Title: Quantum Error Correction 4A

Date: Jan 30, 2007 03:30 PM

URL: http://pirsa.org/07010024

Abstract: Guest lecturer Raymond Laflamme: experimental quantum error correction

Experimental Quantum Error Correction

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30th January 2007

Quantum Error Correction

Accuracy Threshold Theorem

A quantum computation can be as long as required with any desired accuracy as long as the noise level is below a threshold value

$$P < 10^{-4}$$

Knill et al.; Science, 279, 342, 1998

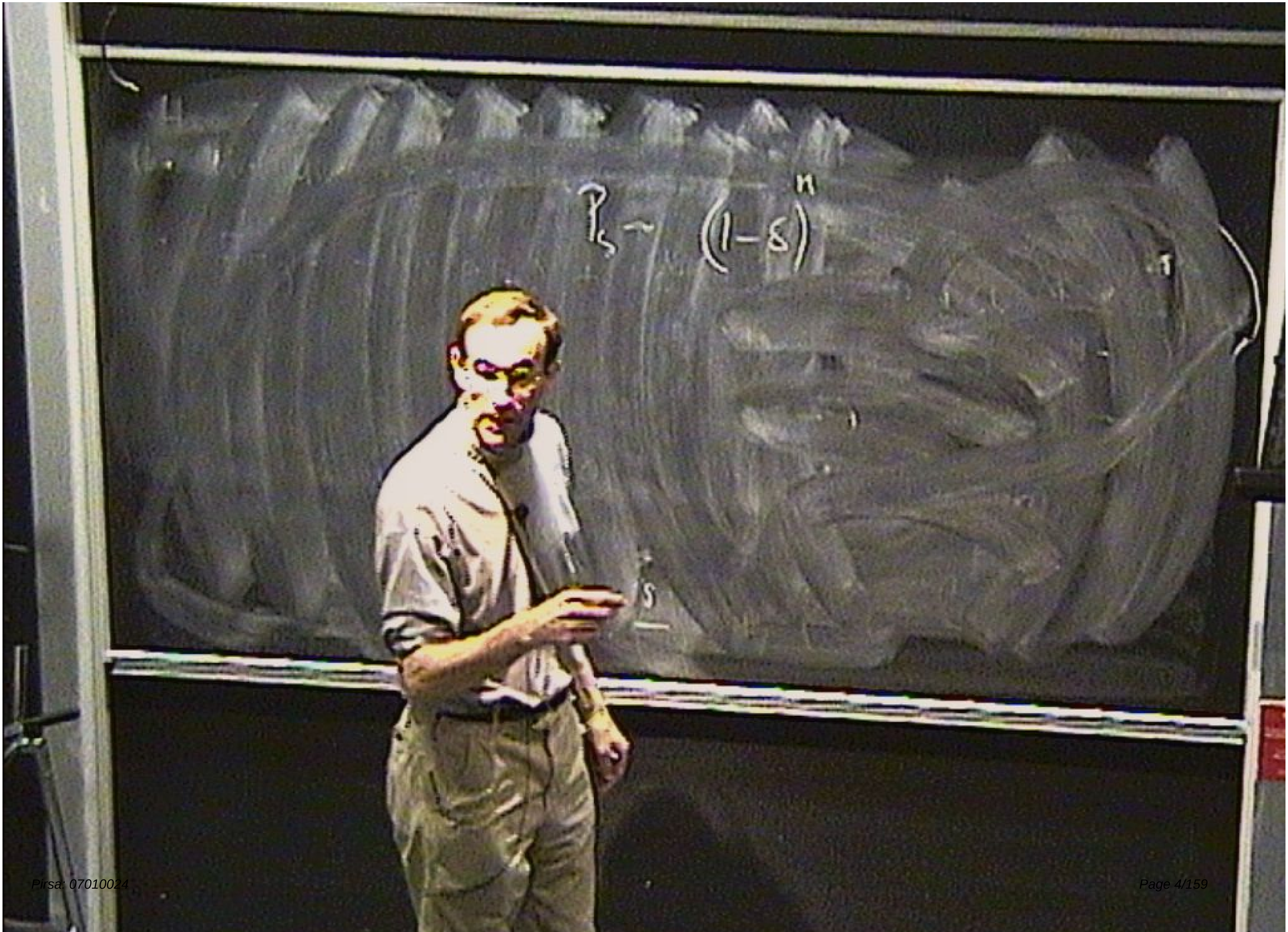
Kitaev, Russ. Math Survey 1997

Aharonov & Ben Or, ACM press

Preskill, PRSL, 454, 257, 1998

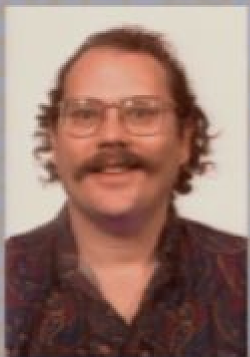
Significance:

- -imperfections and imprecisions are not fundamental objections to quantum computation
- -it gives criteria for scalability
- -its requirements are a guide for experimentalists
- -it is a benchmark to compare different technologies



Quantum Error Correction

Accuracy Threshold Theorem



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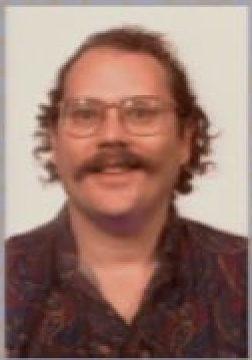

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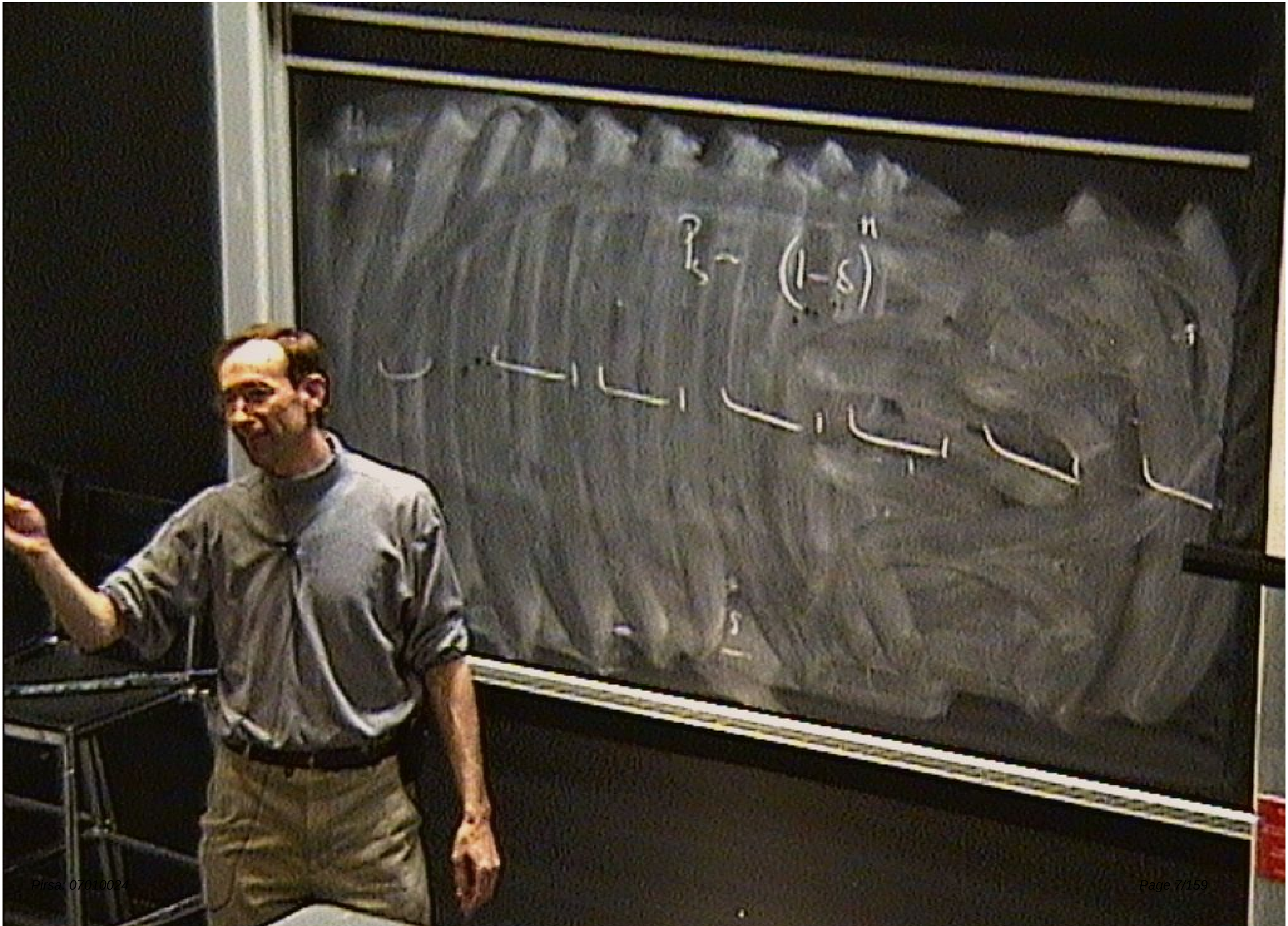

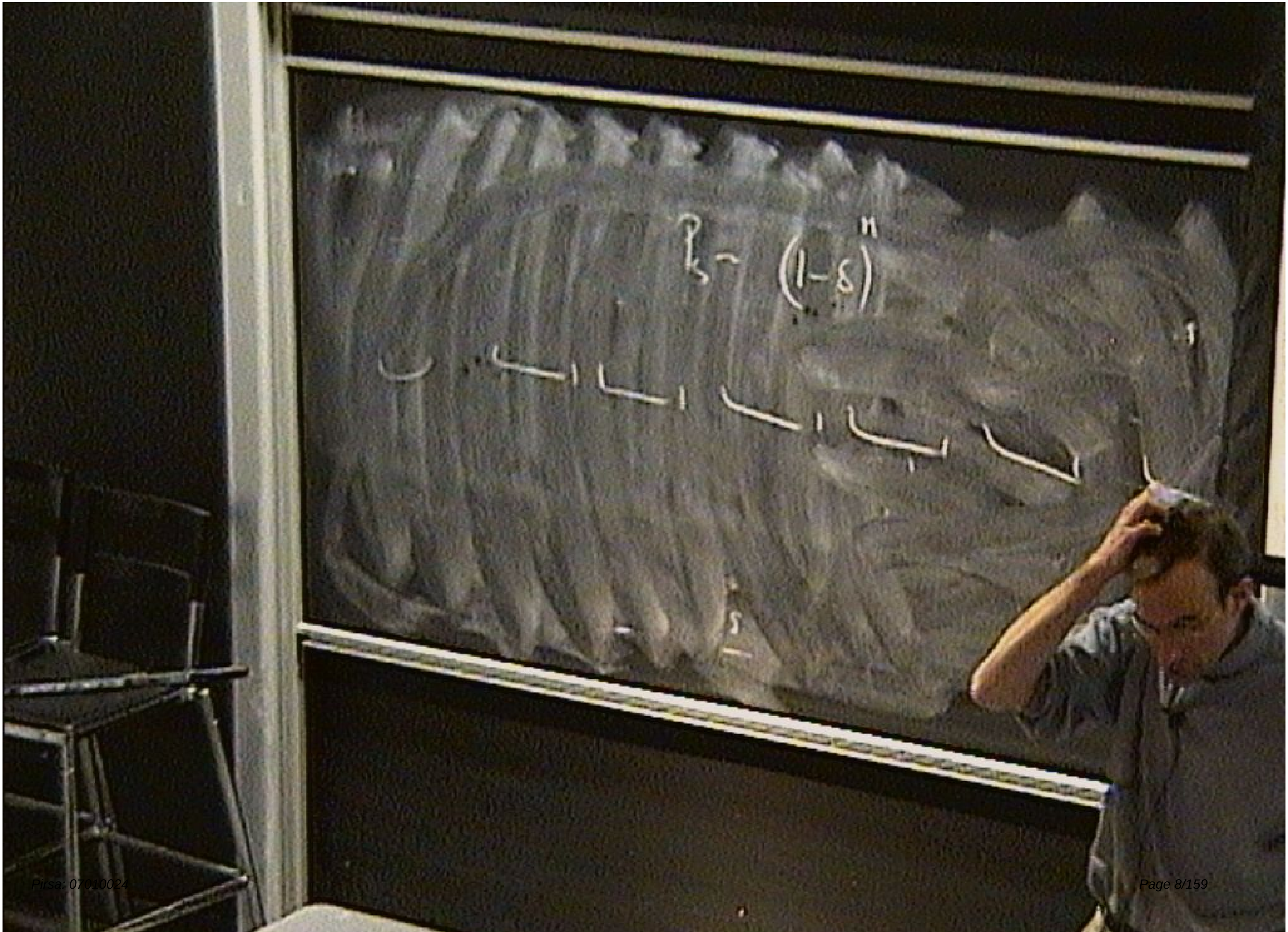
$P_s \sim (1-\delta)^n$

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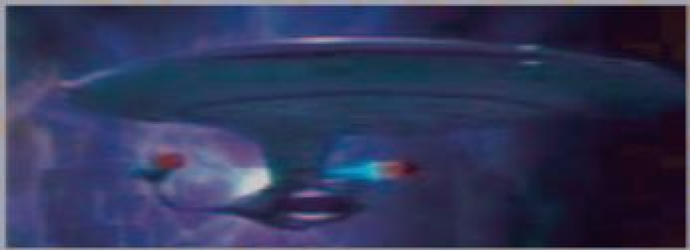
Theory vs experiment

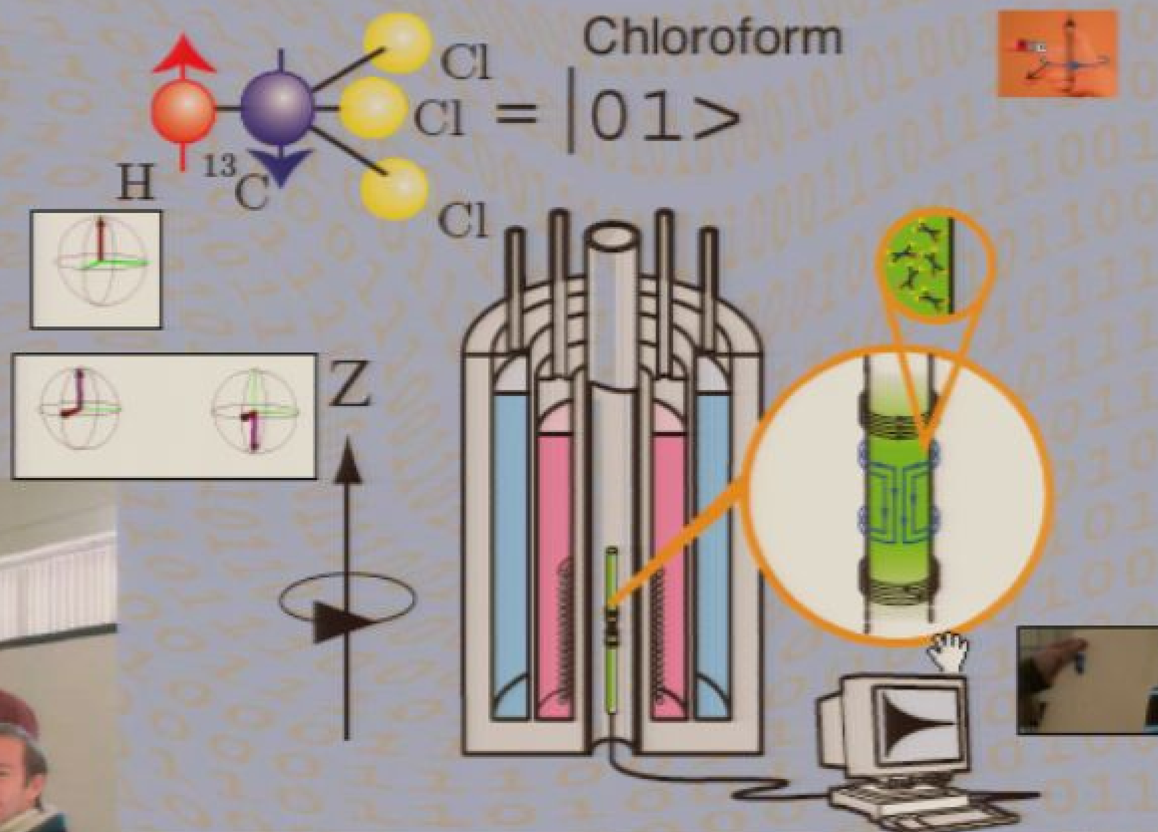
Theory vs experiment



Nuclear Magnetic Resonance

Bloch

Purcell



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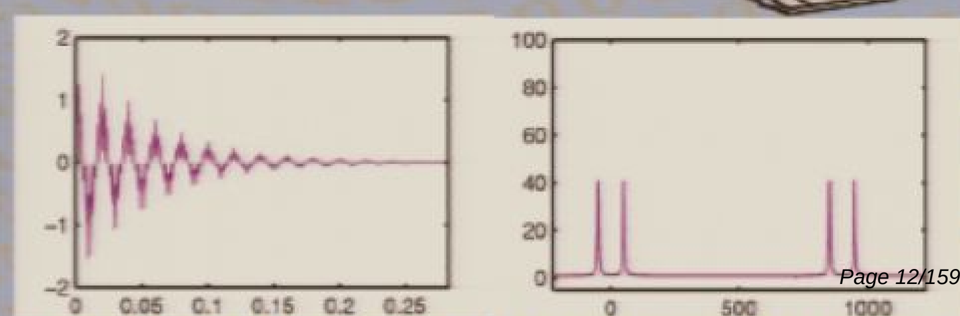

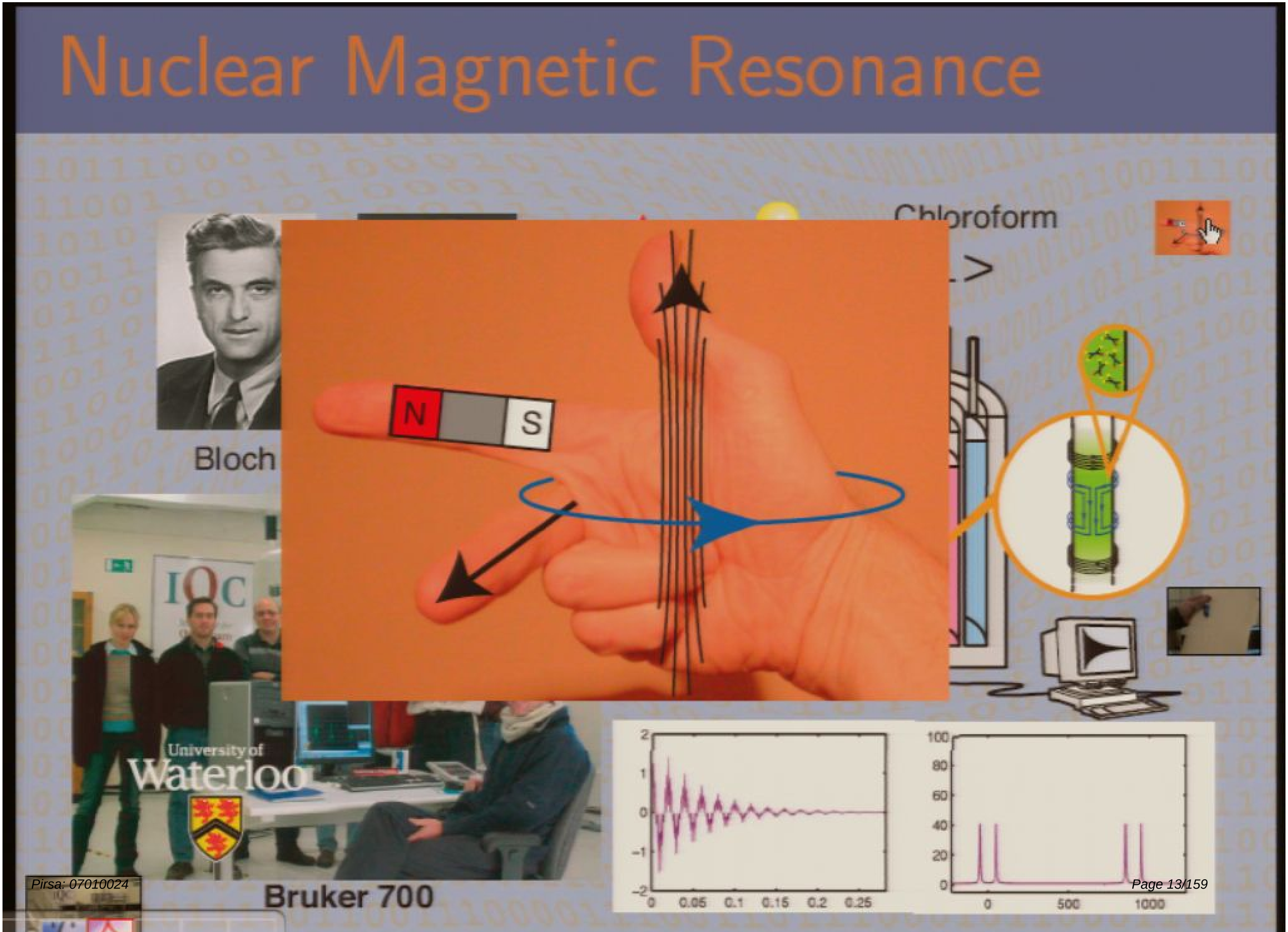
Nuclear Magnetic Resonance
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Nuclear Magnetic Resonance

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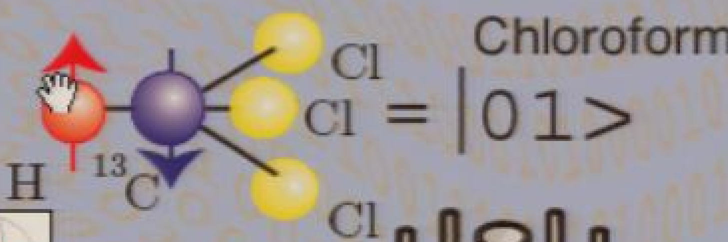

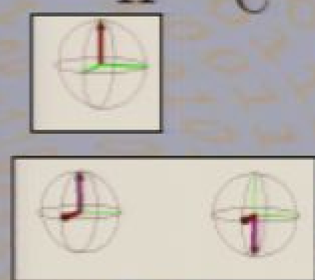

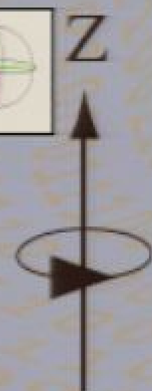

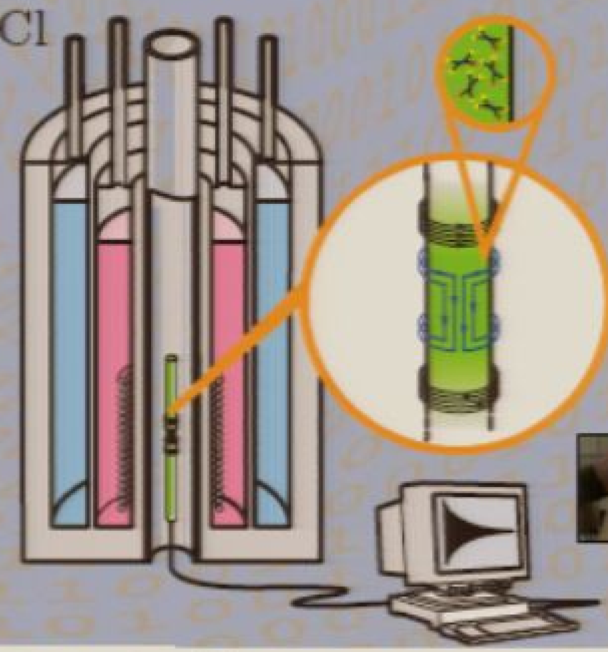

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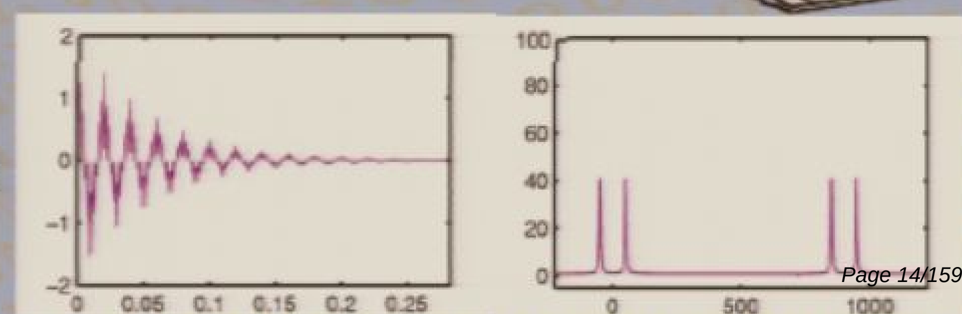
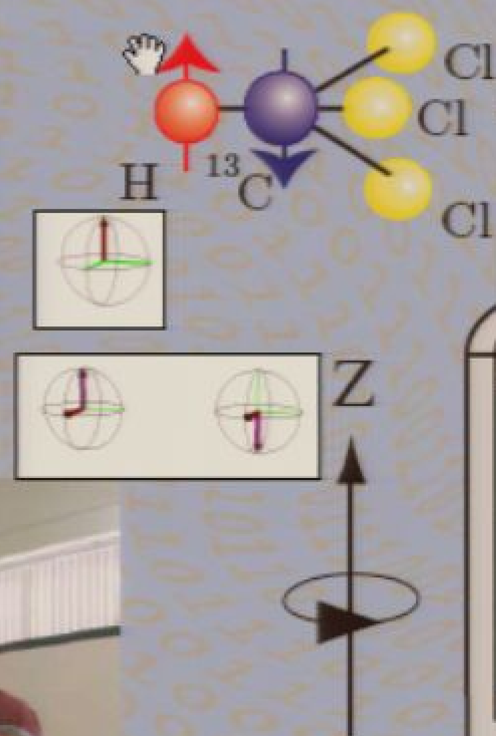


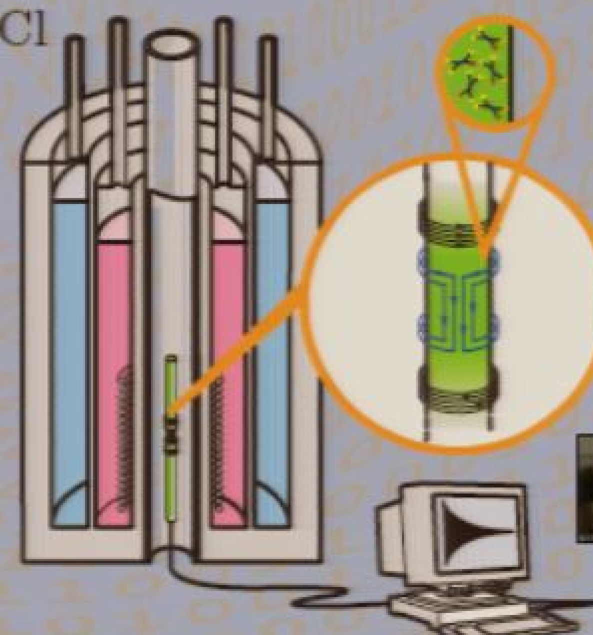
Nuclear Magnetic Resonance

Bloch

Purcell

Chloroform





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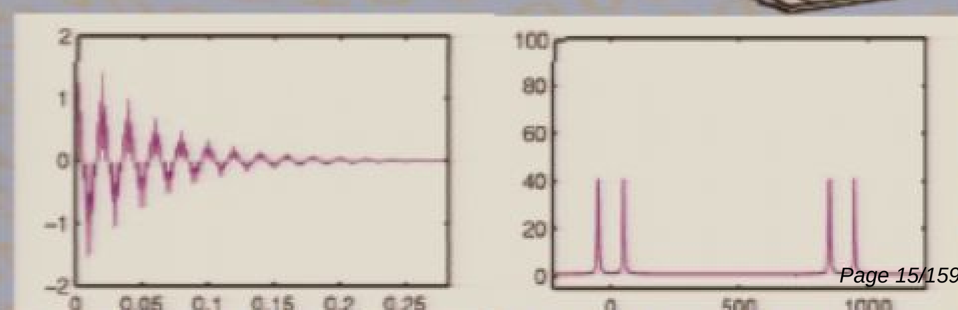

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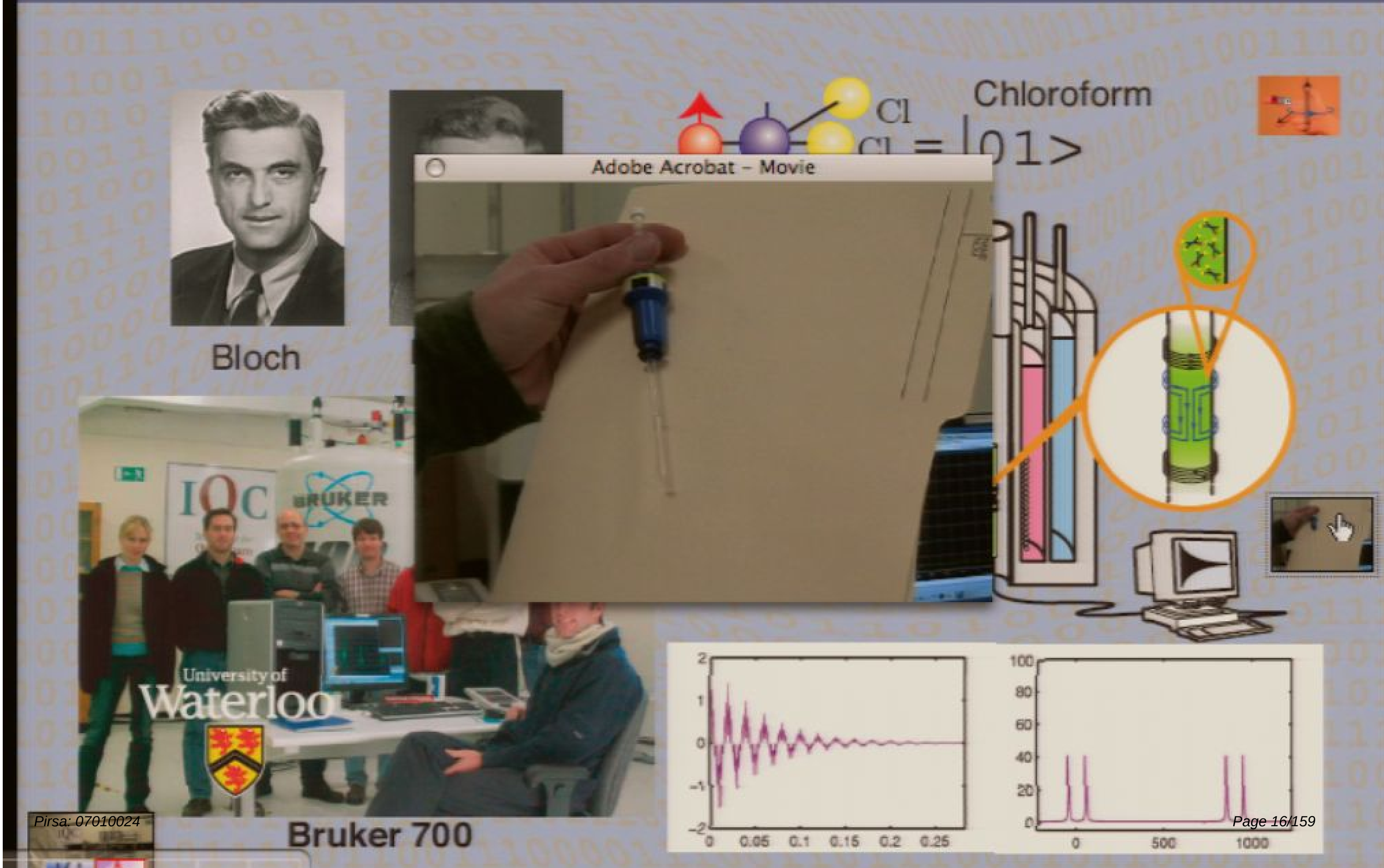

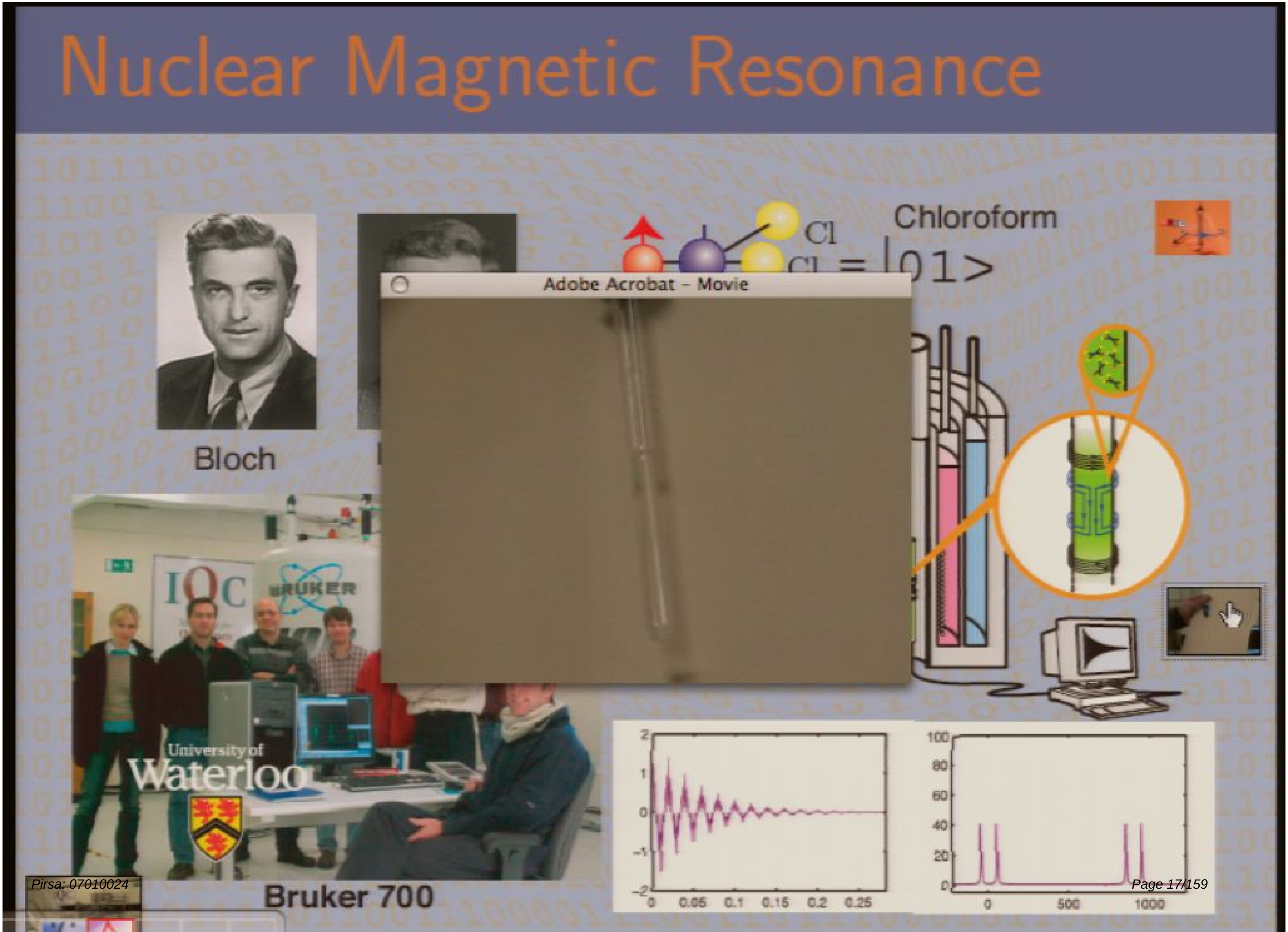
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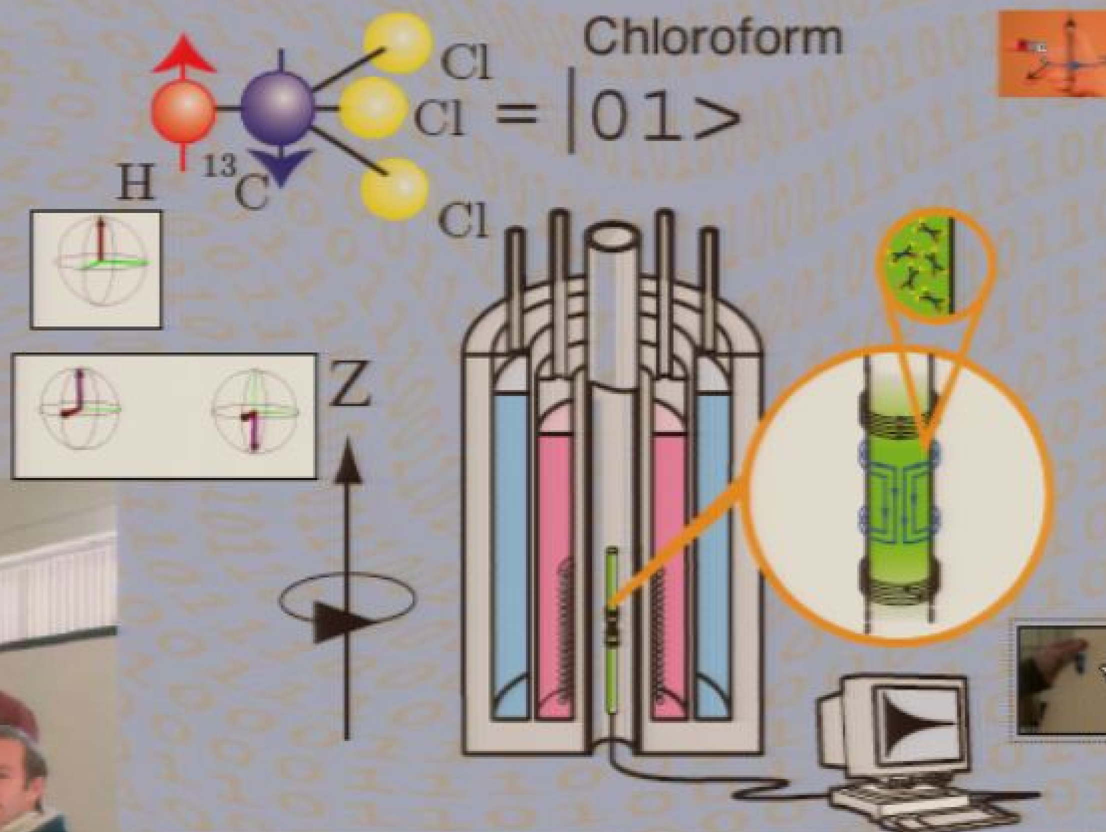

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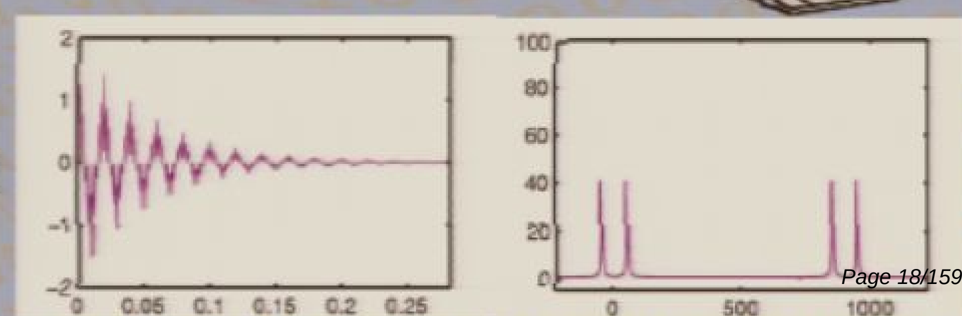

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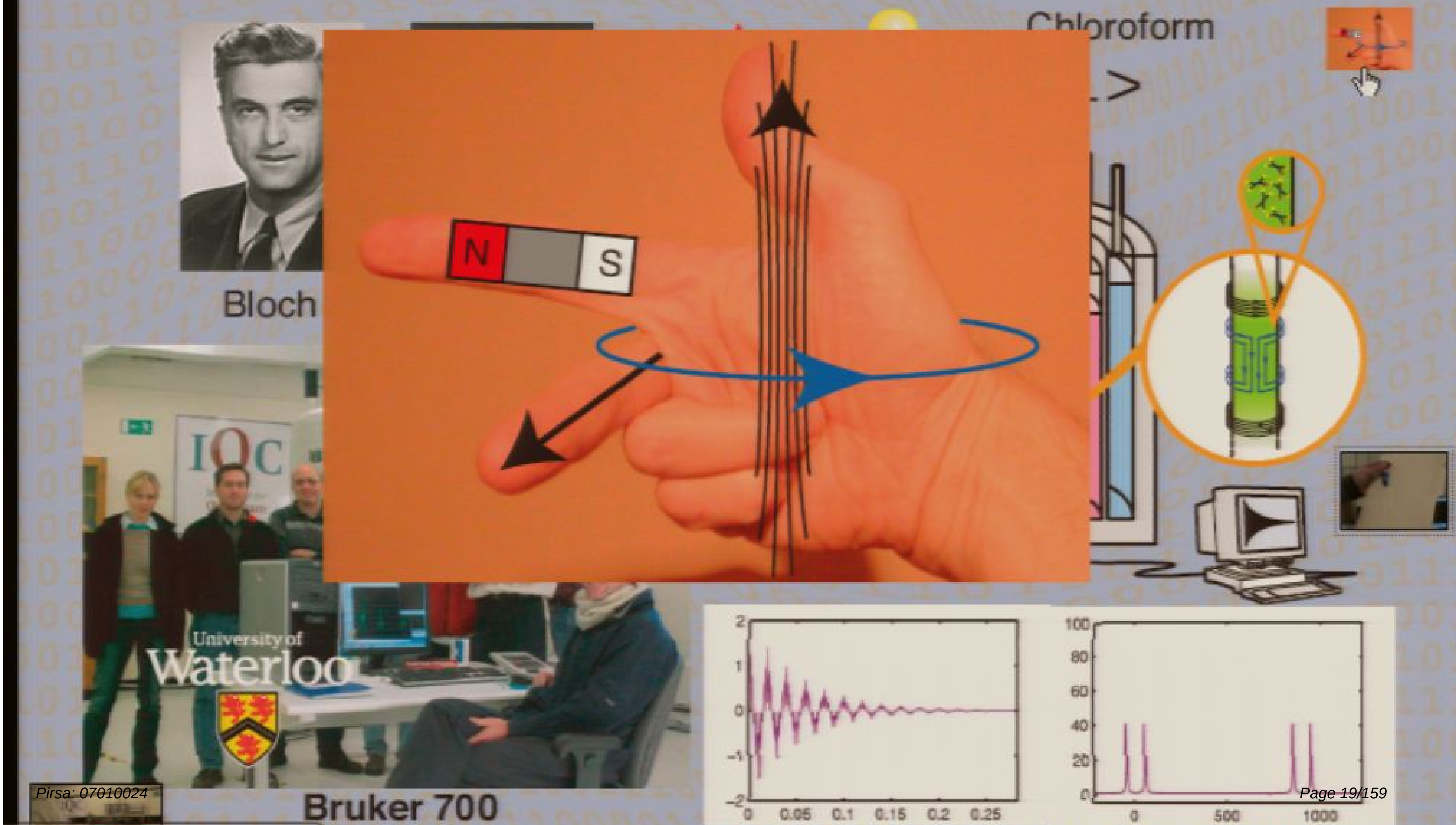

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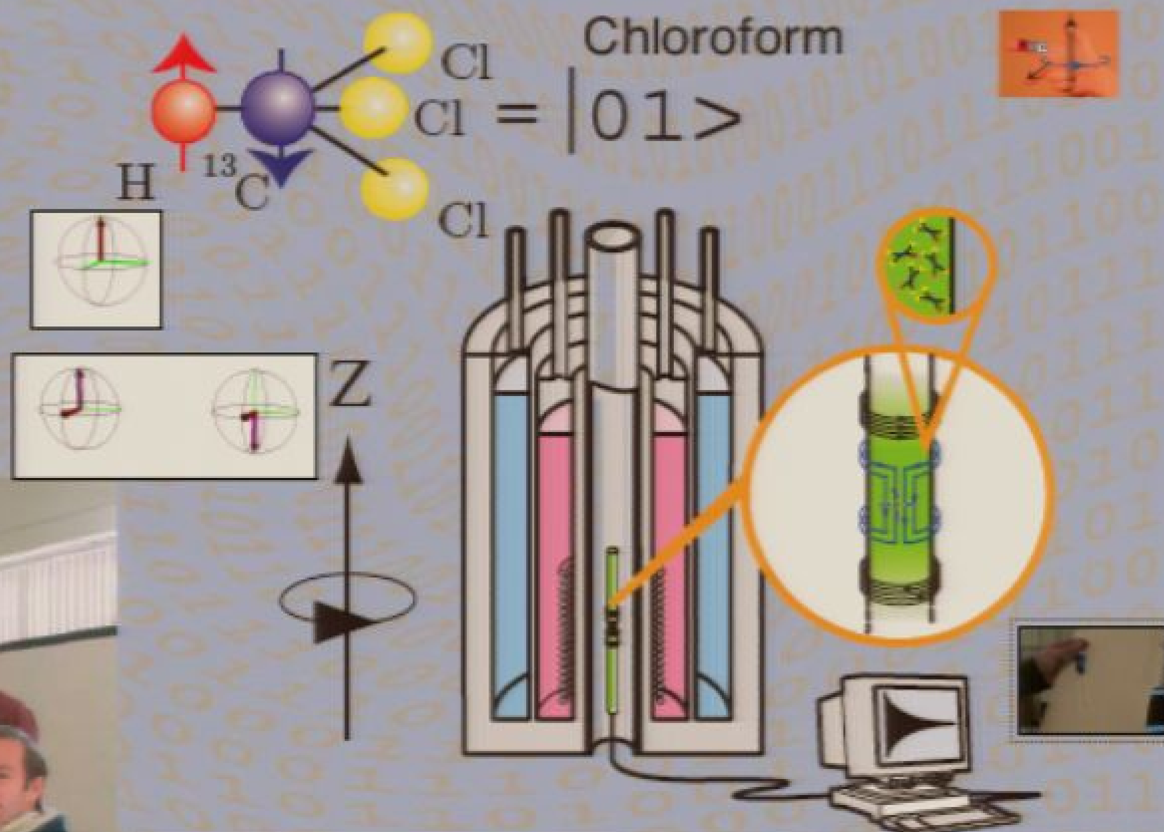

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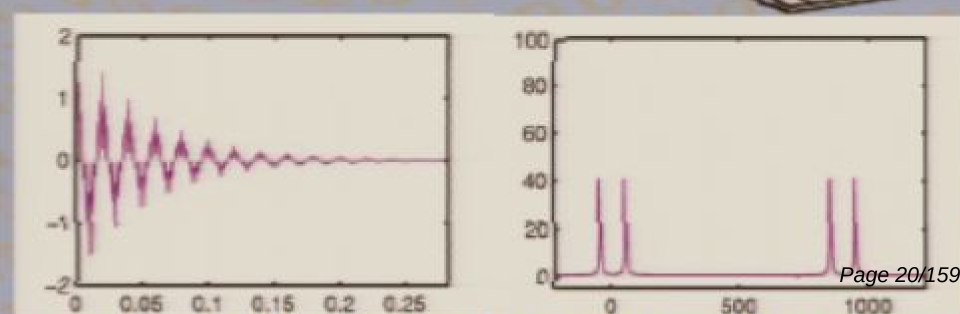

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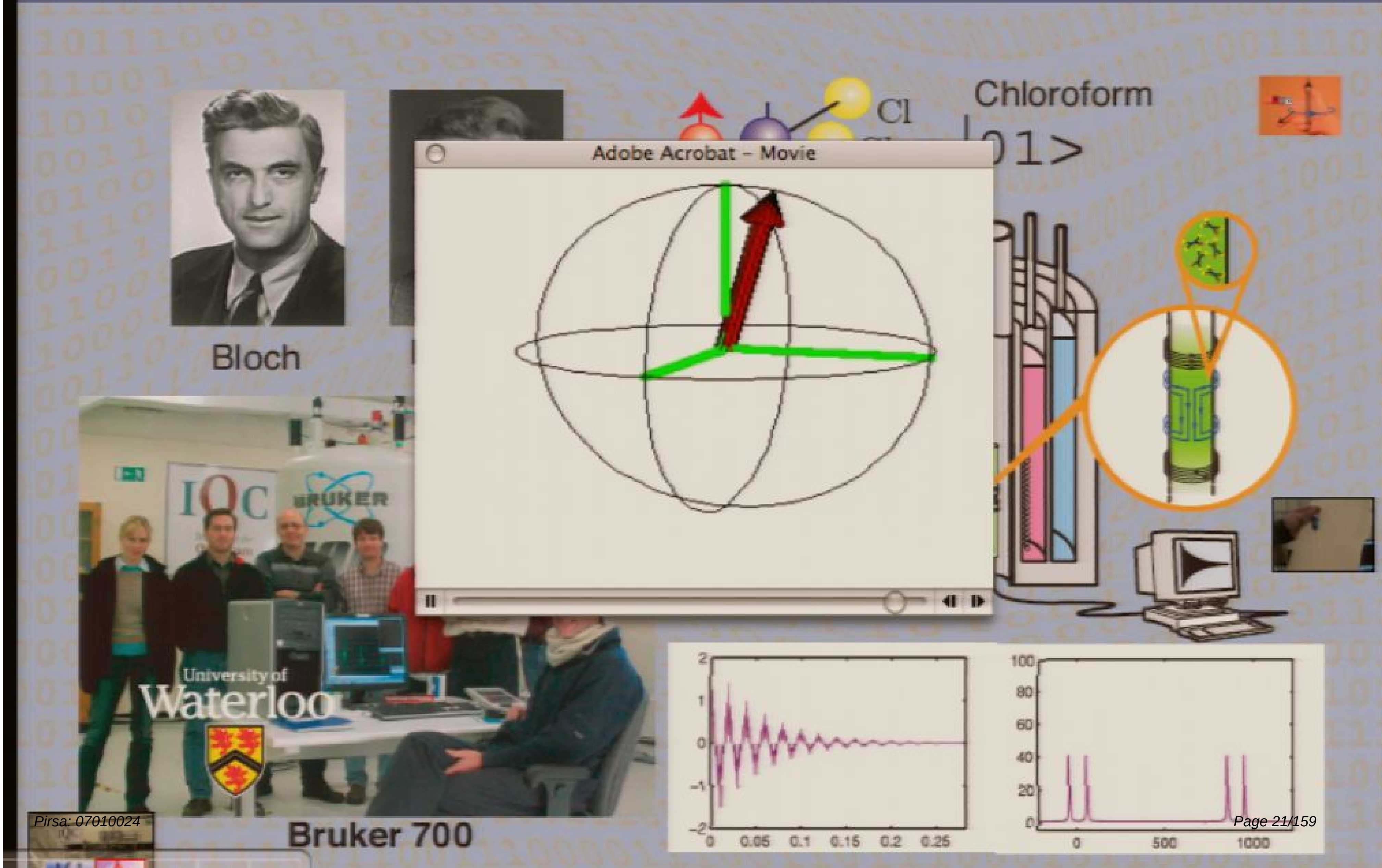

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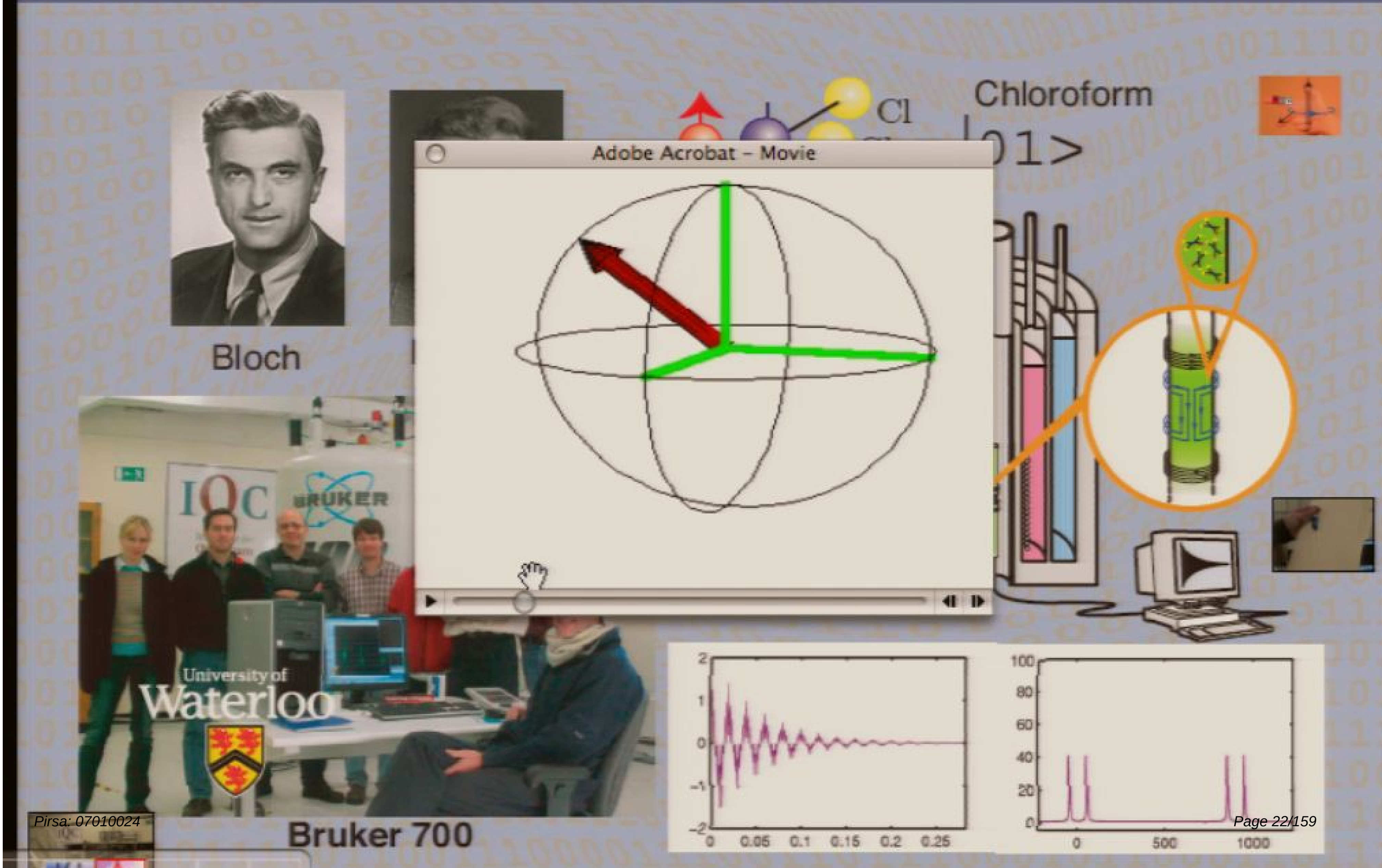

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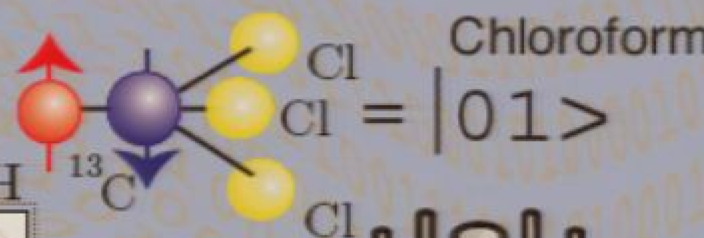

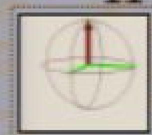

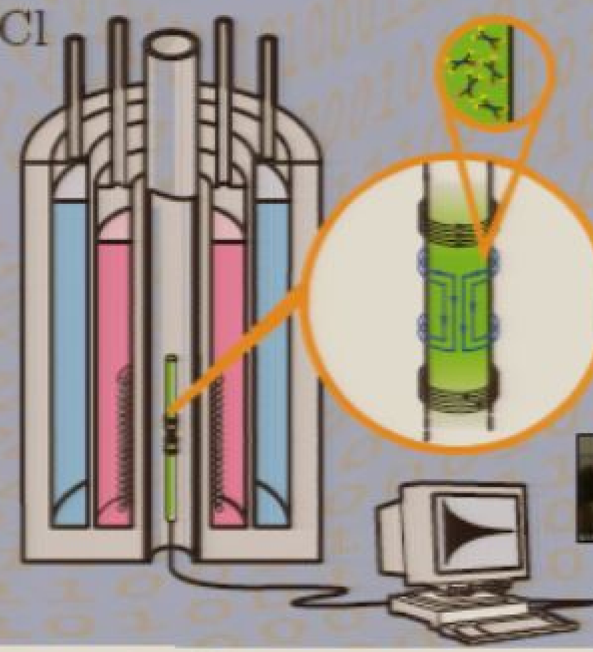

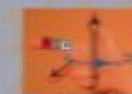

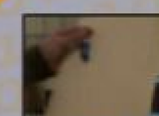

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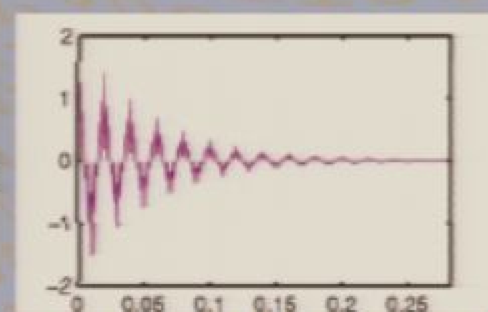

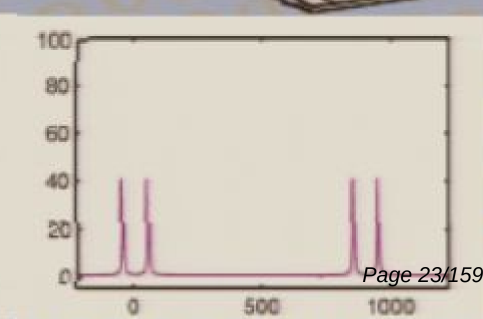

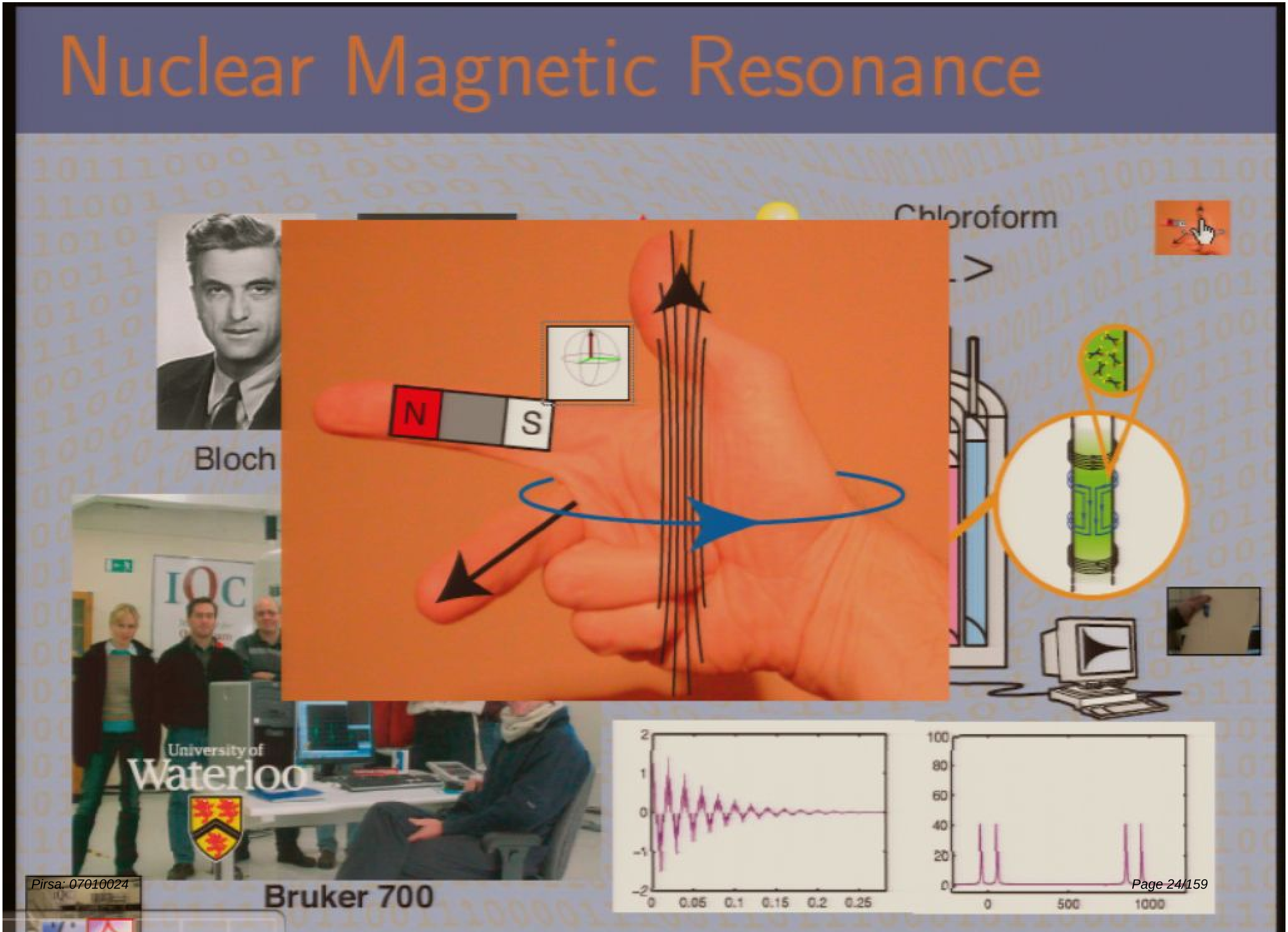

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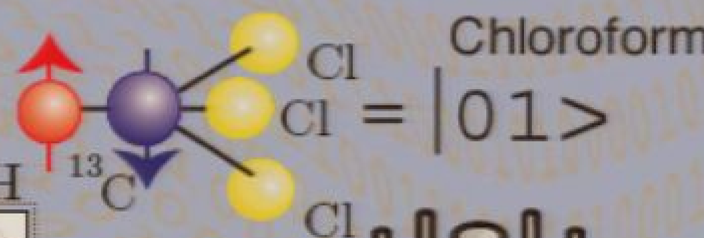

