

Title: Quantum Error Correction 4A

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URL: <http://pirsa.org/07010024>

Abstract: Guest lecturer Raymond Laflamme: experimental quantum error correction

Experimental Quantum Error Correction



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Quantum Error Correction

Accuracy Threshold Theorem



A quantum computation
can be as long as required
with any desired accuracy
as long as the noise level
is below a threshold value

$$P < 10^{-4}$$



Knill et al.; Science, 279, 342, 1998

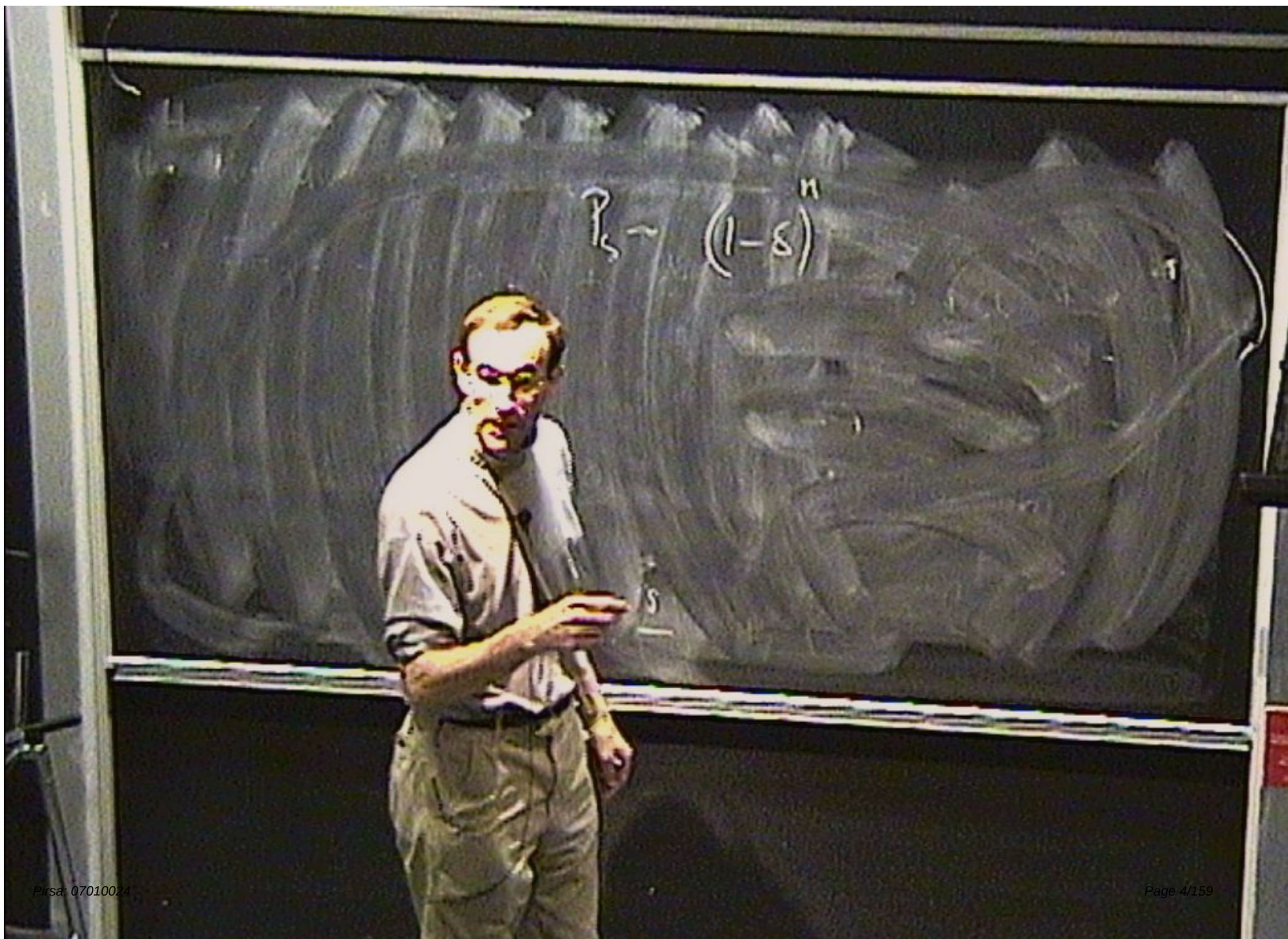
Kitaev, Russ. Math Survey 1997

Aharonov & Ben Or, ACM press

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Significance:

- imperfections and imprecisions are not fundamental objections to quantum computation
- it gives criteria for scalability
- its requirements are a guide for experimentalists
- it is a benchmark to compare different technologies



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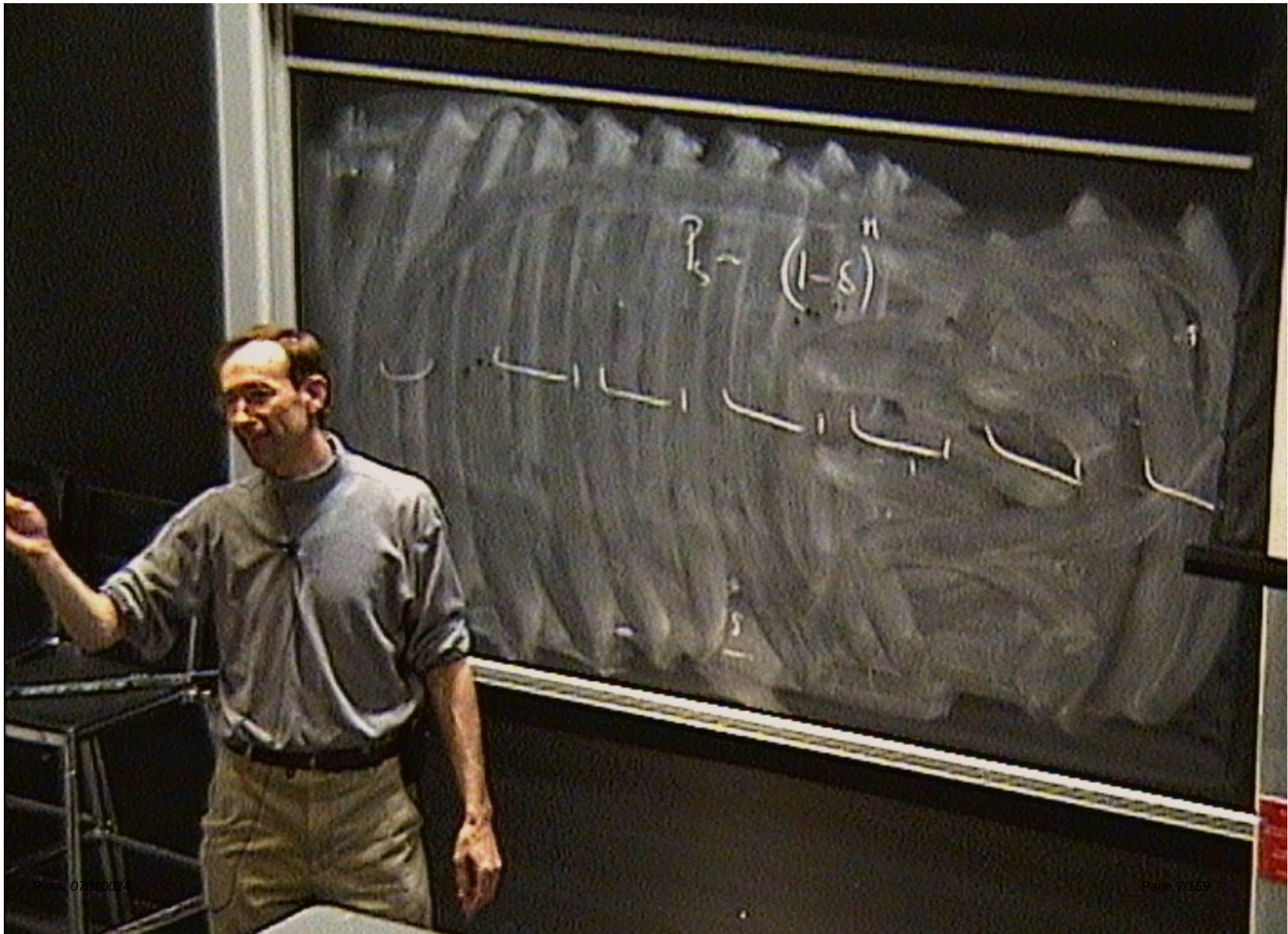
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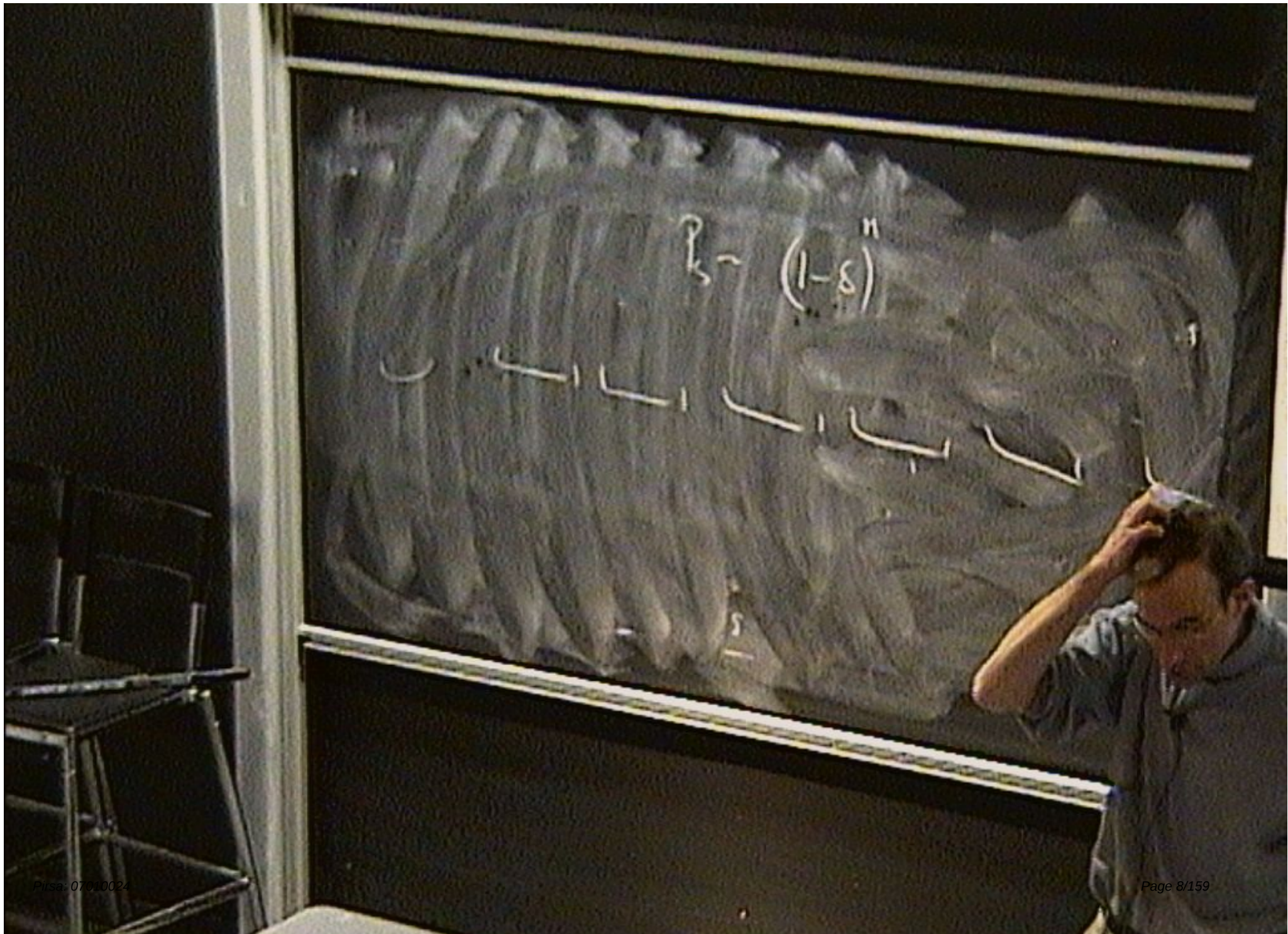
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Theory vs experiment



Theory vs experiment



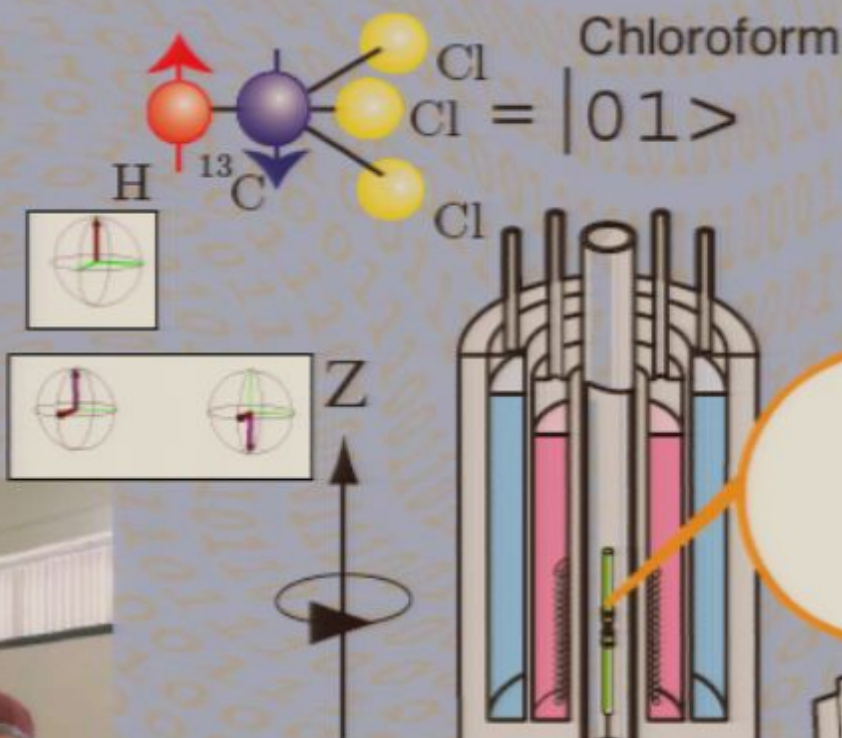
Nuclear Magnetic Resonance



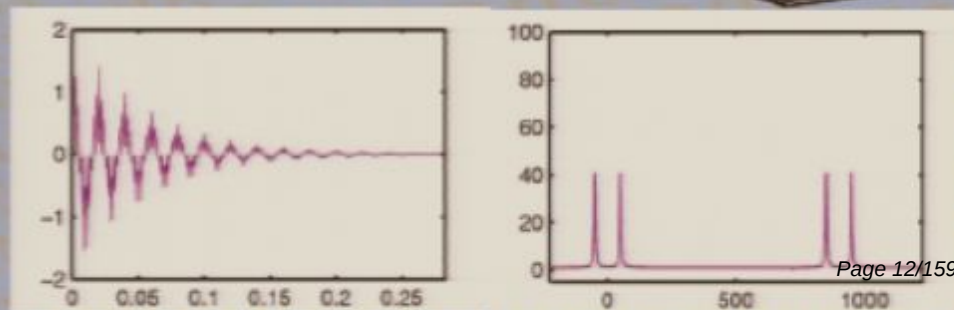
Bloch



Purcell



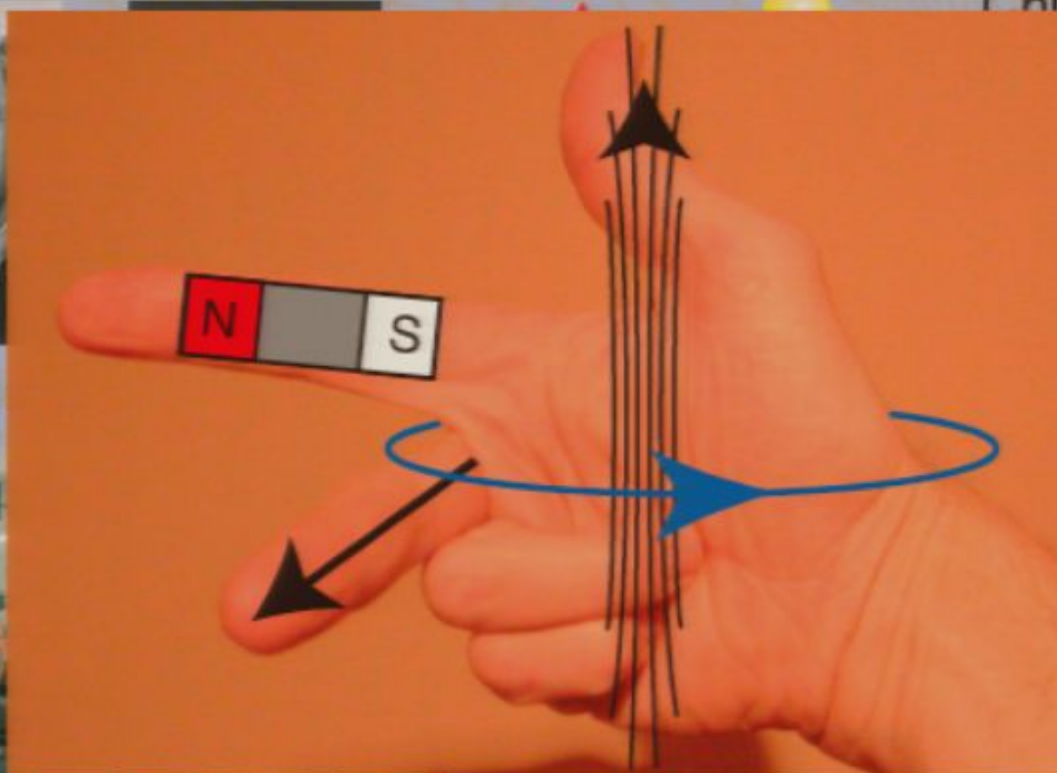
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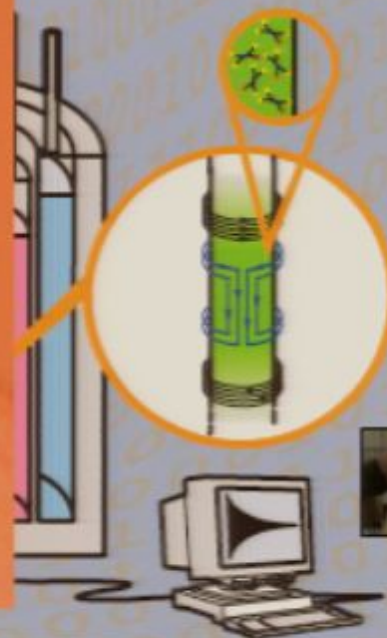
Nuclear Magnetic Resonance



Bloch



Chloroform

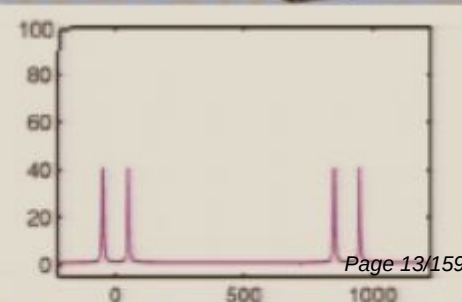
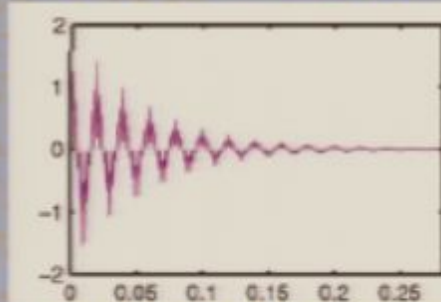


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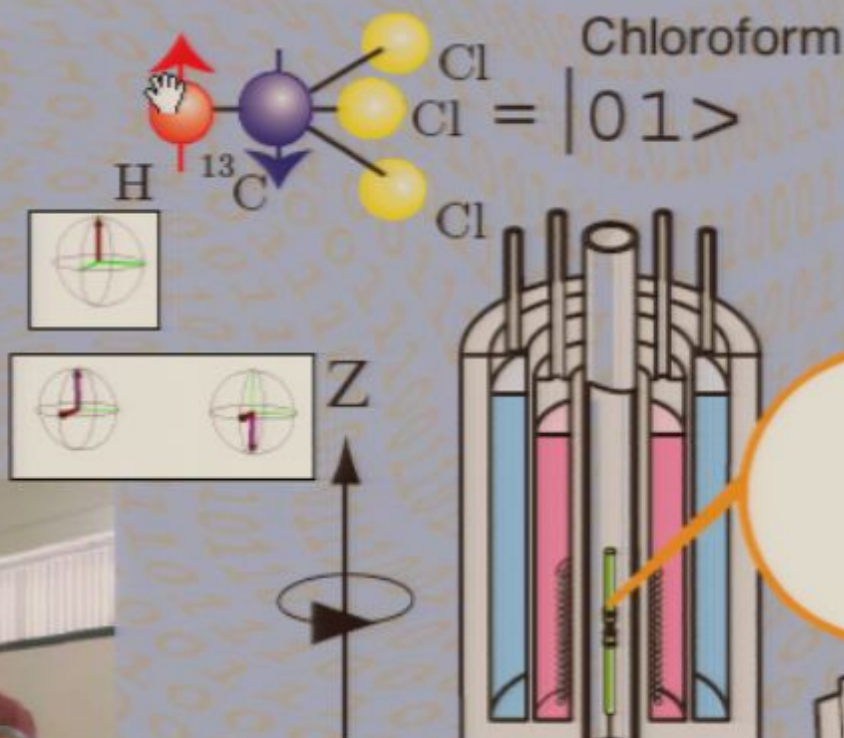
Nuclear Magnetic Resonance



Bloch



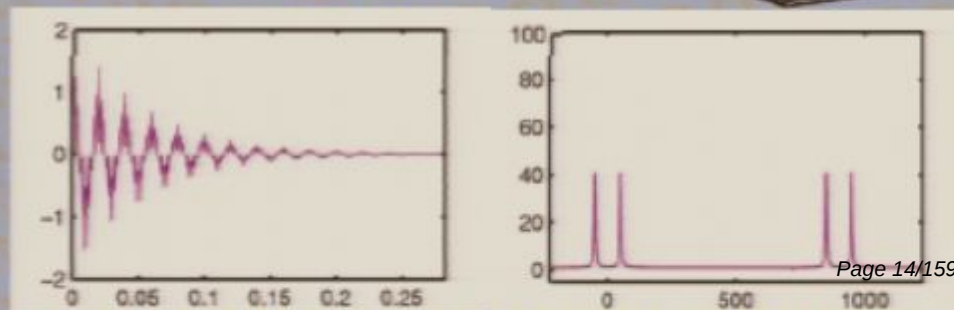
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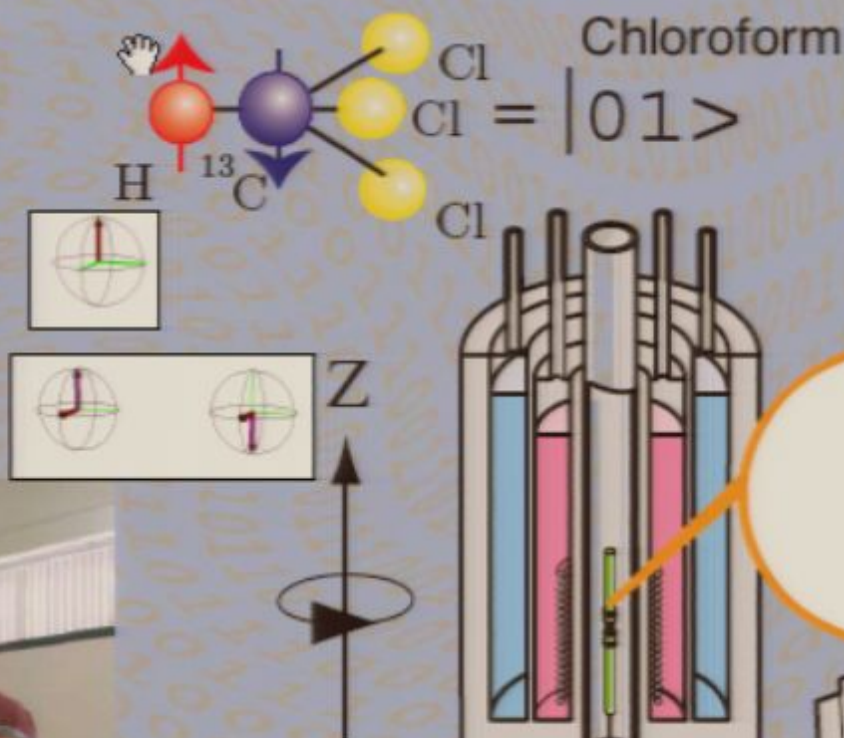
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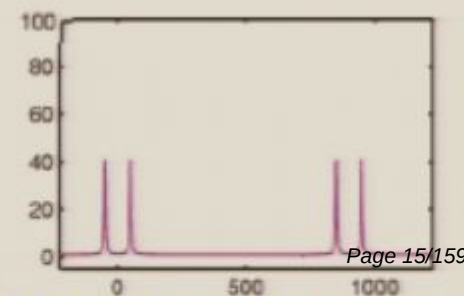
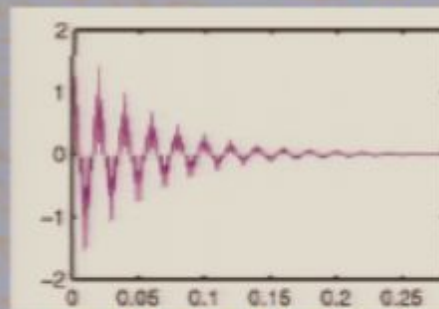
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Nuclear Magnetic Resonance



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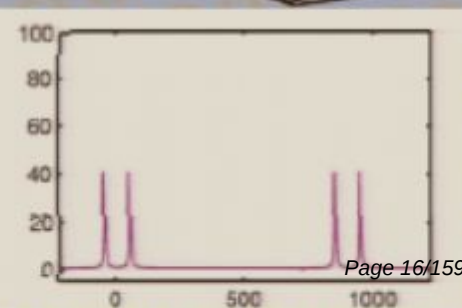
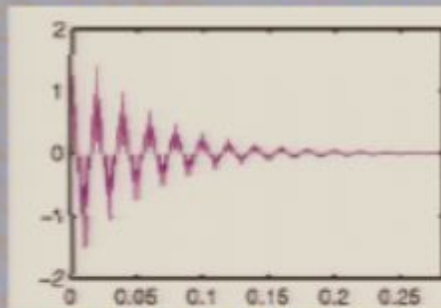
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Nuclear Magnetic Resonance

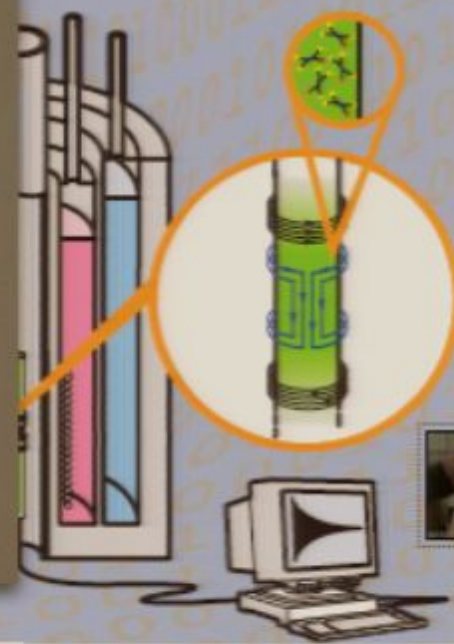
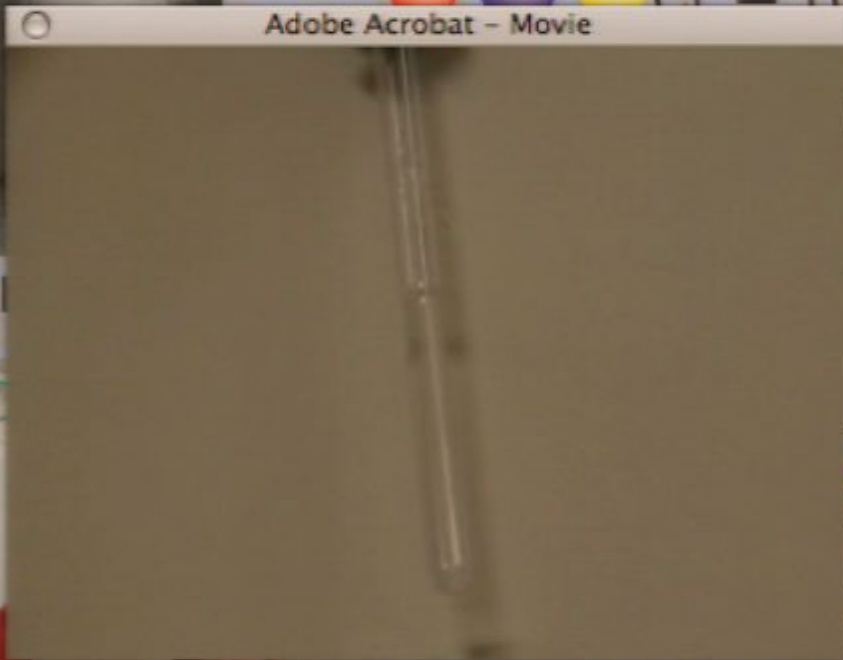


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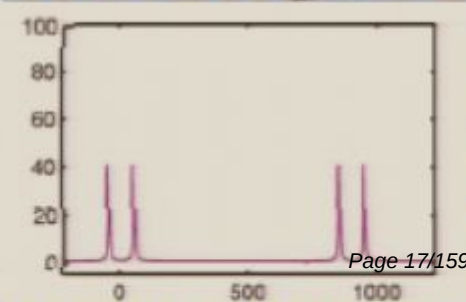
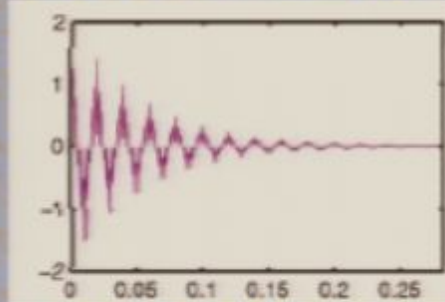
$$|01\rangle$$



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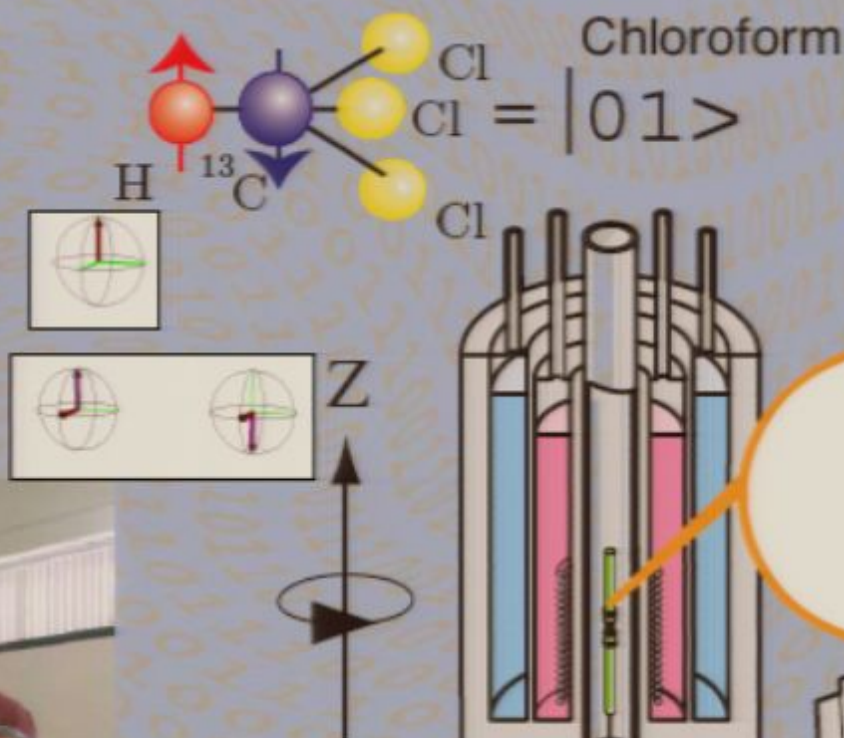
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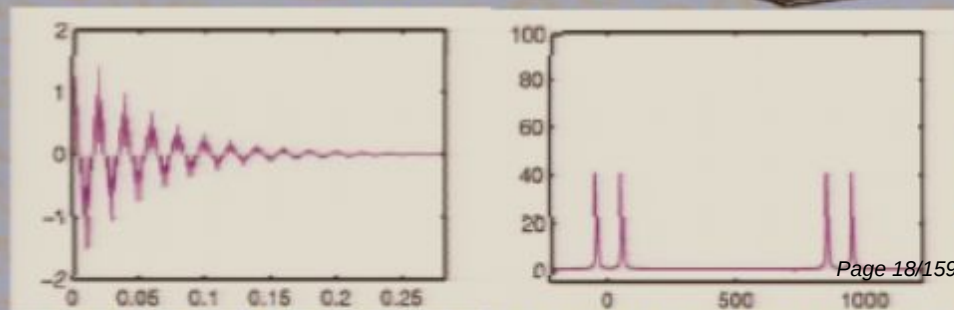
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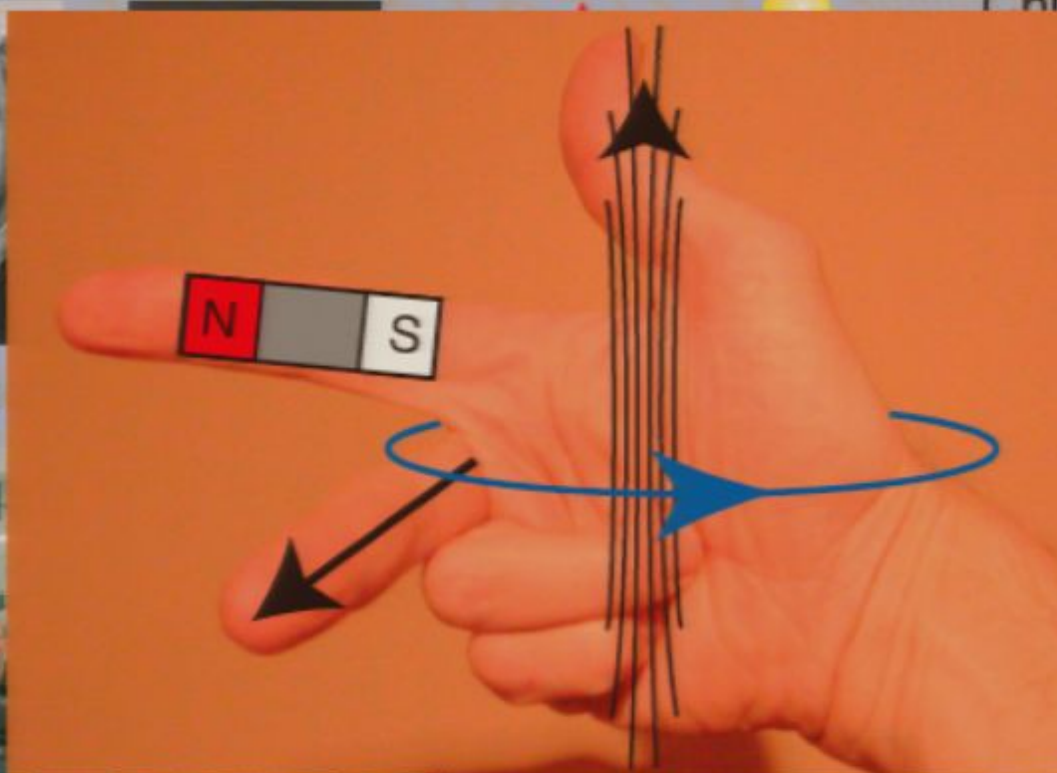
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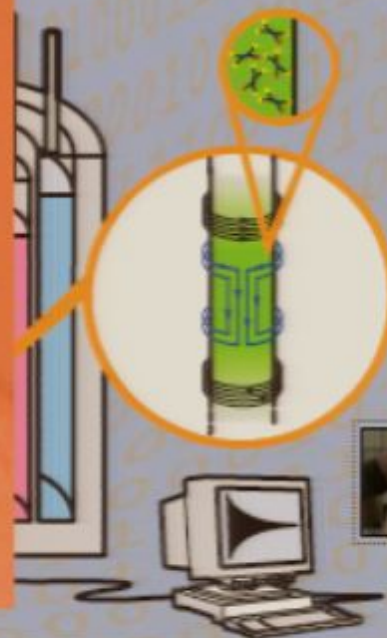
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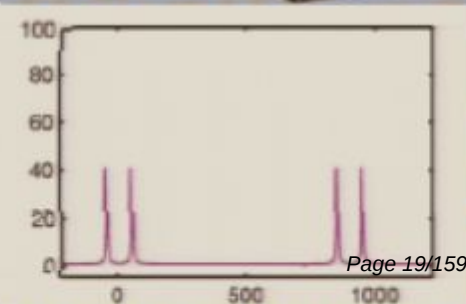
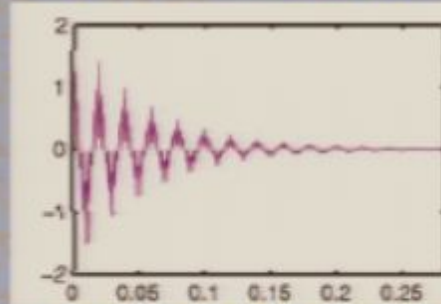


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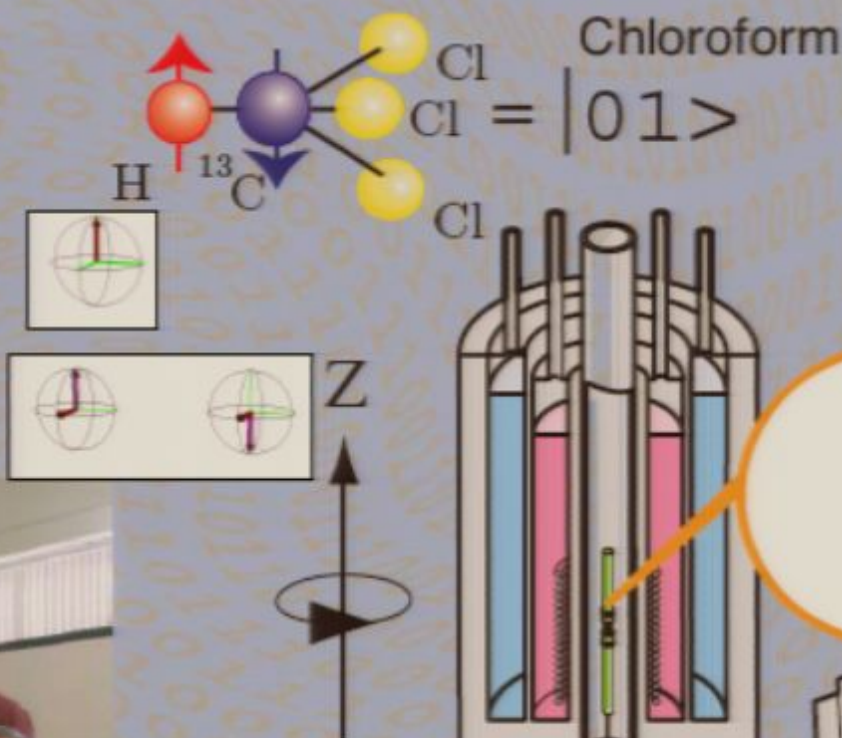
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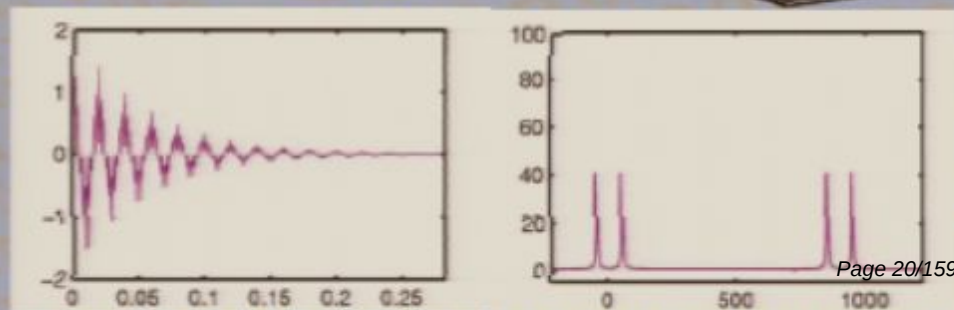
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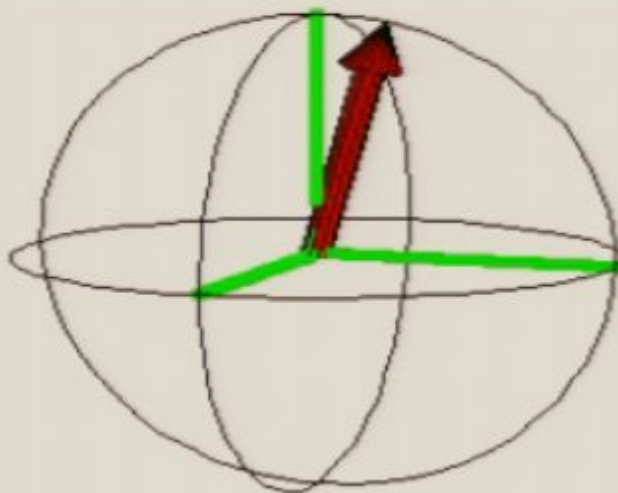
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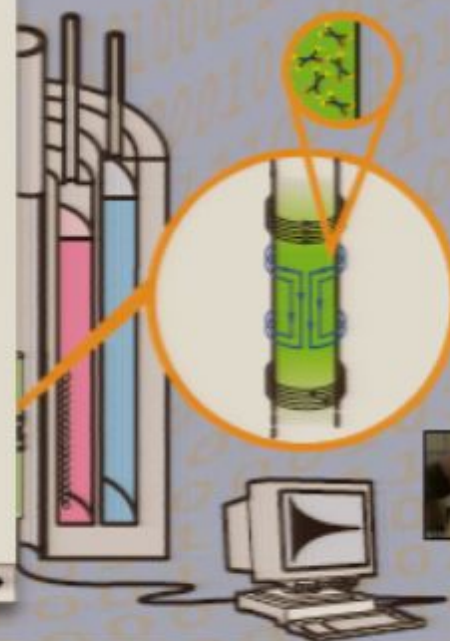
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Adobe Acrobat - Movie



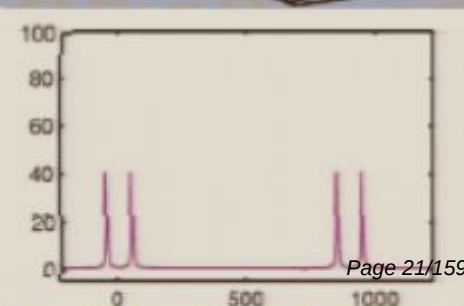
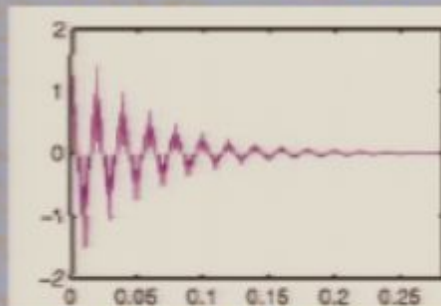
Chloroform



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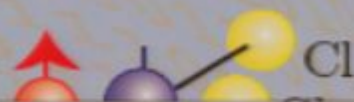
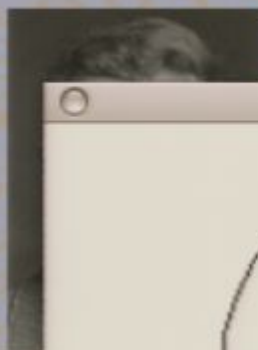
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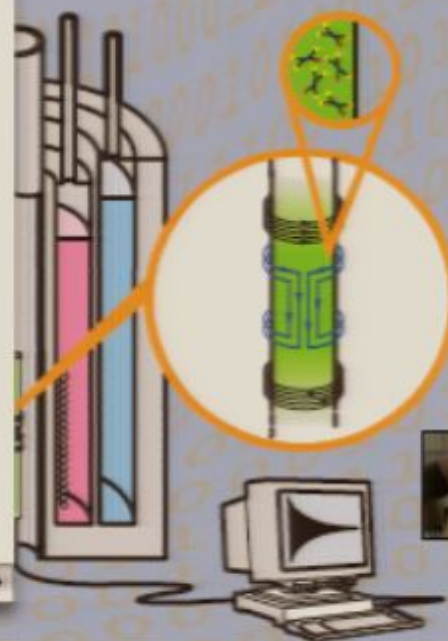
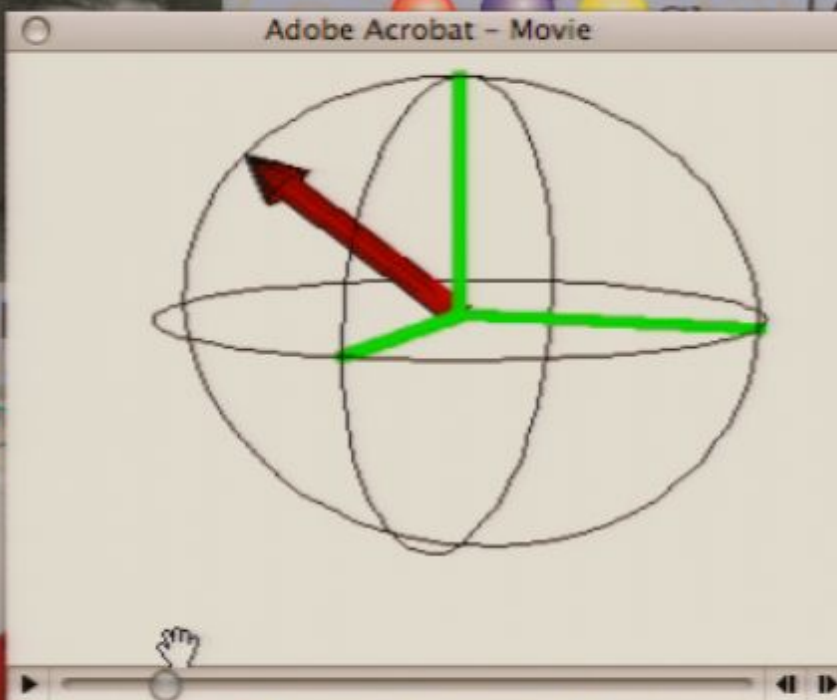
Nuclear Magnetic Resonance



