Title: The strength of crystalline color superconductors

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Abstract: We discuss the properties of matter in the low temperature regime at density that may exist in the core of compact stars.
Assuming that in these conditions quarks are deconfined the attractive
color interaction determines the formation of Cooper pairs of quarks
and the resulting quark matter has properties analogous to standard
superconductors.
We show that under reasonable conditions a state were Cooper pairs
have non-zero total momentum is energetically favored and the
resulting non-homogeneous condensate is characterized by a crystal
symmetry.
Studying the elastic properties of such a state we find that it
behaves like a solid crystal with a very large shear modulus.
Our results raise the possibility that
(some) pulsar glitches may originate within the Crystalline Color
Superconductor core of Neutron stars.

The strength of crystalline color superconductors

Massimo Mannarelli

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hep-ph/0603076 and hep-ph/07... Collaboration with K.Rajagopal and R.Sharma

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Outline of the talk

- Background and motivations
  - Basics of Color Superconductivity (CSC)
- Stressing superconductive matter
  - Mismatching the Fermi surfaces
  - Magnetic instability
- Crystalline color superconductors
  - Two-flavor case
  - Three-flavor case
  - Phonons and Shear modulus
- Conclusions

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Reviews: hep-ph/0011333, hep-ph/0102047, hep-ph/0509068

Technicalities: The High Density Effective Theory (HDET), hep-ph/0202037

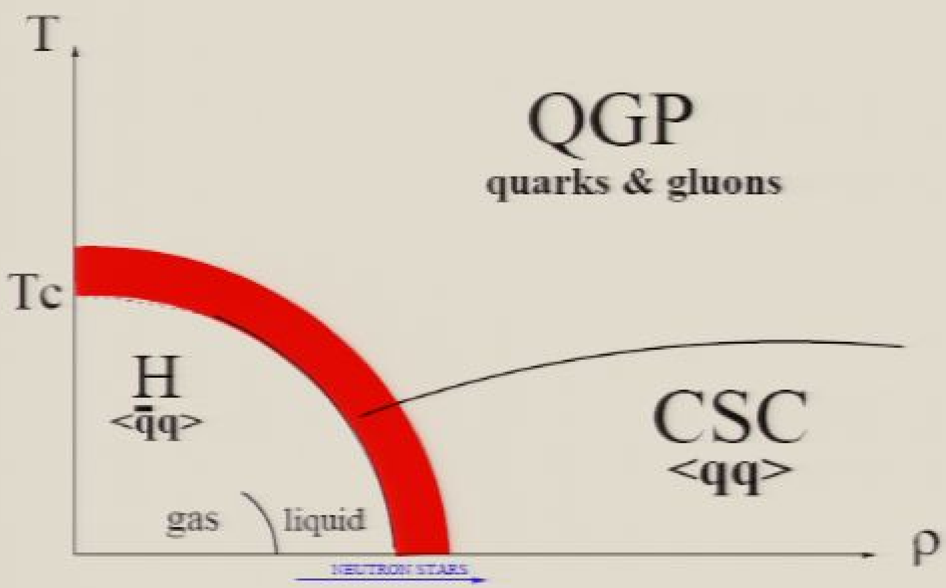
Motivations

Why do we study matter in extreme conditions $\rho >> \rho_0 = 0.17\ \mathrm{fm}^{-3}$ and T$\sim$ KeV ?

- Exploring the entire phase diagram is useful to understand the structure of hadrons and of non perturbative QCD
- QCD simplifies: Asymptotic freedom tells that at high densities quarks and gluons are the correct degrees of freedom.
- These conditions may exist in compact stellar objects

In particular we are interested in explaining and/or predicting neutron star phenomenology

QCD phase diagram



- Three main phases:
  - H Hadronic
  - QGP Quark-gluon plasma
  - CSC Color Superconductor
- Red region: non perturbative effects are relevant.

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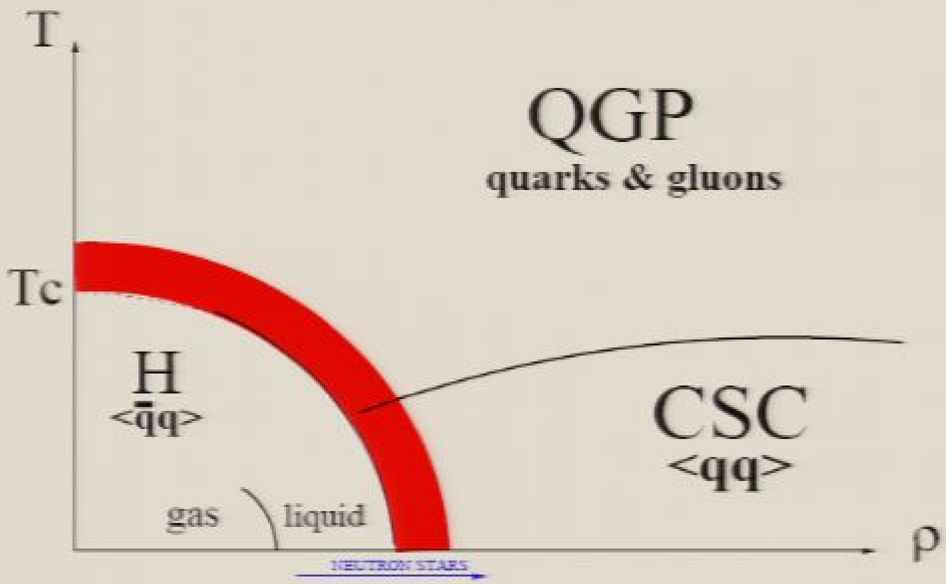

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- We know $T_c \sim 170$ MeV from unquenched lattice simulations.
- At non zero (real) chemical potential lattice simulations are in their early stage.
- The behavior of matter near the deconfinement phase transition is not completely understood. At $T_c < T \lesssim 2T_c$, RHIC data and lattice simulations indicate that matter is strongly coupled

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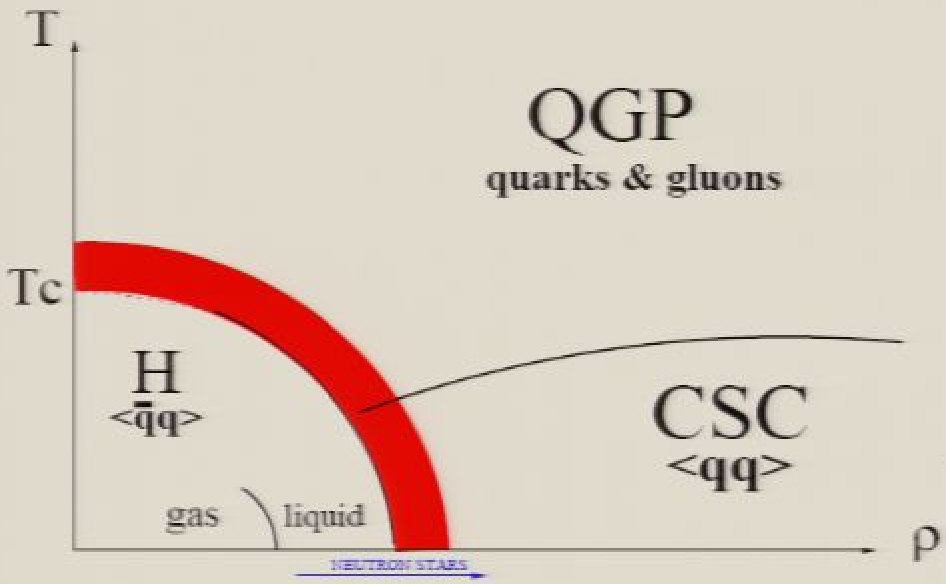

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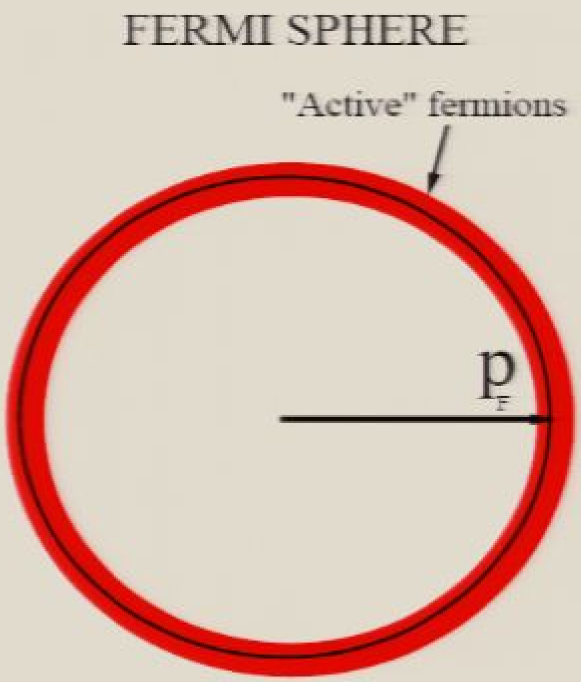
Fermions at high ρ and small T



- Fermions fill all the levels up to the Fermi energy ϵ_F
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  - Antiparticles decouple

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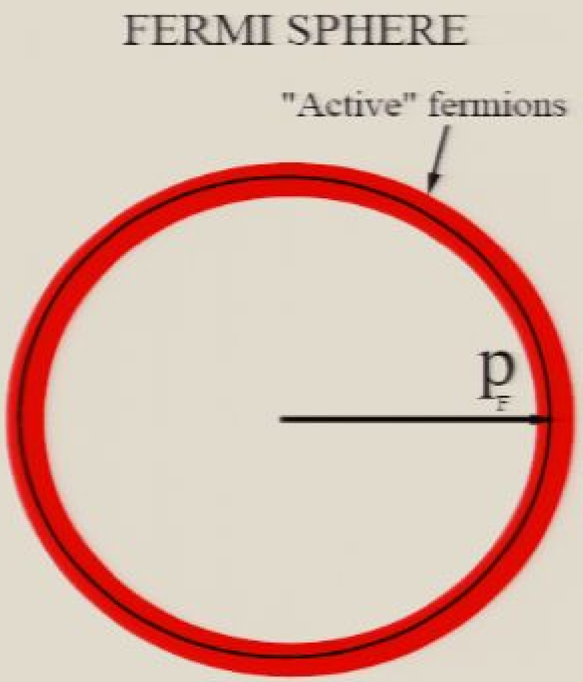

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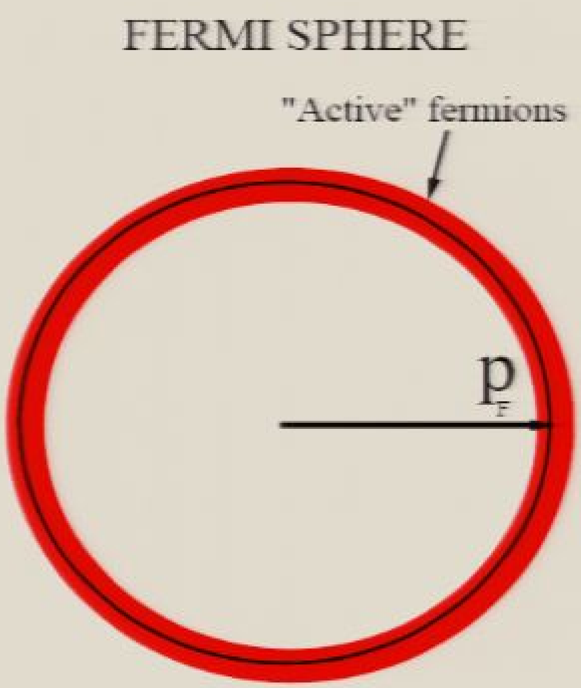

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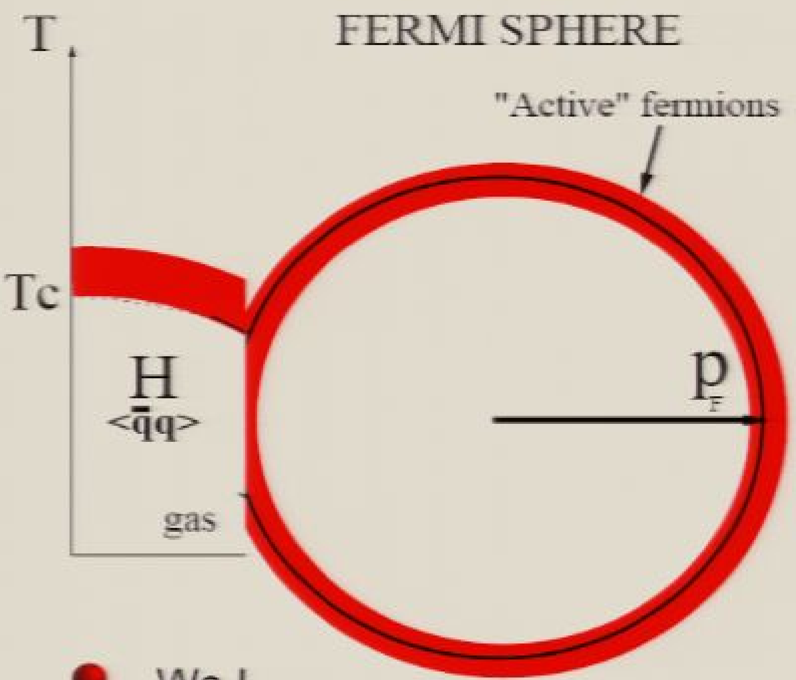