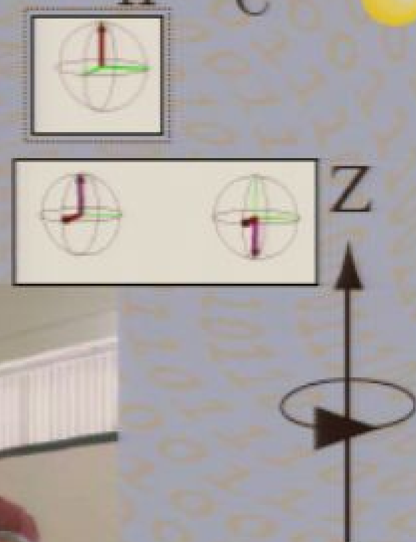

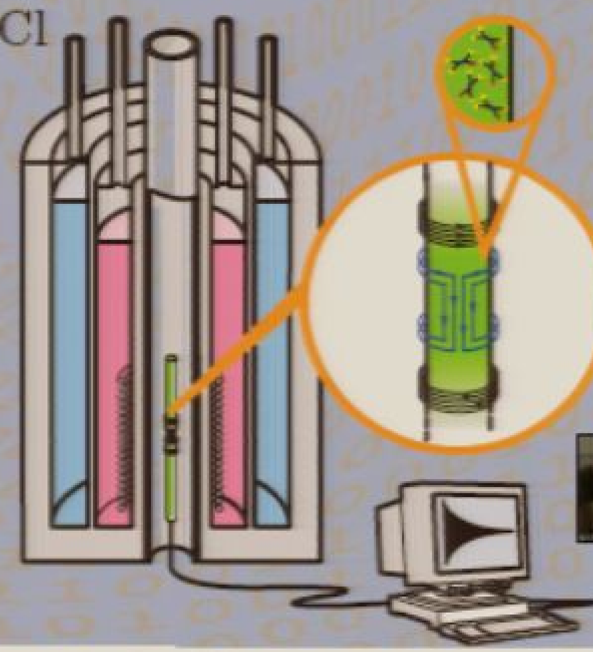

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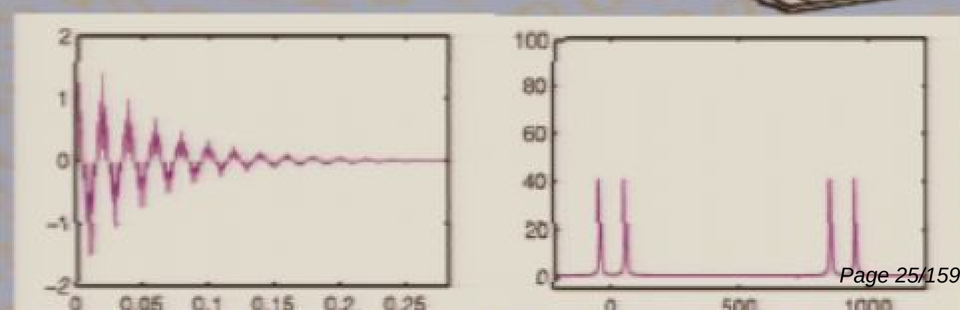
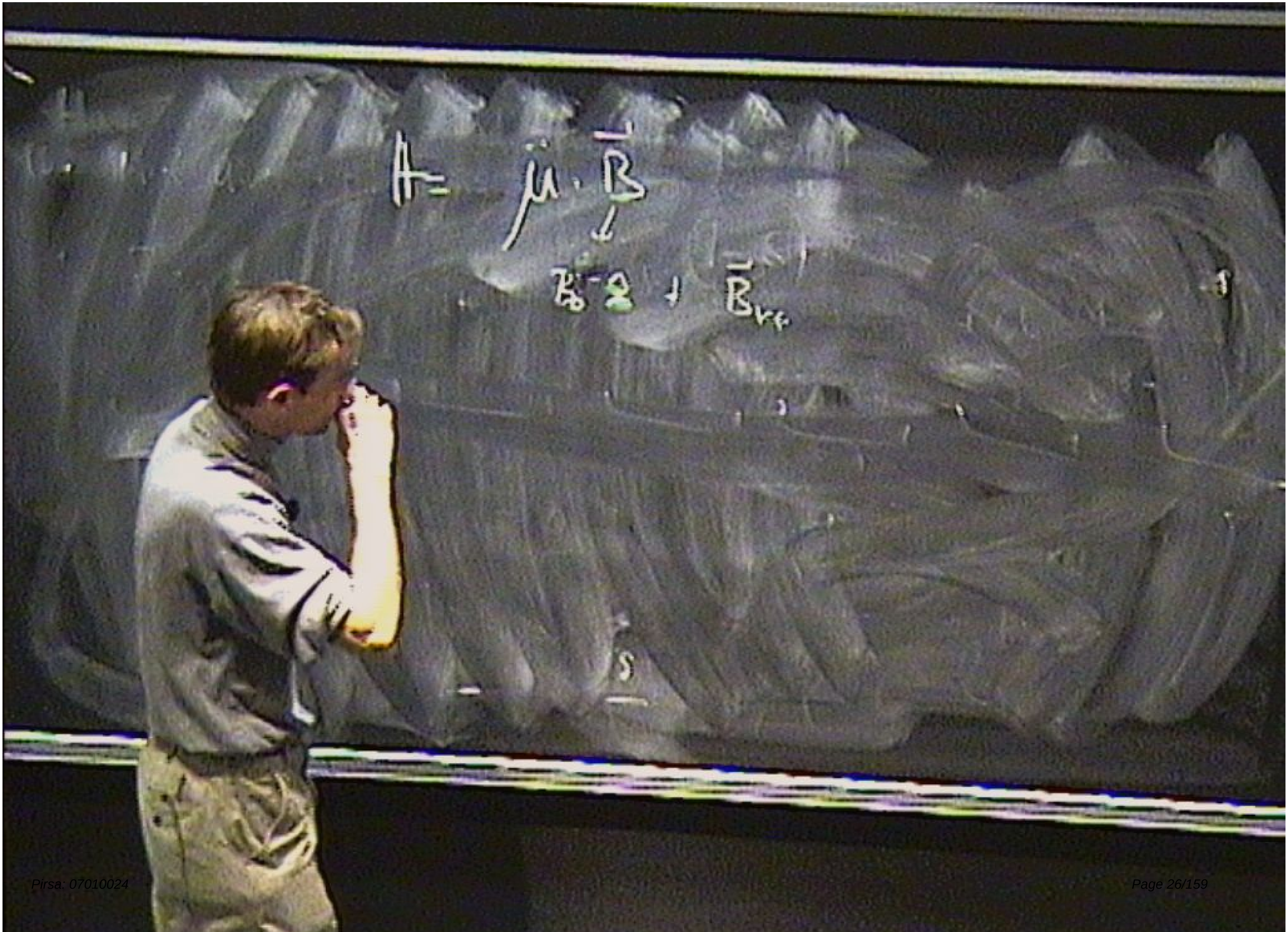
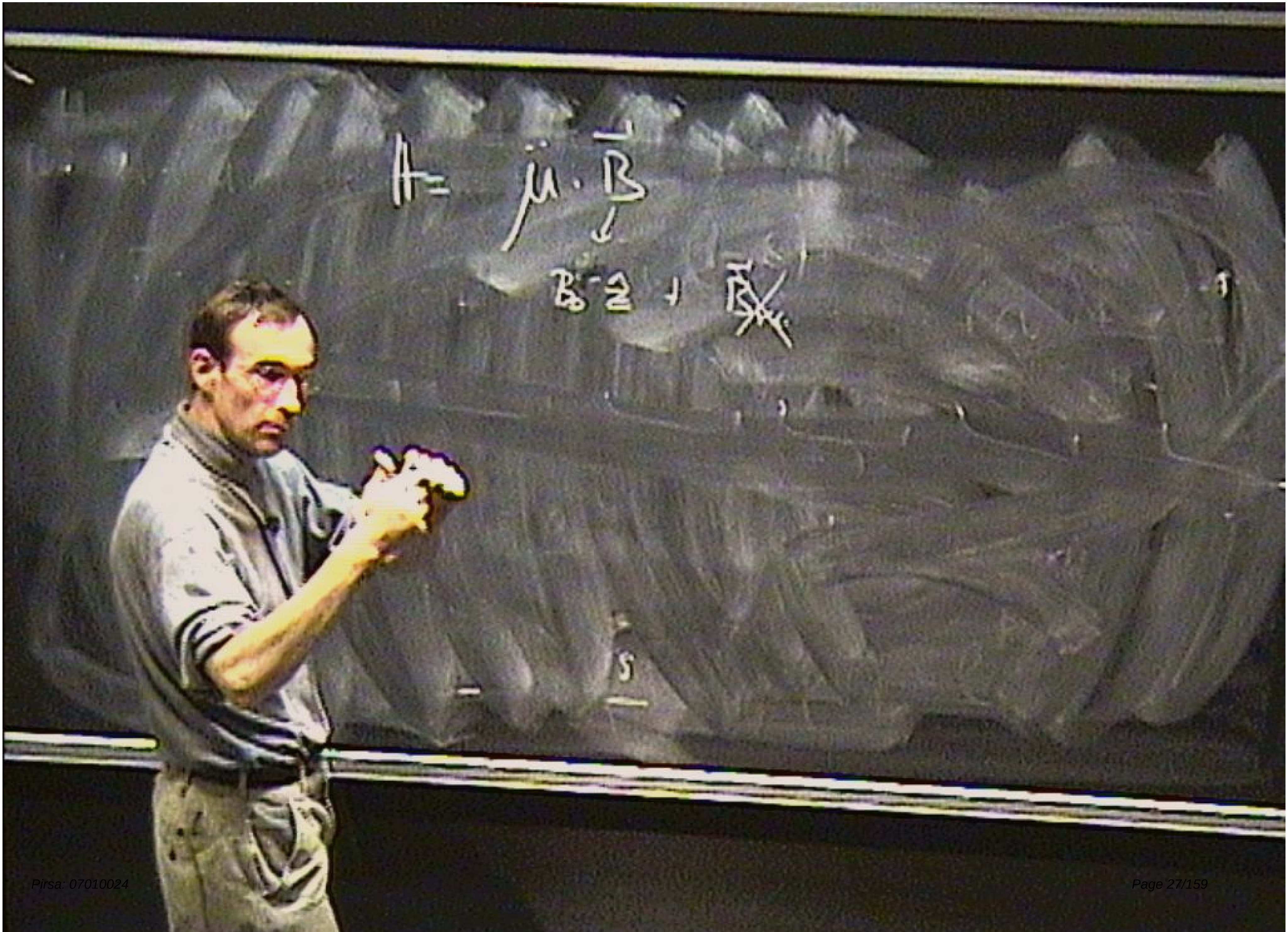
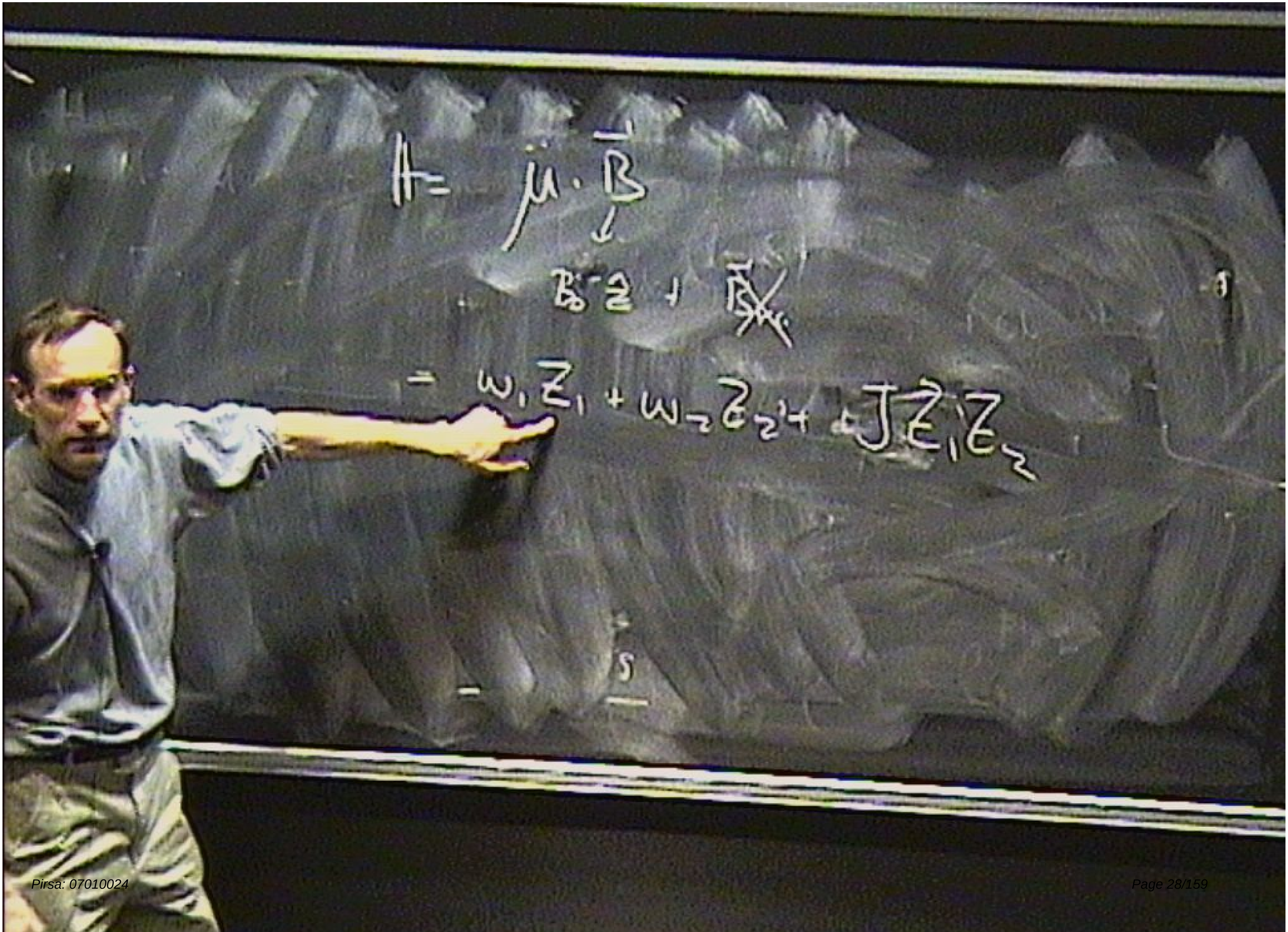
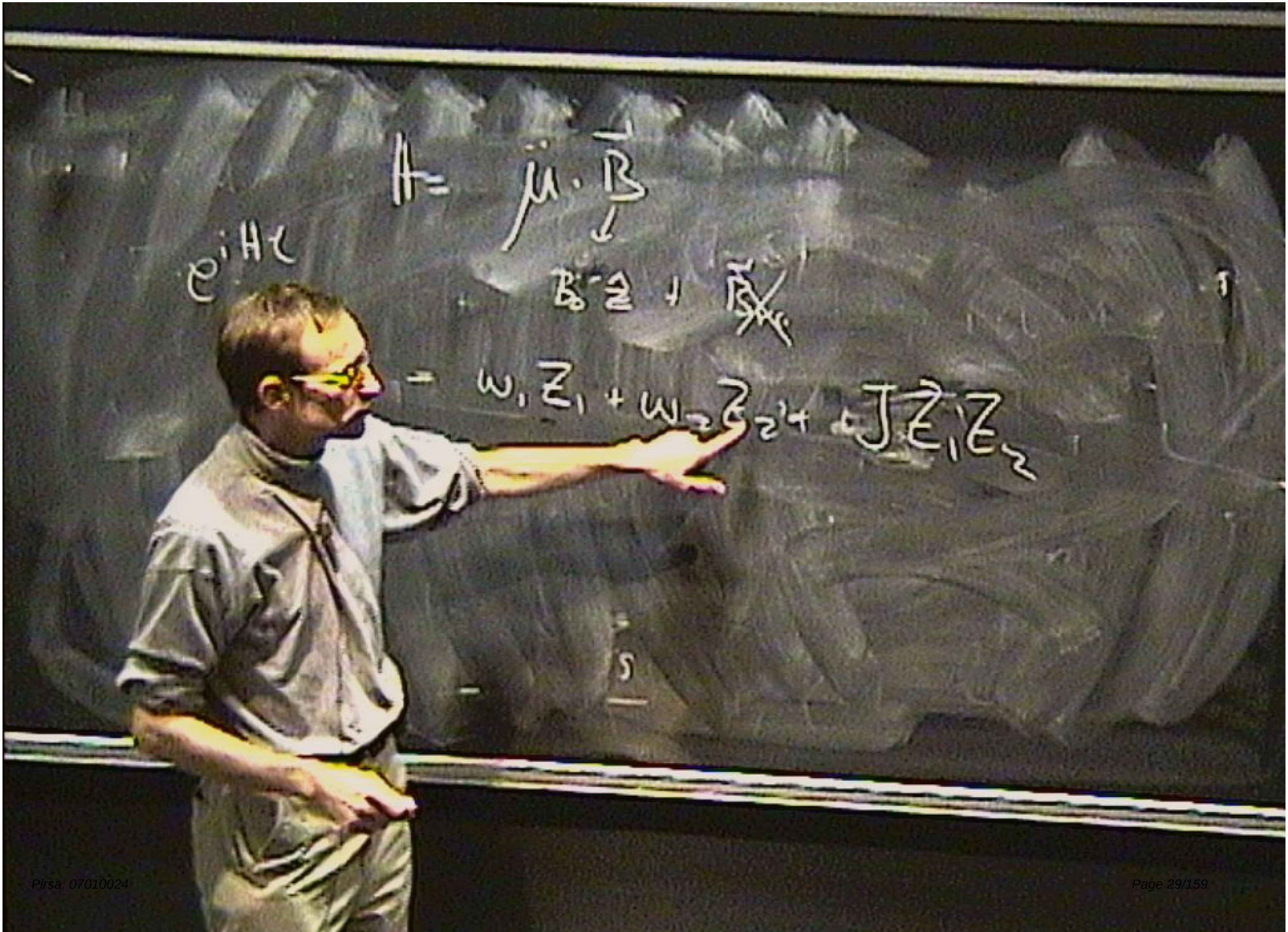
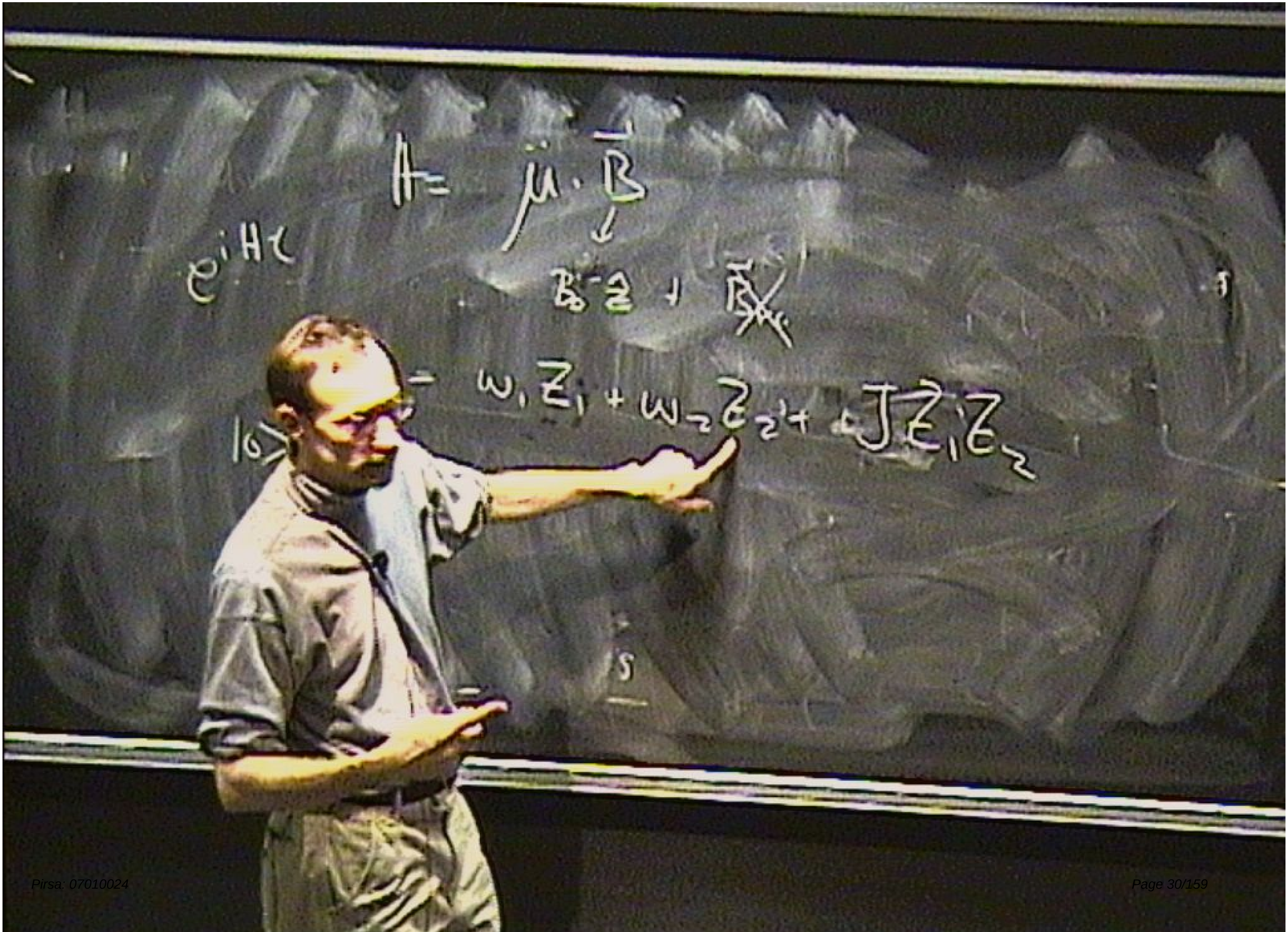



$H = \vec{\mu} \cdot \vec{B}$
$B_0 \hat{z} + \vec{B}_{rf}$


H= μ·B


$H = \vec{\mu} \cdot \vec{B}$
$B_0 \hat{z} + \cancel{\vec{B}_x}$
$= \omega_1 Z_1 + \omega_2 Z_2 + J Z_1 Z_2$


H = μ · B
e^{iHt}
B_0 Z + B_x
= ω_1 Z_1 + ω_2 Z_2 + J Z_1 Z_2


H = μ·B
e^iHt
B₀Z + BX
= ω₁Z₁ + ω₂Z₂ + JZ₁Z₂
|0>

$$H = \vec{\mu}\cdot\vec{B}$$

$$e^{iHt}$$

$$B_0\hat{z} + \cancel{\vec{B}_x}$$

$$= \omega_1 Z_1 + \omega_2 Z_2 + J Z_1 Z_2$$

$$|0\rangle_1 \longrightarrow (\omega_2 + J) Z_2$$

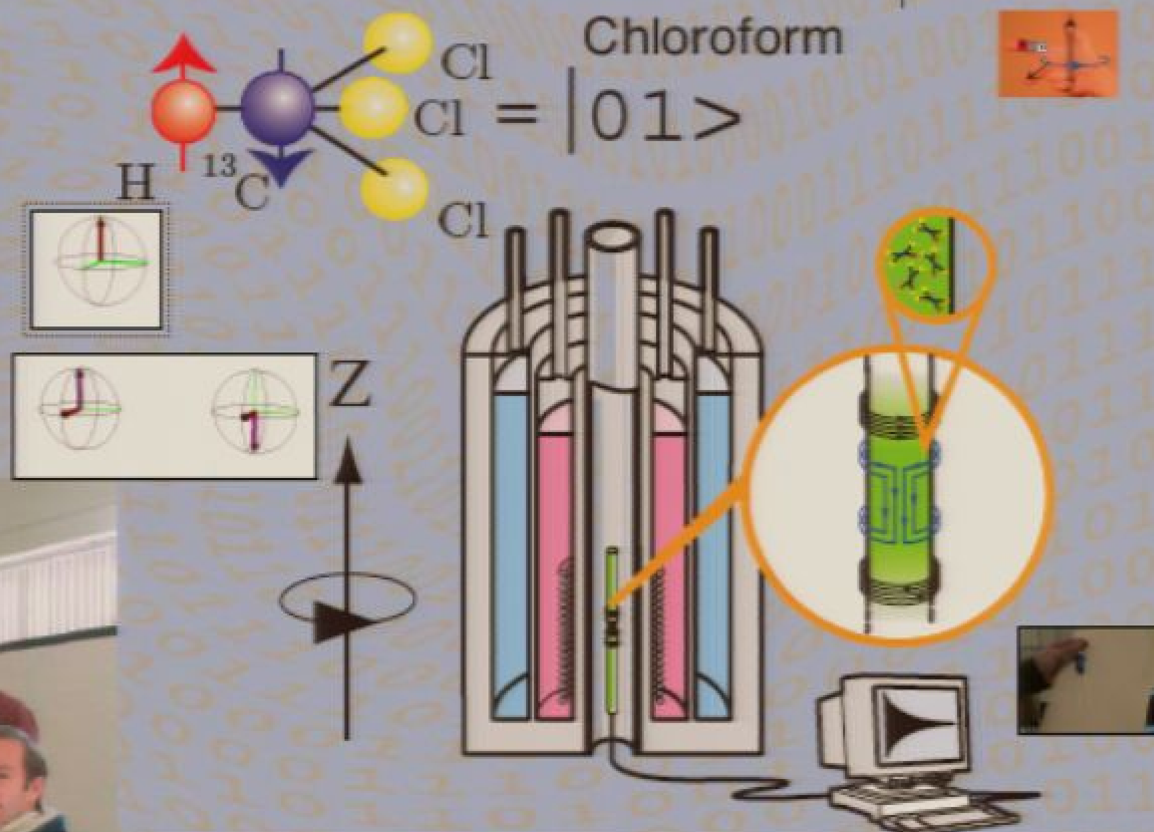
$$|1\rangle \longrightarrow (\omega_2 - J) Z_2$$

$$\underline{S}$$

Nuclear Magnetic Resonance

Bloch

Purcell



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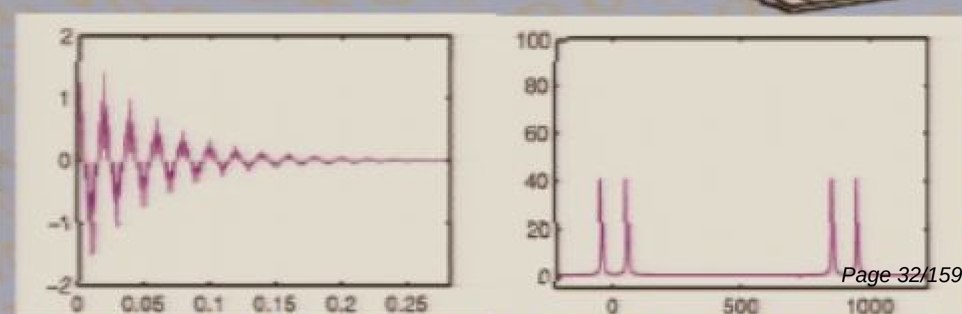

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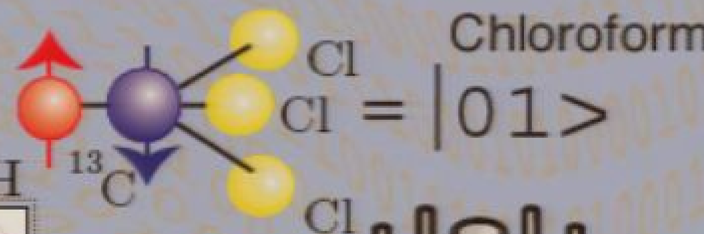

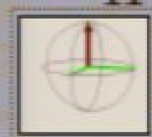

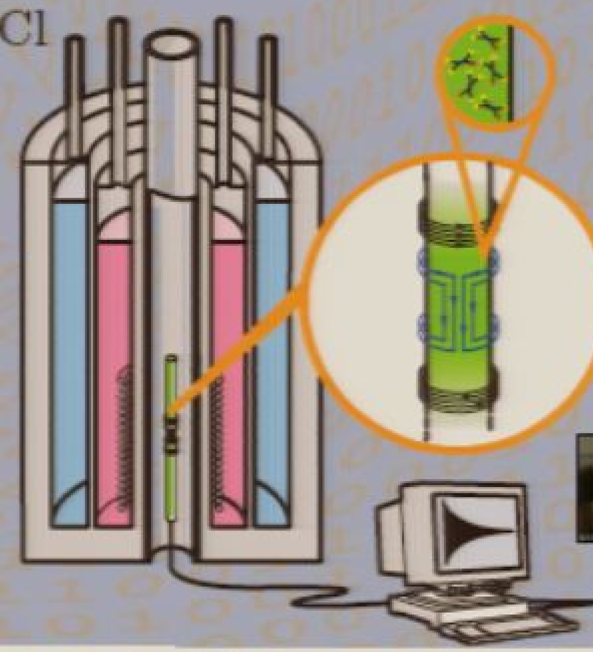

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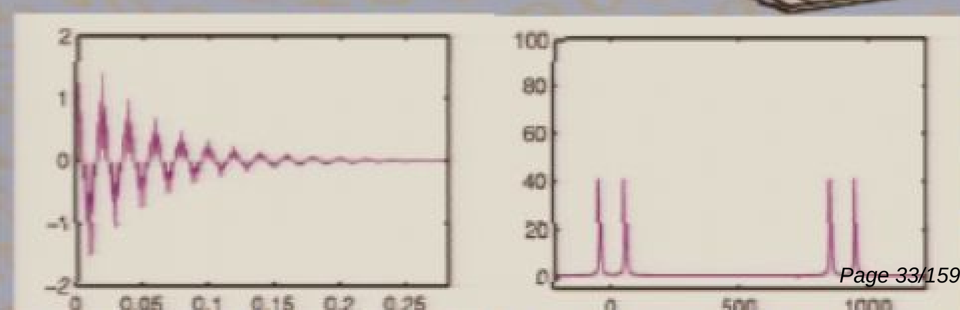

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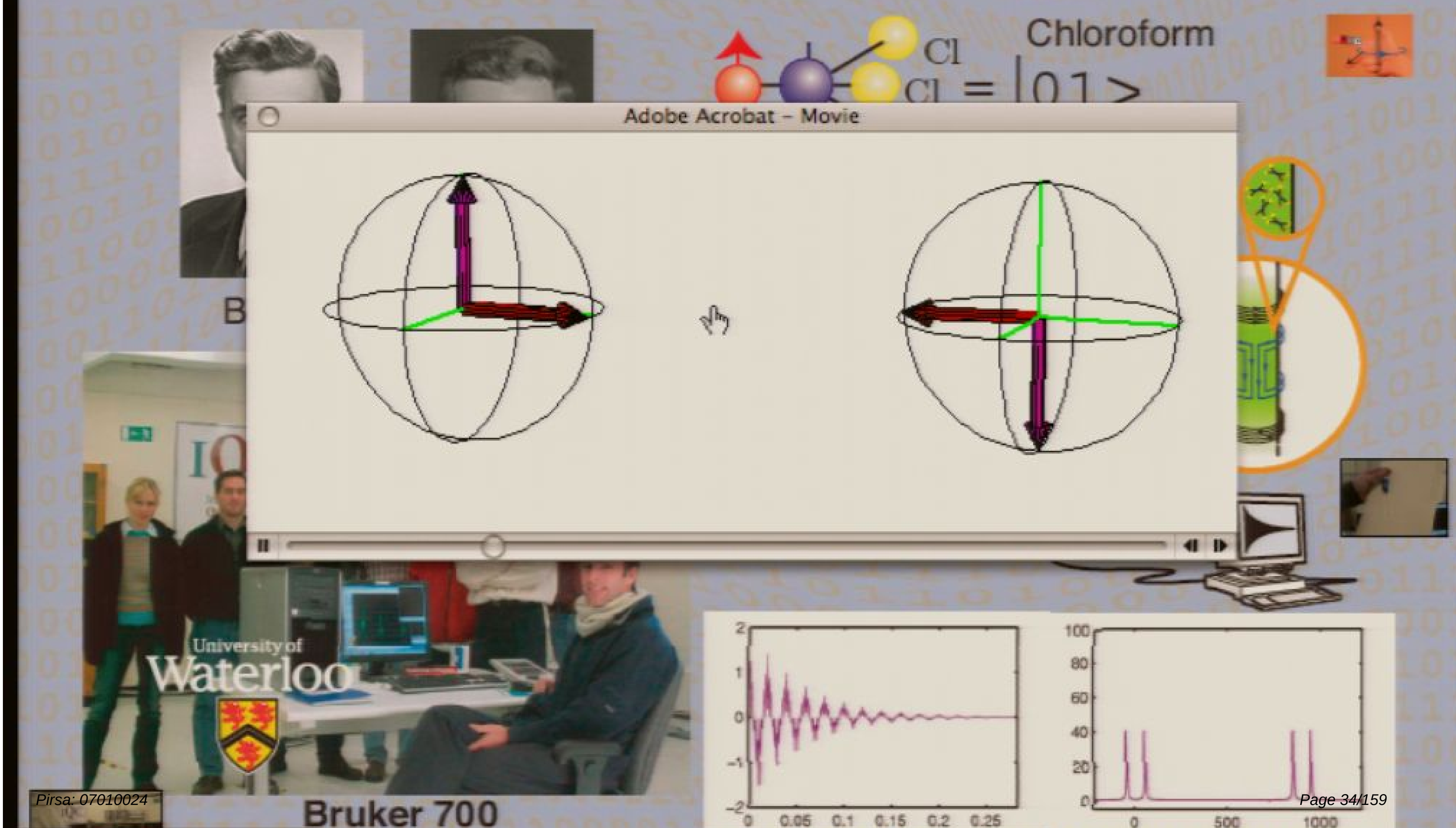

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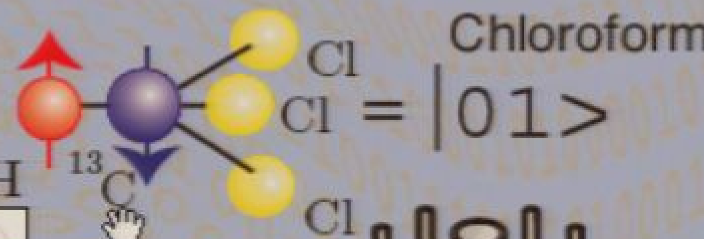

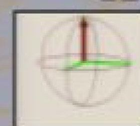

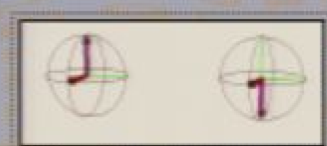

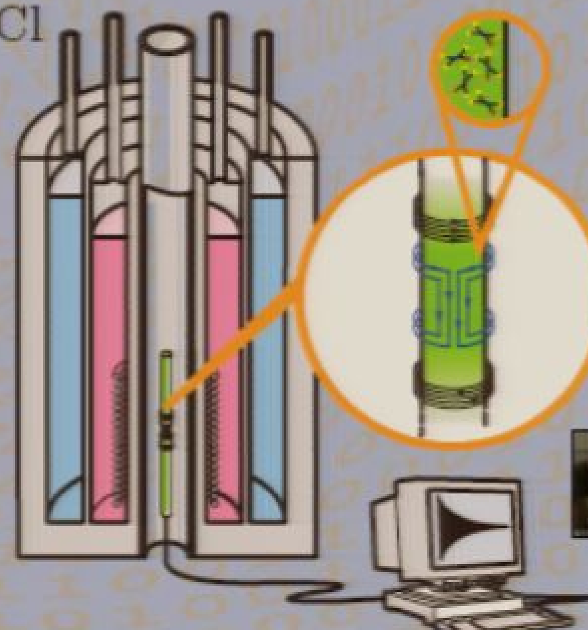

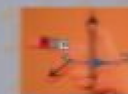

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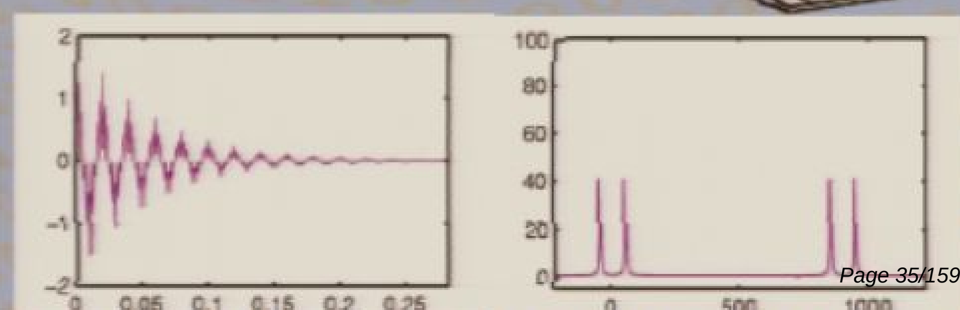


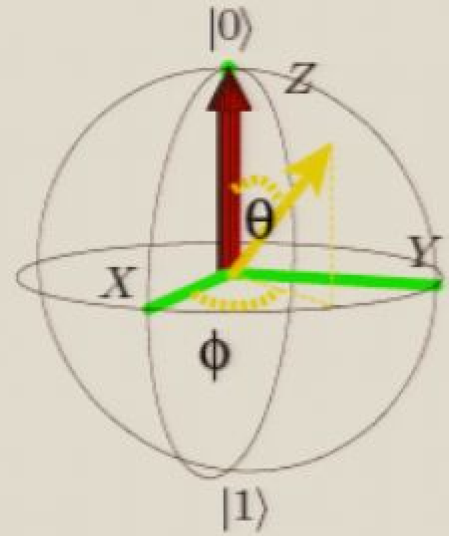
Notation

- Pauli matrices:

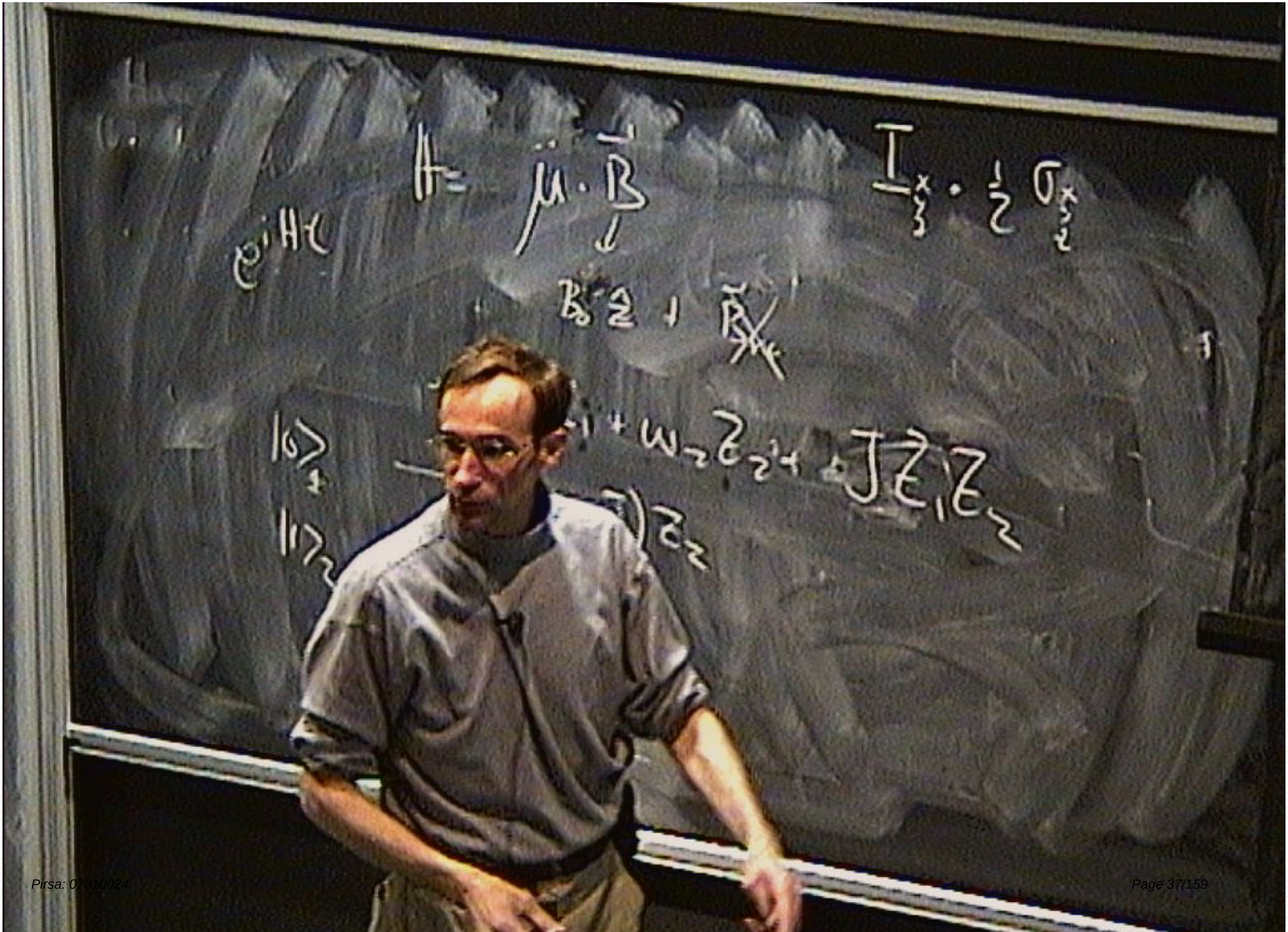
$$\mathbb{1} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}; \sigma_x = X = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}; \sigma_y = Y = \begin{pmatrix}0 & -i\\ 0 & i\end{pmatrix}; \sigma_z = Z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

- Tensor product $X_k = \mathbb{1} \otimes \mathbb{1} \otimes \cdots \otimes X \otimes \cdots \otimes \mathbb{1}$.

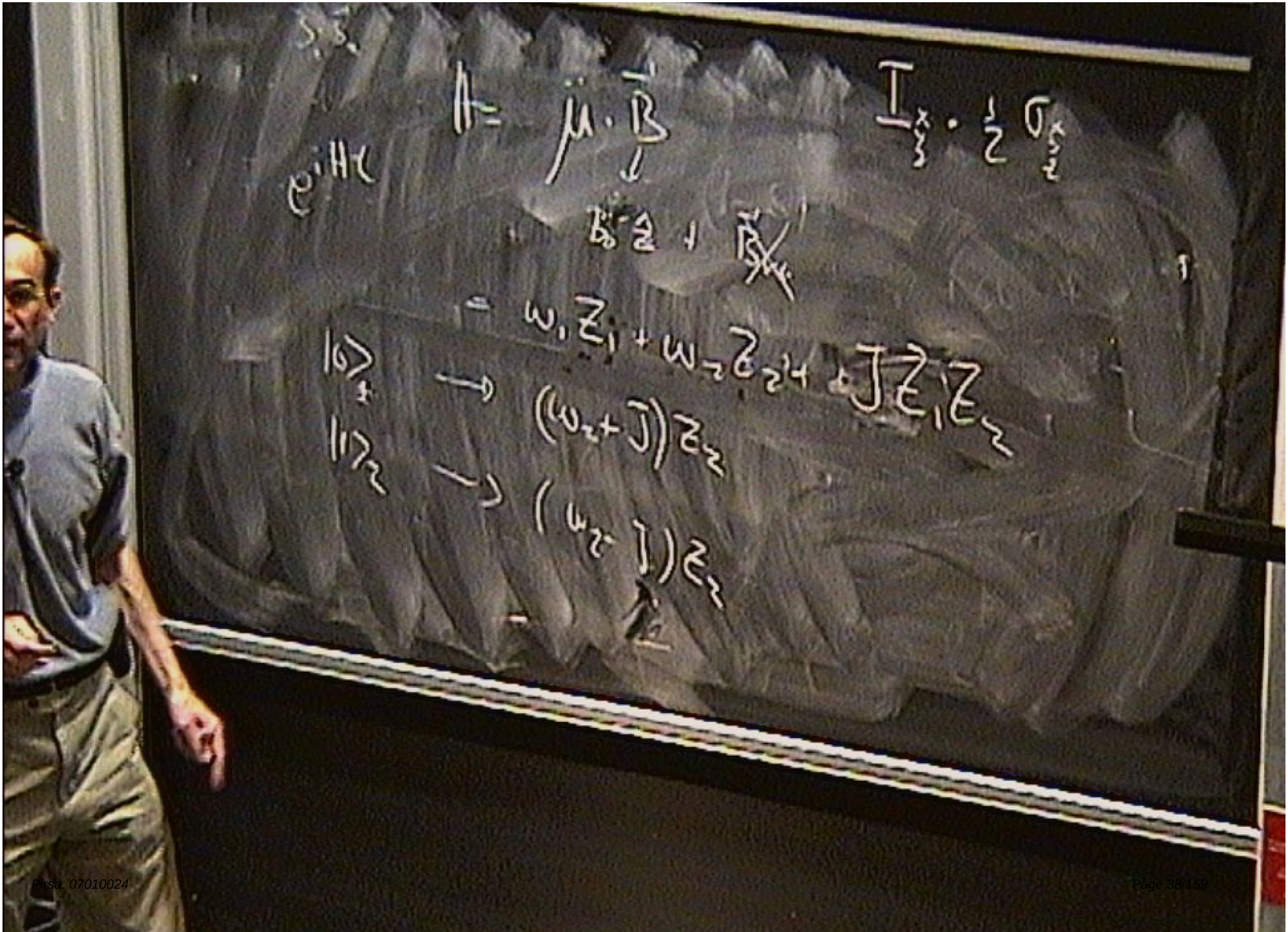
- Bloch sphere: a geometric representation of the state
$\Psi = \cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle$ or
$\rho = \frac{1}{2}(\mathbb{1} + \sin[\theta]\cos[\phi]X + \sin[\theta]\sin[\phi]Y + cos[\theta]Z)$



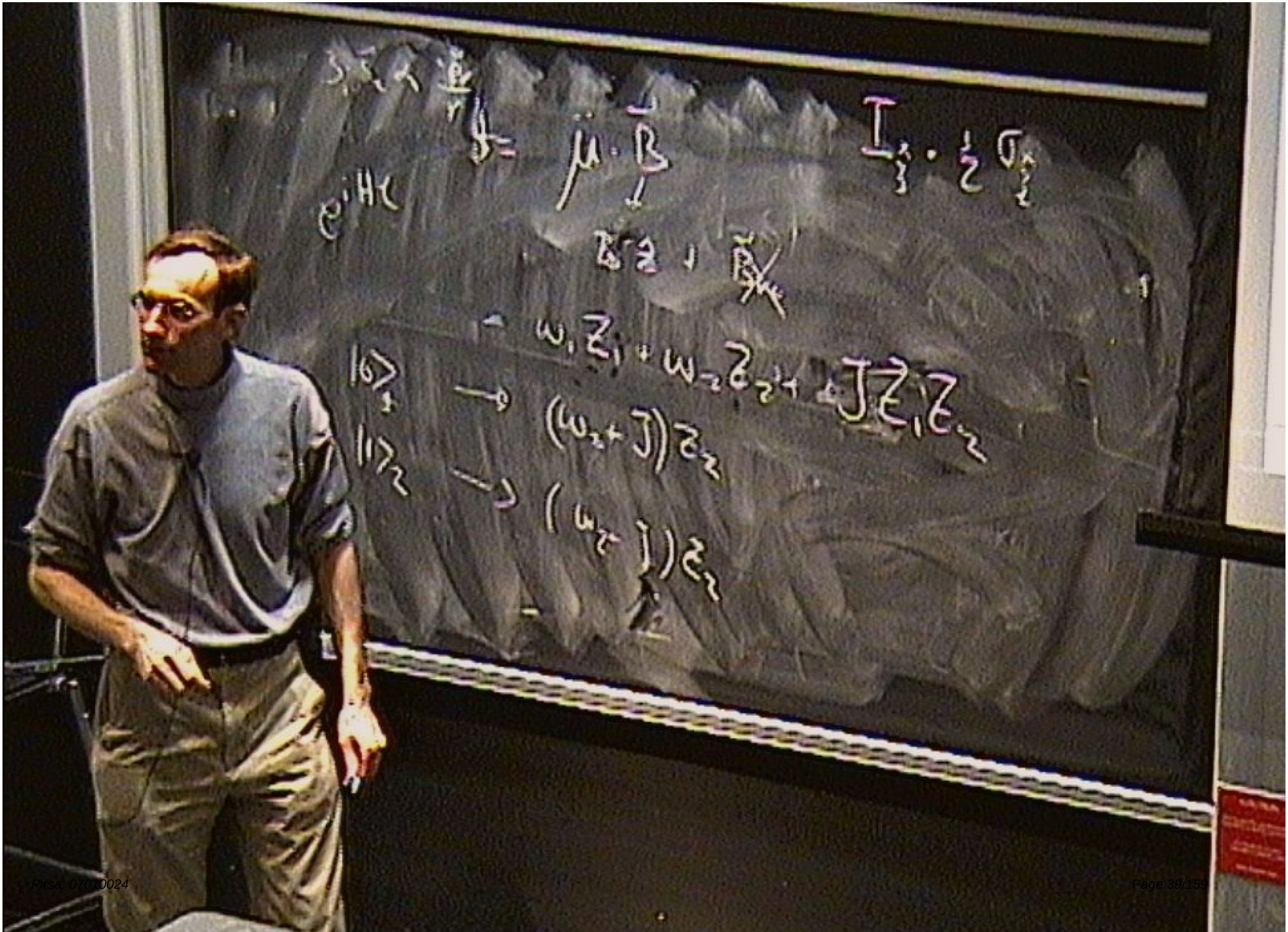
- $\hbar = 1.05 \times 10^{-34}$ Js, $k = 1.38 \times 10^{-23}$ J/K
1 Tesla$= 10^4$ Gauss (Earth Magnetic field=0.5 Gauss)
$\mu_p = 26.75 \times 10^7/(\mathrm{Ts})$
- Freq. of the spectrometer $\nu_L^p = 26.75 \times 10^7 rad/(Ts) * 11.7T/2\pi = 500MHz$
- Corresponding $\beta = \hbar \times \omega_l/kT$
$= 1.05 \times 10^{-34} Js \times 2\pi \times 486MHz/(1.38 \times 10^{-23} J/K \times 300K) \approx 7.8 \times 10^{-5}$
- Definition of T_1 (relaxation time): $M_z(\tau) = M_z^0[1 - 2e^{(-\tau/T_1)}]$
- Definition of T_2 (decoherence time): $M_x(\tau) = M_x^0[exp(-2\tau/T_2)]$

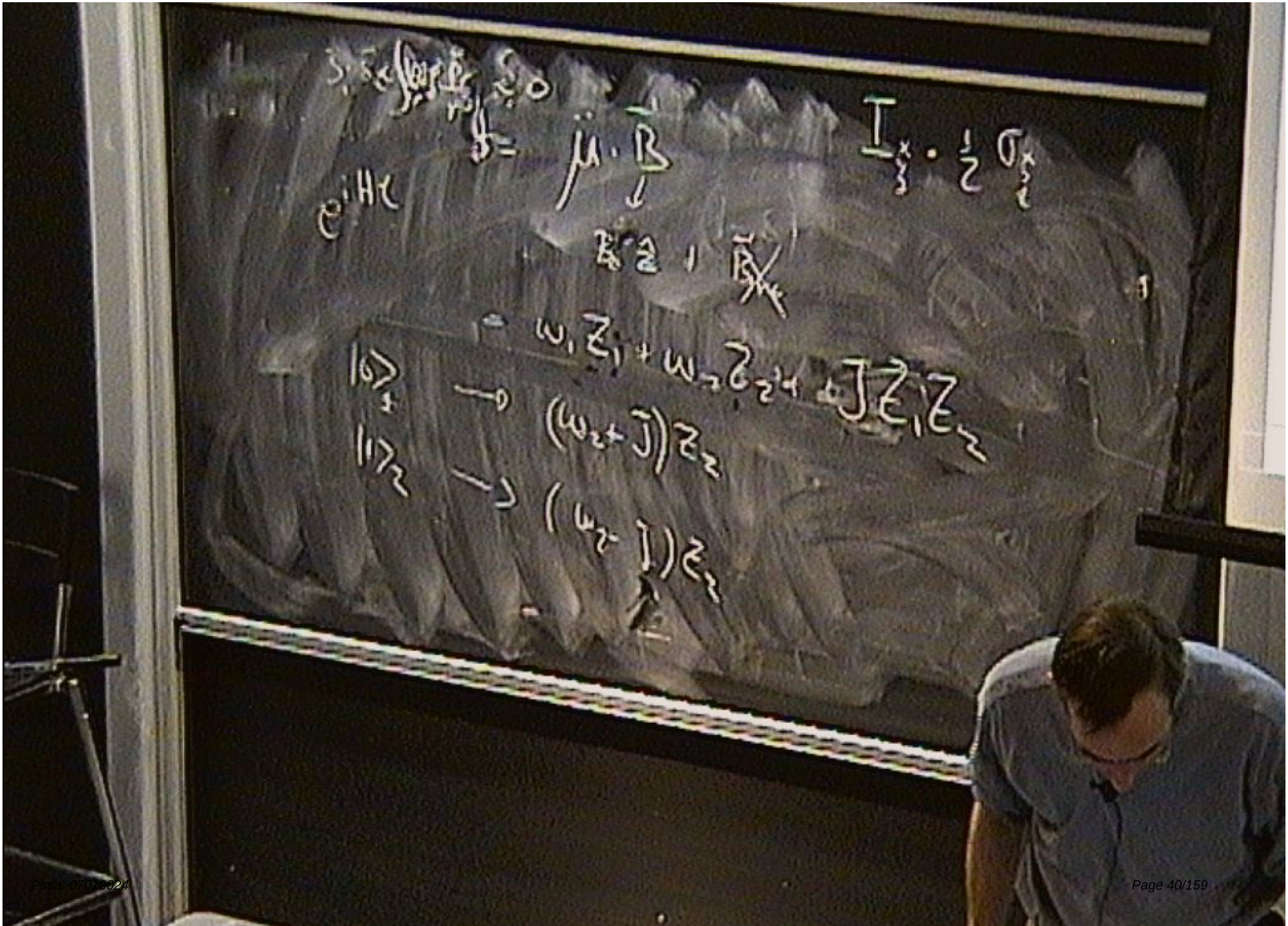


H = μ · B
I ⊗ ½ σ
e^{-iHt}
|0⟩
|1⟩
+ ω₂ Z₂ + J Z₁ Z₂



$H = \vec{\mu} \cdot \vec{B}$
$I_x \cdot \frac{1}{2}\sigma_x$
e^{iHt}
$= \omega_1 Z_1 + \omega_2 Z_2 + J Z_1 Z_2$
$|0\rangle_1 \longrightarrow (\omega_2 + J) Z_2$
$|1\rangle_1 \longrightarrow (\omega_2 - J) Z_2$





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$\omega_1 Z_1 + \omega_2 Z_2 + J Z_1 Z_2$
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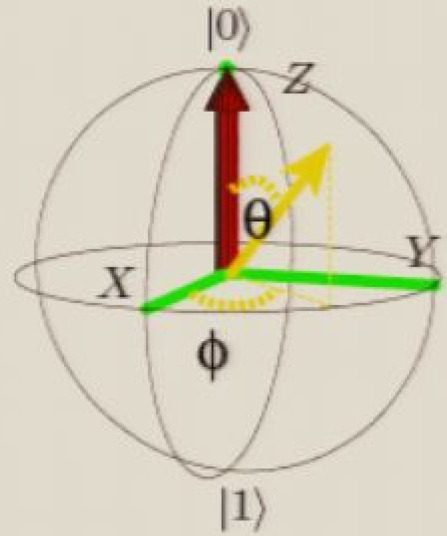
Notation

- Pauli matrices:

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = Y = \begin{pmatrix} 0 & -i \\ 0 & i \end{pmatrix}; \sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Tensor product $X_k = \mathbb{1} \otimes \mathbb{1} \otimes \cdots \otimes X \otimes \cdots \otimes \mathbb{1}$.

