

Title: The Fall and Rise of Lattice QCD: High-Precision Numerical QCD Confronts Experiment

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Abstract: A full analysis of QCD, the fundamental theory of subnuclear structure and interactions, relies upon numerical simulations and the lattice approximation. After being stalled for almost 30 years, recent breakthroughs in lattice QCD allow us for the first time to analyze the low-energy structure of QCD nonperturbatively with few-percent precision. This talk will present a non-technical overview of the history leading up to these breakthroughs, and survey the wide array of applications that have been enabled by them. It will focus in particular on the impact of these new techniques on experiments that explore such areas as heavy-quark and Standard Model physics.

The Fall and Rise of Lattice QCD: High-Precision Lattice QCD Confronts Experiment

Peter Lepage

Cornell University

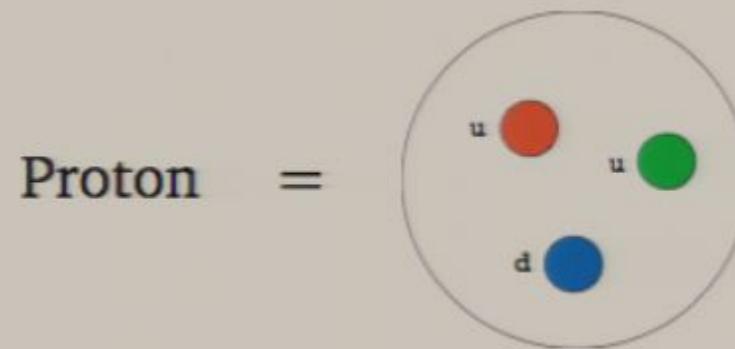
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Strong Interactions — A History

Quark Model (1960s)



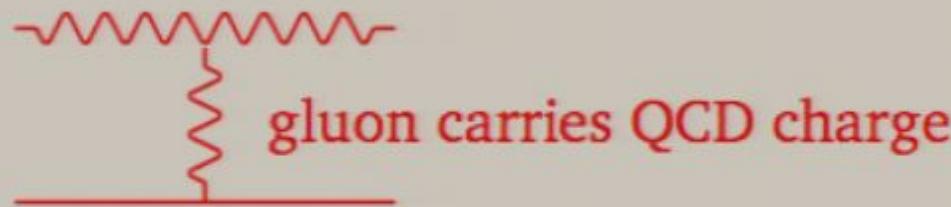
Interactions — QCD (1970s)



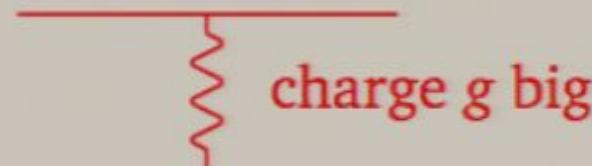
Gauge theory (like QED) \Rightarrow Complete theory!

But...

Nonlinear:

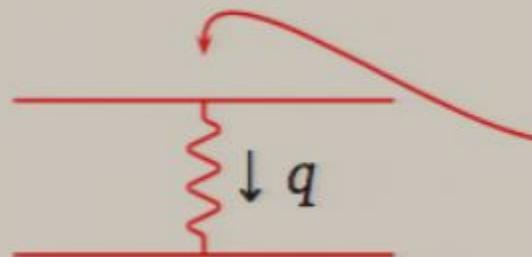


Strongly Interacting:



- ⇒ Couldn't solve QCD.
- ⇒ QCD added nothing to understanding of proton structure.
- ⇒ Theory useless?

Asymptotic Freedom (1973)



$$g_{\text{eff}} = g(q) \rightarrow 0 \text{ as } q \rightarrow \infty.$$

⇒ Solved QCD for high-energy (short-distance) processes by expanding in powers of

$$\alpha_s(q) \equiv \frac{g^2(q)}{4\pi}.$$

⇒ Detailed experimental verification of QCD at high-energy accelerators (1980s–1990s).

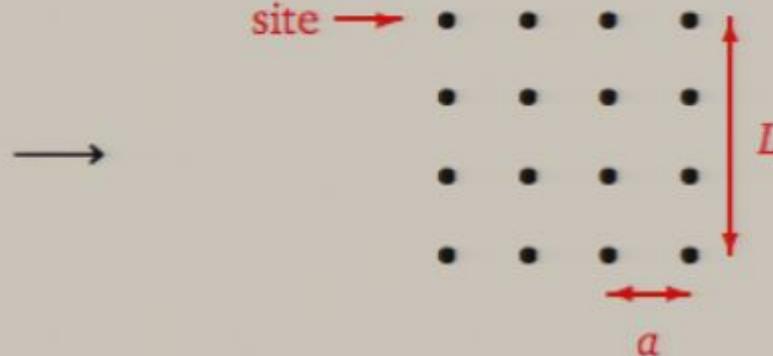
But still no insight into proton, neutron, pion... structure.

- ◊ Low-energy ($< 1 \text{ GeV}$) QCD is non-perturbative.

Lattice QCD

Lattice Approximation

Continuous
Space & Time



⇒ Fields $\psi(x)$, $A_\mu(x)$ specified only at grid sites;
interpolate for other points.

K. Wilson (1974)

⇒ QCD → multidimensional integration.

$$\int \mathcal{D}A_\mu \dots e^{-\int L dt} \rightarrow \int \prod_{x_j \in \text{grid}} dA_\mu(x_j) \dots e^{-a \sum L_j}.$$

⇒ Millions of integration variables.
⇒ Numerical Monte Carlo integration.

Fall & Rise of LQCD

- Invented in 1974; “explains” confinement.
- Stalls for almost 20 years.
 - ◊ Ken Wilson declares it dead! (1986)
- Renaissance in 1990’s.
 - ◊ Perturbation theory fixed.
 - ◊ Effective field theories for c , b quarks.
 - ◊ Improved discretizations \Rightarrow larger a .
 - ◊ First precise results; Ken Wilson retracts. (1995)
 - ◊ Unquenching! (2000)

Two QCD Breakthroughs

1) Larger a (1992)

Before \Rightarrow need $a \leq 0.05$ fm.

Now $\Rightarrow a = 0.1\text{--}0.4$ fm works.

Simulation cost $\propto (1/a)^6$

\Rightarrow new simulations cost $10^2\text{--}10^6$ times less!

2) Smaller u/d Quark Masses (2000)

Before $\Rightarrow m_{u/d}$ 10–20× too big; vac. pol'n impossible.

Now \Rightarrow 3–5× smaller masses; extrapolate to real QCD.

Vac. pol'n enters at 15–30%

\Rightarrow high-precision (few %) simulations
possible now, for first time.

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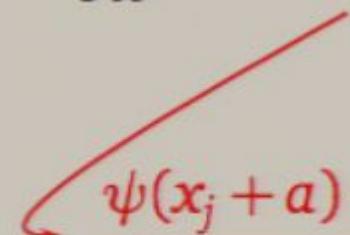
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Quantum Field Theory on a Lattice

Approximate Derivatives

Numerical Analysis \Rightarrow

$$\frac{\partial \psi(x_j)}{\partial x} = \Delta_x \psi(x_j) + \mathcal{O}(a^2)$$


$$\frac{\psi(x_j + a) - \psi(x_j - a)}{2a}$$

\Rightarrow uses only ψ 's at grid sites.

Large $a \Rightarrow$ need *improved discretizations*.