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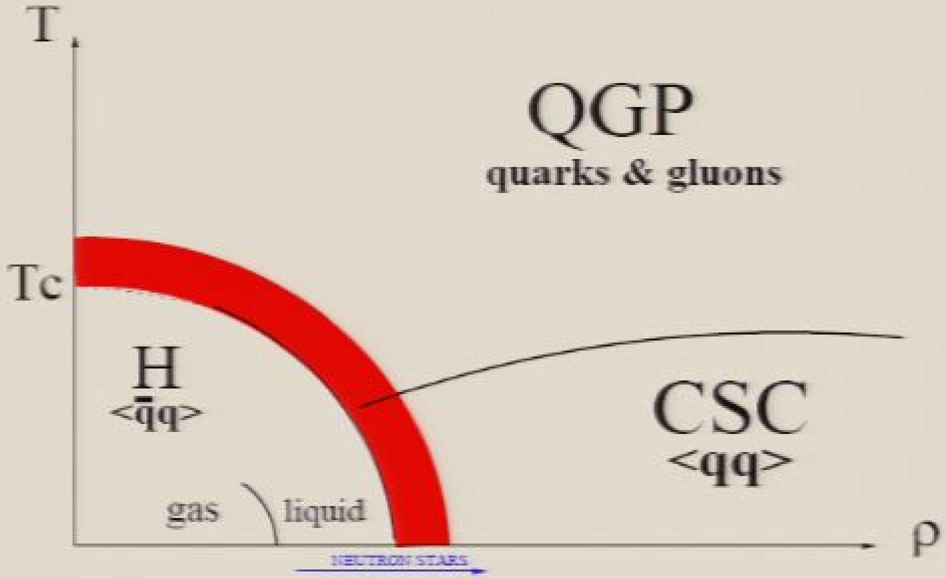

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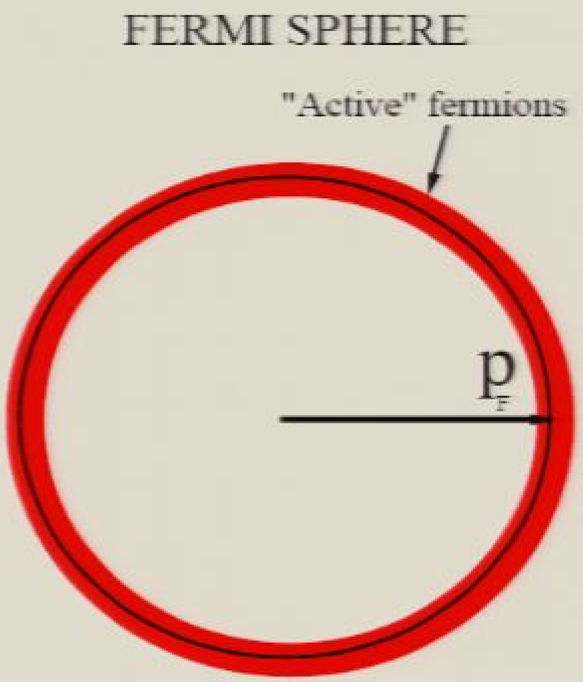

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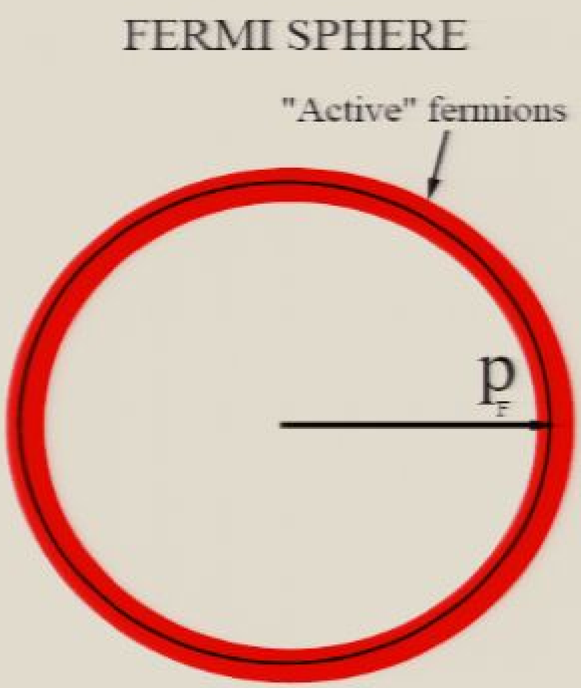

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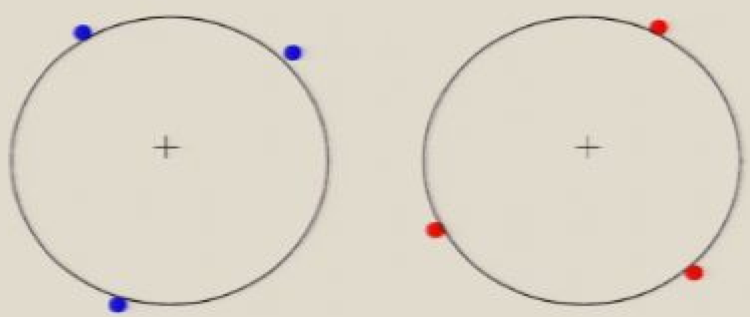
Superconductivity

Cooper Theorem: At high density and sufficient low temperature any (arbitrarily weak) attractive interaction between fermions $\rightarrow$ Cooper pairs.



Cooper pairs: correlated pair of half-integer spin particles with total spin 0 and total momentum 0

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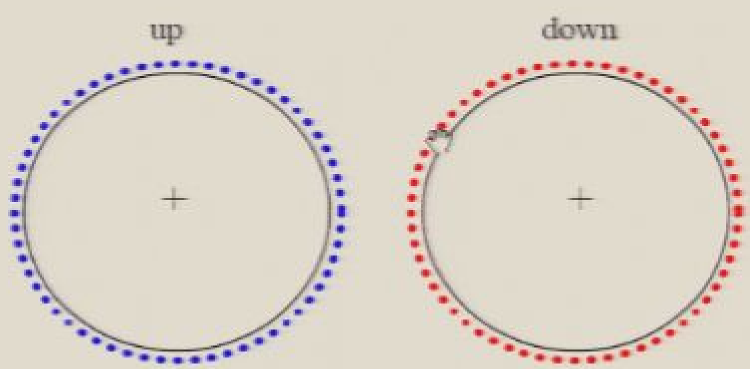

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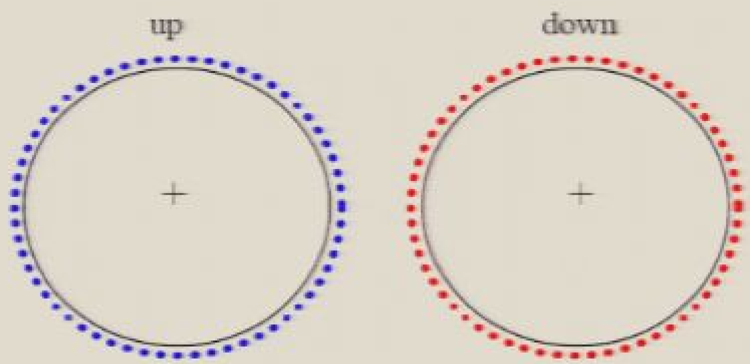


Cooper pairs: correlated pair of half-integer spin particles with total spin 0 and total momentum 0

- Breaking of gauge symmetries
- The magnetic field is expelled: Meissner effect
- The excitation spectrum is described by gapped quasi-particle excitations

Color Superconductivity

This story begins soon after the discovery of asymptotic freedom *J.C. Collins* and *M.J. Perry* Phys. Rev. Lett. **34** 1353 (1975) and *Barrois B.* Nucl. Phys. B **129** 390 (1977).

"New" development: instantons *M. Alford et al.* hep-ph/9711395, *R. Rapp et al.* hep-ph/9711396, long range chromo-magnetic interactions *D.T. Son* hep-ph/9812287

- quark-quark condensate <q q>
- the interaction in color $\bar{3}$ channel is attractive
- it is possible to form a condensate with total momentum 0 and total spin 0
- no need to resort to phonons: Color Superconductivity is driven by the color interaction and is a robust phenomenon

Large gap $\Delta \sim 10 - 100$ MeV

Nambu-Jona Lasinio model

The non-interacting Lagrangian

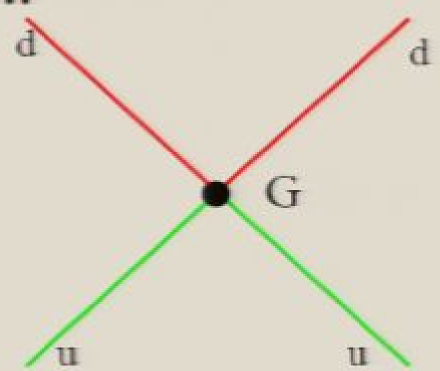
$$\mathcal{L}_0 = \bar{\psi}\,(i\,\partial\!\!\!/ + \mu\,\gamma_0)\,\psi$$

where

$$\mu_{ij}^{\alpha\beta} = (\mu_b\delta_{ij} - \mu_Q Q_{ij})\,\delta^{\alpha\beta} + \delta_{ij}\left(\mu_3 T_3^{\alpha\beta} + \frac{2}{\sqrt{3}}\mu_8 T_8^{\alpha\beta}\right)$$

$i, j = 1, 3$ flavor; $\alpha, \beta = 1, 3$ color. In NJL models the color and electrical chemical potential are introduced by hand. In QCD the gauge fields assure the neutrality conditions.

Interaction:



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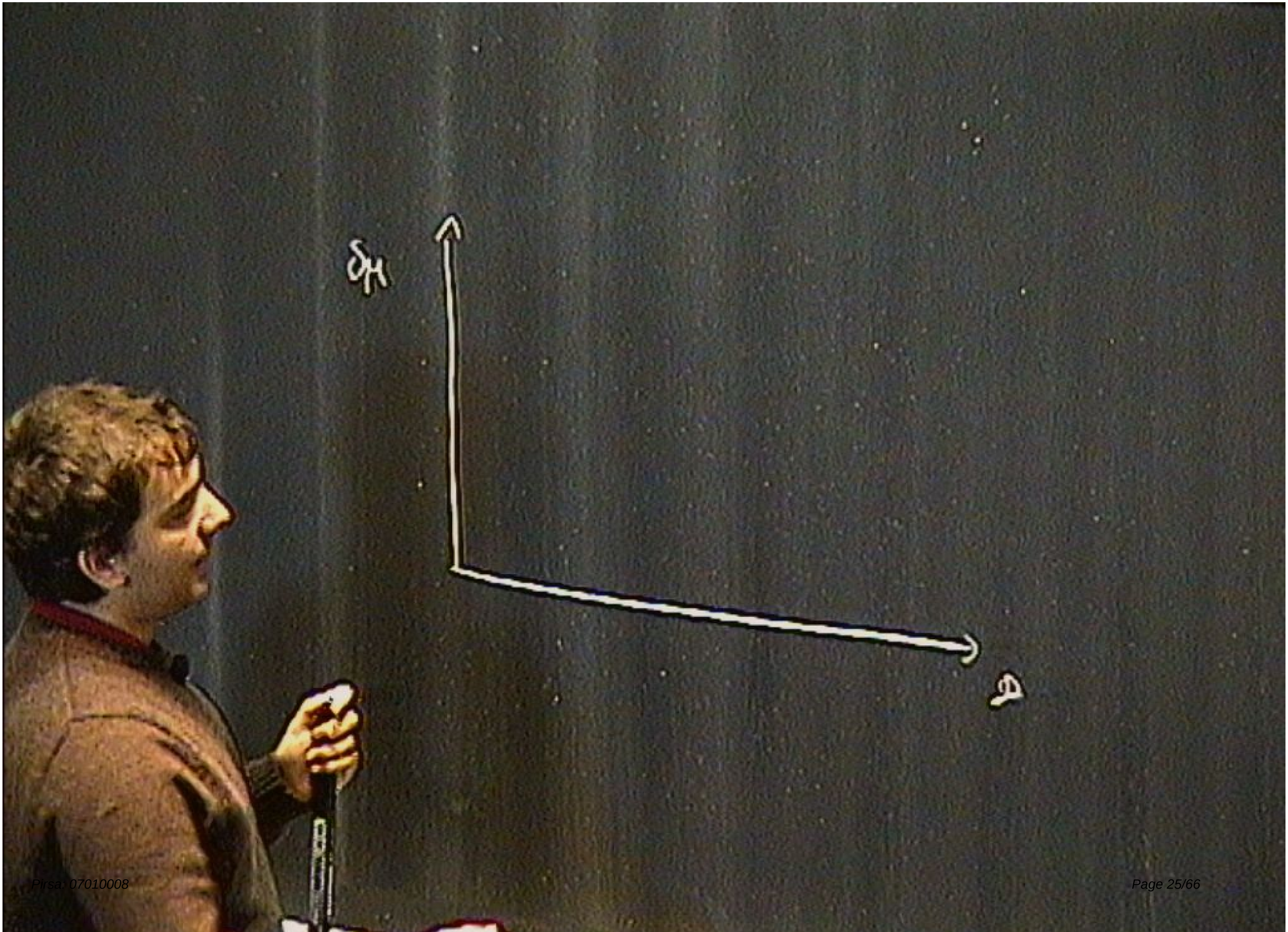
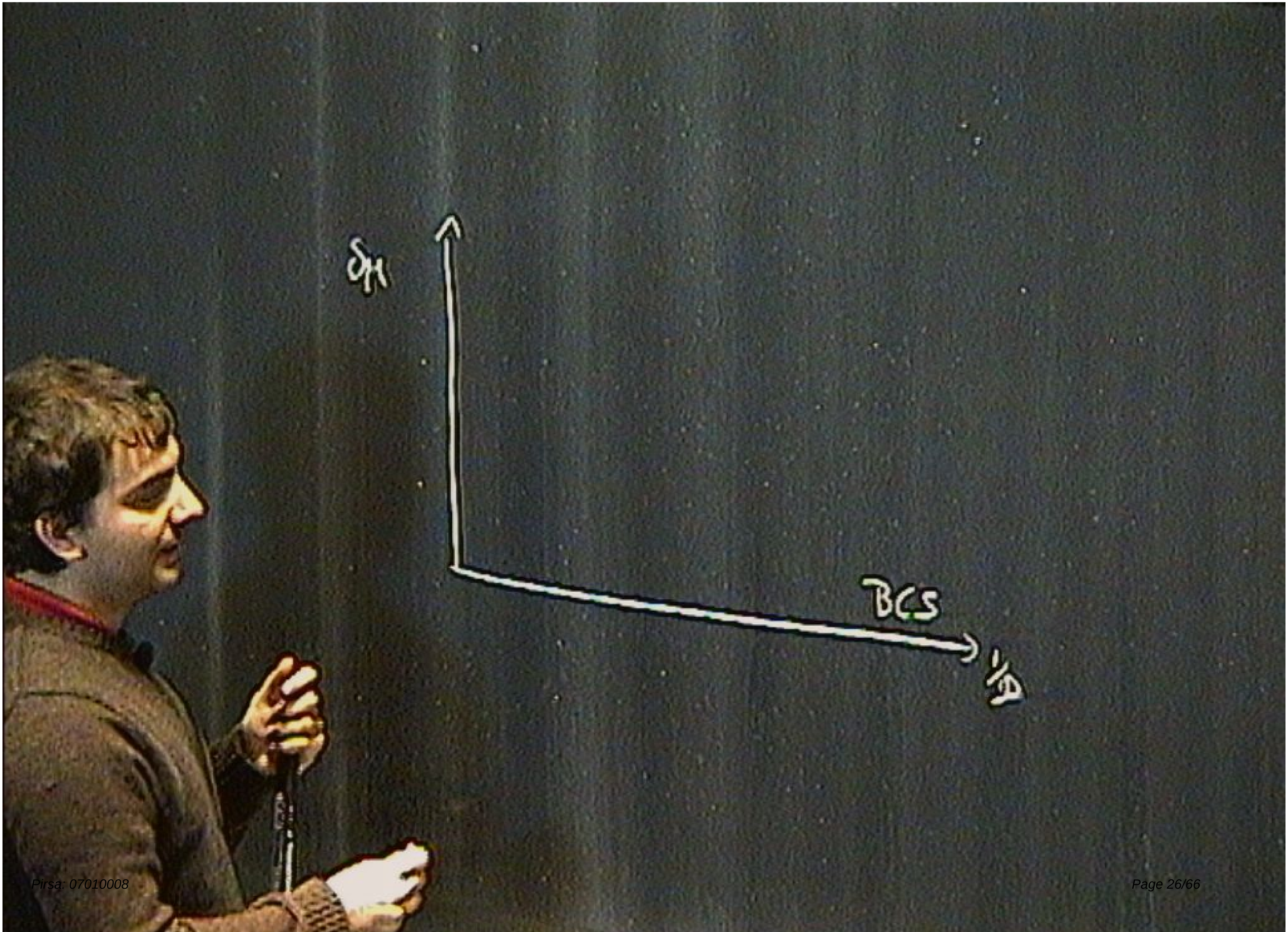
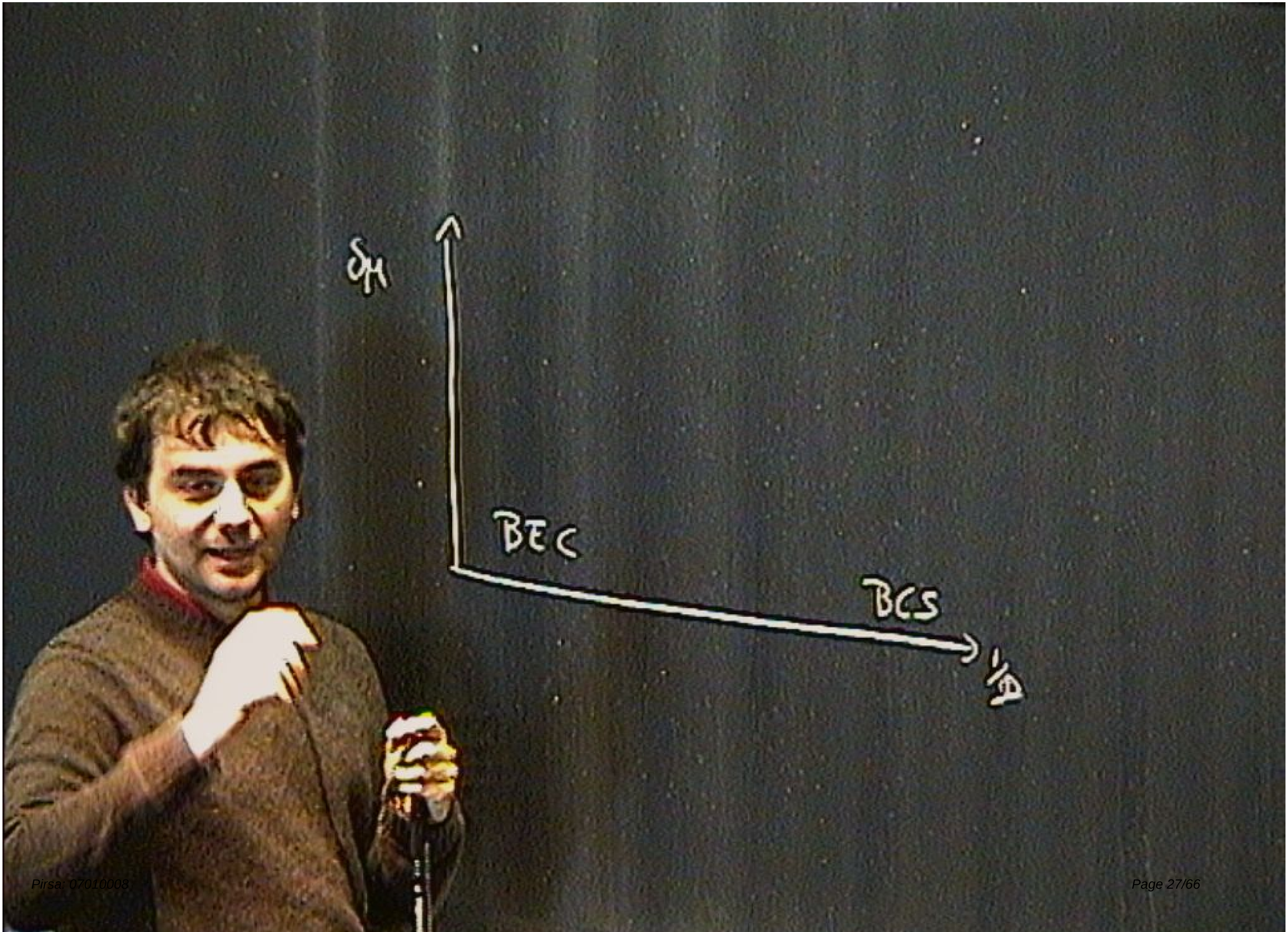


δ_H


BCS


BEC
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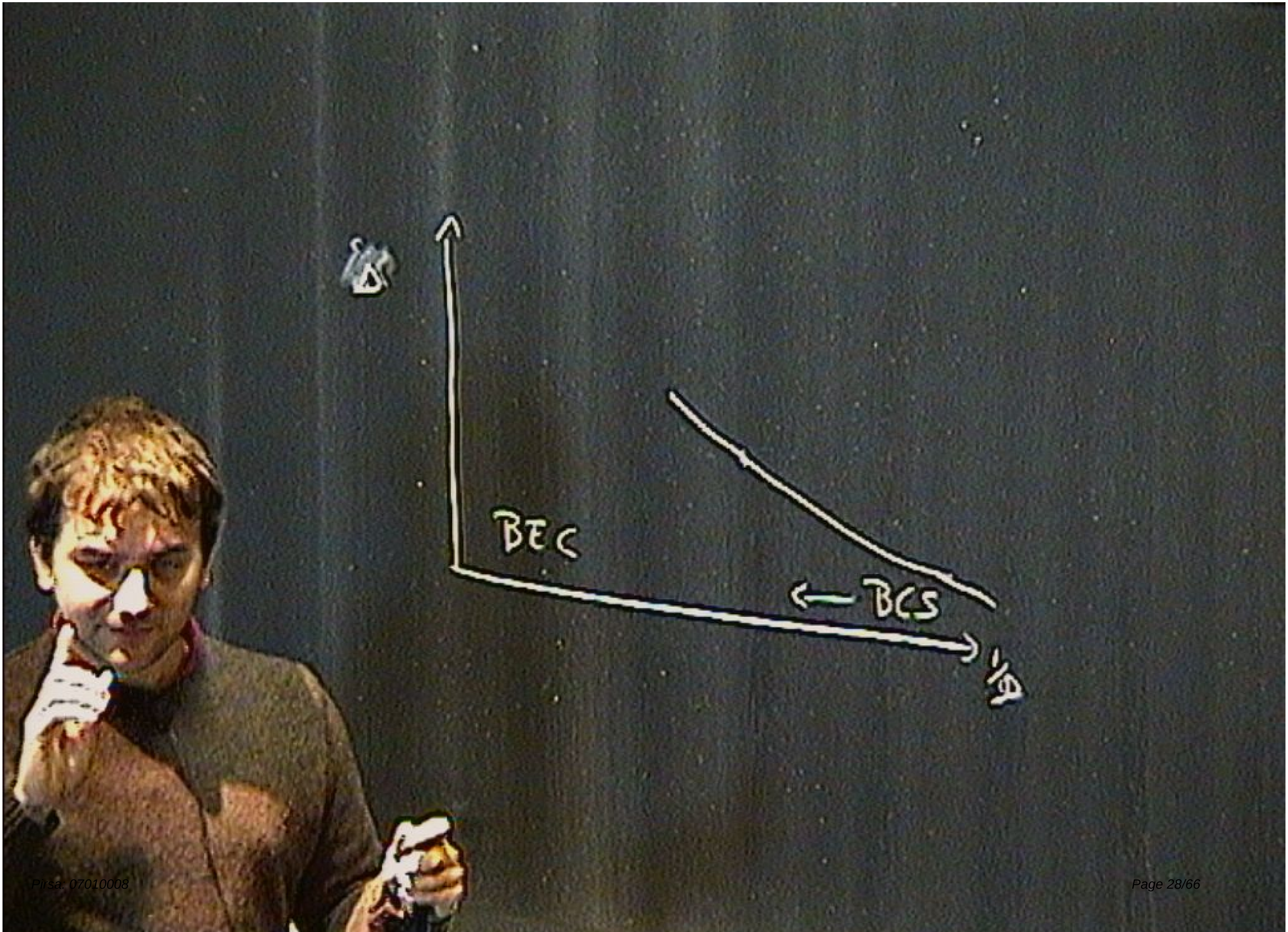