- Bloch sphere: a geometric representation of the state
$\Psi = \cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle$ or
$\rho = \frac{1}{2}(\mathbb{1} + \sin[\theta]\cos[\phi]X + \sin[\theta]\sin[\phi]Y + cos[\theta]Z)$



- $\hbar = 1.05 \times 10^{-34}$ Js, $k = 1.38 \times 10^{-23}$ J/K
1 Tesla= 10^4 Gauss (Earth Magnetic field=0.5 Gauss)
$\mu_p = 26.75 \times 10^7/(\mathrm{Ts})$
- Freq. of the spectrometer $\nu_L^p = 26.75 \times 10^7 rad/(Ts) * 11.7T/2\pi = 500MHz$
- Corresponding $\beta = \hbar \times \omega_l/kT$
$= 1.05 \times 10^{-34} Js \times 2\pi \times 486MHz/(1.38 \times 10^{-23} J/K \times 300K) \approx 7.8 \times 10^{-5}$
- Definition of T_1 (relaxation time): $M_z(\tau) = M_z^0[1 - 2e^{(-\tau/T_1)}]$
- Definition of T_2 (decoherence time): $M_x(\tau) = M_x^0[exp(-2\tau/T_2)]$

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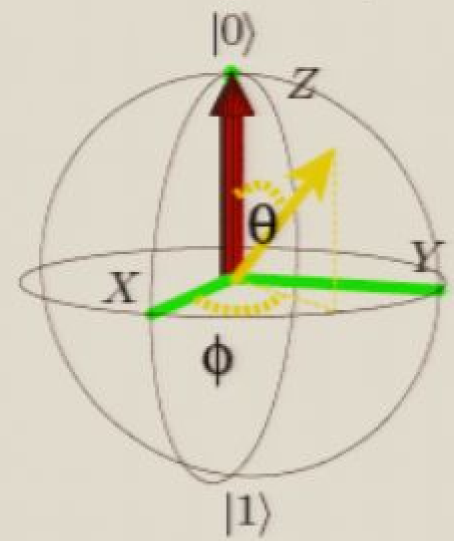

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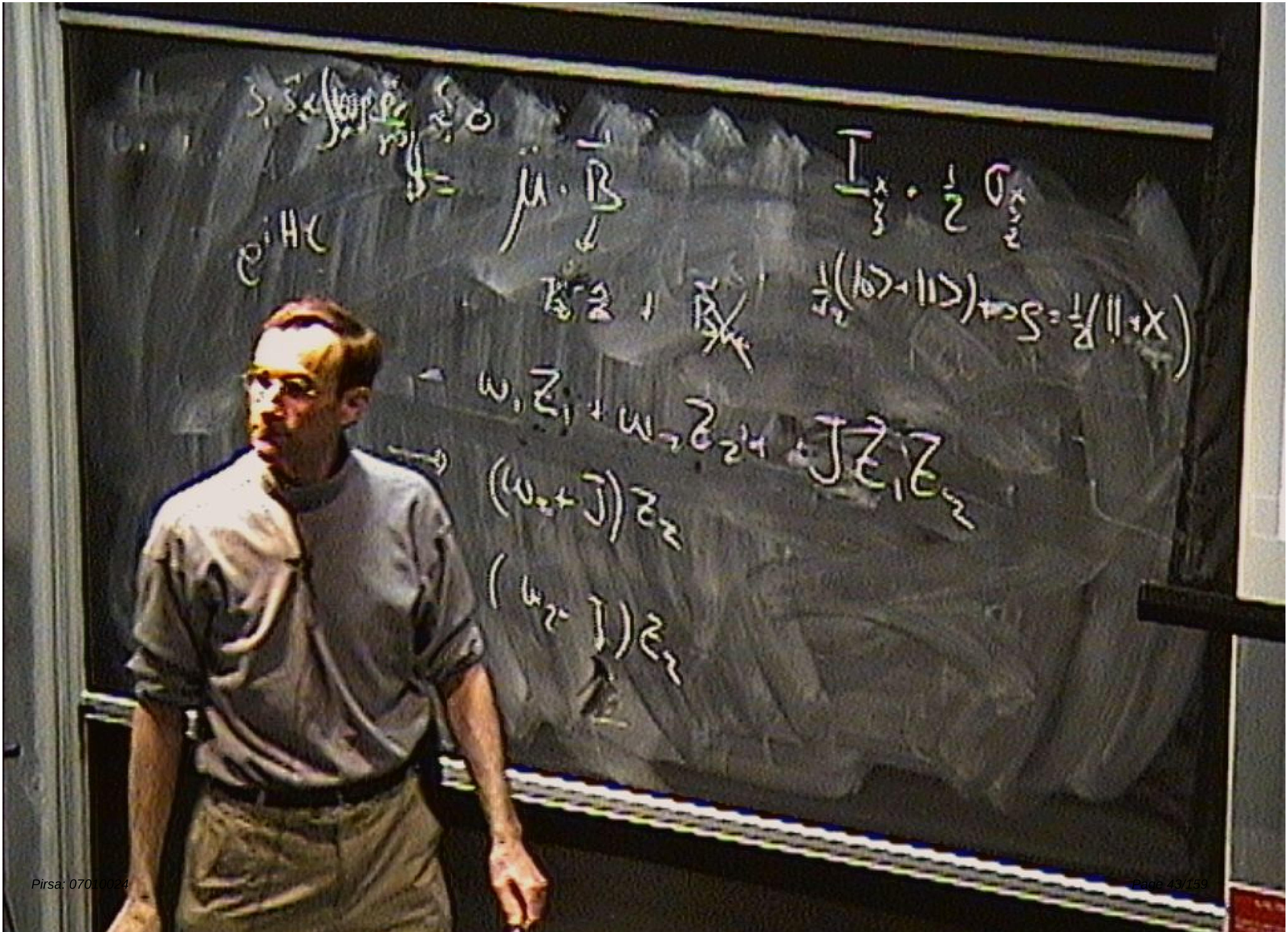
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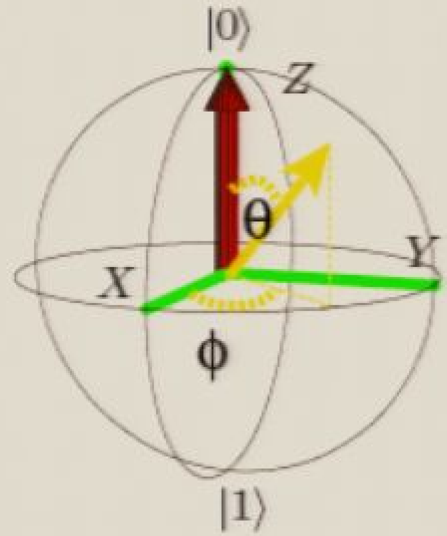

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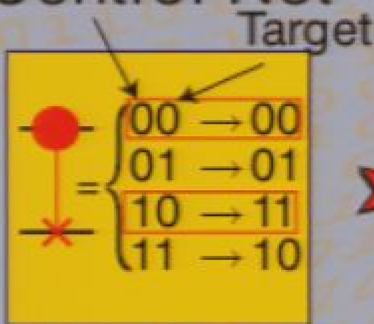
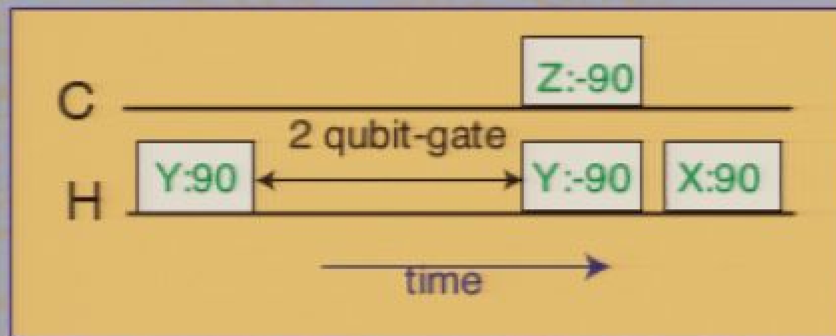
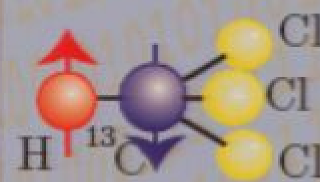
Quantum algorithms 101: the Control-NOT

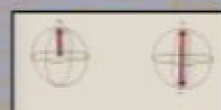
Control-Not



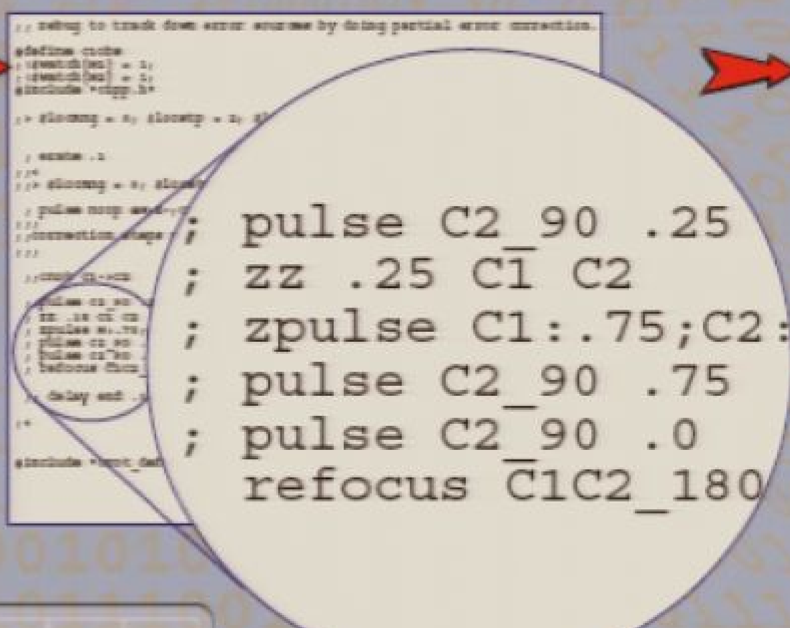
Quantum circuit



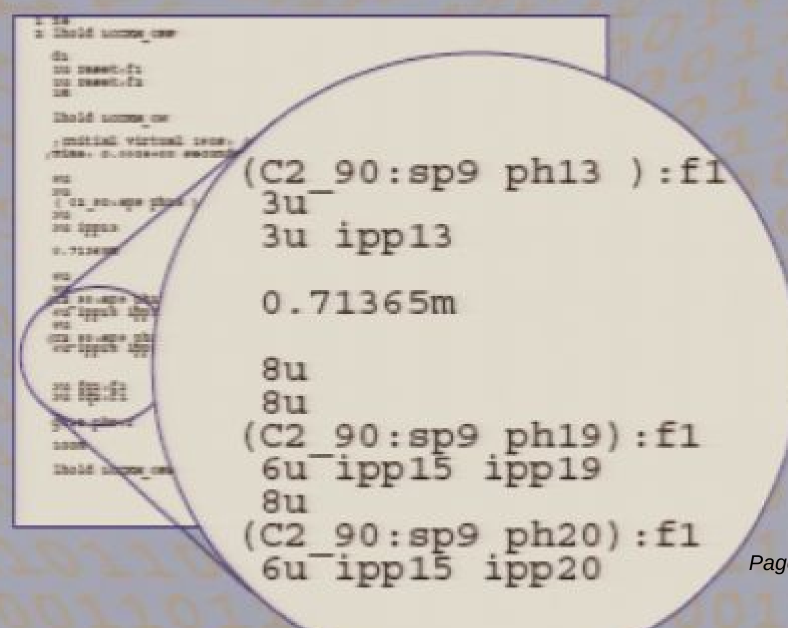


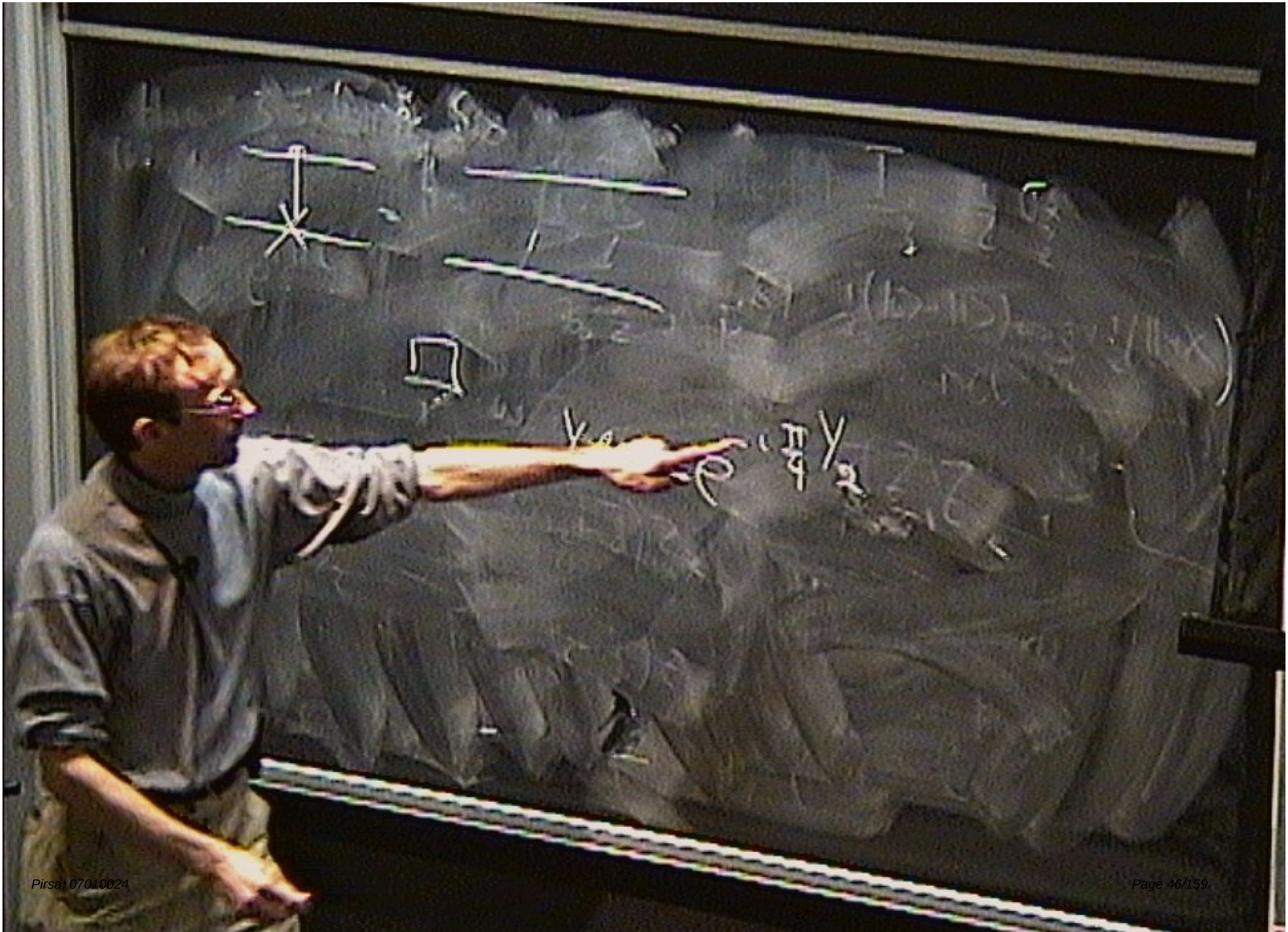


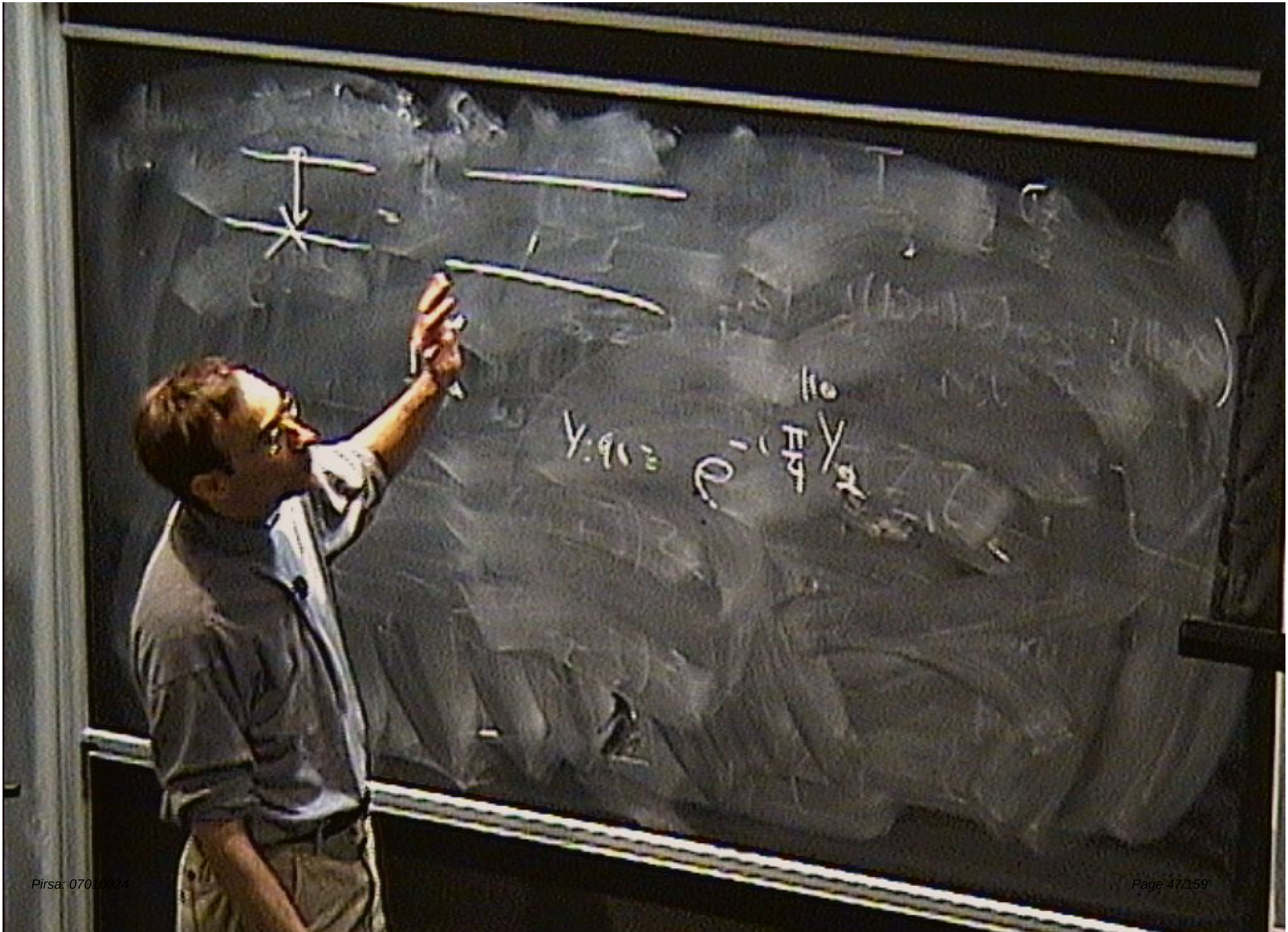
Pre-compiler (Optimizer)

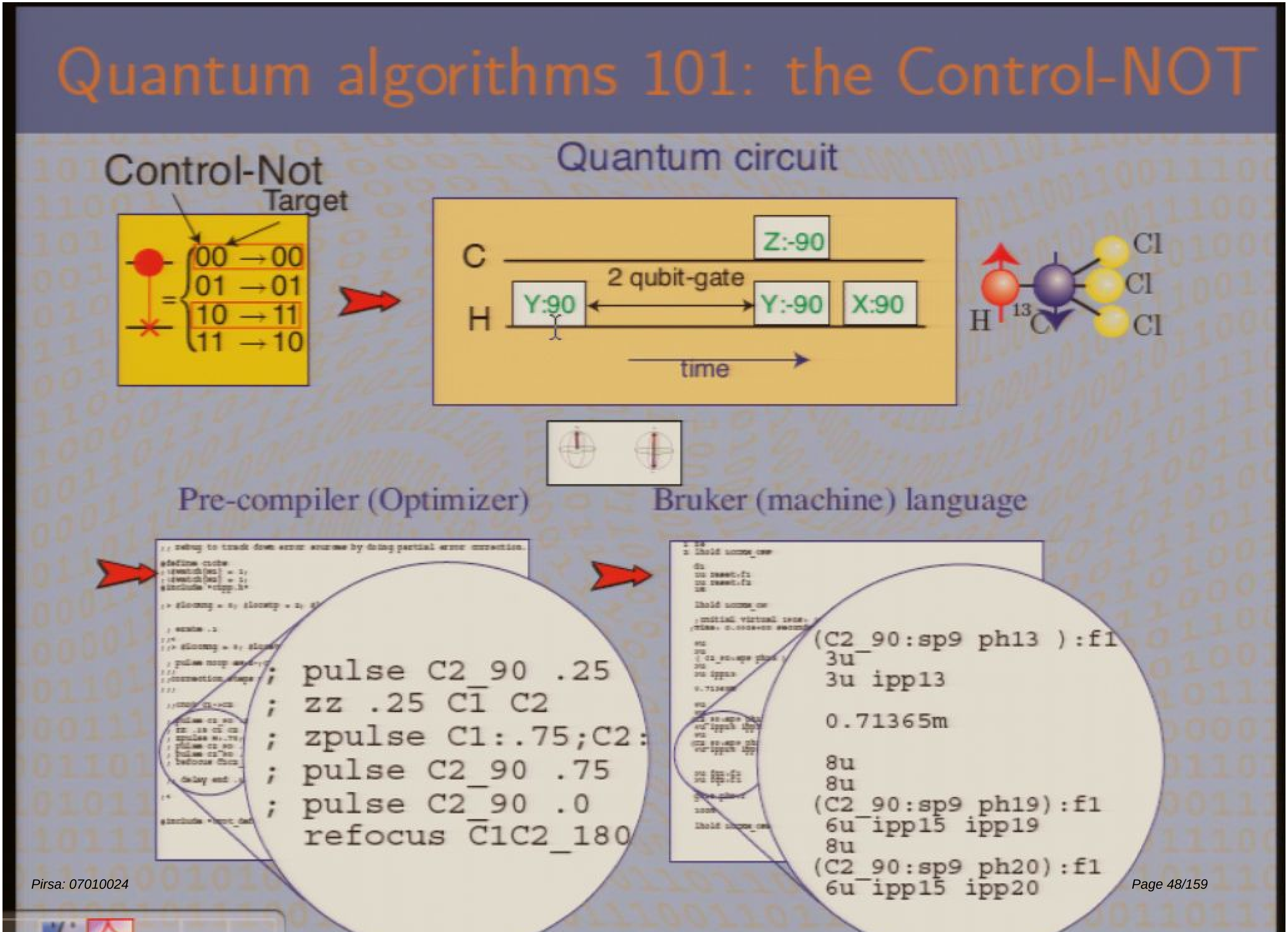


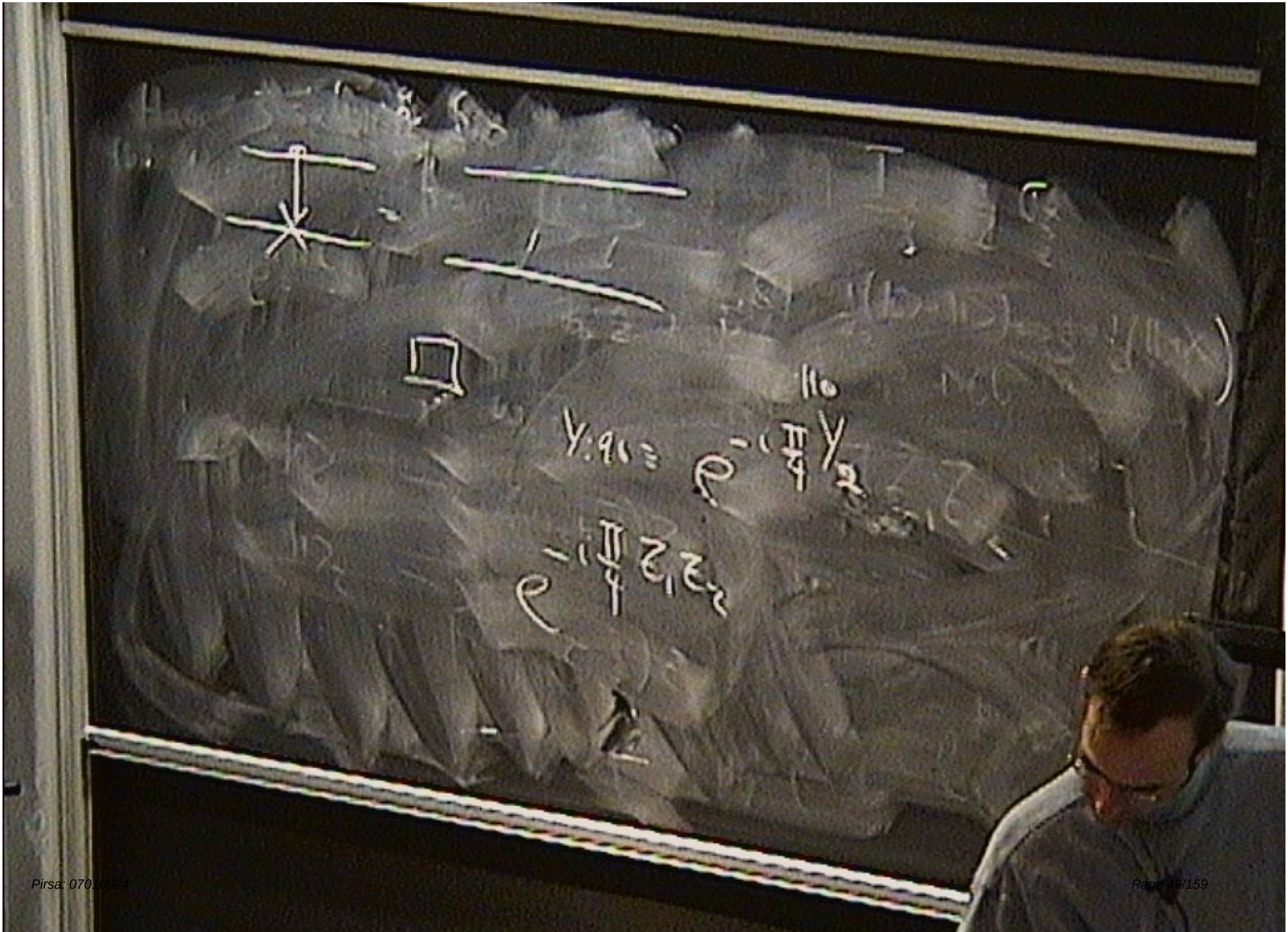
Bruker (machine) language



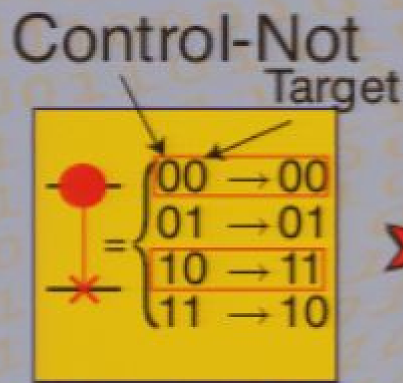




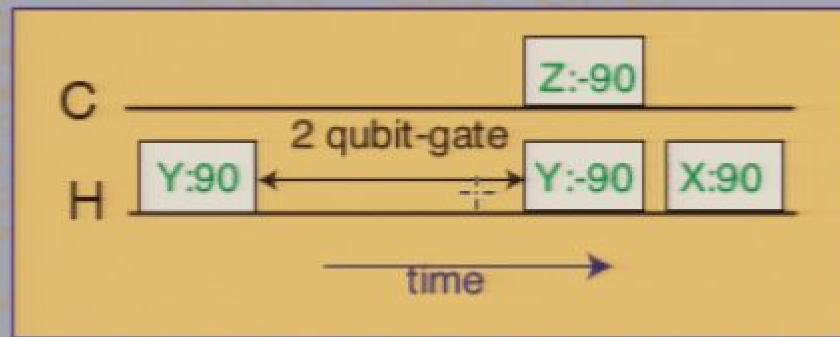

Quantum algorithms 101: the Control-NOT
Control-Not
Target
00 → 00
01 → 01
10 → 11
11 → 10
Quantum circuit
C
H
Z:-90
2 qubit-gate
Y:90
Y:-90
X:90
time
Cl
Cl
Cl
H
13C
Pre-compiler (Optimizer)
Bruker (machine) language
; pulse C2_90 .25
; zz .25 C1 C2
; zpulse C1:.75;C2:
; pulse C2_90 .75
; pulse C2_90 .0
refocus C1C2_180
(C2_90:sp9 ph13):f1
3u
3u ipp13
0.71365m
8u
8u
(C2_90:sp9 ph19):f1
6u ipp15 ipp19
8u
(C2_90:sp9 ph20):f1
6u ipp15 ipp20

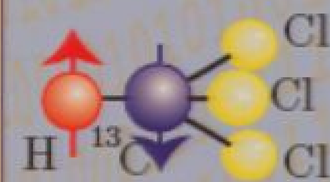


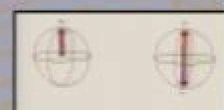
Quantum algorithms 101: the Control-NOT



Quantum circuit





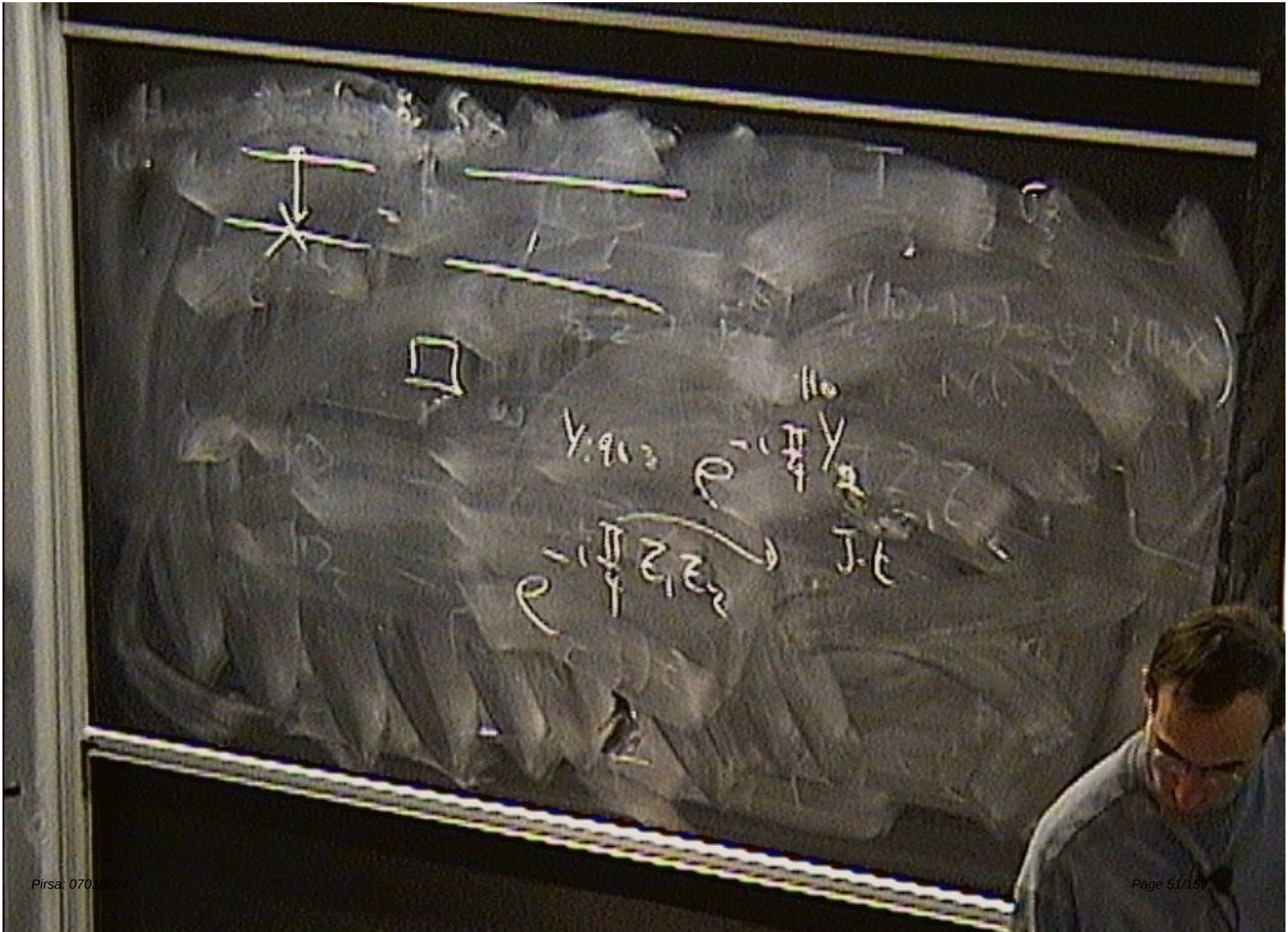


Pre-compiler (Optimizer)

```
; pulse C2_90 .25
; zz .25 C1 C2
; zpulse C1:.75;C2:
; pulse C2_90 .75
; pulse C2_90 .0
  refocus C1C2_180
```

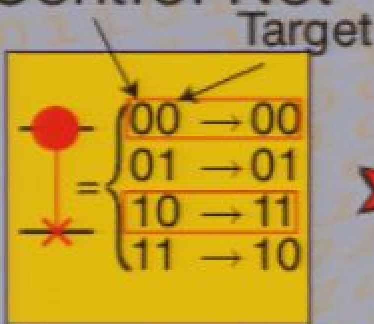
Bruker (machine) language

```
(C2_90:sp9 ph13 ):f1
 3u
 3u ipp13

 0.71365m

 8u
 8u
(C2_90:sp9 ph19):f1
 6u ipp15 ipp19
 8u
(C2_90:sp9 ph20):f1
 6u ipp15 ipp20
```

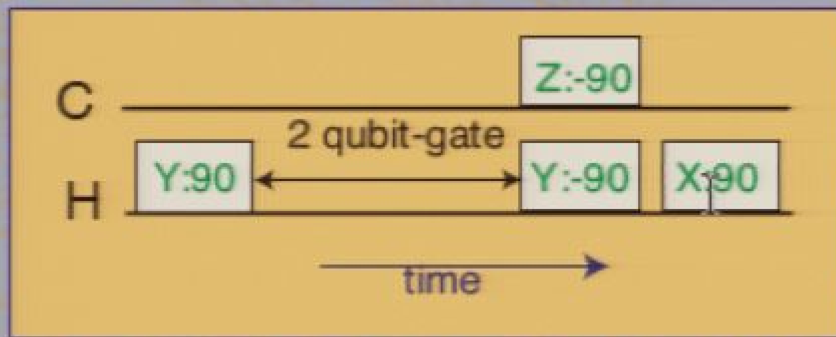


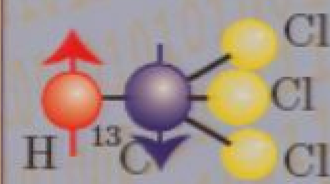
Quantum algorithms 101: the Control-NOT

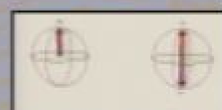
Control-Not



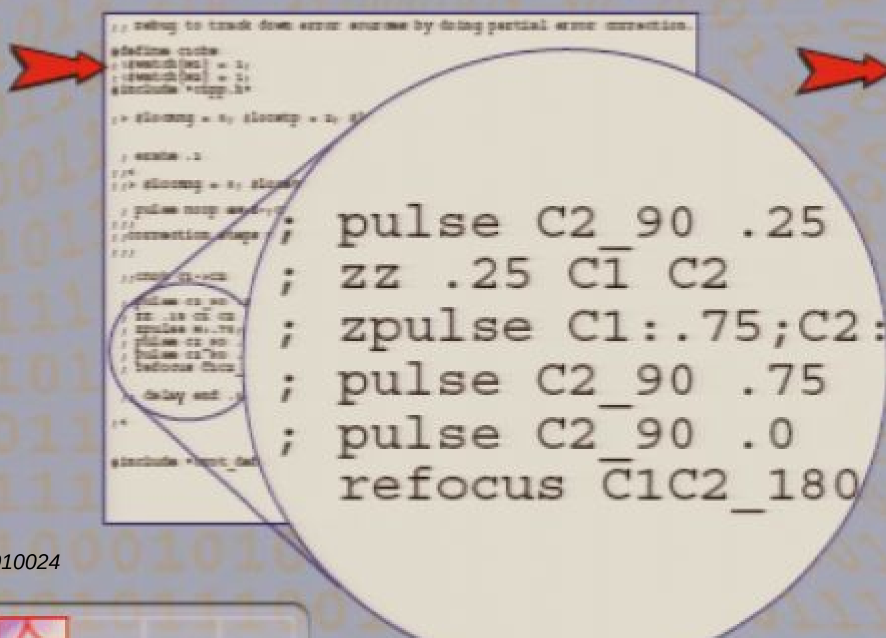
Quantum circuit







Pre-compiler (Optimizer)



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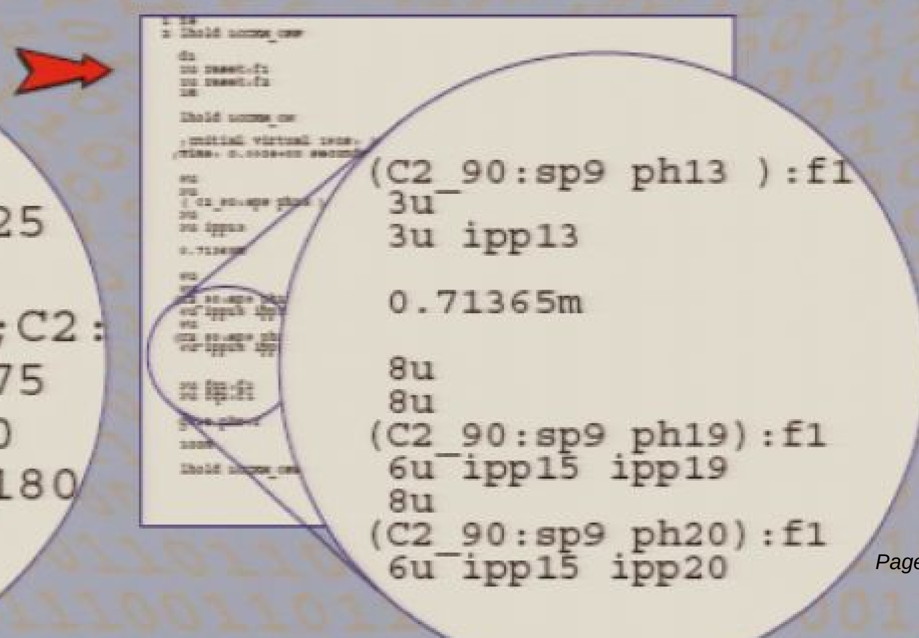
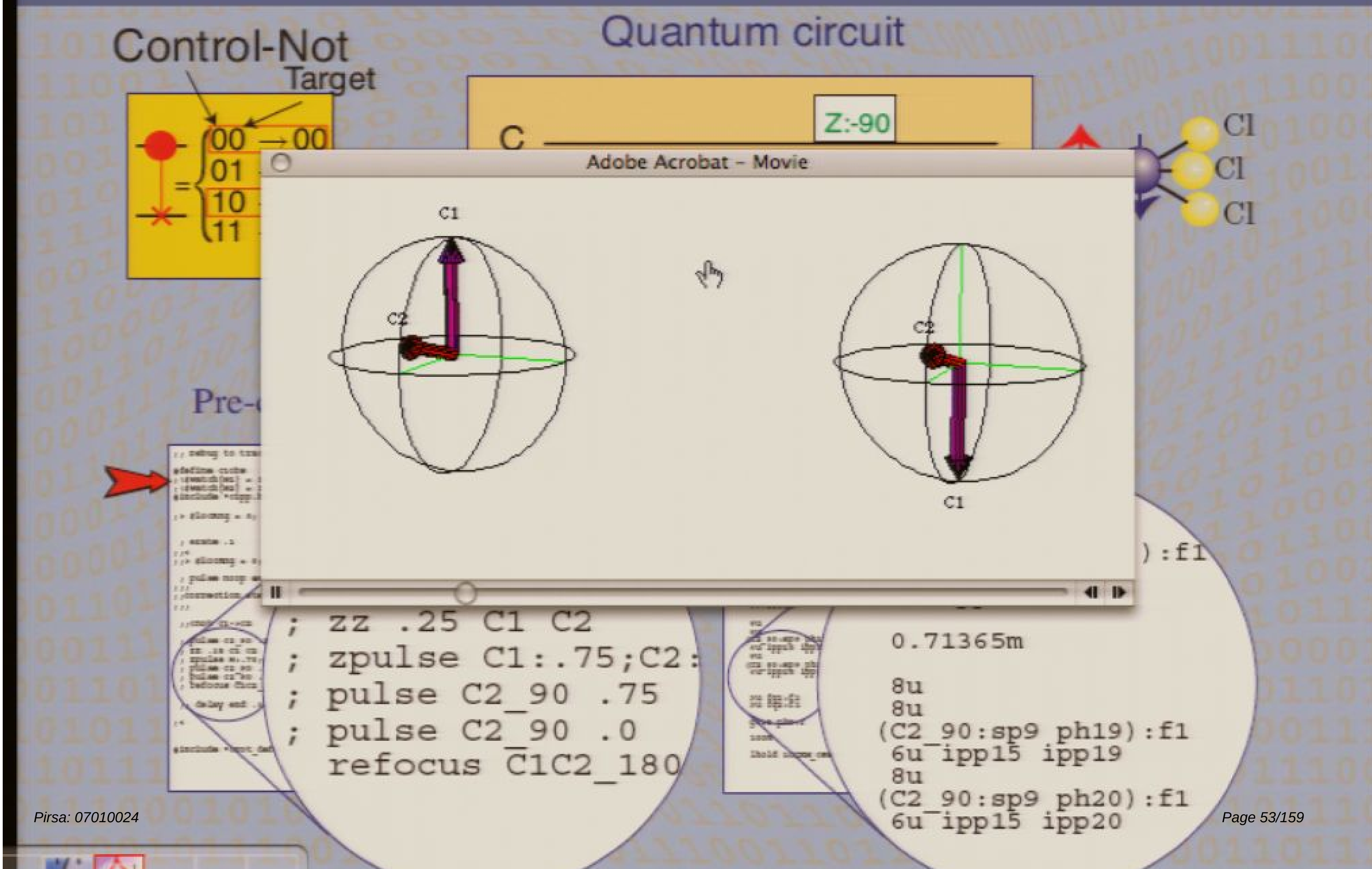


Quantum algorithms 101: the Control-NOT



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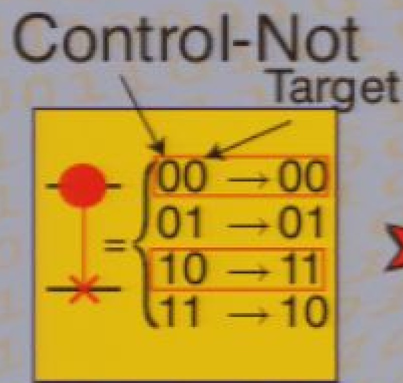

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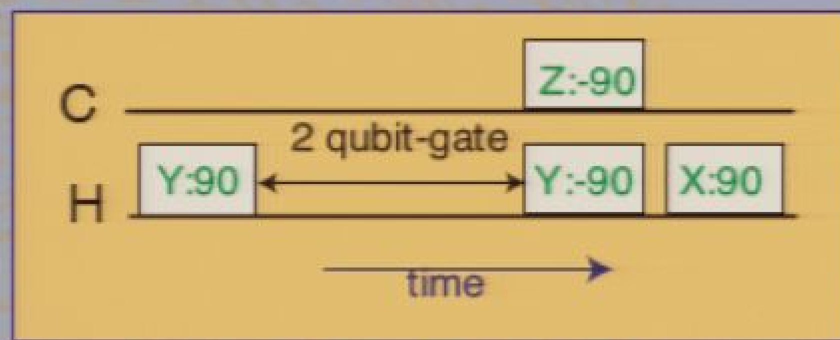

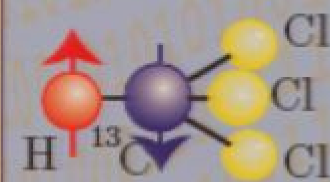

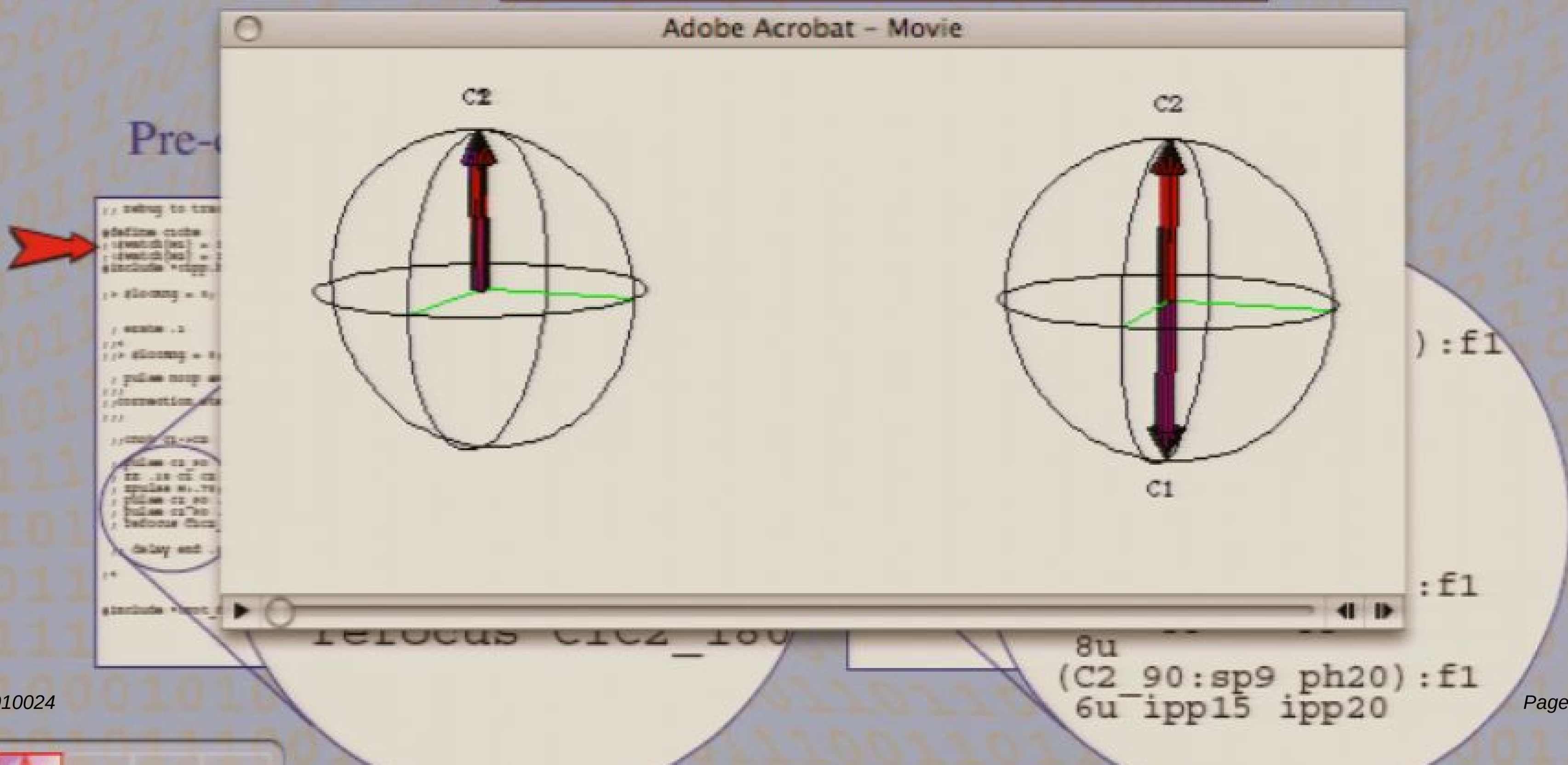

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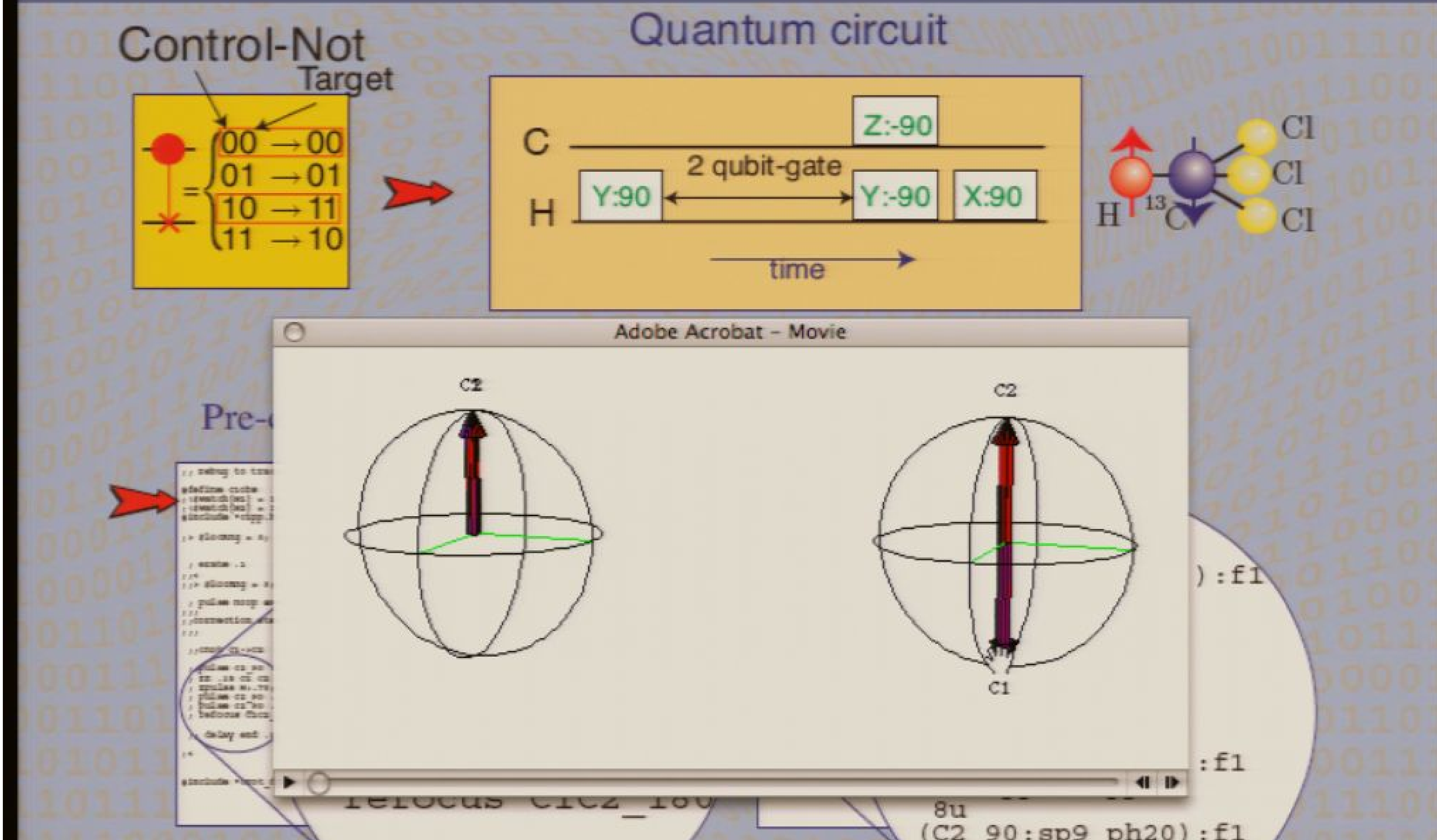

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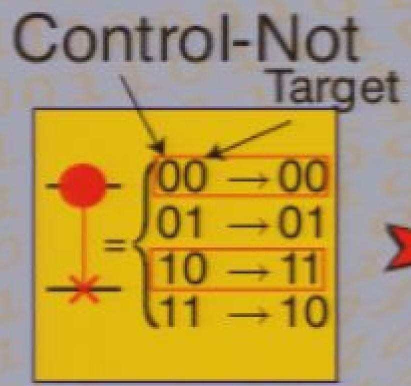

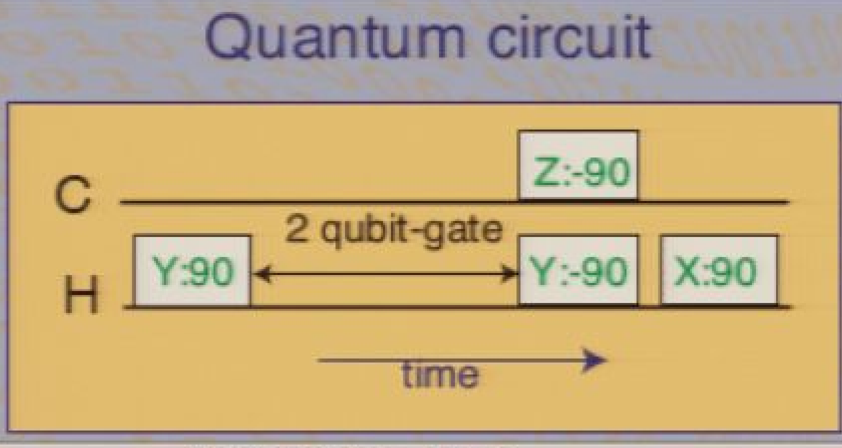

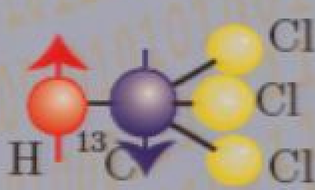

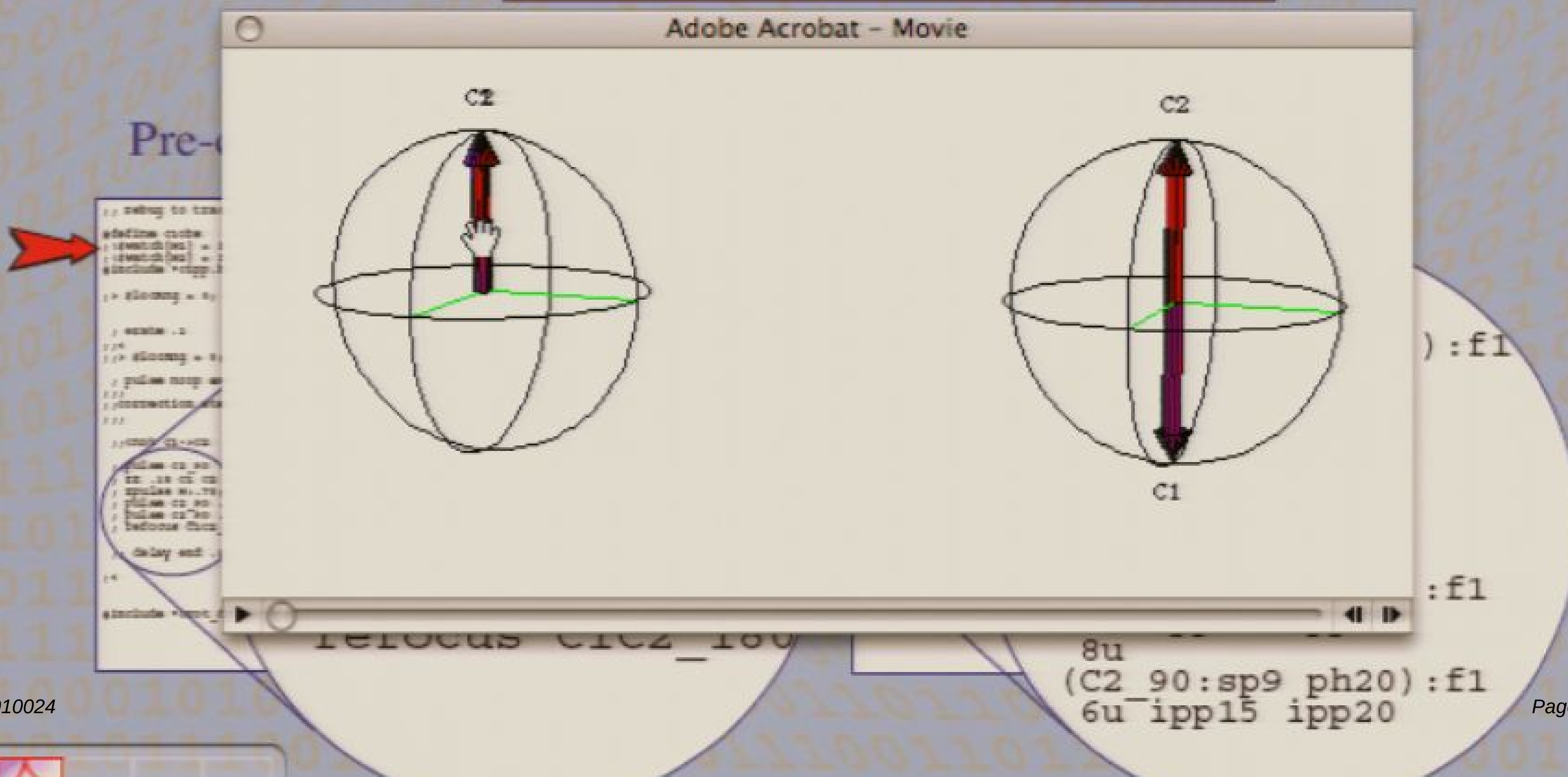

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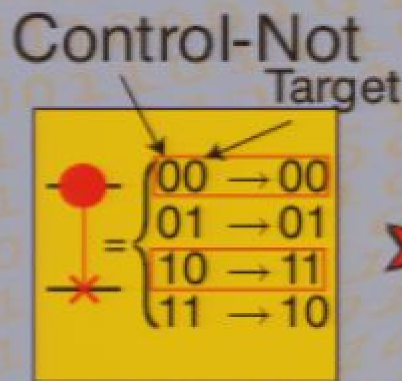
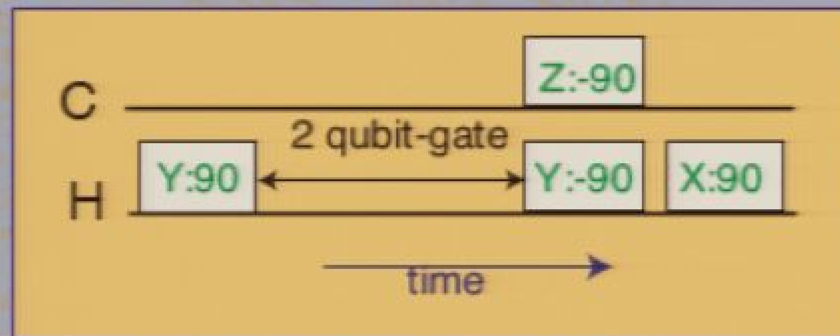
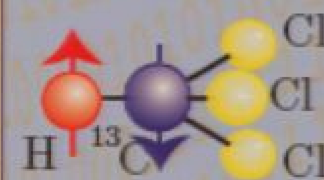
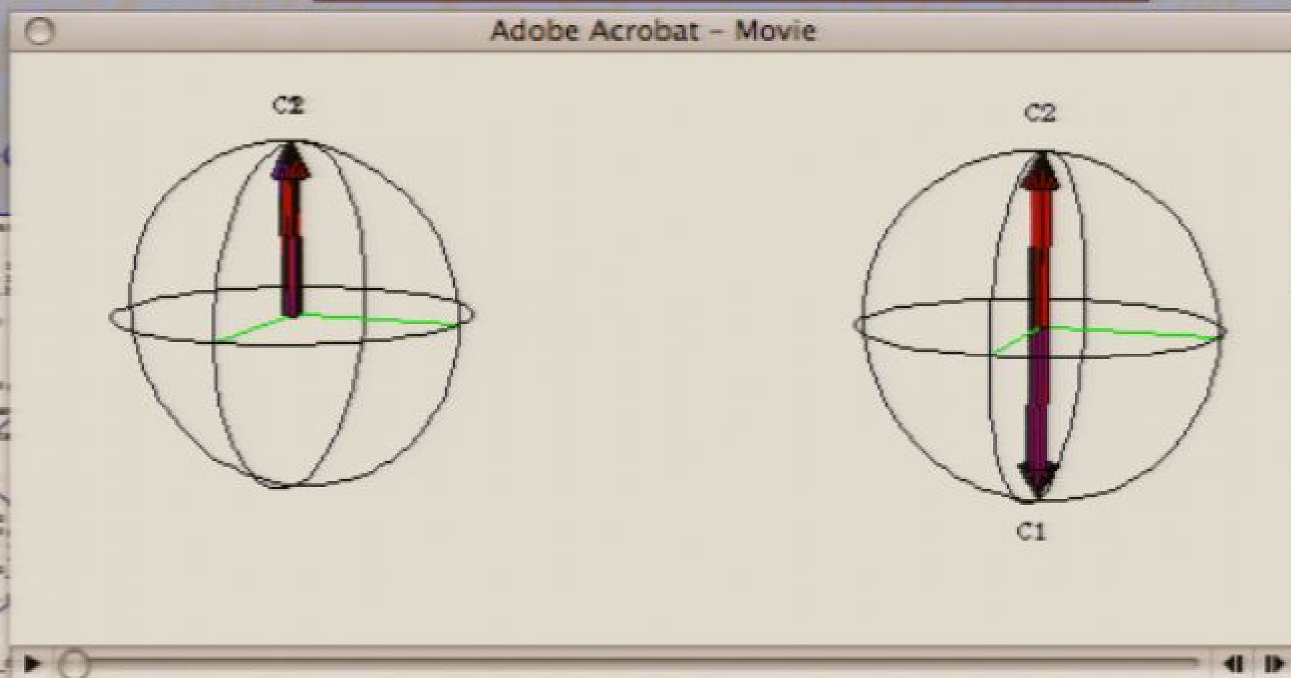


Quantum circuit

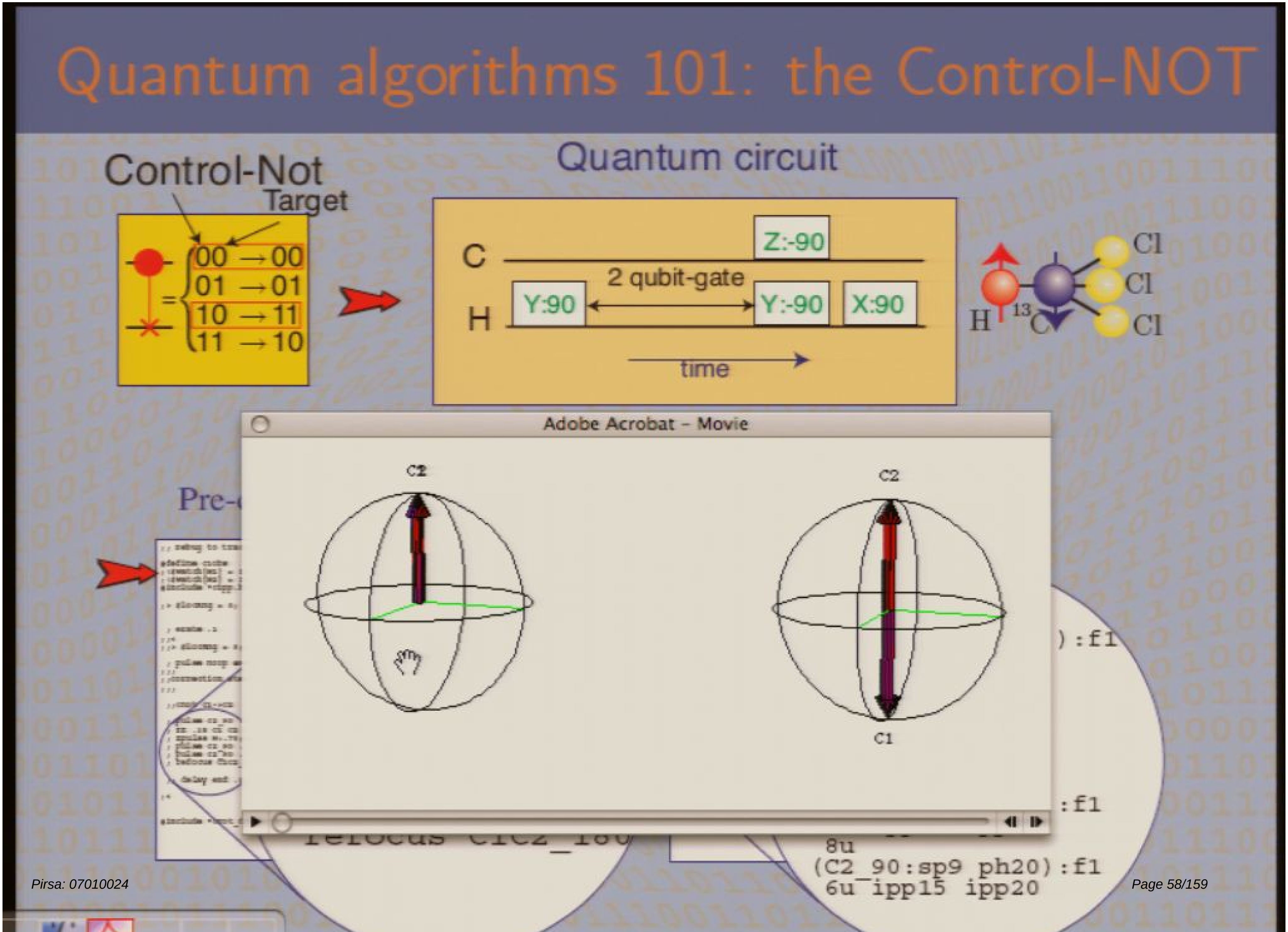


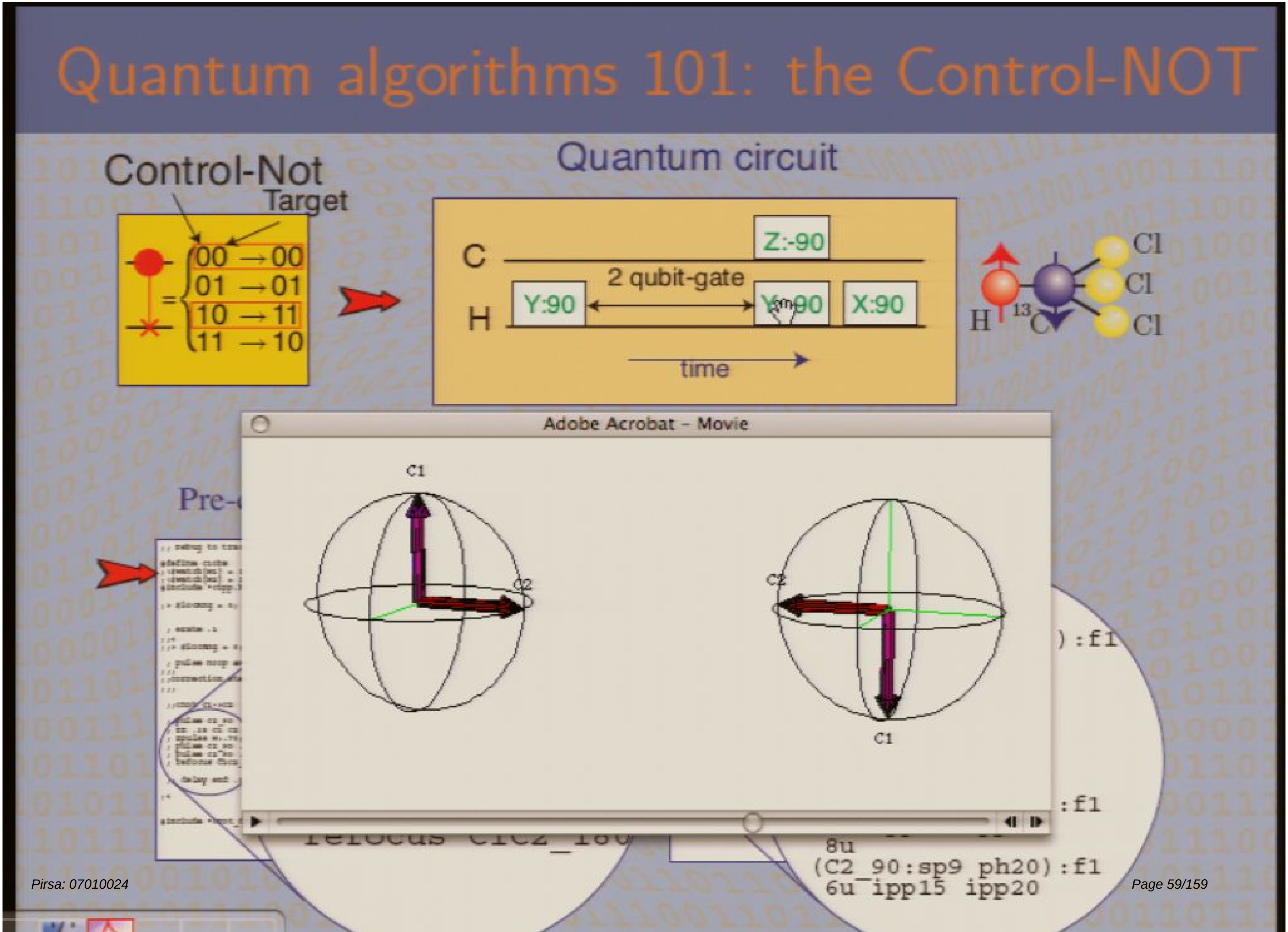


Pre-c




```
):f1
:f1
refocus c1c2_180
8u
(C2_90:sp9 ph20):f1
6u ipp15 ipp20
```