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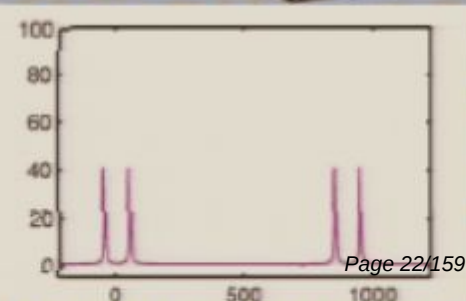
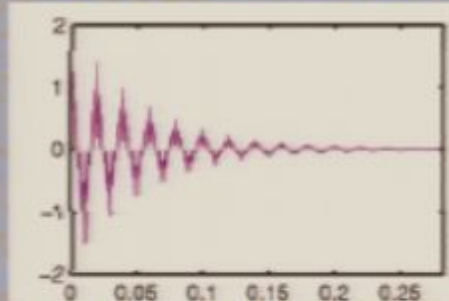
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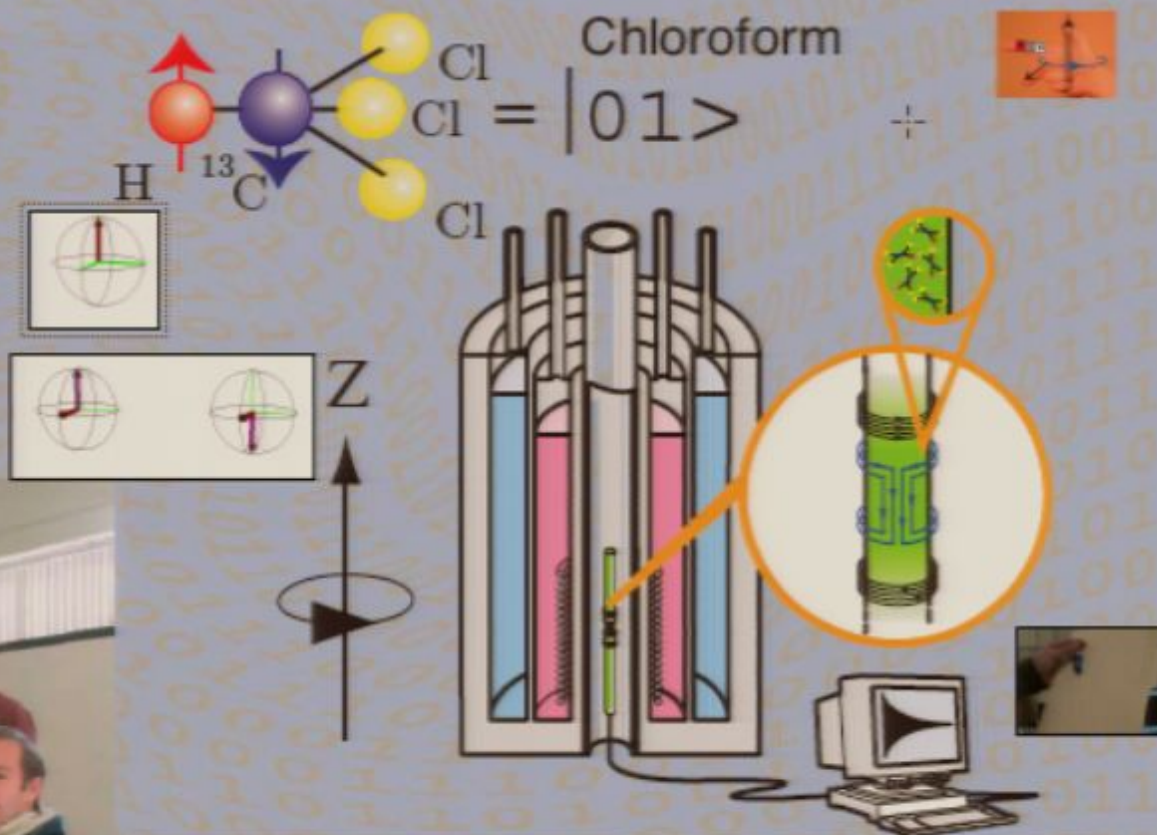
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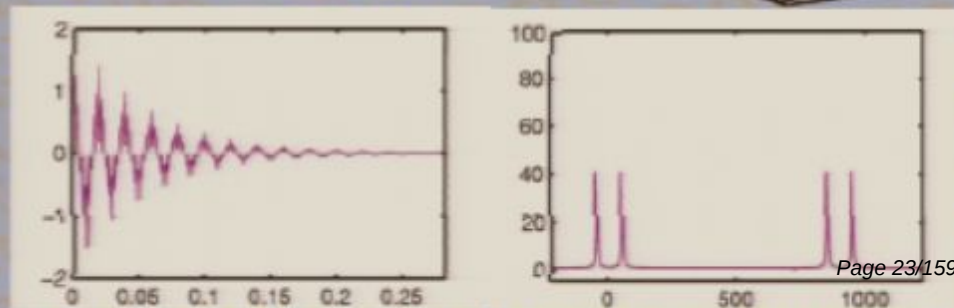
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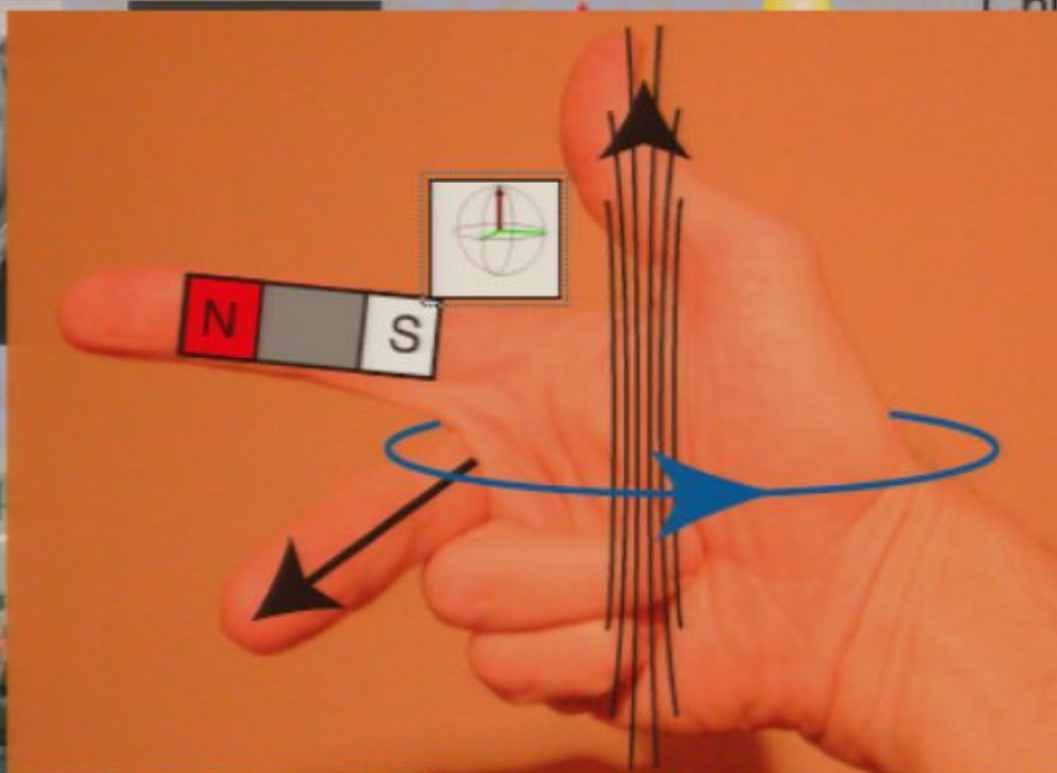
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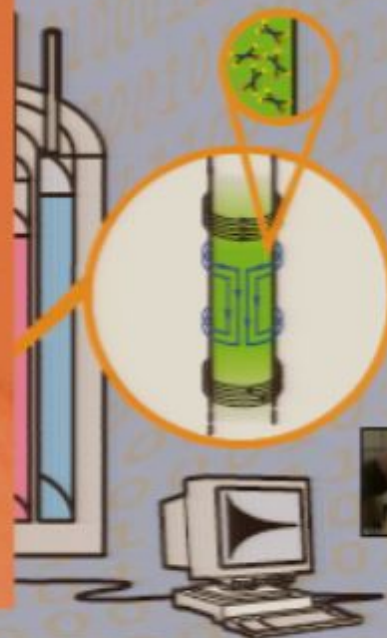
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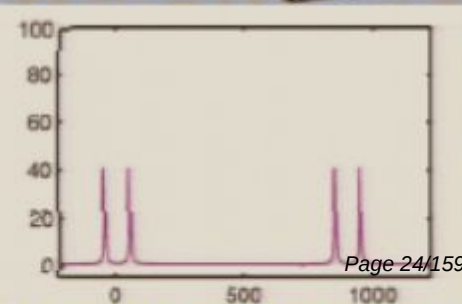
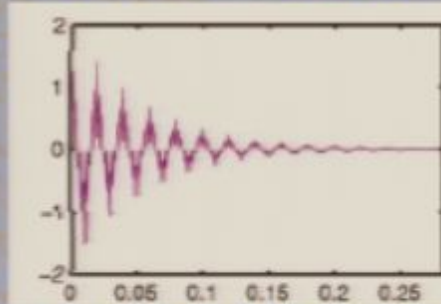
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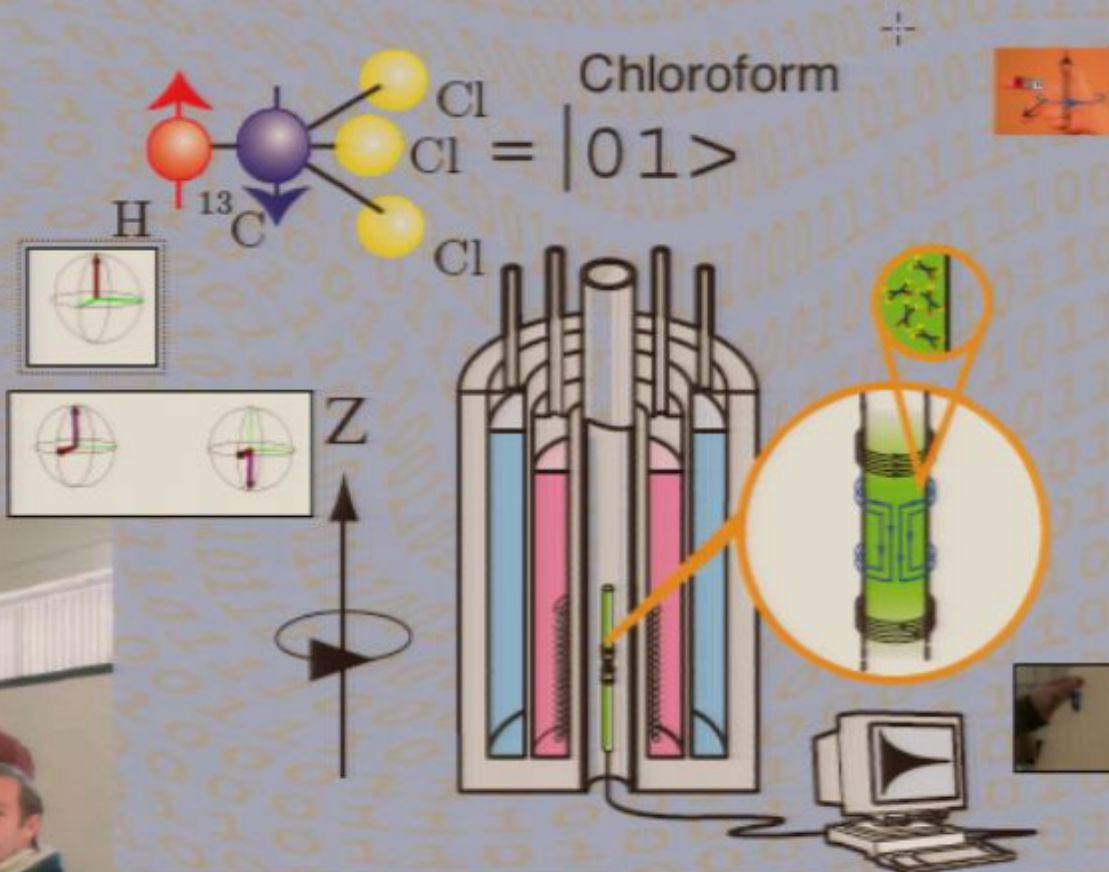
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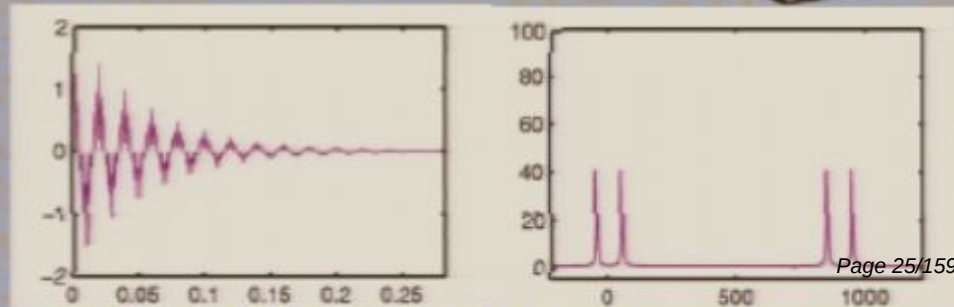
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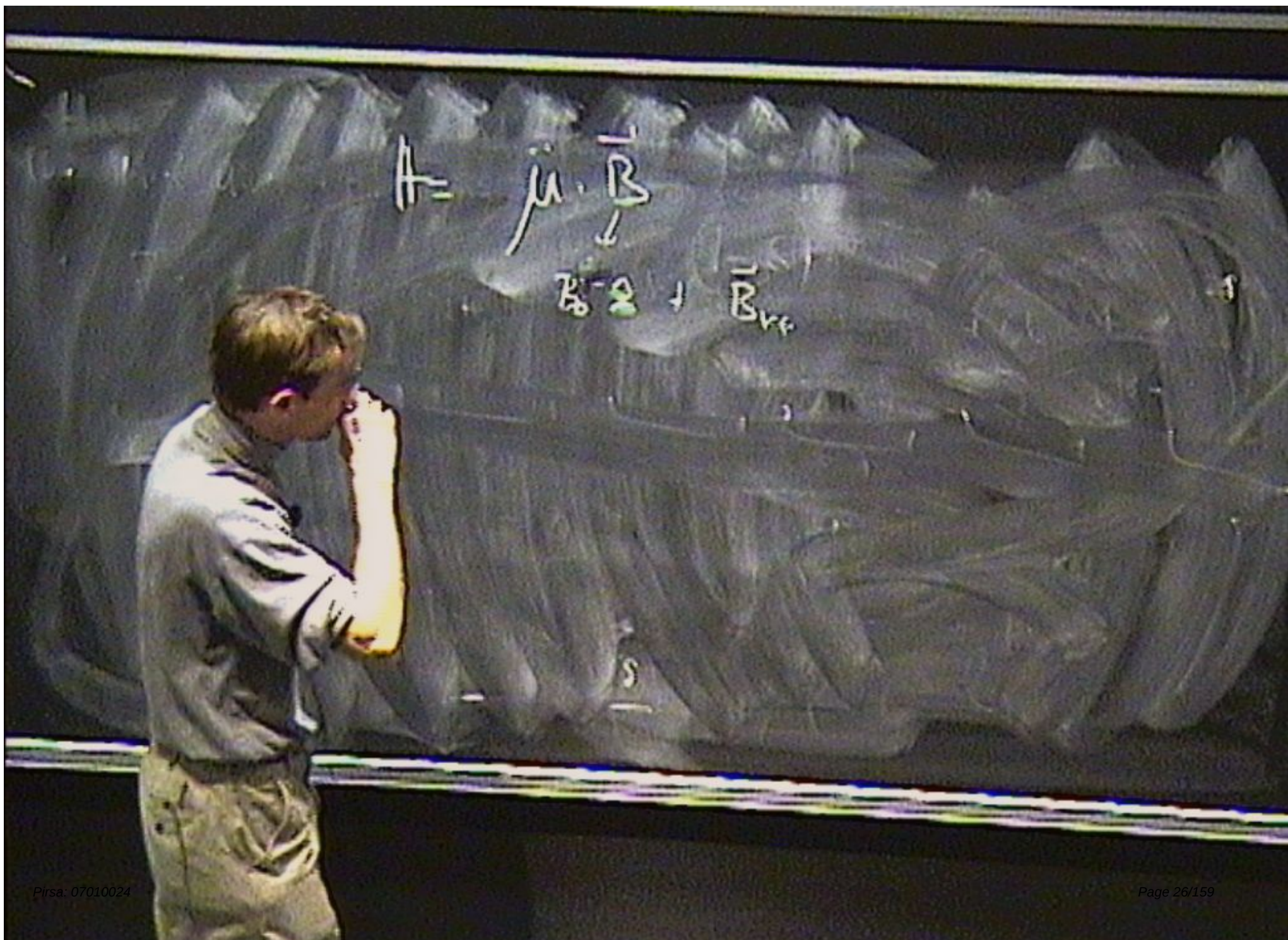


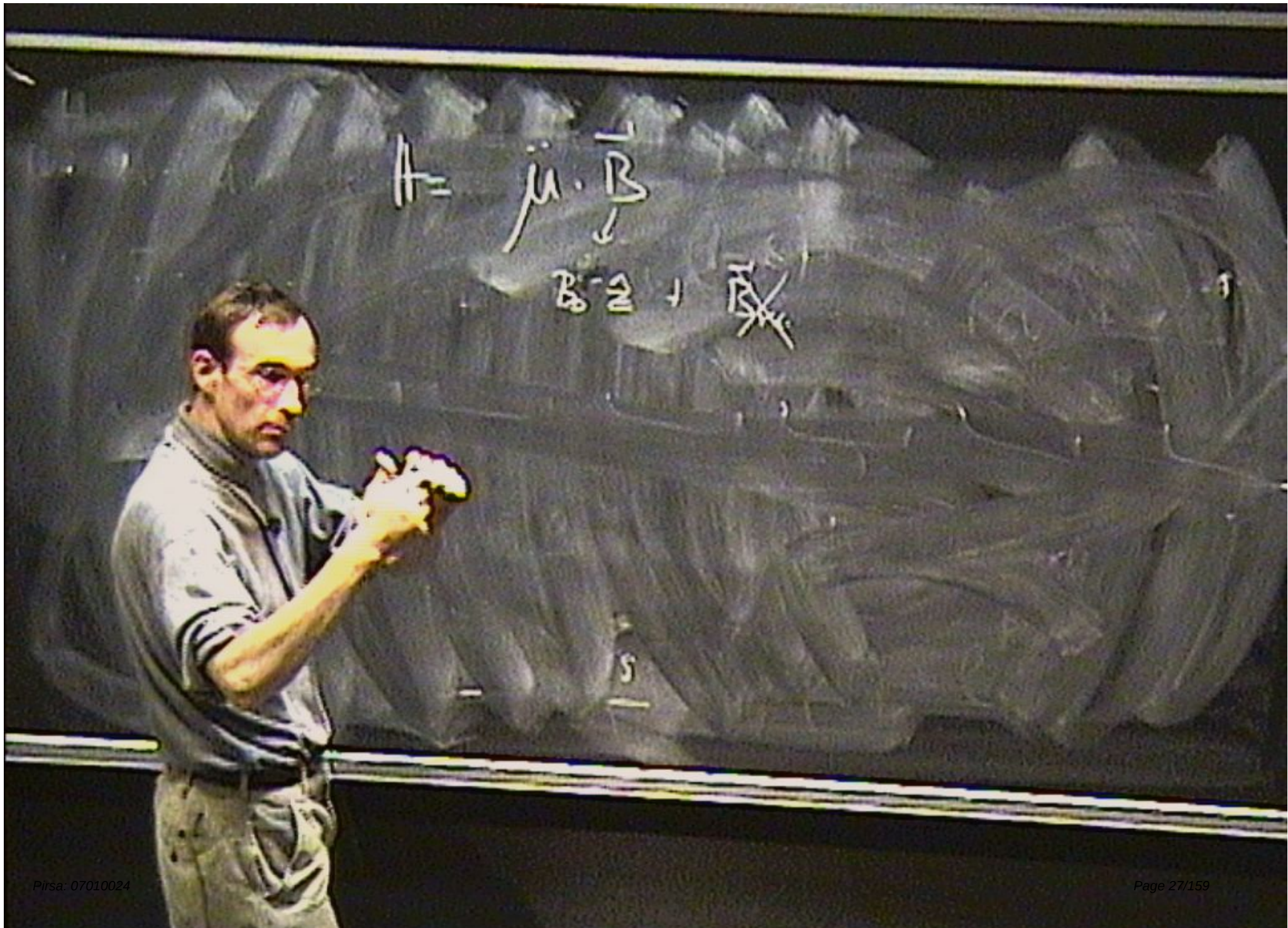
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$$A = \mu \cdot \vec{B}$$

$$B = \frac{1}{\mu_0} \nabla \times \vec{A}$$

$$H = \mu \cdot \vec{B}$$

$$\vec{B} = \vec{B}_0 + \vec{B}_1$$

$$= \omega_1 \hat{z}_1 + \omega_2 \hat{z}_2 + J \hat{z}_1 \hat{z}_2$$

$$H = \mu \cdot \vec{B}$$

$\vec{B} = \frac{e\hbar}{4\pi m_e} (\frac{1}{Z_1} + \frac{1}{Z_2}) \vec{S}$

$$= \omega_1 Z_1 + \omega_2 Z_2 + J Z_1 Z_2$$

$$H = \mu \cdot \vec{B}$$

$$B = \frac{1}{2} + \frac{1}{2} \sigma_x$$

$$H = \omega_1 Z_1 + \omega_2 Z_2 + J Z_1 Z_2$$

$e^{iH\tau}$
 $|0\rangle$

$$H = \mu \cdot \vec{B}$$

$B_z \hat{=} 1$ ~~$\vec{B}_x$~~

$e i \hbar c$

$$= \omega_1 \hat{z}_1 + \omega_2 \hat{z}_2 + J \hat{z}_1 \hat{z}_2$$

$|0\rangle_+$

$$\rightarrow (\omega_2 + J) \hat{z}_2$$

$|0\rangle_-$

$$\rightarrow (\omega_2 - J) \hat{z}_2$$

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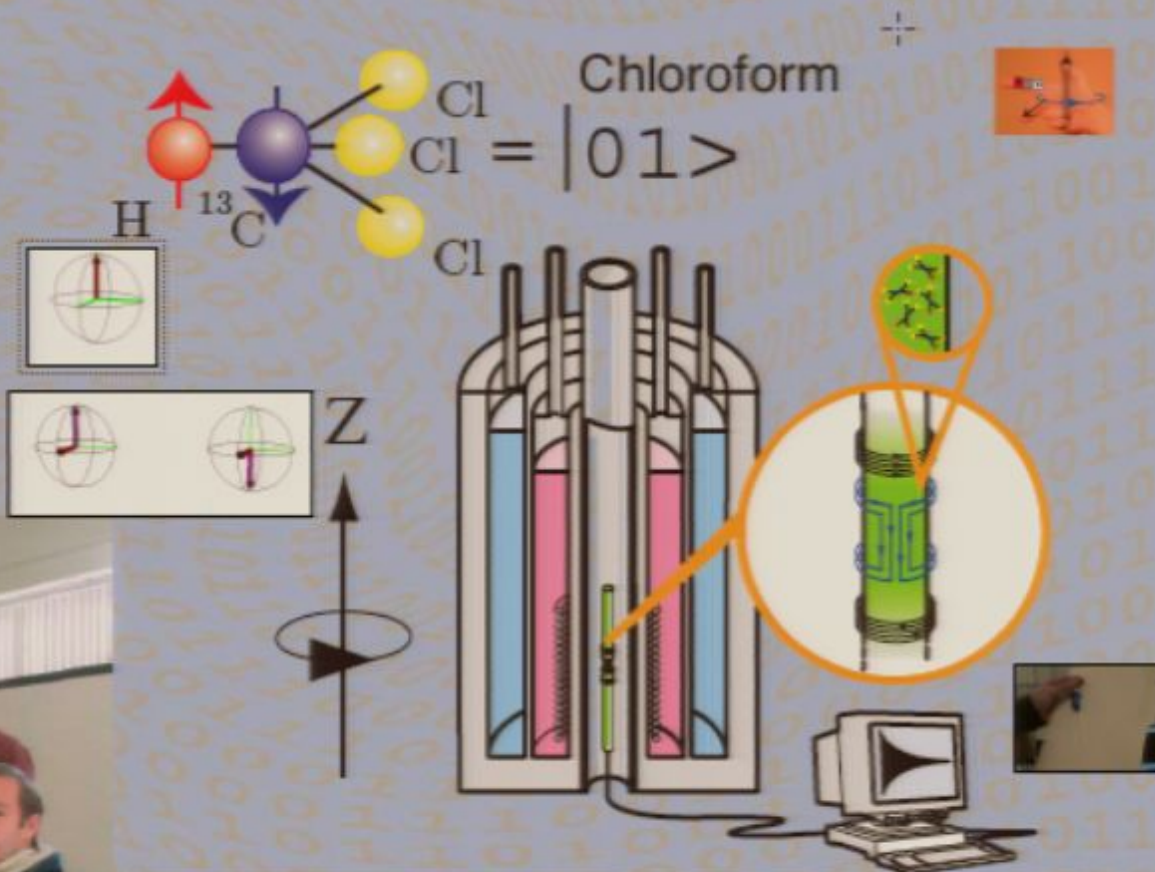
Nuclear Magnetic Resonance



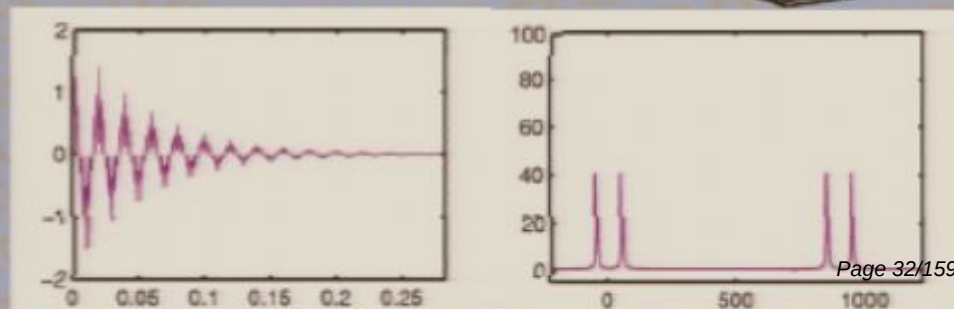
Bloch



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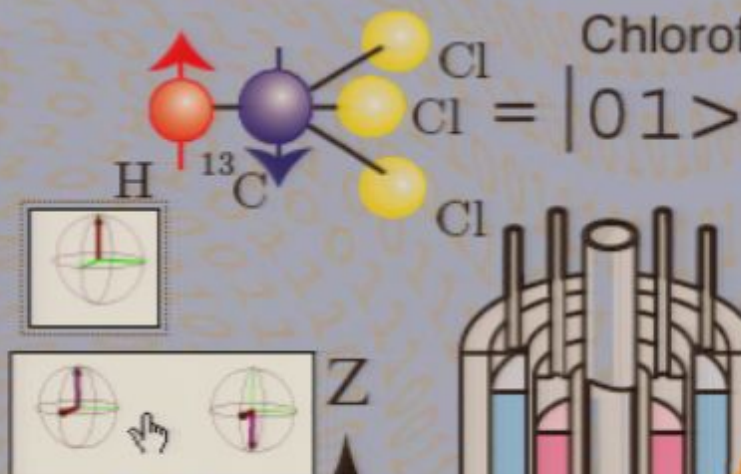
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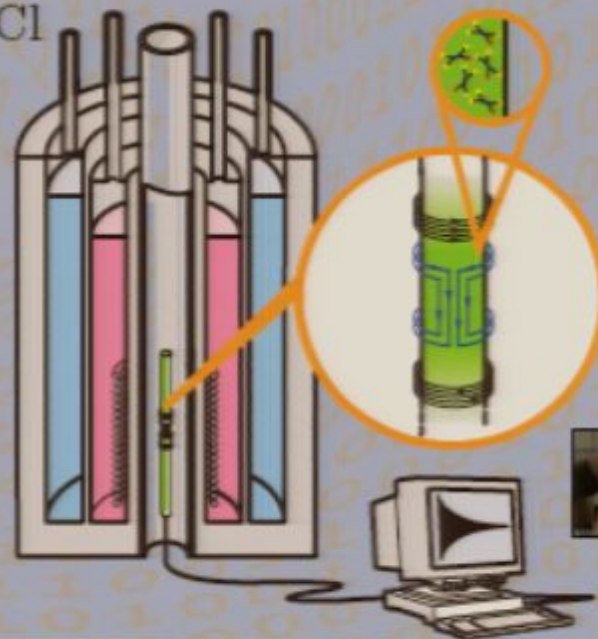
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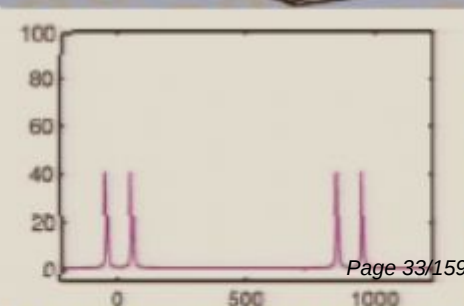
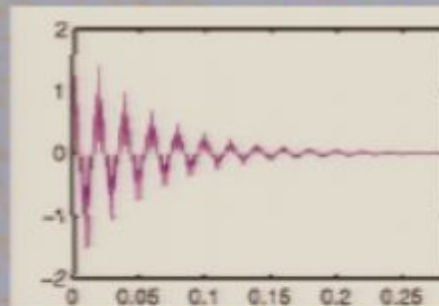
Chloroform



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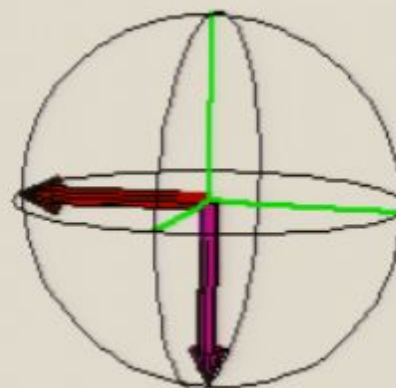
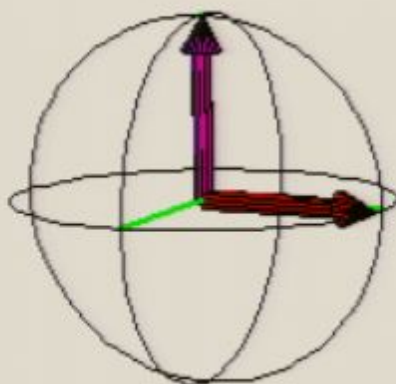
Nuclear Magnetic Resonance



Chloroform



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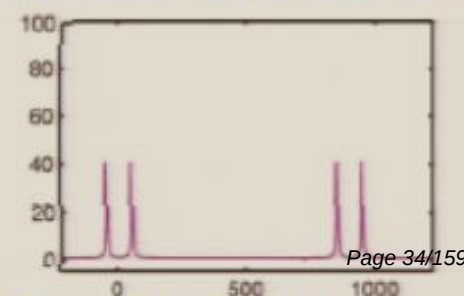
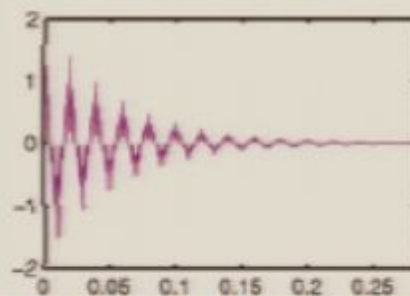


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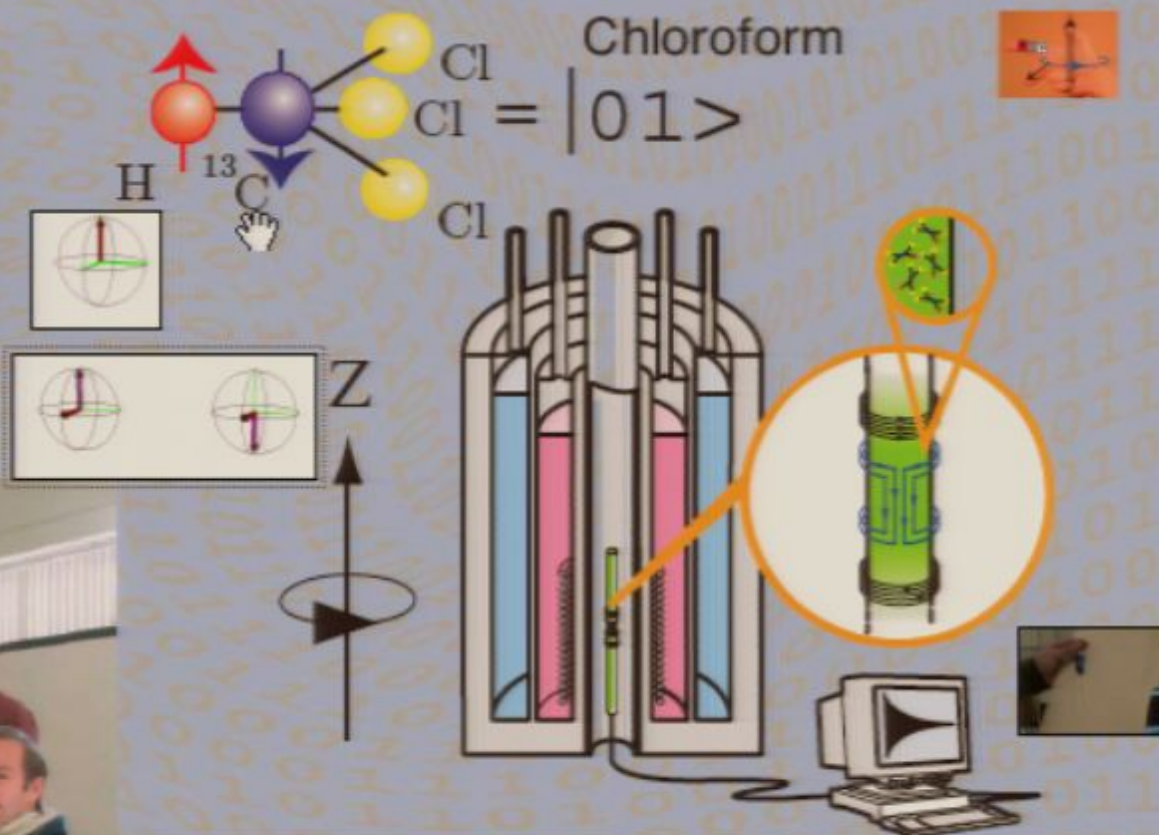
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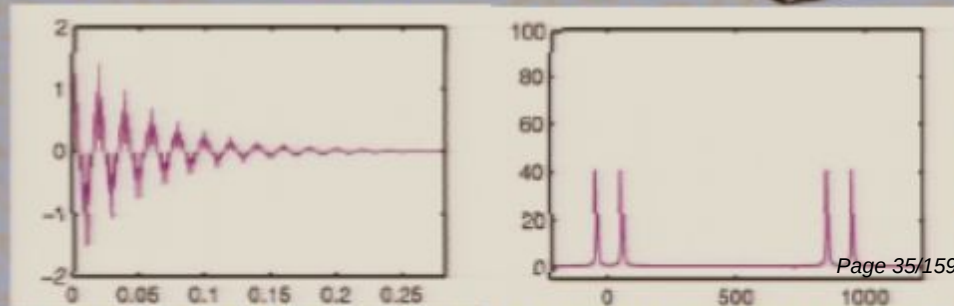
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Notation

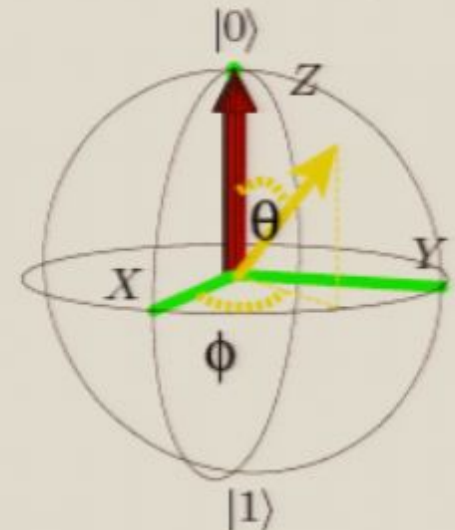
- Pauli matrices:

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Tensor product $X_k = \mathbb{1} \otimes \mathbb{1} \otimes \dots \otimes X \otimes \dots \otimes \mathbb{1}$.

- Bloch sphere: a geometric representation of the state $\Psi = \cos \theta |0\rangle + e^{i\phi} \sin \theta |1\rangle$ or

$$\rho = \frac{1}{2}(\mathbb{1} + \sin[\theta] \cos[\phi] X + \sin[\theta] \sin[\phi] Y + \cos[\theta] Z)$$



- $\hbar = 1.05 \times 10^{-34} \text{ Js}$, $k = 1.38 \times 10^{-23} \text{ J/K}$

1 Tesla = 10^4 Gauss (Earth Magnetic field = 0.5 Gauss)

$$\mu_p = 26.75 \times 10^7 / (\text{Ts})$$

- Freq. of the spectrometer $\nu_L^p = 26.75 \times 10^7 \text{ rad/(Ts)} * 11.7 \text{ T} / 2\pi = 500 \text{ MHz}$

- Corresponding $\beta = \hbar \times \omega_l / kT$

$$= 1.05 \times 10^{-34} \text{ Js} \times 2\pi \times 486 \text{ MHz} / (1.38 \times 10^{-23} \text{ J/K} \times 300 \text{ K}) \approx 7.8 \times 10^{-5}$$

- Definition of T_1 (relaxation time): $M_z(\tau) = M_z^0 [1 - 2e^{(-\tau/T_1)}]$

- Definition of T_2 (decoherence time): $M_x(\tau) = M_x^0 [\exp(-2\tau/T_2)]$

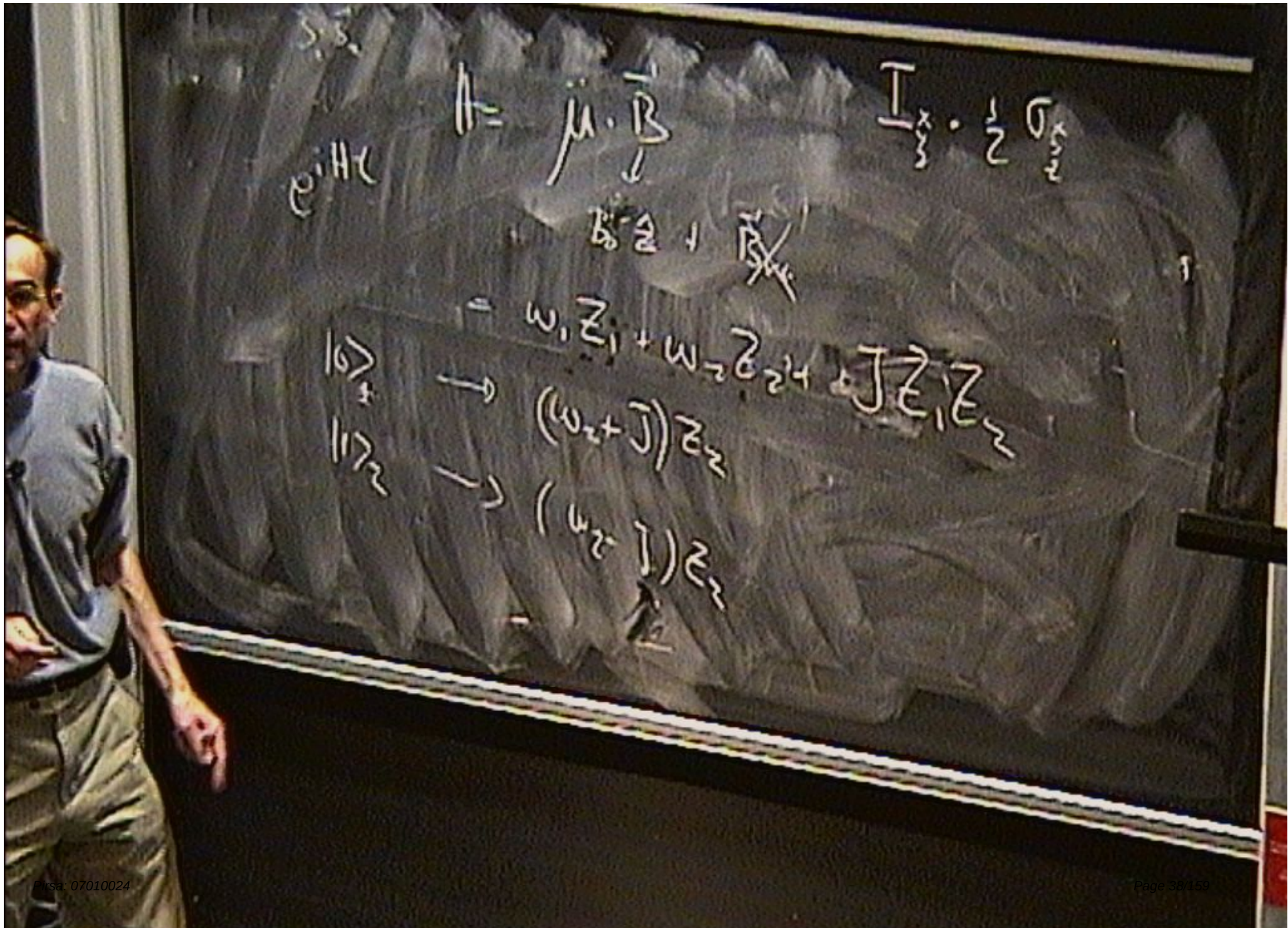
$$H = \mu \cdot \vec{B} \quad I_x \cdot \frac{1}{2} \sigma_x$$

$$B_0 \hat{z} + \vec{B}_1$$

$$|0\rangle_1$$

$$|1\rangle_2$$

$$1 + \omega_2 \hat{z}_2 + J \hat{z}_1 \hat{z}_2$$

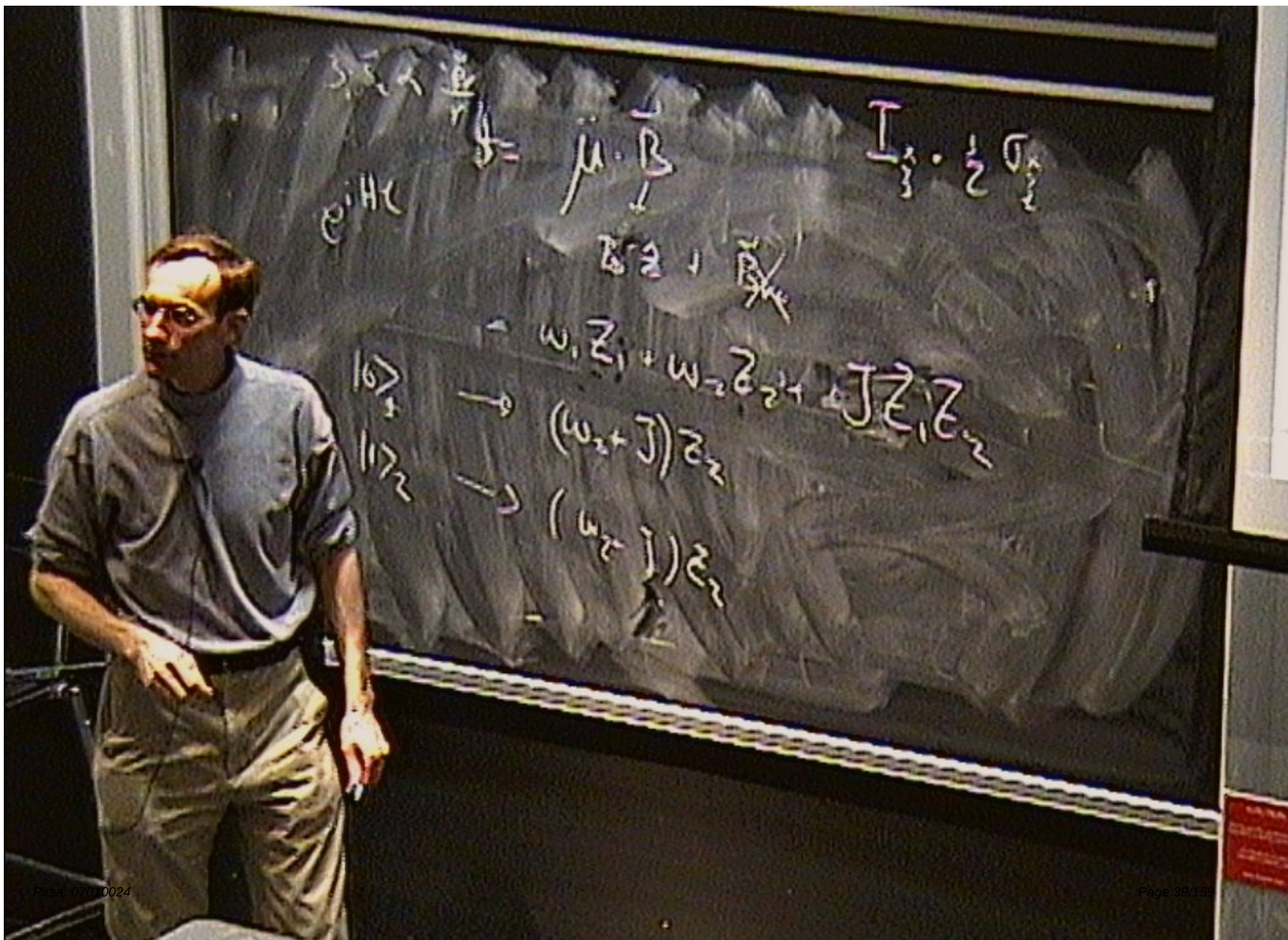


$S_1 \cdot S_2$

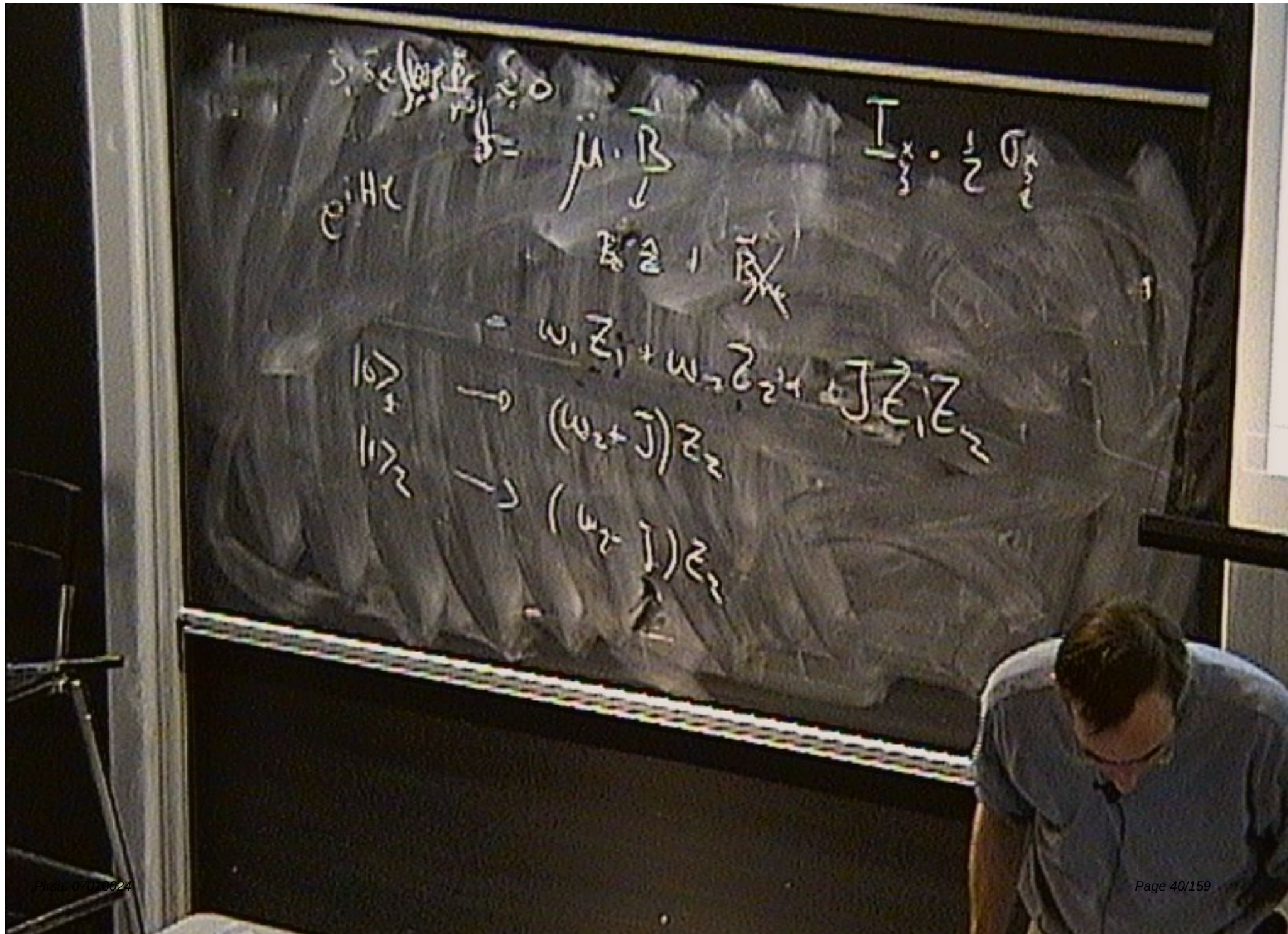
$$H = \mu \cdot \vec{B} \quad I_x \cdot \frac{1}{2} \sigma_x$$
$$B = B_z + B_x$$
$$= \omega_1 z_1 + \omega_2 z_2 + J z_1 z_2$$

$|0\rangle_+$ $\rightarrow (\omega_2 + J) z_2$

$|1\rangle_+$ $\rightarrow (\omega_2 - J) z_2$



$$\begin{aligned} \psi &= \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \\ e^{iHt} &= e^{-i\mu \cdot \vec{B}} \\ &= e^{-i\omega_1 \tau_1 - i\omega_2 \tau_2 - iJ \tau_1 \tau_2} \\ &\rightarrow (\omega_1 + J) \tau_1 \\ &\rightarrow (\omega_2 + J) \tau_2 \end{aligned}$$



Notation

- Pauli matrices:

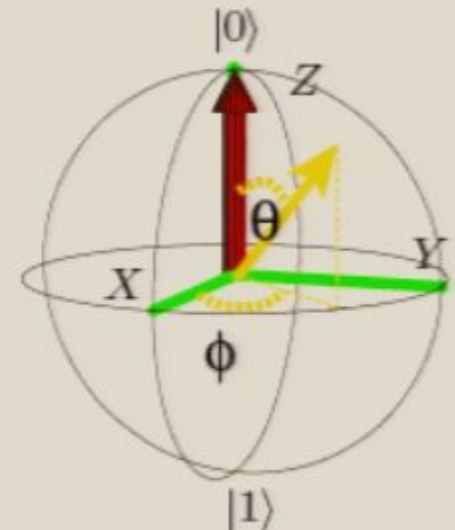
$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Tensor product $X_k = \mathbb{1} \otimes \mathbb{1} \otimes \dots \otimes X \otimes \dots \otimes \mathbb{1}$.

