Title: Landau-Ginzburg perspective on Gregory-Laflamme instability

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Abstract: I begin with a brief description of the black strings in backgrounds with compact circle, the Gregory-Laflamme instability and the resulting phase transition, and the critical dimensions.Then I describe a Landau-Ginzburg thermodynamic perspective on the instability and on the order of the phase transition. Next, the approach is generalized from a circle compactification to an arbitrary torus compactification. It is shown that the transition order depends only on the number of extended dimensions. I end up with outlining several open questions and puzzles related to the outcome of the Gregory-Laflamme instability.

LG (Landau-Ginzburg) in GL (Gregory-Laflamme)

Based on: B Kol and E Sorkin, hep-th/0604015

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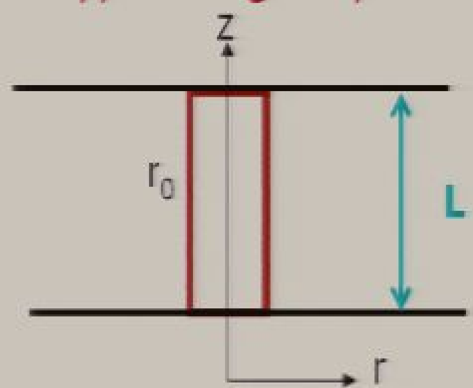
Outline:

- Gregory-Laflamme instability
- Phase transition, its order, and a critical dimension
- Ginzburg-Landau theory of phase transitions
- Application to black string
- Arbitrary torus compactification
  The critical dimension depends only on number of extended dimensions
- Some open questions

Gregory-Laflamme instability of a black string

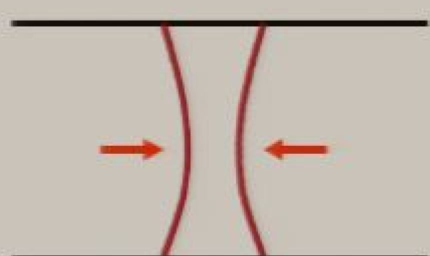
Spacetime topology - cylinder $R^{D-2,1} \times S^1$ **D=d+1**

Uniform Black-string (UBS):
wrapped along compact z-direction $\mathrm{Schw}_d + dz^2$



A single dimensionless - control -parameter

$$\mu \equiv \frac{G_N m}{L^{D-3}}$$

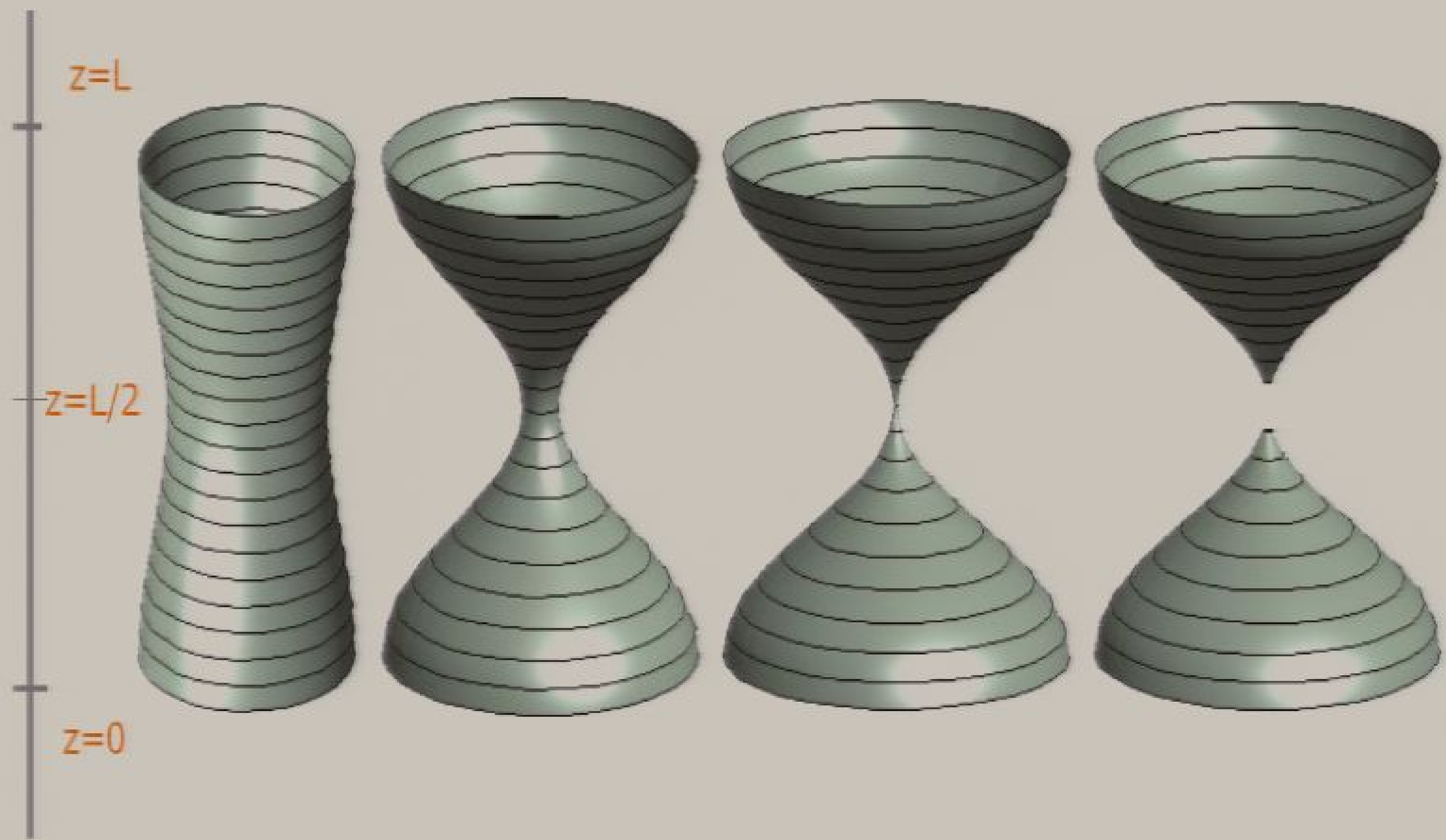


However...

$$\mu < \mu_{GL}$$

Classical growing mode – Gregory&Laflamme Instability ('93)

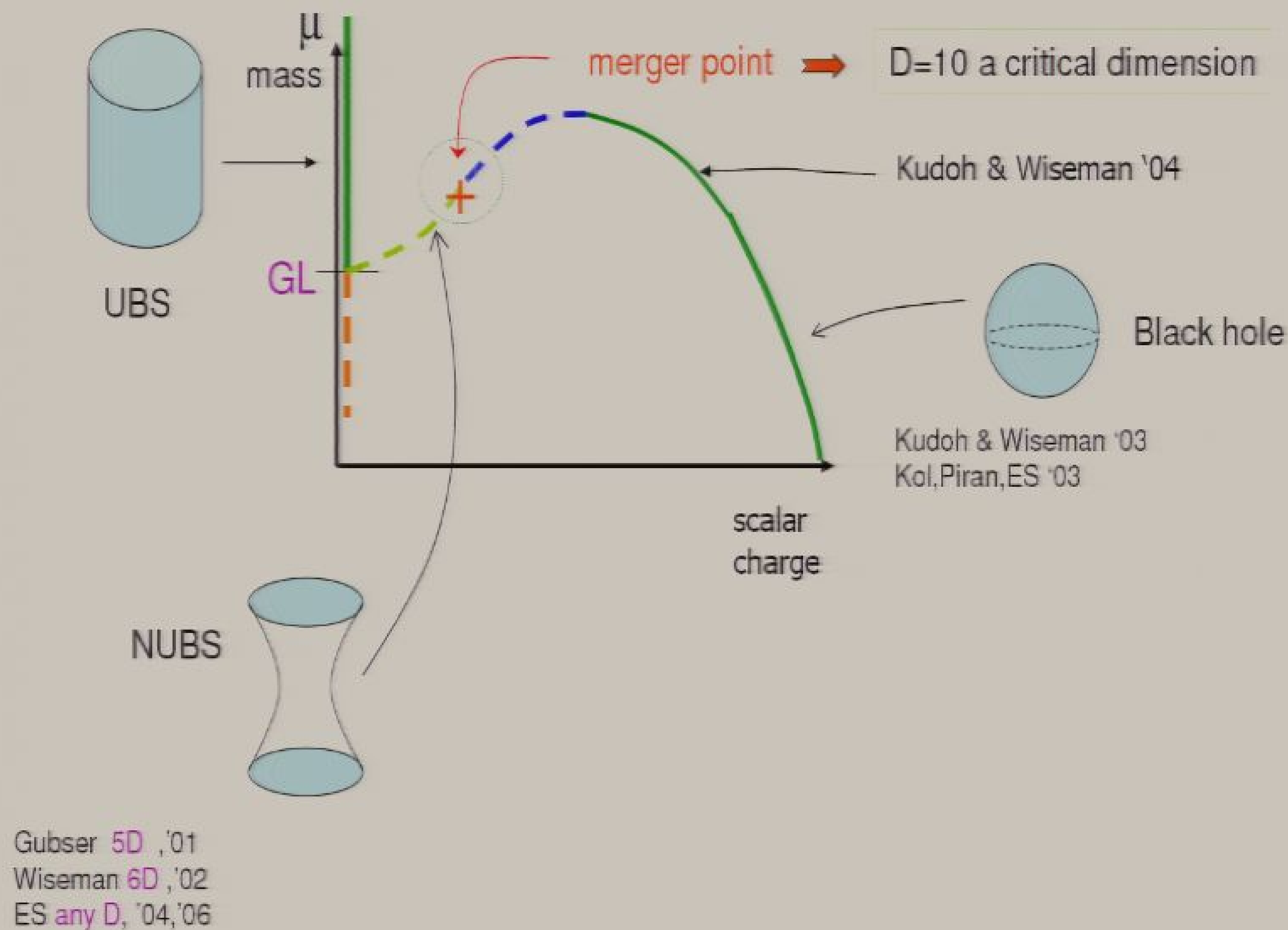
Non-uniform Black-string (NUBS) $\mu = \mu_{GL}$ marginally tachyonic (zero) mode

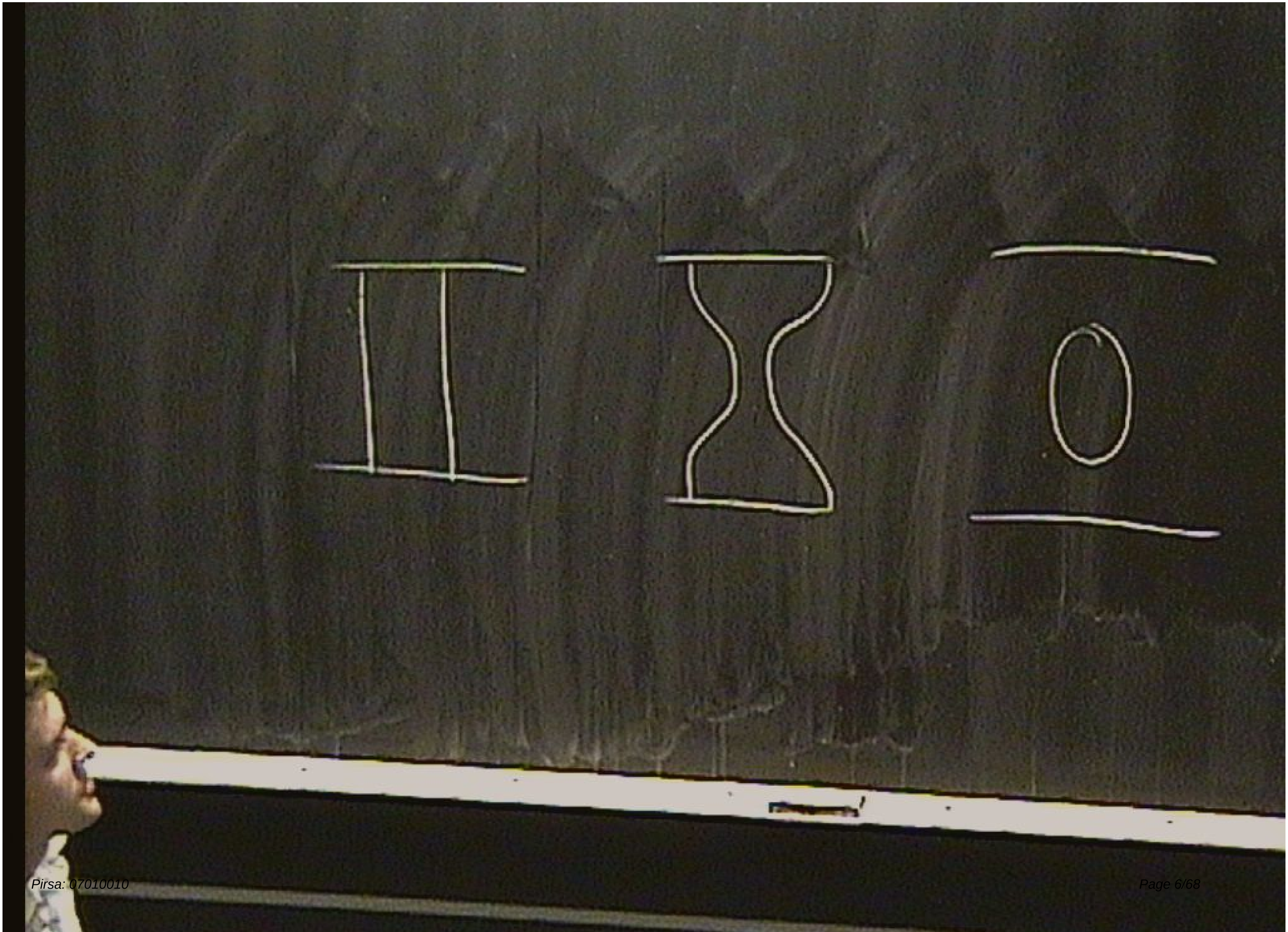


What is the endstate of GL instability?
Use dynamical evolution (Choptuik et al '03 + in progress).
Does the pinch off occur at finite (asymptotic) time?

In order to understand the phase transition and the end-state it suffices to find all static solutions, namely one has to construct a phase diagram.

A phase diagram in D<13





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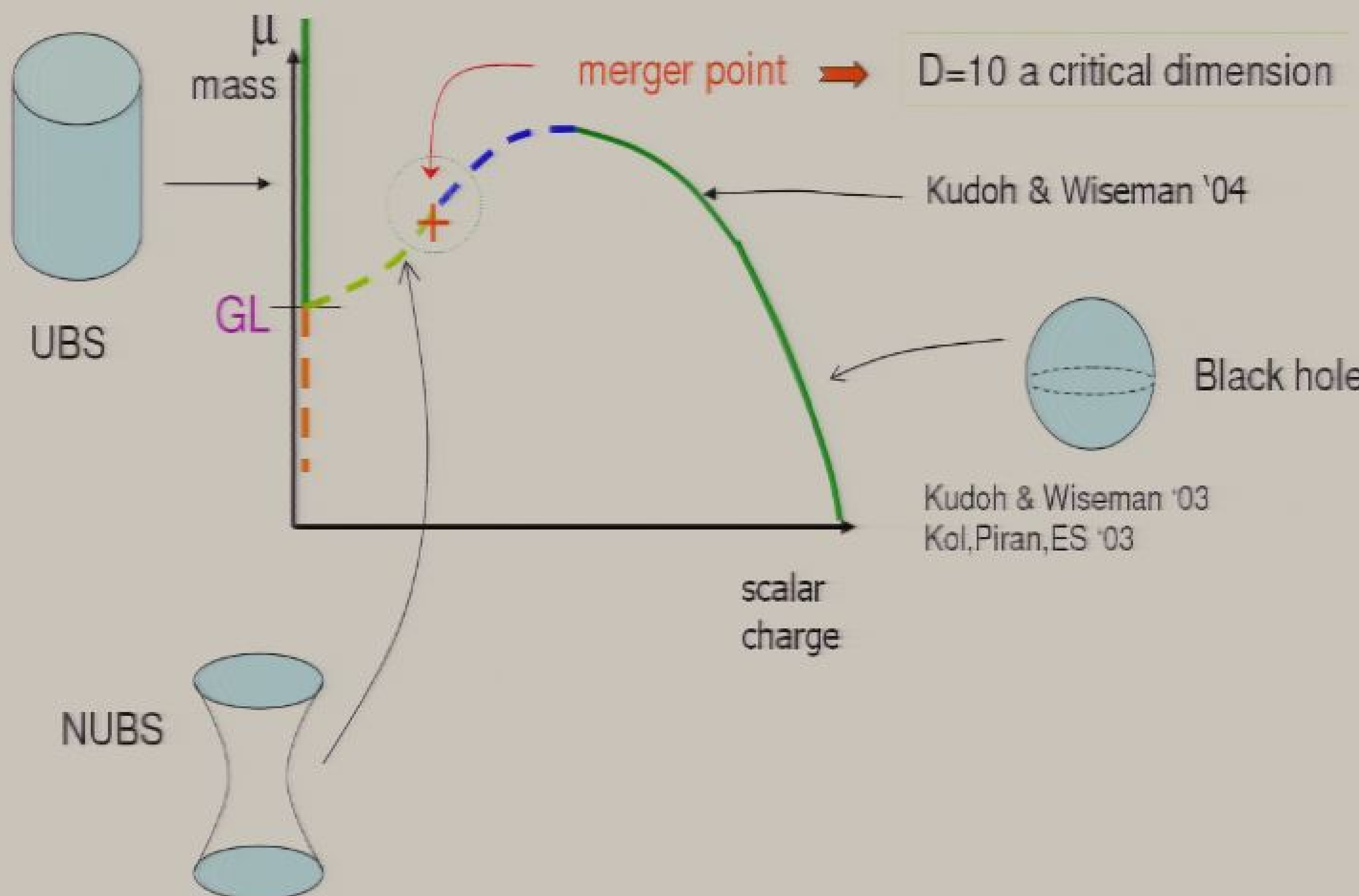
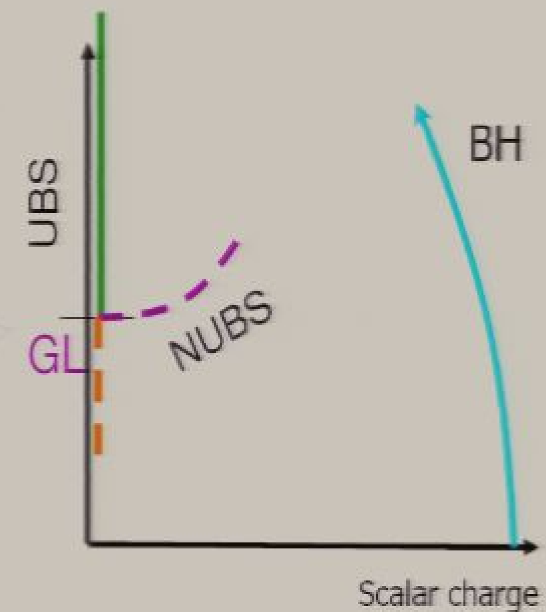


Gubser 5D ,'01
Wiseman 6D ,'02
ES any D, '04,'06

For this range of dimensions (D<13):

Perturbation analysis in 5D (Gubser): First order black-strings phase transition. Namely, the nonuniform phase develops **non-smoothly** from the GL string.

