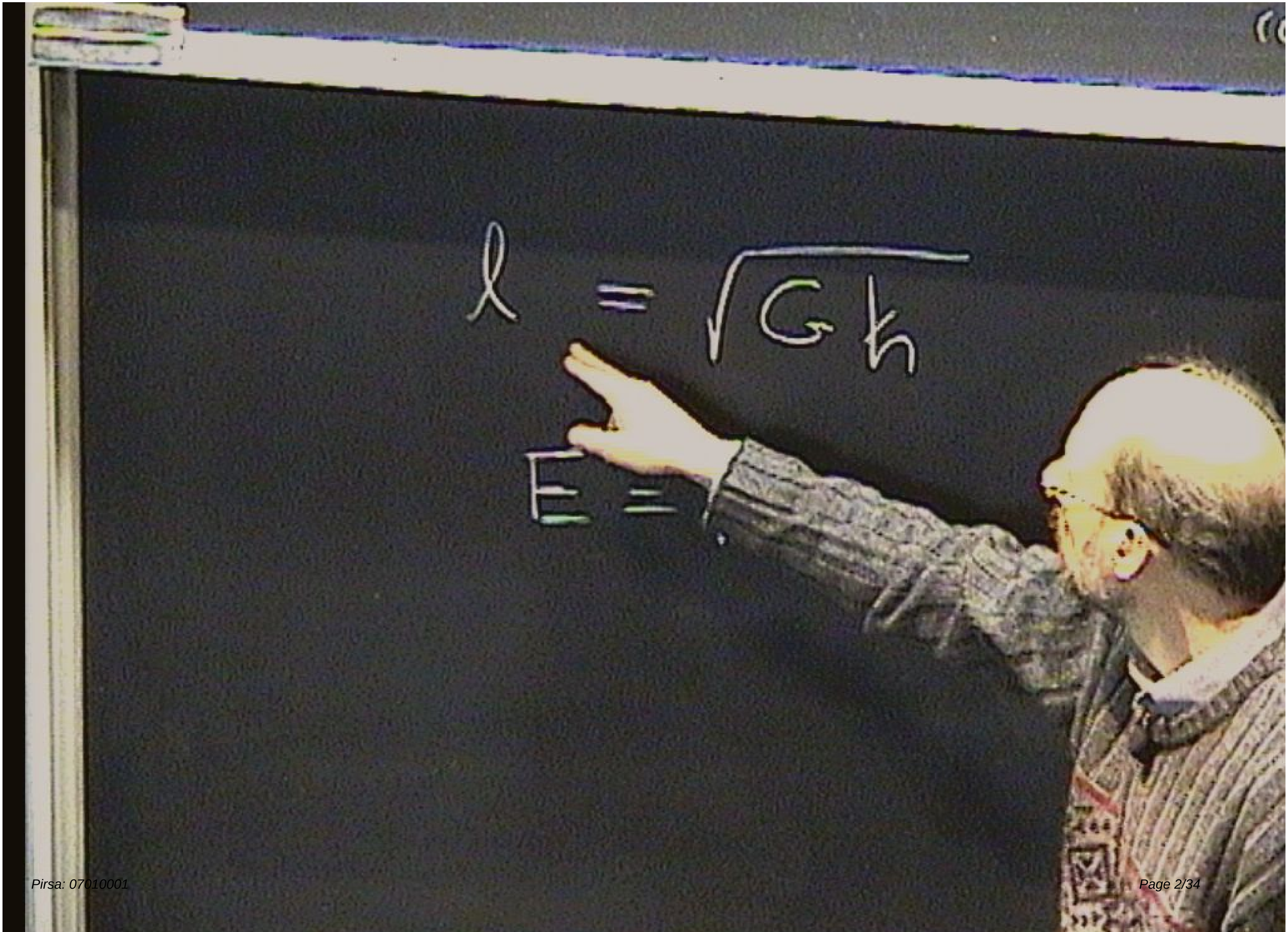
Title: Does quantum gravity give rise to an observable nonlocality?