BEC
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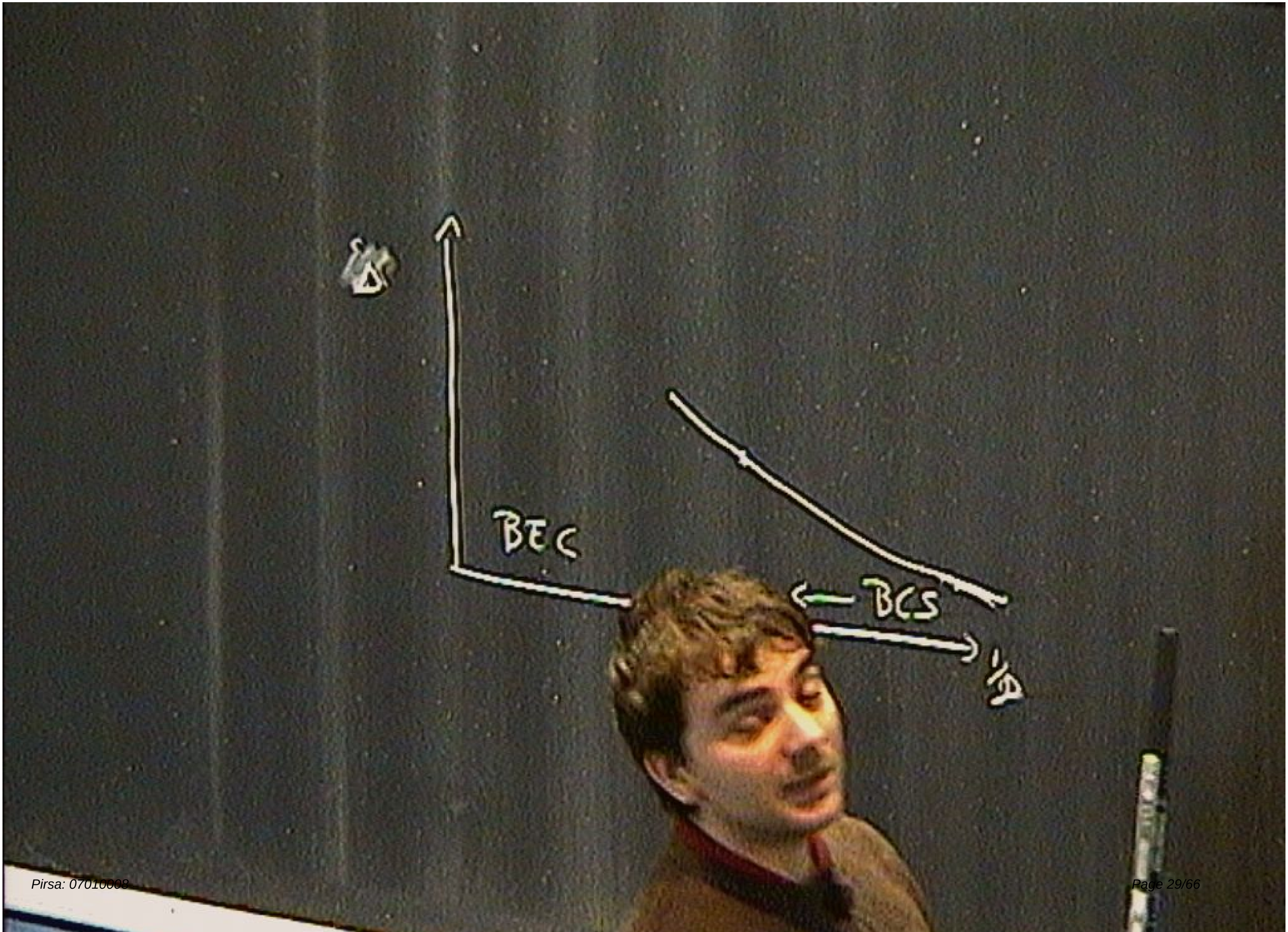
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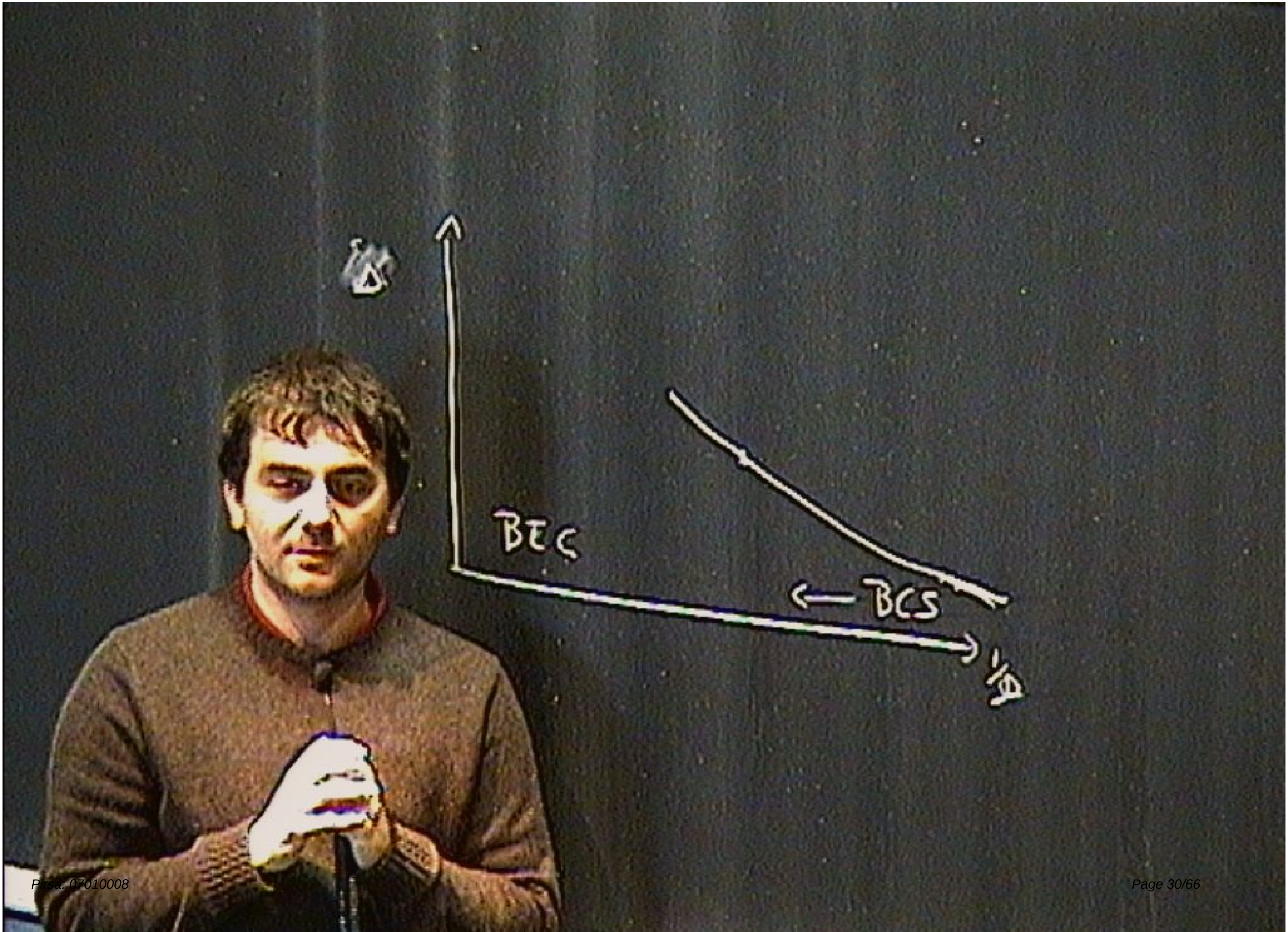
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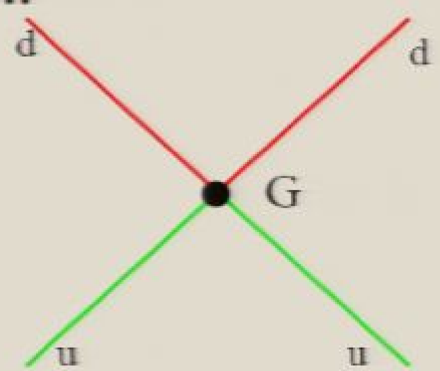

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Mean field and CSC phases

The interaction Lagrangian term in mean field approximation

$$\mathcal{L}_\Delta = \frac{1}{2}\,\Delta_{ij}^{\alpha\beta}\,\psi_{i,\alpha}\,C\,\gamma_5\,\psi_{j\beta} + h.c.\,.$$

where the gap matrix is

$$\Delta_{ij}^{\alpha\beta} \sim <\psi_{i\alpha}\,C\,\gamma_5\,\psi_{\beta j}> = \sum_{I,K=1}^{3}\,\Delta_{IK}\,\epsilon^{\alpha\beta I}\,\epsilon_{ijK}$$

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- $\Delta_3 = \Delta_2 = \Delta_1 > 0$ CFL (*M. Alford et al.* hep-ph/9804403)
- $\Delta_3 > \Delta_2 > \Delta_1 > 0$ gCFL (*M. Alford et al.* hep-ph/0311286)
- $\Delta_3 > 0\; \Delta_2 = \Delta_3 = 0$ 2SC (g2SC)

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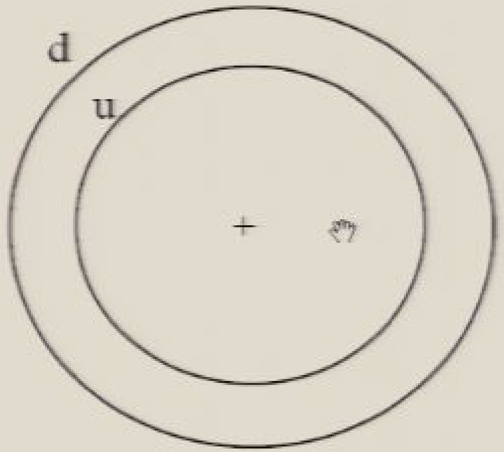
Breaking patterns:

CFL $SU(3)_C \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_{C+L+R}$

2SC $SU(3)_C \times SU(2)_L \times SU(2)_R \rightarrow SU(2)_C \times SU(2)_L \times SU(2)_R$

Stressing the superconductor

Mismatched Fermi spheres: $\mu_1 = \mu + \delta\mu$, $\mu_2 = \mu - \delta\mu$



Clogstone limit (weak coupling): for $\delta\mu < \Delta_0/\sqrt{2}$ the BCS superconductor is stable
For larger $\delta\mu$ several possibilities

- Phase separation *P.F. Bedaque et al.* cond-mat/0306694, For 2SC *S. Reddy and G. Rupak* nucl-th/0405054 however for three-flavors does not work *M. Alford et al.* hep-ph/0407257
- Deformed Fermi spheres *H. Muther and A. Sedrakian* cond-mat/0202409
- Breached Pair *M.M. Forbes et al.* hep-ph/0405059
- LOFF (or FFLO) *A.I. Larkin and Y.N. Ovchinnikov '65, P. Fulde and R.A. Ferrel '64*

Mismatch driven by neutrality

The Fermi momenta of the quarks and electrons in quark matter that is electrically and color neutral and in weak equilibrium are given in the absence of pairing by

$$\begin{aligned} p_F^d &= \mu + \frac{\mu_e}{3} \\ p_F^u &= \mu - \frac{2\mu_e}{3} \\ p_F^s &= \sqrt{\left(\mu + \frac{\mu_e}{3}\right)^2 - M_s^2} \approx \mu + \frac{\mu_e}{3} - \frac{M_s^2}{2\mu} \\ p_F^e &= \mu_e \ , \end{aligned}$$

for M_s and μ_e much smaller than μ electric neutrality requires $\mu_e = \frac{M_s^2}{4\mu}$, yielding

$$p_F^d = p_F^u + \frac{M_s^2}{4\mu}; \quad p_F^u = \mu - \frac{M_s^2}{6\mu}; \quad p_F^s = p_F^u - \frac{M_s^2}{4\mu}; \quad p_F^e = \frac{M_s^2}{4\mu} \ .$$

Note that for $M_s = 0$ one gets no electrons

The gapless CFL phase

Ansatz: Gap matrix $\Delta_{IK} = \Delta_I \delta_{IK}$. Note that Δ_I describes pairing between quarks whose flavor and color is not I.