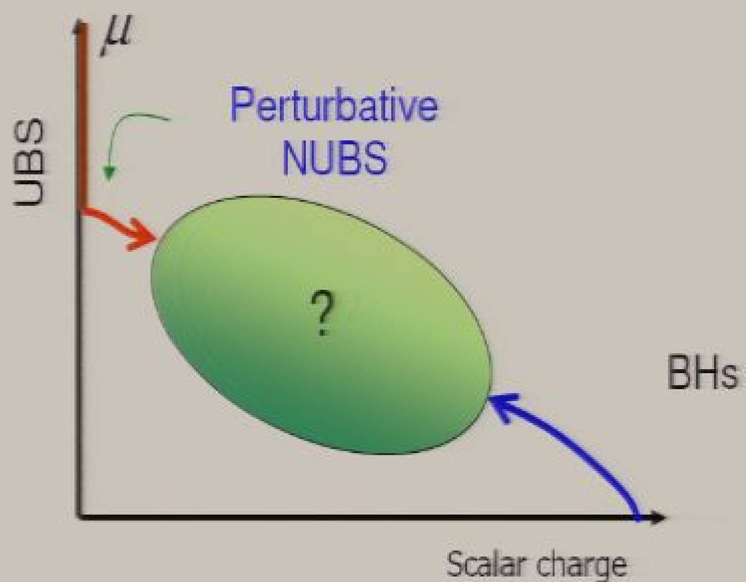


$$\mu_{NUBS} > \mu_{UBS}$$

$$S_{NUBS}(\mu) < S_{UBS}(\mu)$$

These are the two variables (mass and entropy) computed in the perturbation theory

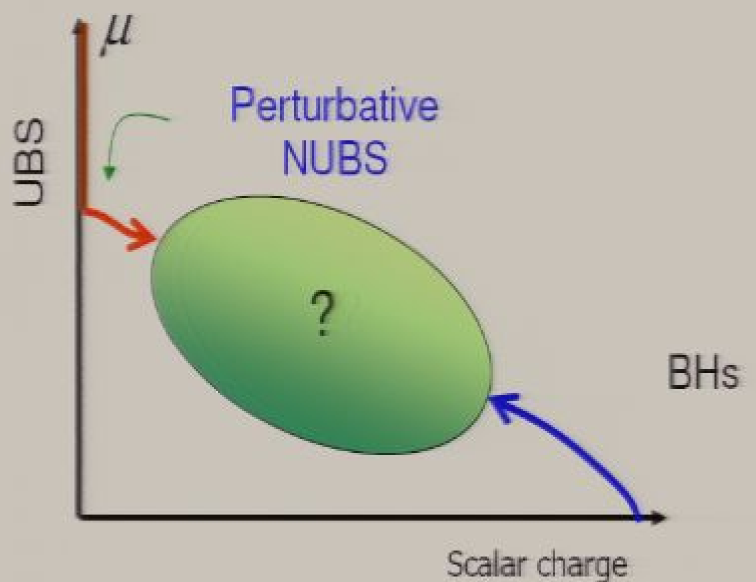
However



For **D*>13.5** a sudden change in the order of the phase transition. It becomes smooth (ES '04)

Surprising **critical dimension**: What is the physical scale?

However



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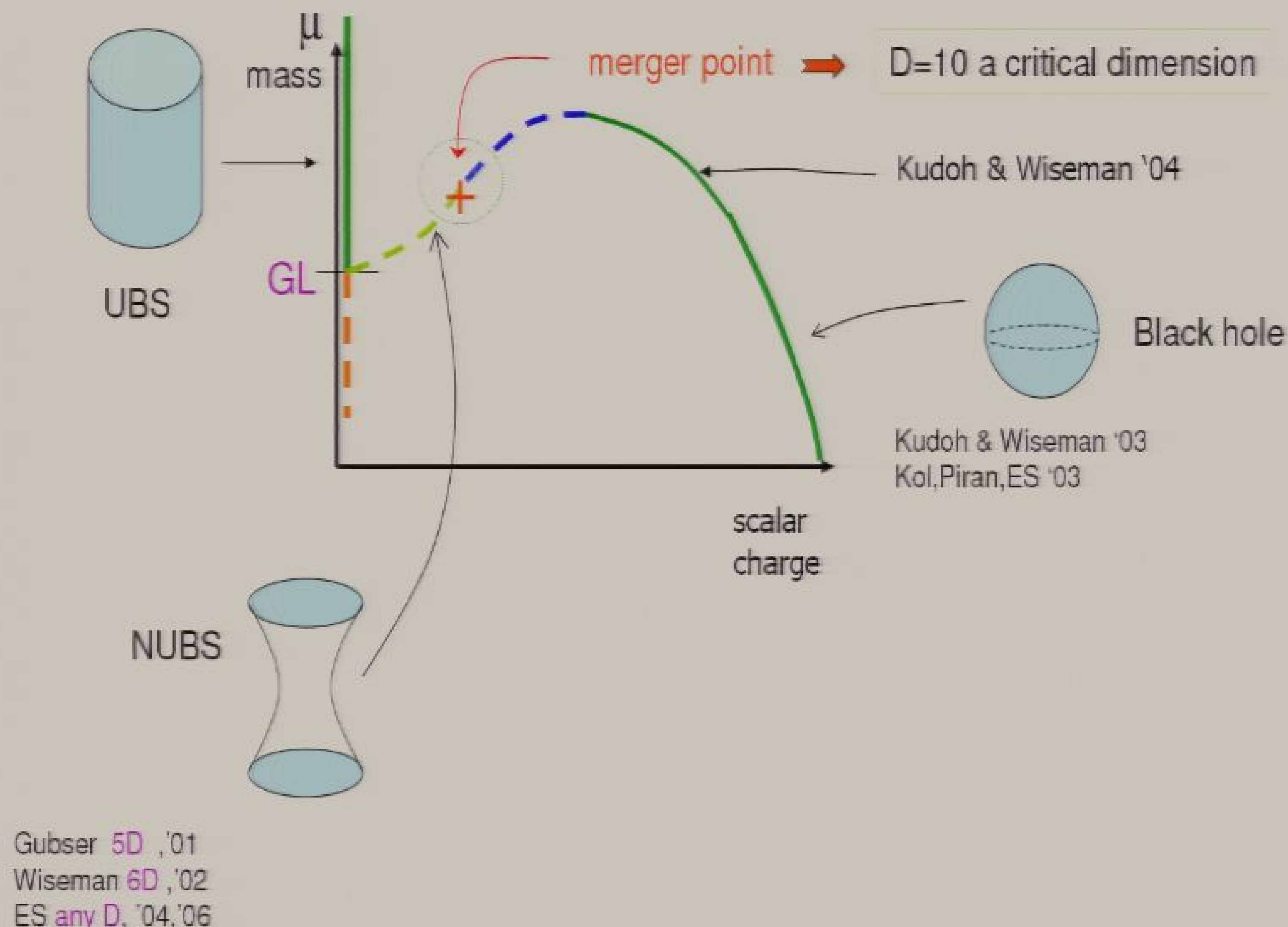

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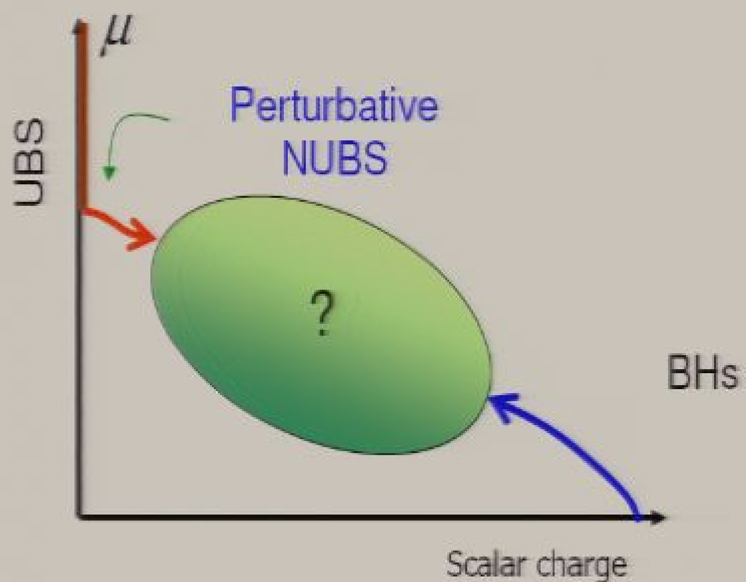

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In conclusion there is the GL instability that leads to a phase transition and order of this transition depends on the spacetime dimension.

The conclusion is based on the perturbative construction of static NUBS emerging from the GL point. Namely, the marginally static GL mode induces non-uniformity, then the back-reaction due to this mode is solved up to the **third order**, where one computes mass and entropy variation. This is the direct Gubser's method

Here I'd like to describe another perspective of the phase transition and its order analysis: Ginzburg-Landau theory

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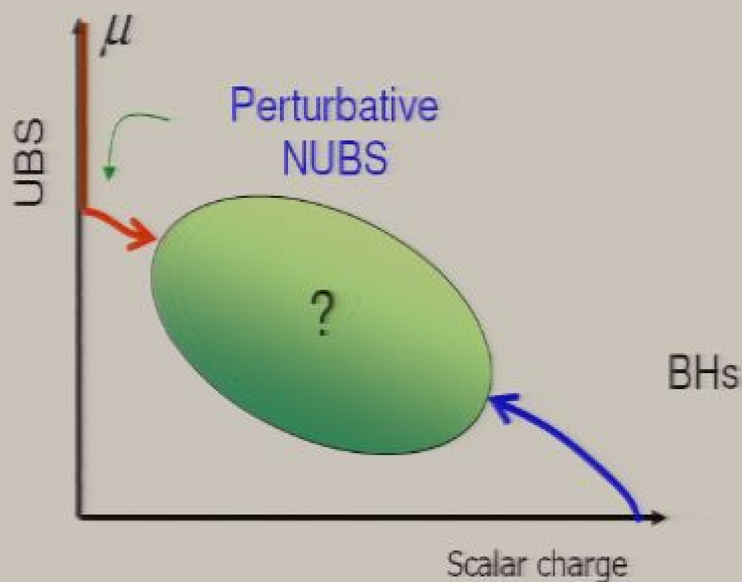

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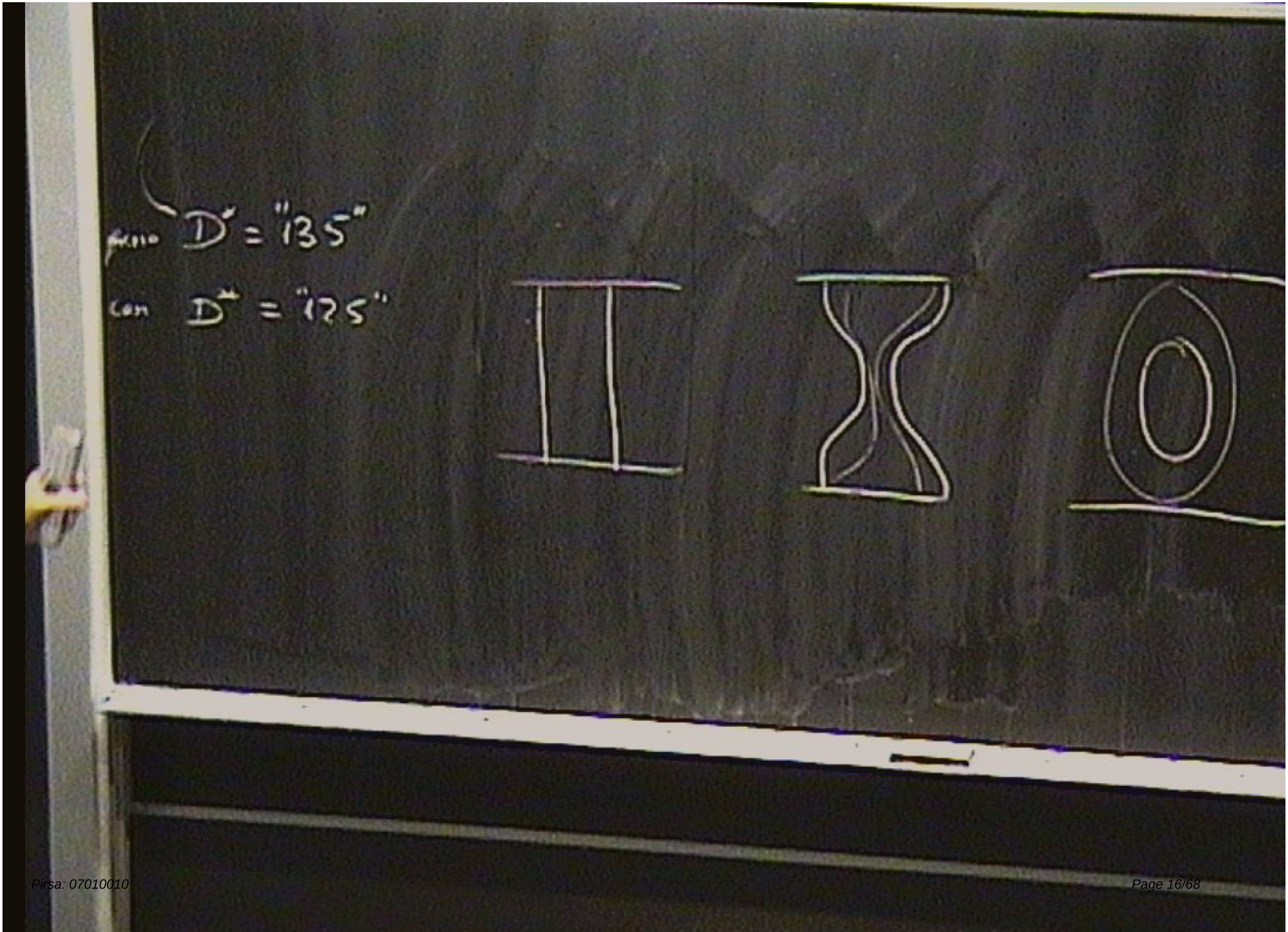
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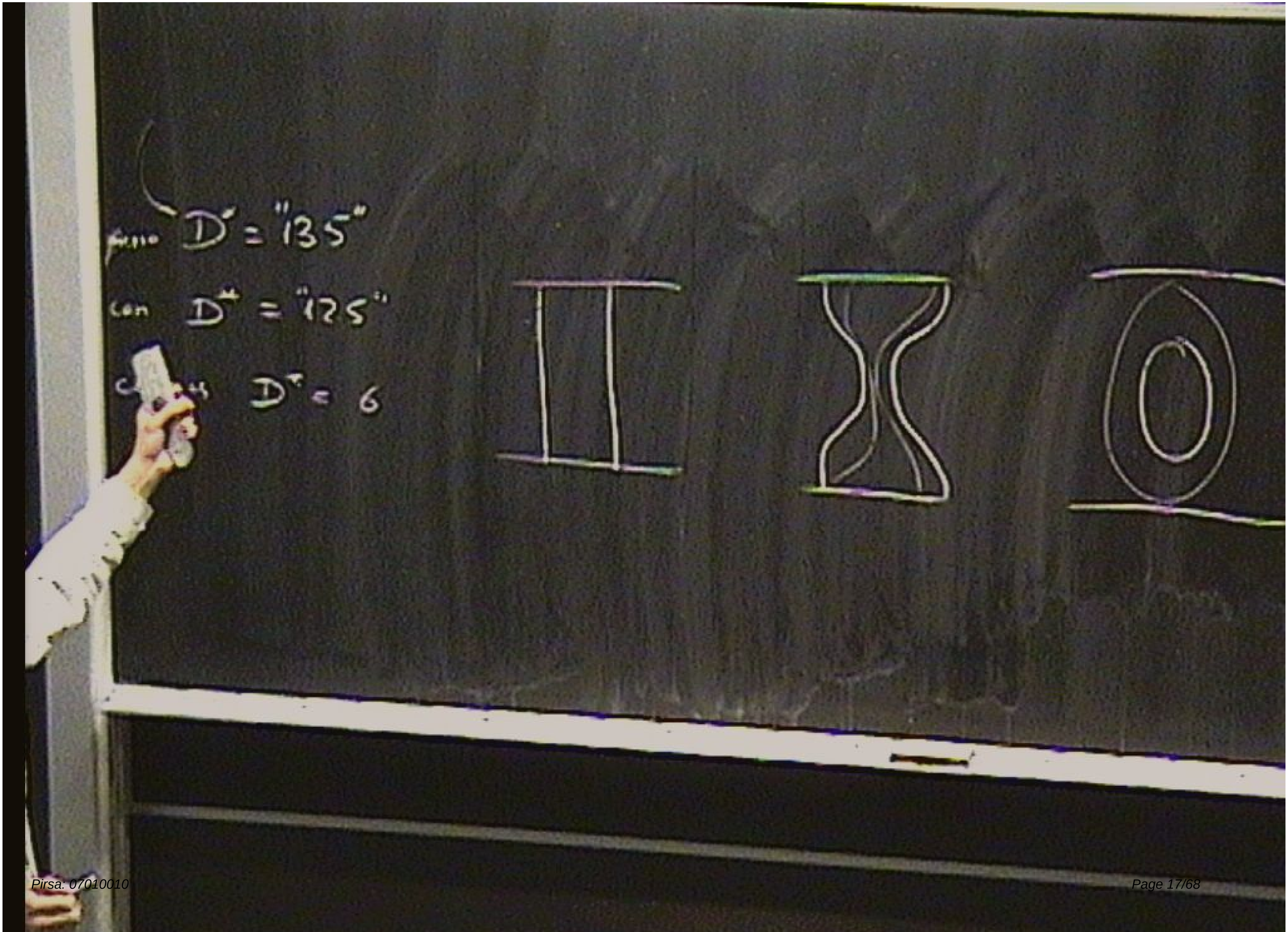
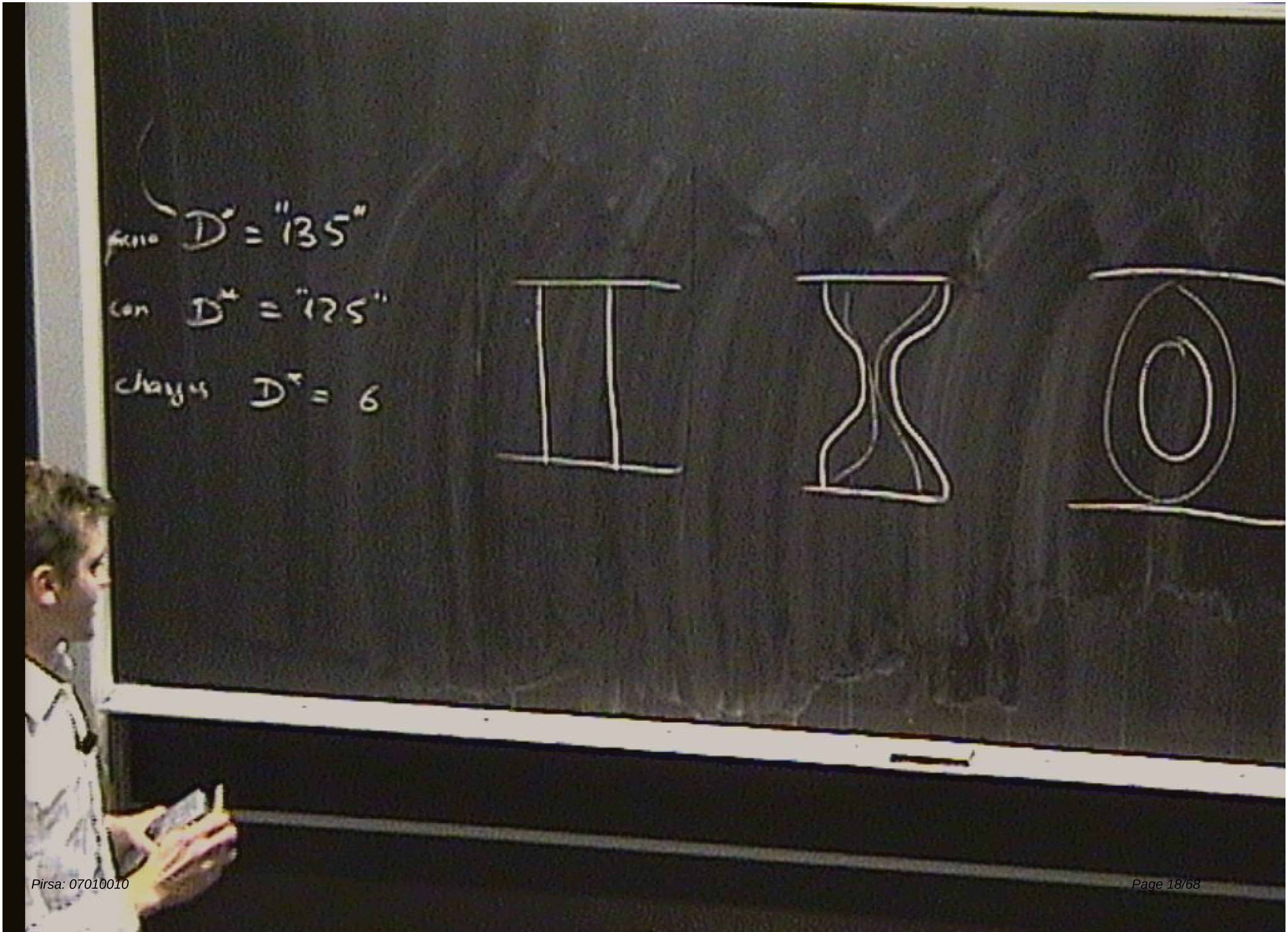
con D'' = "125"