- Bloch sphere: a geometric representation of the state

$\Psi = \cos \theta |0\rangle + e^{i\phi} \sin \theta |1\rangle$ or

$$\rho = \frac{1}{2}(\mathbb{1} + \sin[\theta] \cos[\phi] X + \sin[\theta] \sin[\phi] Y + \cos[\theta] Z)$$



- $\hbar = 1.05 \times 10^{-34} \text{ Js}$, $k = 1.38 \times 10^{-23} \text{ J/K}$

1 Tesla = 10^4 Gauss (Earth Magnetic field = 0.5 Gauss)

$$\mu_p = 26.75 \times 10^7 / (\text{Ts})$$

- Freq. of the spectrometer $\nu_L^p = 26.75 \times 10^7 \text{ rad/(Ts)} * 11.7 \text{ T} / 2\pi = 500 \text{ MHz}$

- Corresponding $\beta = \hbar \times \omega_l / kT$

$$= 1.05 \times 10^{-34} \text{ Js} \times 2\pi \times 486 \text{ MHz} / (1.38 \times 10^{-23} \text{ J/K} \times 300 \text{ K}) \approx 7.8 \times 10^{-5}$$

- Definition of T_1 (relaxation time): $M_z(\tau) = M_z^0 [1 - 2e^{(-\tau/T_1)}]$

- Definition of T_2 (decoherence time): $M_x(\tau) = M_x^0 [\exp(-2\tau/T_2)]$

Notation

- Pauli matrices:

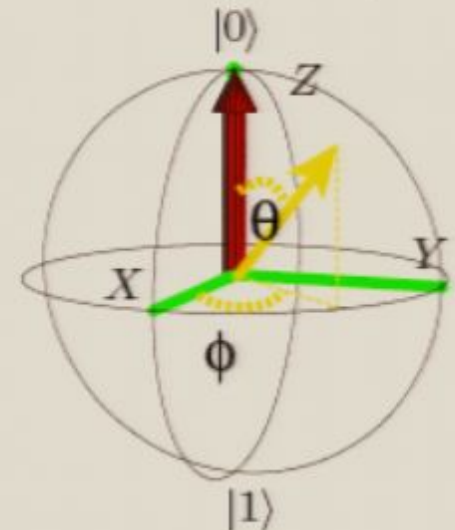
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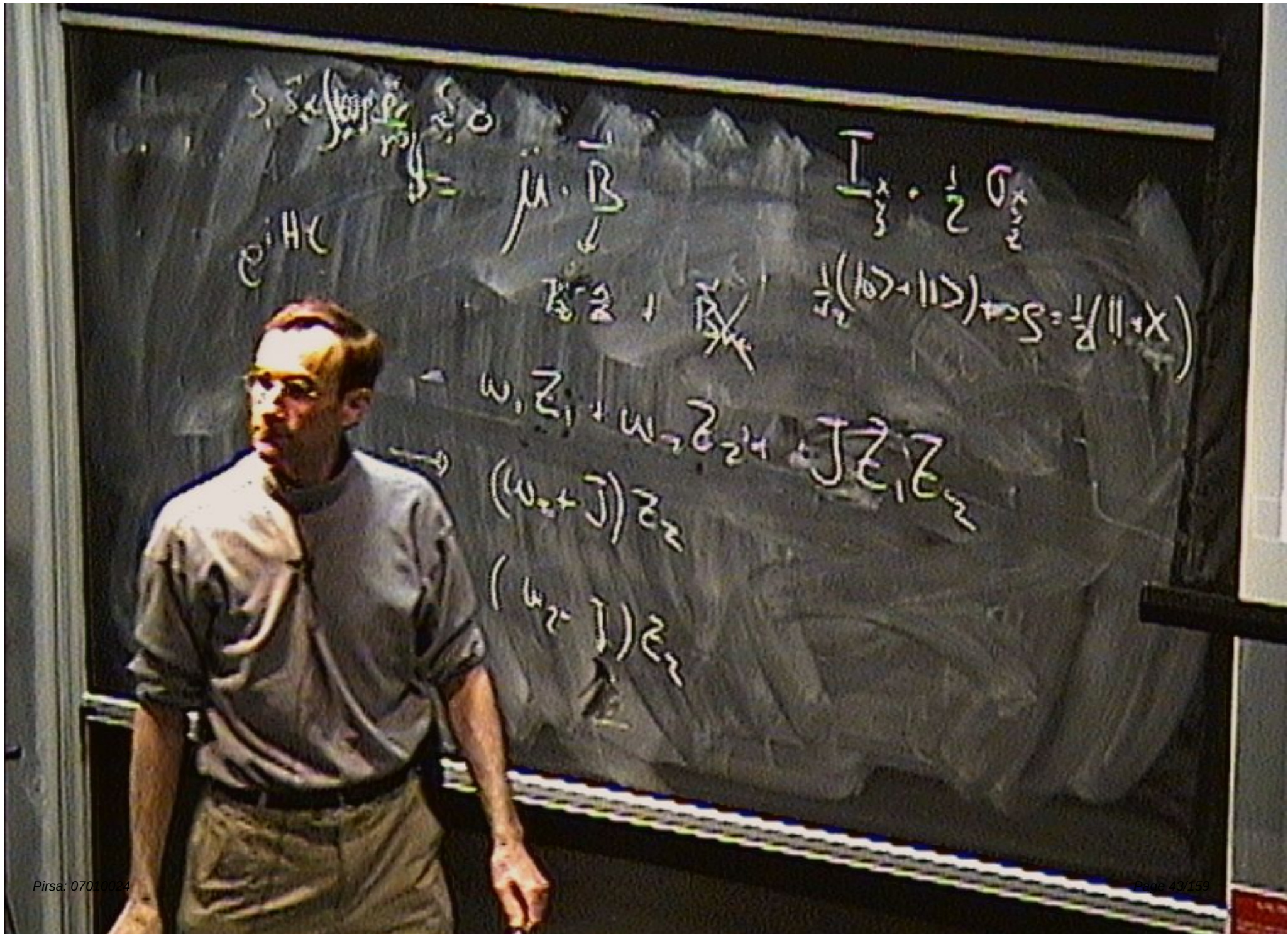
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Notation

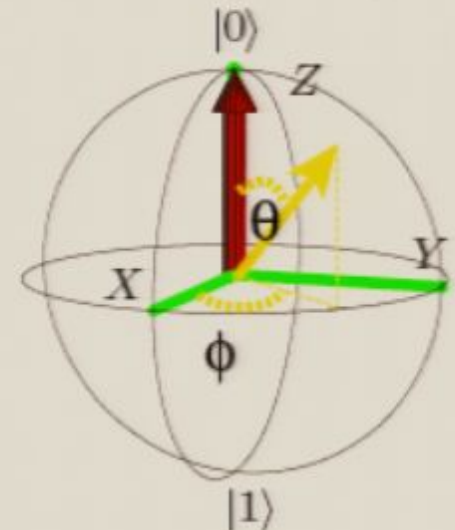
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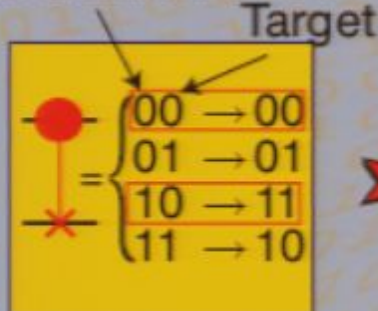
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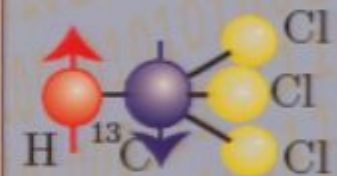
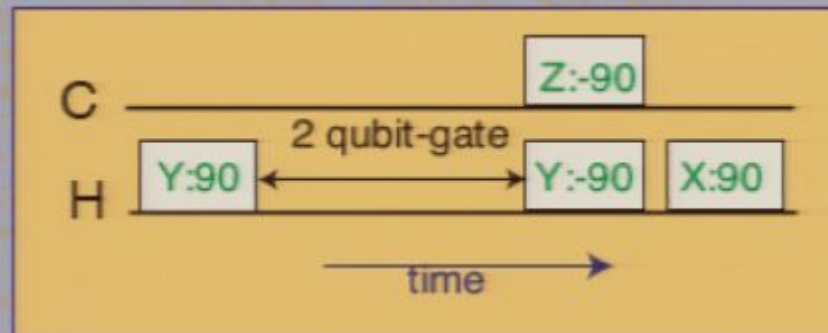
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- Definition of T_2 (decoherence time): $M_x(\tau) = M_x^0 [\exp(-2\tau/T_2)]$

Control-Not



Quantum circuit



Pre-compiler (Optimizer)

```
;; Refring to track down error sources by doing partial error correction.

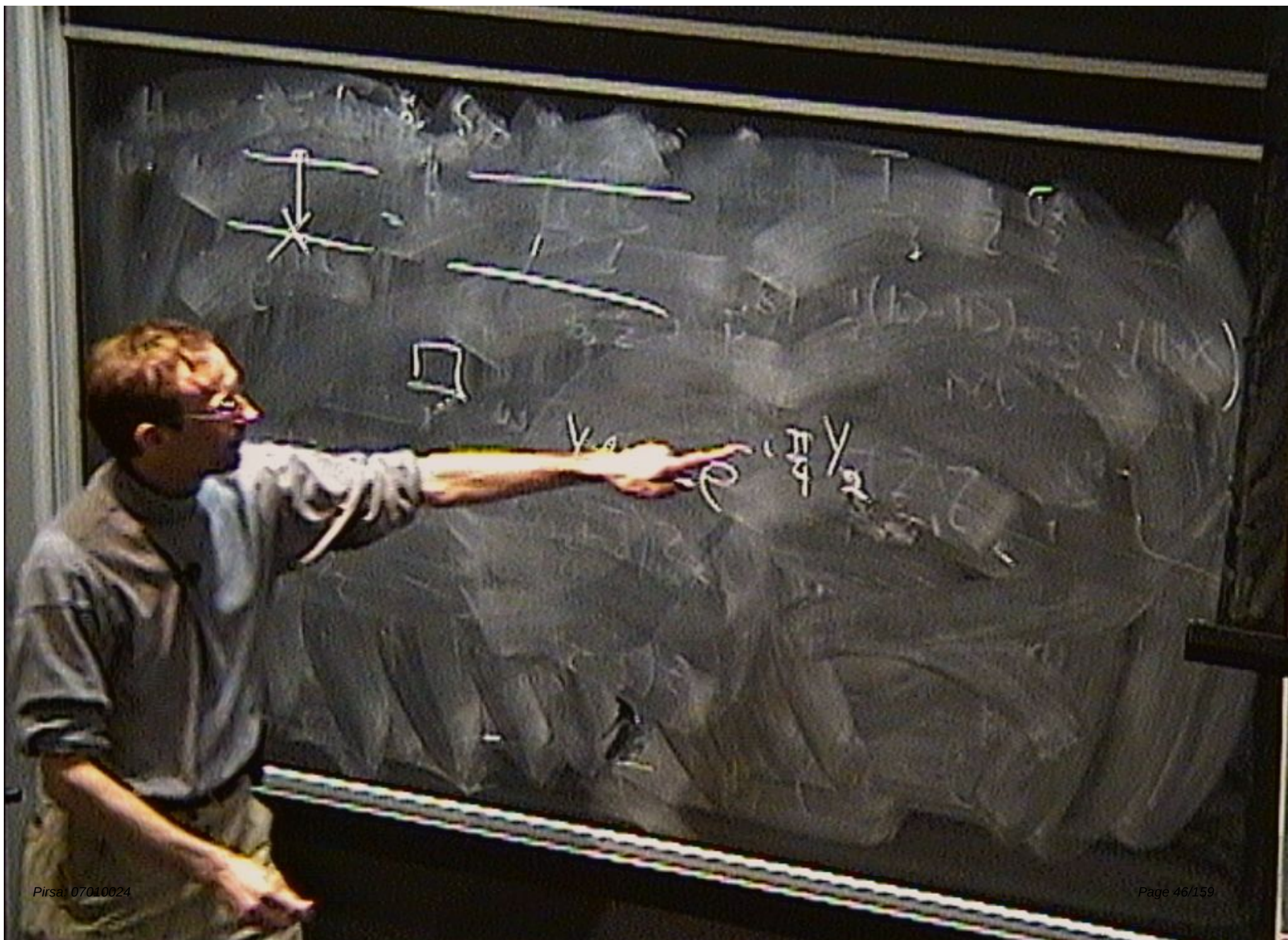
(define cube
  (switch (ex) = 1)
  (switch (ex) = 2)
  (include "cnp.h")

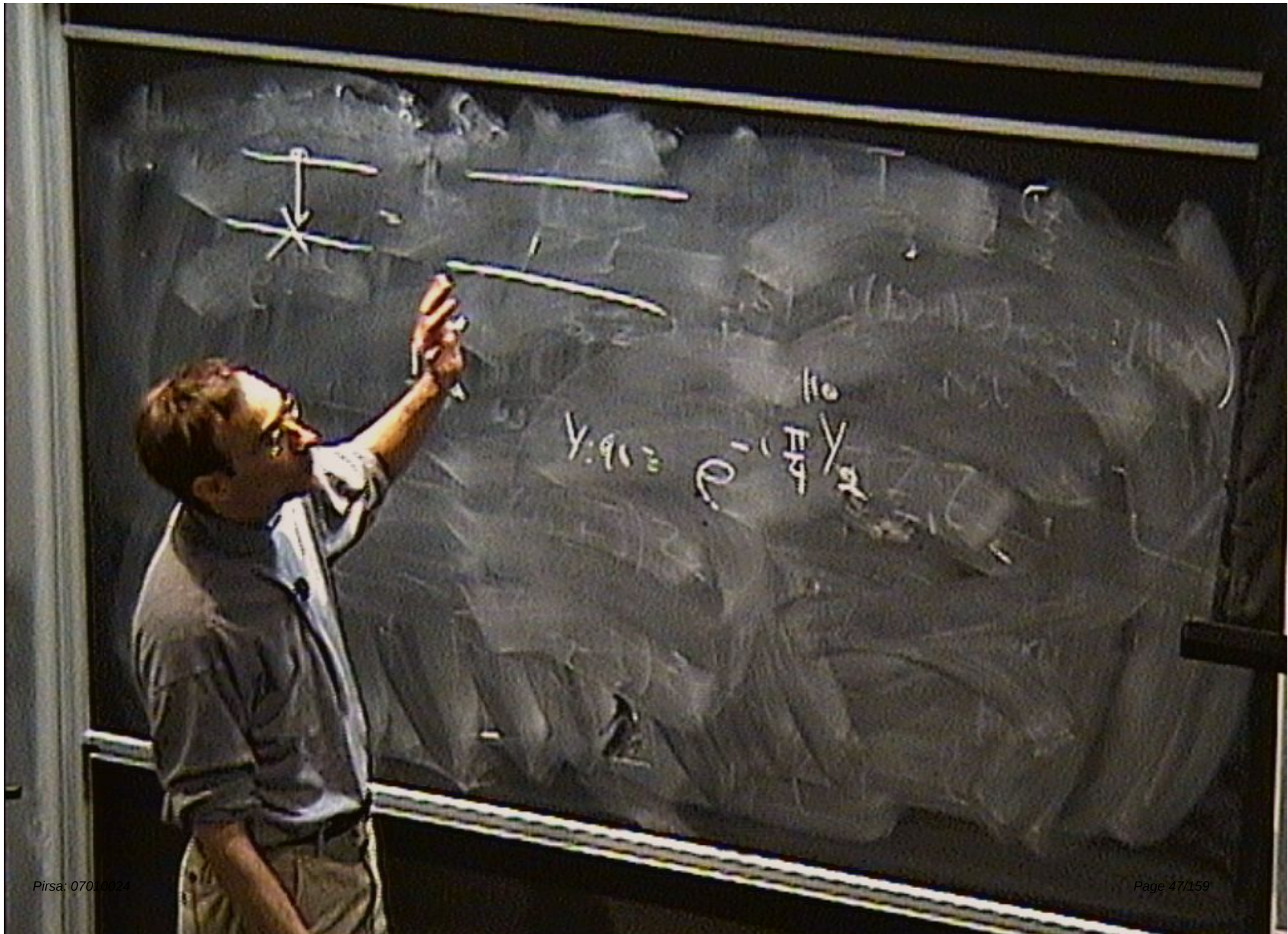
  > slowing = 0; slowing = 1; 4)

; echo . 1
;;
;; > slowing = 0; slowing
; pulse cnp w=0.7;
; correction shape ; pulse C2_90
;
; > cnp c1 = 0.05 ; zz .25 C1 C
;
; pulse c1 90 ;
; ex . 10 C1 C ; zpulse C1:..
; pulse c1 90 ;
; pulse c1 90 ;
; because then ; pulse C2_90
; delay end ;
;
; ; pulse C2_90
;
; include "cnp_def"
; refocus C1C
```

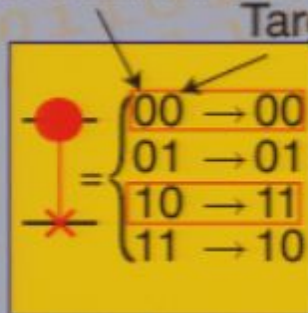
Brüker (machine) language

[illegible]

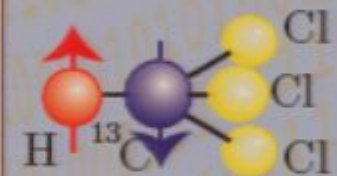
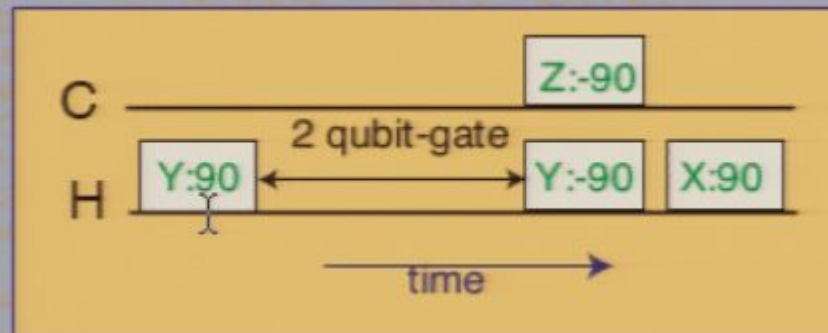




Control-Not



Quantum circuit



Pre-compiler (Optimizer)

[illegible]

Brüker (machine) language

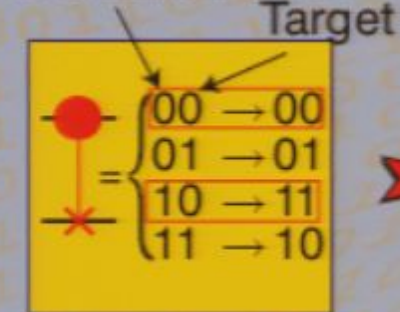
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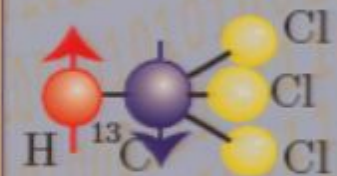
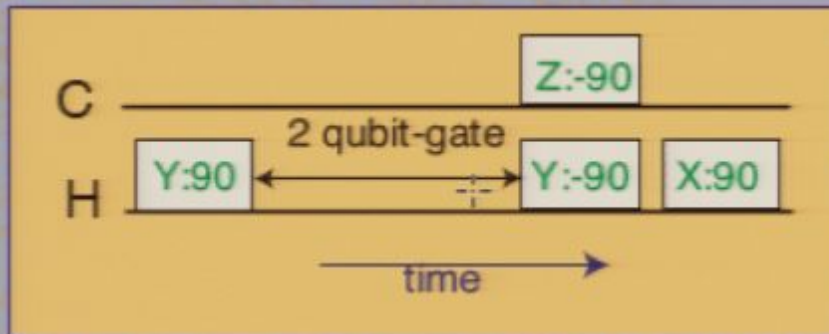


$$y: q_1 = \rho^{-i\frac{\pi}{4}} \gamma_2$$

$$\rho^{-i\frac{\pi}{4}} z_1 z_2$$



Target



Pre-compiler (Optimizer)

Brüker (machine) language

```

; trying to track down error source by doing partial error correction.
#define cube
; (search[0]) = 1;
; (search[1]) = 1;
#include "capp.h"

; slowexp = 0; slowexp = 1; 0

; scale .1
; 1/4
; slowexp = 0; slowexp
; pulse ramp exp=0;
; correction ramp
; 1/2
; ramp 0.1
; pulse c2_90
; rz .25 c1 c
; pulse c1_90;
; pulse c2_90;
; pulse c2_90;
; before flip
; delay and
; 1/4
; pulse c2_90
; refocus c1c

```

```
; pulse C2_90 .25
; zz .25 C1 C2
; zpulse C1:.75;C2:
; pulse C2_90 .75
; pulse C2_90 .0
refocus C1C2 180
```

```

1 0x
2 fhold ucnme_cow

du
du swent.fz
du swent.fz
du

fhold ucnme_cow
;initial virtual zero.
;TUNE: 0.000e+00 seconds

vu
vu
( C2_90:sp9 ph13 ):f1
3u_
3u ipp13
0.71365m
0.71365m
8u
8u
(C2_90:sp9 ph19):f1
6u_ipp15 ipp19
8u
(C2_90:sp9 ph20):f1
6u_ipp15 ipp20

```



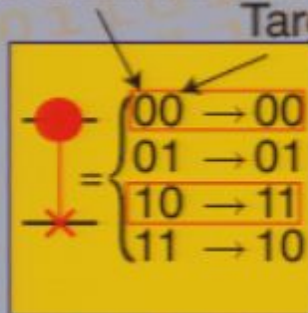
$\gamma: q_1, q_2$

$\rho^{-1}(\frac{1}{4}\gamma)$

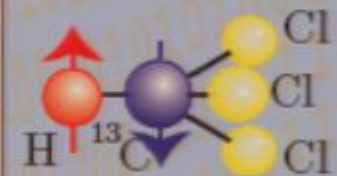
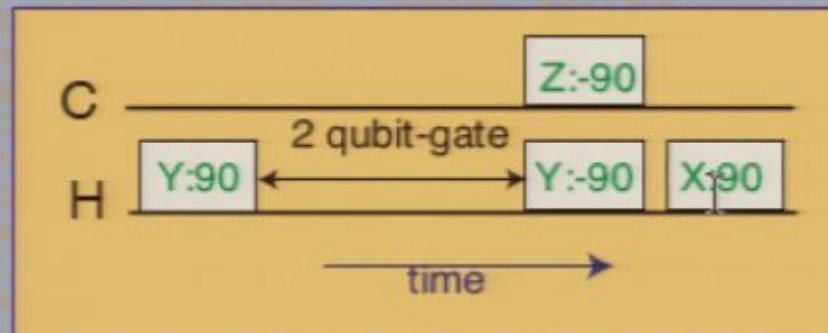
$\rho^{-1}(\frac{1}{4}\gamma) \rightarrow J \cdot t$

$J \cdot t$

Control-Not



Quantum circuit



Pre-compiler (Optimizer)

[illegible]

Bruker (machine) language

```

1  $*
2  $hold source_code

da
da reset.fz
da reset.fz
da

$hold source_code

initial virtual time
time 0.000000 second

da
da ( da_esp_esp_13 ) :f1
3u_
3u_ ipp13

0.71365m

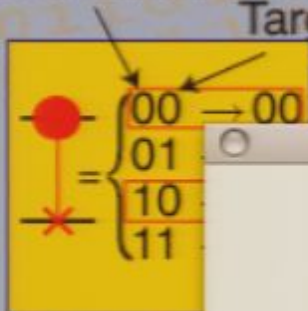
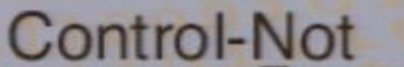
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da da_esp_esp_13
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da da_esp_esp_13
da_ipp13_ipp13

da da_esp_esp_13
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da da_esp_esp_13
da da_ipp13_ipp13

$hold source_code

```



Target

Quantum circuit

C

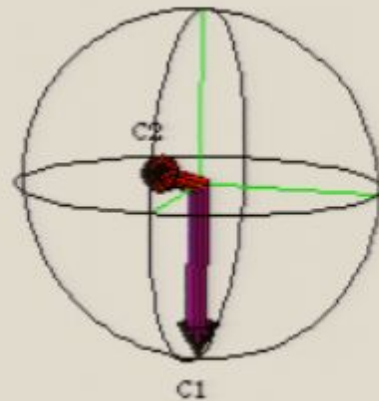
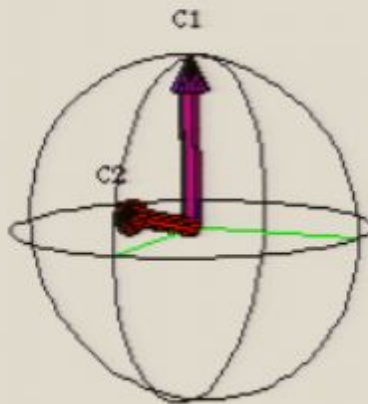
Z:-90

Adobe Acrobat – Movie

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Cl



Pre-

```

12 delay to time
define cube
1 search (a) = 1
2 search (a) = 2
3 search (a) = 3
4 glowing = 0
5
6 scale .1
7
8 glowing = 0
9 pulse ramp on
10 connection on
11
12 scale 0.1-0.9
13 pulse 0.1 0.1
14   .15 0.1 0.1
15   .16 0.1 0.1
16   .17 0.1 0.1
17   .18 0.1 0.1
18   .19 0.1 0.1
19   .20 0.1 0.1
20 before this
21 delay and .1
22
23 include "temp_d

```

```
; zz .25 C1 C2
; zpulse C1:.75;C2:
; pulse C2_90 .75
; pulse C2_90 .0
refocus C1C2 180
```

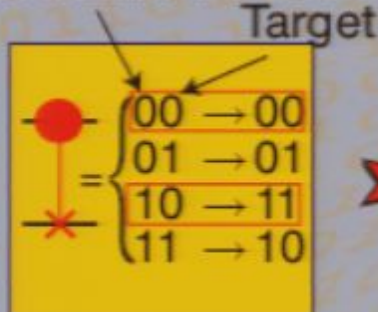
0.71365m

```

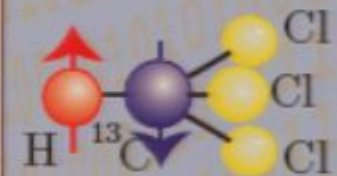
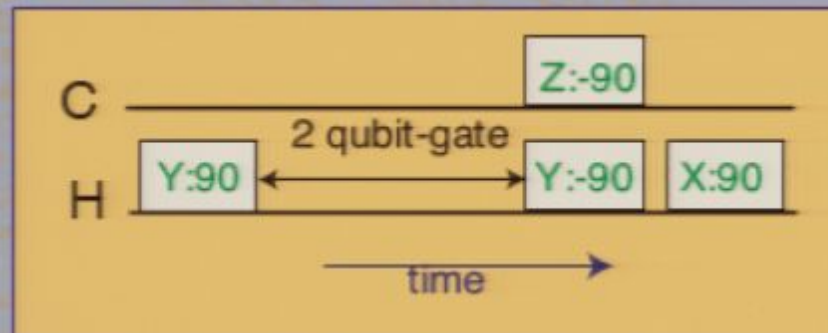
8u
8u
(C2_90:sp9 ph19):f1
6u-ipp15 ipp19
8u
(C2_90:sp9 ph20):f1
6u-ipp15 ipp20

```

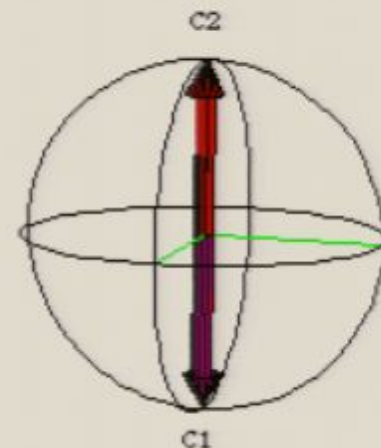
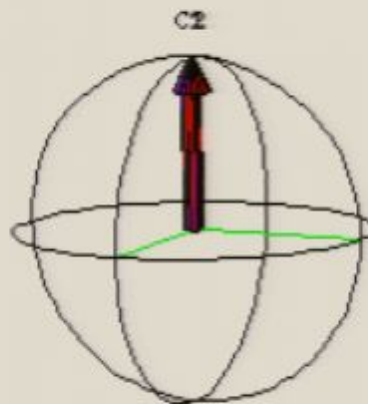
Control-Not



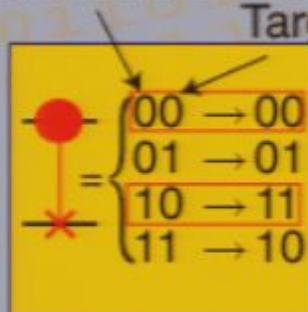
Quantum circuit



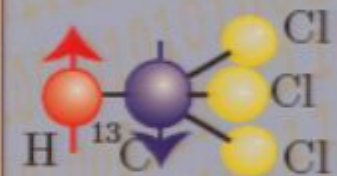
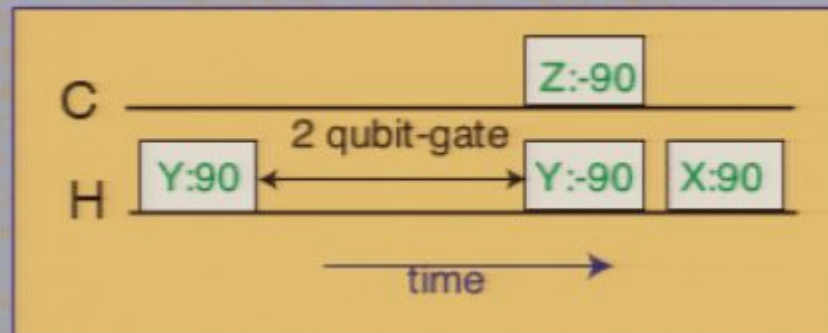
Pre-

[illegible]

Control-Not



Quantum circuit

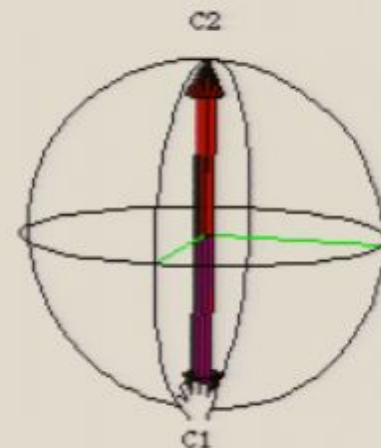
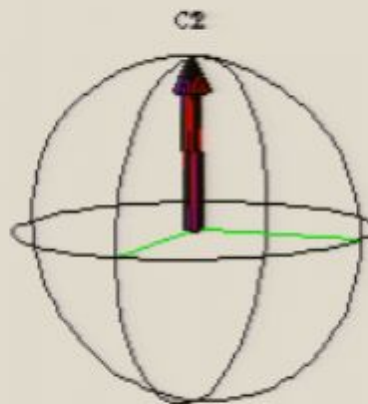


Pre-

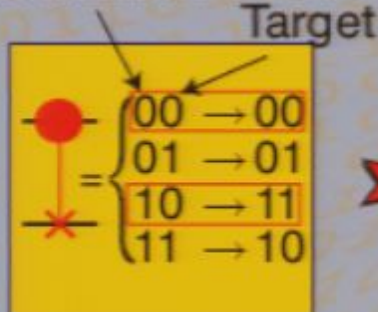
```

11 return to top
endifine code
( search [0] ) =
( search [6] ) =
endifine code
) > glowing = 0
) scale = 2
116 glowing = 0
) pulse ramp up
117 connection str
118
) state 0 - end
) gl = 0.90
) gl = 0.80
) gl = 0.70
) gl = 0.60
) gl = 0.50
) gl = 0.40
) gl = 0.30
) gl = 0.20
) gl = 0.10
) delay end
)
endifine code

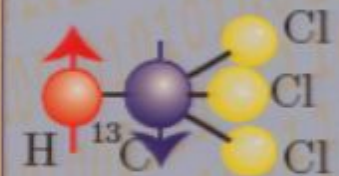
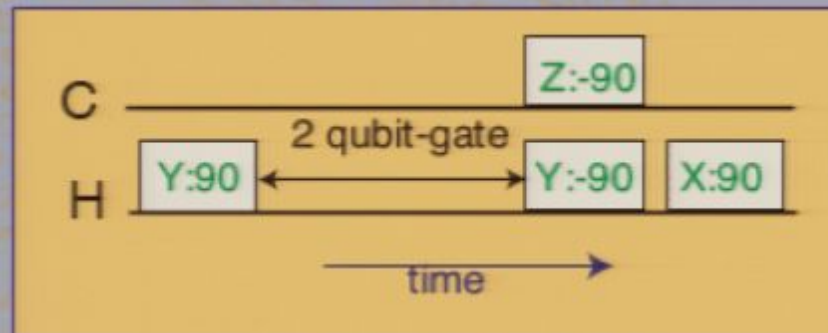
```



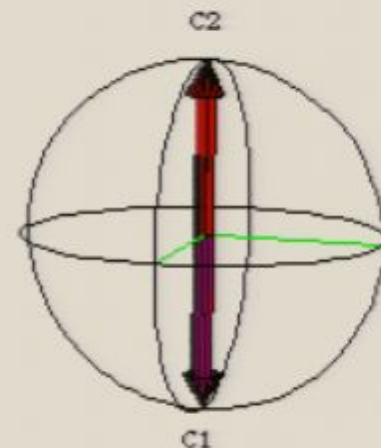
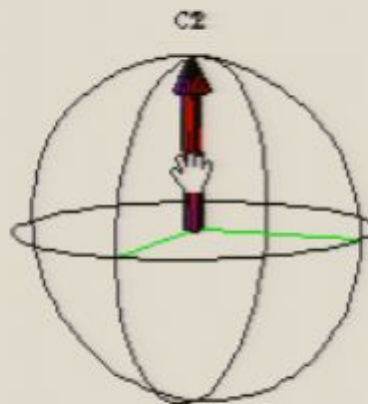
Control-Not



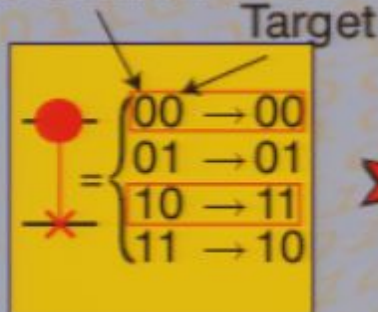
Quantum circuit



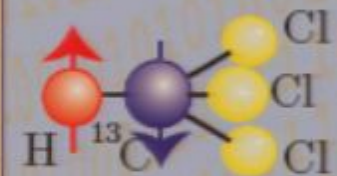
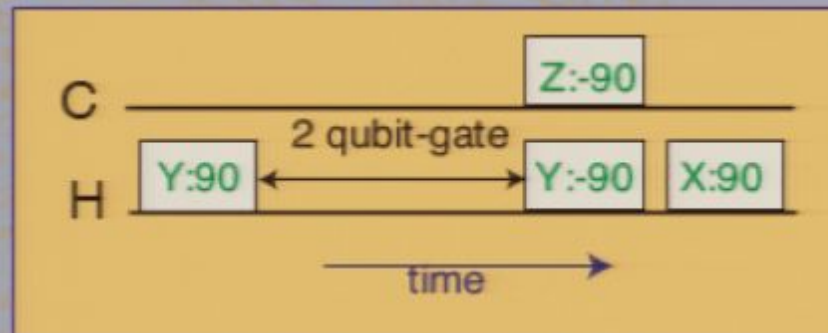
Pre-

[illegible]

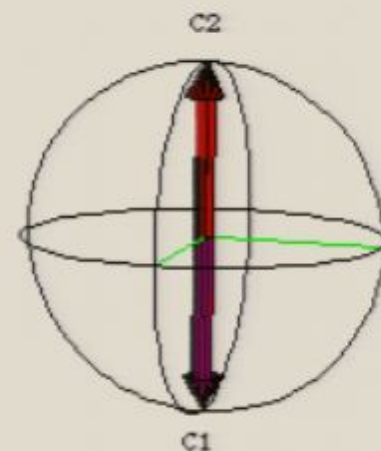
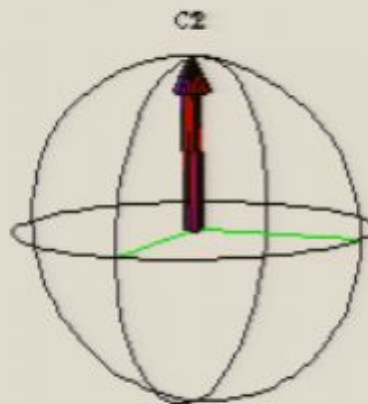
Control-Not



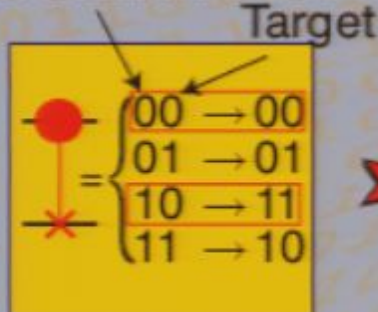
Quantum circuit



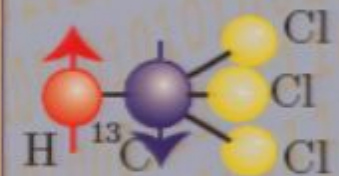
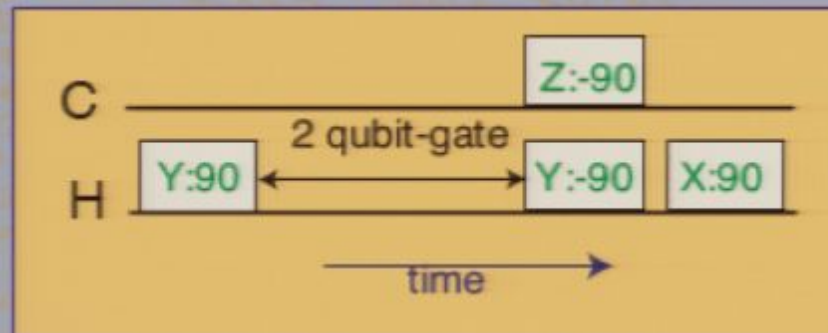
Pre-

[illegible]

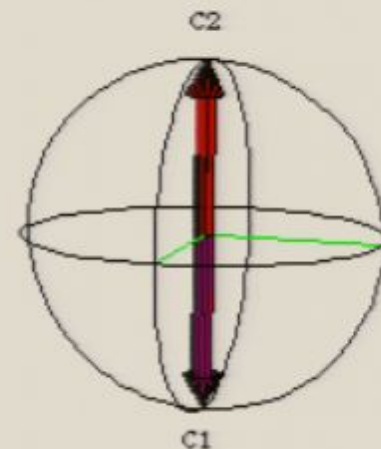
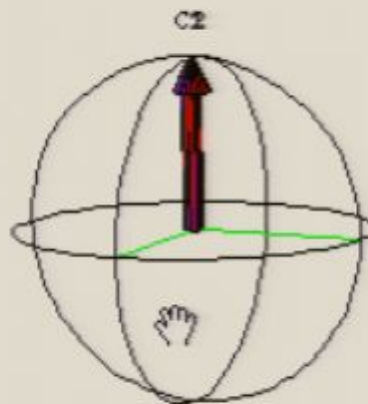
Control-Not



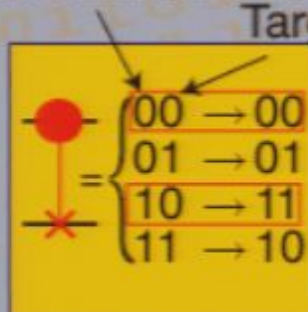
Quantum circuit



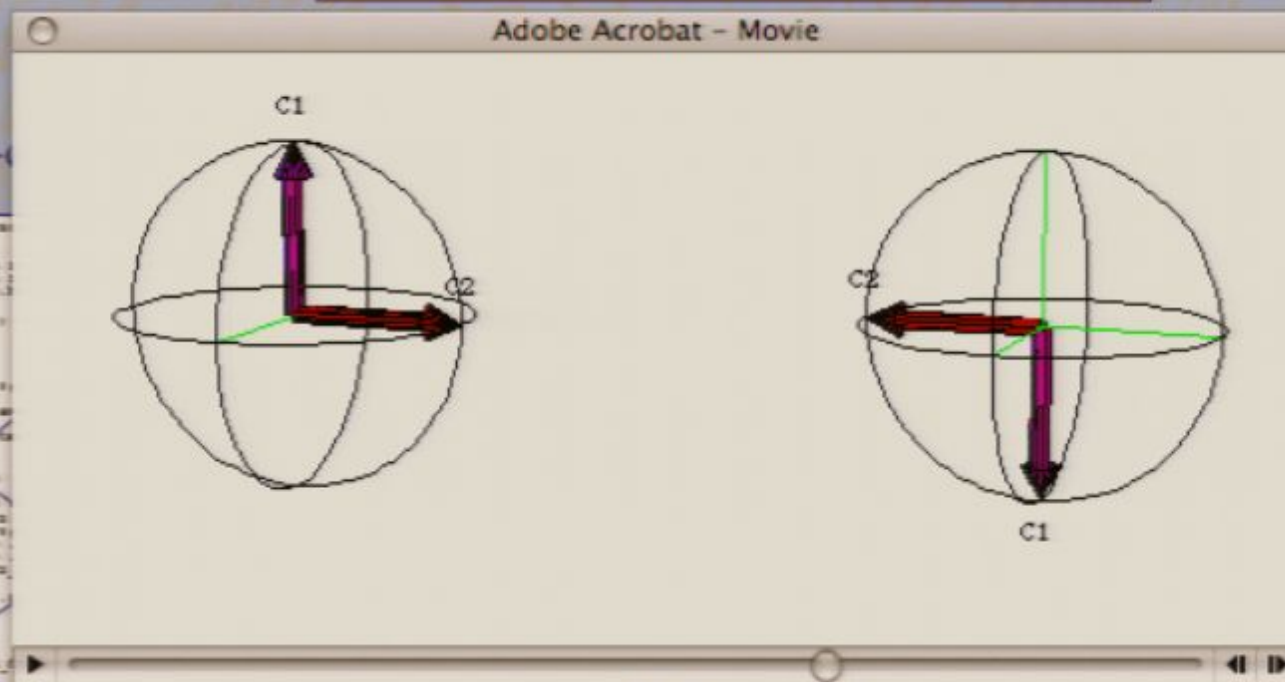
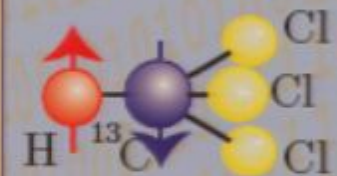
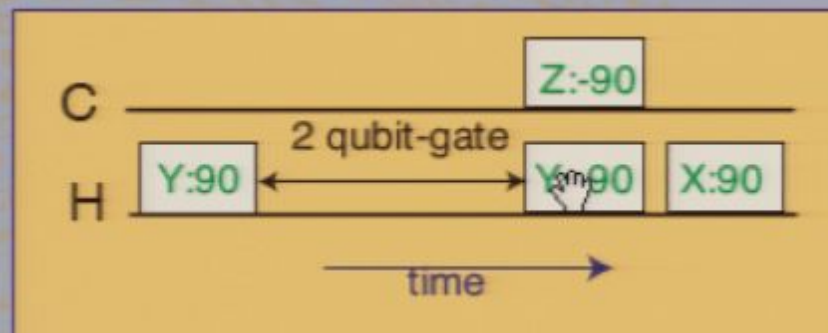
Pre-

[illegible]

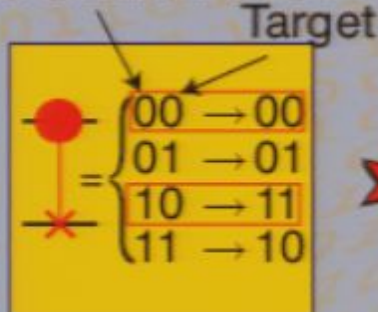
Control-Not



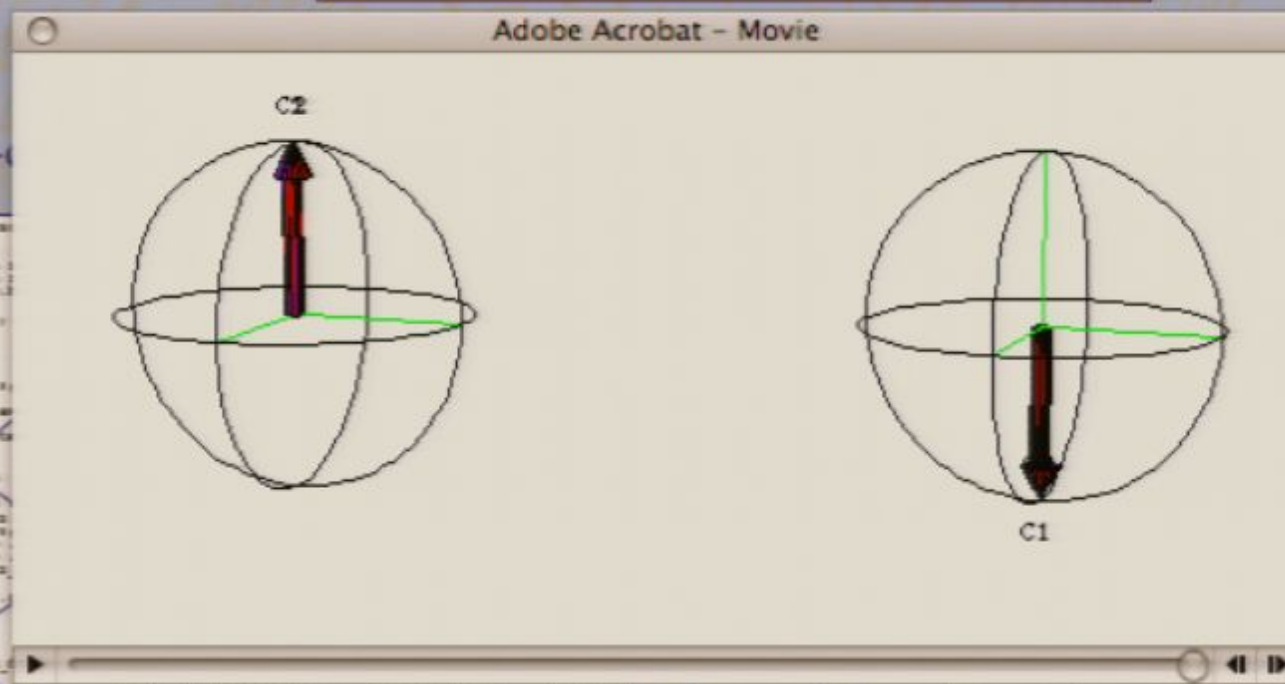
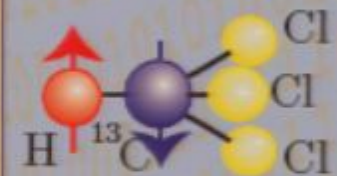
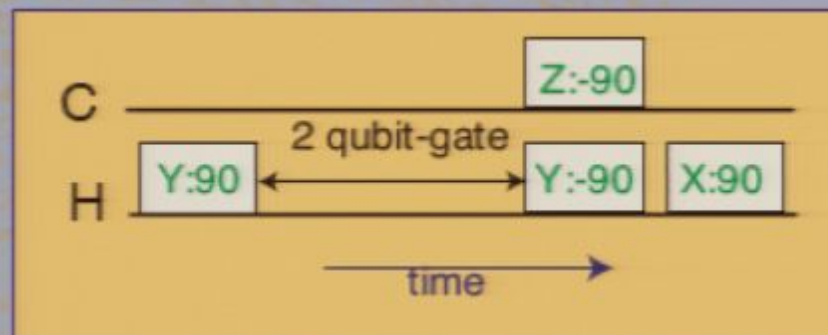
Quantum circuit

