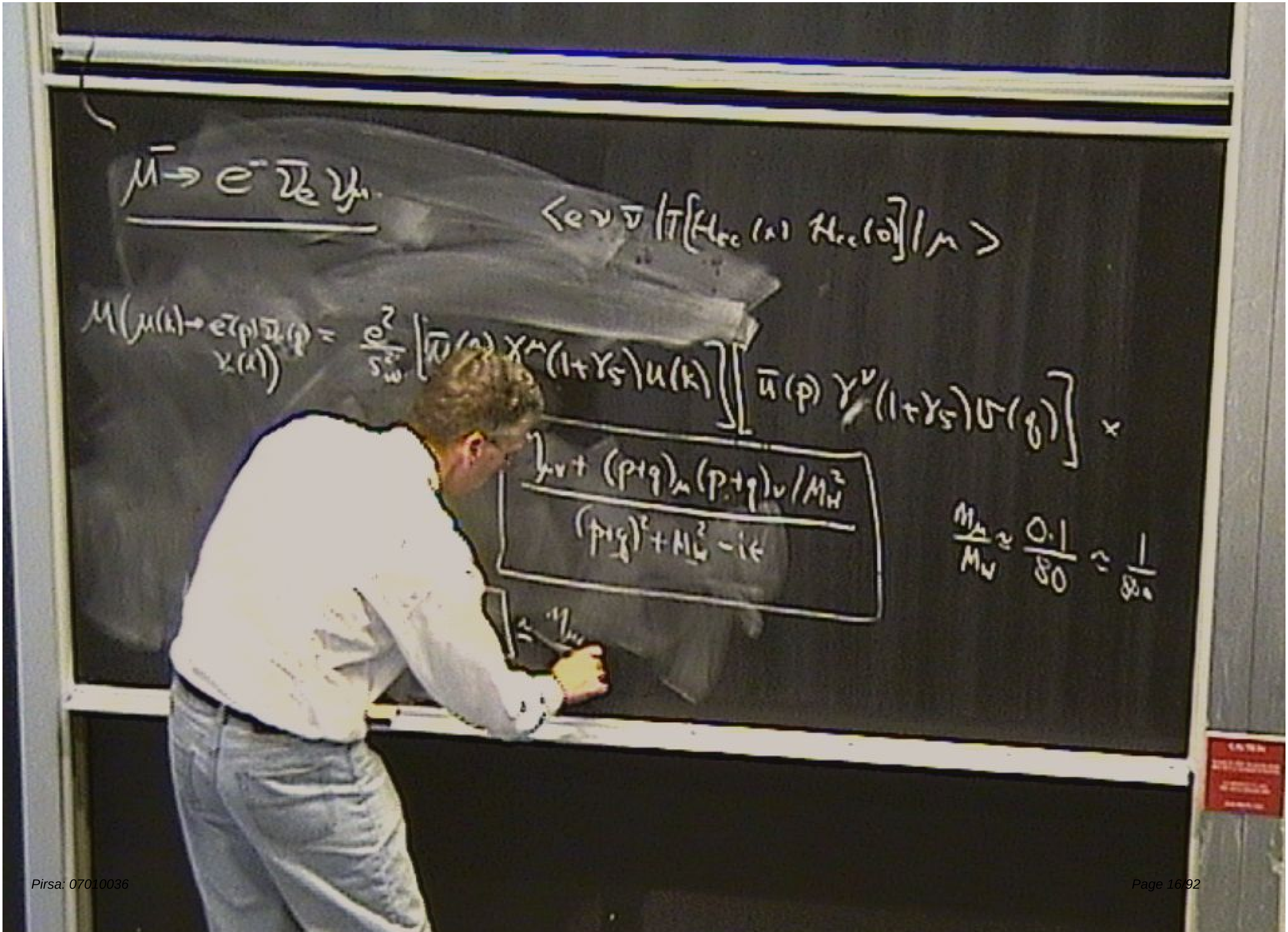
$$\mu^- \to e^- \bar{\nu}_e \nu_\mu$$

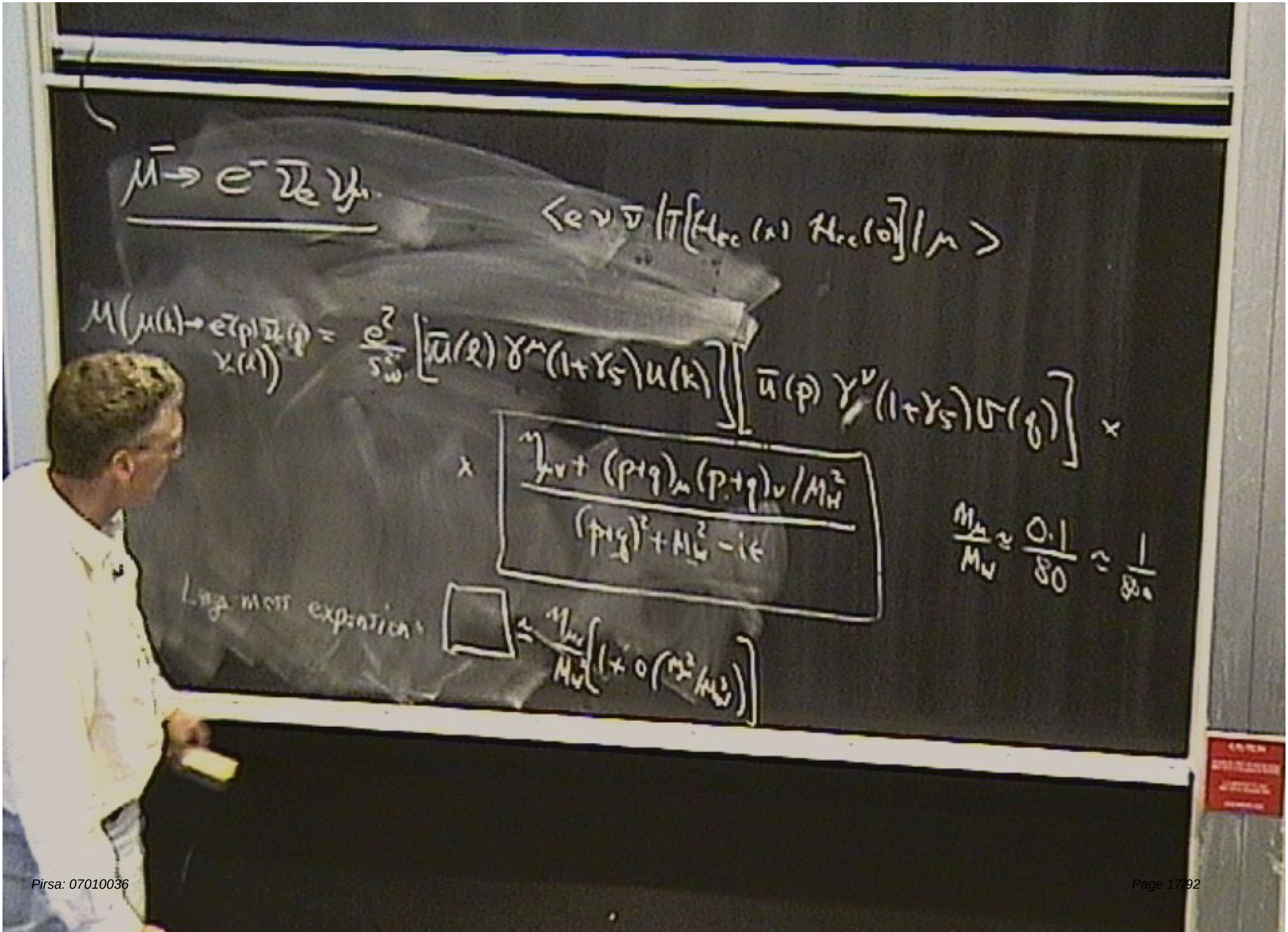
$$\langle e \nu \bar{\nu} | T[H_{cc}(x) \, H_{cc}(0)] | \mu \rangle$$

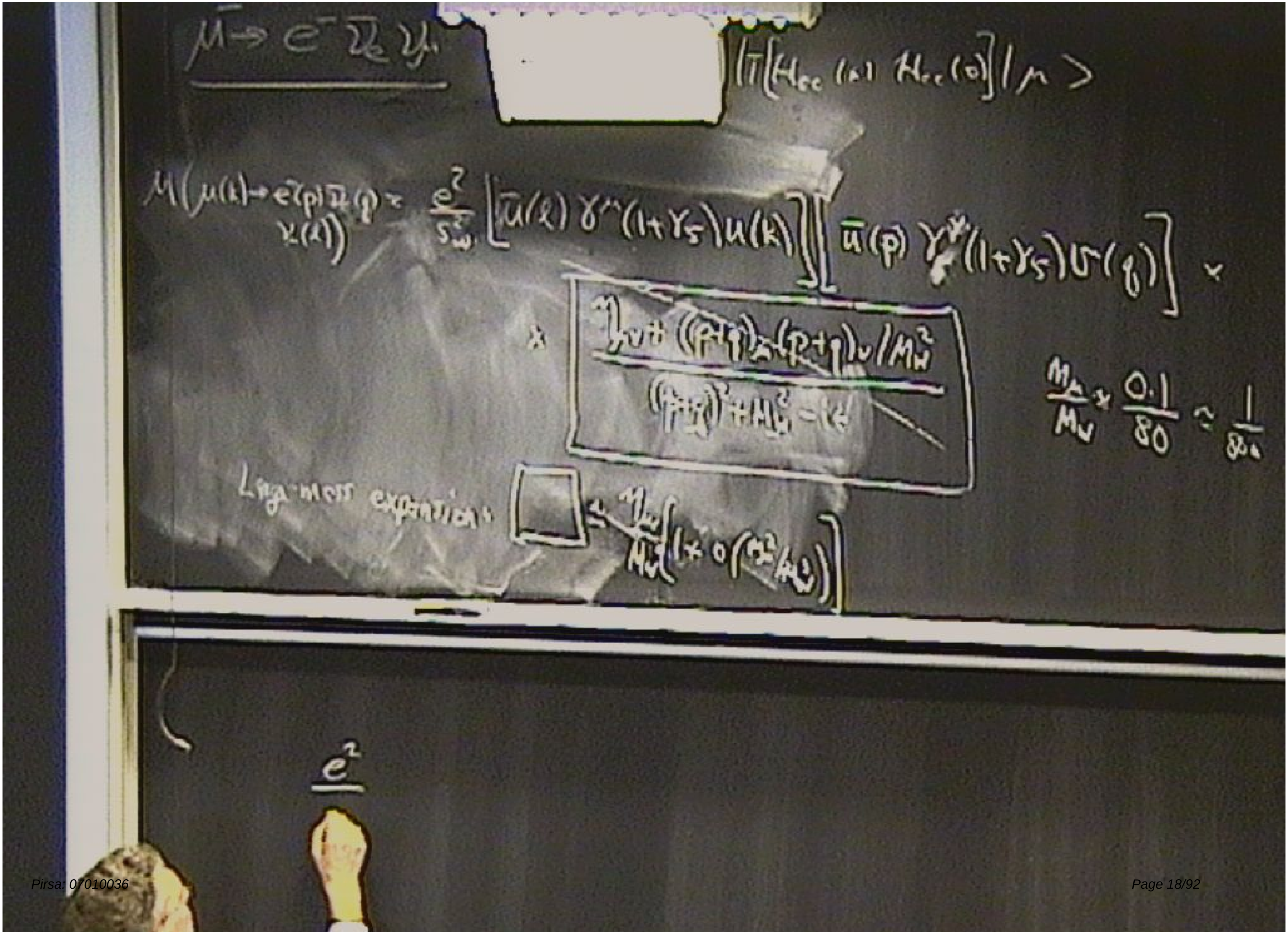
$$\mathcal{M}(\mu(k) \to e^-(p) \bar{\nu}_e(q) \nu(k')) = \frac{e^2}{s_w^2} \left[\bar{u}(k') \gamma^\mu (1+\gamma_5) u(k)\right] \left[\bar{u}(p) \gamma^\nu (1+\gamma_5) v(q)\right] \frac{\eta_{\mu\nu} + (p+q)_\mu (p+q)_\nu / M_W^2}{(p+q)^2 + M_W^2 - i\epsilon}$$

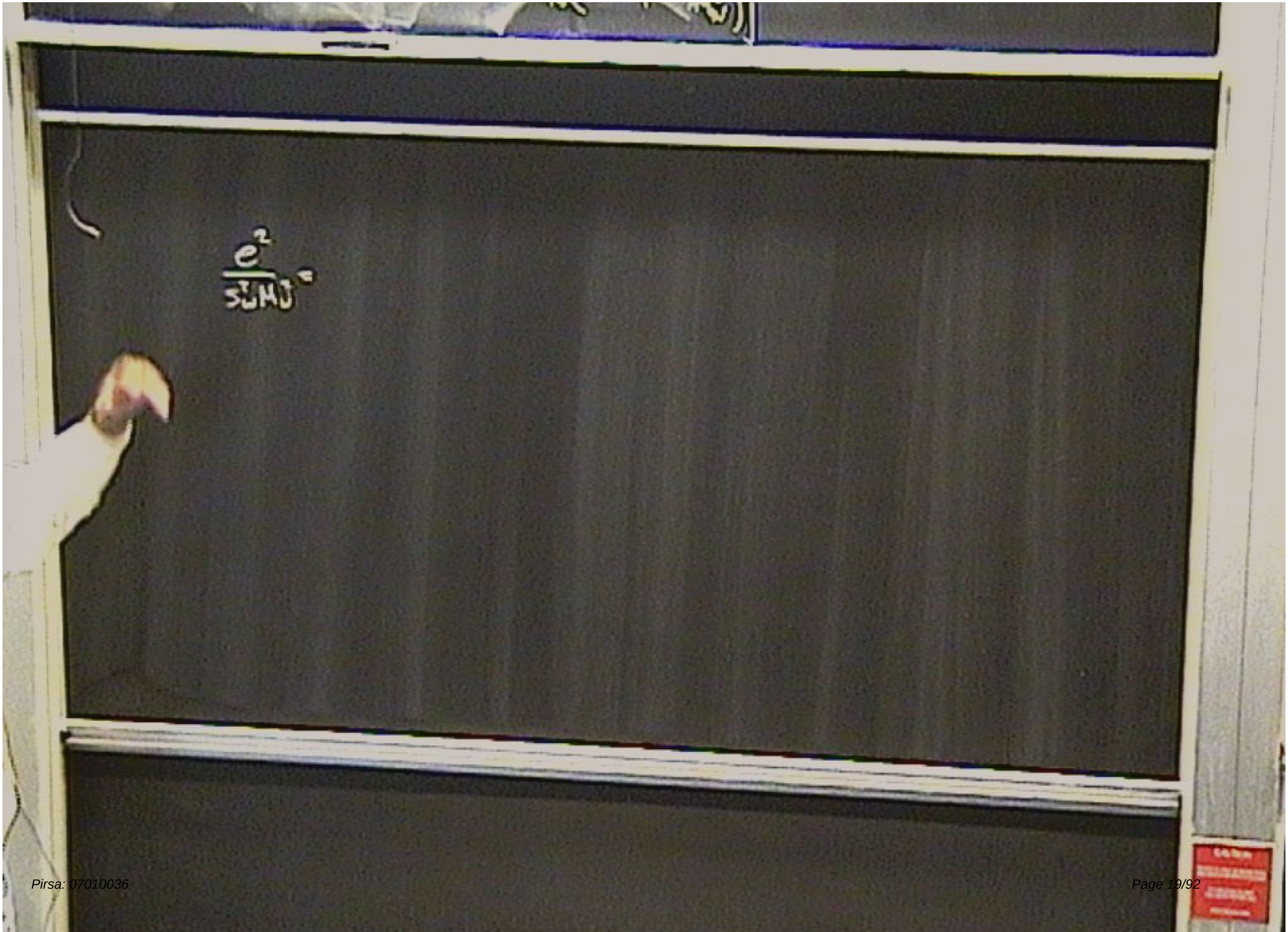

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$
$\langle e \nu \bar{\nu} | T[H_{cc}(x)\, H_{cc}(0)] | \mu \rangle$
$e^-(p)\,\bar{\nu}_e(q)\,\nu_\mu(l)) = \frac{e^2}{s_W^2}\left[\bar{u}(l)\gamma^\mu(1+\gamma_5)u(k)\right]\left[\bar{u}(p)\gamma^\nu(1+\gamma_5)v(q)\right] \times$
$\times\ \boxed{\frac{\eta_{\mu\nu} + (p+q)_\mu (p+q)_\nu / M_W^2}{(p+q)^2 + M_W^2 - i\epsilon}}$


$\mu^- \to e^- \bar{\nu}_e \nu_\mu$
$\langle e \nu \bar{\nu} | T[H_{cc}(x)\, H_{cc}(0)] | \mu \rangle$
$\mathcal{M}(\mu(k) \to e^-(p) \bar{\nu}_e(q) \nu(l)) = \frac{e^2}{s_W^2} [\bar{u}(l) \gamma^\mu (1+\gamma_5) u(k)] [\bar{u}(p) \gamma^\nu (1+\gamma_5) v(q)] \times$
$\times \boxed{\frac{\eta_{\mu\nu} + (p+q)_\mu (p+q)_\nu / M_W^2}{(p+q)^2 + M_W^2 - i\epsilon}}$
$\frac{m_\mu}{M_W} \approx \frac{0.1}{80} \approx \frac{1}{800}$


$\mu^- \to e^- \bar{\nu}_e \nu_\mu$
$\langle e \nu \bar{\nu} | T[H_{cc}(x)\, H_{cc}(0)] | \mu \rangle$
$\mathcal{M}(\mu(k) \to e^-(p)\, \bar{\nu}_e(q)\, \nu(k')) = \frac{e^2}{s_w^2} [\bar{u}(q)\gamma^\mu(1+\gamma_5)u(k)][\bar{u}(p)\gamma^\nu(1+\gamma_5)v(q)] \times$
$\frac{g_{\mu\nu} + (p+q)_\mu (p+q)_\nu / M_W^2}{(p+q)^2 + M_W^2 - i\epsilon}$
$\approx g_{\mu\nu}$
$\frac{m_\mu}{M_W} \approx \frac{0.1}{80} \approx \frac{1}{800}$


μ⁻ → e⁻ ν̄_e ν_μ
⟨e ν ν̄ | T[H_cc(x) H_cc(0)] | μ⟩
M(μ(k) → e(p) ν̄_e(q) ν(l)) = e²/s_w² [ū(l) γ^μ (1+γ₅) u(k)] [ū(p) γ^ν (1+γ₅) v(q)] ×
× [η_μν + (p+q)_μ (p+q)_ν / M_W²] / [(p+q)² + M_W² − iε]
m_μ/M_W ≈ 0.1/80 ≈ 1/800
Large mass expansion: ≈ η_μν/M_W² [1 + O(m²/M_W²)]


μ⁻ → e⁻ ν̄e νμ
<|T[Hcc(x) Hcc(0)]|μ>
M(μ(k) → e⁻(p) ν̄e(q) νμ(l)) = e²/s²W [ū(l) γ^μ (1+γ5) u(k)] [ū(p) γ^ν (1+γ5) v(q)] ×
Long-mass expansion
e²

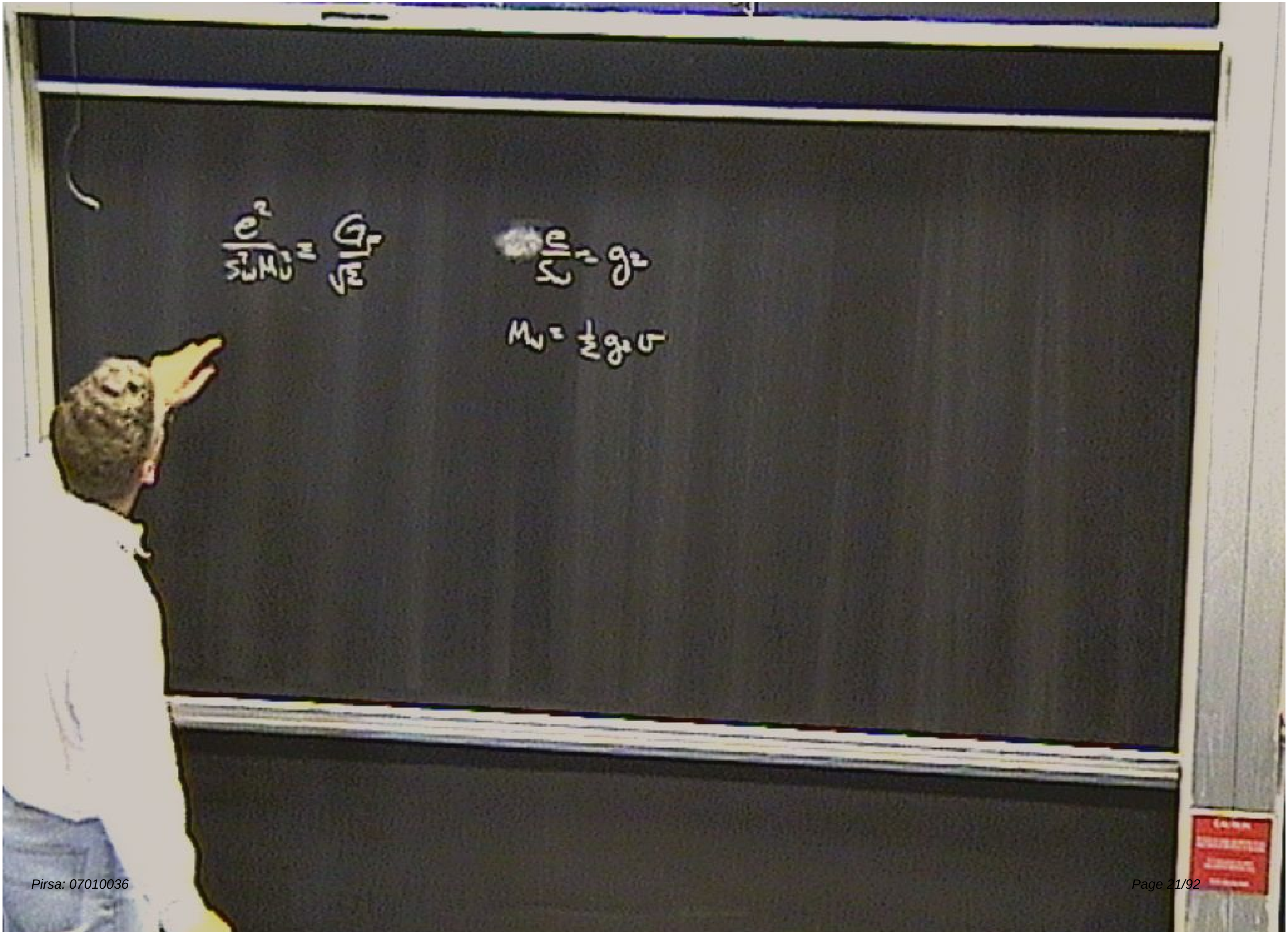


$$\frac{\eta_{\mu\nu} + (p+q)_\mu (p+q)_\nu / M_W^2}{(p+q)^2 + M_W^2 - i\epsilon}$$