Date: Jan 17, 2007 02:00 PM

URL: http://pirsa.org/07010001

Abstract: If spacetime is "quantized" (discrete), then any equation of motion compatible with the Lorentz transformations is necessarily non-local. I will present evidence that this sort of nonlocality survives on length scales much greater than Planckian, yielding for example a nonlocal effective wave-equation for a scalar field propagating on an underlying causal set. Nonlocality of our effective field theories may thus provide a characteristic signature of quantum gravity.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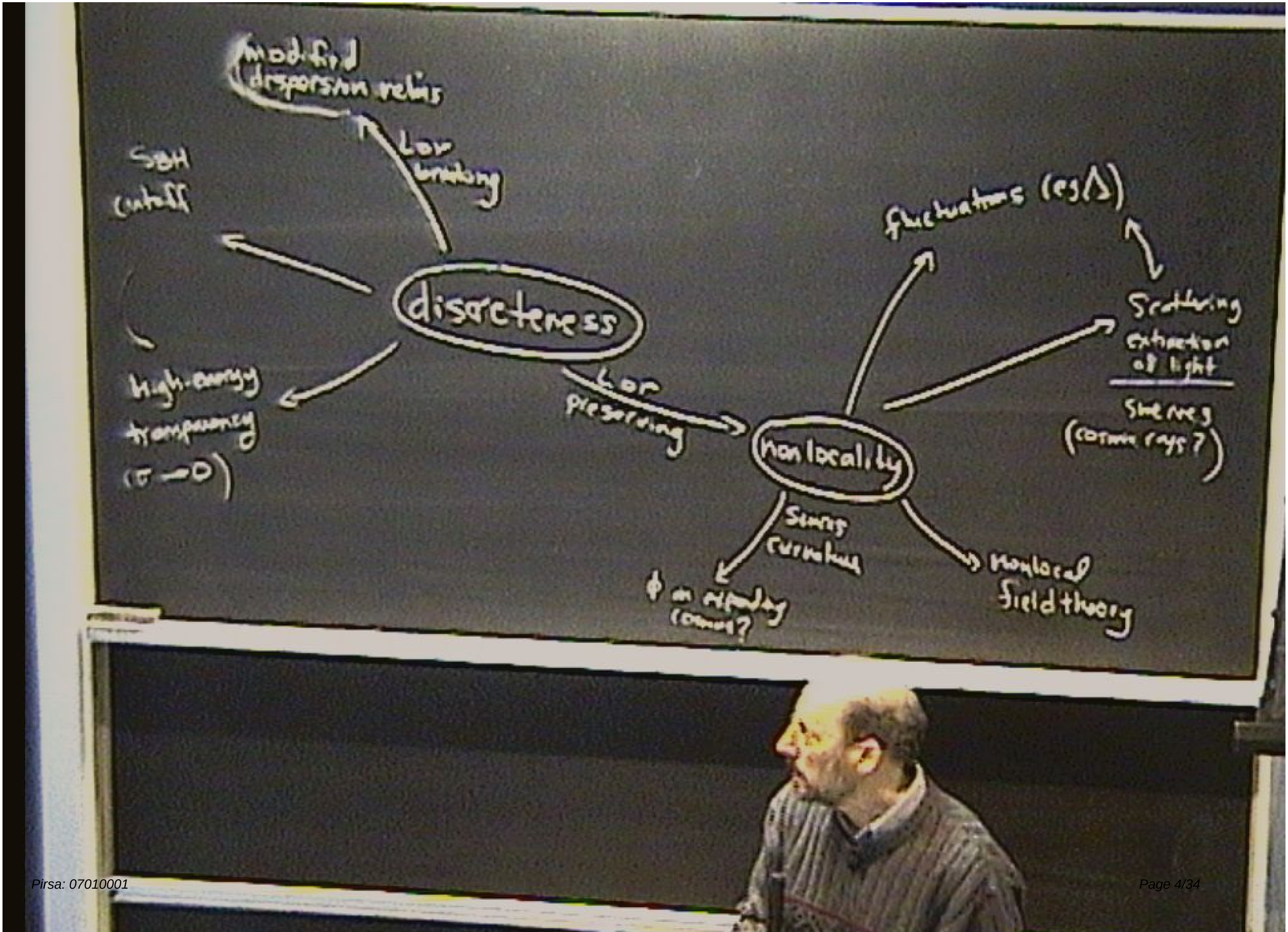
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Lor breaking
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Swerves
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Sources
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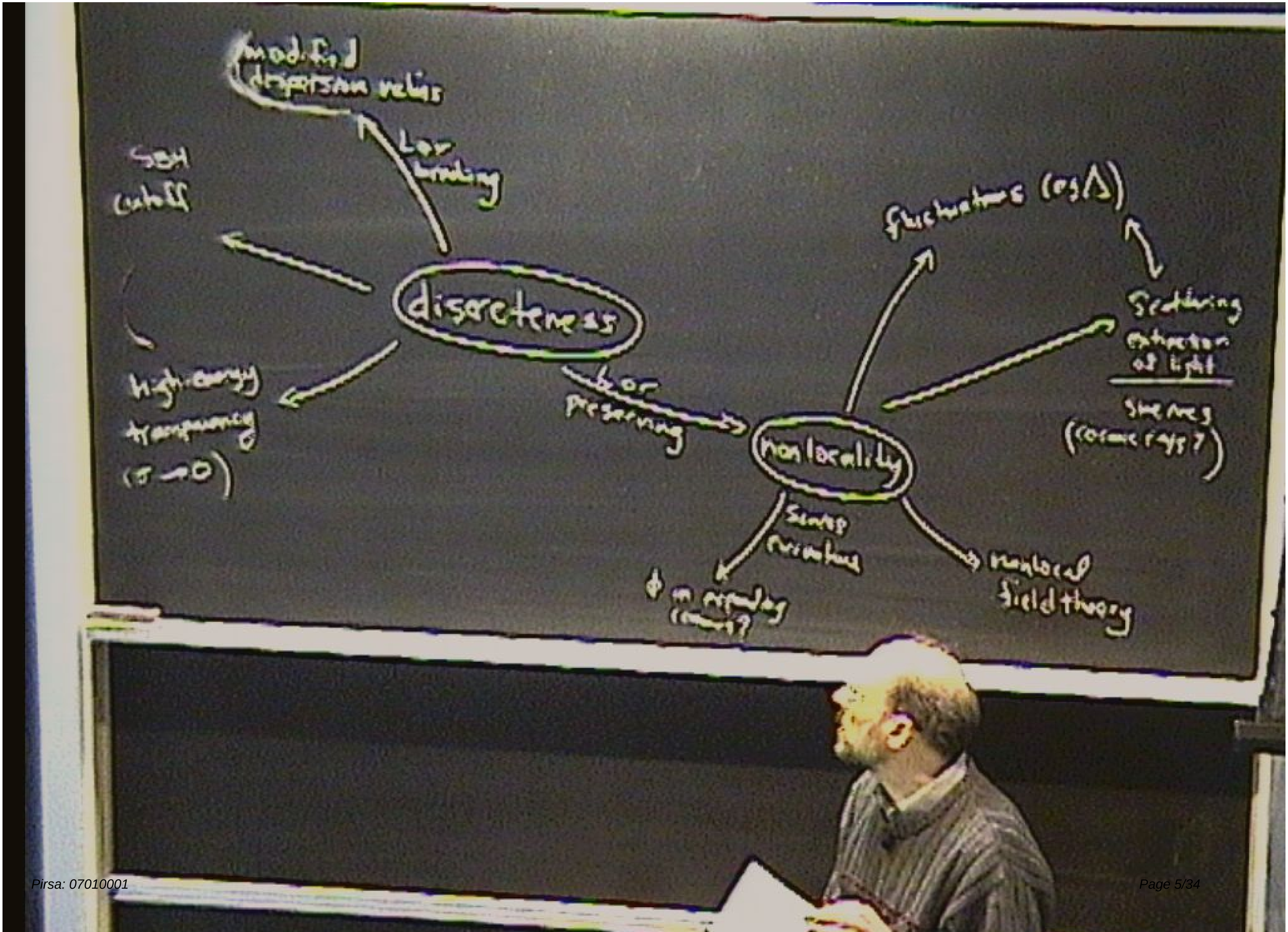
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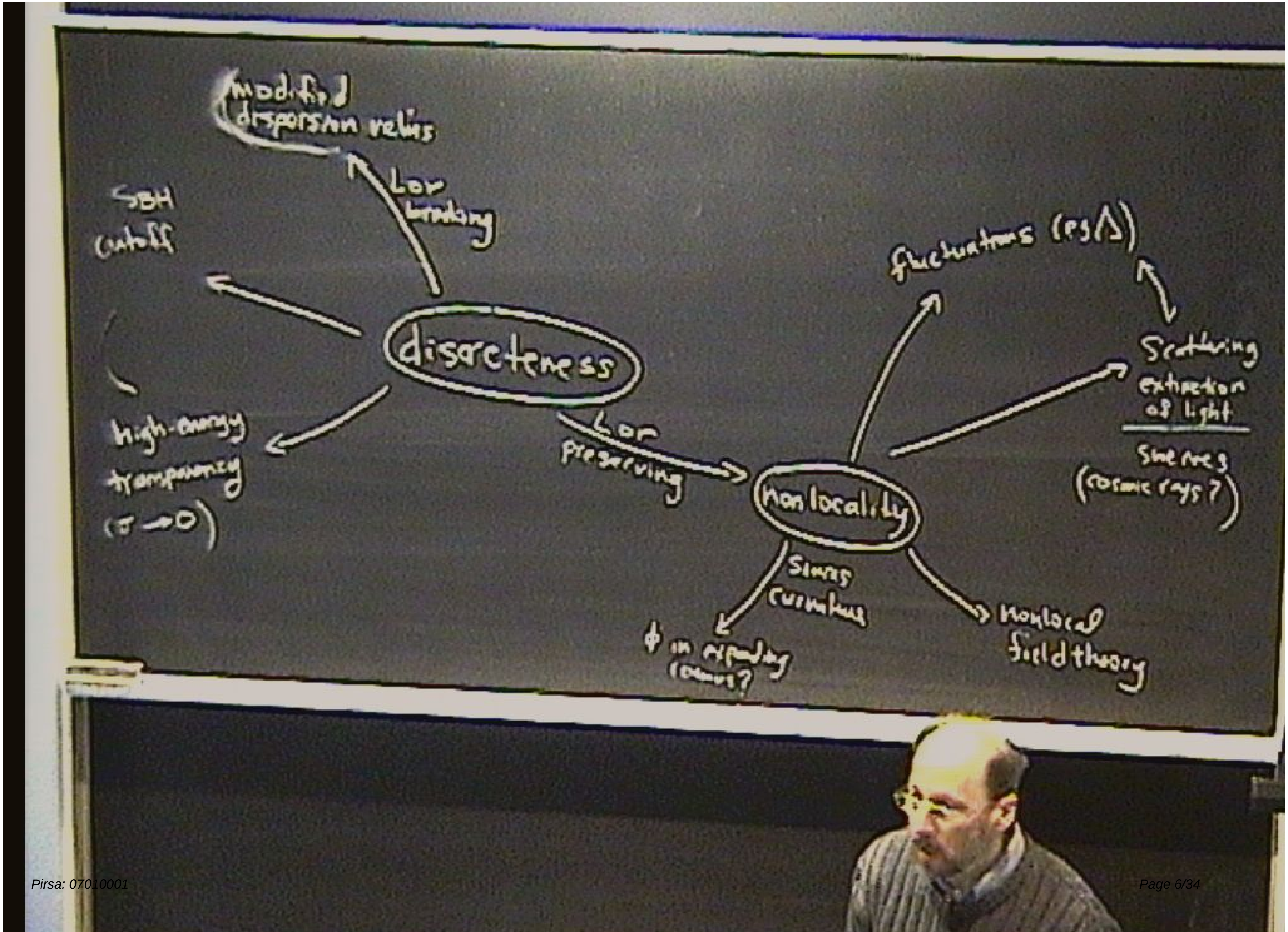
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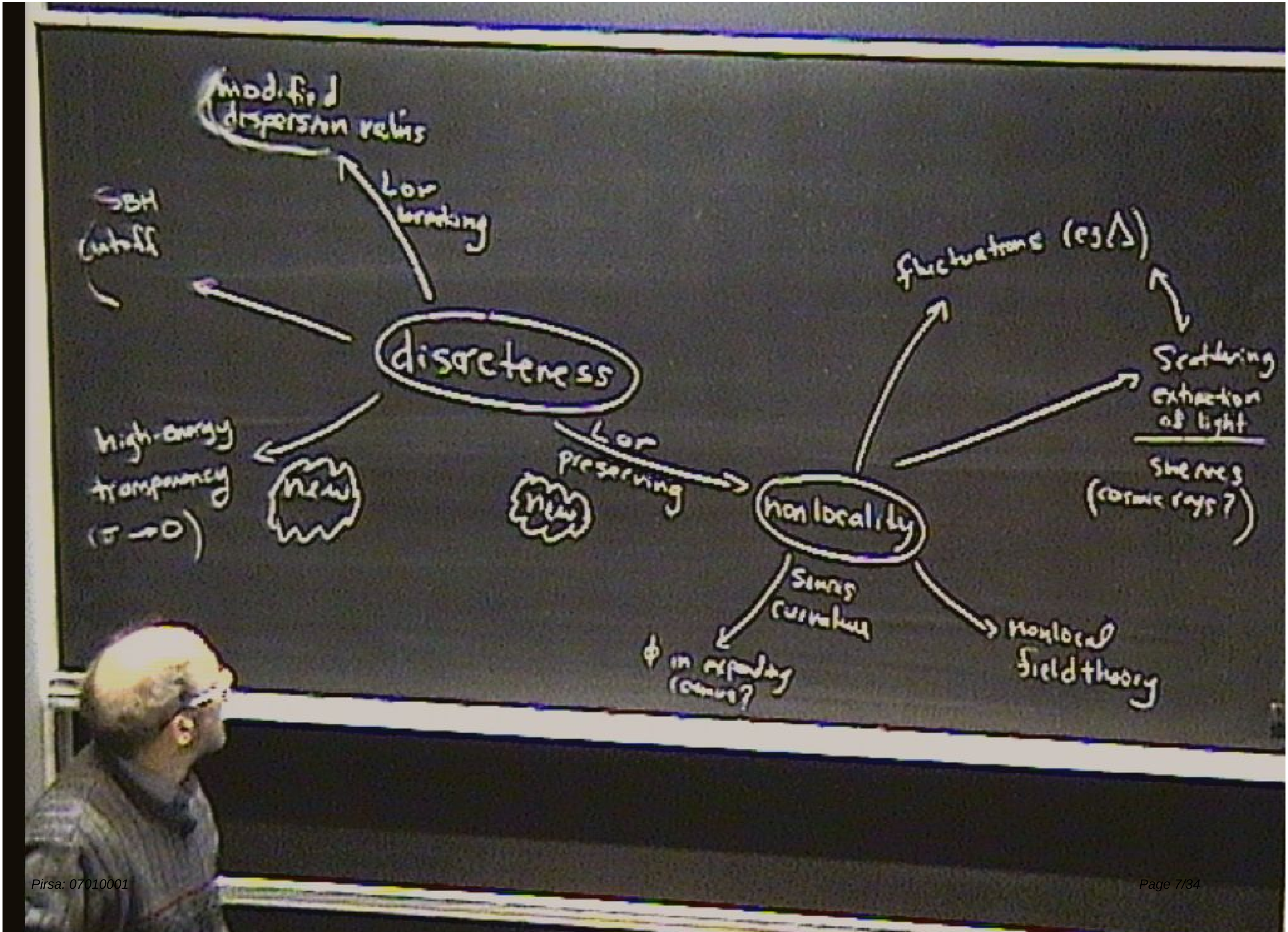
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Discreteness can respect the Lorentz group