Quantum algorithms 101: the Control-NOT
Control-Not
Target
00 → 00
01 → 01
10 → 11
11 → 10
Quantum circuit
C
H
Z:-90
2 qubit-gate
Y:90
Y:-90
X:90
time
Cl
Cl
Cl
H
13C
Adobe Acrobat – Movie
C2
C2
C1
Pre-
):f1
:f1
8u
(C2_90:sp9 ph20):f1
6u ipp15 ipp20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C1
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C2
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Pre-
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:f1
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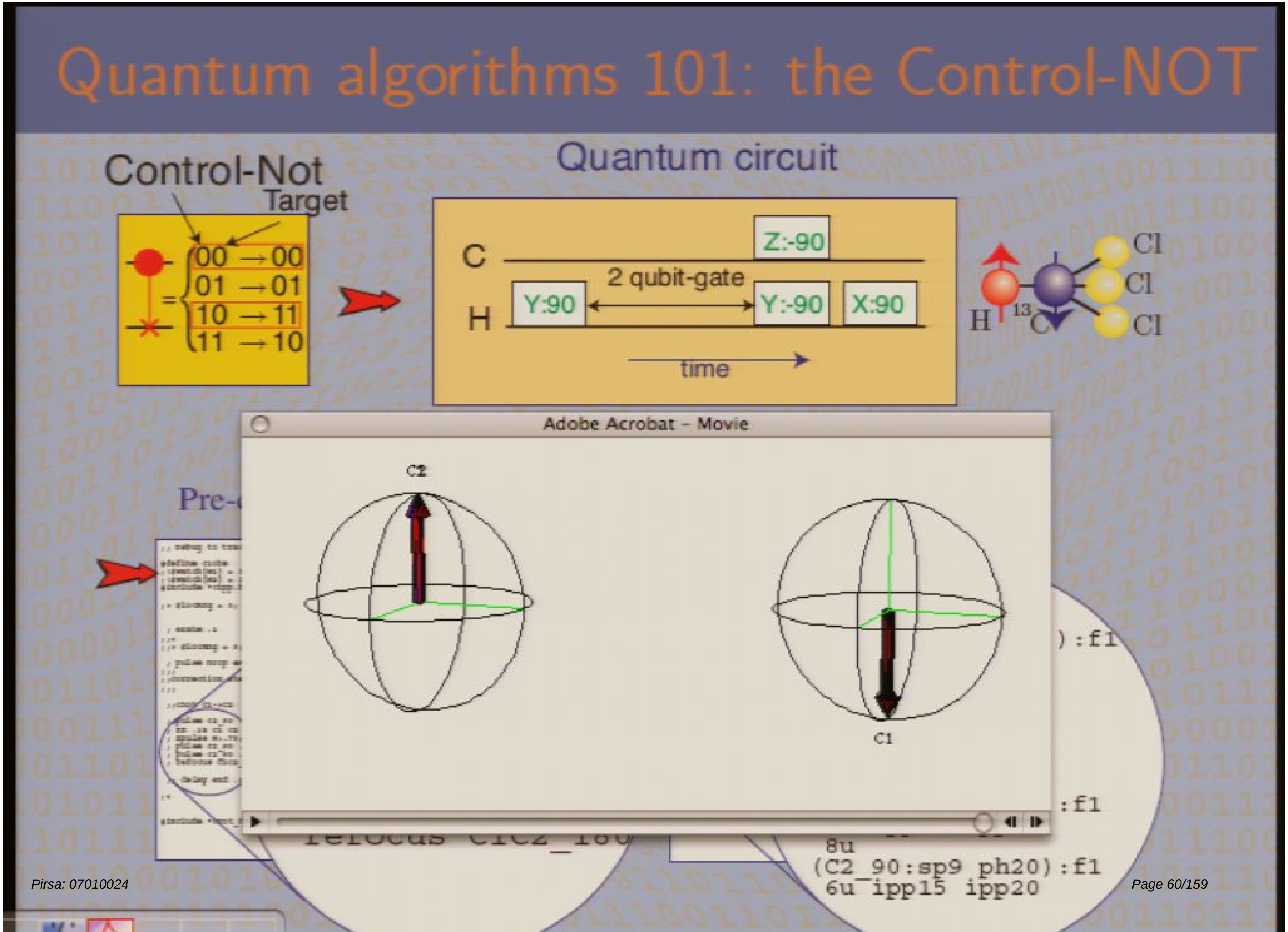

Quantum algorithms 101: the Control-NOT
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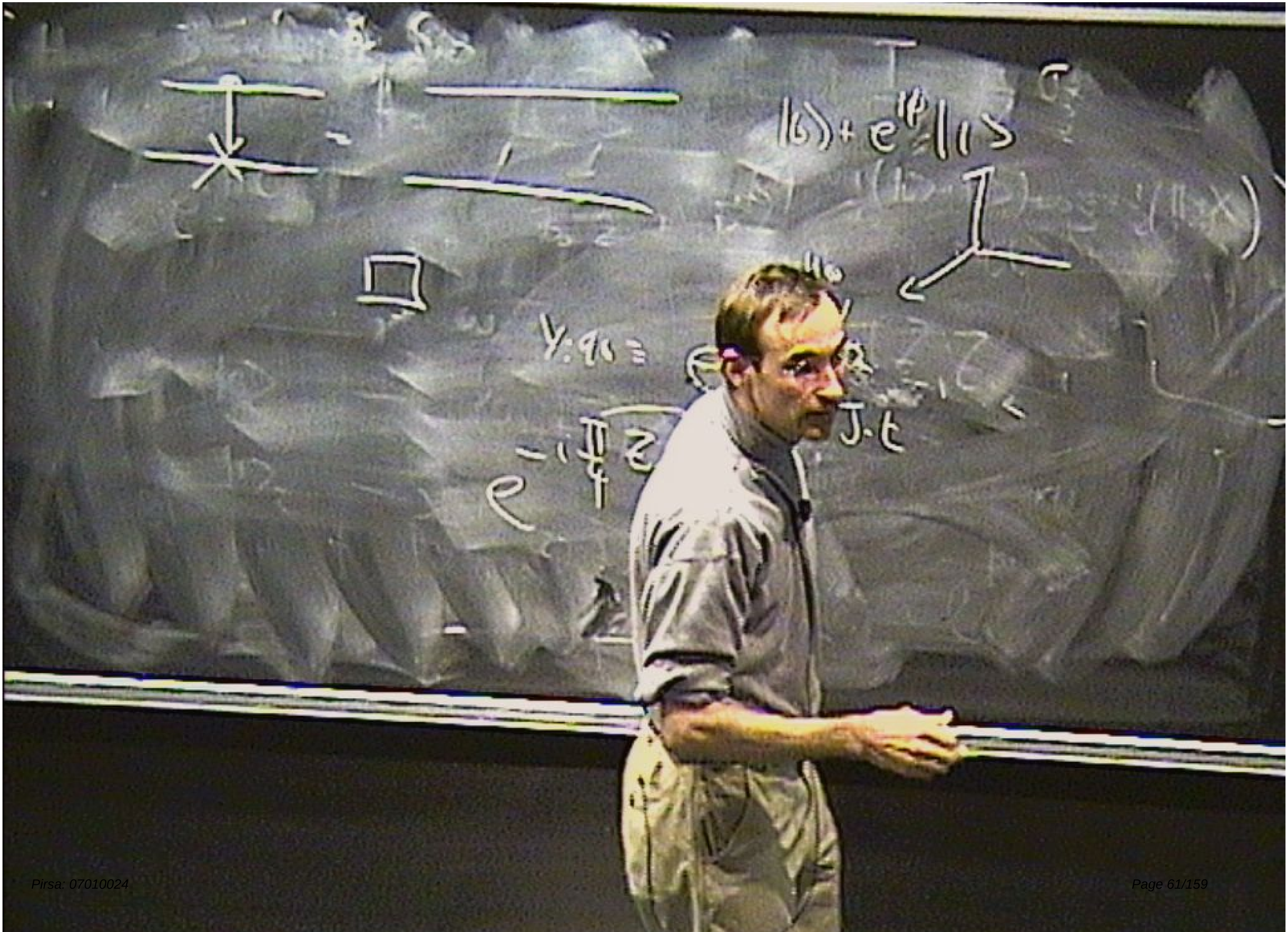
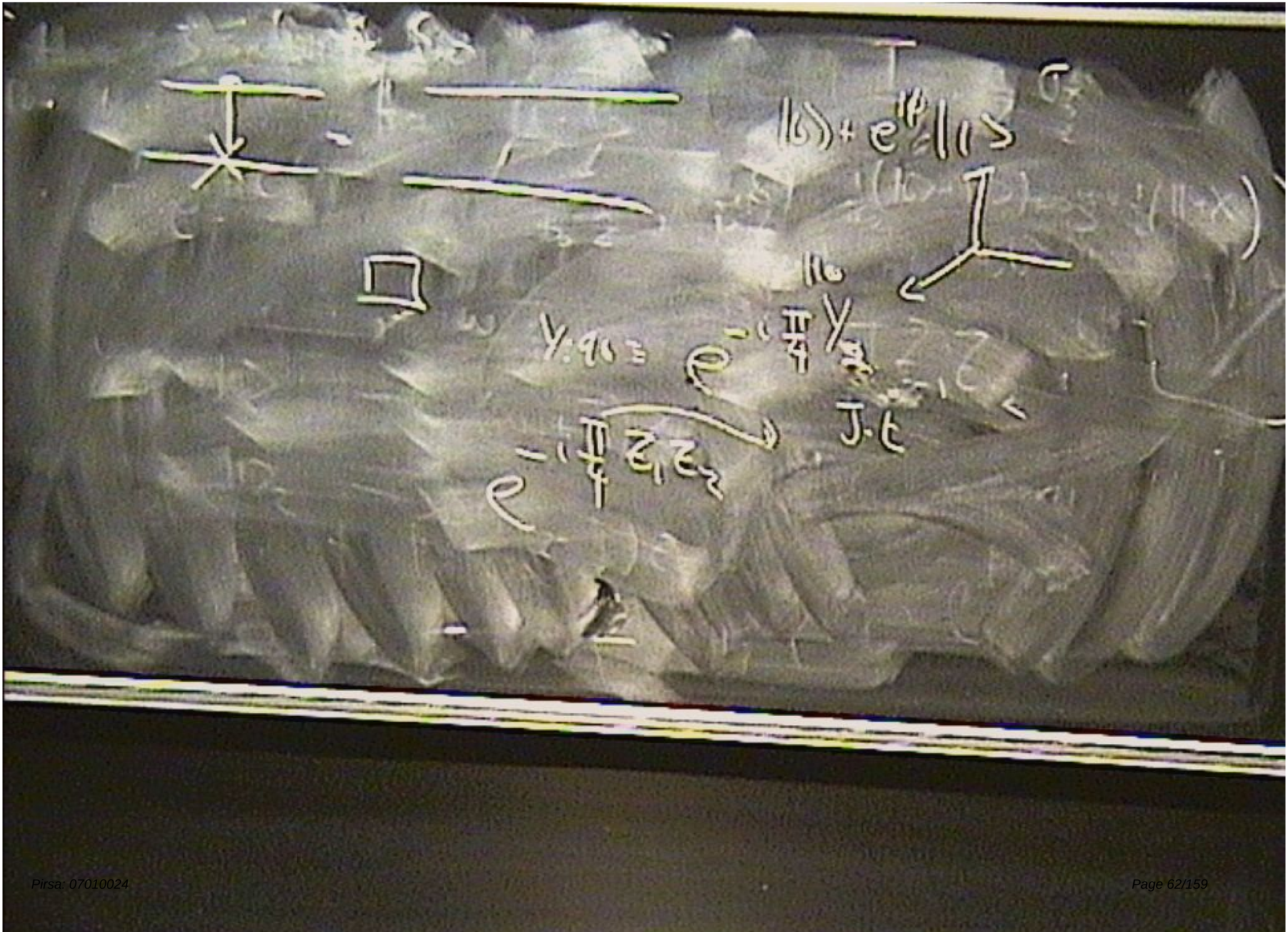
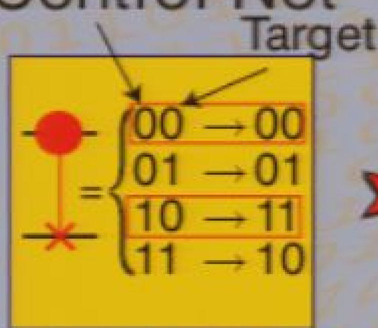
time
H
13C
Cl
Cl
Cl
Pre-c
Adobe Acrobat – Movie
C2
C1
):f1
:f1
8u
(C2_90:sp9 ph20):f1
6u ipp15 ipp20



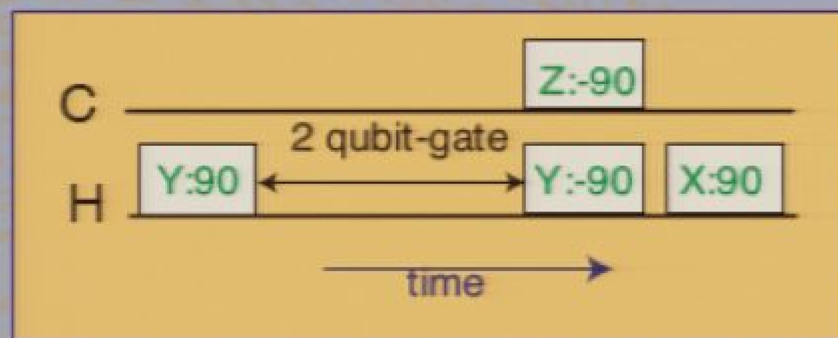

|0⟩+e^{iφ}|1⟩
|0⟩
Y:90° = e^{-i π/4 Y_2}
e^{-i π/4 Z_1 Z_2}
J·t

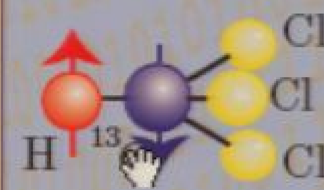
Quantum algorithms 101: the Control-NOT

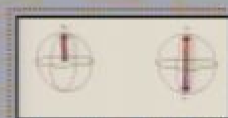
Control-Not



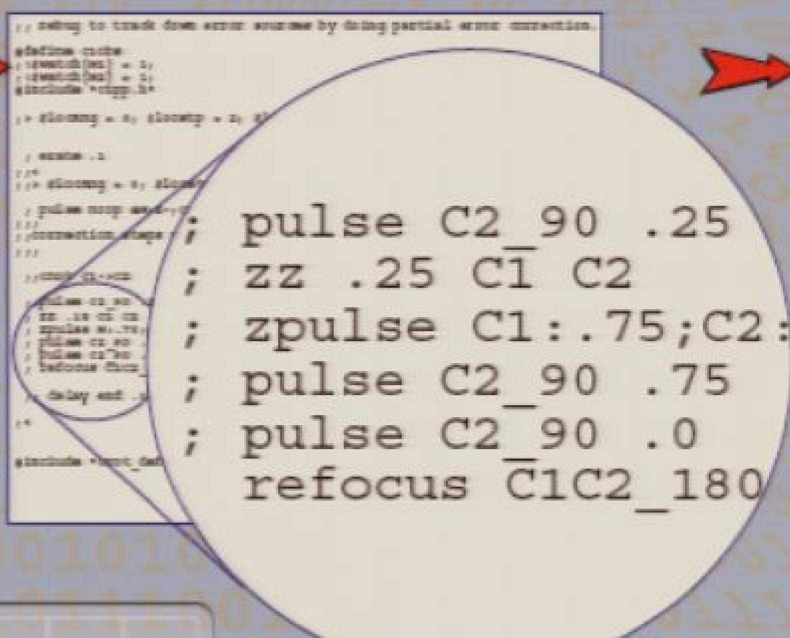
Quantum circuit



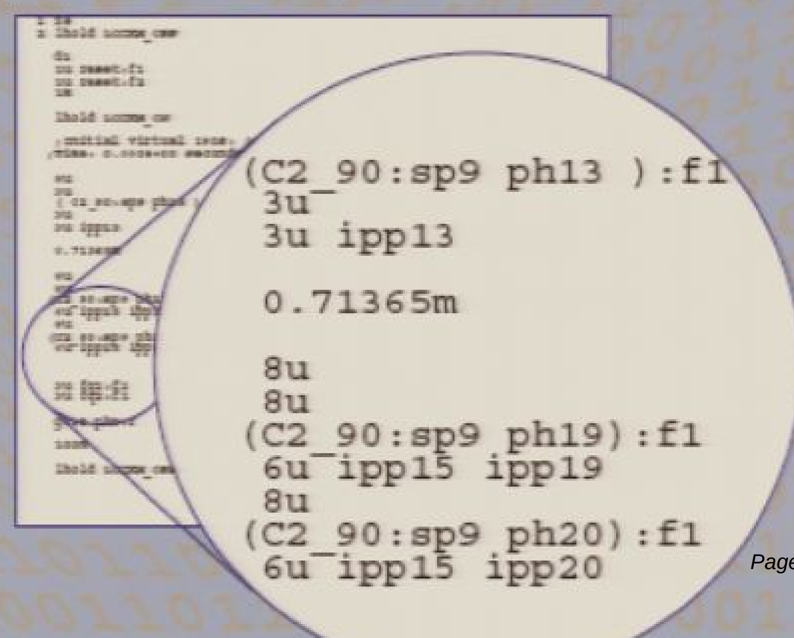




Pre-compiler (Optimizer)



Bruker (machine) language

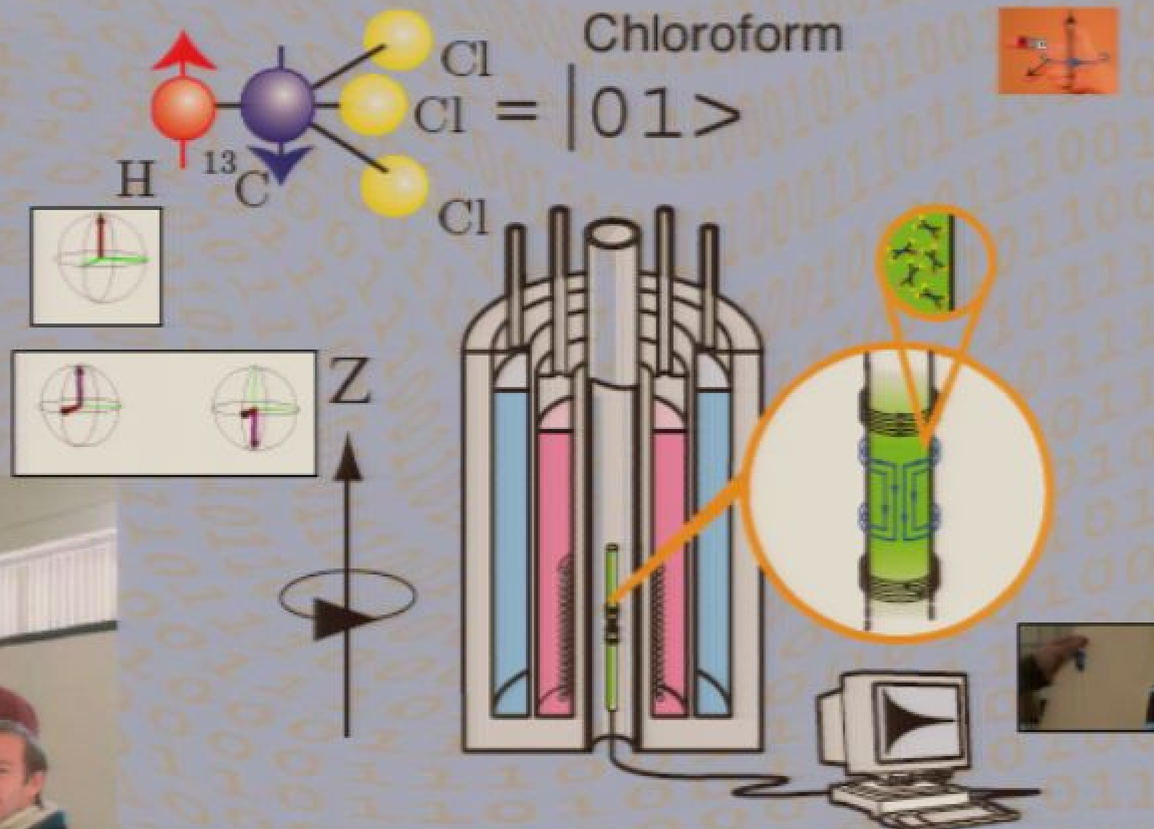


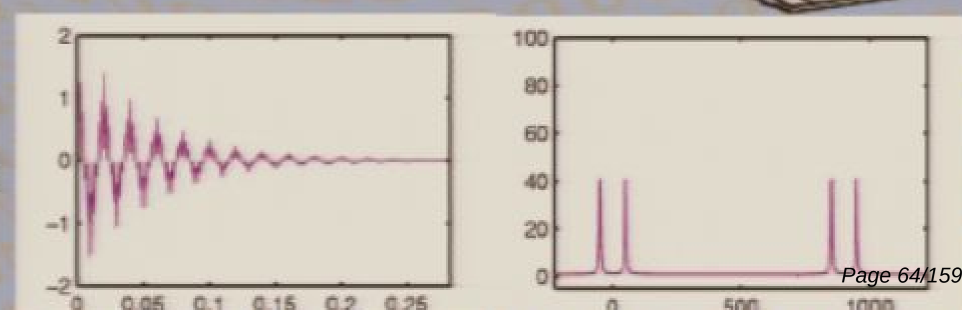
Nuclear Magnetic Resonance

Bloch

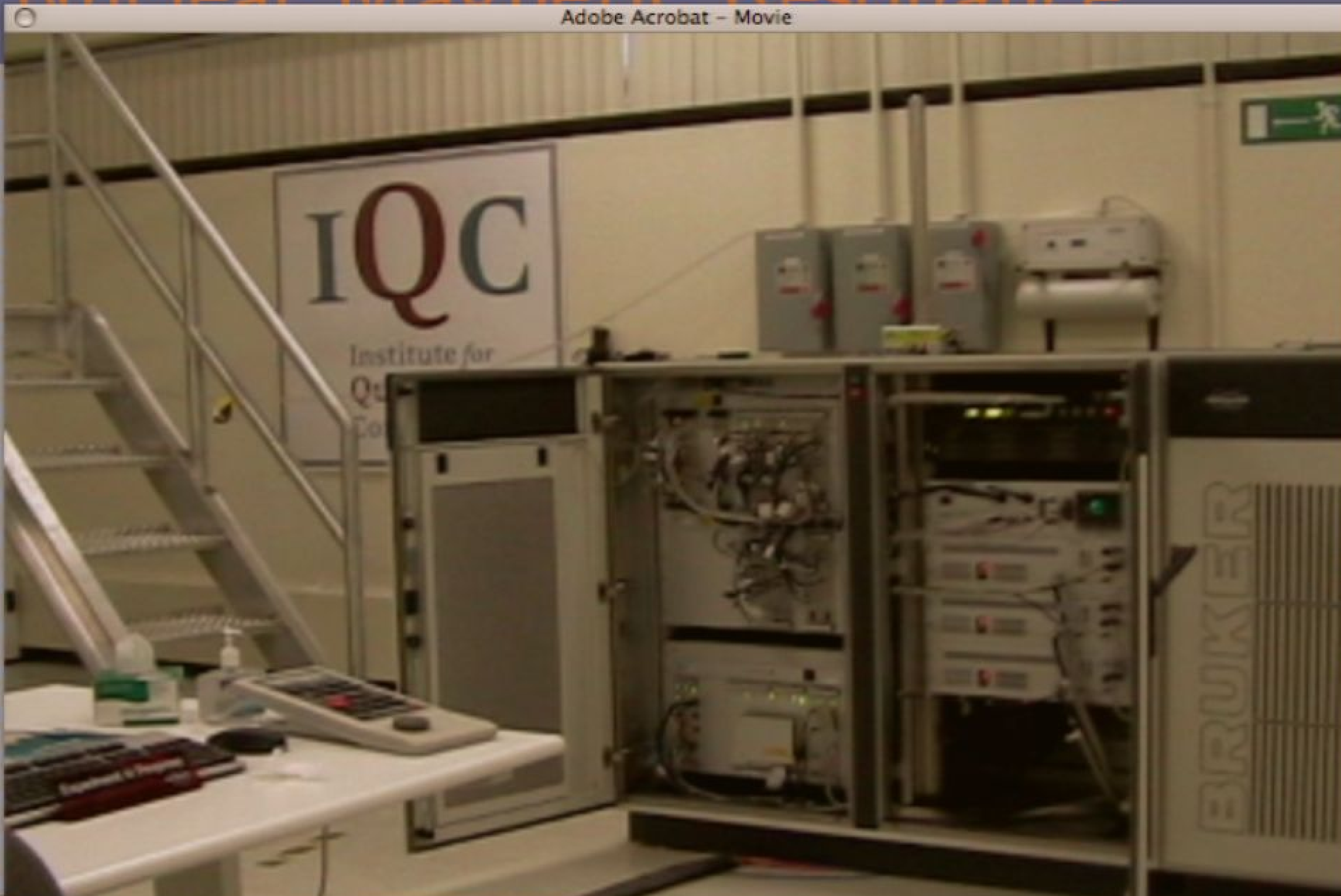
Purcell

Bruker 700





Nuclear Magnetic Resonance



Bruker 700

Nuclear Magnetic Resonance

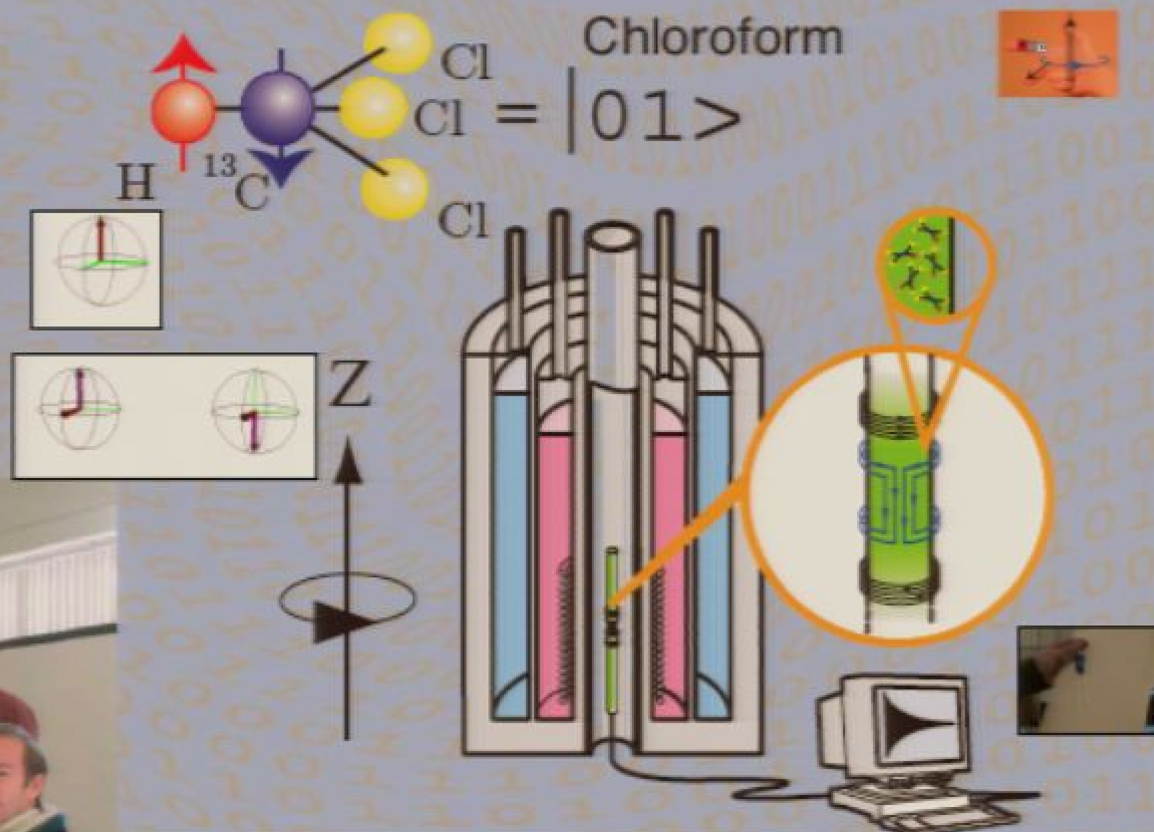
Adobe Acrobat – Movie

Bruker 700

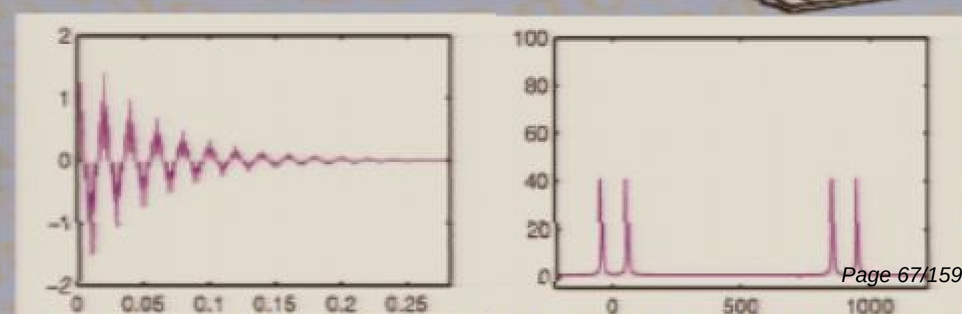
Nuclear Magnetic Resonance

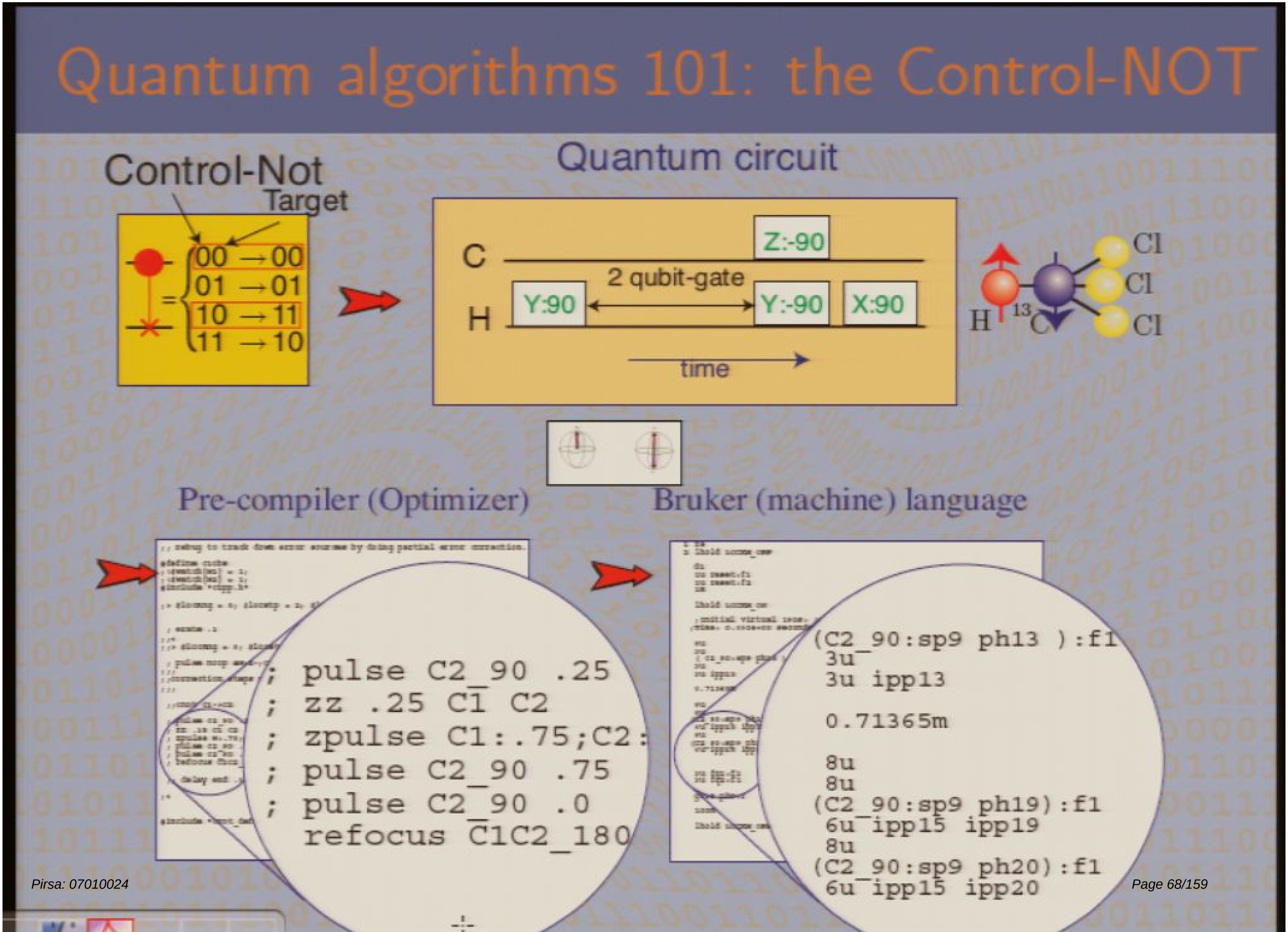
Bloch

Purcell



Bruker 700

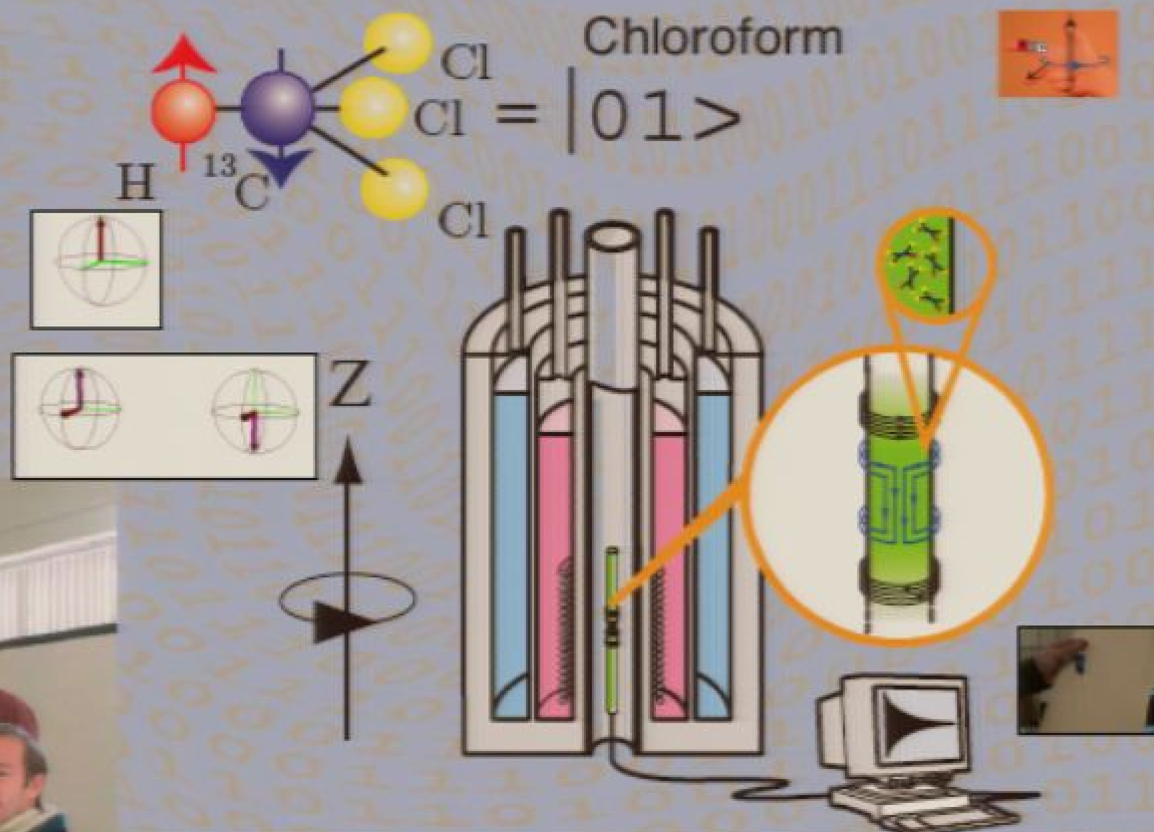



Quantum algorithms 101: the Control-NOT
Control-Not
Target
00 → 00
01 → 01
10 → 11
11 → 10
Quantum circuit
C
H
Z:-90
2 qubit-gate
Y:90
Y:-90
X:90
time
Cl
Cl
Cl
H
13C
Pre-compiler (Optimizer)
Bruker (machine) language
; pulse C2_90 .25
; zz .25 C1 C2
; zpulse C1:.75;C2:
; pulse C2_90 .75
; pulse C2_90 .0
refocus C1C2_180
(C2_90:sp9 ph13):f1
3u
3u ipp13
0.71365m
8u
8u
(C2_90:sp9 ph19):f1
6u ipp15 ipp19
8u
(C2_90:sp9 ph20):f1
6u ipp15 ipp20

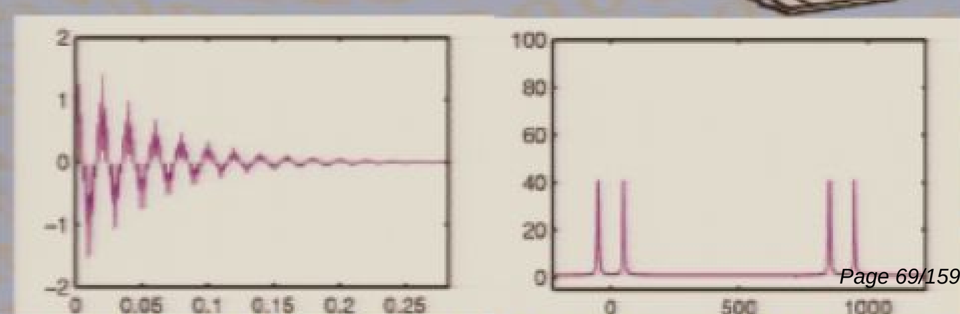
Nuclear Magnetic Resonance

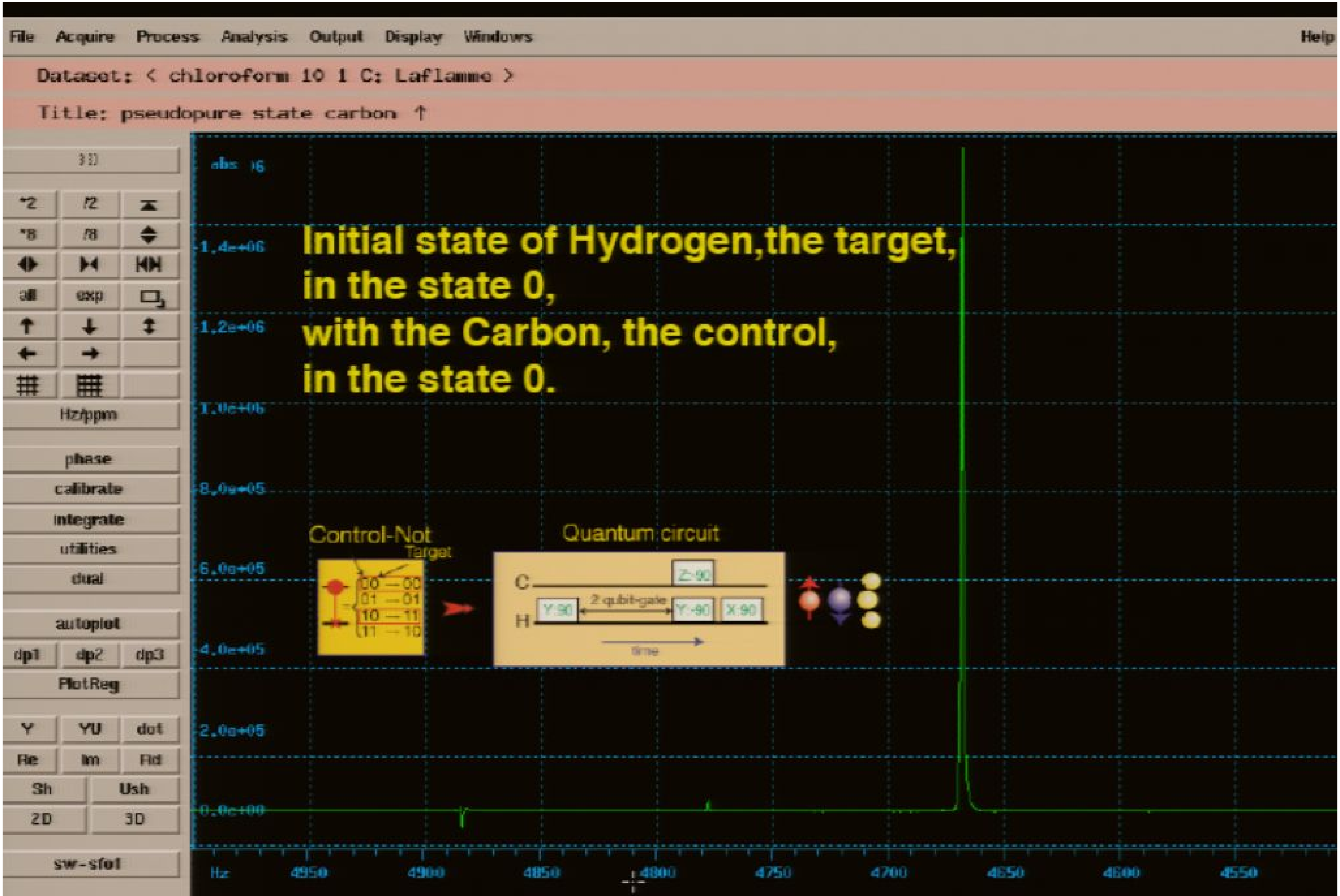
Bloch

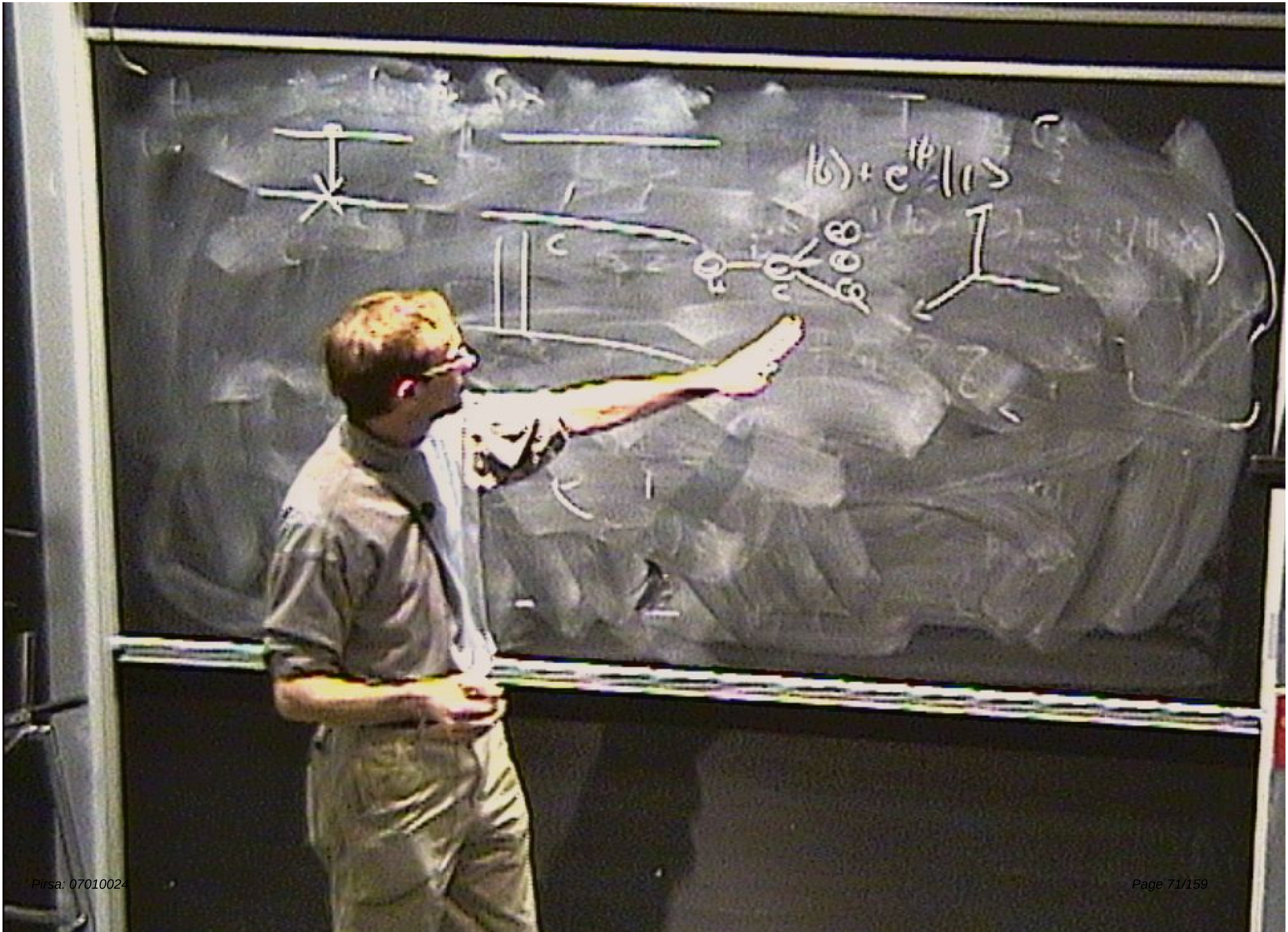
Purcell

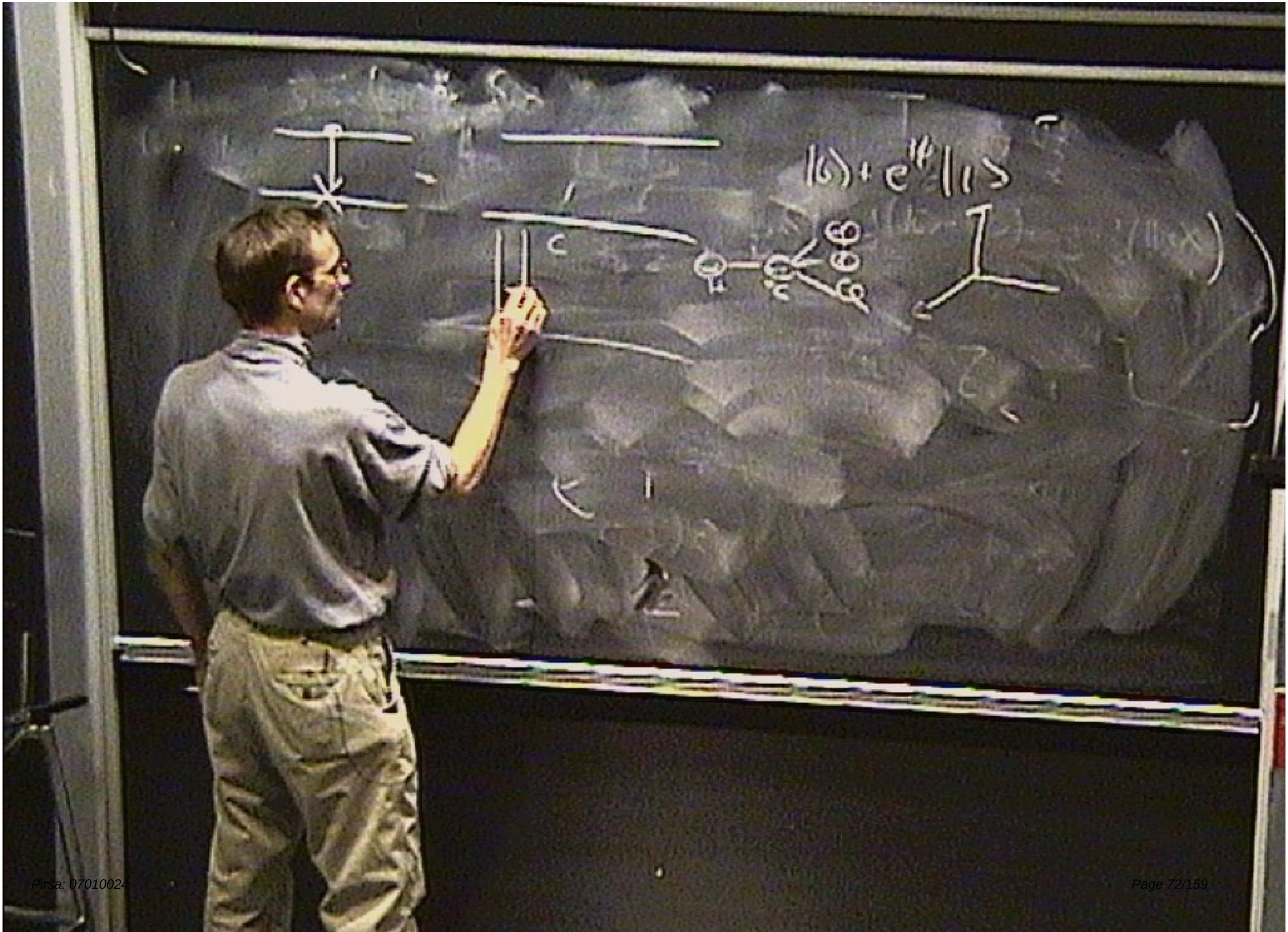


Bruker 700

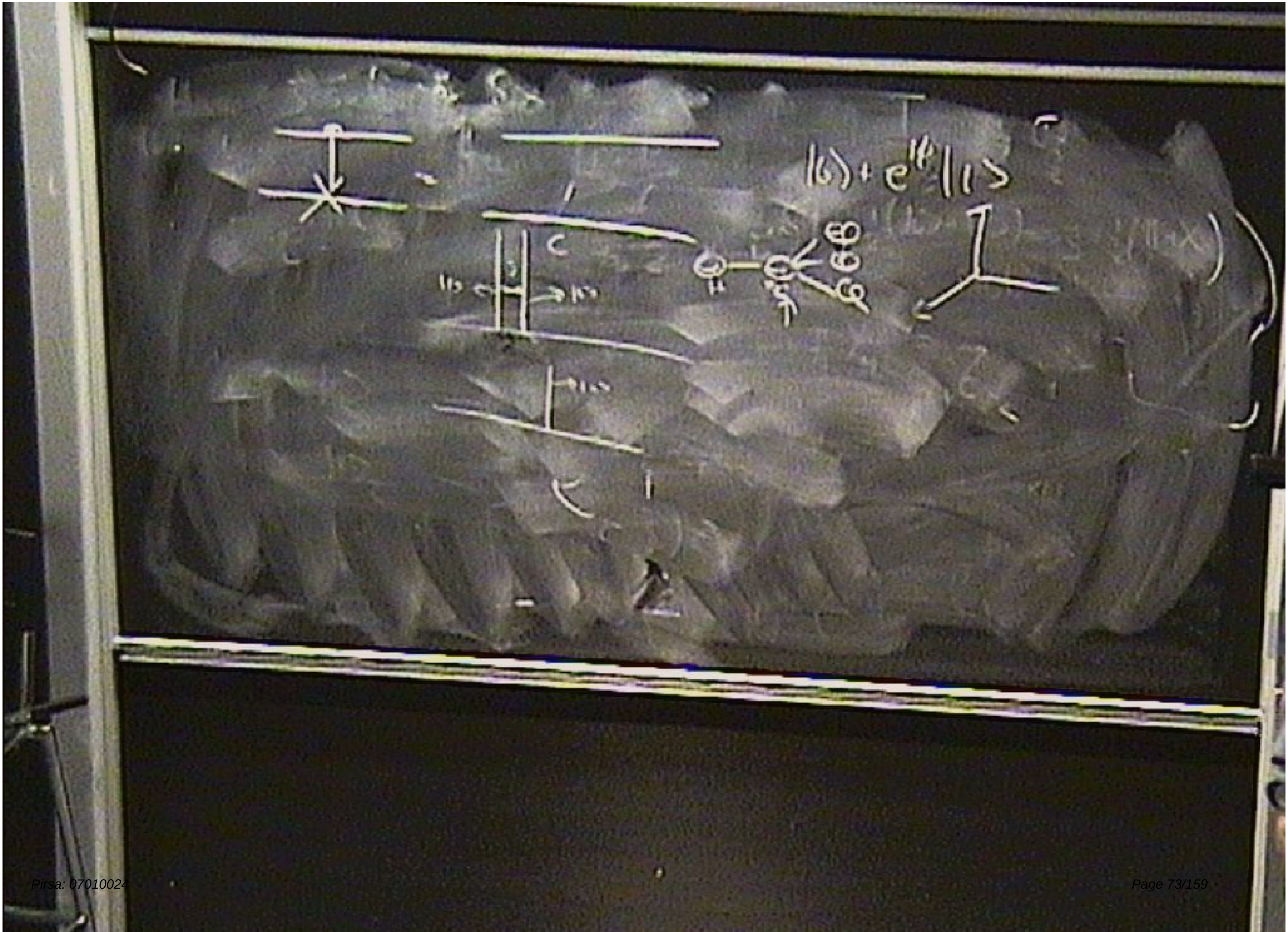


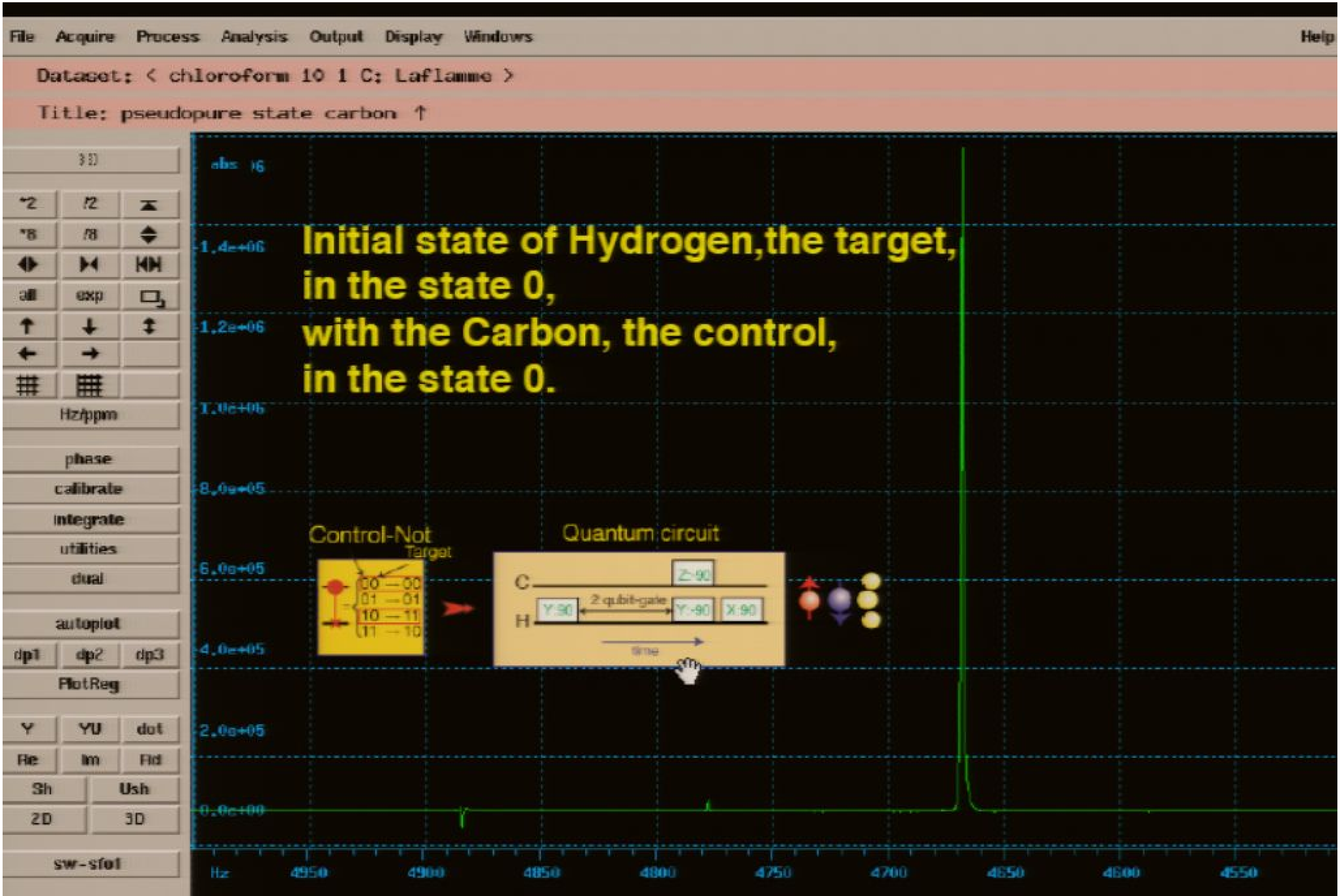

File Acquire Process Analysis Output Display Windows Help
Dataset: < chloroform 10 1 C: Laflamme >
Title: pseudopure state carbon
Initial state of Hydrogen,the target,
in the state 0,
with the Carbon, the control,
in the state 0.
Control-Not
Target
Quantum circuit
C
H
Z-90
Y-90
2 qubit-gate
Y-90
X-90
phase
calibrate
integrate
utilities
dual
autoplot
dp1 dp2 dp3
PlotReg
Hz/ppm
sw-sfof
Hz 4950 4900 4850 4800 4750 4700 4650 4600 4550