Ex. Δ_1 describes $< ds >$ and $< sd >$ pairing

With increasing splitting (proportional to M_s) between the chemical potentials

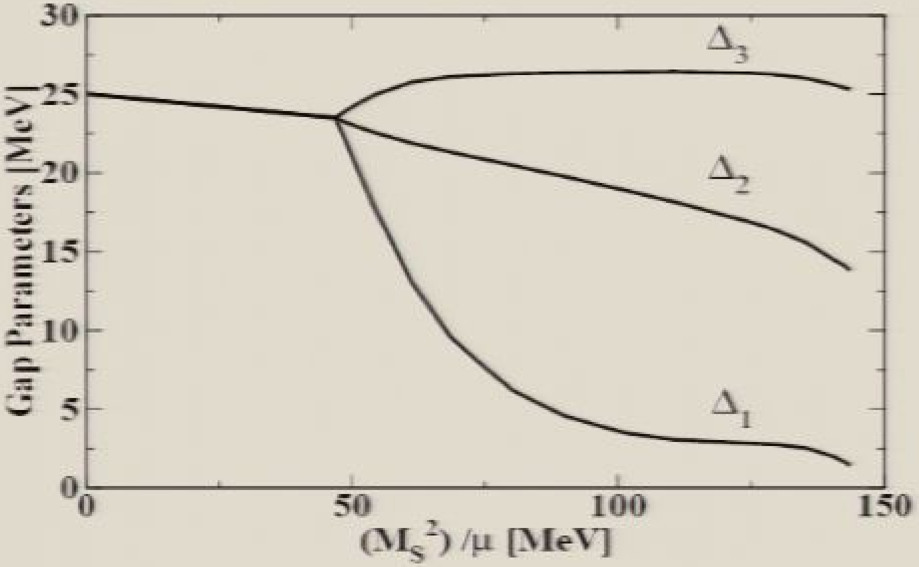
- $0 < M_s^2/\mu_b \lesssim 2\Delta_0$ CFL ($\Delta_3 = \Delta_2 = \Delta_1$) is stable
- $2\Delta_0 \lesssim M_s^2/\mu_b \lesssim \Lambda$ gCFL ($\Delta_3 > \Delta_2 > \Delta_1$) is favored
- $\Lambda \lesssim M_s^2/\mu_b$ Unpaired quark matter is favored

Where Δ_0 is the gap at zero mismatch and Λ depends on the value Δ_0. For $\Delta_0 = 25$ MeV, we get $\Lambda = 135$ MeV;

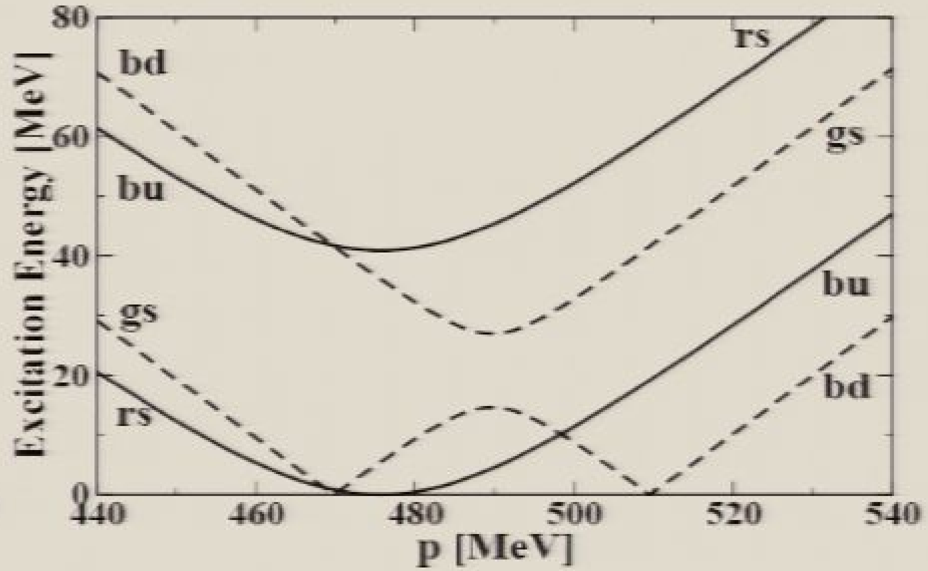
The gapless CFL phase

Some results for the gapless CFL phase of Alford et al. '04.

Gaps



Dispersion laws $M_s^2/\mu_b = 80$ MeV

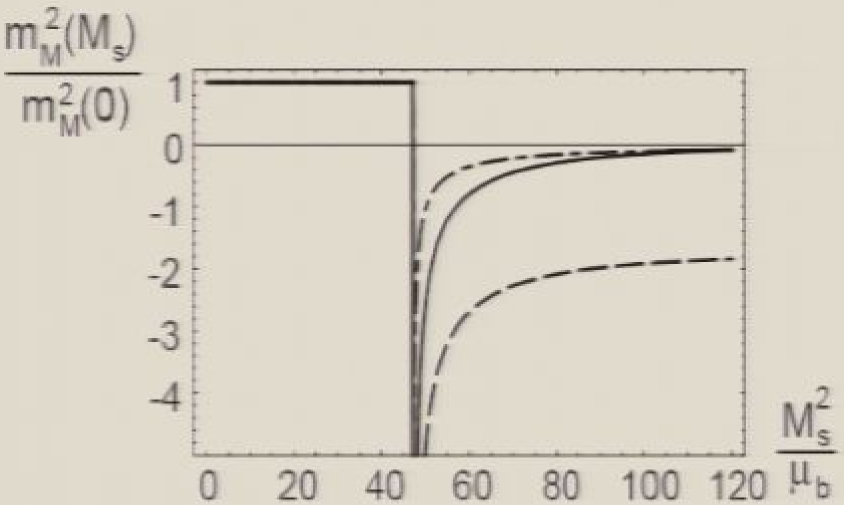


Instability of gCFL

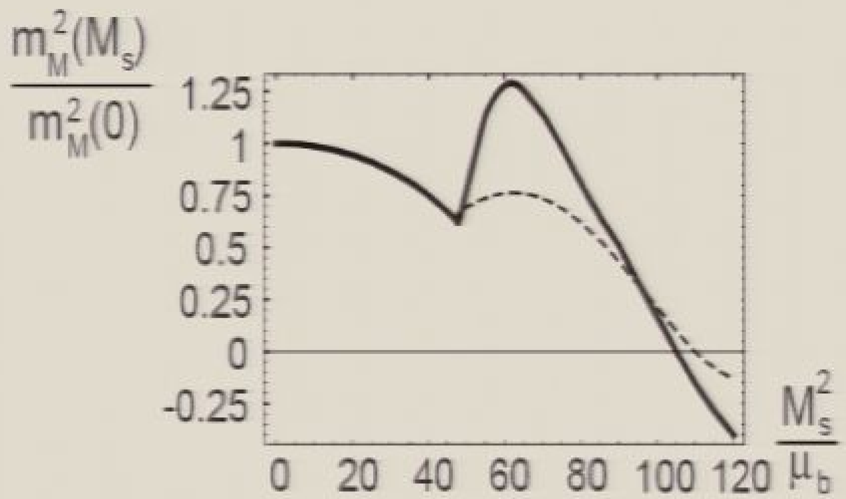
The gapless CFL phase is characterized by imaginary Meissner masses m_M for some gauge fields. For $M_s = 0$ (in CFL)

$$m_M^2(0) = \frac{\mu^2 g^2}{\pi^2}\left(-\frac{11}{36} - \frac{2}{27}\ln 2 + \frac{1}{2}\right)$$

With a non vanishing strange quark mass *Casalbuoni et al.* hep-ph/0410401 see also *M. Alford and Q. Wang* hep-ph/0501078



Solid $a = 1, 2$; dashed $a = 3$; dot-dashed $a = 8$



Dashed $a = 4, 5$; solid $a = 6, 7$

Possible solutions

Several possible solutions of the instability have been proposed. In general in terms of a phase transition.

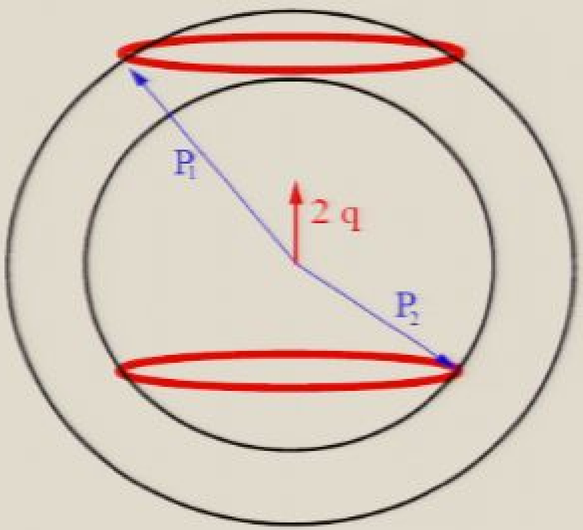
- Phase separation
- Transition to an anisotropic (LOFF) superconductor *Giannakis et al.* hep-ph/0412015.
- Transition to a state with a spontaneously generated baryon current *Huang et al. '05.*
- "Gluonic phase": Vector condensation of gluons *E.V. Gorbar et al.* hep-ph/0507303
- Secondary condensate *D.K. Hong* hep-ph/0506097, however *M. Alford and Q. Wang* hep-ph/0507269 find that this does not work.

2-flavor LOFF phase

Larkin-Ovchinnikov and Fulde-Ferrel (LOFF) have shown that for $\delta\mu_1 < \delta\mu < \delta\mu_2$ ($\delta\mu_1 \simeq \Delta_0/\sqrt{2}$, $\delta\mu_2 \simeq 0.754\Delta_0$) an anisotropic phase is favored with Cooper pairs of total momentum $2\mathbf{q}$.

1 wave ansatz

$$< \psi(x)\psi(x) >\sim \Delta e^{2i\mathbf{q}\cdot\mathbf{x}}$$



dispersion laws

$$\epsilon_1 = +\delta\mu - \mathbf{q}\cdot\mathbf{v} + \sqrt{\xi^2+\Delta^2}$$
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- The dispersion laws are gapless
- Breaking of space symmetry
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3-flavor LOFF

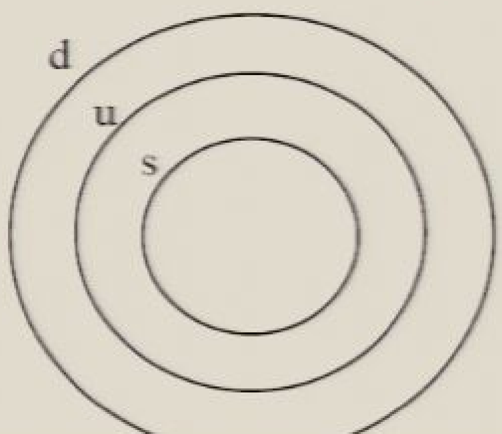
We generalize the LOFF condensate (*R. Casalbuoni et. al* hep-ph/050724, *M. M., K.Rajagopal and R. Sharma* hep-ph/0603076)

$$\langle\psi_{\alpha i}(x)C\gamma_5\psi_{\beta j}(x)\rangle \propto \sum_{I=1}^{3}\Delta_I \sum_{\mathbf{q_I^a}\in\{\mathbf{q_I^a}\}} e^{2i\mathbf{q_I^a}\cdot\mathbf{r}}\epsilon_{I\alpha\beta}\epsilon_{Iij}$$

where $\{\mathbf{q_I^a}\}$ is a set of vectors. For each value of I, the vectors $\mathbf{q_I^a}$ with $a = 1, n$ describe a crystalline structure. In general one has to employ a Ginzburg-Landau expansion in the gap parameter.

Simplifying assumption: Near the transition point to the normal phase

$p_F^d = p_F^u + 2\delta\mu; \quad p_F^u = \mu - \frac{M_s^2}{6\mu}; \quad p_F^s = p_F^u - 2\delta\mu$ with $\delta\mu = \frac{M_s^2}{8\mu}$



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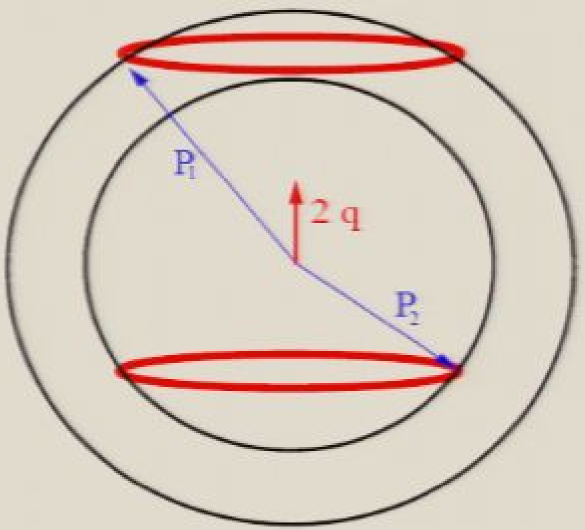

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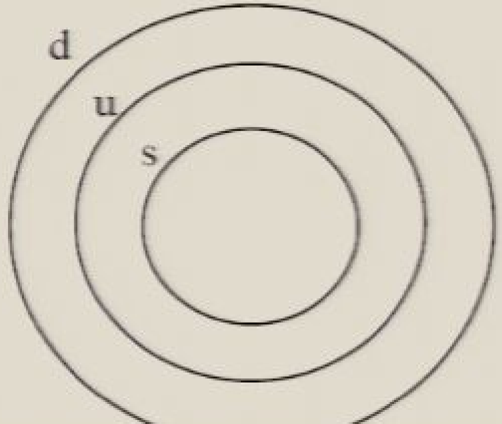

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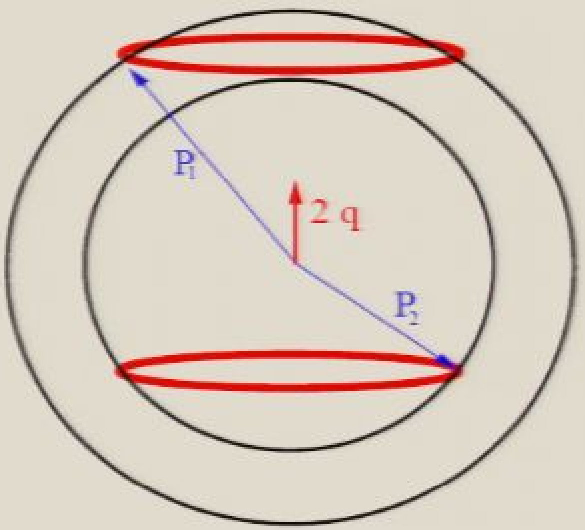

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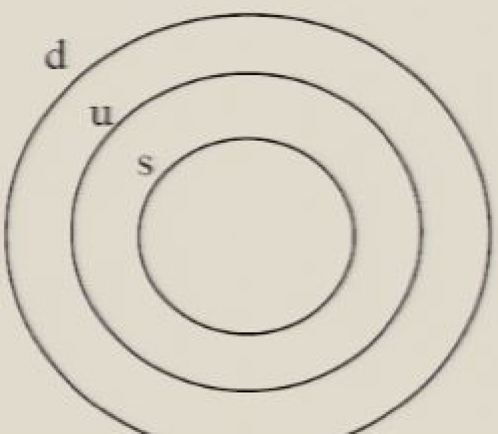

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Two plane waves

Consider the case where $n = 1$. The condensate is determined by 2 vectors $\mathbf{q_2}$ and $\mathbf{q_3}$. The free parameters are Δ and $\phi = \widehat{q_2 q_3}$. This case can be solved without employing the Ginzburg-Landau expansion.