E.g.

$$\frac{\partial \psi}{\partial x} = \Delta_x \psi - \frac{a^2}{6} \Delta_x^3 \psi + \mathcal{O}(a^4)$$



10–15% for
 $a = 0.4 \text{ fm}$

1–2% for
 $a = 0.4 \text{ fm}$

$\Rightarrow a = 0.4 \text{ fm}$ okay?

But quantum numerical analysis \neq classical numerical analysis!

Ultraviolet Cutoff

$\lambda_{\min} = 2a$ is smallest wavelength.

E.g.) $\psi = \begin{array}{cccccc} +1 & -1 & +1 & -1 & +1 \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$

\Rightarrow all quark and gluon states with $p > \pi/a$ are excluded by the lattice since $p = 2\pi/\lambda$.

N.B. Lattice QCD \equiv QCD + lattice UV regulator
 \equiv real QCD.

But $\forall ps$ important in quantum field theory!
(Consider ultraviolet divergences.)

Renormalization Theory \Rightarrow mimic effects of $p > \pi/a$
excluded states by adding extra a -dependent *local* terms to the
field equations, Lagrangian, currents, operators, etc.

$$\Rightarrow \partial\psi \rightarrow \Delta\psi + c(a)a^2\Delta^3\psi + \dots$$

where

$$c(a) = -\frac{1}{6} + \text{Contribution for } p > \pi/a \text{ physics}$$

Numerical
Analysis

Theory & context specific
 \Rightarrow not universal!

Bad News: Need a^2 corrections when a large, but *Numerical Recipes* won't tell you values of $c(a)$...

Good News: $p > \pi/a$ QCD is perturbative if a small enough (asymptotic freedom).

- ⇒ compute $c(a)$... using perturbation theory.
- ⇒ Perturbation theory fills in gaps in lattice.
- ⇒ Continuum results without $a \rightarrow 0$!

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E.g.,

$$\mathcal{L}^{(\Lambda)} = Z(a) \bar{\psi} (\Delta \cdot \gamma - m(a)) \psi + c(a) a^2 \bar{\psi} \Delta^3 \cdot \gamma \psi + \dots$$

Renormalization constant.

Finite- a correction.

where

$$c(a) = -\frac{1}{6} + c_1 \alpha_s(\pi/a) + \dots$$

Numerical
Analysis

Mimics effects of $p > \pi/a$
states excluded by grid.

Lattice QCD Strategy

Asymptotic freedom in QCD \Rightarrow

- short-distance physics simple (perturbative);
- long-distance physics difficult (nonperturbative).

Lattice separates “short” from “long”:

- $p > \pi/a$ QCD \rightarrow corrections $\delta \mathcal{L}$ computed in perturbation theory (determines a);
- $p < \pi/a$ QCD \rightarrow nonperturbative, numerical Monte Carlo integration.

Perturbation Theory

Improved discretizations and larger as — old ideas.

But perturbation theory is essential.

- ⇒ a small enough so that $p \approx \pi/a$ QCD is perturbative.
- ⇒ Before 1992: $a < 0.05$ fm.
- ⇒ After 1992: $a < 0.4$ fm works.

G.P. Lepage and P.B. Mackenzie (1992).

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Test by comparing short-distance quantities from:

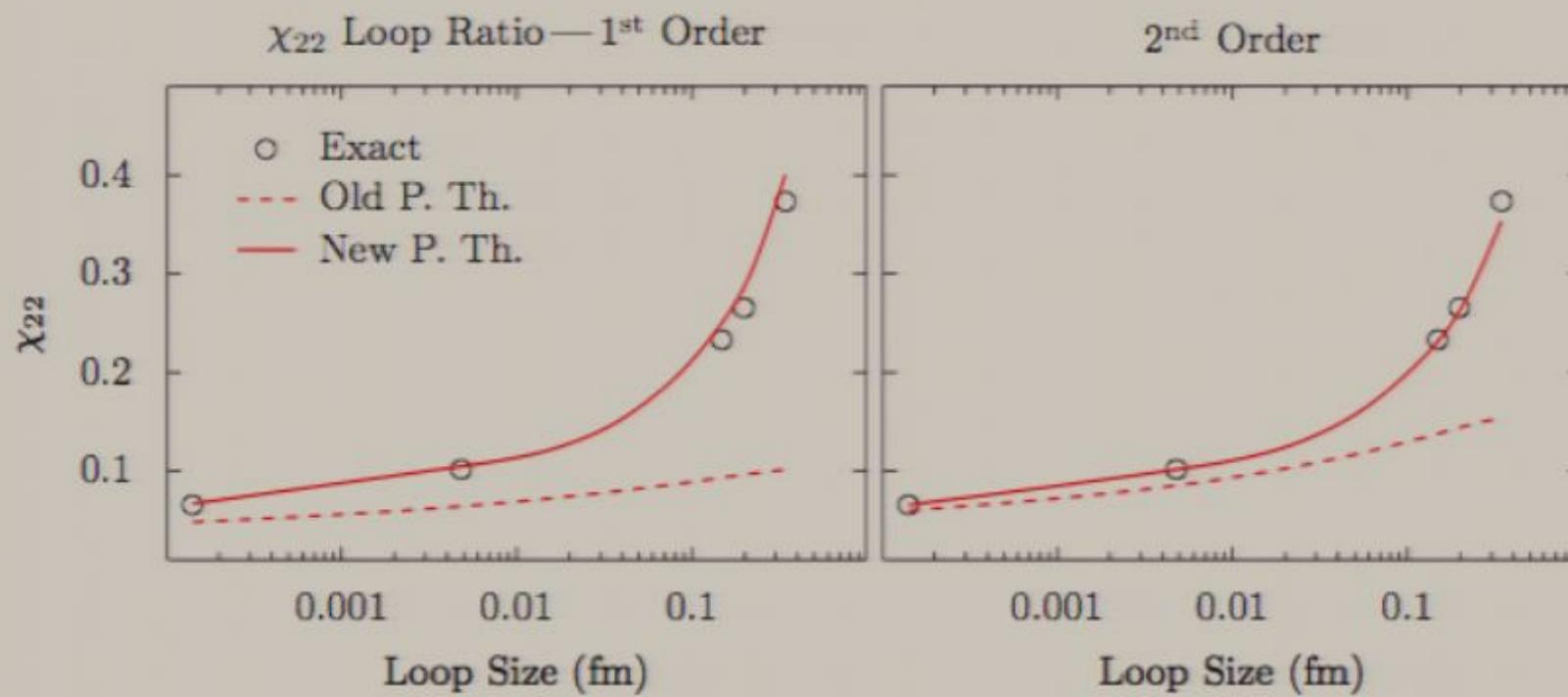
- perturbation theory;
- numerical Monte Carlo integration (\Rightarrow exact result).

E.g., Wilson loops:

$$W(\mathcal{C}) \equiv \langle 0 | \frac{1}{3} \text{Re} \text{Tr} P e^{-ig \oint_{\mathcal{C}} A \cdot dx} | 0 \rangle,$$



\mathcal{C} = small, closed path.



Lepage and Mackenzie (1992).

Does It Work?