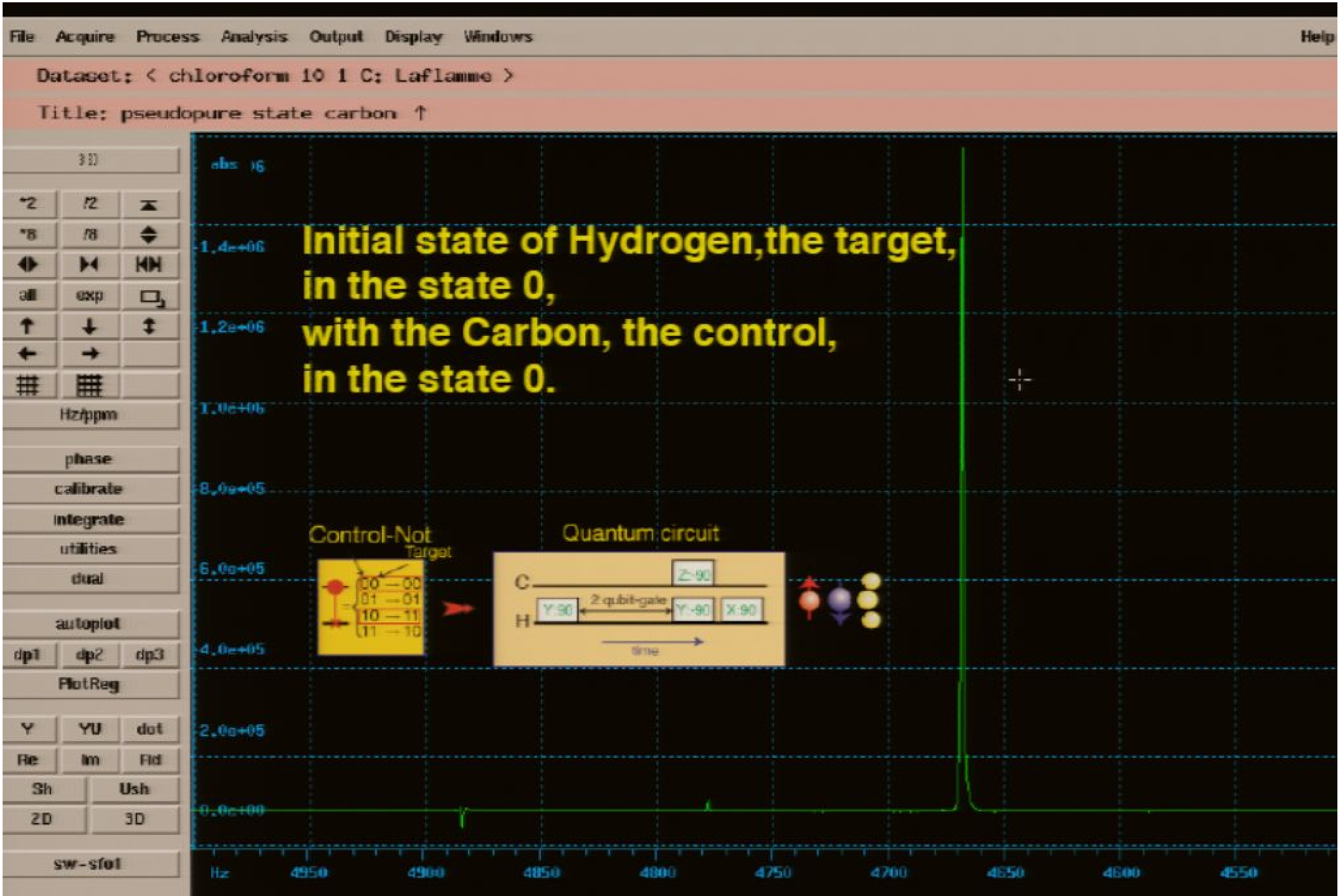


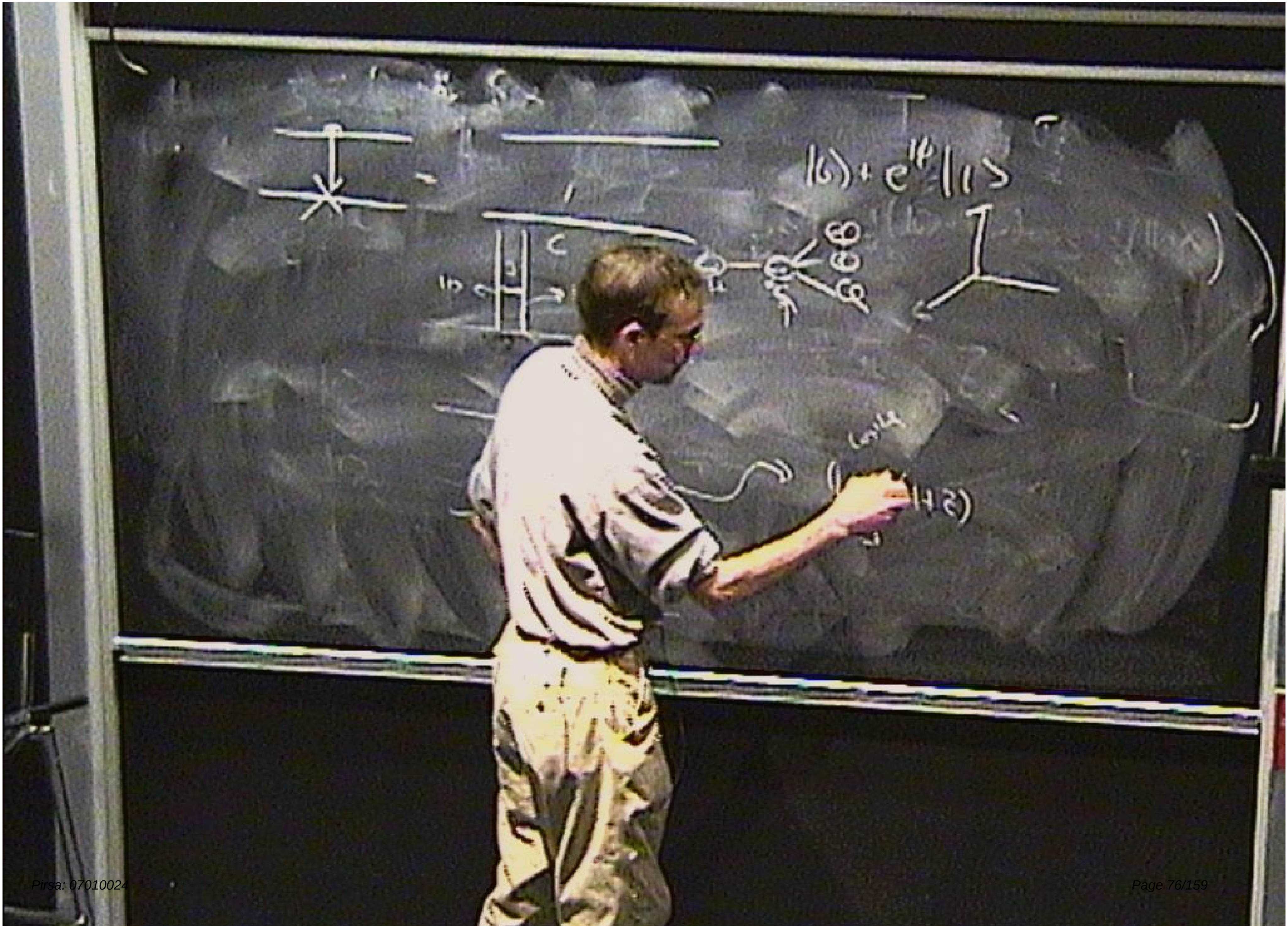
$|0\rangle + e^{i\phi}|1\rangle$



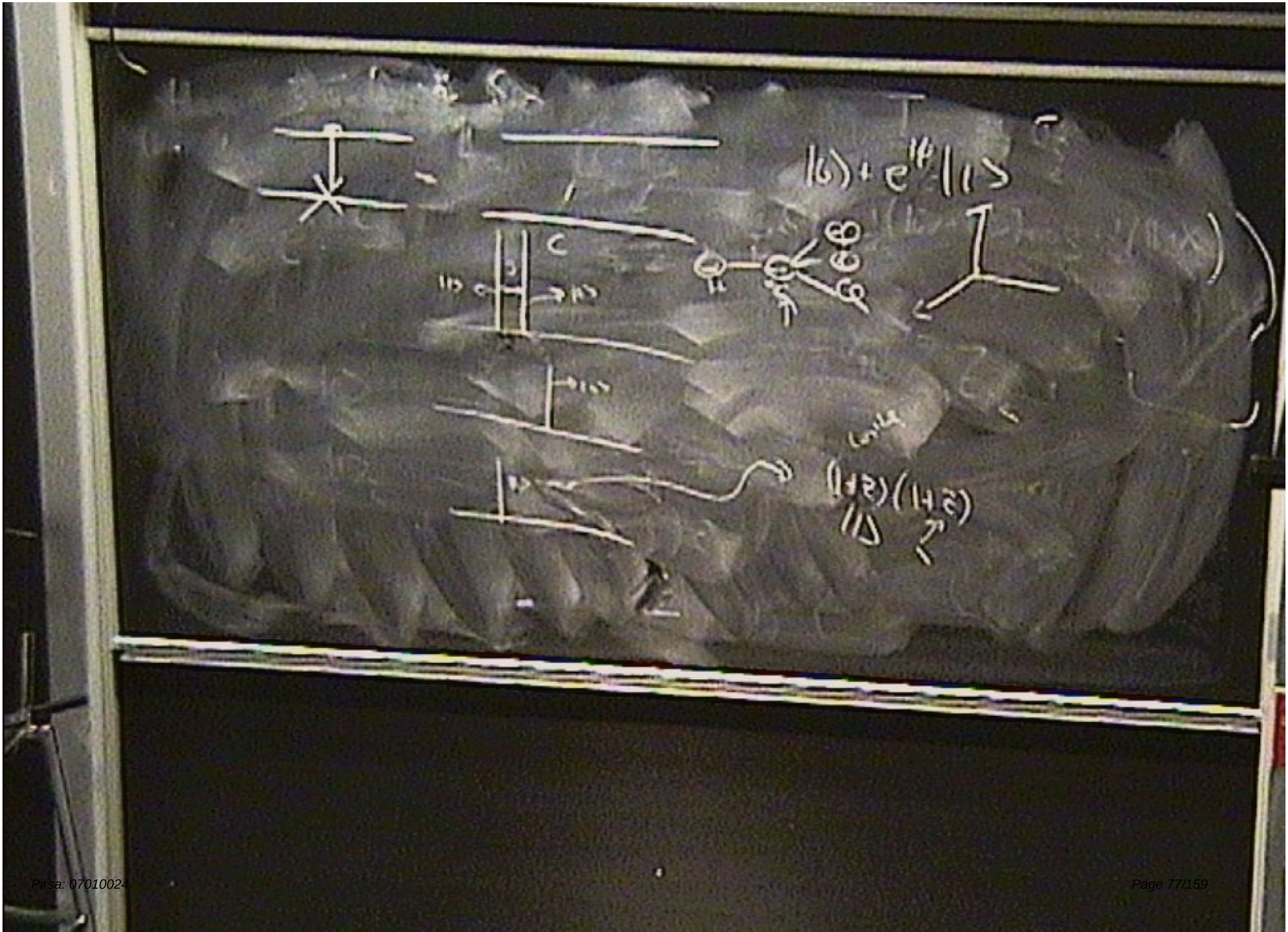


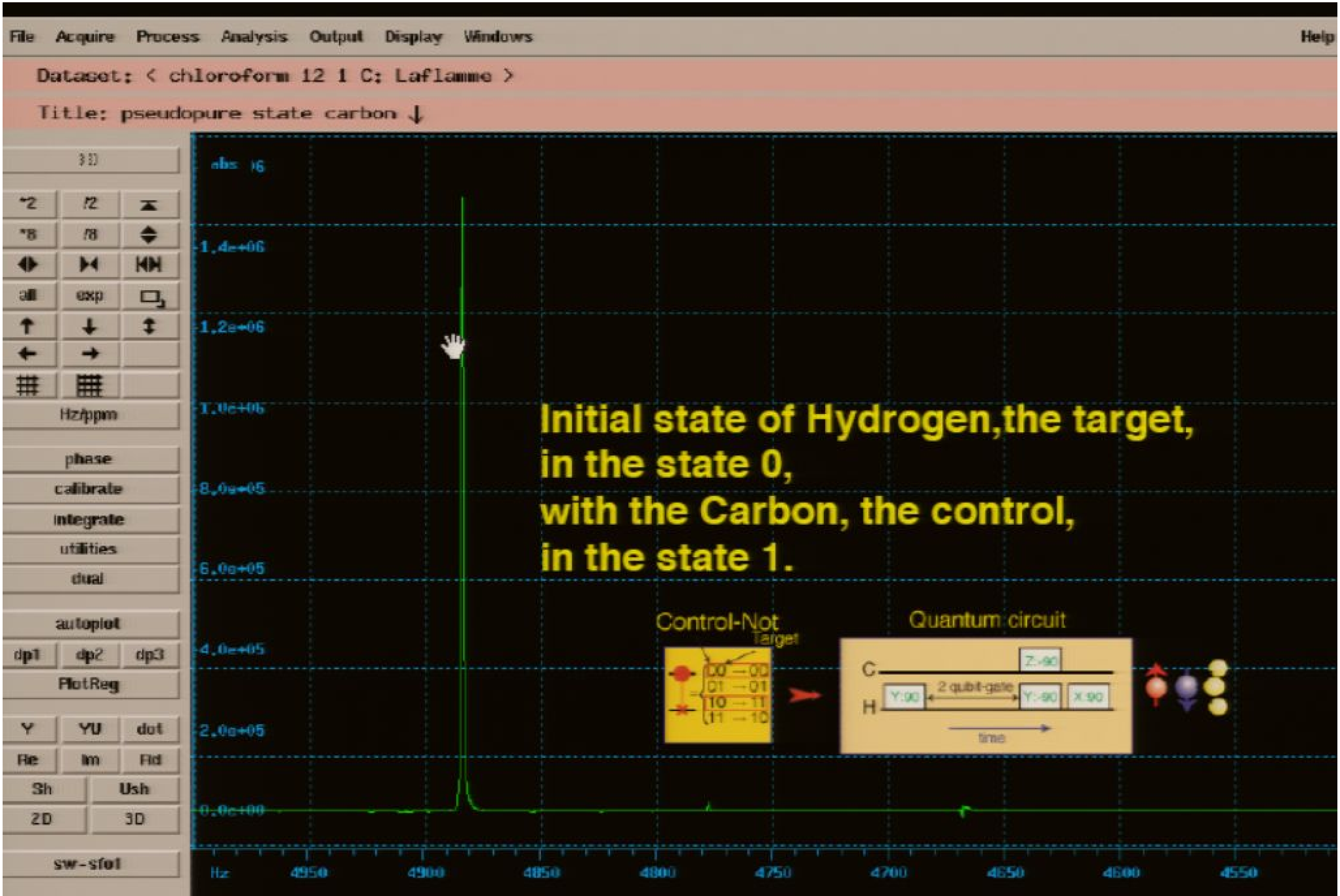
File Acquire Process Analysis Output Display Windows Help
Dataset: < chloroform 10 1 C: Laflamme >
Title: pseudopure state carbon ↑
Initial state of Hydrogen,the target,
in the state 0,
with the Carbon, the control,
in the state 0.
Control-Not
Target
00 — 00
01 — 01
10 — 11
11 — 10
Quantum circuit
C
H
Z-90
Y-90
2 qubit-gate
Y-90
X-90
time
phase
calibrate
integrate
utilities
dual
autoplot
dp1 dp2 dp3
PlotReg
Y YU dot
Re Im Fid
Sh Ush
2D 3D
sw-sfo1
Hz 4950 4900 4850 4800 4750 4700 4650 4600 4550

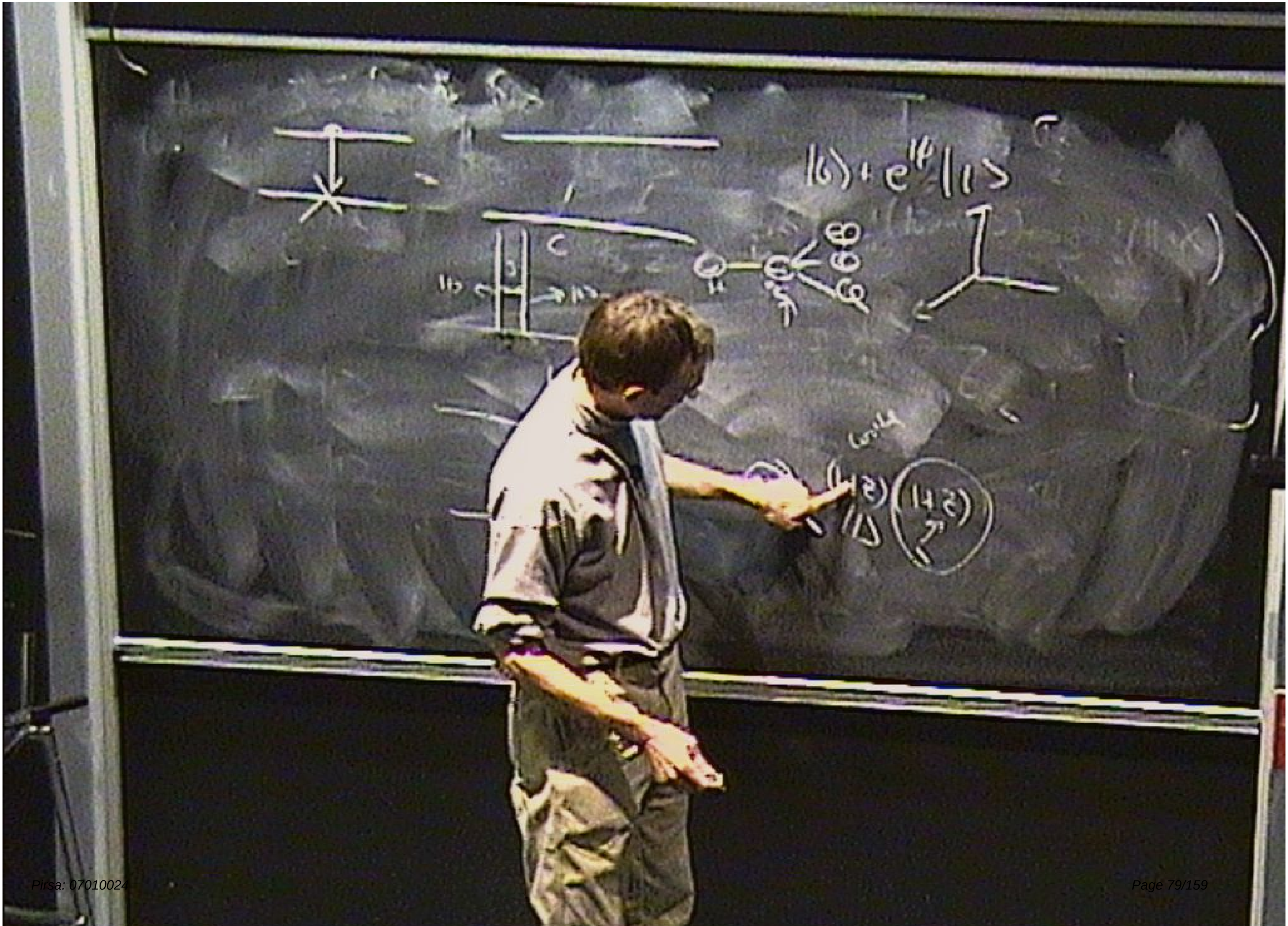

File Acquire Process Analysis Output Display Windows Help
Dataset: < chloroform 10 1 C: Laflamme >
Title: pseudopure state carbon ↑
Initial state of Hydrogen,the target,
in the state 0,
with the Carbon, the control,
in the state 0.
Control-Not
Target
00 — 00
01 — 01
10 — 11
11 — 10
Quantum circuit
C
H
Z-90
Y-90
2 qubit-gate
Y-90
X-90
time
Hz/ppm
phase
calibrate
integrate
utilities
dual
autoplot
dp1 dp2 dp3
PlotReg
Y YU dot
Re Im Fid
Sh Ush
2D 3D
sw-sfo1
Hz 4950 4900 4850 4800 4750 4700 4650 4600 4550

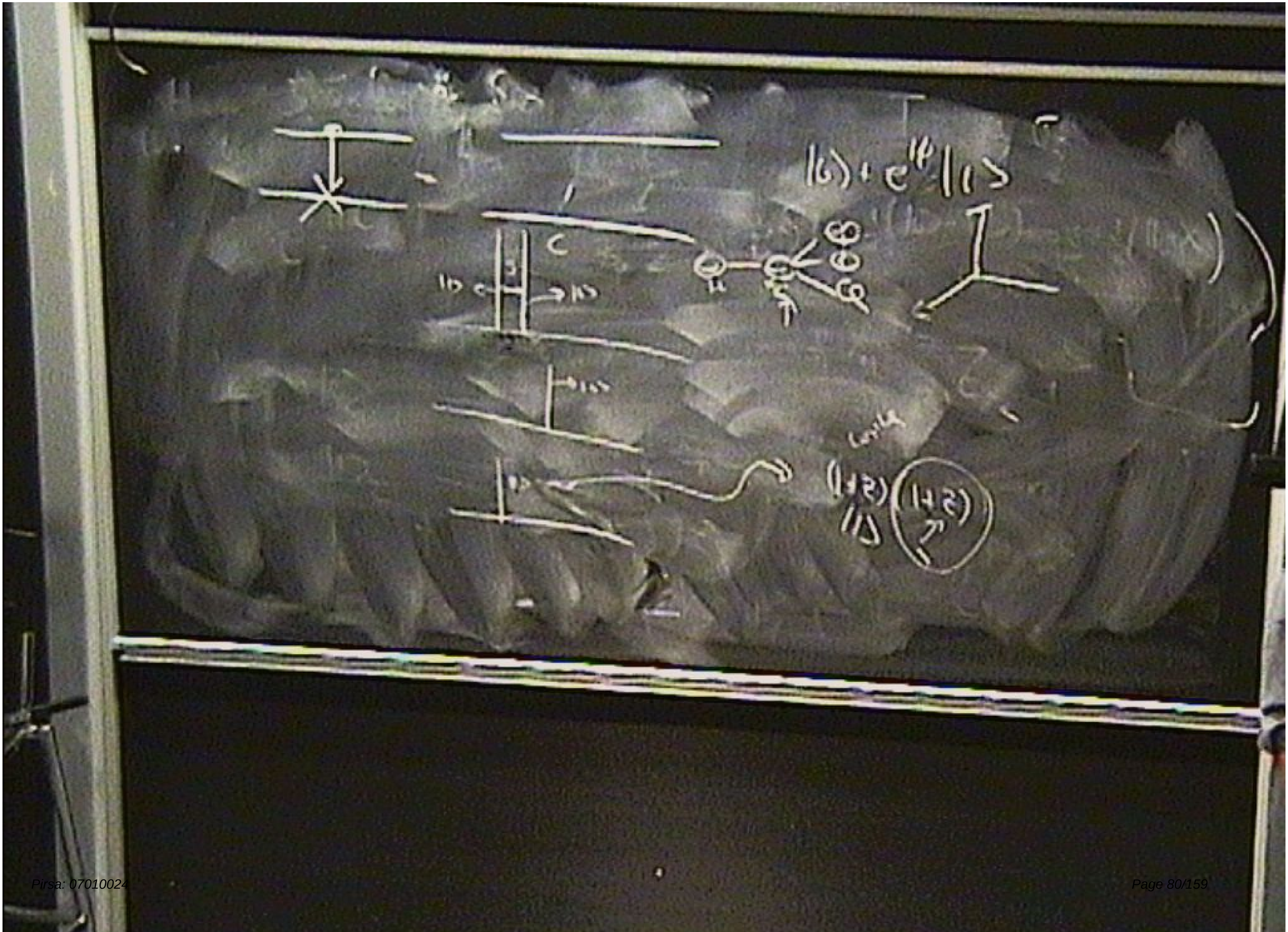


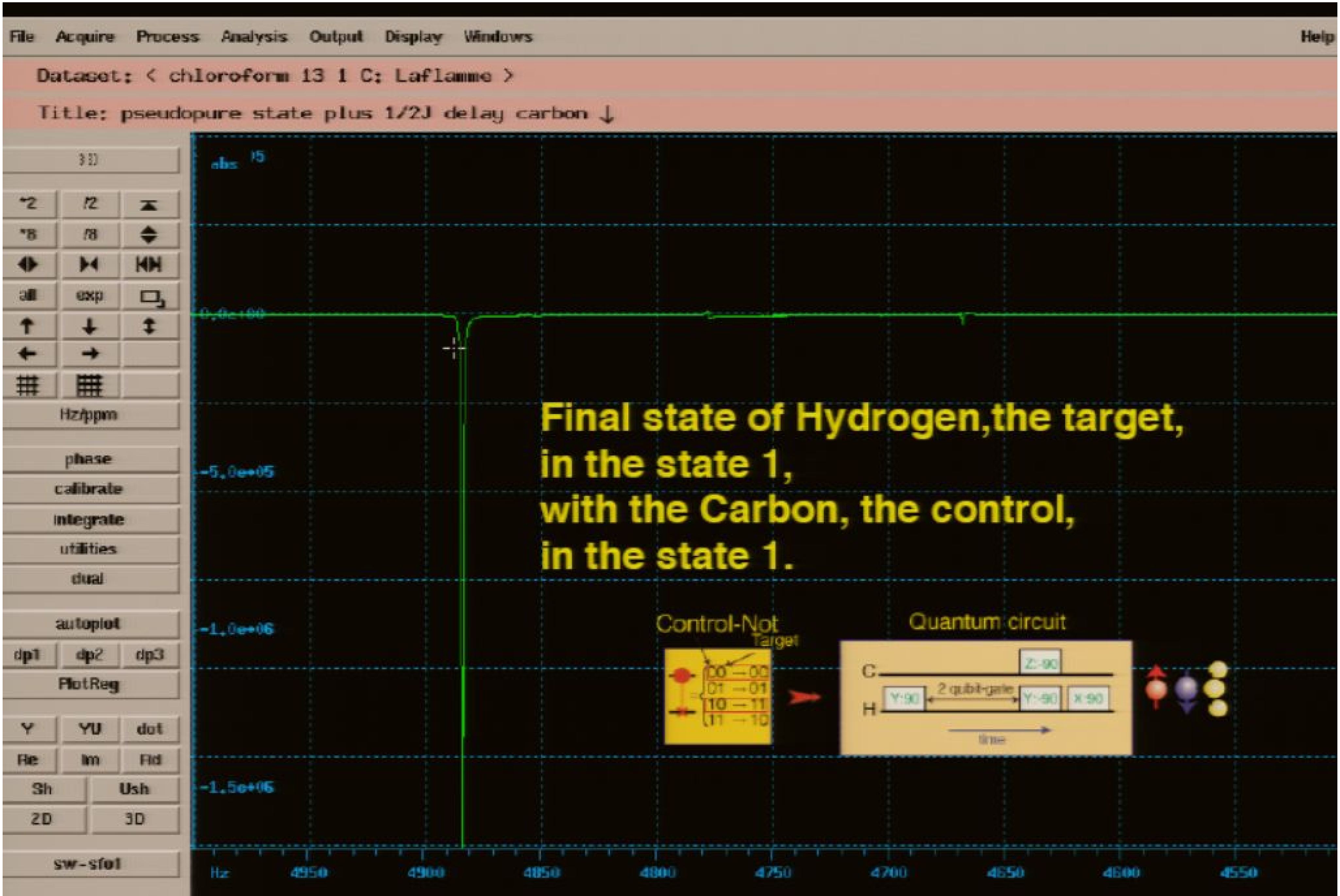
$|0\rangle + e^{i\phi}|1\rangle$


|0⟩ + e^{iθ}|1⟩
(1+ε)(1+ε)


File Acquire Process Analysis Output Display Windows Help
Dataset: < chloroform 12 1 C: Laflamme >
Title: pseudopure state carbon ↓
Initial state of Hydrogen,the target,
in the state 0,
with the Carbon, the control,
in the state 1.
Control-Not
Target
00 → 00
01 → 01
10 → 11
11 → 10
Quantum circuit
C
H
Z-90
Y:90
2 qubit-gate
Y:-90
X:90
time
Hz
4950
4900
4850
4800
4750
4700
4650
4600
4550
phase
calibrate
integrate
utilities
dual
autoplot
dp1 dp2 dp3
PlotReg
Y YU dot
Re Im Fid
Sh Ush
2D 3D
sw-sfo1
Hz/ppm






File Acquire Process Analysis Output Display Windows Help
Dataset: < chloroform 13 1 C: Laflamme >
Title: pseudopure state plus 1/2J delay carbon ↓
Final state of Hydrogen,the target,
in the state 1,
with the Carbon, the control,
in the state 1.
Control-Not
Target
00 → 00
01 → 01
10 → 11
11 → 10
Quantum circuit
C
H
Z:90
Y:90
2 qubit-gate
Y:-90
X:90
time
Hz/ppm
phase
calibrate
integrate
utilities
dual
autoplot
dp1 dp2 dp3
PlotReg
Y YU dot
Re Im Fid
Sh Ush
2D 3D
sw-sfo1
0.0e+00
-5.0e+05
-1.0e+06
-1.5e+06
Hz 4950 4900 4850 4800 4750 4700 4650 4600 4550

Quantum error correction

P. Shor, PRA, 52, 2493, 1995; A. Steane, RSLA 452, 2551, 1996

	Noise model	Redundancy	Majority voting
Classical	bit flip $0 \leftrightarrow 1$	copy information $0 \longrightarrow 000$ $1 \longrightarrow 111$	observe and compare $110 \longrightarrow 111$
Quantum	bit flip phase flip $\alpha\|0\rangle + \beta\|1\rangle$ $\alpha\|0\rangle - \beta\|1\rangle$	No cloning theorem (Wootters & Zurek, 1982) Encode $\alpha\|000\rangle + \beta\|111\rangle$	Detect the error without finding the message $\alpha\|001\rangle + \beta\|110\rangle$

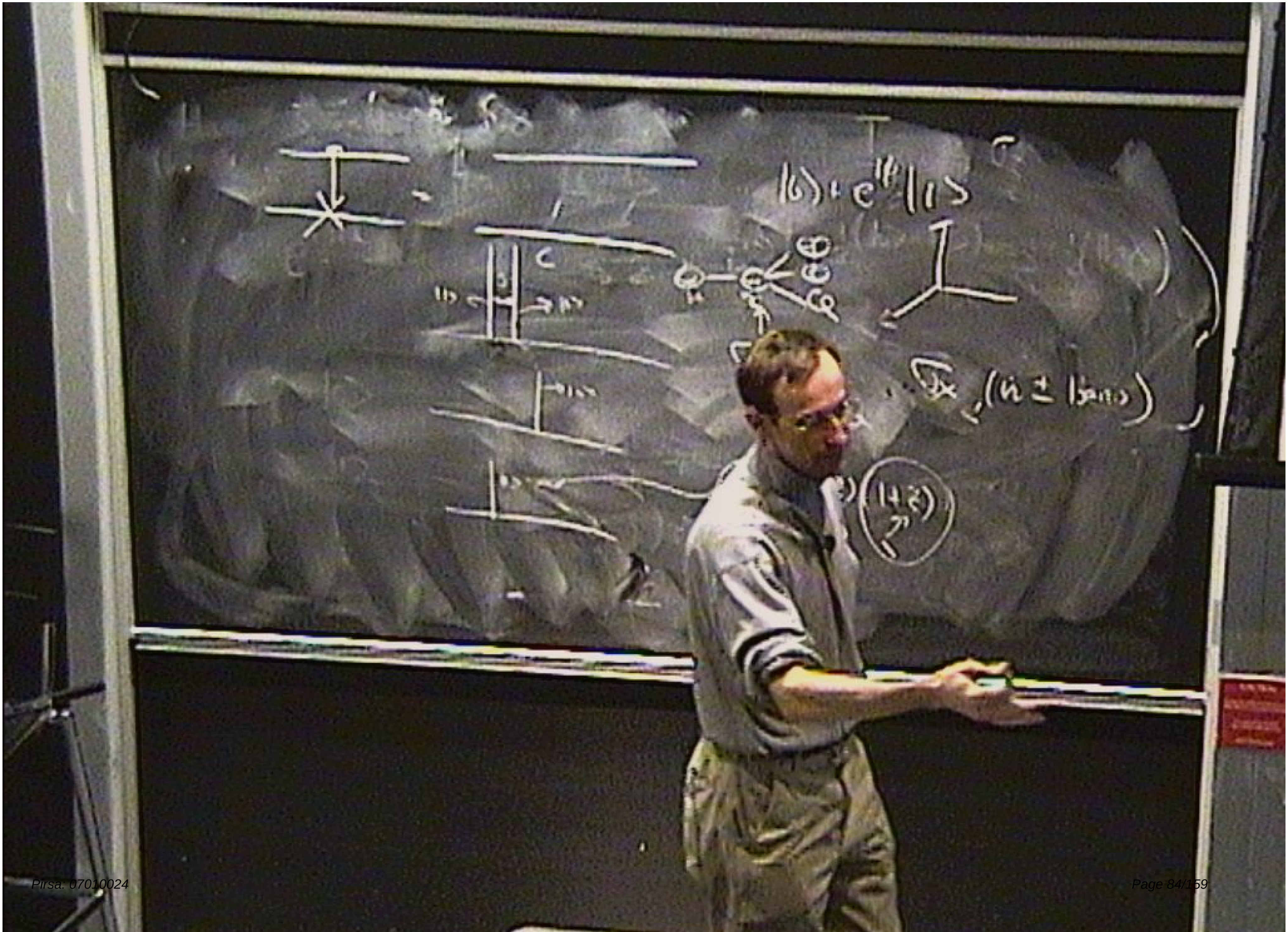
Quantum error correction

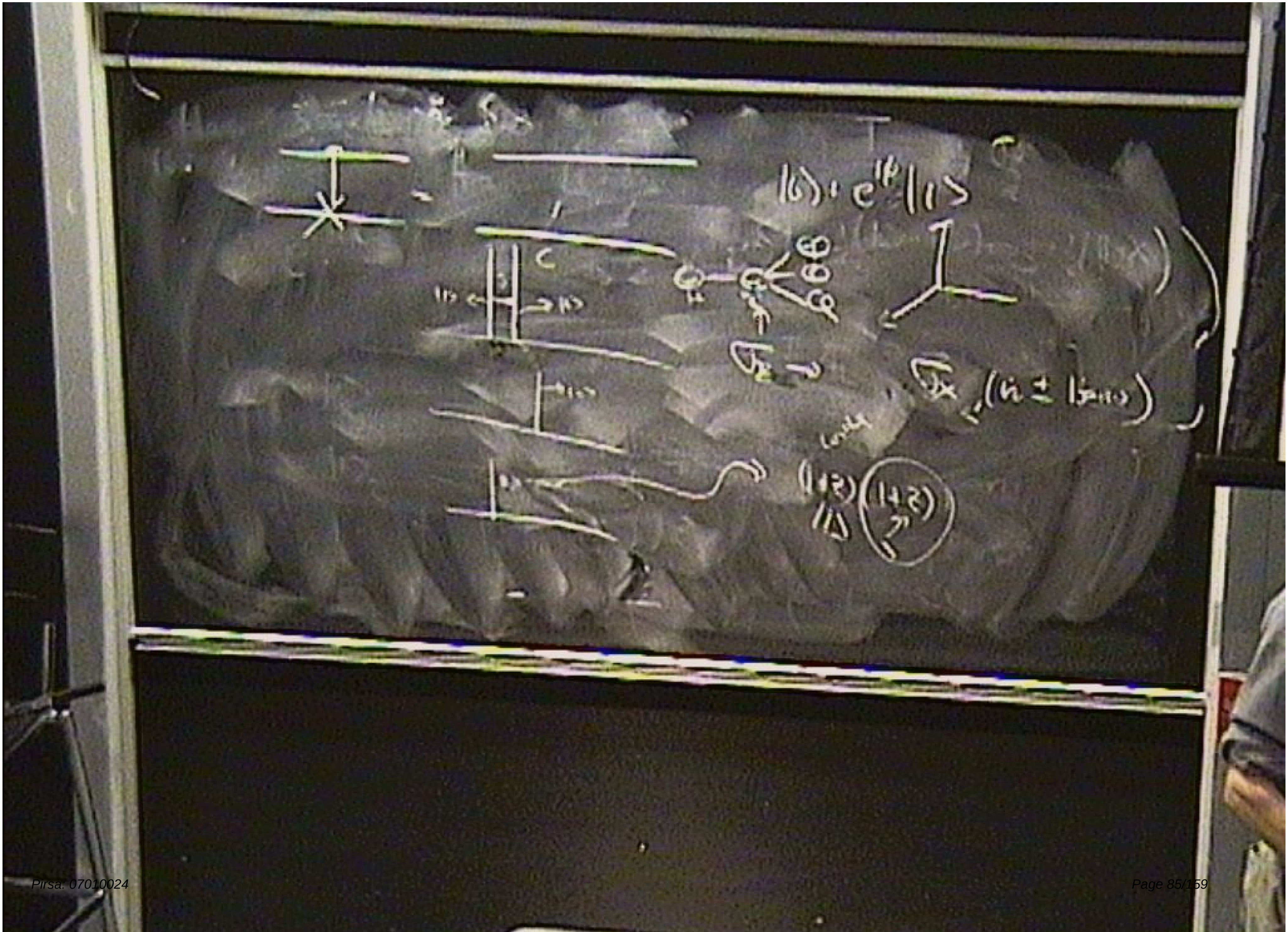
P. Shor, PRA, 52, 2493, 1995; A. Steane, RSLA 452, 2551, 1996

	Noise model	Redundancy	Majority voting
Classical	bit flip $0 \leftrightarrow 1$	copy information $0 \longrightarrow 000$ $1 \longrightarrow 111$	observe and compare $110 \longrightarrow 111$
Quantum	b ph $\alpha\|0$ $\alpha\|0$	N	D

$$\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix} \longrightarrow \begin{pmatrix}\rho_{00} & \rho_{01}e^{-t/T_2}\\ \rho_{10}e^{-t/T_2} & \rho_{11}\end{pmatrix}$$

$$= \frac{(1+e^{-t/T_2})}{2}\mathbb{1}\begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix}\mathbb{1} + \frac{(1-e^{-t/T_2})}{8} I_z \begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix} I_z$$





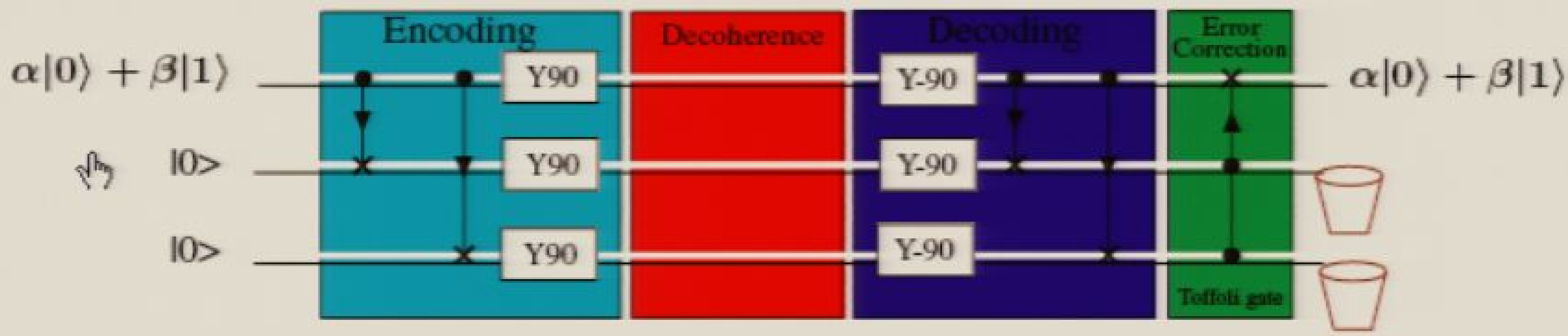
Quantum error correction

P. Shor, PRA, 52, 2493, 1995; A. Steane, RSLA 452, 2551, 1996

$$I_z^1 \longrightarrow \frac{1}{4}\left(I_x^1 + I_x^2 + I_x^3 + 4\, I_x^1 I_x^2 I_x^3\right)$$

			Majority voting
Classi	bit flip $0 \leftrightarrow 1$	copy information $0 \longrightarrow 000$ $1 \longrightarrow 111$	observe and compare $110 \longrightarrow 111$
Quantum	bit flip phase flip $\alpha\|0\rangle + \beta\|1\rangle$ $\alpha\|0\rangle - \beta\|1\rangle$	No cloning theorem (Wootters & Zurek, 1982) Encode $\alpha\|000\rangle + \beta\|111\rangle$	Detect the error without finding the message $\alpha\|001\rangle + \beta\|110\rangle$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\begin{array}{l} \alpha|+++\rangle + \beta|---\rangle \\ \alpha|-++\rangle + \beta|+--\rangle \\ \alpha|+-+\rangle + \beta|-+-\rangle \\ \alpha|++-\rangle + \beta|--+\rangle \end{array}$$

$$\begin{array}{l} (\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ (\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ (\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ (\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{array}$$

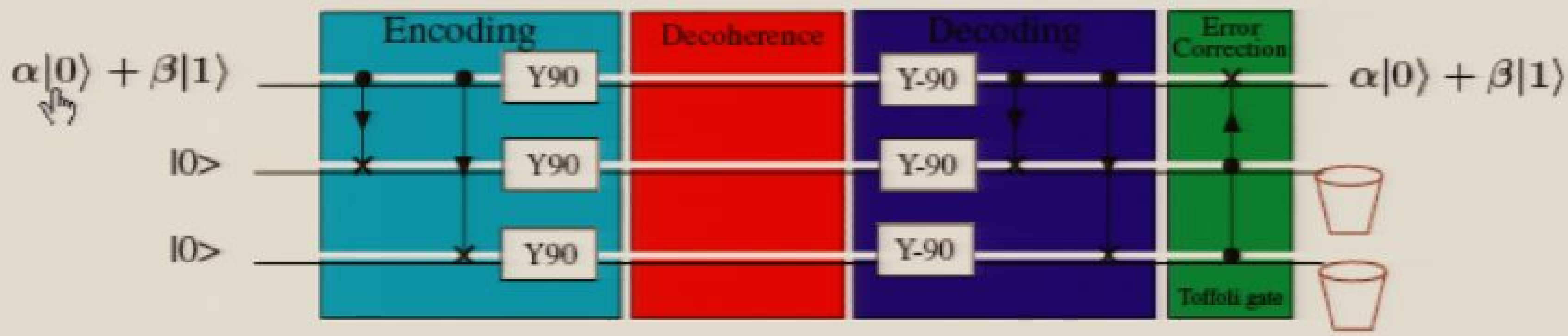
Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$\alpha|+++\rangle + \beta|---\rangle$

$$\rho_{\text{init}} = I^1_{x,y,z}\frac{1}{2}(\mathbb{1} + 2I^2_z)\frac{1}{2}(\mathbb{1} + 2I^3_z)$$

$\alpha|++-\rangle + \beta|--+\rangle$

$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$
$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

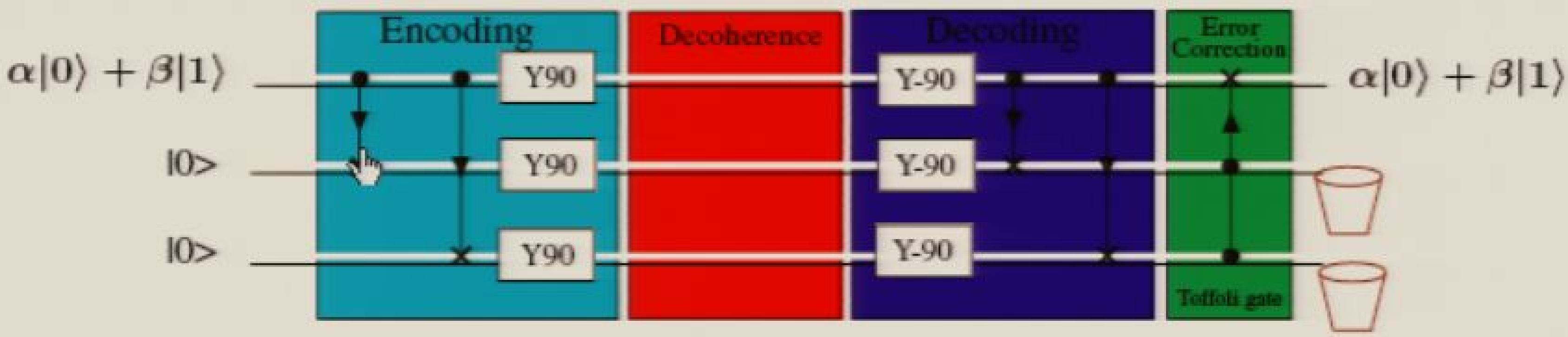
Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$\alpha|+++\rangle + \beta|---\rangle$
$\alpha|-++\rangle + \beta|+--\rangle$
$\alpha|+-+\rangle + \beta|-+-\rangle$
$\alpha|++-\rangle + \beta|--+\rangle$

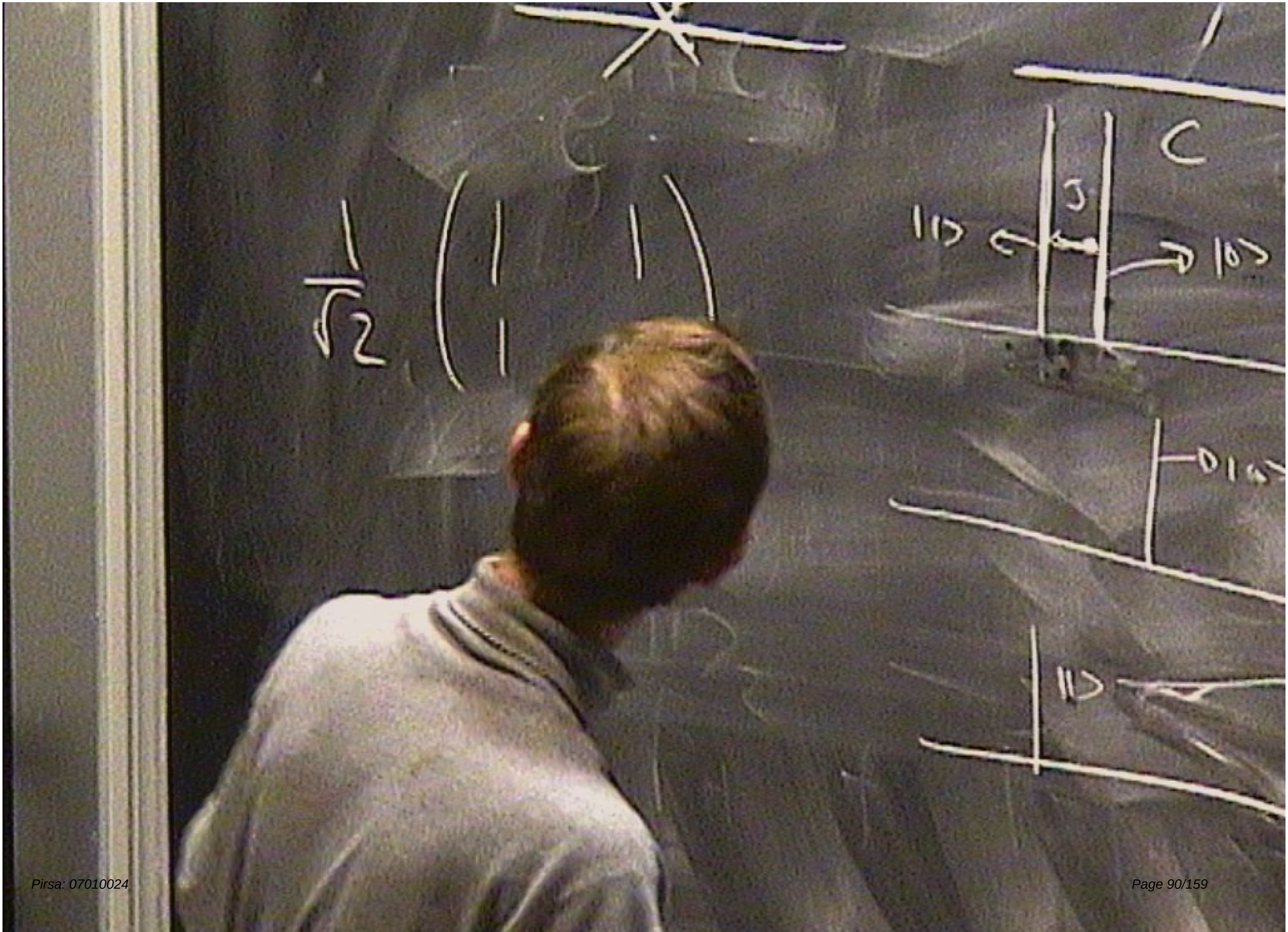
$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$
$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

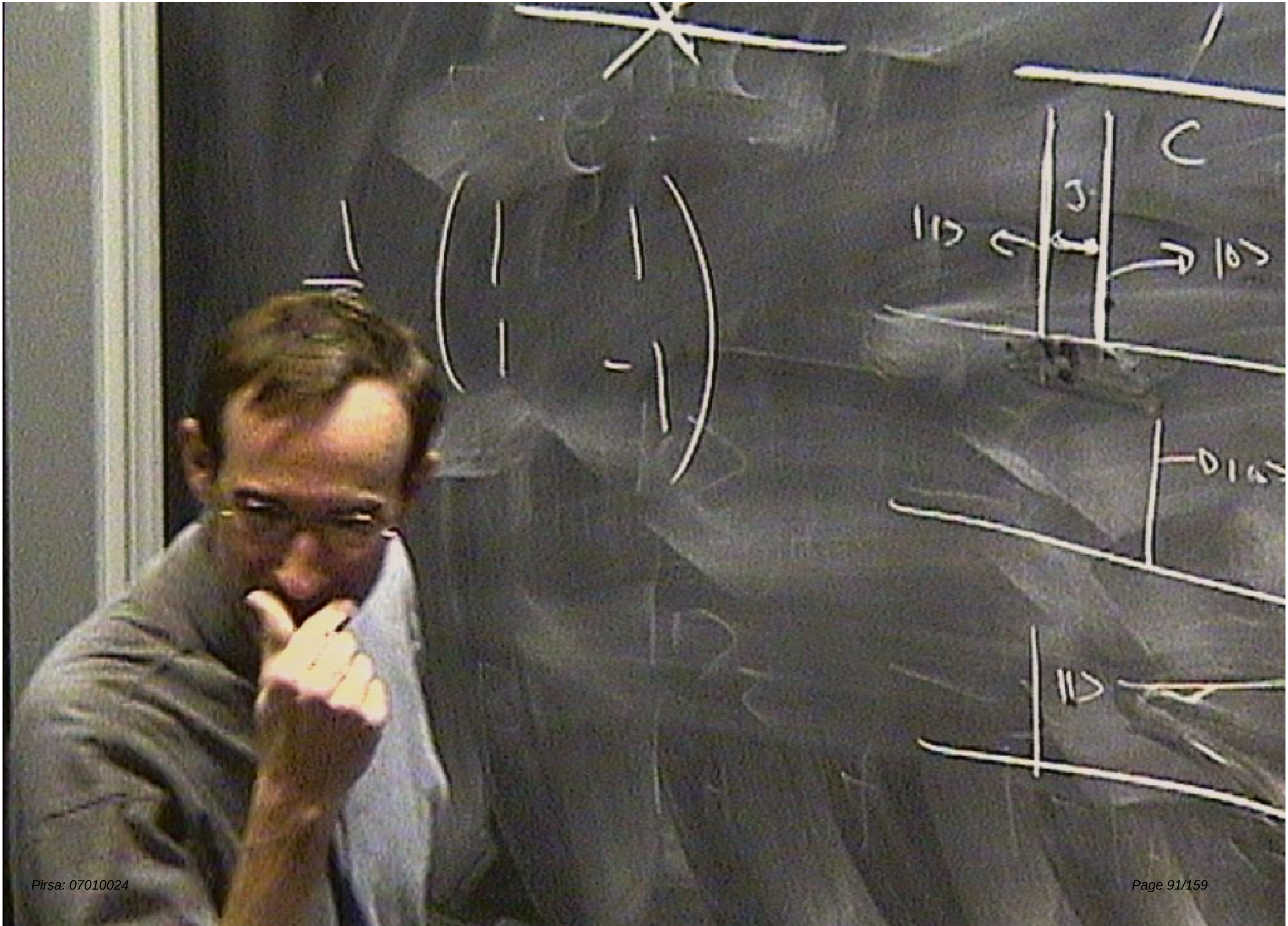
$$\rho_B \equiv \frac{1}{4}\left(I_x^1 + I_x^2 + I_x^3 + 4 I_x^1 I_x^2 I_x^3\right)$$

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

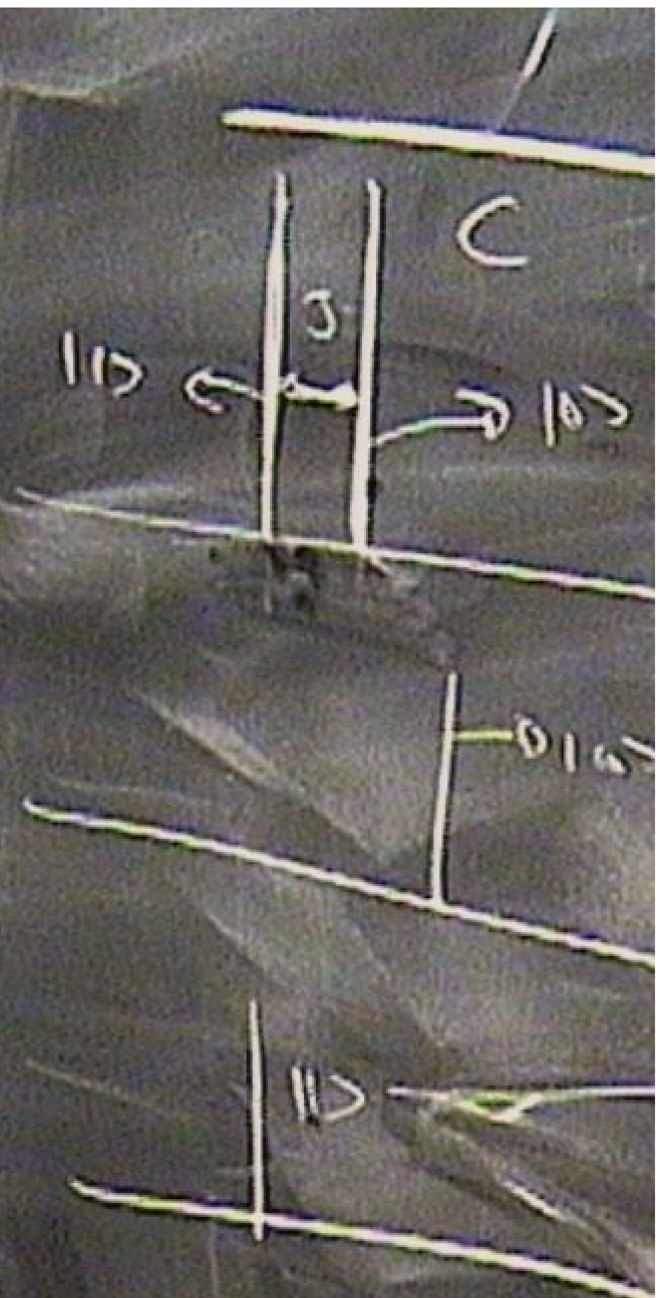
$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

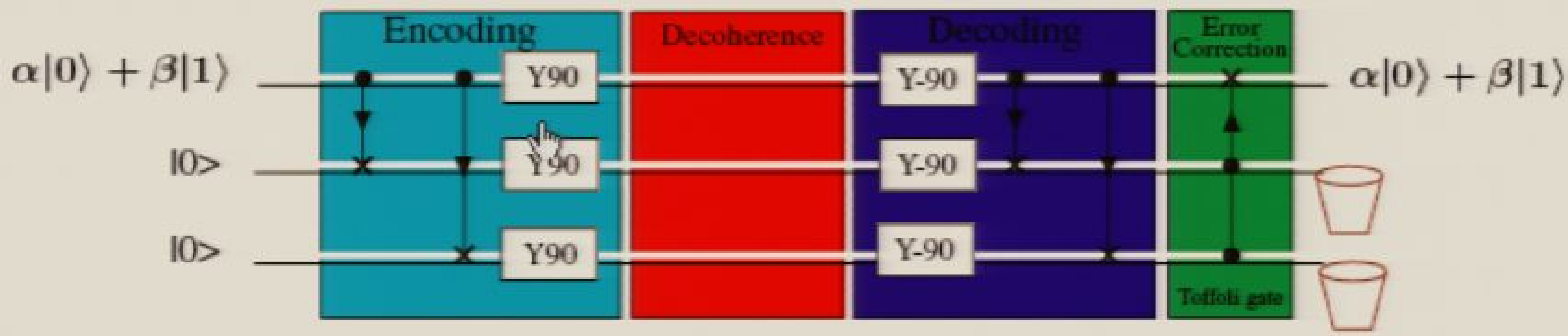

C
|1⟩
|0⟩
|1⟩


C
J
|1⟩
|0⟩
|1⟩

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$\alpha|+++\rangle + \beta|---\rangle$
$\alpha|-++\rangle + \beta|+--\rangle$
$\alpha|+-+\rangle + \beta|-+-\rangle$
$\alpha|++-\rangle + \beta|--+\rangle$

$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$
$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

$$\rho_B \equiv \frac{1}{4}\left(I_x^1 + I_x^2 + I_x^3 + 4 I_x^1 I_x^2 I_x^3\right)$$

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

$\sim 1 - 3\gamma^2$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase

$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
$$\alpha|++-\rangle + \beta|--+\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$
$$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

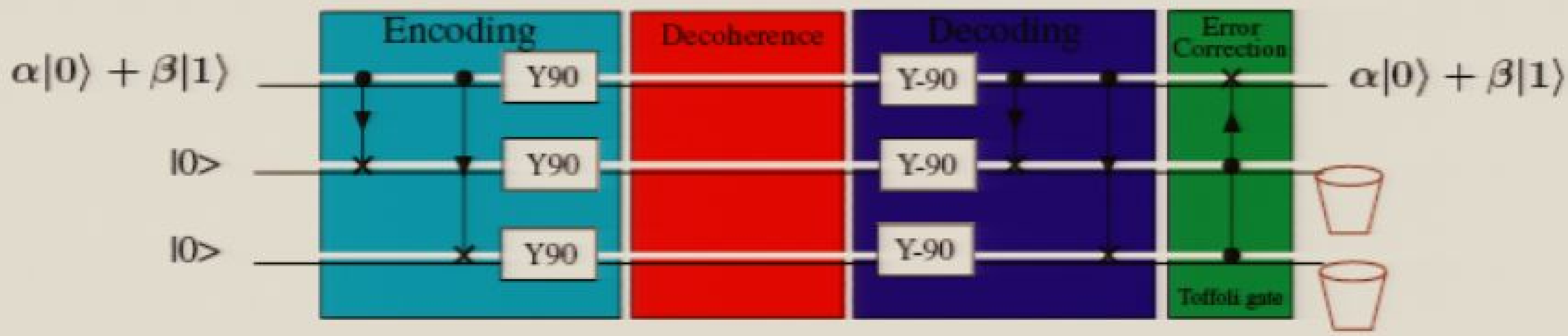
$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\begin{array}{l} \alpha|+++\rangle + \beta|---\rangle \\ \alpha|-++\rangle + \beta|+--\rangle \\ \alpha|+-+\rangle + \beta|-+-\rangle \\ \alpha|++-\rangle + \beta|--+\rangle \end{array}$$

$$\begin{array}{l} (\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ (\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ (\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ (\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{array}$$

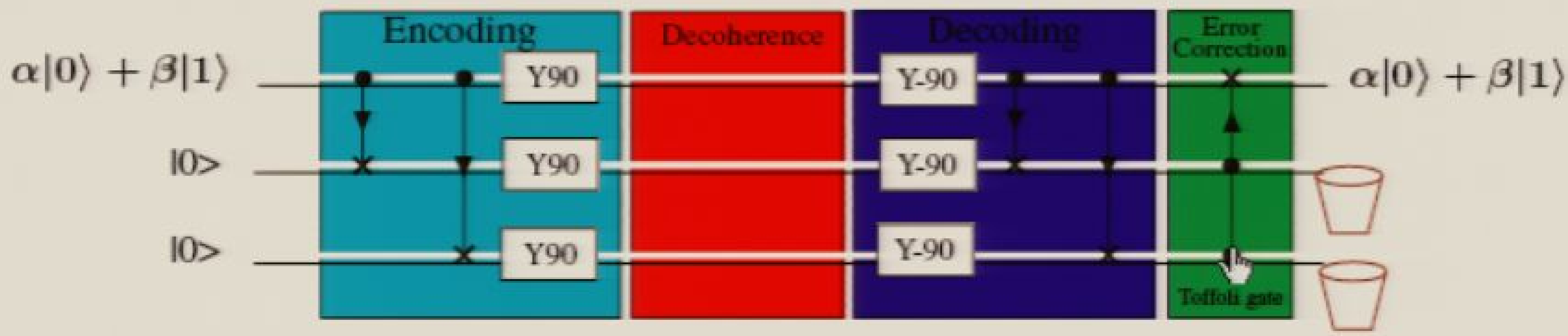
Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
$$\alpha|++-\rangle + \beta|--+\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

$$\rho_E^1 \equiv \tfrac{1}{8} I_z^1 (9e^{-t/\tau} - e^{-9t/\tau}) \approx I_z^1 (1 - \frac{9}{2} t^2/\tau^2 + \cdots)$$

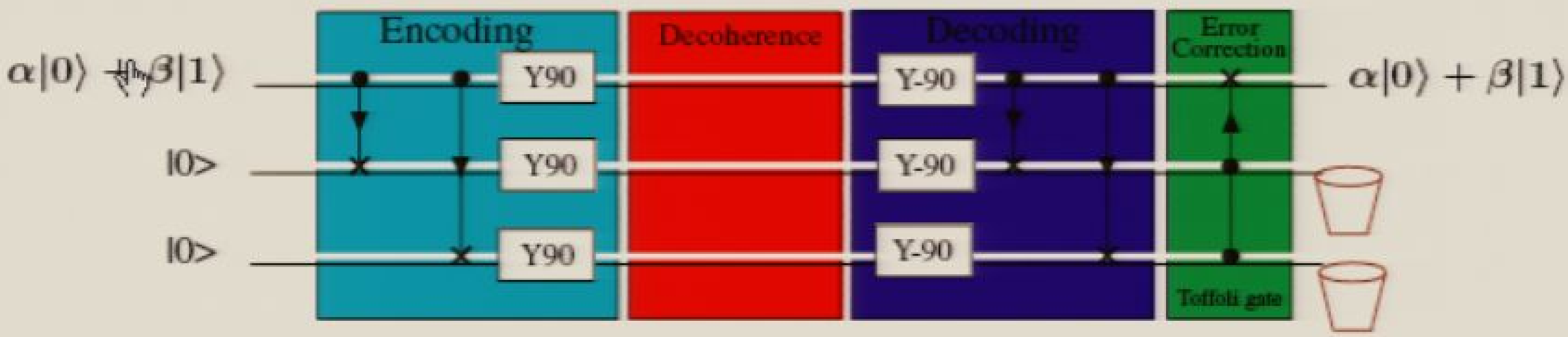
$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$\alpha|+++\rangle + \beta|---\rangle$

$$\rho_{\text{init}} = I^1_{x,y,z}\frac{1}{2}(\mathbb{1} + 2I^2_z)\frac{1}{2}(\mathbb{1} + 2I^3_z)$$

$\alpha|++-\rangle + \beta|--+\rangle$

$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$
$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$
$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

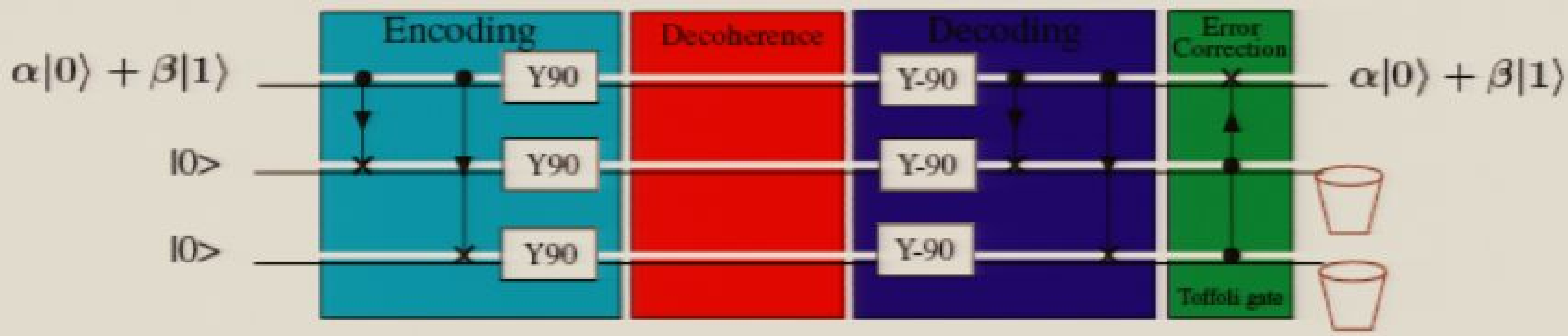
$\sim 1 - 3\gamma^2$

Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

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$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
$$\alpha|++-\rangle + \beta|--+\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$
$$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$$
$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

$$\sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase

$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

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$\alpha|+++\rangle + \beta|---\rangle$

$$\rho_{\text{init}} = I^1_{x,y,z}\frac{1}{2}(\mathbb{1} + 2I^2_z)\frac{1}{2}(\mathbb{1} + 2I^3_z)$$

$\alpha|++-\rangle + \beta|--+\rangle$

$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$

$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$

$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$

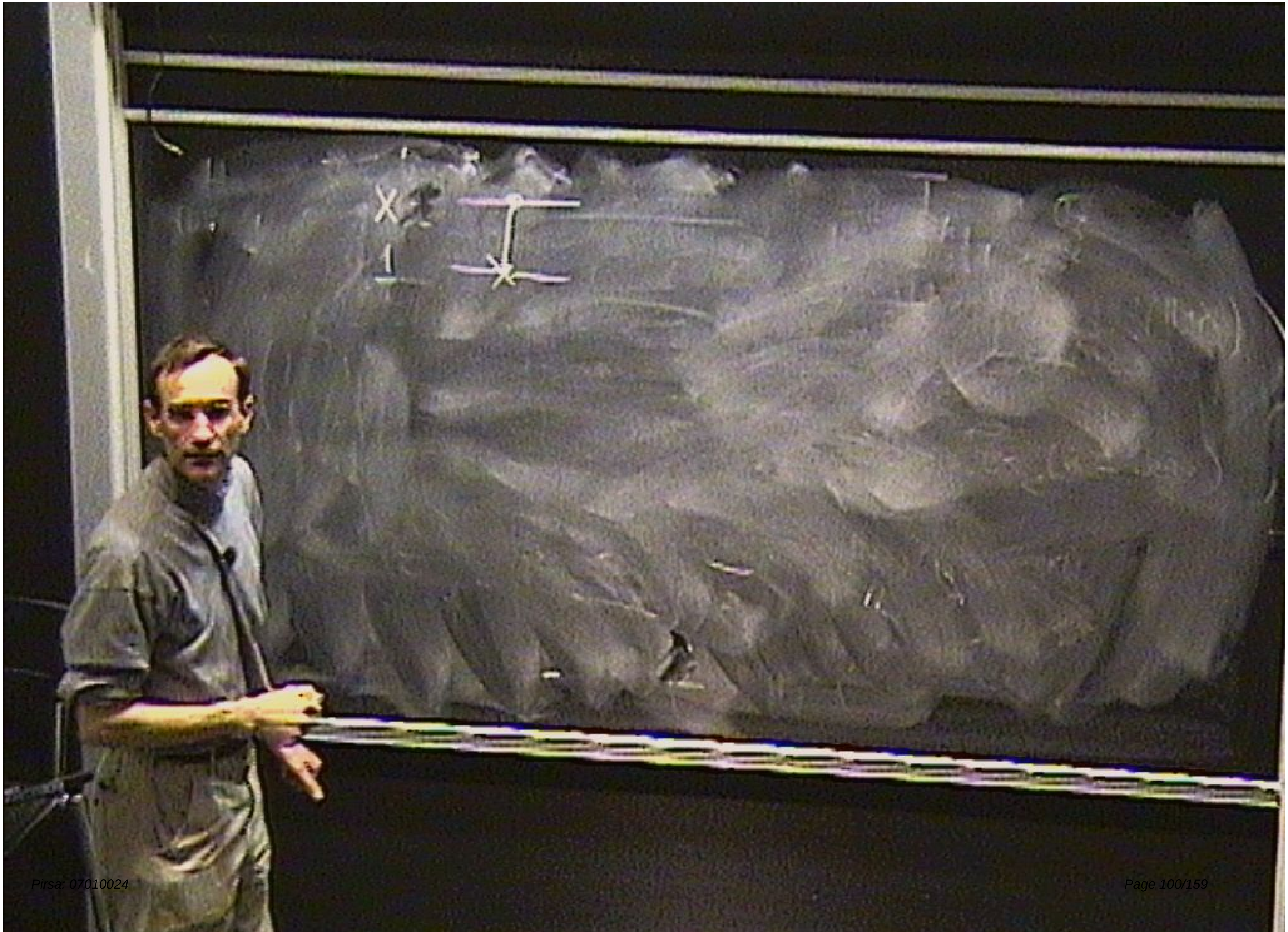
$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

