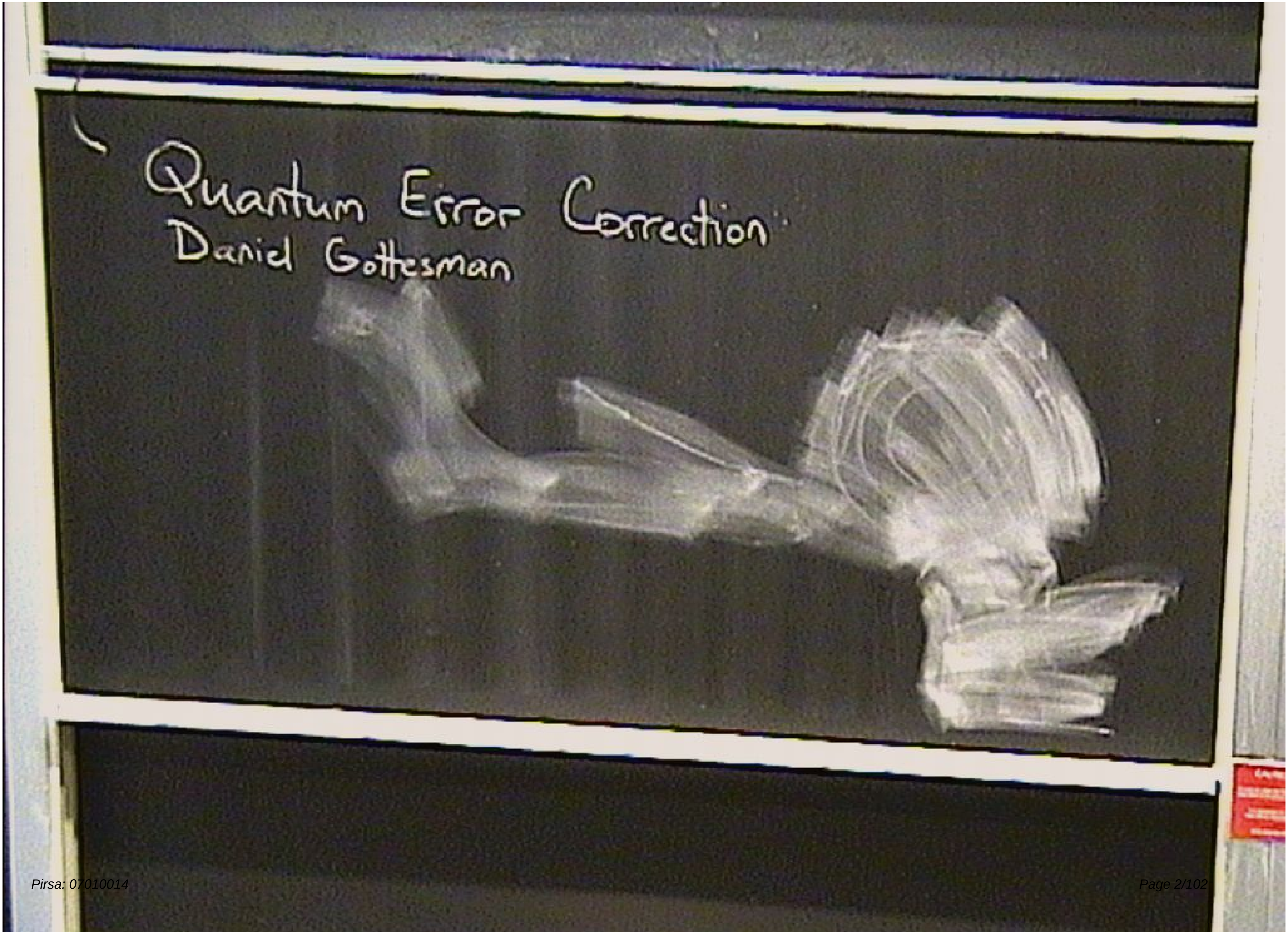
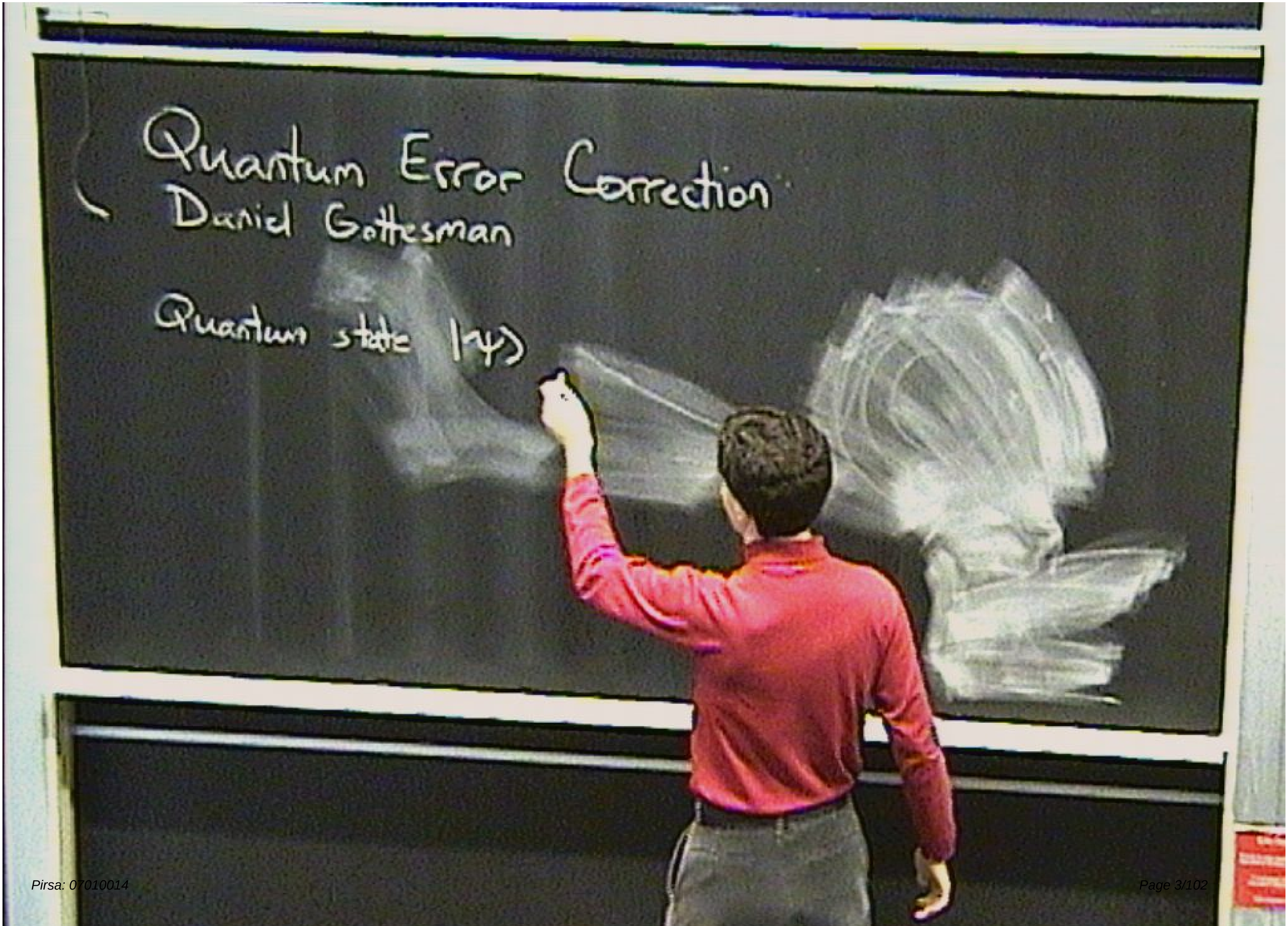
Title: Quantum Error Correction 1A

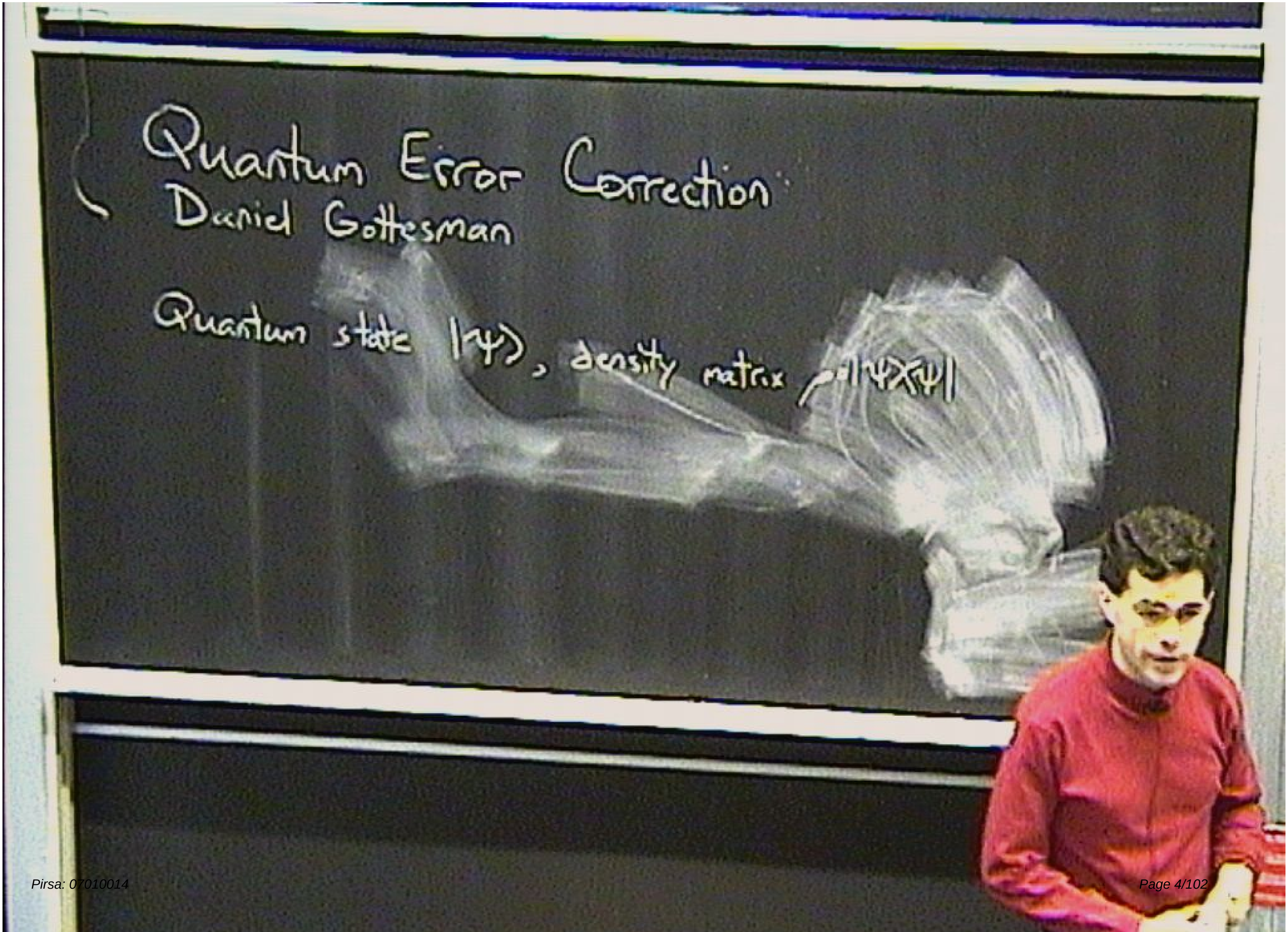
Date: Jan 09, 2007 03:30 PM

URL: http://pirsa.org/07010014

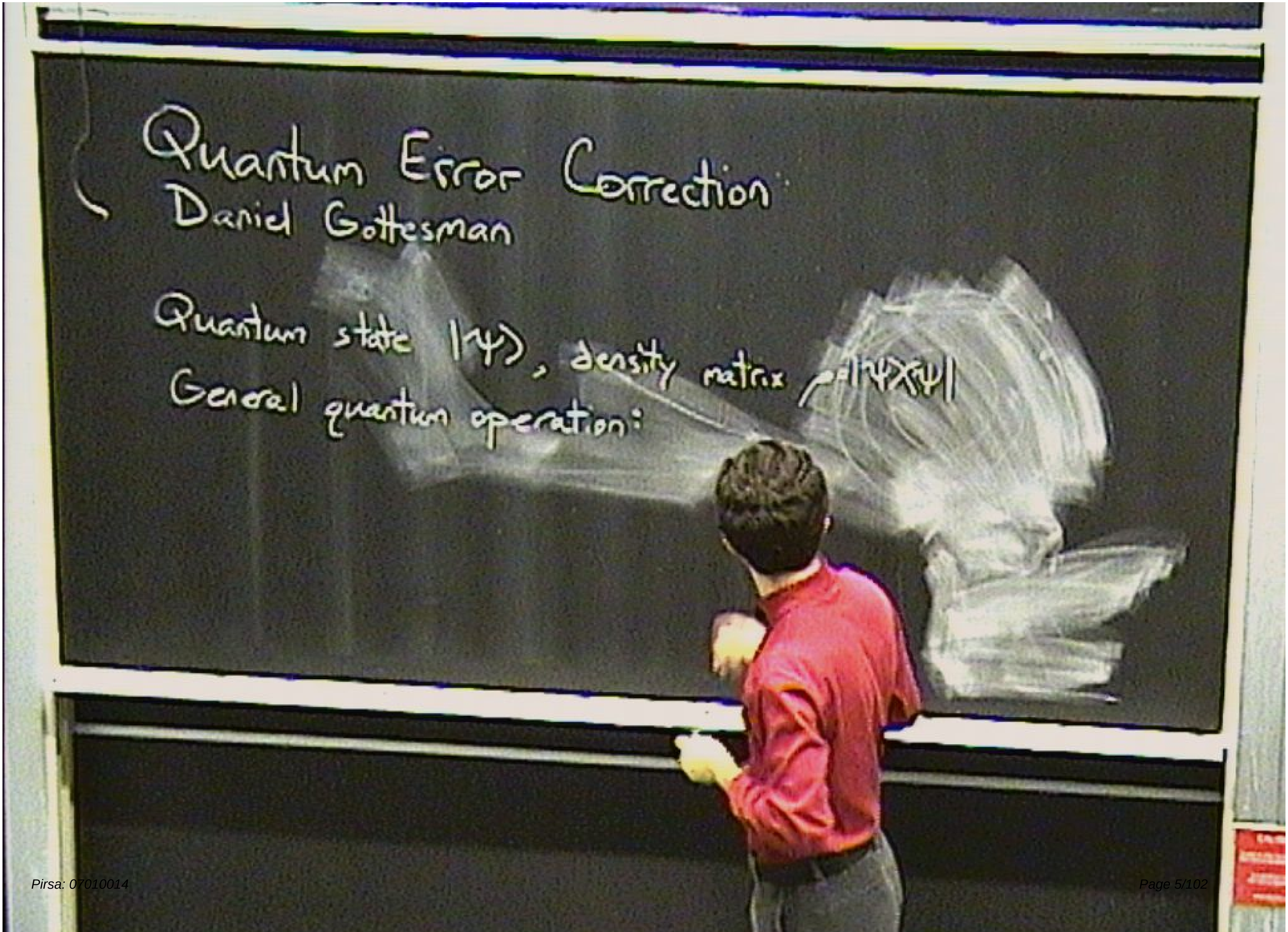
Abstract: Administrative introduction, quantum operations, examples of quantum channels, quantum code correcting bit flip errors, quantum code correcting phase errors

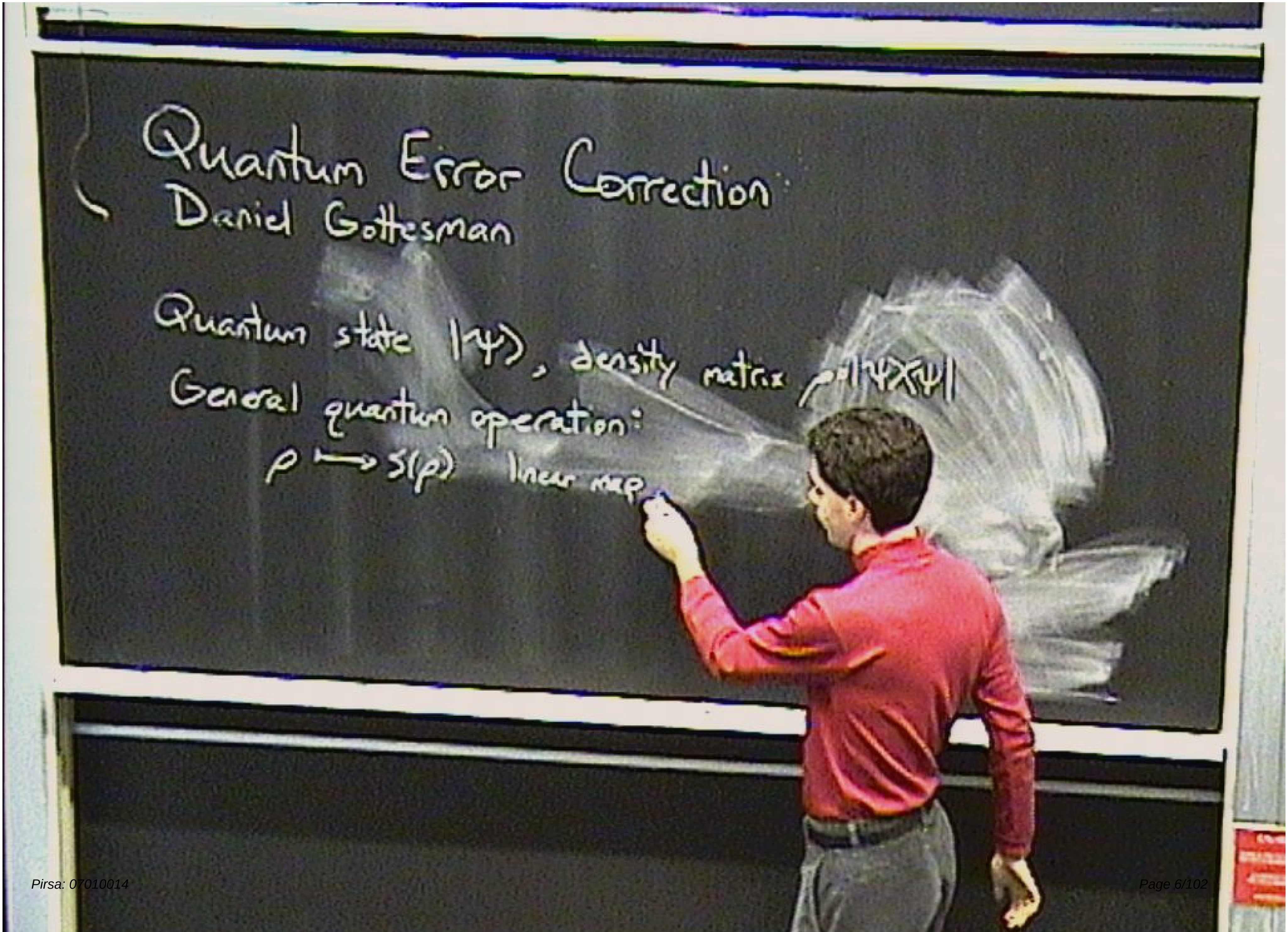

Quantum Error Correction
Daniel Gottesman


Quantum Error Correction
Daniel Gottesman
Quantum state $|\psi\rangle$



Quantum Error Correction
Daniel Gottesman
Quantum state $|\psi\rangle$, density matrix $\rho = |\psi\rangle\langle\psi|$


Quantum Error Correction
Daniel Gottesman
Quantum state $|\psi\rangle$, density matrix $\rho = |\psi\rangle\langle\psi|$
General quantum operation:


Quantum Error Correction:
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Quantum state $|\psi\rangle$, density matrix $\rho = |\psi\rangle\langle\psi|$
General quantum operation:
$\rho \longmapsto S(\rho)$ linear map

Quantum Error Correction:

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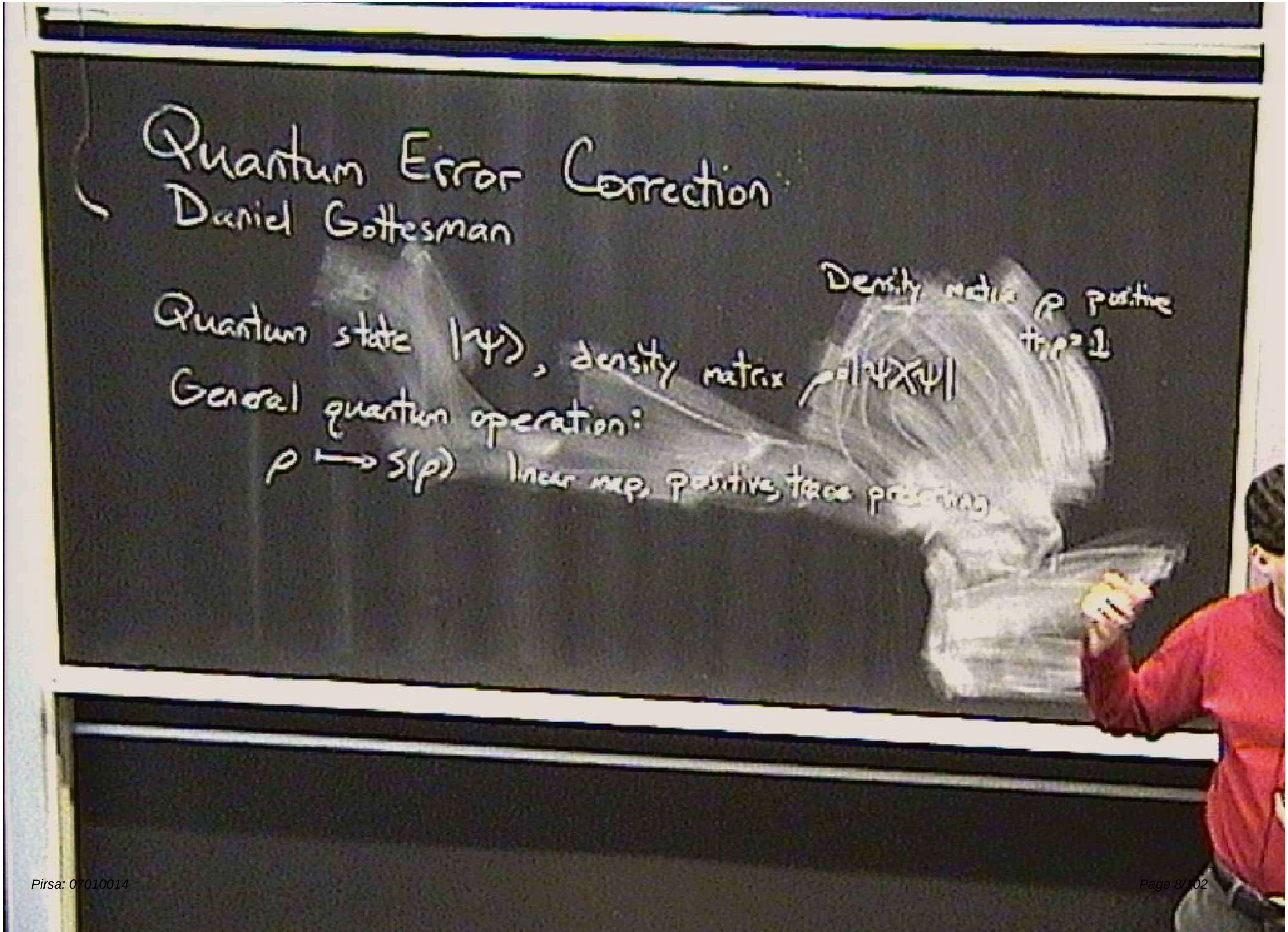
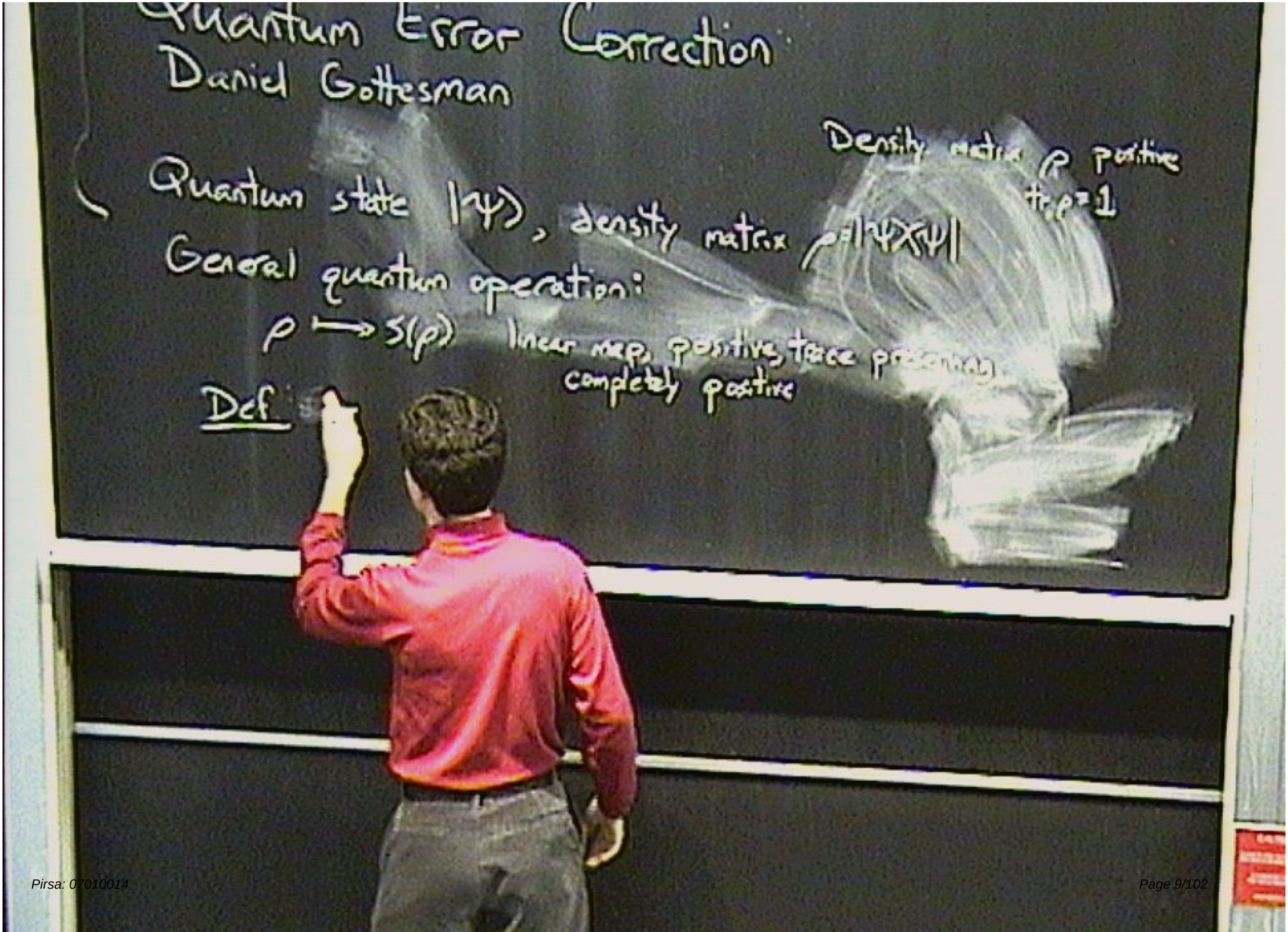
Quantum state $|\psi\rangle$, density matrix $\rho = |\psi\rangle\langle\psi|$

General quantum operation:

$\rho \longmapsto S(\rho)$ linear map, positive, trace preserving



Quantum Error Correction
Daniel Gottesman
Density matrix ρ positive
tr ρ = 1
Quantum state |ψ⟩, density matrix ρ = |ψ⟩⟨ψ|
General quantum operation:
ρ ⟼ S(ρ) linear map, positive, trace


Quantum Error Correction
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Density matrix ρ positive
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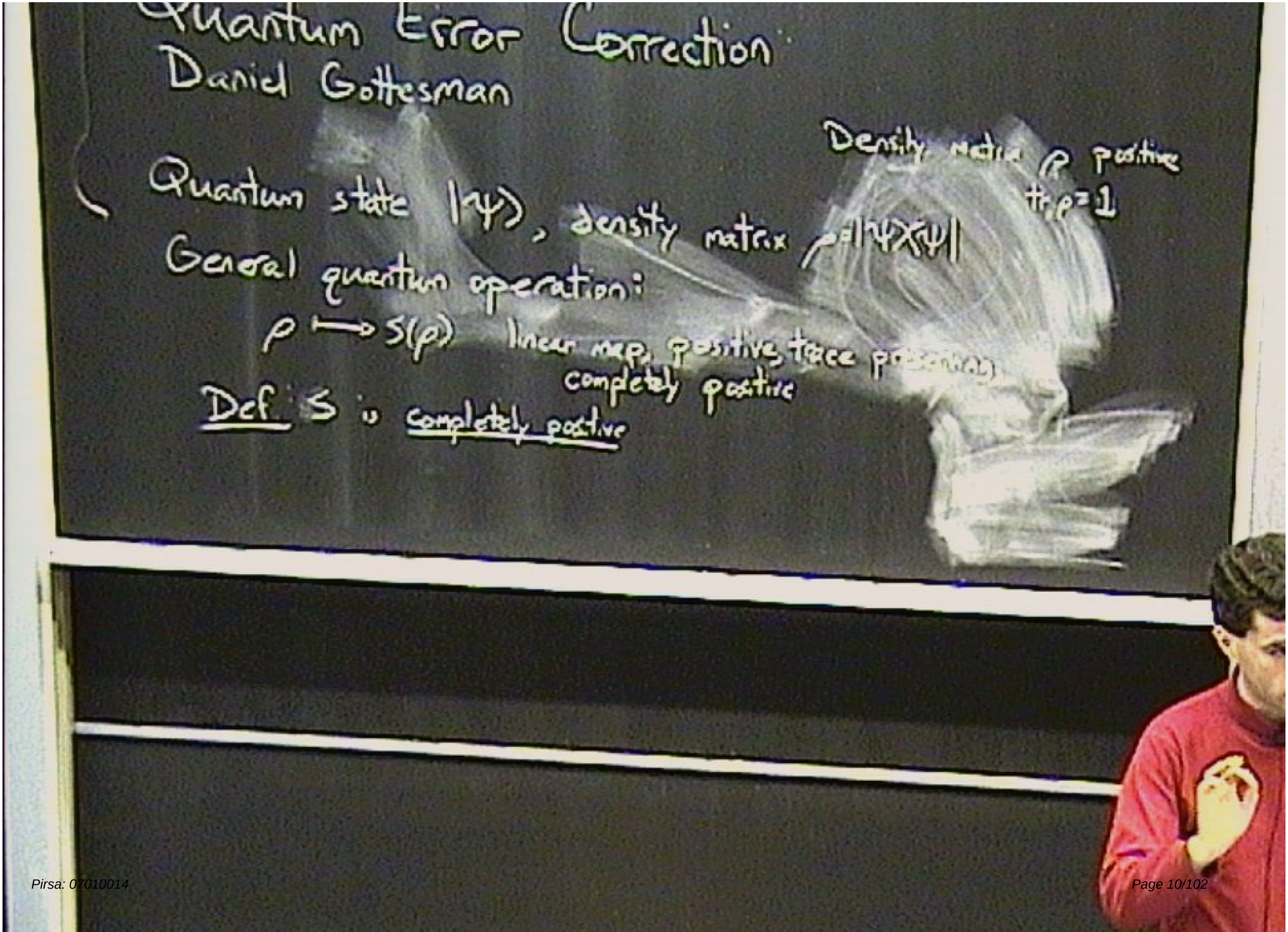
Quantum state |ψ⟩, density matrix ρ=|ψ⟩⟨ψ|
General quantum operation:
ρ ⟶ S(ρ) linear map, positive, trace preserving,
completely positive
Def