D = "135"
con D = "125"
D = 6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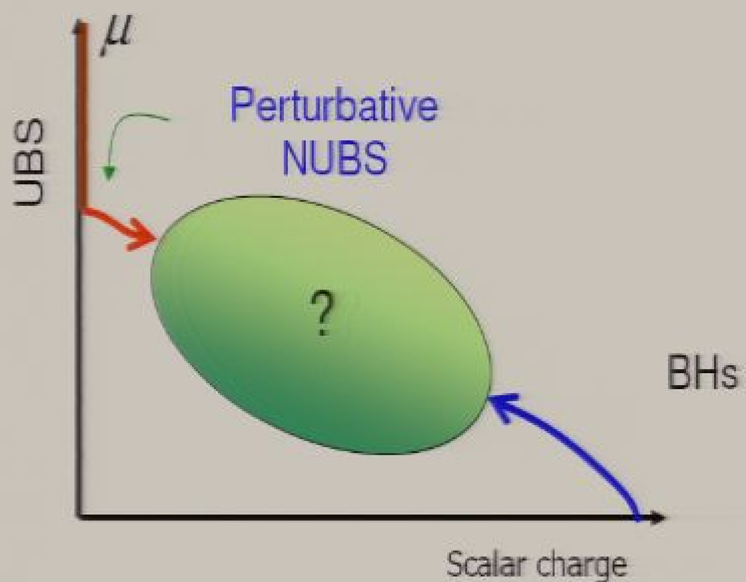

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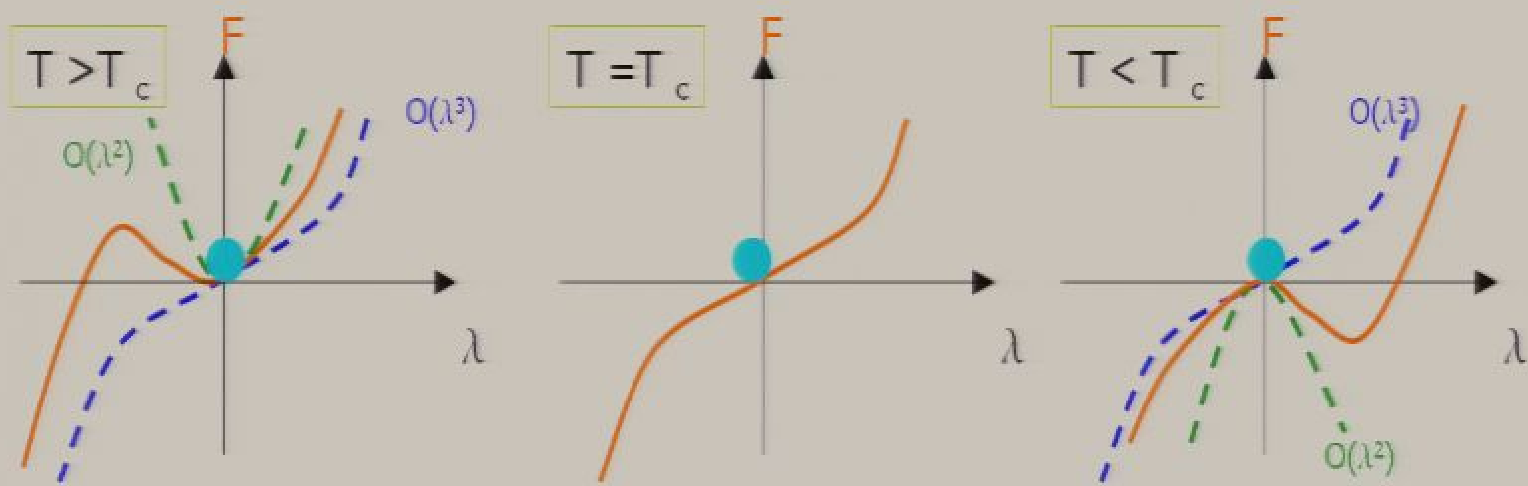
Ginzburg-Landau theory of phase transitions

Local analysis: focusing on low energy modes and zooming.
Consider the expansion of the free energy F around critical temperature T_c in powers of an order parameter λ

$$F(T,\lambda) = F_0(T) + (T - T_c) A \lambda^2 + B(T_c)\lambda^3 + C(T_c)\lambda^4 + \ldots$$

A>0 since for $T>T_c$ the free energy has a minimum; Generically B≠0; Let B>0. and we assumed the existence of critical solution with zero mode.



However, in some cases, symmetries set B=0 identically;
In our case this will be the parity $\lambda \to -\lambda$ symmetry, such that
$F(\lambda)= F(-\lambda) \Rightarrow F(\lambda^2)$

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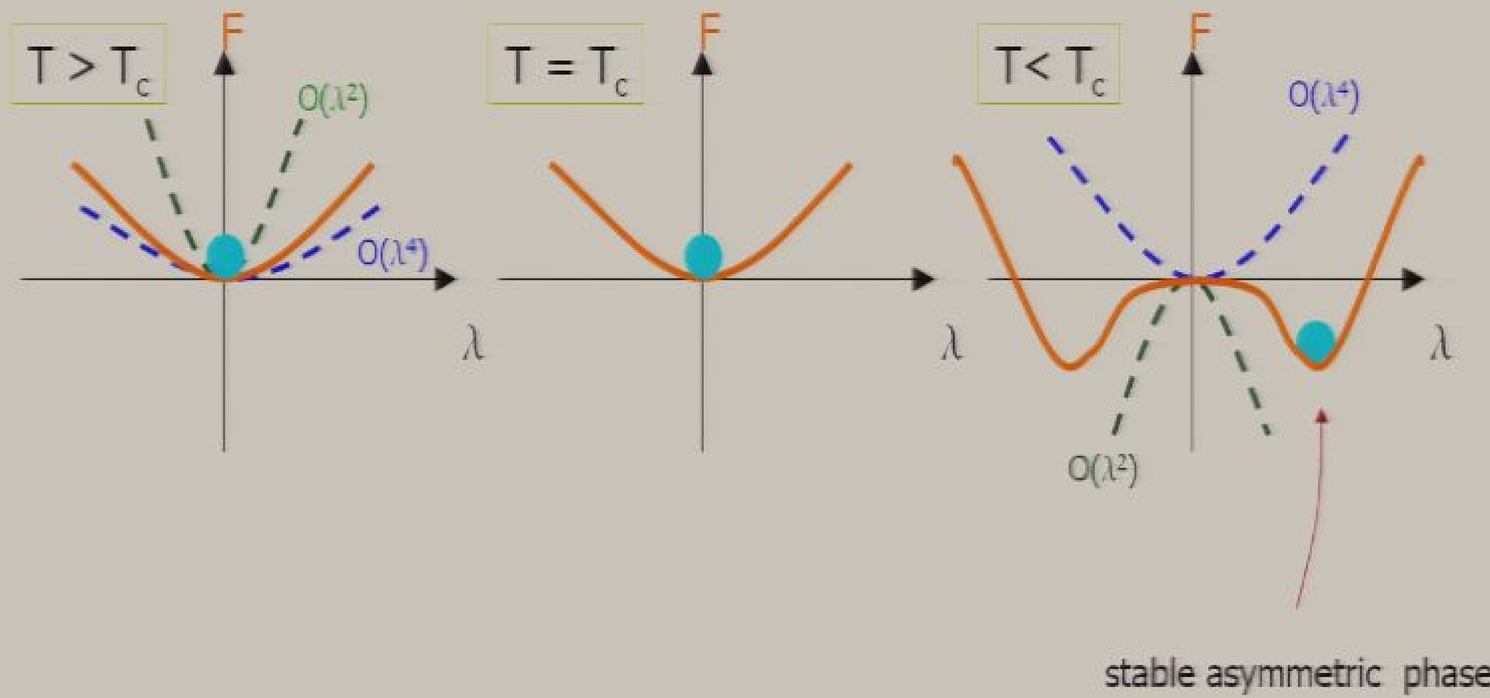
The extrema of the free energy $dF/d\lambda=0$ are $\lambda=0$ and $\lambda_b^2 = -\frac{A(T-T_c)}{2C}$

so a new branch exists if $\mathrm{sign}(T-T_c) = -\,\mathrm{sign}(C)$

There are two possibilities: $C>0$ or $C<0$

$$F(T;\lambda) = A(T - T_c)\lambda^2 + C\lambda^4 + \ldots$$

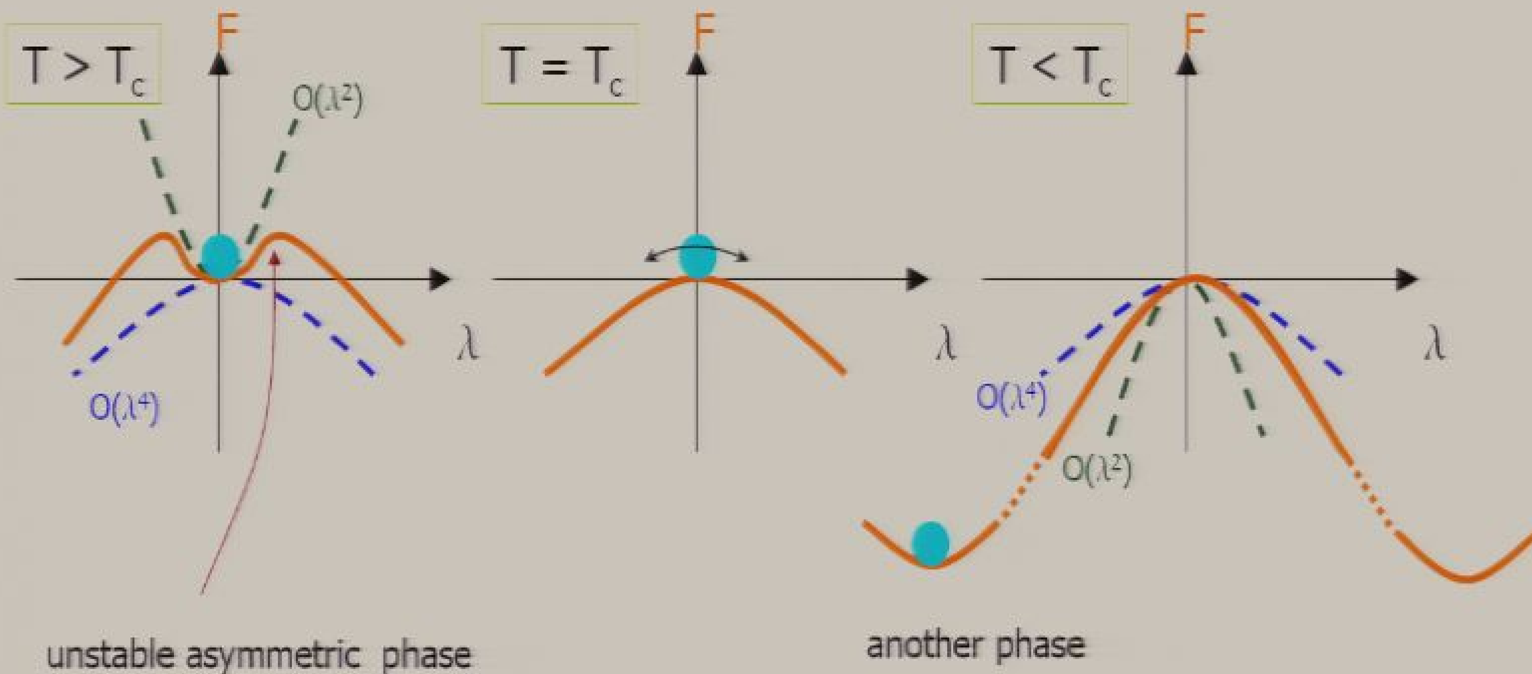
C>0



Second order phase transition

$$F(T;\lambda) = A(T - T_c)\lambda^2 + C\lambda^4 + \ldots$$

C<0



First order phase transition

$$F(T,\lambda) = F_0(T) + (T - T_c) A \lambda^2 + C(T_c) \lambda^4 + \ldots$$

In summary:

C<0 first order phase transition
C>0 second order phase transition
(C=0 higher order)

Thermodynamics is encoded by F

Reviewed

- GL instability of black string and the associated phase transition
- LG theory of phase transitions

Next

- LG in GL
- Torus compactification

York-Gibbons-Hawking action (canonical ensamble)

$$I(\beta) = -\beta F = \frac{1}{16\pi G_D}\int dV_D R + \frac{1}{8\pi G_D}\int_{\partial} dV_{D-1}\left[K - K^0\right]$$

In Euclidean signature β is the period of the imaginary time.
It is related to the temperature $T=\hbar/\beta$

The program: We will compute the variation of this action around the GL point: namely looking at the perturbation of the uniform black string background due to marginally tachyonic GL mode.

$$h_{ab}(r,z) = \lambda\, H_{\alpha\beta}(r)\exp(ik_{GL} z)$$

Non-uniform black-strings

The most general static black string background on cylinder is

$$ds^2 = e^{2A(r,z)} f(r) dt^2 + e^{2B(r,z)} \left[f(r)^{-1} dr^2 + dz^2 \right] + e^{2C(r,z)} r^2 d\Omega_{d-2}$$

$f(r) = 1 - 1/r^{d-3}$ horizon at r_0=1

For A,B,C=0 we get a *uniform BS*

We denote the fields collectively by x
and consider the expansion

$$x = x^{(0)} + \lambda X^{(1)} + \lambda^2 X^{(2)} + \ldots$$

$$X^{(1)} = X_{GL}$$

$$X^{(2)} = X_{BR}$$

The expansion of the free energy up to the fourth order in λ

$$F(x) = F_0 + F_2(X,X) + F_3(X,X,X) + F_4(X,X,X,X) + ...$$
$$+ \delta k \frac{\partial F}{\partial k} + ...$$

Since in the perturb. theory X is decomposed into GL mode and BR, we get

$$F(\lambda; X; \beta) = F_0(\beta) + A\,\delta\beta\,\lambda^2 + \underbrace{F_2(X_{BR}, X_{BR}) + \lambda^2 G(X_{BR}) + F_{4GL}\lambda^4}_{C\lambda^4 = \left[F_{4GL} - F_2(X_{BR})\right]\lambda^4}$$

because of the eqn for back reaction: $\delta F / \delta X = 0 \;\Rightarrow\; 2F_2 X_{BR} + G = 0$

The computation steps:

1. Solve Einstein equations for X_{GL}: $\mathbf{L}X_{GL}=0$, get k_{GL}
2. Solve Einstein equations for X_{BR} : $\mathbf{L}X_{BR} = Src(X_{GL}{}^2)$
3. Substitute into the action $F(T,\lambda) = F_0(T) + (T-T_c)A\lambda^2 + C(T_c)\lambda^4 + ...$
   and by integration compute the coefficients A and C
4. Calculate thermodynamical variables M and S

Result

In either the direct Gubser's or LG methods there are 2 "bottom line" numbers {M,S} and {A,C}.