Momentum shift and "Nonstandard-HDET" decomposition, $i, j = 1, 2, 3$

$$\psi_i(x) \to e^{i\mathbf{k_i}\cdot\mathbf{x}}\psi_i(x) = e^{i\mathbf{k_i}\cdot\mathbf{x}} \int \frac{d\mathbf{v}}{4\pi} e^{-i\mu\mathbf{v}\cdot\mathbf{x}} \left(\psi_{i,\mathbf{v}}(x) + \psi^-_{i,\mathbf{v}}(x)\right)$$

The interaction Lagrangian becomes

$$\mathcal{L}_\Delta = \Delta e^{\mathbf{2i}(\mathbf{q_2}+\mathbf{k_1}+\mathbf{k_3})\cdot\mathbf{x}}\psi_1\psi_3 + \Delta e^{\mathbf{2i}(\mathbf{q_3}+\mathbf{k_1}+\mathbf{k_2})\cdot\mathbf{x}}\psi_1\psi_2$$

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Get rid of space dependence in gap term with

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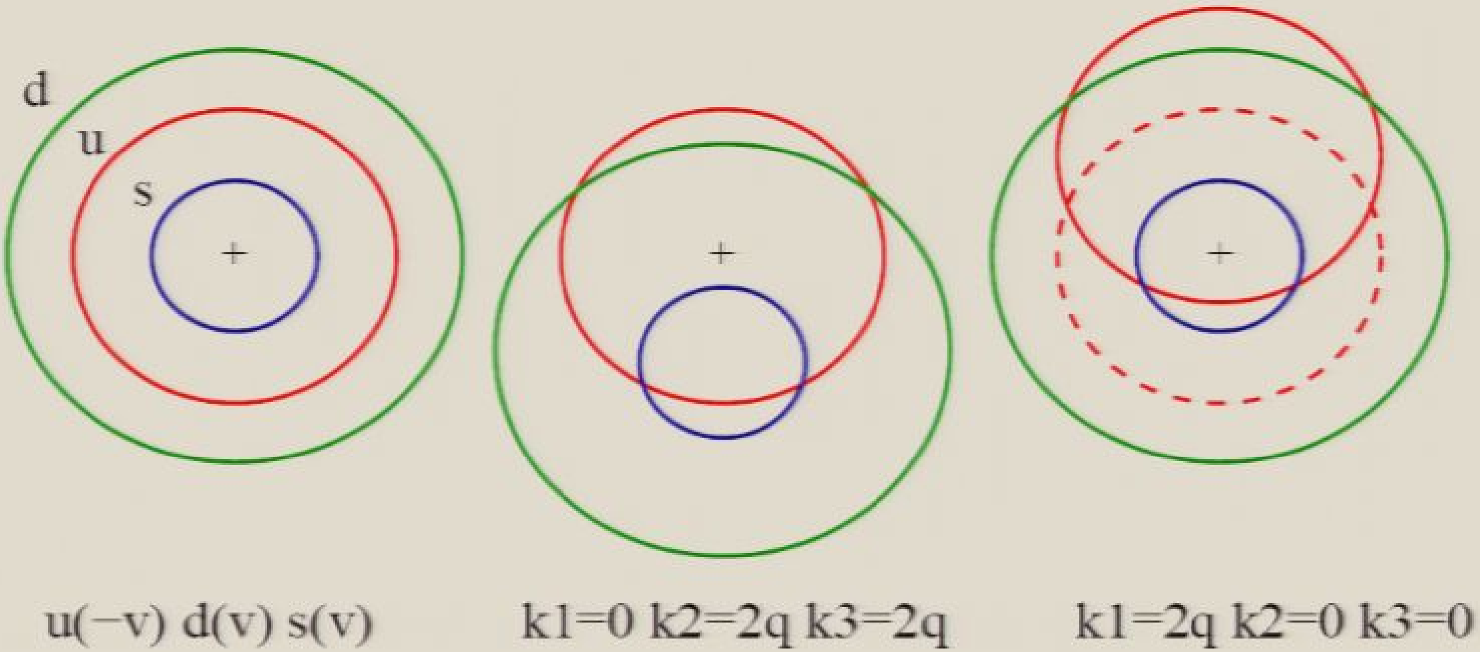
We can graphically see the effect of the shifts. For $\mathbf{q_1} = \mathbf{q_2} = \mathbf{q}$



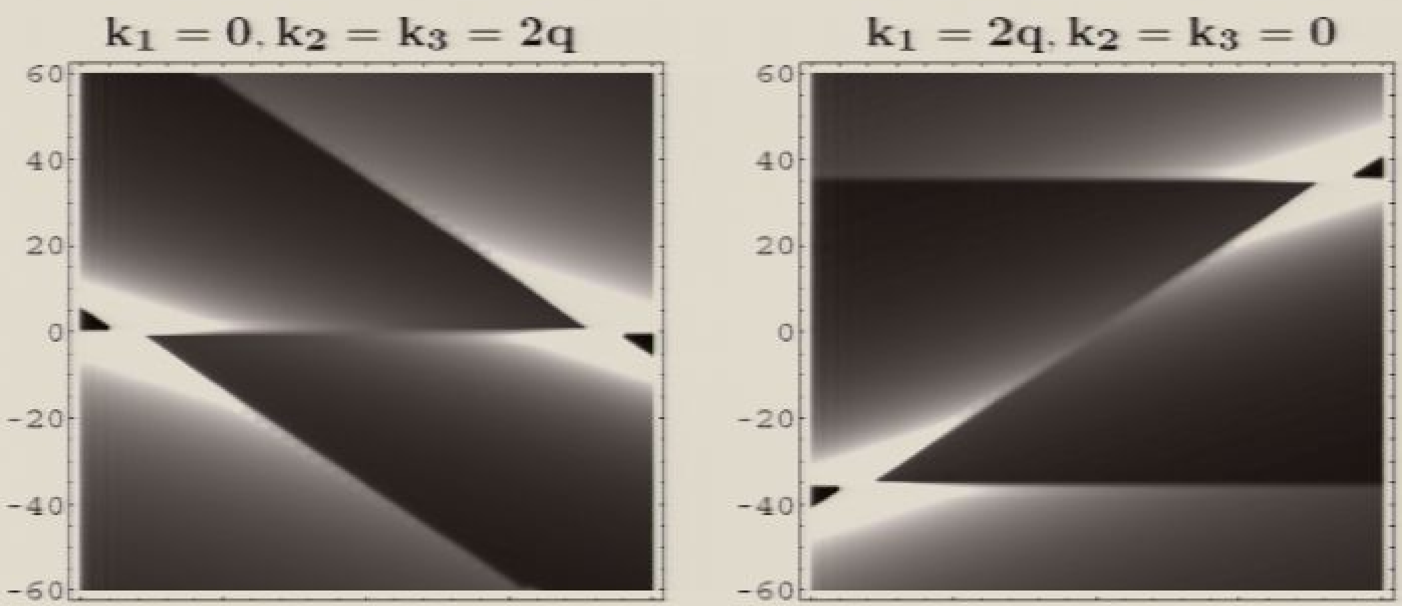
Free-energy and Gap Equation

Graphical representation of the “pairing regions”: region that gives the largest contribution to the gap parameter

$$\Delta \propto \int_{-\delta}^{+\delta} dl_{\parallel} \int d\mathbf{v}\, f(\mathbf{v}, l_{\parallel})$$

for the “parallel case” we plot ($\mu = 500$ MeV, $\Delta = 1$ MeV, $\delta\mu = 17.7$ MeV)

$$f(\mathbf{v}, l_{\parallel}) = \sum_{a=1}^{9} \frac{\partial E_a}{\partial \Delta} \mathrm{Sign}(E_a)$$

