Title: Domain Lines as Fractional Strings

Date: Jan 23, 2007 11:00 AM

URL: http://pirsa.org/07010007

Abstract: We consider N=2 supersymmetric quantum electrodynamics (SQED) with 2 flavors, the Fayet--Iliopoulos parameter, and a mass term $beta$ which breaks the extended supersymmetry down to N=1. The bulk theory has two vacua; at $beta=0$ the BPS-saturated domain wall interpolating between
them has a moduli space parameterized by a U(1) phase $sigma$ which can
be promoted to a scalar field in the effective low-energy theory on the
wall world-volume. At small nonvanishing $beta$ this field gets a
sine-Gordon potential. As a result, only two discrete degenerate BPS
domain walls survive. We find an explicit solitonic solution for domain lines -- string-like objects living on the surface of the domain wall which separate wall I from wall II. The domain line is seen as a BPS kink in the world-volume effective theory. The domain line carries the magnetic flux which is exactly 1/2 of the flux carried by the flux tube living in the bulk on each side of the wall. Thus, the domain lines on the wall confine charges living on the wall, resembling Polyakov's
three-dimensional confinement.

Domain Lines as Fractional Vortices

R. Auzzi, M. Shifman & A. Yung

Introduction

Domain walls natural objects in supersymmetric theories

There are the following alternatives (without taking in account the trivial translational modulus):

a) A **single** domain wall interpolating two vacua.
b) A **moduli space** of solution. The modulus can be promoted to a massless field on the wall world-volume (for example $\mathcal{N} = 2$ SQED)
c) A **discrete number** $\neq 1$ of solutions (expected in $\mathcal{N} = 1$ Super-Yang-Mills)

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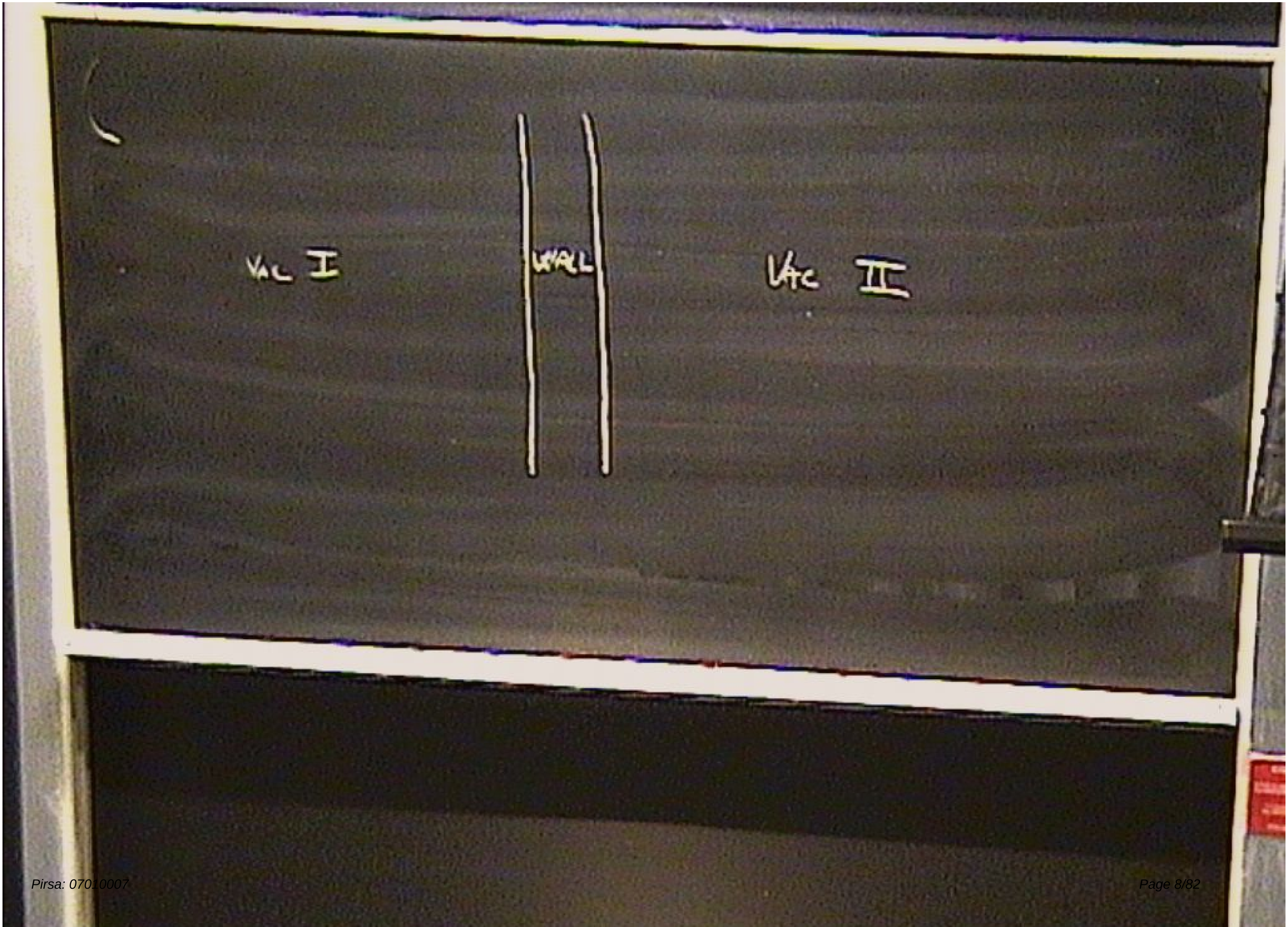
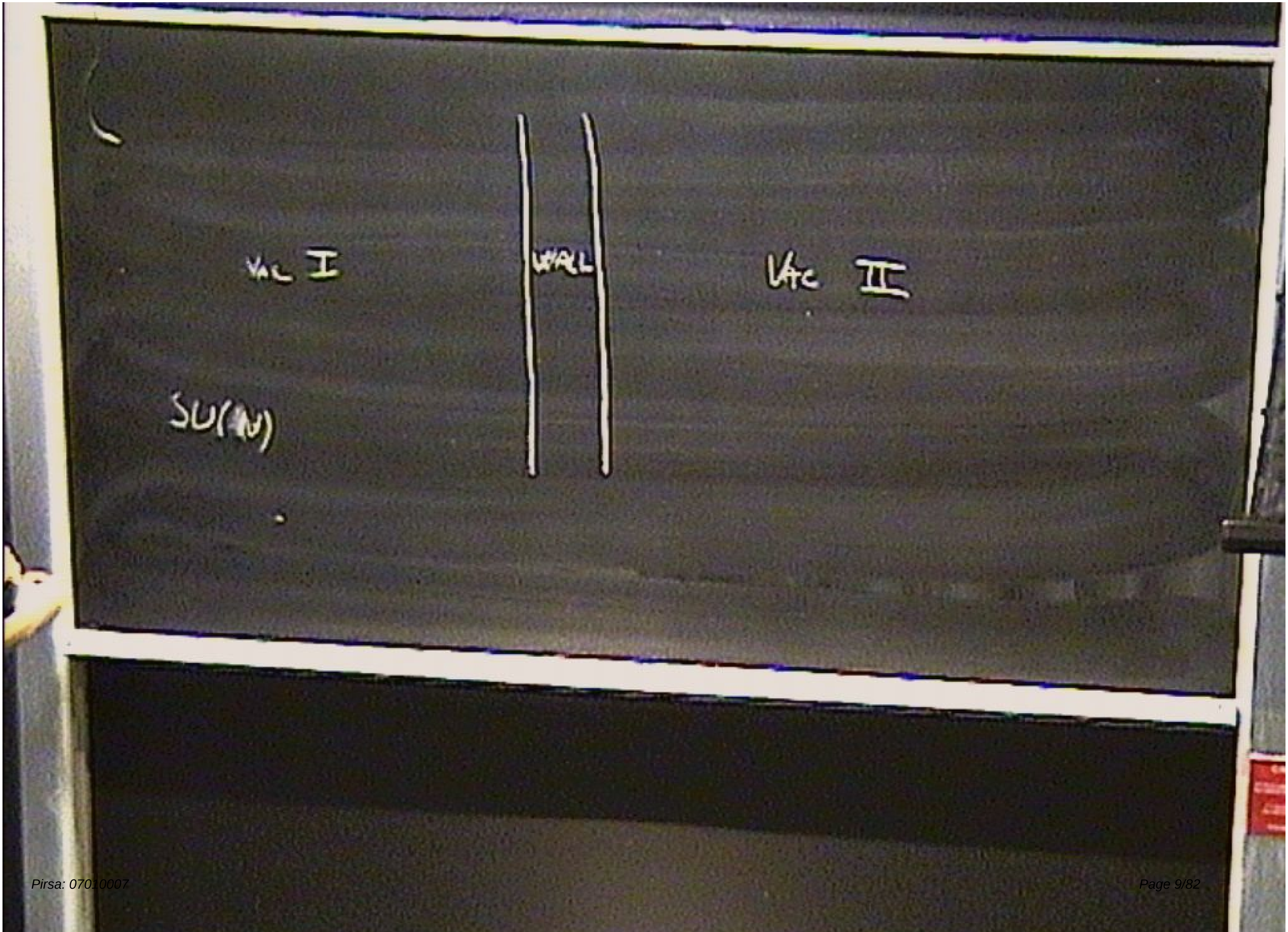

Vac I
WALL
Vac II


VAC I
WALL
VAC II
SU(N)

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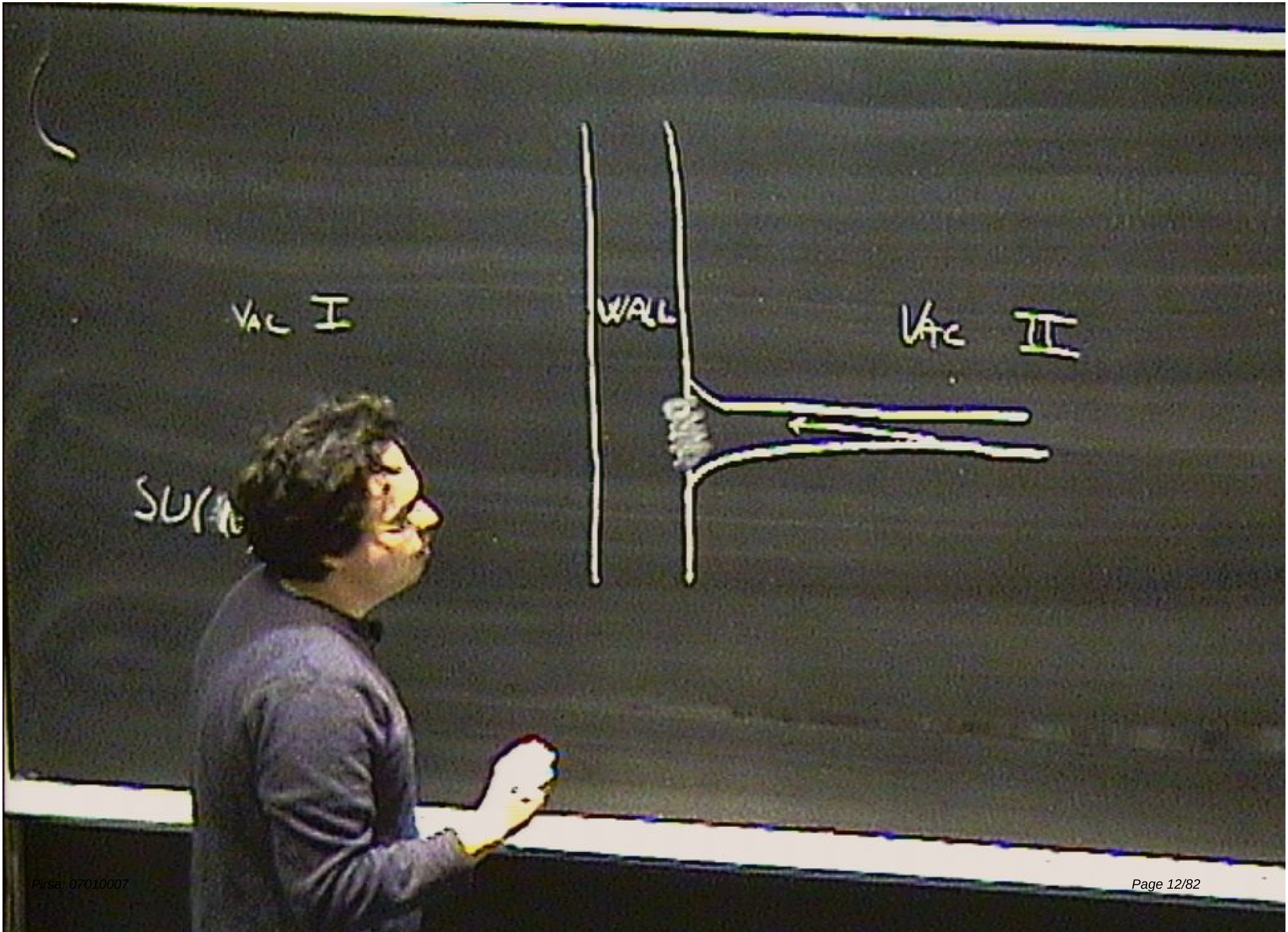
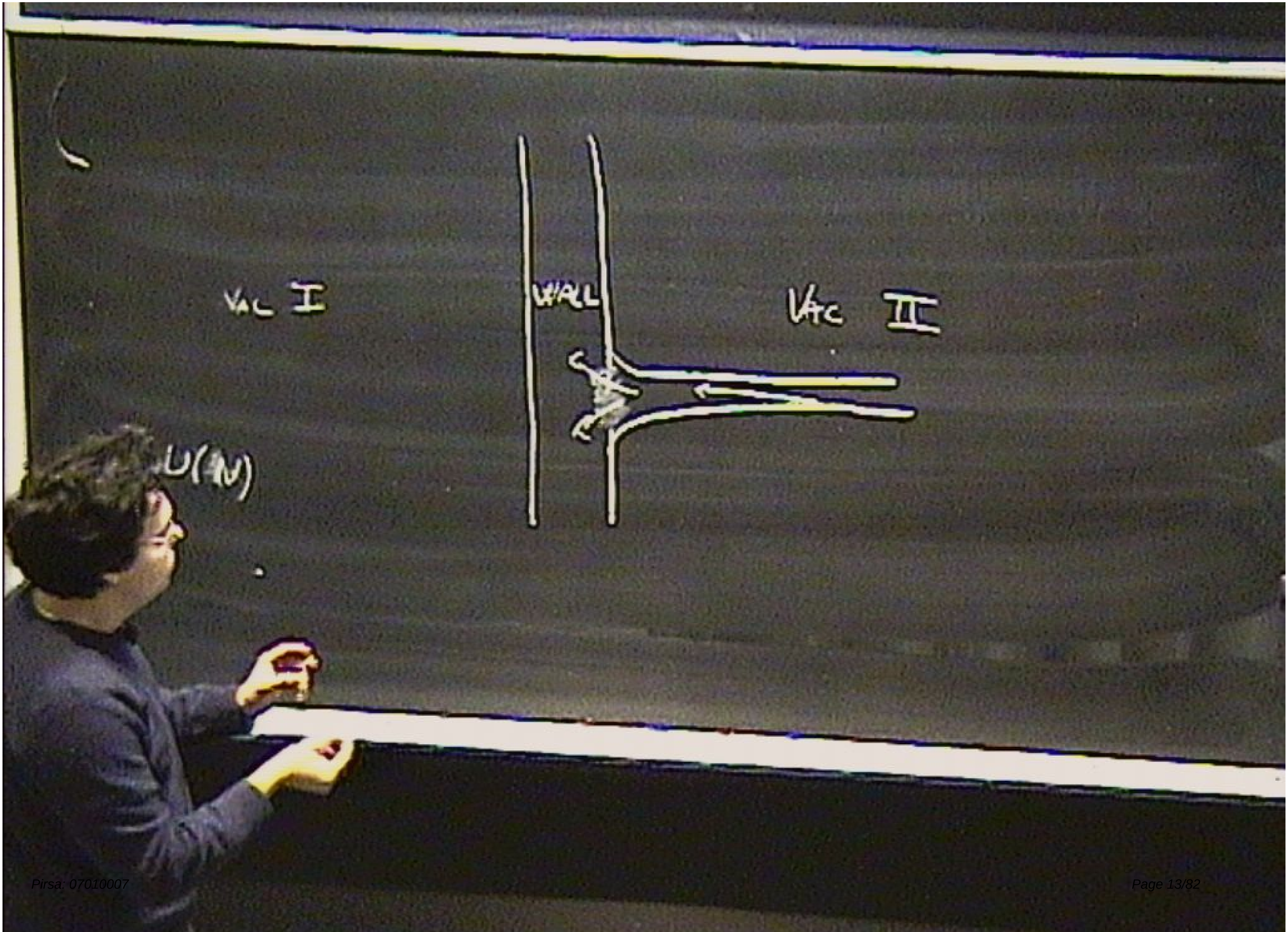
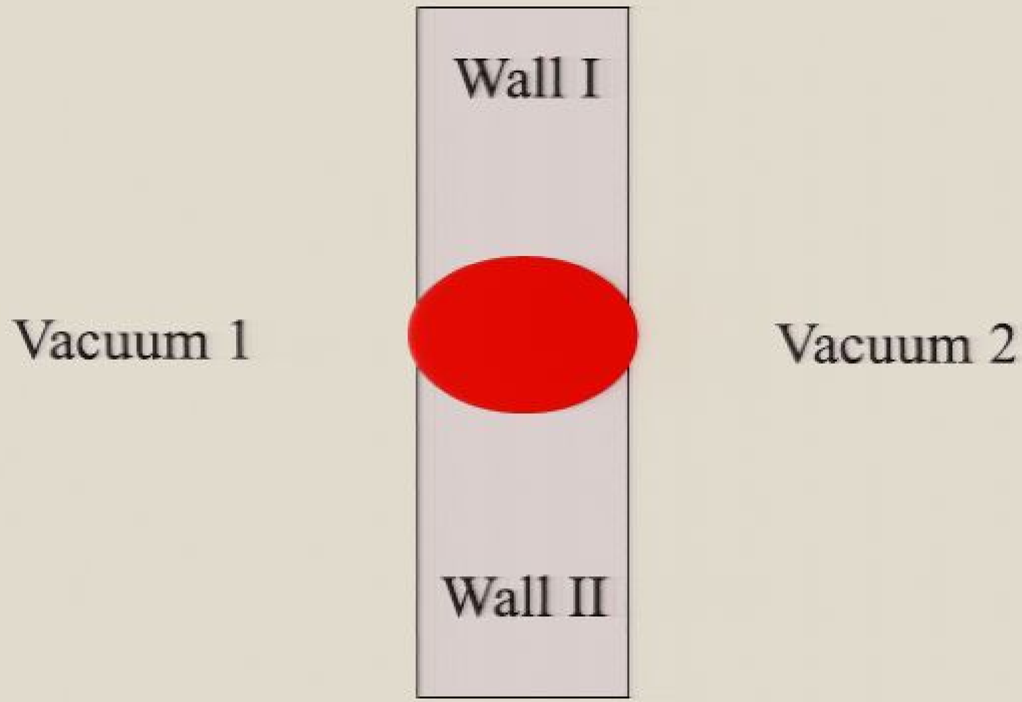