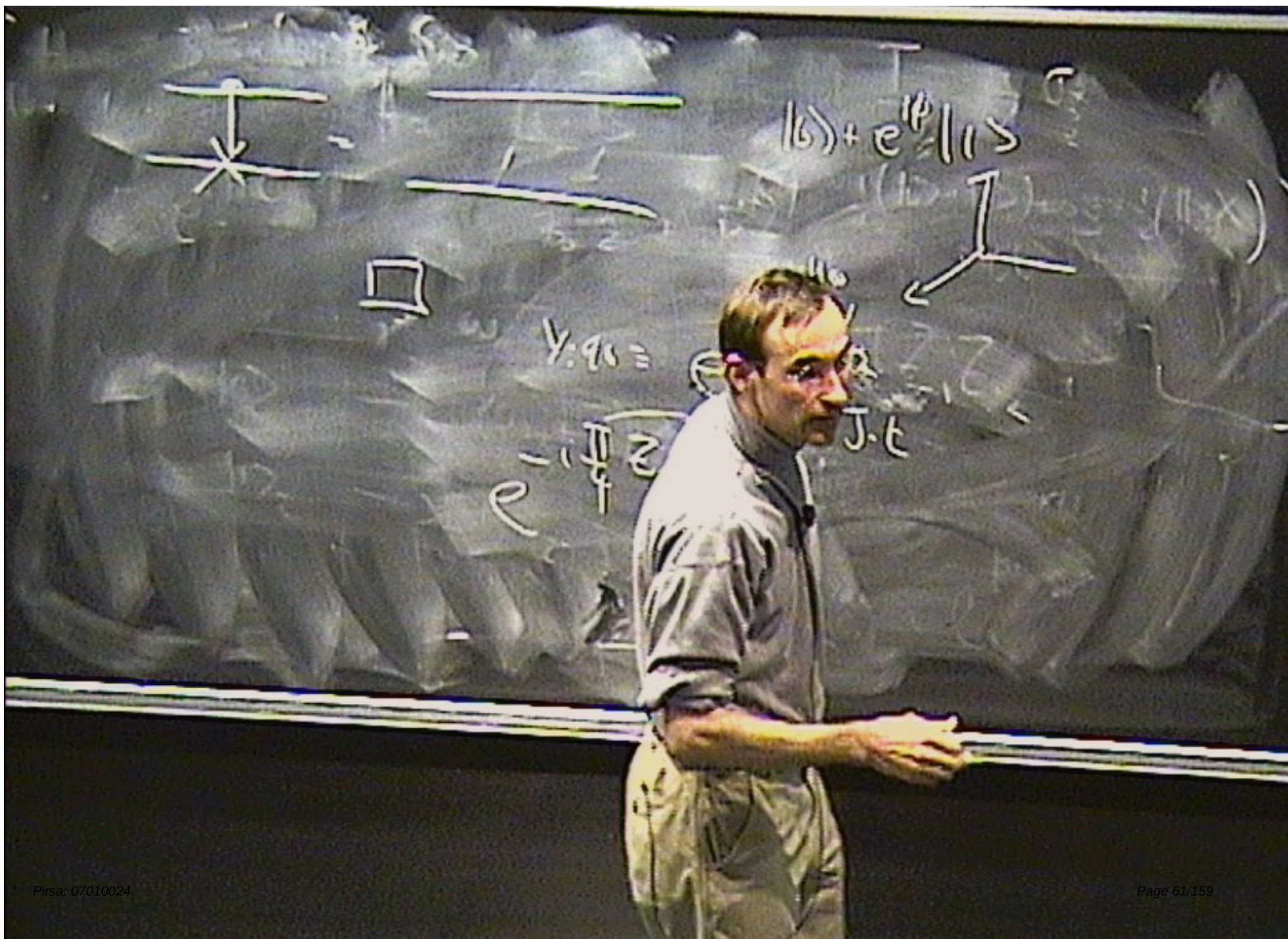


Control-Not



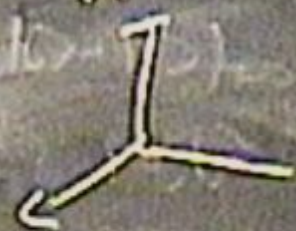
Quantum circuit







$$|b\rangle + e^{i\phi} |1\rangle$$



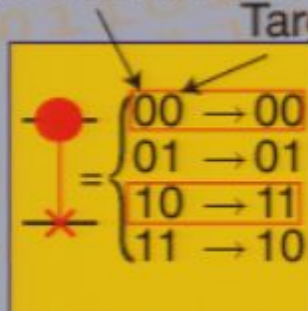
$$Y: q_1 = e^{-i\frac{\pi}{4}} Y$$

$$e^{-i\frac{\pi}{4}} z_1 z_2$$

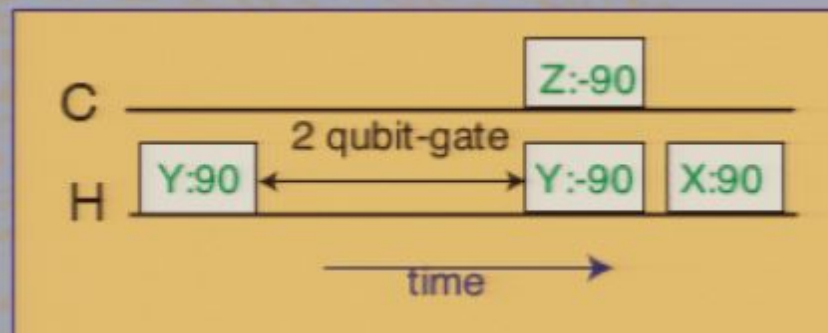
J. t

Quantum algorithms 101: the Control-NOT

Control-Not



Quantum circuit



Pre-compiler (Optimizer)

```

// setting to track down error sources by doing partial error correction.
#define ctrl
#define nctrl 1
#define nctrl2 2
#define nctrl3 3
#define nctrl4 4
#define nctrl5 5
#define nctrl6 6
#define nctrl7 7
#define nctrl8 8
#define nctrl9 9
#define nctrl10 10
#define nctrl11 11
#define nctrl12 12
#define nctrl13 13
#define nctrl14 14
#define nctrl15 15
#define nctrl16 16
#define nctrl17 17
#define nctrl18 18
#define nctrl19 19
#define nctrl20 20
#define nctrl21 21
#define nctrl22 22
#define nctrl23 23
#define nctrl24 24
#define nctrl25 25
#define nctrl26 26
#define nctrl27 27
#define nctrl28 28
#define nctrl29 29
#define nctrl30 30
#define nctrl31 31
#define nctrl32 32
#define nctrl33 33
#define nctrl34 34
#define nctrl35 35
#define nctrl36 36
#define nctrl37 37
#define nctrl38 38
#define nctrl39 39
#define nctrl40 40
#define nctrl41 41
#define nctrl42 42
#define nctrl43 43
#define nctrl44 44
#define nctrl45 45
#define nctrl46 46
#define nctrl47 47
#define nctrl48 48
#define nctrl49 49
#define nctrl50 50
#define nctrl51 51
#define nctrl52 52
#define nctrl53 53
#define nctrl54 54
#define nctrl55 55
#define nctrl56 56
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#define nctrl74 74
#define nctrl75 75
#define nctrl76 76
#define nctrl77 77
#define nctrl78 78
#define nctrl79 79
#define nctrl80 80
#define nctrl81 81
#define nctrl82 82
#define nctrl83 83
#define nctrl84 84
#define nctrl85 85
#define nctrl86 86
#define nctrl87 87
#define nctrl88 88
#define nctrl89 89
#define nctrl90 90
#define nctrl91 91
#define nctrl92 92
#define nctrl93 93
#define nctrl94 94
#define nctrl95 95
#define nctrl96 96
#define nctrl97 97
#define nctrl98 98
#define nctrl99 99

```

Bruker (machine) language

```

1 10
2 10
3 10
4 10
5 10
6 10
7 10
8 10
9 10
10 10
11 10
12 10
13 10
14 10
15 10
16 10
17 10
18 10
19 10
20 10
21 10
22 10
23 10
24 10
25 10
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27 10
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29 10
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32 10
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36 10
37 10
38 10
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64 10
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67 10
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77 10
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80 10
81 10
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86 10
87 10
88 10
89 10
90 10
91 10
92 10
93 10
94 10
95 10
96 10
97 10
98 10
99 10

```

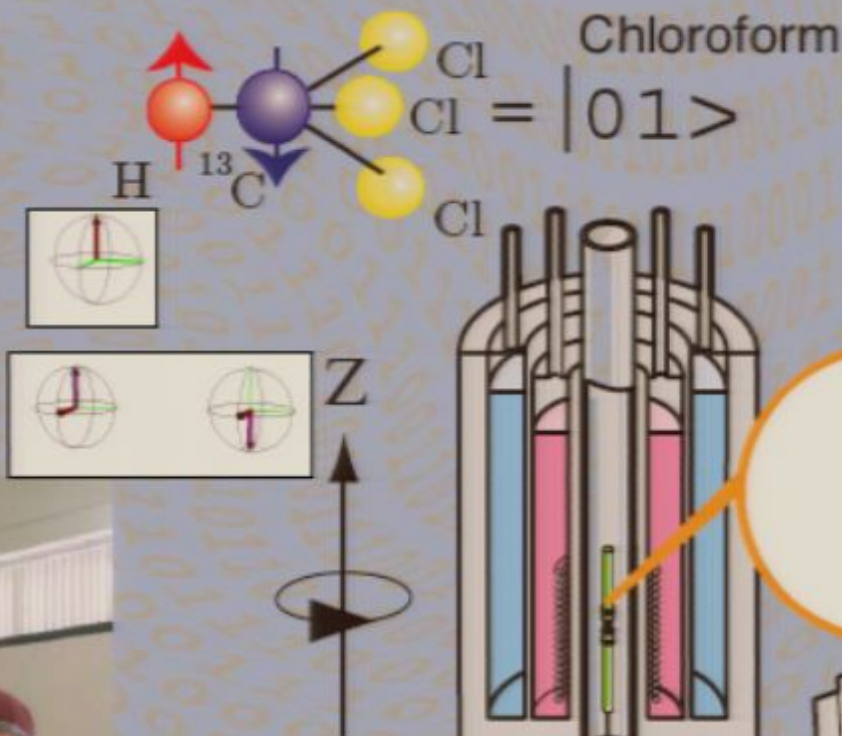
Nuclear Magnetic Resonance



Bloch



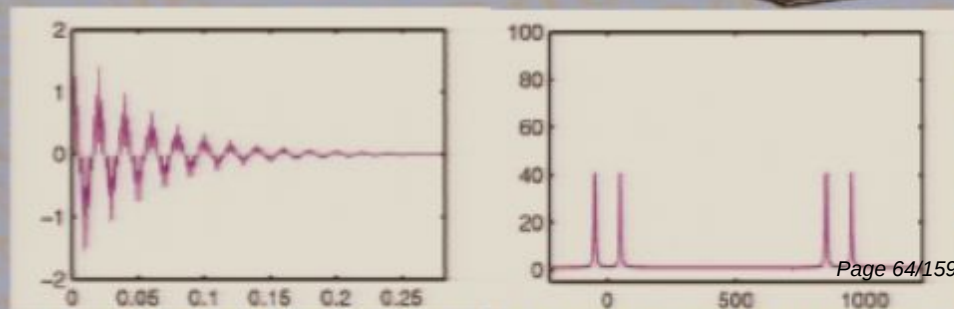
Purcell



University of
Waterloo

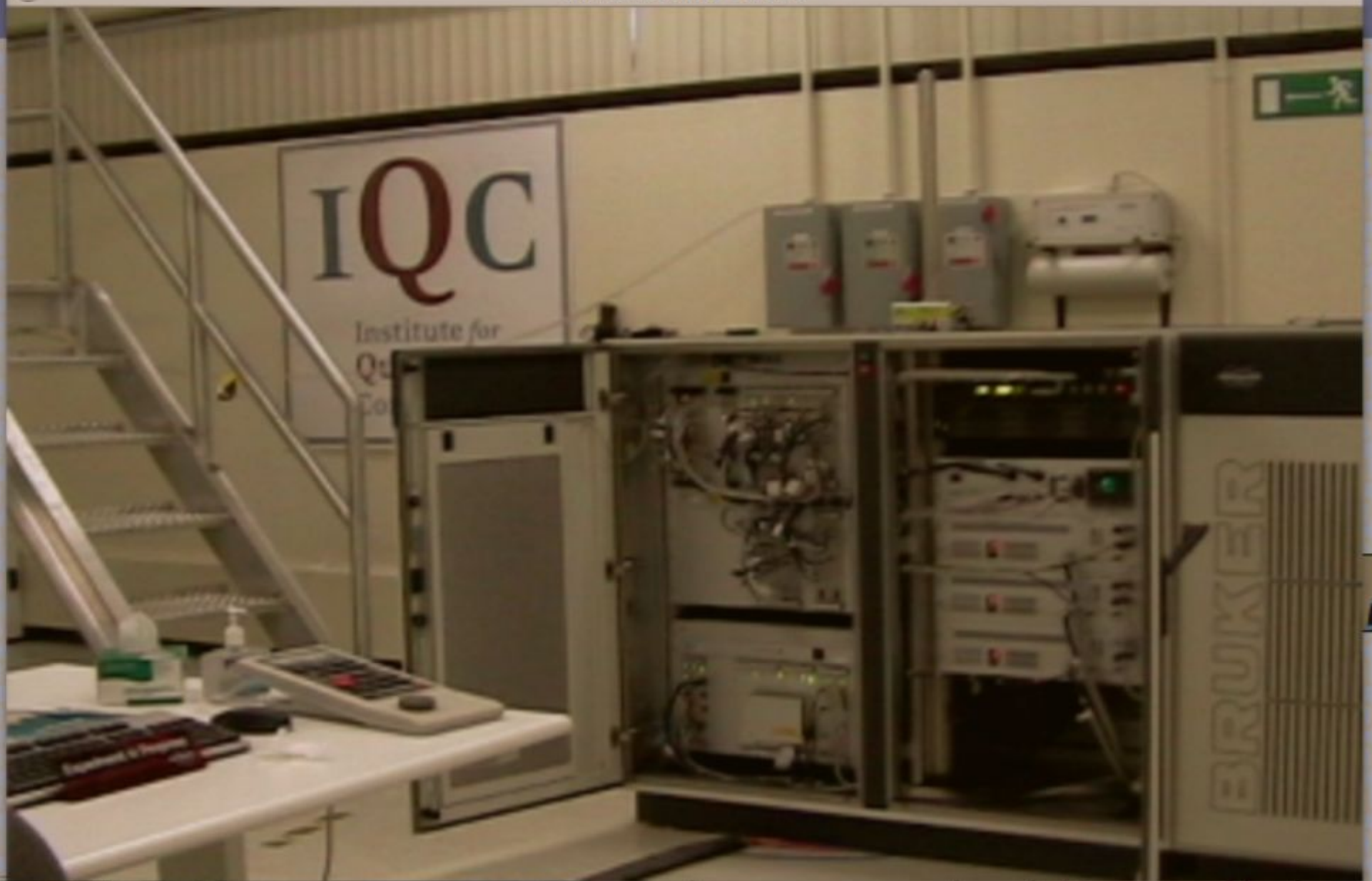


Bruker 700



Nuclear Magnetic Resonance

Adobe Acrobat - Movie



Pirsa: 07010024

Bruker 700

-2 0 0.05 0.1 0.15 0.2 0.25

0 500 1000

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Nuclear Magnetic Resonance

Adobe Acrobat - Movie



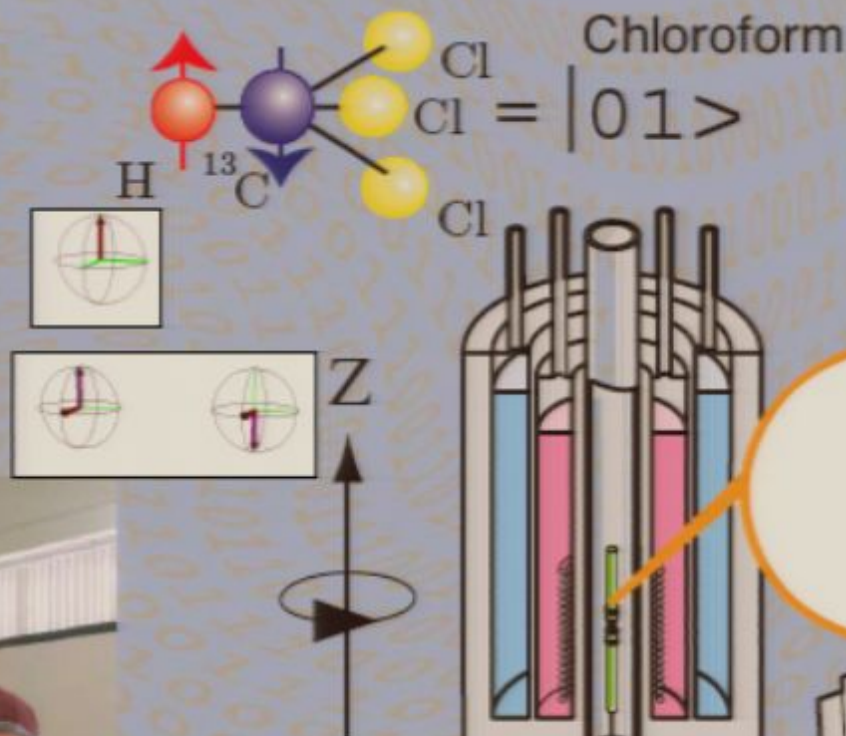
Nuclear Magnetic Resonance



Bloch



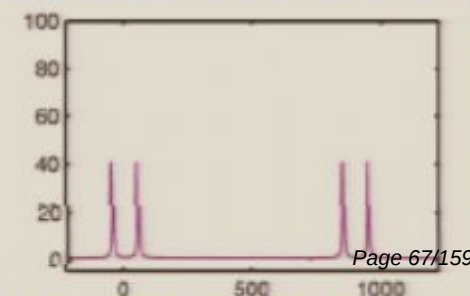
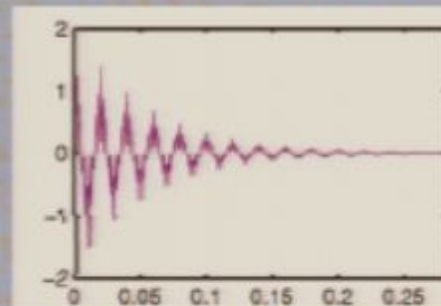
Purcell



University of
Waterloo

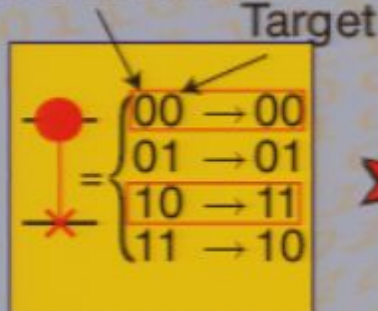


Bruker 700

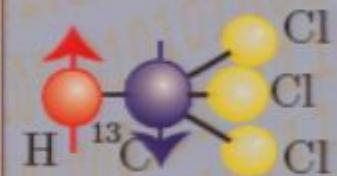
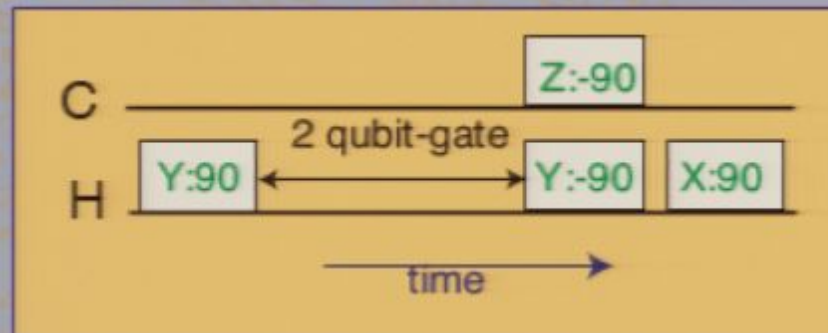


Quantum algorithms 101: the Control-NOT

Control-Not



Quantum circuit



Pre-compiler (Optimizer)

Bruker (machine) language

```

// setting to track down error sources by doing partial error correction.
#define cube
#define nch1 = 1;
#define nch2 = 2;
#include "cupp.h"

// slowamp = 0, slowup = 1, sl
// scale .1
// slowamp = 0, slowup
// pulse ramp and
// correction shape
//
// (C1-C2)
// pulse C2 90 .25
// zz .25 C1 C2
// zpulse C1:.75;C2:
// pulse C2_90 .75
// pulse C2_90 .0
// refocus C1C2_180

```

```

; pulse C2_90 .25
; zz .25 C1 C2
; zpulse C1:.75;C2:
; pulse C2_90 .75
; pulse C2_90 .0
; refocus C1C2_180

```

```

1 10
2 1000 1000 1000
3 1000 1000 1000
4 1000 1000 1000
5 1000 1000 1000
6 1000 1000 1000
7 1000 1000 1000
8 1000 1000 1000
9 1000 1000 1000
10 1000 1000 1000
11 1000 1000 1000
12 1000 1000 1000
13 1000 1000 1000
14 1000 1000 1000
15 1000 1000 1000
16 1000 1000 1000
17 1000 1000 1000
18 1000 1000 1000
19 1000 1000 1000
20 1000 1000 1000
21 1000 1000 1000
22 1000 1000 1000
23 1000 1000 1000
24 1000 1000 1000
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27 1000 1000 1000
28 1000 1000 1000
29 1000 1000 1000
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31 1000 1000 1000
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66 1000 1000 1000
67 1000 1000 1000
68 1000 1000 1000
69 1000 1000 1000
70 1000 1000 1000
71 1000 1000 1000
72 1000 1000 1000
73 1000 1000 1000
74 1000 1000 1000
75 1000 1000 1000
76 1000 1000 1000
77 1000 1000 1000
78 1000 1000 1000
79 1000 1000 1000
80 1000 1000 1000
81 1000 1000 1000
82 1000 1000 1000
83 1000 1000 1000
84 1000 1000 1000
85 1000 1000 1000
86 1000 1000 1000
87 1000 1000 1000
88 1000 1000 1000
89 1000 1000 1000
90 1000 1000 1000
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96 1000 1000 1000
97 1000 1000 1000
98 1000 1000 1000
99 1000 1000 1000
100 1000 1000 1000

```

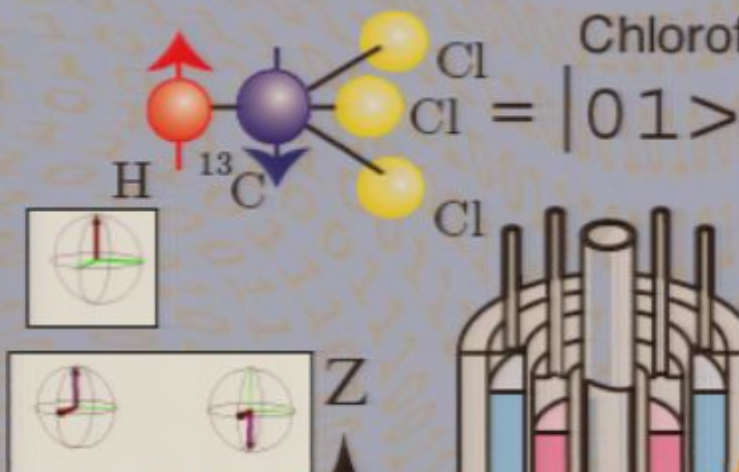
Nuclear Magnetic Resonance



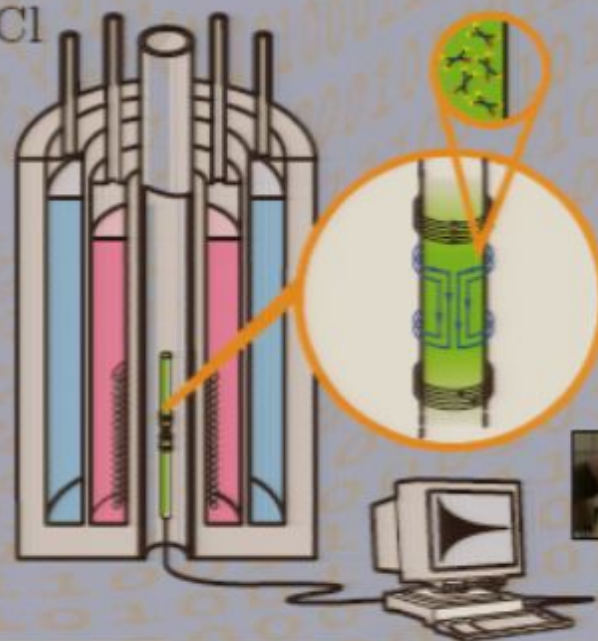
Bloch



Purcell



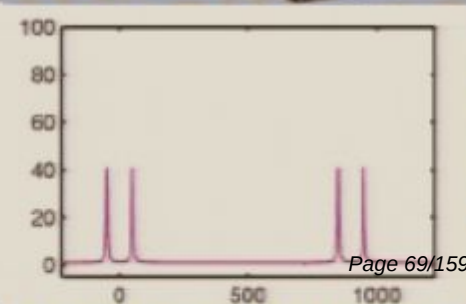
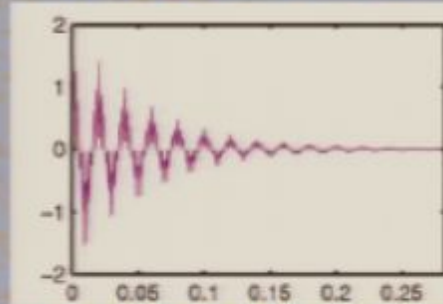
Chloroform

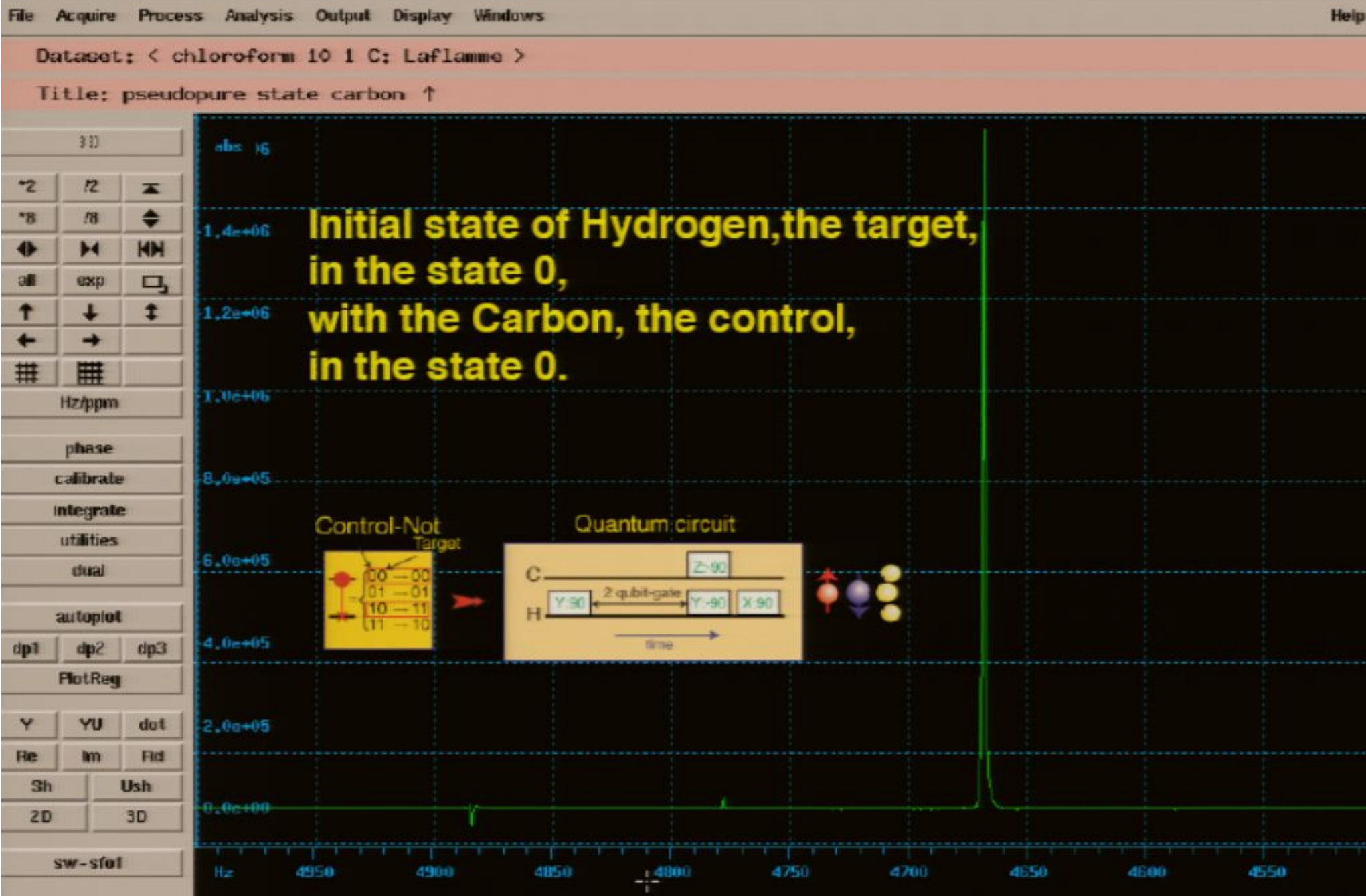


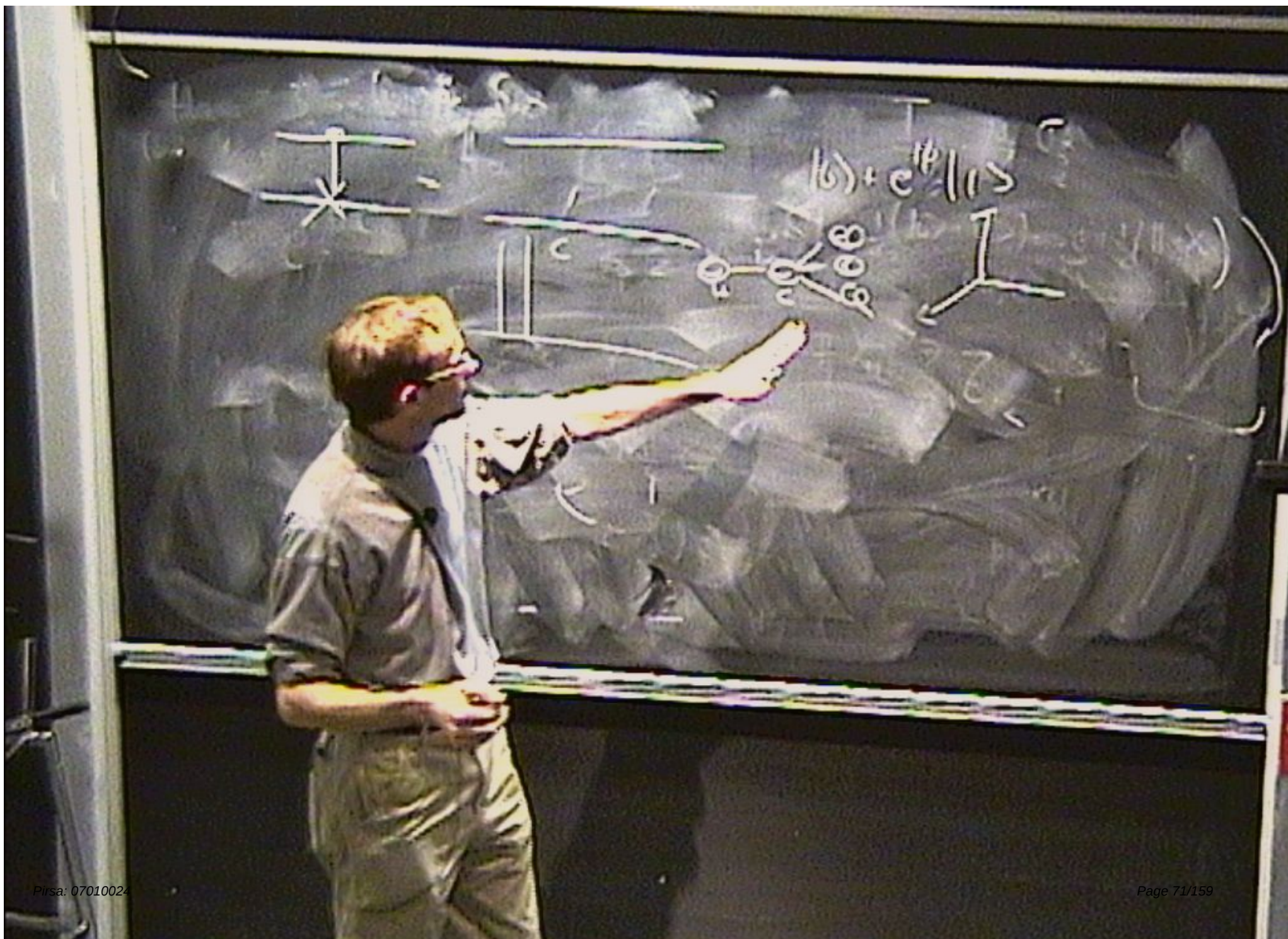
University of
Waterloo



Bruker 700

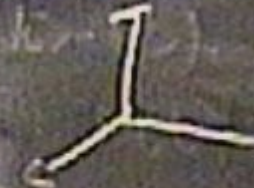






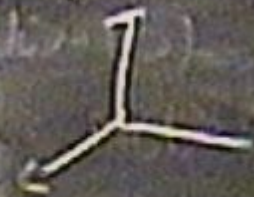


$$|b\rangle + e^{i\phi} |1\rangle$$



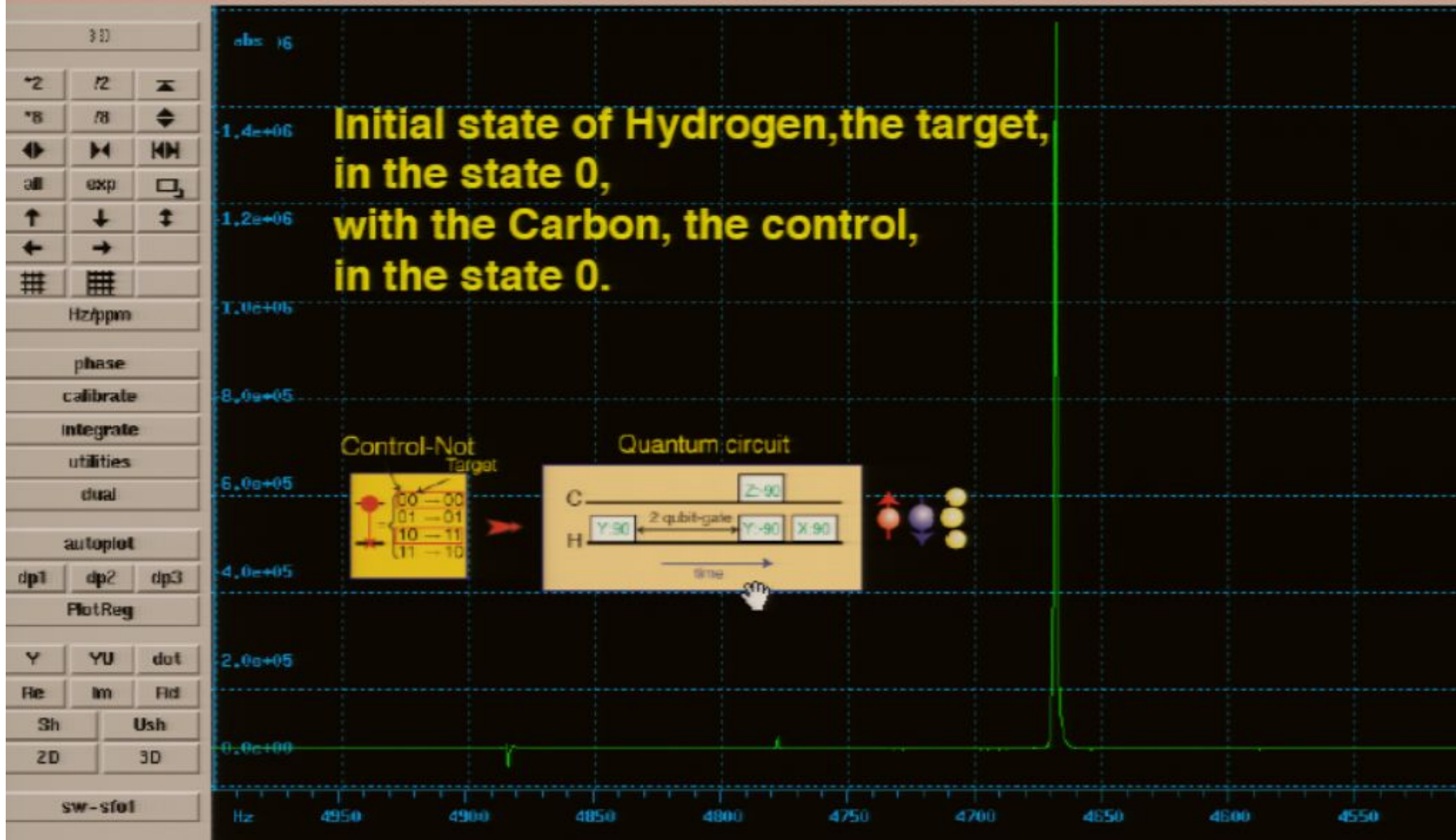


$$|b\rangle + e^{i\phi} |1\rangle$$



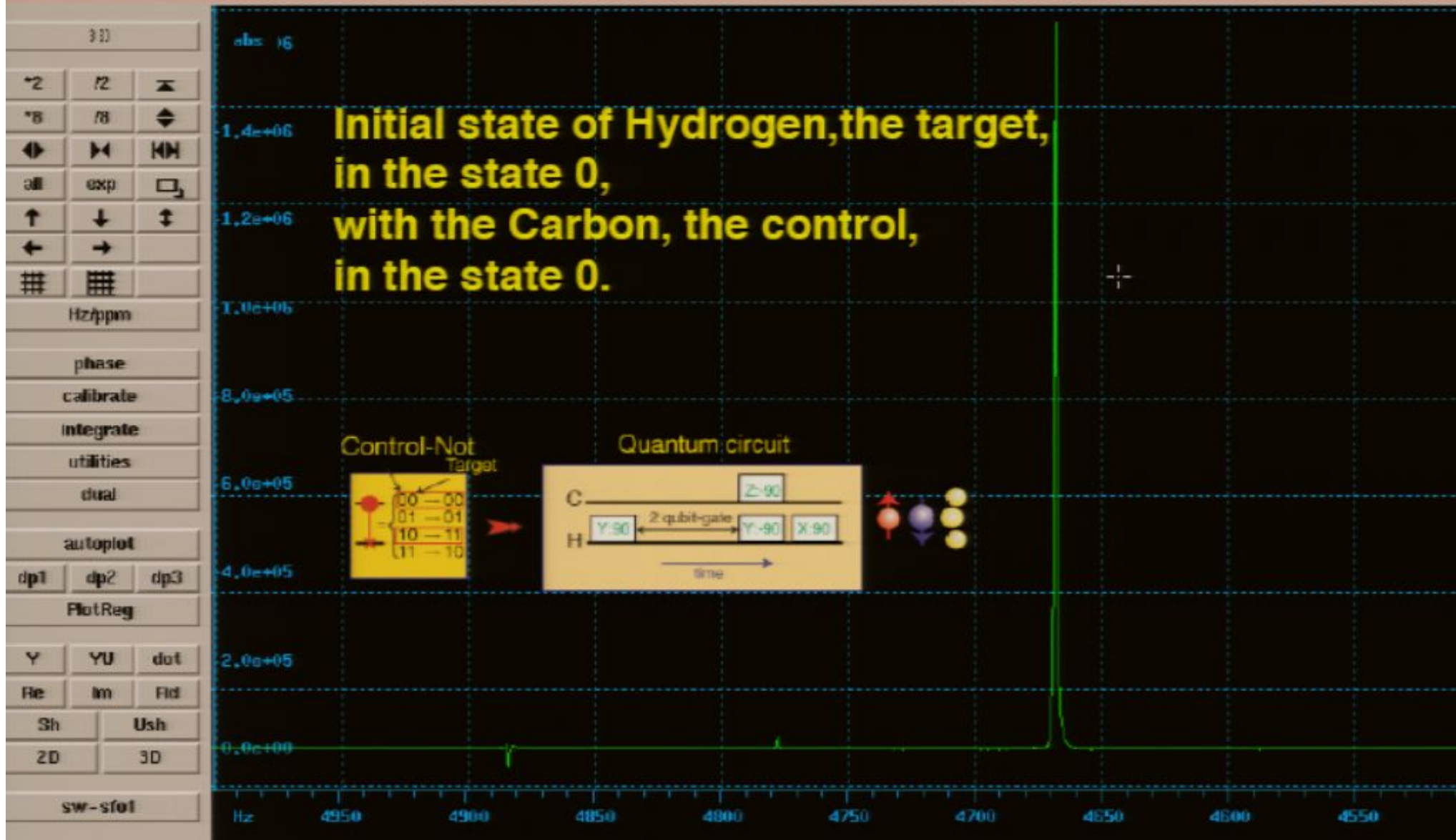
Dataset: < chloroform 10 1 C; Laflamme >