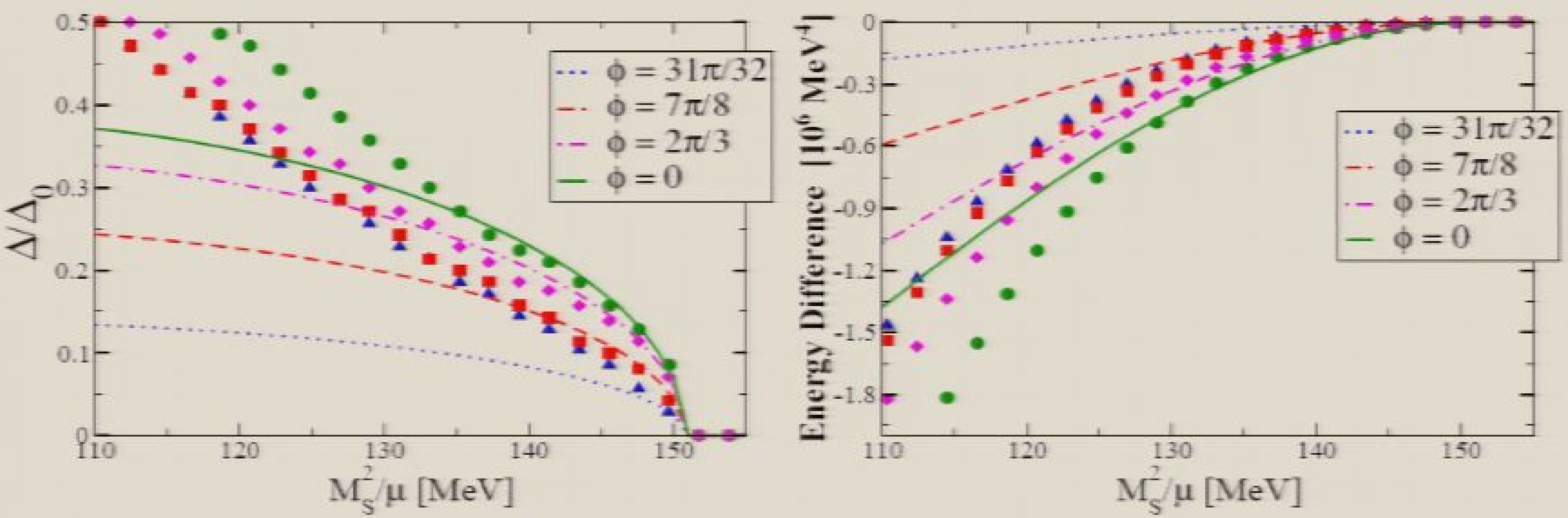


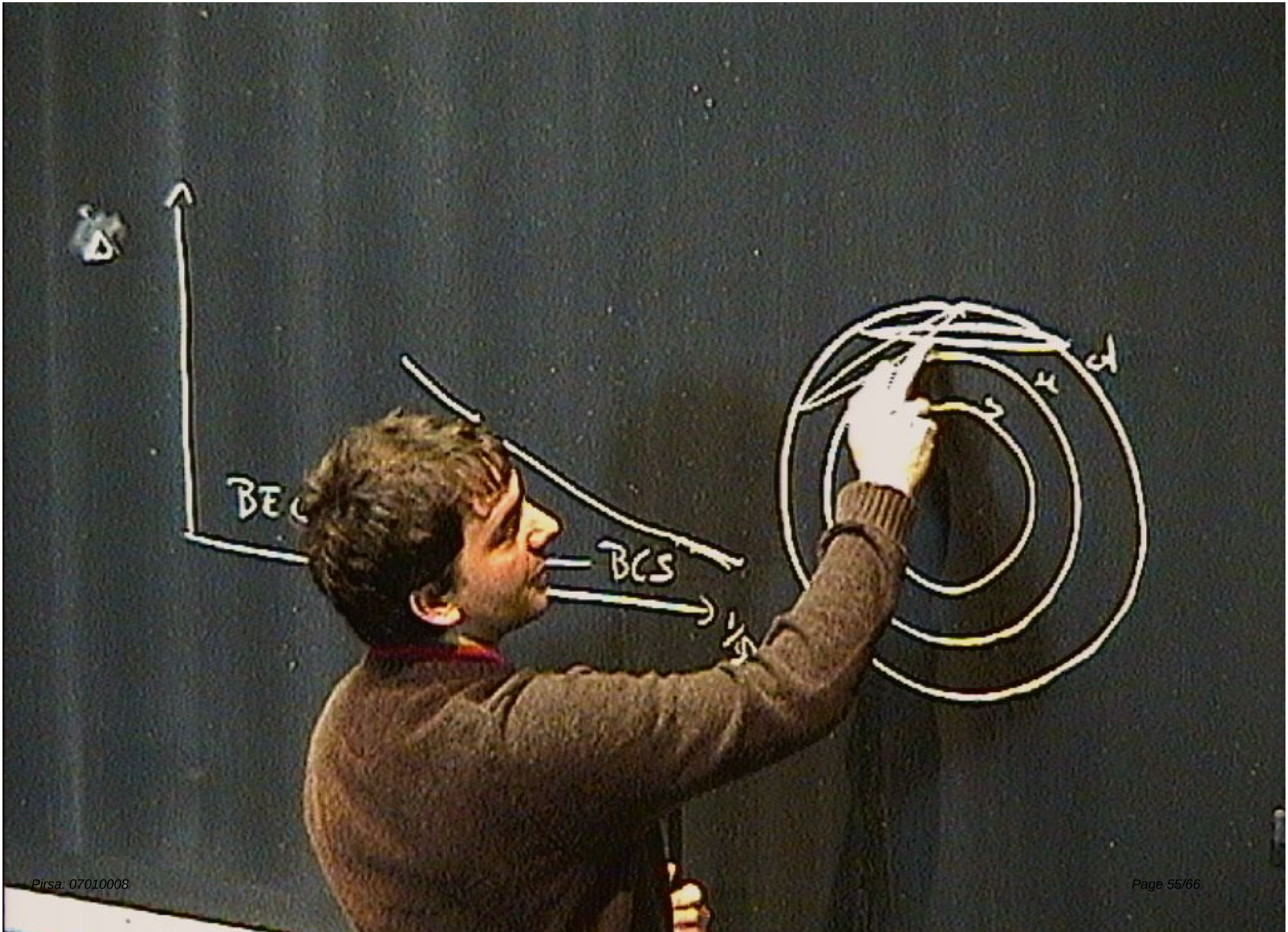
Free-energy and Gap Equation

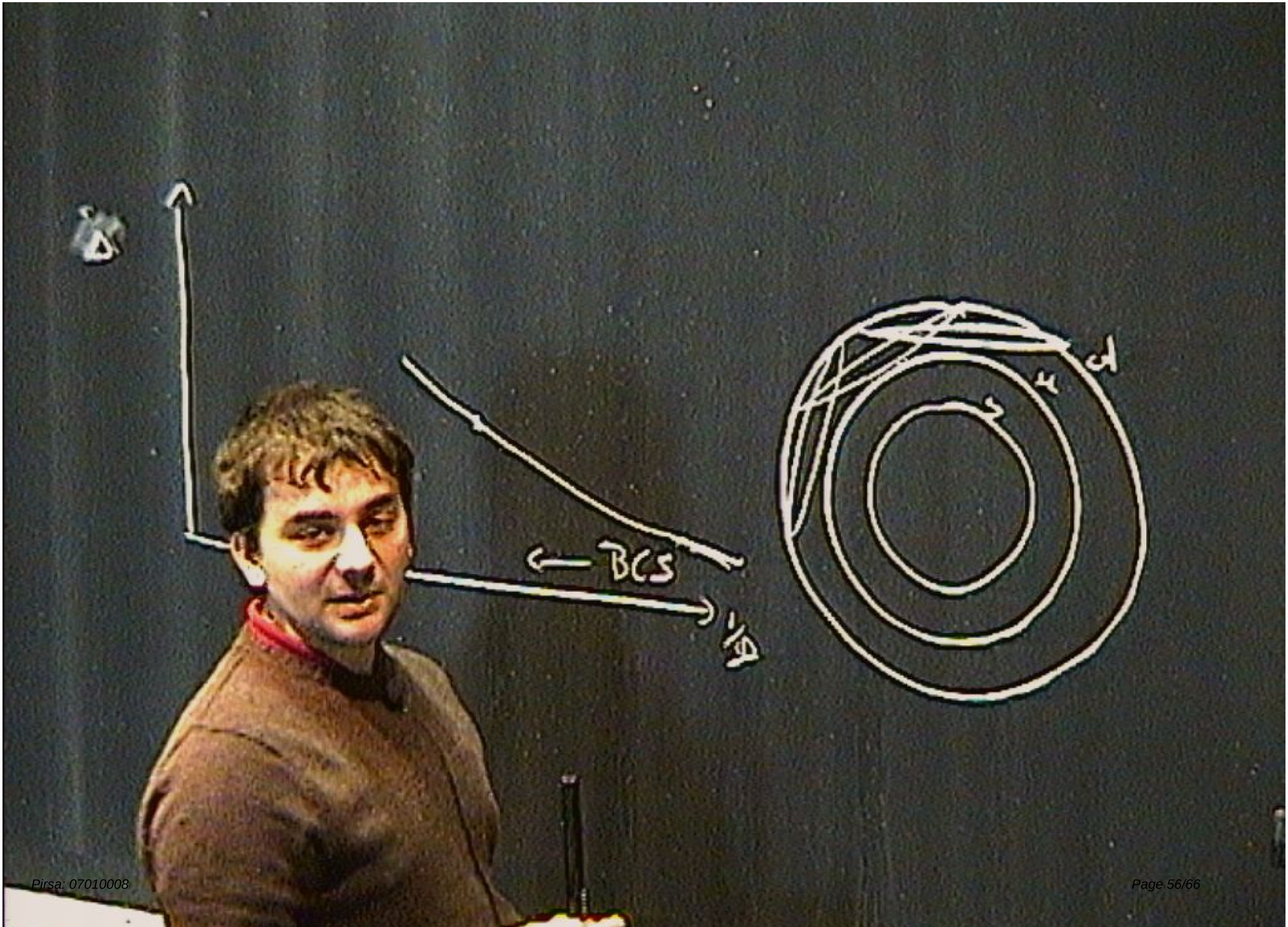
Free energy for the two plane wave case

$$\Omega[\phi, \Delta] = -\frac{\mu^2}{4\pi^2} \sum_{a=1,18} \int_{-\delta}^{+\delta} dl_{\parallel} \int \frac{d\mathbf{v}}{4\pi} |E_a(\mathbf{v}, l_{\parallel})| + \frac{2\Delta^2}{G} - \frac{\mu_e^4}{12\pi^2}$$

Gap equation: $\frac{\partial\Omega}{\partial\Delta} = 0$




BCS


← BCS

Crystal structures

We consider now the crystal structures CubeX and 2Cube45z.

CubeX: eight vectors that belong to two sets of four vectors $\{\hat{\mathbf{q}}_2\}$ and $\{\hat{\mathbf{q}}_3\}$.
$\{\hat{\mathbf{q}}_2\} = (1/\sqrt{3})(\pm\sqrt{2}, 0, \pm 1)$ (pink rectangle)
$\{\hat{\mathbf{q}}_3\} = (1/\sqrt{3})(0, \pm\sqrt{2}, \pm 1)$ (purple rectangle)

2Cube45z: sixteen vectors that belong to two sets $\{\hat{\mathbf{q}}_2\}$ and $\{\hat{\mathbf{q}}_3\}$
$\{\hat{\mathbf{q}}_2\} = (1/\sqrt{3})(\pm 1, \pm 1, \pm 1)$ (black cube)
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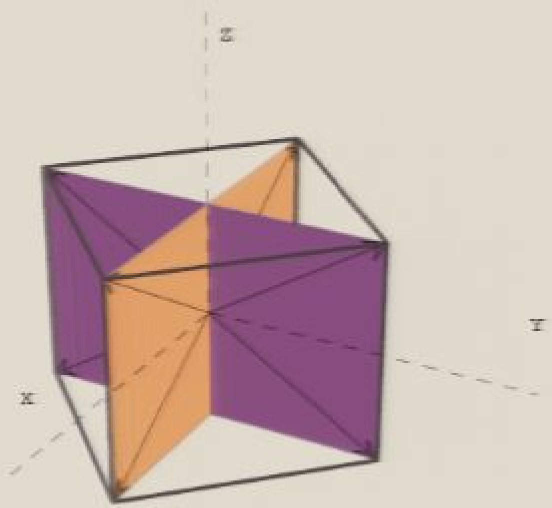

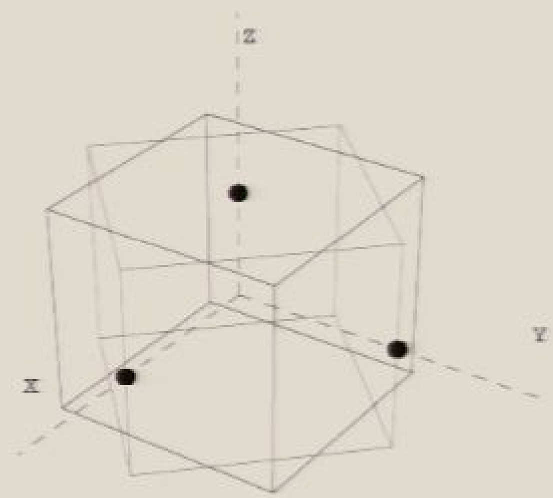

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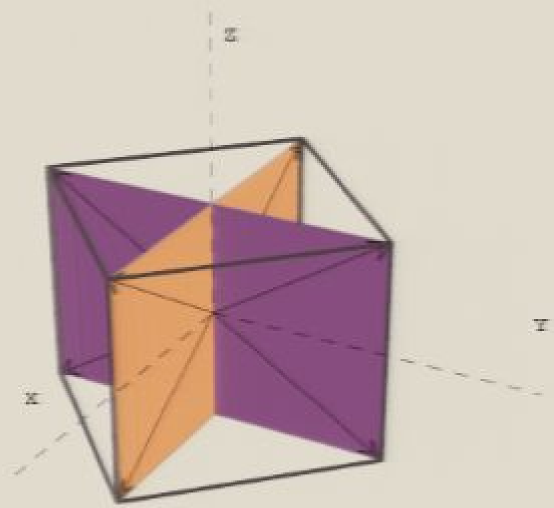
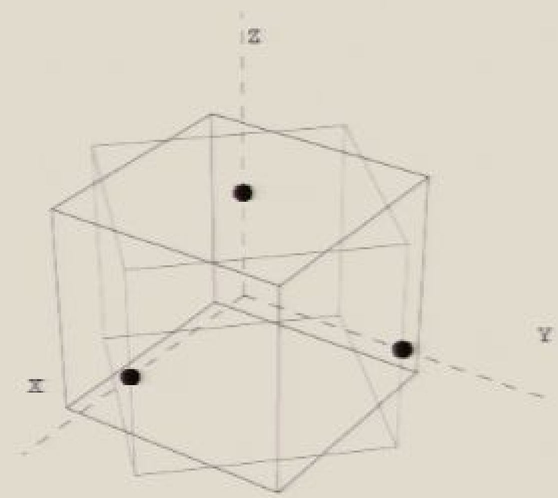
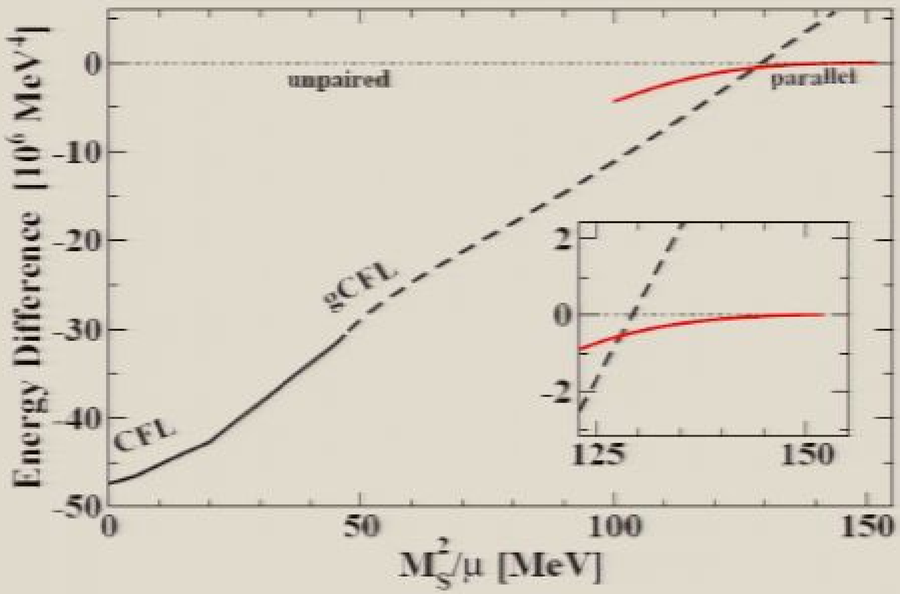