(Kinematic randomness plays a role – Poisson processes – and causets require this)

But locality must be abandoned

$\Rightarrow$ radical nonlocality at fundamental level (micro-scale ℓ)

One can recover locality approximately at large scales (*macro*-scale L)

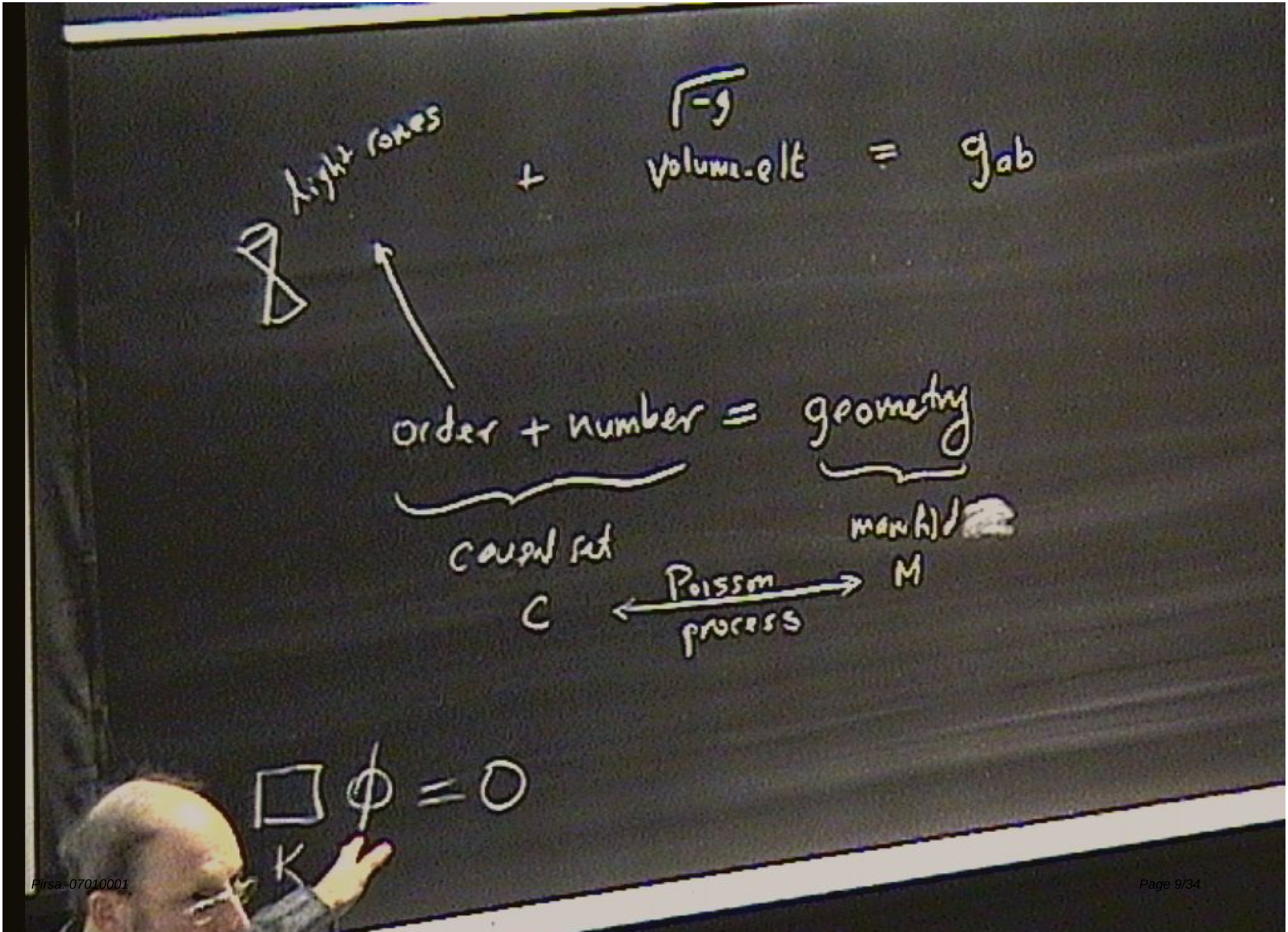
But residual nonlocality survives at *intermediate* scales

(*meso*-scale λ_0)

An effective meso-theory would be continuous but nonlocal

Illustrate these claims with scalar field ϕ on a fixed causet C: Recovery of $\Box\,\phi$.