Quantum Error Correction:
David Gottesman
Density matrix ρ positive
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General quantum operation:
ρ ⟼ S(ρ) linear map, positive, trace preserving
completely positive
Def. S is completely positive

Quantum Error Correction

Daniel Gottesman

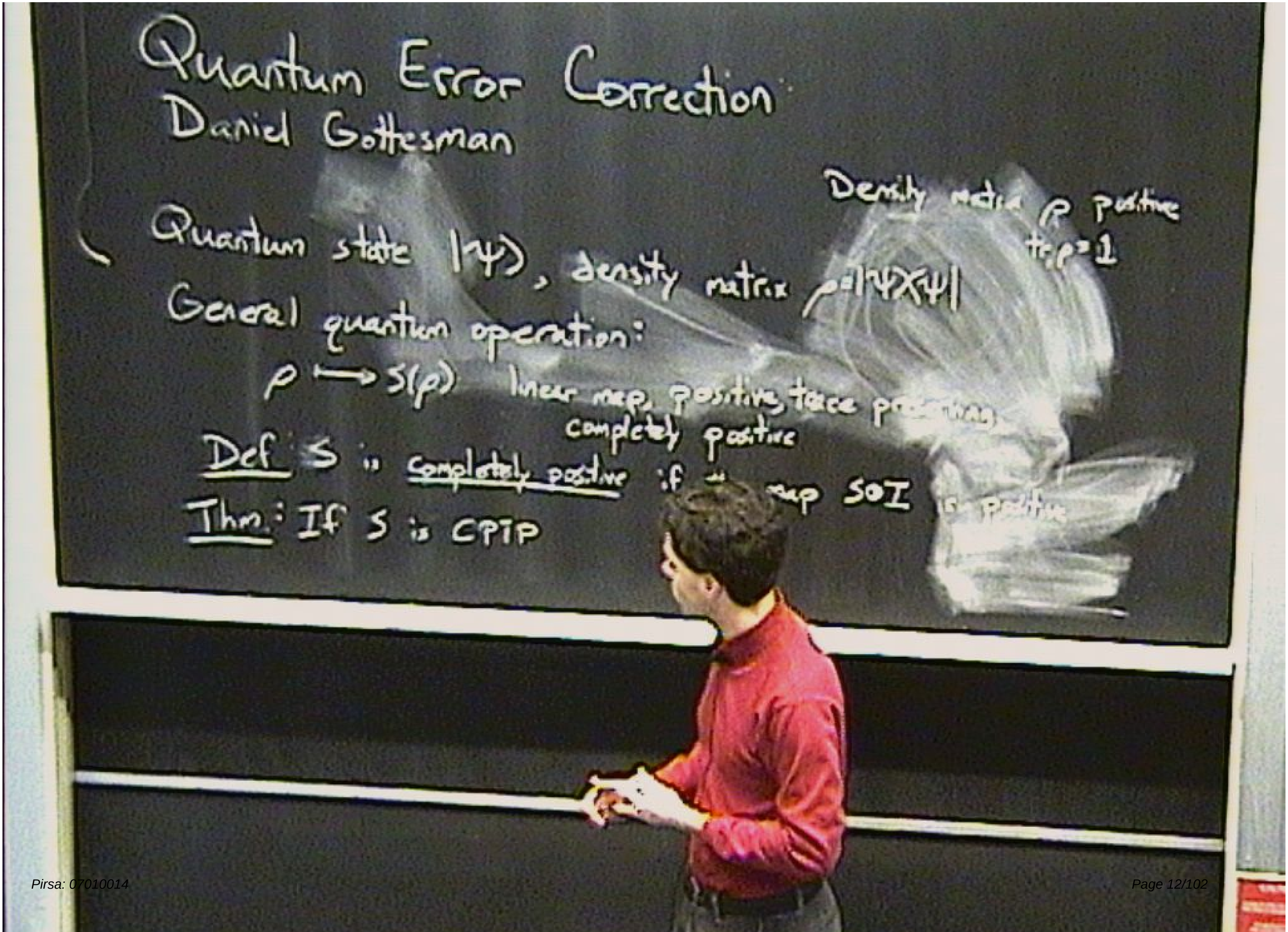
Density matrix ρ positive
$\mathrm{tr}\,\rho = 1$

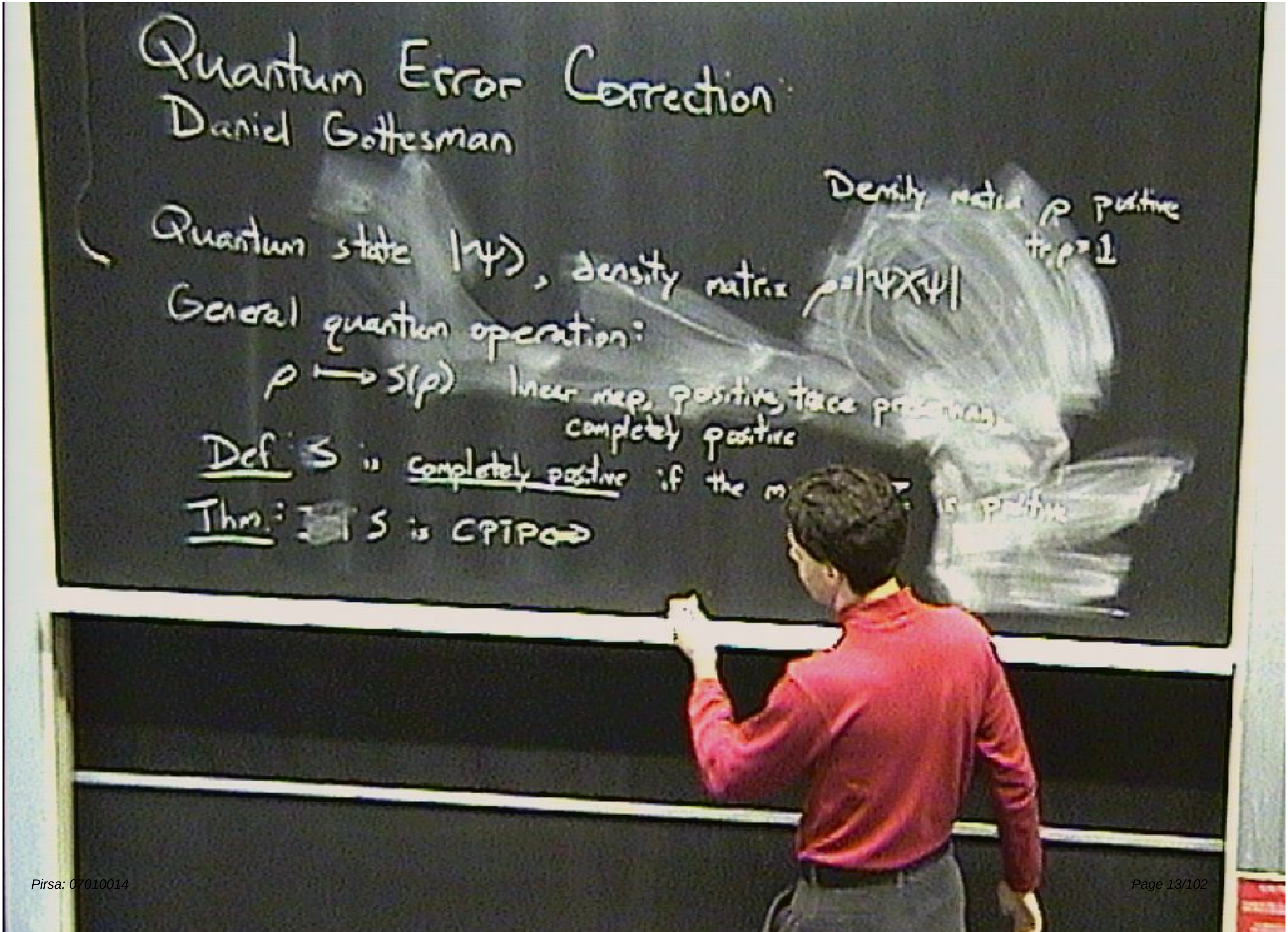
Quantum state $|\psi\rangle$, density matrix $\rho = |\psi\rangle\langle\psi|$

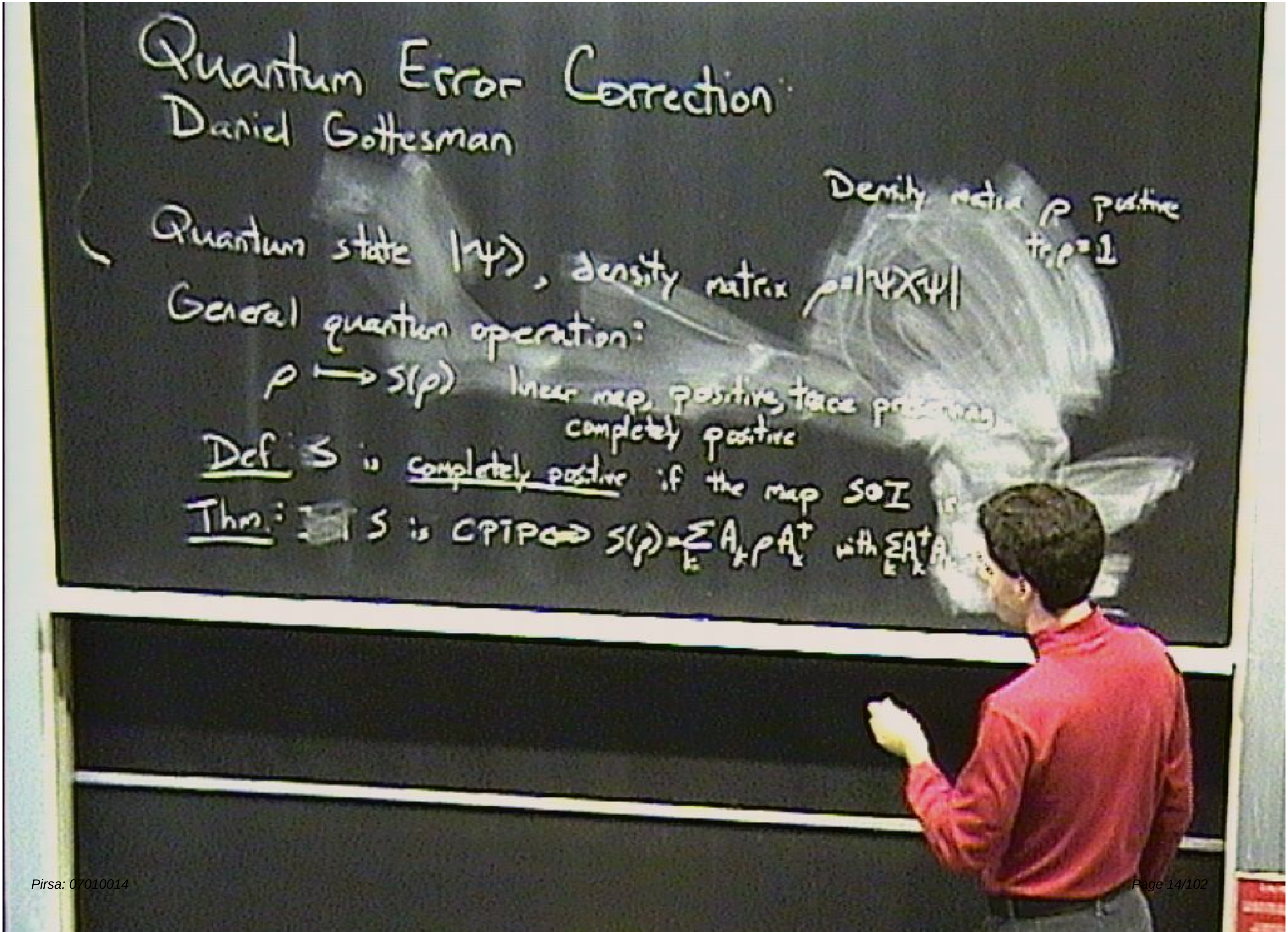
General quantum operation:

$$\rho \longmapsto S(\rho)$$ linear map, positive, trace preserving
completely positive

<u>Def</u>: S is <u>completely positive</u> if the map $S \otimes I$ is positive


Quantum Error Correction
Daniel Gottesman
Density matrix ρ positive
tr ρ = 1
Quantum state |ψ⟩, density matrix ρ=|ψ⟩⟨ψ|
General quantum operation:
ρ ⟼ S(ρ) linear map, positive, trace preserving
completely positive
Def: S is completely positive if
map S⊗I is positive
Thm: If S is CPTP


Quantum Error Correction
Daniel Gottesman
Density matrix ρ positive
tr ρ = 1
Quantum state |ψ⟩, density matrix ρ = |ψ⟩⟨ψ|
General quantum operation:
ρ ⟼ S(ρ) linear map, positive, trace preserving
completely positive
Def: S is completely positive if the m
is positive
Thm: S is CPTP ⟺


Quantum Error Correction
Daniel Gottesman
Density matrix ρ positive
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General quantum operation:
ρ ⟼ S(ρ) linear map, positive, trace preserving
completely positive
Def: S is completely positive if the map S⊗I
Thm: S is CPTP ⟺ S(ρ)=Σ_k A_k ρ A_k† with Σ_k A_k† A_k

Quantum Error Correction

Daniel Gottesman

Density matrix ρ positive

$\mathrm{tr}\,\rho = 1$

Quantum state $|\psi\rangle$, density matrix $\rho = |\psi\rangle\langle\psi|$

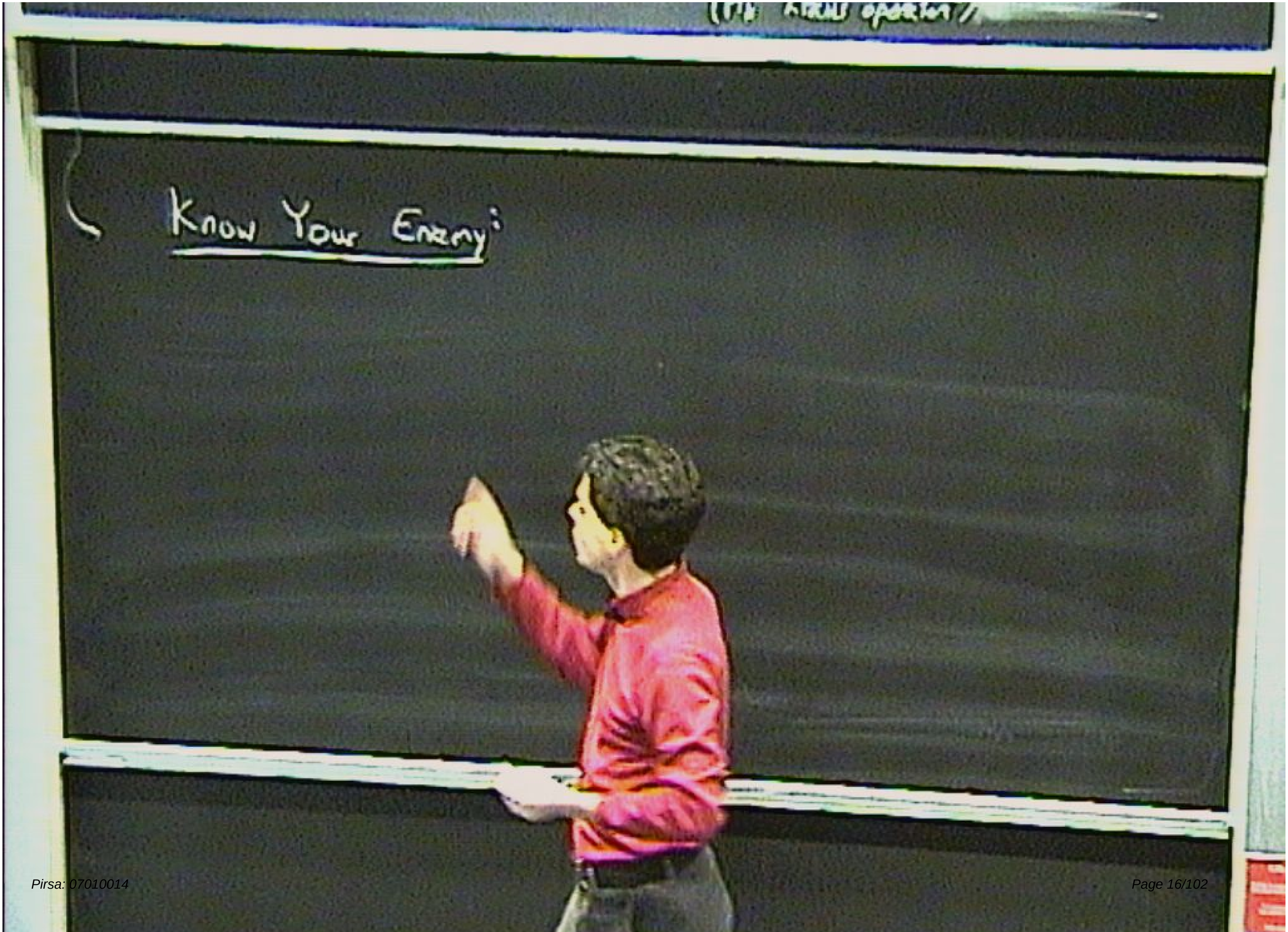
General quantum operation:

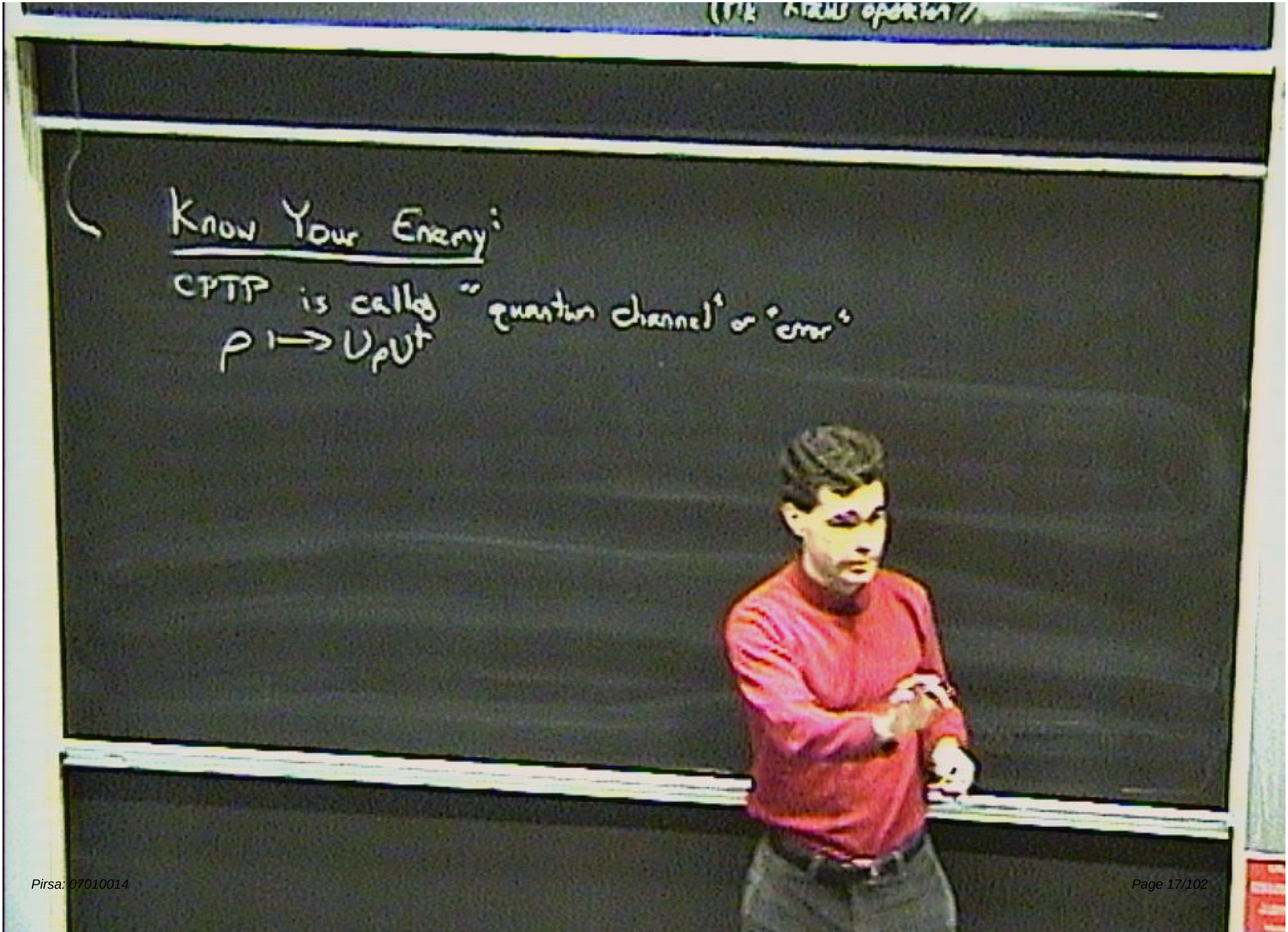
$$\rho \longmapsto S(\rho)$$ linear map, positive, trace preserving, completely positive

<u>Def</u>: S is <u>completely positive</u> if the map $S \otimes I$ is positive

<u>Thm</u>: S is CPTP $\Longleftrightarrow$ $S(\rho) = \sum_k A_k \rho A_k^\dagger$ with $\sum_k A_k^\dagger A_k = I$

(A_k "Kraus operators")


Know Your Enemy:

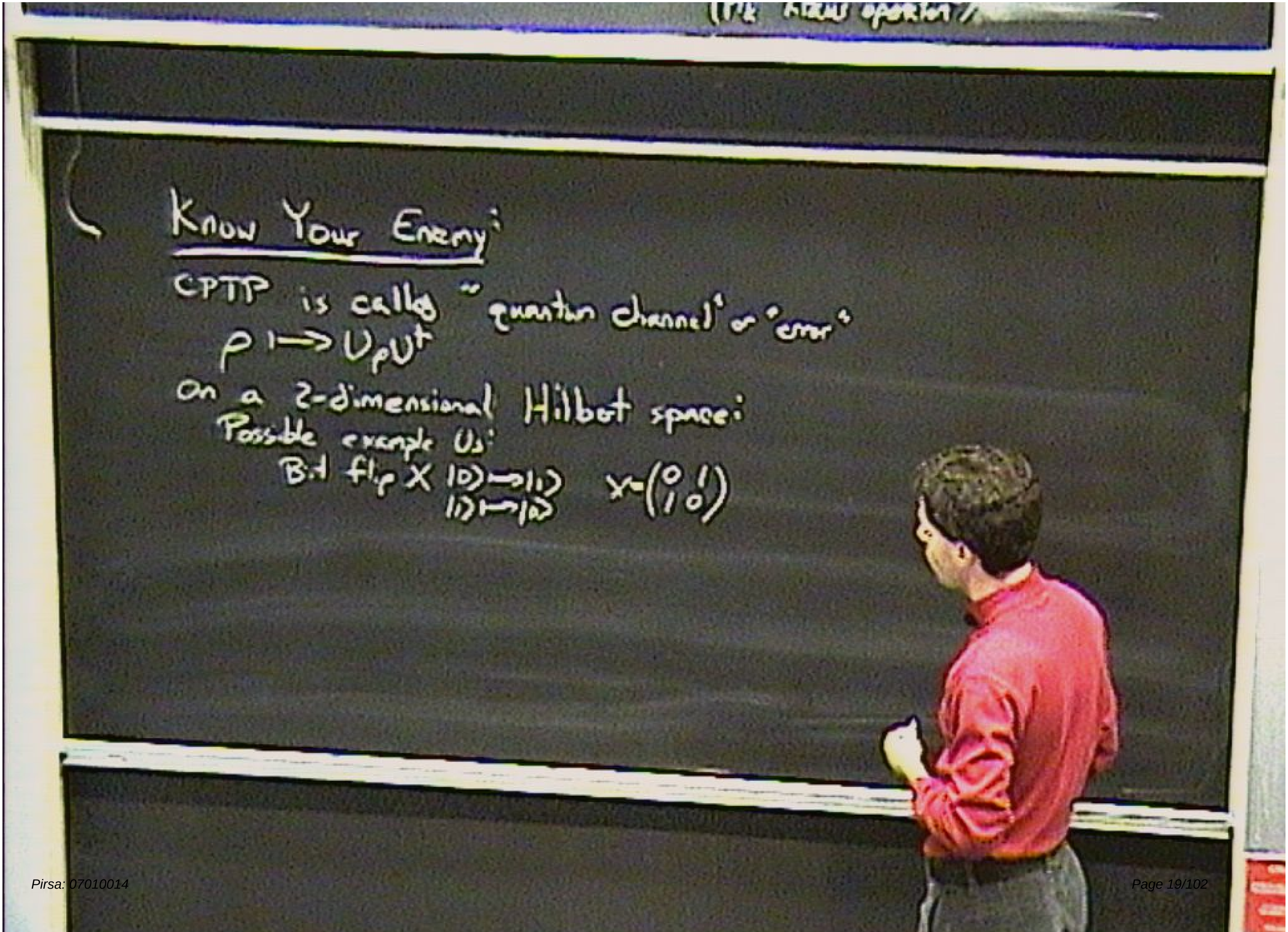

Know Your Enemy:
CPTP is called "quantum channel" or "error"
ρ ↦ UρU†

(the noise operator)

Know Your Enemy:

CPTP is called "quantum channel" or "error"

$\rho \longmapsto U \rho U^\dagger$


Know Your Enemy:
CPTP is called "quantum channel" or "error"
$\rho \mapsto U\rho U^{\dagger}$
On a 2-dimensional Hilbert space:
Possible example Us:
Bit flip X $|0\rangle \mapsto |1\rangle$ $|1\rangle \mapsto |0\rangle$ $X=\begin{pmatrix}0&1\\1&0\end{pmatrix}$

Know Your Enemy:

CPTP is called "quantum channel" or "error"

$\rho \longmapsto U\rho U^\dagger$

On a 2-dimensional Hilbert space:

Possible example U's:

Bit flip $X: |0\rangle \mapsto |1\rangle$, $|1\rangle \mapsto |0\rangle$ $\quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Phase flip $Z: |0\rangle \mapsto |0\rangle$, $|1\rangle \mapsto -|1\rangle$

(The [illegible] operator)

Know Your Enemy:

CPTP is called "quantum channel" or "error"

$\rho \mapsto U \rho U^\dagger$

On a 2-dimensional Hilbert space:

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No error $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(i.e. Kraus operator)

Know Your Enemy:

CPTP is called "quantum channel" or "error"

$\rho \longmapsto U \rho U^\dagger$

On a 2-dimensional Hilbert space:

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$X^2 = Y^2 = Z^2$

$XY = -YX$, $XZ = -ZX$, $YZ = -ZY$

Know Your Enemy:

CPTP is called "quantum channel" or "error"

$\rho \longmapsto U \rho U^{\dagger}$

On a 2-dimensional Hilbert space:

Possible example Us:

Bit flip $X: |0\rangle \mapsto |1\rangle$, $|1\rangle \mapsto |0\rangle$ $\quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

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Phase rotation $R_\theta = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

Know Your Enemy:

CPTP is called "quantum channel" or "error"

$\rho \mapsto U \rho U^\dagger$

On a 2-dimensional Hilbert space:

Possible example U's:

Bit flip $X: |0\rangle \mapsto |1\rangle$, $|1\rangle \mapsto |0\rangle$ $\quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