They are related by usual thermodynamics relations
$S = -dF/dT$ and $M = F + TS$

Numerical values of S, M computed directly (Gubser's) or derived from A and C (LG) are comparable within 5% in the checked range of dimensions $4 \le d \le 14$

√ LG works in black string case.
The benefits: more economic method;
It computes the action integral instead of solving more ODEs at 3d order

The computation steps:

1. Solve Einstein equations for X_{GL}: $\mathbf{L}X_{GL}=0$, get k_{GL}
2. Solve Einstein equations for X_{BR} : $\mathbf{L}X_{BR} = Src(X_{GL}{}^2)$
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Torus compactification $R^d \times T^p$ D=d+p

For a general p-torus the uniform brane: $\mathcal{B}_p = Schw_d \times \mathbf{T}^p$

Instability: Zero mode $\Delta_L h_{\mu\nu}(r,z)=0$, $h_{\mu\nu}(r,z)=h_{\mu\nu}(r)\, e^{ik_j z^j}$ => $\lambda_{Schw} + \lambda_{T^p} = 0$

$$\lambda_{T^p} = \left|\vec{k}\right|^2$$

critical GL mass ⟺ shortest vector in the reciprocal lattice

Options:

#(marginal tachyons)=
= #(reciprocal lattice vectors closest to the origin)

One can view the space of torii as having two boundaries:

•Highly asymmetrical torii, where one or more dimensions are much larger then others (*First GL mode along the shortest* k_i)

•Highly symmetrical torii, such as the square torus (*several simult. modes*)

By studying the second case we achieve understanding of both limits and therefore of the intermediate region of general torii.

For simplicity we focus on the square torus

For square T^p there are p simultaneous tachyons

Torus directions $z^i \sim z^i + L$

So at the linear order the GL instability generalizes to

$$X^{(1)} = \lambda X_{GL} \rightarrow X^{(1)} = \lambda_i X^i_{GL}$$

Back-reactions is sourced by $\sim \lambda_i X^i_{GL} \lambda_j X^j_{GL}$ and hence

$$X_{BR} = \lambda_i \lambda_j X^{ij}_{BR}$$

In addition there are more metric fields than in S^1 compactification

Square torus symmetries $z_i \leftrightarrow z_j$ for any i,j imply that X_{BR}^{ij} or more generally any tensor T^{ij} has only 2 distinct components:

$$T^{ij} = \begin{cases} T^{=} & i = j \\ T^{\neq} & i \neq j \end{cases}$$

$X_{BR}^{=}$ was computed already for T^1, while $X_{BR}^{\neq}$ is novel

We consider expansion of the free energy along $X = \lambda_i X_{GL}^i + \lambda_i \lambda_j X_{BR}^{ij}$

As a result the quadratic coefficient the free energy expansion is proportional to that computed for T¹ and the quartic coefficient becomes tensor

$$C \rightarrow C(\lambda^i) = C^{ij} |\lambda^i|^2 |\lambda^j|^2$$

$$C^{ij} = \begin{cases} C^{=} & i = j \\ C^{\neq} & i \neq j \end{cases}$$

Determination of the order of phase transition

Consider how $C(\lambda^i)$ varies over all possible directions in tachyon space.

It's enough to take $\sum|\lambda_i|^2 = 1$

It is possible to show that C varies in the range

$$C \in \left[C^{=}, \bar{C}(p)\right] \subseteq \left[C^{=}, C^{\neq}\right]$$

$$\bar{C}(p) \equiv \frac{C^{=} + (p-1)C^{\neq}}{p} \qquad \text{"average"}$$

The transition is second order iff C is positive for all directions in λ^i space,

iff $C^{=}, \bar{C}(p) \geq 0$

By definition $C^{=}$ and $C^{\neq}$ doesn't depend on p. $C^{=}$ was actually found in the p=1 case and, moreover suffices it solve only p=2 case in order to compute $C^{\neq}$ and to infer the order of the p.t. for any p.

Rather than determining $C^{\neq}$ directly we look along the "diagonal" direction

$$\bar{\lambda}_i = \lambda / \sqrt{p}$$

In this case we compute $C(\bar{\lambda}_i) = \bar{C}$

and for p=2 we derive $C^{\neq} = 2\bar{C}\big|_{p=2} - C^{=}$

Results for T^p

Second order p.t. iff $C^{=}, \bar{C}(p) \geq 0$

- A necessary condition $C^{=} > 0$; so for all $D < D^{*} = 12.5$ where the p.t. is first order for p=1 it is first order for any p

- It turns out that the converse is true as well, for d>11, where $C^{=} > 0$, then also $C^{\neq} > 0$ and therefore $\bar{C}(p) > 0$ for all p and hence the transition is second order in this range of dimensions for all p.

- In T^p compactification: The transition order depends only on d

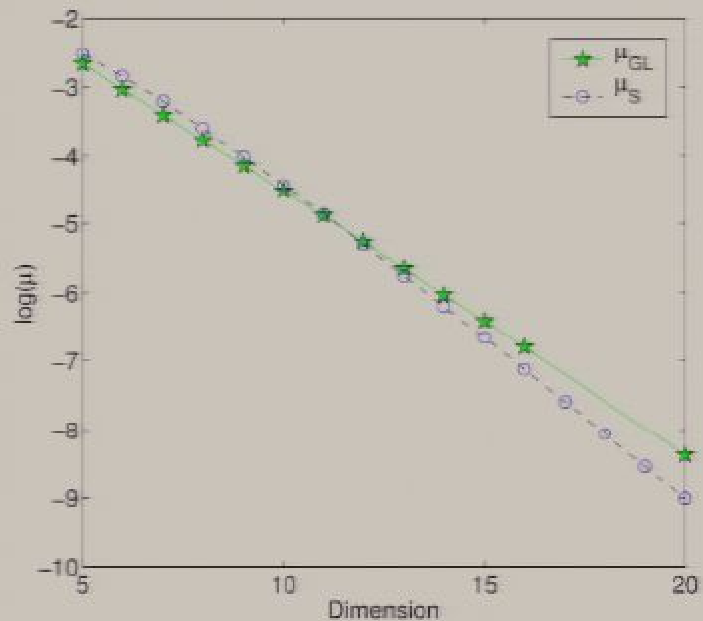
Results for T^p

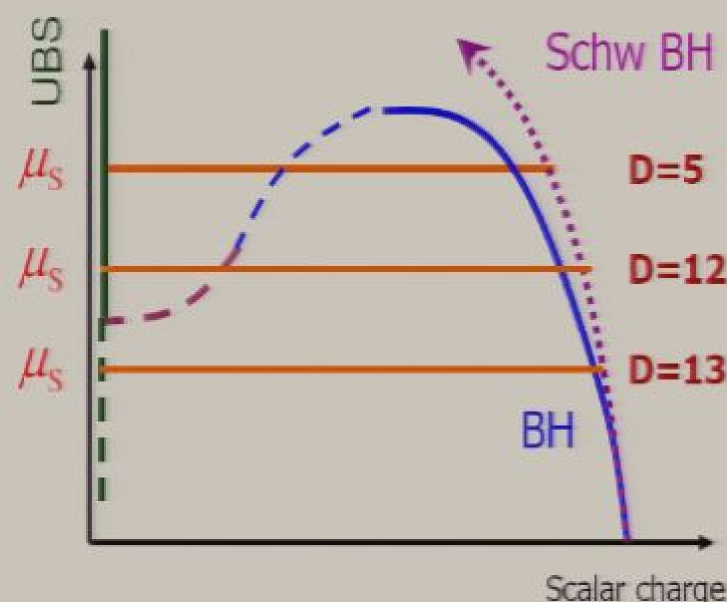
- From analysis of $C^{=}, \bar{C}$ it follows that (for d>4) the diagonal direction is disfavored relative to turning on a "single tachyon"

Hence in a case of 1st order p.t. the decay will proceed (initially) through a single tachyon, while for a 2nd order p.t. the system will re-settle into a slightly non-uniform along one of the directions brane.

Failure of `Equal-area for equal-mass *estimator* '

$$A_{BH}^{(Schw)}(\mu_S) = A_{BStr}(\mu_S)$$





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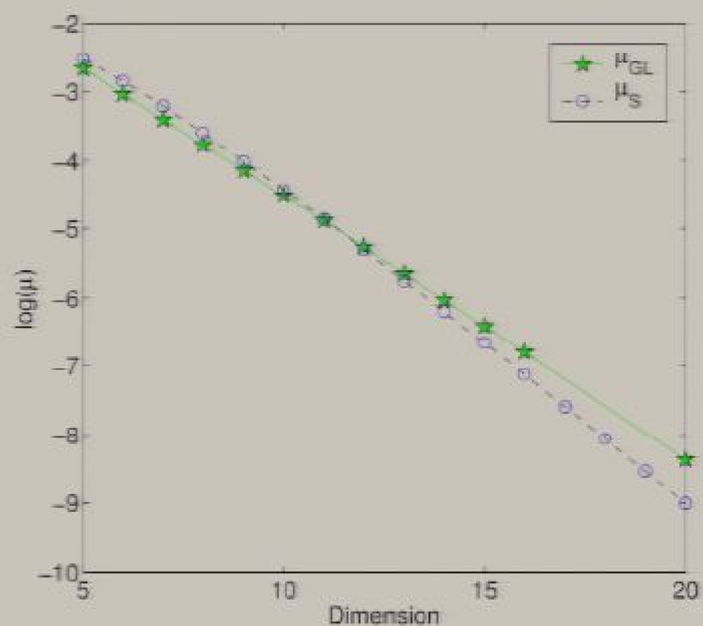

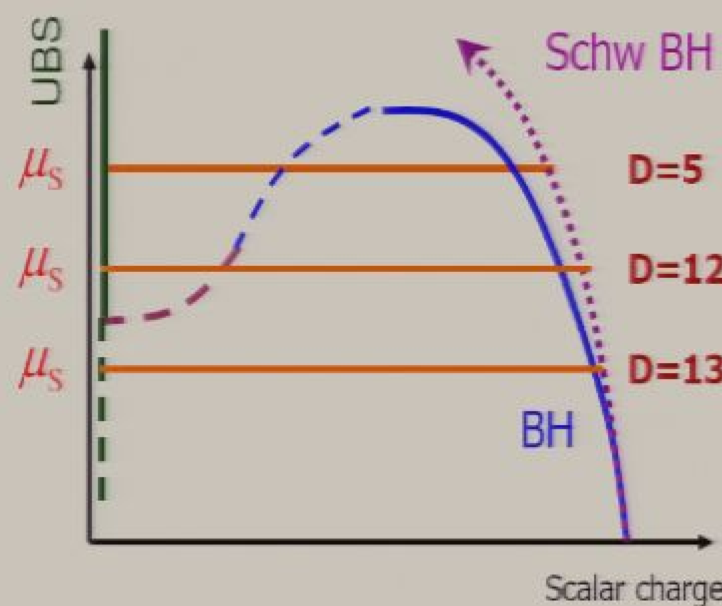

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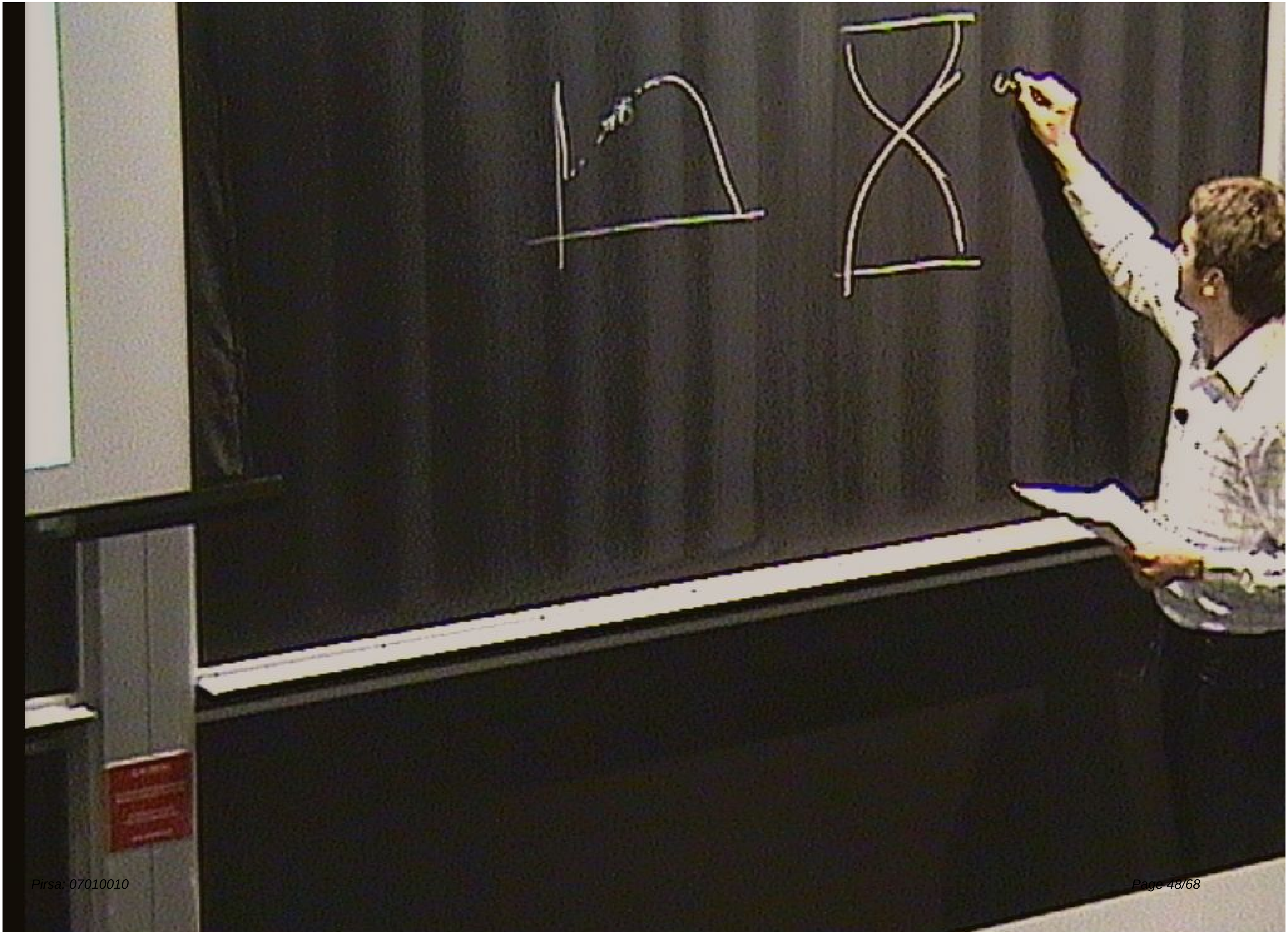
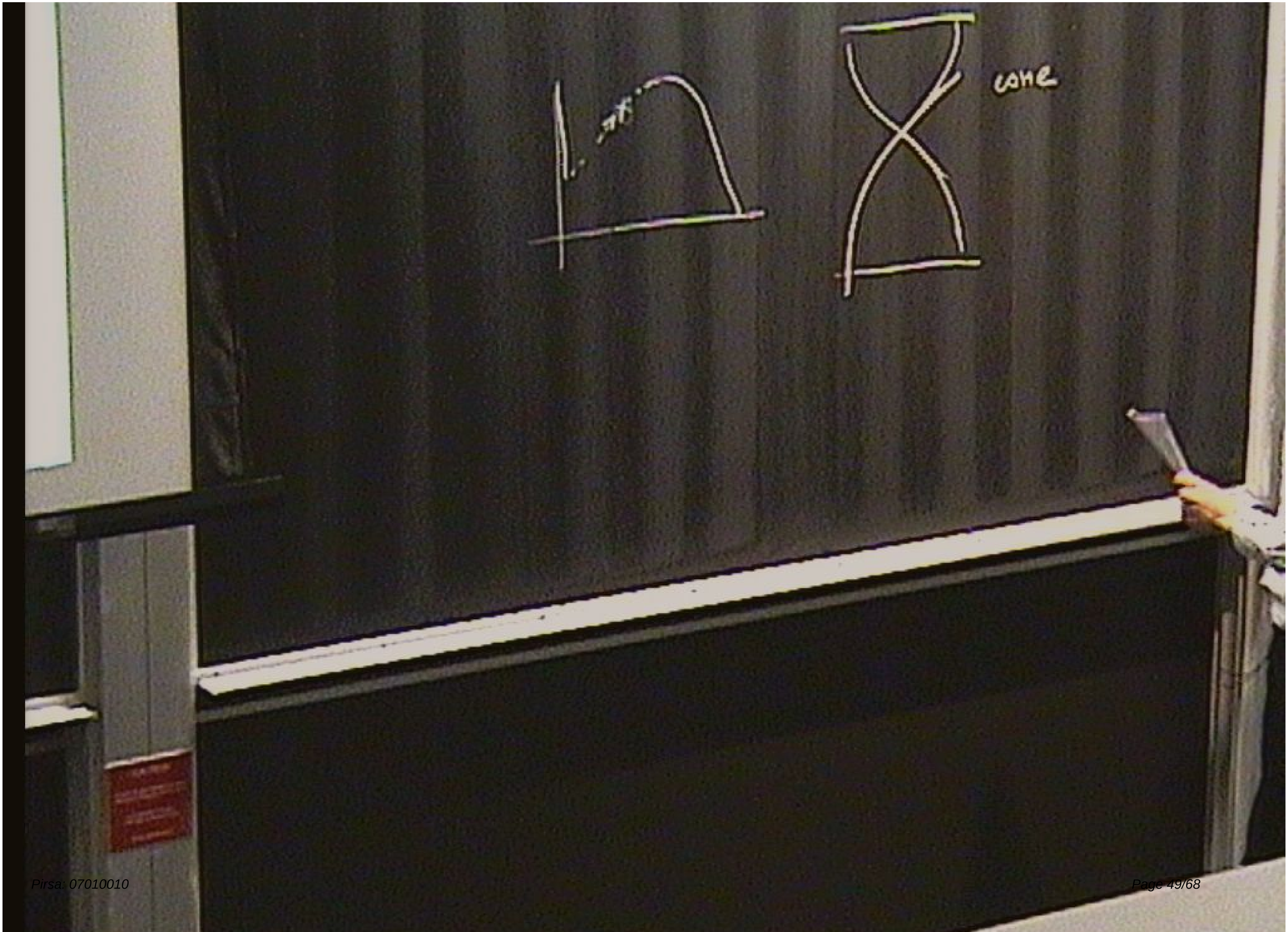
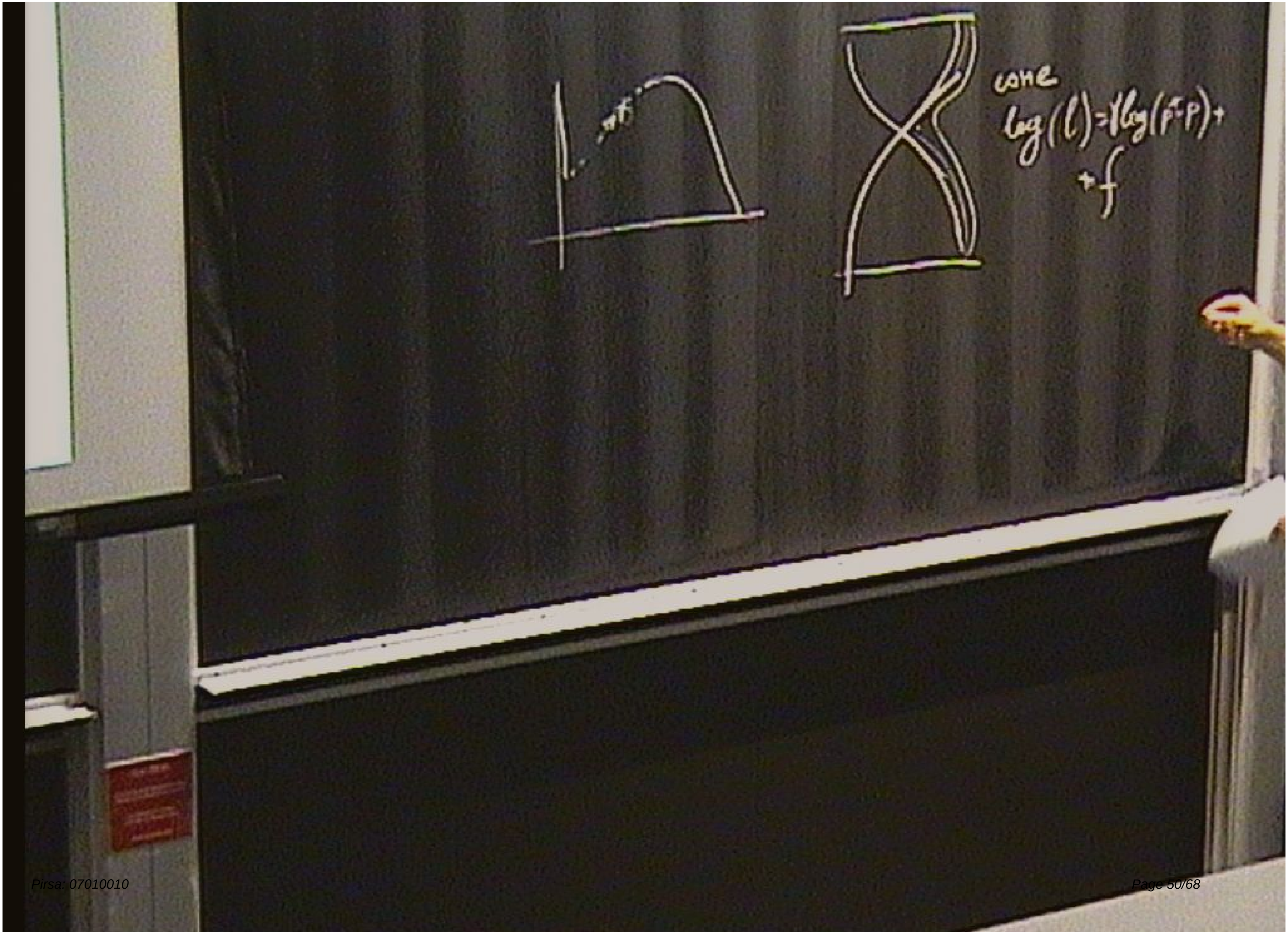
Some open questions on the GL phase transition