($\delta\Lambda$ is also a nonlocal effect of discreteness; I'll not discuss it)



light cones
+
√-g
volume-elt = gab
order + number = geometry
causal set
manifold
C
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light cones

$$+ \quad \underset{\text{volume-elt}}{\sqrt{-g}} \quad = \quad g_{ab}$$

$$\underbrace{\text{order} + \text{number}}_{\text{causal set}} = \underbrace{\text{geometry}}_{\text{manifld}}$$

$$C \xleftrightarrow[\text{process}]{\text{Poisson}} M$$

$$\underset{K}{\Box}\,\phi = 0$$

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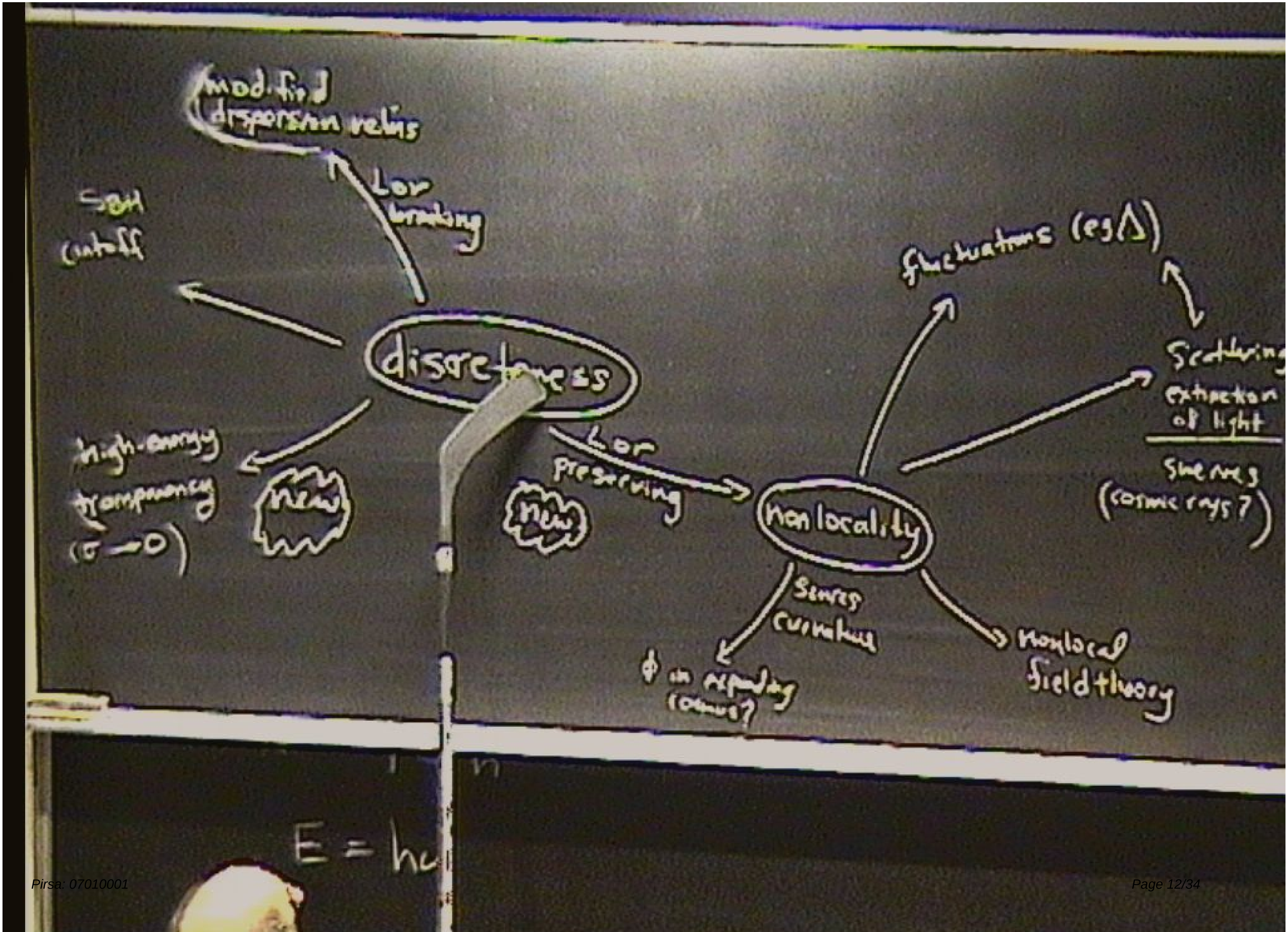
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$E = h$

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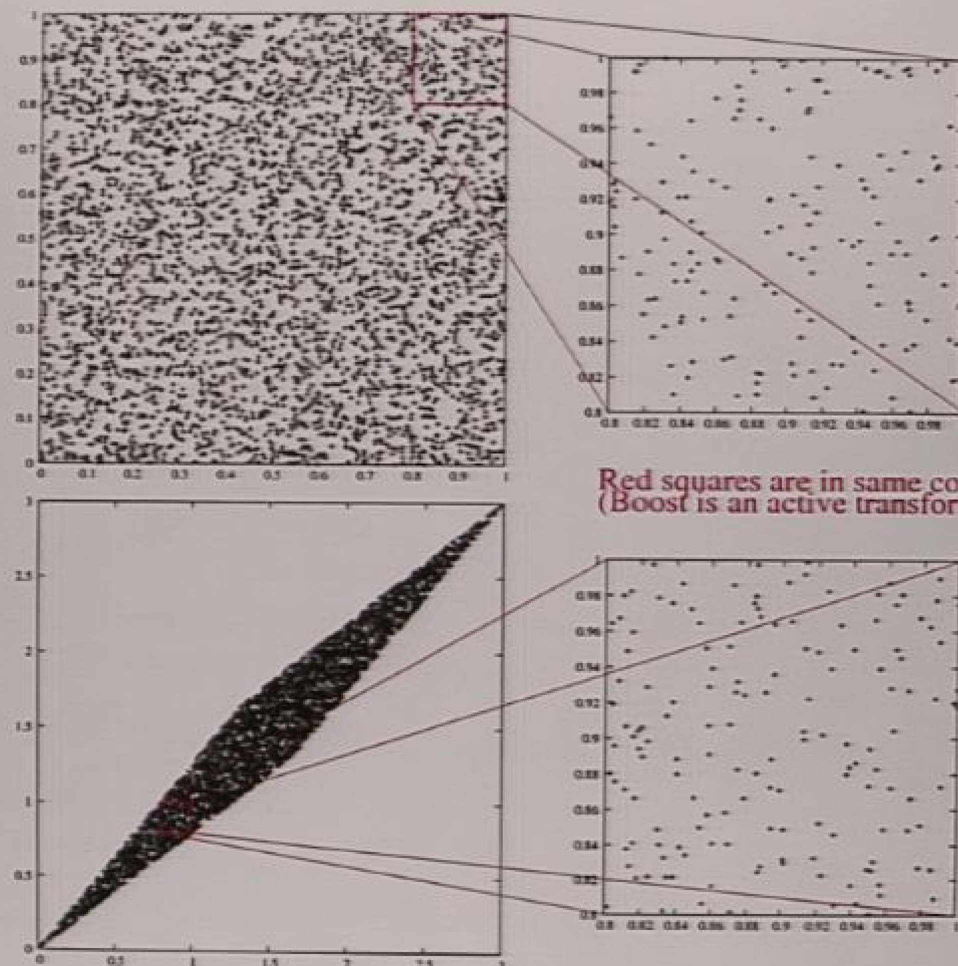

Red squares are in same coordinate location.
(Boost is an active transformation.)