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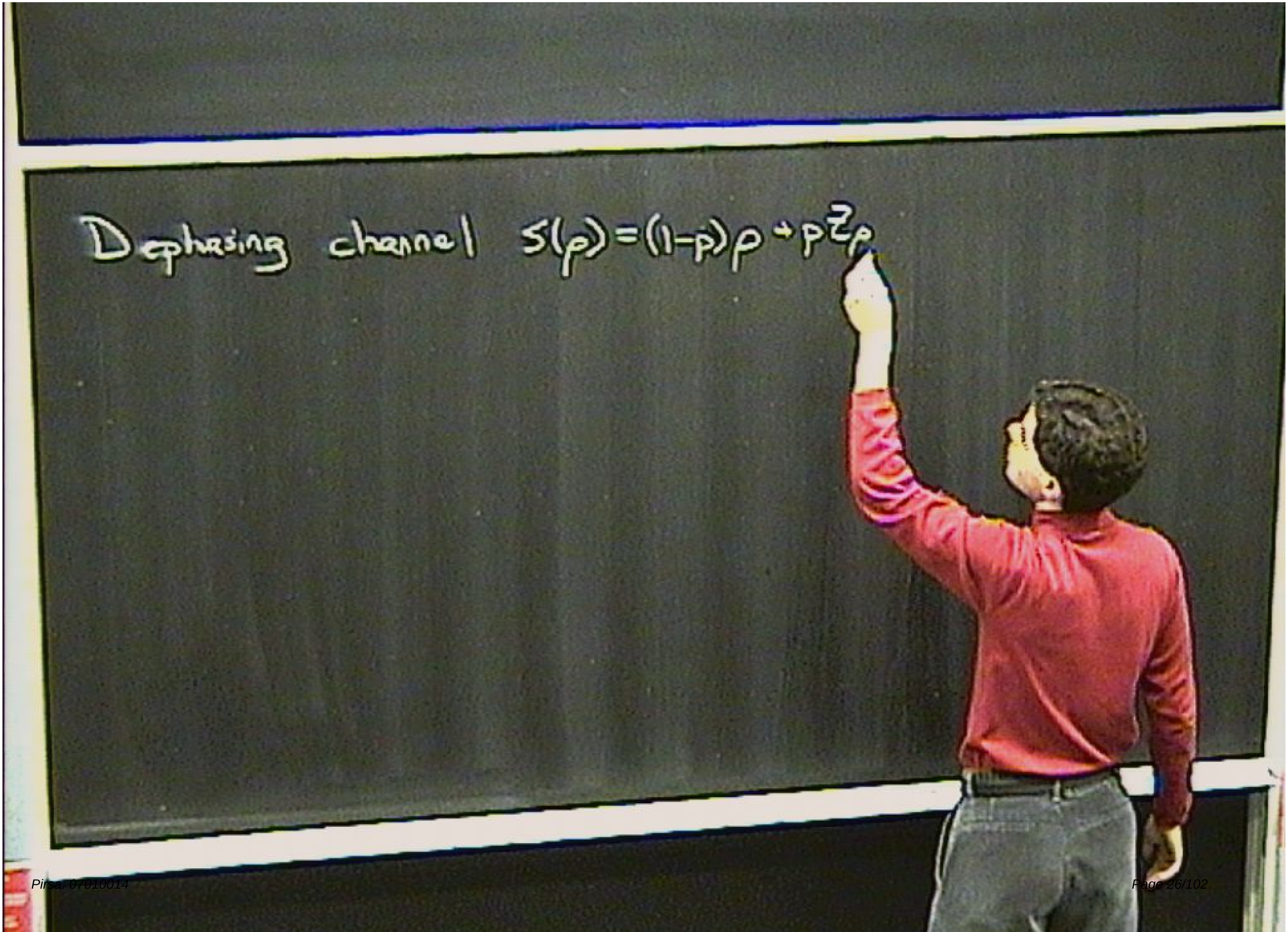
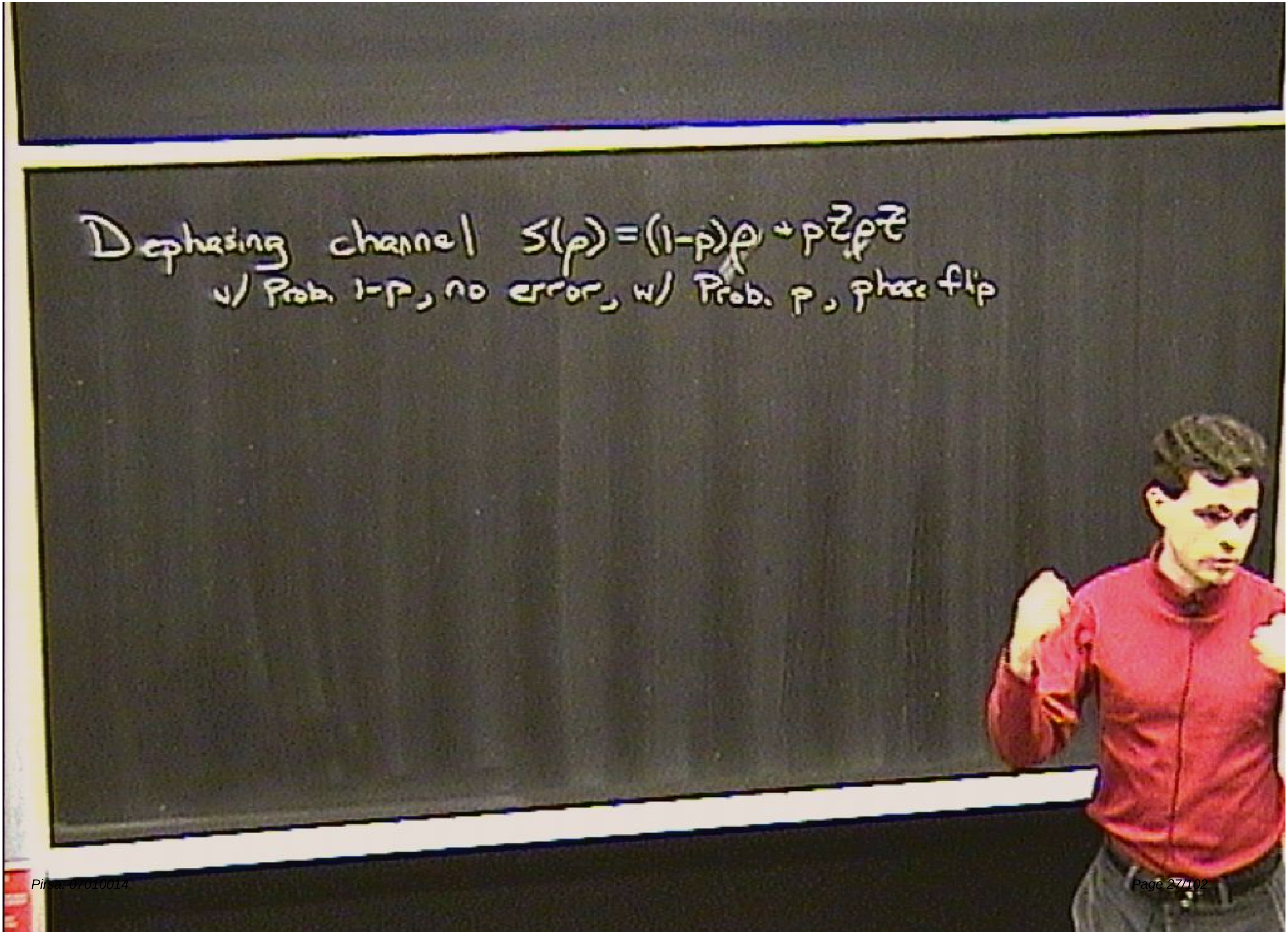
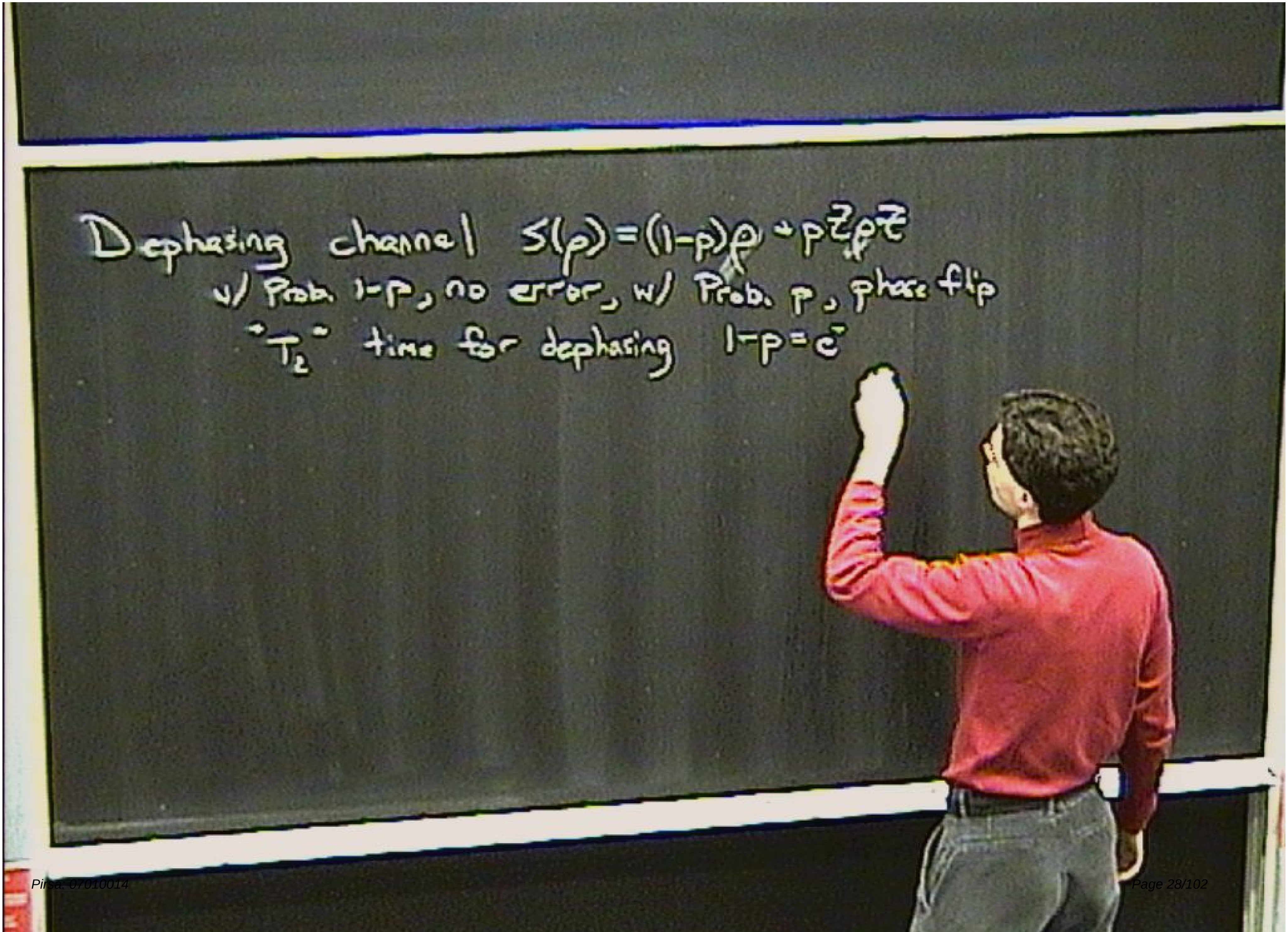
$X^2 = Y^2 = Z^2$ $\quad XY = -YX,\ XZ = -ZX,\ YZ = -ZY$

Phase rotation $R_\theta = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{-i\theta} \begin{pmatrix} 1 & 0 \\ 0 & e^{2i\theta} \end{pmatrix}$


Dephasing channel $S(\rho)=(1-p)\rho+pZ\rho$


Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$
w/ Prob. 1-p, no error, w/ Prob. p, phase flip


Dephasing channel S(ρ) = (1-p)ρ + pZρZ
w/ Prob. 1-p, no error, w/ Prob. p, phase flip
"T2" time for dephasing 1-p = e

Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$

w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-Tt}$

Know Your Enemy:

CPTP is called "quantum channel" or "error"

$$\rho \longmapsto U \rho U^\dagger$$

On a 2-dimensional Hilbert space:

Possible example U's:

I, X, Y, Z "Pauli operators"

- Bit flip $X: |0\rangle \mapsto |1\rangle$, $|1\rangle \mapsto |0\rangle$ $\quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- No error $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- $Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Phase flip $Z: |0\rangle \mapsto |0\rangle$, $|1\rangle \mapsto -|1\rangle$ $\quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

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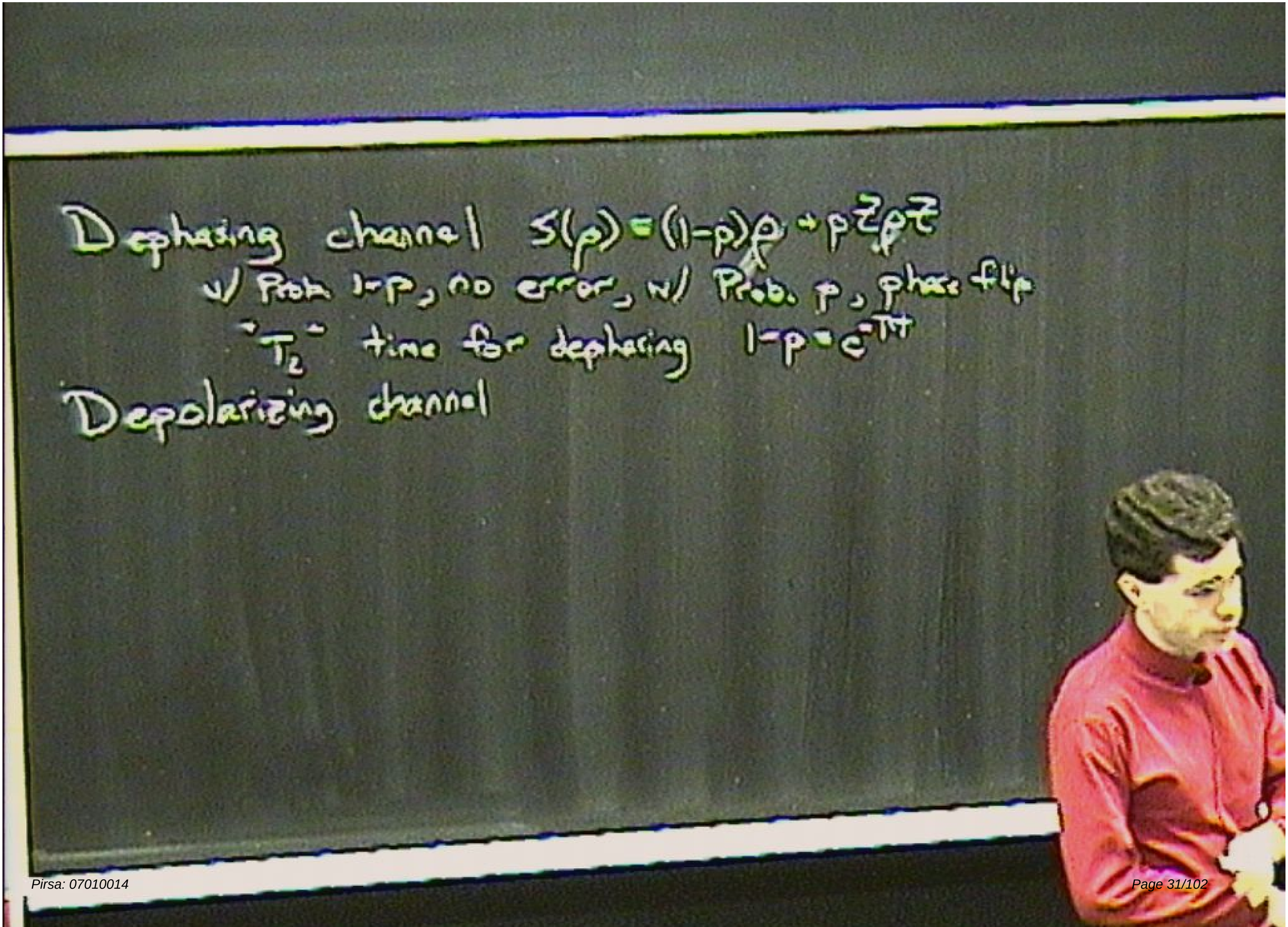
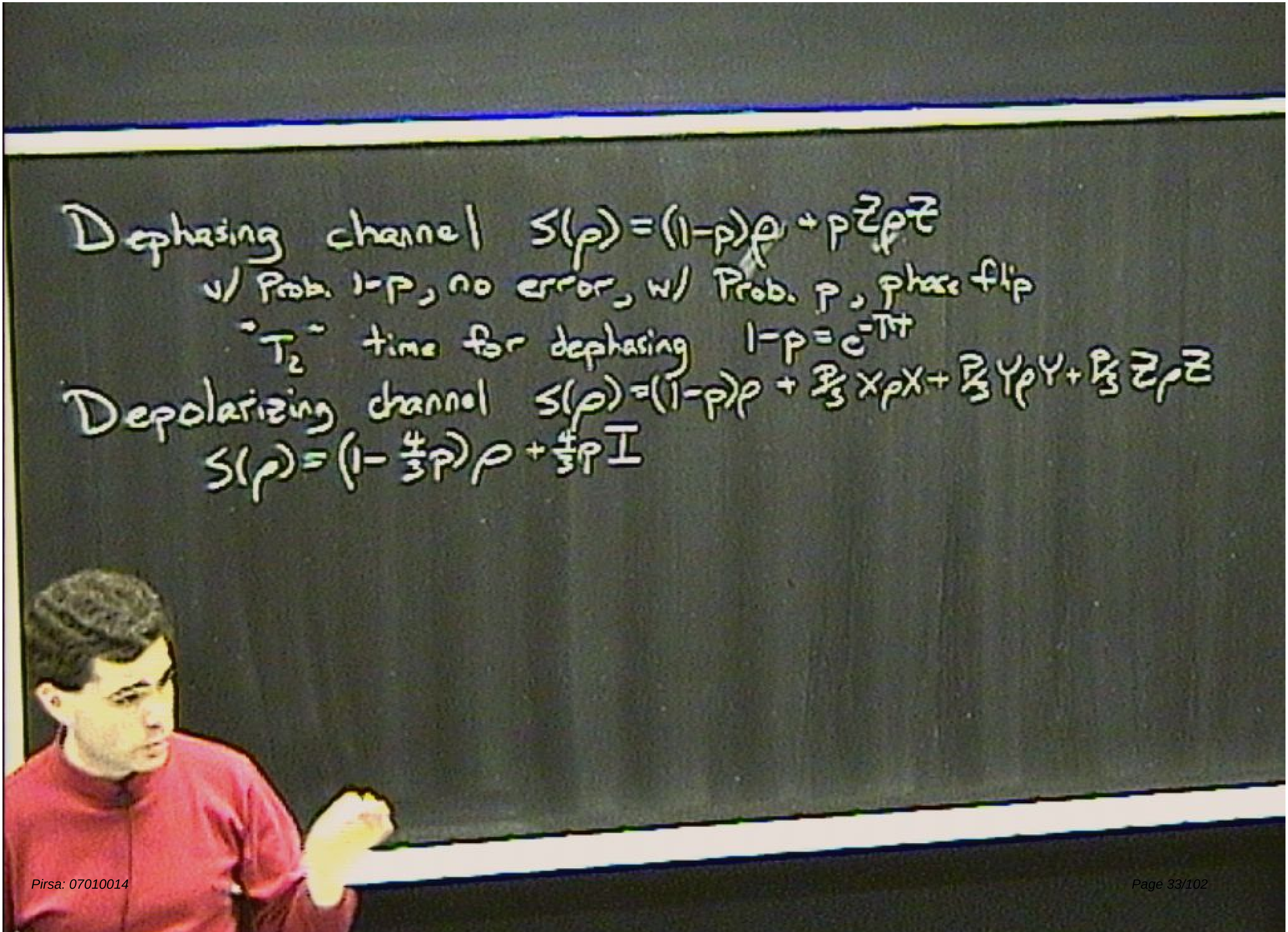

Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$
w/ Prob. $1-p$, no error, w/ Prob. p, phase flip
"T_2" time for dephasing $1-p = e^{-t/T}$
Depolarizing channel

Dephasing channel $S(\rho) = (1-p)\rho + p Z \rho Z$

w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-T/t}$

Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3} X\rho X + \frac{p}{3} Y\rho Y + \frac{p}{3} Z\rho Z$


Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$
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Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$
$S(\rho) = (1-\frac{4}{3}p)\rho + \frac{4}{3}p I$

Dephasing channel $S(\rho) = (1-p)\rho + p Z \rho Z$

w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-\Gamma t}$

Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3} X\rho X + \frac{p}{3} Y\rho Y + \frac{p}{3} Z\rho Z$

$$S(\rho) = \left(1 - \tfrac{4}{3}p\right)\rho + \tfrac{4}{3}p\left(I/2\right)$$

Dephasing channel $S(\rho) = (1-p)\rho + p Z \rho Z$

w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-\Gamma t}$

Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3} X\rho X + \frac{p}{3} Y\rho Y + \frac{p}{3} Z\rho Z$

$S(\rho) = (1-\frac{4}{3}p)\rho + \frac{4}{3}p(I/2)$

$I/2 = \frac{1}{4} I -$

Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$

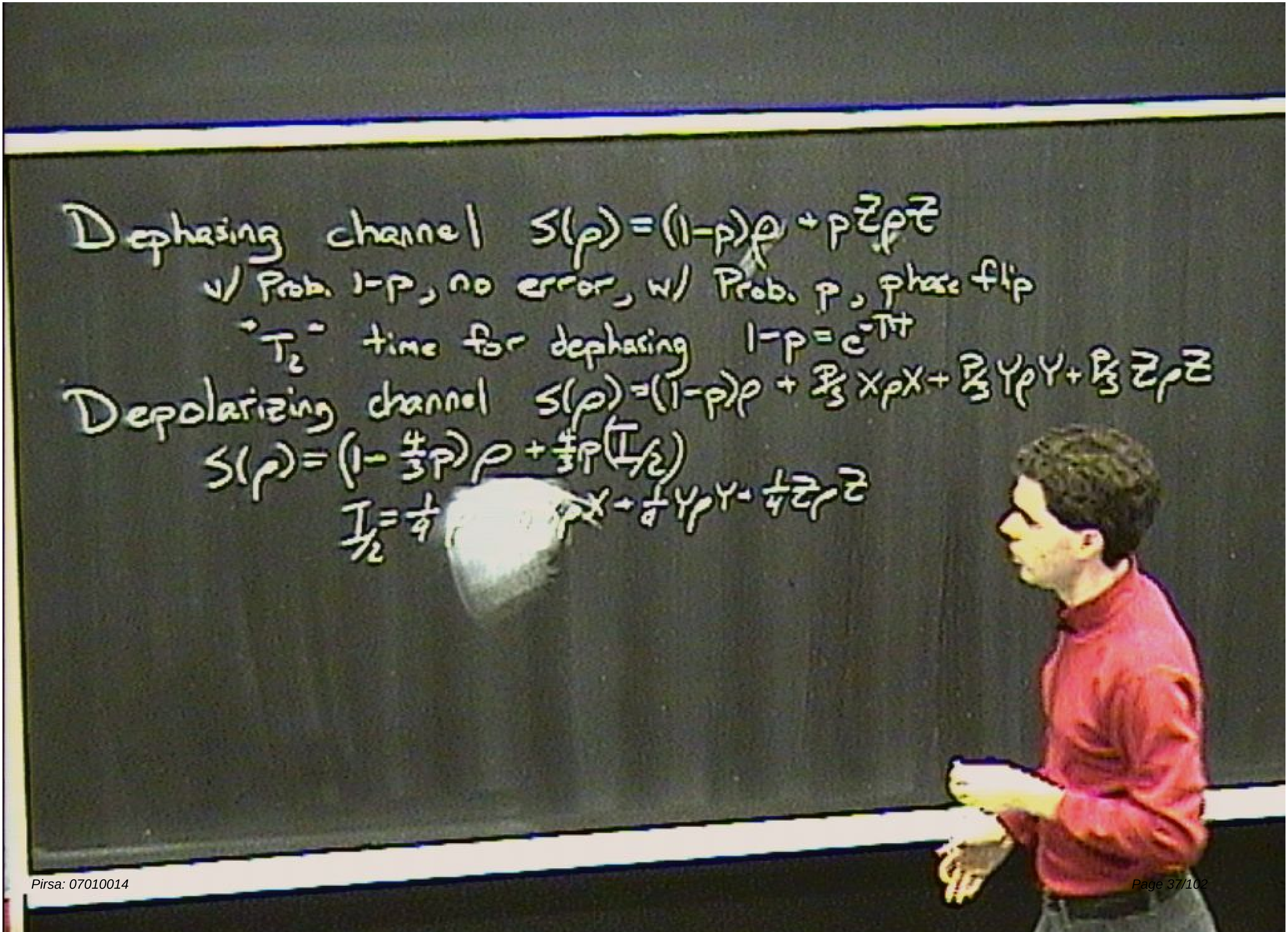
w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-Tt}$

Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

$S(\rho) = (1-\frac{4}{3}p)\rho + \frac{4}{3}p(I/2)$

$I/2 = \frac{1}{4}$


Dephasing channel S(ρ) = (1−p)ρ + pZρZ
w/ Prob. 1−p, no error, w/ Prob. p, phase flip
"T2" time for dephasing 1−p = e^−Γt
Depolarizing channel S(ρ) = (1−p)ρ + p/3 XρX + p/3 YρY + p/3 ZρZ
S(ρ) = (1 − 4/3 p)ρ + 4/3 p (I/2)
I/2 = 1/4 … ρX + 1/4 YρY + 1/4 ZρZ

Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$

w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-Tt}$

Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

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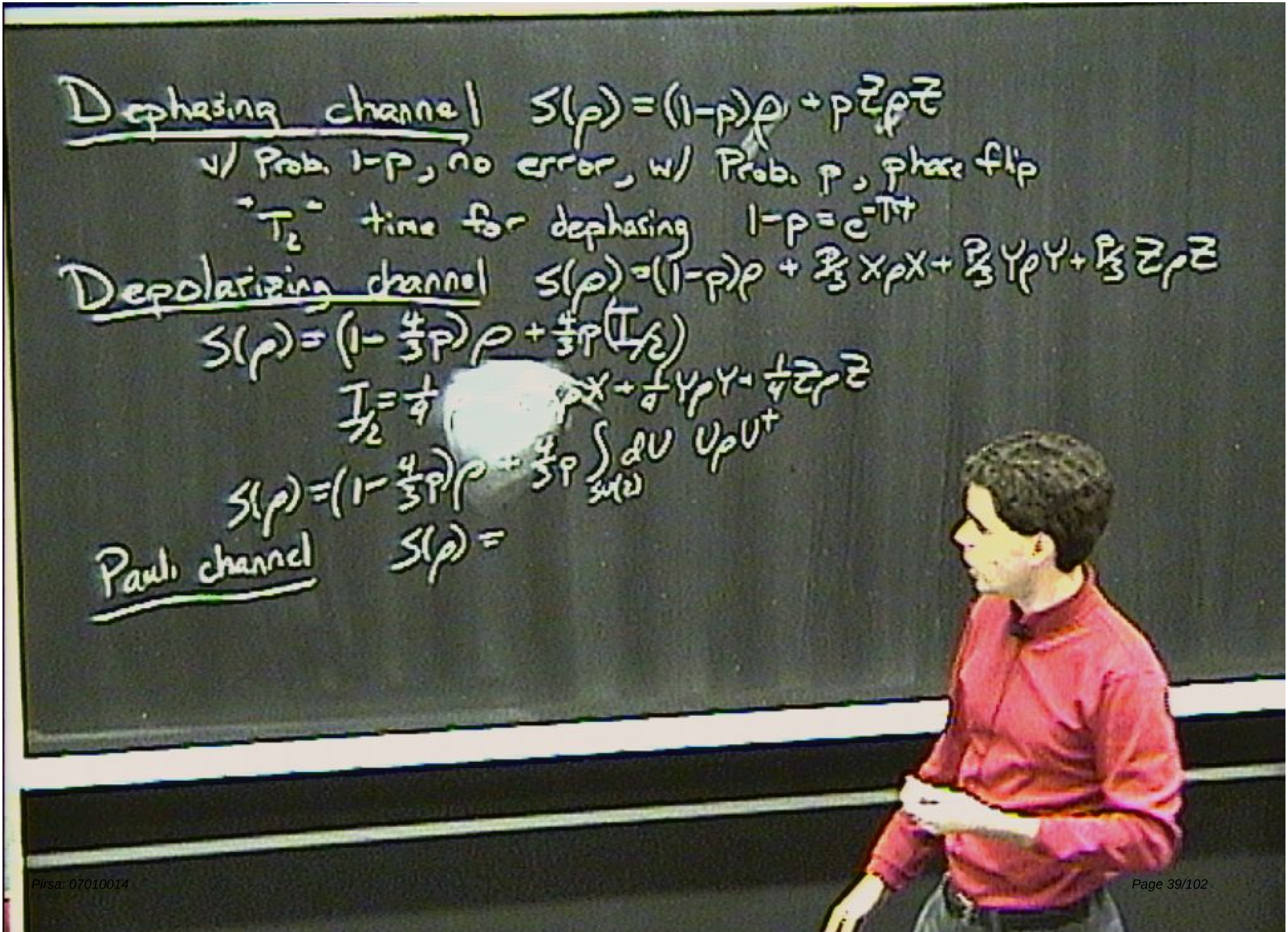
$I/2 = \frac{1}{4} \ldots \rho X + \frac{1}{4}Y\rho Y + \frac{1}{4}Z\rho Z$

$S(\rho) = (1-\frac{4}{3}p)\rho + \frac{4}{3}p\int_{SU(2)} dU\, U\rho U^{+}$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w/ Prob. 1-p, no error, w/ Prob. p, phase flip
"T_2" time for dephasing $1-p = e^{-T/t}$
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$S(\rho) = (1-\frac{4}{3}p)\rho + \frac{4}{3}p(\frac{I}{2})$
$\frac{I}{2} = \frac{1}{4}$ … $\rho X + \frac{1}{4}Y\rho Y + \frac{1}{4}Z\rho Z$
$S(\rho) = (1-\frac{4}{3}p)\rho + \frac{4}{3}p\int_{SU(2)} dU\, U\rho U^{\dagger}$
Pauli channel $S(\rho) =$

Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$

w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-\Gamma t}$

Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

$$S(\rho) = \left(1-\tfrac{4}{3}p\right)\rho + \tfrac{4}{3}p\left(\tfrac{I}{2}\right)$$

$$\tfrac{I}{2} = \tfrac{1}{4} \ldots \rho X + \tfrac{1}{4}Y\rho Y + \tfrac{1}{4}Z\rho Z$$

$$\tfrac{I}{2} = \int_{SU(2)} dU\, U\rho U^\dagger$$

$$S(\rho) = \left(1-\tfrac{4}{3}p\right)\rho + \tfrac{4}{3}p \int_{SU(2)} dU\, U\rho U^\dagger$$

Pauli channel $S(\rho) = (1-p_X-p_Y-p_Z)\rho + p_X X\rho X + p_Y Y\rho Y + p_Z Z\rho Z$

Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$

w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-T/t}$

Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

$$S(\rho) = (1 - \tfrac{4}{3}p)\rho + \tfrac{4}{3}p\left(\tfrac{I}{2}\right)$$

$$\tfrac{I}{2} = \tfrac{1}{4} \ldots X + \tfrac{1}{4}Y\rho Y + \tfrac{1}{4}Z\rho Z$$

$$S(\rho) = (1 - \tfrac{4}{3}p)\rho + \tfrac{4}{3}p\int_{SU(2)} dU\, U\rho U^{+}$$

Pauli channel $S(\rho) = (1 - p_x - p_y - p_z)\rho + p_x X\rho X + p_y Y\rho Y + p_z Z\rho Z$

Amplitude damping $S(\rho) = A_0\rho A_0^{+} + A_1\rho A_1^{+}$ $A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma^2} \end{pmatrix}$, $A_1 = \begin{pmatrix} 0 & \gamma \\ 0 & 0 \end{pmatrix}$

<u>Dephasing channel</u> $S(\rho) = (1-p)\rho + pZ\rho Z$

w/ Prob. $1-p$, no error, w/ Prob. p, phase flip

"T_2" time for dephasing $1-p = e^{-T/t}$

<u>Depolarizing channel</u> $S(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$