Quarks

The standard discretization of the quark action has $\mathcal{O}(a^2)$ errors:

$$\mathcal{L}_{\text{lat}} \approx \bar{\psi} (D \cdot \gamma + m) \psi + \frac{a^2}{6} \sum_{\mu} \bar{\psi} D_{\mu}^3 \gamma^{\mu} \psi + \dots$$



$\mathcal{O}(a^2)$ error violates rotation/Poincaré invariance; removed by adding correction term.

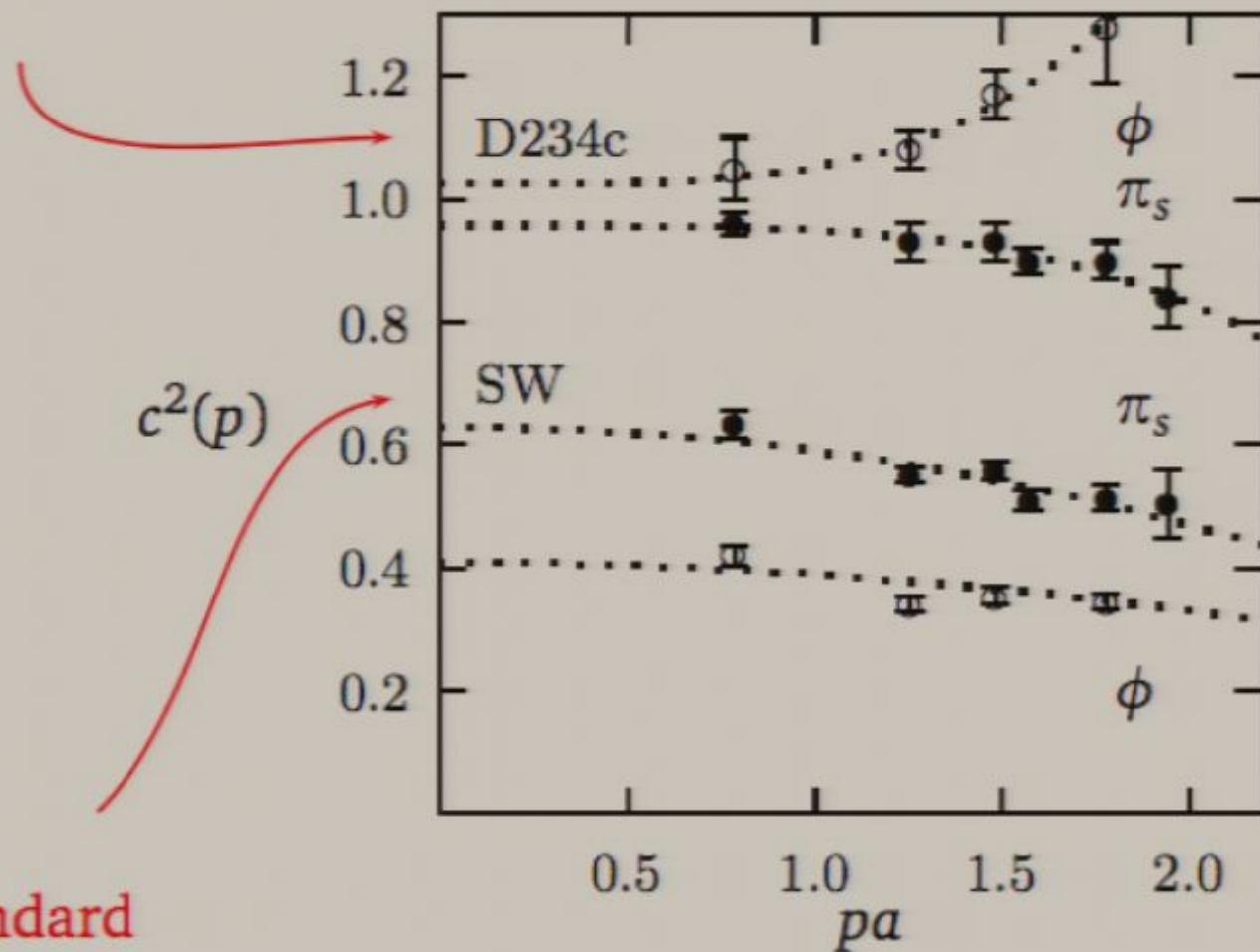
Test by computing

$$c^2(\mathbf{p}) \equiv \frac{E^2(\mathbf{p}) - m^2}{\mathbf{p}^2}.$$

Lorentz invariance implies:

$$c^2(\mathbf{p}) = 1 \quad \forall \mathbf{p}.$$

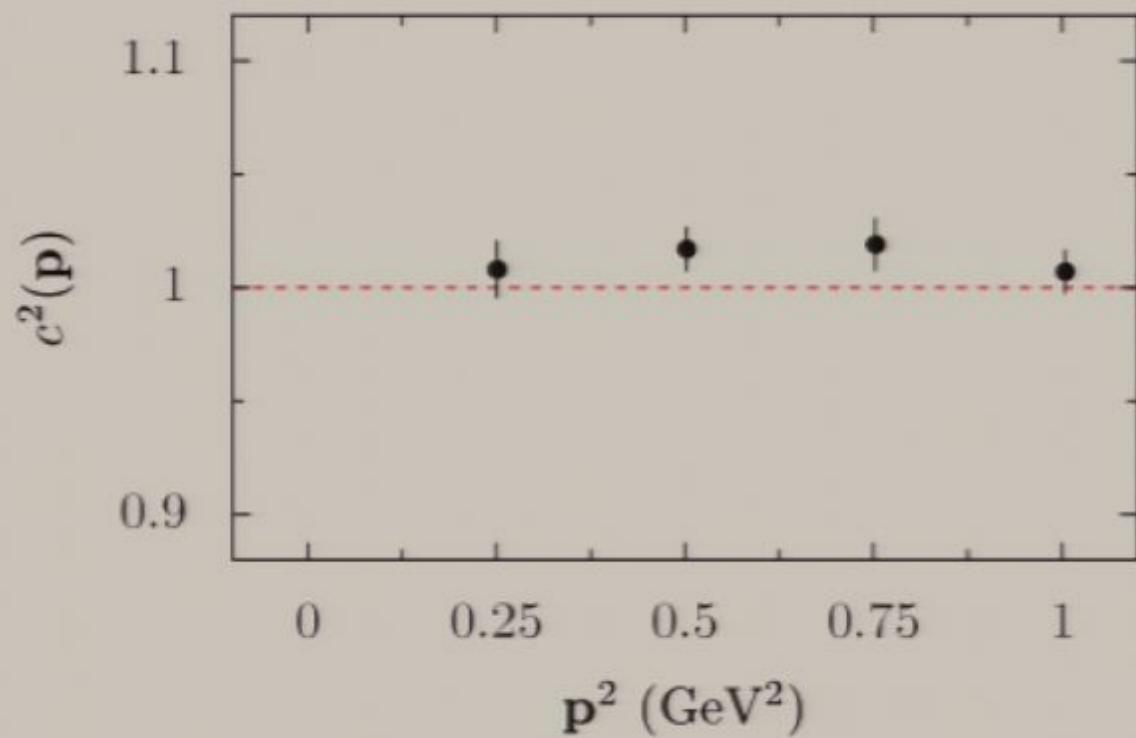
Improved



Alford et al (1997).

N.B. Much **higher standards** today.