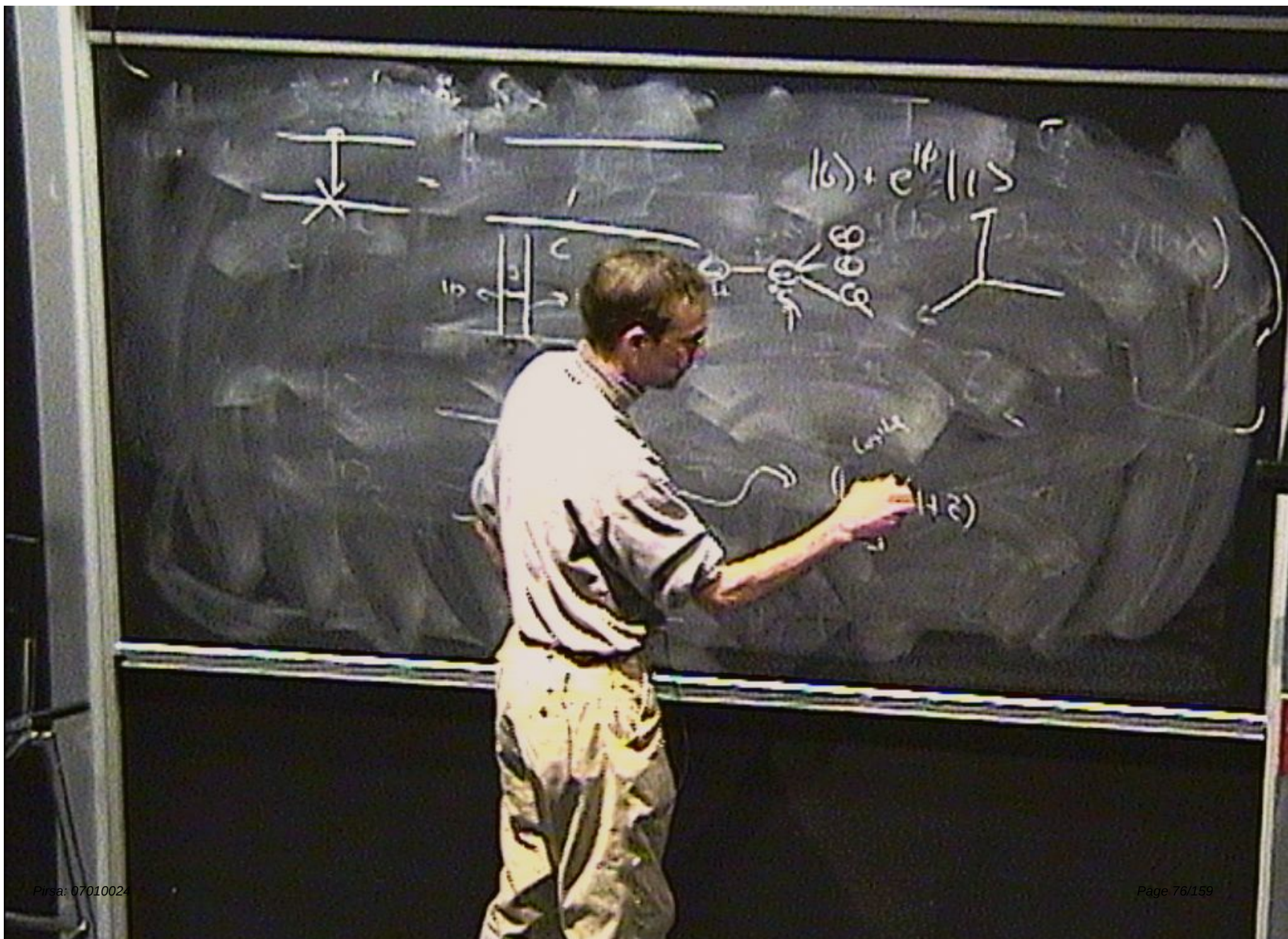
Title: pseudopure state carbon ↑



Dataset: < chloroform 10 1 C; Laflamme >

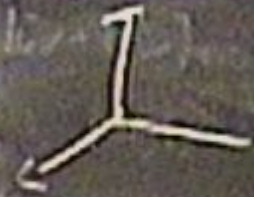
Title: pseudopure state carbon ↑



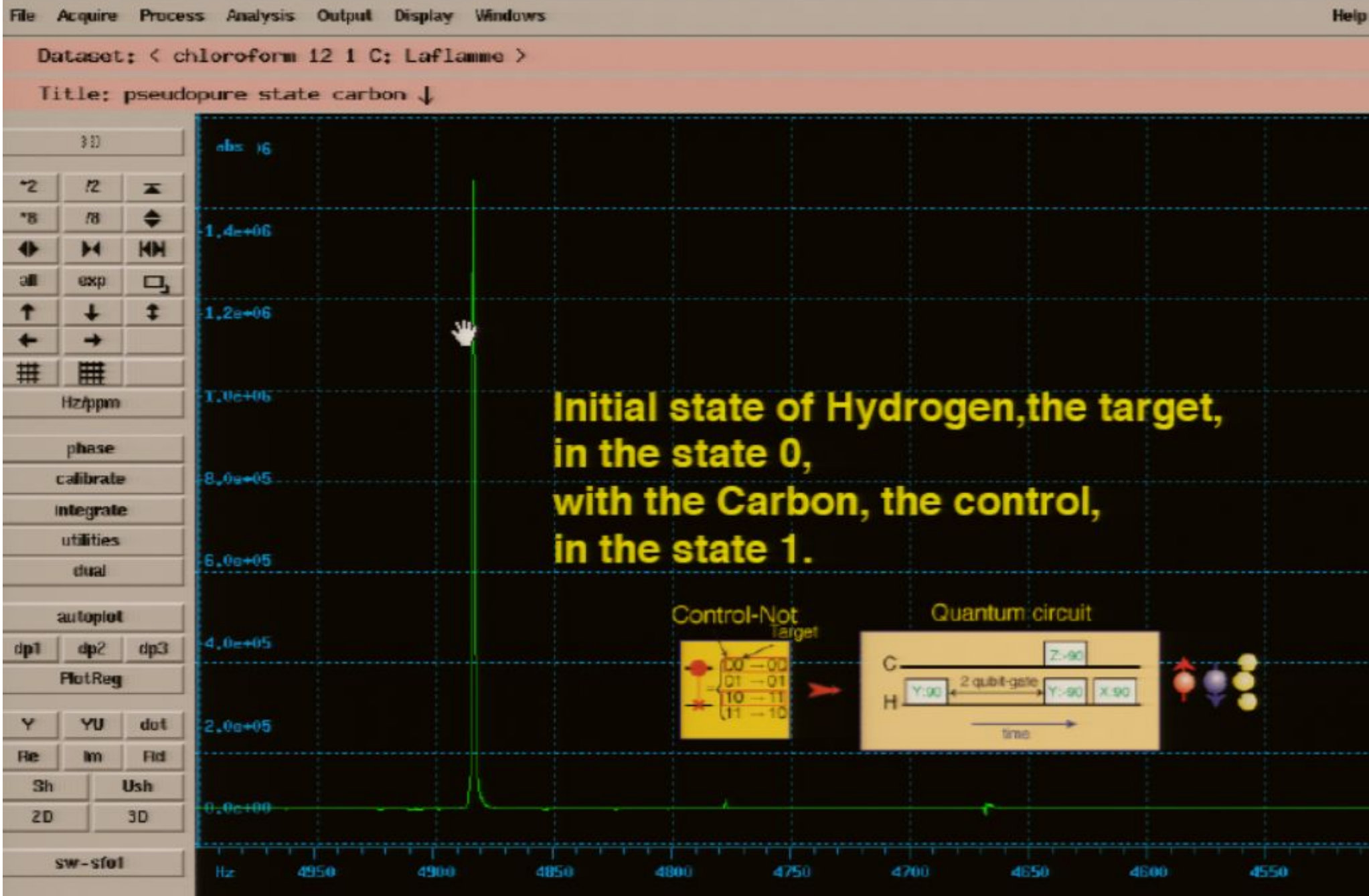


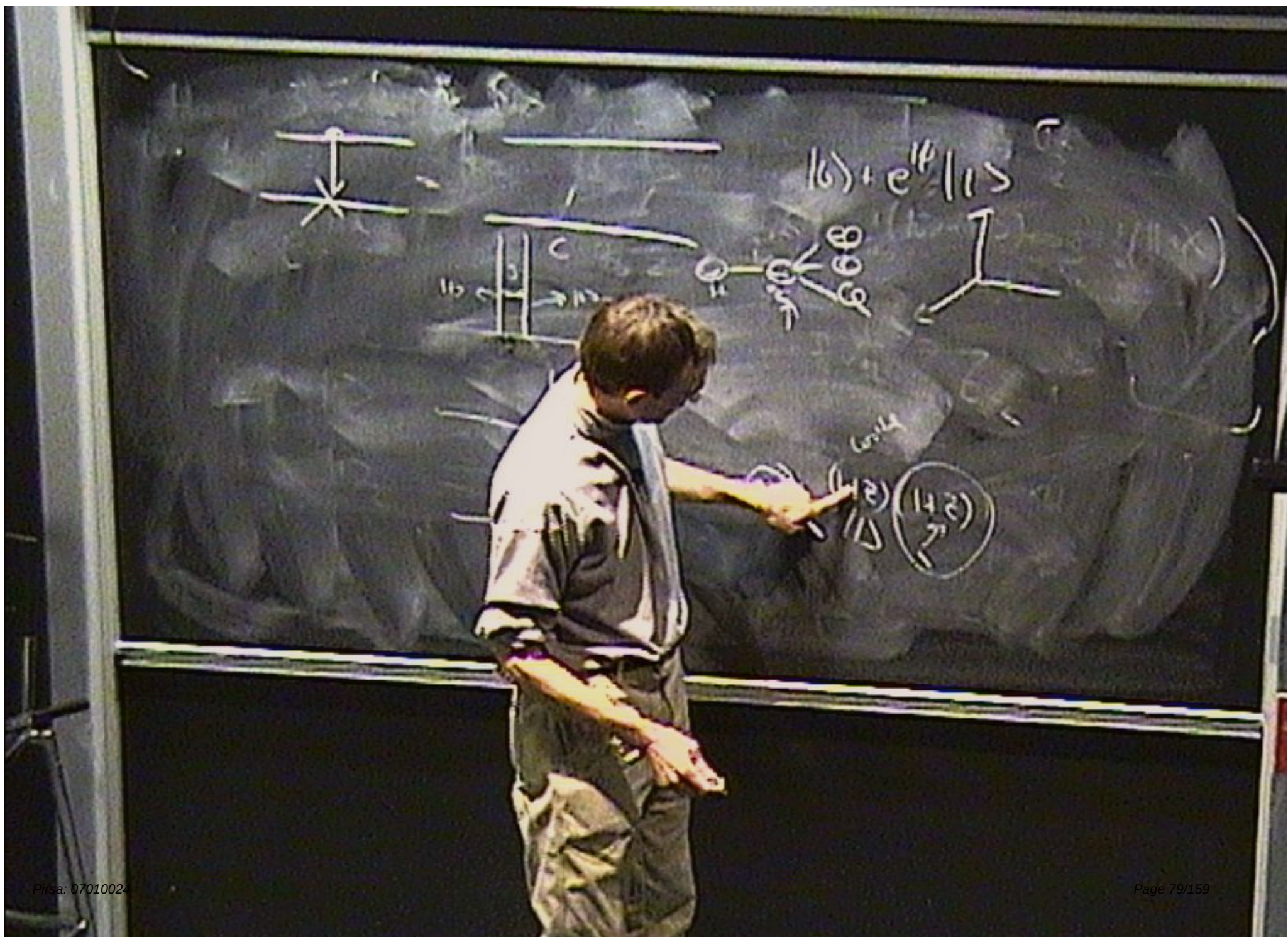


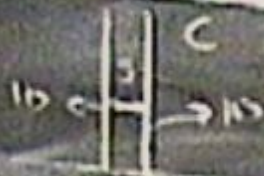
$$16) + e^{14} 115$$



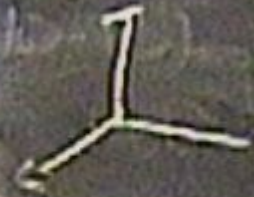
$$(1+2)(142)$$

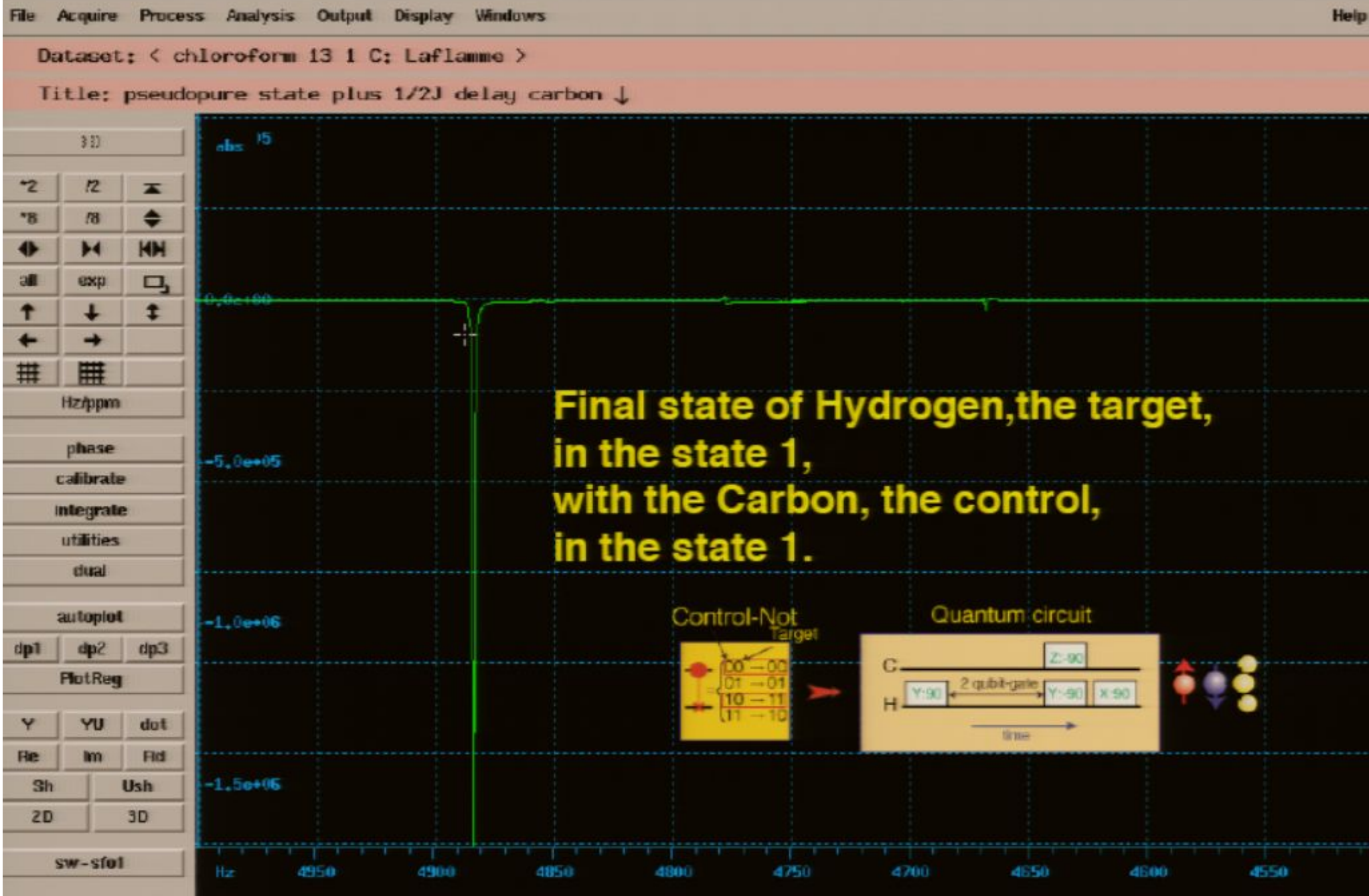






$16) + e^{14} | 15$





Quantum error correction

P. Shor, PRA, 52, 2493, 1995; A. Steane, RSLA 452, 2551, 1996

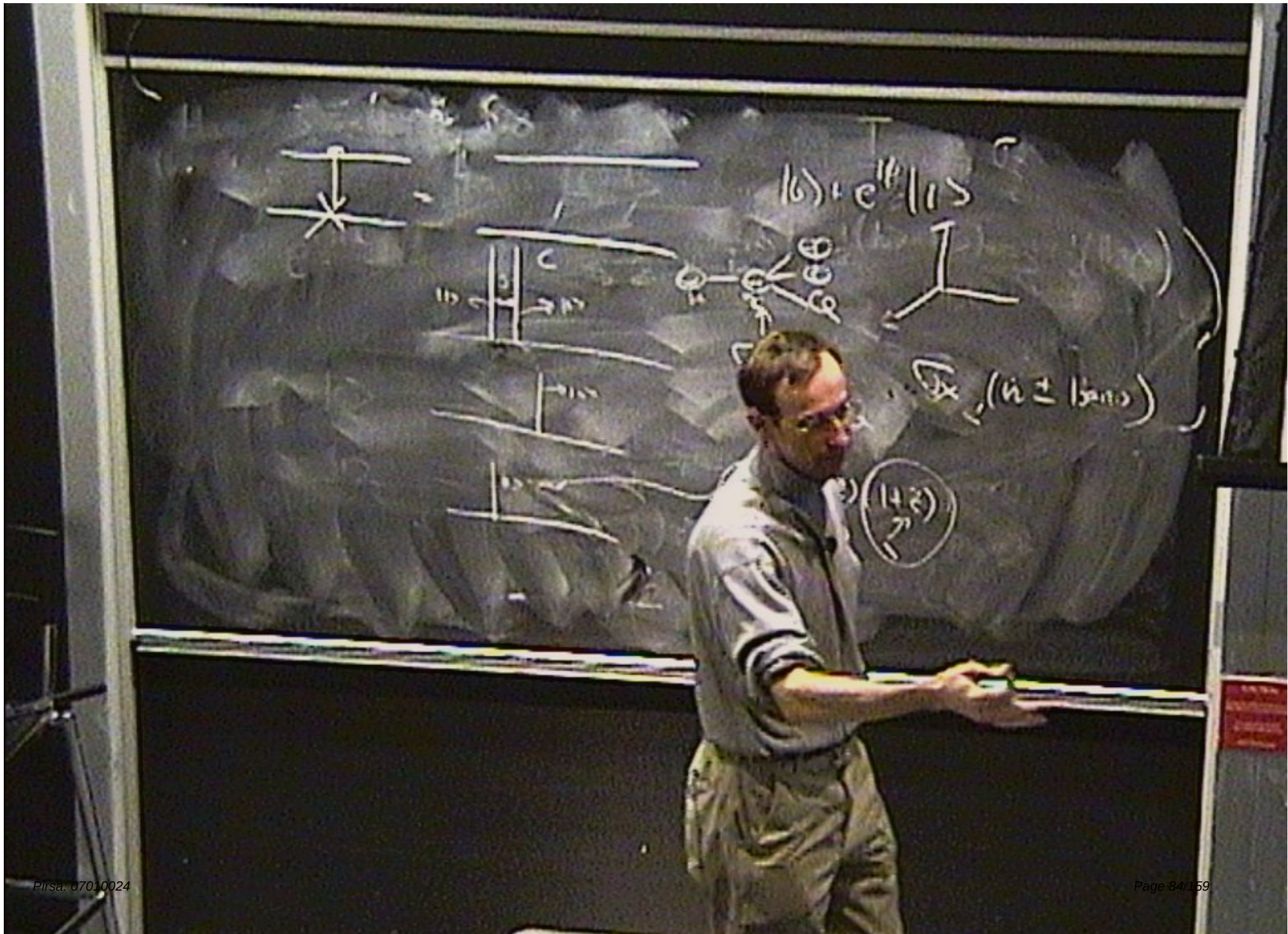
	Noise model ¹	Redundancy	Majority voting
Classical	bit flip $0 \leftrightarrow 1$	copy information $0 \rightarrow 000$ $1 \rightarrow 111$	observe and compare $110 \rightarrow 111$
Quantum	bit flip phase flip $\alpha 0\rangle + \beta 1\rangle$ $\alpha 0\rangle - \beta 1\rangle$	No cloning theorem (Wootters & Zurek, 1982) Encode $\alpha 000\rangle + \beta 111\rangle$	Detect the error without finding the message $\alpha 001\rangle + \beta 110\rangle$

Quantum error correction

P. Shor, PRA, 52, 2493, 1995; A. Steane, RSLA 452, 2551, 1996

	Noise model	Redundancy	Majority voting
Classical	bit flip $0 \leftrightarrow 1$	copy information $0 \rightarrow 000$ $1 \rightarrow 111$	observe and compare $110 \rightarrow 111$
Quantum	bit flip phase flip $\alpha 0\rangle + \beta 1\rangle \rightarrow \alpha 0\rangle - \beta 1\rangle$	Neutronics Neutronics Neutronics	Neutronics Neutronics Neutronics

$$\begin{aligned}
 & \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \rightarrow \begin{pmatrix} \rho_{00} & \rho_{01} e^{-t/T_2} \\ \rho_{10} e^{-t/T_2} & \rho_{11} \end{pmatrix} \\
 & = \frac{(1 + e^{-t/T_2})}{2} \mathbb{I} \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \mathbb{I} + \frac{(1 - e^{-t/T_2})}{8} I_z \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} I_z
 \end{aligned}$$



Quantum error correction

P. Shor, PRA, 52, 2493, 1995; A. Steane, RSLA 452, 2551, 1996

$$I_z^1 \rightarrow \frac{1}{4} (I_x^1 + I_x^2 + I_x^3 + 4 I_x^1 I_x^2 I_x^3)$$

Classical
Quantum

bit flip

$0 \leftrightarrow 1$

bit flip
phase flip
 $\alpha|0\rangle + \beta|1\rangle$
 $\alpha|0\rangle - \beta|1\rangle$

copy
information

$0 \rightarrow 000$

$1 \rightarrow 111$

No cloning theorem
(Wootters & Zurek, 1982)

Encode

$\alpha|000\rangle + \beta|111\rangle$

Majority
voting

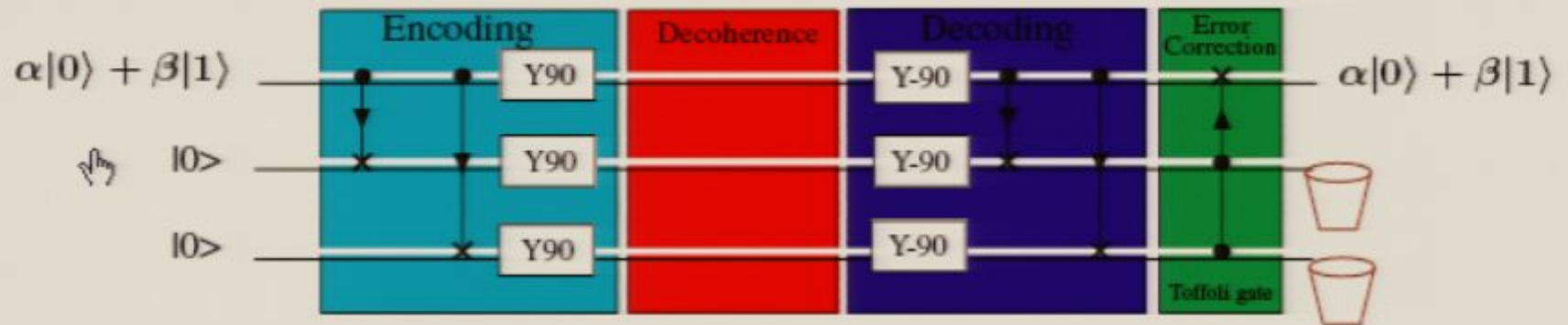
observe
and
compare

$110 \rightarrow 111$

Detect the error
without finding
the message

$\alpha|001\rangle + \beta|110\rangle$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} &\alpha|+++ \rangle + \beta|--- \rangle \\ &\alpha| - ++ \rangle + \beta| + -- \rangle \\ &\alpha| + - + \rangle + \beta| - + - \rangle \\ &\alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

Control-Not

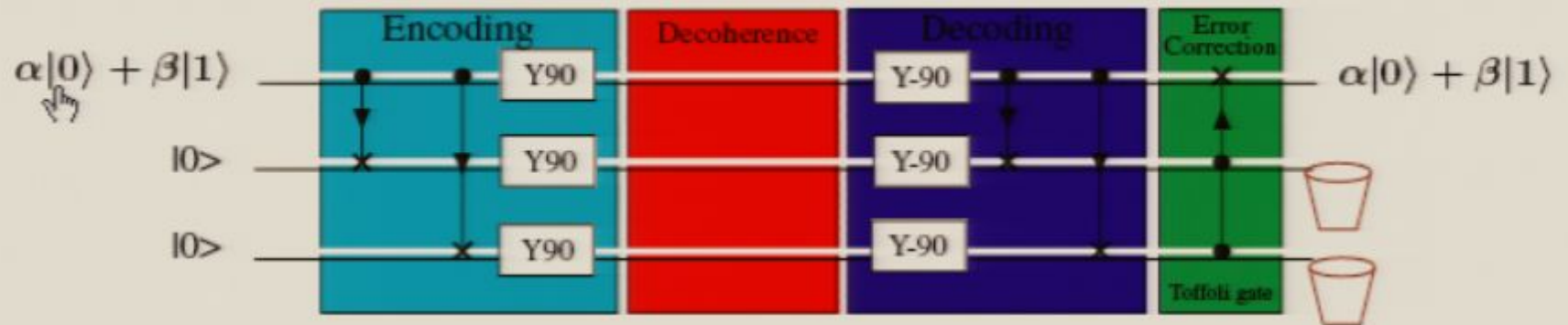
$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\rho_{\text{init}} = I_{x,y,z}^1 \frac{1}{2} (\mathbb{I} + 2I_z^2) \frac{1}{2} (\mathbb{I} + 2I_z^3)$$

Control-Not

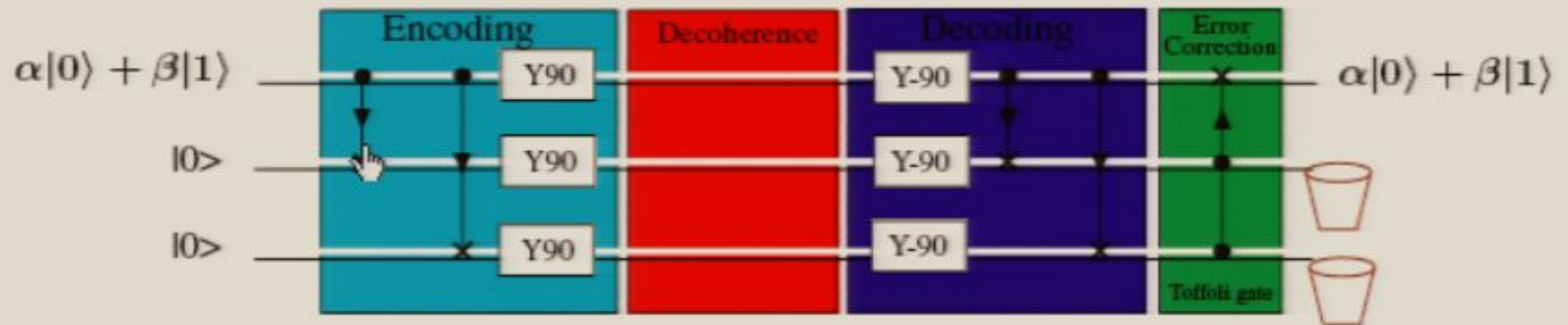
$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} &\alpha|+++ \rangle + \beta|--- \rangle \\ &\alpha| - ++ \rangle + \beta| + -- \rangle \\ &\alpha| + - + \rangle + \beta| - + - \rangle \\ &\alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

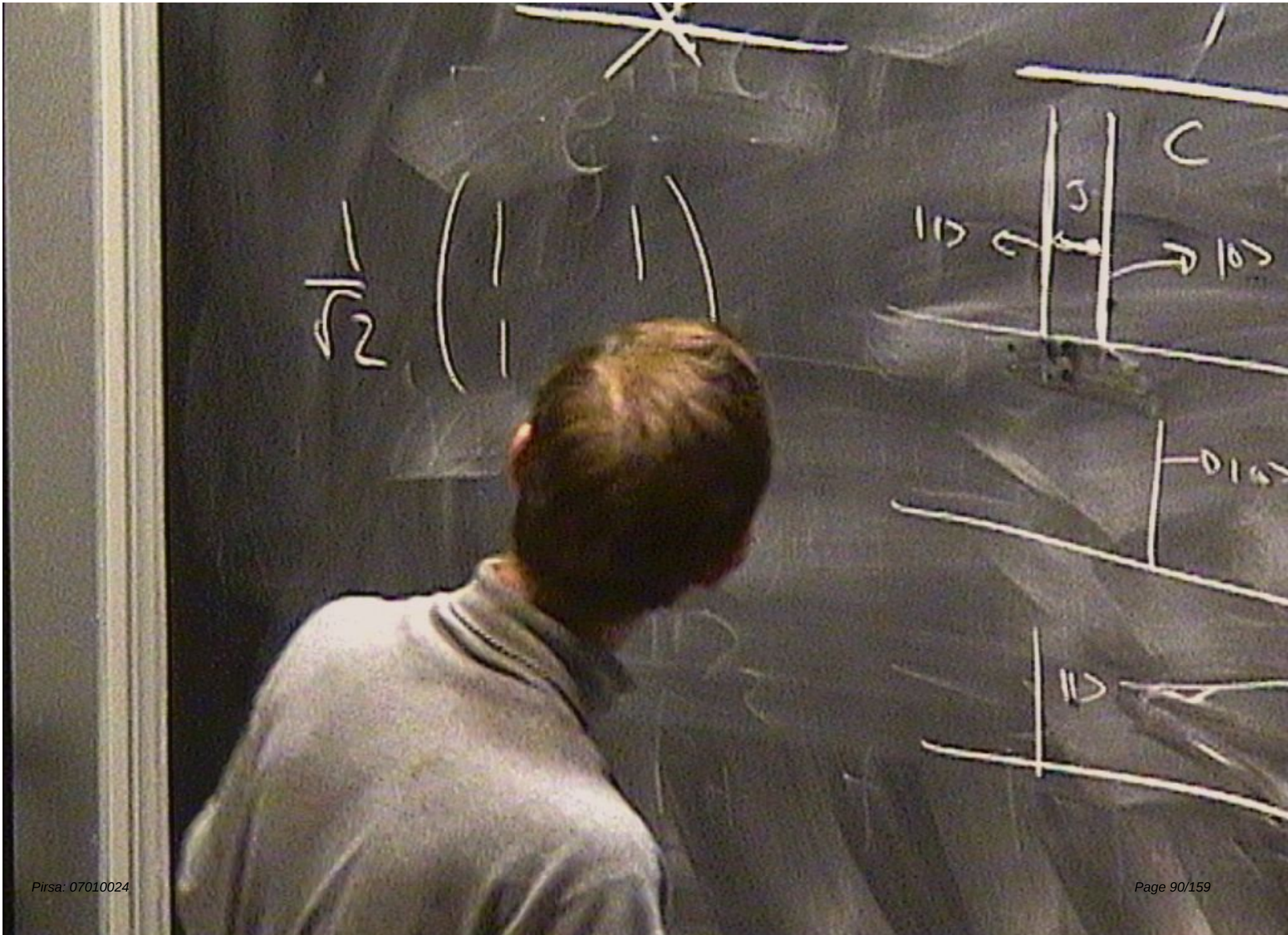
$$\rho_B \equiv \frac{1}{4} (I_x^1 + I_x^2 + I_x^3 + 4I_x^1 I_x^2 I_x^3)$$

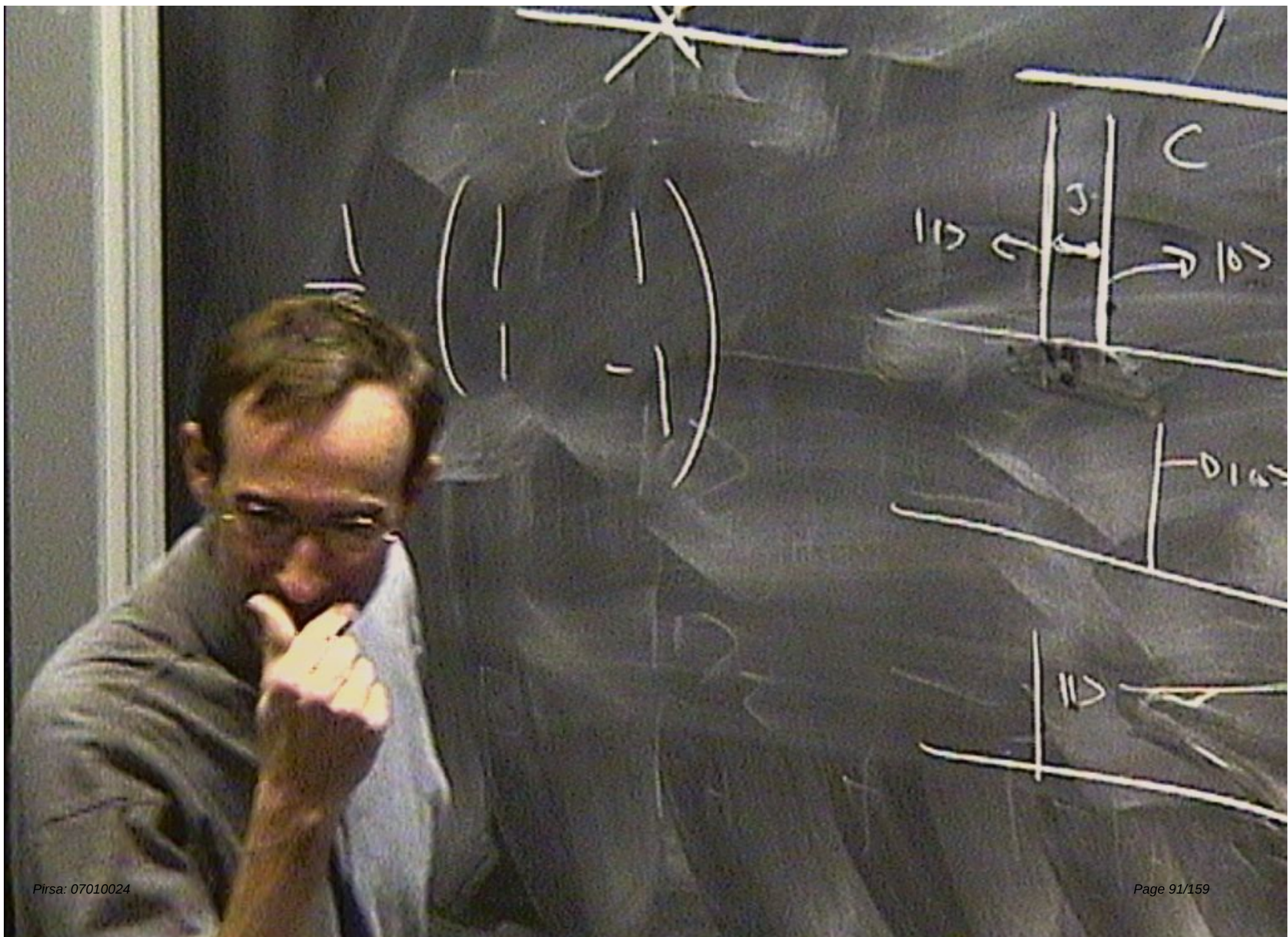
$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$





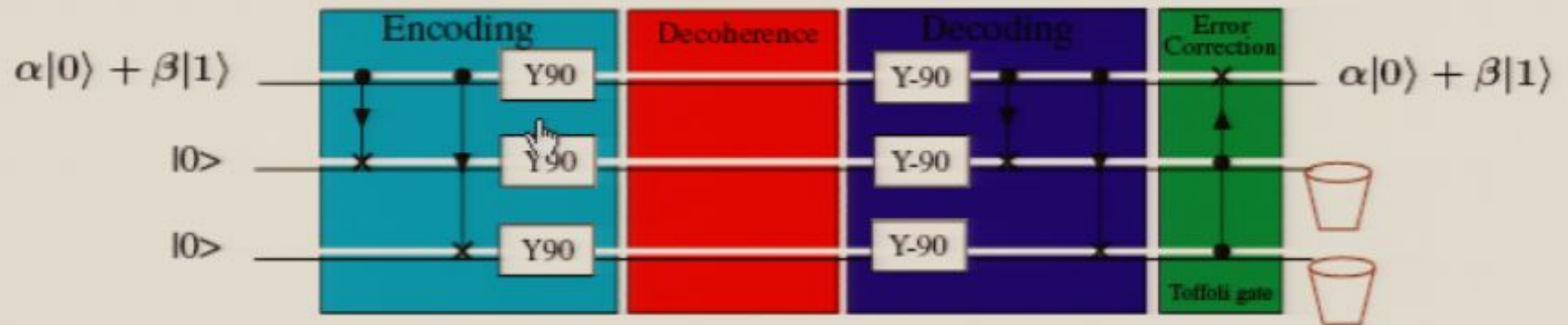
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$110 \leftarrow \begin{array}{c|c} & C \\ \hline 3 & \\ \hline \end{array} \rightarrow 100$$

$$010$$

$$110$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} &\alpha|+++ \rangle + \beta|--- \rangle \\ &\alpha| - ++ \rangle + \beta| + -- \rangle \\ &\alpha| + - + \rangle + \beta| - + - \rangle \\ &\alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

$$\rho_B \equiv \frac{1}{4} (I_x^1 + I_x^2 + I_x^3 + 4I_x^1 I_x^2 I_x^3)$$

$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} & \alpha|+++ \rangle + \beta|--- \rangle \\ & \alpha| - ++ \rangle + \beta| + -- \rangle \\ & \alpha| + - + \rangle + \beta| - + - \rangle \\ & \alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

$$\begin{aligned} & (\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ & (\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ & (\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ & (\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

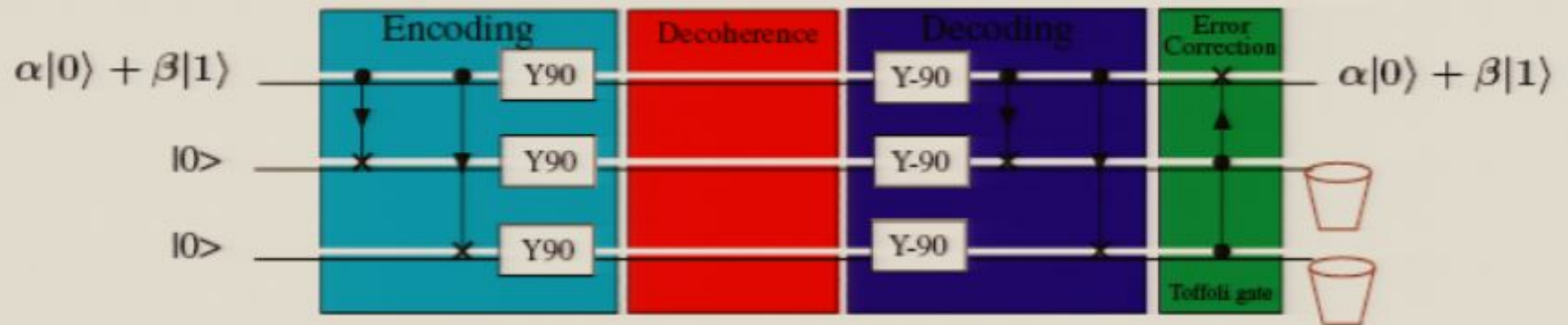
Control-Not

$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} &\alpha|+++ \rangle + \beta|--- \rangle \\ &\alpha| - ++ \rangle + \beta| + -- \rangle \\ &\alpha| + - + \rangle + \beta| - + - \rangle \\ &\alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

Control-Not

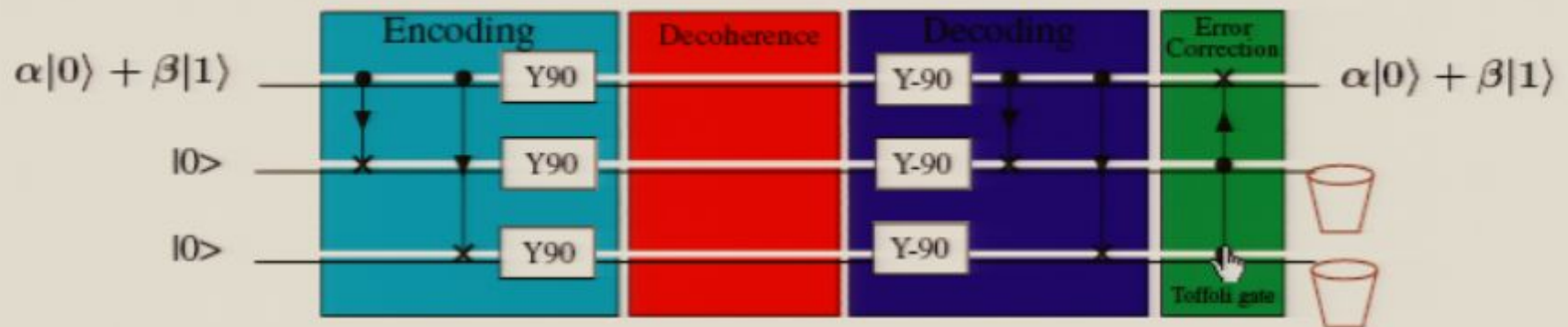
$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} &\alpha|+++ \rangle + \beta|--- \rangle \\ &\alpha| - ++ \rangle + \beta| + -- \rangle \\ &\alpha| + - + \rangle + \beta| - + - \rangle \\ &\alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$

Control-Not

$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

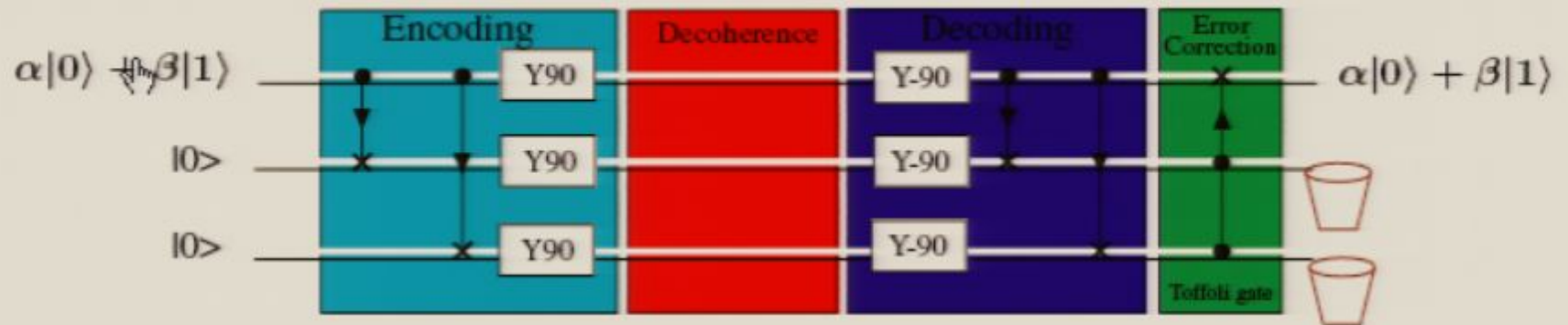
$$\rho_E^1 \equiv \frac{1}{8}I_z^1(9e^{-t/\tau} - e^{-9t/\tau}) \approx I_z^1(1 - \frac{9}{2}t^2/\tau^2 + \dots)$$

$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\rho_{\text{init}} = I_{x,y,z}^1 \frac{1}{2} (\mathbb{I} + 2I_z^2) \frac{1}{2} (\mathbb{I} + 2I_z^3)$$

Control-Not

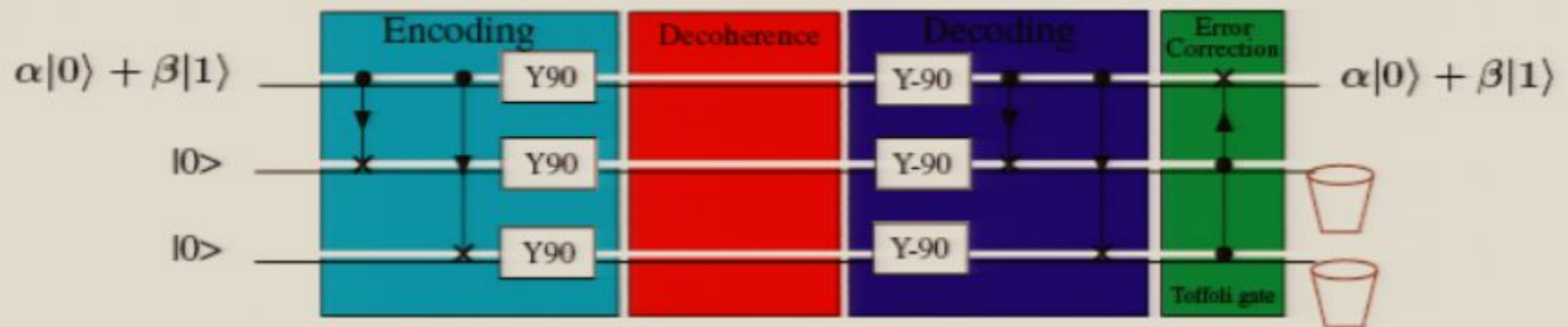
$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

Q. Error Correction for Phase



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Control-Not

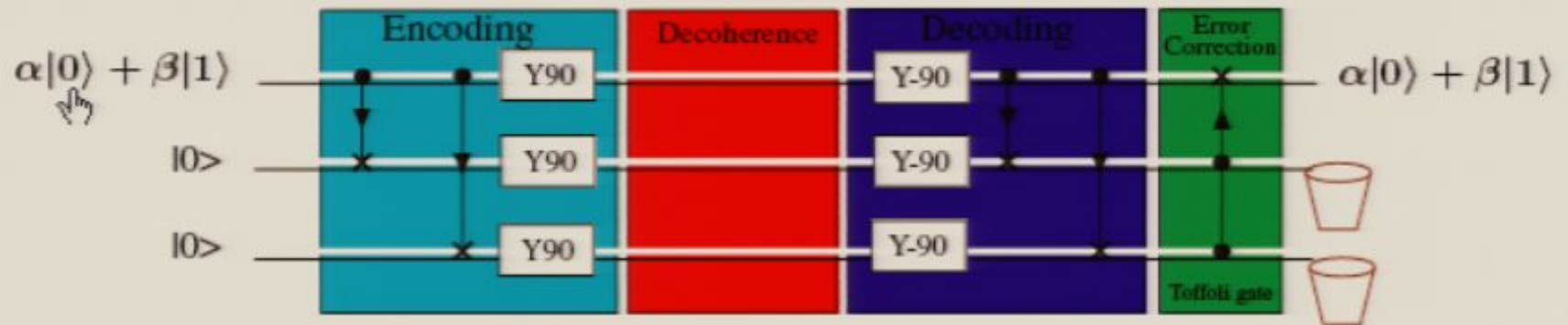
$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\rho_{\text{init}} = I_{\mathbf{x},\mathbf{y},\mathbf{z}}^1 \frac{1}{2} (\mathbb{I} + 2I_z^2) \frac{1}{2} (\mathbb{I} + 2I_z^3)$$

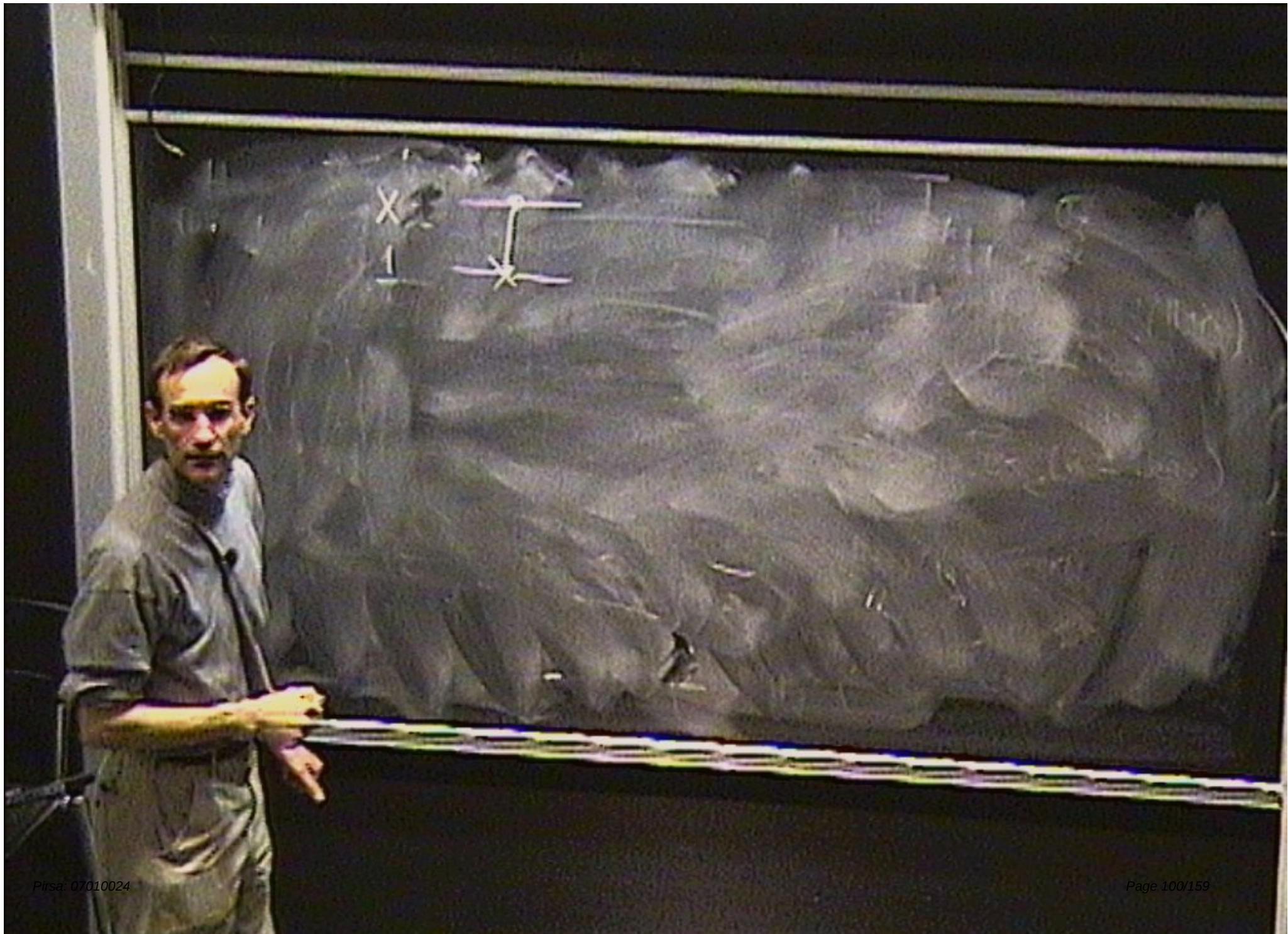
Control-Not

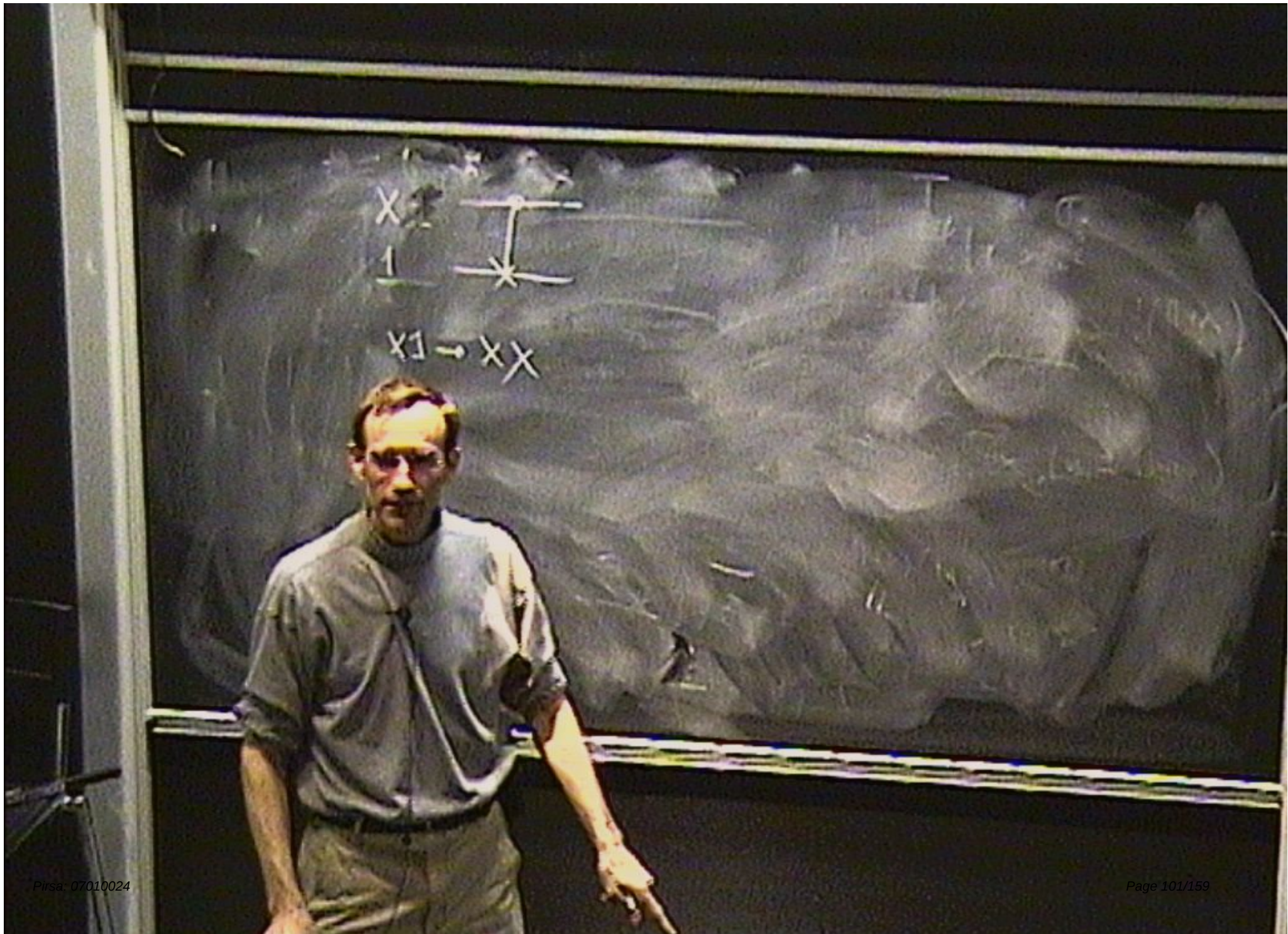
$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

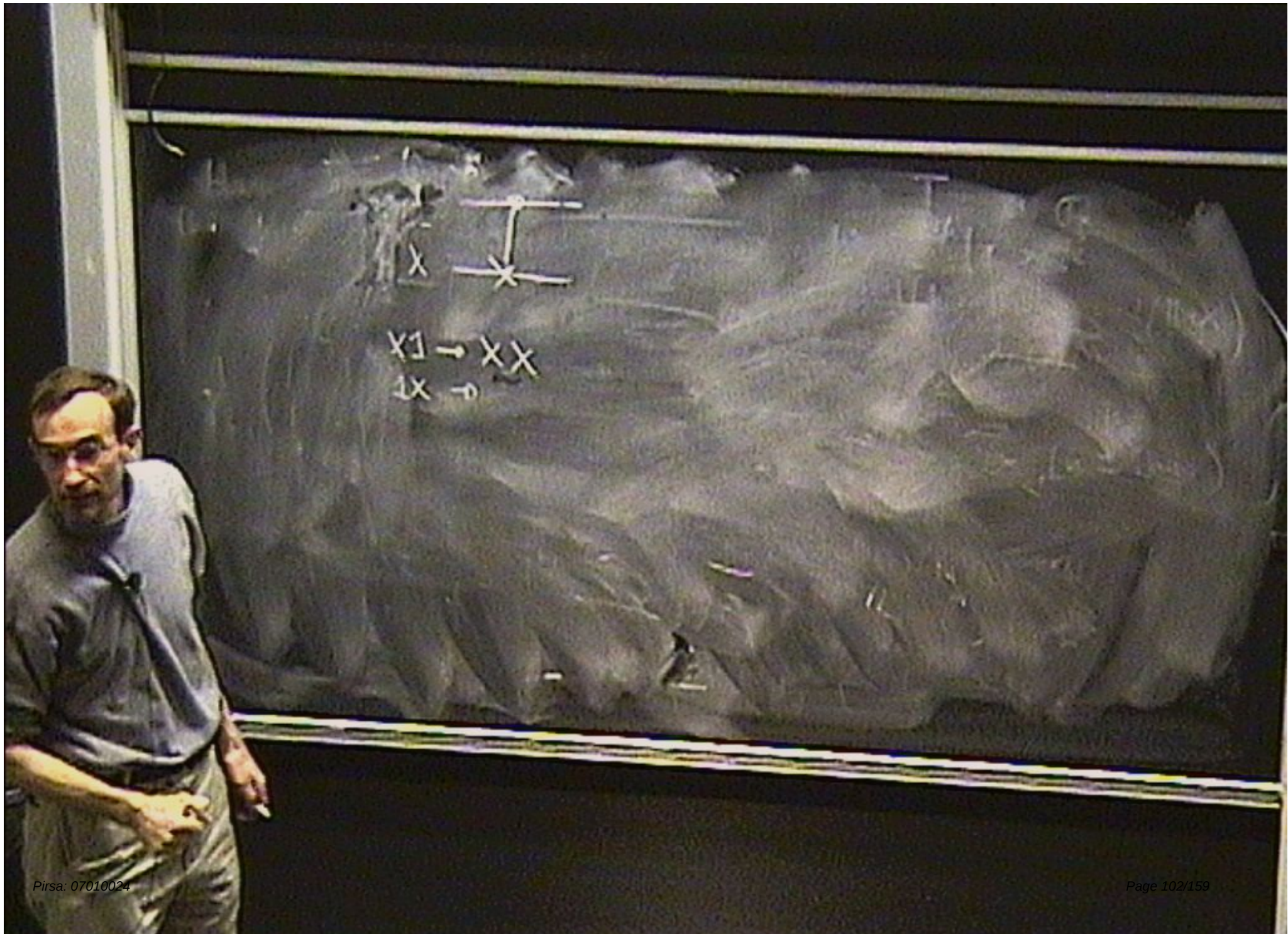
$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