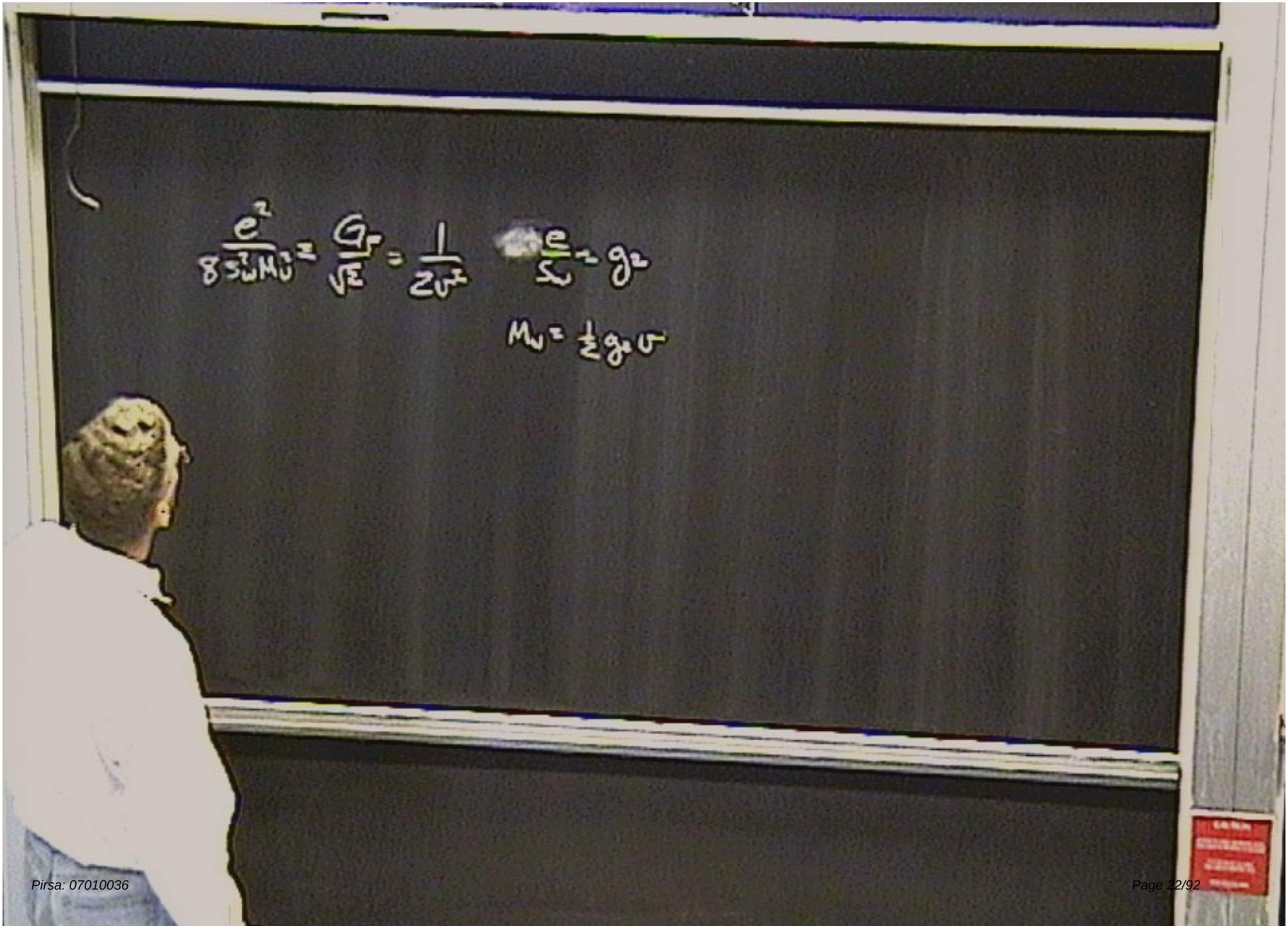
Low mom expansion $\boxed{} \approx \frac{\eta_{\mu\nu}}{M_W^2}\left[1 + O(p^2/M_W^2)\right]$

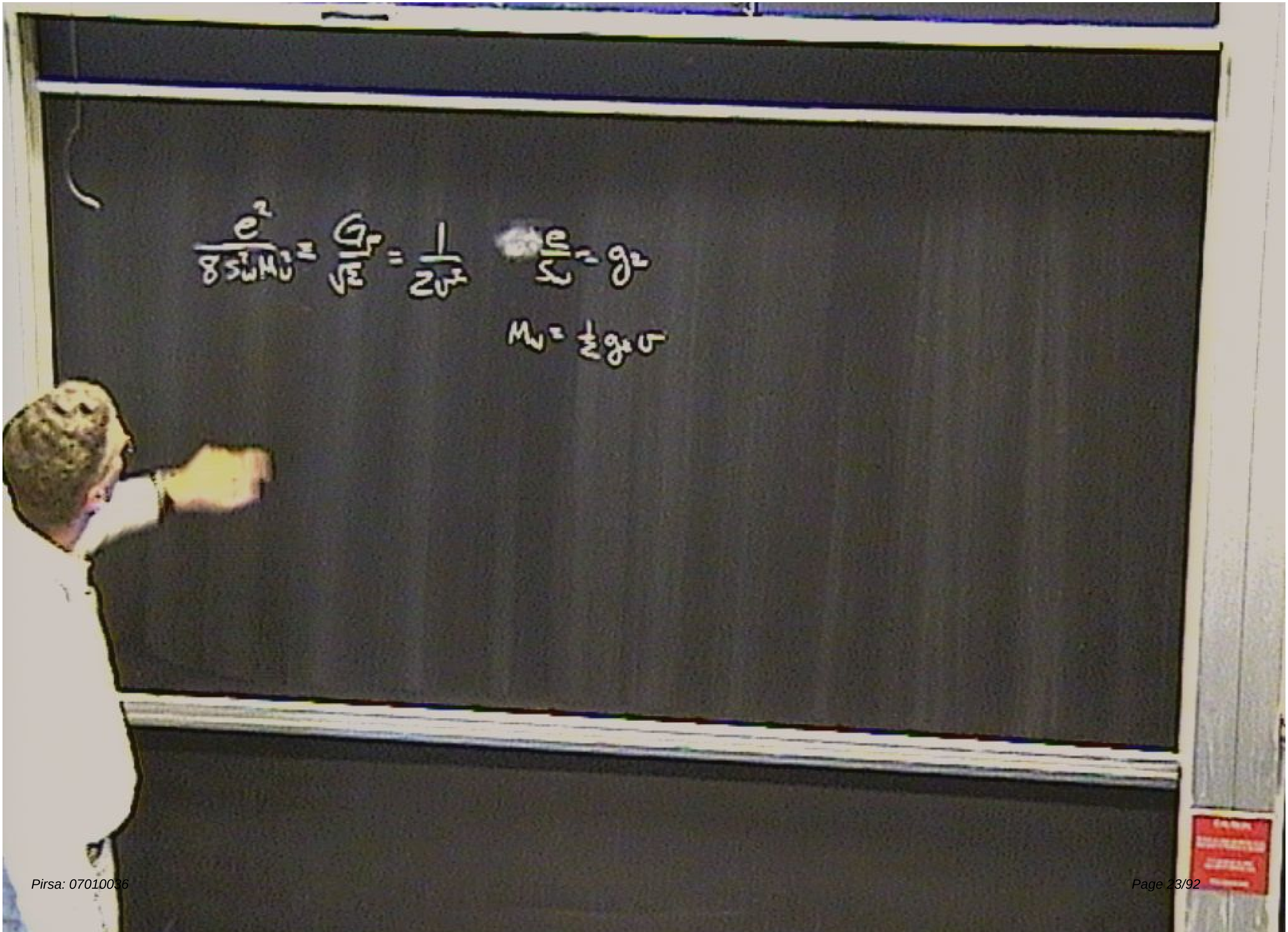
$$\frac{m_\mu}{M_W} \approx \frac{0.1}{80} \approx \frac{1}{800}$$

$$\frac{e^2}{s_W^2 M_W^2} \approx \frac{G_F}{\sqrt{2}}$$



$\frac{e^2}{s_w^2 M_W^2} = \frac{G_F}{\sqrt{2}}$
$\frac{e}{s_w} = g_2$
$M_W = \frac{1}{2} g_2 v$


$\frac{e^2}{8 s_W^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$ $\frac{e}{s_W} = g_2$
$M_W = \frac{1}{2} g_2 v$


$\frac{e^2}{8 s_w^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$
$\frac{e}{s_w} = g_2$
$M_W = \frac{1}{2} g_2 v$

$$\frac{e^2}{8 s_w^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad \frac{e}{s_w} = g_2$$

$$v = 246 \text{ GeV}$$

$$M_W = \frac{1}{2} g_2 v$$

$$\frac{e^2}{8 s_W^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad \frac{e}{s_W} = g_2 \qquad v = 246 \text{ GeV}$$

$$M_W = \frac{1}{2} g_2 v$$

Use this $|\mathcal{M}|^2$ in $\Gamma(\mu \to e \nu \bar{\nu})$:

$$\frac{e^2}{8 s_w^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad \frac{e}{s_w} = g_2$$

$$v = 246 \text{ GeV}$$

$$M_W = \tfrac{1}{2} g_2 v$$

Use this $|M|^2$ in $\Gamma(\mu \to e \nu \bar{\nu})$.

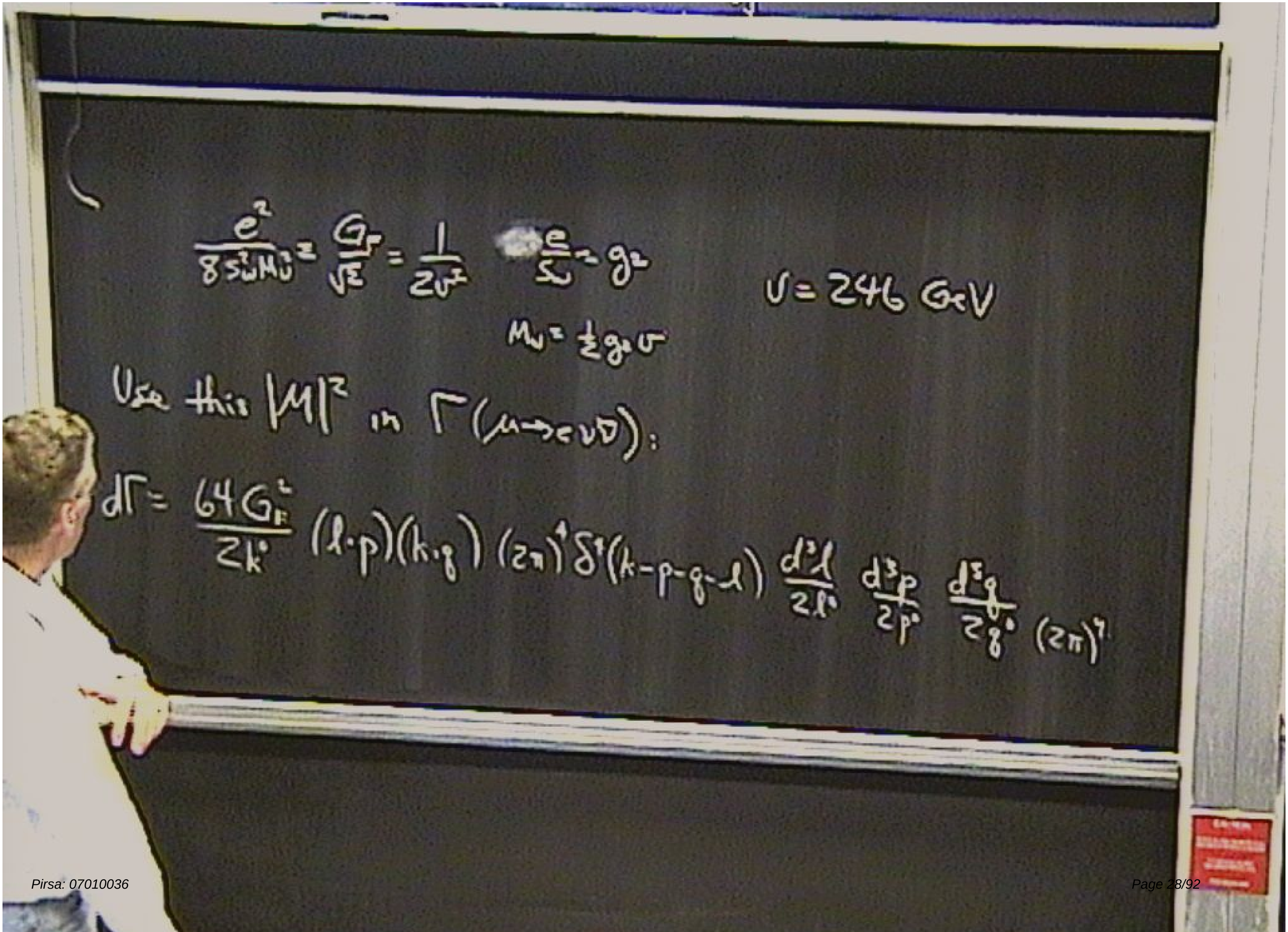
$$d\Gamma = \frac{64 G_F^2}{2k^0}$$

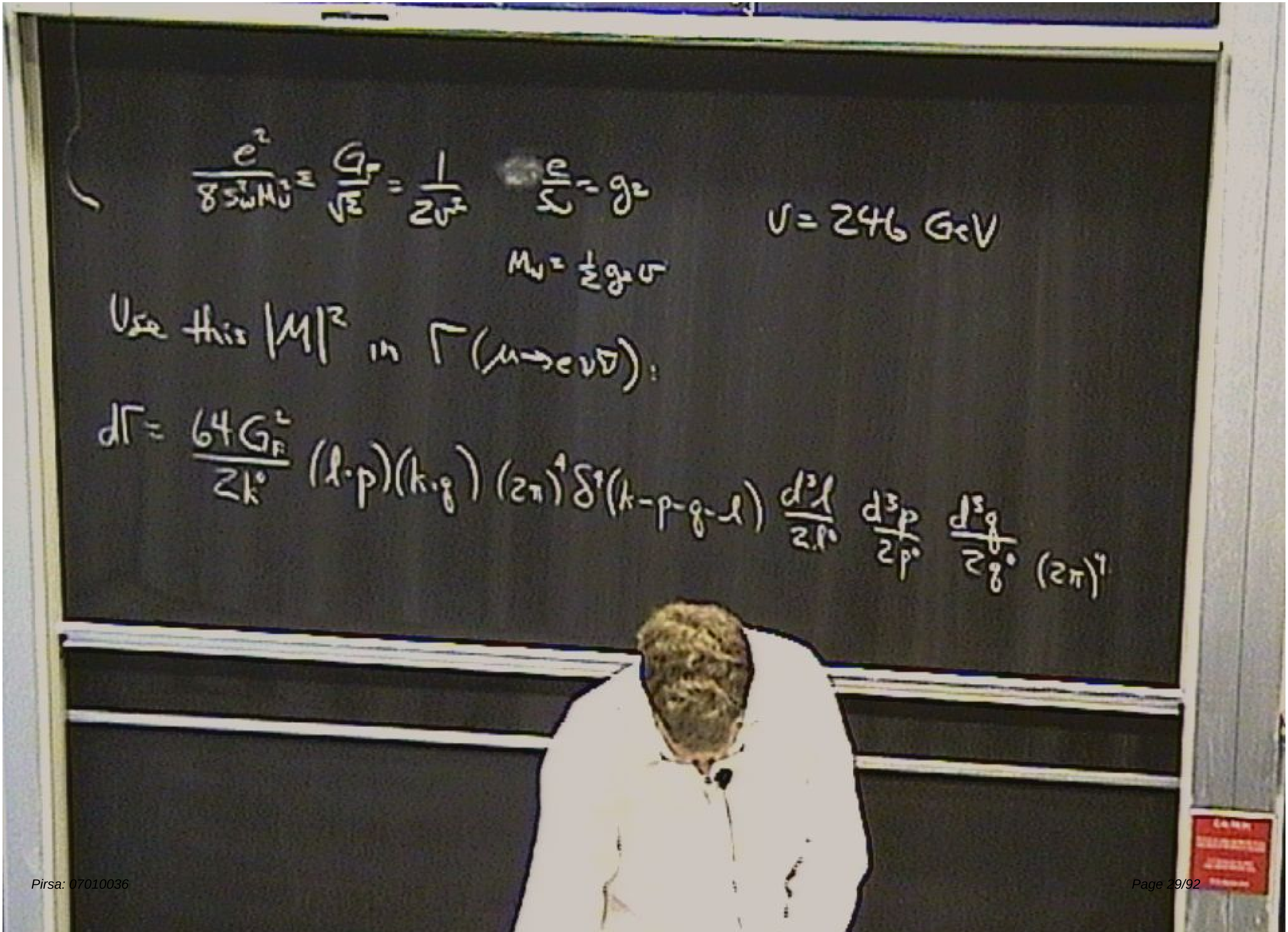
$$\frac{e^2}{8 s_w^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad \frac{e}{s_w} = g_2 \qquad v = 246\ \text{GeV}$$

$$M_W = \tfrac{1}{2} g_2 v$$

Use this $|\mathcal{M}|^2$ in $\Gamma(\mu \to e \nu \bar{\nu})$:

$$d\Gamma = \frac{64 G_F^2}{2k^0} (l \cdot p)(k \cdot q)\,(2\pi \text{[illegible]} -l)\, \frac{d^3 l}{2l^0}\, \frac{d^3 p}{2p^0}\, \frac{d^3 q}{2q^0}\, (2\pi)^{7}$$


$\frac{e^2}{8 s_W^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$ $\frac{e}{s_W} = g_2$
$v = 246$ GeV
$M_W = \frac{1}{2} g_2 v$
Use this $|M|^2$ in $\Gamma(\mu \to e \nu \bar{\nu})$:
$d\Gamma = \frac{64 G_F^2}{2k^0} (l \cdot p)(k \cdot q) (2\pi)^4 \delta^4(k-p-q-l) \frac{d^3 l}{2 l^0} \frac{d^3 p}{2 p^0} \frac{d^3 q}{2 q^0} (2\pi)^9$



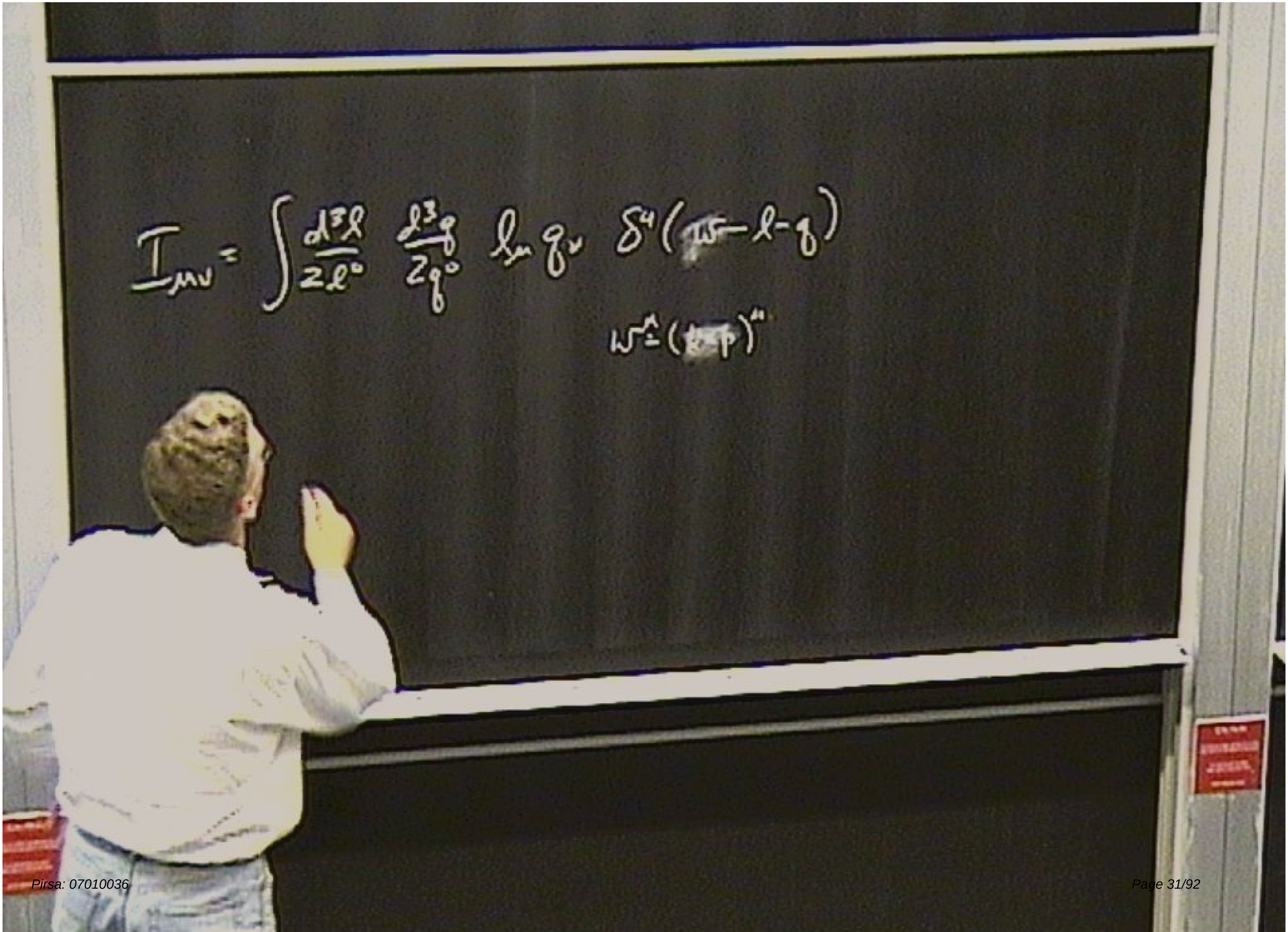
$\frac{e^2}{8 s_w^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$ $\frac{e}{s_w} = g_2$
$v = 246$ GeV
$M_W = \frac{1}{2} g_2 v$
Use this $|M|^2$ in $\Gamma(\mu \to e \nu \bar{\nu})$:
$d\Gamma = \frac{64 G_F^2}{2k^0} (l\cdot p)(k\cdot q)\,(2\pi)^4 \delta^4(k-p-q-l)\,\frac{d^3 l}{2l^0}\,\frac{d^3 p}{2p^0}\,\frac{d^3 q}{2q^0\,(2\pi)^9}$

$$\frac{e^2}{8 s_w^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad \frac{e}{s_w} = g_2 \qquad v = 246\ \text{GeV}$$

$$M_W = \tfrac{1}{2} g_2 v$$

Use this $|M|^2$ in $\Gamma(\mu \to e \nu \bar{\nu})$:

$$d\Gamma = \frac{64 G_F^2}{2k^0} (l \cdot p)(k \cdot q)\, (2\pi)^4 \delta^4(k-p-q-l)\, \frac{d^3 l}{2l^0} \frac{d^3 p}{2p^0} \frac{d^3 q}{2q^0 (2\pi)^9}$$