VAC I
WALL
VAC II


VAC I
WALL
VAC II
U(N)

Domain Lines



Domain Lines interpolating between discrete Domain Wall solutions: an explicit solitonic example at weak coupling

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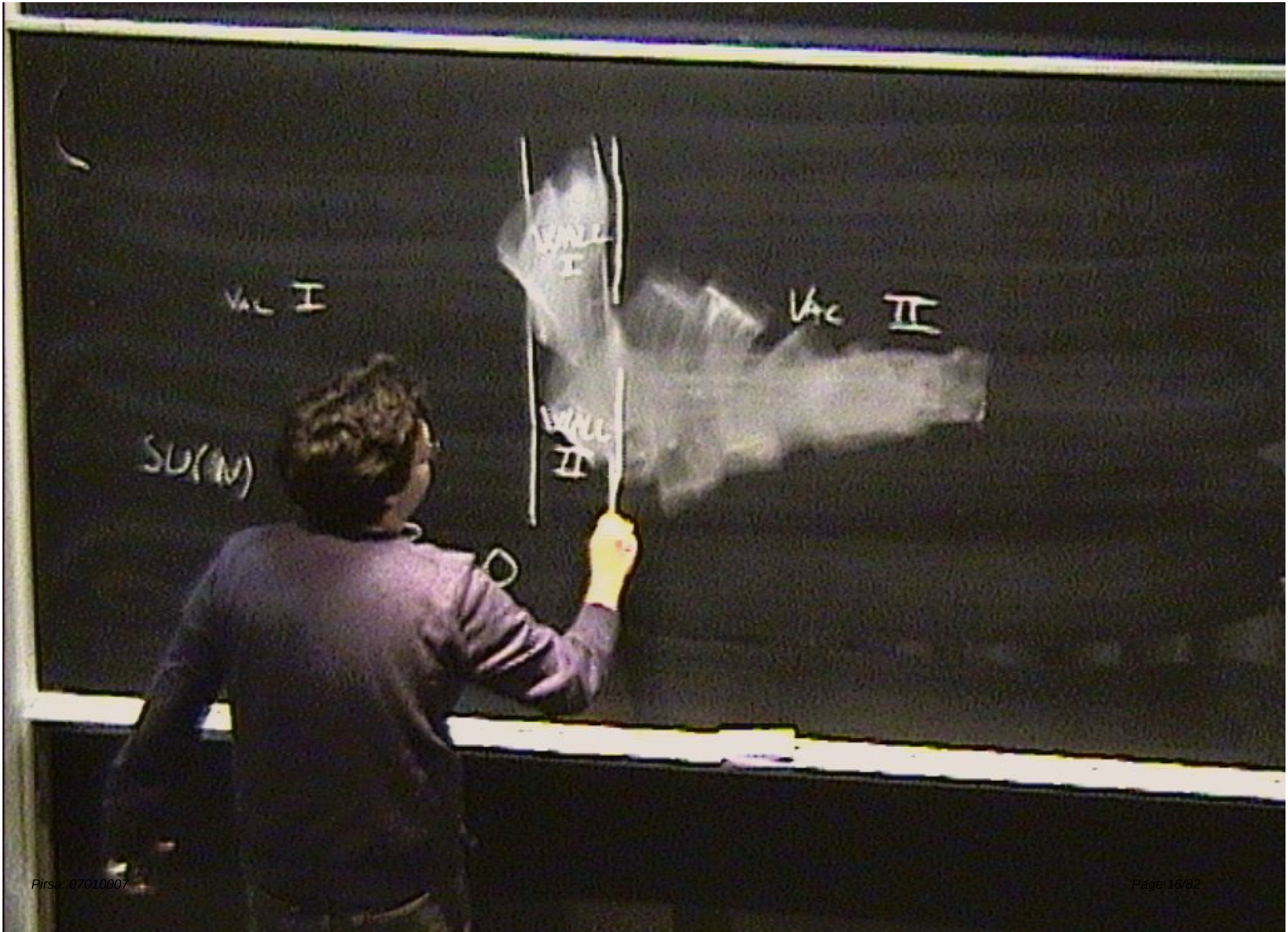
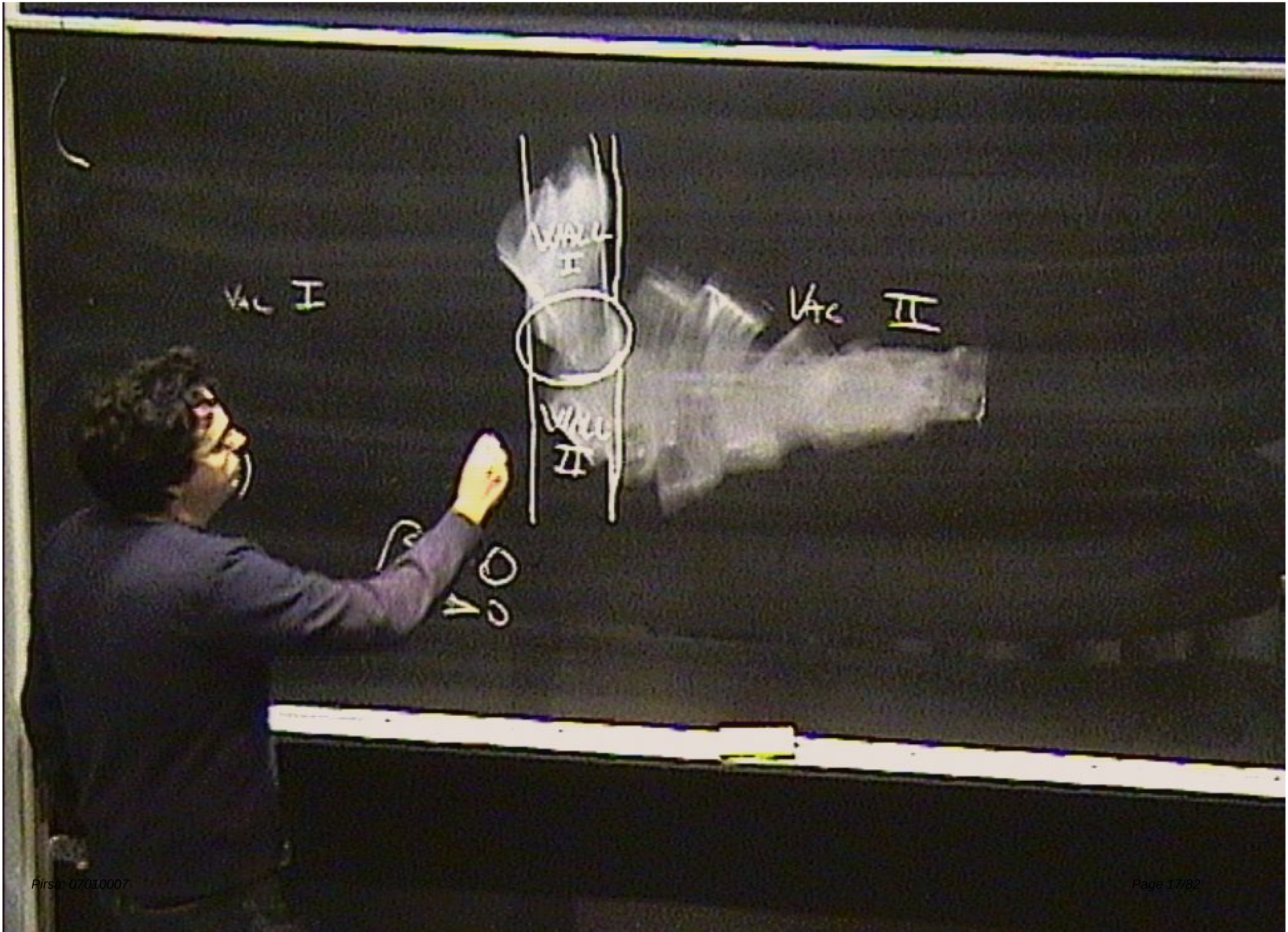
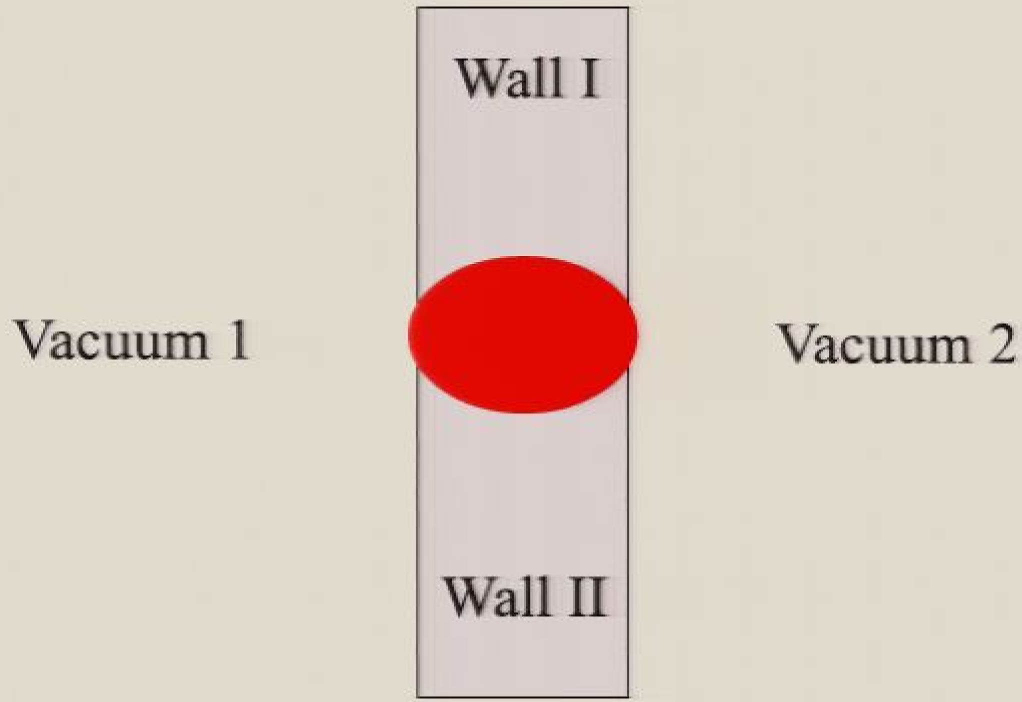

WALL I
VAC I
VAC II
WALL II
SU(N)


VAC I
WALL II
VAC II
WALL II

Domain Lines



Domain Lines interpolating between discrete Domain Wall solutions: an explicit solitonic example at weak coupling

Fields & Superpotential

$\mathcal{N} = 2$ U(1) gauge theory

Bosonic Fields:

* U(1) gauge vector superfield

* superfield A with zero charge

* $N_f = 2$ squark superfields Q_B,$\tilde{Q}_B^\dagger$ from hypermultiplets

$$W = \frac{1}{\sqrt{2}} A Q_B \tilde{Q}_B + m_B Q_B \tilde{Q}_B + \frac{\beta}{\sqrt{2}} (Q_1 \tilde{Q}_2 - \tilde{Q}_1 Q_2) - \frac{1}{2\sqrt{2}} \xi A,$$

$$m_1 = -m_2 = m$$

$\beta \neq 0$ we are breaking the extended supersymmetry

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Potential

$$V_D = \frac{g^2}{8}(|q_B|^2 - |\tilde{q}_B|^2)^2$$

$$V_F = \frac{1}{2}|q_1(a + \sqrt{2}m_1) - \beta q_2|^2 + \frac{1}{2}(|\tilde{q}_2(a + \sqrt{2}m_2) - \beta \tilde{q}_1|^2 +$$

$$+\frac{1}{2}|(a + \sqrt{2}m_1)\tilde{q}_1 + \beta \tilde{q}_2|^2 + \frac{1}{2}|(a + \sqrt{2}m_2)q_2 + \beta q_1|^2 + \frac{g^2}{2}|\tilde{q}_A q_A - \frac{\xi}{2}|^2.$$