$S(\rho) = (1 - \frac{4}{3}p)\rho + \frac{4}{3}p(\frac{I}{2})$

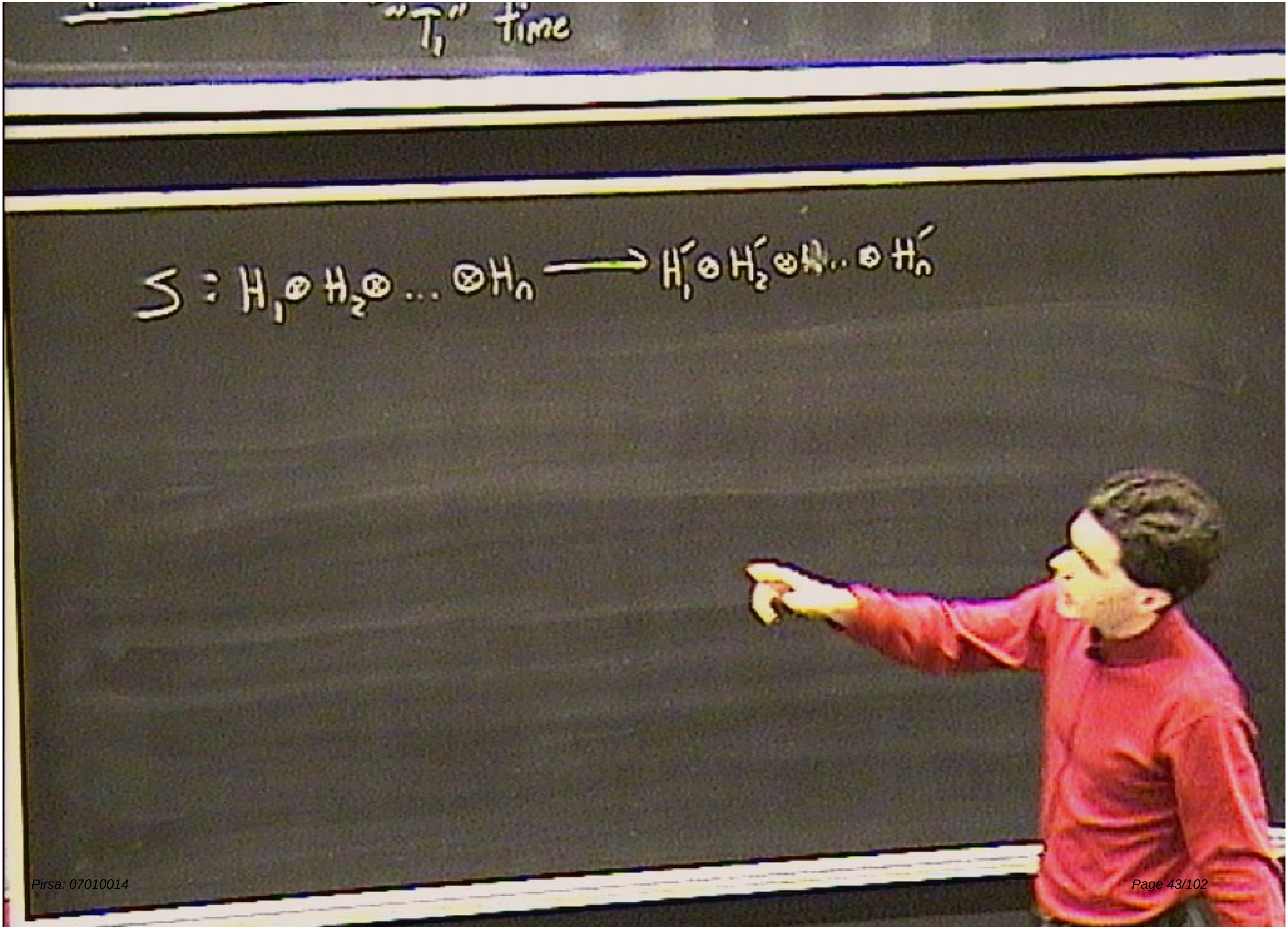
$\frac{I}{2} = \frac{1}{4}$ [illegible] $\rho X + \frac{1}{4}Y\rho Y + \frac{1}{4}Z\rho Z$

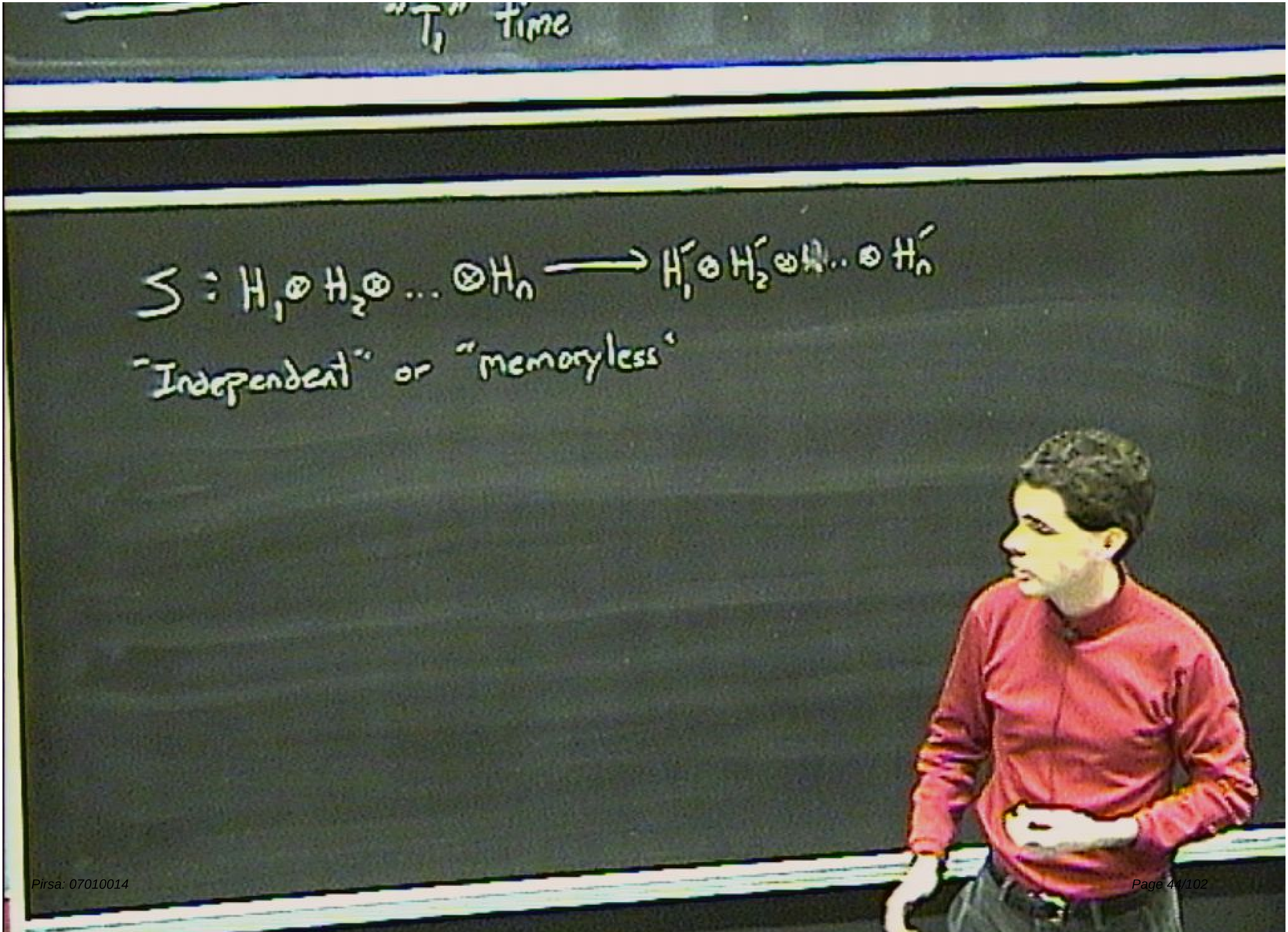
$S(\rho) = (1 - \frac{4}{3}p)\rho + \frac{4}{3}p \int_{SU(2)} dU\, U\rho U^{+}$

<u>Pauli channel</u> $S(\rho) = (1 - p_x - p_y - p_z)\rho + p_x X\rho X + p_y Y\rho Y + p_z Z\rho Z$

<u>Amplitude damping</u> $S(\rho) = A_0\rho A_0^{+} + A_1\rho A_1^{+}$ $A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma^2} \end{pmatrix}$, $A_1 = \begin{pmatrix} 0 & \gamma \\ 0 & 0 \end{pmatrix}$

"T_1" time


"T_1" time
$S : H_1 \otimes H_2 \otimes \ldots \otimes H_n \longrightarrow H_1' \otimes H_2' \otimes \ldots \otimes H_n'$


"T_1" time
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"Independent" or "memoryless"

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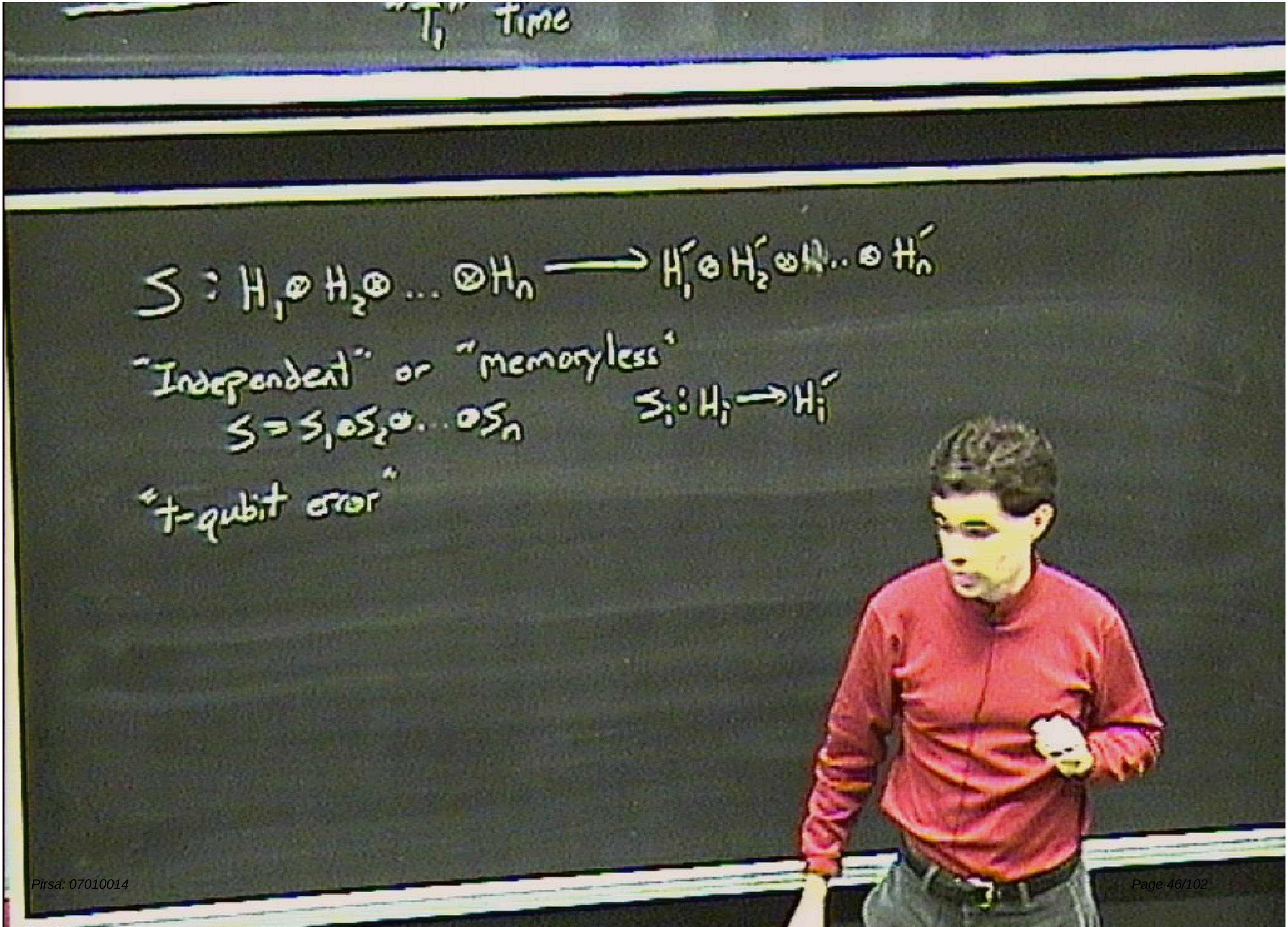
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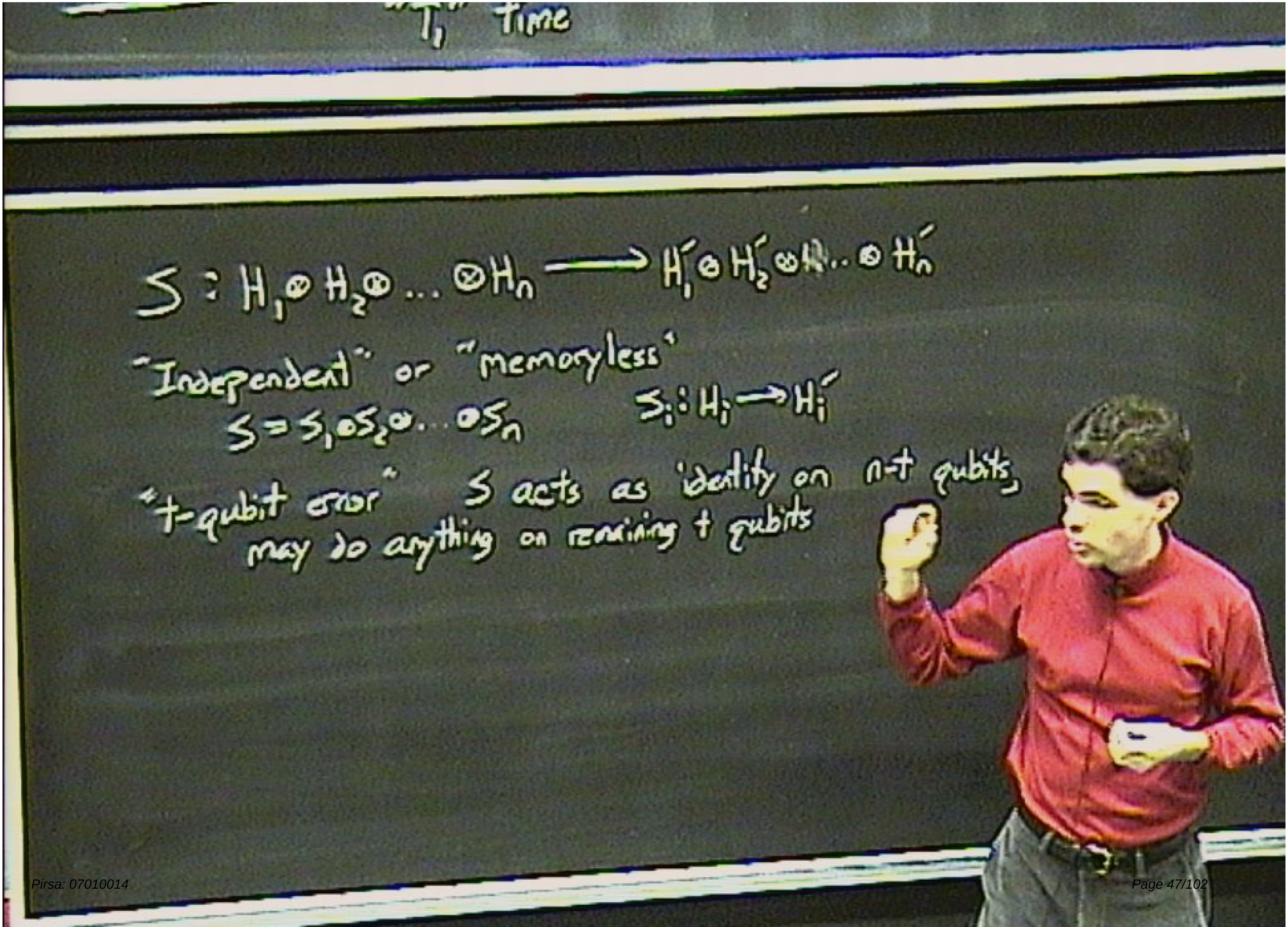
"t-qubit error"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"t-qubit error" S acts as 'identity on n-t qubits,
may do anything on remaining t qubits

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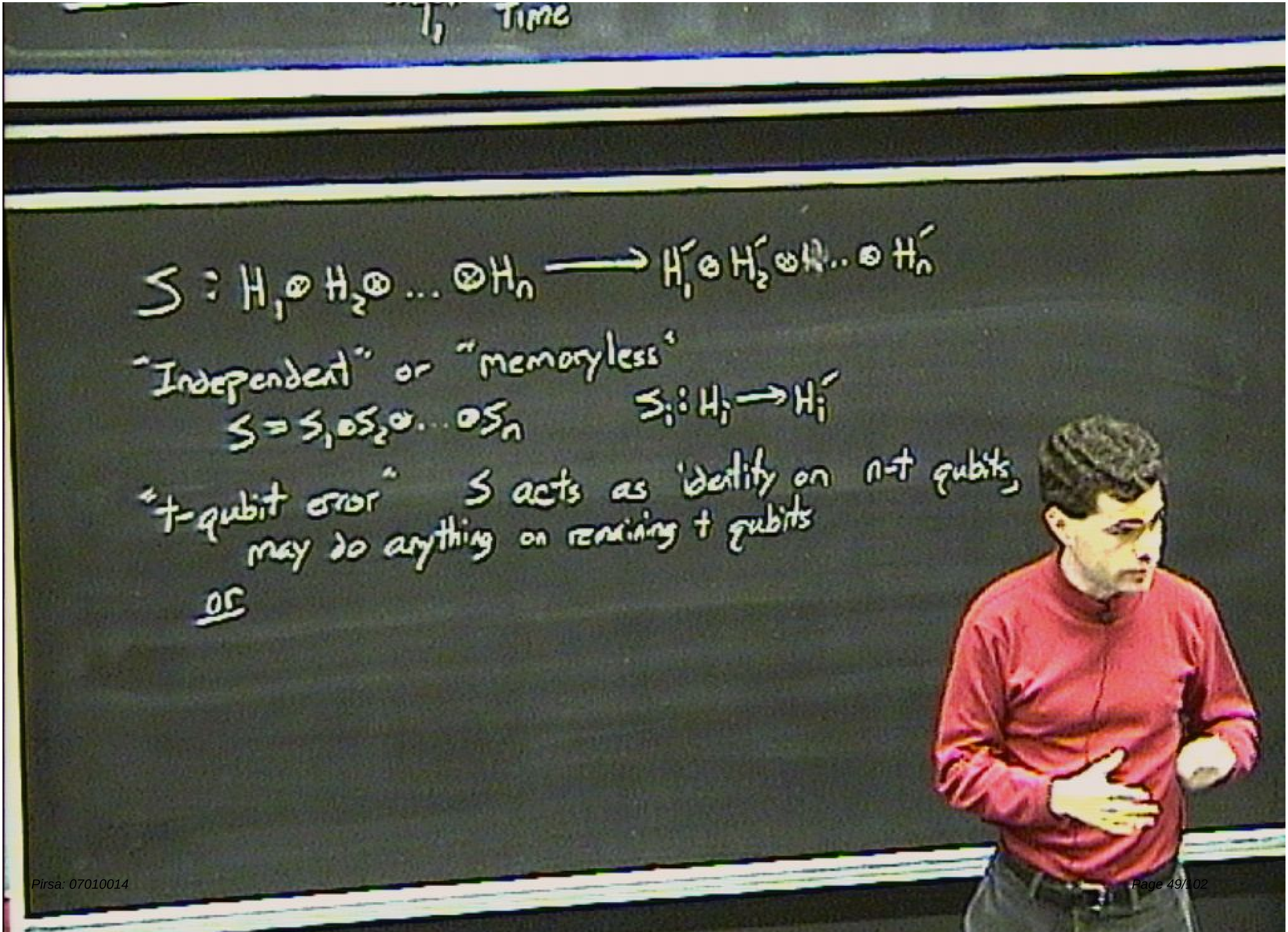

T_1 Time
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or

Time

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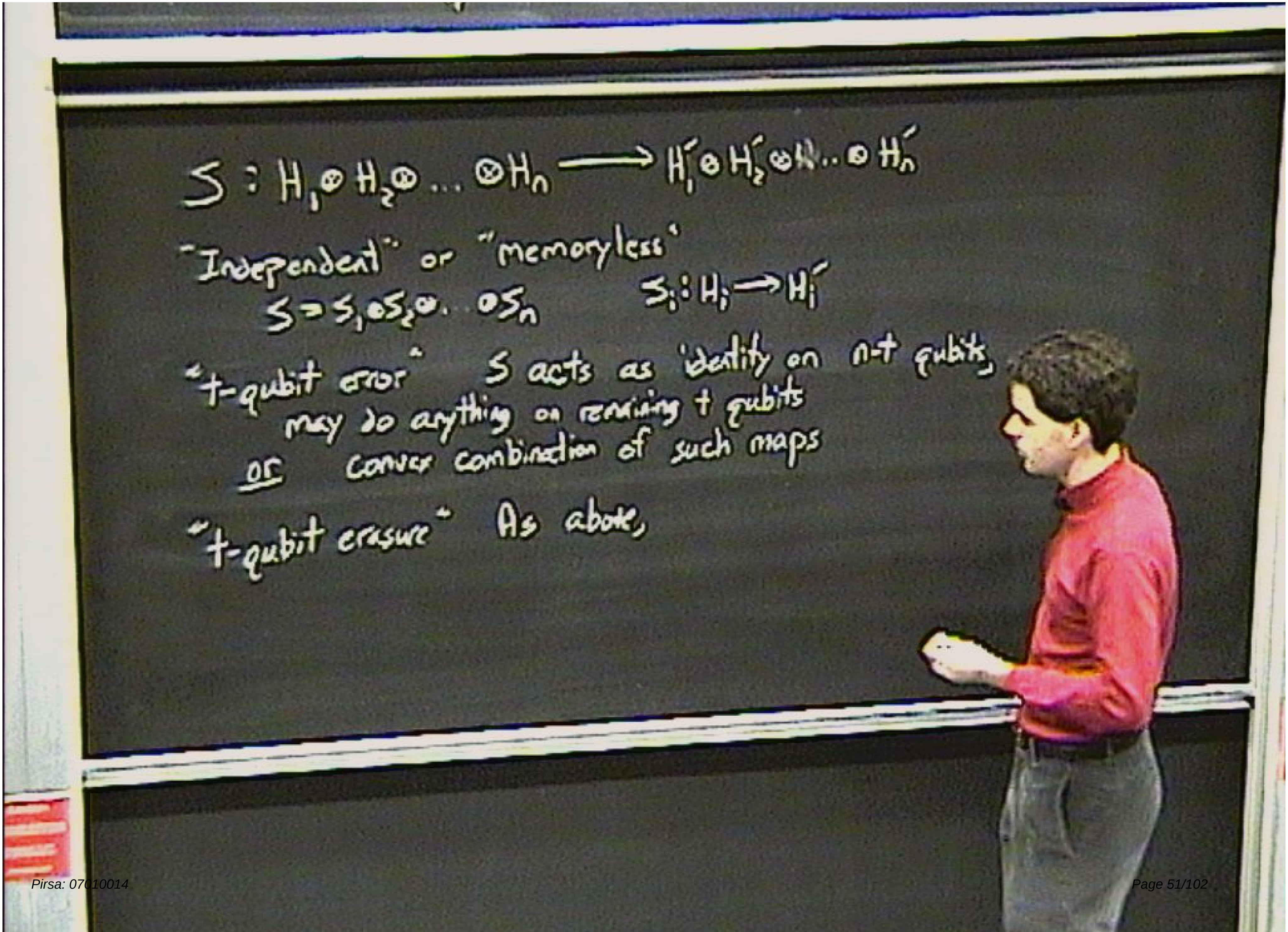
or convex combination of such maps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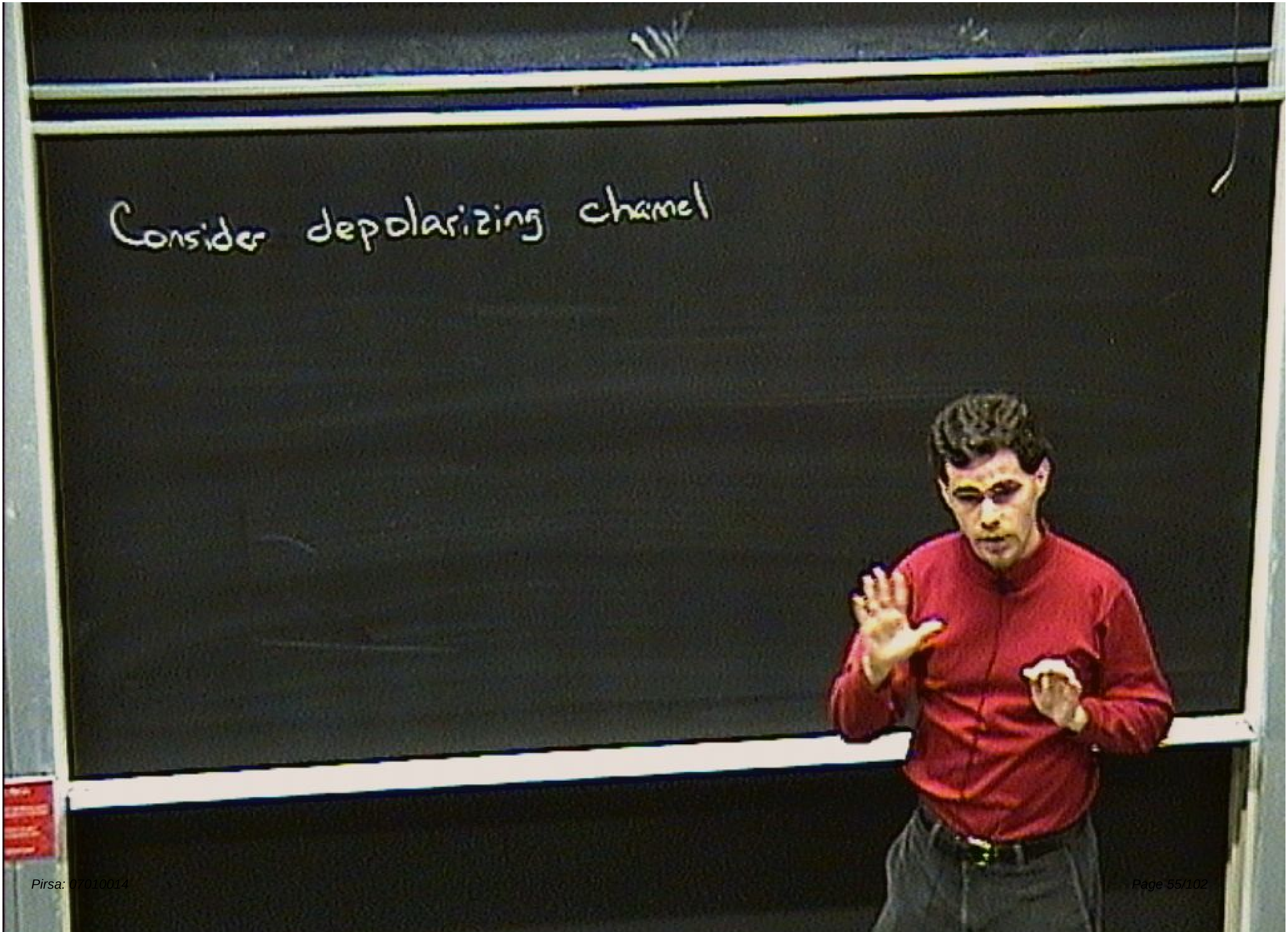
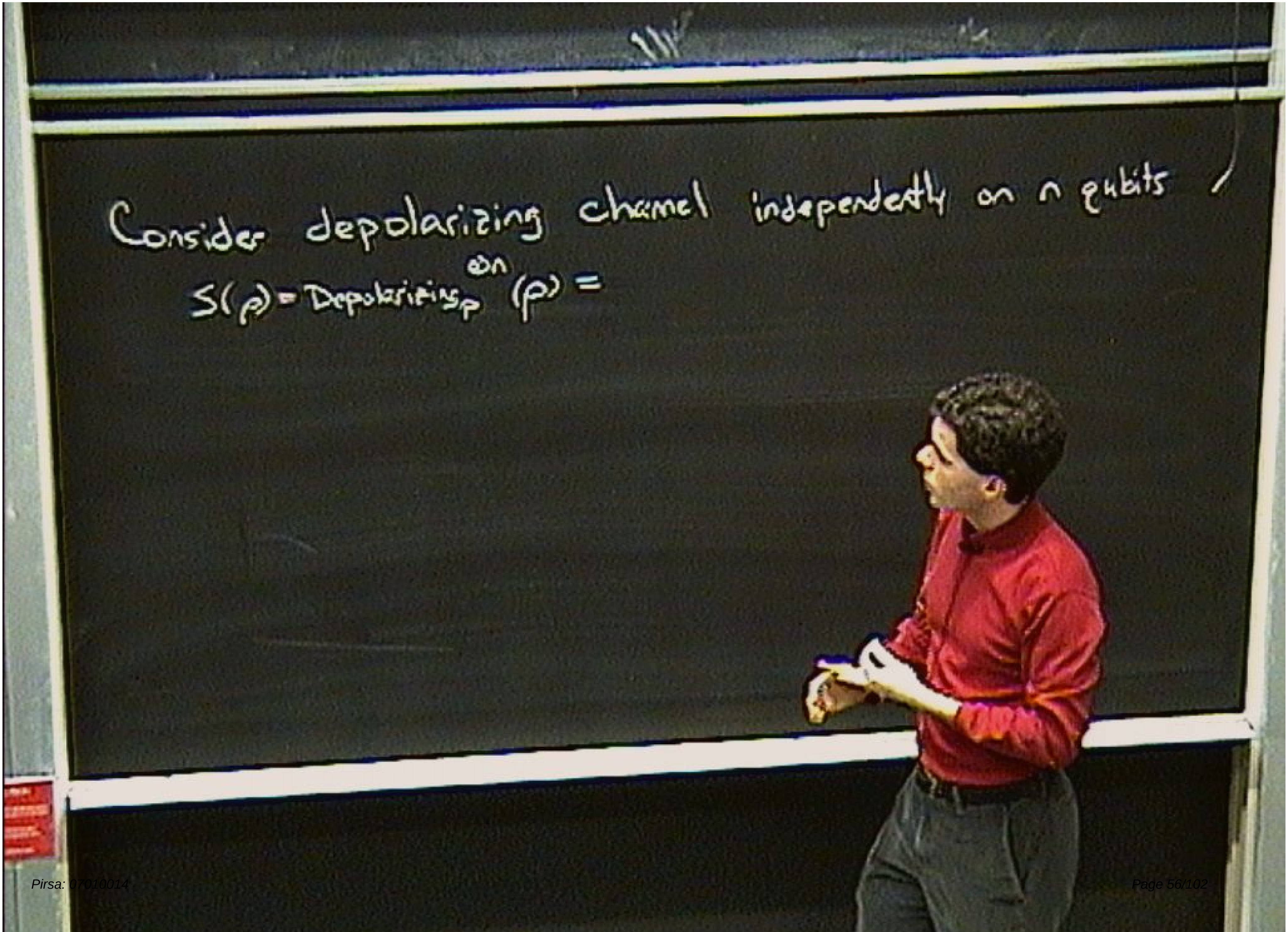

Consider depolarizing channel


Consider depolarizing channel independently on n qubits
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$$S(\rho) = \mathrm{Depolarizing}_p^{\otimes n}(\rho) = (1-p)^n \rho + (1-p)^{n-1} p \Big(\underset{X^{(1)}\rho X^{(1)} + X^{(2)}\rho X^{(2)} + \dots}{\text{one Pauli error}}$$