$I_{\mu\nu} = \int \frac{d^3\ell}{2\ell^0} \frac{d^3q}{2q^0} \ell_\mu q_\nu \, \delta^4(w-\ell-q)$
$w^\mu = (k-p)^\mu$

$$I_{\mu\nu}(w) = \int \frac{d^3\ell}{2\ell^0}\,\frac{d^3q}{2q^0}\,\ell_\mu q_\nu\,\delta^4(w - \ell - q)$$

$$w^\mu = (\quad - p)^\mu$$

$$= A(w^2)\,\eta_{\mu\nu} + B(w^2)\,w_\mu w_\nu$$

$$I_{\mu\nu}(w) = \int \frac{d^3 l}{2l^0} \frac{d^3 q}{2q^0} \, l_\mu q_\nu \, \delta^4(w - l - q)$$

$$w^\mu = (k - p)^\mu$$

$$= A(w^2)\, \eta_{\mu\nu} + B(w^2)\, w_\mu w_\nu$$

$$I^\mu{}_\mu = 4A + Bw^2 = \int \frac{d^3 l}{2l^0} \frac{d^3 q}{2q^0} \, l \cdot q \, \delta^4(w - l - q)$$

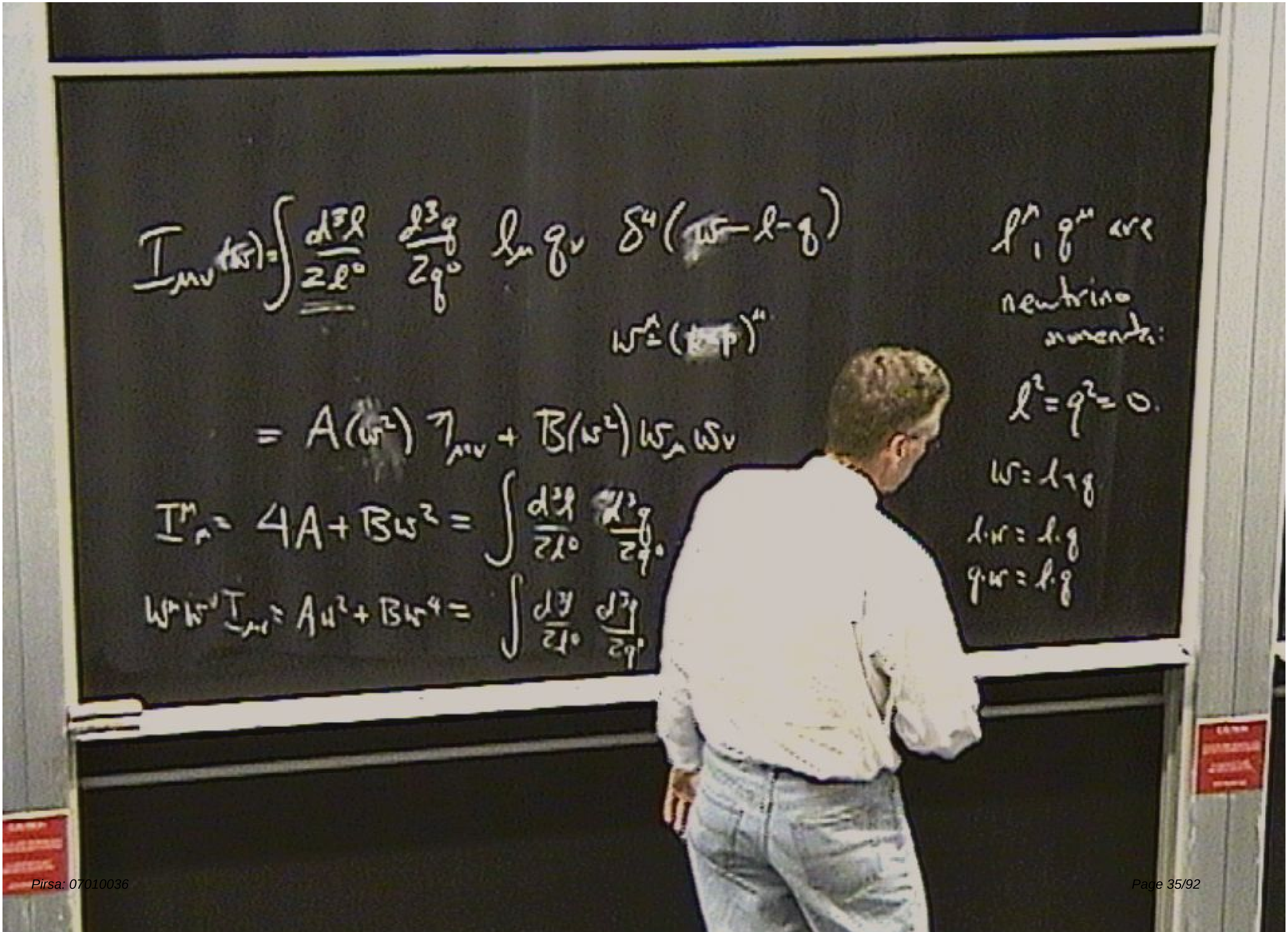
$$I_{\mu\nu}(w) = \int \frac{d^3 l}{2l^0}\,\frac{d^3 q}{2q^0}\, l_\mu q_\nu \,\delta^4(w - l - q)$$

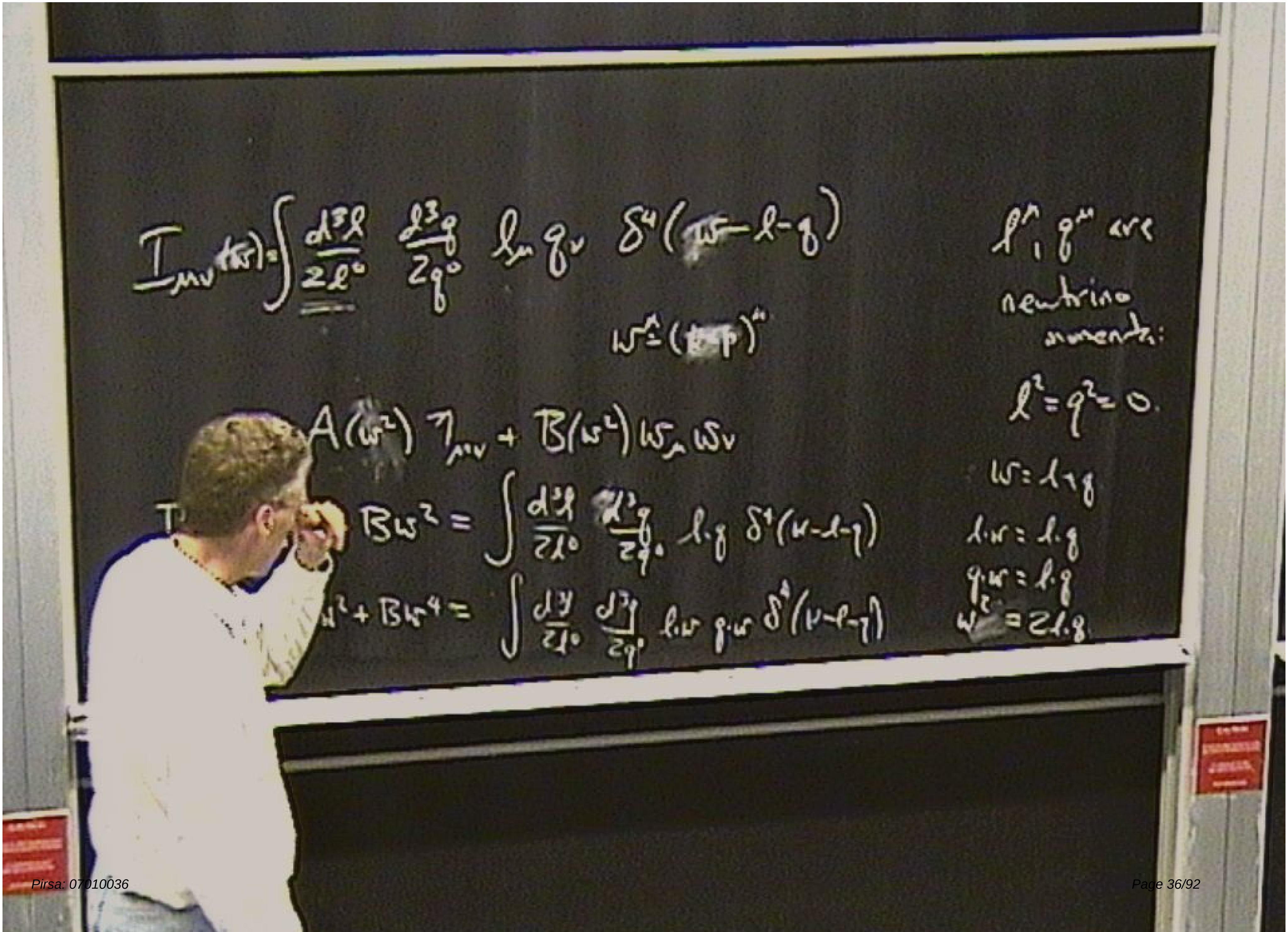
$$w^\mu = (l+q)^\mu$$

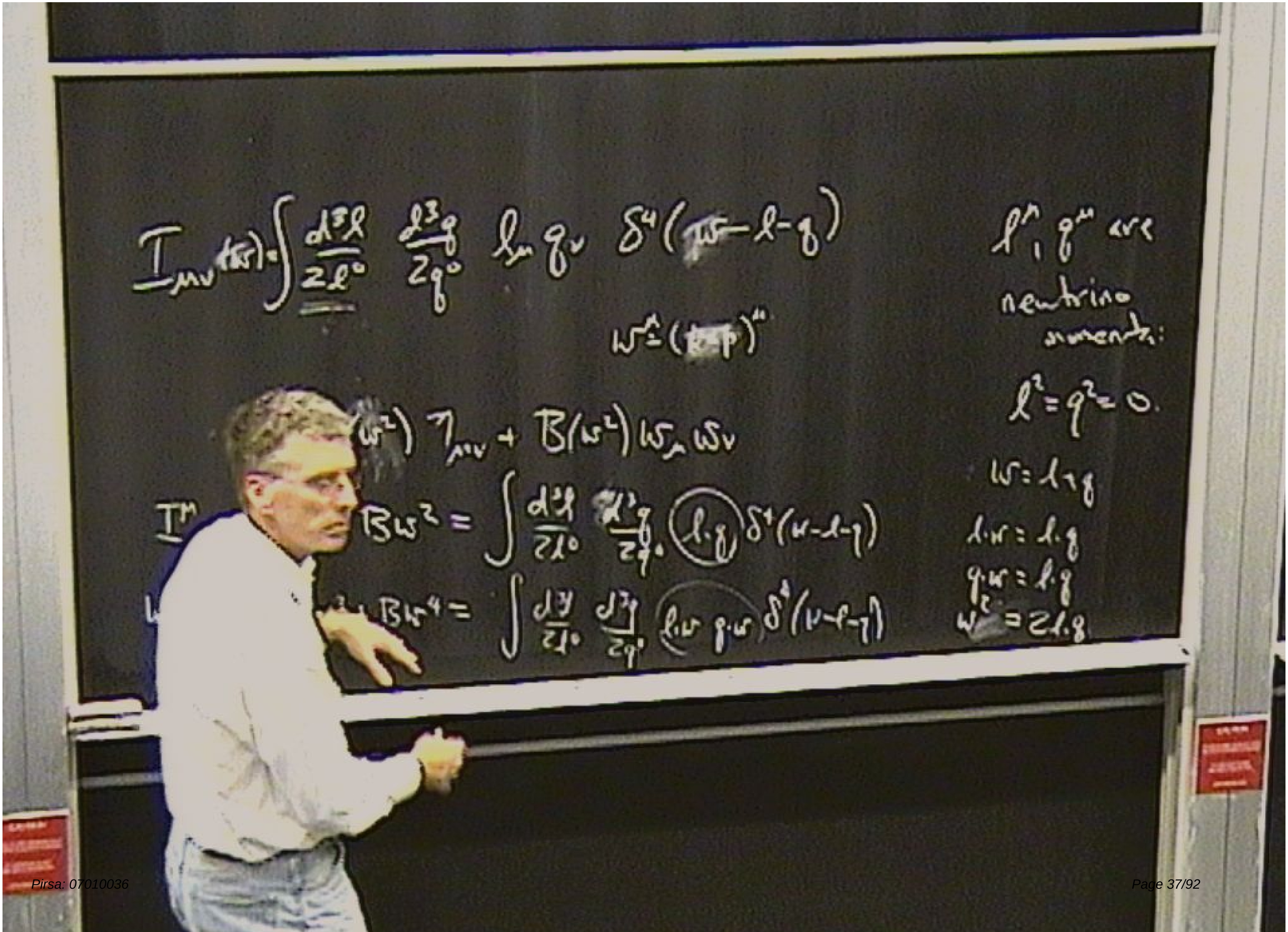
$$= A(w^2)\,\eta_{\mu\nu} + B(w^2)\, w_\mu w_\nu$$

$$I^\mu_\mu = 4A + Bw^2 = \int \frac{d^3 l}{2l^0}\,\frac{d^3 q}{2q^0}\, l\cdot q \,\delta^4(w - l - q)$$

$$w^\mu w^\nu I_{\mu\nu} = Aw^2 + Bw^4 = \int \frac{d^3 l}{2l^0}\,\frac{d^3 q}{2q^0}\, l\cdot w \; q\cdot w \,\delta^4(w - l - q)$$


$I_{\mu\nu}(w) = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0} l_\mu q_\nu \delta^4(w - l - q)$
$w^\mu = (k-p)^\mu$
$= A(w^2)\eta_{\mu\nu} + B(w^2) w_\mu w_\nu$
$I^\mu_\mu = 4A + Bw^2 = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0}$
$w^\mu w^\nu I_{\mu\nu} = Aw^2 + Bw^4 = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0}$
l^μ, q^μ are neutrino momenta:
$l^2 = q^2 = 0.$
$w = l + q$
$l \cdot w = l \cdot q$
$q \cdot w = l \cdot q$


$I_{\mu\nu}(w) = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0} l_\mu q_\nu \delta^4(w - l - q)$
l^μ, q^μ are neutrino momenta:
$w^\mu = (k-p)^\mu$
$A(w^2)\eta_{\mu\nu} + B(w^2) w_\mu w_\nu$
$Bw^2 = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0} l \cdot q\, \delta^4(w-l-q)$
$w^2 + Bw^4 = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0} l\cdot w\, q\cdot w\, \delta^4(w-l-q)$
$l^2 = q^2 = 0.$
$w = l + q$
$l \cdot w = l \cdot q$
$q \cdot w = l \cdot q$
$w^2 = 2l \cdot q$


$I_{\mu\nu}(w) = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0} l_\mu q_\nu \delta^4(w - l - q)$
$w^\mu = (k - p)^\mu$
$(w^2)\eta_{\mu\nu} + B(w^2) w_\mu w_\nu$
$Bw^2 = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0} l\cdot q\, \delta^4(w-l-q)$
$Bw^4 = \int \frac{d^3l}{2l^0} \frac{d^3q}{2q^0} l\cdot w\, q\cdot w\, \delta^4(w-l-q)$
l^μ, q^μ are neutrino momenta:
$l^2 = q^2 = 0.$
$w = l + q$
$l\cdot w = l\cdot q$
$q\cdot w = l\cdot q$
$w^2 = 2l\cdot q$

$$I_{\mu\nu}(w) = \int \frac{d^3\ell}{2\ell^0}\,\frac{d^3q}{2q^0}\,\ell_\mu q_\nu\,\delta^4(w-\ell-q)$$

$$w^\mu = (k-p)^\mu$$

$$= A(w^2)\,\eta_{\mu\nu} + B(w^2)\,w_\mu w_\nu$$

$$4A + Bw^2 = \int \frac{d^3\ell}{2\ell^0}\,\frac{d^3q}{2q^0}\,\overbrace{\ell\cdot q}^{\frac{1}{2}w^2}\,\delta^4(w-\ell-q)$$

$$w^\mu w^\nu I_{\mu\nu} = Aw^2 + Bw^4 = \int \frac{d^3\ell}{2\ell^0}\,\frac{d^3q}{2q^0}\,\overbrace{\ell\cdot w\; q\cdot w}^{\frac{1}{4}(w^2)^2}\,\delta^4(w-\ell-q)$$

ℓ^μ, q^μ are neutrino momenta:

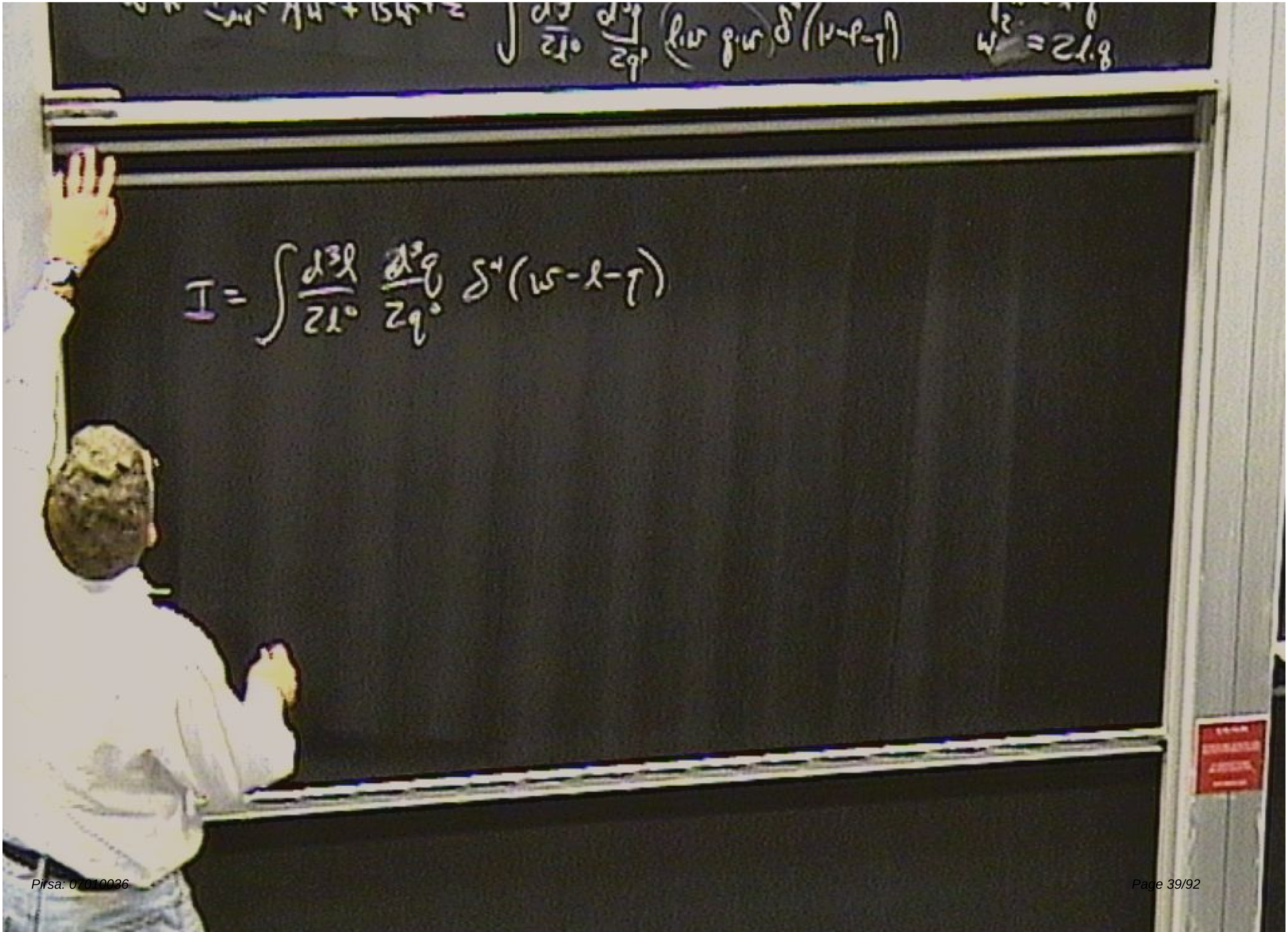
$$\ell^2 = q^2 = 0.$$

$$w = \ell + q$$

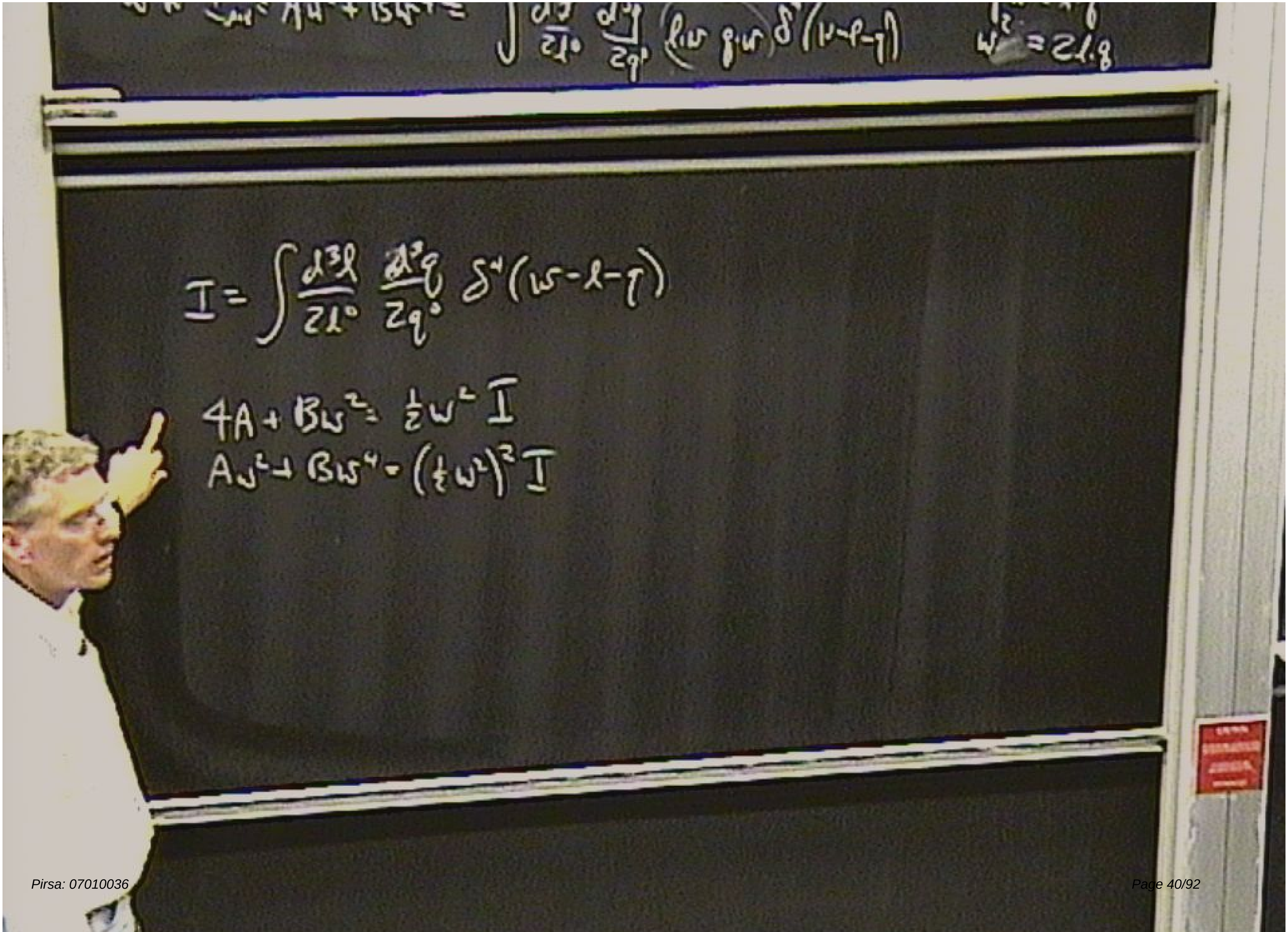
$$\ell\cdot w = \ell\cdot q$$

$$q\cdot w = \ell\cdot q$$

$$w^2 = 2\ell\cdot q$$



$I = \int \frac{d^3 l}{2l^0} \frac{d^3 q}{2q^0} \delta^4(w - l - q)$


$\int \frac{d^3l}{2l^0}\frac{d^3q}{2q^0}\, l\cdot w\; q\cdot w\; \delta^4(w-l-q)$
$w^2 = 2l\cdot q$
$I = \int \frac{d^3l}{2l^0}\frac{d^3q}{2q^0}\,\delta^4(w-l-q)$
$4A + Bw^2 = \frac{1}{2}w^2 I$
$Aw^2 + Bw^4 = \left(\frac{1}{2}w^2\right)^2 I$

$$\int \frac{d^3 l}{2l^0}\frac{d^3 q}{2q^0}\,(l\cdot w\; q\cdot w)\,\delta^4(w-l-q) \qquad w^2 = 2l\cdot q$$

$$I = \int \frac{d^3 l}{2l^0}\frac{d^3 q}{2q^0}\,\delta^4(w-l-q)$$

$$4A + Bw^2 = \tfrac{1}{2}w^2 I$$

$$Aw^2 + Bw^4 = \left(\tfrac{1}{2}w^2\right)^2 I$$

given I, solve for A, B.