Symmetry breaking

Flavor $SU(2)_F \to U(1)_F$
by the parameter m

$U(1)_F$ is broken by β
$SU(2)_R$ is also broken by β

A $\mathbb{Z}_2$ subgroup of
$U(1)_R \times U(1)_F$,
generated by a π rotation in both the $U(1)$ factors
is left unbroken by β, m
This $\mathbb{Z}_2$ is spontaneously broken by the wall
(exchanges **Wall I** with **Wall II**)

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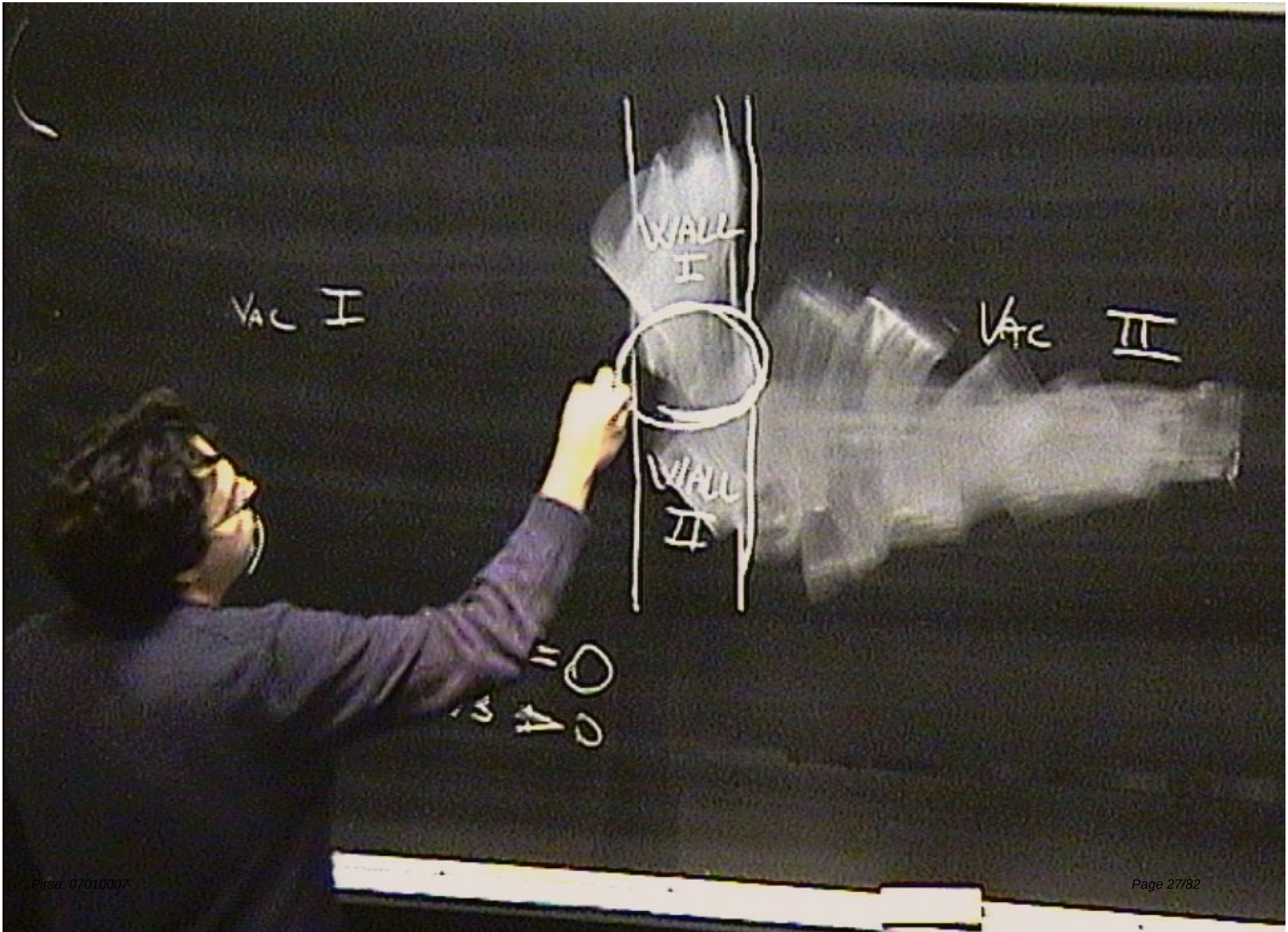
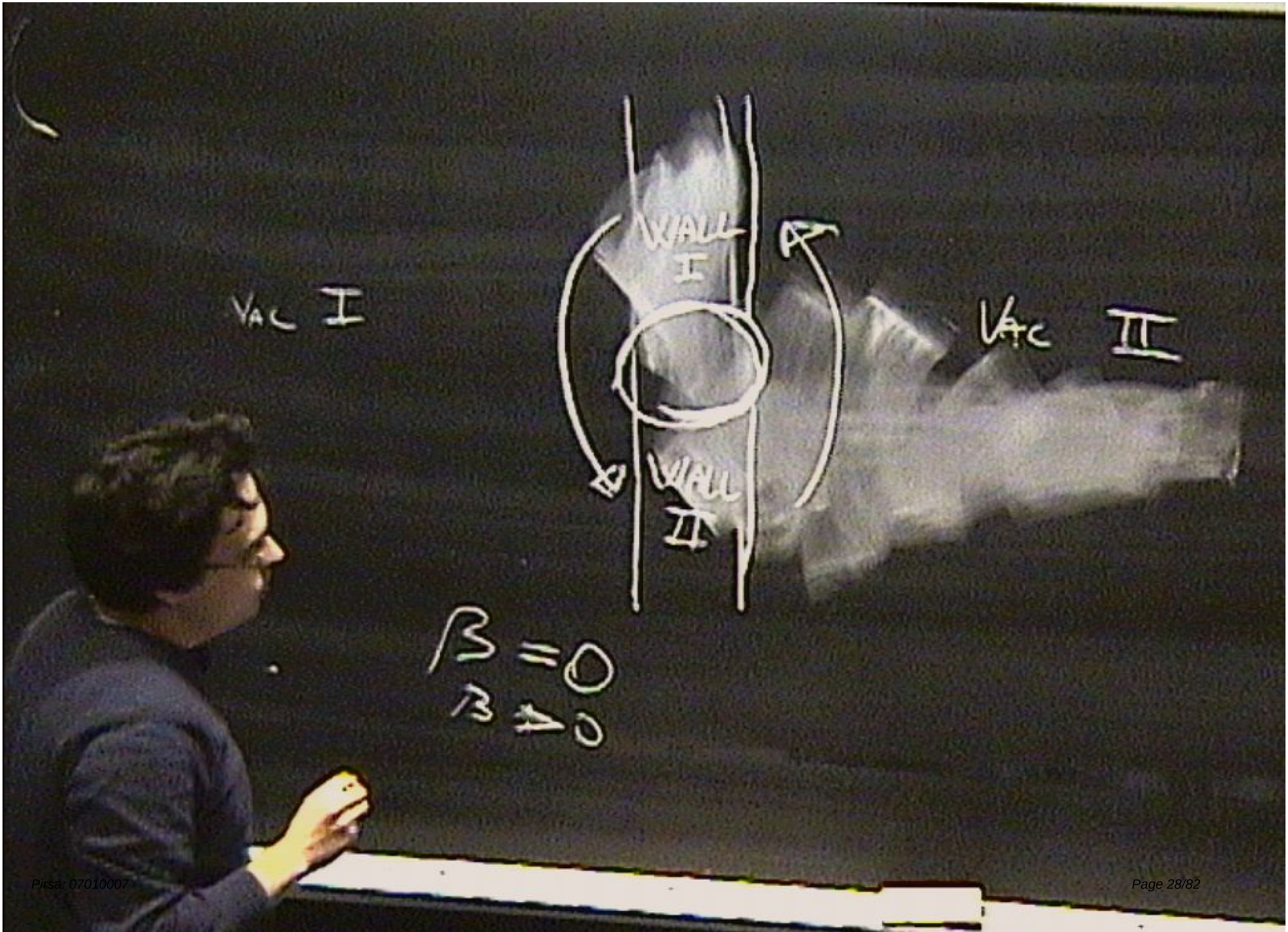


WALL
I
VAC I
VAC II
WALL
II
=0
≠0


WALL I
VAC I
VAC II
WALL II
$\beta = 0$
$\beta \neq 0$

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Sigma Model description

$g\sqrt{\xi} >> m, \beta$ we can integrate out a, A_μ

Constraints:

$$|q_1|^2 + |q_2|^2 = |\tilde{q}_1|^2 + |\tilde{q}_2|^2, \qquad \tilde{q}_A q_A = \frac{\xi}{2}$$

Eguchi-Hanson manifold (four real dimensions)

$$A_\mu = \frac{i\left(\bar{q}\overleftrightarrow{\partial}_\mu q - \bar{\tilde{q}}\overleftrightarrow{\partial}_\mu \tilde{q}\right)}{\bar{q}q + \bar{\tilde{q}}\tilde{q}}.$$

$$\frac{\sqrt{2}m\left(|q^2|^2 + |\tilde{q}_2|^2 - |q^1|^2 - |\tilde{q}_1|^2\right) + \beta\left(\bar{q}_1 q^2 - q^1\bar{q}_2 + \tilde{q}_1\bar{\tilde{q}}^2 - \bar{\tilde{q}}^1\tilde{q}_2\right)}{\sum\left(|q^A|^2 + |\tilde{q}_A|^2\right)}$$

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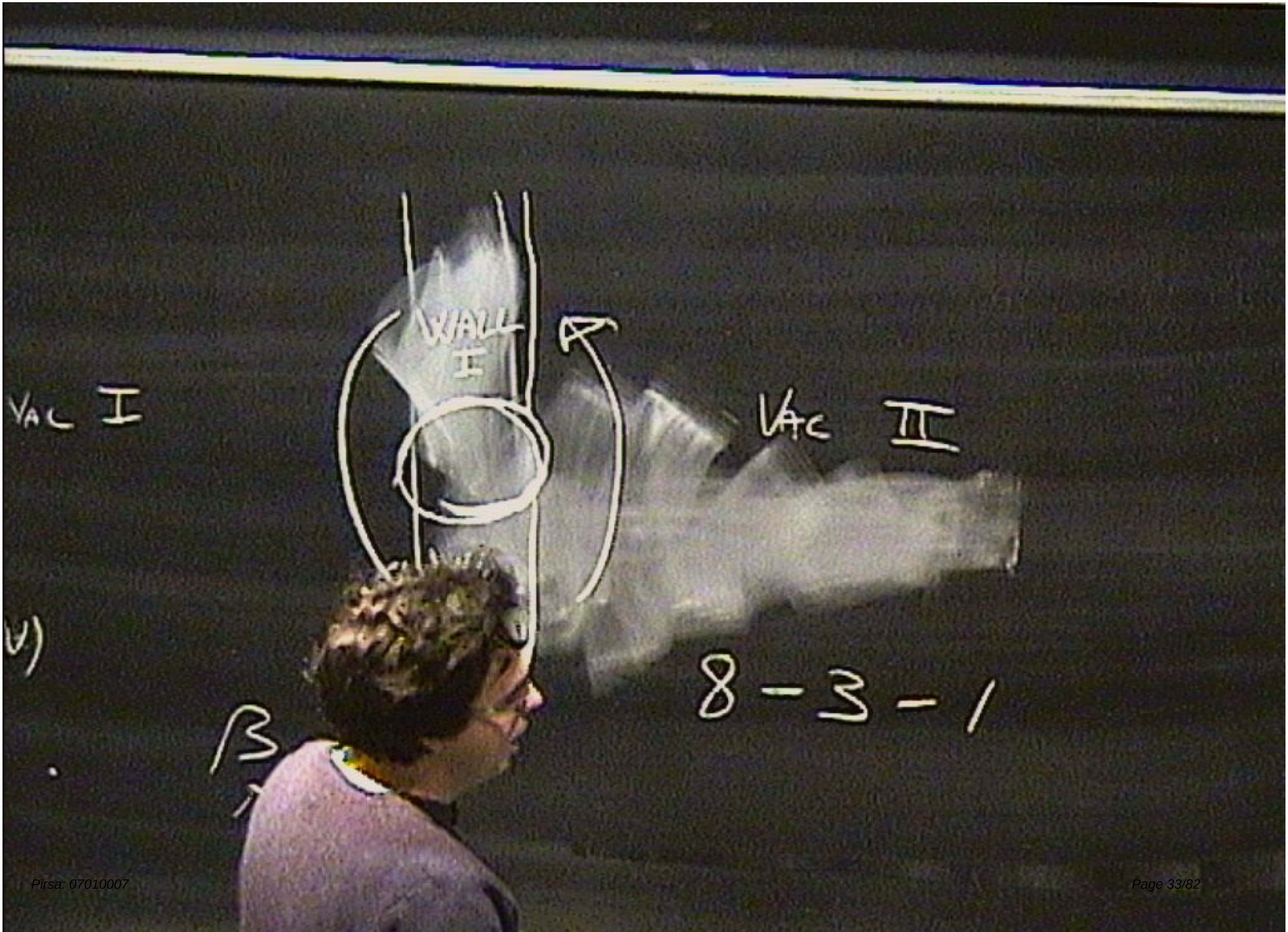
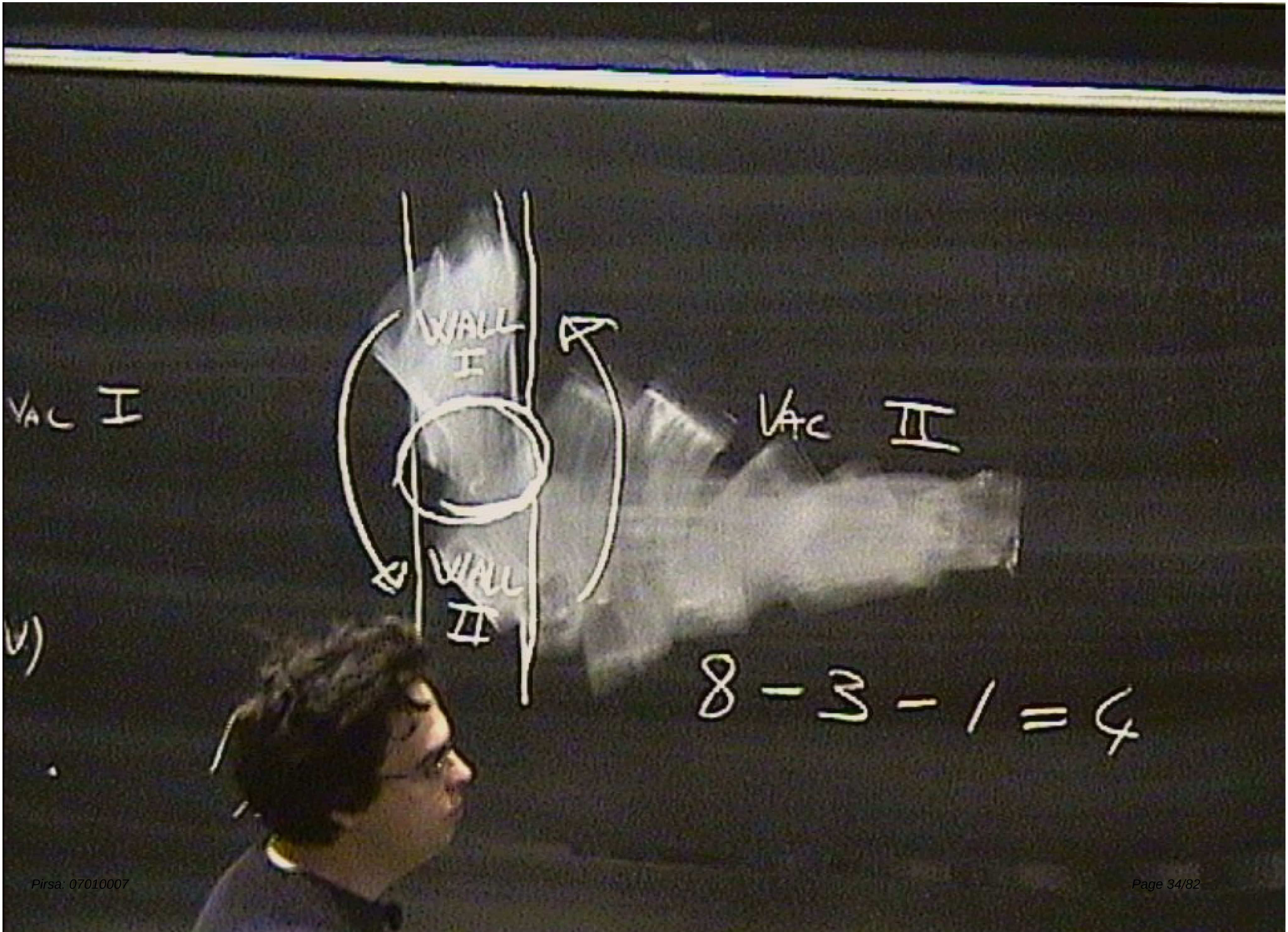
a