A theorem on Poisson processes

$\Omega =$ space of all sprinklings of $\mathbb{M}^d$ (sample space)

Poisson process induces a measure μ on Ω

Let f be a rule for deducing a direction from a sprinkling $f : \Omega \to H =$ unit vectors in $\mathbb{M}^d$

Require f *equivariant* ($f\Lambda = \Lambda f$, $\Lambda \in$ Lorentz)

Assume that f is measurable (hardly an assumption)

THEOREM *No such f exists* (not even on a partial domain of positive measure)

(So with probability 1, a sprinkling will not determine a frame.)

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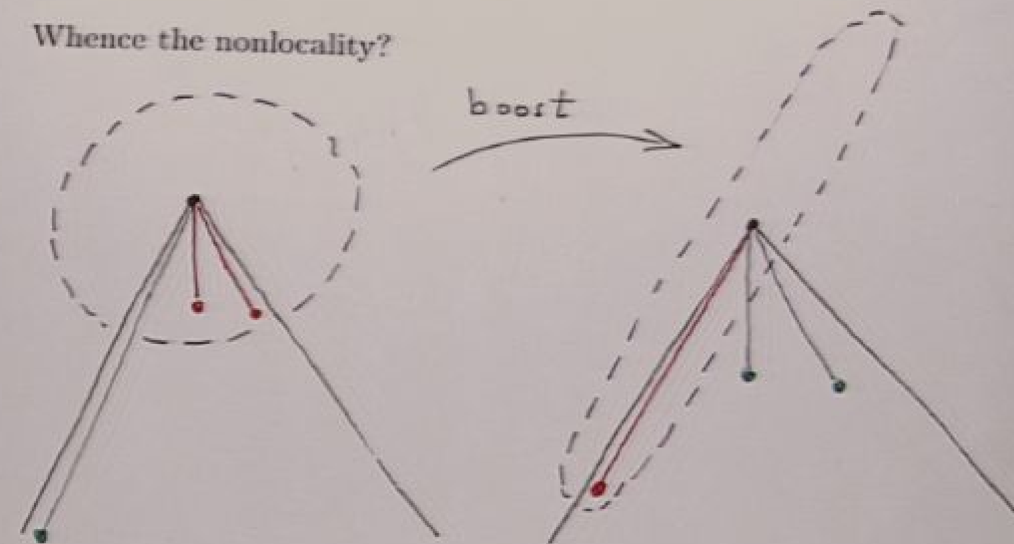
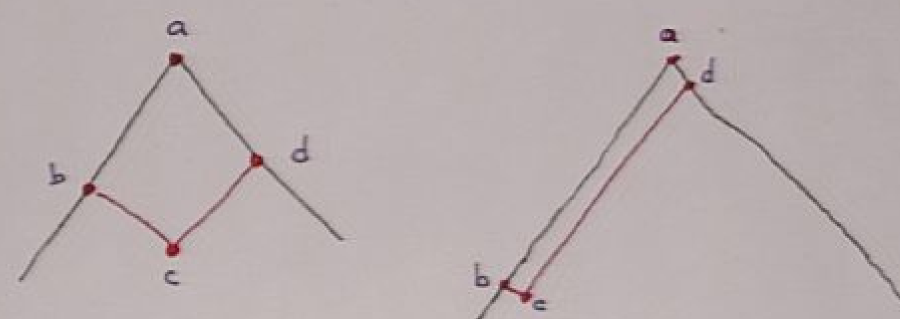
Whence the nonlocality?

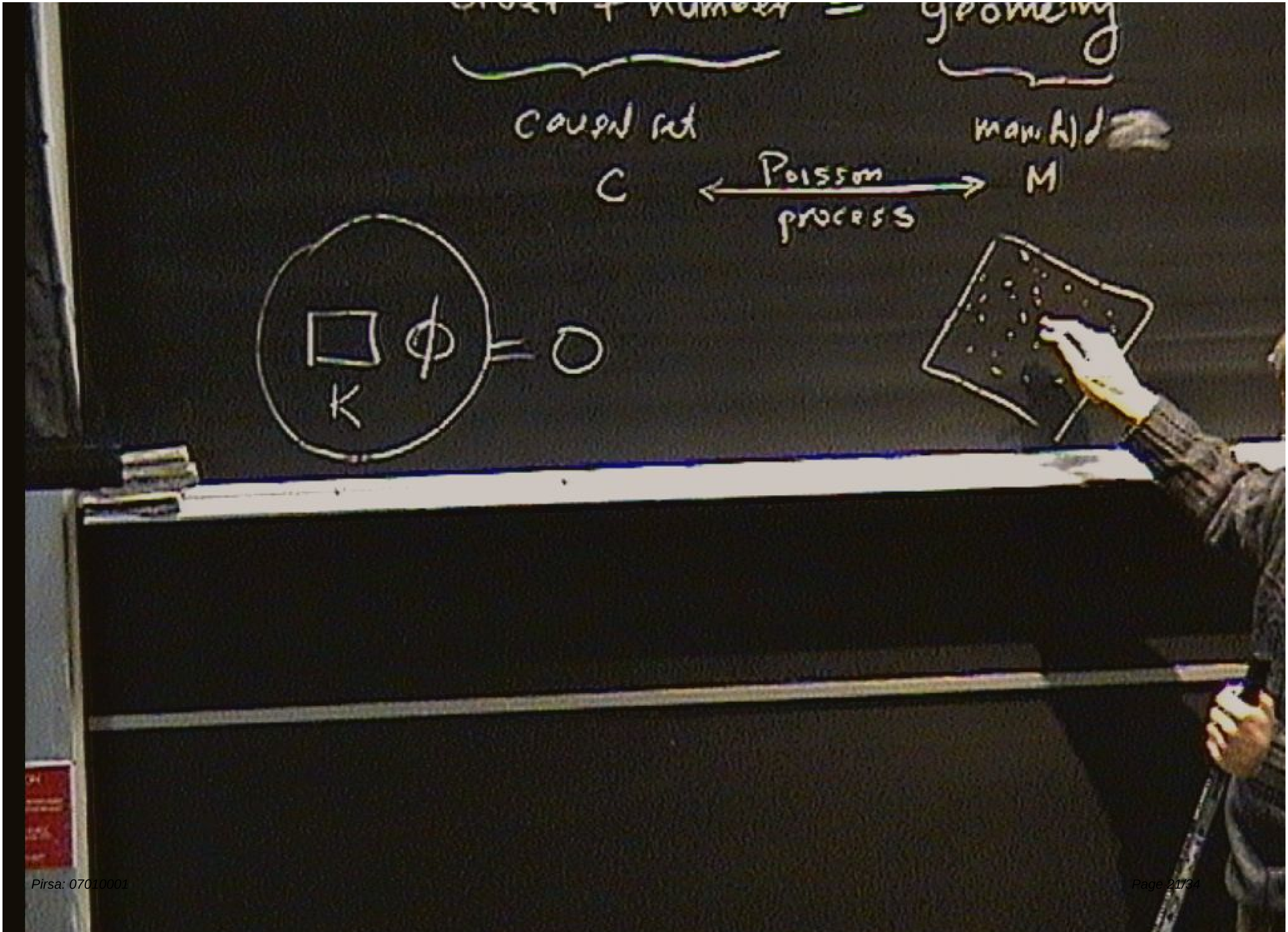


Needs a miracle. (consider eg $\phi = t^2 - x^2$, invariance $\Rightarrow \infty$?)