$$I_{\mu\nu}(w) = \int \frac{d^3\ell}{2\ell^0}\,\frac{d^3q}{2q^0}\,\ell_\mu q_\nu\,\delta^4(w-\ell-q)$$

ℓ^μ, q^μ are neutrino momenta:

$$= A(w^2)\,\eta_{\mu\nu} + B(w^2)\,w_\mu w_\nu$$

$\ell^2 = q^2 = 0.$

$$I^\mu_\mu = 4A + Bw^2 = \int \frac{d^3\ell}{2\ell^0}\,\frac{d^3q}{2q^0}\,\overset{\frac{1}{2}w^2}{\ell\cdot q}\,\delta^4(w-\ell-q)$$

$$w^\mu w^\nu I_{\mu\nu} = Aw^2 + Bw^4 = \int \frac{d^3\ell}{2\ell^0}\,\frac{d^3q}{2q^0}\,\overset{\frac{1}{4}(w^2)^2}{\ell\cdot w\; q\cdot w}\,\delta^4(w-\ell-q)$$

$w = \ell + q$

$\ell\cdot w = \ell\cdot q$

$q\cdot w = \ell\cdot q$

$w^2 = 2\ell\cdot q$

$$I = \int \frac{d^3\ell}{2\ell^0}\,\frac{d^3q}{2q^0}\,\delta^4(w-\ell-q) = I(w^2)$$

$$4A + Bw^2 = \tfrac{1}{2}w^2 I$$

given I, solve for A, B.

$$I = \int \frac{d^3 l}{2l^0}\,\frac{d^3 q}{2q^0}\,\delta^4(w-l-q) = I(w^2)$$

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$$Aw^2 + Bw^4 = \left(\tfrac{1}{2}w^2\right)^2 I$$

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Since $l^2 = q^2 = 0$ and $w = l+q$, $w^2 \leq 0$

$$w^2 = 2l\cdot q = -2l^0 q^0 + 2\vec{q}\cdot\vec{l} = 2|\vec{l}||\vec{q}|(-1+\cos\theta)$$

$\longrightarrow w^\mu$ is timelike.

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Since $\underline{\underline{I}}$ is $\underline{\underline{\text{invariant}}}$, we can evaluate it in any frame: Choose rest frame for $W^\mu = \begin{pmatrix} W \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$$W^\mu W_\mu = -W^2$$

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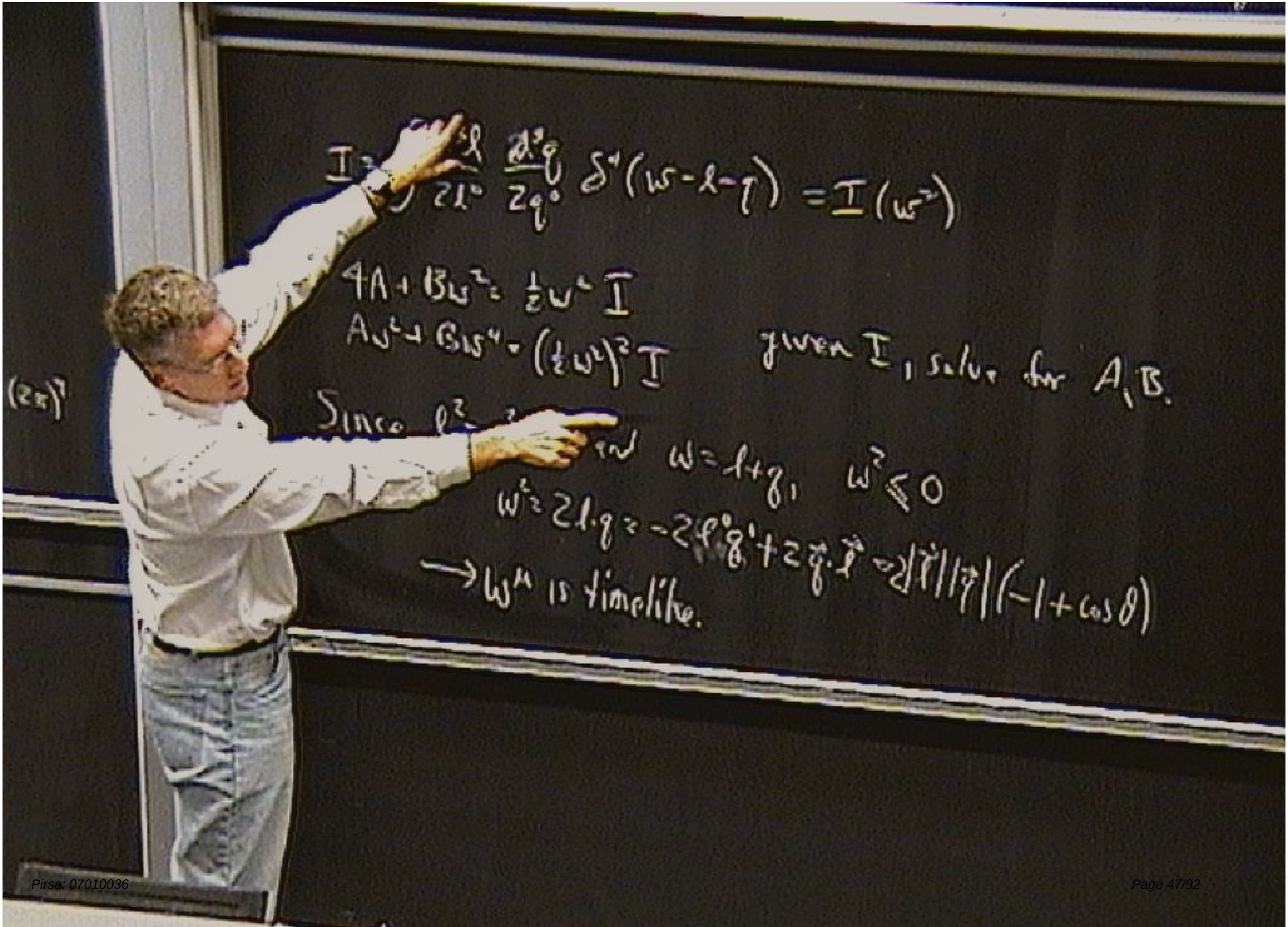
$$\delta^4(W-l-q) = \delta(W-l^0-q^0)\,\delta^3(\vec{l}+\vec{q})$$


(2π)^3
I = ∫ d^3l/2l^0 d^3q/2q^0 δ^4(w − l − q) = I(w^2)
4A + Bw^2 = ½w^2 I
Aw^2 + Bw^4 = (½w^2)^2 I
given I, solve for A, B.
Since
w = l+q, w^2 ≤ 0
w^2 = 2l·q = −2l^0q^0 + 2q·l = 2|l||q|(−1 + cos θ)
→ w^μ is timelike.

Since I is invariant, we can evaluate it in any frame: Choose rest frame for $W^\mu = \begin{pmatrix} W \\ 0 \\ 0 \\ 0 \end{pmatrix}$

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$$\delta^4(W-l-q) = \underbrace{\delta(W-l^0-q^0)}_{\delta(W-l-q)}\,\delta^3(\vec{l}+\vec{q})$$

$$\underline{I}(W^2) = \frac{\pi}{2}$$

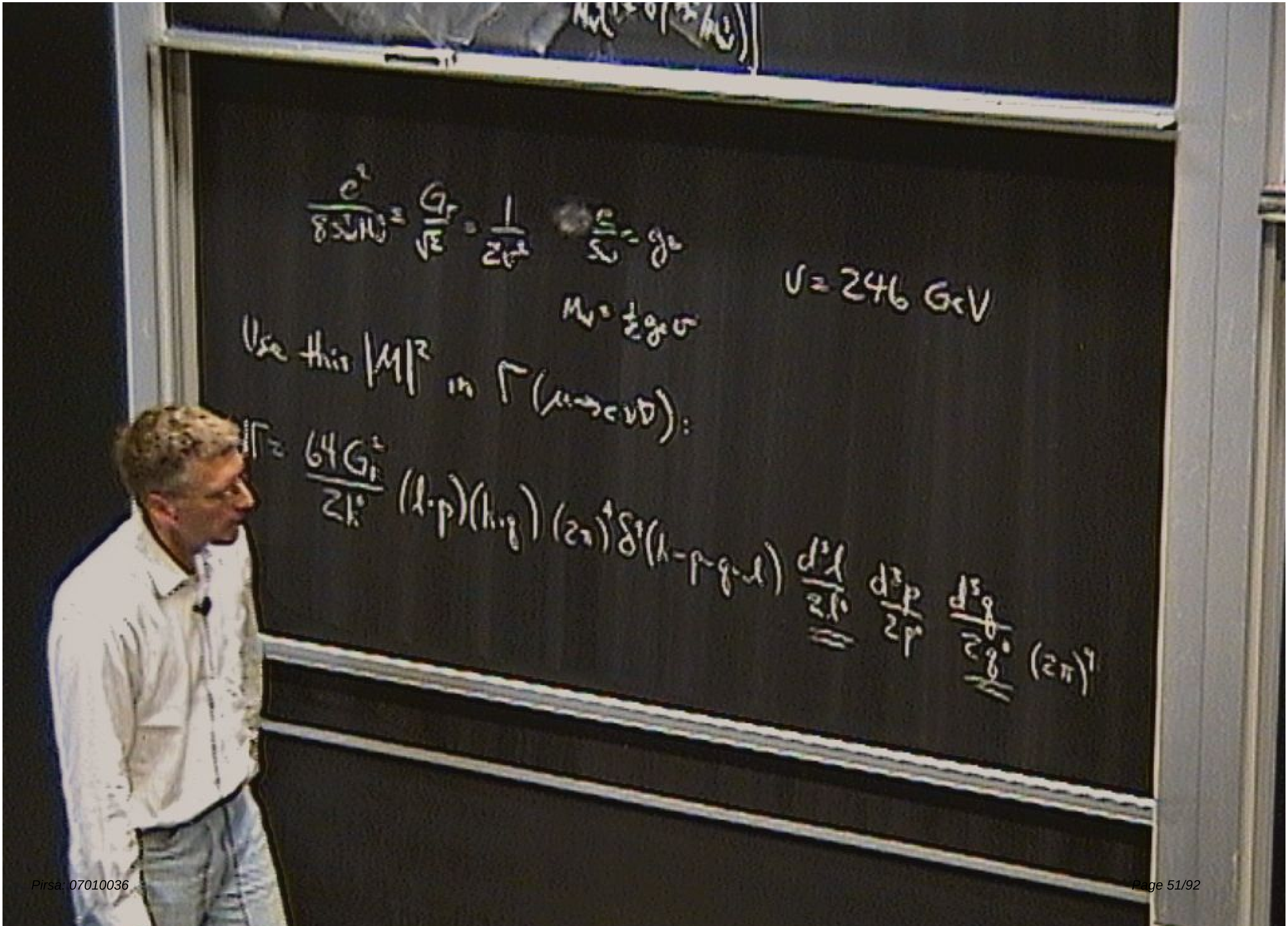
Since $\underline{I}$ is invariant, we can evaluate it in any frame: Choose rest frame for $w^\mu = \begin{pmatrix} W \\ 0 \\ 0 \\ 0 \end{pmatrix}$

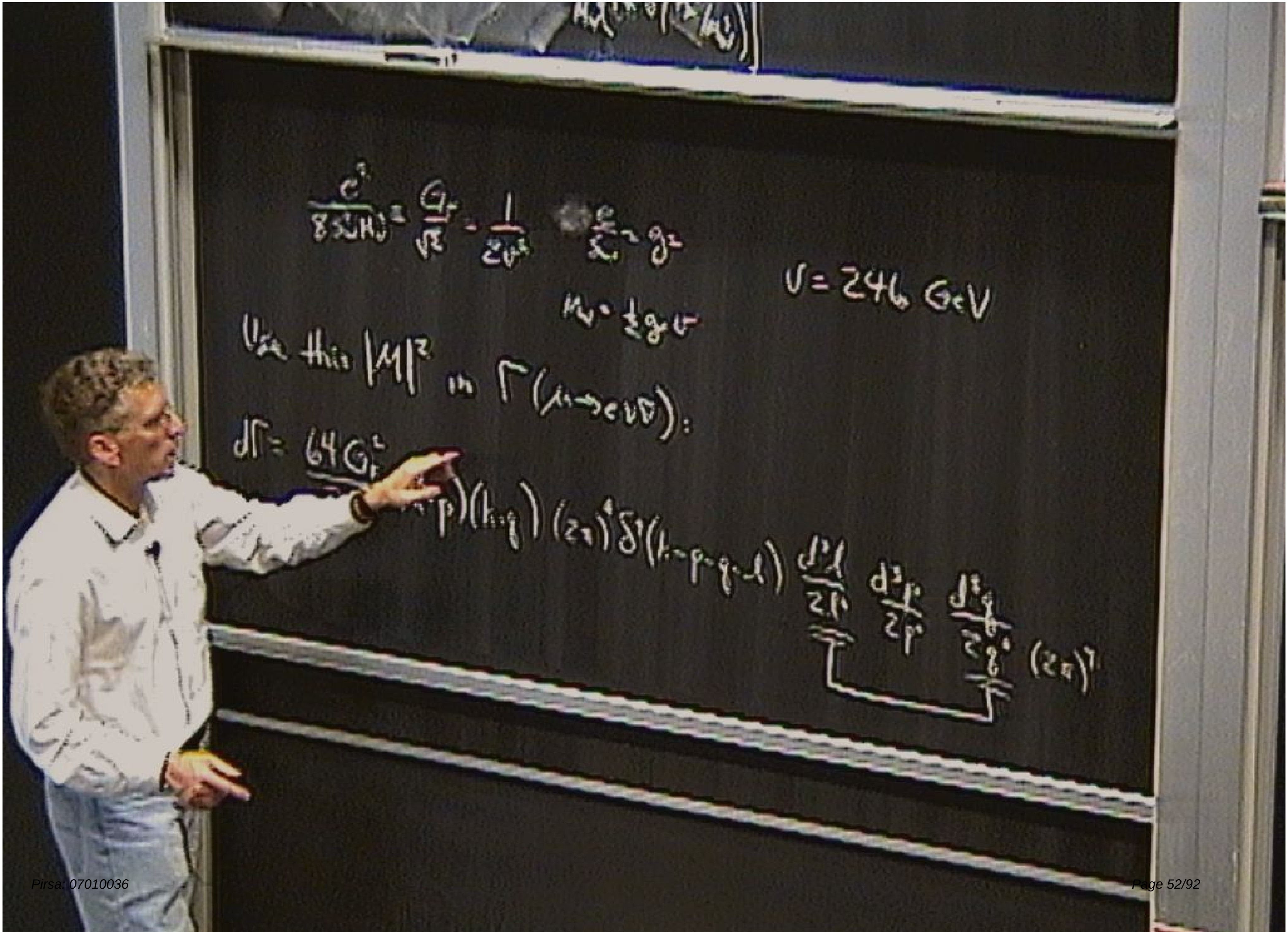
$$w^\mu w_\mu = -W^2$$

$$\delta^4(w-l-q) = \underbrace{\delta(W-l^0-q^0)}_{\delta(W-l-q)}\,\delta^3(\vec{l}+\vec{q})$$

$$\underline{I}(w^2) = \frac{\pi}{2}\,\Theta(-w^2)$$

w^μ must be timelike.


$\frac{e^2}{8\sin^2\theta_W M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$
$\frac{e}{\sin\theta_W} = g_W$
$v = 246$ GeV
$M_W = \frac{1}{2} g_W v$
Use this $|M|^2$ in $\Gamma(\mu \to e \nu \bar{\nu})$:
$d\Gamma = \frac{64 G_F^2}{2 l^0} (l\cdot p)(k\cdot q)\,(2\pi)^4 \delta^4(l - p - q - k)\,\frac{d^3 k}{2k^0}\,\frac{d^3 p}{2p^0}\,\frac{d^3 q}{2q^0}\,(2\pi)^9$


$\frac{e^2}{8\sin^2\theta_W M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$
$v = 246\ \mathrm{GeV}$
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Use this $|M|^2$ in $\Gamma(\mu \to e\nu\bar{\nu})$:
$d\Gamma = 64 G_F^2$
$(2\pi)^4 \delta^4(k-p-q-l)$
$(2\pi)^9$

$$\frac{e^2}{8 s_W^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad \frac{e}{s_W} = g_2$$