- The pinch-off in both the dynamical and the static contexts; topology changing phase transition
- Local geometry at the pinch-off; scaling and universality
- The cosmic censorship




cone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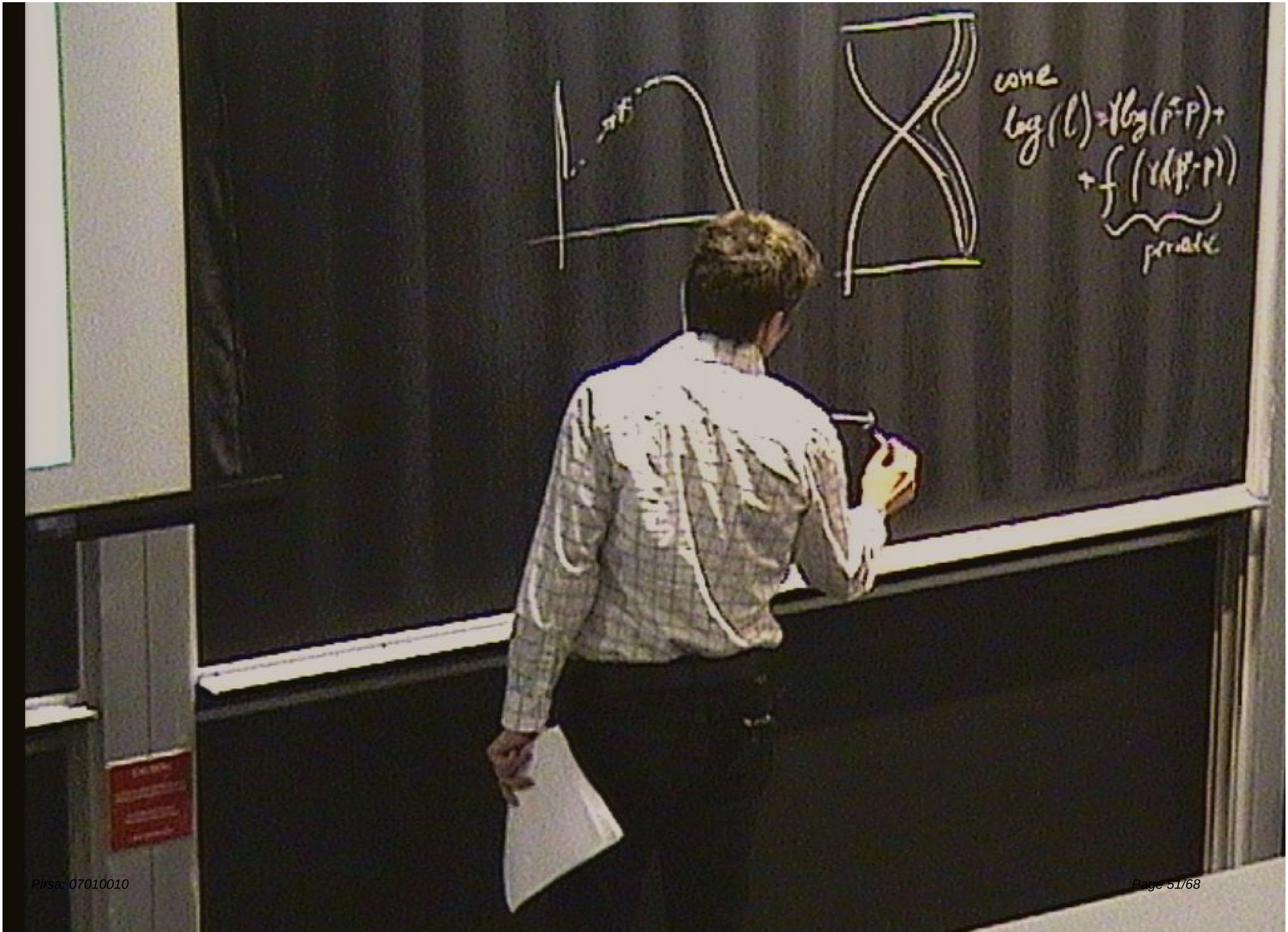
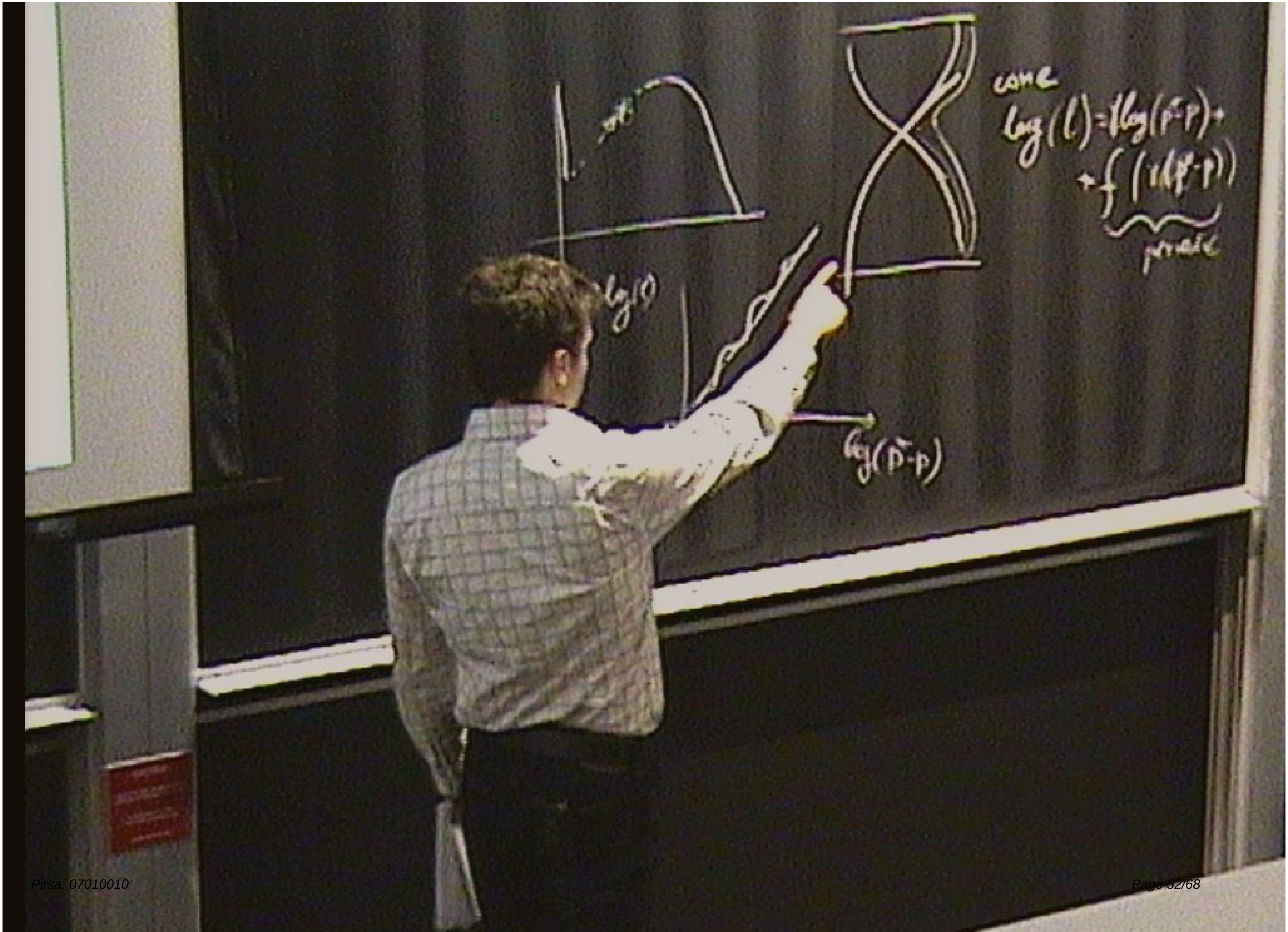
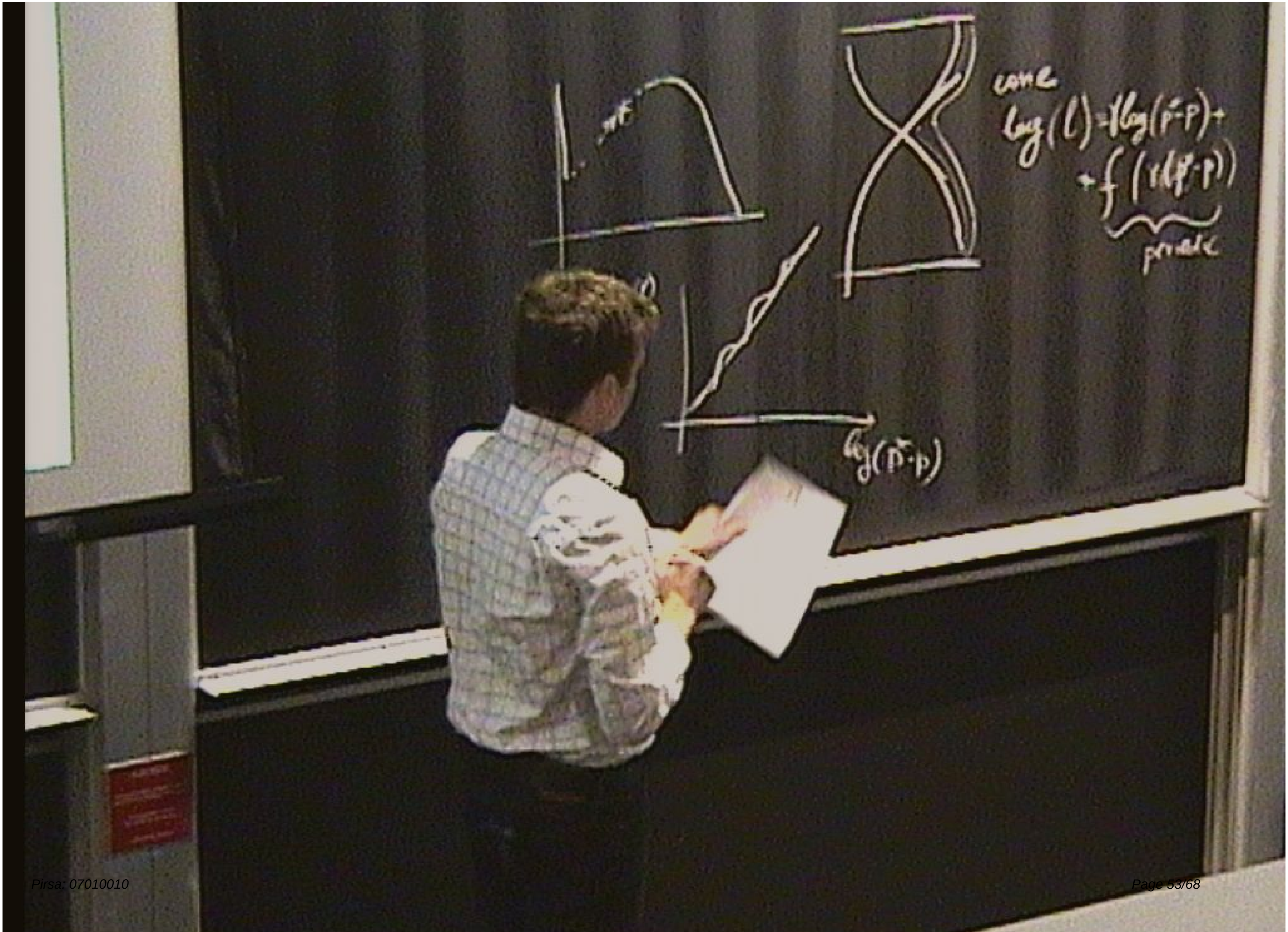