$(a+c) - (b+d) \longrightarrow \Box\phi$

$(a+c) - (b+d)$
$= (a-d) - (b-c) \longrightarrow 0$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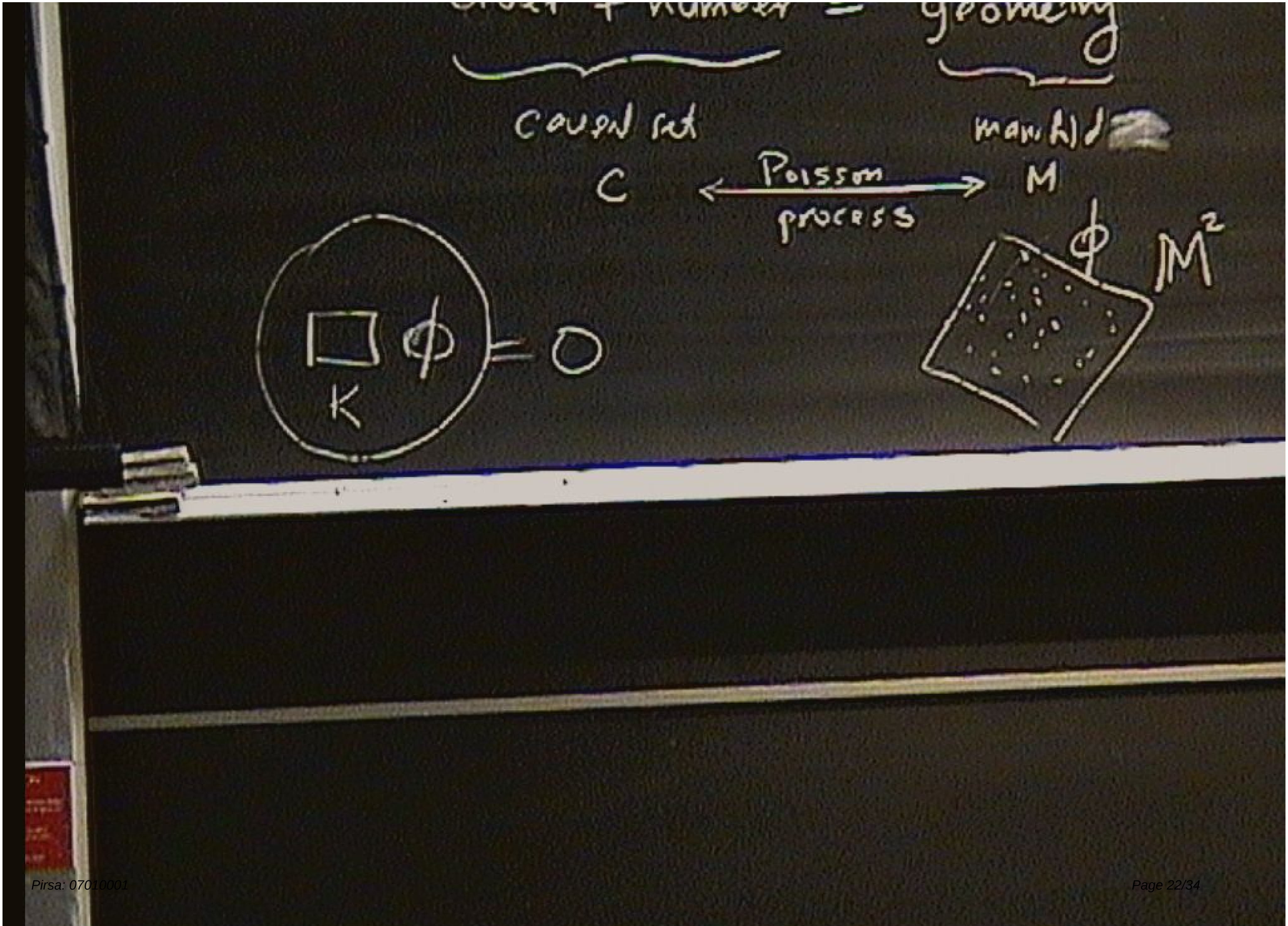
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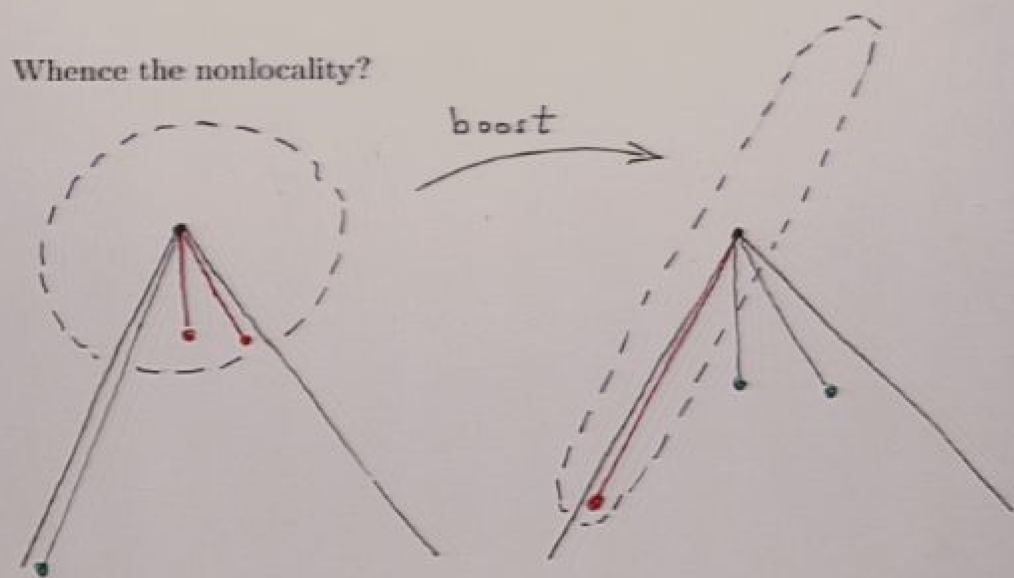

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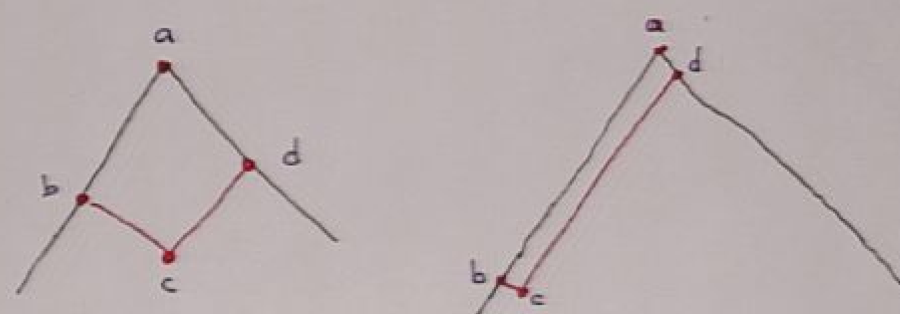

$(a+c)-(b+d) \longrightarrow \Box\phi$

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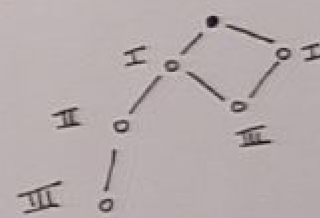
These ideas lead to expressions like

$$\frac{4}{\ell^2}\left(-\frac{1}{2}\phi(0)+\sum_{x\in I}\phi(x)-2\sum_{x\in II}\phi(x)+\sum_{x\in III}\phi(x)\right)$$

i.e.