Eg., c^2 for η_c , with $m_c = 0.67/a$, using HISQ action:



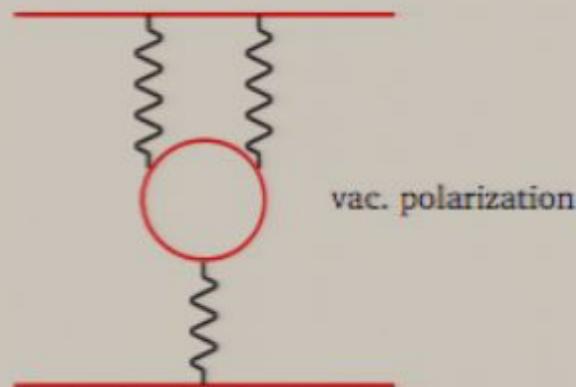
Follana et al (2007).

“Unquenching” and Quark Masses

Unquenched LQCD

“Quenched” QCD \equiv QCD without quark vacuum polarization.

- \Rightarrow 15–30% errors in most calculations;
- \Rightarrow *the major limitation of LQCD until 2000.*



Naive/staggered quarks + improved discretization

⇒ 50–1000 times faster
& smallest finite- a errors
& best behavior in chiral limit!

⇒ High-precision (few %) LQCD possible *now!*
⇒ Already have thousands of configurations (MILC):

- ◊ $n_f = 3$;
- ◊ smallest ($m_u = m_d$) ever: $m_s \dots m_s/5, m_s/7$;
- ◊ small $a s$: 1/8 fm, 1/11 fm;
- ◊ large Ls : 2.5 fm, 3.0 fm.

High-Precision Test

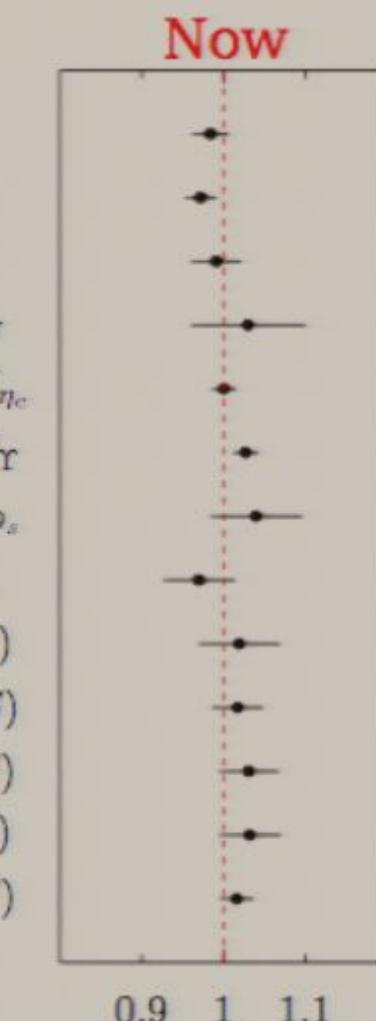
- 1) Tune 5 free parameters (bare $m_u = m_d$, m_s , m_c , m_b and α_s) using m_π , m_K , m_ψ , m_Υ , and $\Delta E_\Upsilon(1P - 1S)$.
- 2) Compute other quantities and compare with experiment.

Davies et al, Phys. Rev. Lett. 92:022001, 2004. (HPQCD, MILC, Fermilab, UKQCD)

Lattice QCD/Experiment (no free parameters!):



LQCD/Exp't ($n_f = 0$)



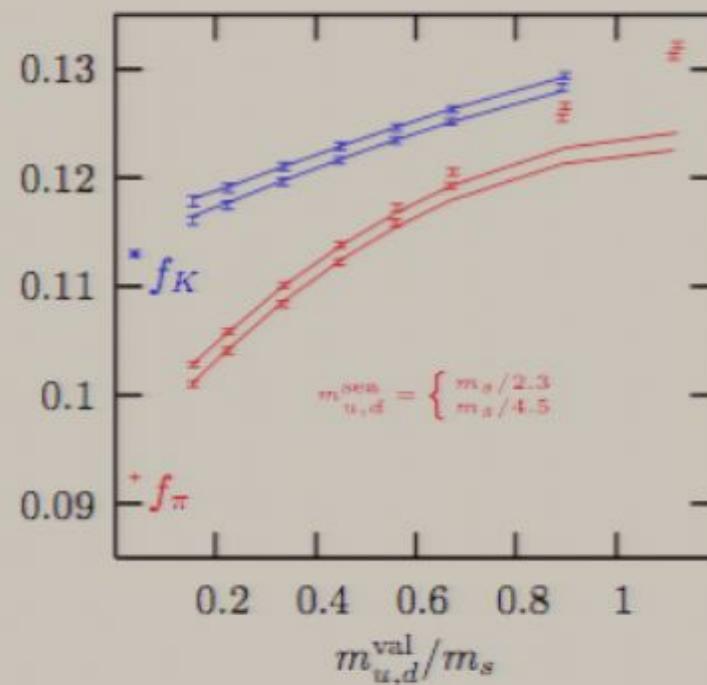
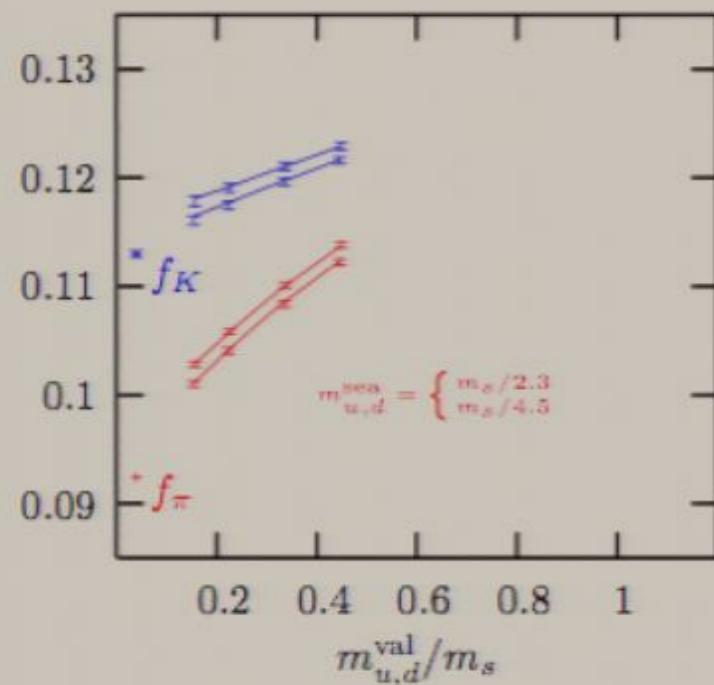
LQCD/Exp't ($n_f = 3$)

Tests:

- $m_{u,d}$ extrapolation;
 - masses and wavefunctions;
 - s quark;
 - light-quark baryons;
 - light-heavy mesons;
 - heavy quarks (no potential model...);
 - improved staggered quark vacuum polarization.
- ⇒ Most accurate strong interaction calculation in history!