VAC I
WALL
I
VAC II
8 - 3 - 1


VAC I
WALL I
WALL II
VAC II
8 - 3 - 1 = 4

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Sigma Model description

$$q^1 = \frac{e^{i\varphi/2}}{2}\left(e^{i\psi/2}g(r)\cos\frac{\theta}{2} + e^{-i\psi/2}f(r)\sin\frac{\theta}{2}\right),$$

$$q^2 = \frac{e^{-i\varphi/2}}{2}\left(e^{i\psi/2}g(r)\sin\frac{\theta}{2} - e^{-i\psi/2}f(r)\cos\frac{\theta}{2}\right),$$

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$$f(r) = \sqrt{-\frac{\xi}{2} + \sqrt{r^2 + (\xi/2)^2}}\,, \qquad g(r) = \sqrt{\frac{\xi}{2} + \sqrt{r^2 + (\xi/2)^2}}\,.$$

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$$a = \frac{\sqrt{2}m\left(|q^2|^2 + |\tilde{q}_2|^2 - |q^1|^2 - |\tilde{q}_1|^2\right) + \beta\left(\bar{q}_1 q^2 - q^1\bar{q}_2 + \tilde{q}_1\bar{\tilde{q}}^2 - \bar{\tilde{q}}^1\tilde{q}_2\right)}{\sum\left(|q^A|^2 + |\tilde{q}_A|^2\right)}$$

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Eguchi-Hanson sigma model

Coordinates: $(r, \theta, \varphi, \psi)$

$$\mathcal{L}_{\mathrm{k}} = \frac{1}{\sqrt{4r^2+\xi^2}}\left\{-(\partial_\mu r)^2 - (\partial_\mu \theta)^2\left(r^2+\left(\frac{\xi}{2}\right)^2\right)\right.$$

$$\left. -(\partial_\mu \varphi)^2\left(r^2+\left(\frac{\xi}{2}\right)^2 \sin^2\theta\right) - (\partial_\mu \psi)^2 r^2 - (\partial_\mu \varphi)(\partial_\mu \psi)(2r^2\cos\theta)\right\}.$$

Potential induced by m, β

$$V = \frac{m^2(4r^2+\xi^2\sin^2\theta)}{\sqrt{4r^2+\xi^2}} + 2\sqrt{2}m\beta r(\cos\theta\cos\varphi\cos\psi - \sin\varphi\sin\psi) +$$

$$+\frac{\beta^2(4r^2+3(\xi/2)^2+2\xi^2\cos(2\theta)\sin^2\varphi+\xi^2\cos(2\varphi))}{2\sqrt{4r^2+\xi^2}}$$

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Wall at $\beta = 0$

Both the vacua and the wall are living in the S^2 subspace of Eguchi-Hanson manifold at $\beta = 0$ Vacua: $r = 0$, $\theta = 0, \pi$
Moduli space of BPS domain walls parameterized by an $U(1)$ phase $e^{i\sigma}$

$$\varphi(z) = \sigma, \;\; r(z) = 0$$

$$\theta(z) = 2\tan^{-1}(\exp(2mz))$$

Modulus promoted to a field on the wall world-volume:

$$S = \int d^3x \left(\frac{\xi}{4m} (\partial_n \sigma)^2 \right).$$

Eguchi-Hanson sigma model

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$$\mathcal{L}_{\mathrm{k}} = \frac{1}{\sqrt{4r^2+\xi^2}} \left\{ -(\partial_\mu r)^2 - (\partial_\mu \theta)^2 \left(r^2 + \left(\frac{\xi}{2}\right)^2 \right) \right.$$

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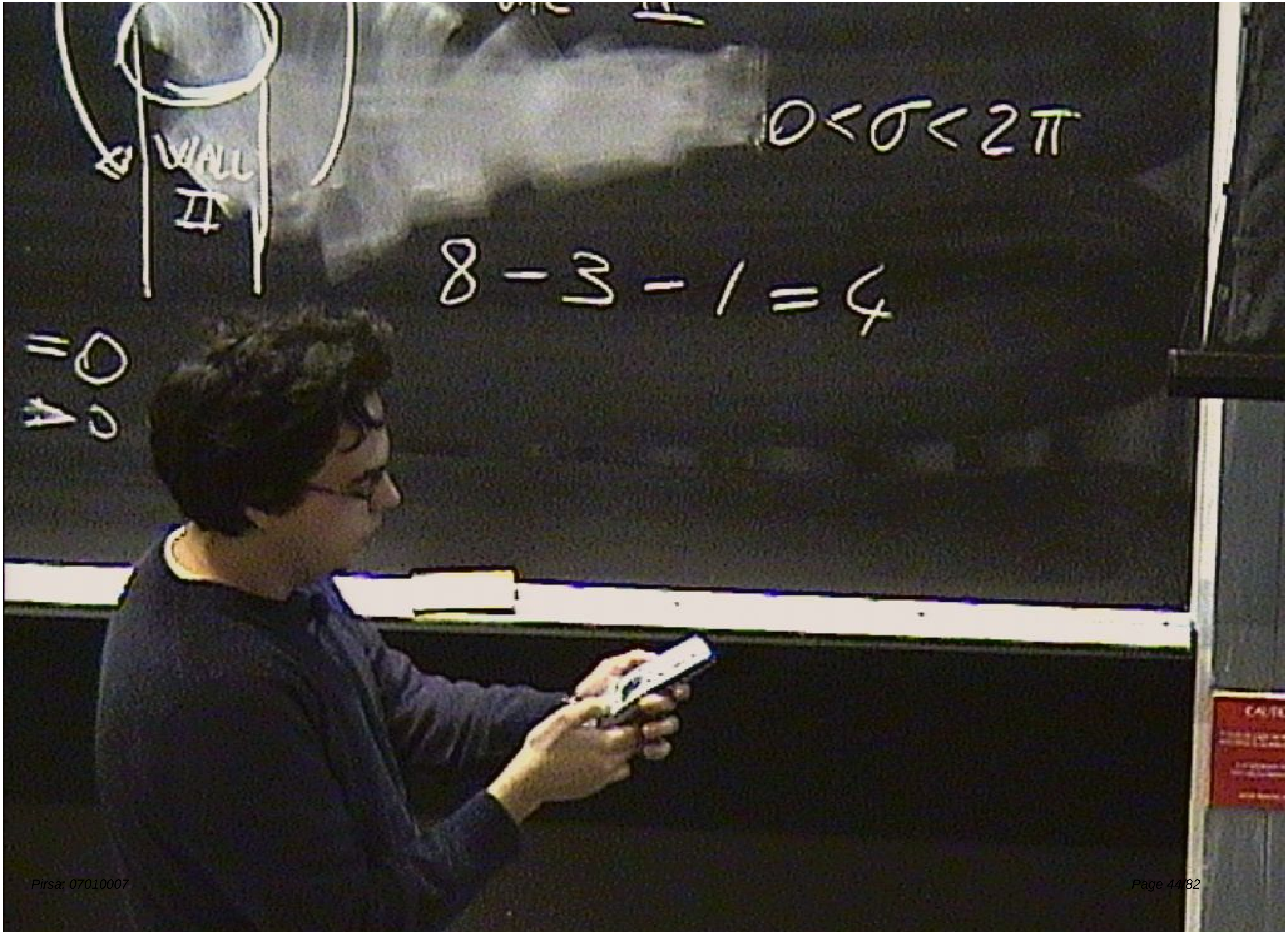
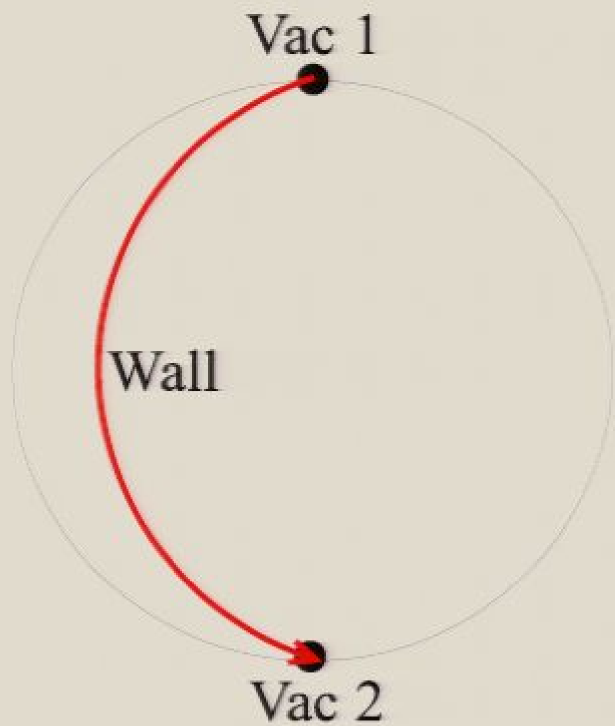

WALL
II
0<σ<2π
8-3-1=4
=0

Wall in the sigma model description

Coordinates (θ, φ)



σ is the longitude of the wall

At $\beta \neq 0$ we get a potential with a nontrivial dependence on the longitude of the sphere (just $\sigma = \pm\pi/2$ will survive)

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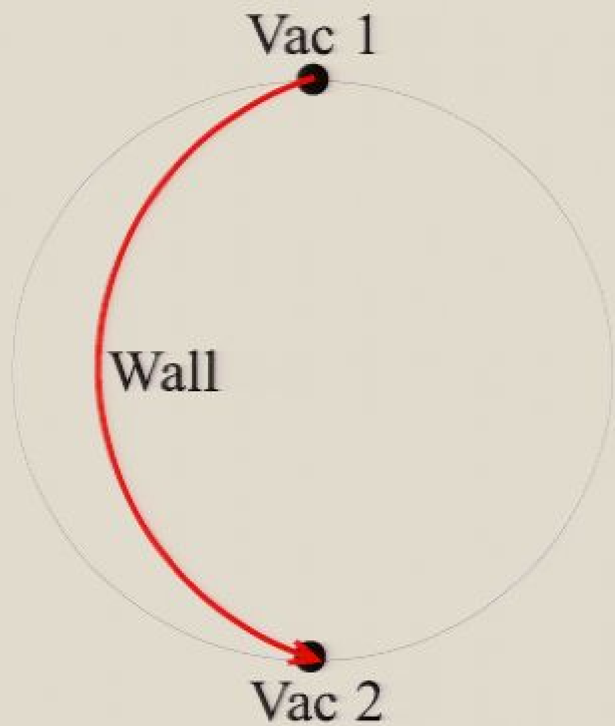
Modulus promoted to a field on the wall world-volume:

$$S = \int d^3x \left(\frac{\xi}{4m} (\partial_n \sigma)^2 \right).$$

Wall in the sigma model description

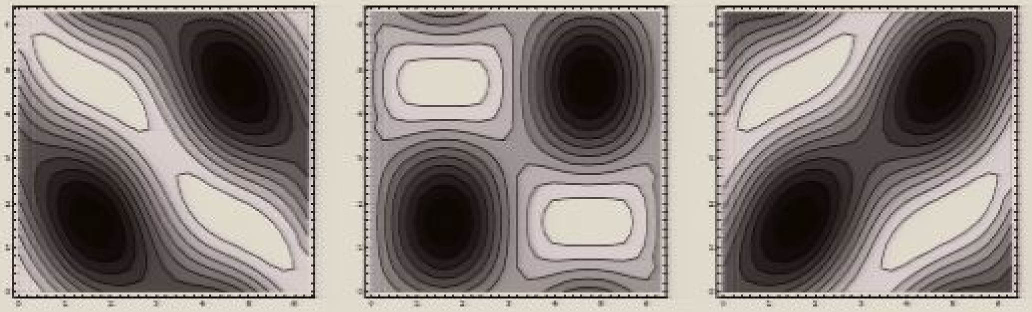
Coordinates (θ, φ)



σ is the longitude of the wall

At $\beta \neq 0$ we get a potential with a nontrivial dependence on the longitude of the sphere (just $\sigma = \pm\pi/2$ will survive)

Wall at $\beta \neq 0$



Plot of the potential at some sections at constant r and at $\theta = \pi/3, \pi/2, 2/3\pi$. There are always two symmetric minima, one at $\varphi = \psi = \pi/2$ and the other at $\varphi = \psi = 3\pi/2$.