$$v = 246 \text{ GeV}$$

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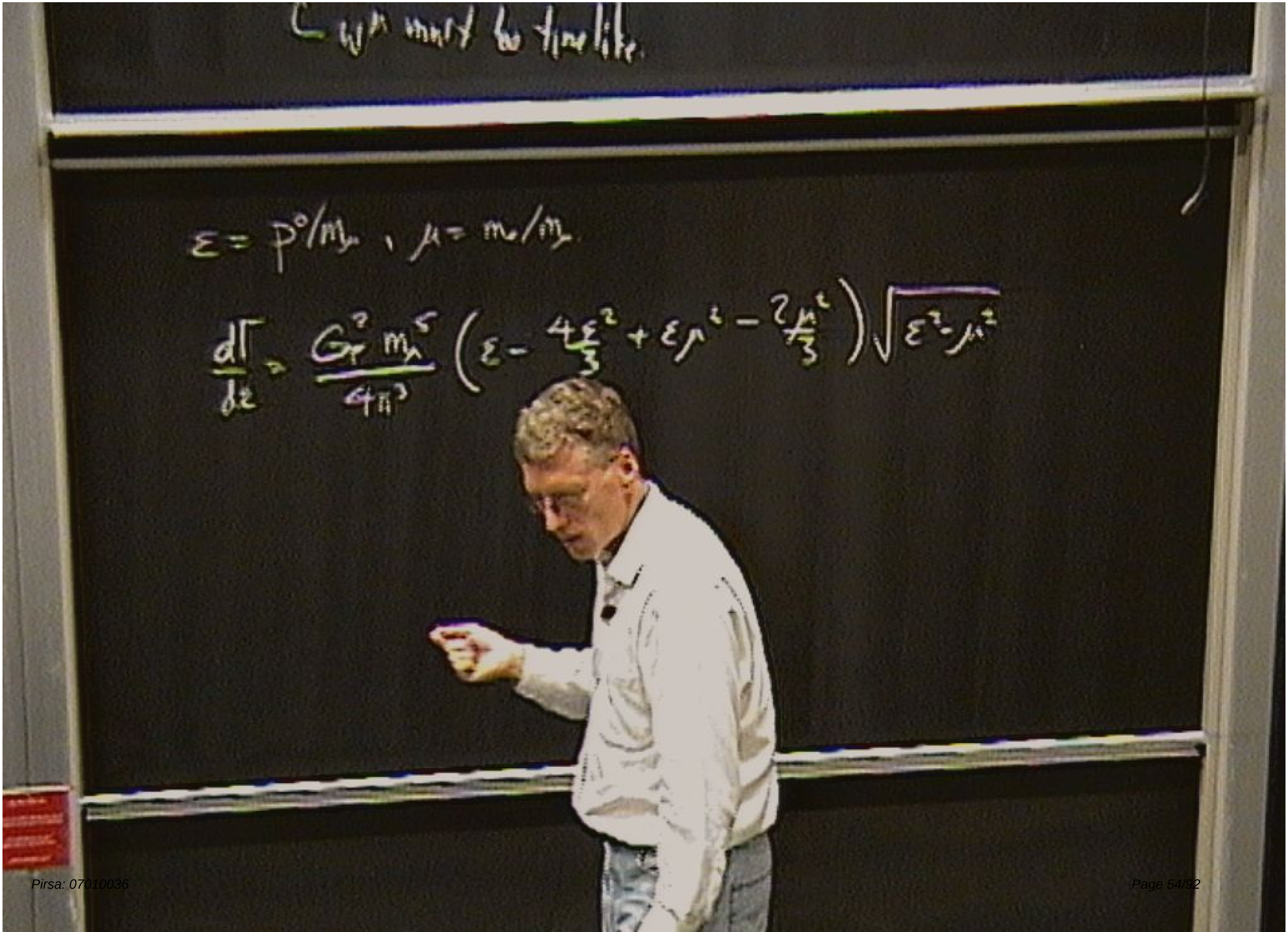
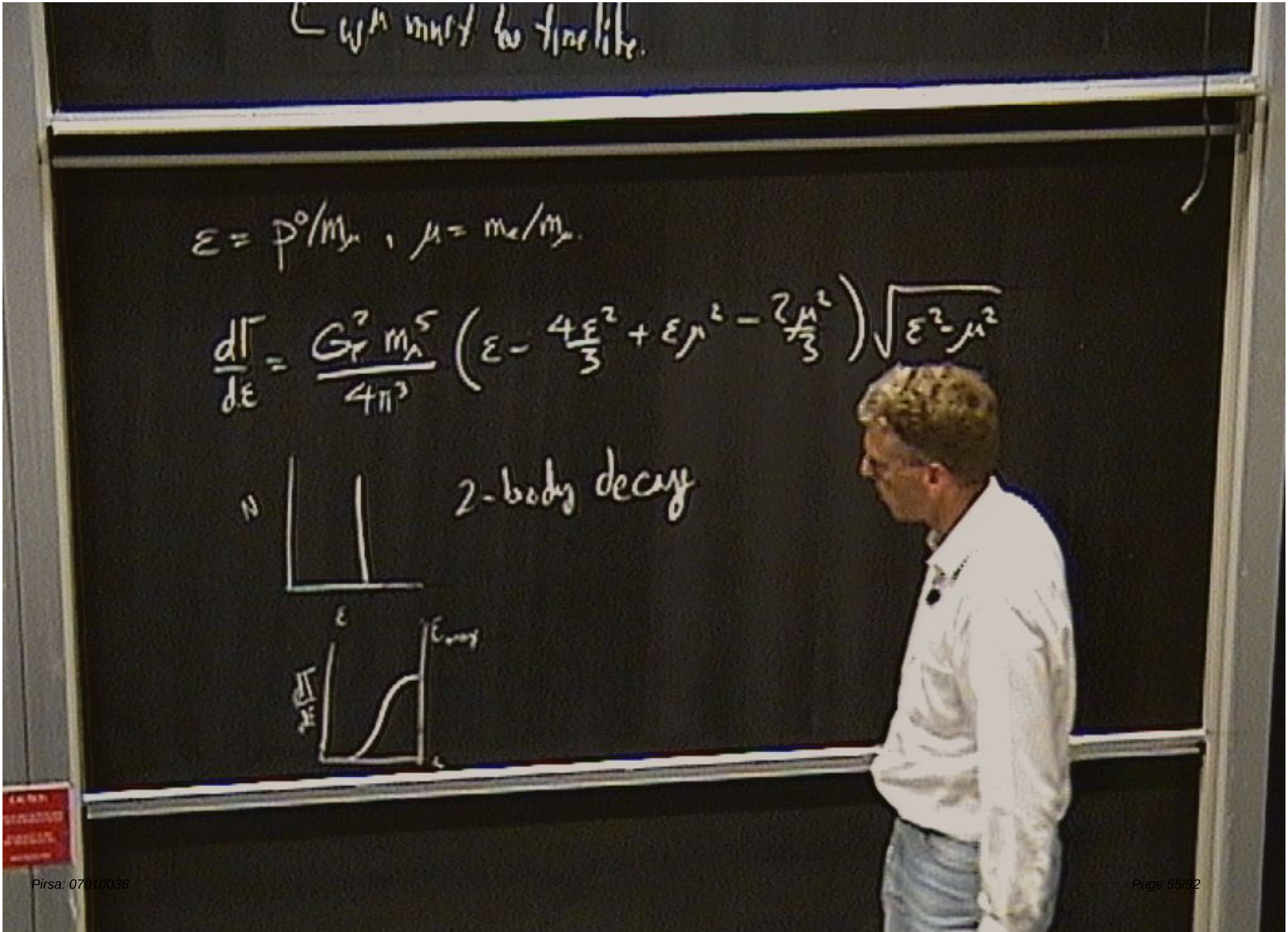
Use this $|\mathcal{M}|^2$ in $\Gamma(\mu \to e \nu \bar{\nu})$:

$$d\Gamma = \frac{64 G_F^2}{2k^0} (l\cdot p)(k \cdot q)(2\pi)^4 \delta^4(k-p-q-l) \frac{d^3 l}{2l^0} \frac{d^3 p}{2p^0} \frac{d^3 q}{2q^0} (2\pi)^9$$


w^μ must be timelike.
$\varepsilon = p^0/m_\mu$, $\mu = m_e/m_\mu$
$\frac{d\Gamma}{d\varepsilon} = \frac{G_F^2 m_\mu^5}{4\pi^3}\left(\varepsilon - \frac{4\varepsilon^2}{3} + \varepsilon\mu^2 - \frac{2\mu^2}{3}\right)\sqrt{\varepsilon^2-\mu^2}$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N
2-body decay
ε
ε_{max}
$\frac{d\Gamma}{d\varepsilon}$

$\llcorner$ w^μ must be timelike.

$\varepsilon = p^0/m_\mu$, $\mu = m_e/m_\mu$.

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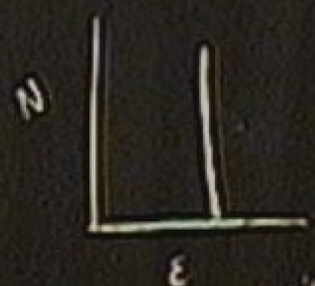
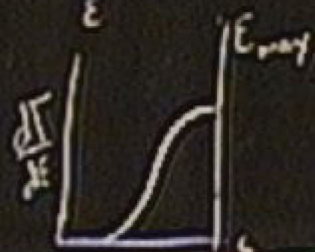


2-body decay



3-body (+ higher)

$\subset$ w^μ must be timelike.

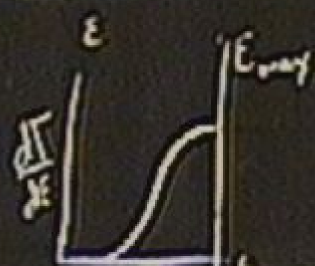
$\varepsilon = p^0/m_\mu$, $\mu = m_e/m_\mu$.

$$\frac{d\Gamma}{d\varepsilon} = \frac{G_F^2\, m_\mu^5}{4\pi^3}\left(\varepsilon - \frac{4\varepsilon^2}{3} + \varepsilon\mu^2 - \frac{2\mu^2}{3}\right)\sqrt{\varepsilon^2-\mu^2}$$

$m_e/m_\mu \to 0$

$$\boxed{\Gamma = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}}$$

2-body decay



3-body (+ photon)

$$\frac{e^2}{8 s_W^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad \frac{e}{s_W} = g_2$$

$$M_W = \tfrac{1}{2} g_2 v$$

$$v = 246 \text{ GeV}$$

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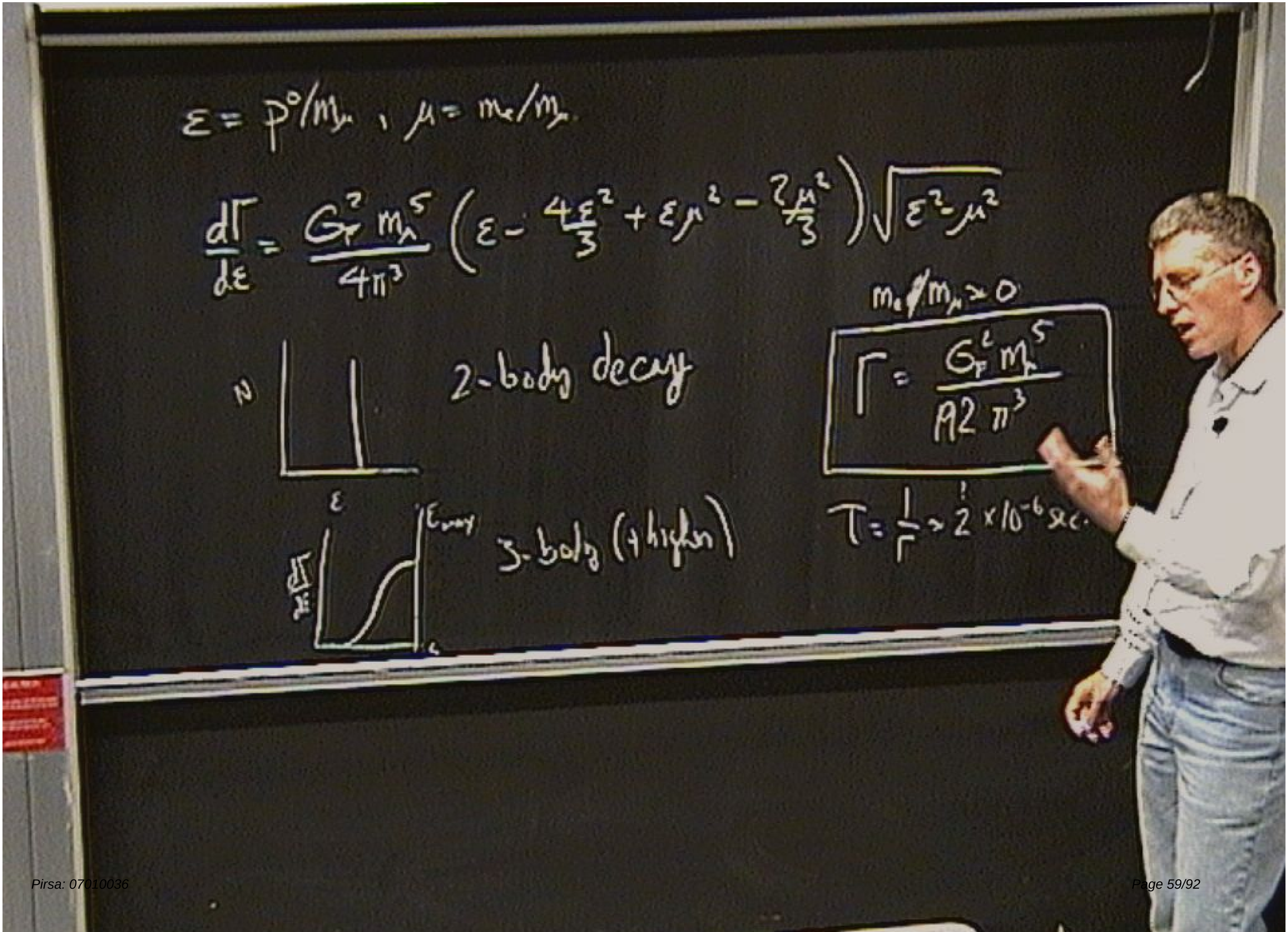
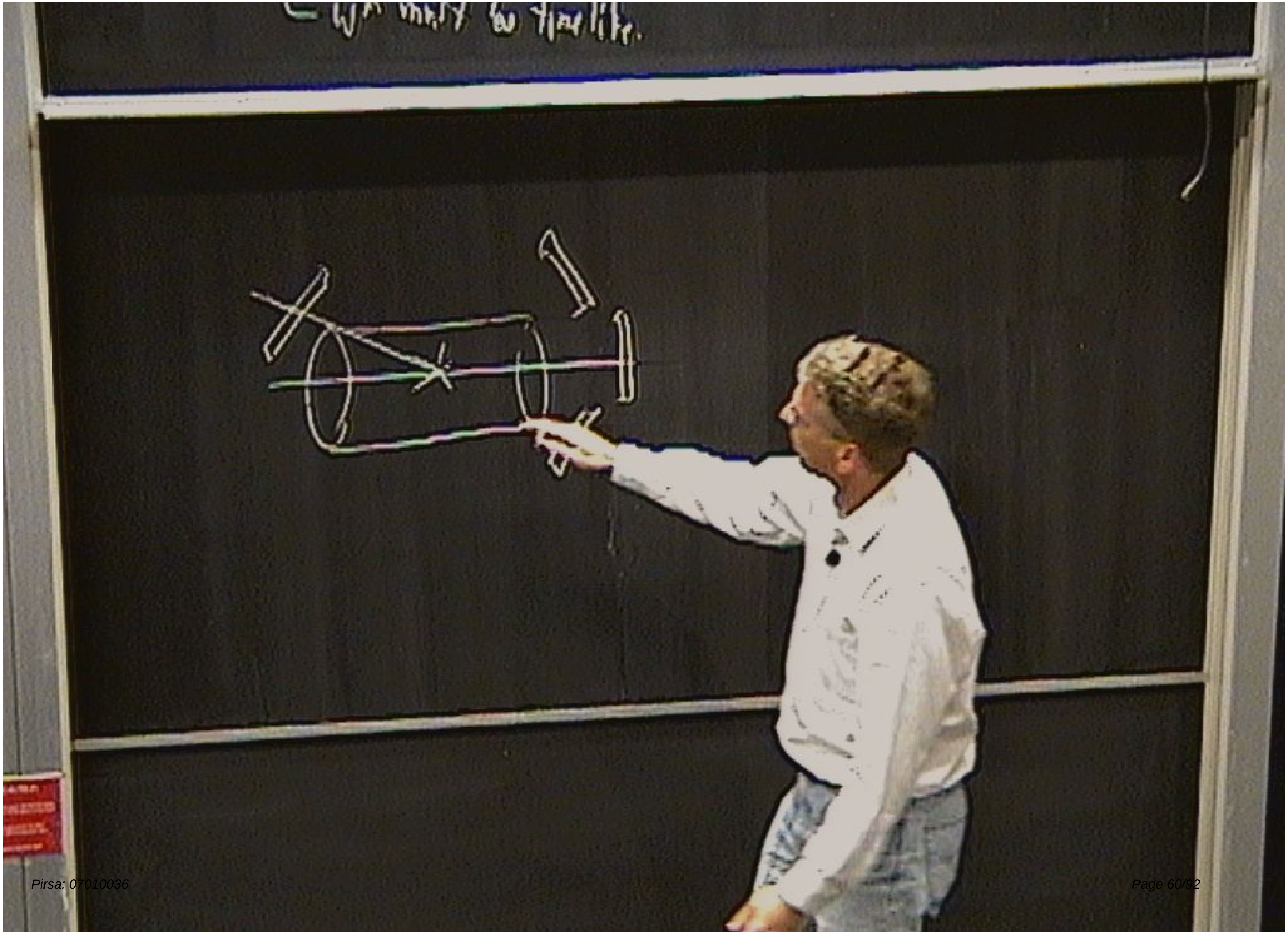
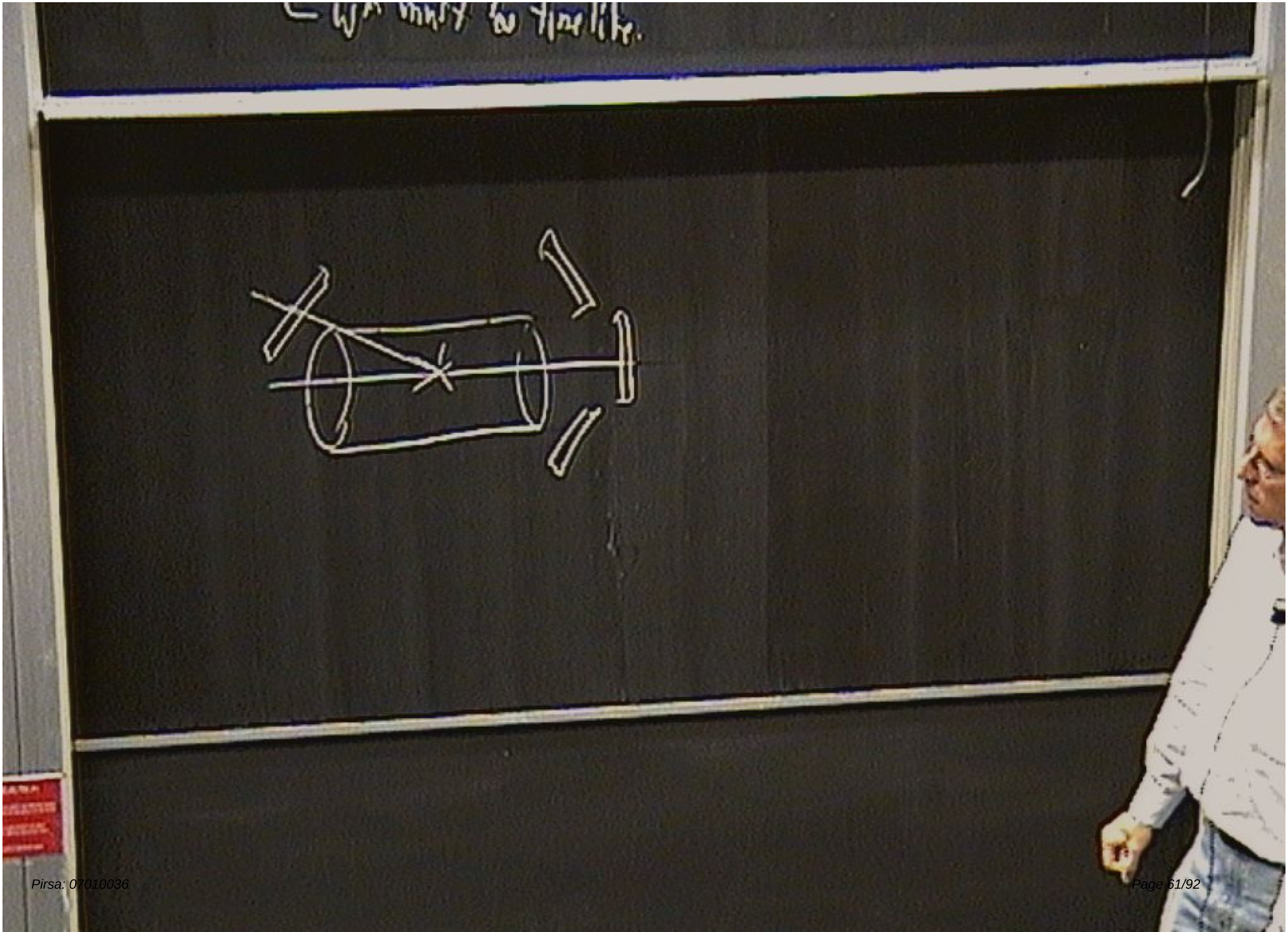
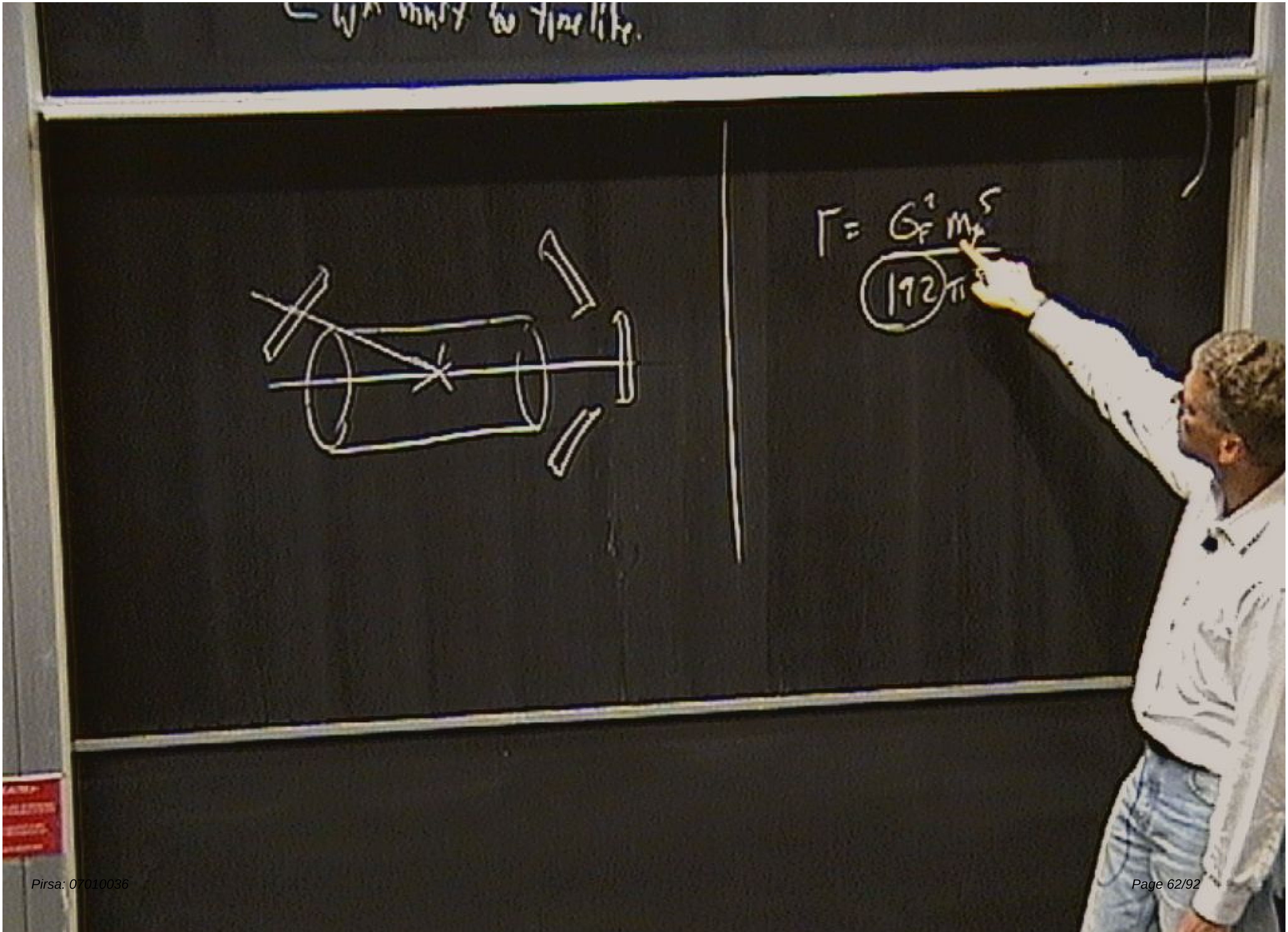
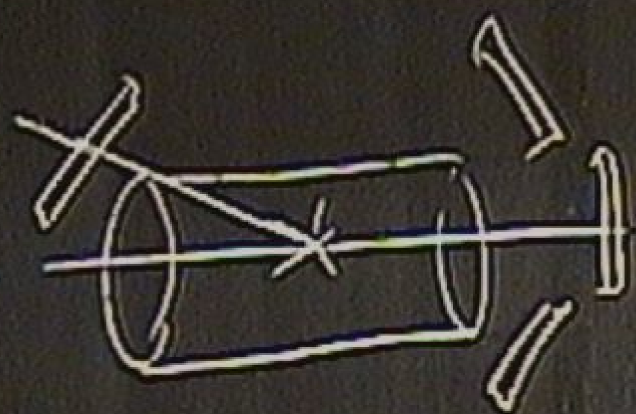


$\varepsilon = p^0/m_\mu \,,\ \mu = m_e/m_\mu$
$\frac{d\Gamma}{d\varepsilon} = \frac{G_F^2 m_\mu^5}{4\pi^3}\left(\varepsilon - \frac{4\varepsilon^2}{3} + \varepsilon\mu^2 - \frac{2\mu^2}{3}\right)\sqrt{\varepsilon^2-\mu^2}$
$m_e/m_\mu \to 0$
$\boxed{\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}}$
N
ε
2-body decay
$\frac{d\Gamma}{d\varepsilon}$
ε_{max}
3-body (+ higher)
$\tau = \frac{1}{\Gamma} \approx 2 \times 10^{-6}$ sec.