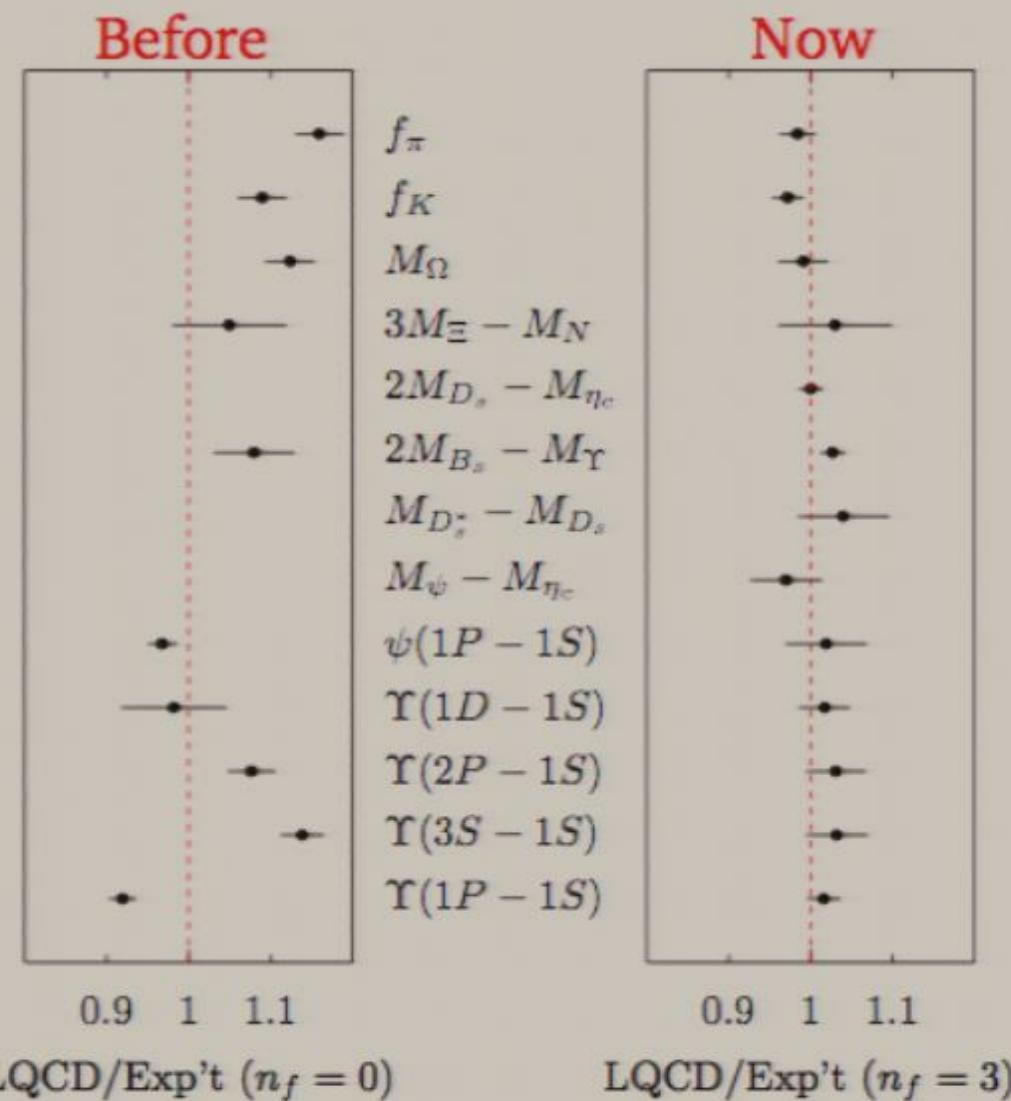
Example Analysis:

f_π and f_K fits versus valence u,d mass:



N.B. Quark mass problem solved!

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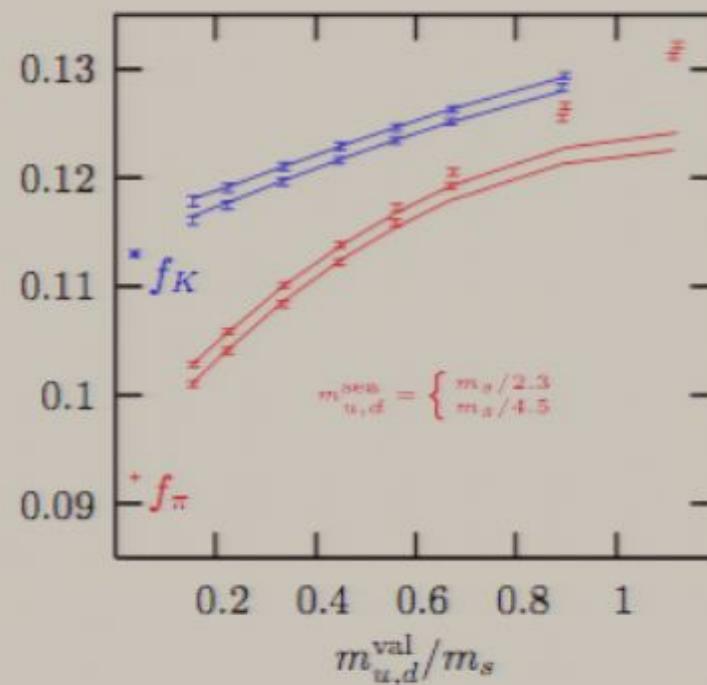
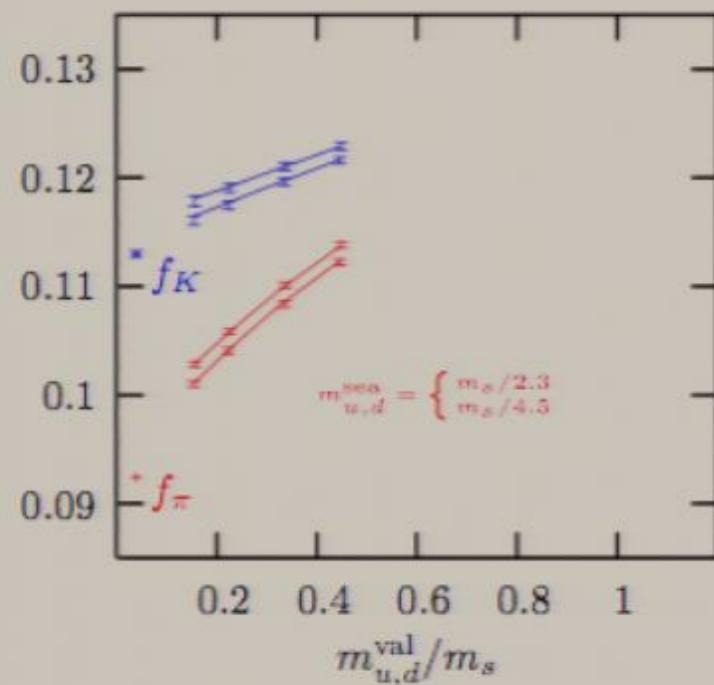