BPS wall equations can be solved in correspondence of these two values.

$$q_1 = \pm i\Omega\cos(\eta/2) + \omega\sin(\eta/2), \;\; q_2 = \pm i\omega\cos(\eta/2) + \Omega\sin(\eta/2)$$

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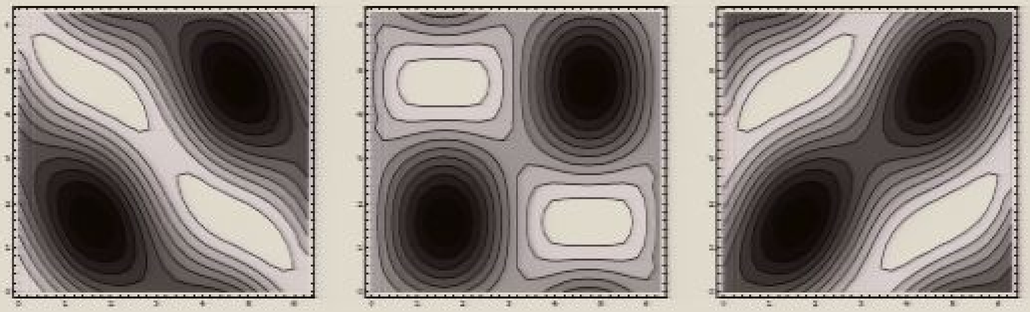

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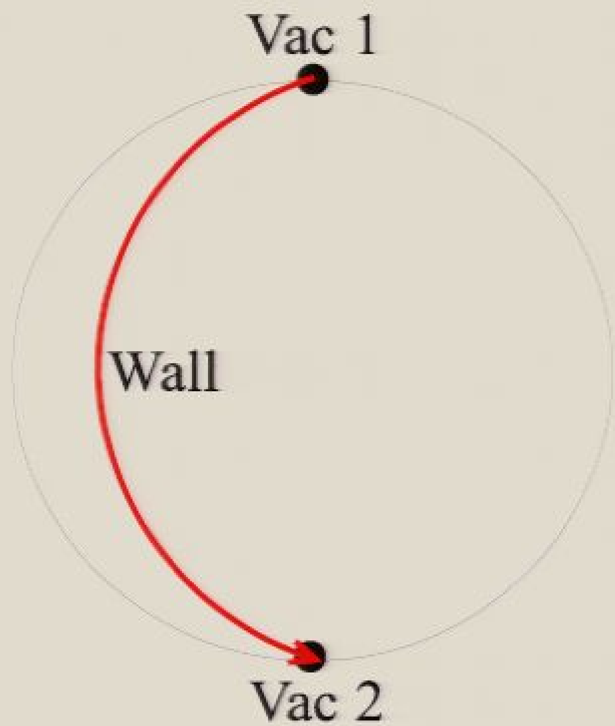

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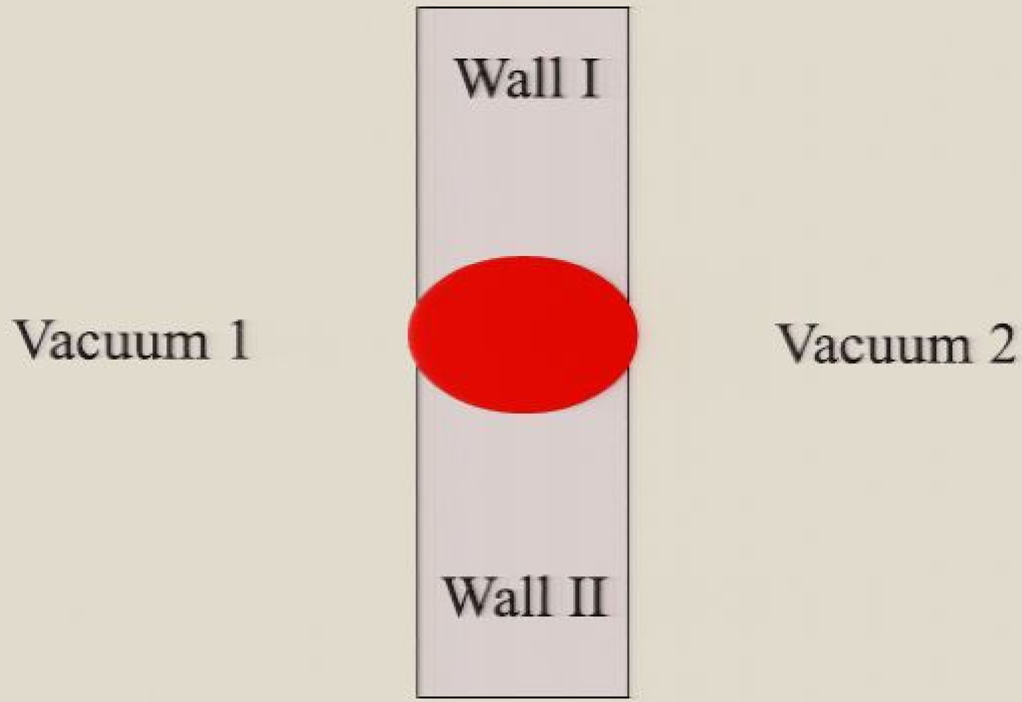
Domain Lines



Domain Lines interpolating between discrete Domain Wall solutions: an explicit solitonic example at weak coupling

Solution for η and world-volume potential

$$\eta(z) = 2\arctan(e^{2mz}) - \frac{\beta^2}{m^2}\frac{e^{2mz}\left(15\cos^2\sigma + 8(1+e^{4mz})mz\right)}{4(1+e^{4mz})^2} + \mathcal{O}(\beta^3)\,.$$

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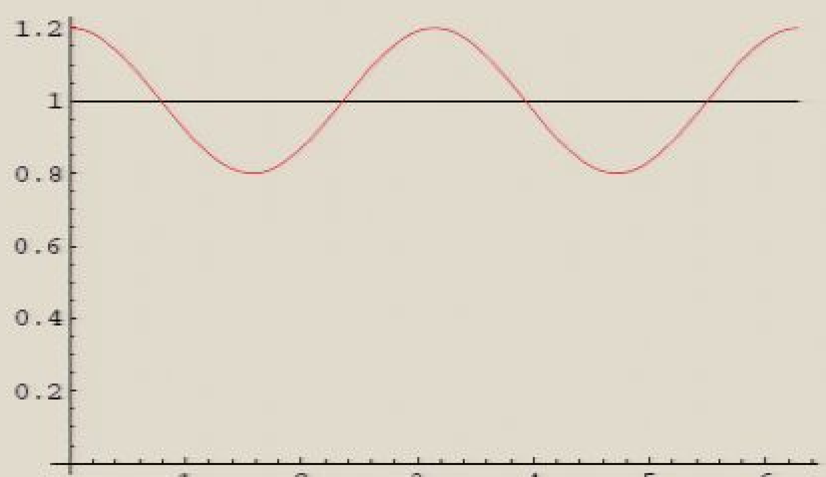
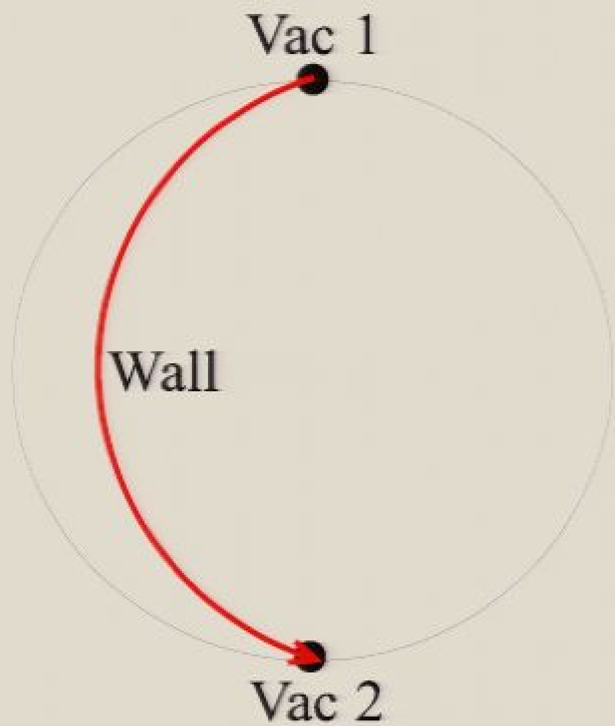


Wall in the sigma model description

Coordinates (θ, φ)



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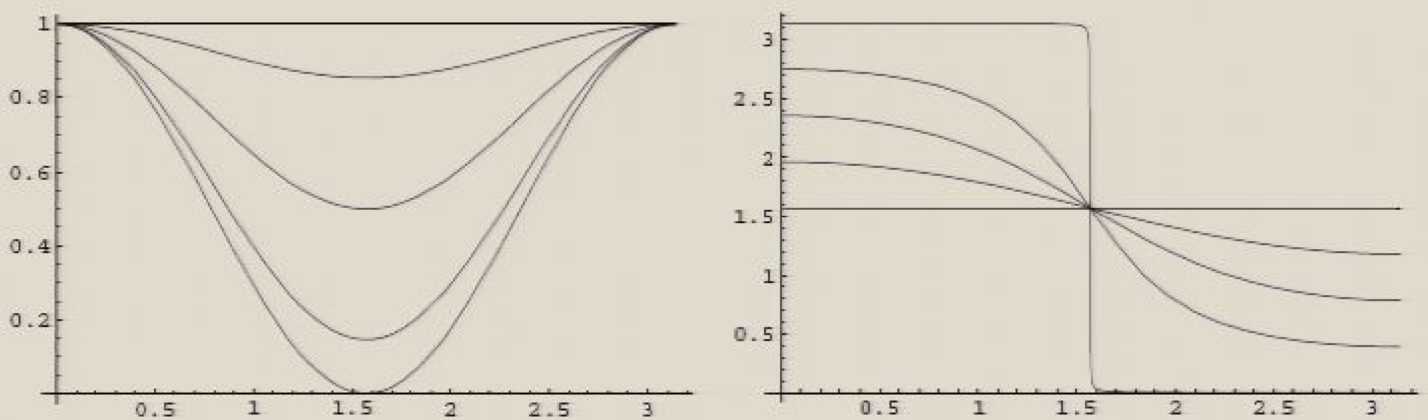
Unstable wall: sigma model

Coordinates: $(r, \theta, \varphi, \psi)$

$$\theta(\eta, \sigma) \approx \eta$$

$$\varphi(\eta, \sigma) \approx \sigma$$



Left: $r(\eta)$, Right: $\psi(\eta)$ for different σ.
They are both constant at $\sigma = \pm\pi/2$.

Unstable walls

The BPS wall solutions at $\beta \neq 0$ correspond to $\sigma = \pi/2, 3\pi/2$.
Ansatz for a “meta-solution“ at generic σ:

$$q_1 = (e^{i\sigma}\Omega\cos(\eta/2) + \omega\sin(\eta/2))/A,$$

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where $\eta(z)$ is a profile function which is calculated by minimization of the action and the factor A is introduced in order to maintain the sigma model constraints.

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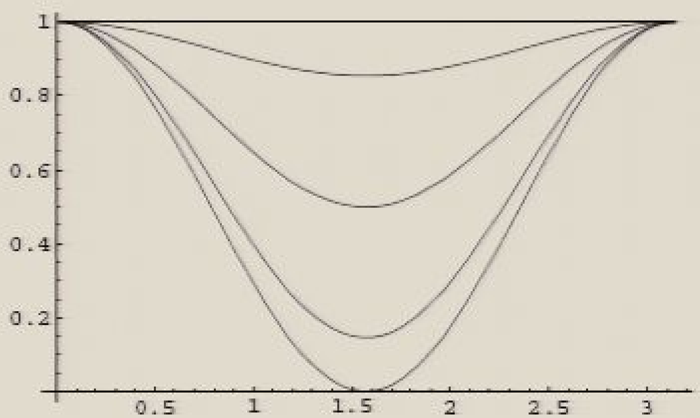

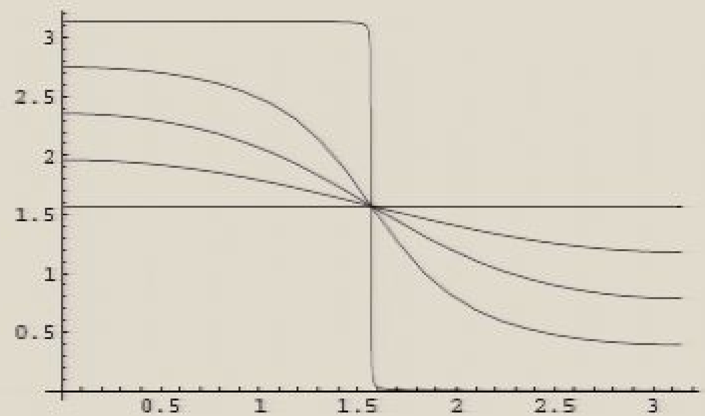

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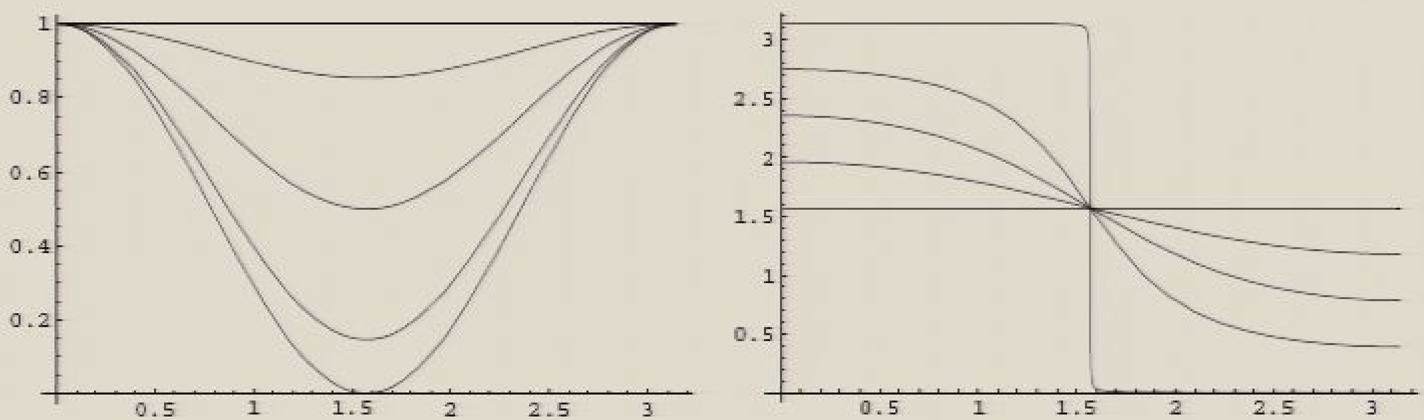

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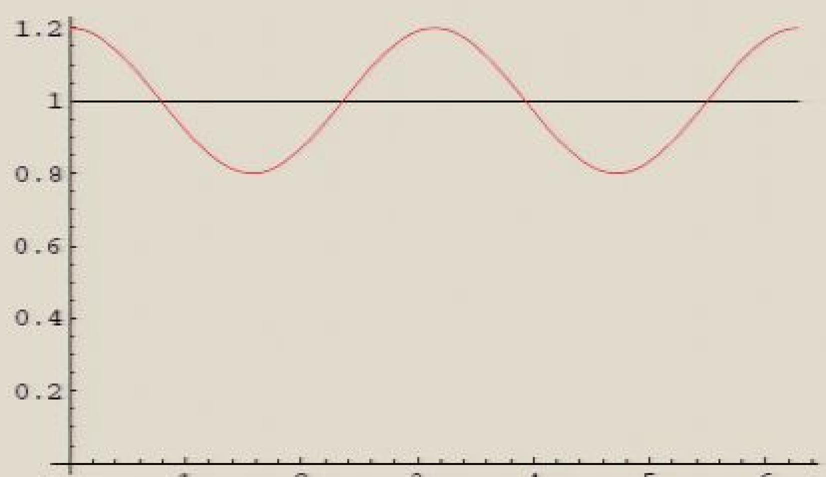

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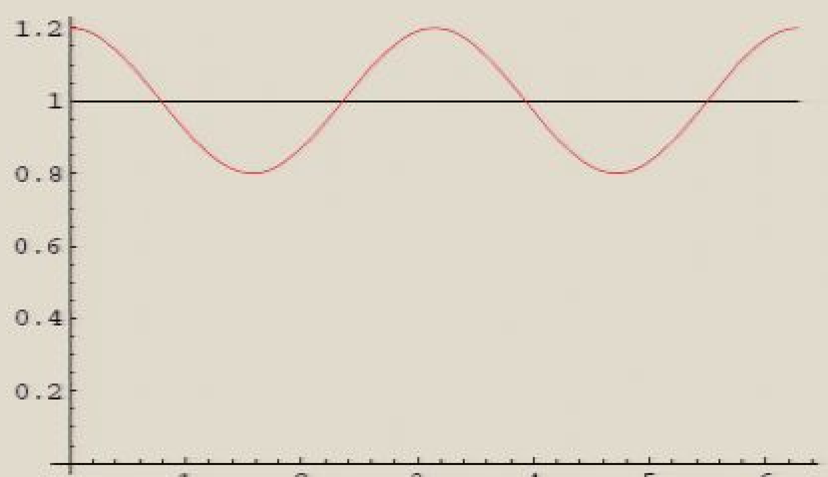