cone
periodic


cone
log(p̄-p)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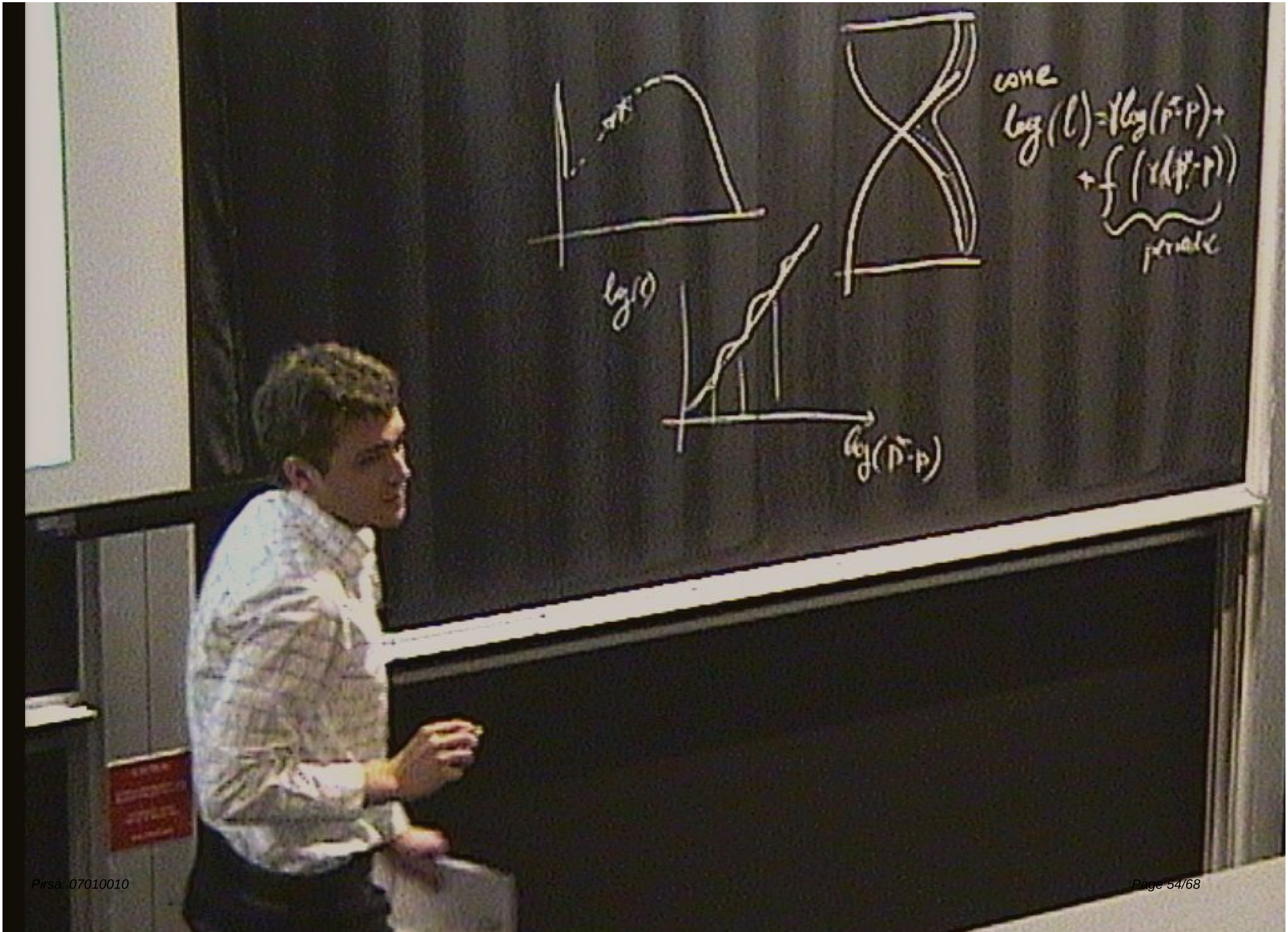
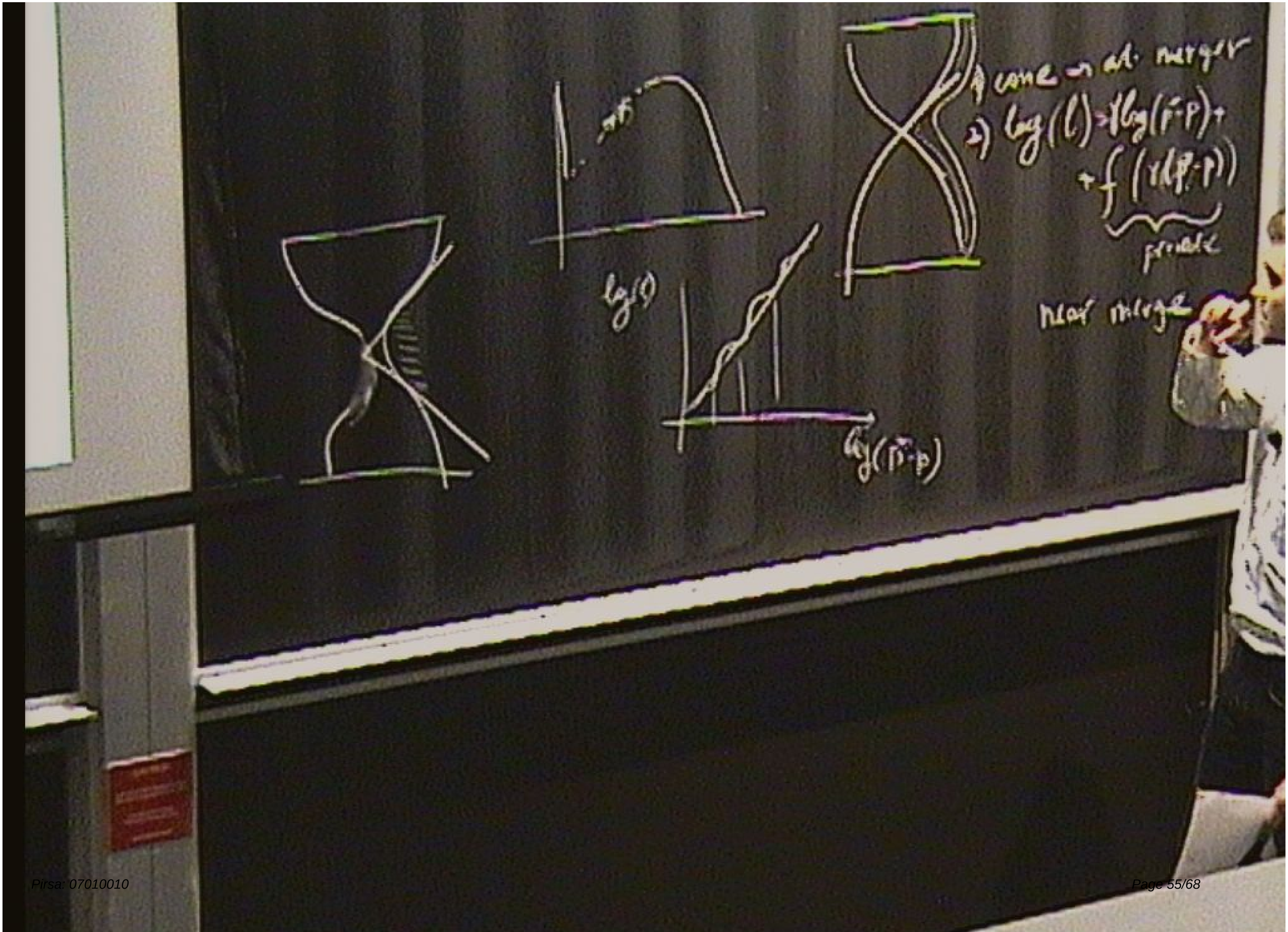
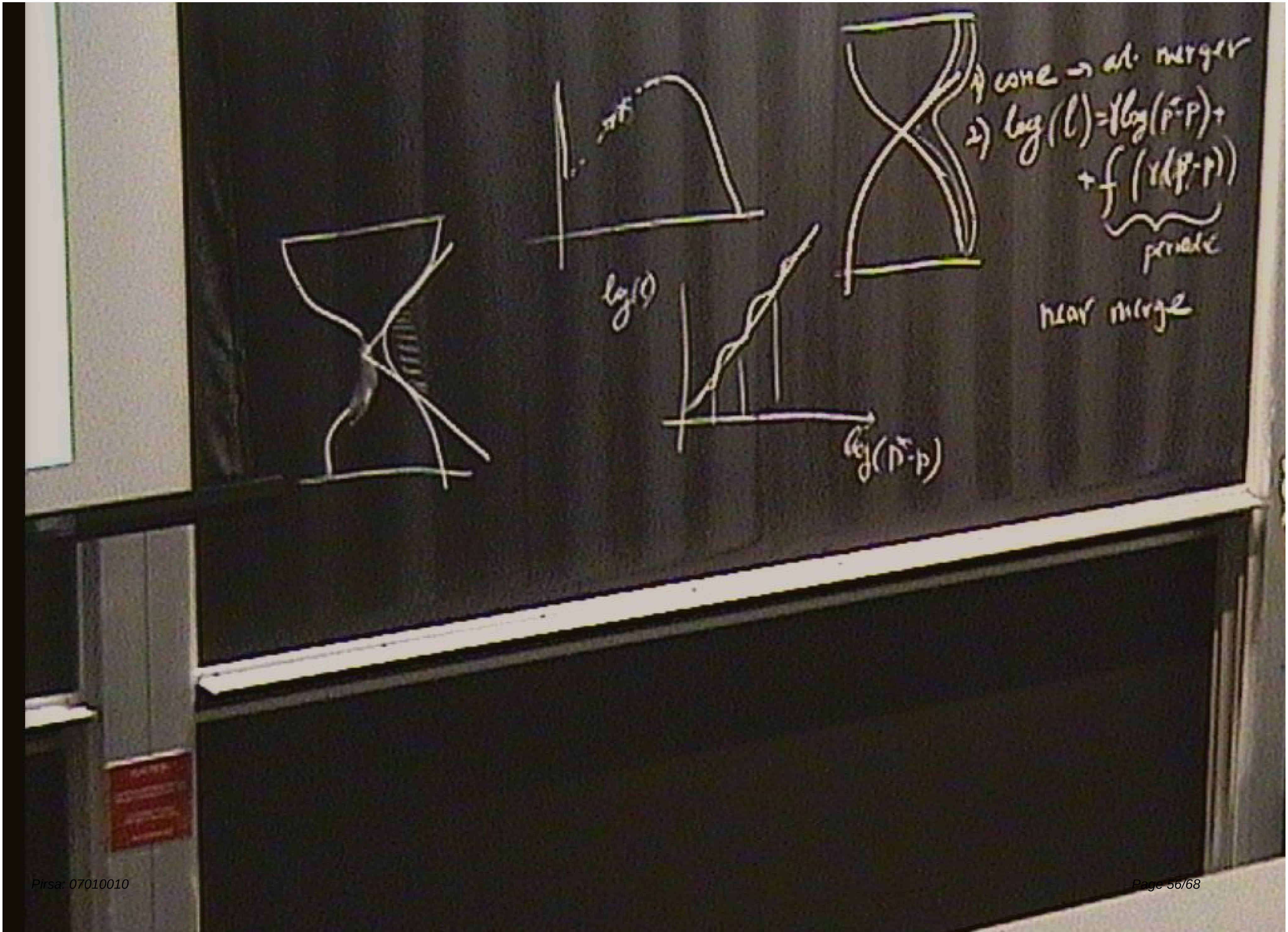
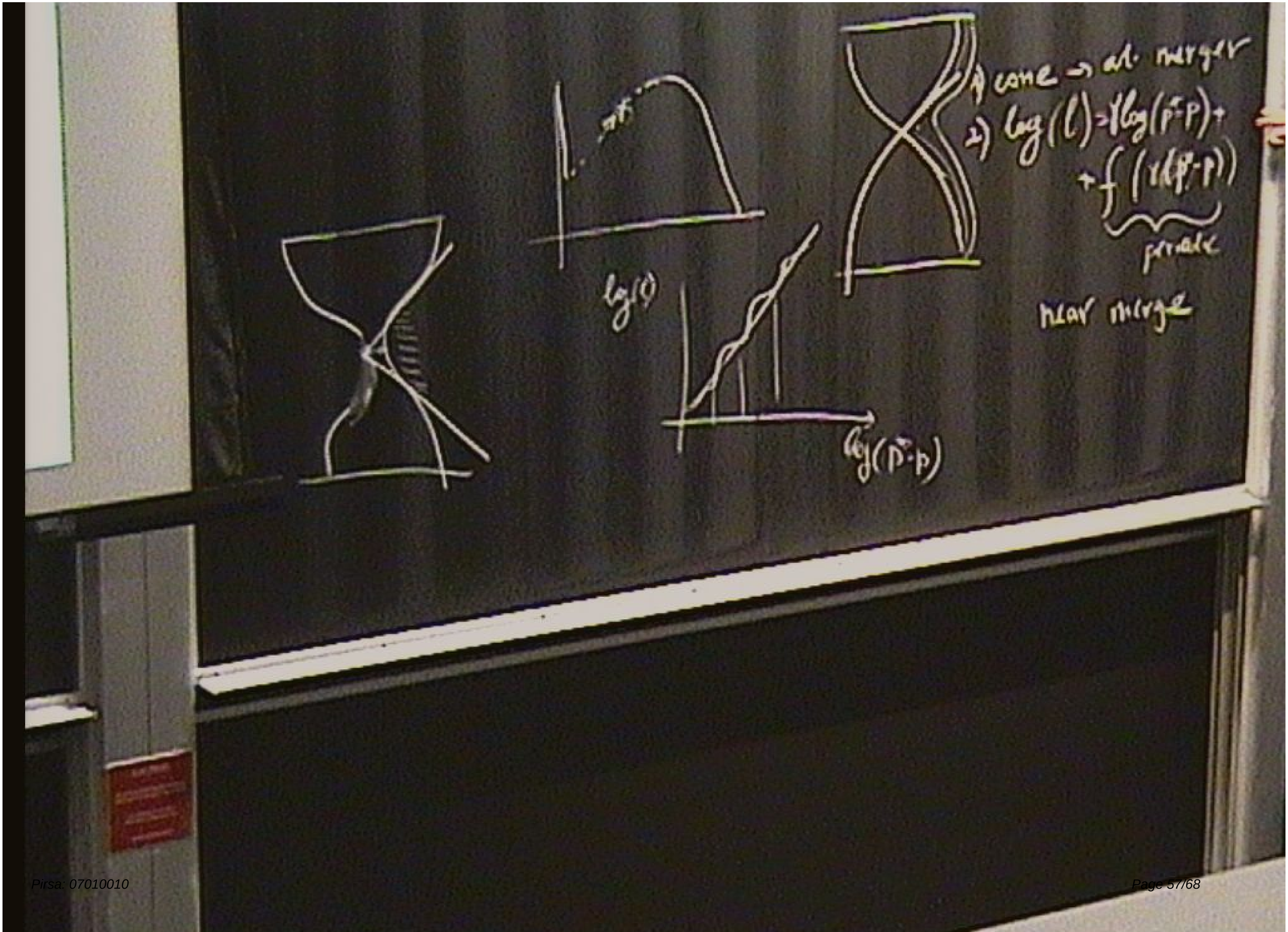
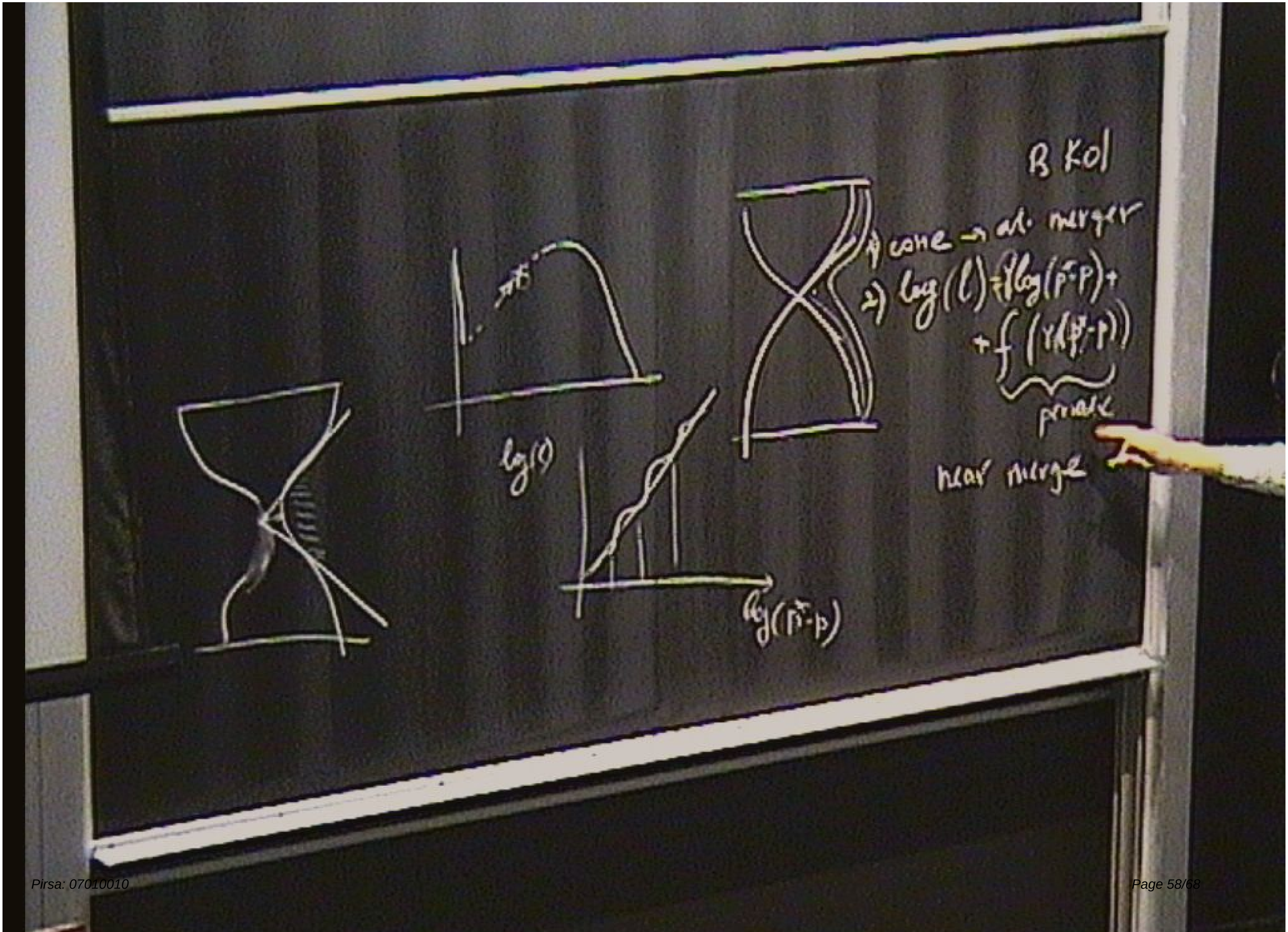
log(p*-p)

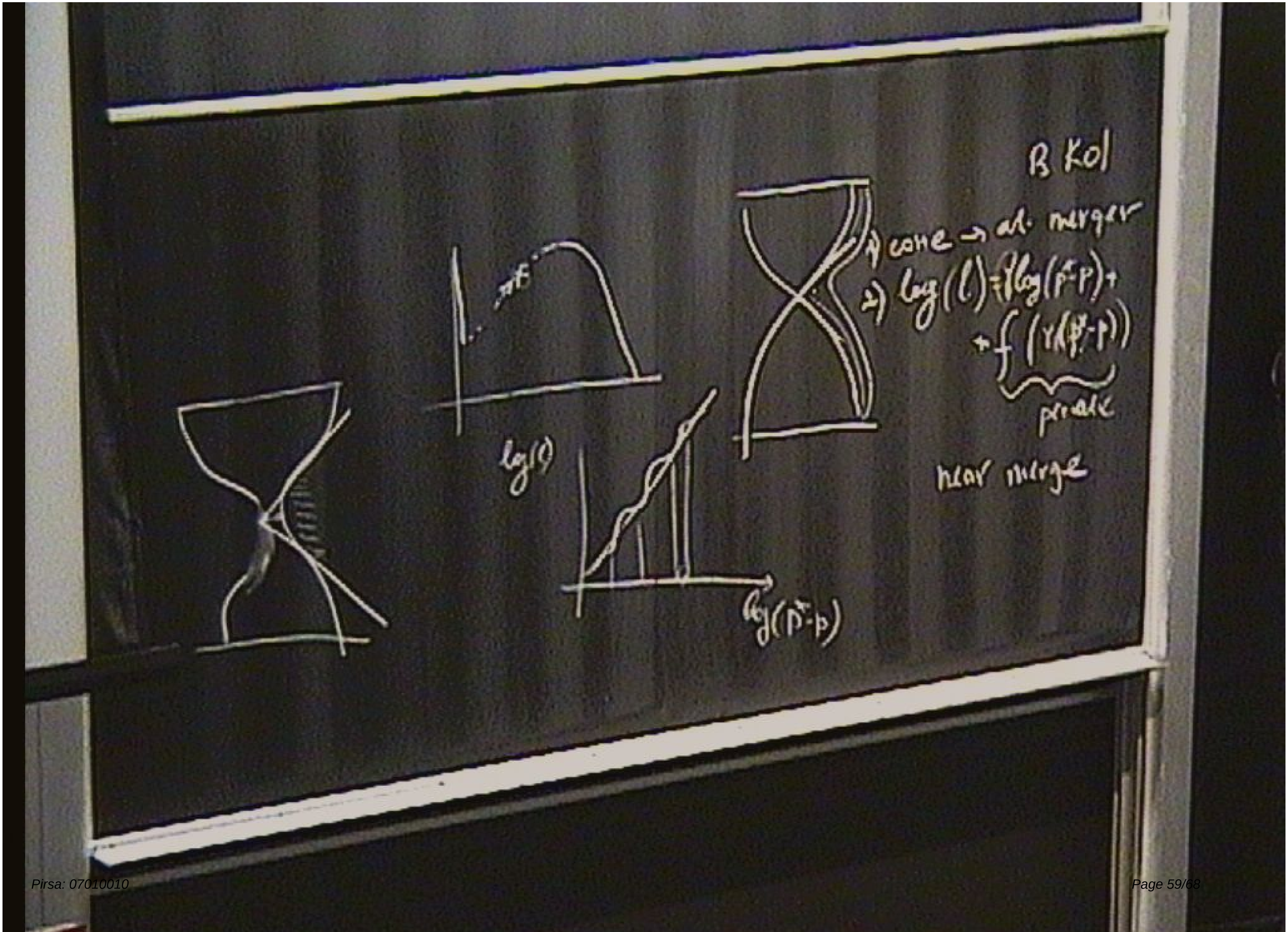

cone
log(l) = γ log(p*-p) +
+ f((γ(p*-p))
periodic
log(l)
log(p*-p)