$$\Box\,\phi(i)\leftrightarrow\sum_k B(i,k)\phi(k)$$

where



$$\frac{\ell^2}{4}B(i,k)=\begin{cases}-\frac{1}{2}\text{ if } i=k\\ 1\text{ if } i\prec k\text{ and } |\langle i,k\rangle|=0 \ \ \text{(NN)(link)}\\ -2\text{ if } i\prec k\text{ and } |\langle i,k\rangle|=1 \ \ \text{(NNN)}\\ 1\text{ if } i\prec k\text{ and } |\langle i,k\rangle|=2 \ \ \text{(NNNN)}\end{cases}$$

One can prove that, as $\ell\to 0$

$$S\equiv\mathbf{E}\sum_k B_{ik}\phi_k\ \to\ \Box\,\phi(x_i)$$

using e.g.

$$\mathbf{E}\sum_{x\in I}\phi(x)=\int\frac{du dv}{\ell^2}\exp\{-uv/\ell^2\}\ \phi(u,v)$$

Problem: $\Delta S\to\infty$ (fluctuations) as $\ell\to 0$!

$$\langle x, y \rangle = \{ z \mid x < z < y \}$$

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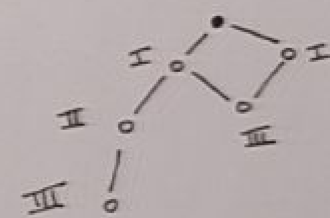

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IDEA: Our averaged sum is a *continuum* expression,

$$\int B(x-x')\,\phi(x')\,d^2x' \;,$$

where

$$B(x) = \theta(x)\left(-2K\delta(x) + 4K^2\,p(\xi)\,e^{-\xi}\right) \;,$$

with $p(\xi) = 1 - 2\xi + \frac{1}{2}\xi^2$, $\xi = Kuv$, and $K = 1/\ell^2$.

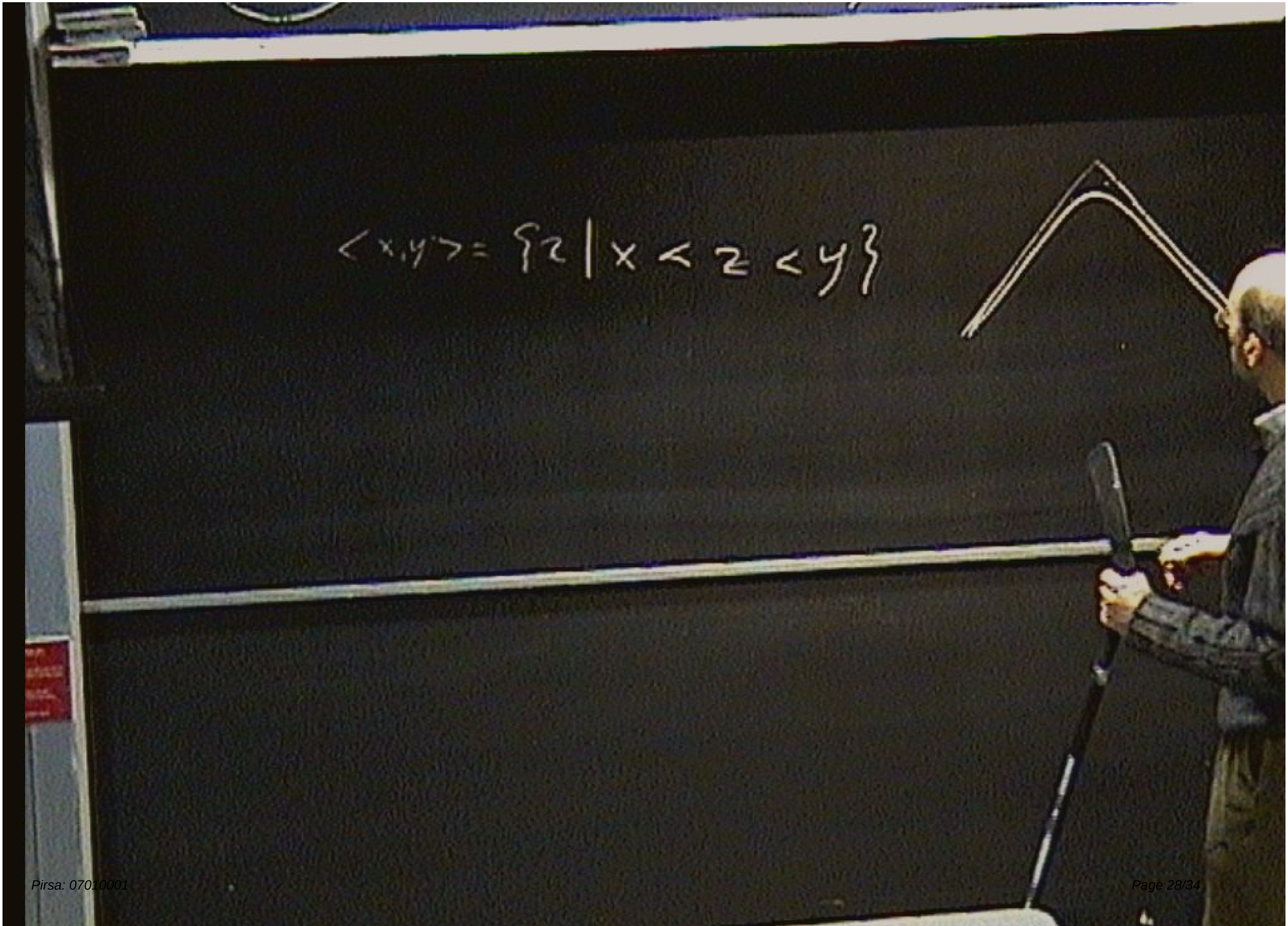
But can *decouple* K from ℓ^2. We get a nonlocal continuum analog of the D'alembertian! Call it $\underset{K}{\Box}$.

Umkehren: replace the integral by a sum over causet elements whose sprinkling-average is just $\underset{K}{\Box}$ itself! This produces the causet expression:

$$\frac{4\varepsilon}{\ell^2}\left(-\frac{1}{2}\phi(y) + \varepsilon \sum_{x \prec y} f(|\langle x,y\rangle|,\varepsilon)\;\phi(x)\right) \;,$$

where $\varepsilon = \ell^2 K$ and
$f(n,\varepsilon) \equiv (1-\varepsilon)^n - 2\varepsilon n(1-\varepsilon)^{n-1} + \frac{1}{2}\varepsilon^2 n(n-1)(1-\varepsilon)^{n-2}$.