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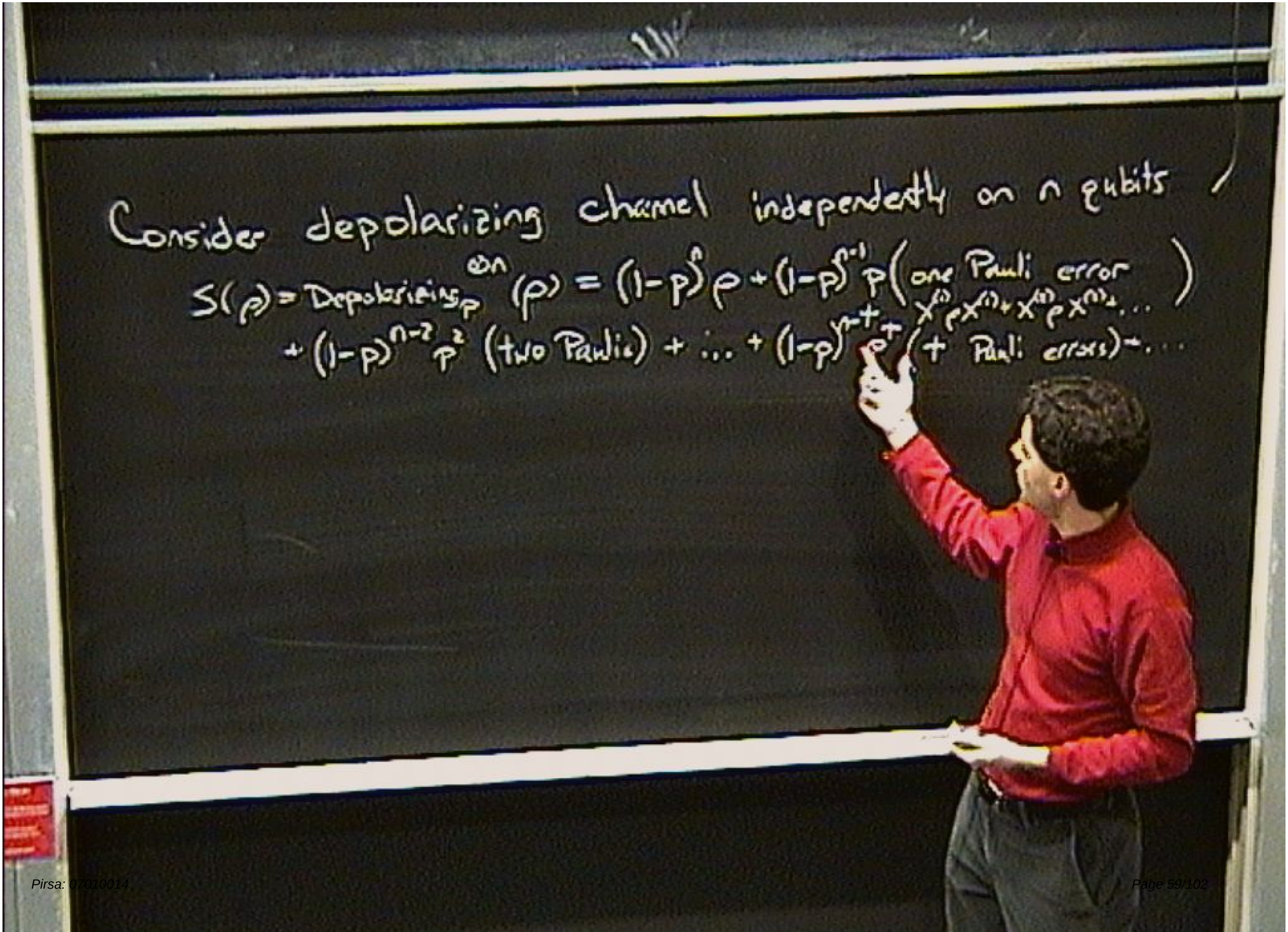
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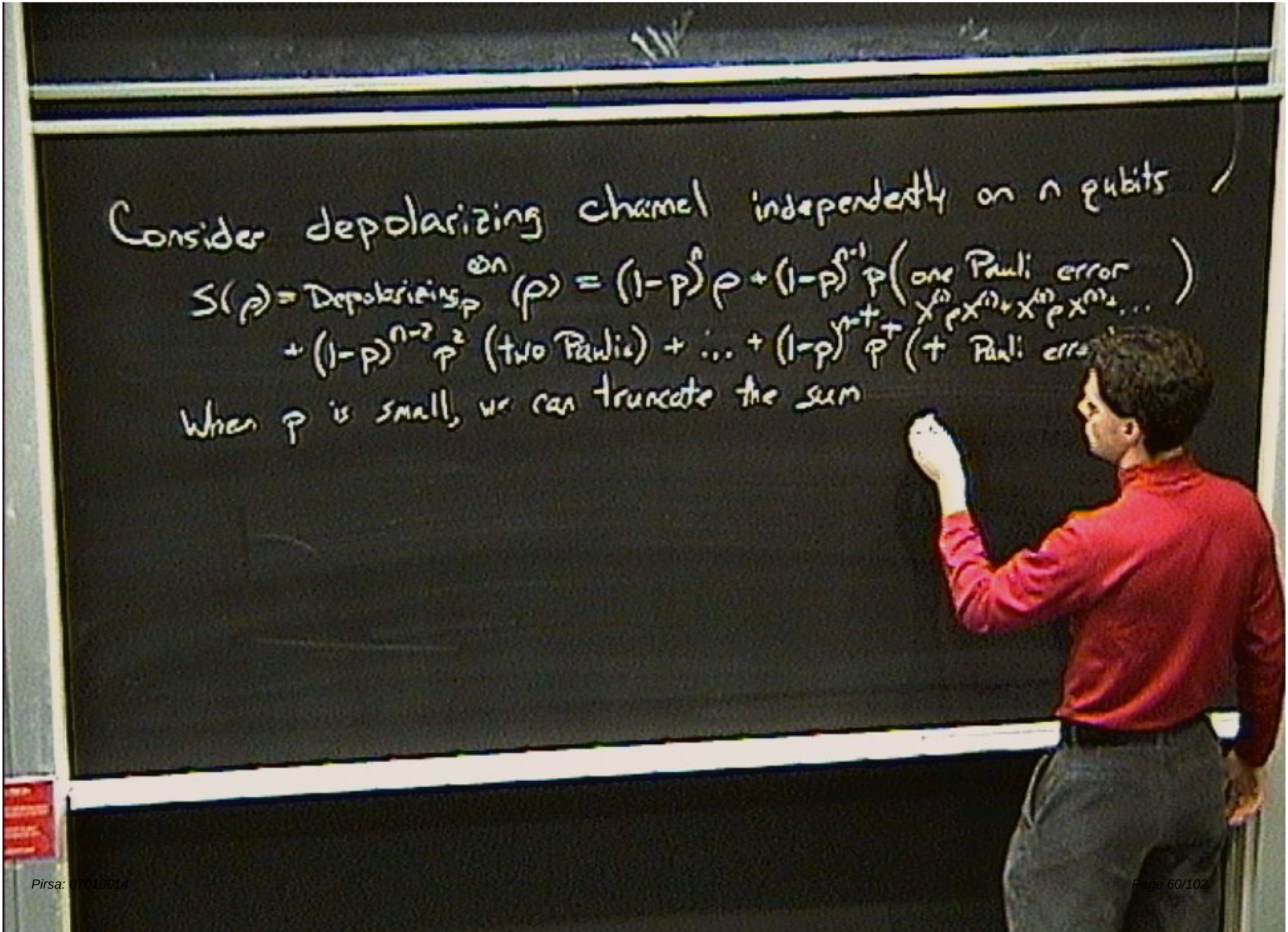
$+ (1-p)^{n-2} p^2$ (two Pauli) $+ \dots + (1-p)^{n-t} p^t$ (t Pauli errors) $+ \dots$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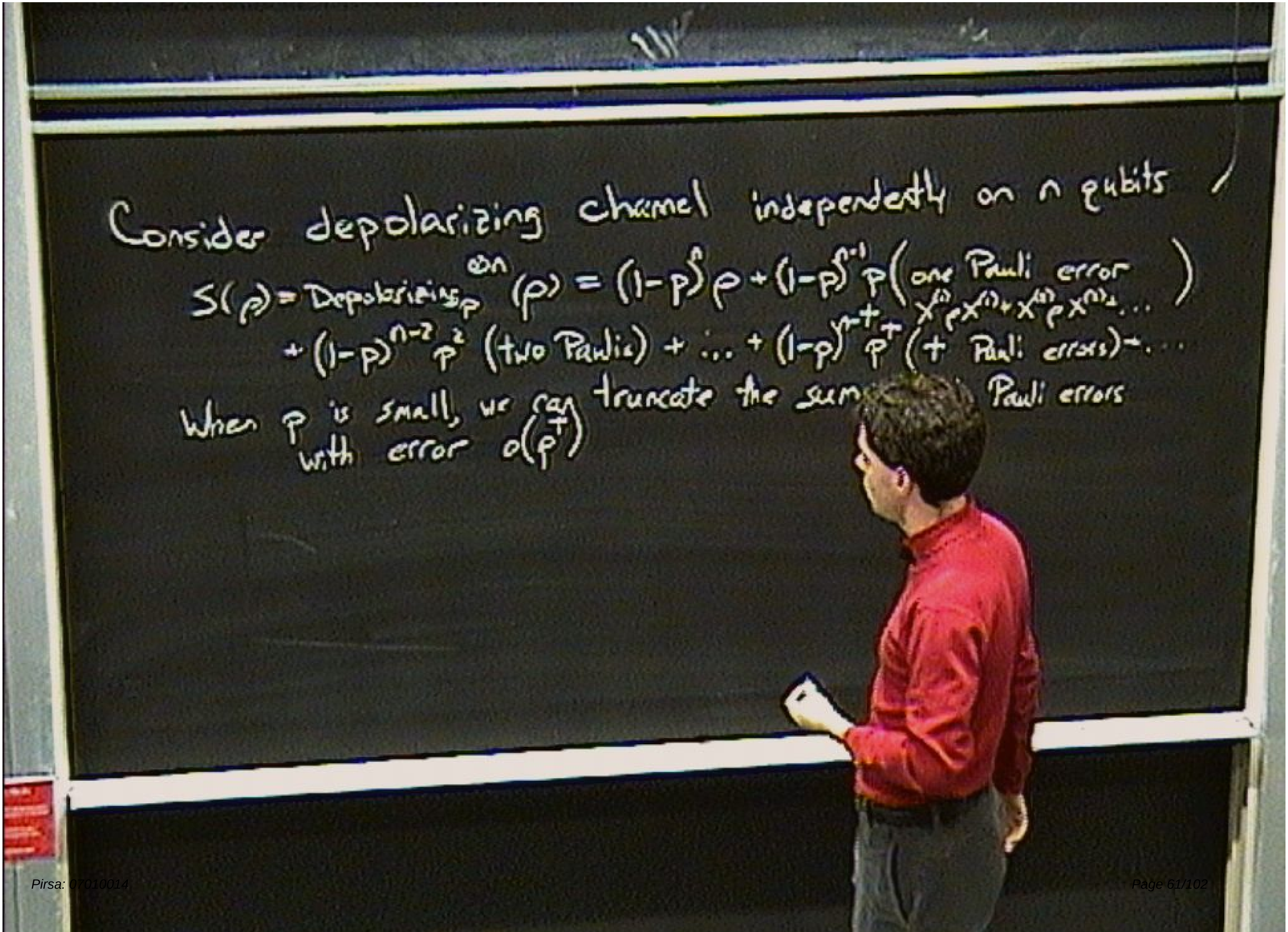
When p is small, we can truncate the sum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When p is small, we can truncate the sum Pauli errors
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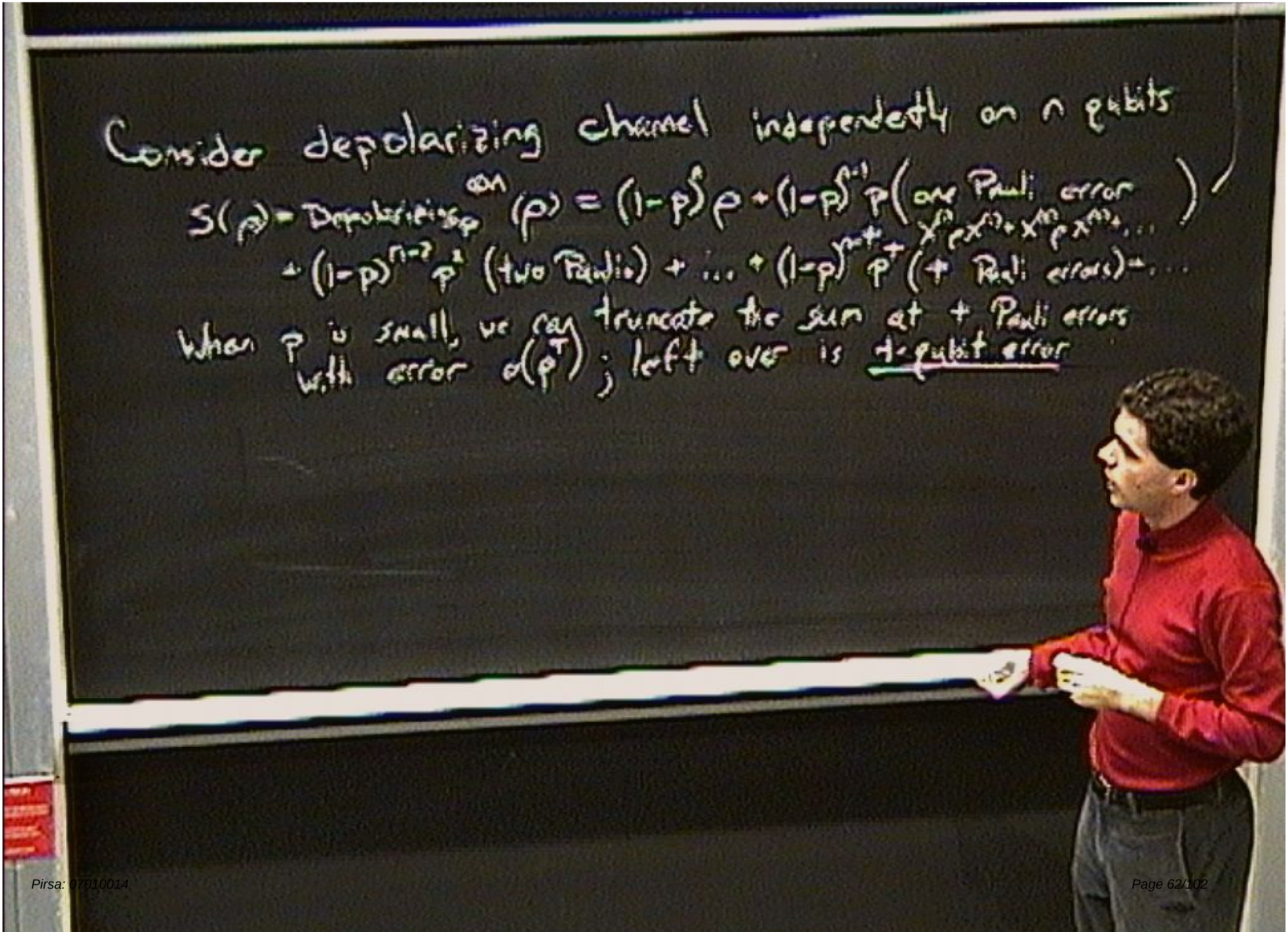
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When p is small, we can truncate the sum at t Pauli errors with error $O(p^T)$; left over is t-qubit error

Consider depolarizing channel independently on n qubits

$$S(\rho) = \text{Depolarizing}_p^{\otimes n}(\rho) = (1-p)^n \rho + (1-p)^{n-1} p \left(\text{one Pauli error} \quad X^{(1)}\rho X^{(1)} + X^{(2)}\rho X^{(2)} + \dots \right),$$

$$+ (1-p)^{n-2} p^2 (\text{two Paulis}) + \dots + (1-p)^{n-t} p^t (t \text{ Pauli errors}) + \dots$$

When p is small, we can truncate the sum at t Pauli errors with error $O(p^t)$; left over is <u>t-qubit error</u>

Consider depolarizing channel independently on n qubits

$$S(\rho) = \text{Depolarizing}_p^{\otimes n}(\rho) = (1-p)^n \rho + (1-p)^{n-1} p \left(\begin{matrix} \text{one Pauli error} \\ X^{(1)}\rho X^{(1)} + X^{(2)}\rho X^{(2)} + \dots \end{matrix} \right),$$

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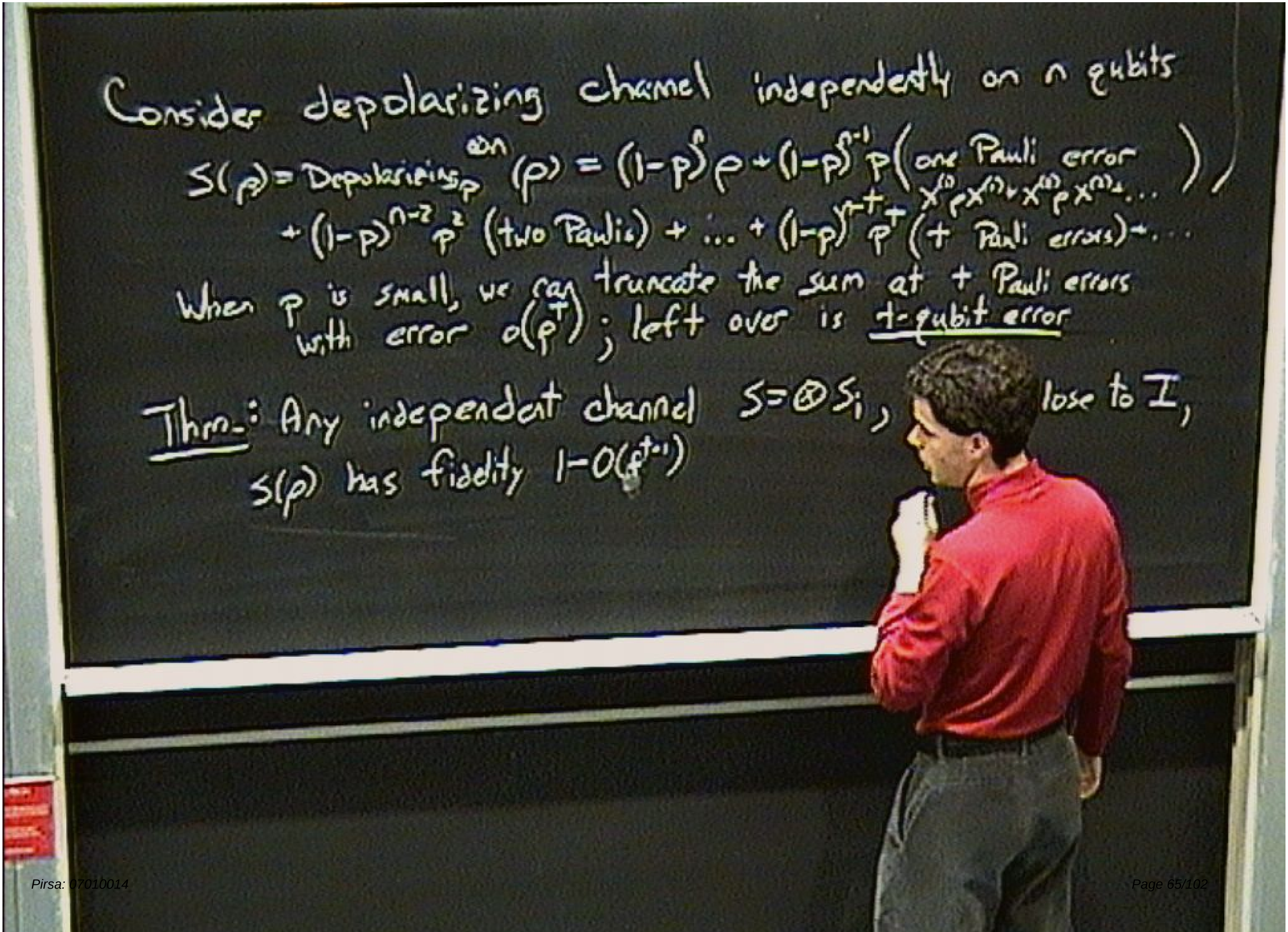
<u>Thm.</u>: Any independent channel $S = \bigotimes S_i$,


Consider depolarizing channel independently on n qubits
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Thm: Any independent channel $S = \otimes S_i$, … lose to I,
$S(\rho)$ has fidelity $1 - O(p^{t+1})$

Consider depolarizing channel independently on n qubits

$$S(\rho) = \text{Depolarizing}_p^{\otimes n}(\rho) = (1-p)^n \rho + (1-p)^{n-1} p \left(\text{one Pauli error } X^{(1)} \rho X^{(1)} + X^{(2)} \rho X^{(2)} + \ldots \right),$$
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When p is small, we can truncate the sum at t Pauli errors with error $O(p^t)$; left over is <u>t-qubit error</u>

<u>Thm.</u>: Any independent channel $S = \otimes S_i$, S_i is cl… I, $S(\rho)$ has fidelity $1 - O(f^{t+1})$ to a t-qubit error ch…
where fidelity $(\rho, S_i(\rho)) \geq 1 - f$

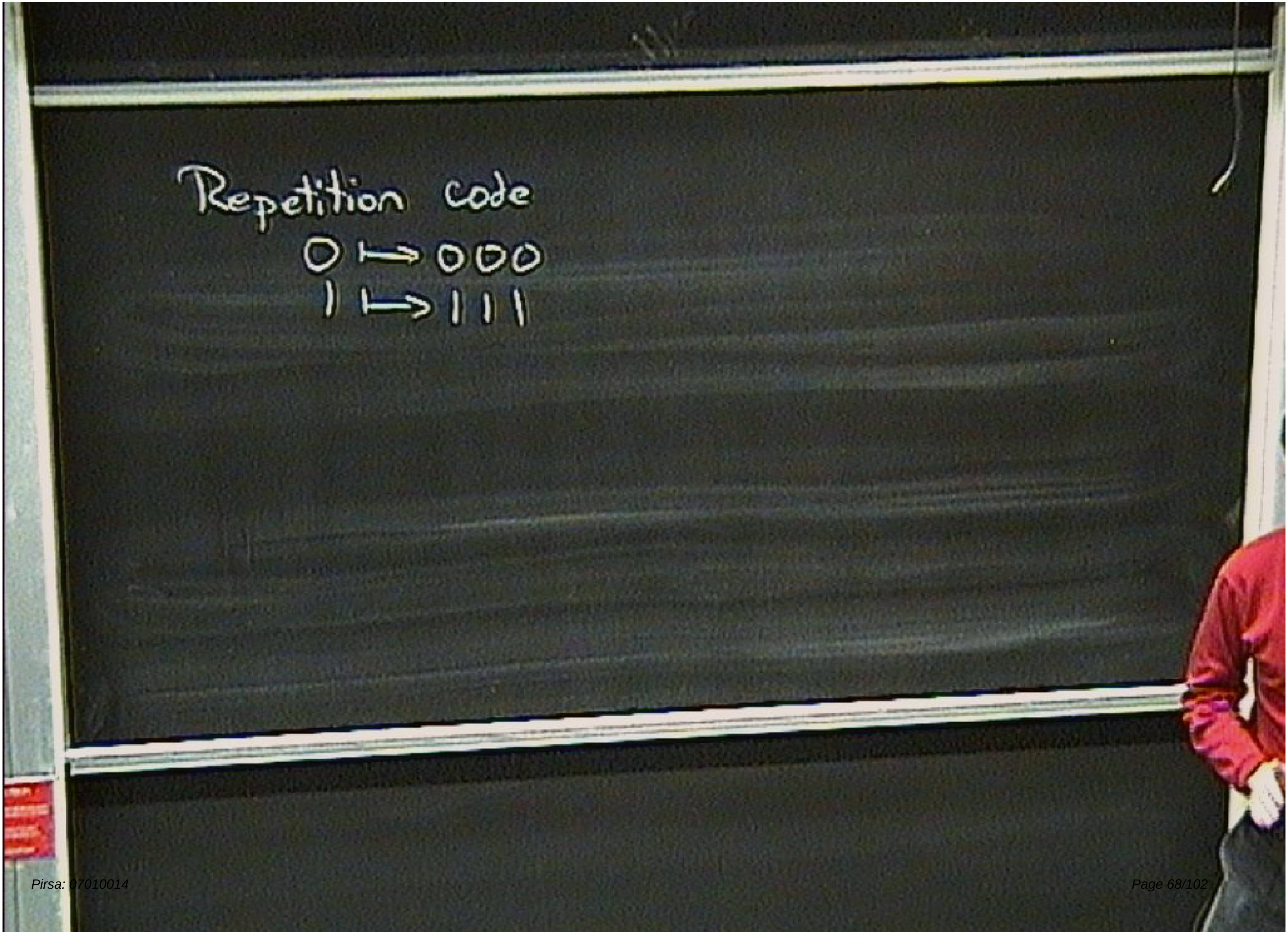
Consider depolarizing channel independently on n qubits

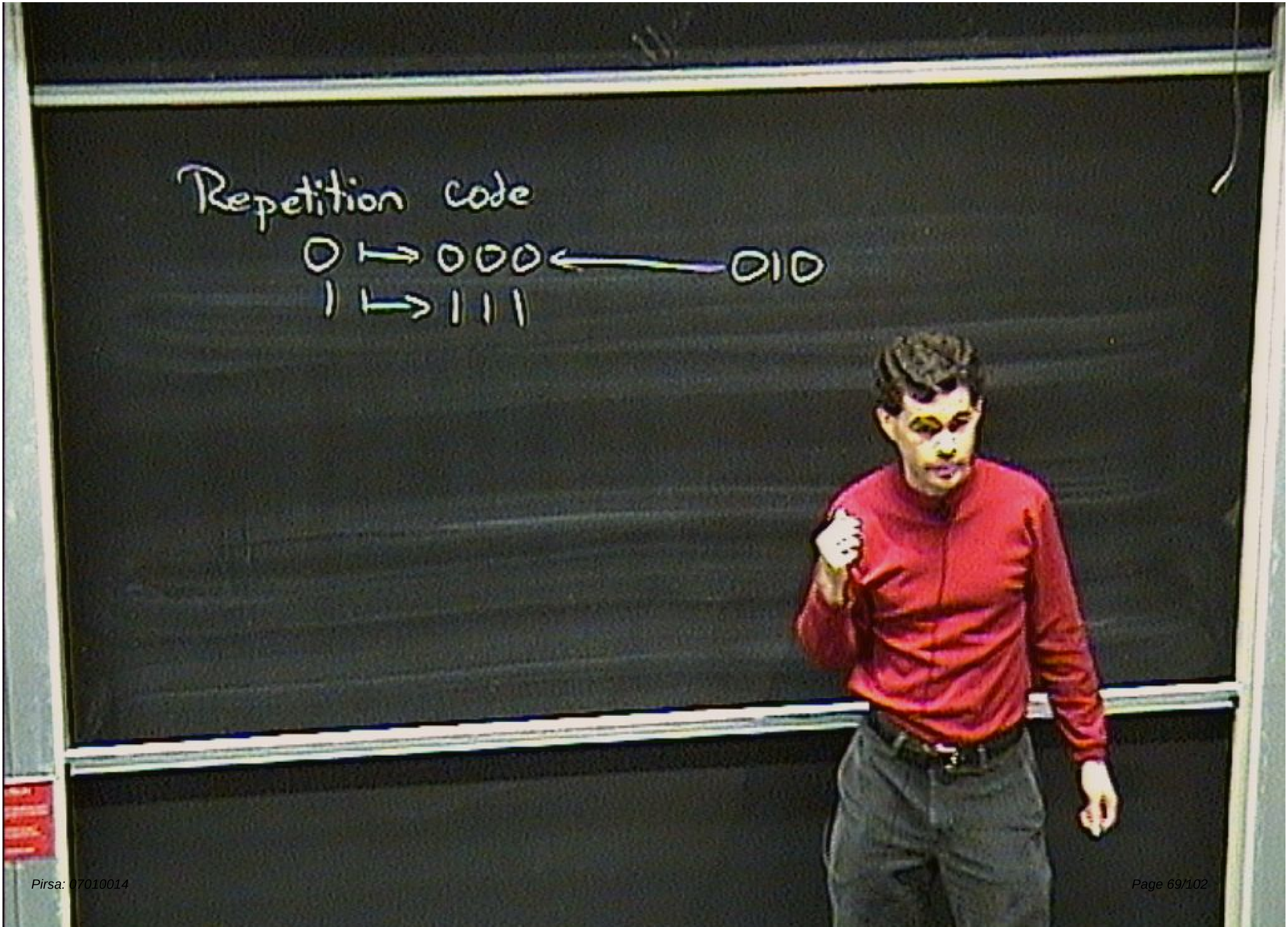
$$S(\rho) = \text{Depolarizing}_p^{\otimes n}(\rho) = (1-p)^n \rho + (1-p)^{n-1} p \left(\text{one Pauli error} \;\; X^{(1)}\rho X^{(1)} + X^{(2)}\rho X^{(2)} + \dots \right)$$

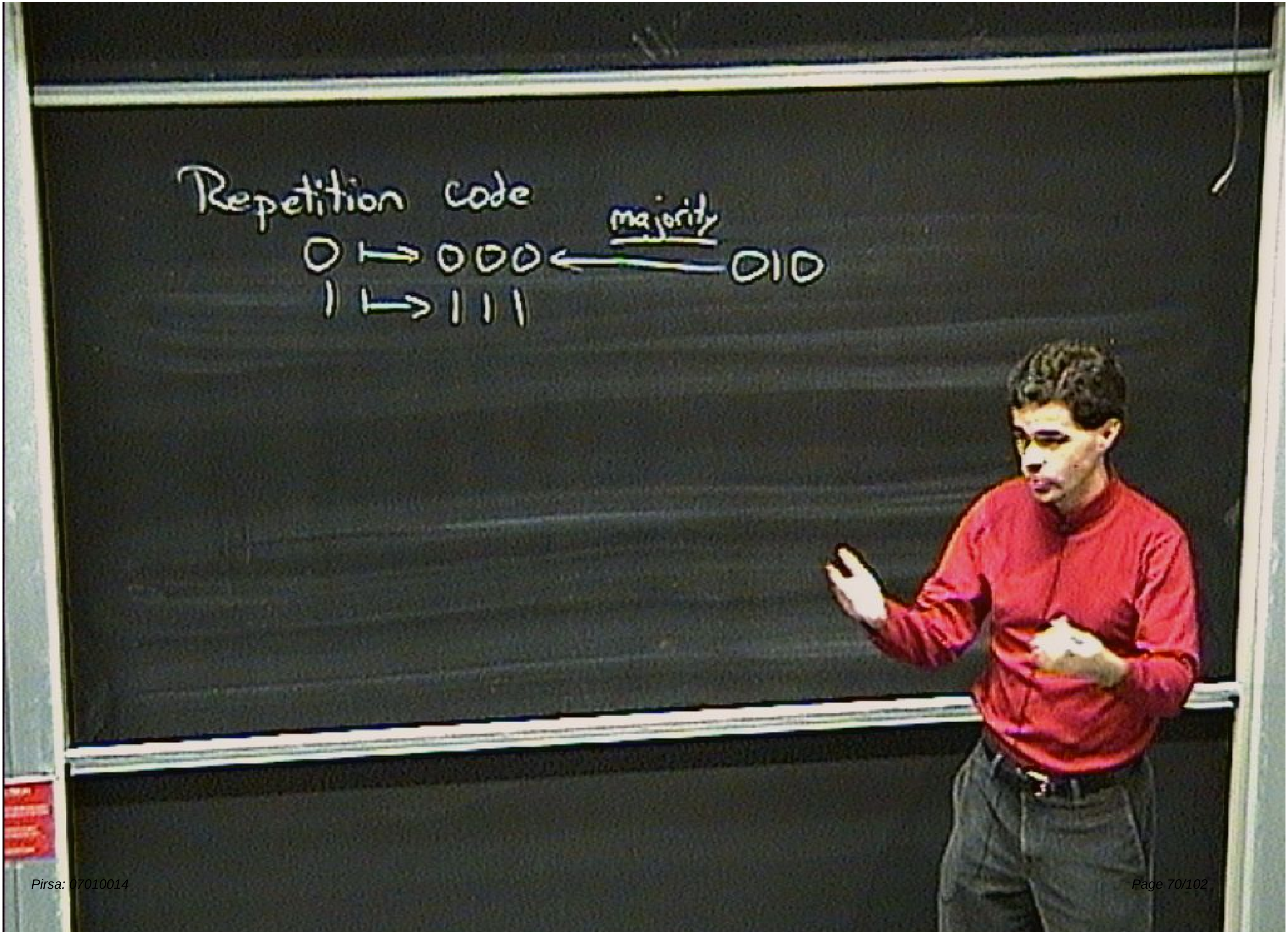
$$+ (1-p)^{n-2} p^2 (\text{two Paulis}) + \dots + (1-p)^{n-t} p^t (t \text{ Pauli errors}) + \dots$$