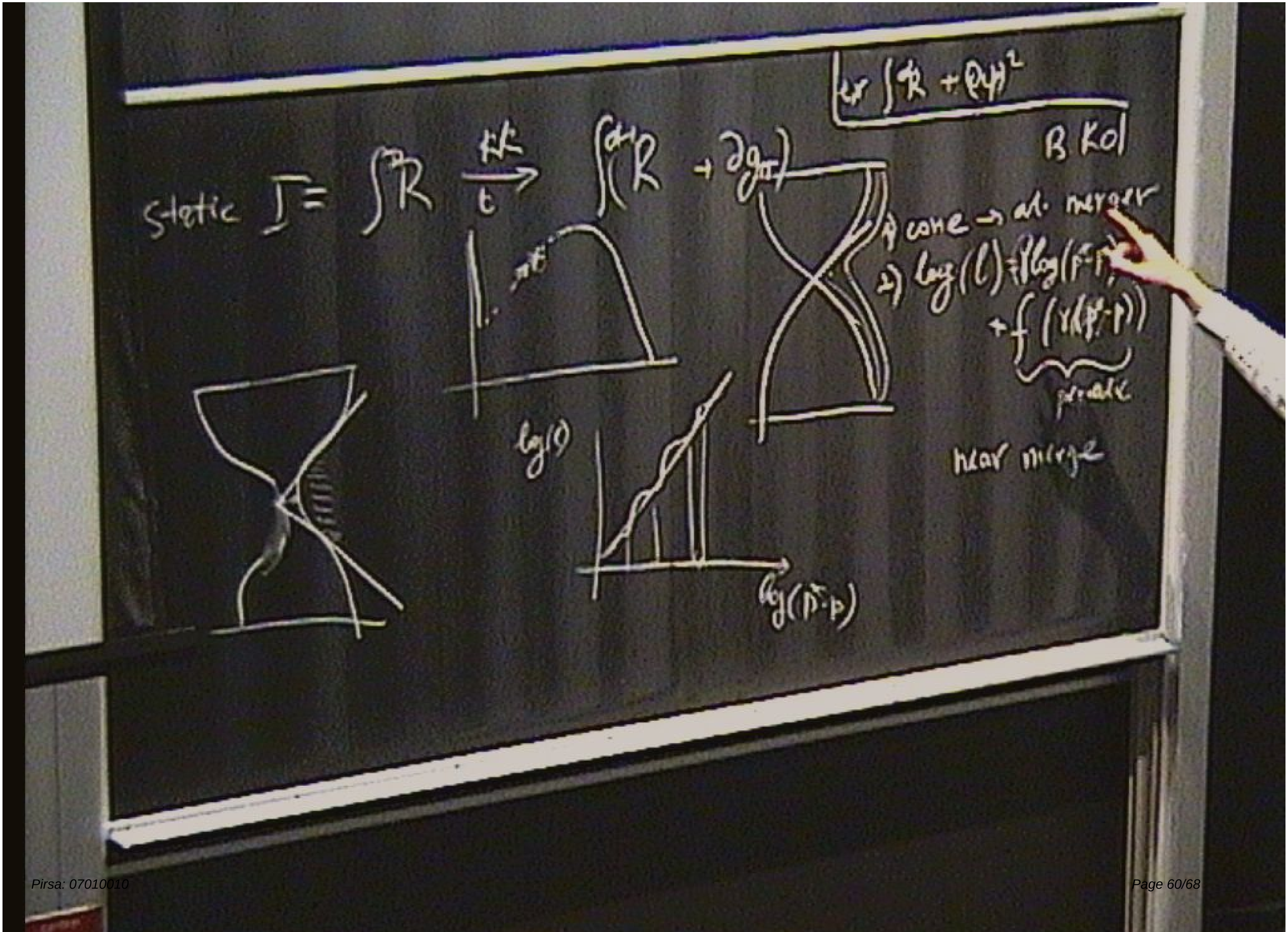

1) cone in at. merger
2) log(l) = γ log(p*-p) + f(γ(p*-p))
periodic
near merge
log(l)
log(p*-p)

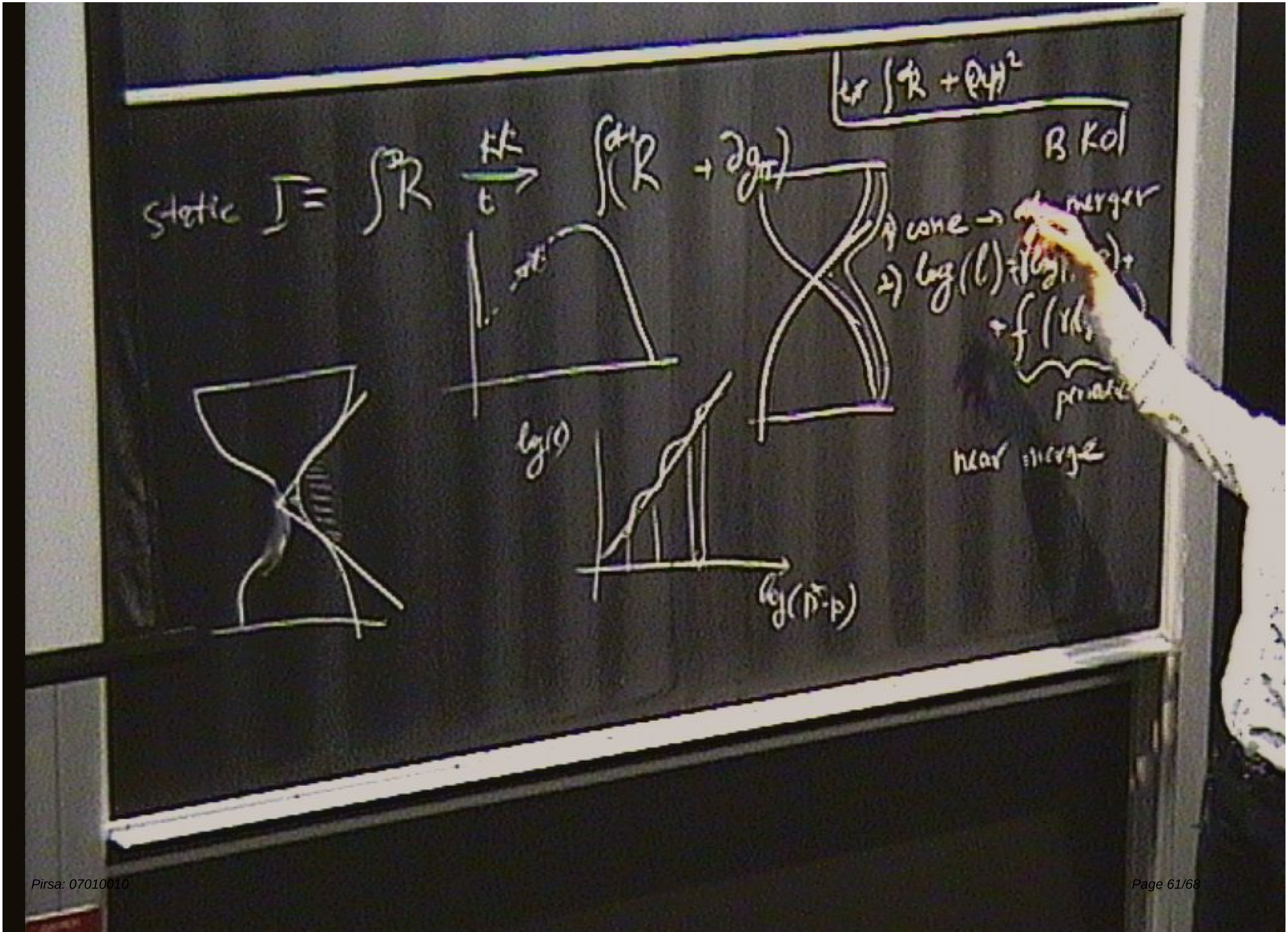

1) cone → at merger
2) log(l) = γ log(p* − p) + f(γ(p* − p))
periodic
near merge
log(l)
log(p* − p)

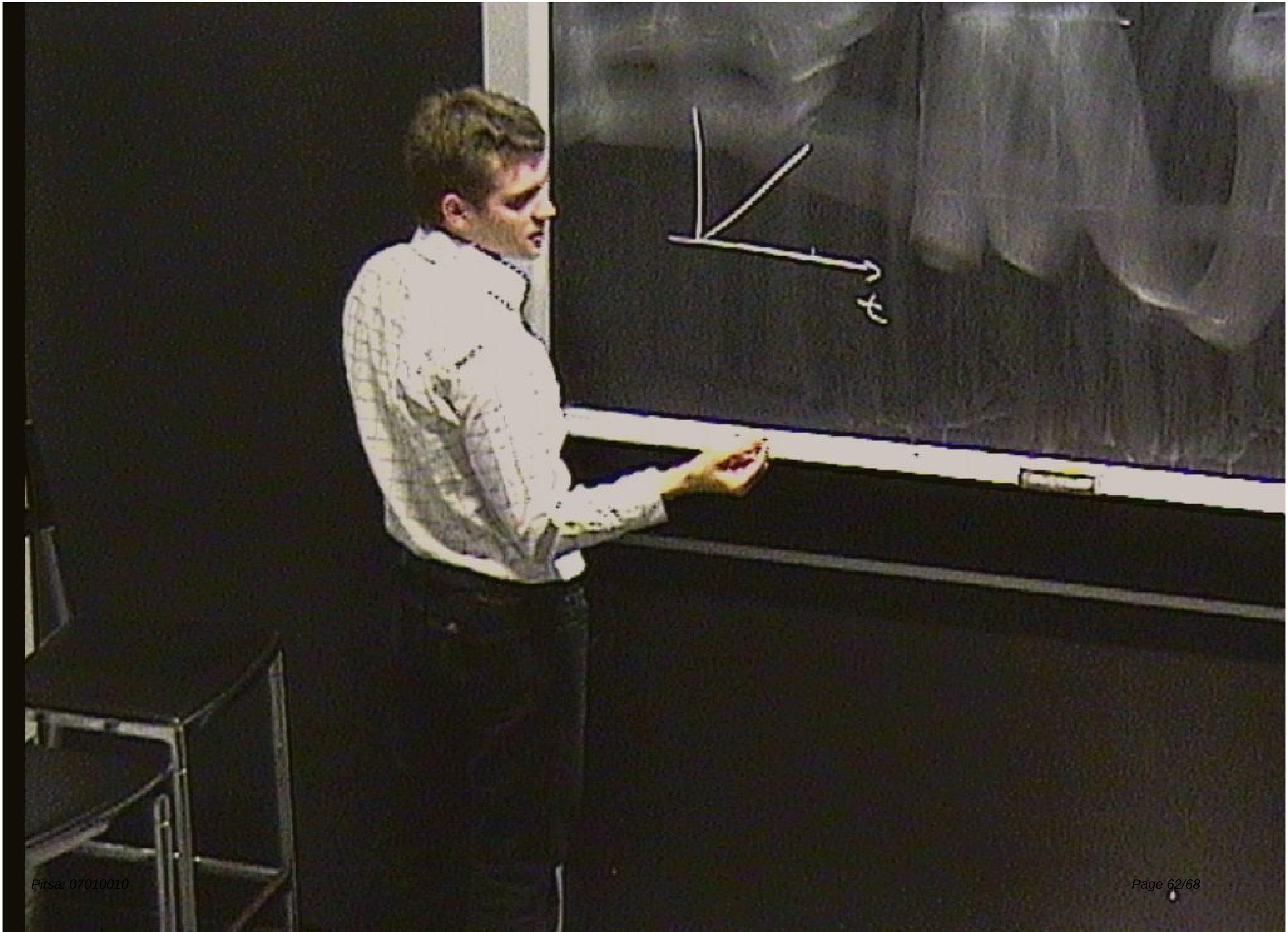

1) cone → at merger
near merge

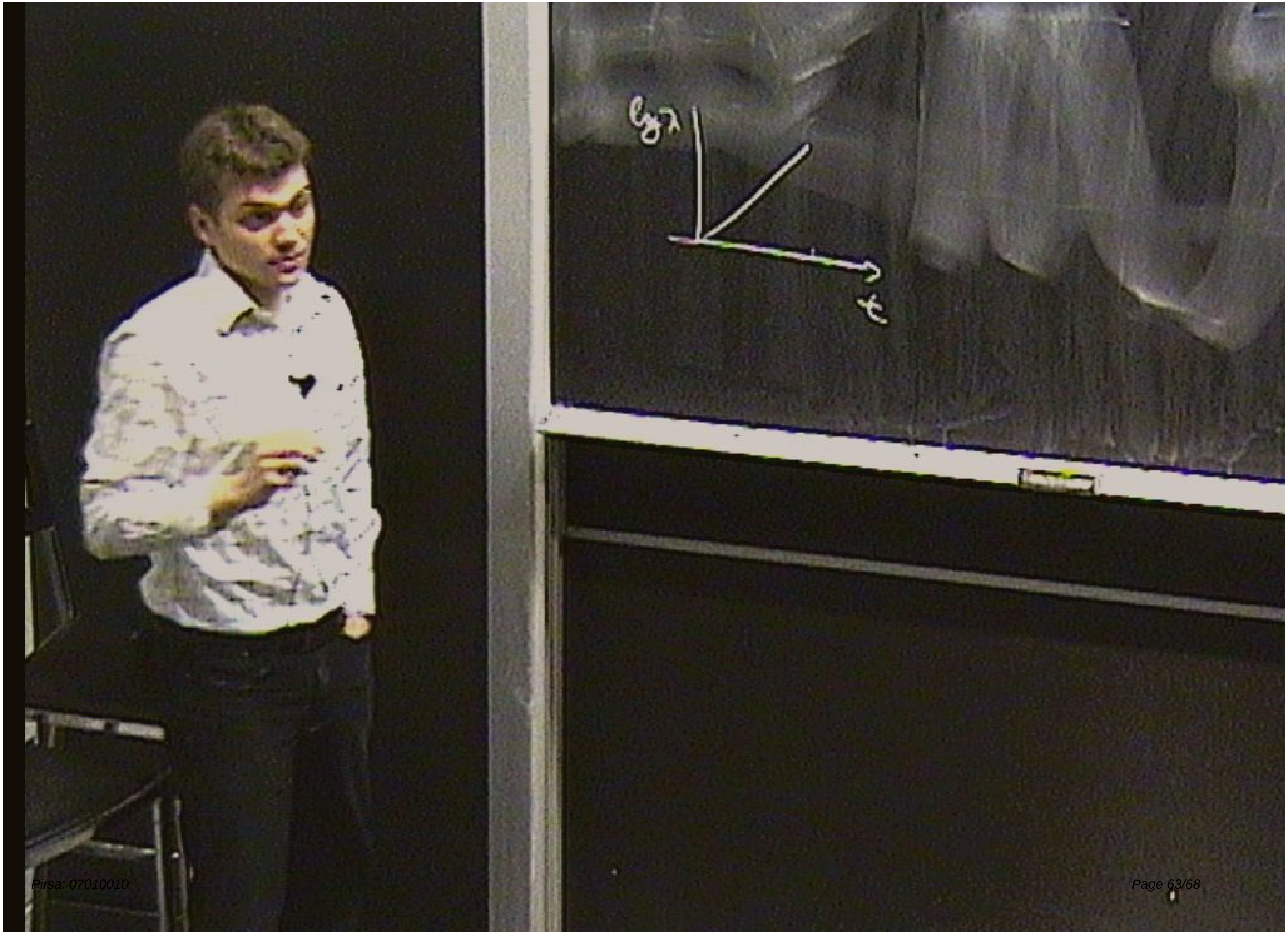

B Kol
cone → al. merger
private
near merge
log(ℓ)
log(p*-p)