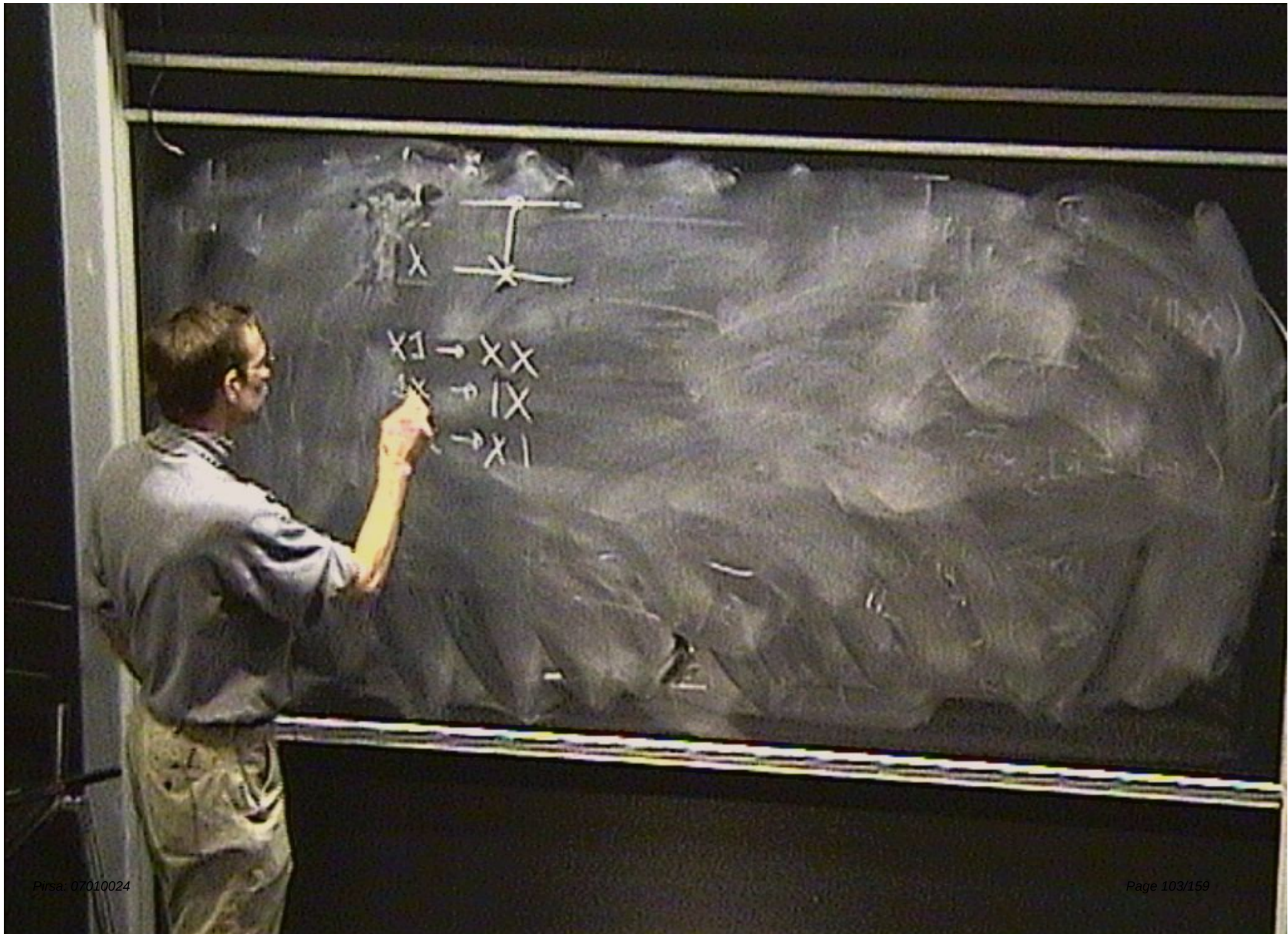
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

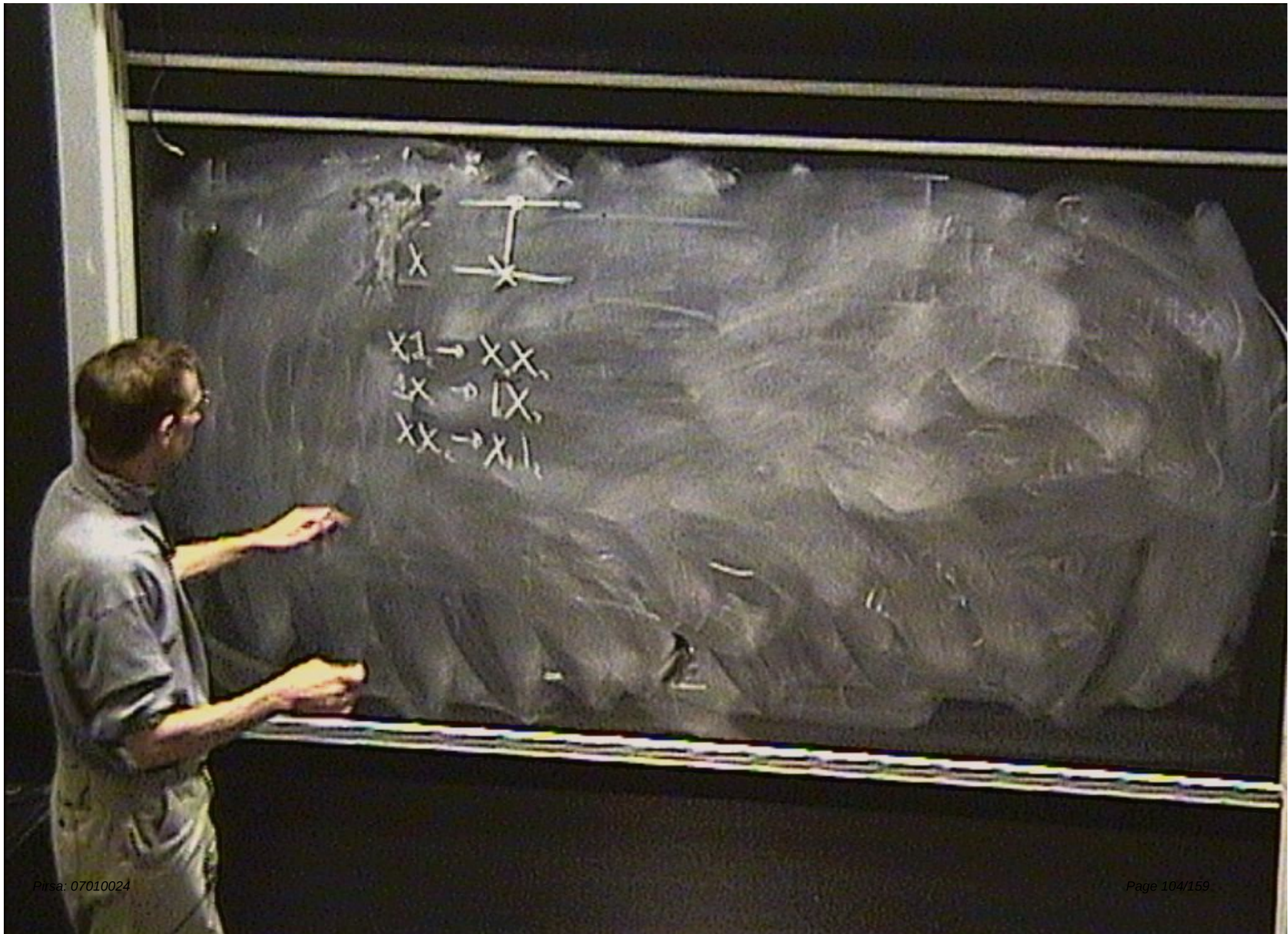




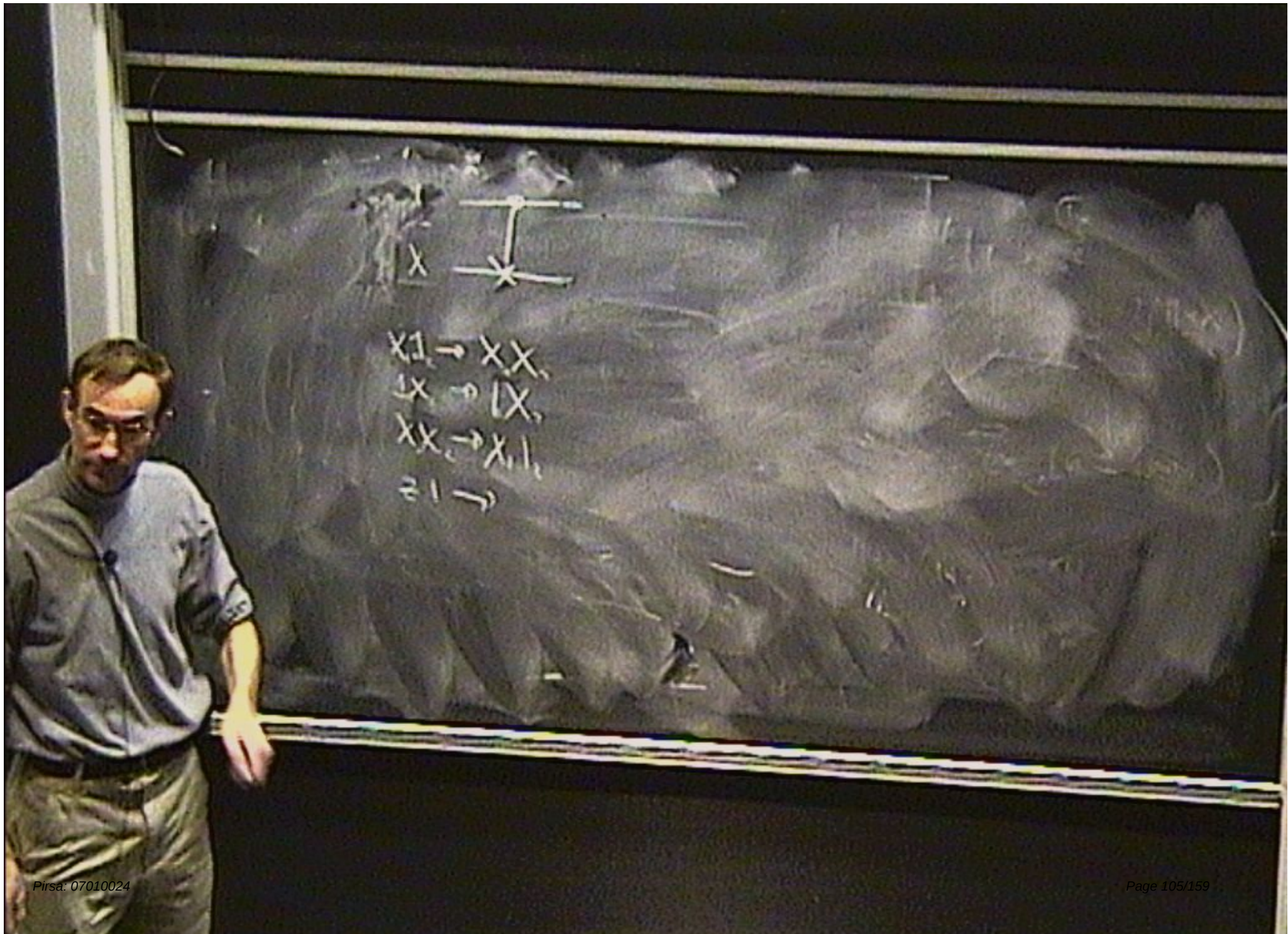


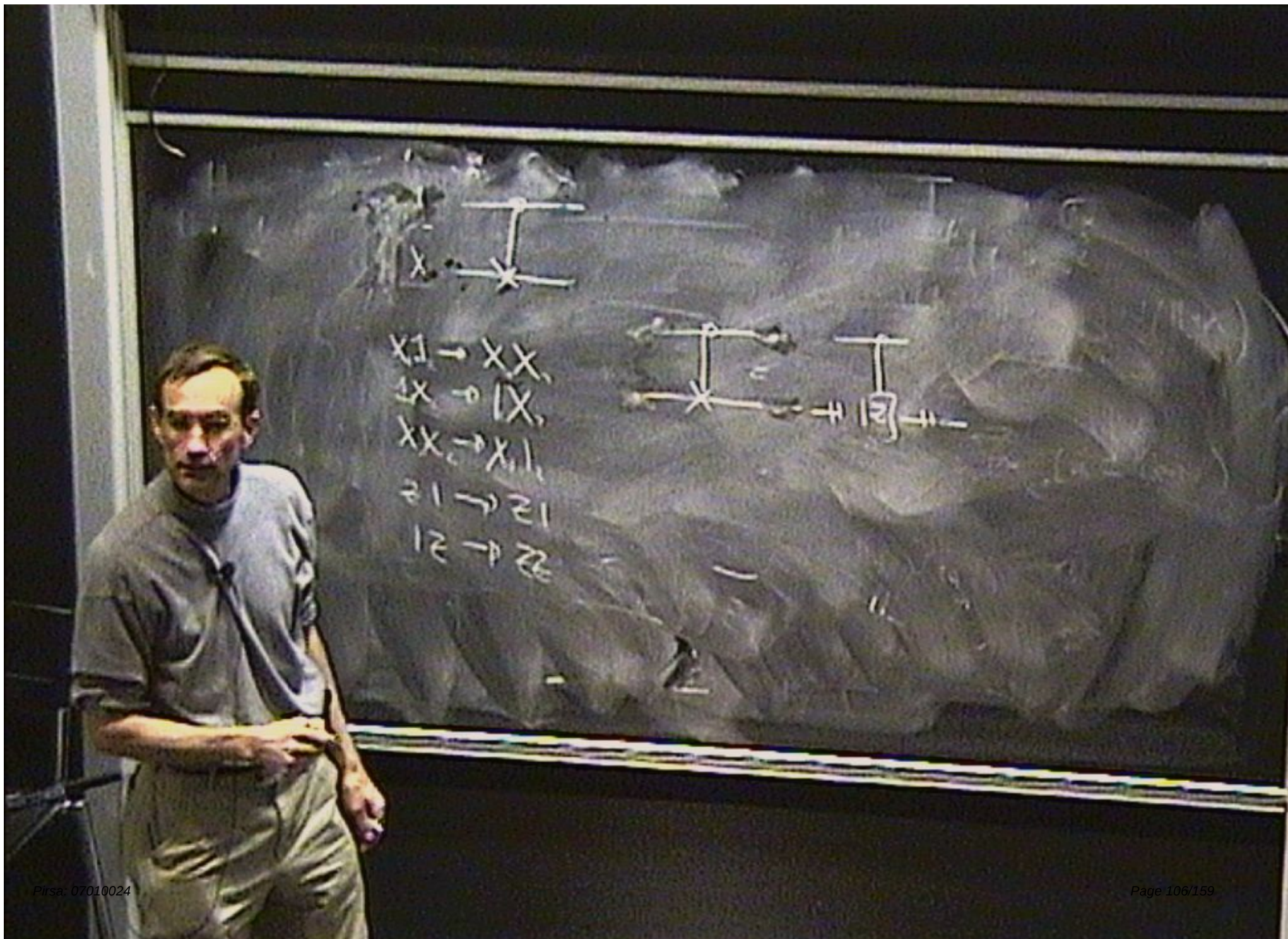


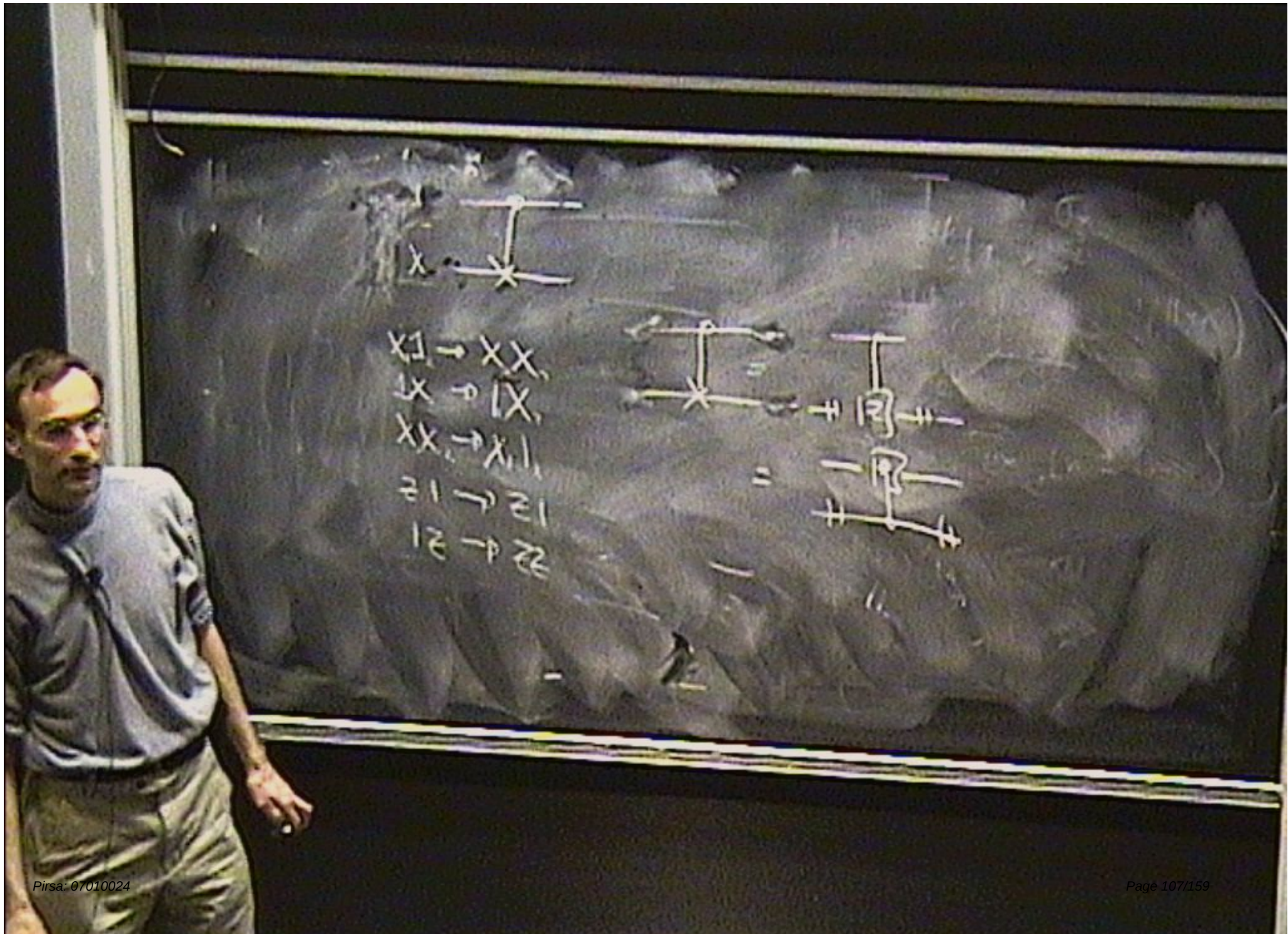
$$\begin{aligned} x1 &\rightarrow xx \\ x &\rightarrow |x \\ |x &\rightarrow x1 \end{aligned}$$



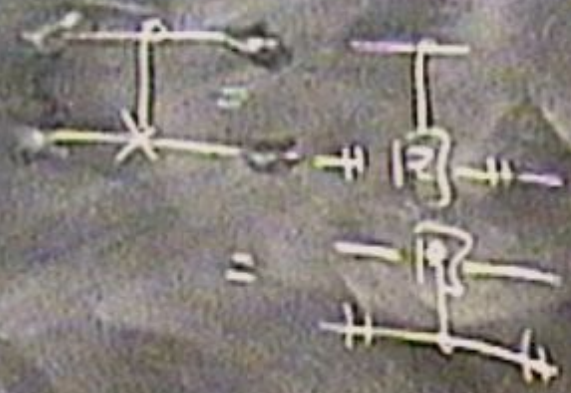
$x1 \rightarrow xx$
 $1x \rightarrow 1x$
 $xx \rightarrow x1$

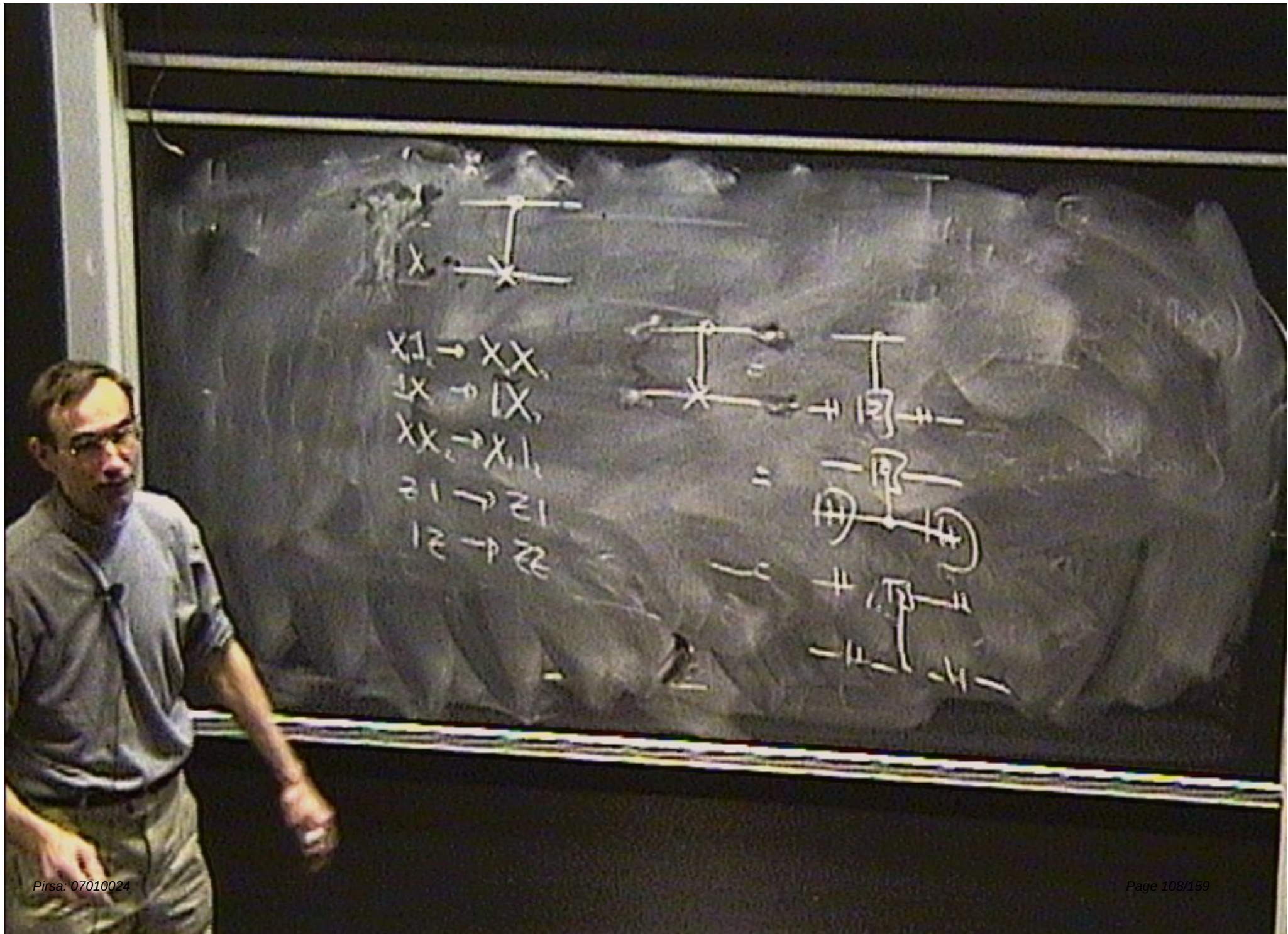




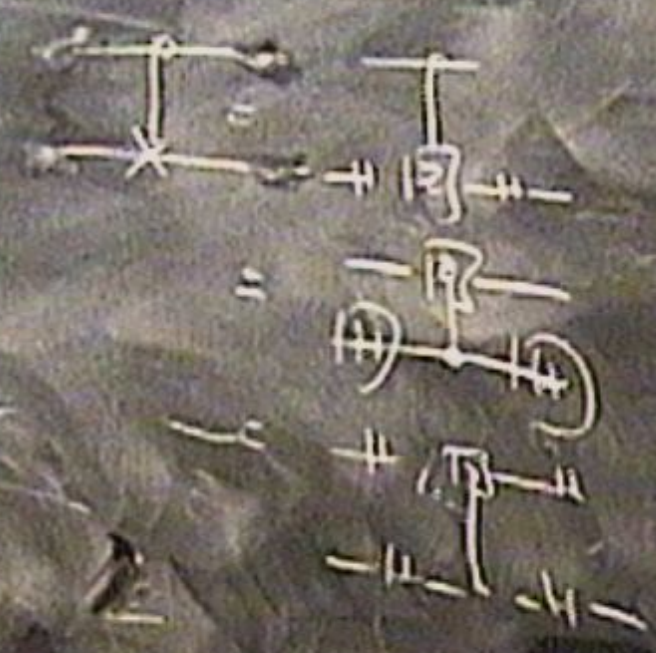


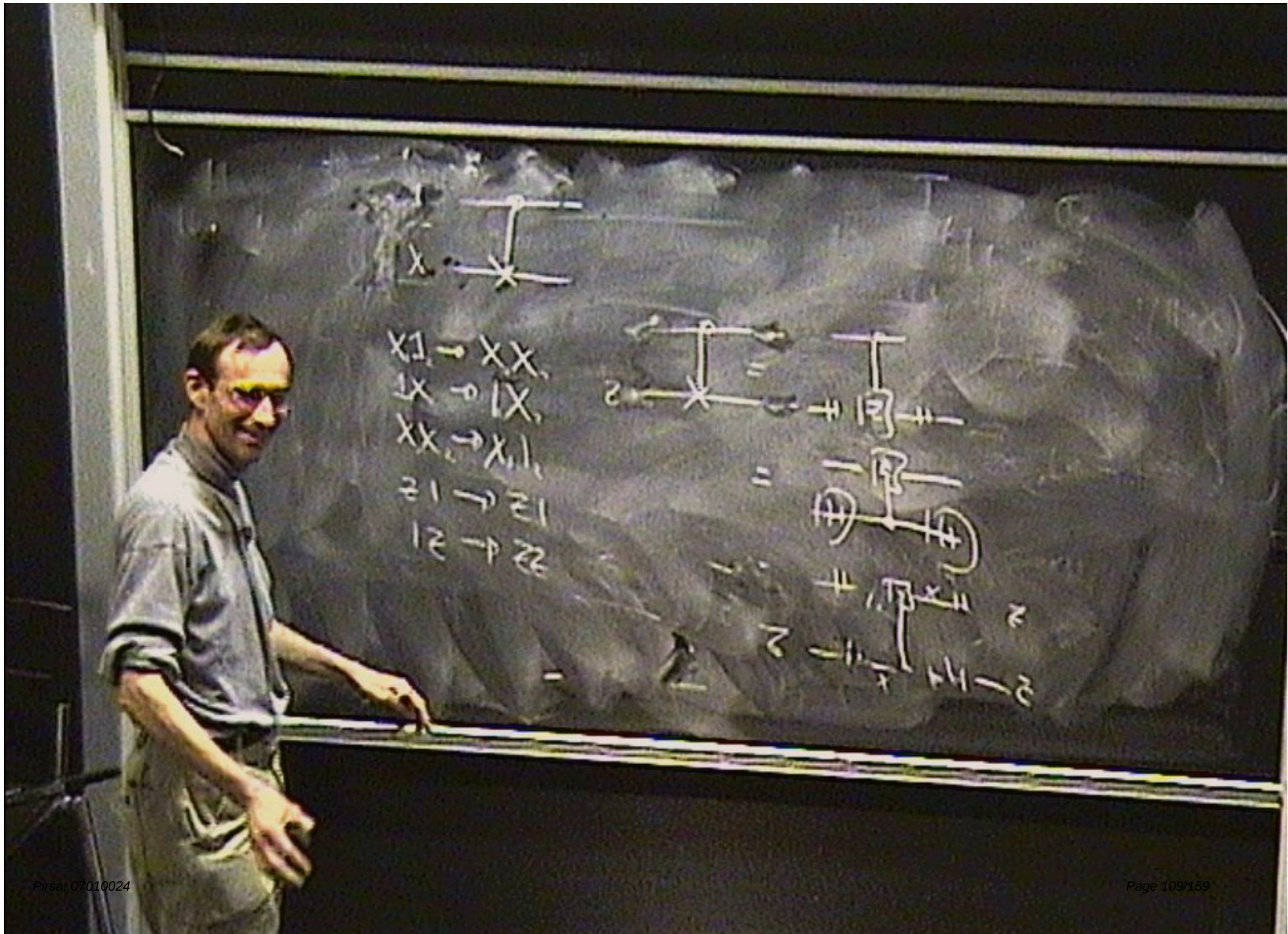
$$\begin{aligned}x_1 &\rightarrow x_2 \\x_2 &\rightarrow x_1 \\x_3 &\rightarrow x_4 \\z_1 &\rightarrow z_2 \\z_2 &\rightarrow z_1\end{aligned}$$





$x_1 \rightarrow x_2$
 $1x \rightarrow 1x$
 $xx \rightarrow x_1$
 $z_1 \rightarrow z_1$
 $1z \rightarrow z_2$

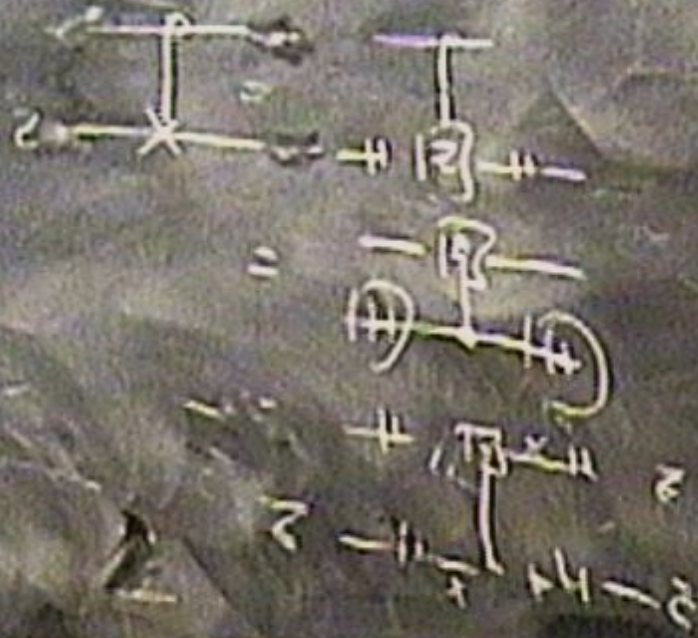






$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$x_1 \rightarrow x_2$
 $x_2 \rightarrow x_1$
 $x_1 \rightarrow x_2$
 $x_2 \rightarrow x_1$
 $z_1 \rightarrow z_2$
 $z_2 \rightarrow z_1$



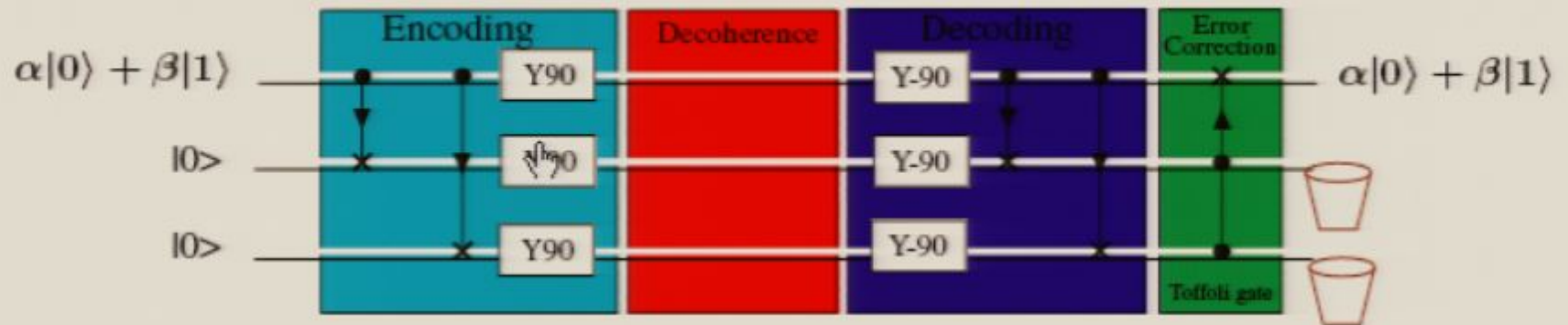
$$x(1+z)(1+z)$$

$$x11 + xz1 + xz2 + xz2$$

$$2(1+z)(1+z)$$

$$2 + 2z + 2z + 2z^2$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} &\alpha|+++ \rangle + \beta|--- \rangle \\ &\alpha| - ++ \rangle + \beta| + -- \rangle \\ &\alpha| + - + \rangle + \beta| - + - \rangle \\ &\alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

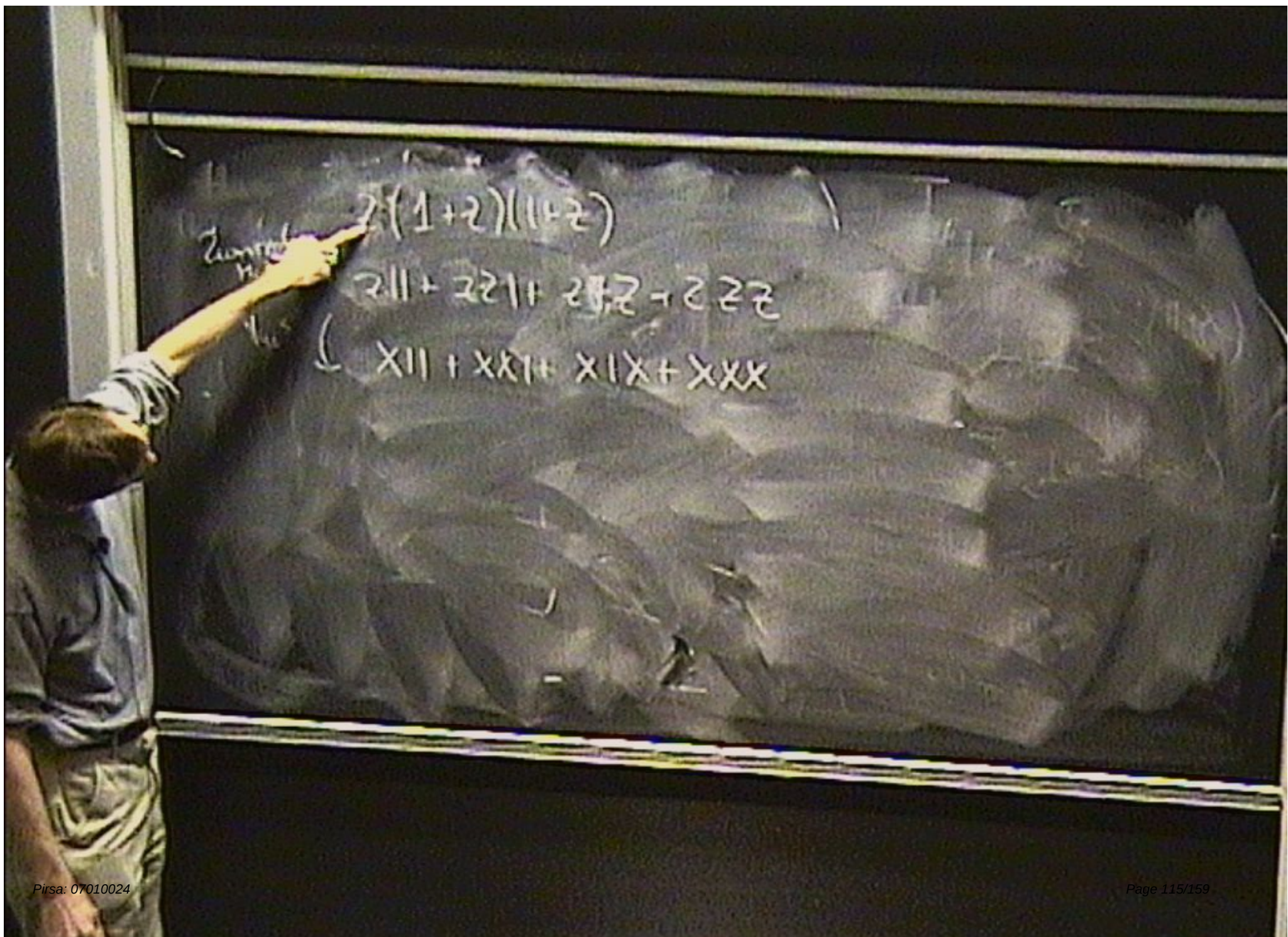
$$\rho_B \equiv \frac{1}{4} (I_x^1 + I_x^2 + I_x^3 + 4I_x^1 I_x^2 I_x^3)$$

$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle)|00\rangle \\ &(\alpha|1\rangle + \beta|0\rangle)|11\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|01\rangle \\ &(\alpha|0\rangle + \beta|1\rangle)|10\rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 00 \\ 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

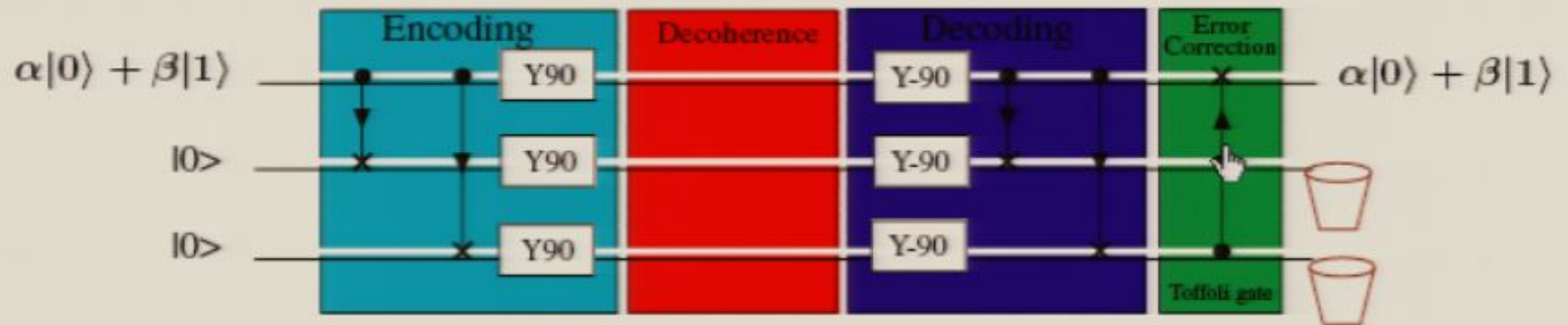


$$z(1+z)(1+z)$$

$$z11 + z21 + z32 + z22$$

$$\downarrow$$
$$x11 + x21 + x12 + x22$$

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} &\alpha|+++ \rangle + \beta|--- \rangle \\ &\alpha| - ++ \rangle + \beta| + -- \rangle \\ &\alpha| + - + \rangle + \beta| - + - \rangle \\ &\alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$

Control-Not

$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

$$\rho_E^1 \equiv \frac{1}{8} I_z^1 (9e^{-t/\tau} - e^{-9t/\tau}) \approx I_z^1 (1 - \frac{9}{2} t^2 / \tau^2 + \dots)$$

$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

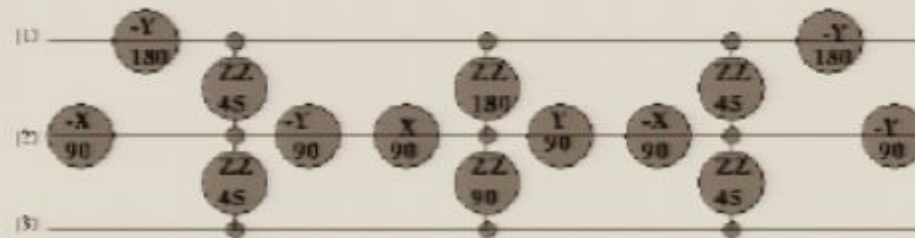
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

Phase QEC NMR circuit

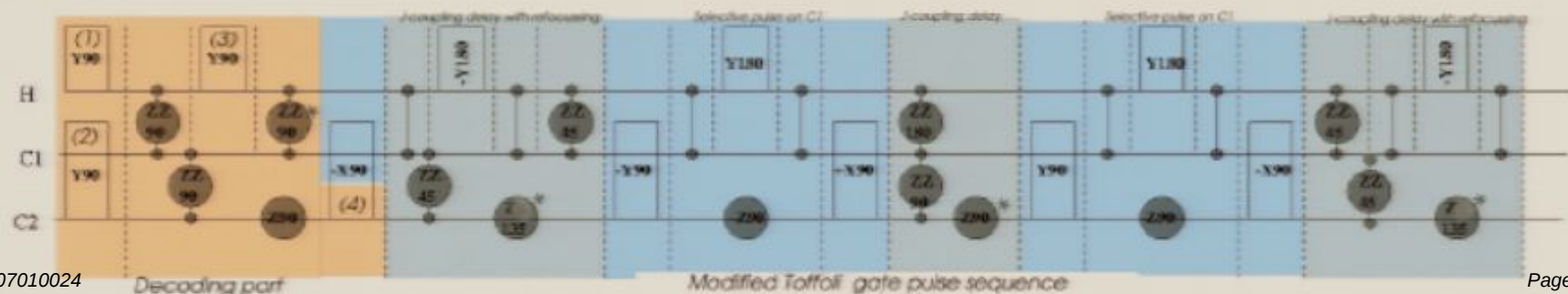
NMR implementation of the decoding and error correction:



Toffoli gate:



and the full decoding and Toffoli, including some optimization



Zumrot
mit

$$2(1+z)(1+z)$$

$$\downarrow 211 + 221 + 232 + 222$$

Yas

$$\downarrow x11 + xx1 + x1x + xxx$$



Zusatz
not

$$2(1+z)(1+z^2)$$

$$\downarrow 2 \cdot 1 + 2 \cdot z + 2 \cdot z^2 + 2 \cdot z \cdot z^2$$

χ_{rel}

$$\downarrow x \cdot 1 + x \cdot x + x \cdot x^2 + x \cdot x^2 \cdot x$$



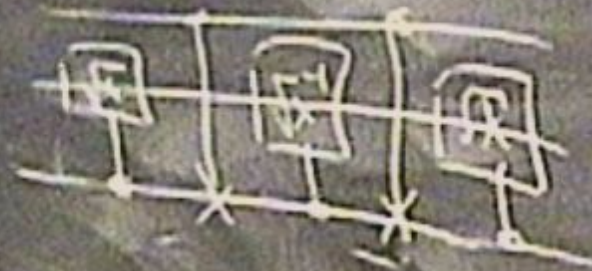
Zusatz
Teil

$$z(1+z)(1+z)$$

$$z \cdot 1 + z \cdot z + z \cdot z + z \cdot z$$

Y₄₄

$$x \cdot 1 + x \cdot x + x \cdot x + x \cdot x$$

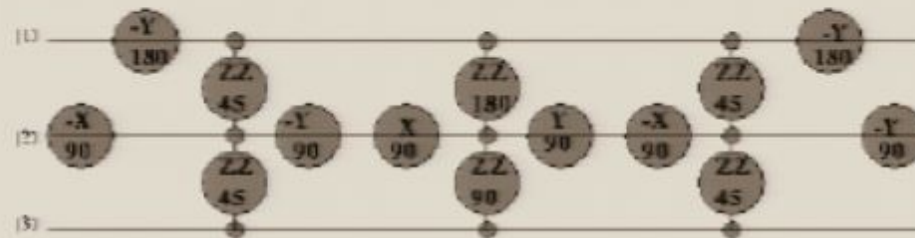


Phase QEC NMR circuit

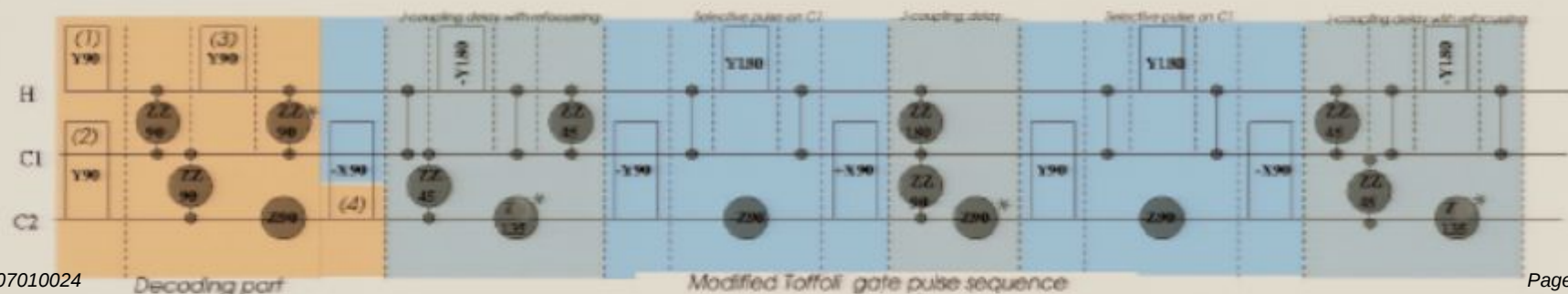
NMR implementation of the decoding and error correction:



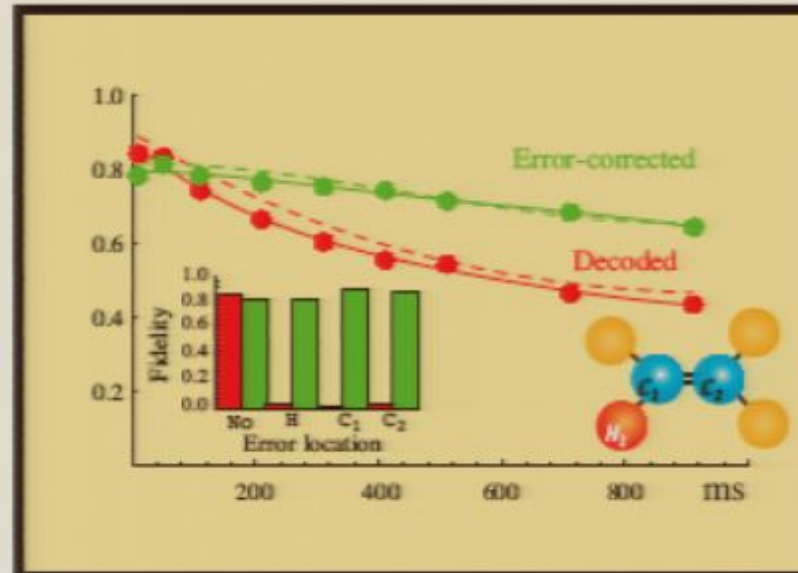
Toffoli gate:



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Experimental results



The demonstration of quantum error correction is in the shape of the green curve which does not have the first order error. The curve is much flatter than the red one.