Γ = G_F^2 m^5
192 π



$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}.$$

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$\langle e \nu \bar{\nu} | T[H_{cc}(x)\, H_{cc}(0)] | \mu \rangle$

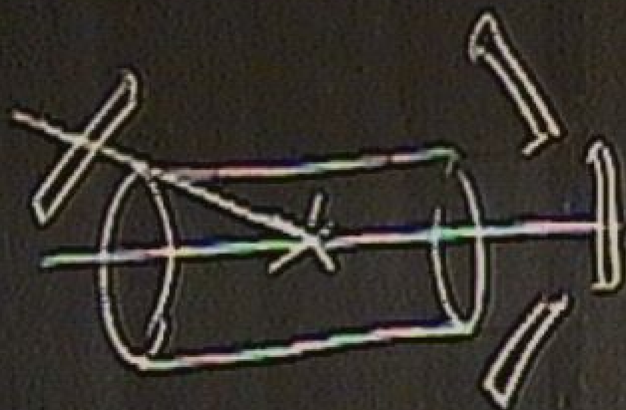
$$\mathcal{M}(\mu(k) \to e^-(p)\bar{\nu}_e(q)\nu_\mu(l)) = \frac{e^2}{s_W^2}\left[\bar{u}(l)\gamma^\mu(1+\gamma_5)u(k)\right]\left[\bar{u}(p)\gamma^\nu(1+\gamma_5)v(q)\right] \times$$

$$\times \boxed{\frac{\eta_{\mu\nu} + (p+q)_\mu (p+q)_\nu / M_W^2}{(p+q)^2 + M_W^2 - i\epsilon}}$$

$$\frac{m_\mu}{M_W} \approx \frac{0.1}{80} \simeq \frac{1}{800}$$

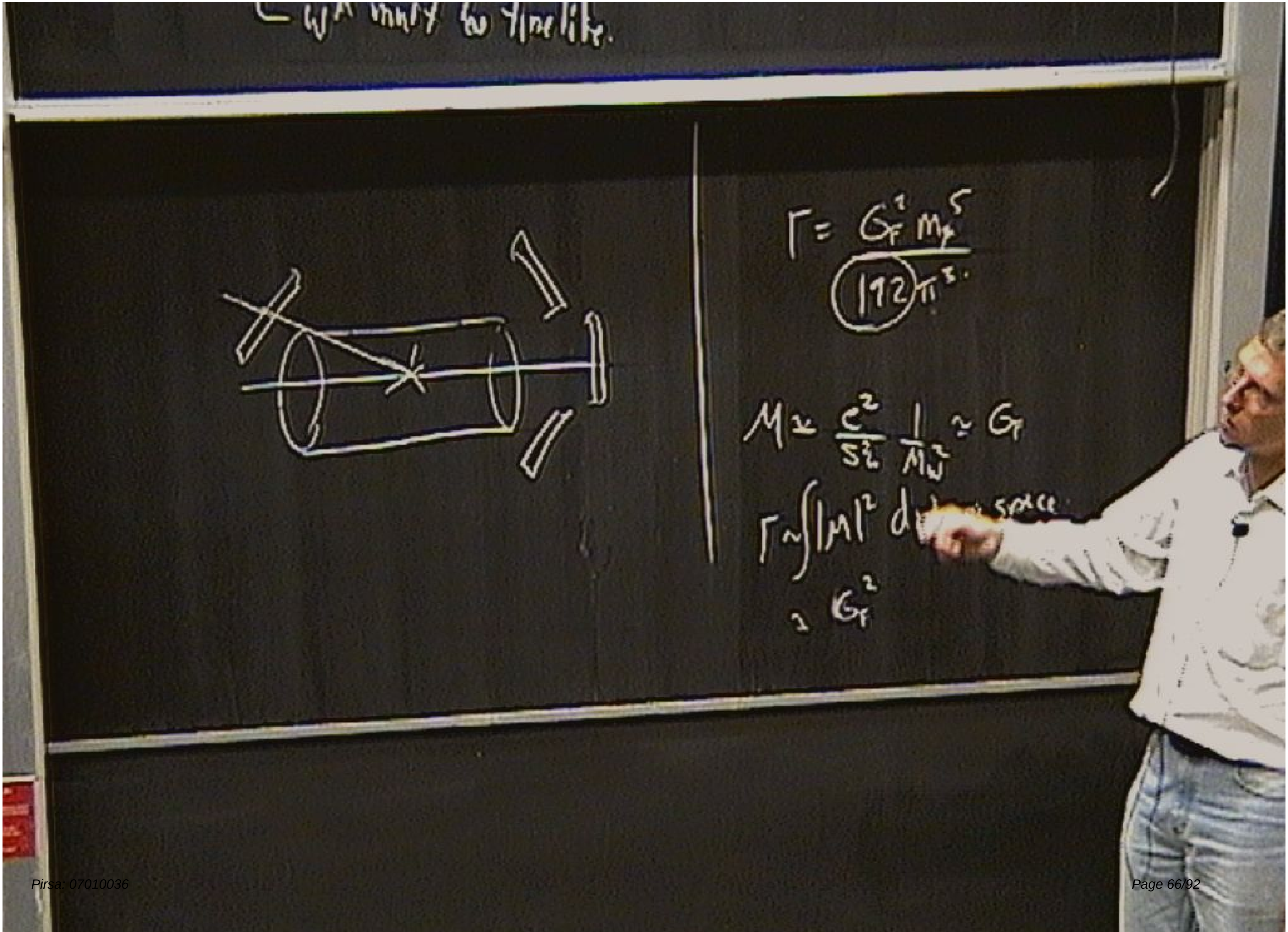
Large-mass expansion: $\boxed{} \approx \frac{\eta_{\mu\nu}}{M_W^2}\left(1 + O(q^2/M_W^2)\right)$

... timelike.



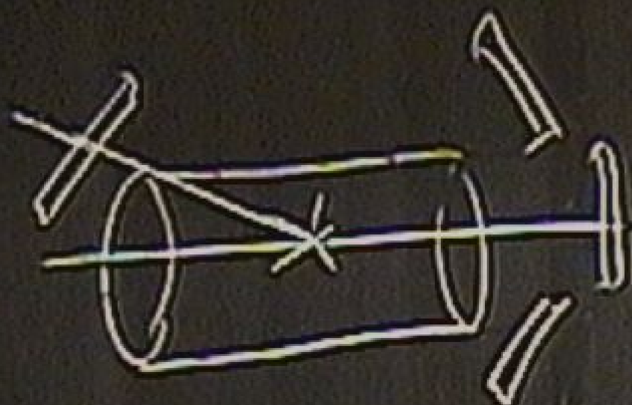
$$\Gamma = \frac{G_F^2 m_\mu^5}{192 \pi^3}.$$

$$M \approx \frac{g^2}{8} \frac{1}{M_W^2}$$



$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$
$M \approx \frac{e^2}{S_W^2} \frac{1}{M_W^2} \approx G_F$
$\Gamma \sim \int |M|^2 d$ space
$\approx G_F^2$

w/ x must be timelike.

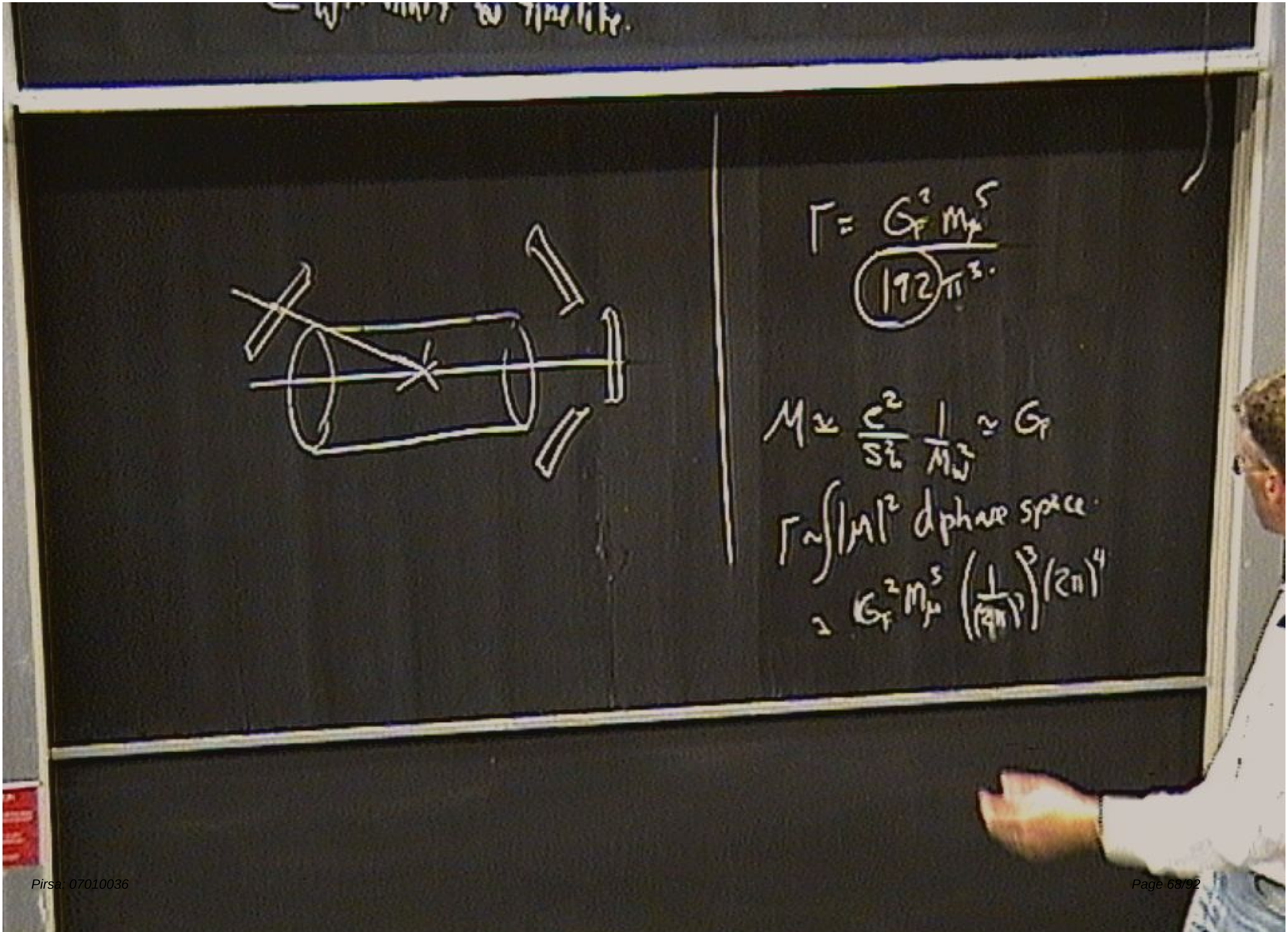


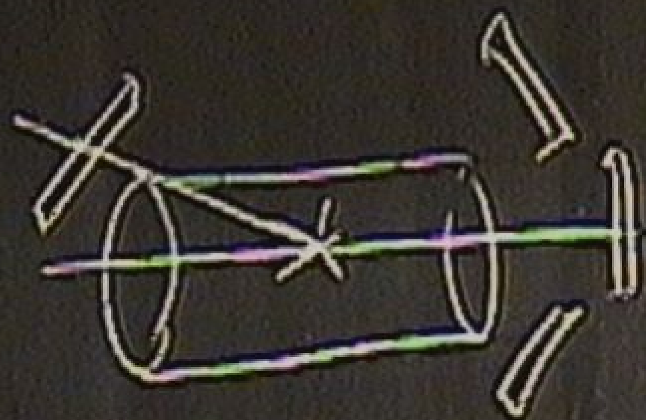
$$\Gamma = \frac{G_F^2 m_\mu^5}{192 \pi^3}$$

$$M \approx \frac{e^2}{s_W^2} \frac{1}{M_W^2} \approx G_F$$

$$\Gamma \sim \int |M|^2 \, d\,\text{phase space}$$

$$\approx G_F^2 m_\mu^5$$


$\Gamma = \frac{G_F^2 m_\mu^5}{192 \pi^3}$.
$M \approx \frac{e^2}{s_W^2} \frac{1}{M_W^2} \approx G_F$
$\Gamma \sim \int |M|^2$ d phase space.
$\approx G_F^2 m_\mu^5 \left(\frac{1}{(4\pi)^2}\right)^3 (2\pi)^4$



$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

$$M \approx \frac{g^2}{8} \frac{1}{M_W^2} \approx G_F$$

$$\Gamma \sim \int |M|^2 \, d\,\text{phase space}$$

$$\sim G_F^2 m_\mu^5 \left(\frac{1}{(2\pi)^3}\right)^3 (2\pi)^4 (2\pi)^3$$

$$\frac{e^2}{8 s_w^2 M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \qquad \frac{e}{s_w} = g_2$$

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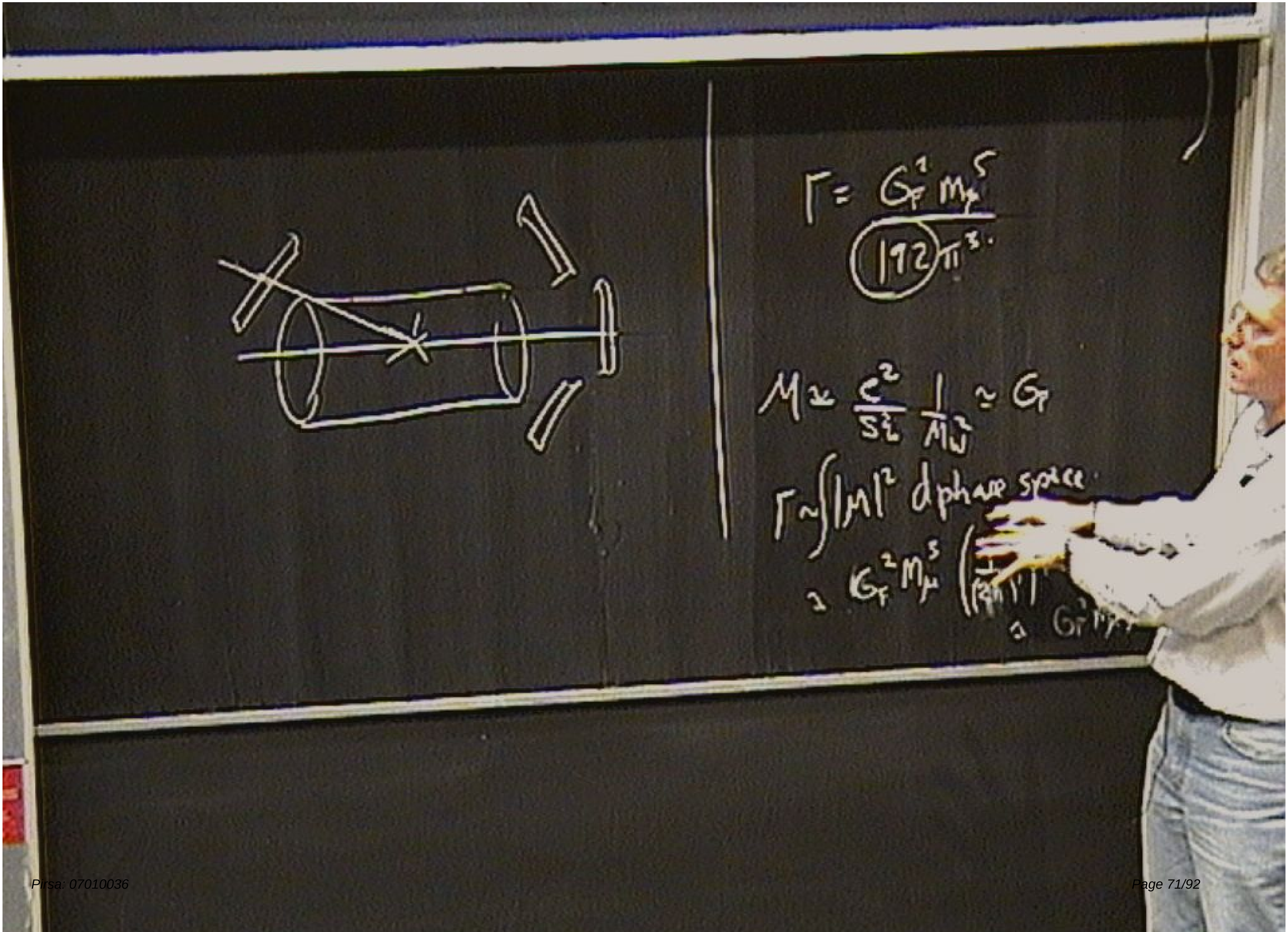
$\Gamma = \frac{G_F^2 m_\mu^5}{192 \pi^3}$
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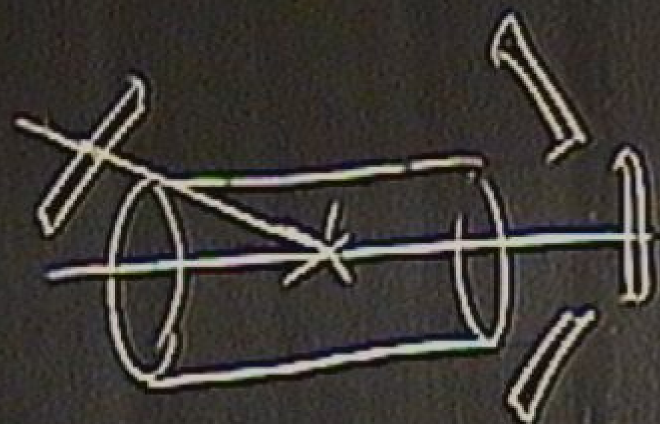


$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}.$$

$$\mathcal{M} \approx \frac{e^2}{s_W^2}\frac{1}{M_W^2} \approx G_F$$

$$\Gamma \sim \int |\mathcal{M}|^2 \, d\,\text{phase space}$$

$$\approx G_F^2 m_\mu^5 \left(\frac{1}{(2\pi)^3}\right)^3 (2\pi)^4 (2\pi)^R$$

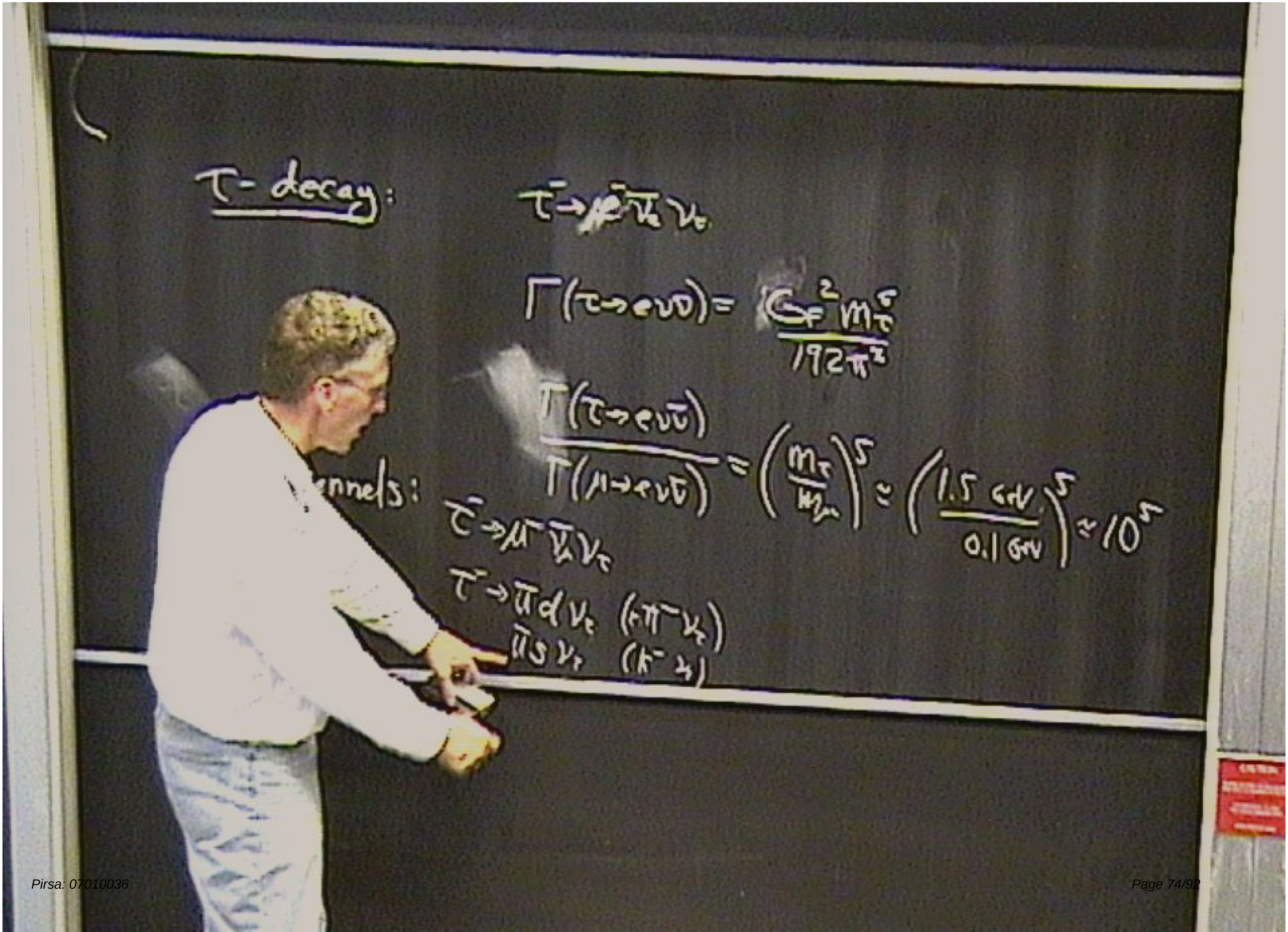
$$\approx G_F^2 m_\mu^5 / (2\pi)^3$$

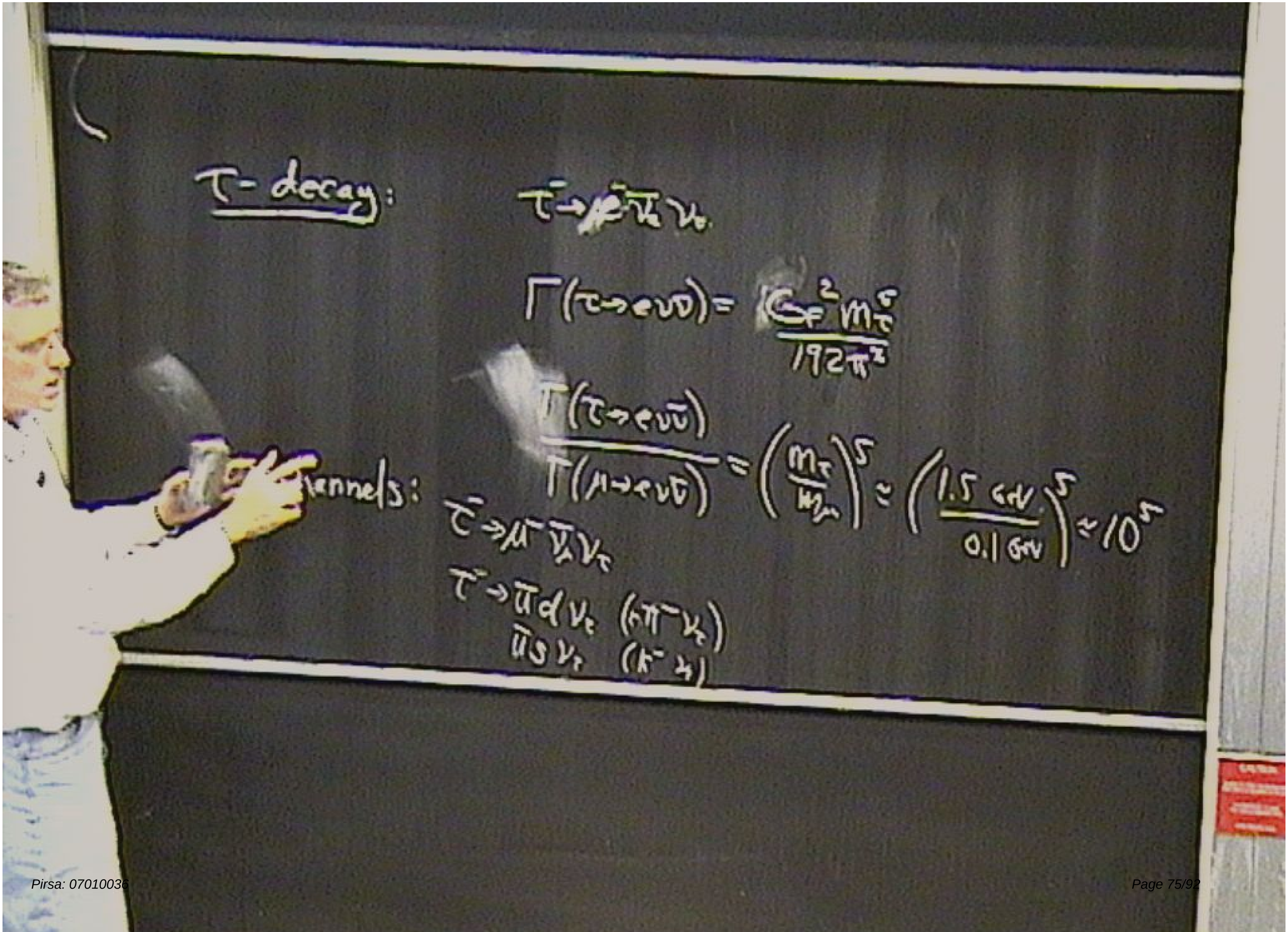
$\underline{\tau\text{- decay}}$:

$$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$$

$$\Gamma(\tau \to e\nu\bar{\nu}) = \frac{G_F^2 m_\tau^5}{192\pi^3}$$

$$\frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})} = \left(\frac{m_\tau}{m_\mu}\right)^5 \simeq \left(\frac{1.5\ \text{GeV}}{0.1\ \text{GeV}}\right)^5 \simeq 10^5$$


τ- decay:
τ⁻ → e⁻ ν̄e ντ
Γ(τ→eνν̄) = GF² mτ⁵ / 192π³
Γ(τ→eνν̄) / Γ(μ→eνν̄) ≈ (mτ/mμ)⁵ ≈ (1.5 GeV / 0.1 GeV)⁵ ≈ 10⁵
…nnels:
τ⁻ → μ⁻ ν̄μ ντ
τ⁻ → ū d ντ (π⁻ ντ)
ū s ντ (K⁻ ντ)


τ- decay:
$\tau^- \to e^- \bar{\nu}_e \nu_\tau$
$\Gamma(\tau \to e\nu\bar{\nu}) = \frac{G_F^2 m_\tau^5}{192\pi^3}$
$\frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})} = \left(\frac{m_\tau}{m_\mu}\right)^5 \approx \left(\frac{1.5\ \text{GeV}}{0.1\ \text{GeV}}\right)^5 \approx 10^5$
hannels:
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$
$\tau^- \to \bar{u} d\, \nu_\tau \quad (\pi^- \nu_\tau)$
$\bar{u} s\, \nu_\tau \quad (K^- \nu_\tau)$

τ- decay:

$\tau^- \to e^- \bar{\nu}_e \nu_\tau$

$$\Gamma(\tau \to e \nu \bar{\nu}) = \frac{G_F^2 m_\tau^5}{192\pi^3}$$

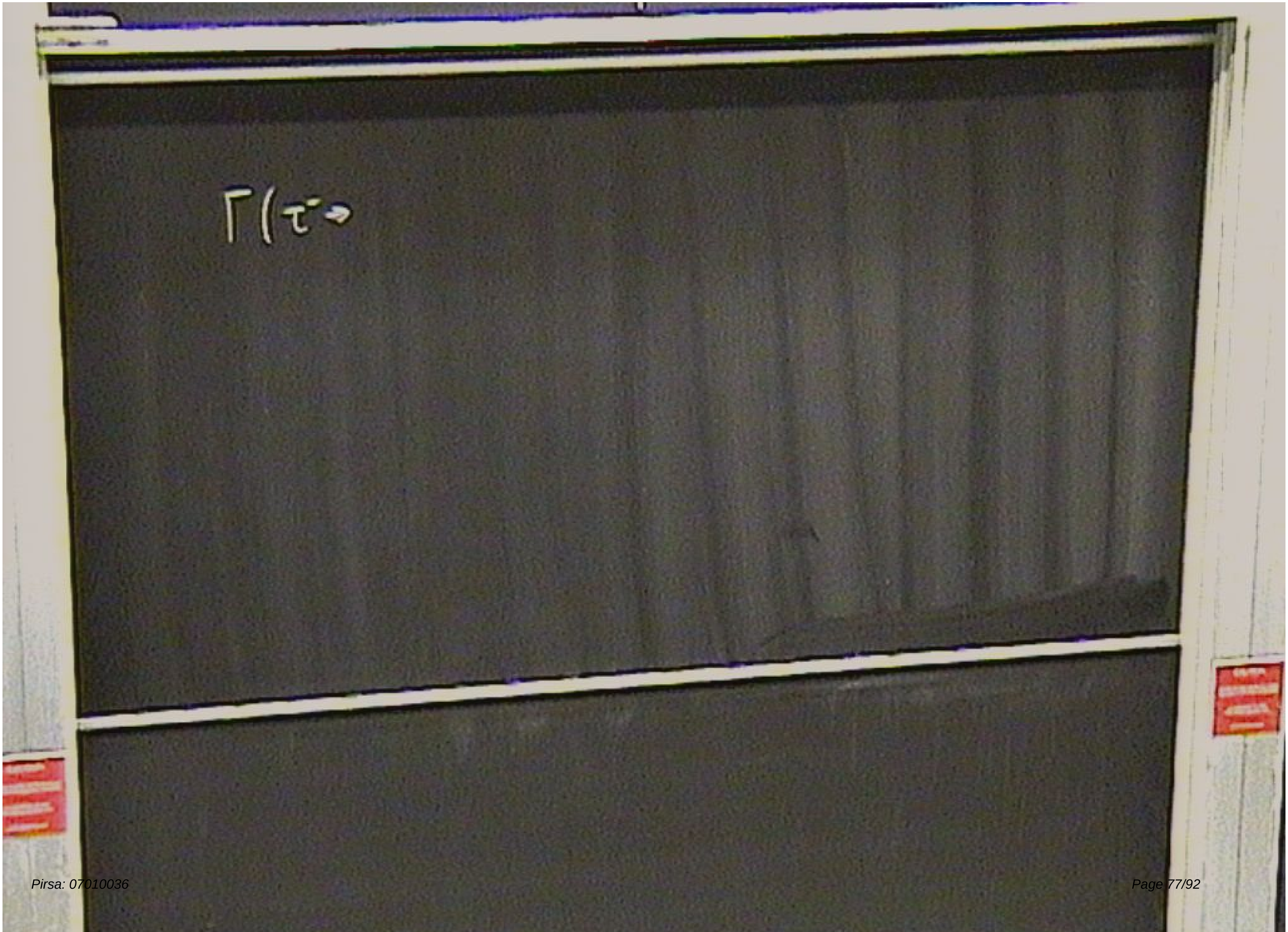
$$\frac{\Gamma(\tau \to e \nu \bar{\nu})}{\Gamma(\mu \to e \nu \bar{\nu})} \approx \left(\frac{m_\tau}{m_\mu}\right)^5 \approx \left(\frac{1.5\ \text{GeV}}{0.1\ \text{GeV}}\right)^5 \approx 10^5$$

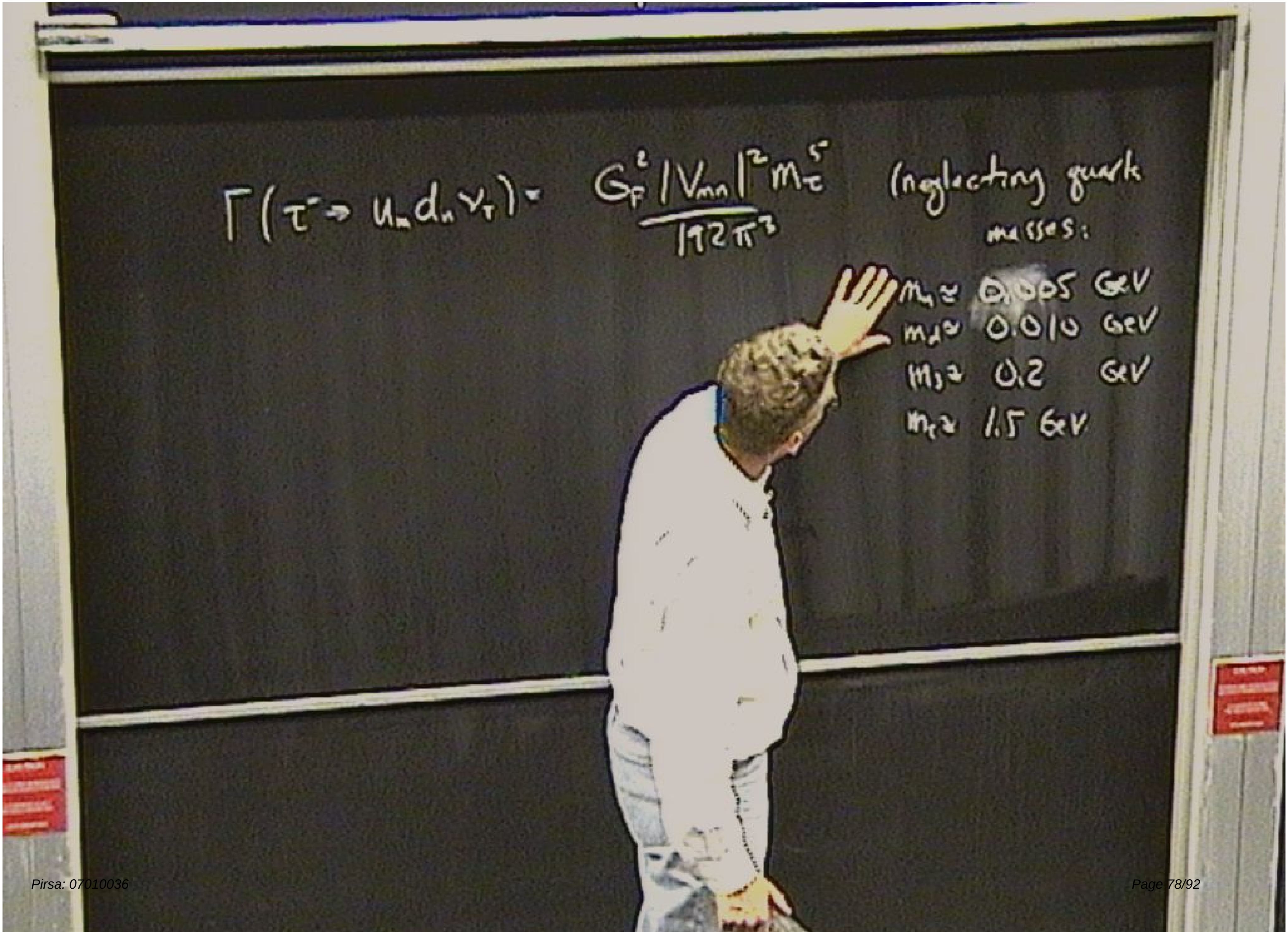
More channels:

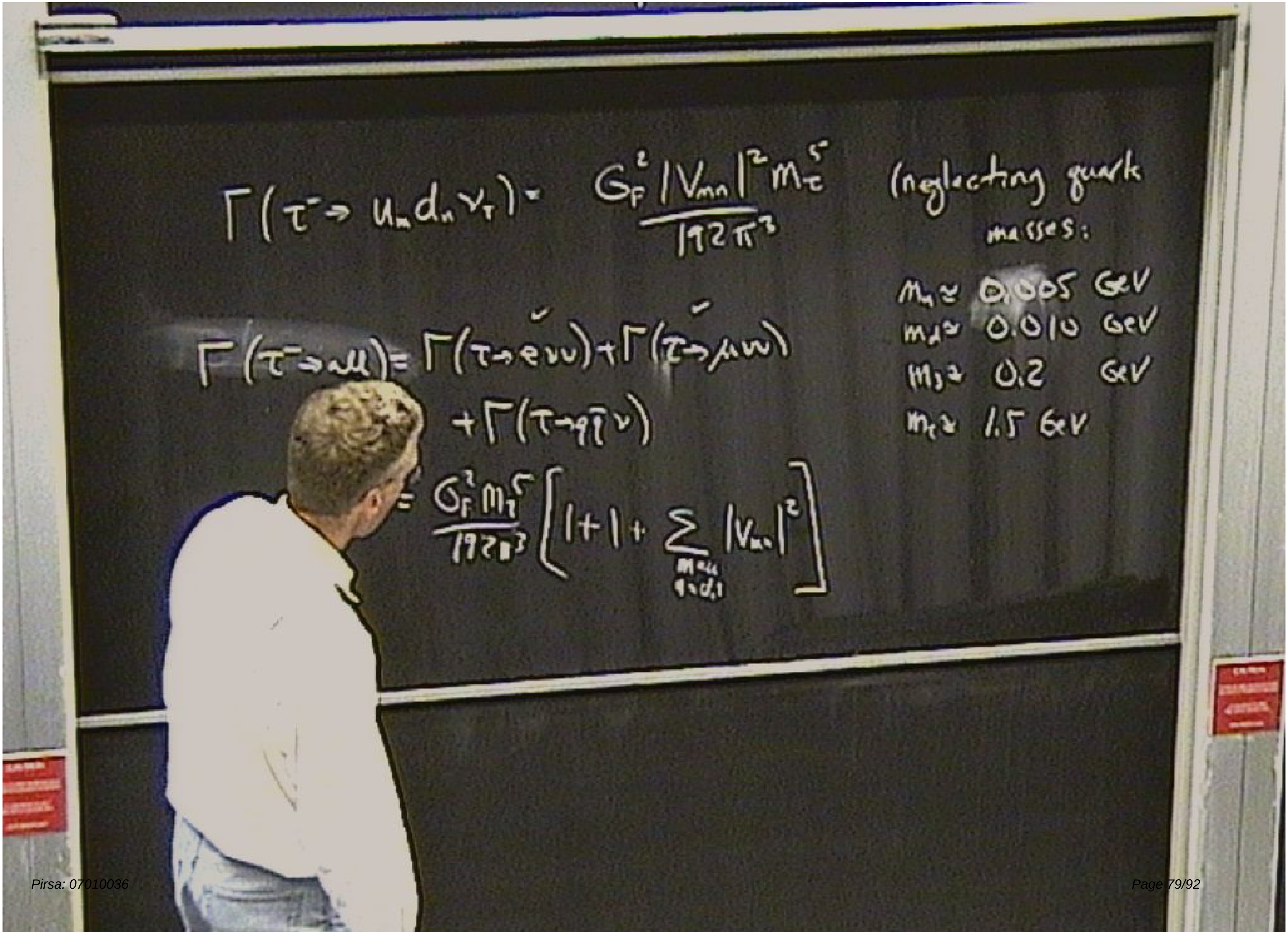
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$

$\tau^- \to \bar{u} d\, \nu_\tau \quad (\pi^- \nu_\tau)$

$\qquad \bar{u} s\, \nu_\tau \quad (K^- \nu_\tau)$


$\Gamma(\tau^- \to$


$\Gamma(\tau^- \to u_m d_n \nu_\tau) = \frac{G_F^2 |V_{mn}|^2 m_\tau^5}{192\pi^3}$
(neglecting quark masses:
$m_u \simeq 0.005$ GeV
$m_d \simeq 0.010$ GeV
$m_s \simeq 0.2$ GeV
$m_c \simeq 1.5$ GeV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$m_u \simeq 0.005$ GeV
$m_d \simeq 0.010$ GeV
$m_s \simeq 0.2$ GeV
$m_c \simeq 1.5$ GeV
$\Gamma(\tau^- \to \text{all}) = \Gamma(\tau \to e\nu\nu) + \Gamma(\tau \to \mu\nu\nu)$
$+ \Gamma(\tau \to q\bar{q}\nu)$
$= \frac{G_F^2 m_\tau^5}{192\pi^3}\left[1 + 1 + \sum_{m=u,\, q=d,s} |V_{mn}|^2\right]$

$$\Gamma(\tau^- \to u_m d_n \nu_\tau) = \frac{G_F^2 |V_{mn}|^2 m_\tau^5}{192\pi^3}$$

(neglecting quark masses:

$m_u \simeq 0.005$ GeV

$m_d \simeq 0.010$ GeV

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$$\Gamma(\tau^- \to \text{all}) = \Gamma(\tau \to e\nu\nu) + \Gamma(\tau \to \mu\nu\nu) + \Gamma(\tau \to q\bar{q}\nu)$$

$$= \frac{G_F^2 m_\tau^5}{192\pi^3}\left[1 + 1 + \sum_{\substack{m=u \\ n=d,s}} |V_{mn}|^2\right]$$

$$\Gamma(\tau^- \to u_m d_n \nu_\tau) = \frac{\overset{\#\text{ colors.}}{3}\, G_F^2 |V_{mn}|^2 m_\tau^5}{192\pi^3}$$

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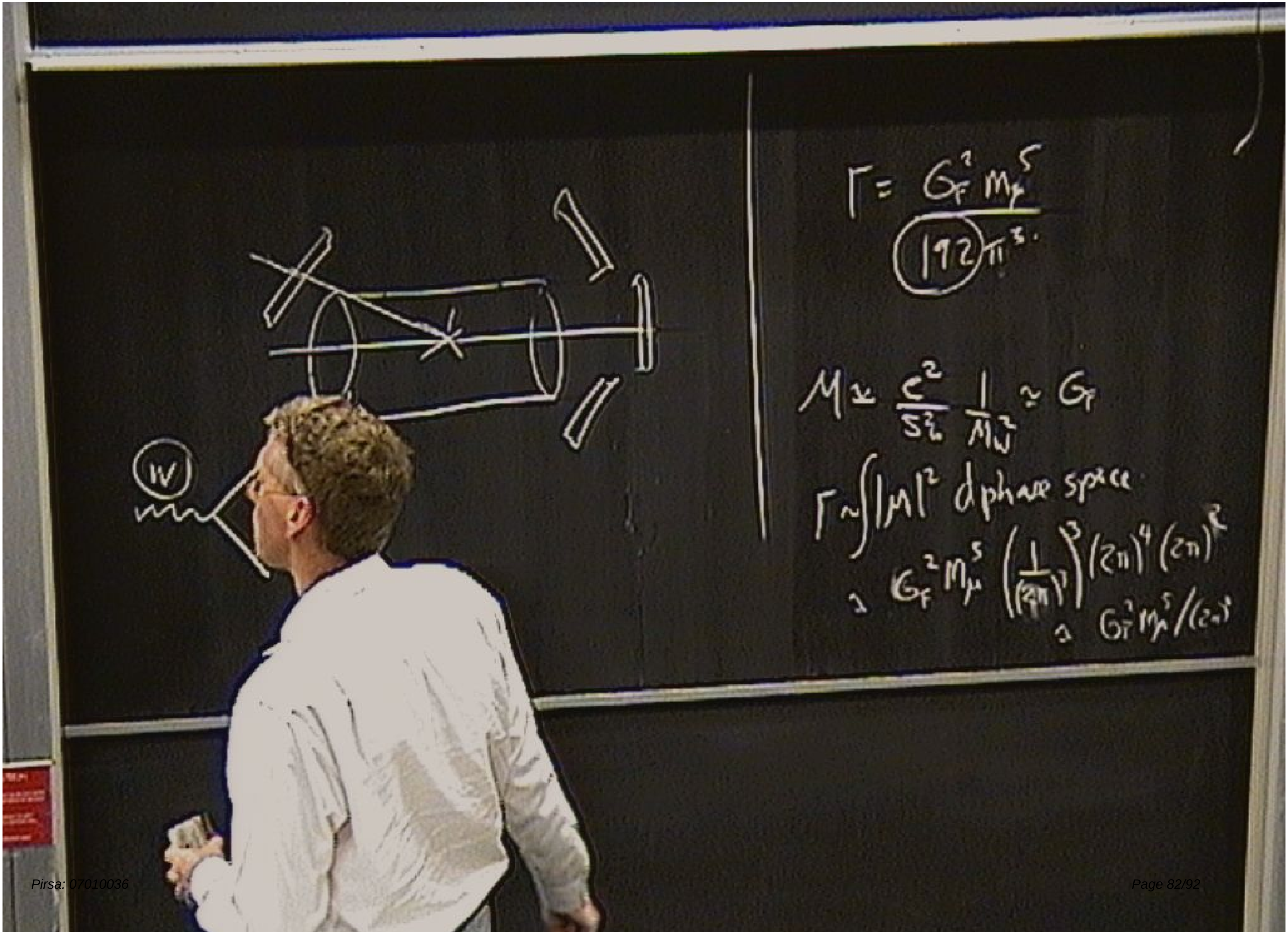
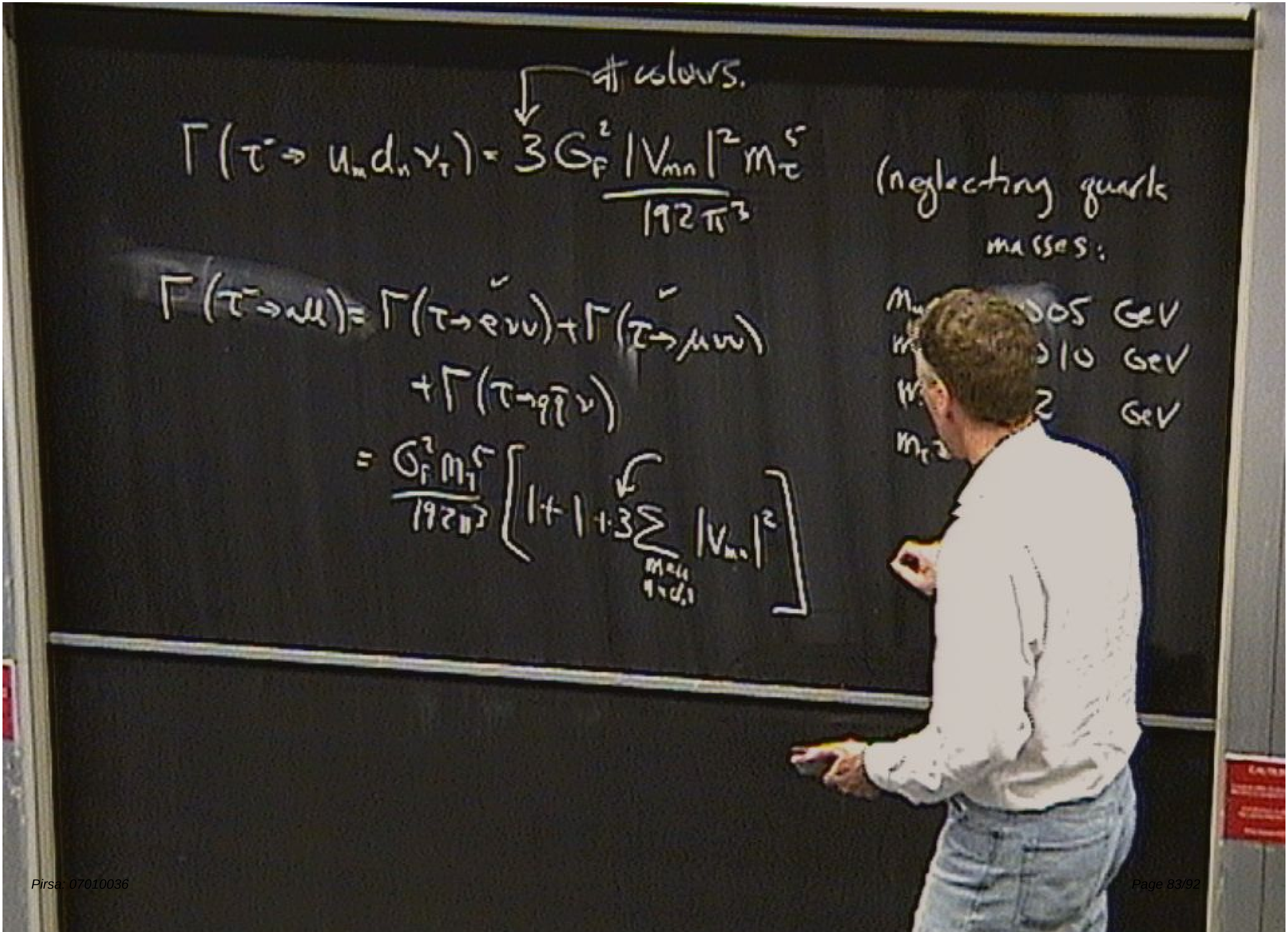
$m_c \simeq 1.5$ GeV