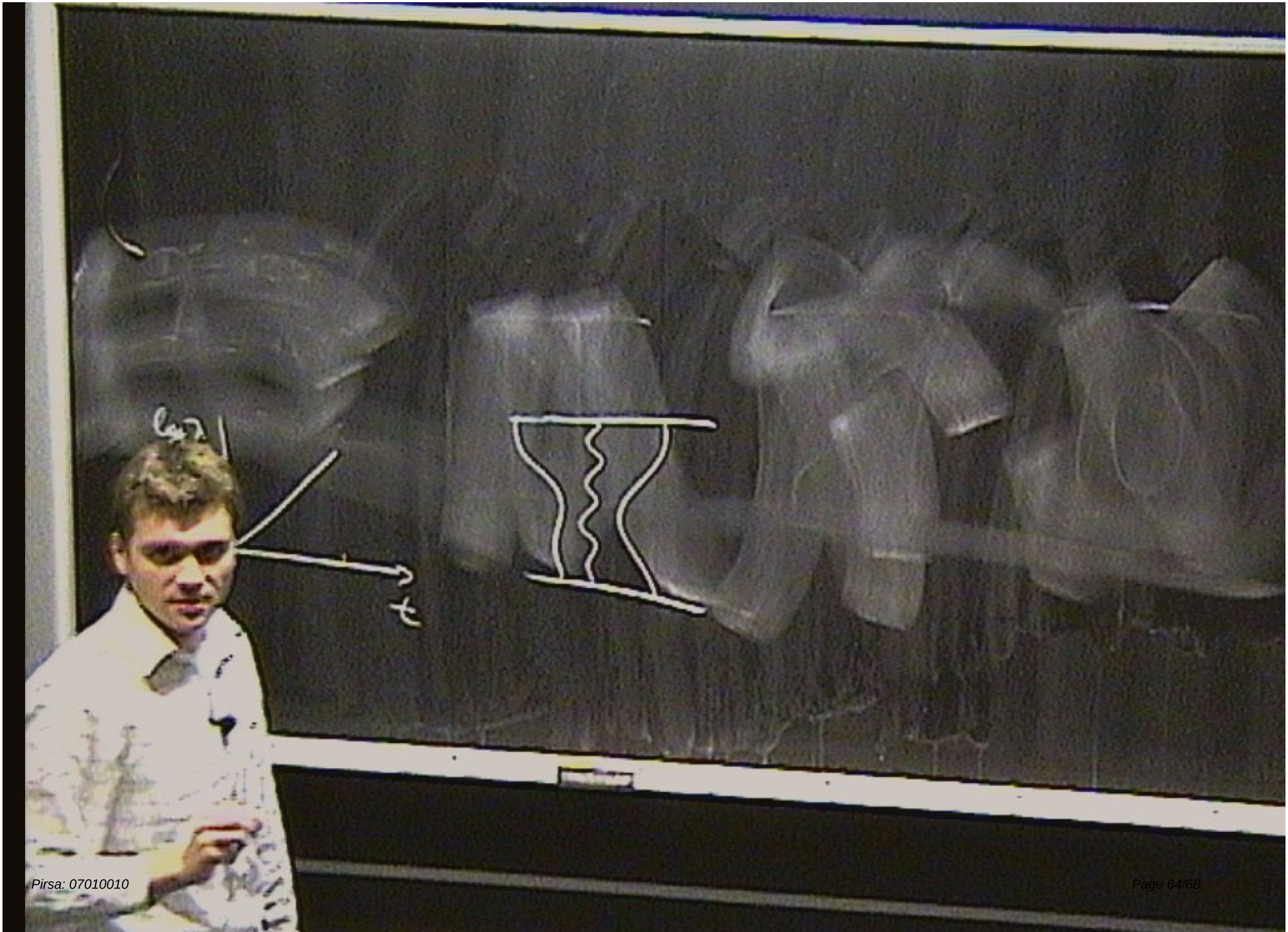

B Kol
1) cone → al. merger
2) log(l) = γ log(p* - p) + f(γ λ(p* - p))
periodic
near merge
log(l)
log(p* - p)

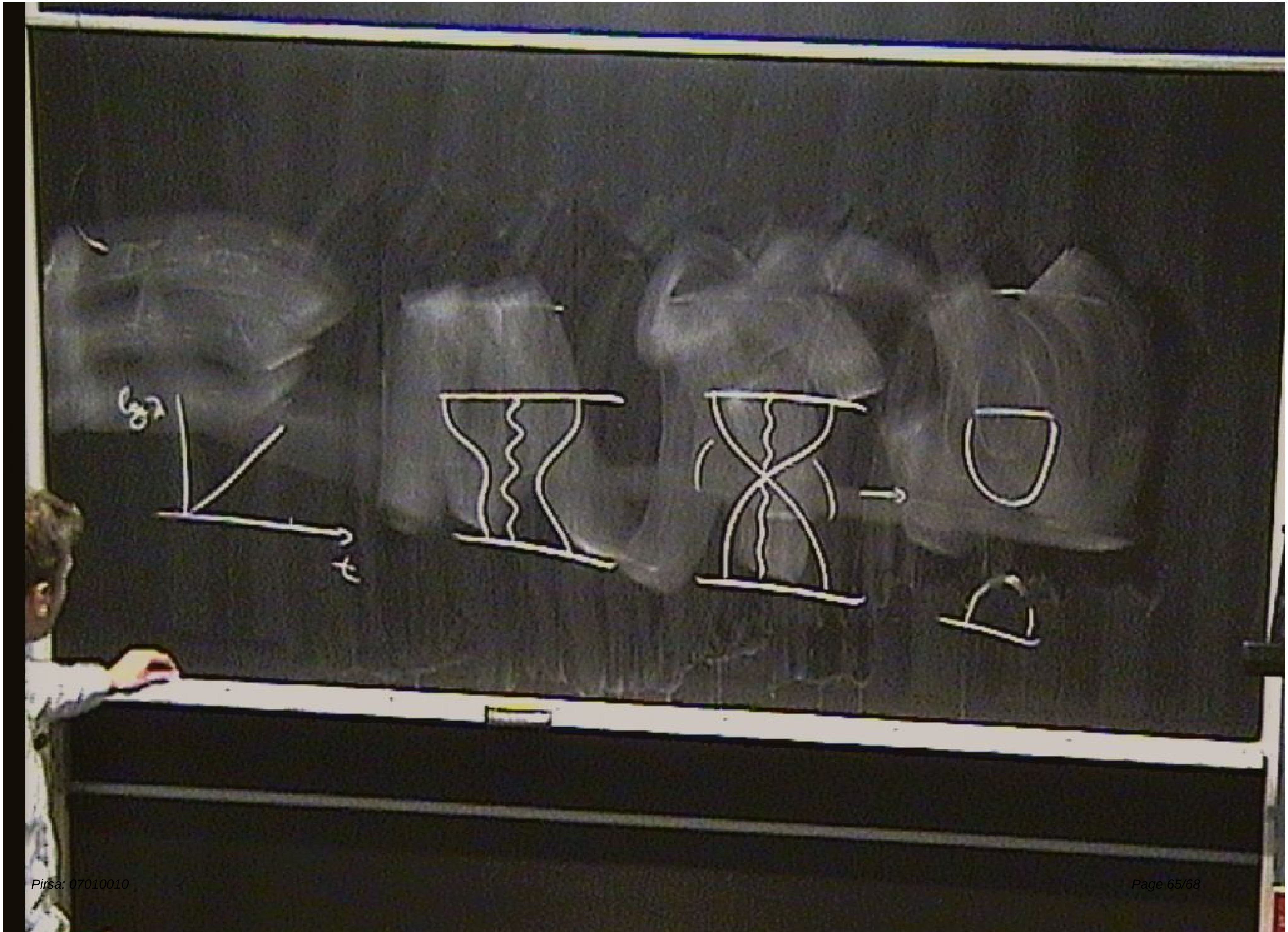


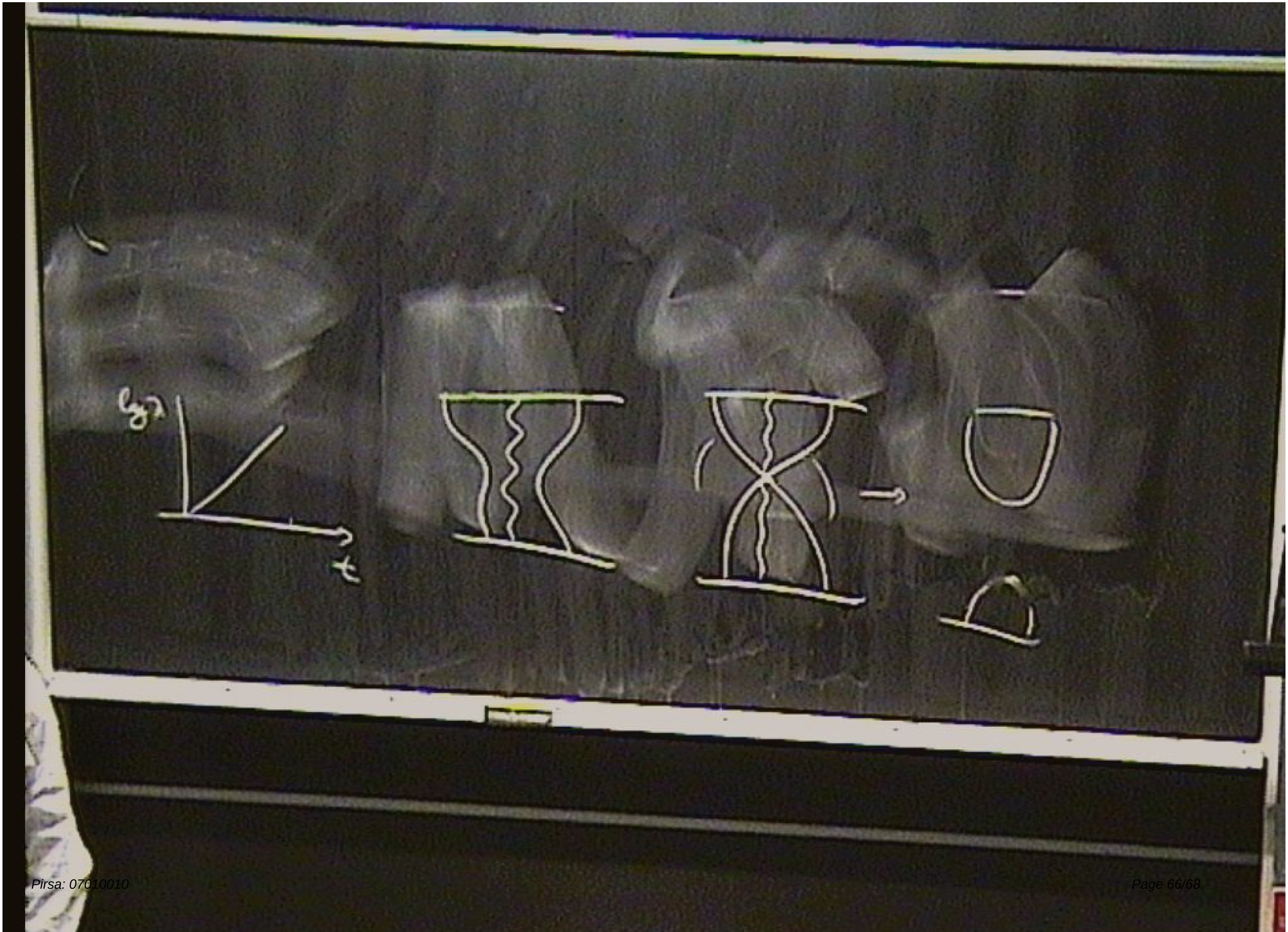

static

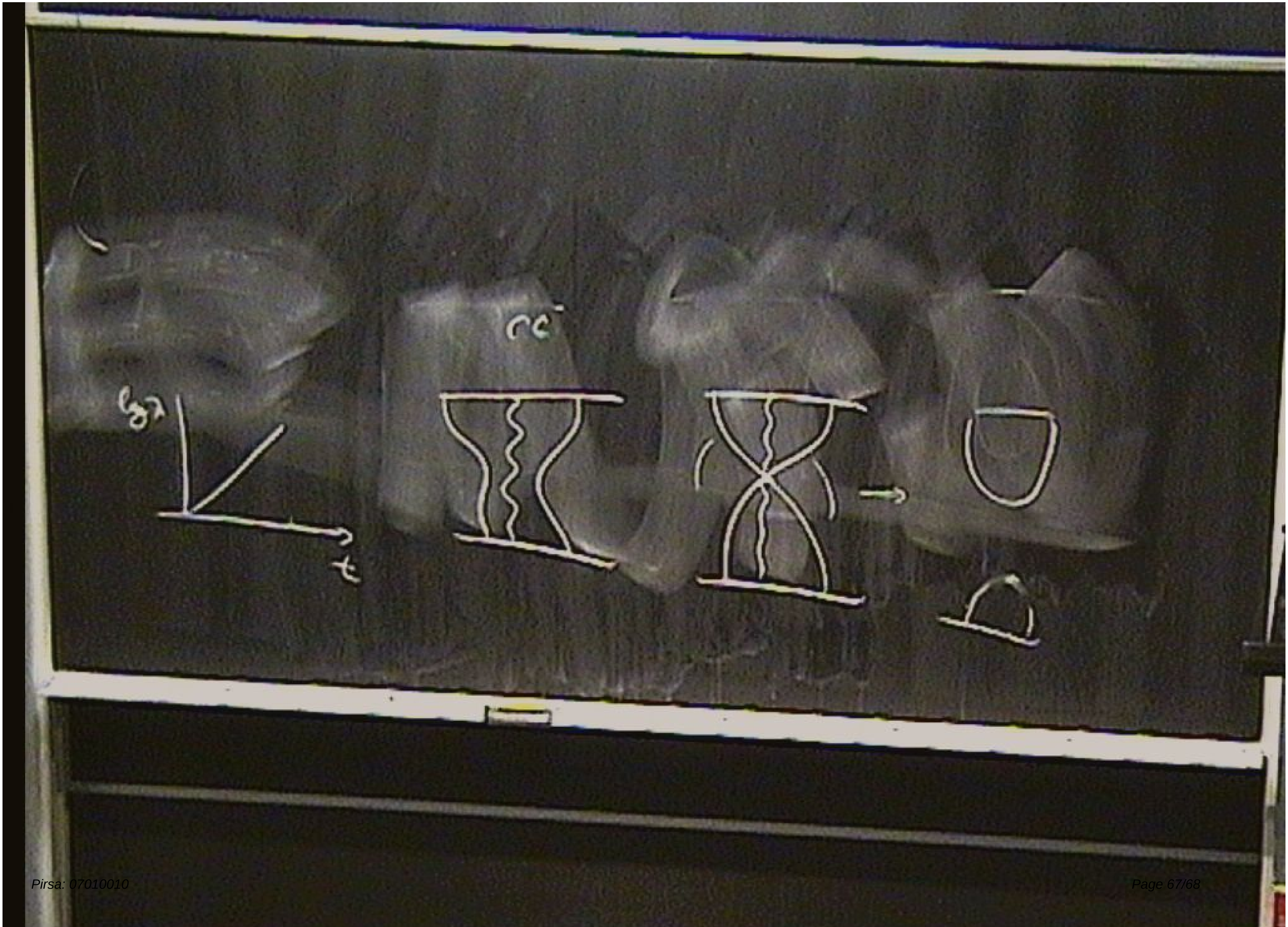

t

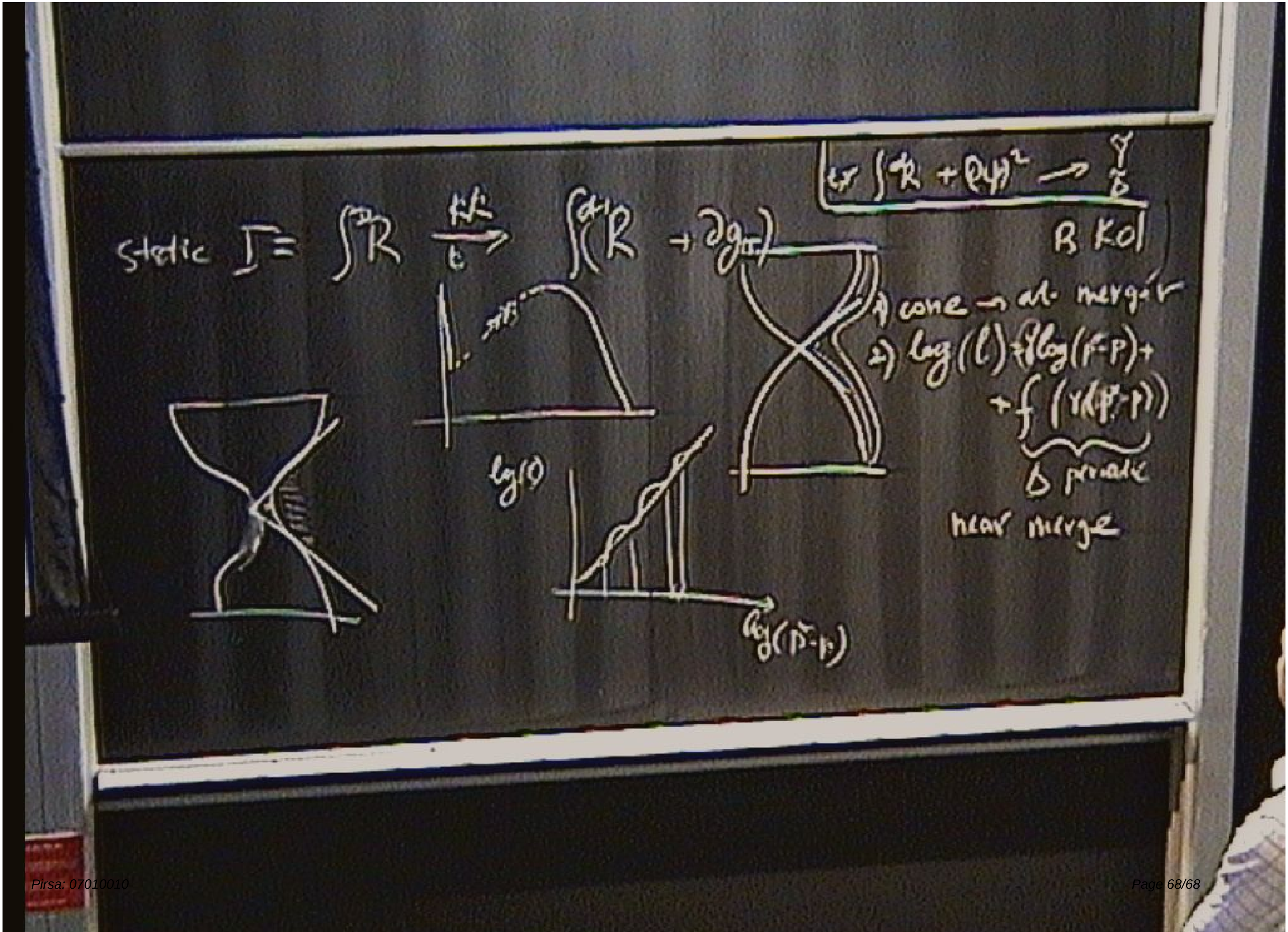

log λ
t








t


Static Γ = ∫R → ∫(R + ∂g)
B KO
1) cone → at merger
2) log(l) = log(p* - p) + f(γ log(p* - p))
Δ periodic
near merge
log(p* - p)