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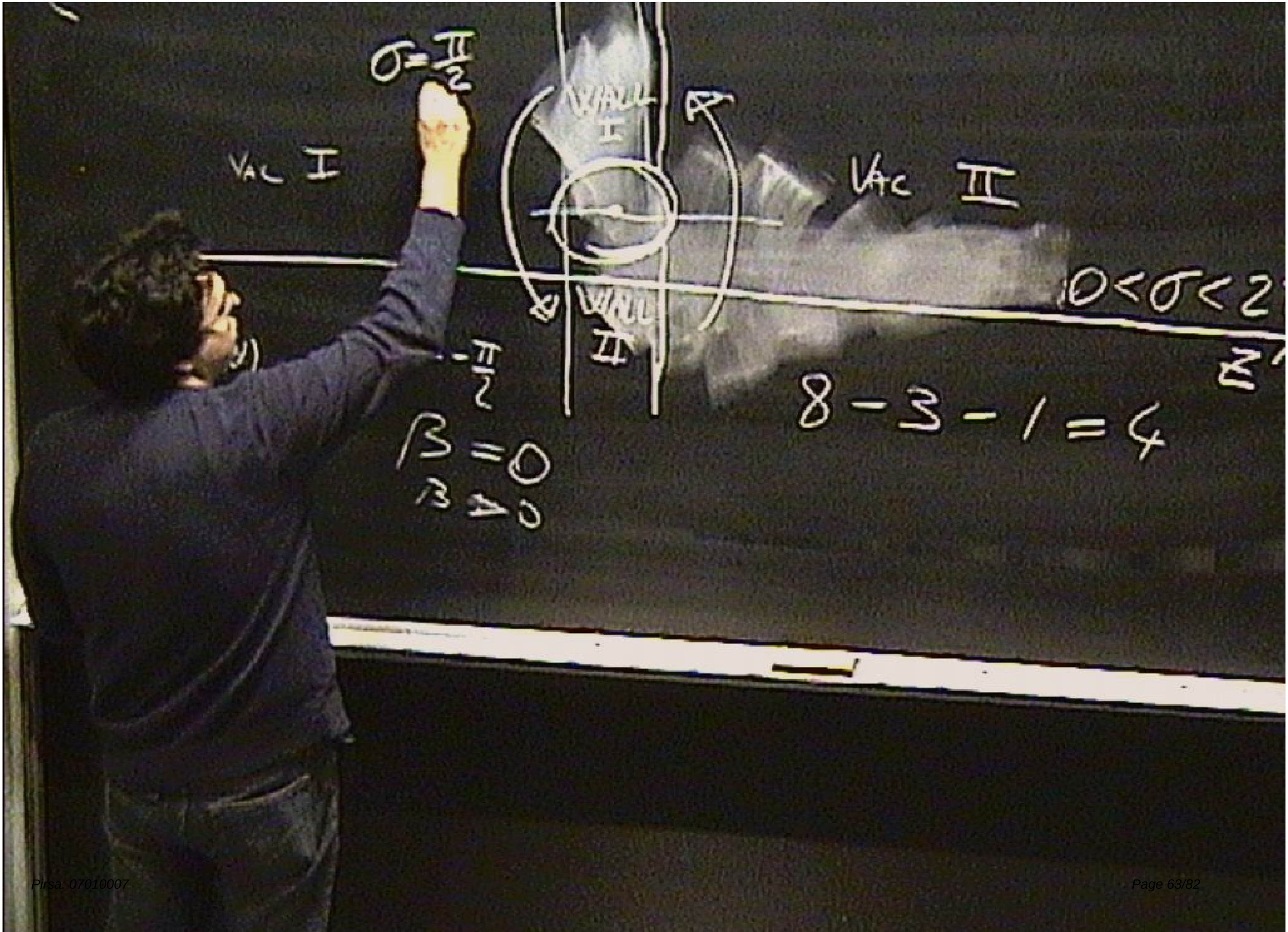
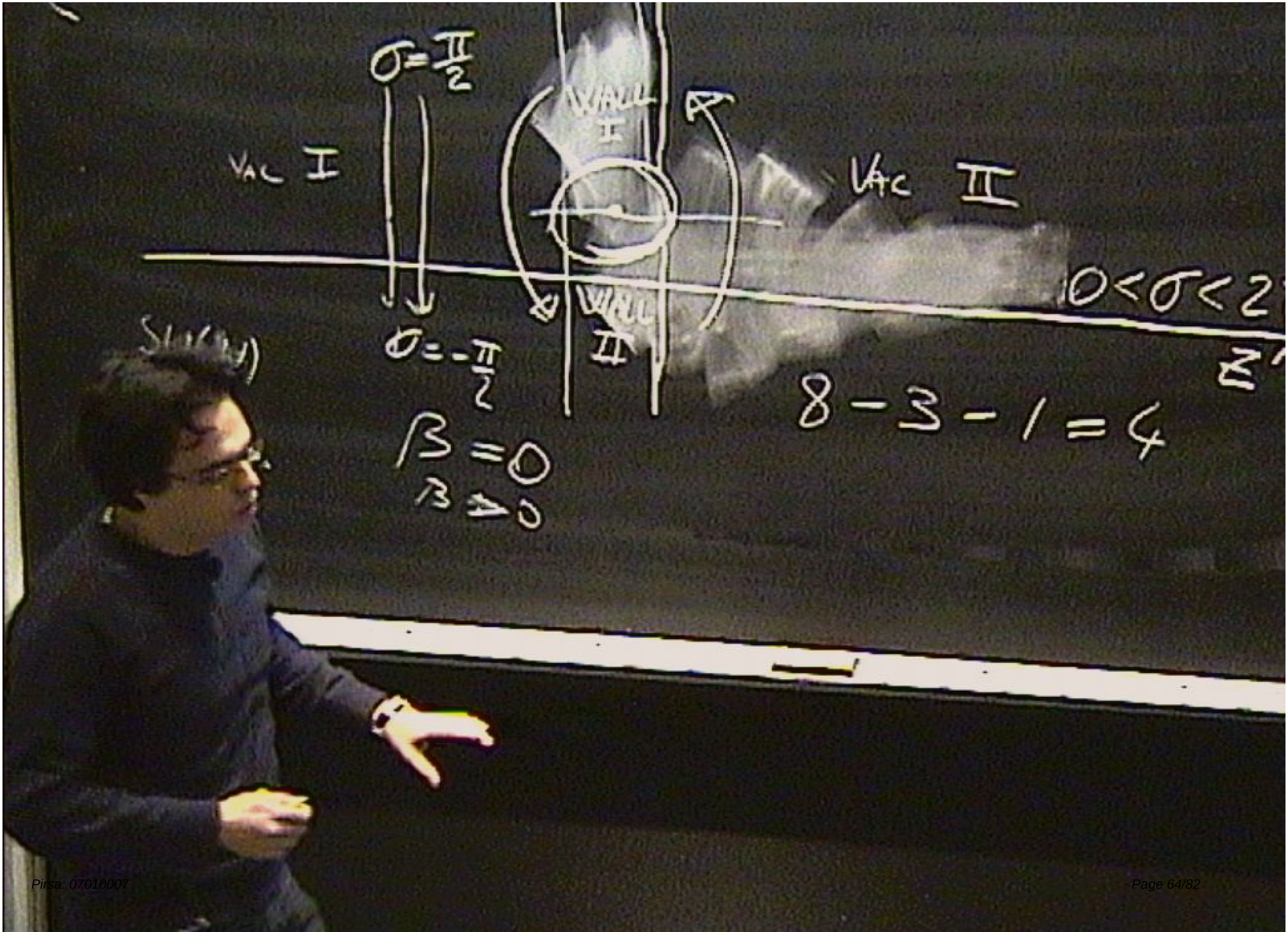

$\sigma = \frac{\pi}{2}$
VAC I
WALL I
VAC II
$0 < \sigma < 2$
WALL II
$-\frac{\pi}{2}$
$8 - 3 - 1 = 4$
$\beta = 0$
$\beta > 0$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VAC I
WALL I
VAC II
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WALL II
8-3-1=4
β=0

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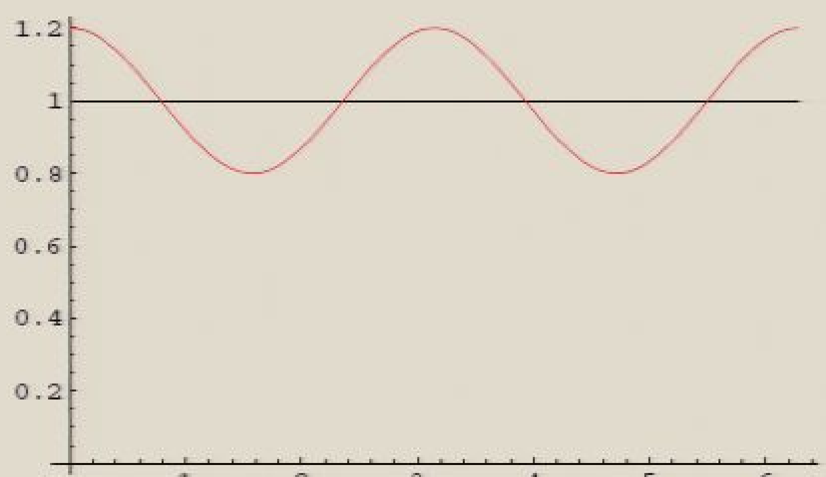
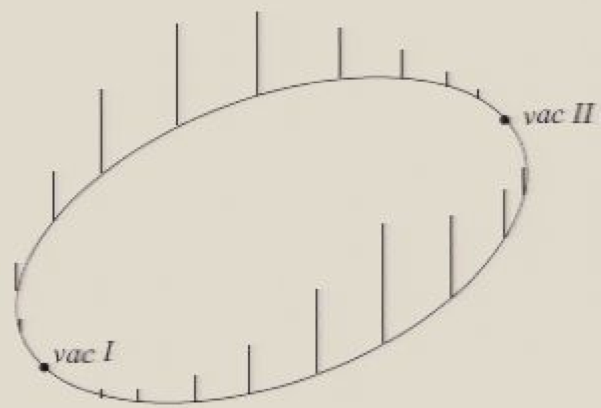


Domain line as a Sine Gordon kink

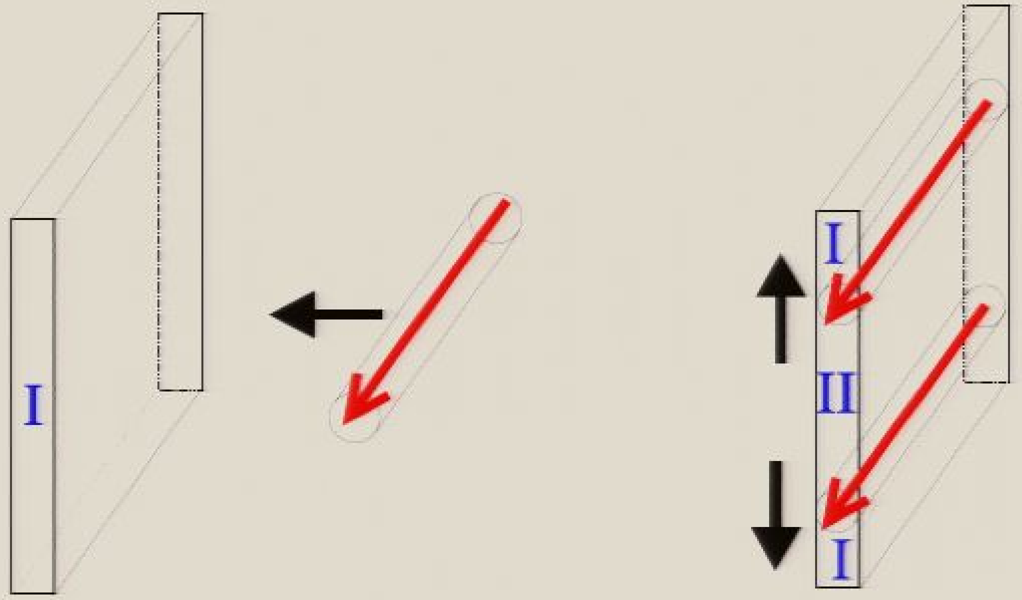


There are two different kinks interpolating the two world-volume vacua

$$\mathcal{H} = \int d^3x \left(\frac{\xi}{4m} (\partial_n \sigma)^2 + \frac{\beta^2 \xi}{2m} \cos^2 \sigma \right) =$$

$$= \int d^3x \left\{ \frac{\xi}{2m} \left(\frac{1}{\sqrt{2}} (\partial_x \sigma) \mp \beta \cos \sigma \right)^2 \mp \frac{\xi \beta}{\sqrt{2} m} \partial (\sin \sigma) \right\}.$$