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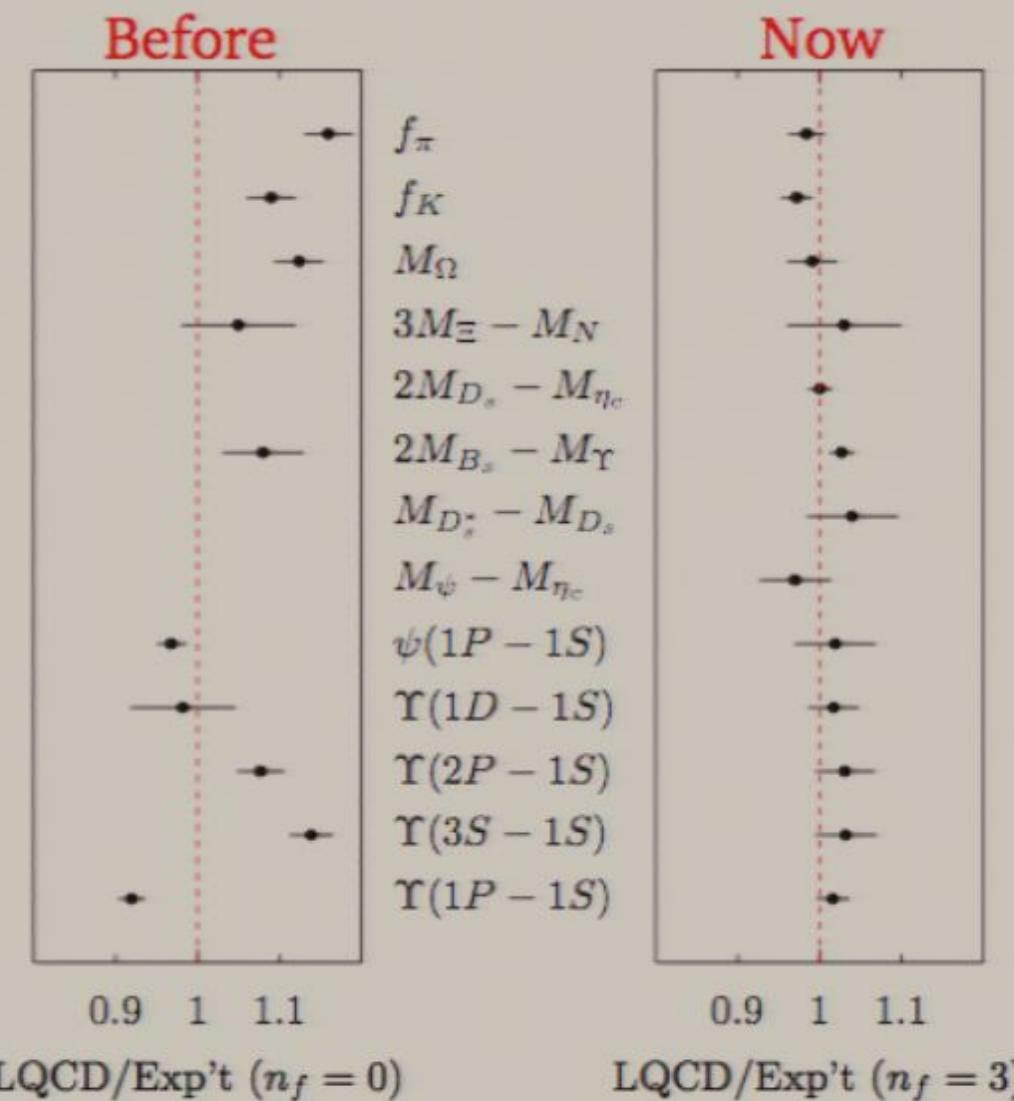
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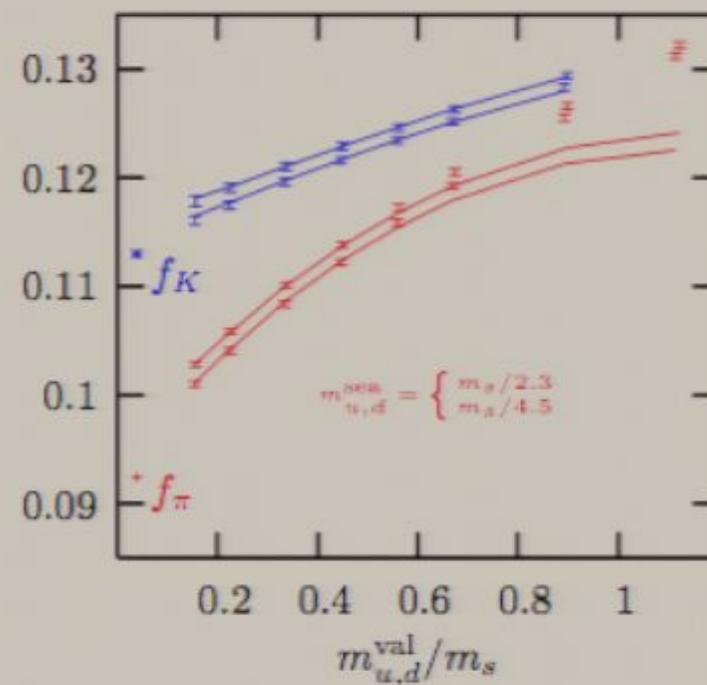
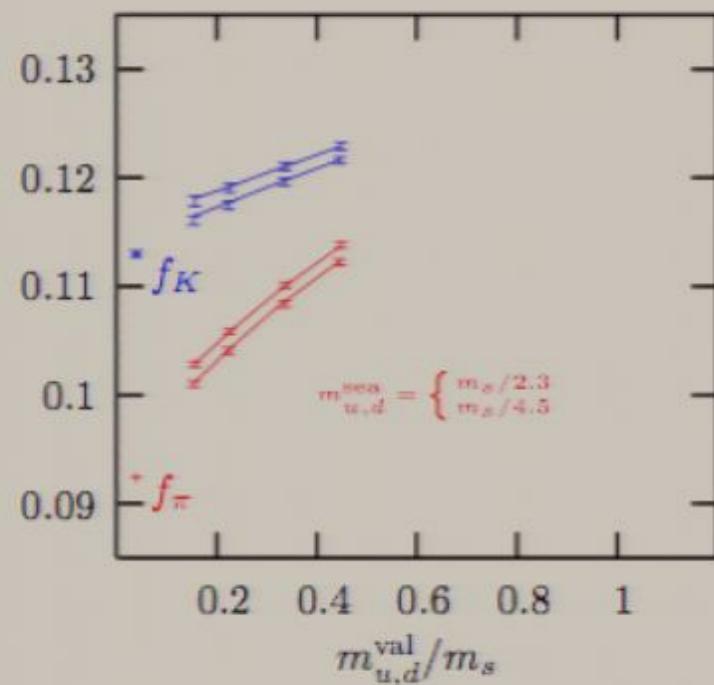