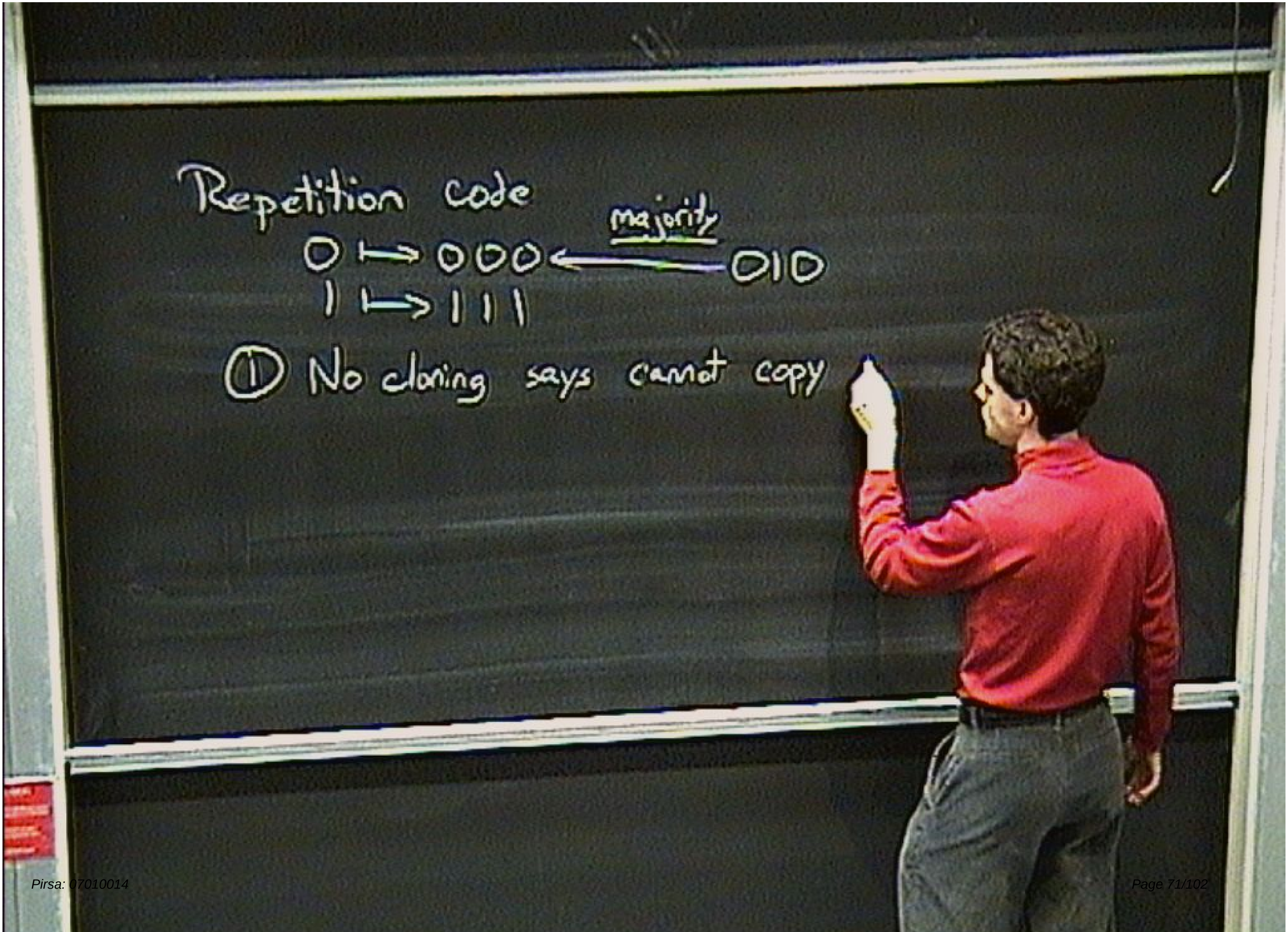
When p is small, we can truncate the sum at t Pauli errors with error $O(p^t)$; left over is <u>t-qubit error</u>

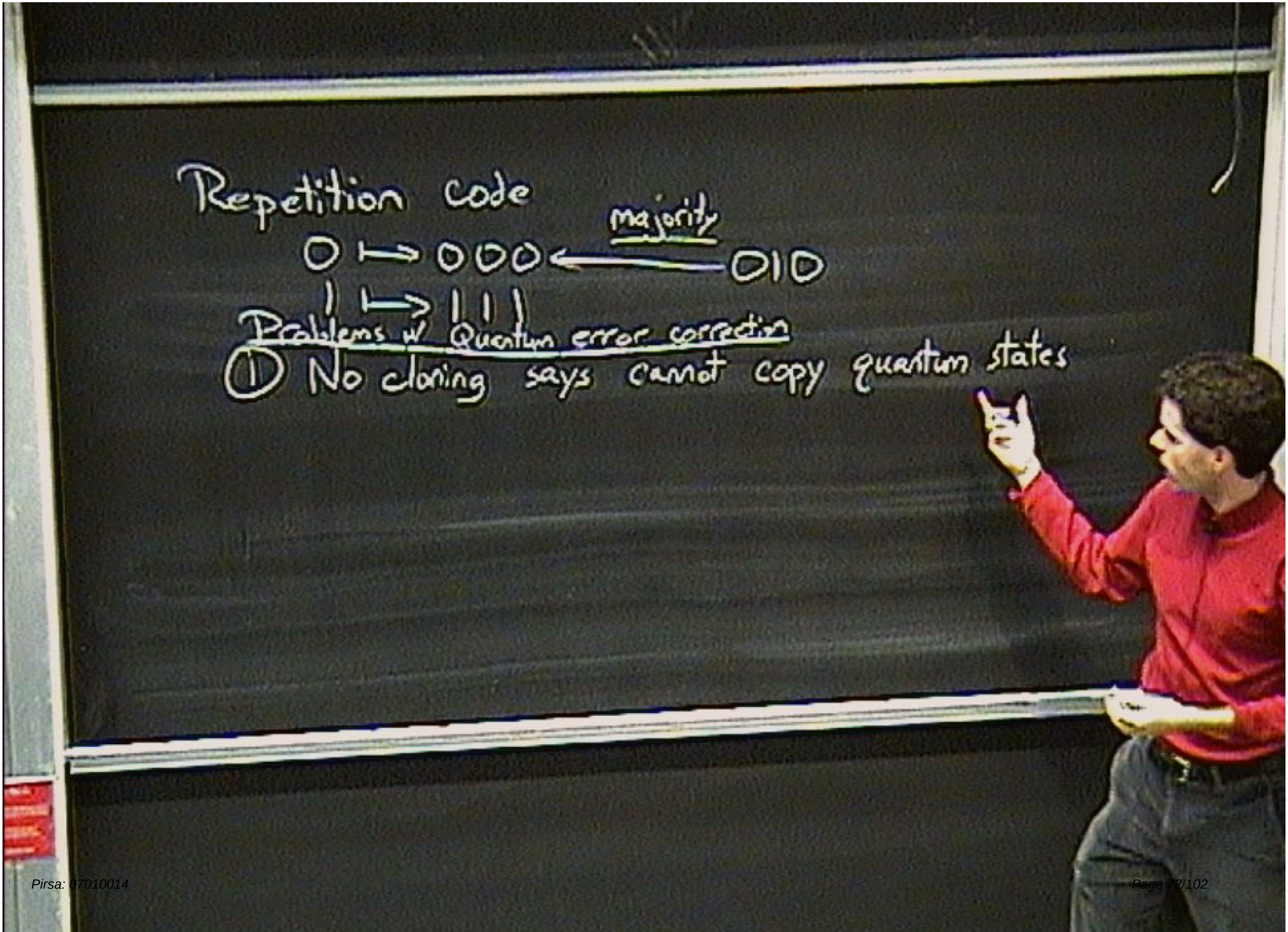
<u>Thm.</u>: Any independent channel $S = \otimes S_i$, S_i is close to I, $S(\rho)$ has fidelity $1 - O(f^{t+1})$ to a t-qubit error channel, where fidelity $(\rho, S_i(\rho)) \geq 1-f$

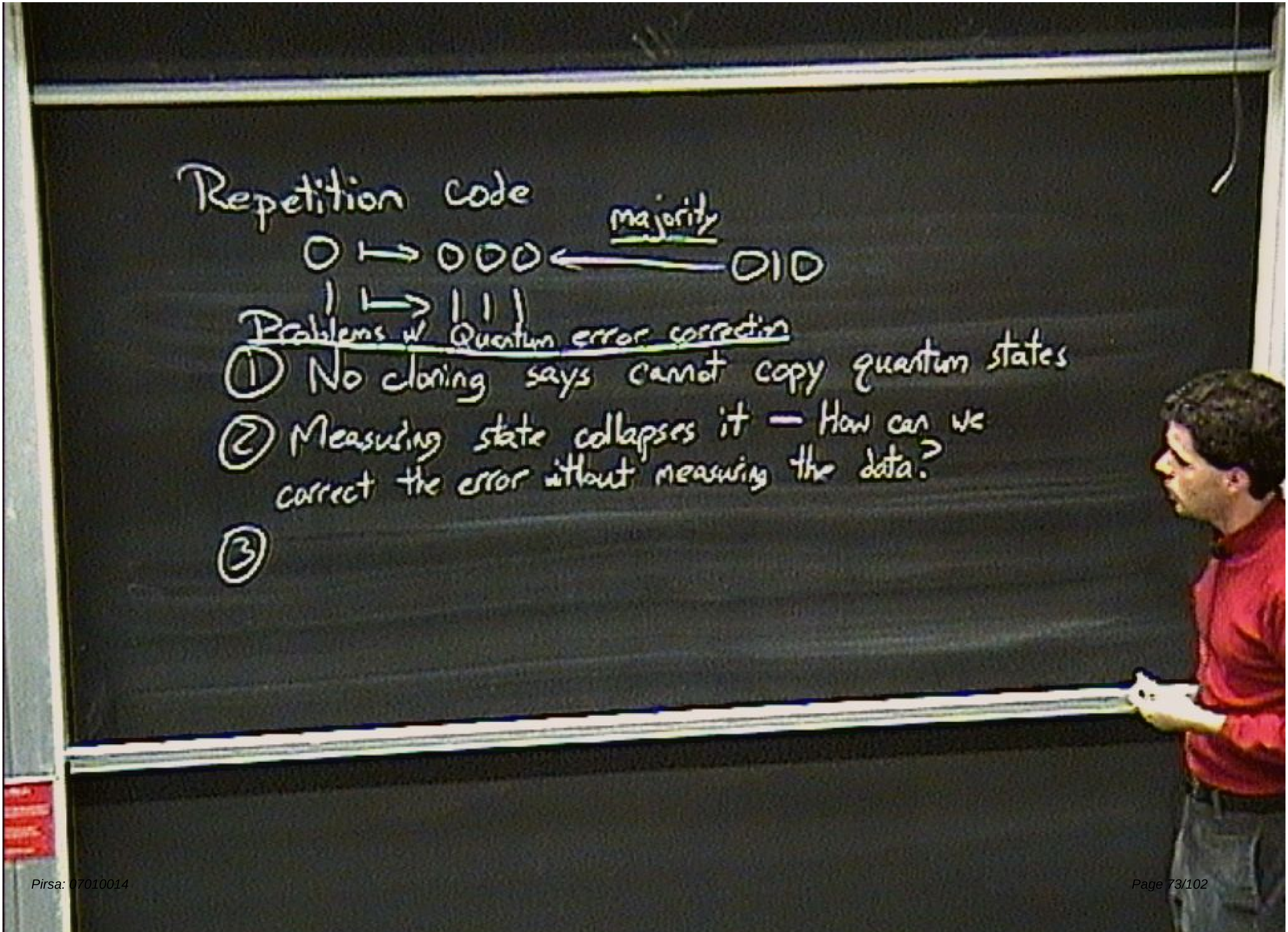

Repetition code
0 ↦ 000
1 ↦ 111

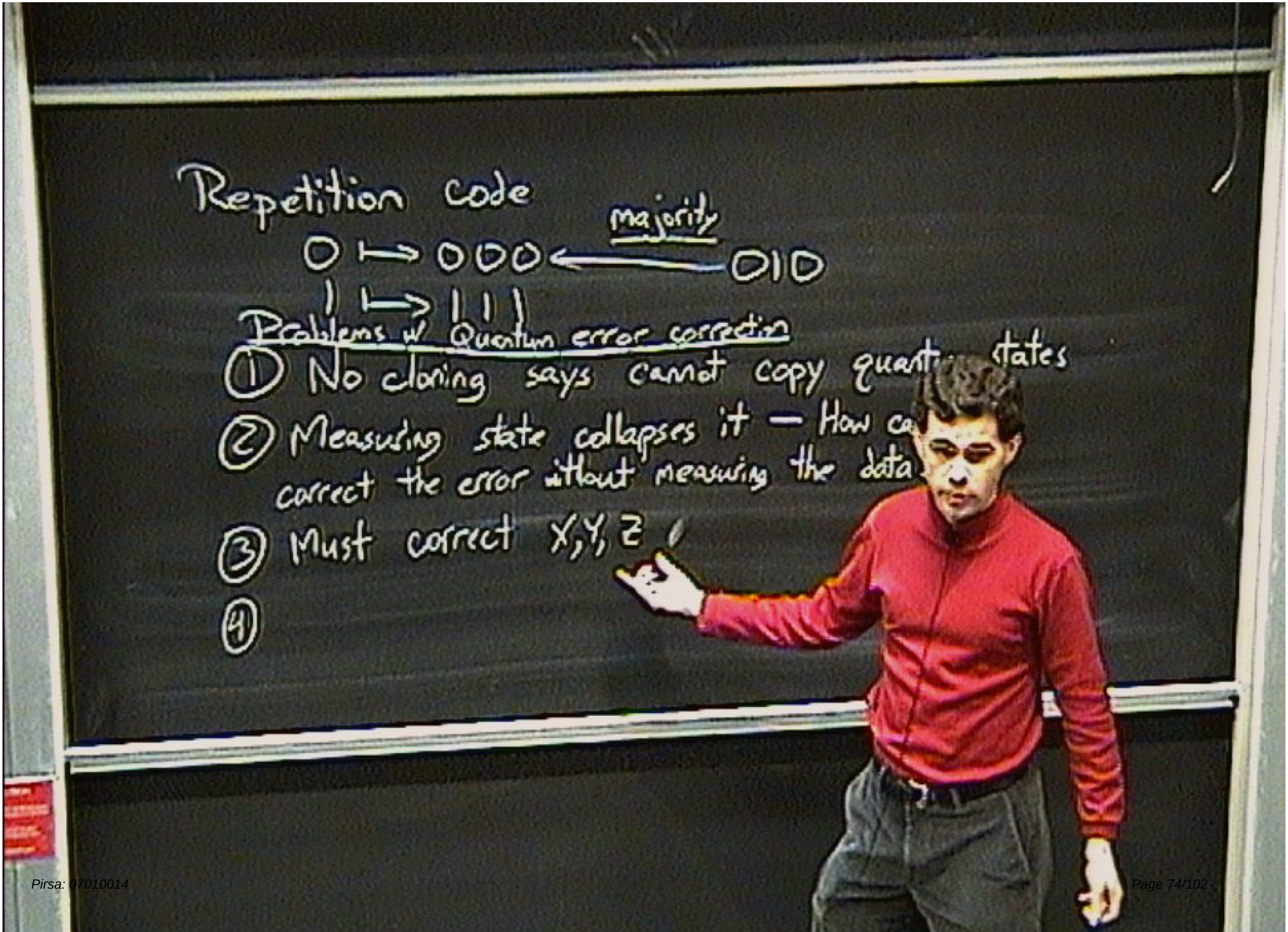

Repetition code
0 ↦ 000 ⟵ 010
1 ↦ 111


Repetition code
majority
0 ↦ 000 ← 010
1 ↦ 111


Repetition code
majority
0 ⟼ 000 ⟵ 010
1 ⟼ 111
① No cloning says cannot copy


Repetition code
majority
0 ↦ 000 ← 010
1 ↦ 111
Problems w/ Quantum error correction
(1) No cloning says cannot copy quantum states


Repetition code
majority
0 ⟼ 000 ⟵ 010
1 ⟼ 111
Problems w/ Quantum error correction
① No cloning says cannot copy quantum states
② Measuring state collapses it — How can we
correct the error without measuring the data?
③


Repetition code
majority
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Problems w/ Quantum error correction
① No cloning says cannot copy quant states
② Measuring state collapses it — How ca
correct the error without measuring the data
③ Must correct X, Y, Z
④

Repetition code

$$0 \mapsto 000 \xleftarrow{\text{majority}} 010$$

$$1 \mapsto 111$$

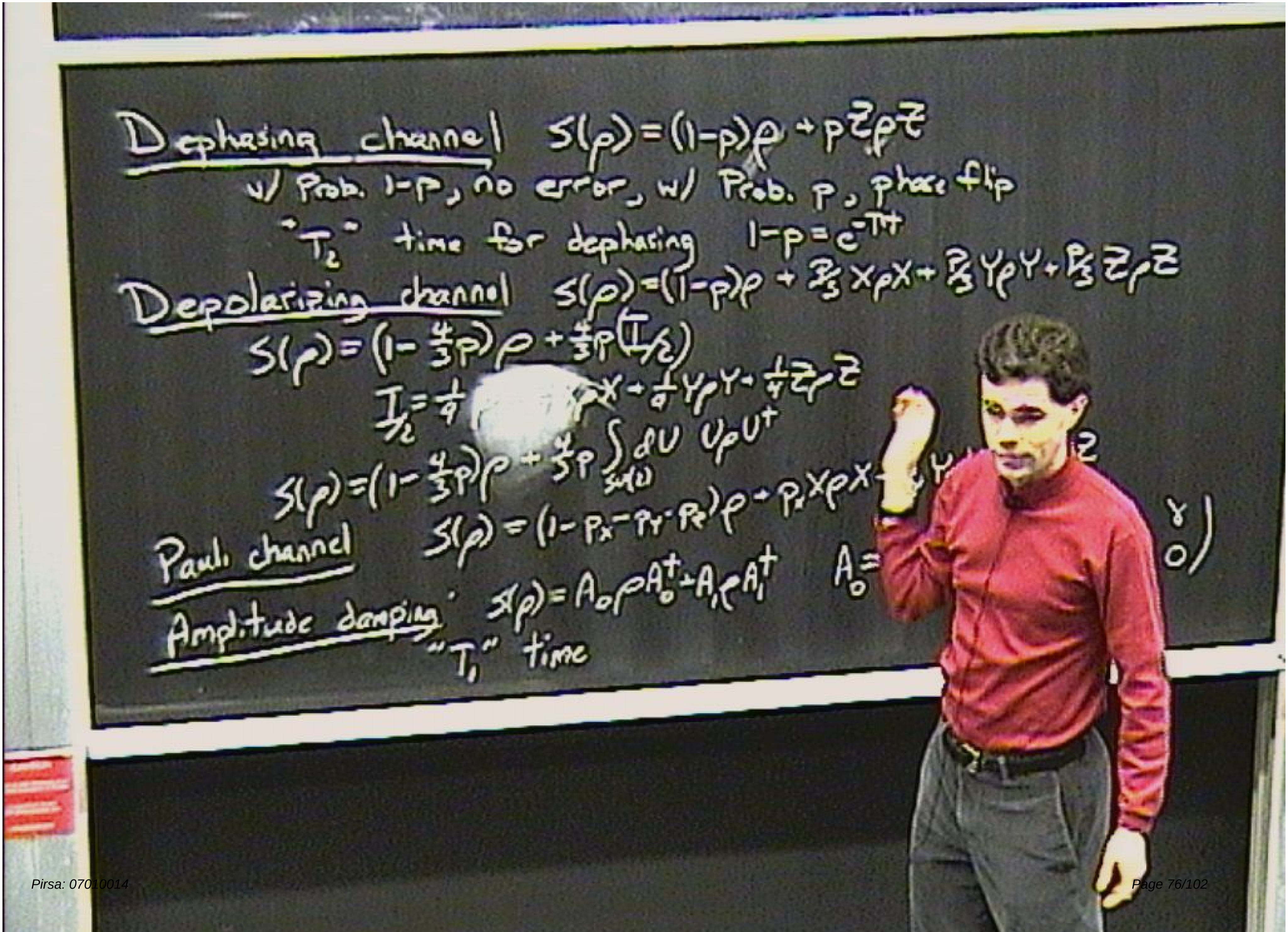
Problems w/ Quantum error correction

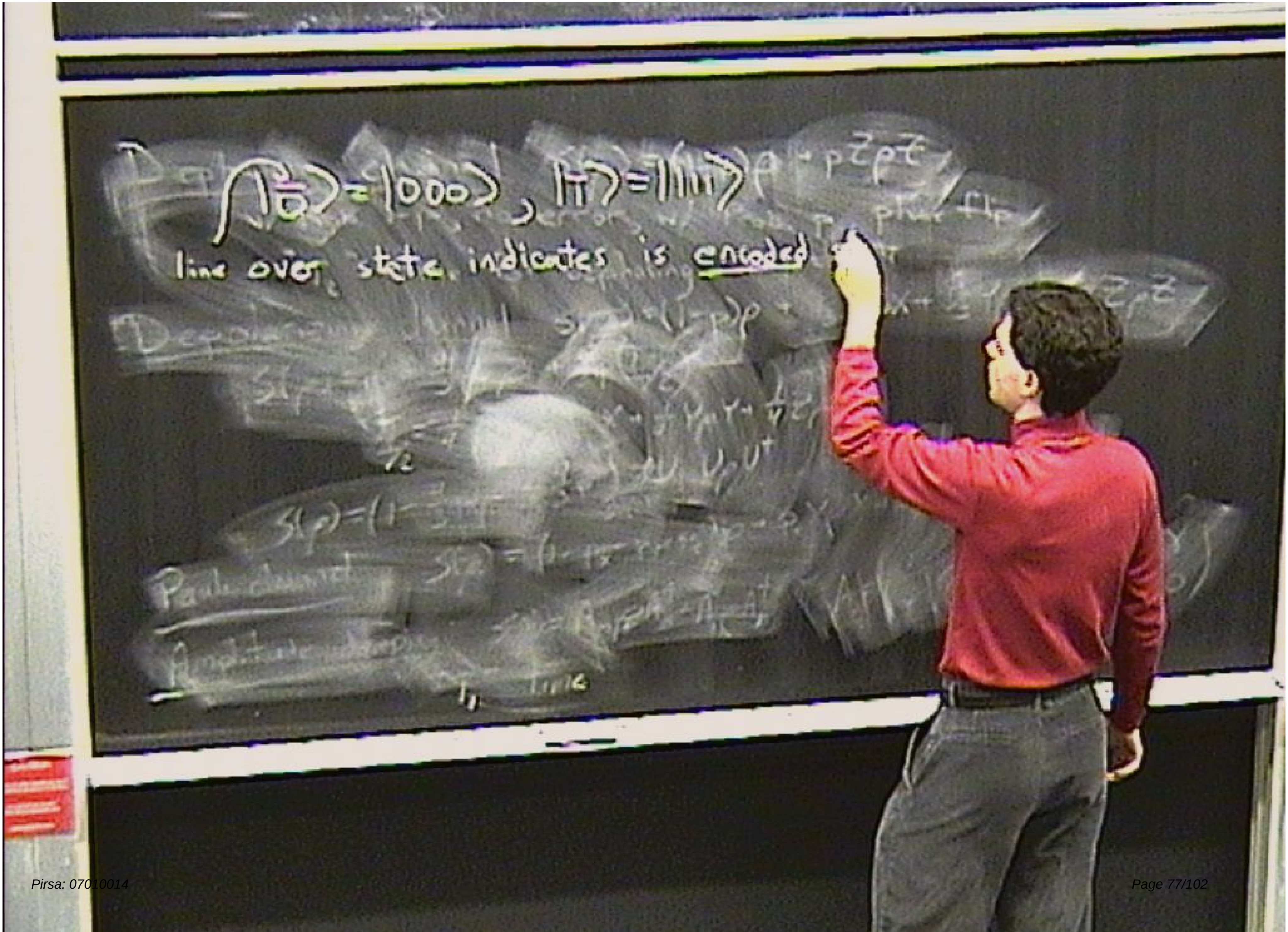
① No cloning says cannot copy quantum states

② Measuring state collapses it — How can we correct the error without measuring the data?

③ Must correct X, Y, Z

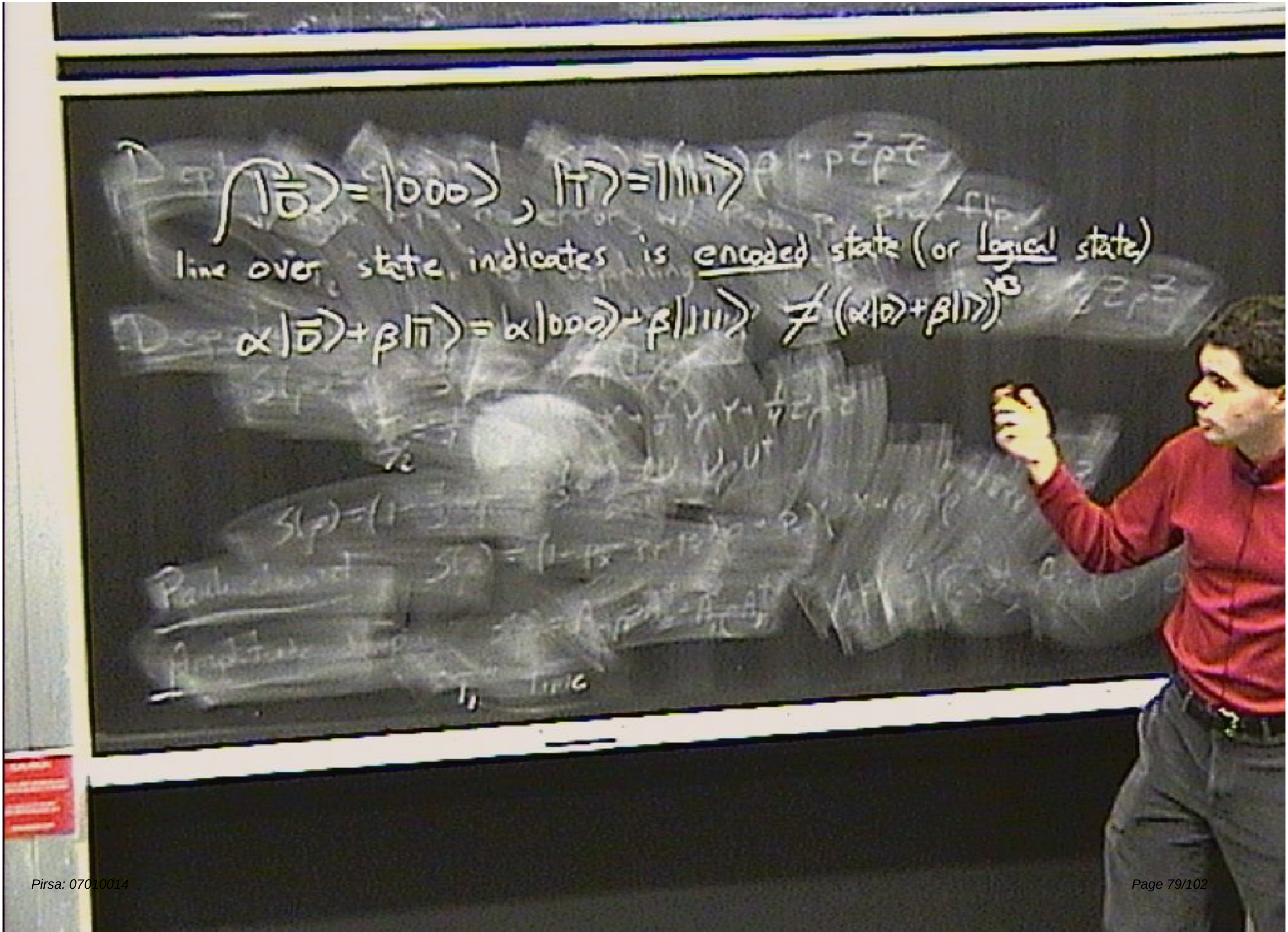
④ What about continuous rotations, decoherence, etc.?