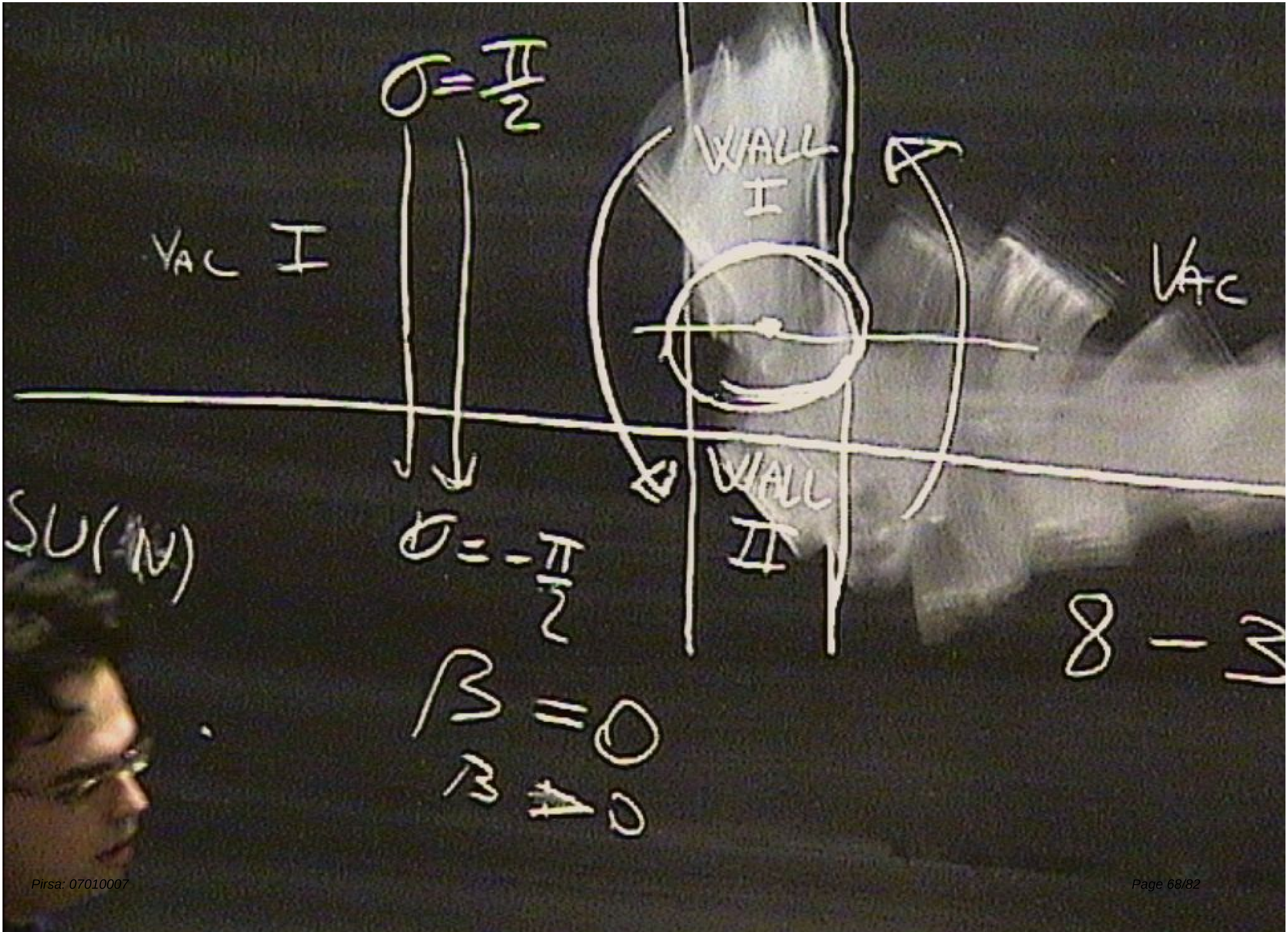
The fate of a bulk vortex



Bulk: $T = 2\pi\xi$

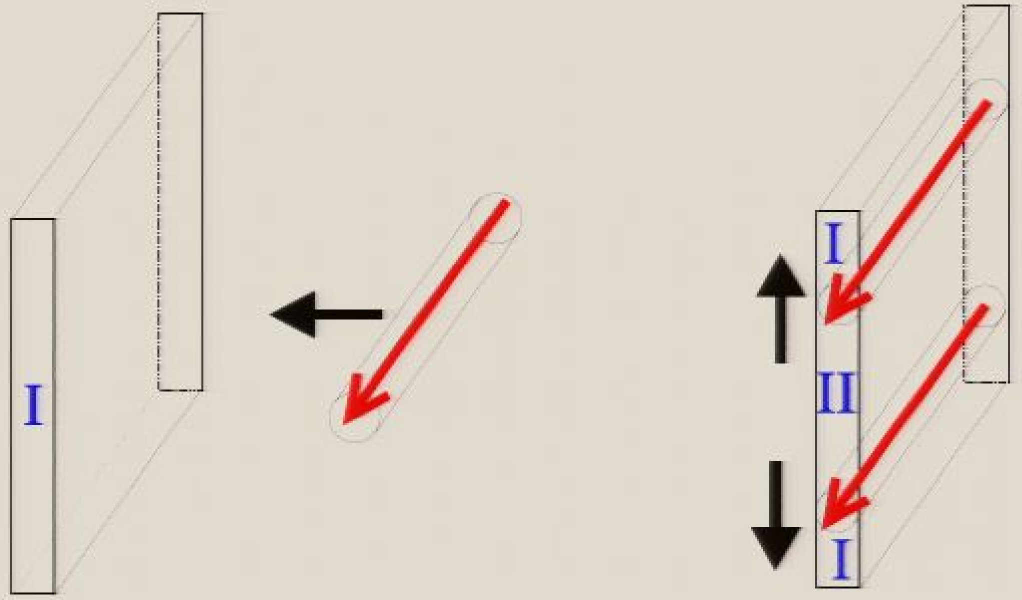
Domain line: $T = \sqrt{2}\beta\xi/m$

The Domain Line carries a magnetic flux which is $1/2$ of the flux carried by a bulk vortex.



σ = π/2
VAC I
WALL I
VAC
WALL II
SU(N)
σ = -π/2
β = 0
β ≥ 0
8 - 3

The fate of a bulk vortex



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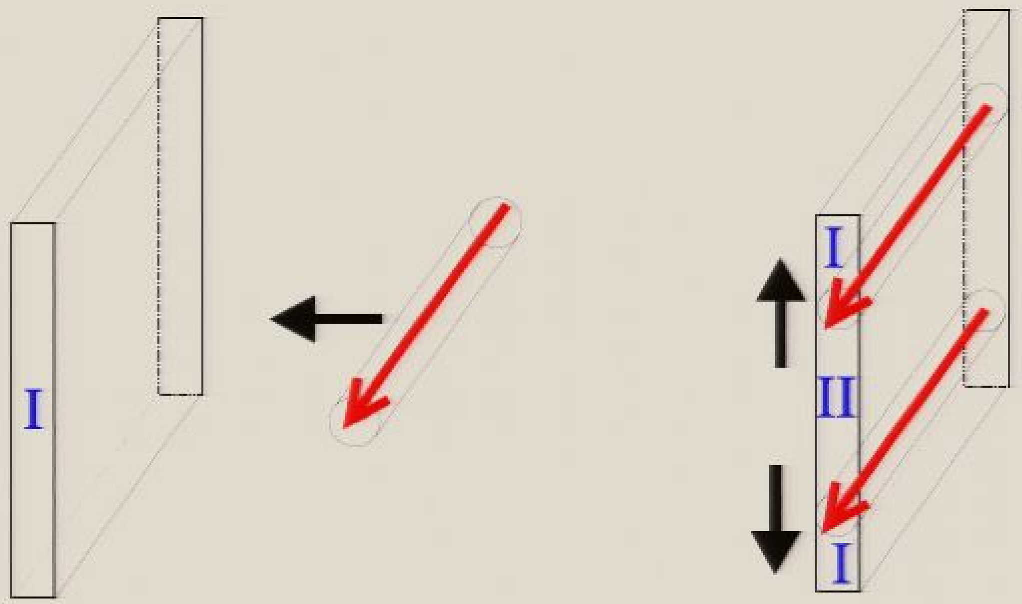

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Sine Gordon vs. Chern-Simon

World-volume scalar field σ:

$$S = \int d^3x \left(\frac{\xi}{4m} (\partial_n \sigma)^2 \right).$$

Duality: $F_{nm}^{(2+1)} = \frac{e_{2+1}^2}{2\pi} \epsilon_{nmk} \partial^k \sigma,$

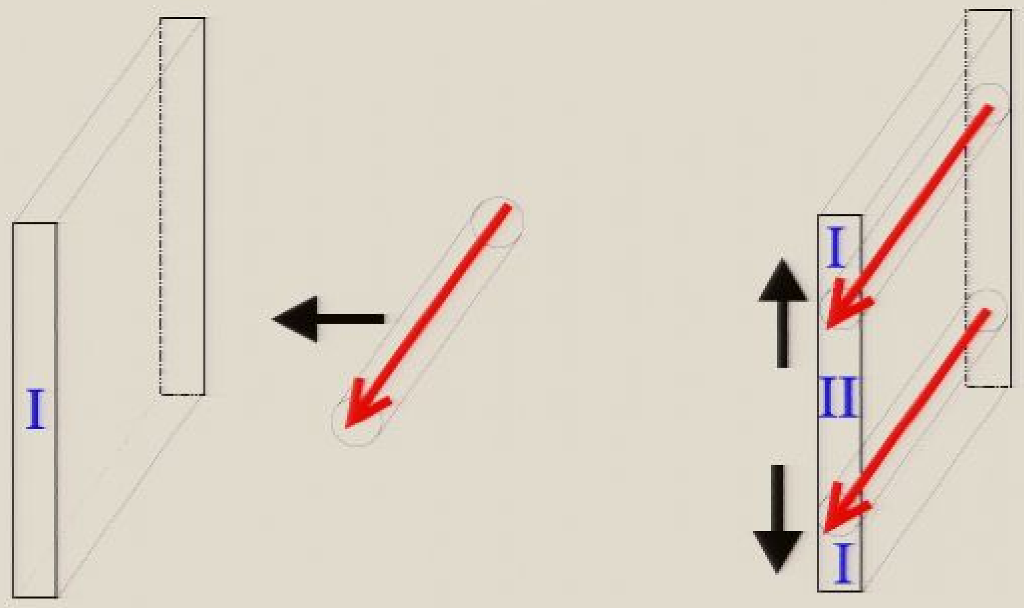
$$S = \int d^3x \left(-\frac{1}{4e^2} (F_{mn}^{2+1})^2 \right).$$

Chern-Simon term (proposed for domain walls of $\mathcal{N} = 1$ Super-Yang-Mills):

$$S_{CS} = \frac{1}{2\pi} \epsilon_{nmk} A_n \partial_m A_k.$$

Different from the Sine-Gordon potential in our weakly coupled example!

The fate of a bulk vortex



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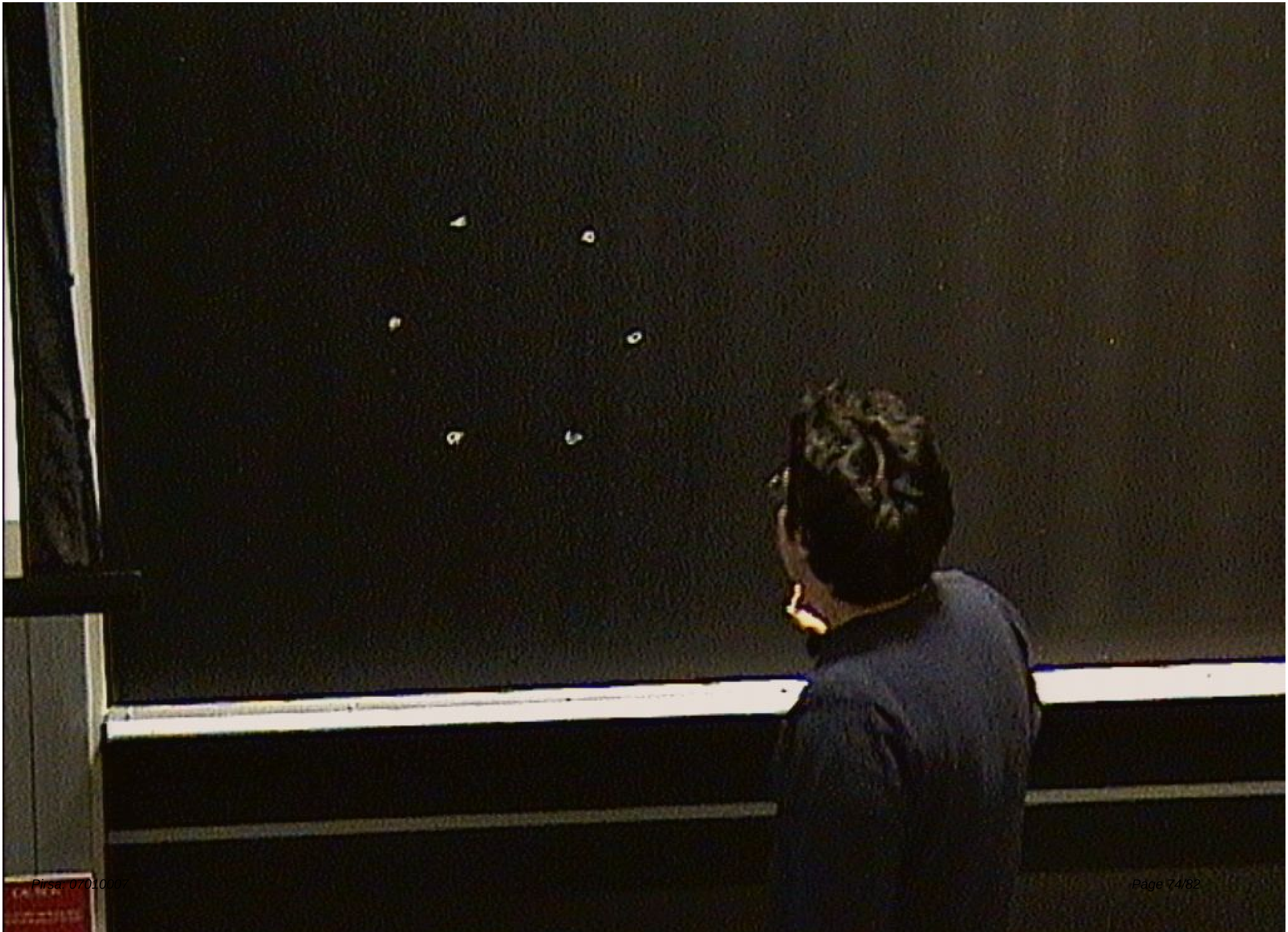

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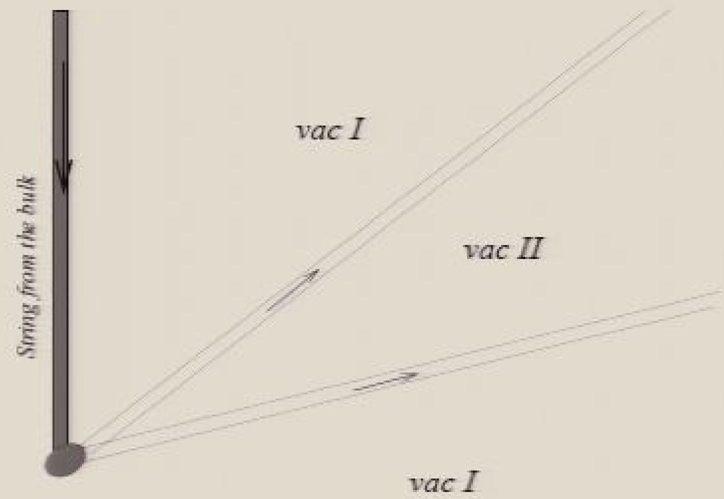
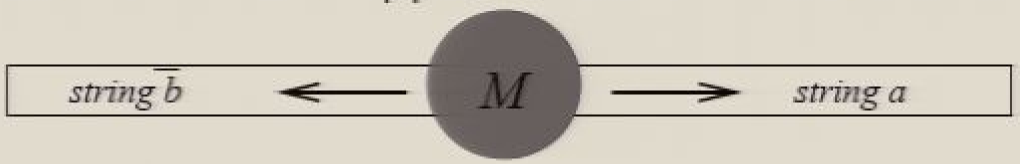
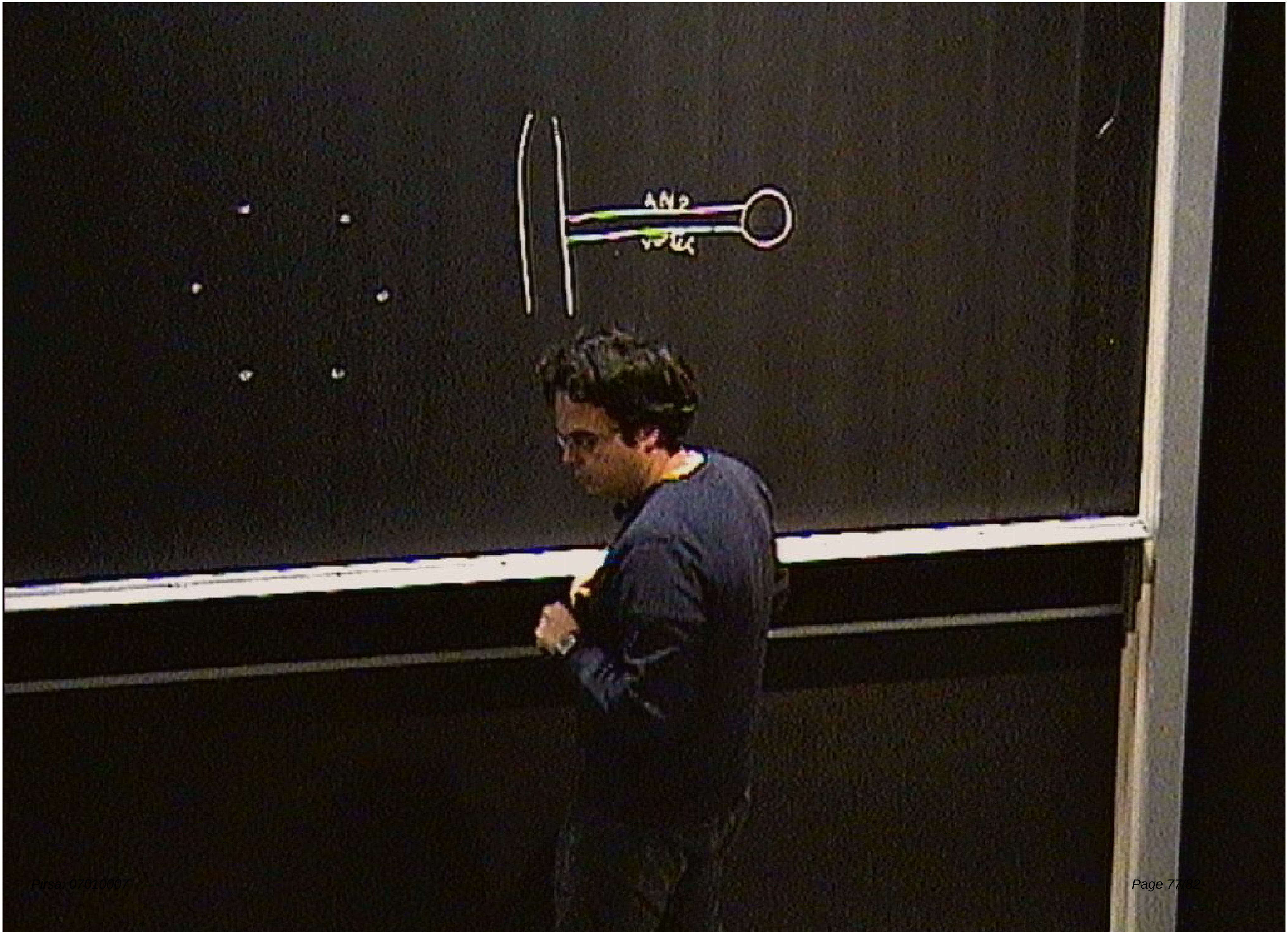
Confined Monopoles