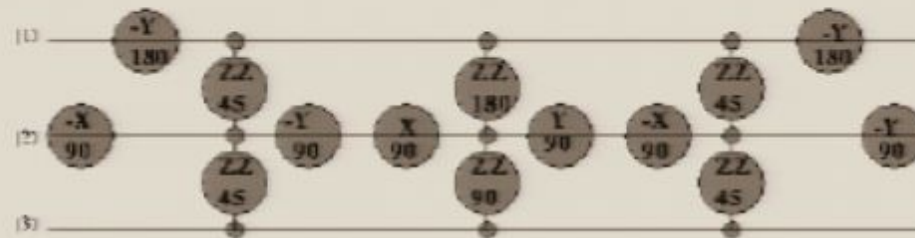
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D. G. Cory, M. D. Price, W. Maas, E. Knill,
R. Laflamme, W. H. Zurek, T. F. Havel and
S. S. Somaroo, PRL 81, 2152, 1998

Phase QEC NMR circuit

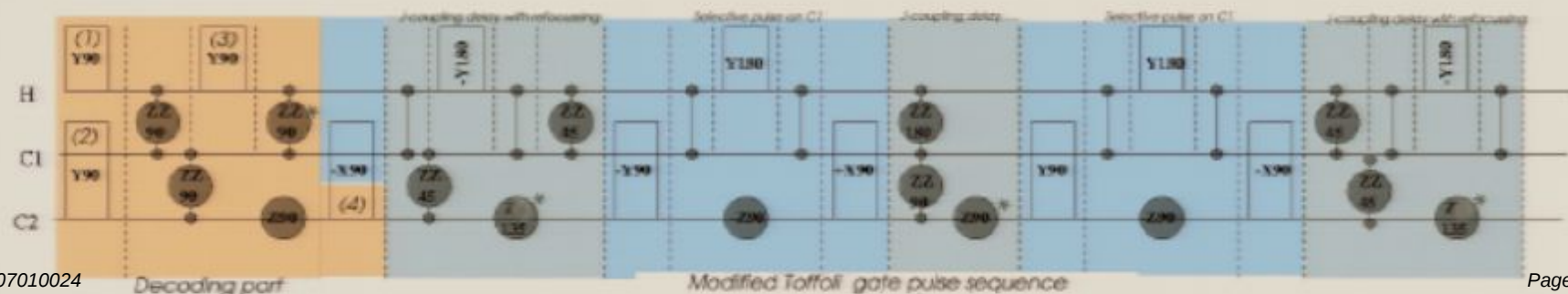
NMR implementation of the decoding and error correction:



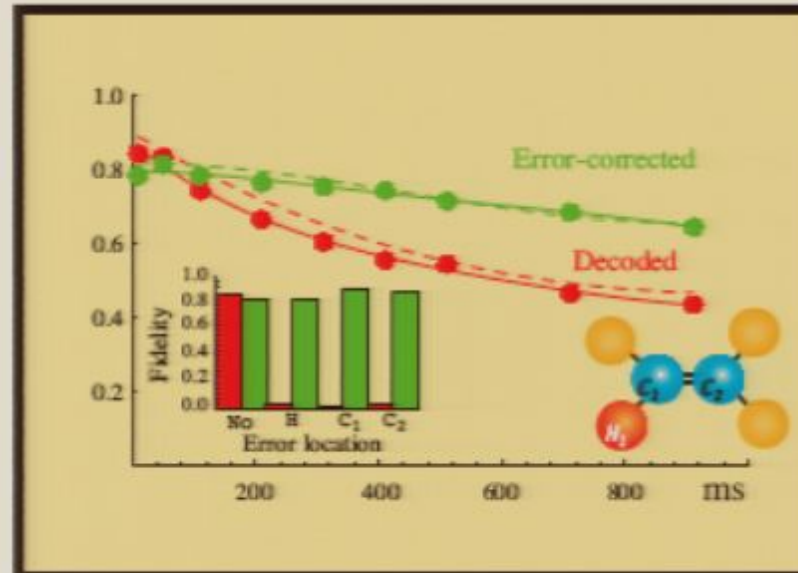
Toffoli gate:



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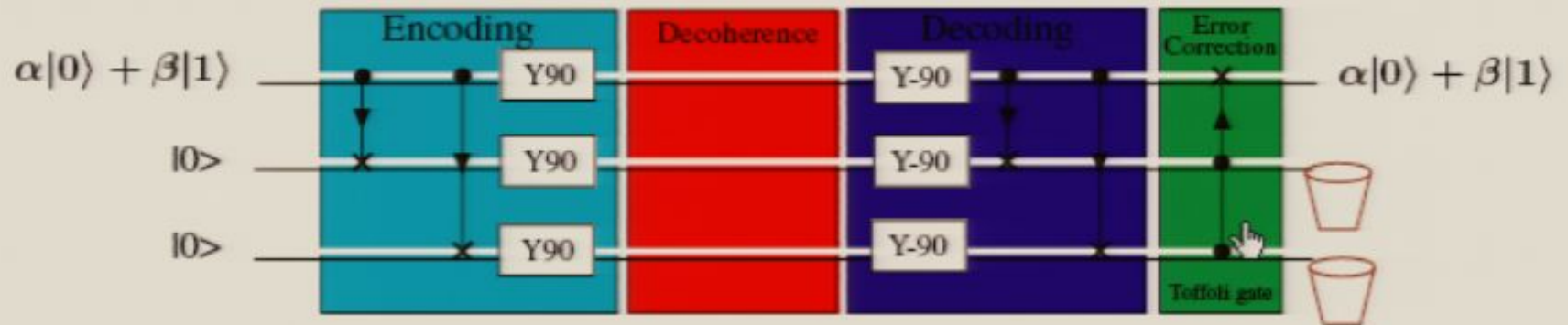
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Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} & \alpha|+++ \rangle + \beta|--- \rangle \\ & \alpha| - ++ \rangle + \beta| + -- \rangle \\ & \alpha| + - + \rangle + \beta| - + - \rangle \\ & \alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$

Control-Not

$$\begin{matrix} \bullet \\ \downarrow \\ \times \end{matrix} = \begin{cases} 00 \rightarrow 00 \\ 01 \rightarrow 01 \\ 10 \rightarrow 11 \\ 11 \rightarrow 10 \end{cases}$$

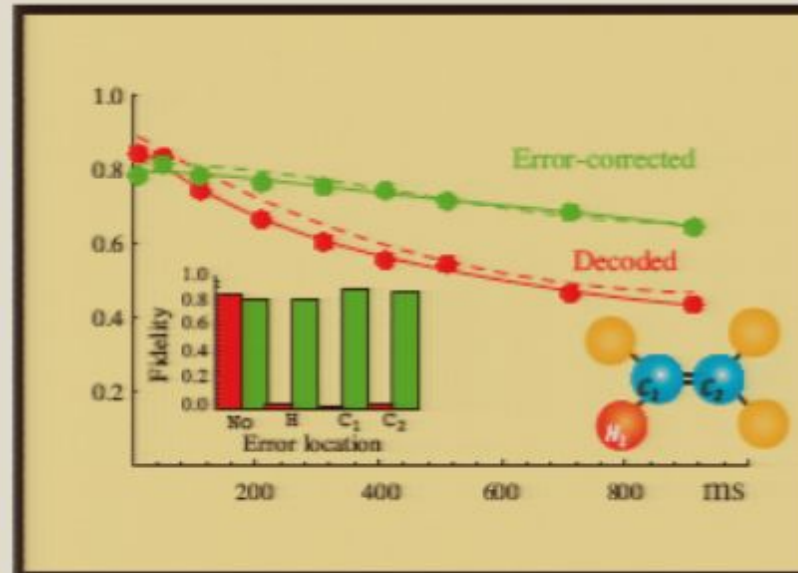
$$\rho_E^1 \equiv \frac{1}{8}I_z^1(9e^{-t/\tau} - e^{-9t/\tau}) \approx I_z^1(1 - \frac{9}{2}t^2/\tau^2 + \dots)$$

$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

$$(\alpha|0\rangle + \beta|1\rangle) \otimes \begin{cases} 11 \\ 01 \\ 10 \end{cases} \sim 1 - 3\gamma^2$$

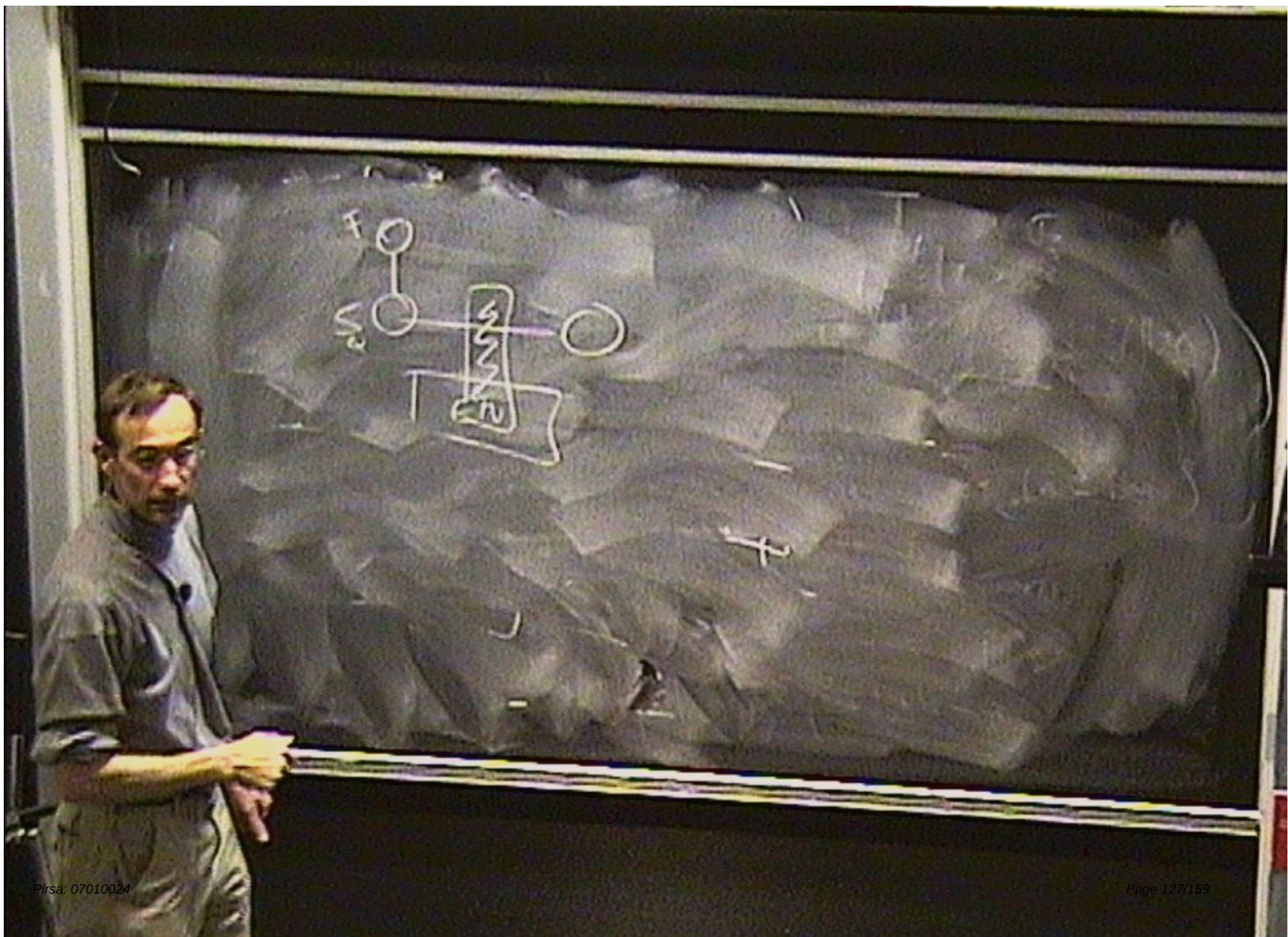
$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

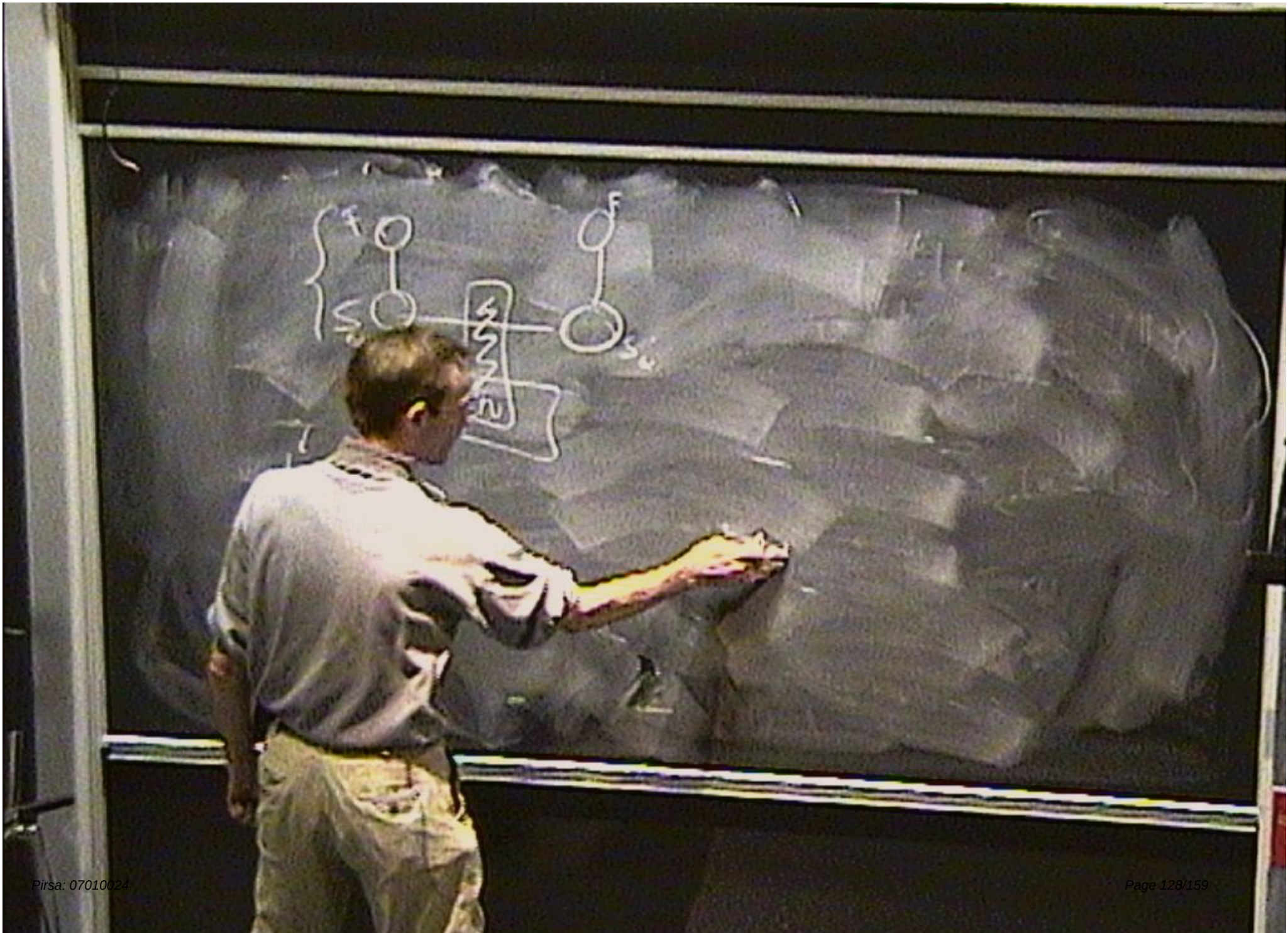
Experimental results

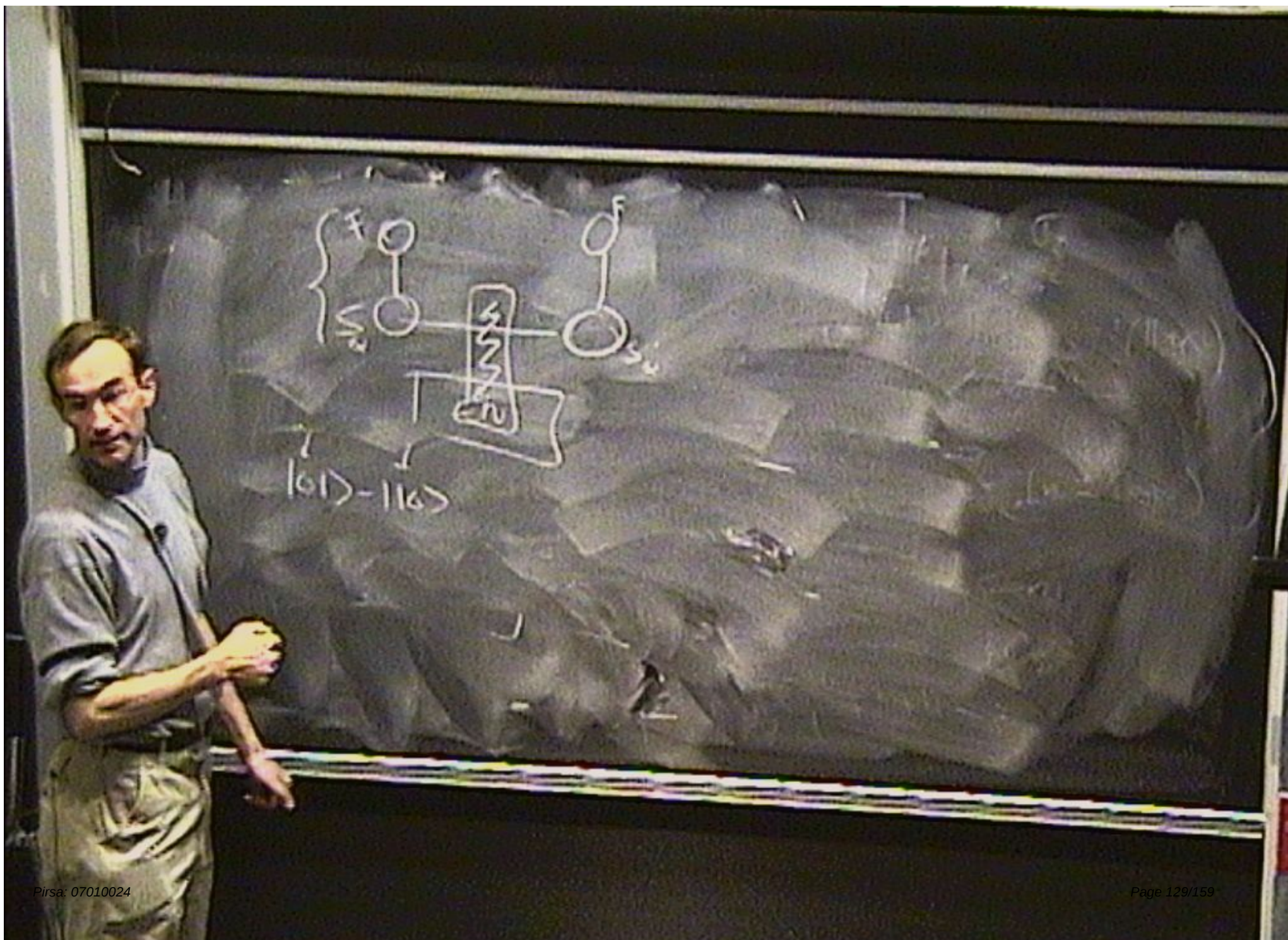


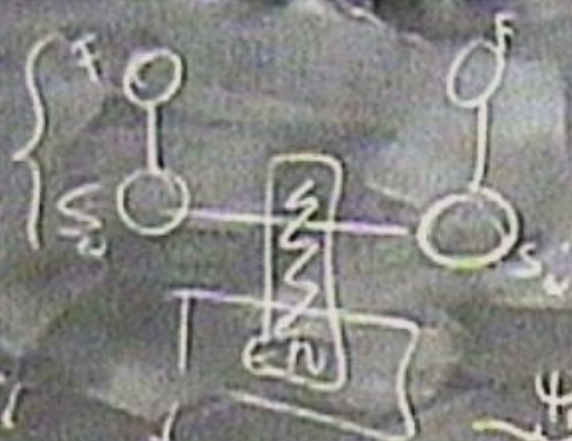
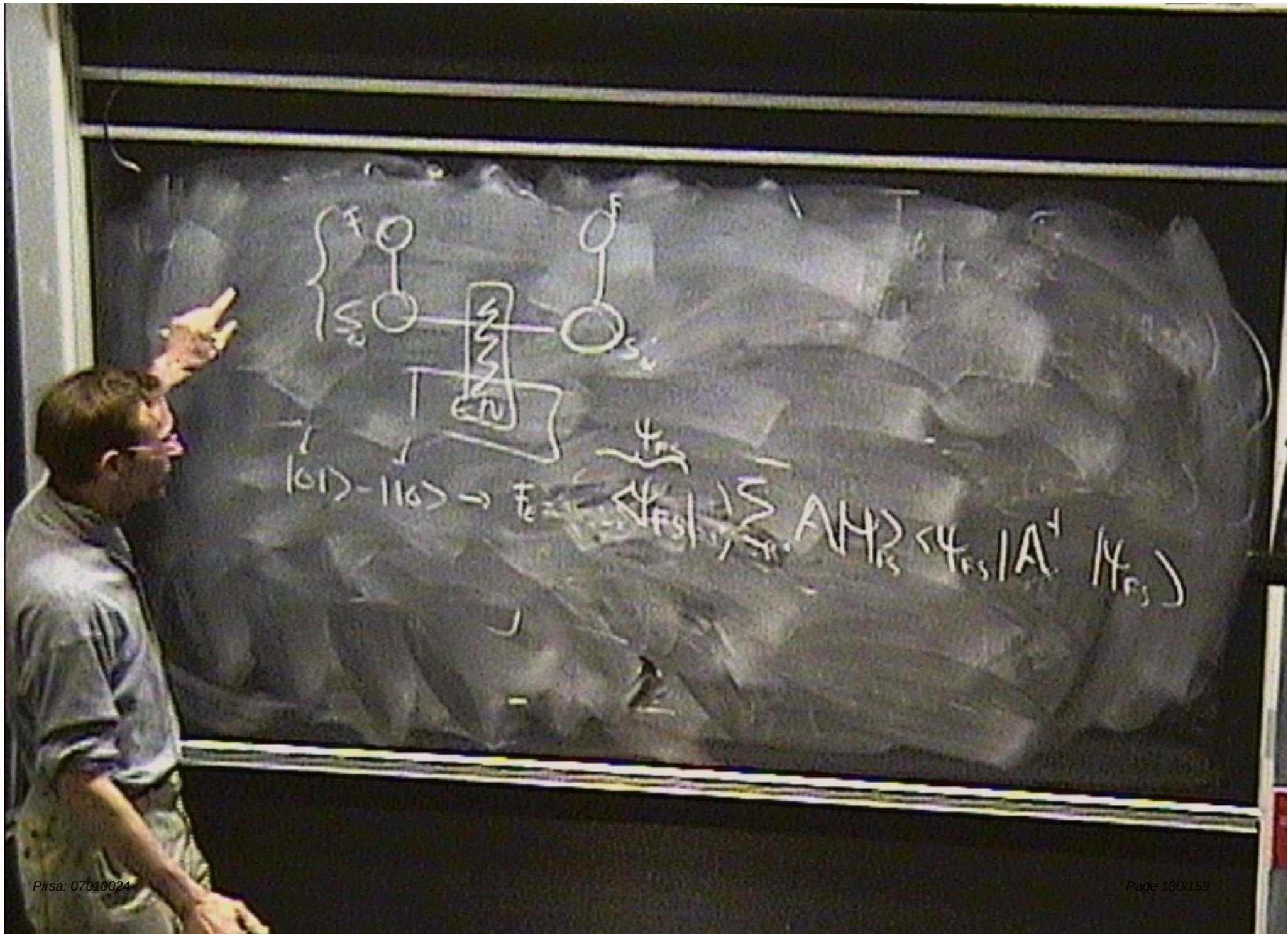
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S. S. Somaroo, PRL 81, 2152, 1998

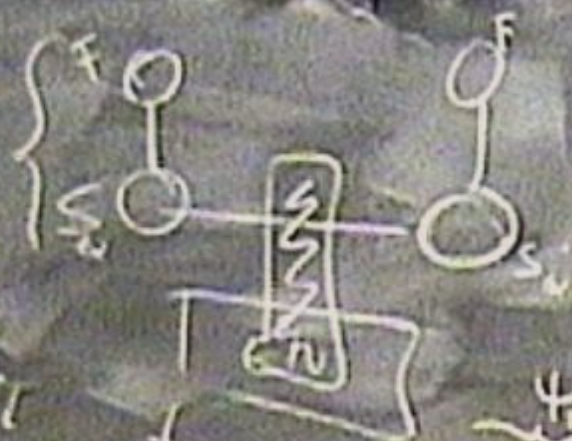






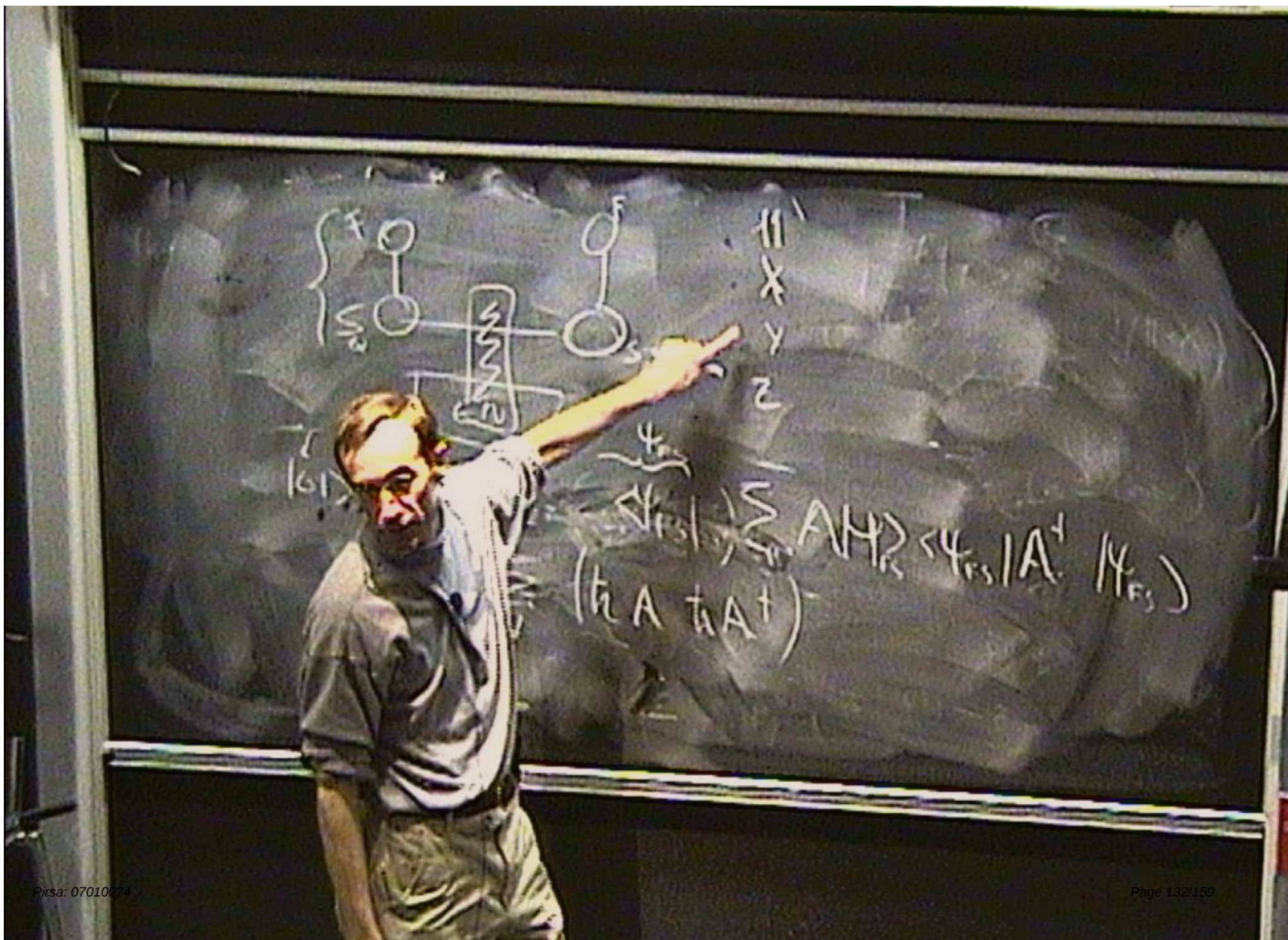


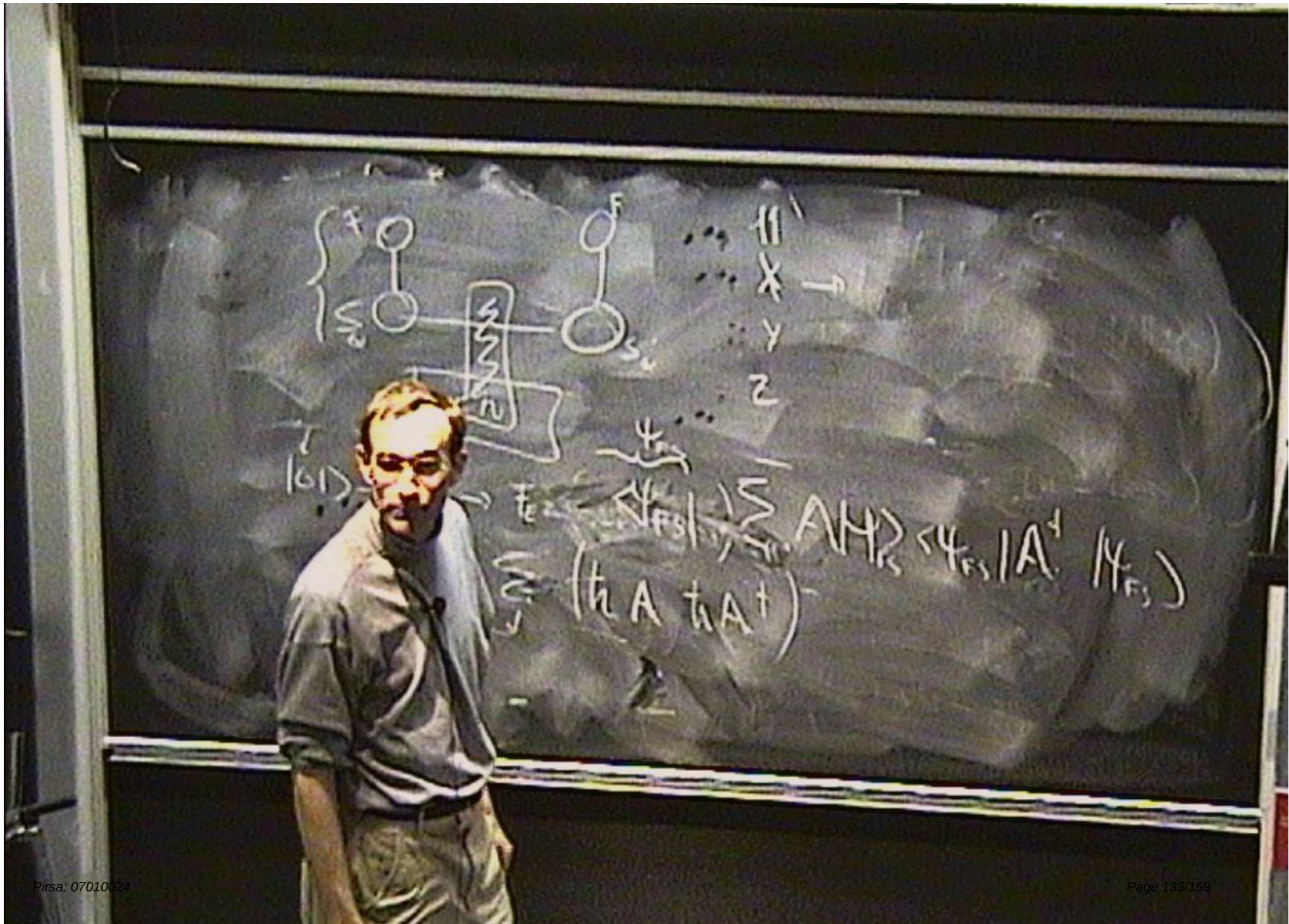
$$|01\rangle - |10\rangle \rightarrow T_2 = \frac{1}{\sqrt{2}} \langle \psi_{FS} | \sum_{i,j} A_{ij} | \psi_{FS} \rangle \langle \psi_{FS} | A^\dagger | \psi_{FS} \rangle$$

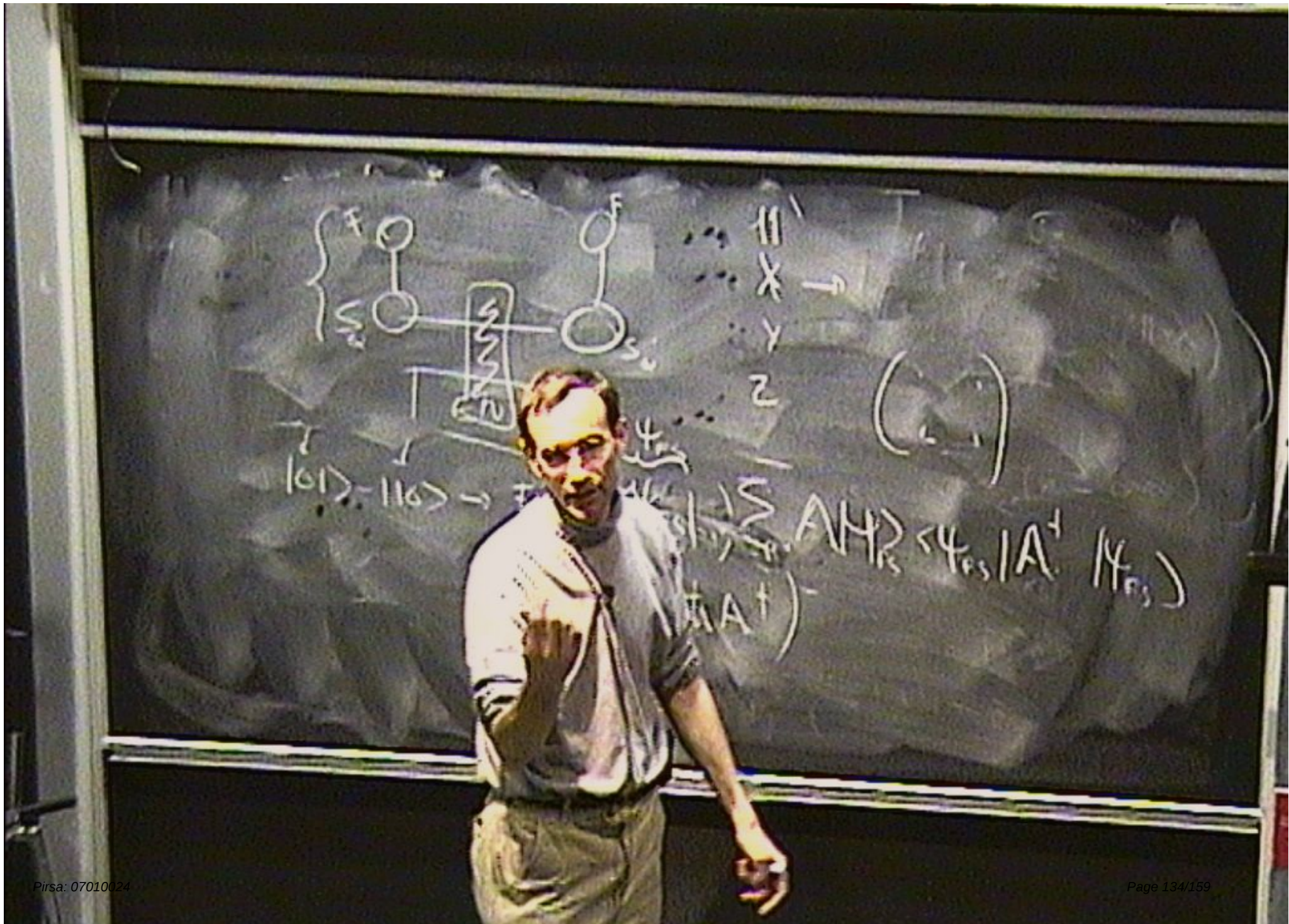


$$|01\rangle, |11\rangle \rightarrow T = \langle \psi_{FS} | \sum_{i,j} A_{ij} | \psi_{FS} \rangle \langle \psi_{FS} | A^\dagger | \psi_{FS} \rangle$$

$$\sum_{i,j} (t A t A^\dagger)$$

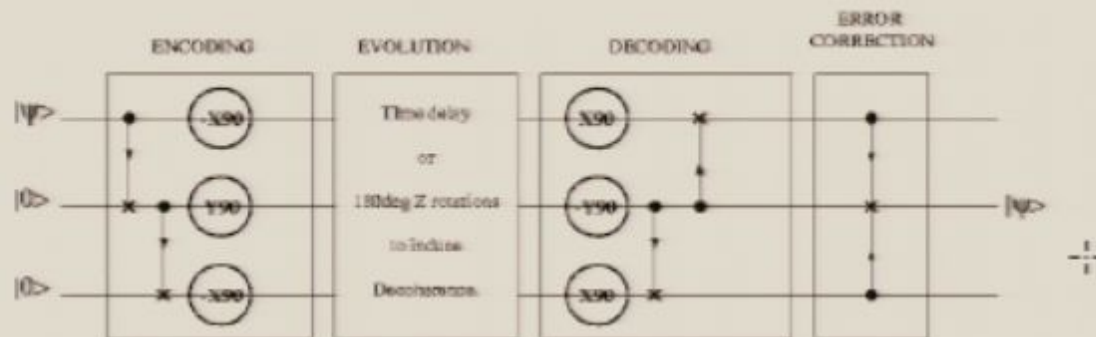




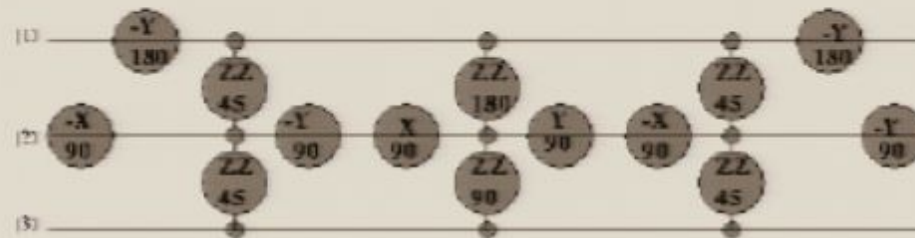


Phase QEC NMR circuit

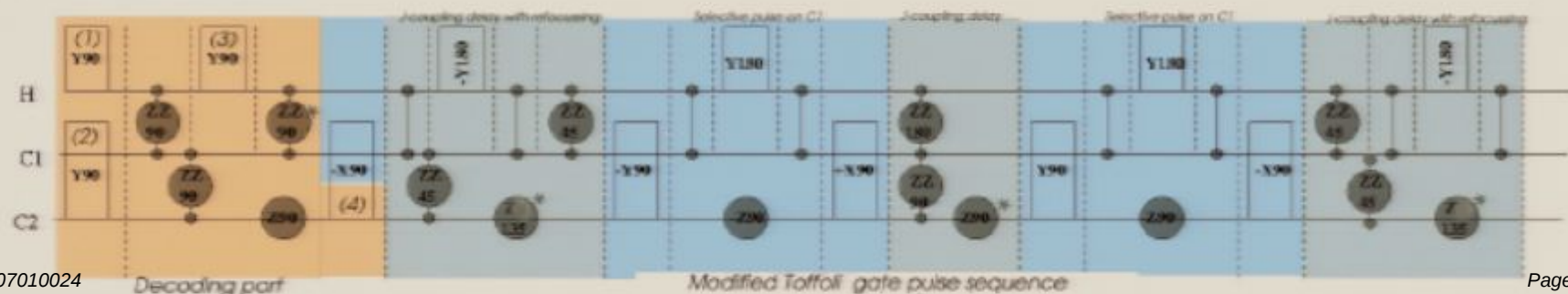
NMR implementation of the decoding and error correction:



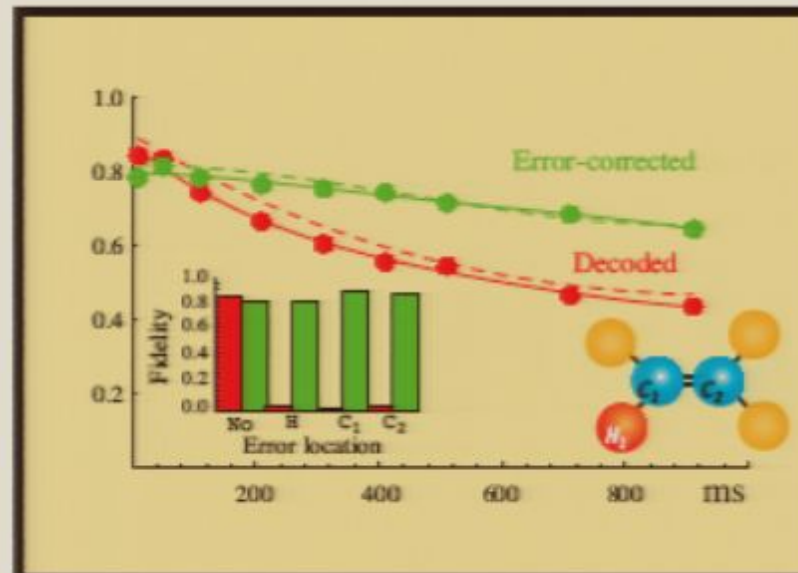
Toffoli gate:



and the full decoding and Toffoli, including some optimization



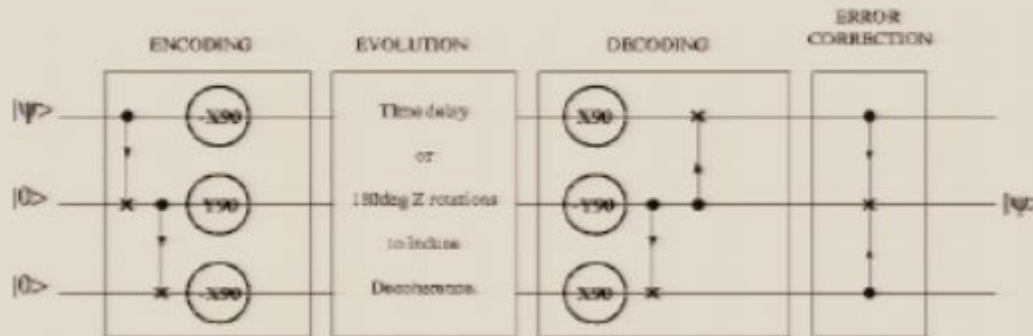
Experimental results



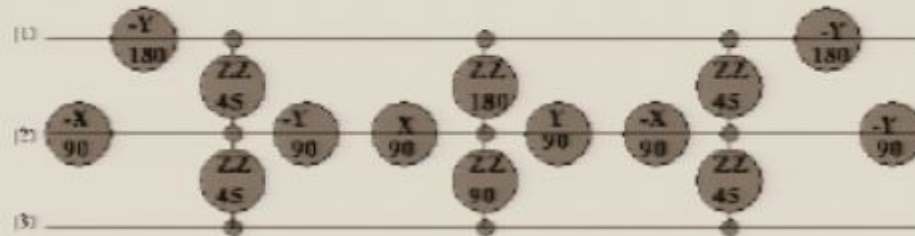
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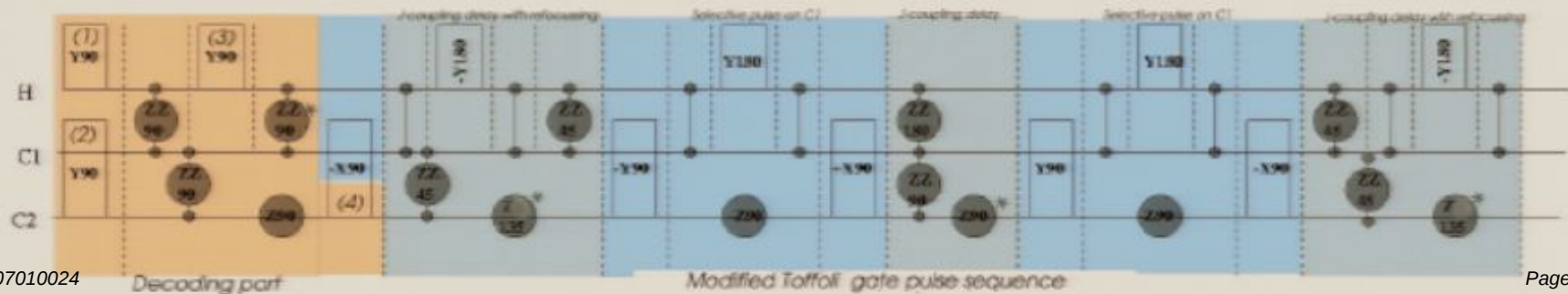
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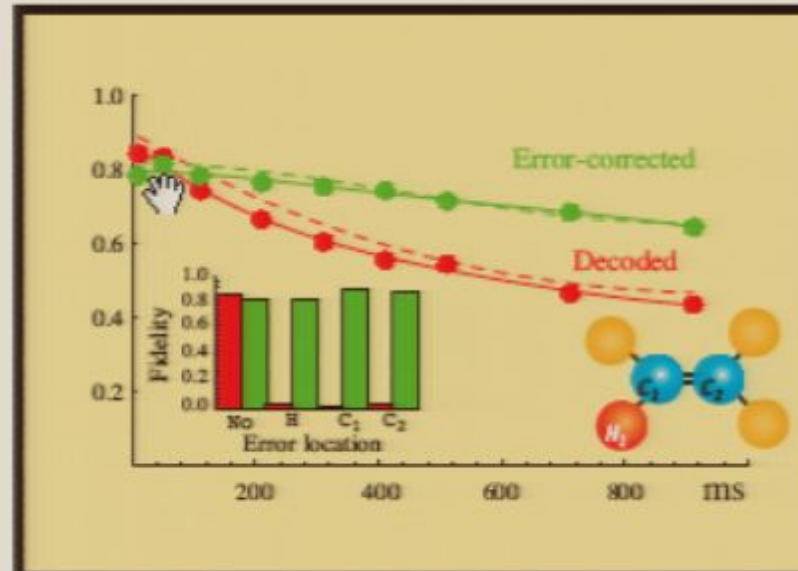
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Q. Error Correction for Phase

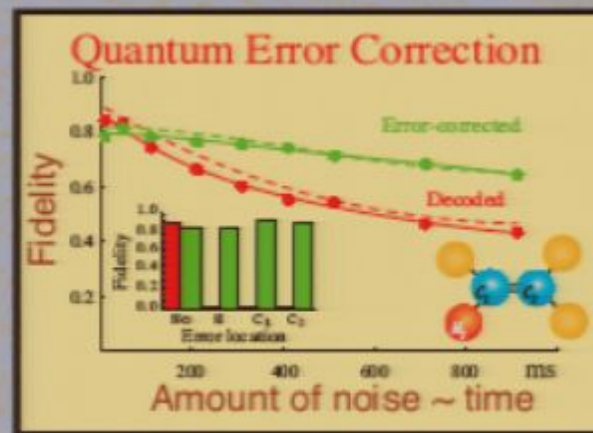
Experiments

$\alpha|0\rangle$

$\beta|1\rangle$

α

Science
Top 10 breakthroughs
of the Year
Surface 282,
2156, 1998



Cont

$\left\{ \begin{array}{l} \bullet \\ \downarrow \\ \times \end{array} \right\}$

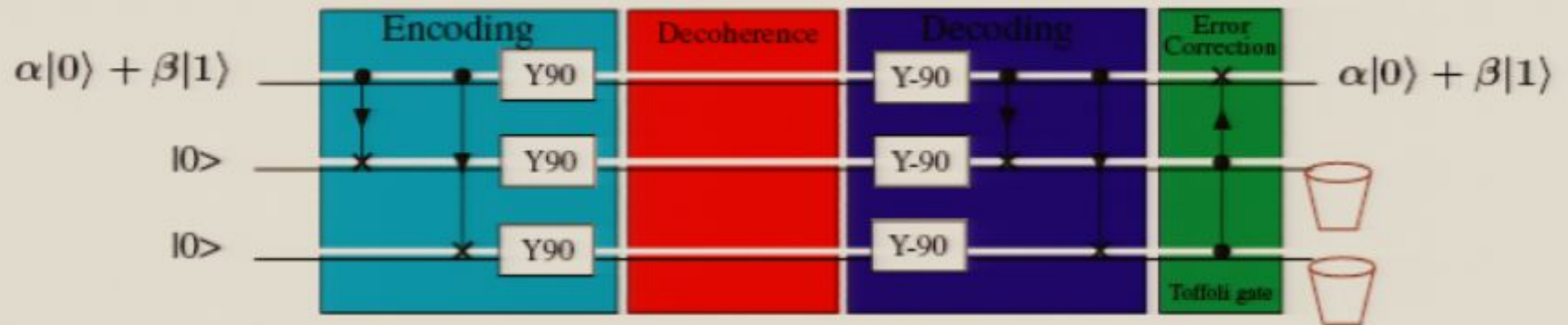
Exper
D. G.
R. La
S. S. Somaroo, PRL 81, 2152, 1998

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$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