Dephasing channel $S(\rho) = (1-p)\rho + pZ\rho Z$
w/ Prob. $1-p$, no error, w/ Prob. p, phase flip
"T_2" time for dephasing $1-p = e^{-T/t}$
Depolarizing channel $S(\rho) = (1-p)\rho + \frac{p}{3}X\rho X + \frac{p}{3}Y\rho Y + \frac{p}{3}Z\rho Z$
$S(\rho) = (1-\frac{4}{3}p)\rho + \frac{4}{3}p(I/2)$
$I/2 = \frac{1}{4}$
$S(\rho) = (1-\frac{4}{3}p)\rho + \frac{4}{3}p\int_{SU(2)} dU\, U\rho U^\dagger$
Pauli channel $S(\rho) = (1-p_X-p_Y-p_Z)\rho + p_X X\rho X$
Amplitude damping $S(\rho) = A_0\rho A_0^\dagger + A_1\rho A_1^\dagger$ $A_0 =$
"T_1" time


|0̄⟩ = |000⟩ , |1̄⟩ = |111⟩
line over state indicates is encoded

$|\bar{0}\rangle = |000\rangle$, $|\bar{1}\rangle = |111\rangle$

line over state indicates is encoded state (or logical state)


$|\bar{0}\rangle = |000\rangle$, $|\bar{1}\rangle = |111\rangle$
line over state indicates is encoded state (or logical state)
$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$

$|\bar{0}\rangle = |000\rangle$, $|\bar{1}\rangle = |111\rangle$

line over state indicates is encoded state (or logical state)

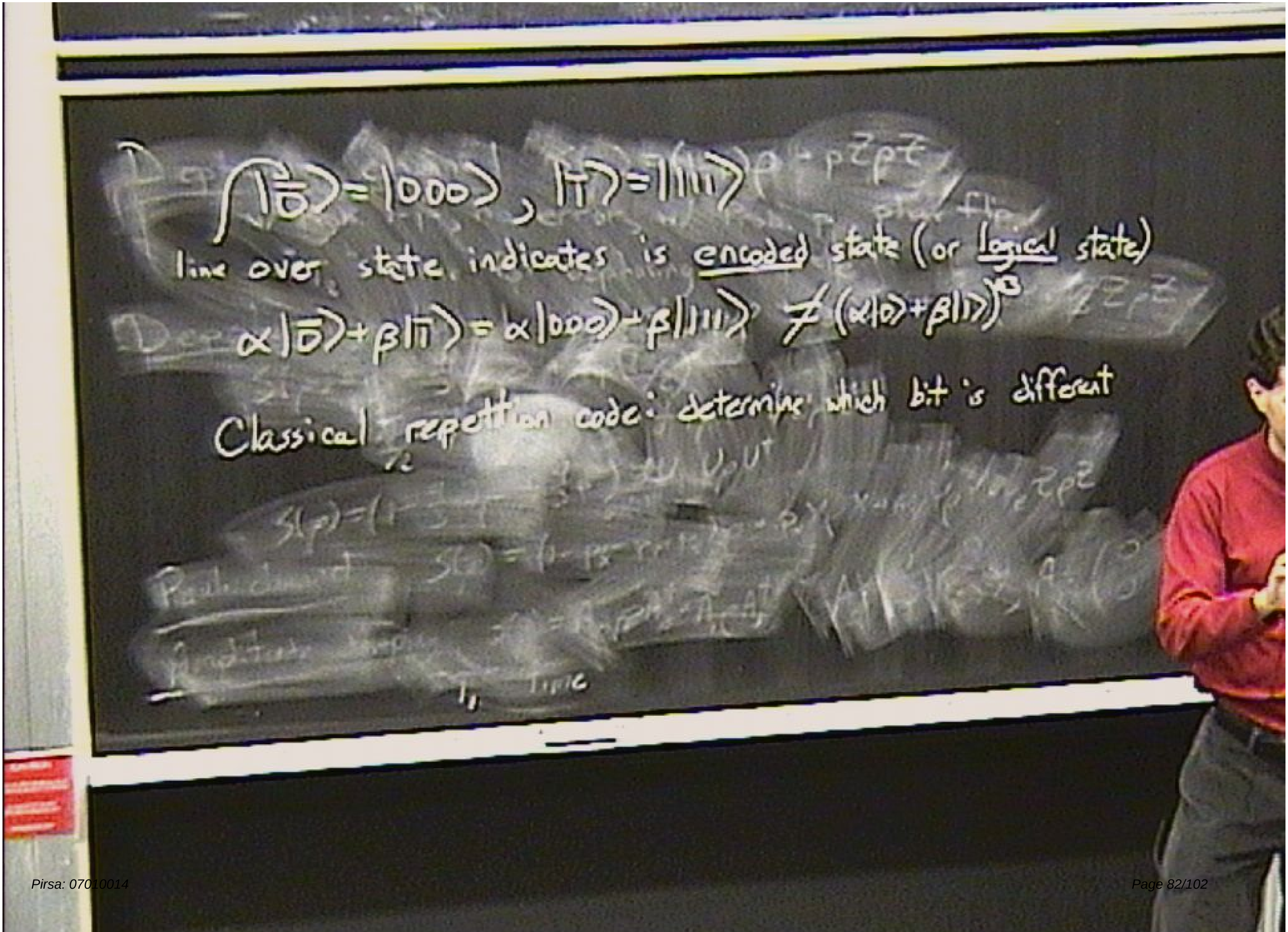
$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$

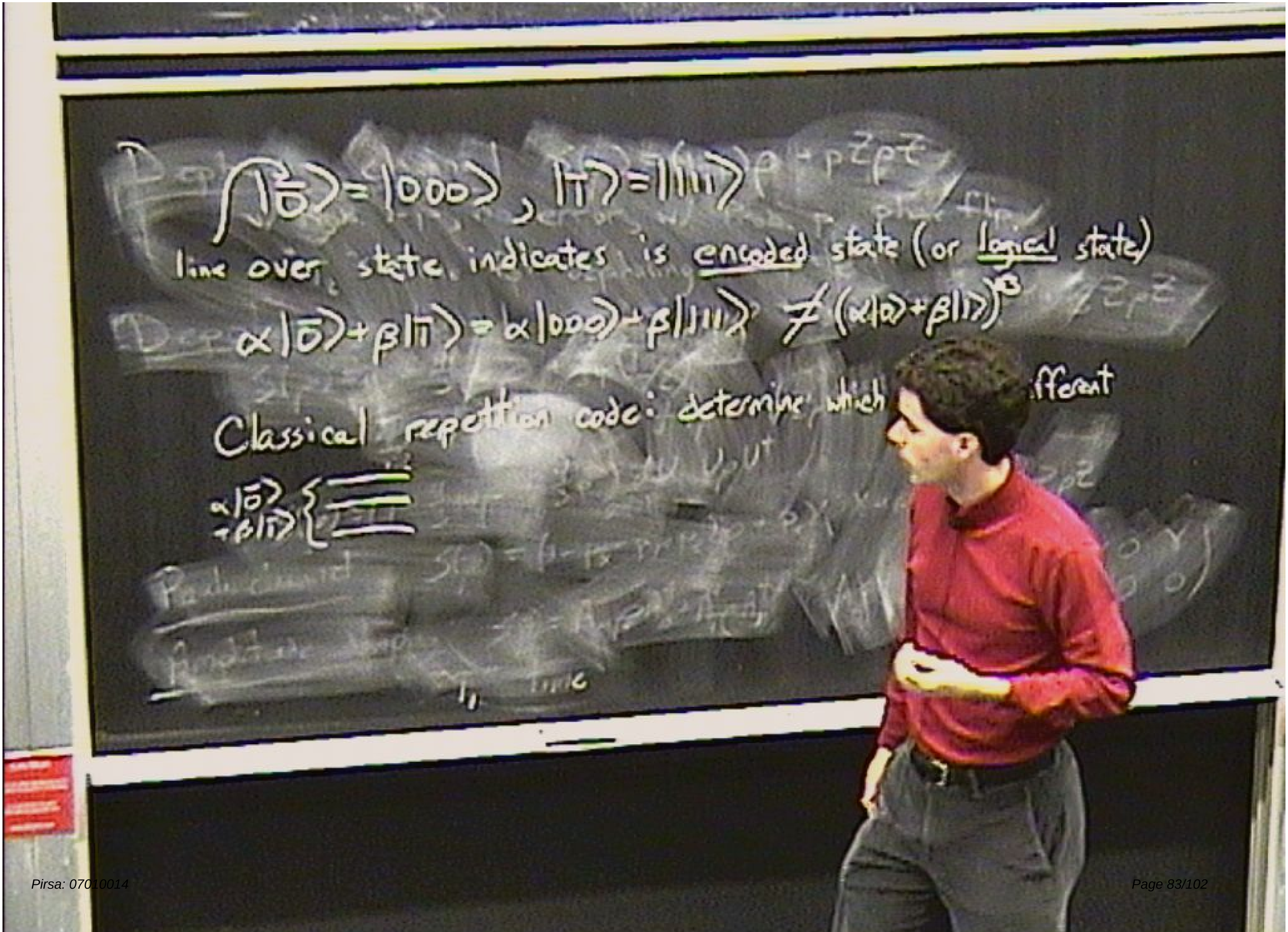
$|\bar{0}\rangle = |000\rangle, \; |\bar{1}\rangle = |111\rangle$

line over state indicates is <u>encoded</u> state (or <u>logical</u> state)

$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$

Classical repetition code:


$|\bar{0}\rangle = |000\rangle$, $|\bar{1}\rangle = |111\rangle$
line over state indicates is encoded state (or logical state)
$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$
Classical repetition code: determine which bit is different



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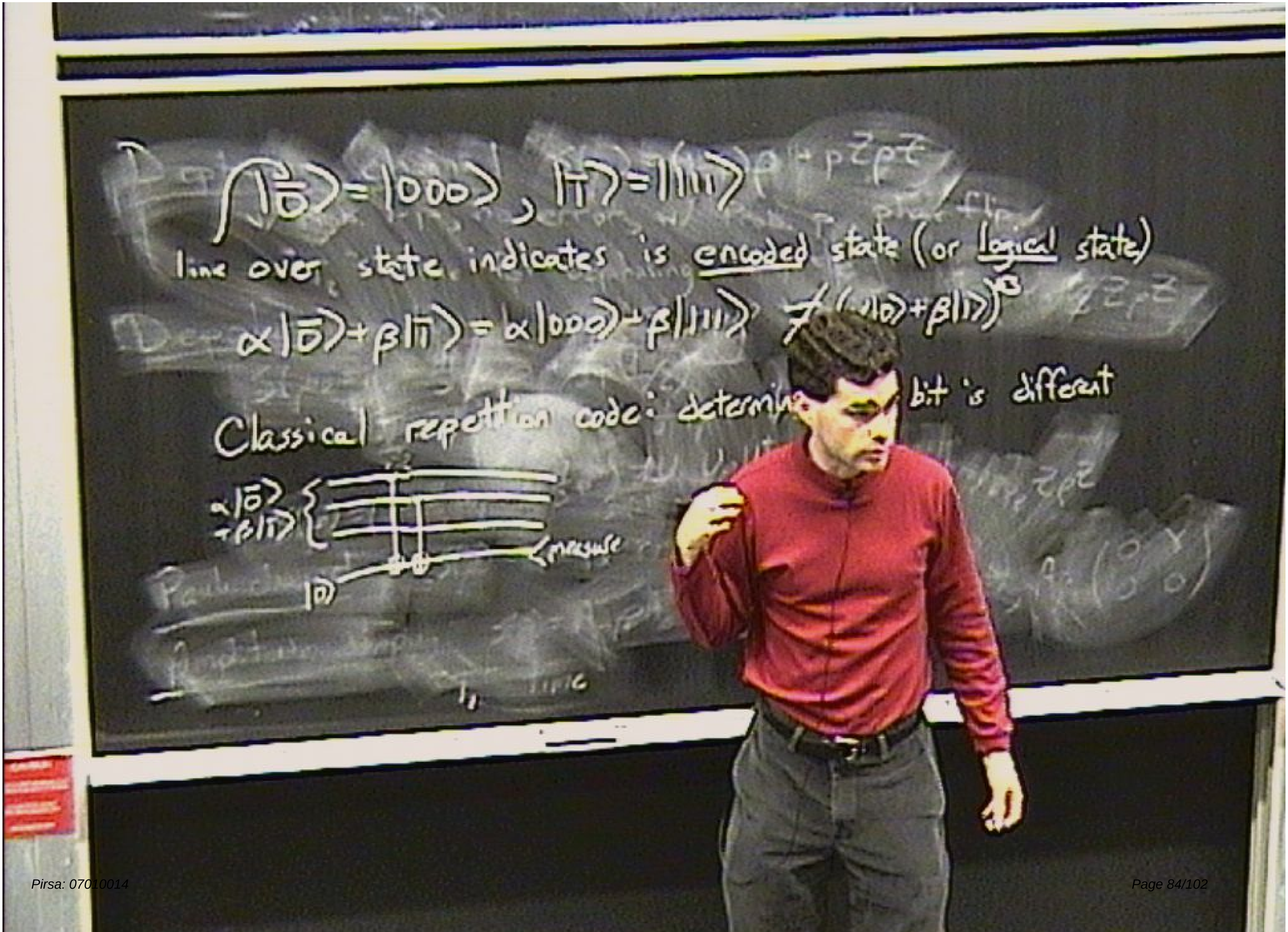
Classical repetition code: determine which
fferent
$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$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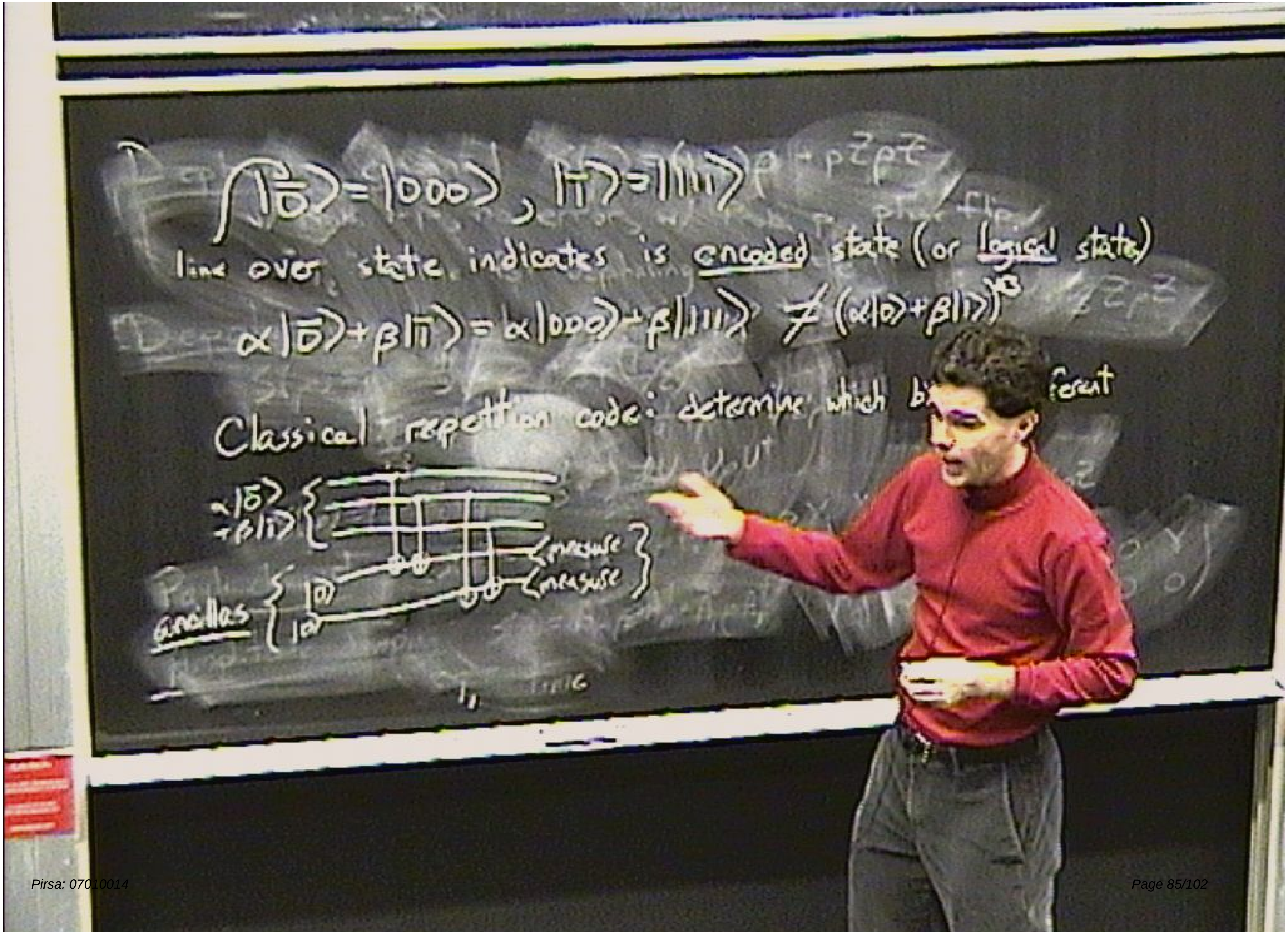
Classical repetition code: determin
bit is different
$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$
measure
$|0\rangle$

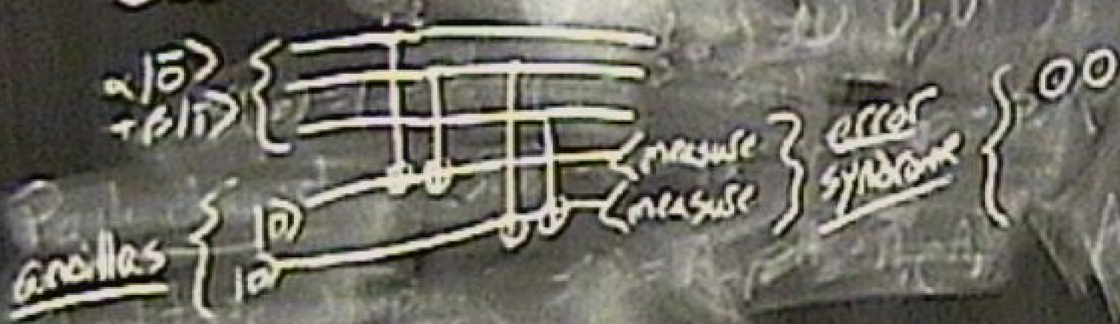

|0̄⟩ = |000⟩ , |1̄⟩ = |111⟩
line over state indicates is encoded state (or logical state)
α|0̄⟩ + β|1̄⟩ = α|000⟩ + β|111⟩ ≠ (α|0⟩ + β|1⟩)⊗3
Classical repetition code: determine which
α|0̄⟩ + β|1̄⟩
measure
measure
ancillas
|0⟩
|0⟩

$|\bar{0}\rangle = |000\rangle$, $|\bar{1}\rangle = |111\rangle$

line over state indicates is encoded state (or logical state)

$$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$$

Classical repetition code: determine which bit is different



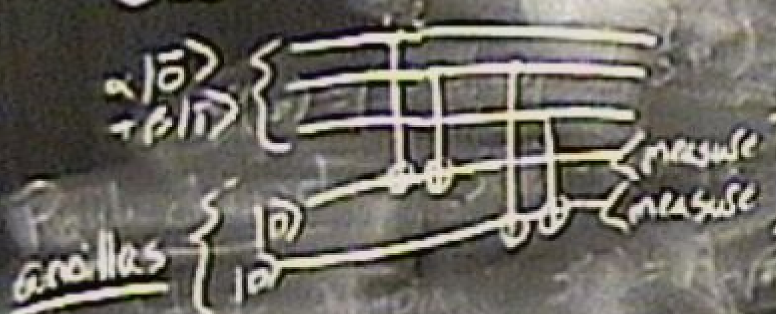
time

$|\bar{0}\rangle = |000\rangle$, $|\bar{1}\rangle = |111\rangle$

line over state indicates is encoded state (or logical state)

$$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$$

Classical repetition code: determine which bit is different



error syndrome

	1-qubit error
00	No error

time

$|\bar{0}\rangle = |000\rangle, \quad |\bar{1}\rangle = |111\rangle$

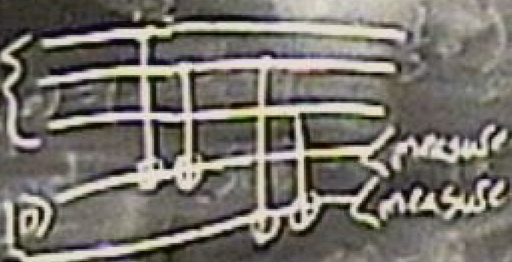
line over state indicates is encoded state (or logical state)

$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$

Classical repetition code: determine which bit is different

$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle$

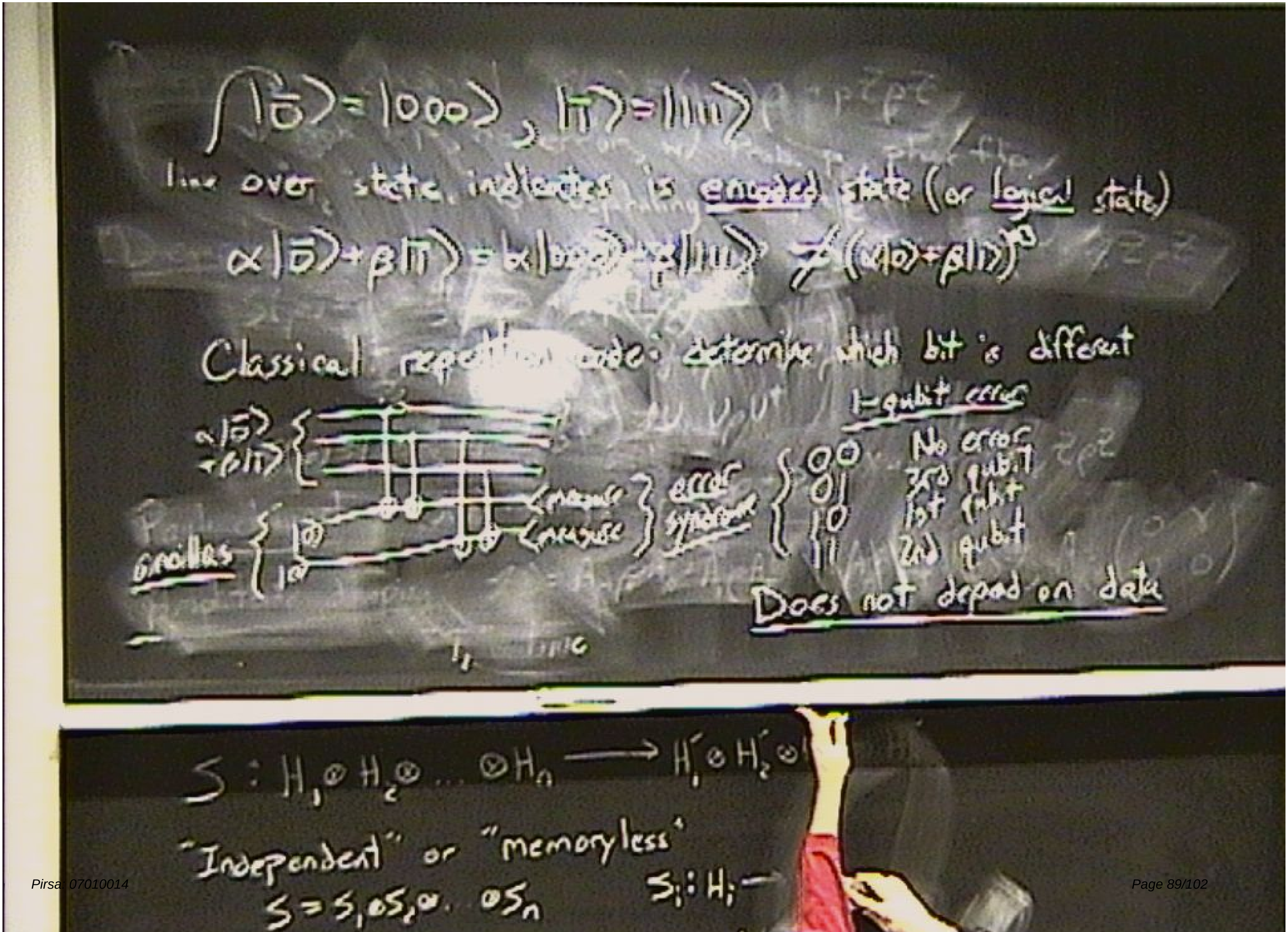
ancillas $|0\rangle$, $|0\rangle$

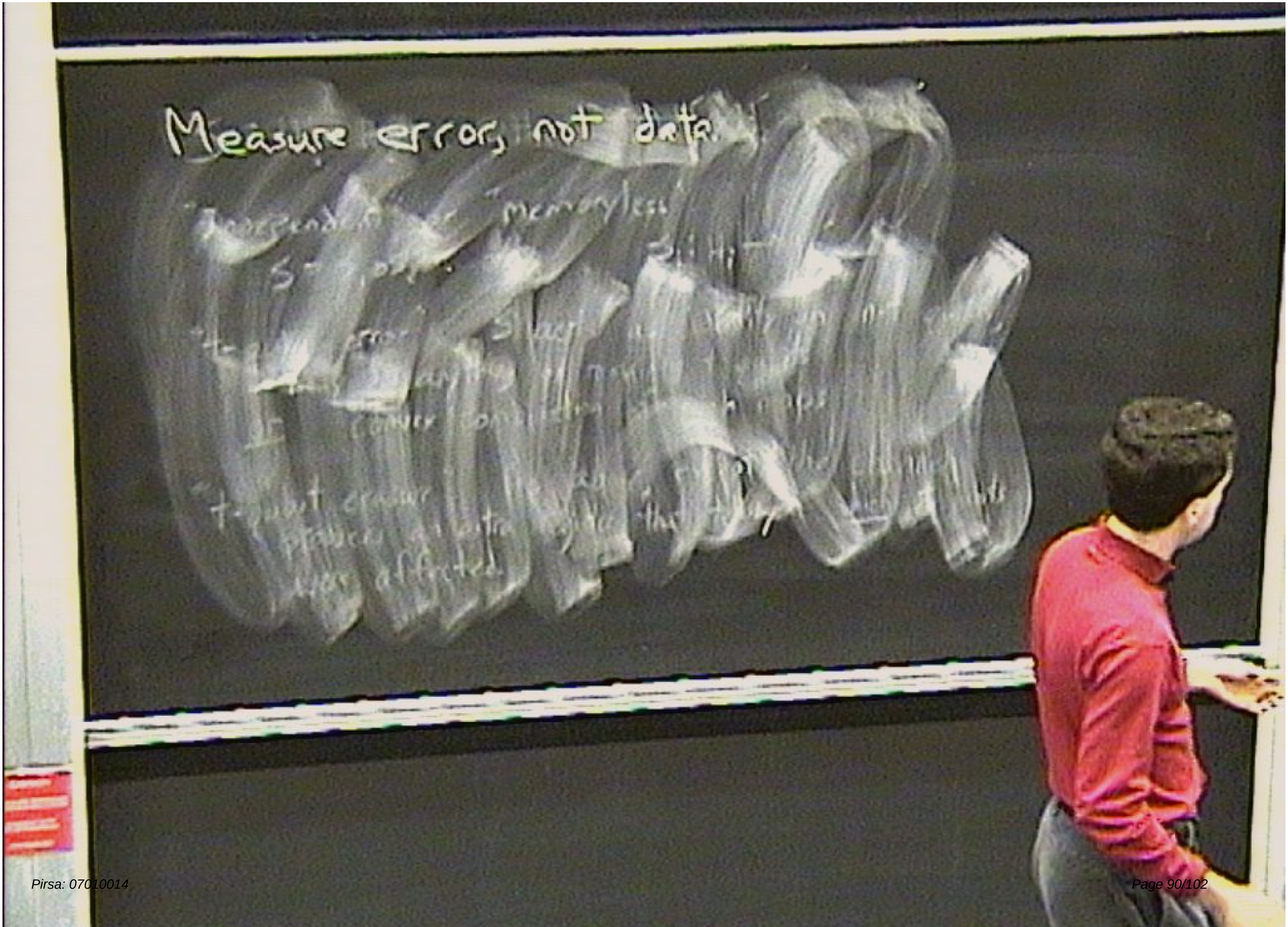


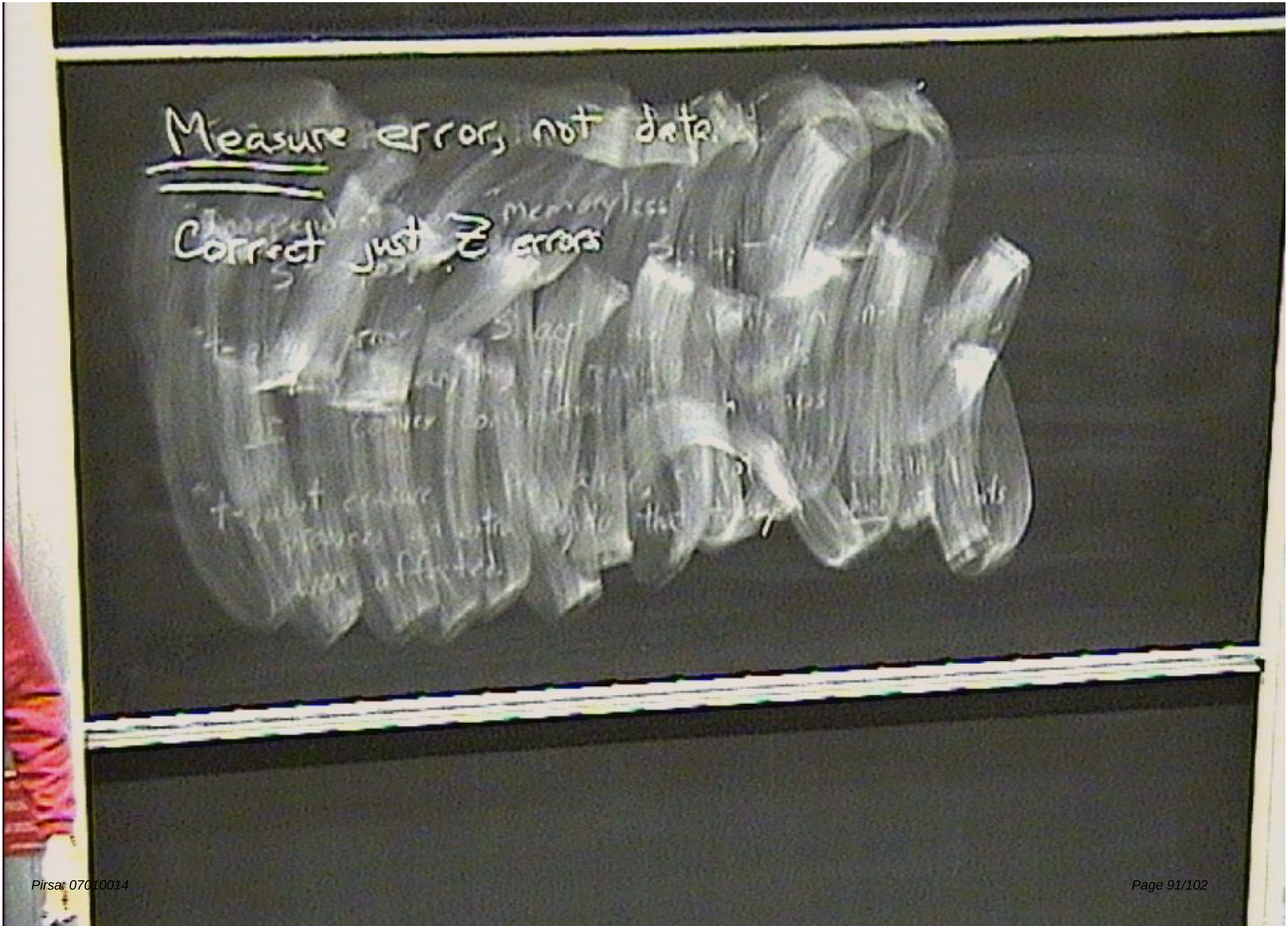
error syndrome

	1-qubit error
00	No error
01	3rd qubit
10	1st qubit
11	2nd qubit

time


|0̄⟩ = |000⟩ , |1̄⟩ = |111⟩
line over state indicates is encoded state (or logical state)
α|0̄⟩ + β|1̄⟩ = α|000⟩ + β|111⟩ ≠ (α|0⟩ + β|1⟩)⊗3
Classical repetition code: determine which bit is different
1-qubit error
00 No error
01 3rd qubit
10 1st qubit
11 2nd qubit
error syndrome
measure
measure
ancillas
Does not depend on data
time
S : H1 ⊗ H2 ⊗ ... ⊗ Hn ⟶ H1′ ⊗ H2′ ⊗
"Independent" or "memoryless"
S = S1 ⊗ S2 ⊗ ... ⊗ Sn
Si : Hi ⟶


Measure error, not data.