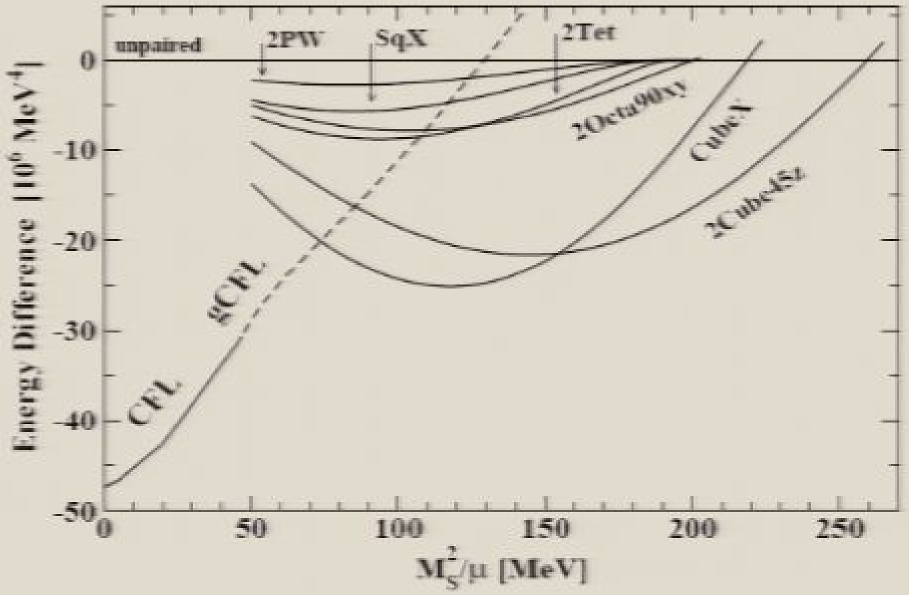
Phase diagram

The free-energies of different phases

Two plane waves



Various Crystals



Crystal fluctuations

Phonons fields $\mathbf{u}_I(x)$ are position and time dependent fluctuation of the condensate

$$\Delta_I(\mathbf{r}) = \Delta_I \sum_{\mathbf{q}_I^a} e^{2i\mathbf{q}_I^a \cdot \mathbf{r}} \rightarrow \Delta_I(\mathbf{r} - \mathbf{u}_I(x))$$

Integrating out the fermionic fields we get the effective action for the phonon fields at the order u^2:

$$\mathcal{S}[\mathbf{u}] \sim \int d^4x \sum_I \mu^2 |\Delta_I|^2 \sum_{\mathbf{q}_I^a} \left[\partial_0(\hat{\mathbf{q}}_I^a \cdot \mathbf{u}_I)\partial_0(\hat{\mathbf{q}}_I^a \cdot \mathbf{u}_I) - (\hat{\mathbf{q}}_I^a \cdot \vec{\partial})(\hat{\mathbf{q}}_I^a \cdot \mathbf{u}_I)(\hat{\mathbf{q}}_I^a \cdot \vec{\partial})(\hat{\mathbf{q}}_I^a \cdot \mathbf{u}_I)\right]$$

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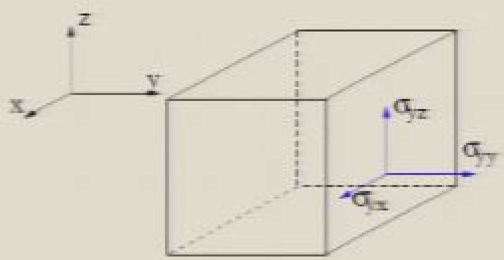


The shear modulus ν describes the response of the crystal to an external stress. It can be read from the phonon potential energy.

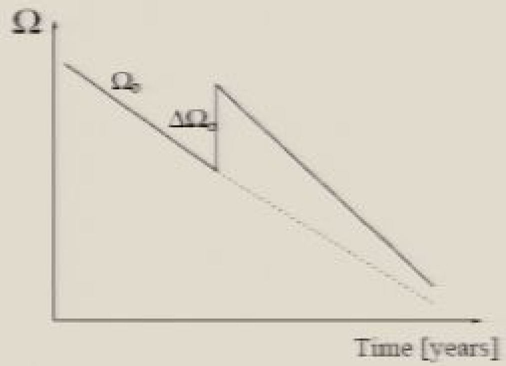
We find

$$\nu = \frac{\text{stress}}{\text{strain}} \sim \mu^2\Delta^2 \gg \nu_{\text{Nuclear}}$$

The crystalline LOFF phase behaves as a very rigid solid. Since $U(1)_B$ is spontaneously broken it is a solid and a superfluid!!

Pulsar glitches

A spinning neutron star gradually spins down but every once in a while it speeds up. Then it relaxes back in days or years.



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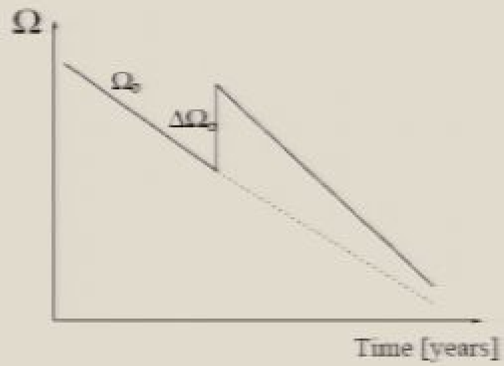

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Mechanism:

1) Density of vortices proportional to the angular momentum
2) The star slows down and vortices want to dilute moving outward
3) Vortices are pinned and they cannot move
4) When magnus force on the vortex equals the pinning force an avalanche is triggered

Conclusion

- The instability of the homogeneous gapless phases leads to alternative condensation patterns
- In cold atoms and 2SC the favored phase is probably the one with phase separation: excess atoms are expelled
- In three-flavor the LOFF phase is a possibility
- The three-flavor LOFF phase behaves as a "superfluid crystal"
- This suggests the possibility that some glitches originate in the core of Neutron stars

Quousque tandem abutere, Catilina, patientia nostra? Cicerone M.T., 1 Catilinaria

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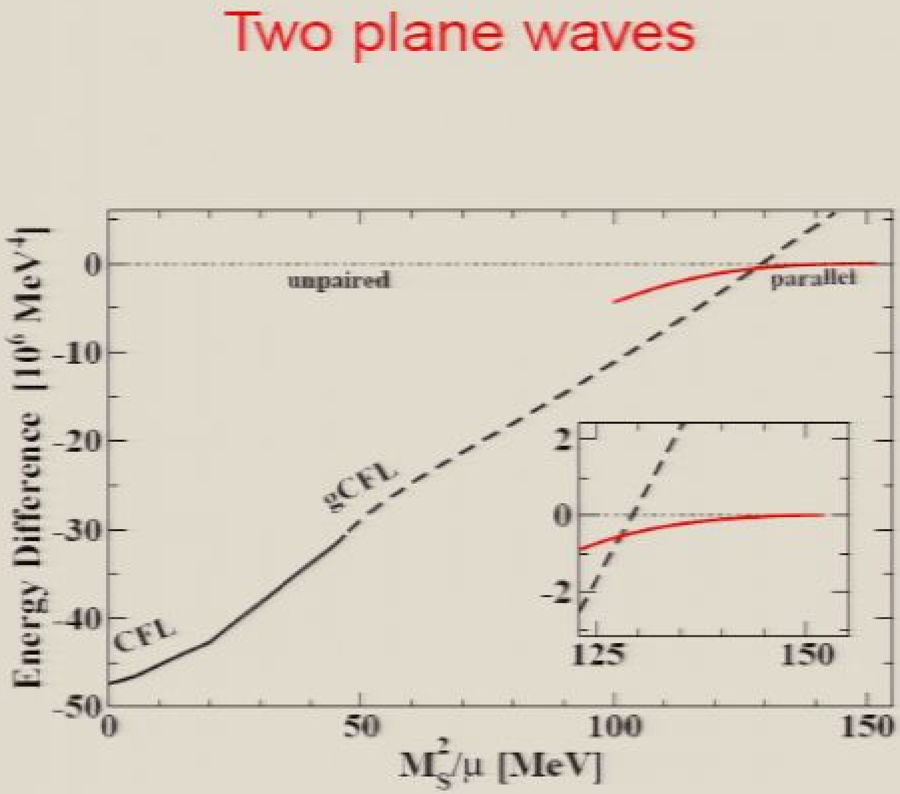

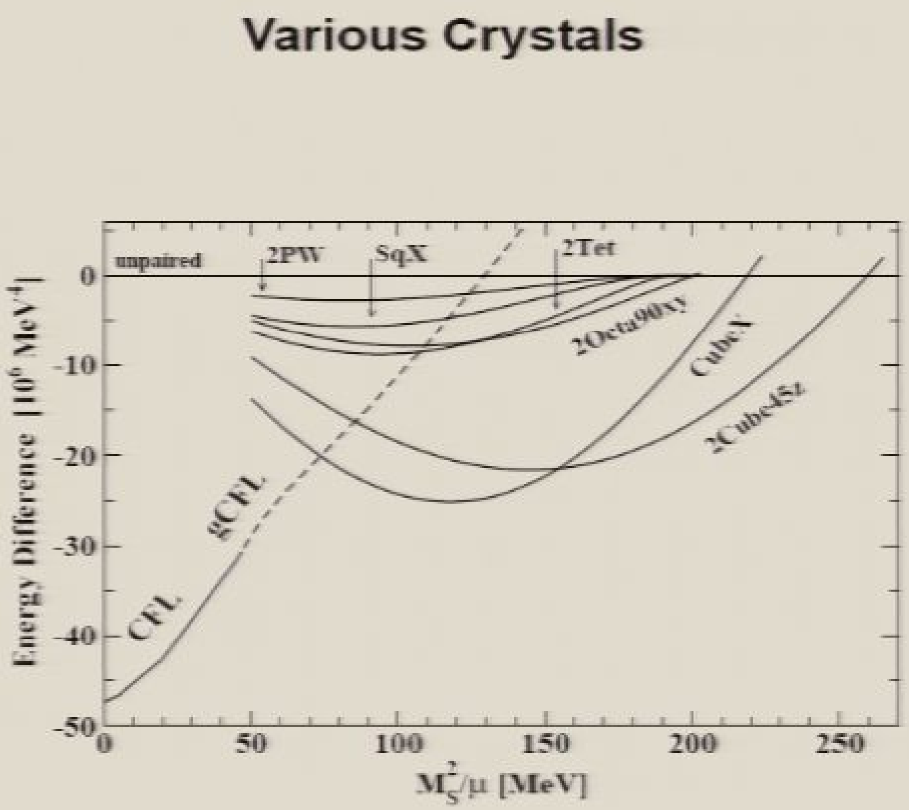

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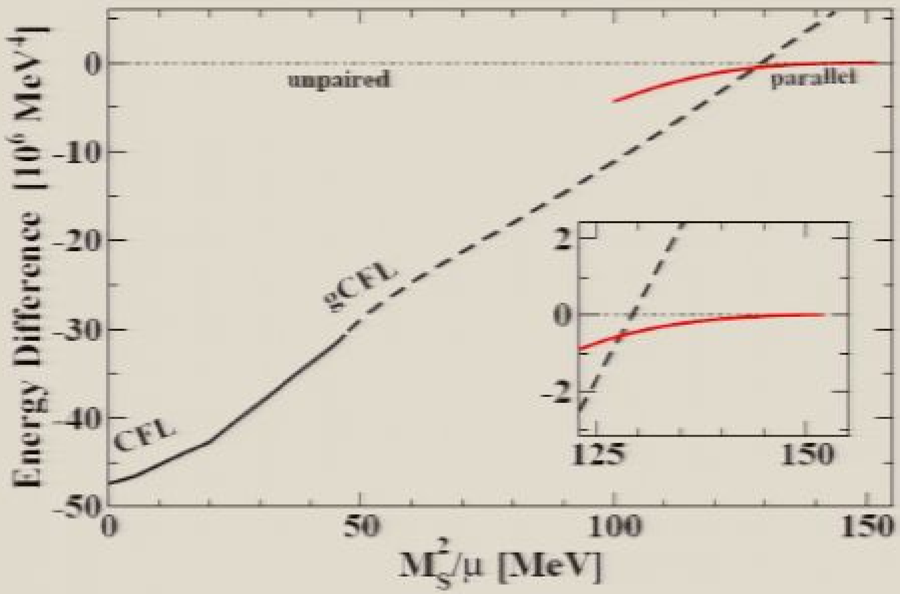

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