This "trick" works. It drives down the fluctuations, but pushes the nonlocality up to the "mesoscopic" length-scale $\lambda_0 = 1/\sqrt{K}$.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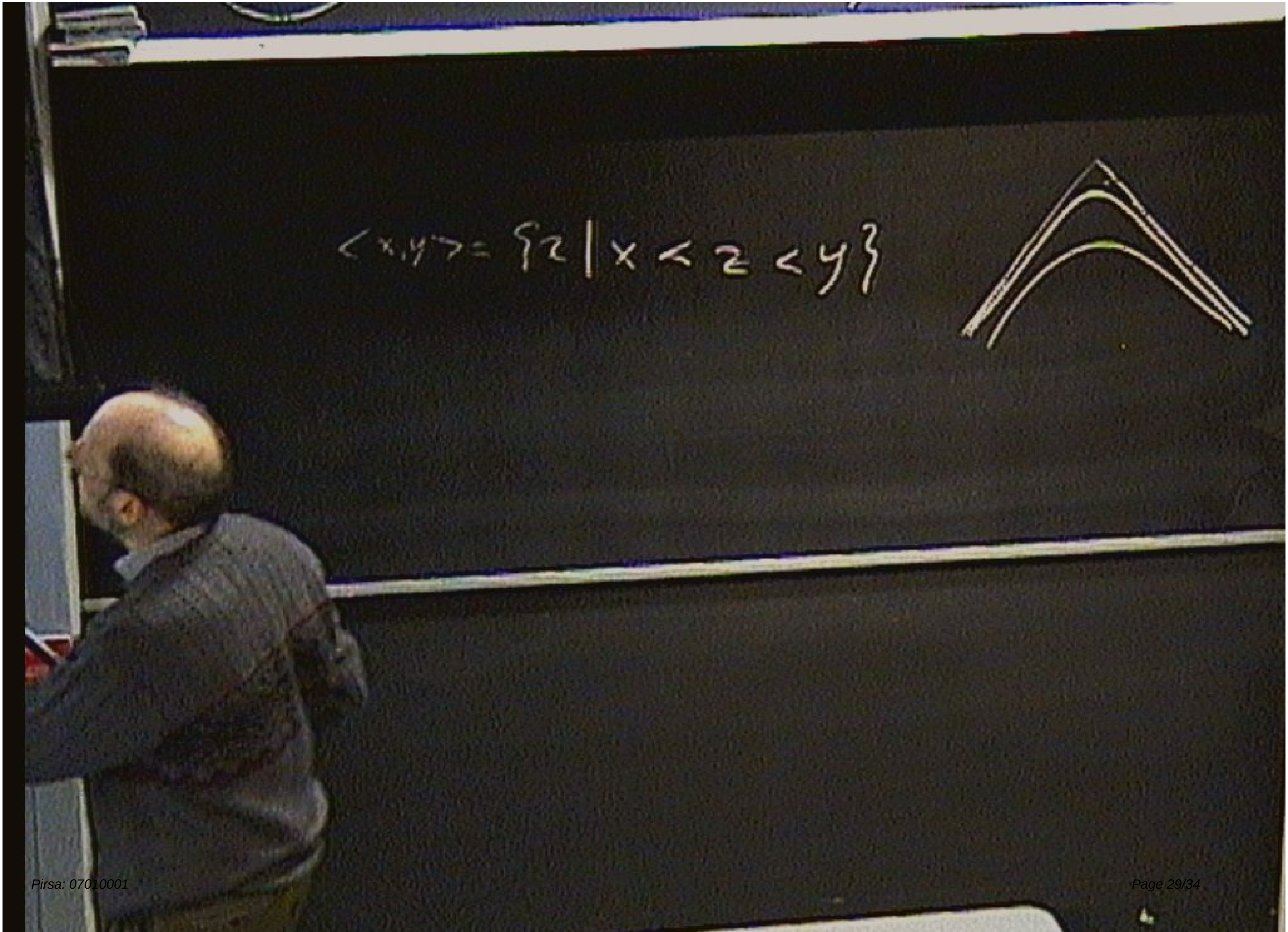
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Remarks and applications

Analogous expressions exist in other dimensions.
In $d=4$

$$p(\xi) = 1 - 3\xi + (3/2)\xi^2 - (1/6)\xi^3$$

$$\leftrightarrow \sum_{I} - 3\sum_{II} + 3\sum_{III} - \sum_{IV}$$

Can now study propagation on sprinkled causet (Rideout) cf. swerves

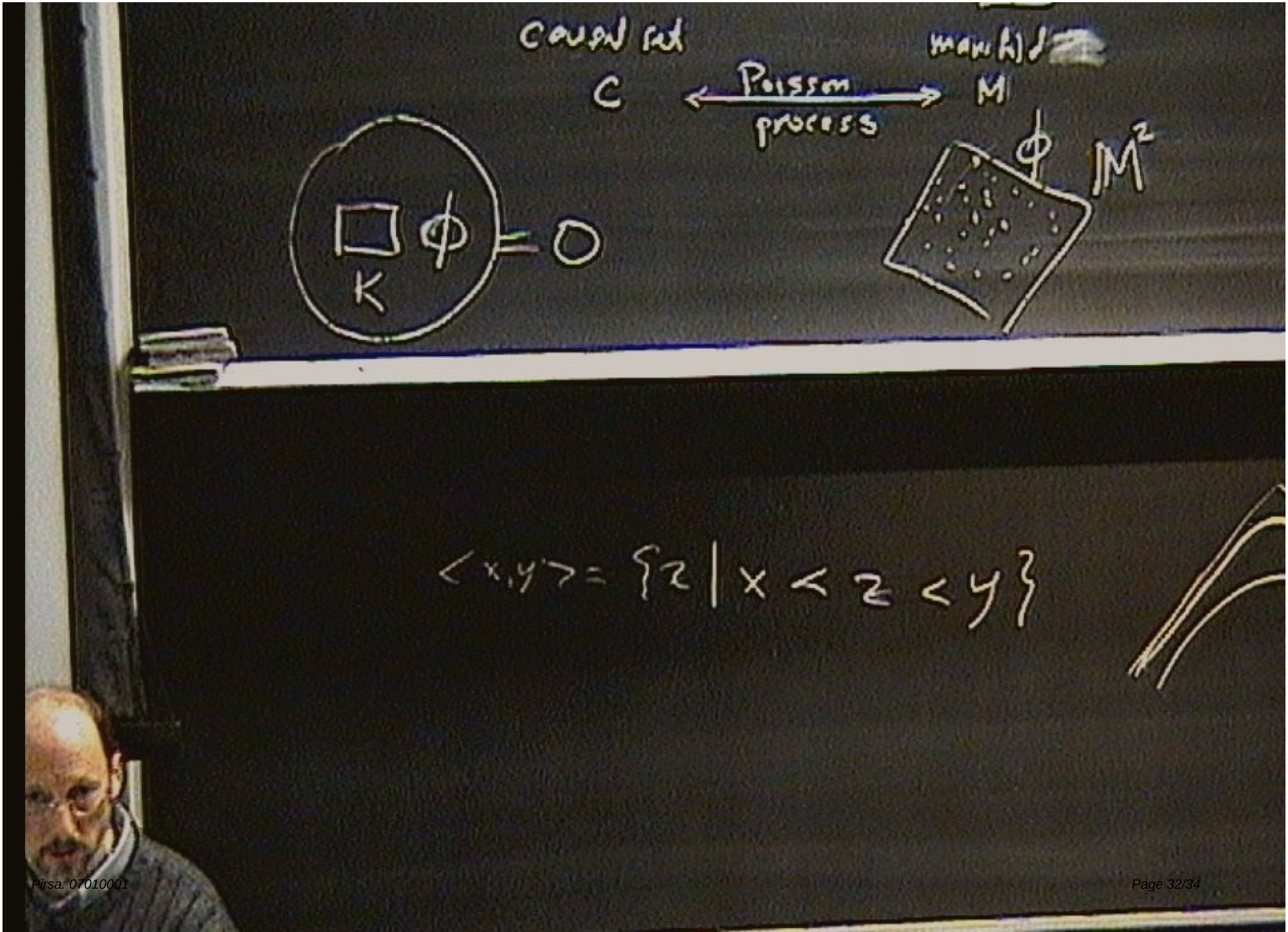
The continuum theory's free field is stable:
($\ker \Box_K = \ker \Box$)
But response to sources and curvature differs

Quantum Field Theory version? New approach to renormalization? Our nonlocality does not remove ∞'s, but perhaps it will allow an invariant (Lorentzian) cutoff.

How big is λ_0? Must balance fluctuations against nonlocality. $L = \text{Hubble}^{-1}$, $\ell =$ Planck length.

$$\lambda_0 \gtrsim (\ell^2 L)^{1/3}$$

if want $\underset{K}{\Box}$ pointwise accurate. $\Rightarrow$ nuclear size!!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