Measure error, not data.
Correct just Z errors

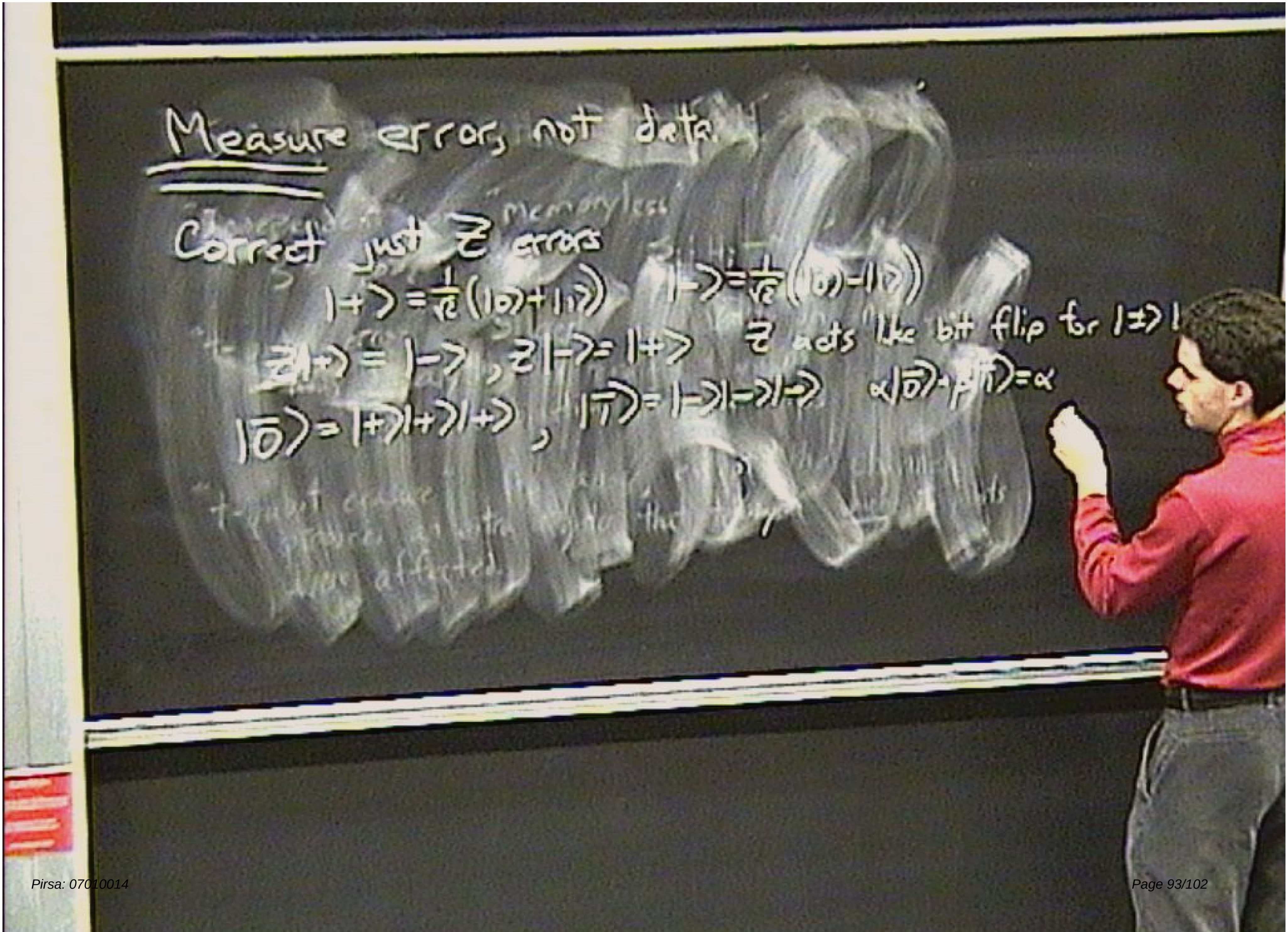
Measure error, not data.

Correct just Z errors

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Z acts like bit flip for $|\pm\rangle$ basis

$$Z|+\rangle = |-\rangle, \quad Z|-\rangle = |+\rangle$$


Measure error, not data
Correct just Z errors
$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
Z acts like bit flip for $|\pm\rangle$
$Z|+\rangle = |-\rangle$, $Z|-\rangle = |+\rangle$
$|\bar{0}\rangle = |+\rangle|+\rangle|+\rangle$, $|\bar{1}\rangle = |-\rangle|-\rangle|-\rangle$ $\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha$

Measure errors, not data.

Correct just Z errors

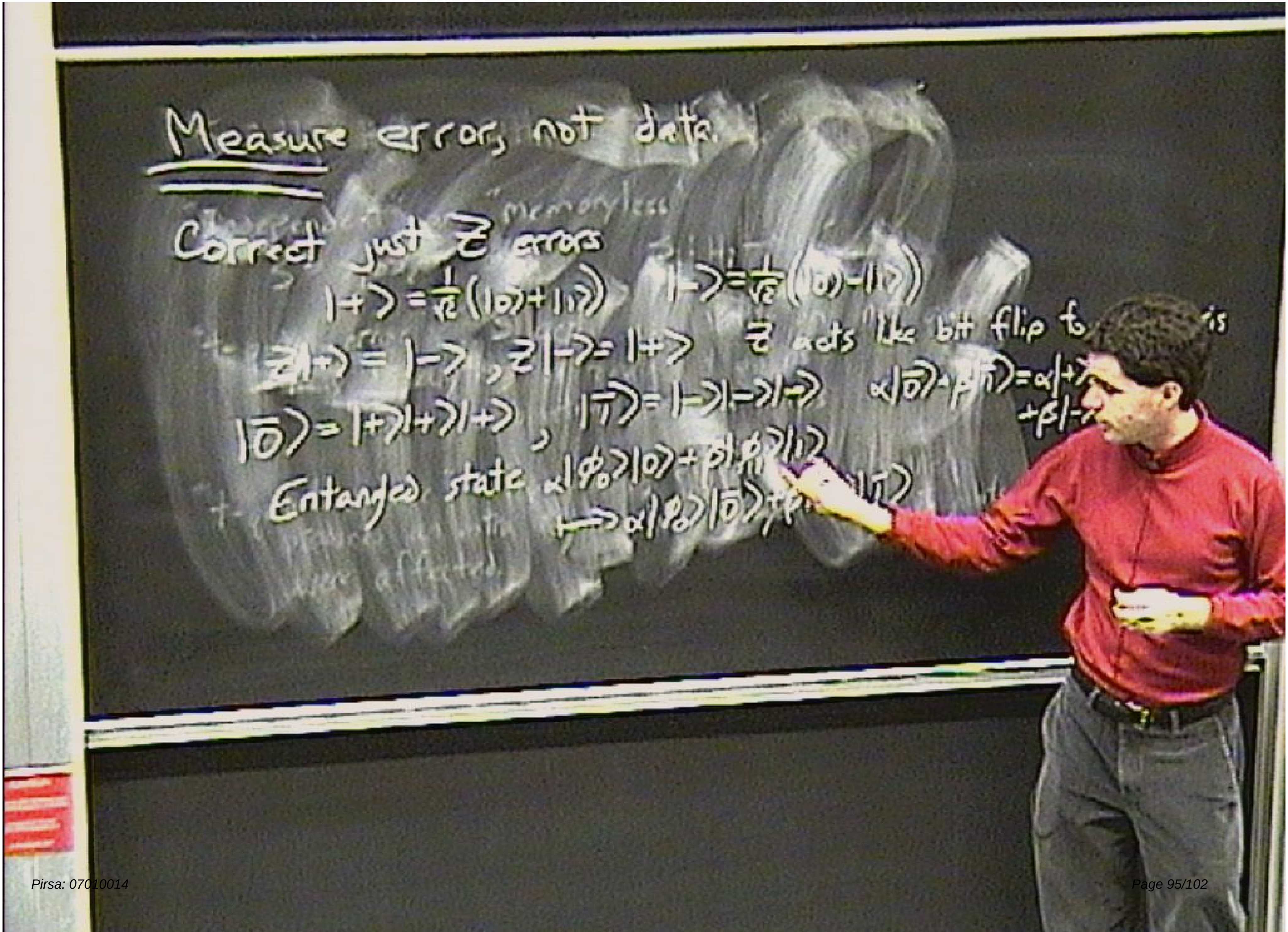
$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

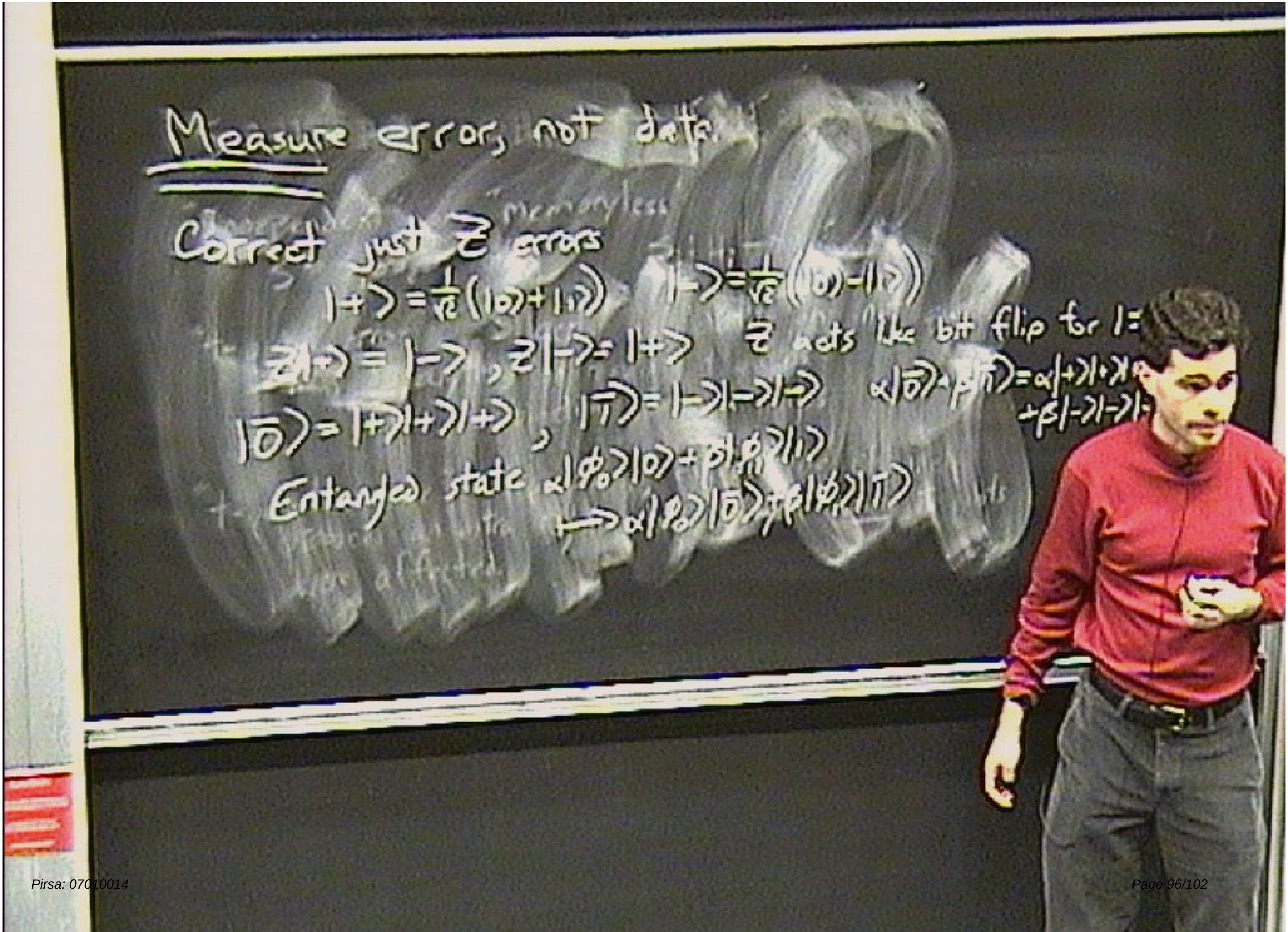
Z acts like bit flip for $|\pm\rangle$ basis

$Z|+\rangle = |-\rangle$, $Z|-\rangle = |+\rangle$

$|\bar{0}\rangle = |+\rangle|+\rangle|+\rangle$, $|\bar{1}\rangle = |-\rangle|-\rangle|-\rangle$

$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|+\rangle|+\rangle|+\rangle + \beta|-\rangle|-\rangle|-\rangle$


Measure error, not data.
memoryless
Correct just Z errors
|+> = 1/√2 (|0>+|1>)
|-> = 1/√2 (|0>-|1>)
Z|+> = |->, Z|-> = |+>
Z acts like bit flip
|0̄> = |+>|+>|+>, |1̄> = |->|->|->
Entangled state α|φ0>|0>+β|φ1>|1>


Measure error, not data.
memoryless
Correct just Z errors
$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
Z acts like bit flip for $1=$
$Z|+\rangle = |-\rangle$, $Z|-\rangle = |+\rangle$
$|\bar{0}\rangle = |+\rangle|+\rangle|+\rangle$, $|\bar{1}\rangle = |-\rangle|-\rangle|-\rangle$
$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|+\rangle|+\rangle$
$+\beta|-\rangle|-\rangle$
Entangled state $\alpha|\phi_0\rangle|0\rangle + \beta|\phi_1\rangle|1\rangle$
$\longmapsto \alpha|\phi_0\rangle|\bar{0}\rangle + \beta|\phi_1\rangle|\bar{1}\rangle$

Measure error, not data

Correct just Z errors

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Z acts like bit flip for $|\pm\rangle$ basis

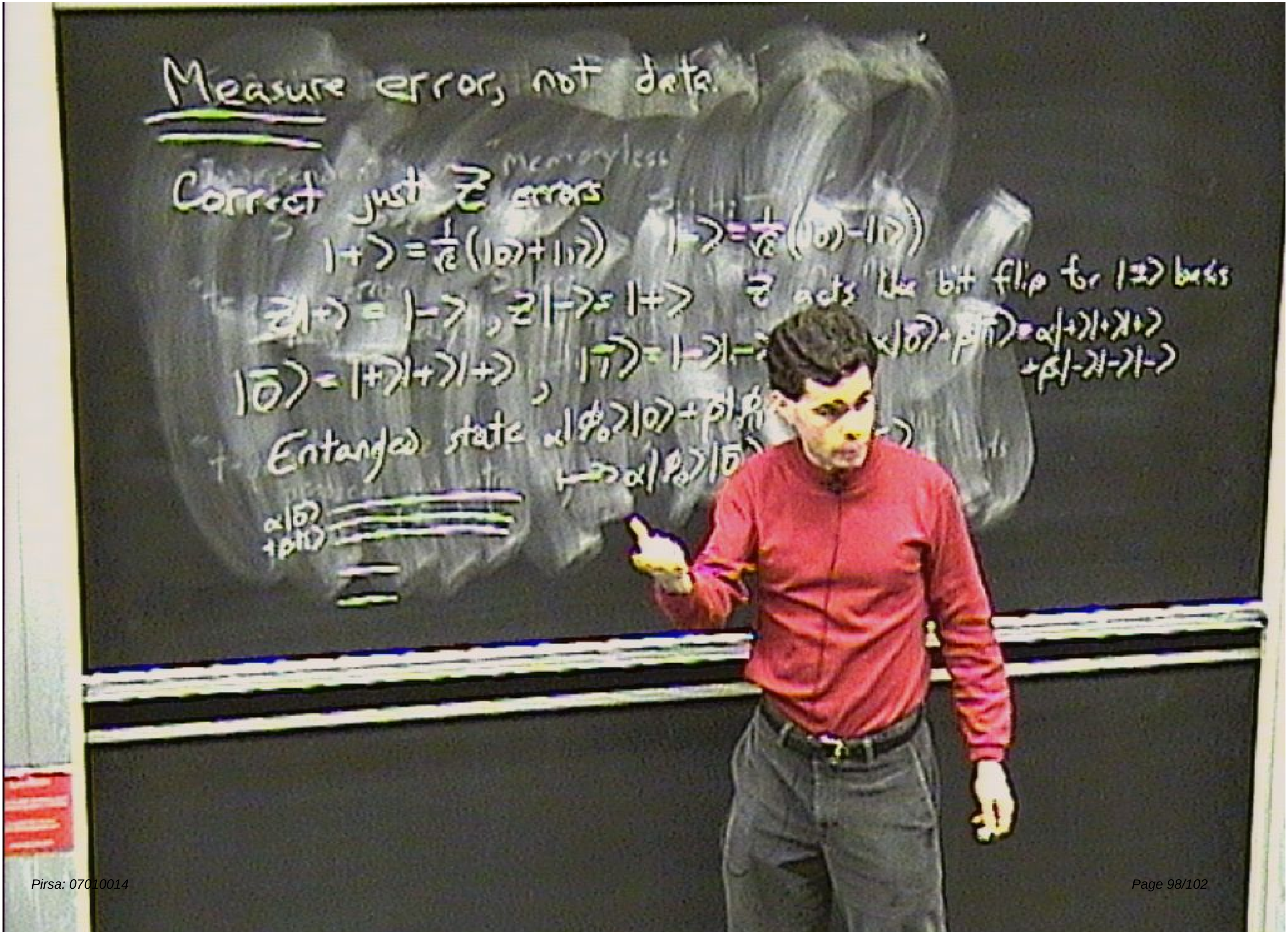
$Z|+\rangle = |-\rangle$, $Z|-\rangle = |+\rangle$

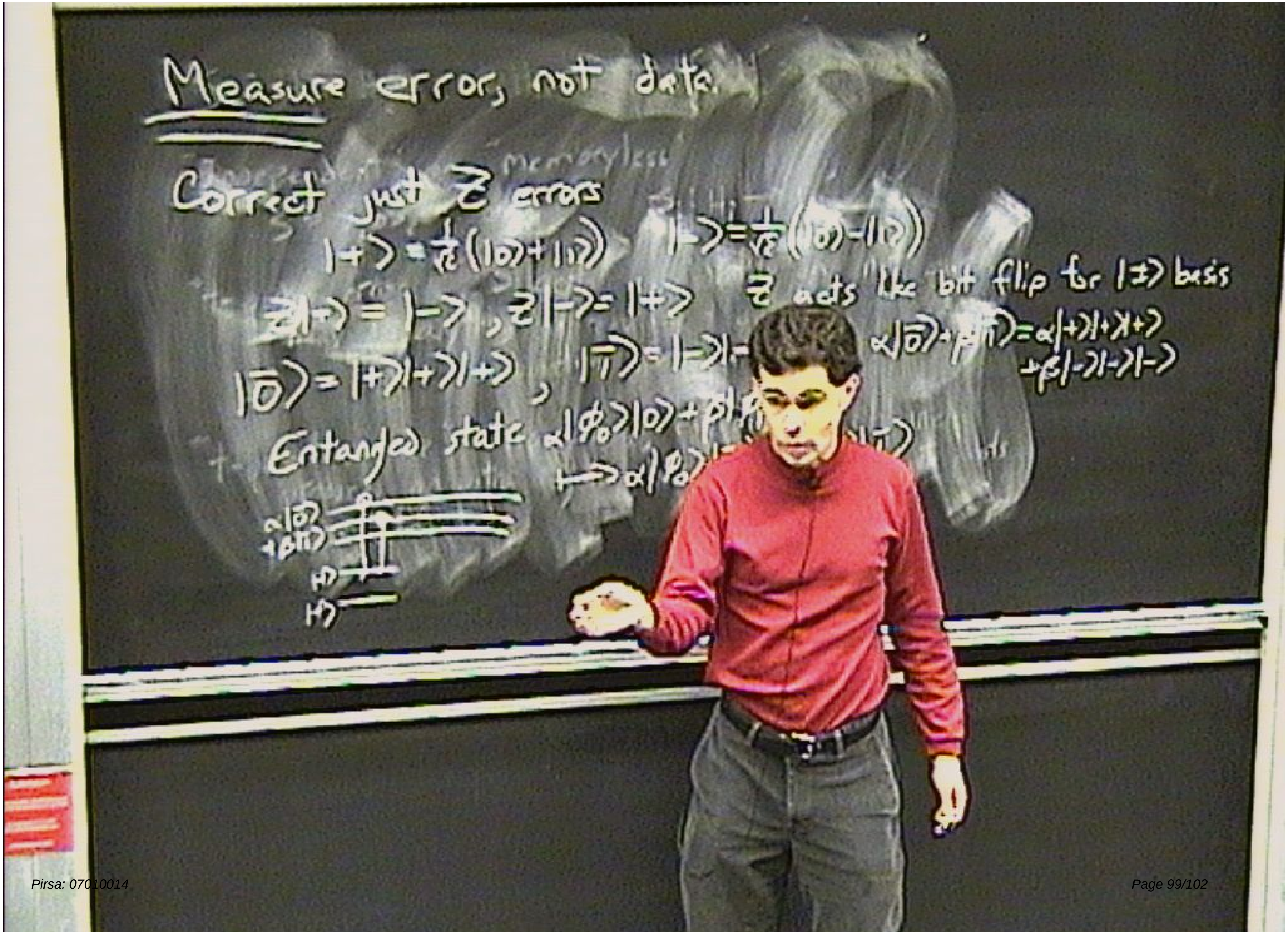
$|\bar{0}\rangle = |+\rangle|+\rangle|+\rangle$, $|\bar{1}\rangle = |-\rangle|-\rangle|-\rangle$

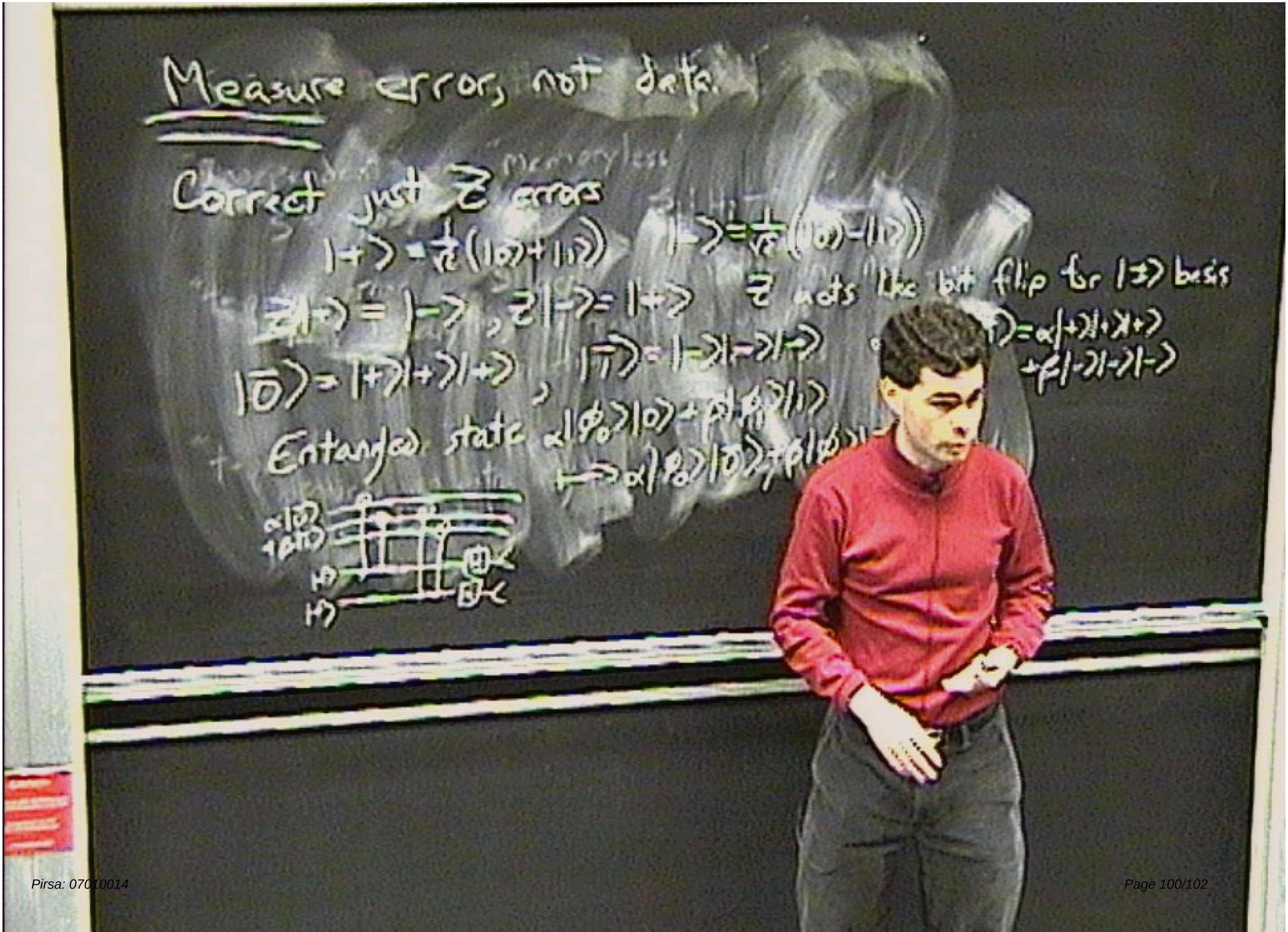
$\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|+\rangle|+\rangle|+\rangle + \beta|-\rangle|-\rangle|-\rangle$

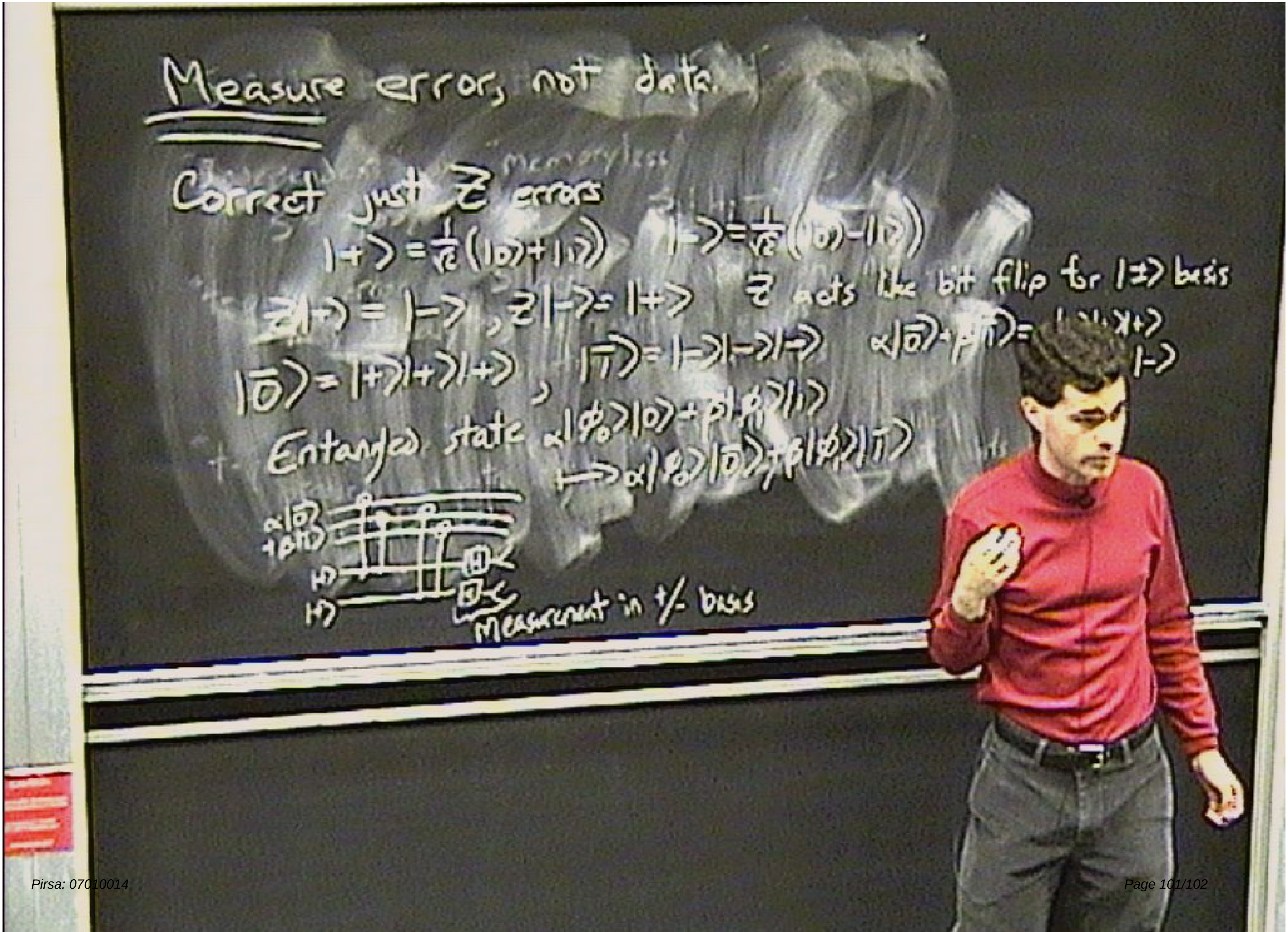
Entangled state $\alpha|\phi_0\rangle|0\rangle + \beta|\phi_1\rangle|1\rangle$

$\longmapsto \alpha|\phi_0\rangle|\bar{0}\rangle + \beta|\phi_1\rangle|\bar{1}\rangle$


Measure error, not data
memoryless
Correct just Z errors
|+> = 1/√2 (|0> + |1>)
|-> = 1/√2 (|0> - |1>)
Z acts like bit flip for |±> basis
Z|+> = |->, Z|-> = |+>
|0> = |+>|+>|+>
Entangled state


Measure error, not data.
Correct just Z errors
|+> = 1/√2 (|0>+|1>)
|-> = 1/√2 (|0>-|1>)
Z acts like bit flip for |±> basis
Z|+> = |->, Z|-> = |+>
|0̄> = |+>|+>|+>
α|0̄>+β|1̄> = α|+>|+>|+>
+β|->|->|->
Entangled state


Measure error, not data.
memoryless
Correct just Z errors
|+> = 1/√2 (|0>+|1>)
|-> = 1/√2 (|0>-|1>)
Z acts like bit flip for |±> basis
Z|+> = |->, Z|-> = |+>
|0̄> = |+>|+>|+>, |1̄> = |->|->|->
Entangled state


Measure error, not data.
Correct just Z errors
|+> = 1/√2 (|0> + |1>) |-> = 1/√2 (|0> - |1>)
Z acts like bit flip for |±> basis
Z|+> = |->, Z|-> = |+>
|0̄> = |+>|+>|+>, |1̄> = |->|->|->
Entangled state α|φ0>|0> + β|φ1>|1>
Measurement in +/- basis

Measure error, not data.

"memoryless"

Correct just Z errors

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad$ $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$Z|+\rangle = |-\rangle$, $Z|-\rangle = |+\rangle$ $\quad$ Z acts like bit flip for $|\pm\rangle$ basis

$|\bar{0}\rangle = |+\rangle|+\rangle|+\rangle$, $|\bar{1}\rangle = |-\rangle|-\rangle|-\rangle$ $\quad$ $\alpha|\bar{0}\rangle + \beta|\bar{1}\rangle = \alpha|+\rangle|+\rangle|+\rangle + \beta|-\rangle|-\rangle|-\rangle$

Entangled state $\alpha|\phi_0\rangle|0\rangle + \beta|\phi_1\rangle|1\rangle \longmapsto \alpha|\phi_0\rangle|\bar{0}\rangle + \beta|\phi_1\rangle|\bar{1}\rangle$

Measurement in +/- basis