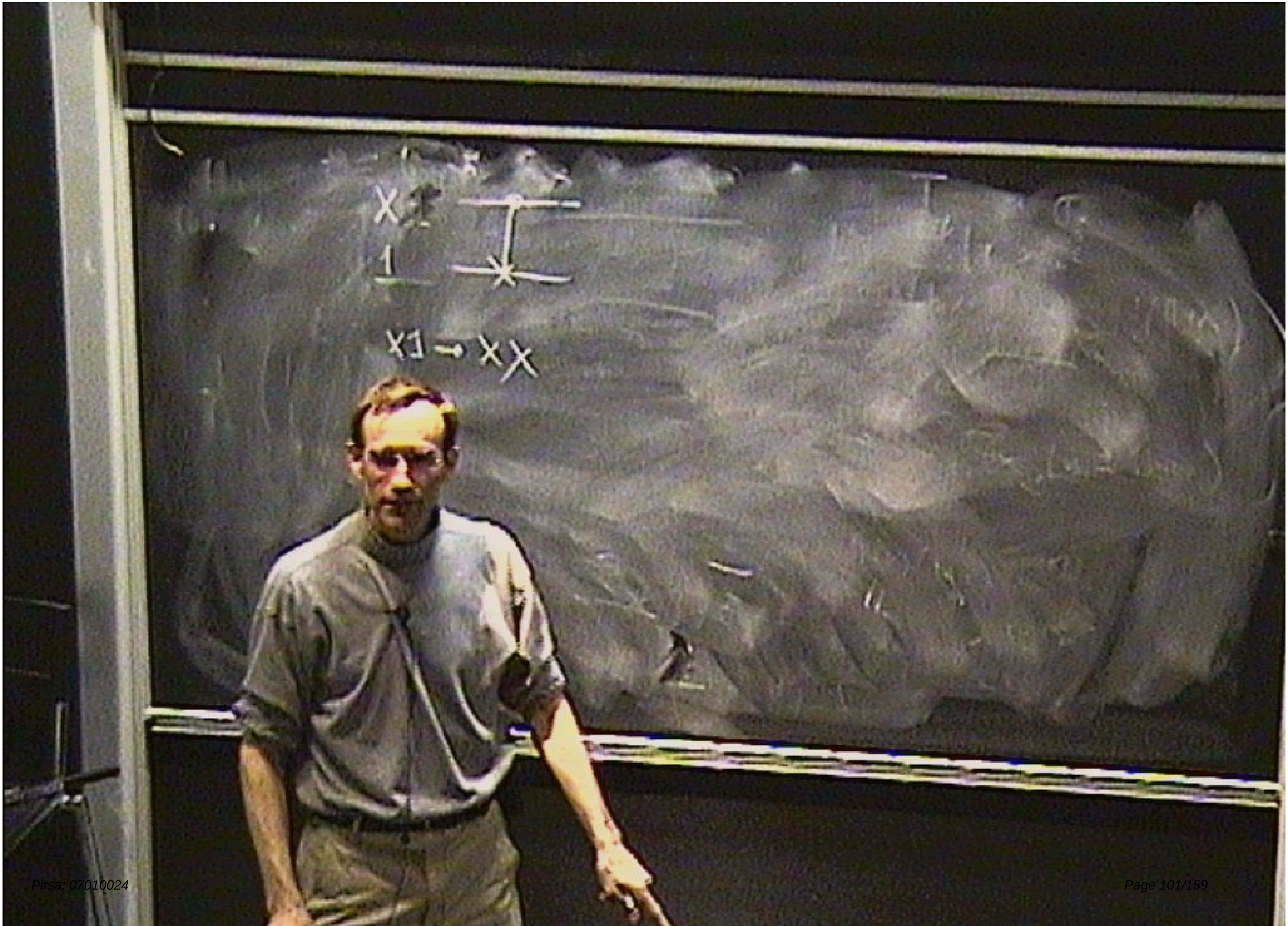
$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

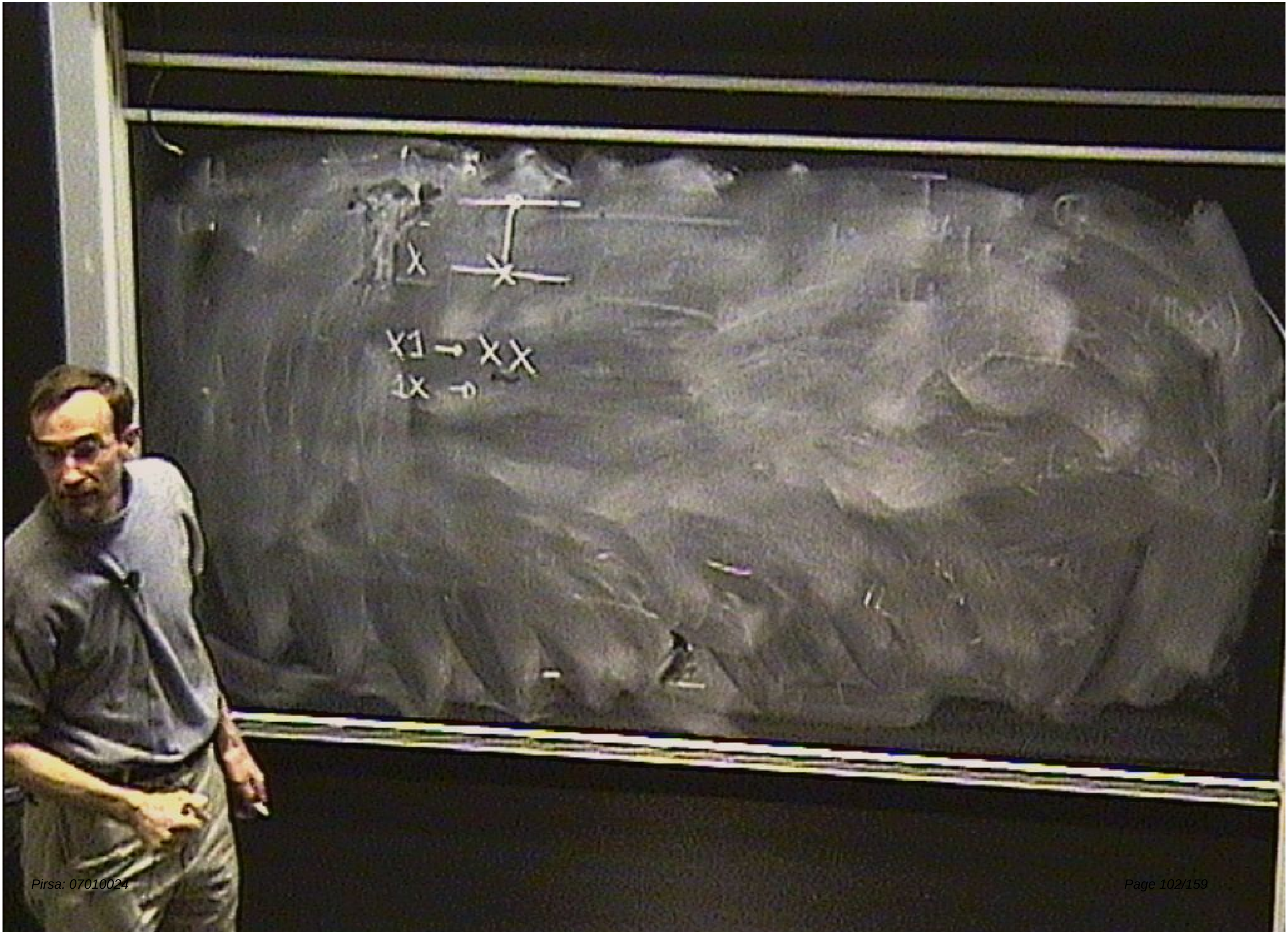
Control-Not

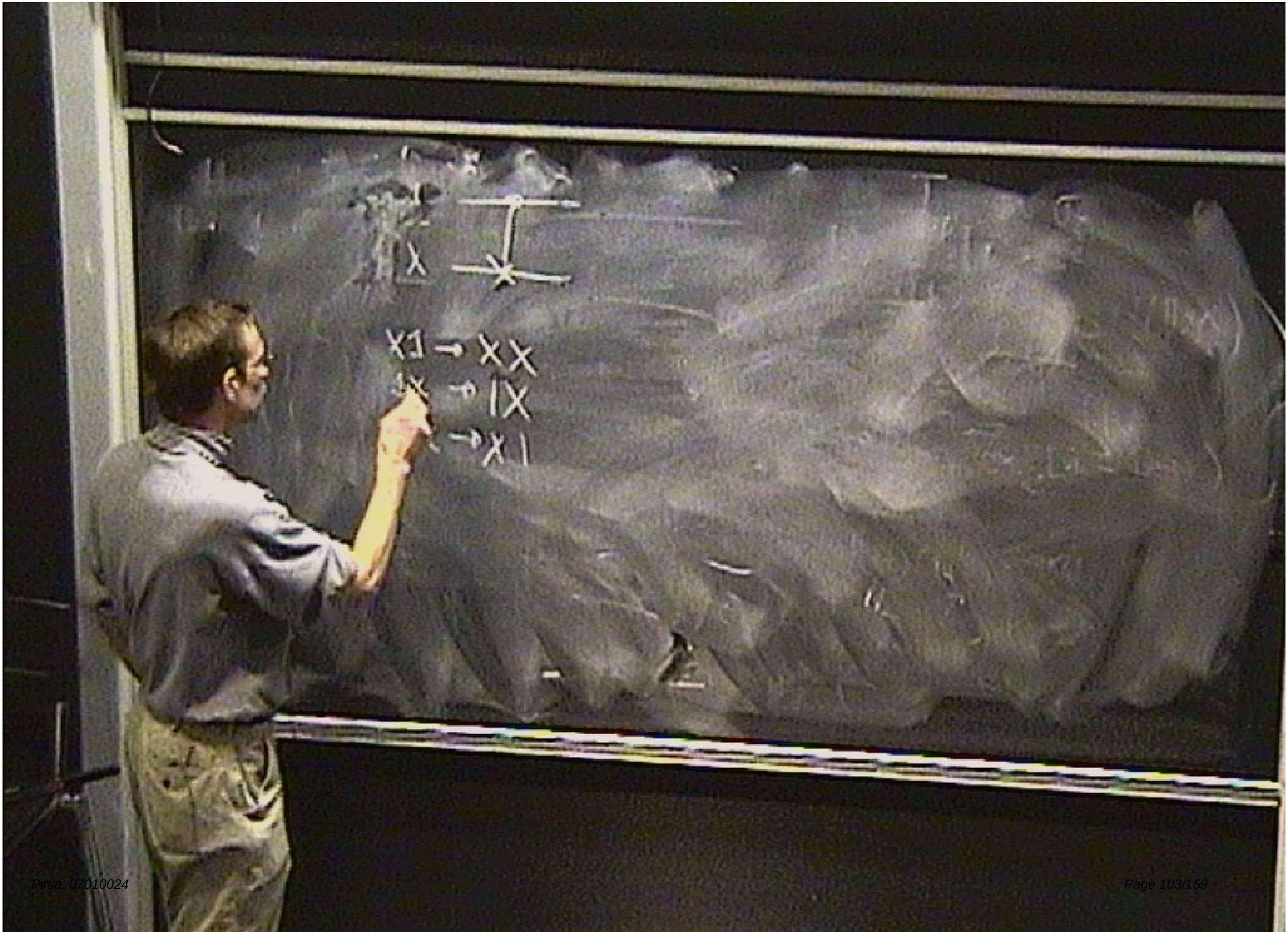
$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

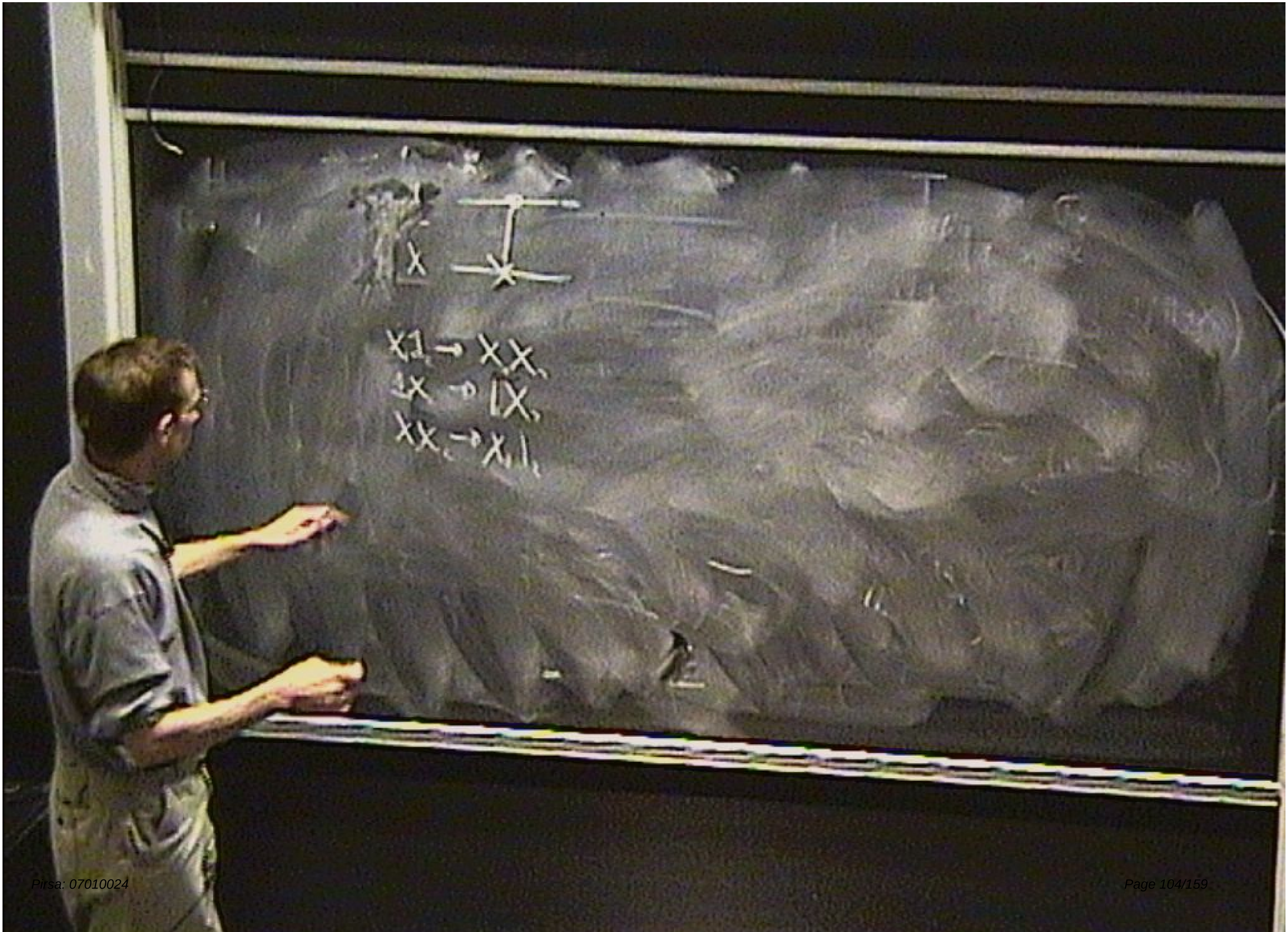
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

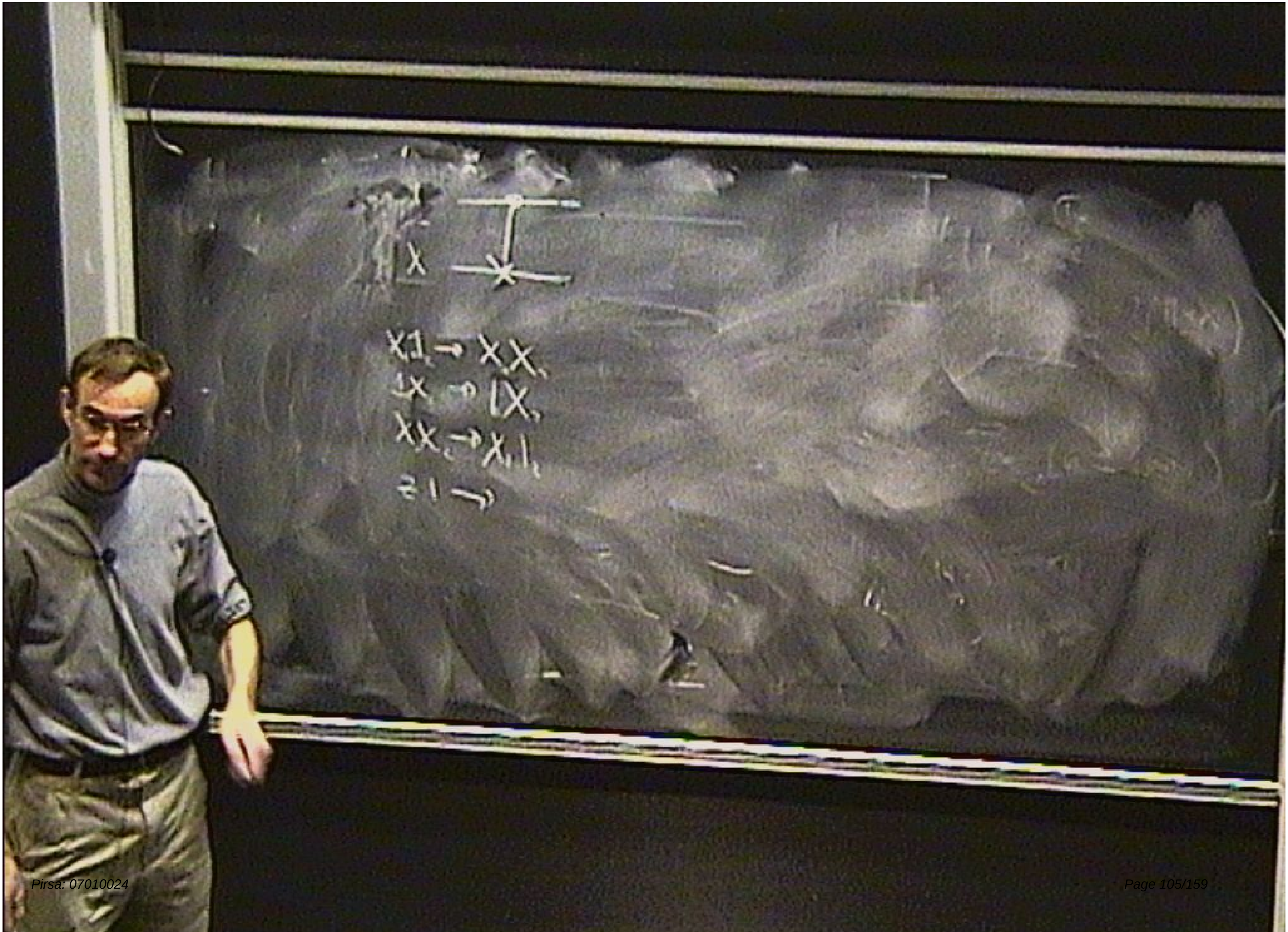


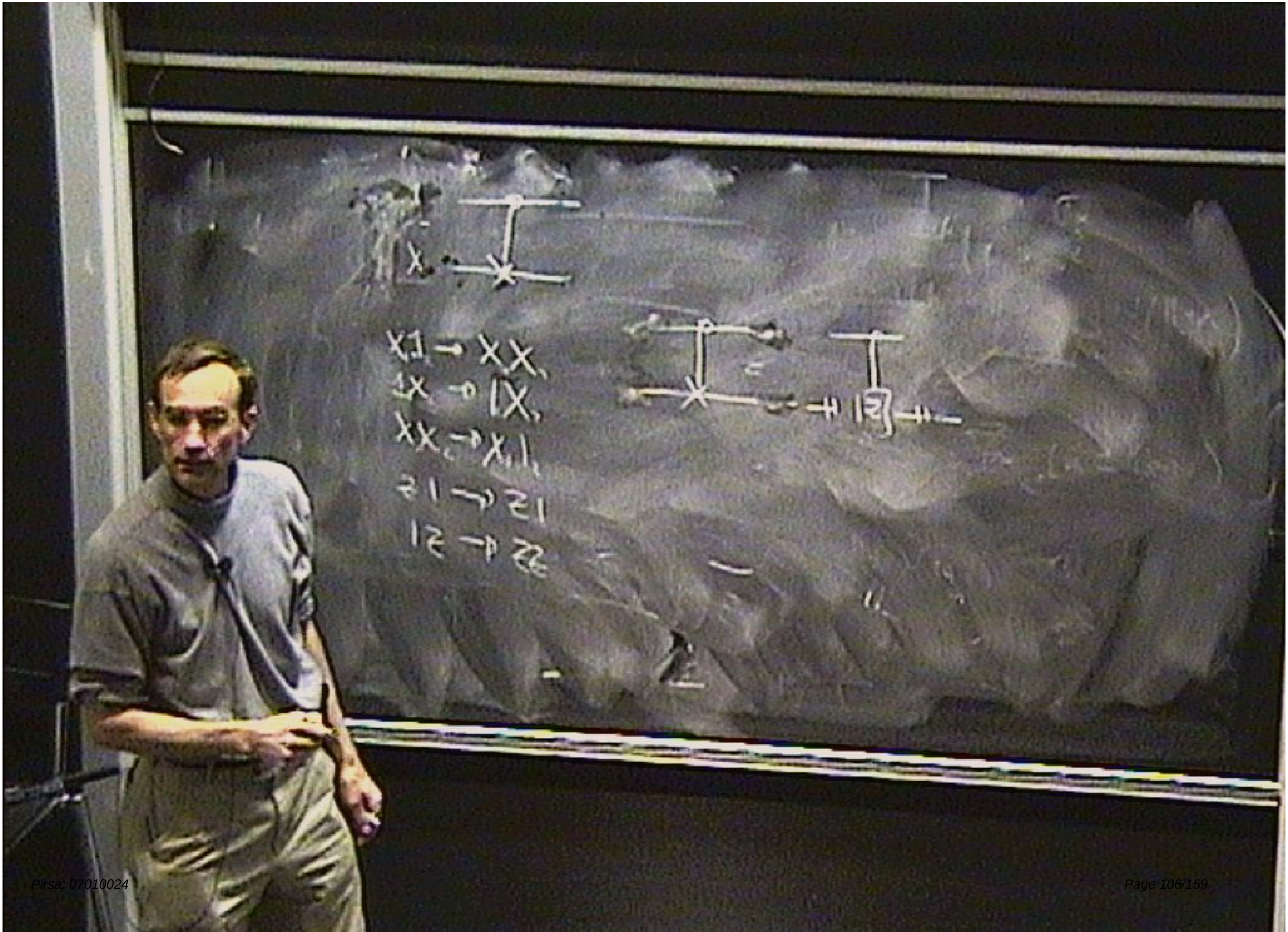


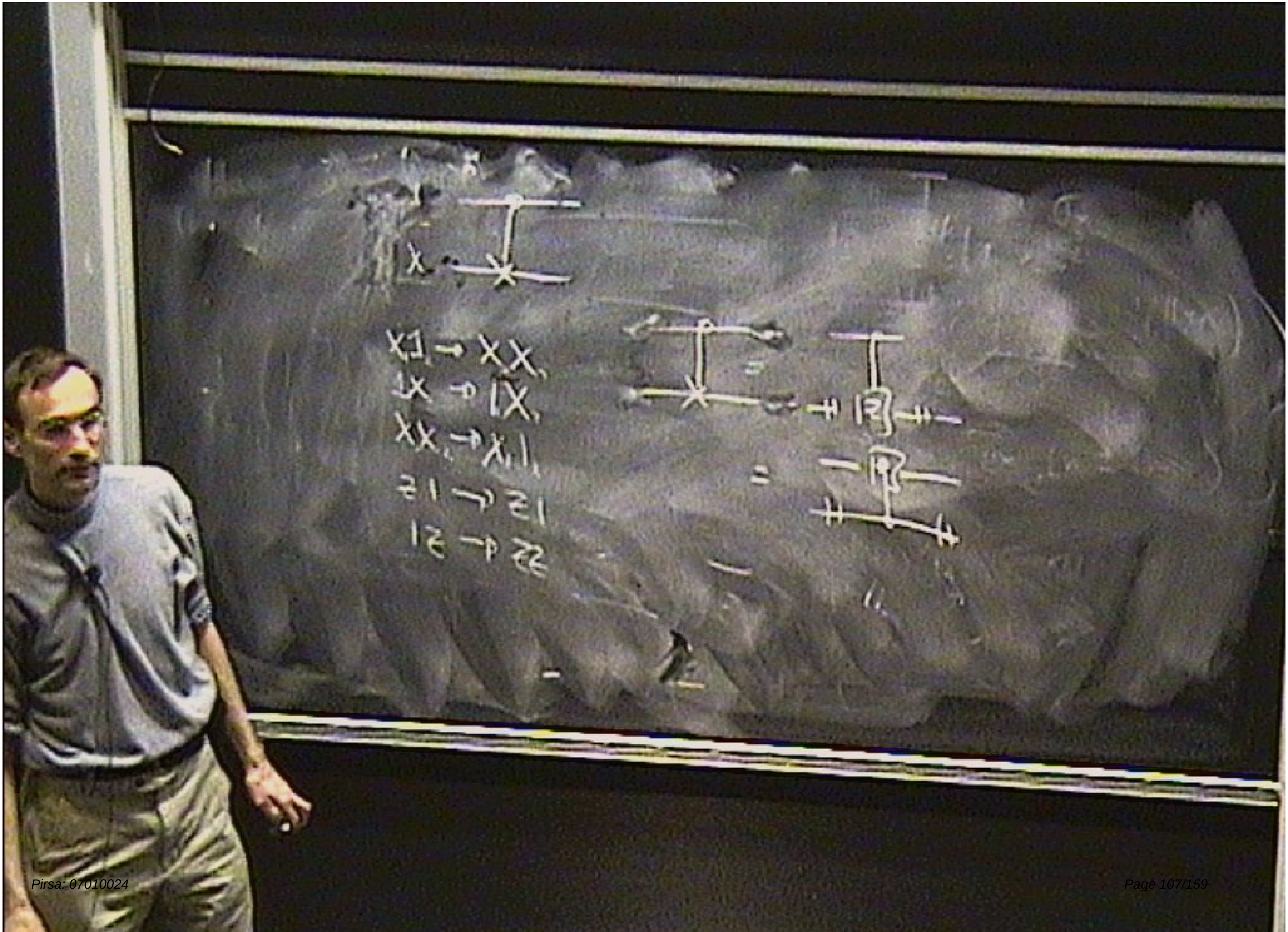


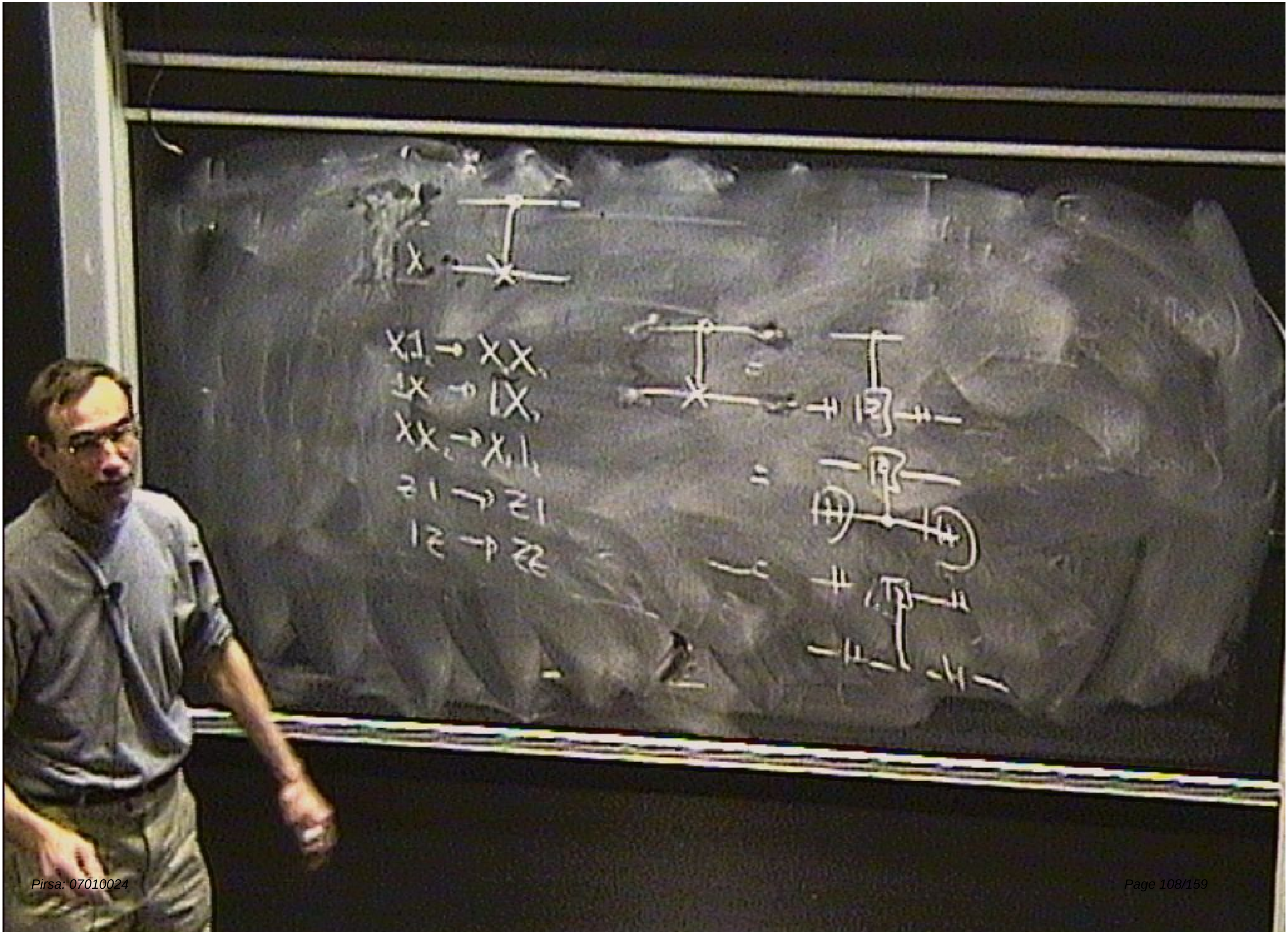


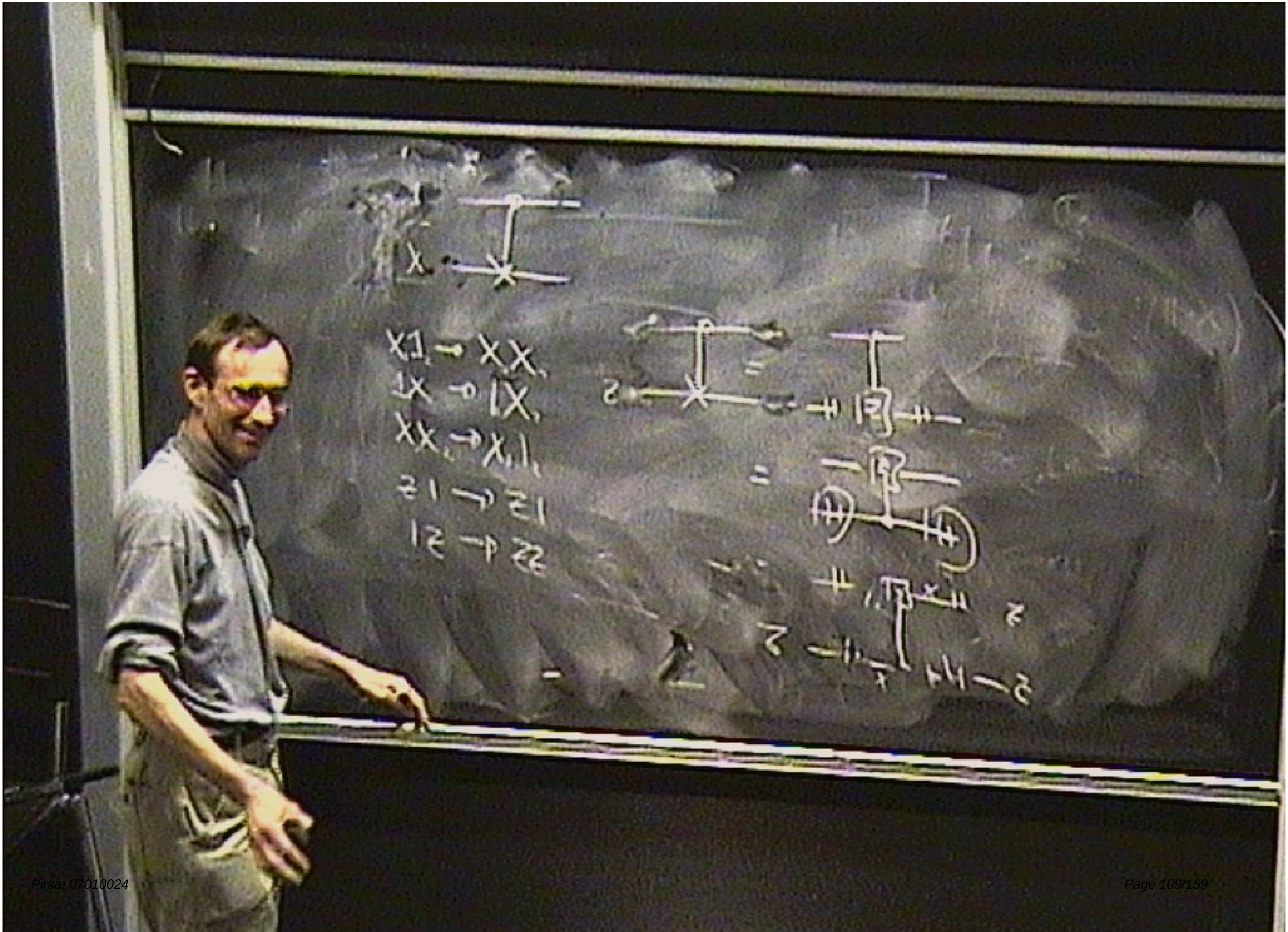

X1 → XX
1X → 1X
XX → X1

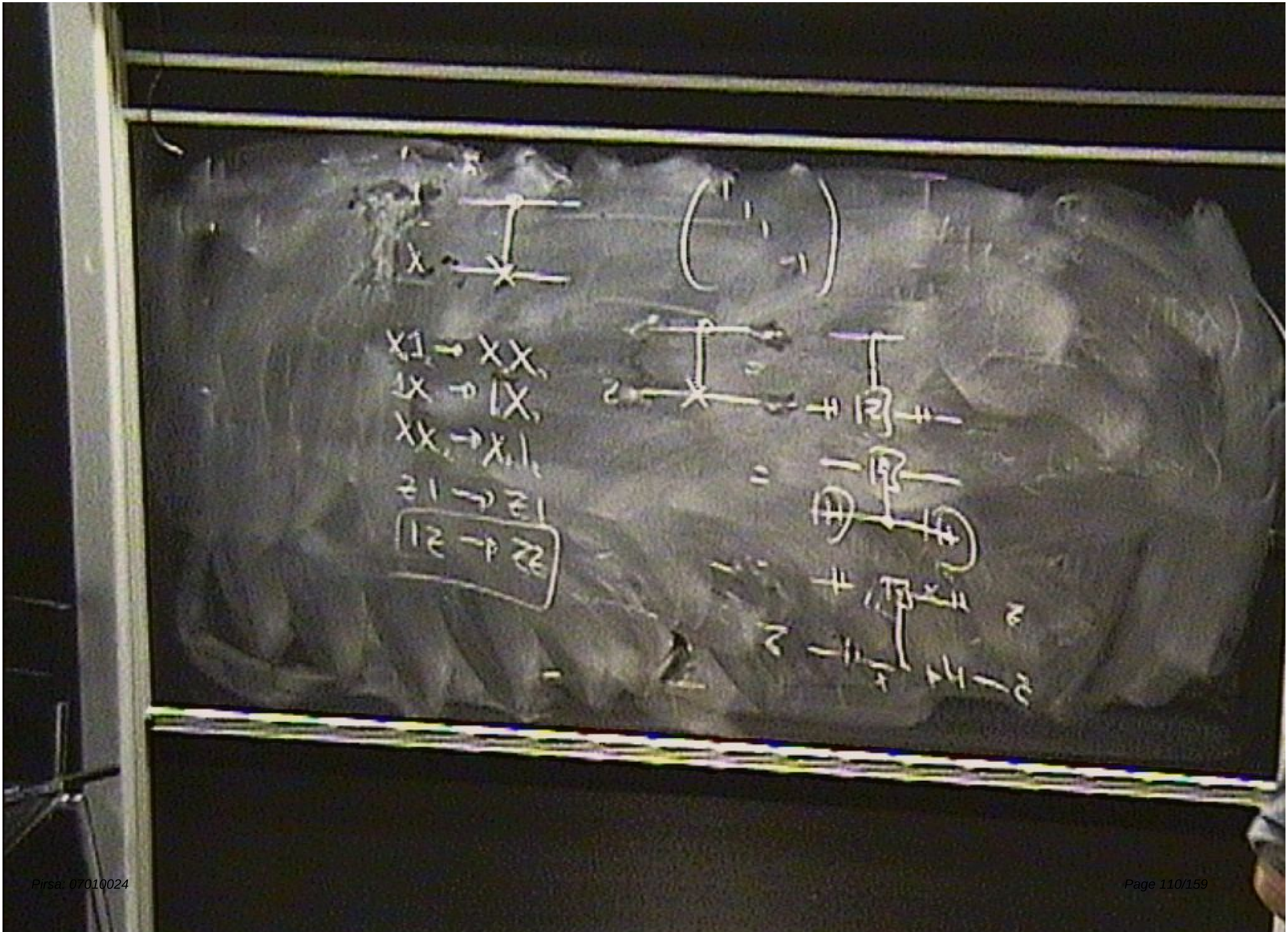

X1₂ → X₁X₂
1X₂ → 1X₂
X₁X₂ → X₁1₂

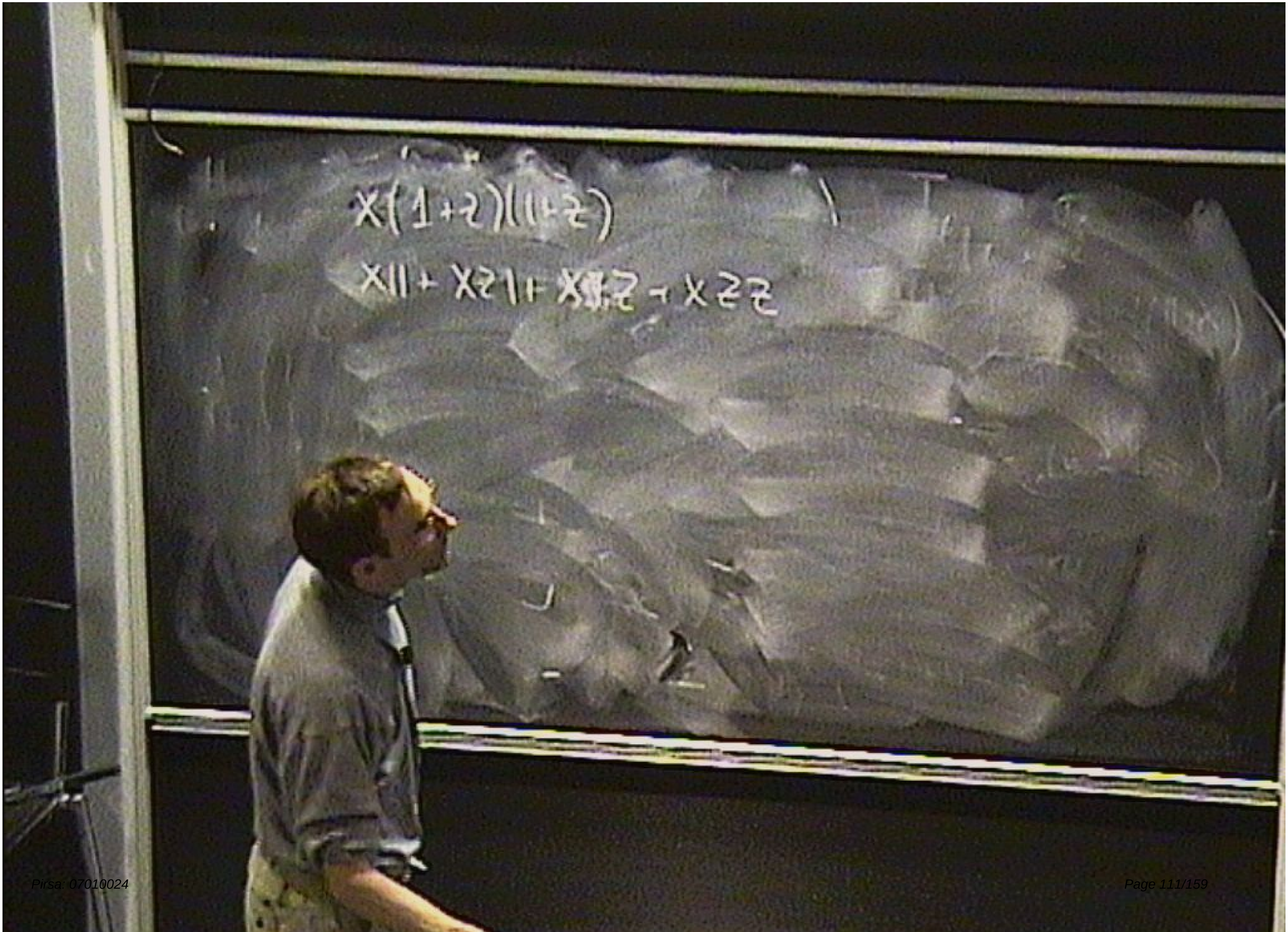

X1 → XX
1X → 1X
XX → X1
Z1 → Z1
1Z → ZZ

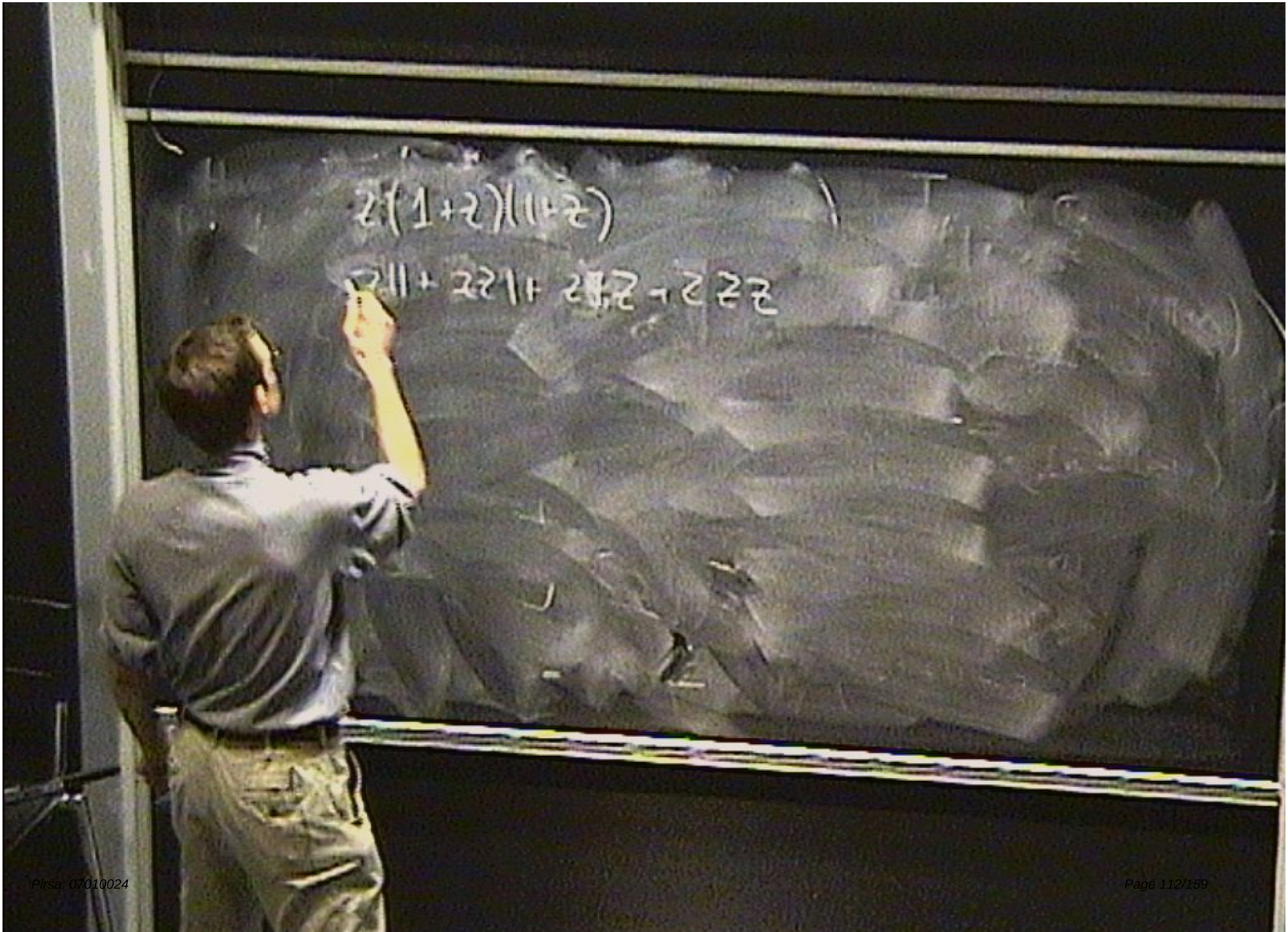

X1 → XX
1X → 1X
XX → X1
Z1 → Z1
1Z → ZZ

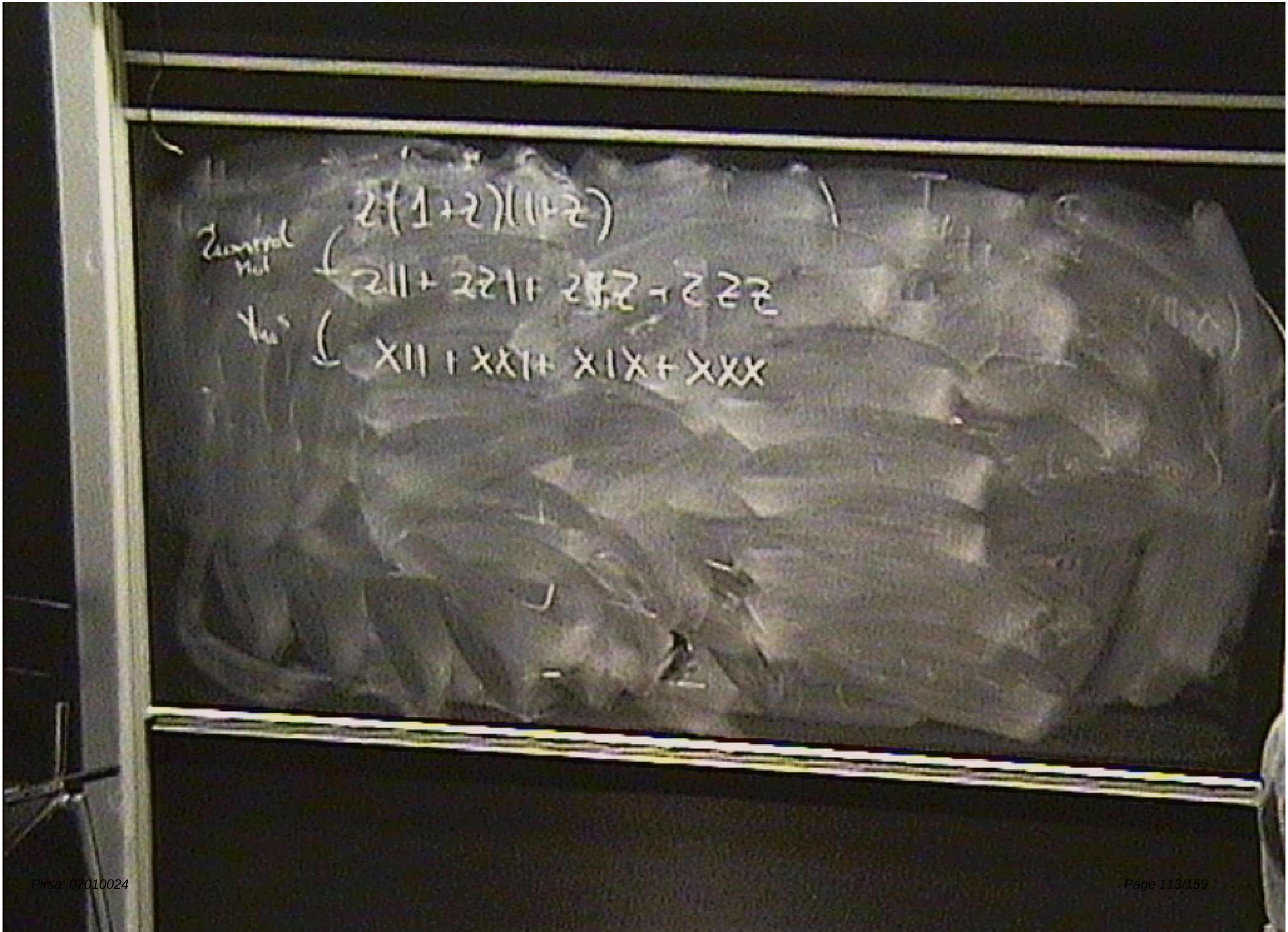

XI → XX
IX → IX
XX → XI
ZI → ZI
IZ → ZZ


X1 → XX
1X → 1X
XX → X1
Z1 → Z1
1Z → ZZ

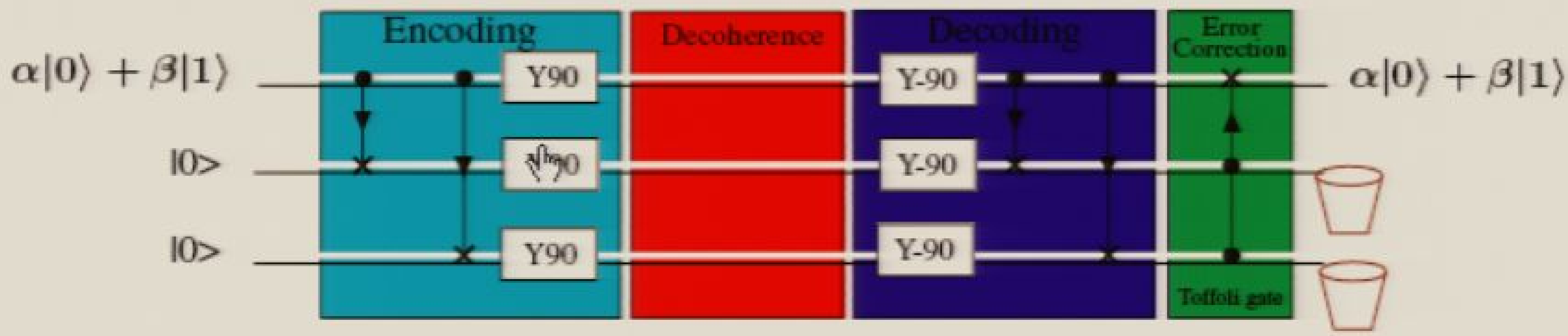

X1 → XX
1X → 1X
XX → X1
Z1 → Z1
1Z → ZZ


X(1+Z)(1+Z)
X11 + XZ1 + X Z + XZZ


z(1+z)(1+z)


z(1+z)(1+z)
z11 + zz1 + z1z + zzz
x11 + xx1 + x1x + xxx

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$\alpha|+++\rangle + \beta|---\rangle$

$\alpha|-++\rangle + \beta|+--\rangle$

$\alpha|+-+\rangle + \beta|-+-\rangle$

$\alpha|++-\rangle + \beta|--+\rangle$

$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$

$(\alpha|1\rangle + \beta|0\rangle)|11\rangle$

$(\alpha|0\rangle + \beta|1\rangle)|01\rangle$

$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

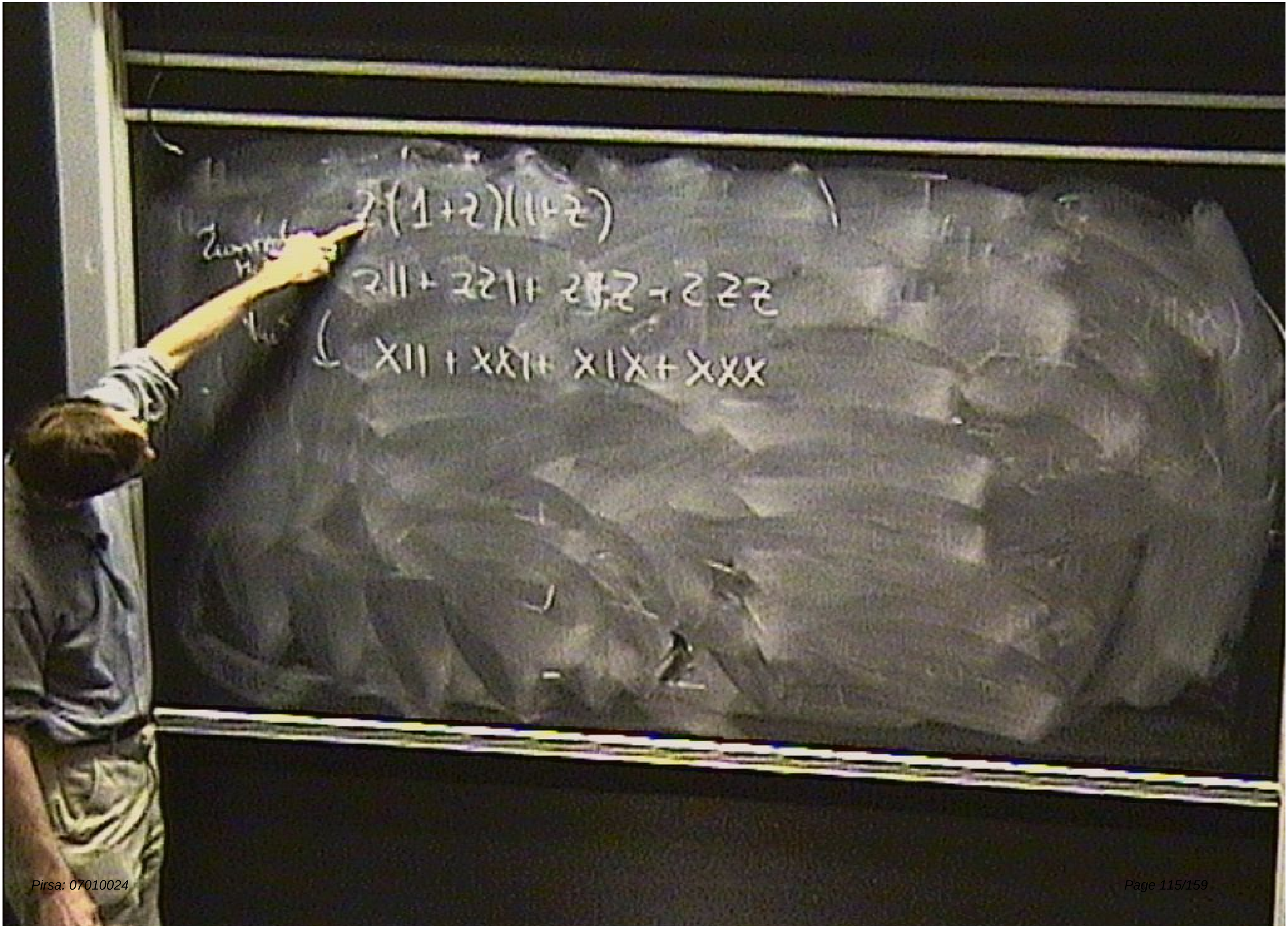
$$\rho_B \equiv \frac{1}{4}\left(I_x^1 + I_x^2 + I_x^3 + 4\, I_x^1 I_x^2 I_x^3\right)$$

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

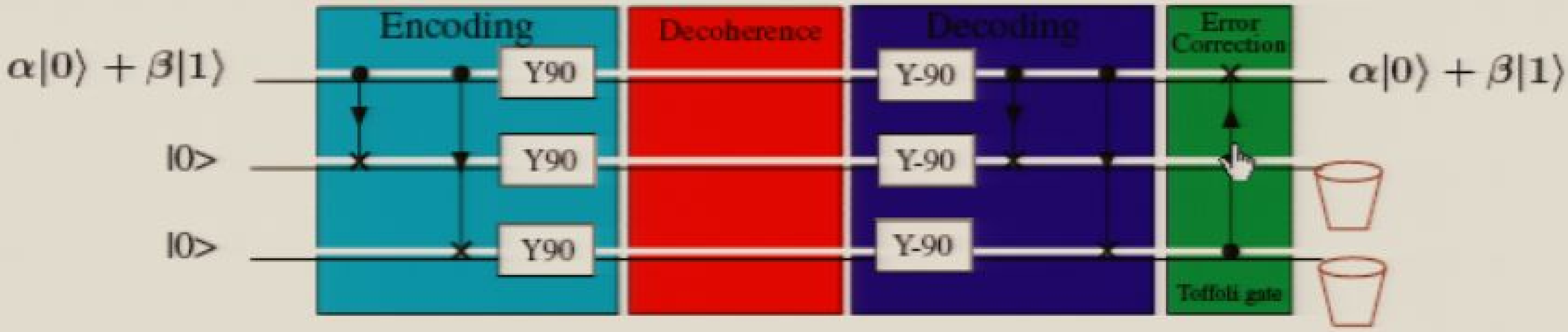
$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases}$$

$\sim 1 - 3\gamma^2$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$


Z(1+Z)(1+Z)
Z11 + ZZ1 + Z1Z + ZZZ
X11 + XX1 + X1X + XXX

Q. Error Correction for Phase



$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$\alpha|++ +\rangle + \beta|---\rangle$
$\alpha|-++\rangle + \beta|+--\rangle$
$\alpha|+-+\rangle + \beta|-+-\rangle$
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$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$

$$\rho_E^1 \equiv \tfrac{1}{8} I_z^1 (9e^{-t/\tau} - e^{-9t/\tau}) \approx I_z^1 (1 - \frac{9}{2} t^2/\tau^2 + \cdots)$$

$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 11 \\ 01 \\ 10 \end{cases}$

$\sim 1 - 3\gamma^2$

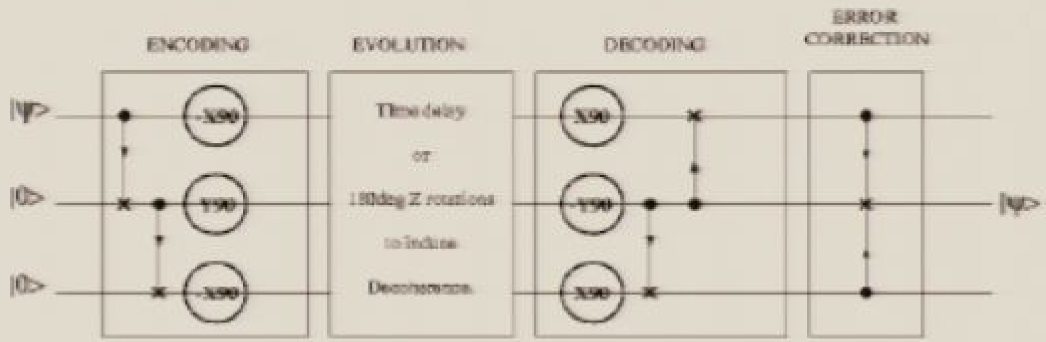
Control-Not

$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$

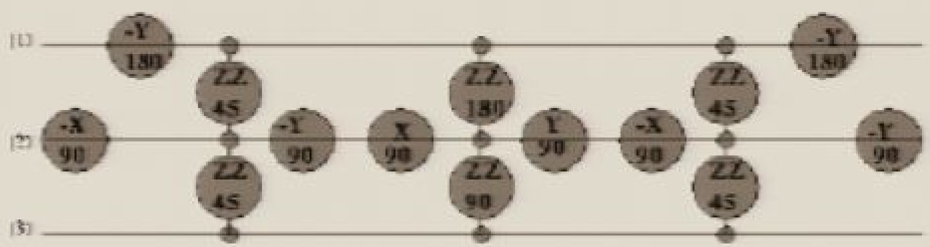
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Phase QEC NMR circuit

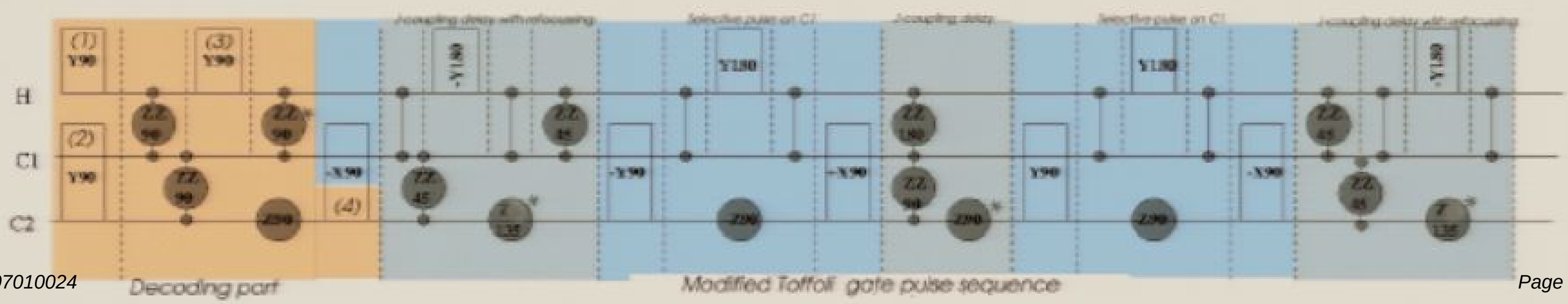
NMR implementation of the decoding and error correction:

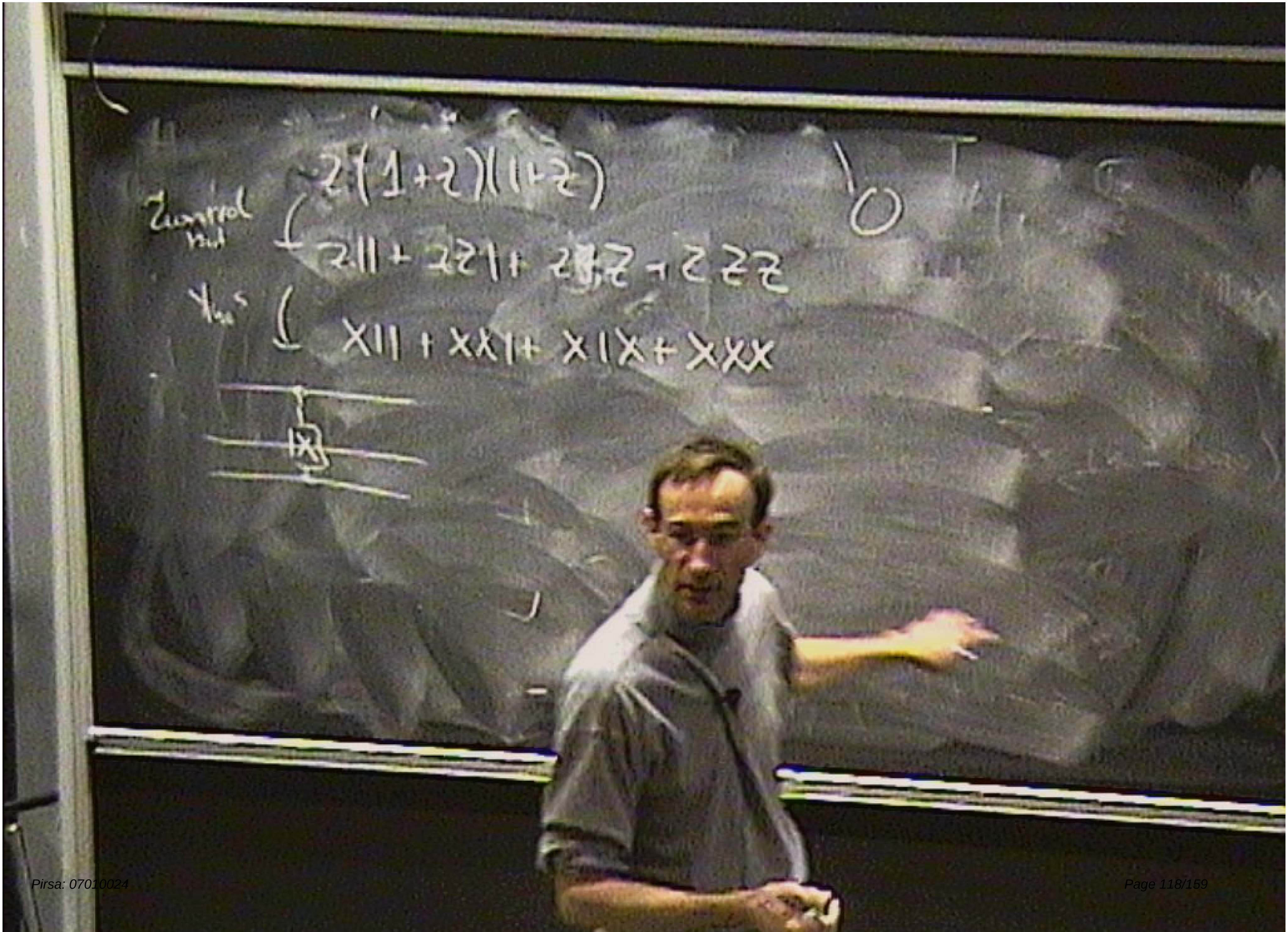


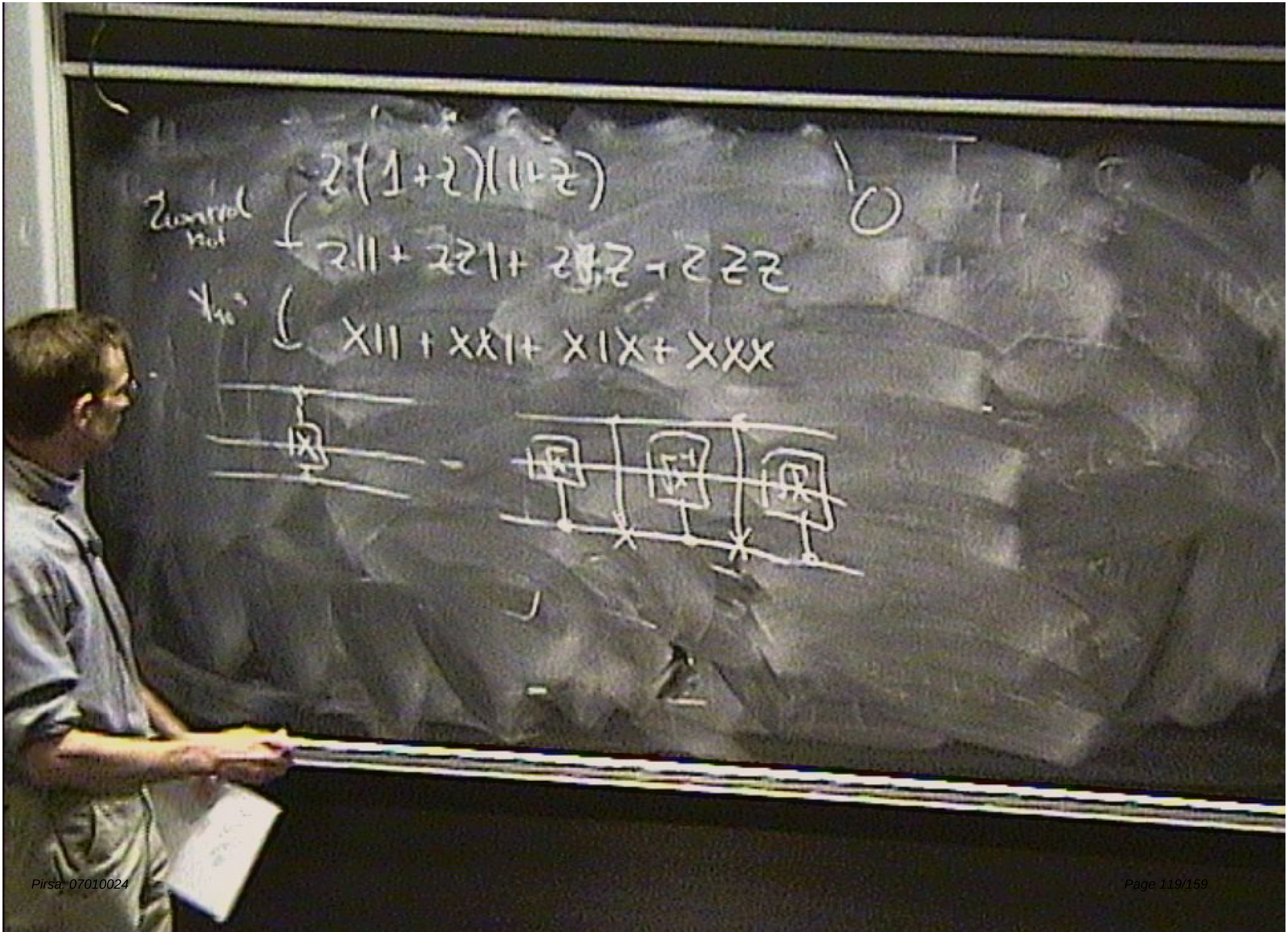
Toffoli gate:

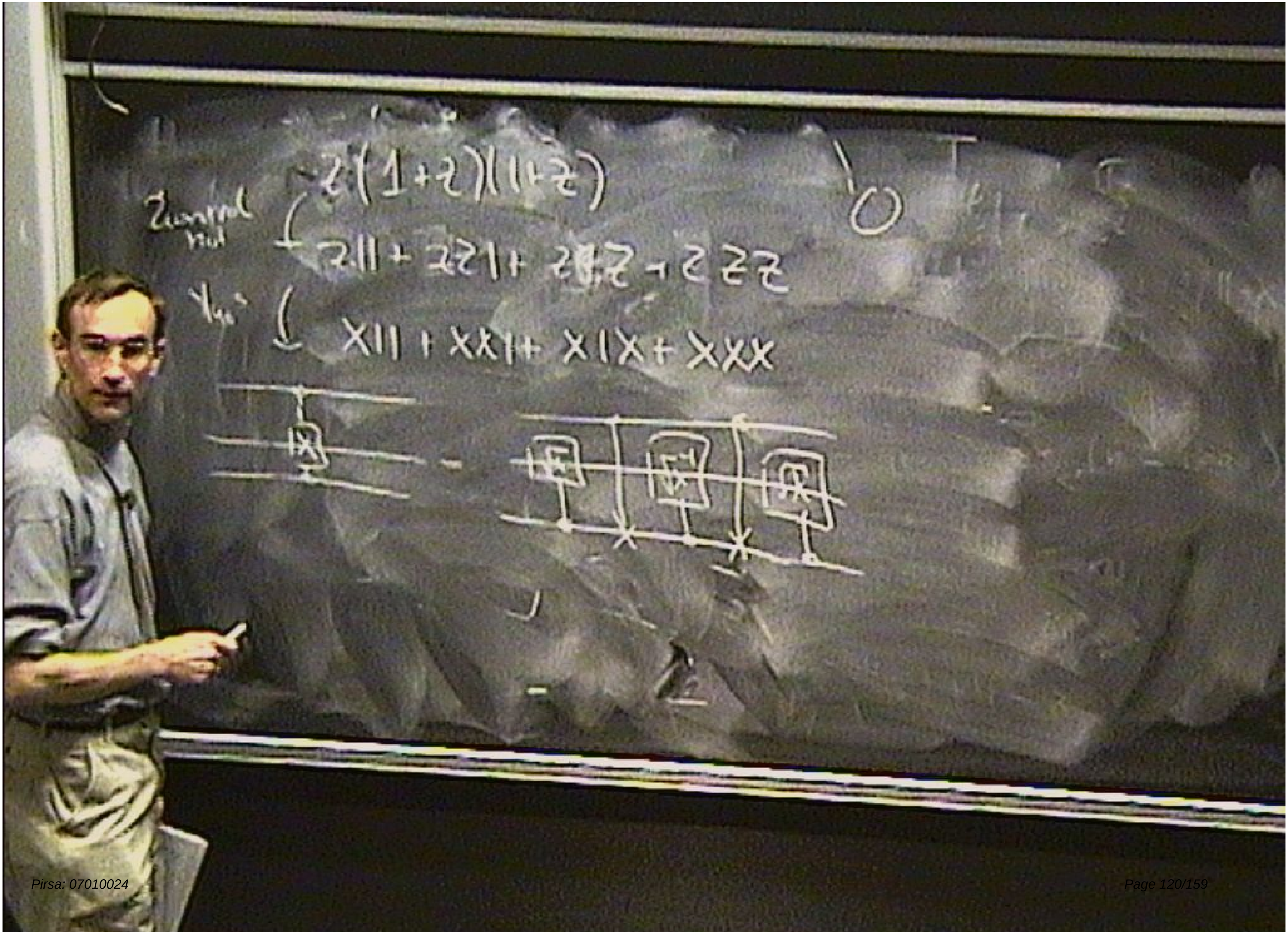


and the full decoding and Toffoli, including some optimization




Z(1+Z)(1+Z)
Z11 + ZZ1 + Z1Z + ZZZ
X11 + XX1 + X1X + XXX


X11 + XX1 + X1X + XXX


z(1+z)(1+z)
z11 + zz1 + z1z + zzz
X11 + XX1 + X1X + XXX

Phase QEC NMR circuit

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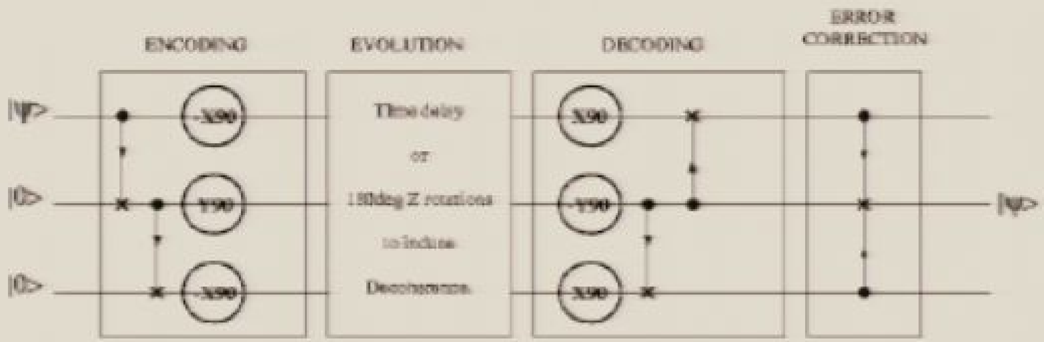
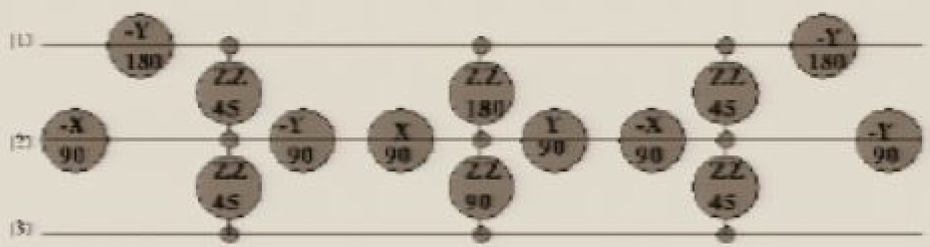


Toffoli gate:



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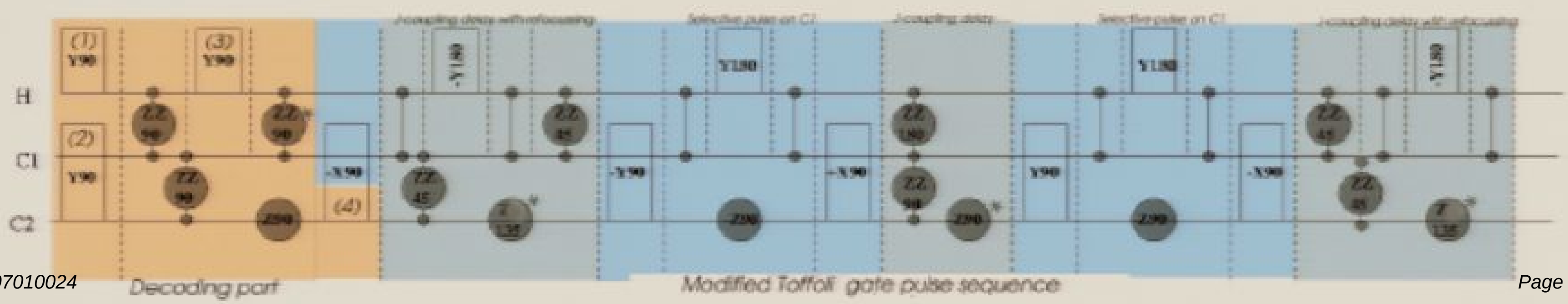
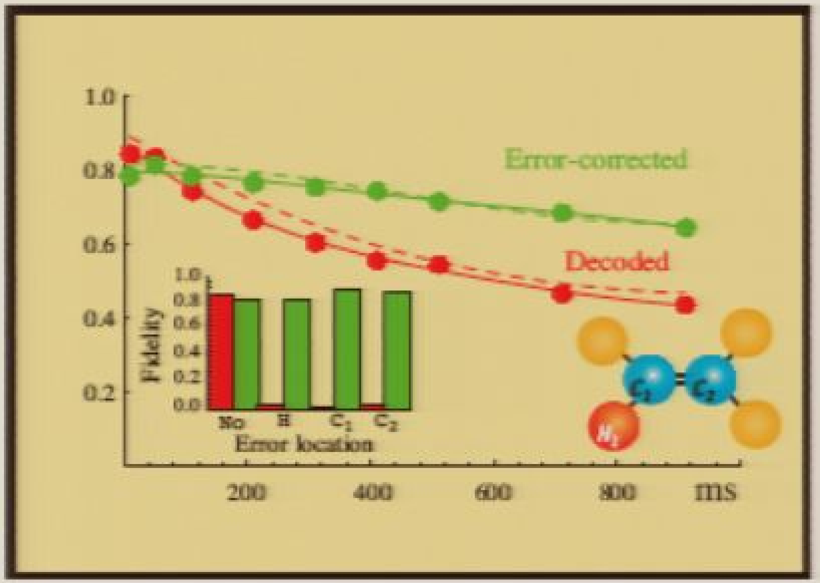


Experimental results



The demonstration of quantum error correction is in the shape of the green curve which does not have the first order error. The curve is much flatter than the red one.

Experimental Quantum Error Correction:
D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek,T. F. Havel and S. S. Somaroo, PRL 81, 2152, 1998

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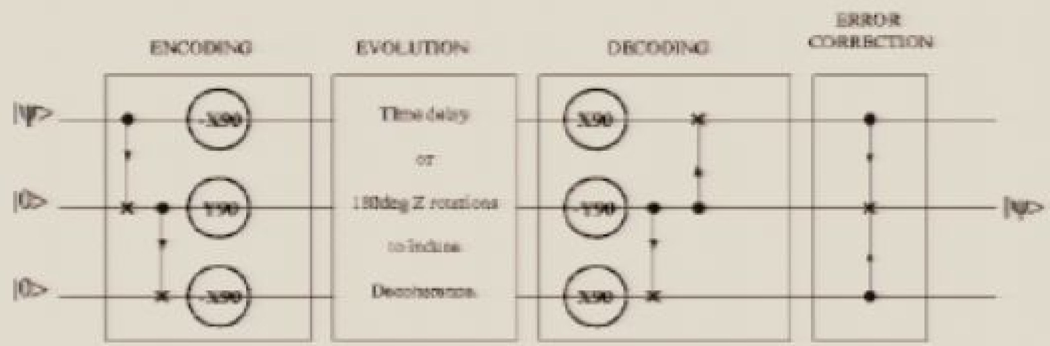
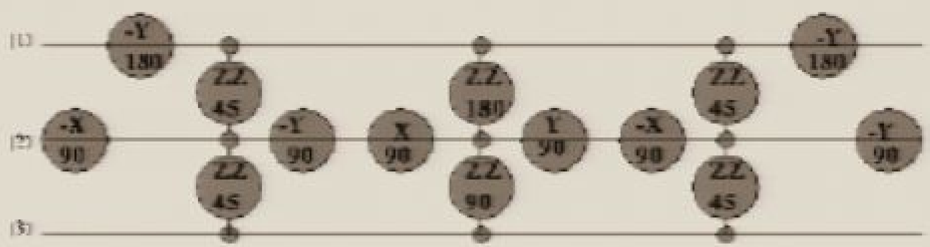


Toffoli gate:



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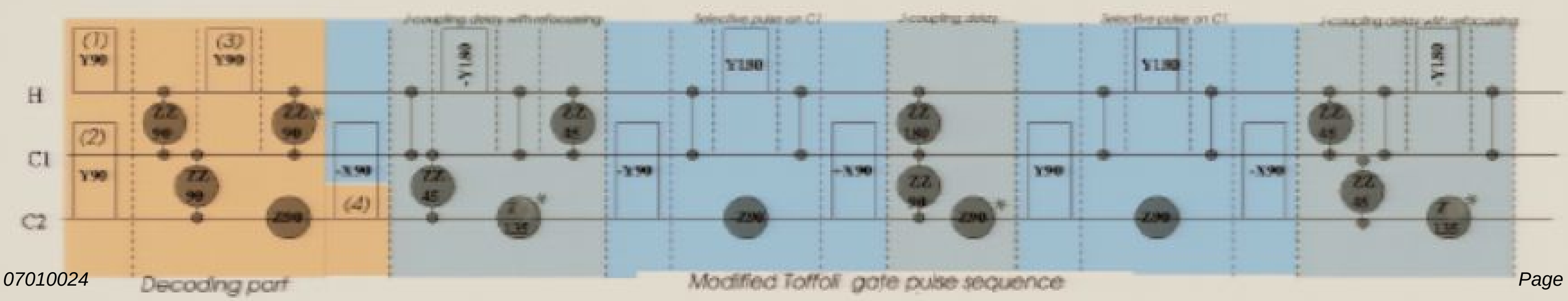

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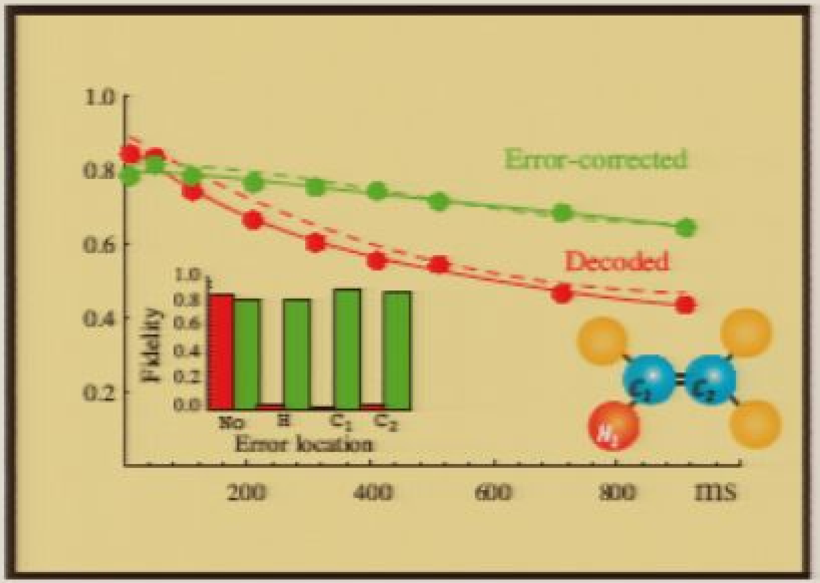

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$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

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$\alpha|+++\rangle + \beta|---\rangle$

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$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$

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$$\rho_E^1 \equiv \tfrac{1}{8} I_z^1 (9e^{-t/\tau} - e^{-9t/\tau}) \approx I_z^1 (1 - \frac{9}{2} t^2/\tau^2 + \cdots)$$

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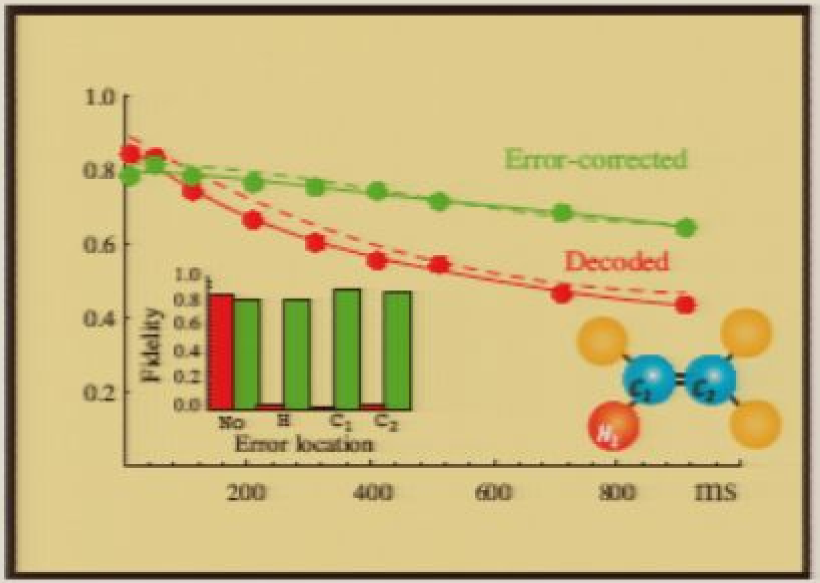
Control-Not

$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Experimental results



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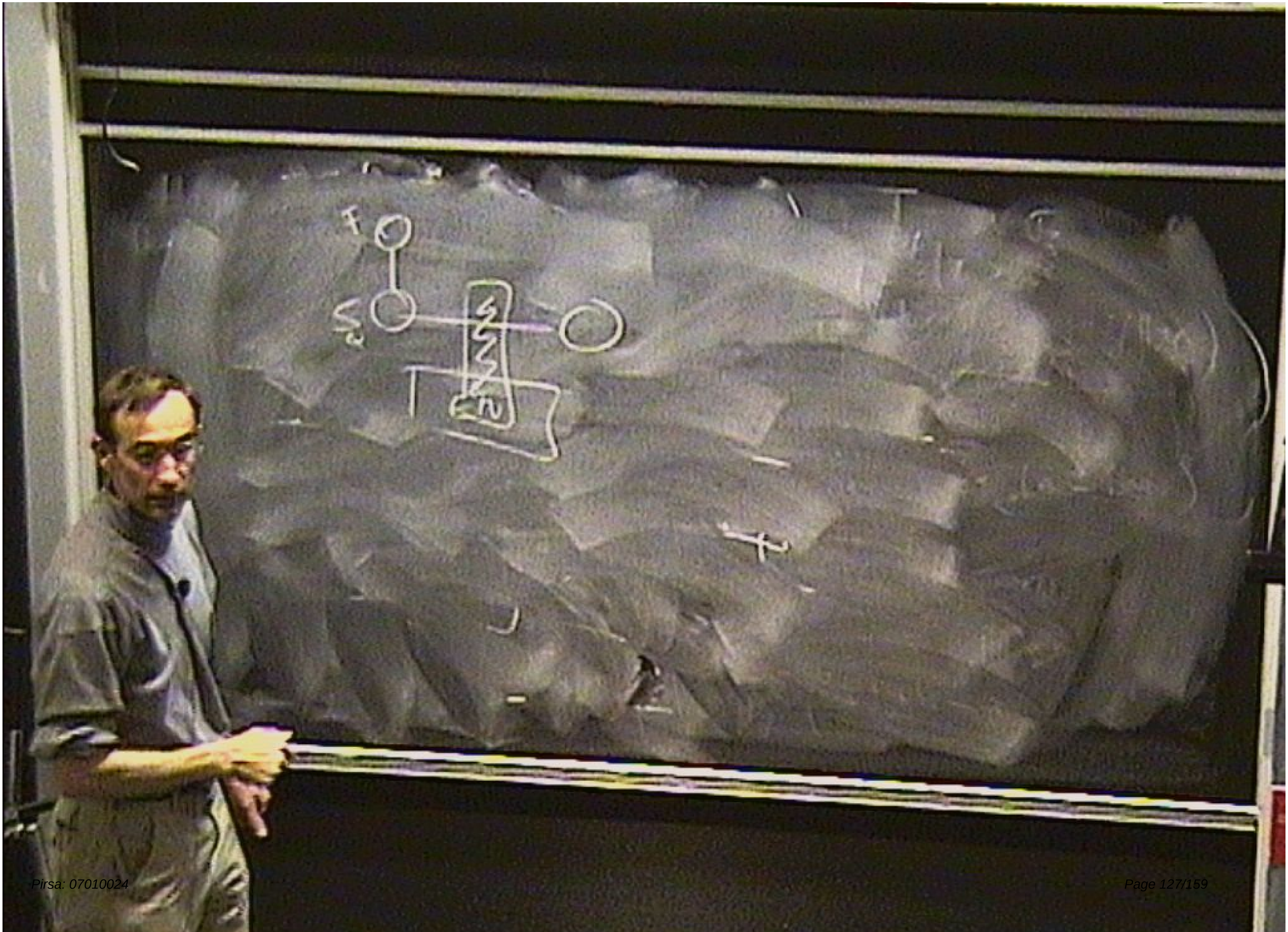
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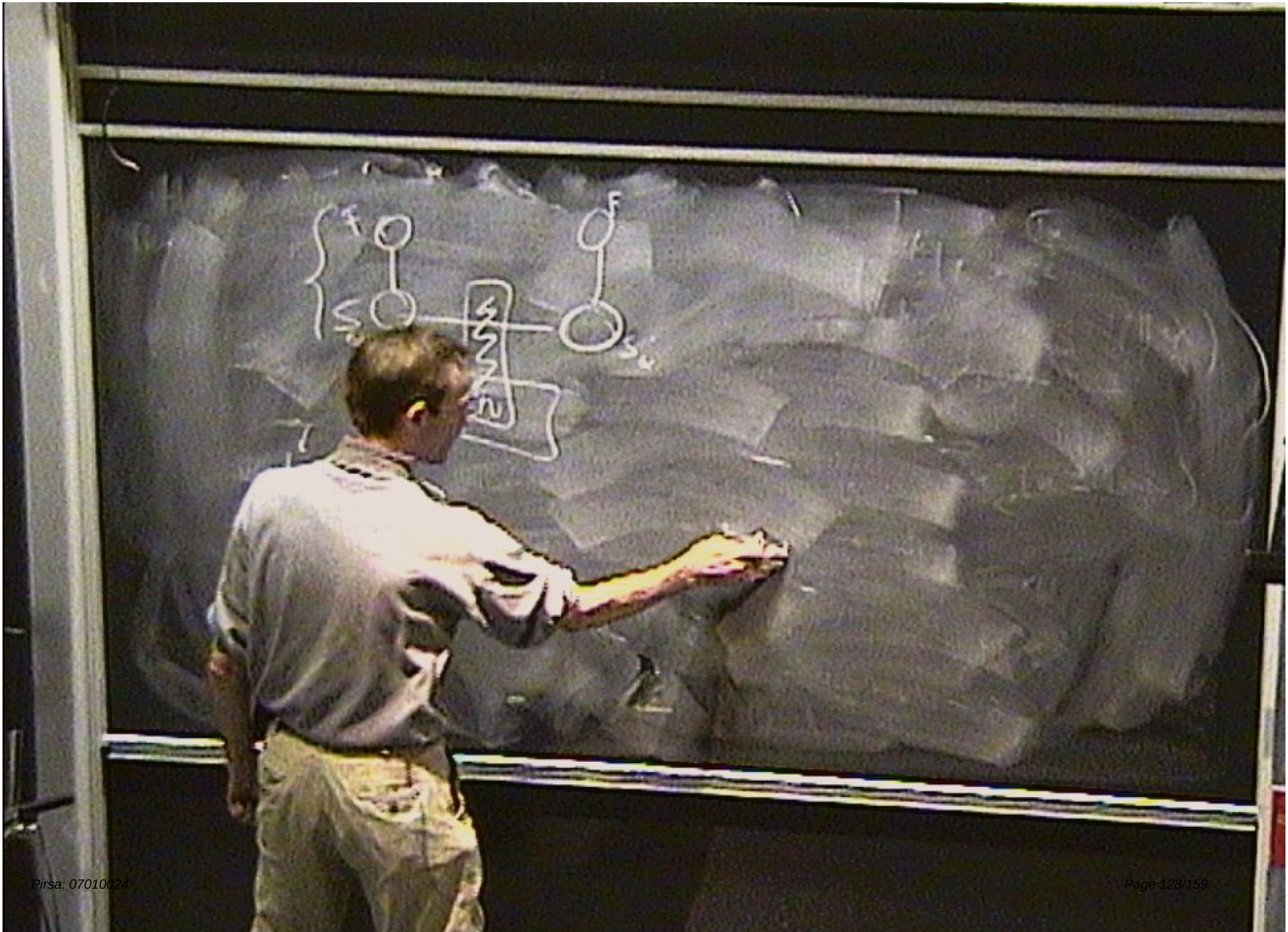
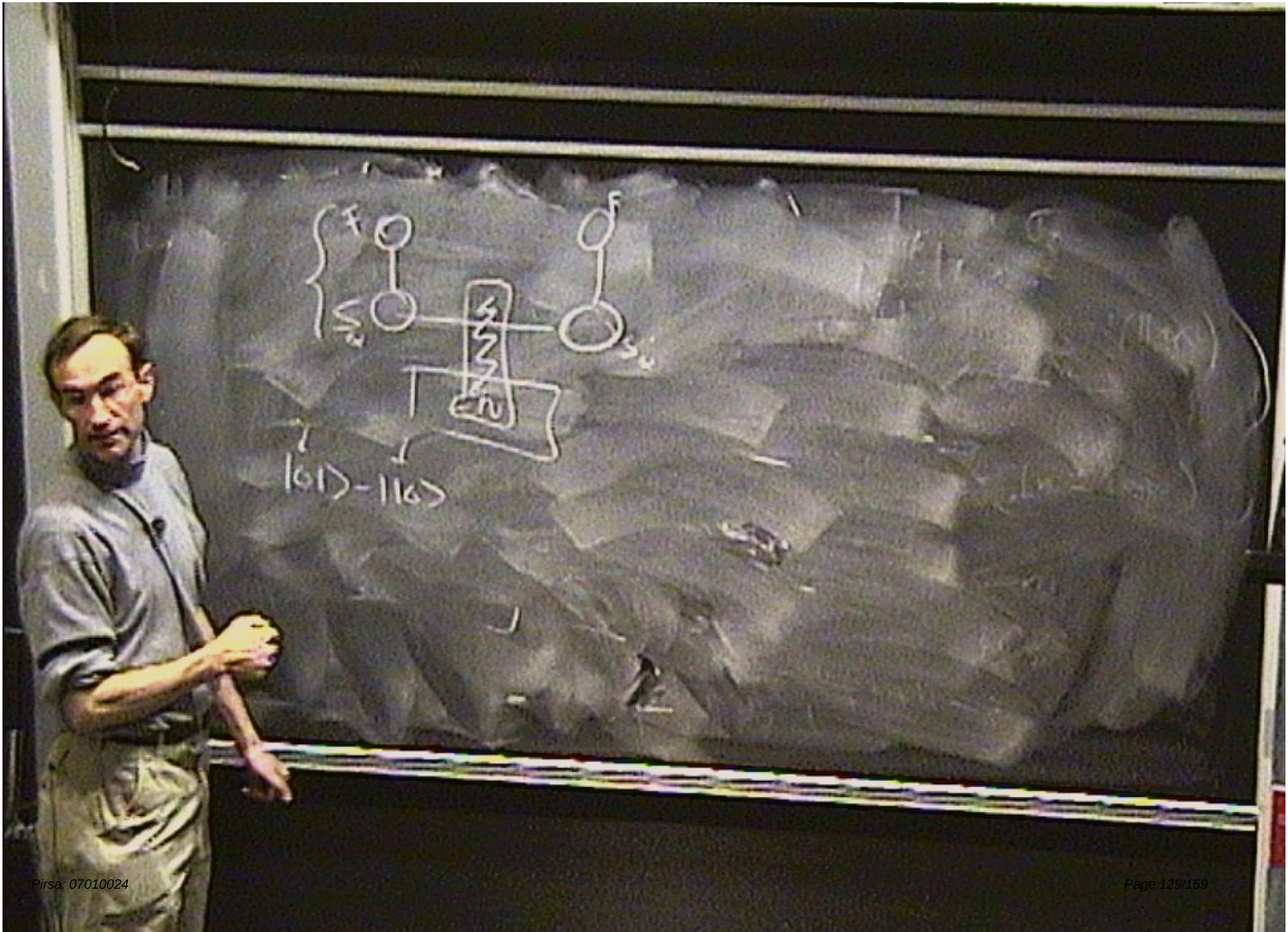

F
F
S
S


F
F
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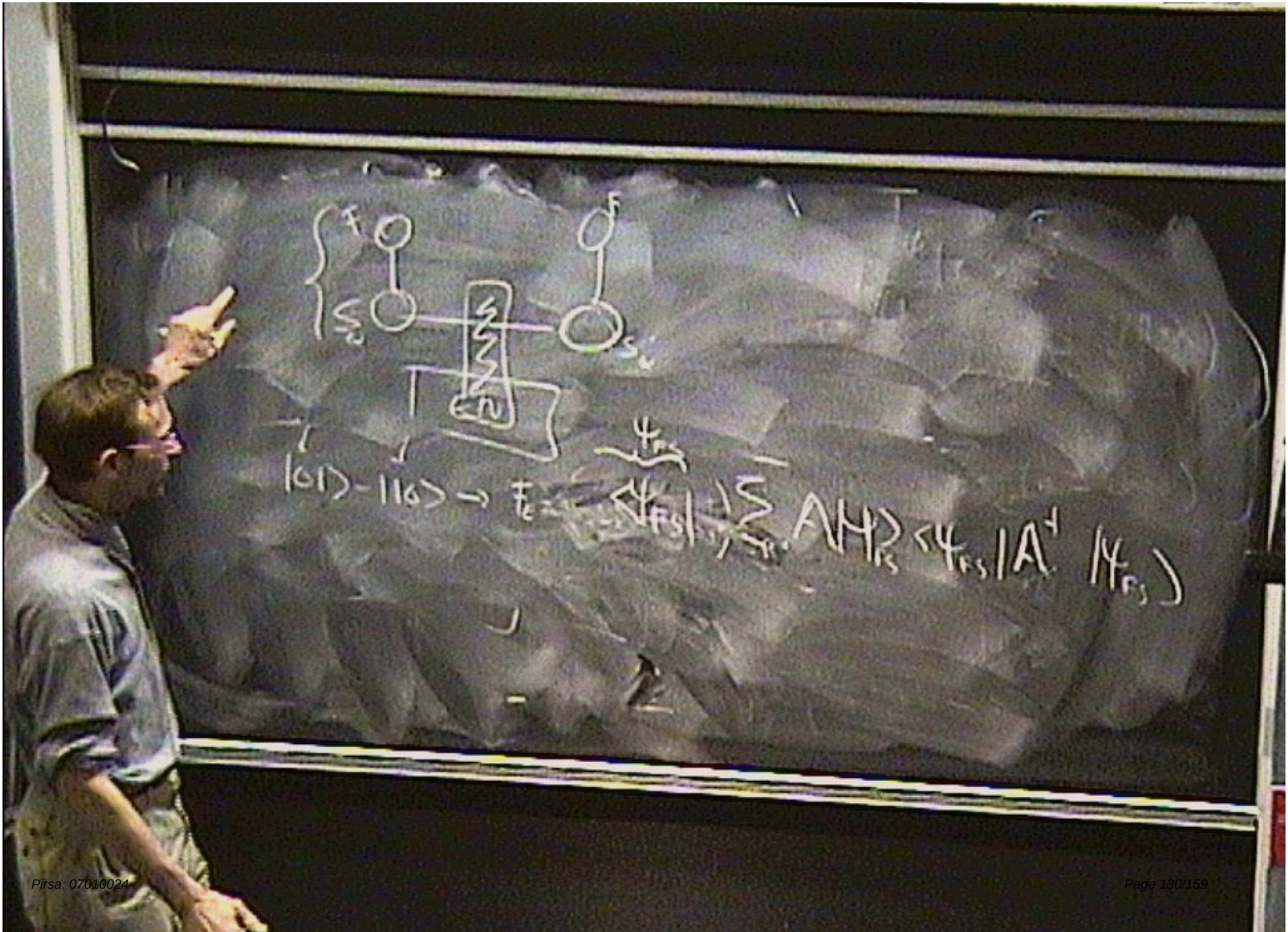
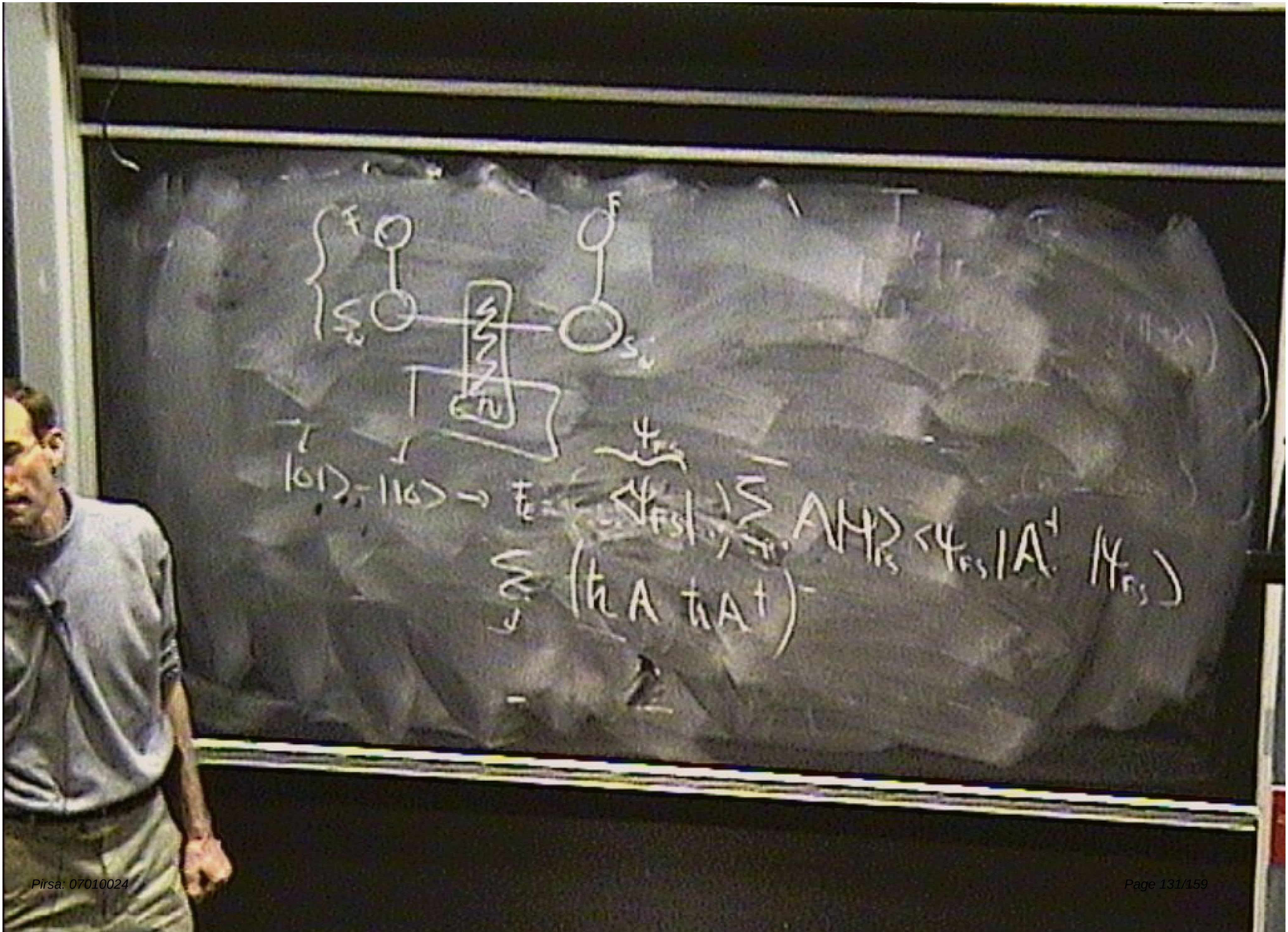
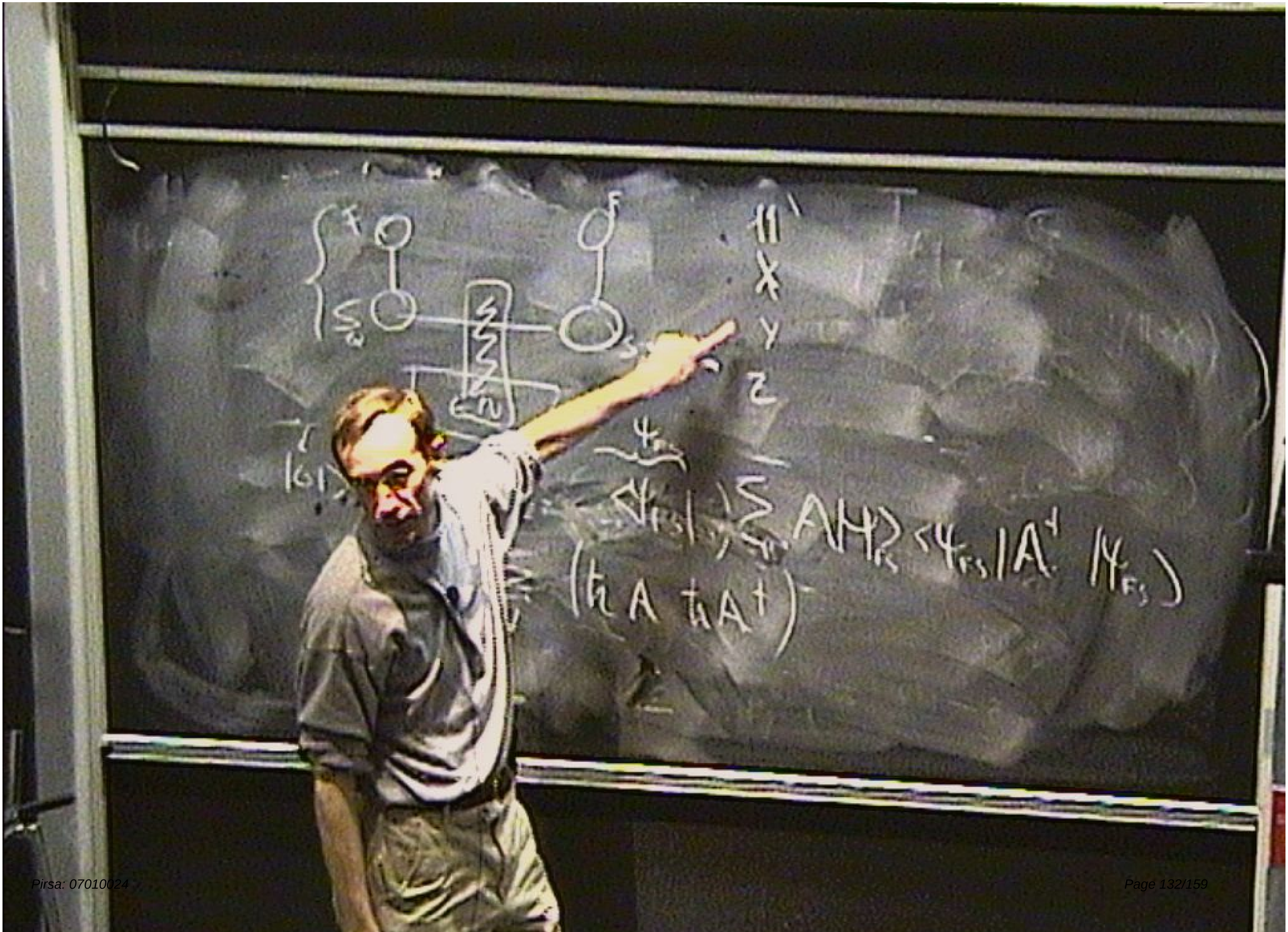
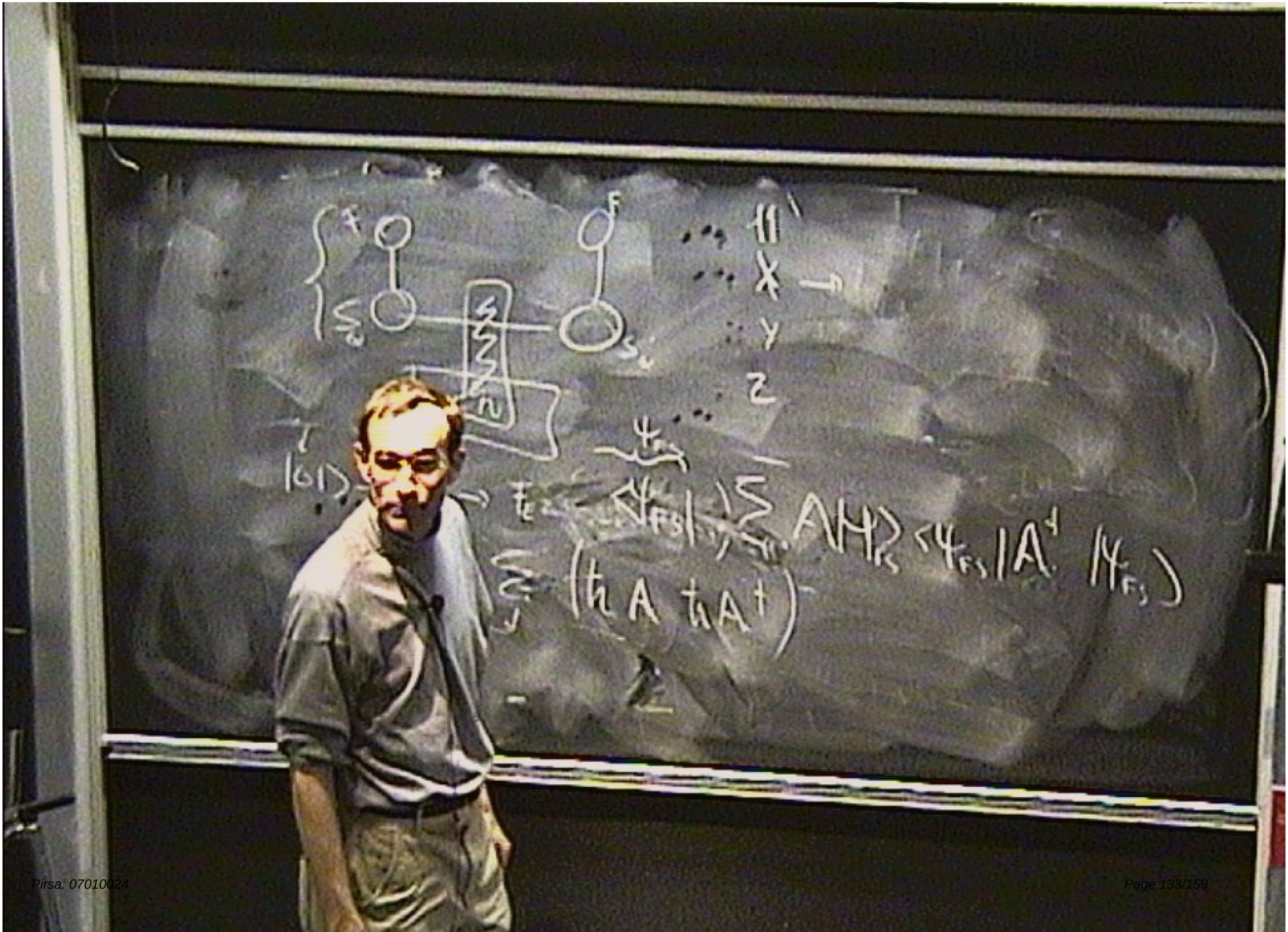
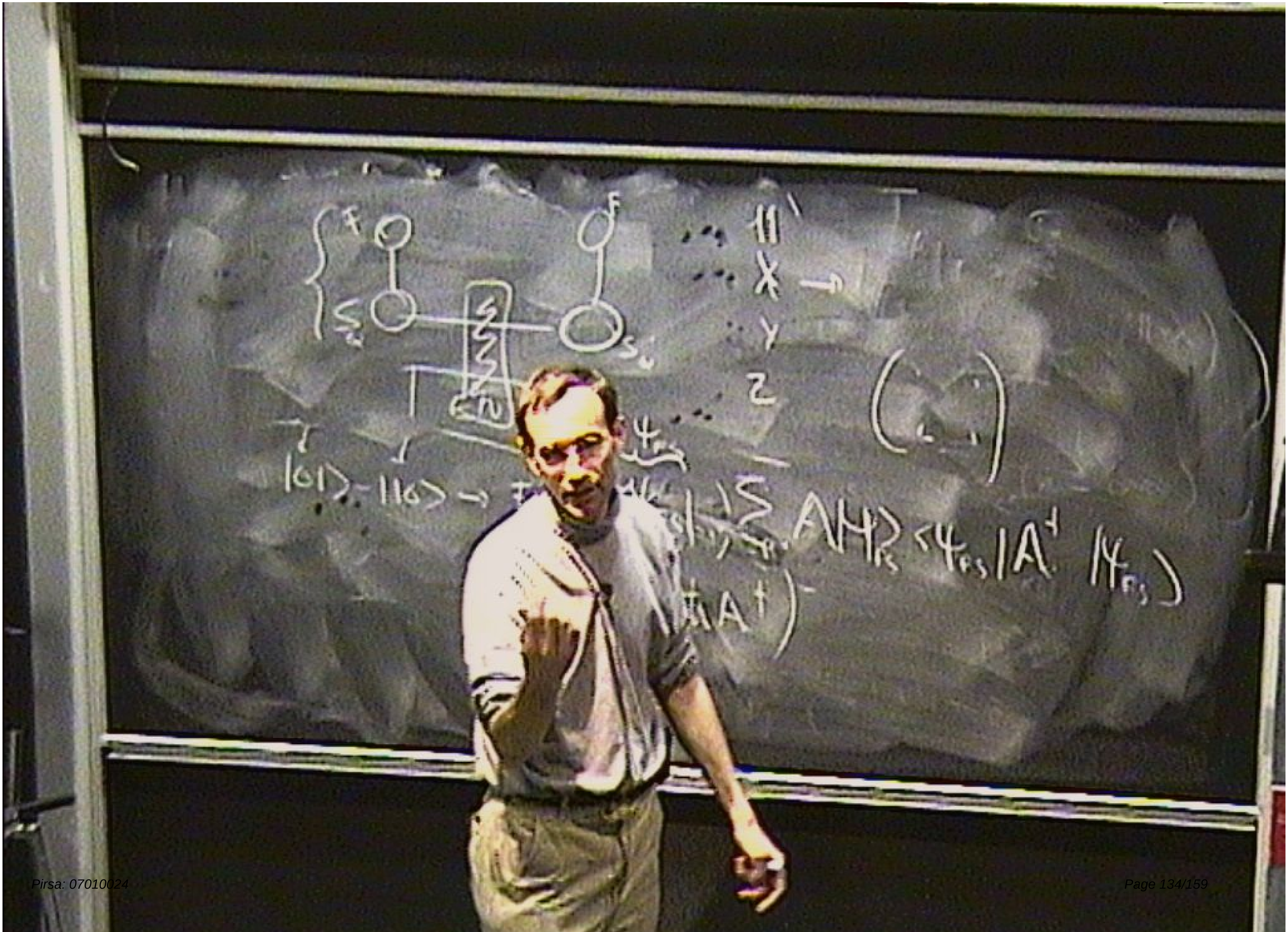
|01> - |10>











Phase QEC NMR circuit

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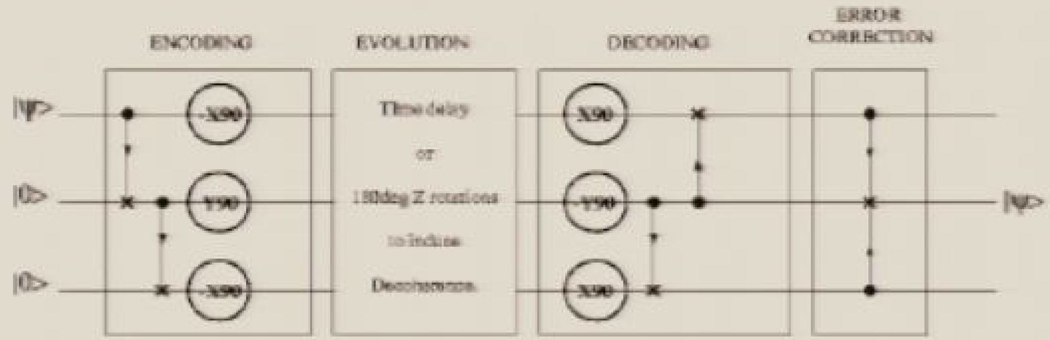
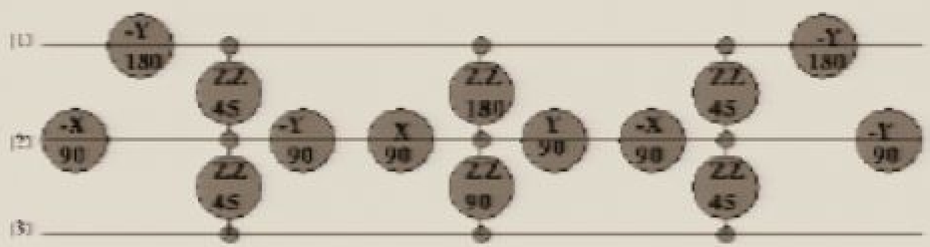


Toffoli gate:



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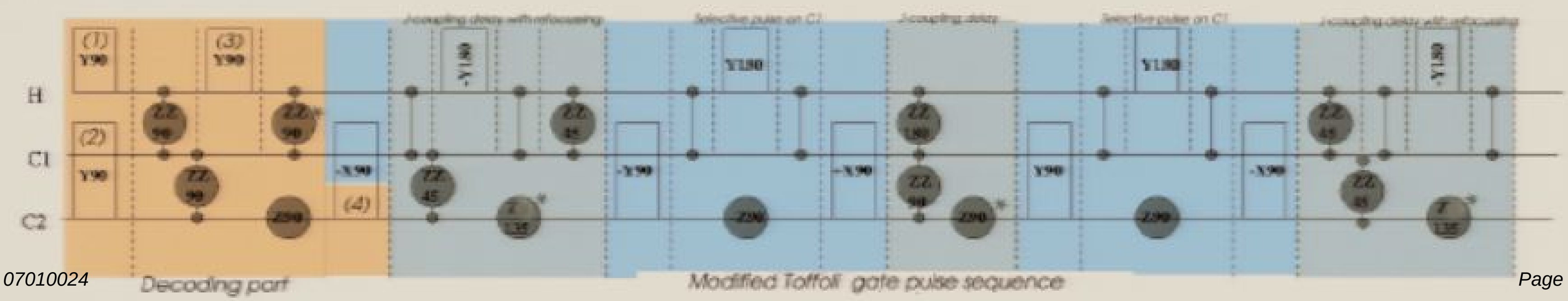
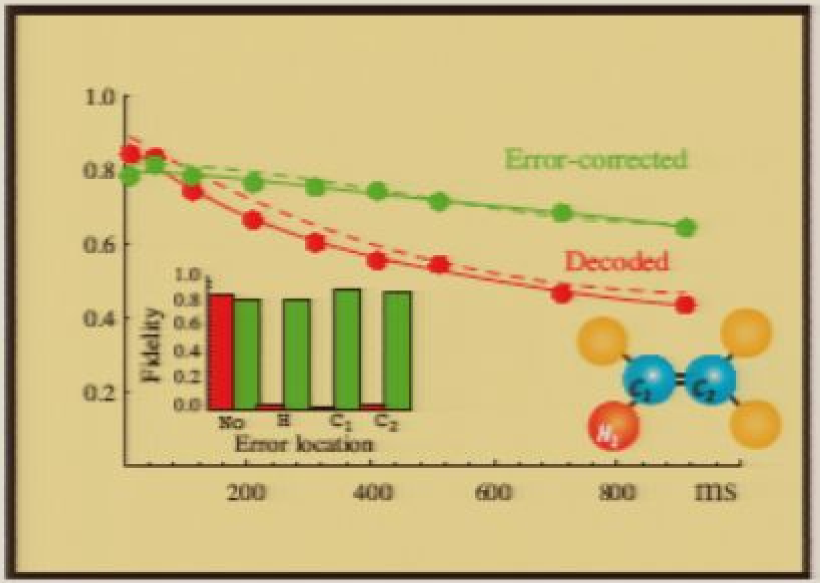


Experimental results



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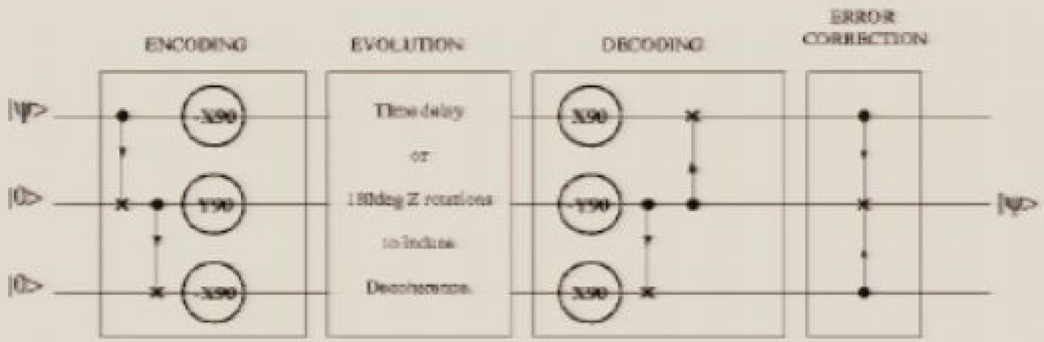
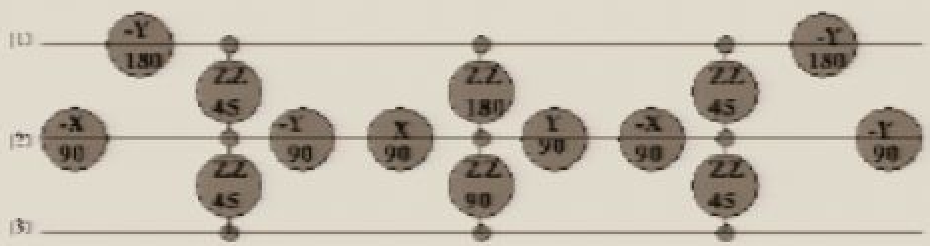
Phase QEC NMR circuit

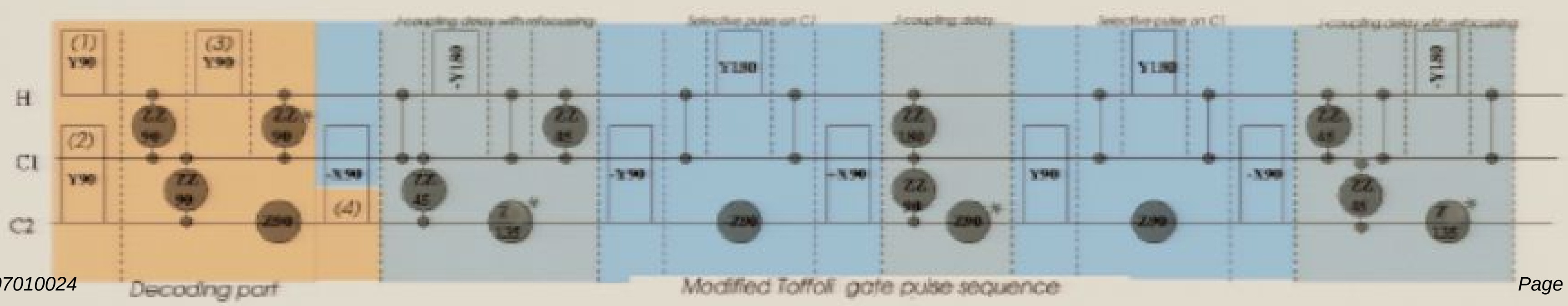
NMR implementation of the decoding and error correction:



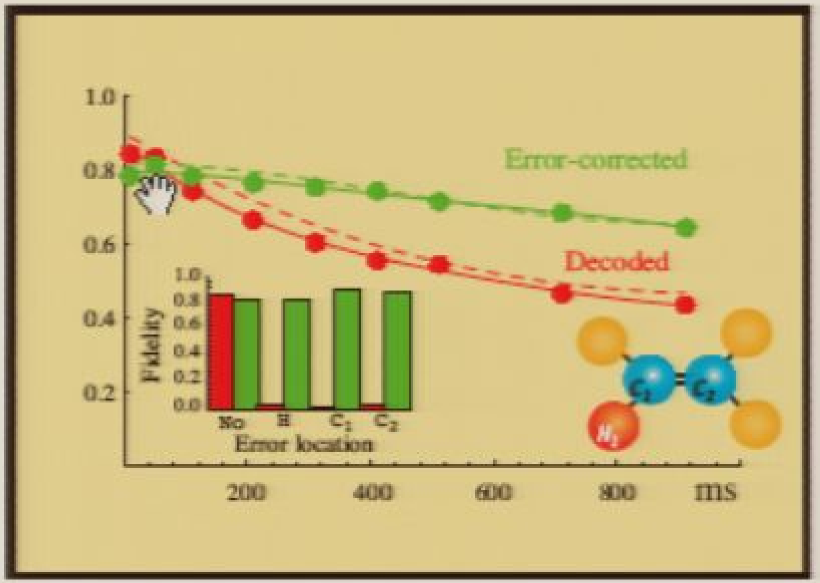
Toffoli gate:



and the full decoding and Toffoli, including some optimization



Experimental results



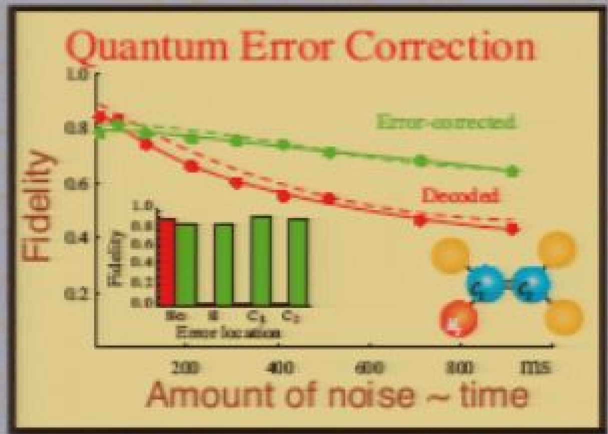
The demonstration of quantum error correction is in the shape of the green curve which does not have the first order error. The curve is much flatter than the red one.