11
01
10

Q. Error Correction for Phase



$$\alpha|0\rangle + \beta|1\rangle$$

Errors: $\begin{matrix} + & \rightarrow & - \\ - & \rightarrow & + \end{matrix}$

$$\begin{aligned} &\alpha|+++ \rangle + \beta|--- \rangle \\ &\alpha| - ++ \rangle + \beta| + -- \rangle \\ &\alpha| + - + \rangle + \beta| - + - \rangle \\ &\alpha| + + - \rangle + \beta| - - + \rangle \end{aligned}$$

$$(\alpha|0\rangle + \beta|1\rangle)|00\rangle$$

Control-Not

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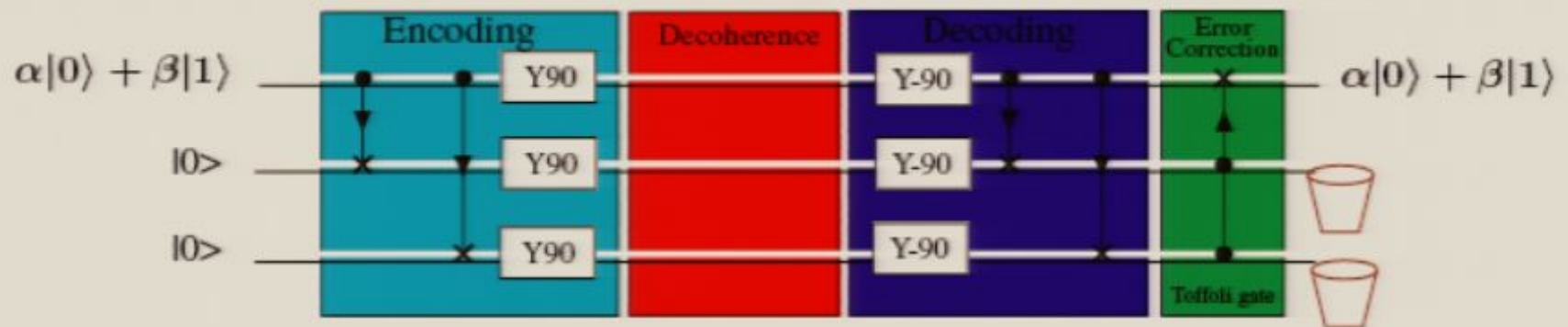
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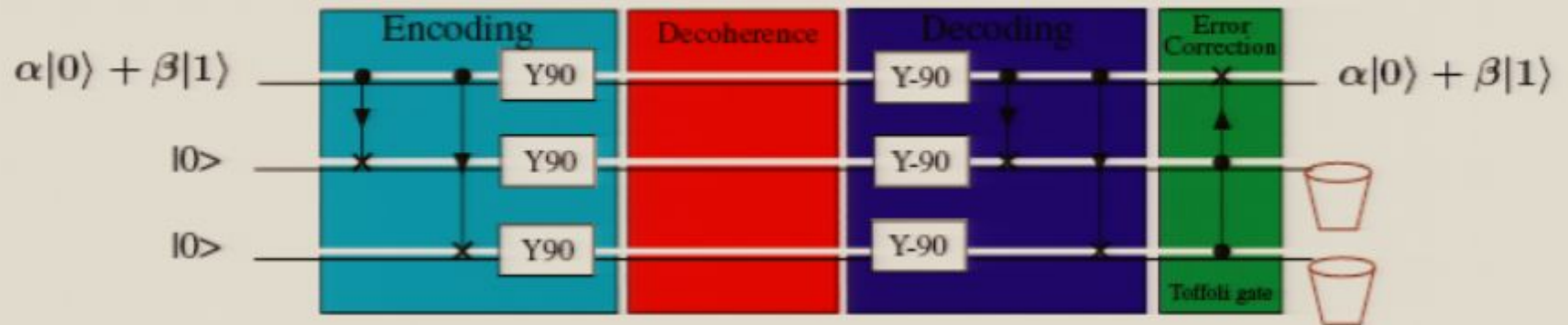
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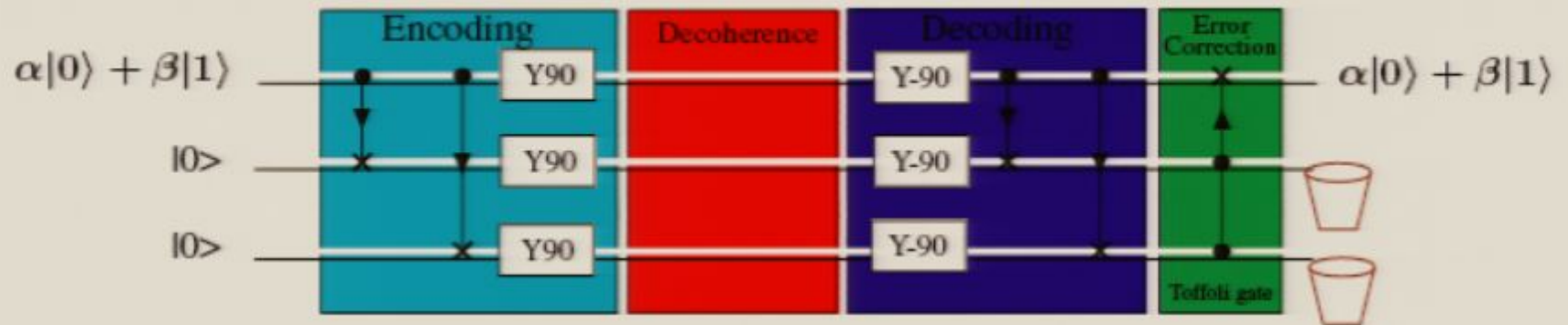
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Control-Not

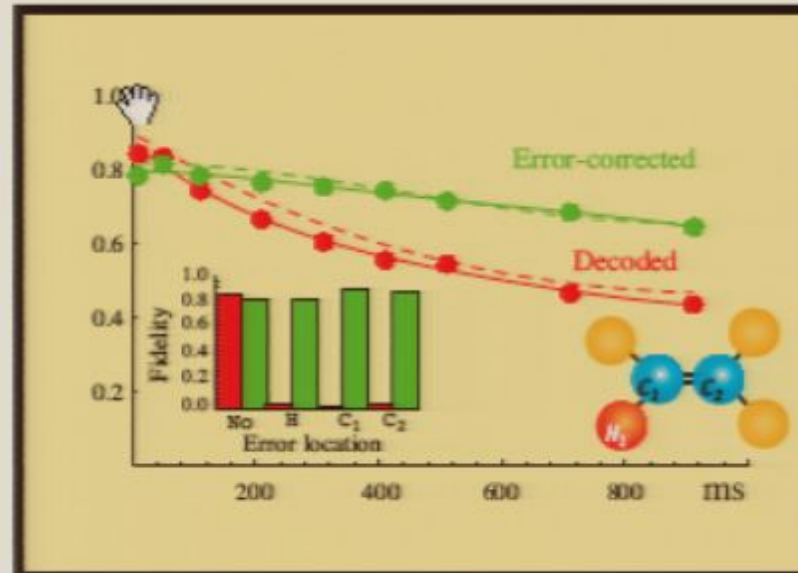
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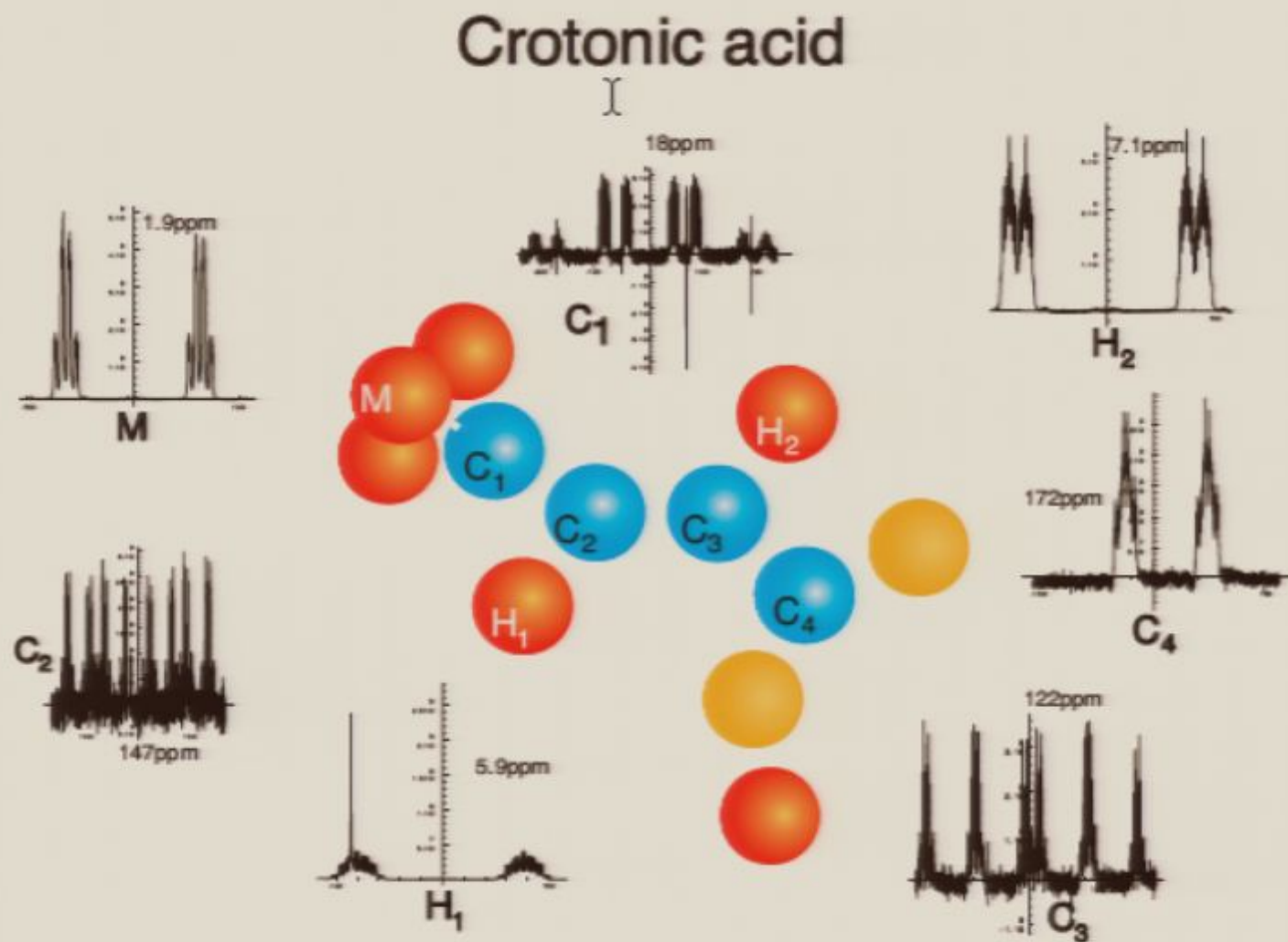
Experimental results



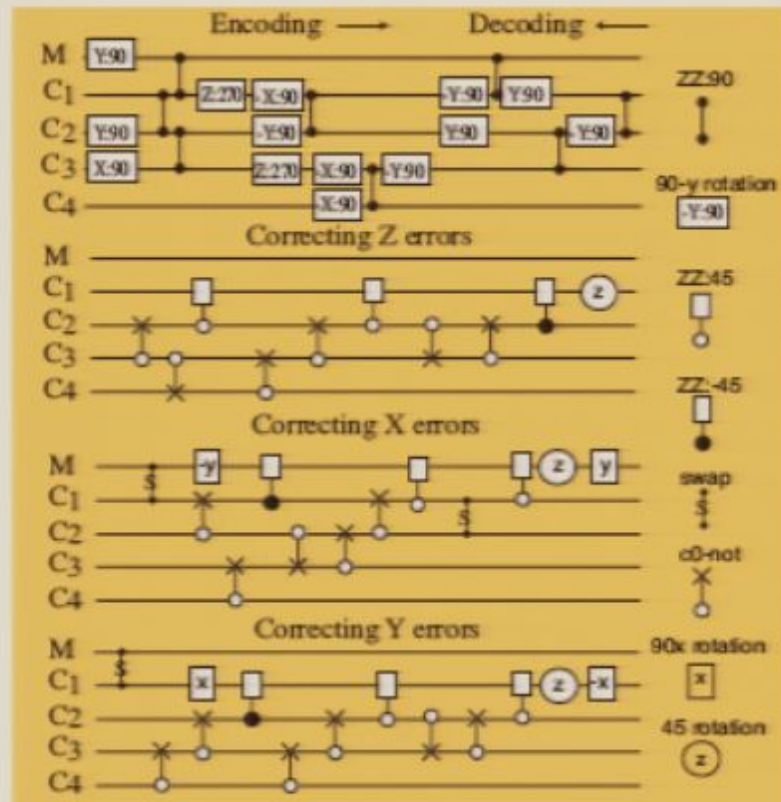
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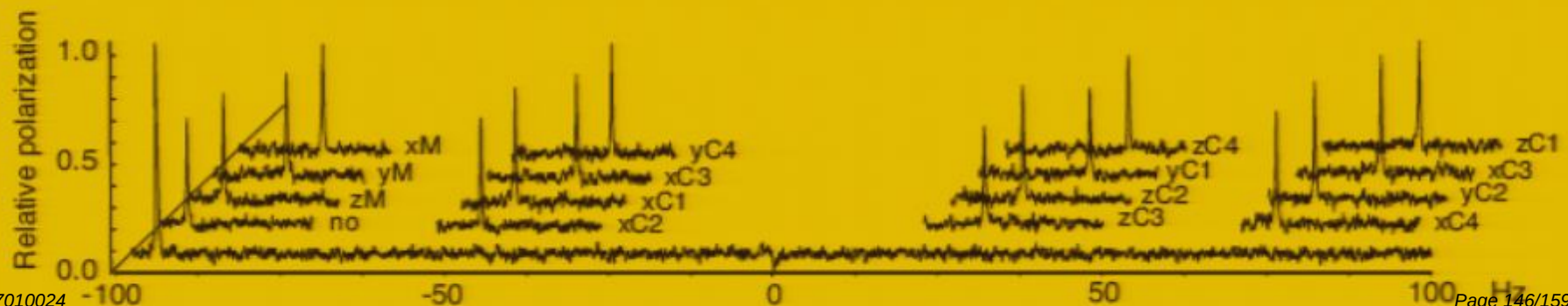
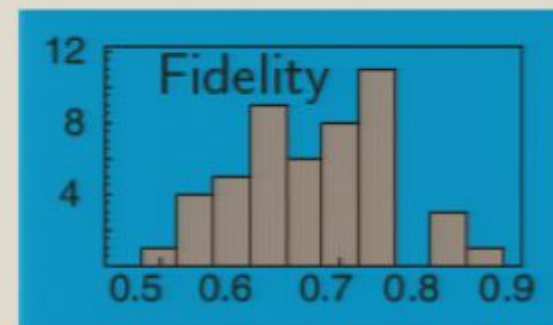
Spectra of crotonic acid



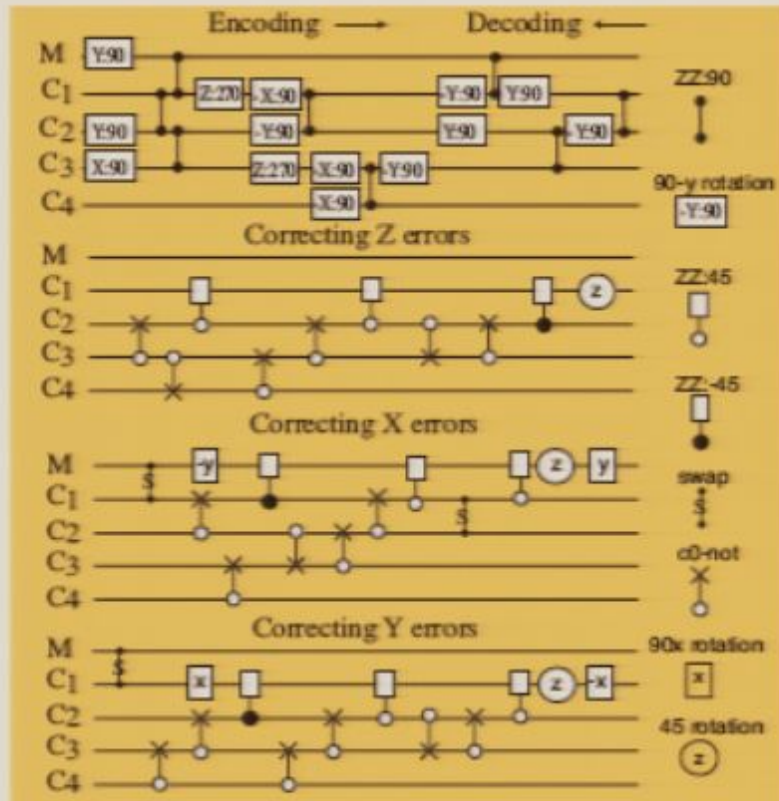
5 bit quantum error correcting code



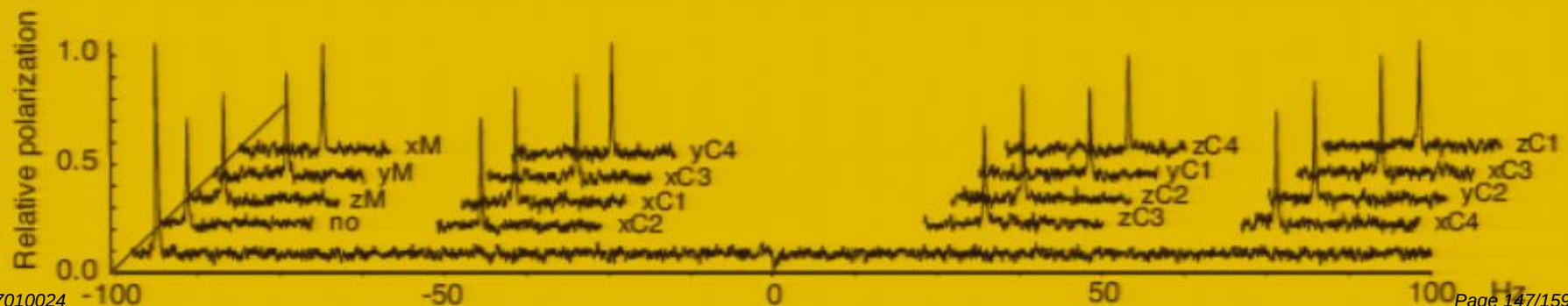
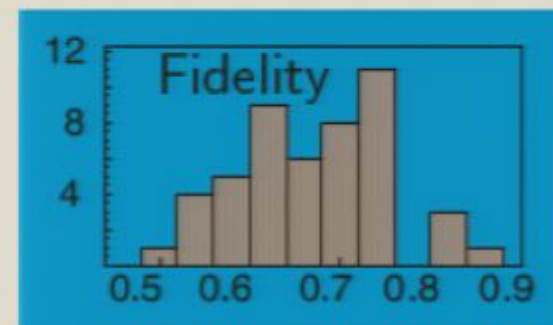
Implementation of the 5 bit code with the stabilizer $Z^2Y^3Y^4X^5$, $Z^1Y^2Y^3X^4$, $Y^2Z^3Z^4Z^5$ and $X^1Z^2X^3Z^4$, including decoding and error correction for a basis of 1 qubit errors [1].

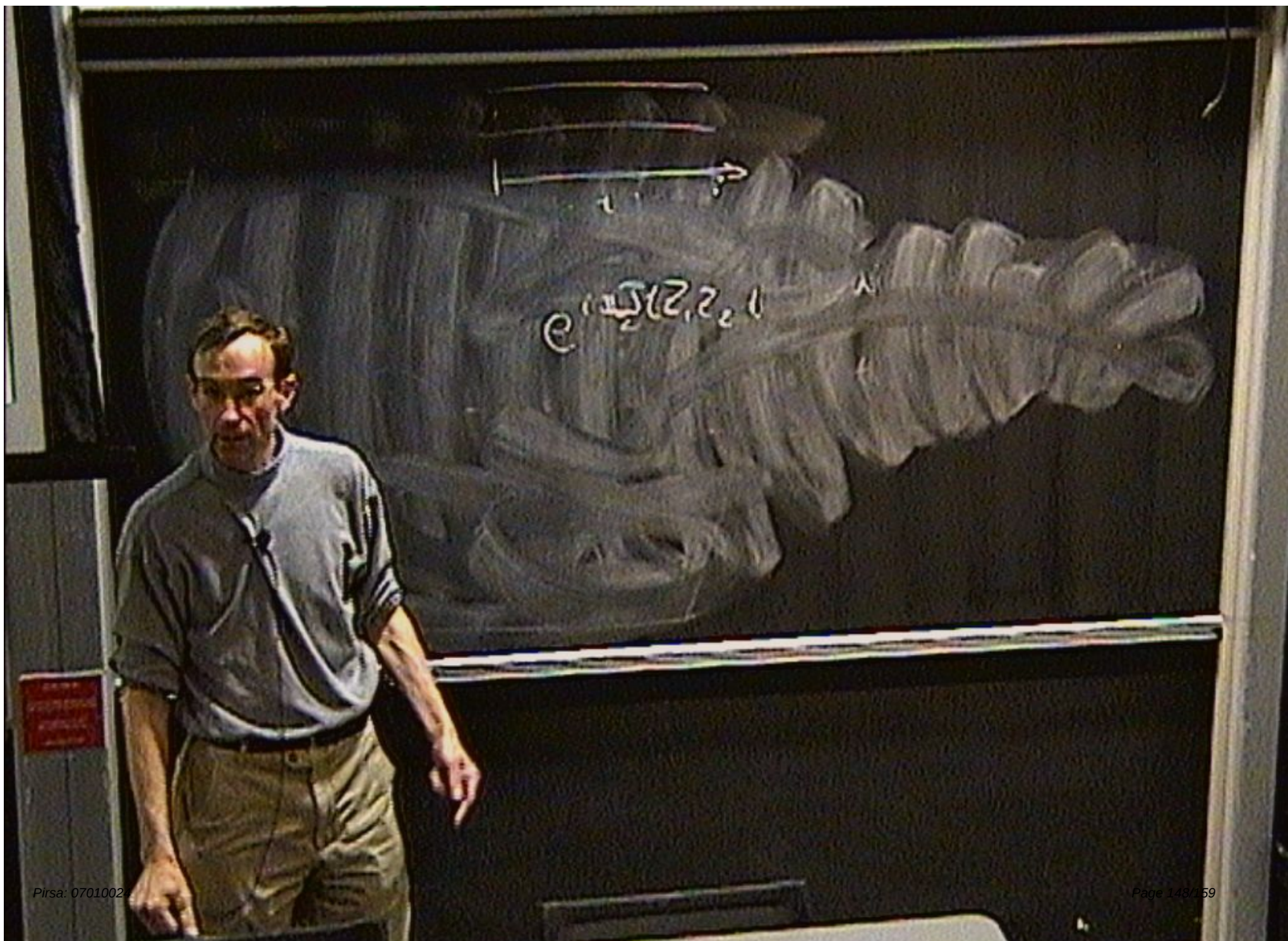


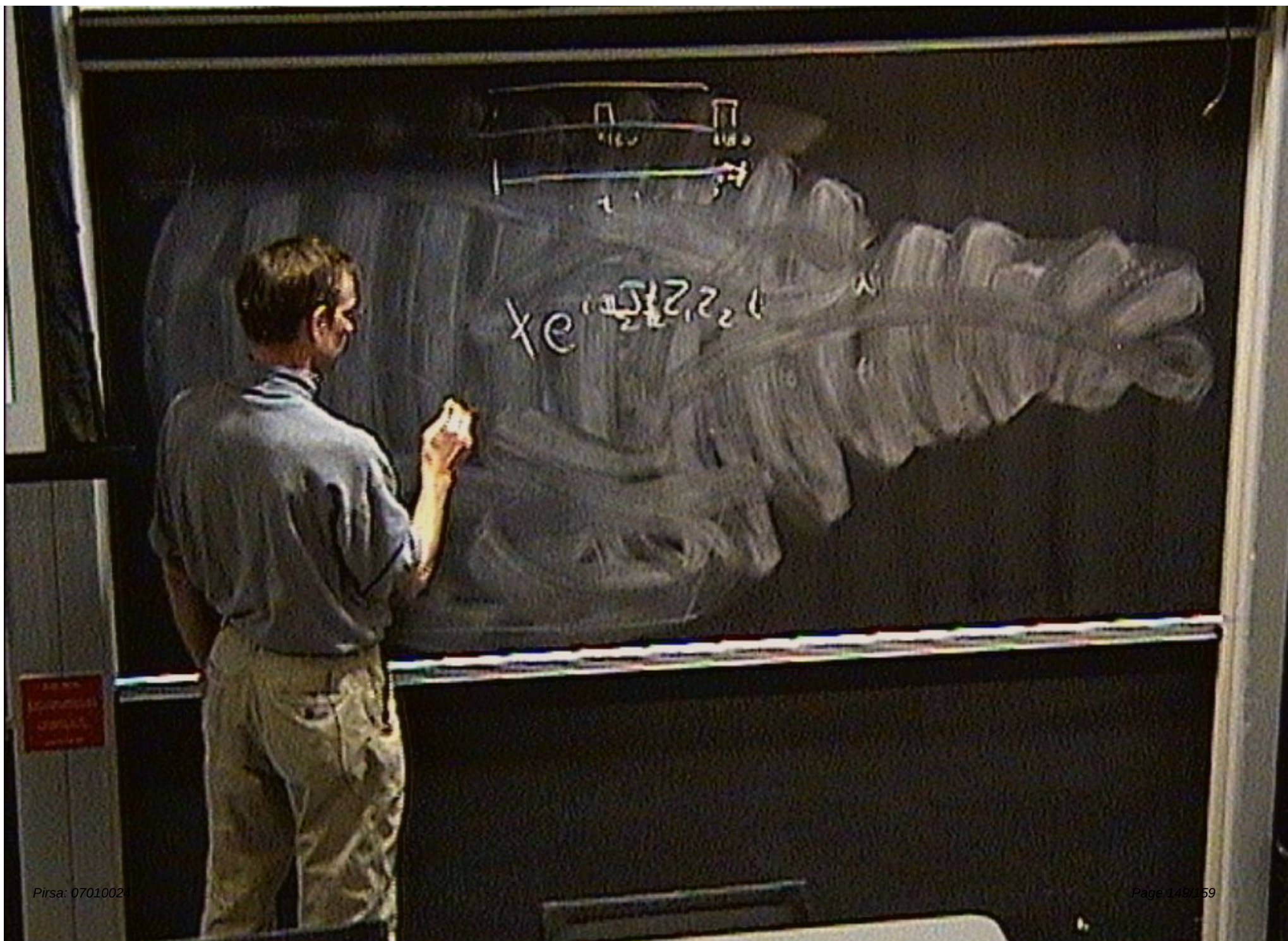
5 bit quantum error correcting code

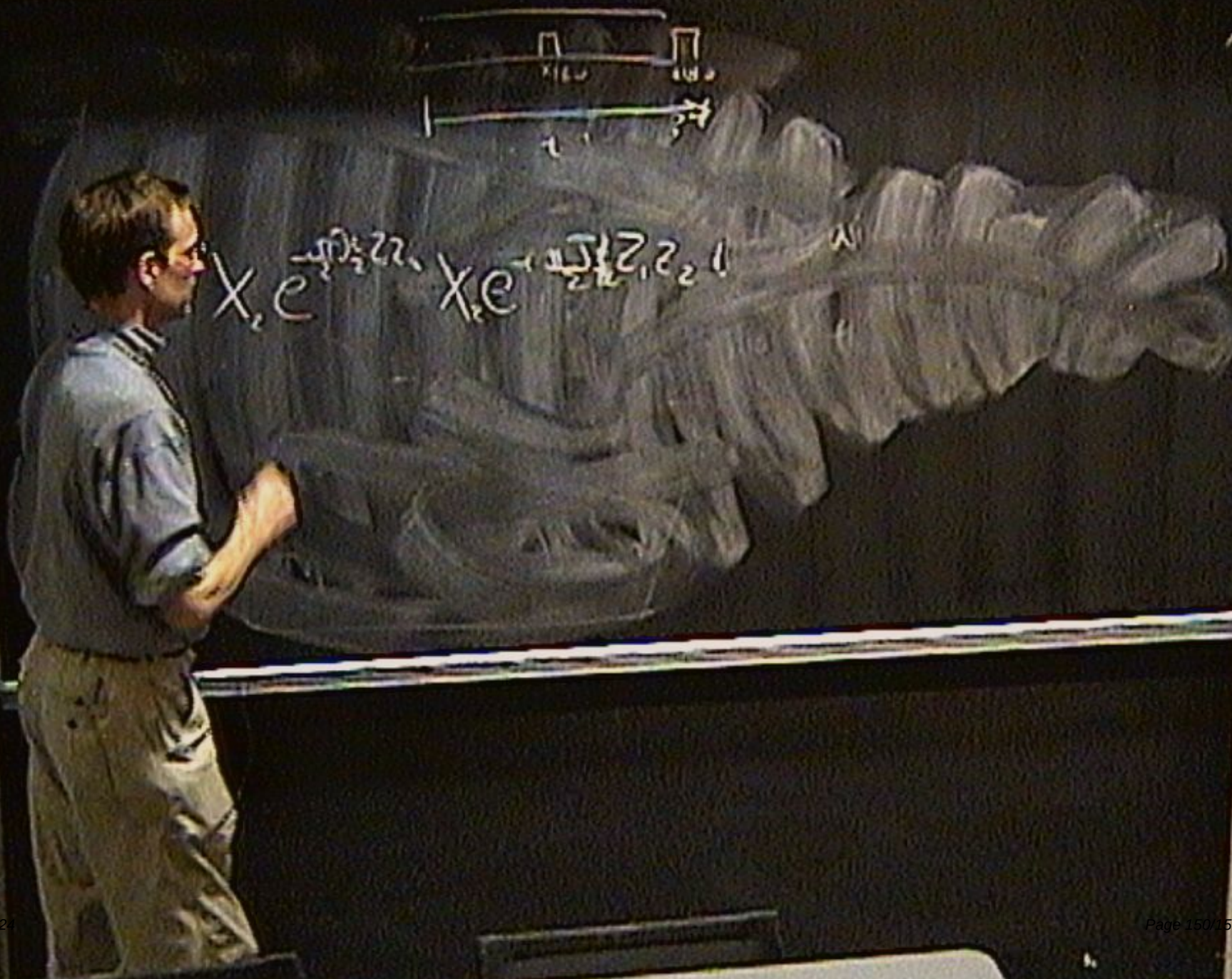


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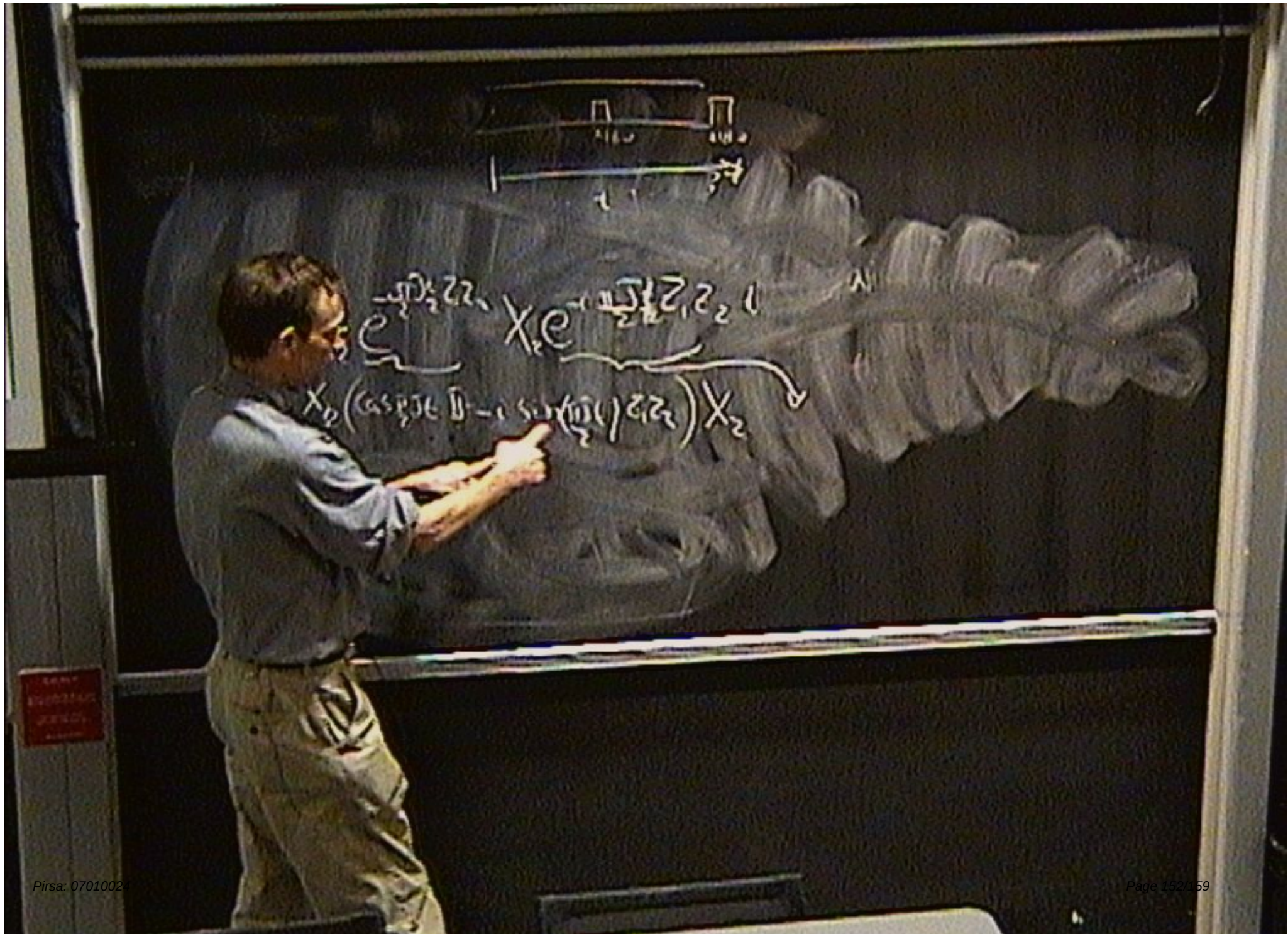


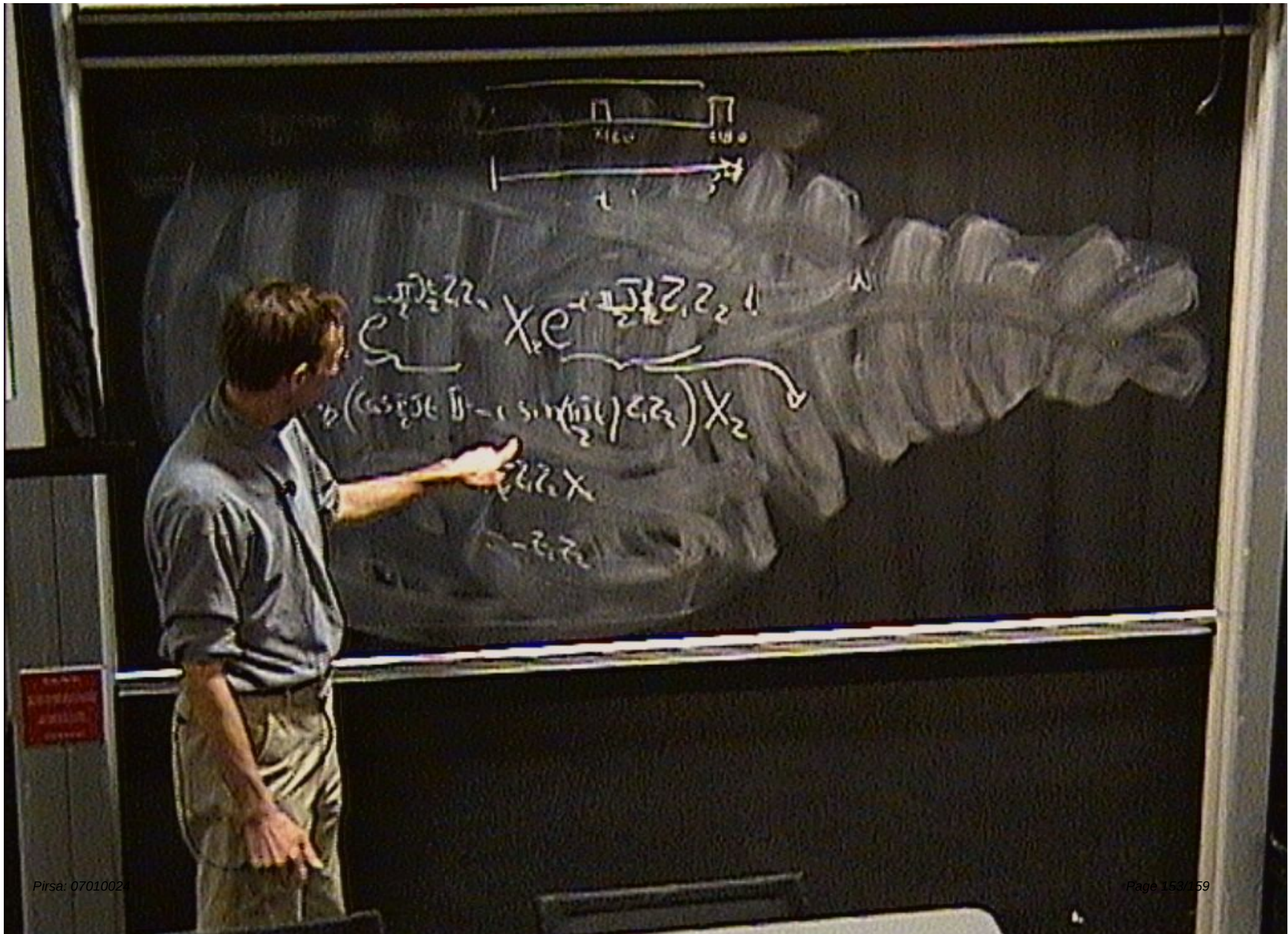




$$x_1 e^{-\frac{\pi}{2} z^{-1}} x_2 e^{-\frac{\pi}{2} z^{-1}}$$

$$x_2 (e^{-\frac{\pi}{2} z^{-1}} - 1) x_2$$





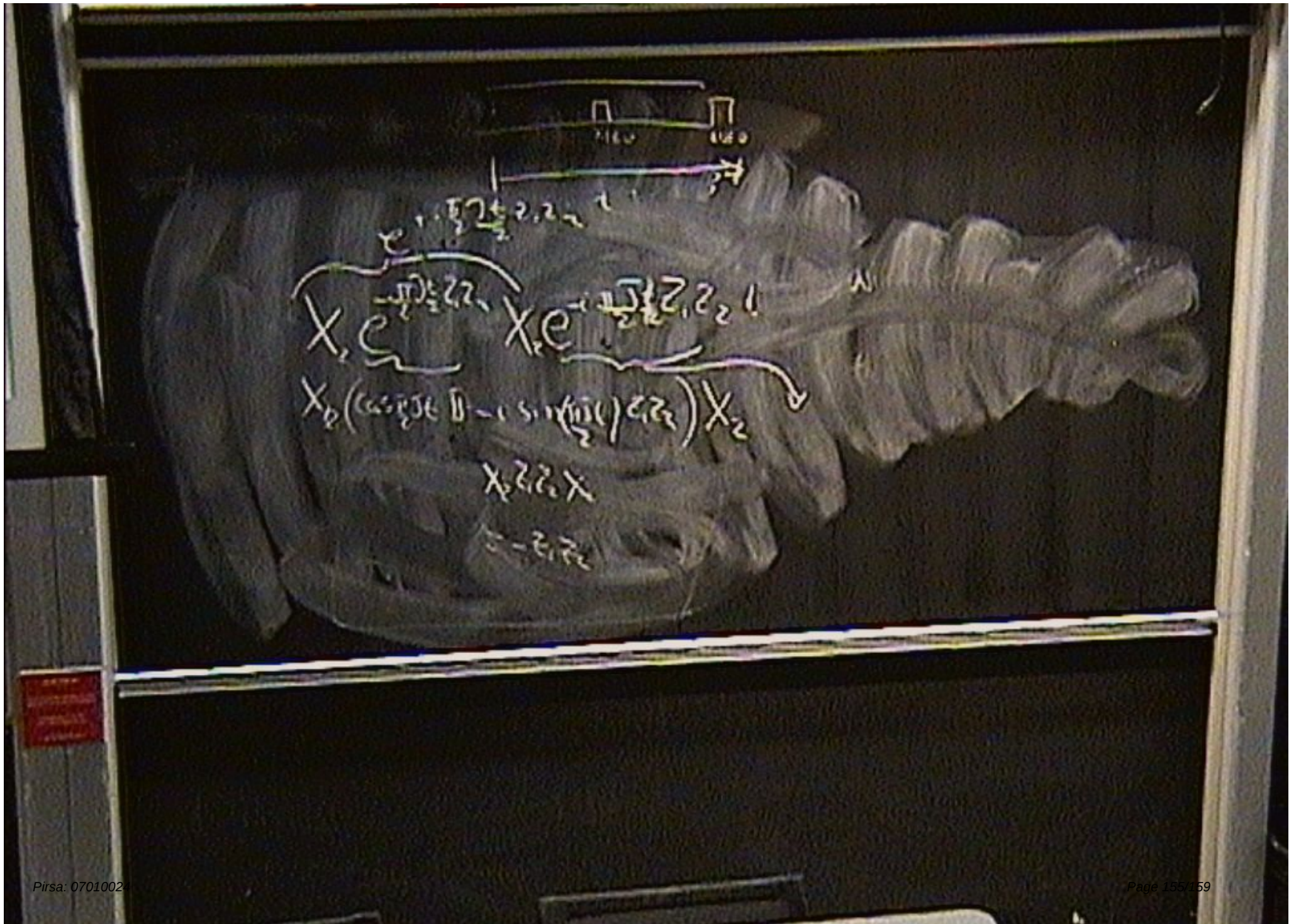
$$e^{-j\frac{\pi}{2}z_2} x_2 e^{-j\frac{\pi}{2}z_2} \quad \text{and} \quad e^{-j\frac{\pi}{2}z_2} x_2 e^{-j\frac{\pi}{2}z_2}$$

$$= (\cos(\frac{\pi}{2}z_2) - j \sin(\frac{\pi}{2}z_2)) x_2$$

$$= \cos(\frac{\pi}{2}z_2) x_2 - j \sin(\frac{\pi}{2}z_2) x_2$$



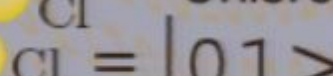
$$\begin{aligned}
 & X_1 e^{-\frac{j\omega}{2} z_2} \quad X_2 e^{-\frac{j\omega}{2} z_1, z_2} \\
 & X_2 \left(\cos\left(\frac{\omega}{2} z_1\right) - j \sin\left(\frac{\omega}{2} z_1\right) z_2 \right) X_1 \\
 & X_1 z_2 X_2 \\
 & z_1 - z_1 z_2
 \end{aligned}$$



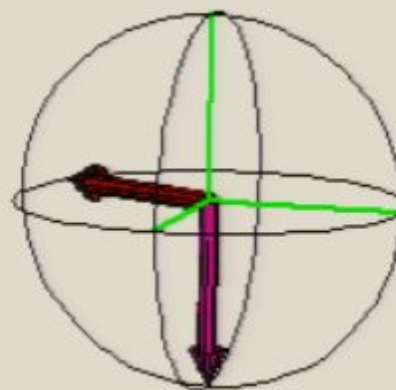
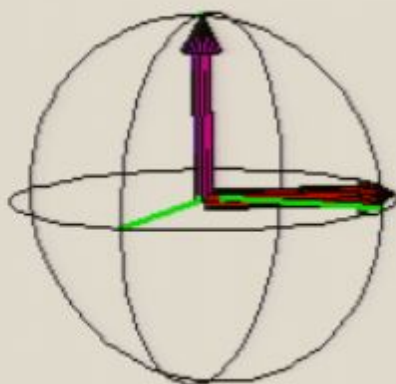
Nuclear Magnetic Resonance



Chloroform



Adobe Acrobat - Movie

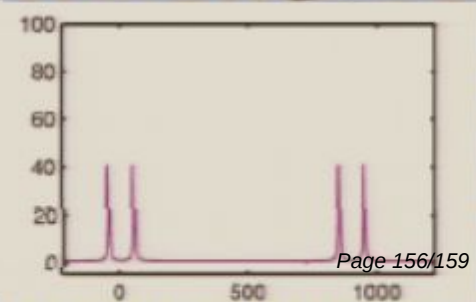
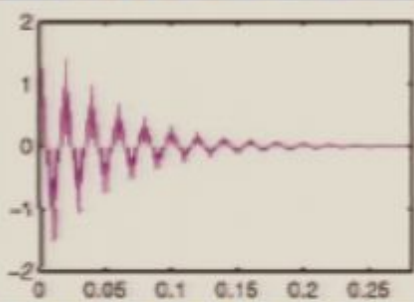


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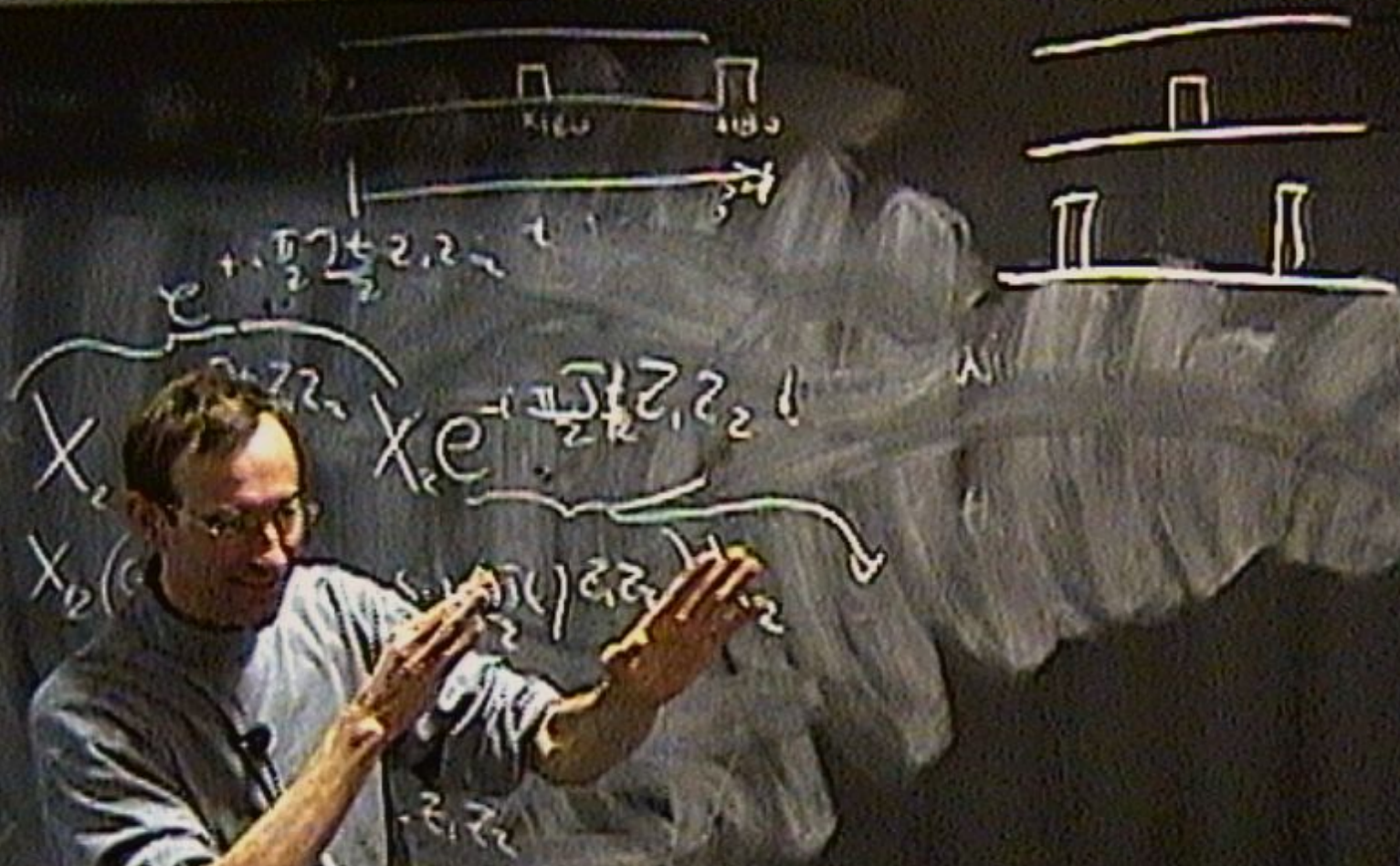


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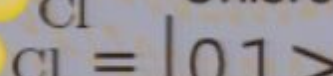




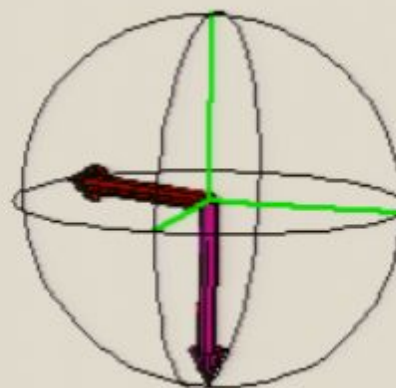
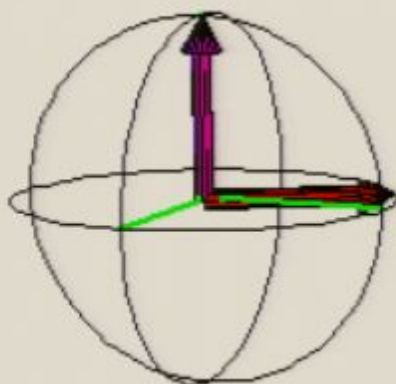
Nuclear Magnetic Resonance



Chloroform



Adobe Acrobat - Movie

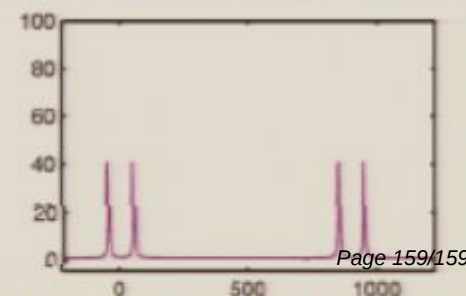
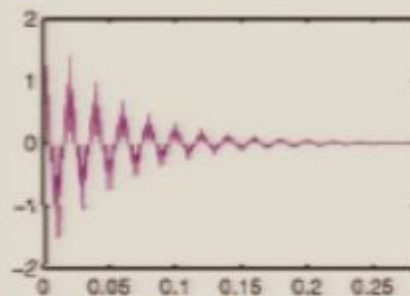


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