$$\Gamma(\tau^- \to \text{all}) = \Gamma(\tau \to e\nu\nu) + \Gamma(\tau \to \mu\nu\nu) + \Gamma(\tau \to q\bar{q}\nu)$$

$$= \frac{G_F^2 m_\tau^5}{192\pi^3}\left[1 + 1 + 3\sum_{\substack{m=u \\ q=d,s}} |V_{mn}|^2\right]$$


W
Γ = G_F^2 m_μ^5 / 192 π^3
M ≈ e^2/s_w^2 · 1/M_W^2 ≈ G_F
Γ ~ ∫|M|^2 d phase space



colours.
Γ(τ⁻ → u_m d_n ν_τ) = 3 G_F² |V_mn|² m_τ⁵ / 192π³
(neglecting quark masses:
Γ(τ⁻ → all) = Γ(τ → eνν) + Γ(τ → μνν)
+ Γ(τ → qq̄ν)
= G_F² m_τ⁵ / 192π³ [1 + 1 + 3 Σ_{m=u, n=d,s} |V_mn|²]
GeV
GeV
GeV

\# colors.

$$\Gamma(\tau^- \to u_m d_n \nu_\tau) = \frac{3\, G_F^2 |V_{mn}|^2 m_\tau^5}{192\pi^3}$$

(neglecting quark masses;

$m_u \simeq 0.005$ GeV
$m_d \simeq 0.010$ GeV
$m_s \simeq 0.2$ GeV
$m_\tau \simeq 1.5$ GeV

$$\Gamma(\tau^- \to \mathrm{all}) = \Gamma(\tau \to e\bar{\nu}\nu) + \Gamma(\tau \to \mu\bar{\nu}\nu) + \Gamma(\tau \to q\bar{q}\nu)$$

$$= \frac{G_F^2 m_\tau^5}{192\pi^3}\left[1 + 1 + 3\sum_{\substack{m=u \\ n=d,s}} |V_{mn}|^2\right]$$

$$\sum_{n=d,s} |V_{mn}|^2 = 1$$

(#)

$\#$ colors.

$$\Gamma(\tau^- \to u_m d_n \nu_\tau) = \frac{3 G_F^2 |V_{mn}|^2 m_\tau^5}{192\pi^3}$$

(neglecting quark masses:

$m_u \simeq 0.005$ GeV
$m_d \simeq 0.010$ GeV
$m_s \simeq 0.2$ GeV
$m_c \simeq 1.5$ GeV

$$\Gamma(\tau^- \to \text{all}) = \Gamma(\tau^- \to e^- \nu\nu) + \Gamma(\tau^- \to \mu^- \nu\nu) + \Gamma(\tau \to q\bar{q}\nu)$$

$$= \frac{G_F^2 m_\tau^5}{192\pi^3}\left[1 + 1 + 3\sum_{\substack{m=u \\ n=d,s}} |V_{mn}|^2\right]$$

$$\sum_{m=d,s,b} |V_{um}|^2 = 1$$

(#)

$$|V_{ud}|^2 + |V_{us}|^2 = 1 - \underbrace{|V_{ub}|^2}$$

$\simeq 1$

$$V_{ub} \simeq (0.2)^3$$

$$|V_{ub}|^2 \simeq (0.2)^6.$$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}.$$

$$\mathcal{M} \simeq \frac{e^2}{s_W^2}\frac{1}{m_W^2} \simeq G_F$$

$$\Gamma \sim \int |\mathcal{M}|^2 \text{ d phase space}$$

$$\simeq G_F^2 m_\mu^5 \left(\frac{1}{(4\pi)^2}\right)^3 (2\pi)^4 (2\pi)^{?}$$

$$\simeq G_F^2 m_\mu^5/(2\pi)^3$$

\# colours

$$\Gamma(\tau^- \to u_m d_n \nu_\tau) = \frac{3\, G_F^2 |V_{mn}|^2 m_\tau^5}{192\pi^3}$$

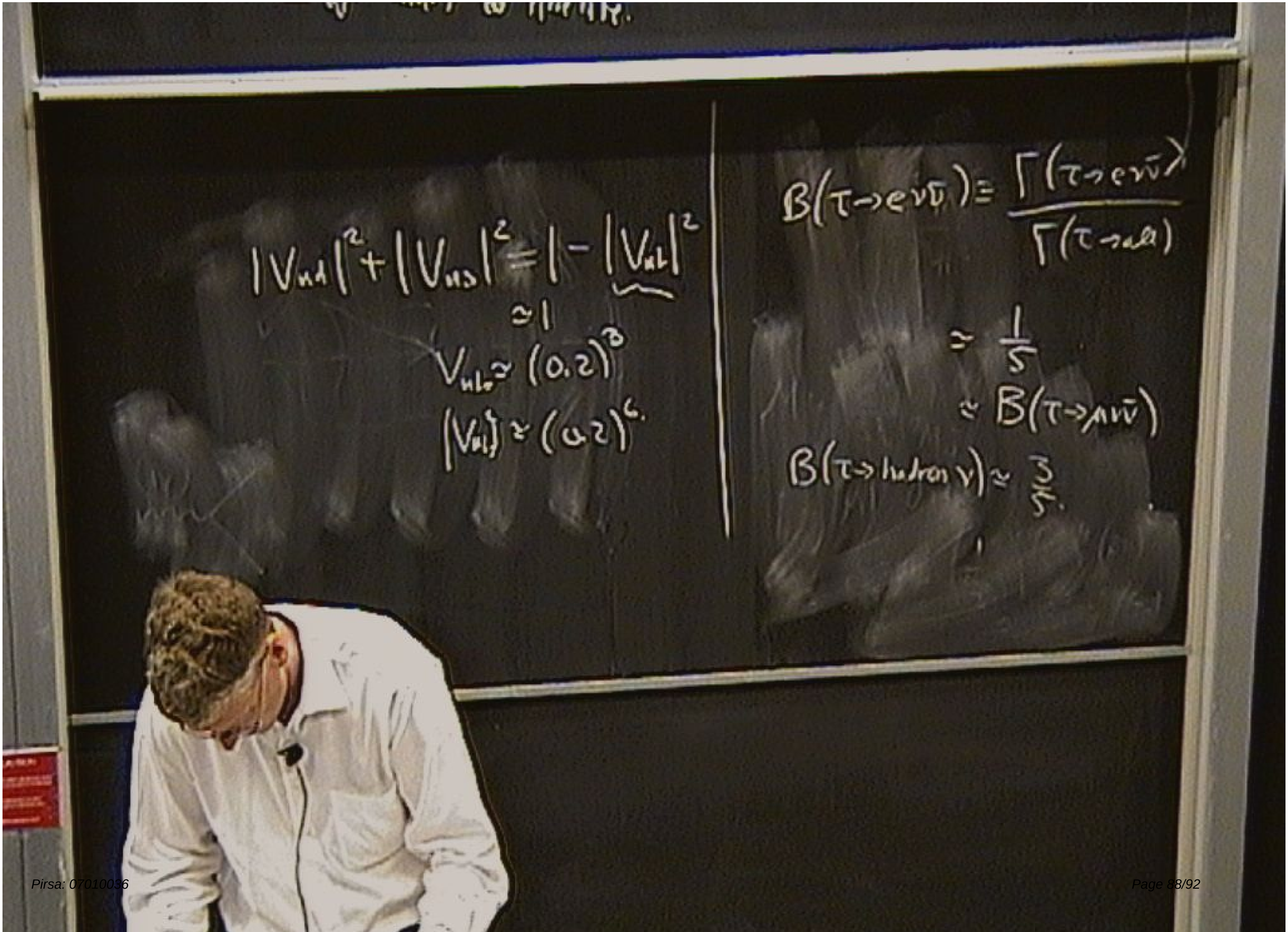
(neglecting quark masses:

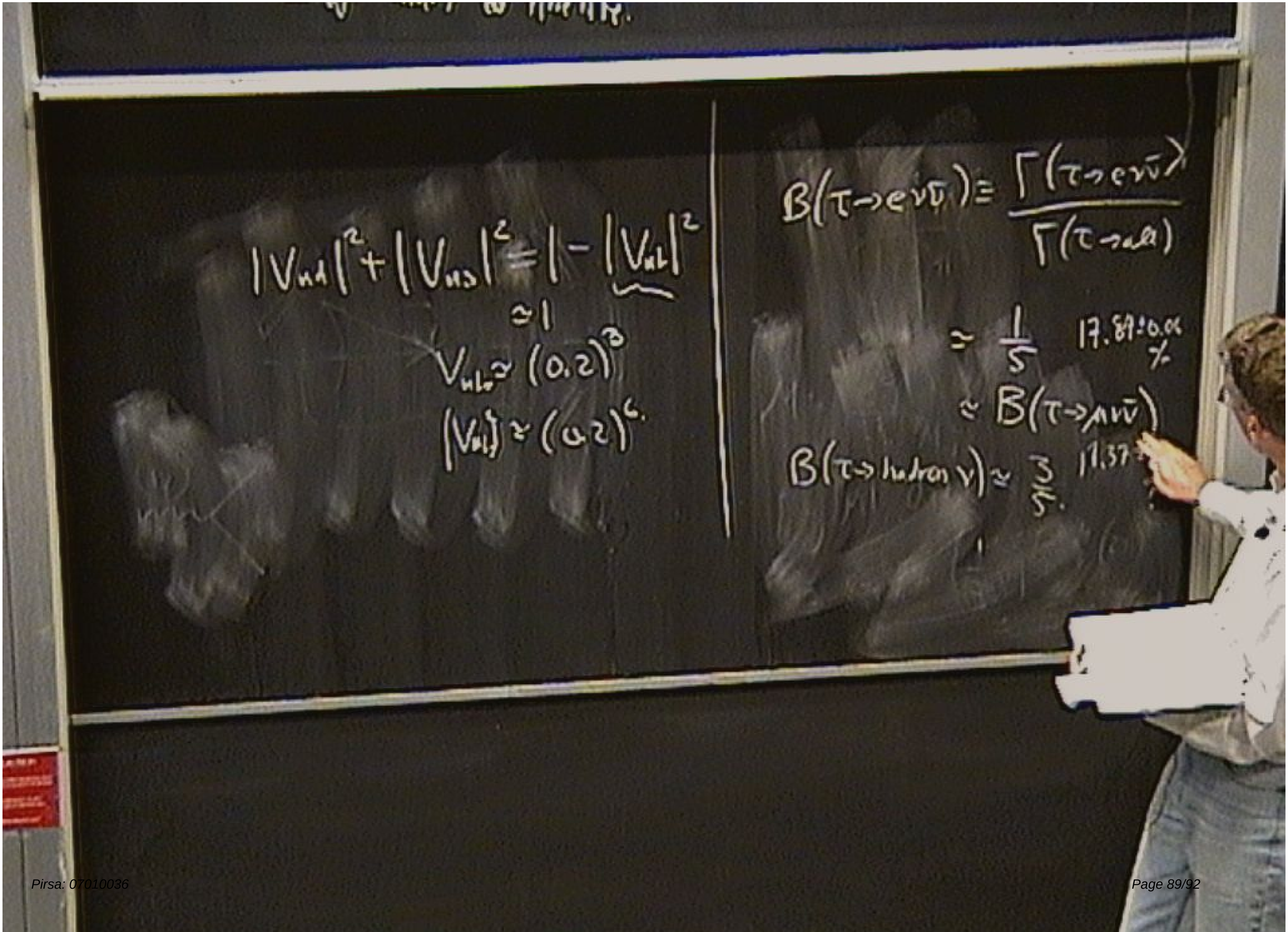
$m_u \simeq 0.005$ GeV
$m_d \simeq 0.010$ GeV
$m_s \simeq 0.2$ GeV
$m_c \simeq 1.5$ GeV

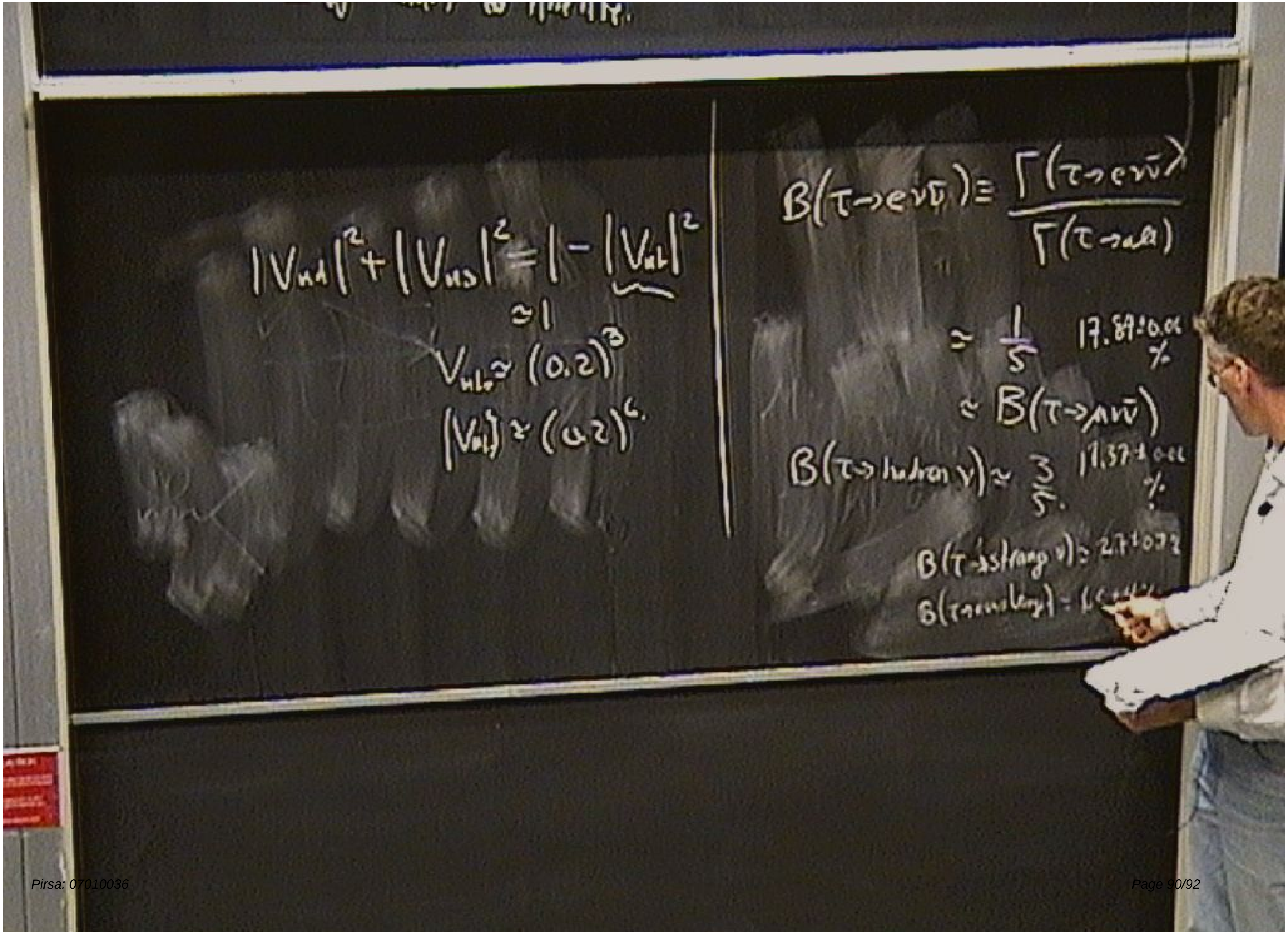
$$\Gamma(\tau \to \text{all}) = \Gamma(\tau \to e\nu\nu) + \Gamma(\tau \to \mu\nu\nu) + \Gamma(\tau \to q\bar{q}\nu)$$

$$= \frac{G_F^2 m_\tau^5}{192\pi^3}\left[\underbrace{1 + 1 + 3\sum_{\substack{m=u\\ n=d,s}} |V_{mn}|^2}_{5}\right]$$

$$\sum_{n=d,s,b} |V_{mn}|^2 = 1 \qquad (\#)$$


$|V_{ud}|^2 + |V_{us}|^2 = 1 - |V_{ub}|^2$
$\simeq 1$
$V_{ub} \simeq (0.2)^3$
$|V_{ub}|^2 \simeq (0.2)^6.$
$B(\tau \to e\nu\bar{\nu}) \equiv \frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\tau \to \text{all})}$
$\simeq \frac{1}{5}$
$\simeq B(\tau \to \mu\nu\bar{\nu})$
$B(\tau \to \text{hadron}\ \nu) \simeq \frac{3}{5}.$


|Vud|² + |Vus|² = 1 − |Vub|²
≃1
Vub ≃ (0.2)³
|Vub|² ≃ (0.2)⁶.
B(τ→eνν̄) ≡ Γ(τ→eνν̄) / Γ(τ→all)
≃ 1/5
17.84±0.05 %
≃ B(τ→μνν̄)
B(τ→hadron ν) ≃ 3/5
17.37


|V_ud|^2 + |V_us|^2 = 1 - |V_ub|^2
≃1
V_ub ≃ (0.2)^3
|V_ub| ≃ (0.2)^6.
B(τ → e ν ν̄) ≡ Γ(τ → e ν ν̄) / Γ(τ → all)
≃ 1/5
17.84 ± 0.05 %
≃ B(τ → μ ν ν̄)
B(τ → hadron ν) ≃ 3/5.
17.37 ± 0.06 %
B(τ → strange ν) ≃ 2.7 ± 0.2

$$|V_{ud}|^2 + |V_{us}|^2 = 1 - |V_{ub}|^2$$

$\simeq 1$

$V_{ub} \simeq (0.2)^3$

$|V_{ub}|^2 \simeq (0.2)^6$

$V_{us} \simeq 0.2$

$V_{ud} \simeq 1$

$$\frac{|V_{us}|^2}{|V_{ud}|^2} \simeq (0.04)$$

$$B(\tau \to e\nu\bar{\nu}) \equiv \frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\tau \to \text{all})}$$

$\simeq \frac{1}{5}$ 17.84 ± 0.06 %

$\simeq B(\tau \to \mu\nu\bar{\nu})$

$B(\tau \to \text{hadron } \nu) \simeq \frac{3}{5}$. 17.37 ± 0.06 %

$B(\tau \to \text{strange } \nu) = 2.7 \pm 0.7\%$

$B(\tau \to \text{non-strange}) = 6 \pm 4\%$

$$|V_{ud}|^2 + |V_{us}|^2 = 1 - |V_{ub}|^2$$

$$\simeq 1$$

$$V_{ub} \simeq (0.2)^3$$

$$|V_{ub}|^2 \simeq (0.2)^6$$

$$V_{us} \simeq 0.2$$

$$V_{ud} \simeq 1$$

$$\frac{|V_{us}|^2}{|V_{ud}|^2} \simeq (0.04)$$

$$B(\tau \to e\nu\bar{\nu}) \equiv \frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\tau \to \text{all})}$$

$$\simeq \frac{1}{5} \qquad 17.84 \pm 0.06\ \%$$

$$\simeq B(\tau \to \mu\nu\bar{\nu})$$

$$B(\tau \to \text{hadron}\ \nu) \simeq \frac{3}{5}. \qquad 17.37 \pm 0.06\ \%$$

$$\frac{0.05}{0.6}$$

$$B(\tau \to \text{strange}\ \nu) \simeq 2.7 \pm 0.3\ \%$$

$$B(\tau \to \text{nonstrange}) = \text{[illegible]} \pm 4\%$$