Experimental Quantum Error Correction:
D. G. Cory, M. D. Price, W. Maas, E. Knill, R. Laflamme, W. H. Zurek, T. F. Havel and S. S. Somaroo, PRL 81, 2152, 1998

Q. Error Correction for Phase

Experiments

$\alpha|0\rangle$ $\beta|1\rangle$



Science
Top 10 breakthroughs of the year
Science 282, 2156, 1998

Cont

Exper
D. G.
R. La
S. S. Somaroo, PRL 81, 2152, 1998

$$\rho^1_E \equiv \frac{1}{8} I^1_z (9e^{-t/\tau} - e^{-9t/\tau}) \approx I^1_z (1 - \frac{9}{2} t^2/\tau^2 + \cdots)$$

$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

11
01
10

Q. Error Correction for Phase

$\alpha|0\rangle + \beta|1\rangle$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\alpha|+++\rangle + \beta|---\rangle$$
$$\alpha|-++\rangle + \beta|+--\rangle$$
$$\alpha|+-+\rangle + \beta|-+-\rangle$$
$$\alpha|++-\rangle + \beta|--+\rangle$$

$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$

$$\rho_E^1 \equiv \tfrac{1}{8} I_z^1 (9e^{-t/\tau} - e^{-9t/\tau}) \approx I_z^1 (1 - \frac{9}{2} t^2/\tau^2 + \cdots)$$

$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$

Control-Not

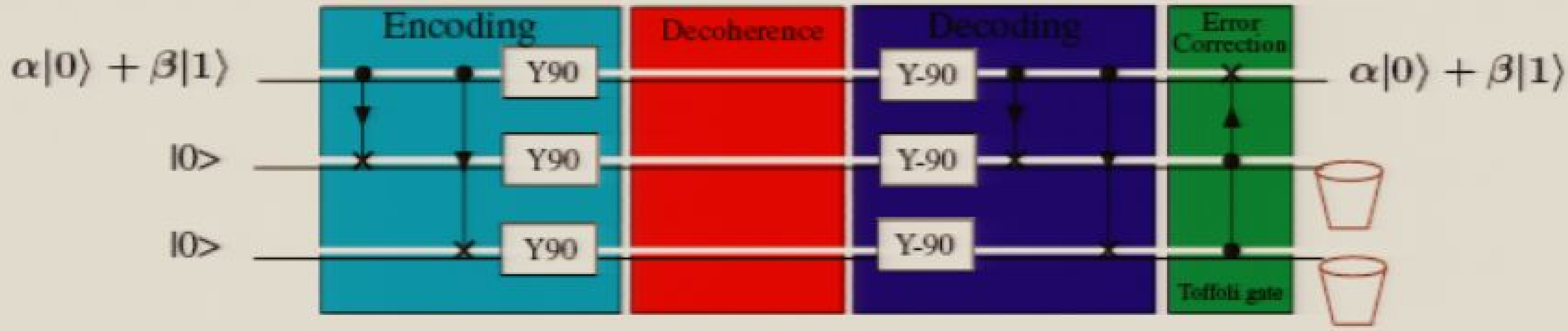
$$= \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 11 \\ 01 \\ 10 \end{cases}$$

$\sim 1 - 3\gamma^2$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



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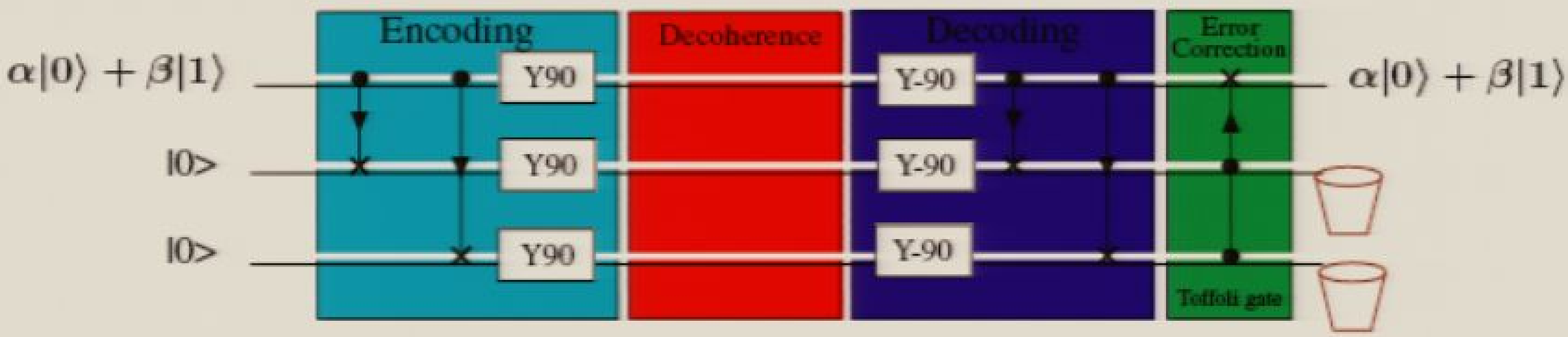

$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $+ \rightarrow -$

$- \rightarrow +$

$$\begin{aligned} &\alpha|+++\rangle + \beta|---\rangle \\ &\alpha|-++\rangle + \beta|+--\rangle \\ &\alpha|+-+\rangle + \beta|-+-\rangle \\ &\alpha|++-\rangle + \beta|--+\rangle \end{aligned}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \quad \sim 1 - 3\gamma^2$$

Control-Not

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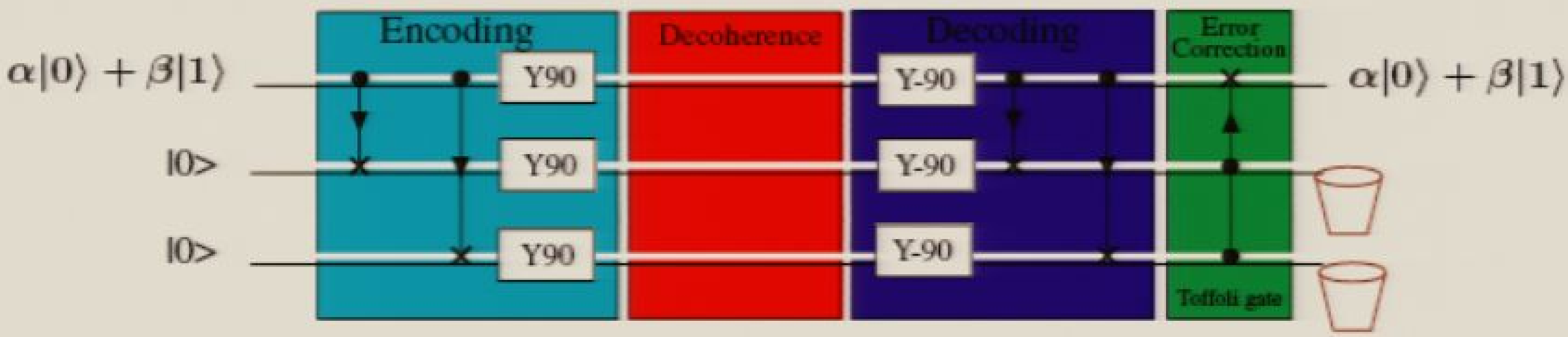
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



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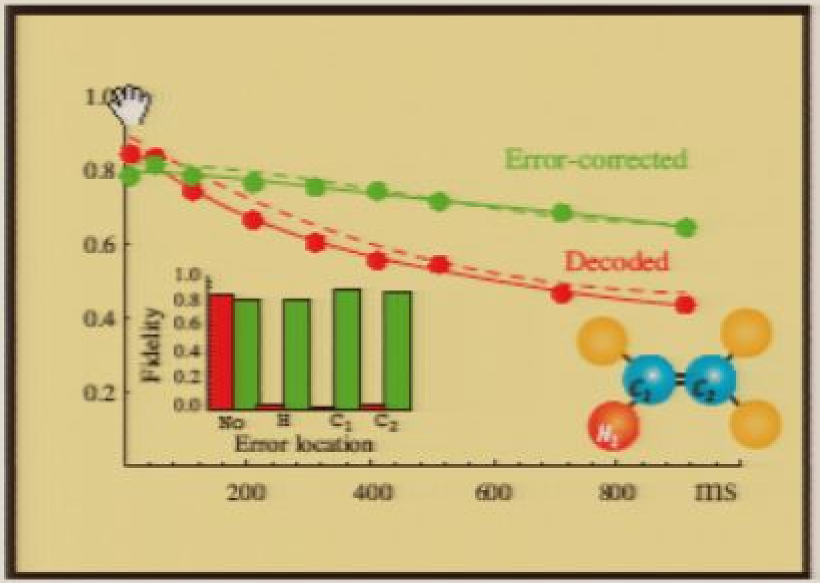
Experimental results



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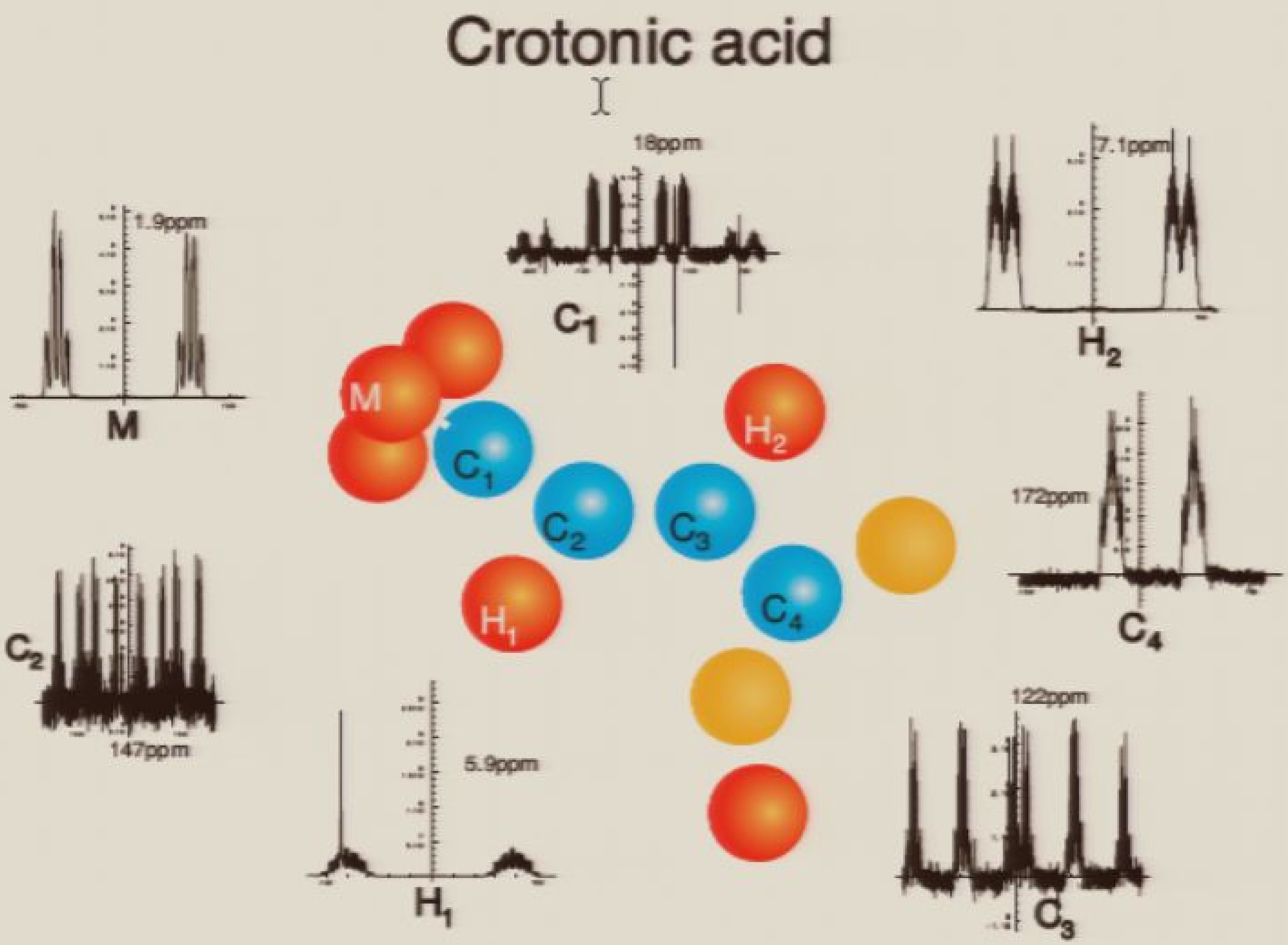
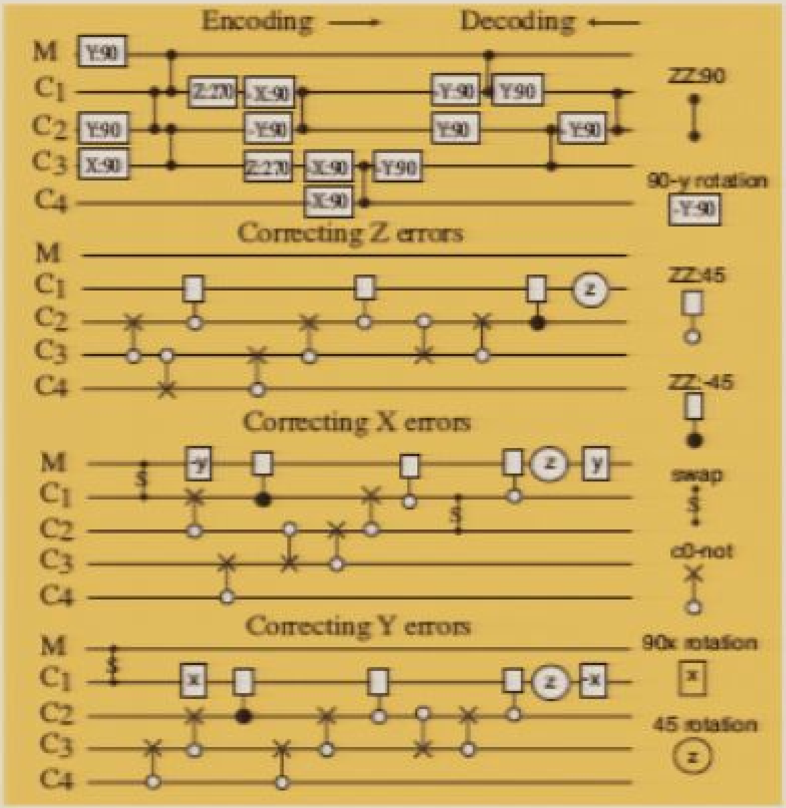
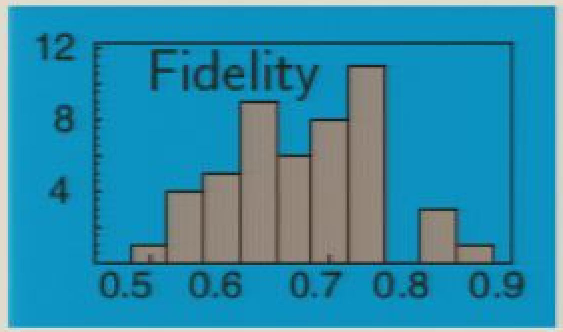
Spectra of crotonic acid

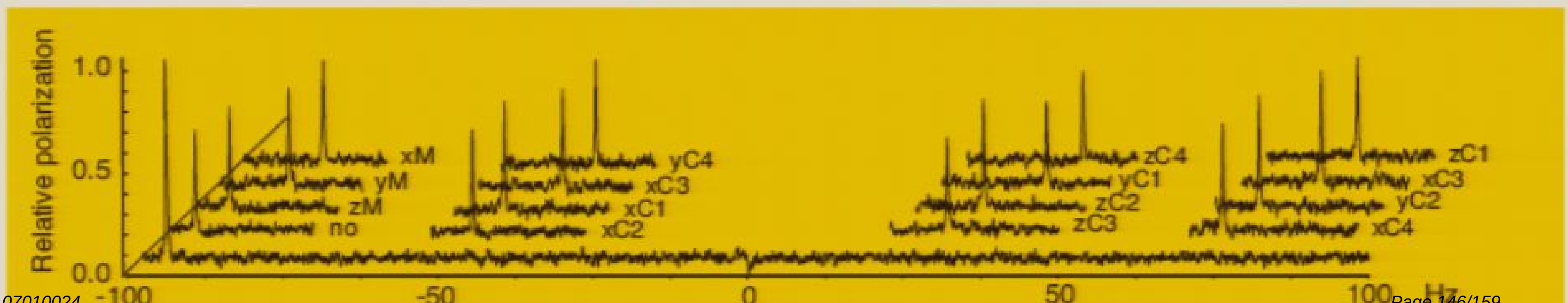


5 bit quantum error correcting code

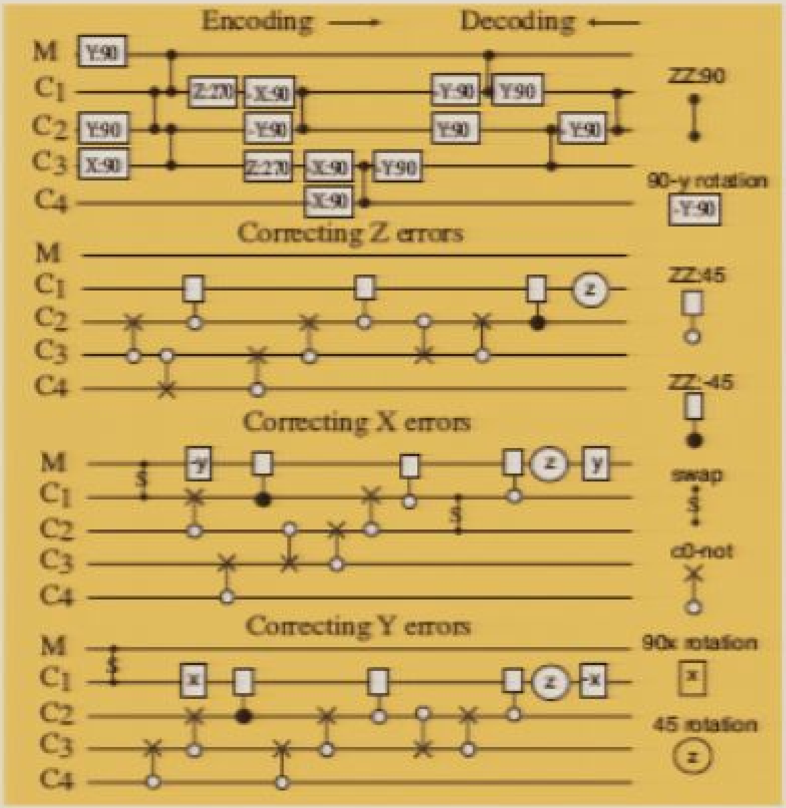


Implementation of the 5 bit code with the stabilizer $Z^2Y^3Y^4X^5$, $Z^1Y^2Y^3X^4$, $Y^2Z^3Z^4Z^5$ and $X^1Z^2X^3Z^4$, including decoding and error correction for a basis of 1 qubit errors [1].





5 bit quantum error correcting code



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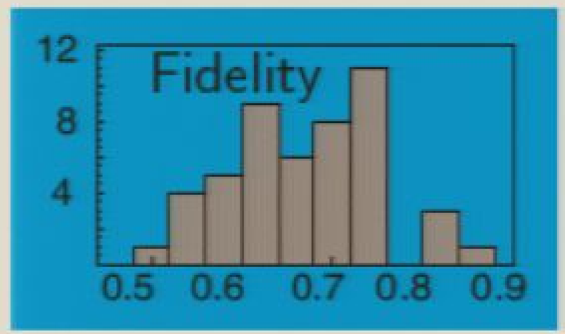
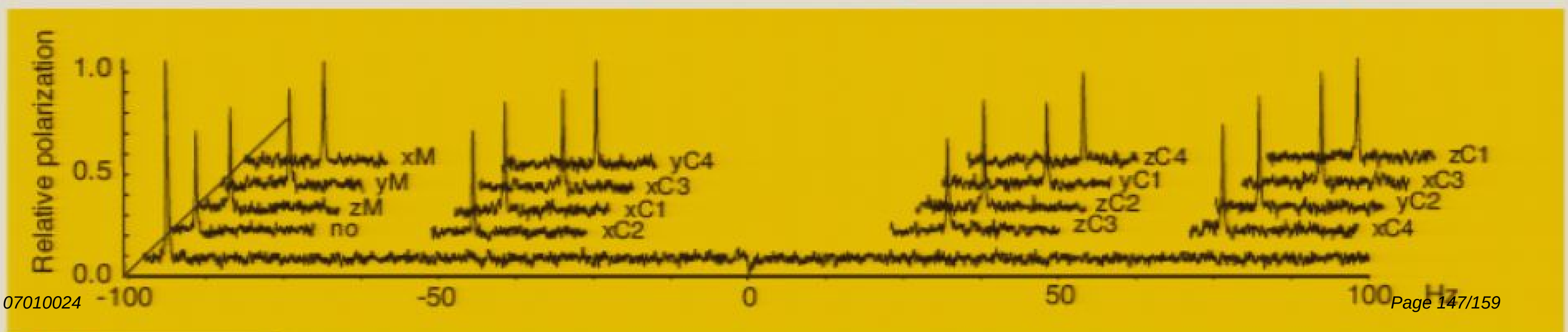
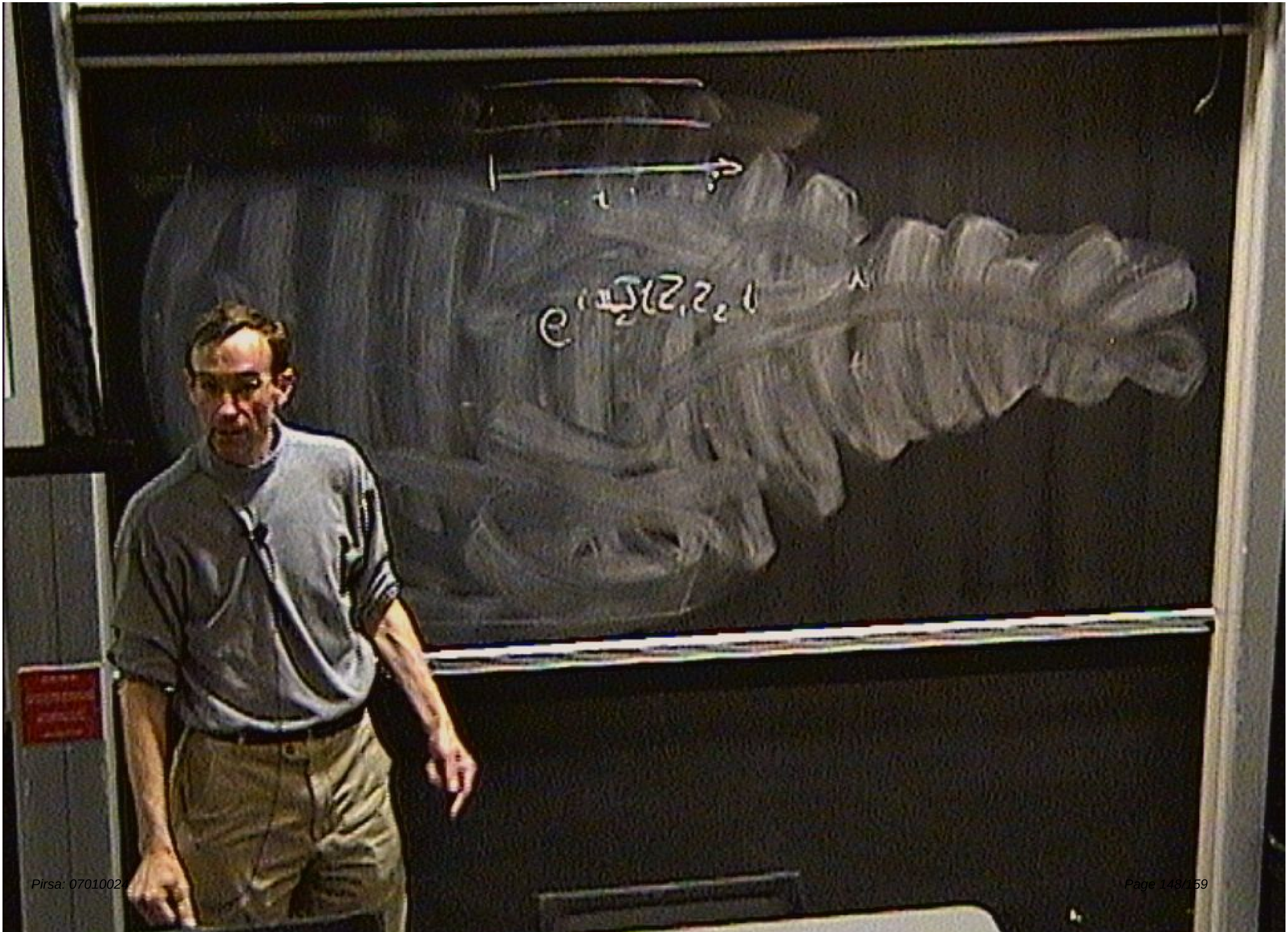
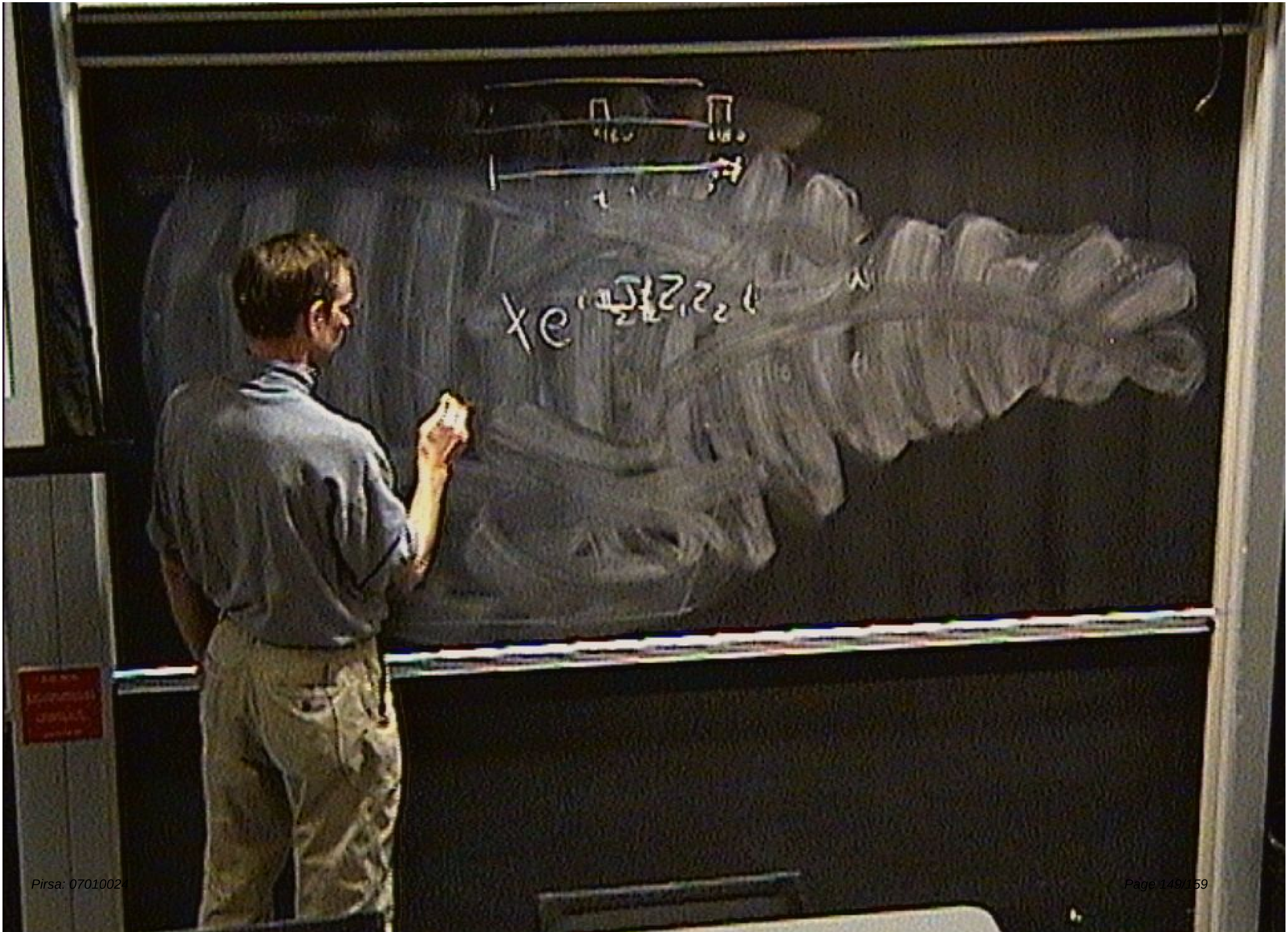
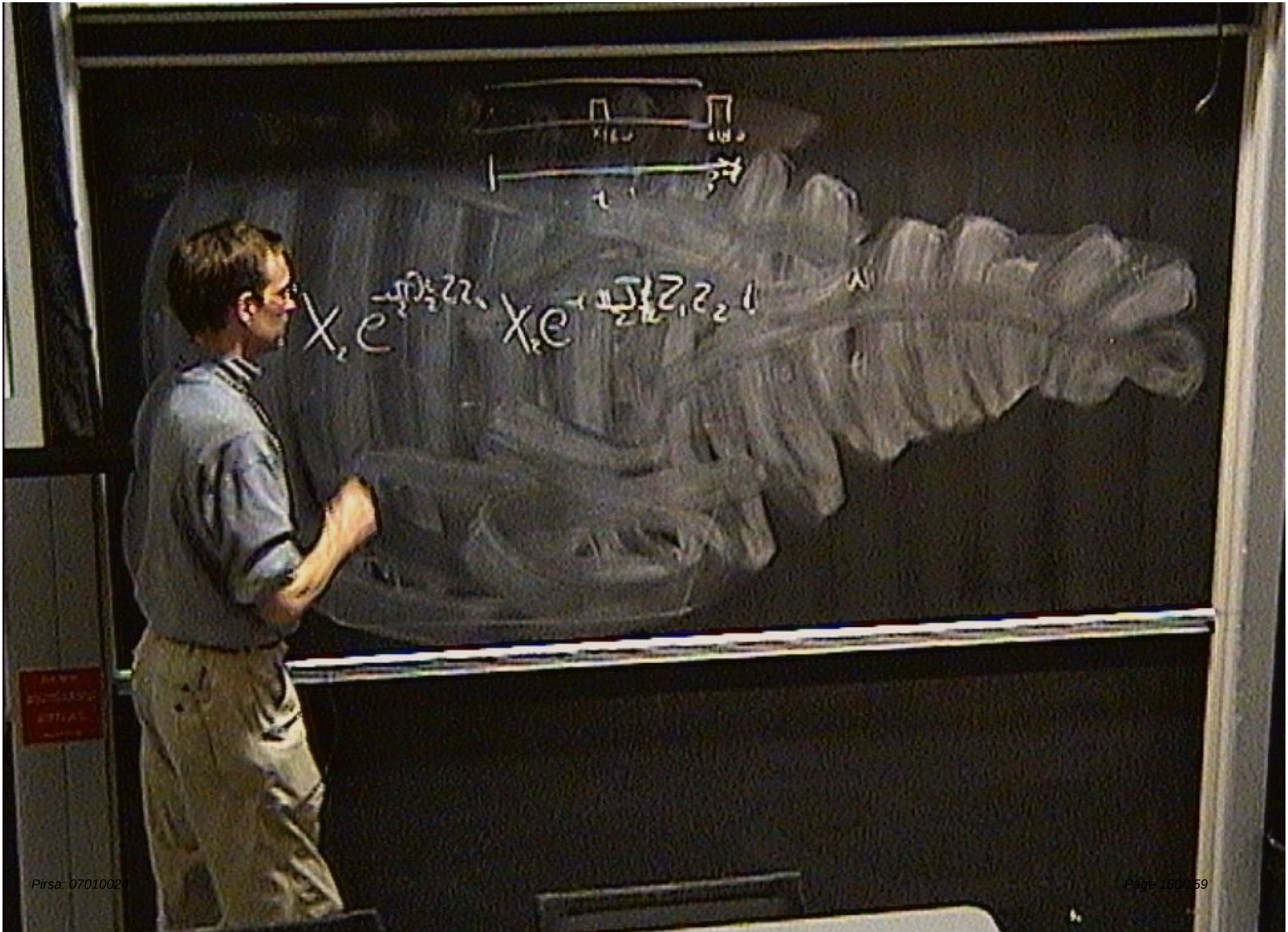
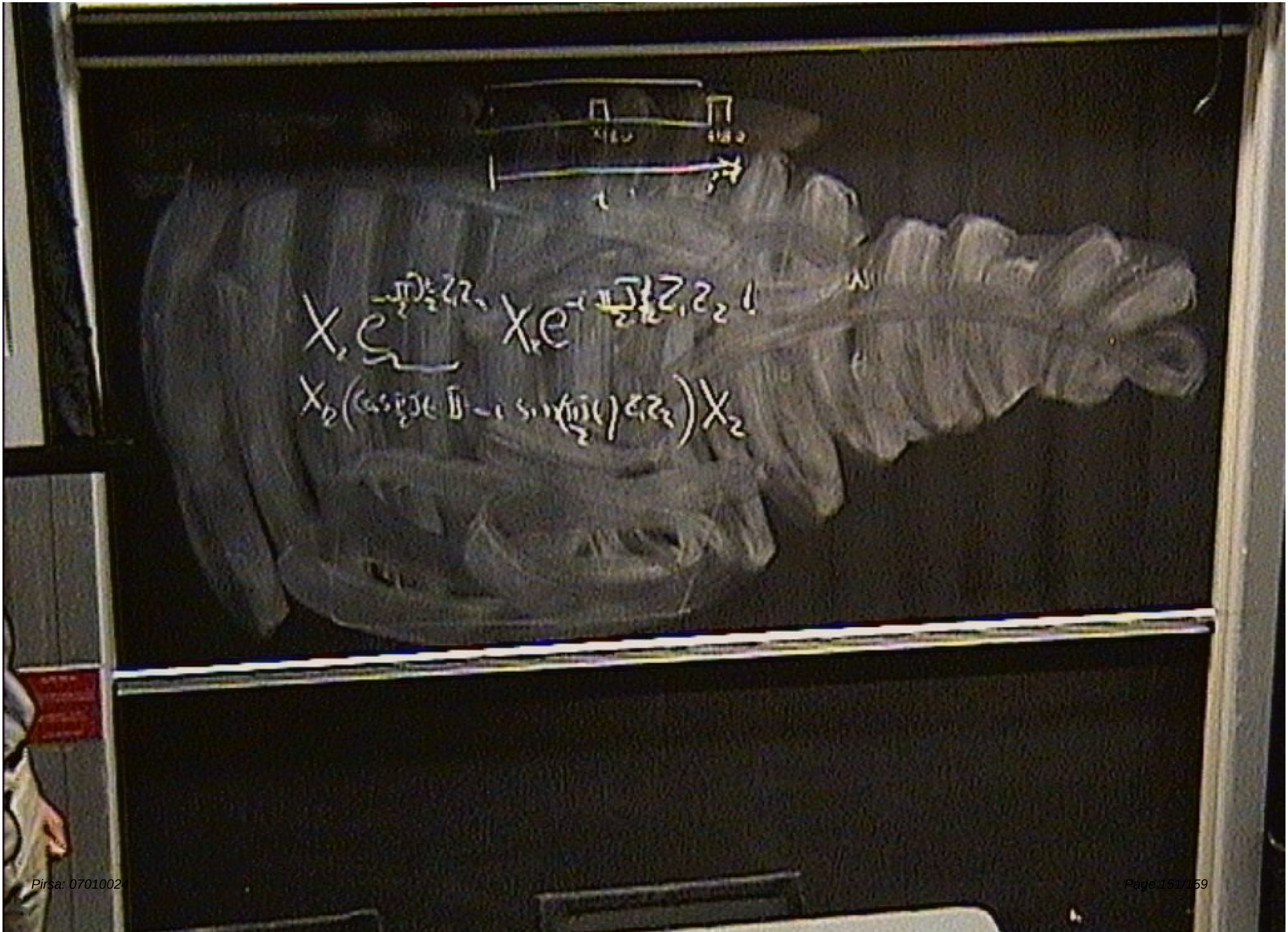


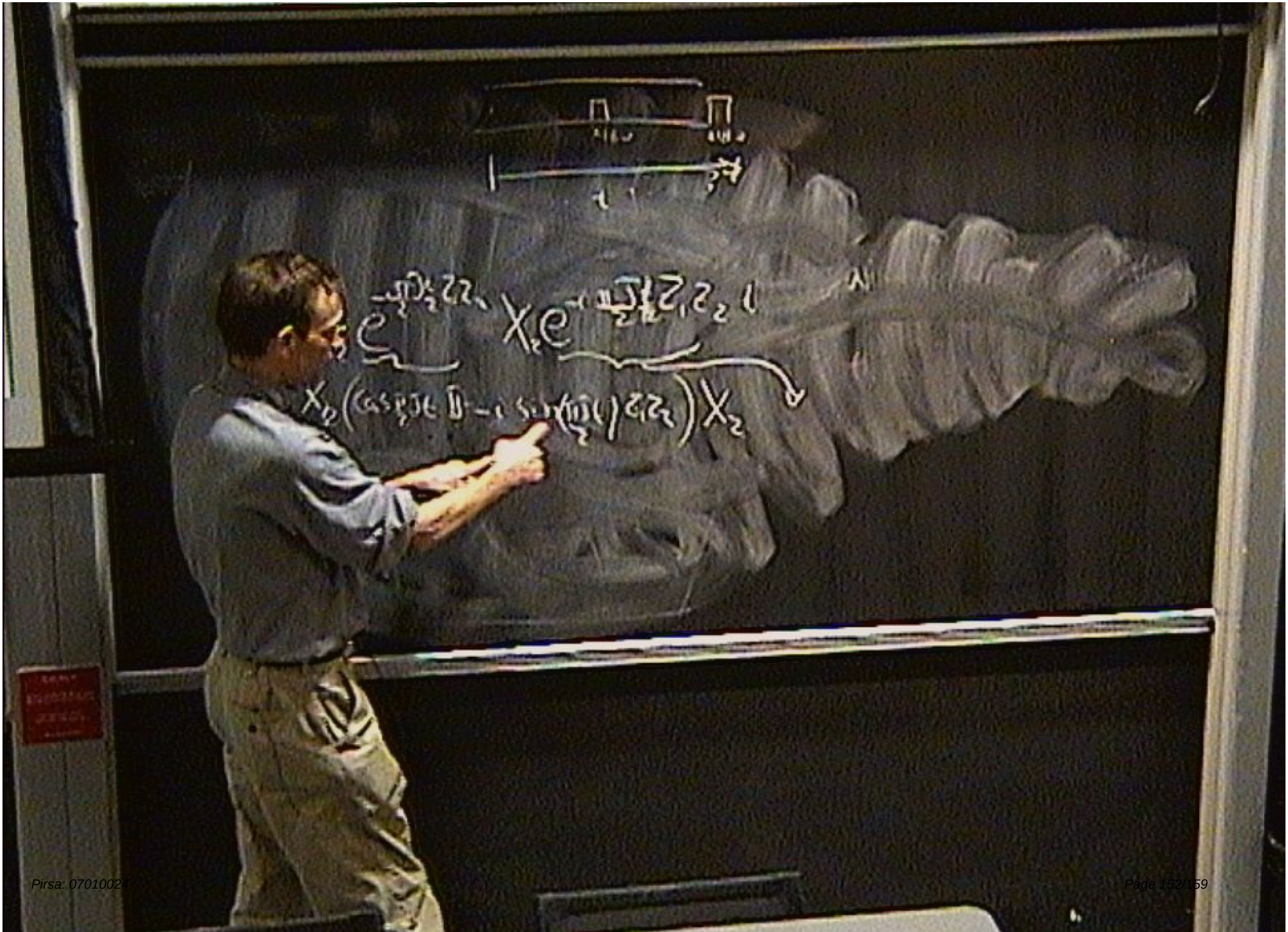


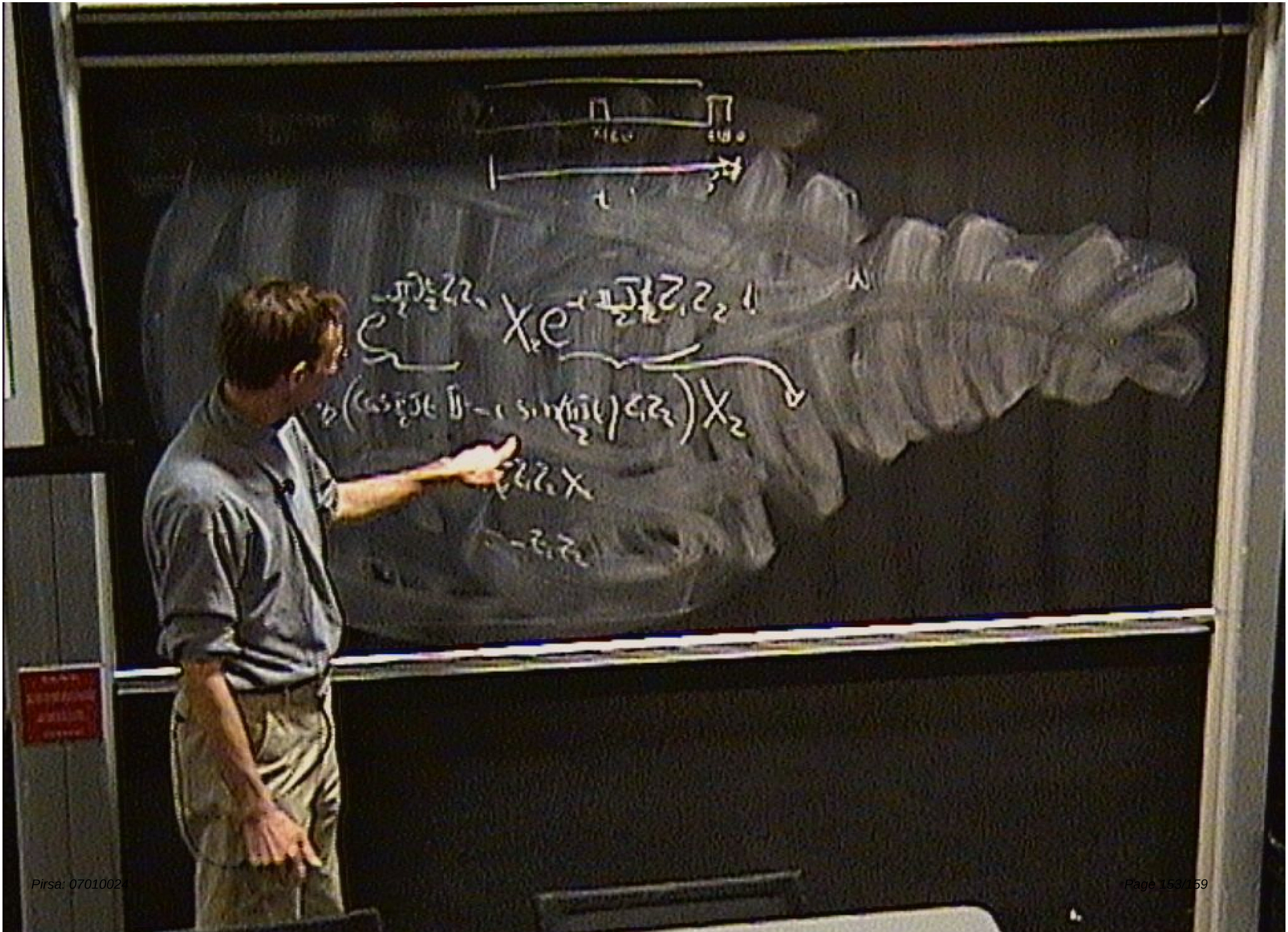


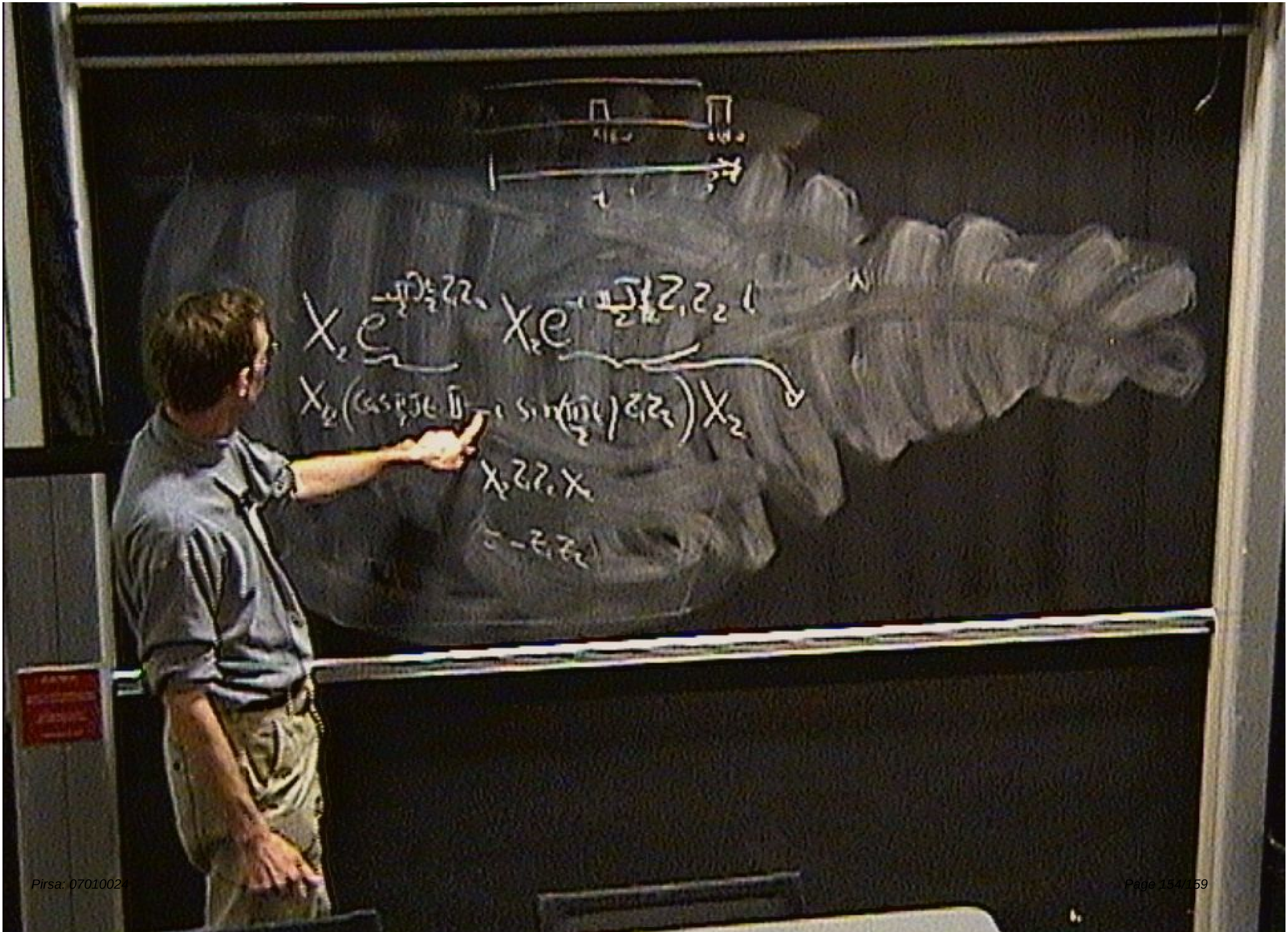


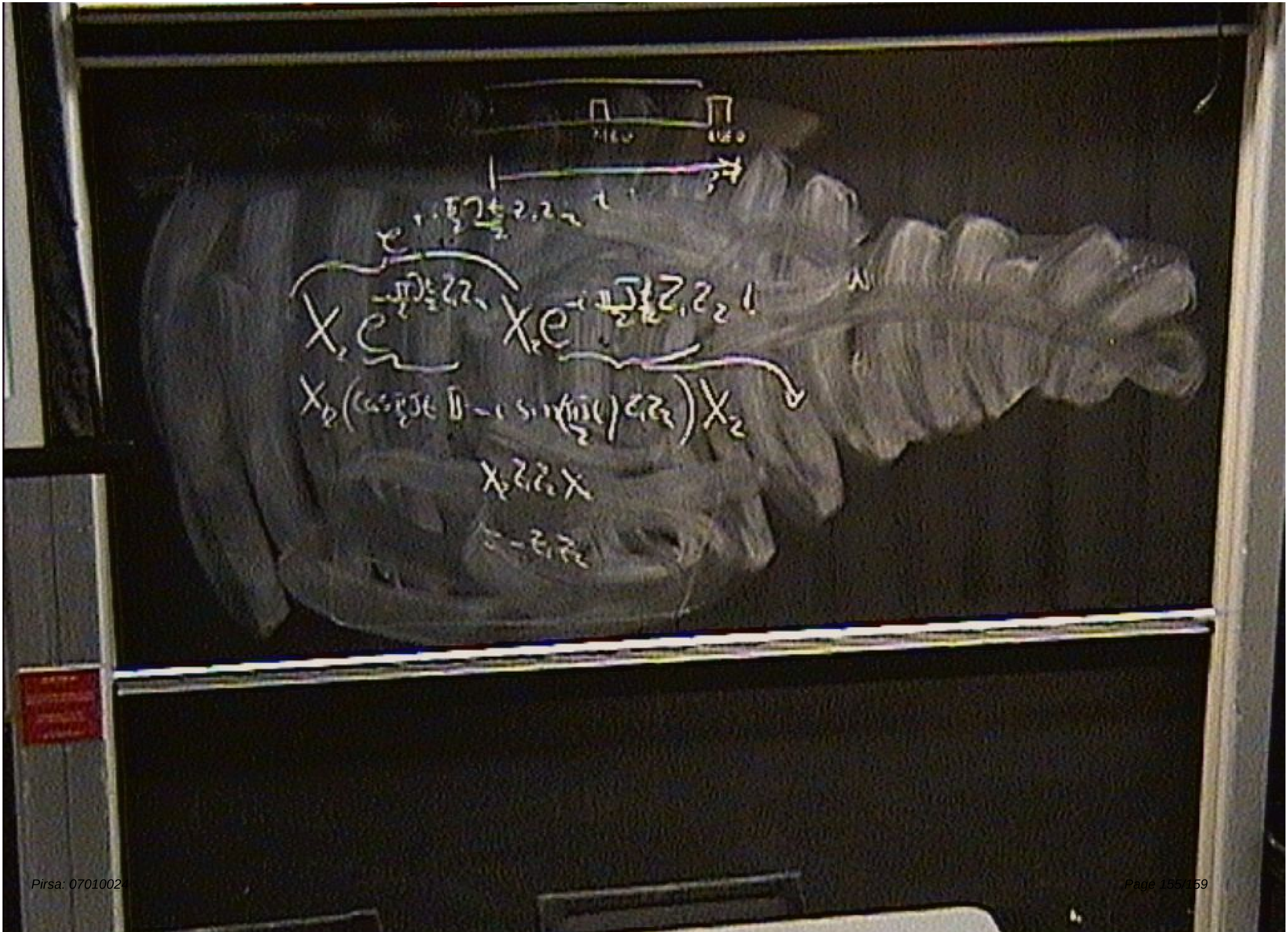




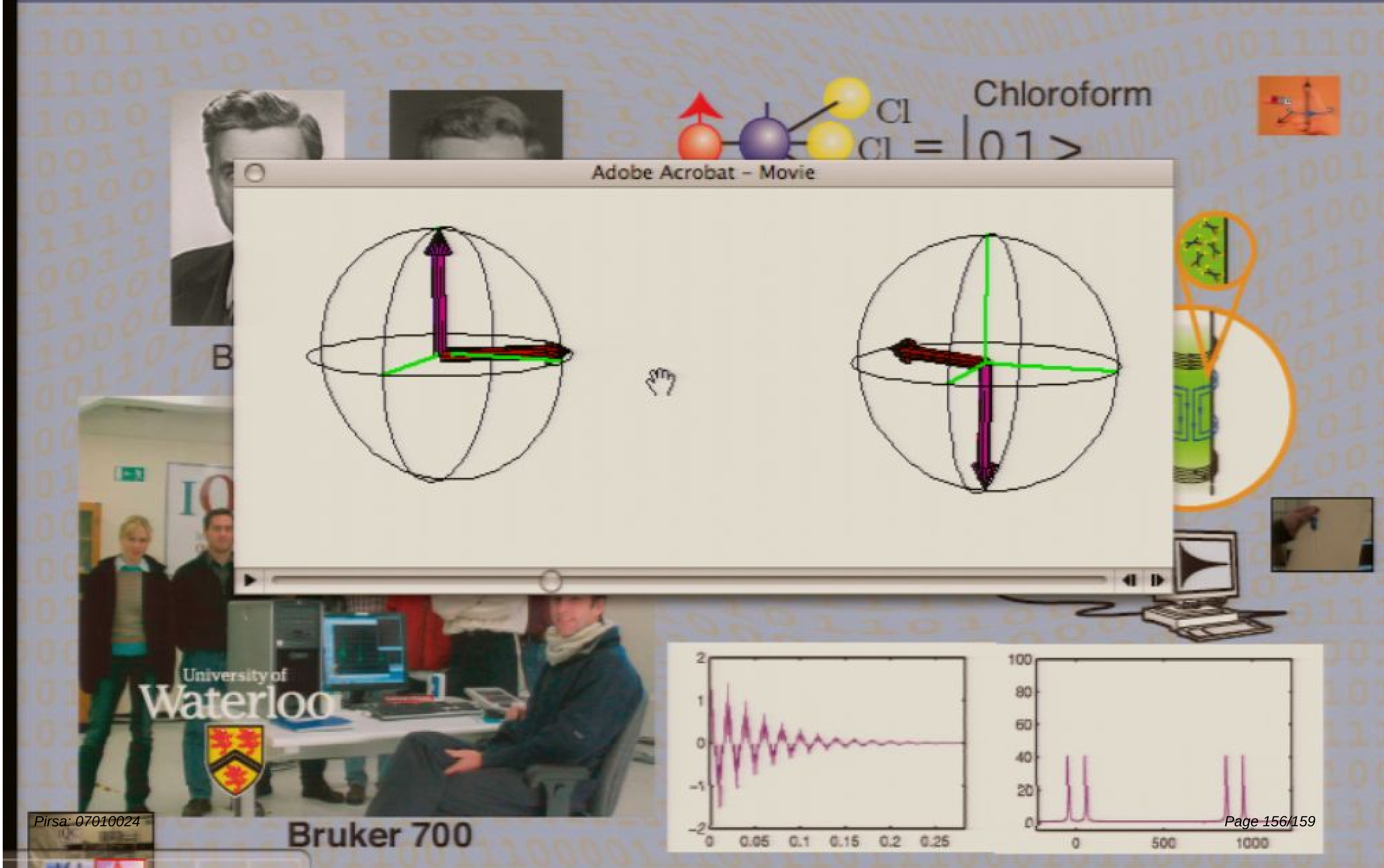


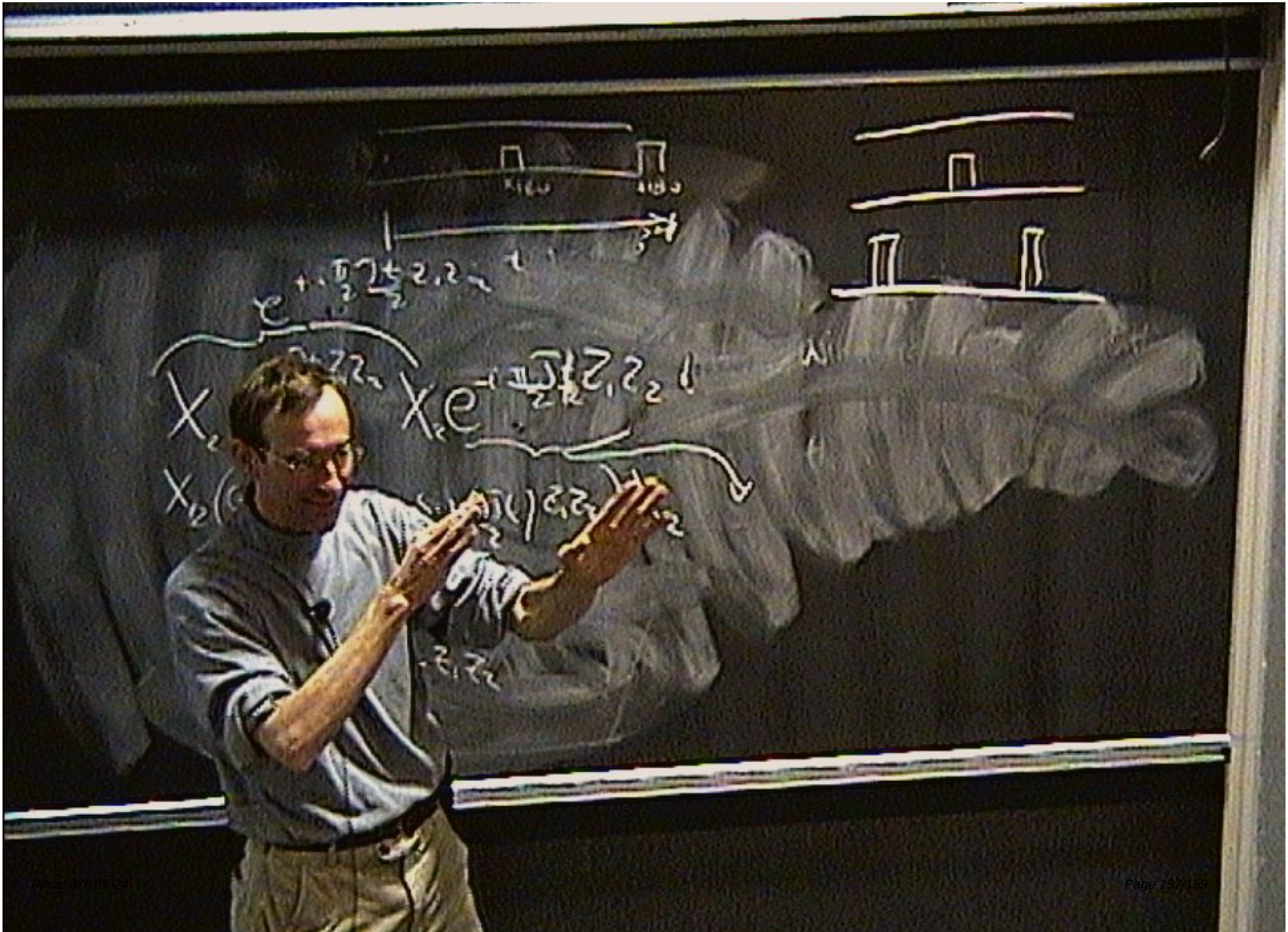


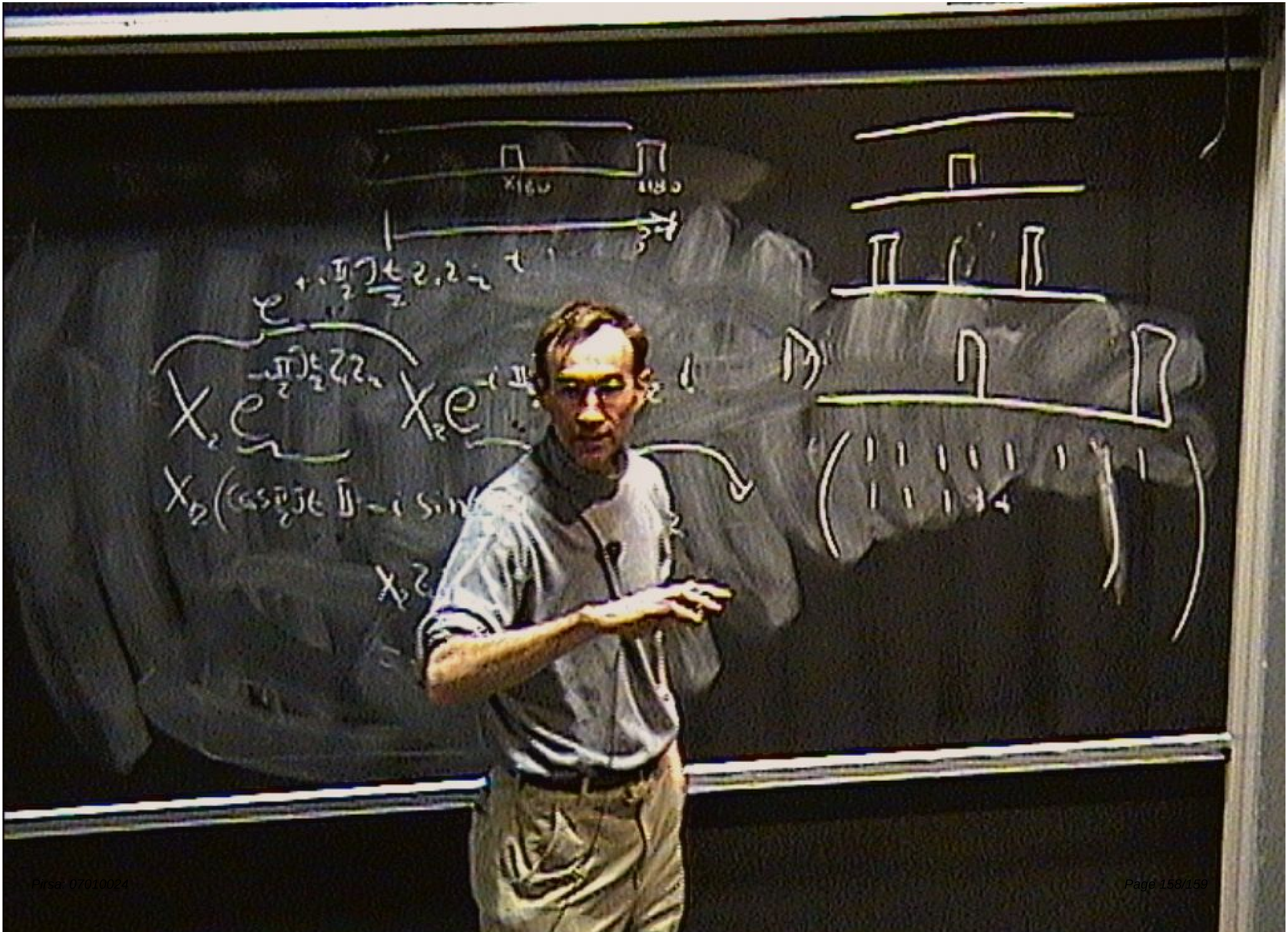




Nuclear Magnetic Resonance







Nuclear Magnetic Resonance