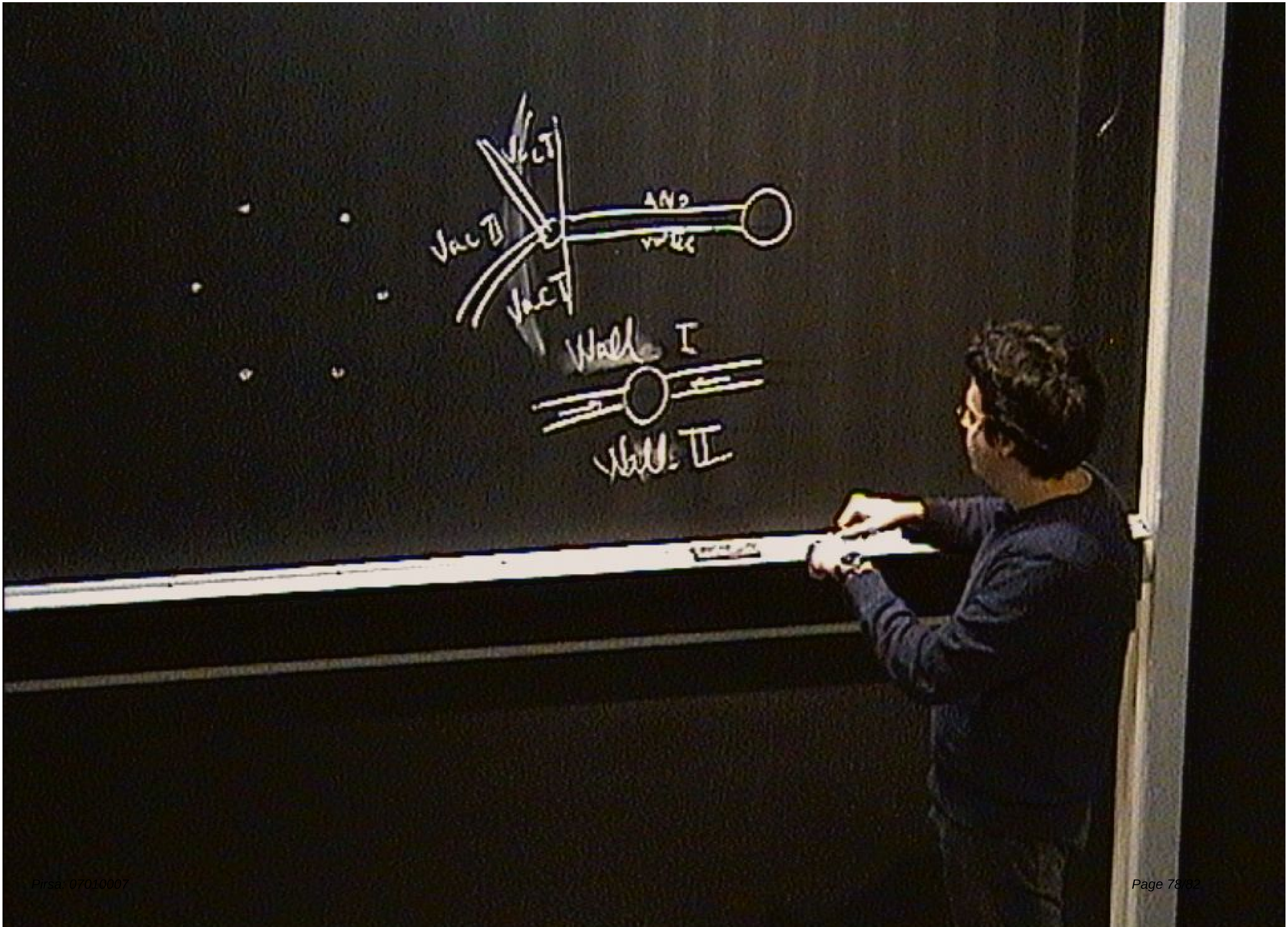


We can introduce external magnetic monopoles embedding the $U(1)$ theory in a $SU(2)$ one

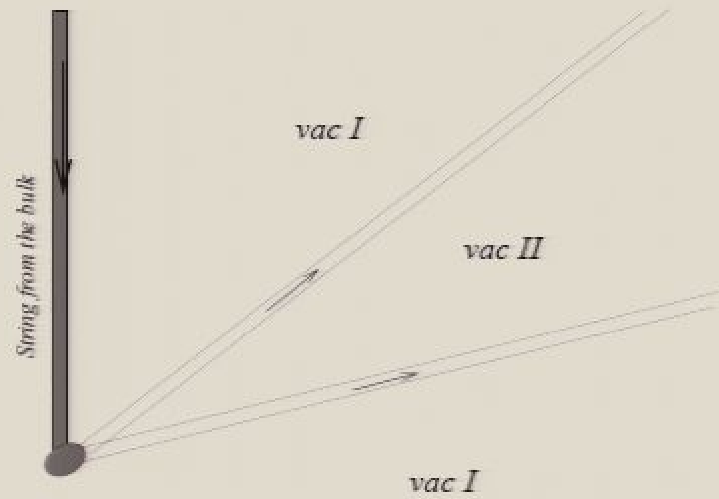
It is possible to build a static configuration where a monopole is a junction of two domain lines (similar to what happen in the bulk for the non-abelian string)






Vac I
Vac II
Vac V
AND
vortex
Wall I
Wall II

Confined Monopoles



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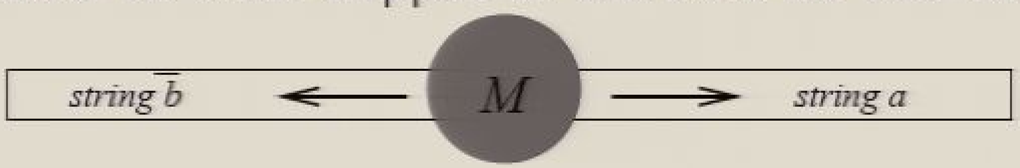
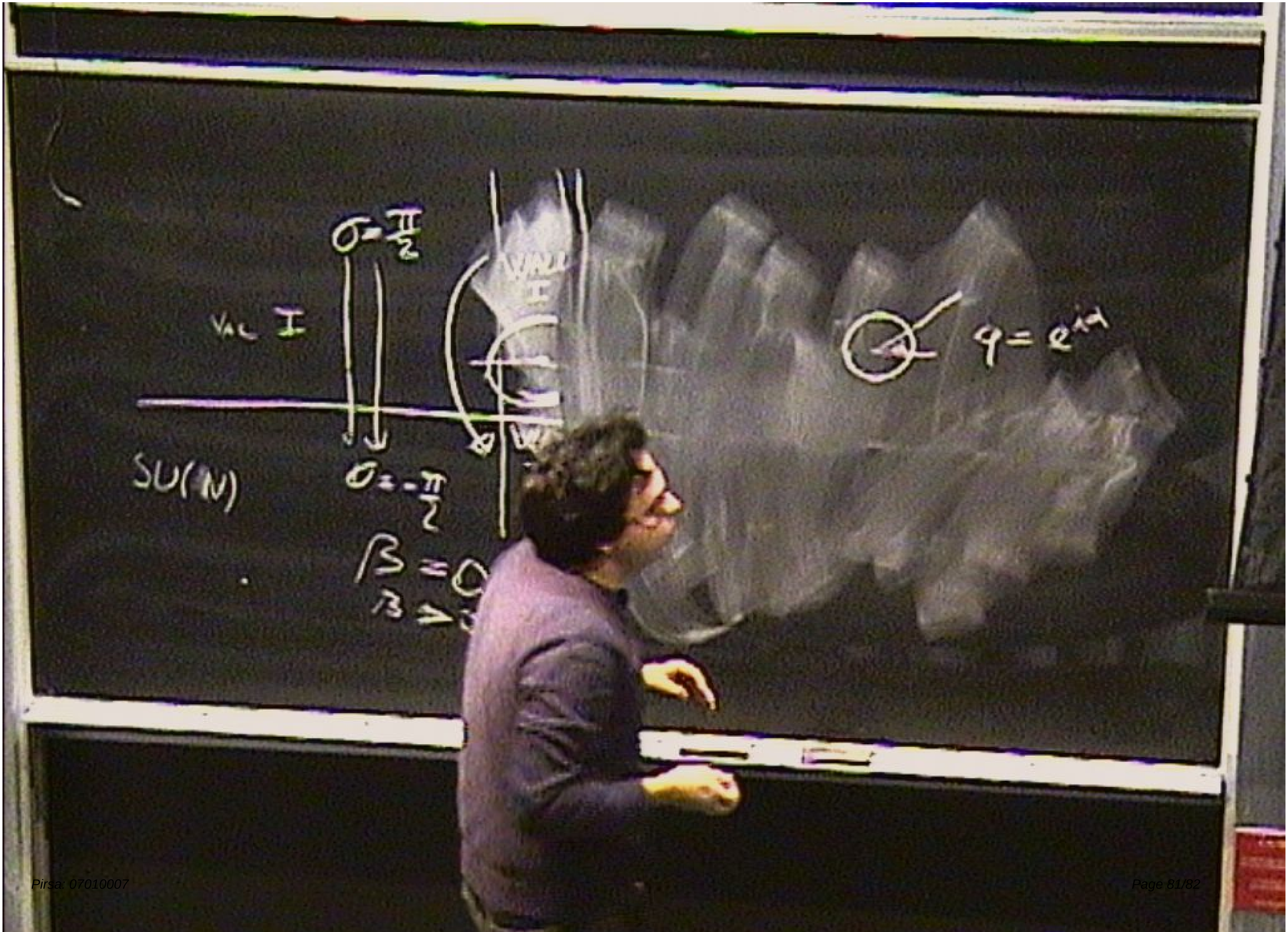


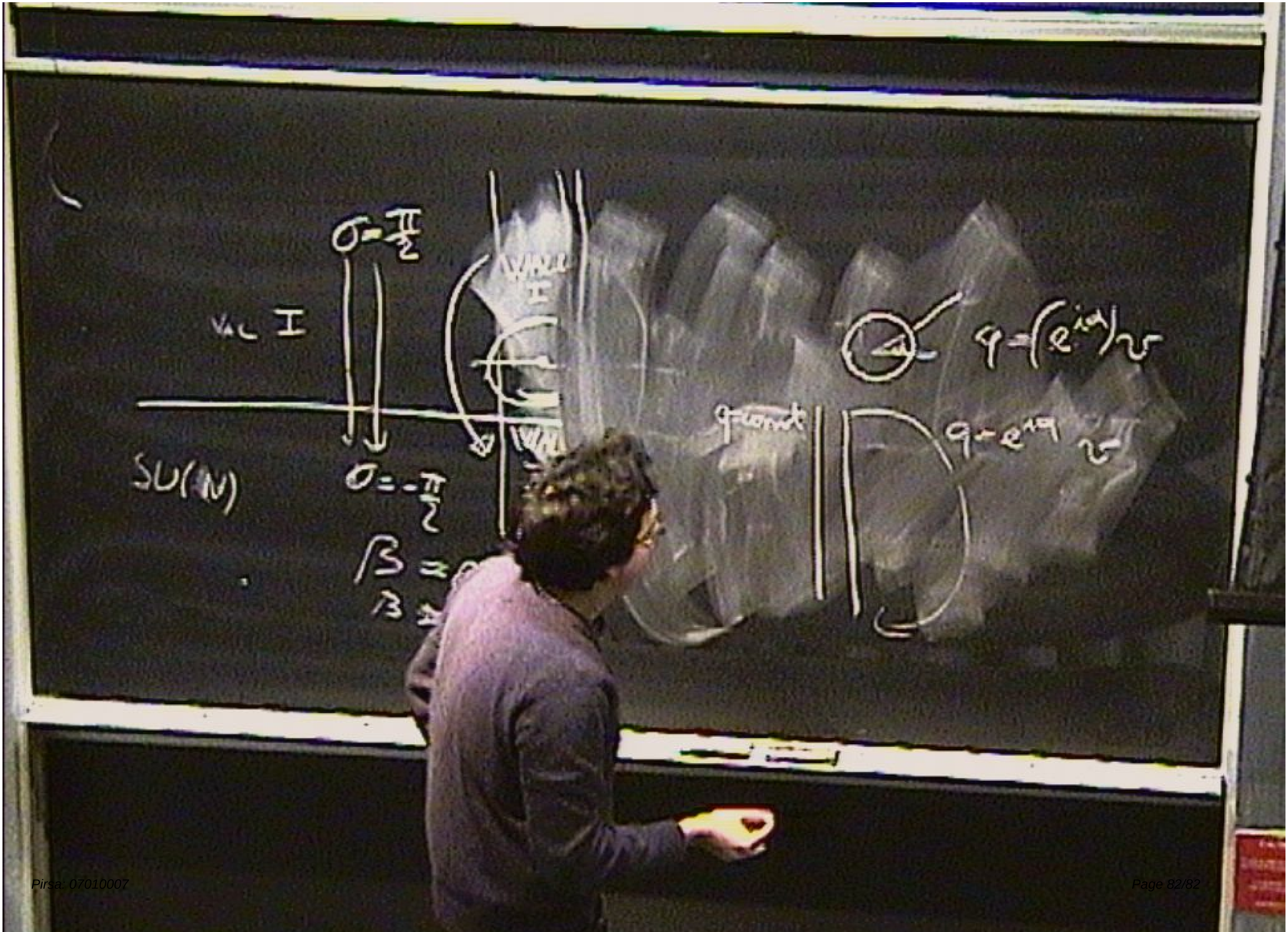
Conclusion

* A Domain-Line soliton has been built in a weakly coupled theory

* The effective world-volume description involves a Sine-Gordon theory with two vacua

* The domain line carries a magnetic flux which is $1/2$ of a bulk Abrikosov-Nielsen-Olesen vortex


σ = π/2
VAC I
SU(N)
σ = -π/2
β = 0
q = e^{iη}


σ = π/2
VAC I
SU(N)
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q = const