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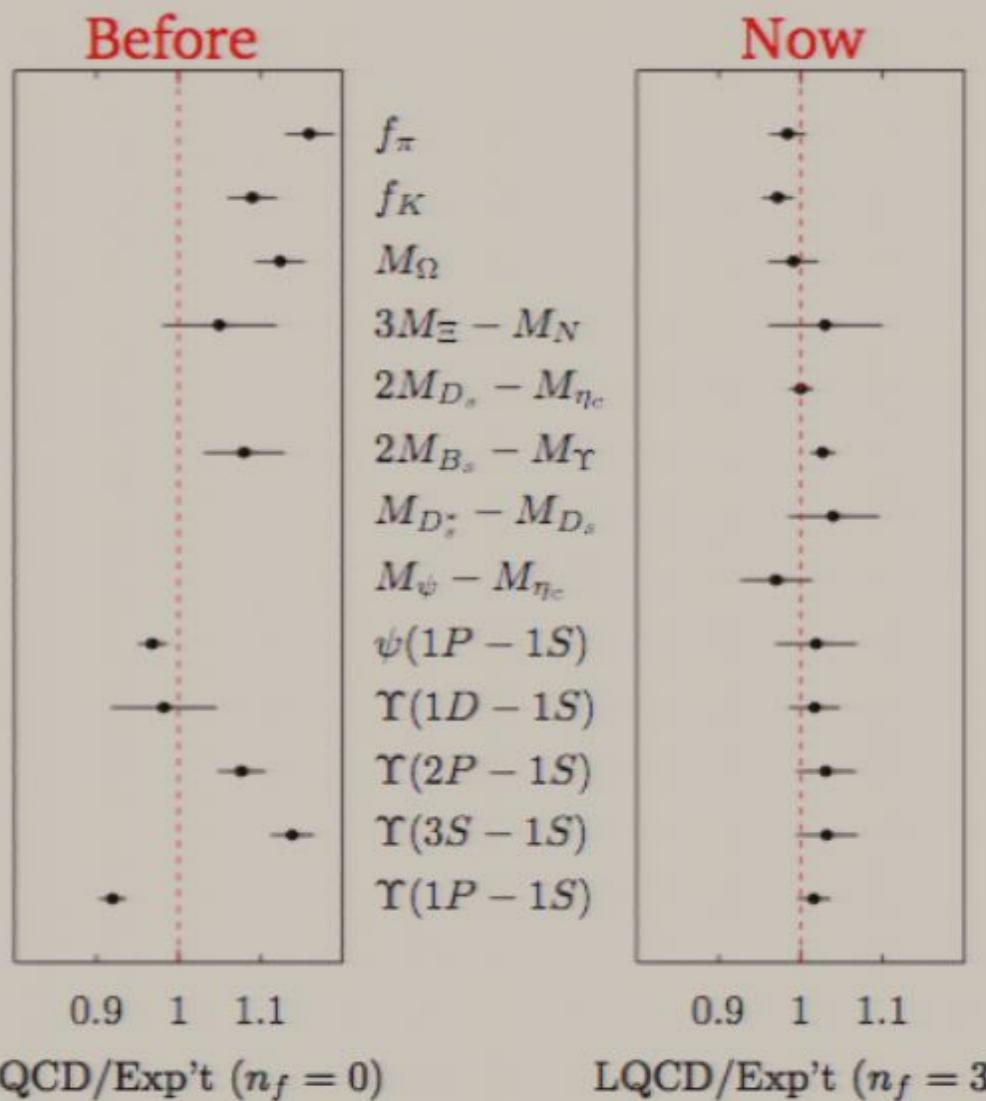
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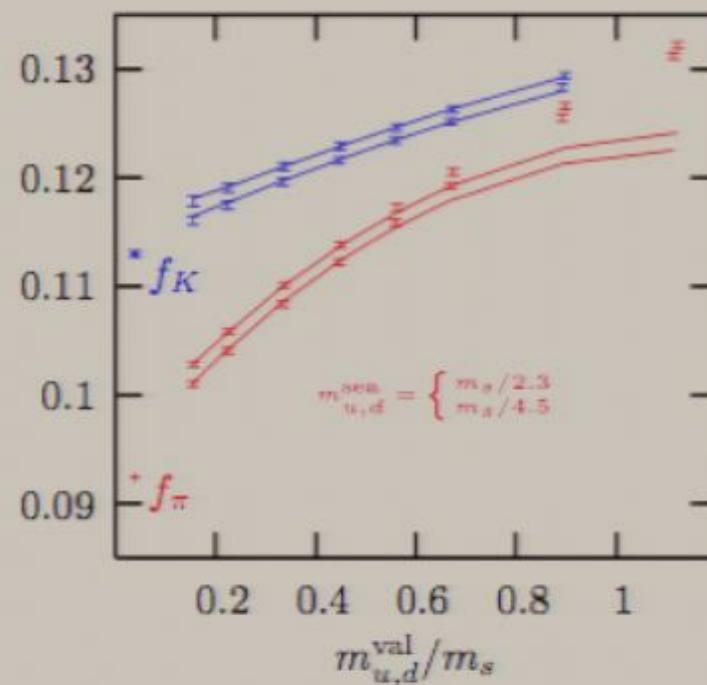
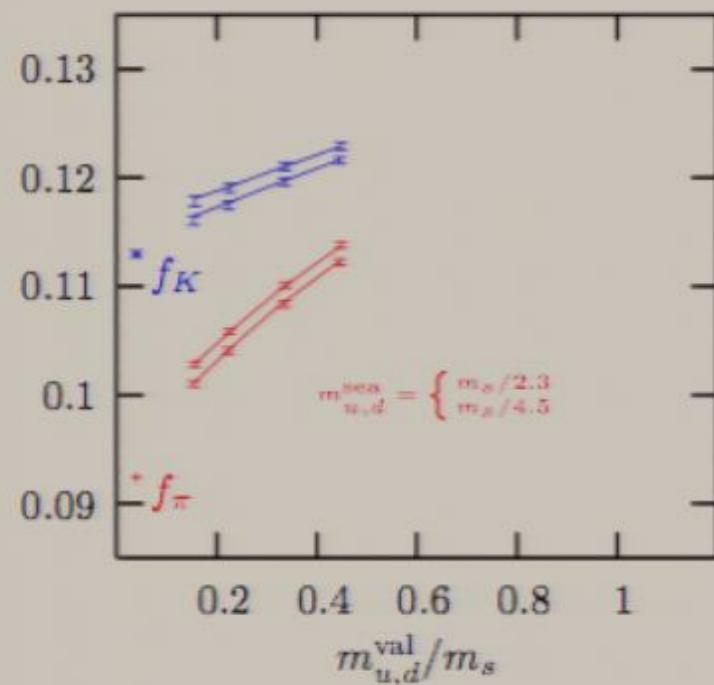
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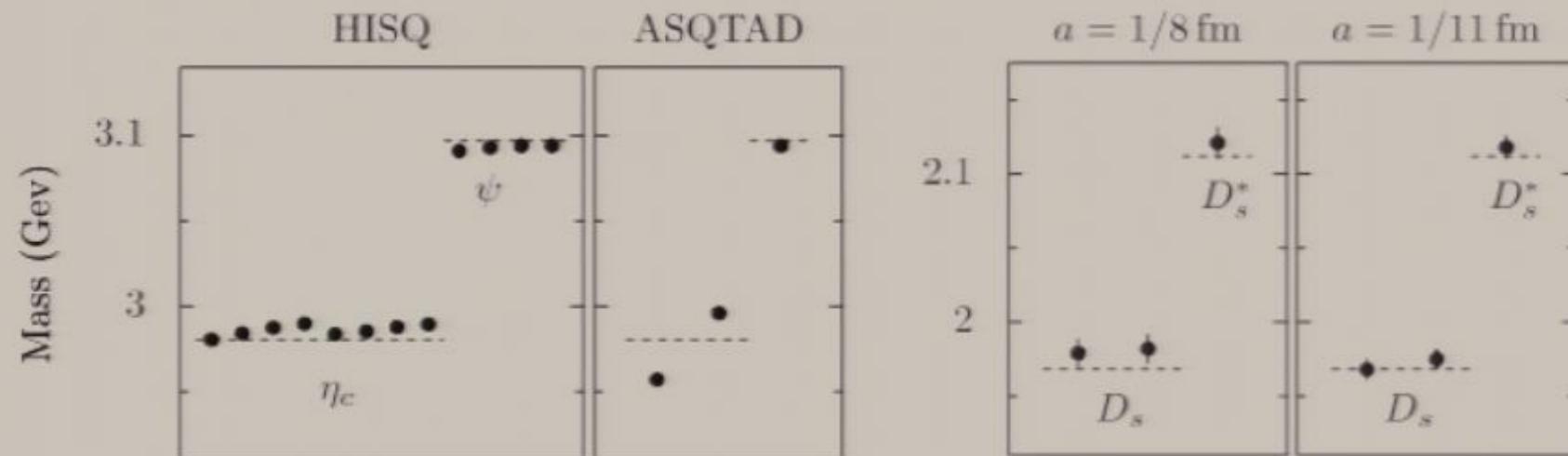
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Example Analysis:

Hyperfine mass splittings for mesons with charmed quarks using HISQ action:



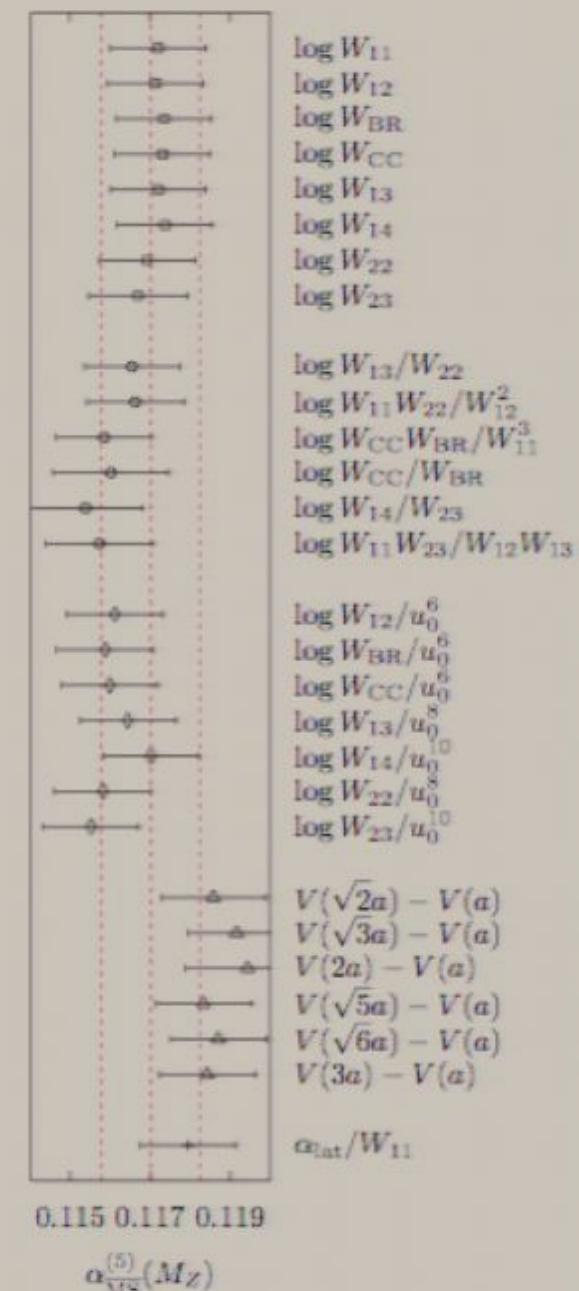
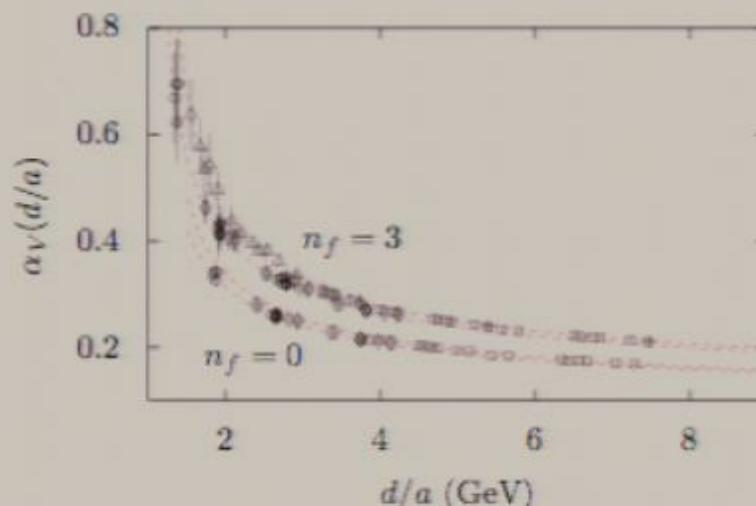
N.B. Few MeV precision with no free parameters!

Follana et al (2007).

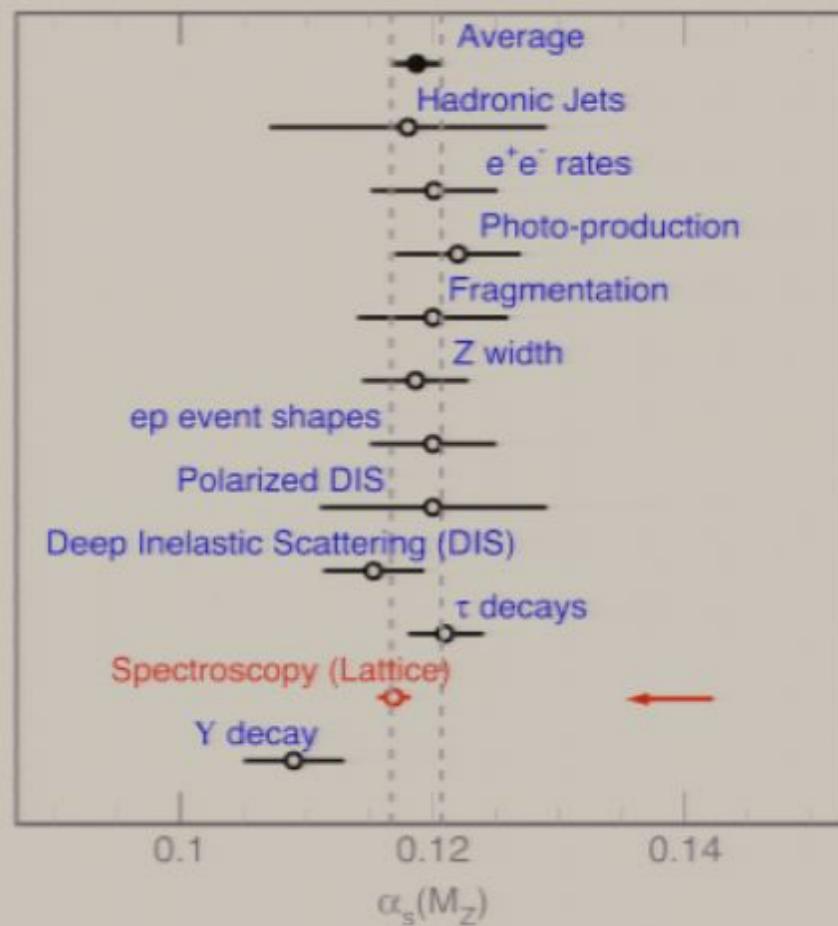
QCD Coupling

- Tuned LQCD simulation \equiv real QCD.
- “Measure” 28 short-distance quantities $Y^{(i)}$ in simulation (nonperturbatively).
- Extract coupling α_s by comparing with perturbative expansions:

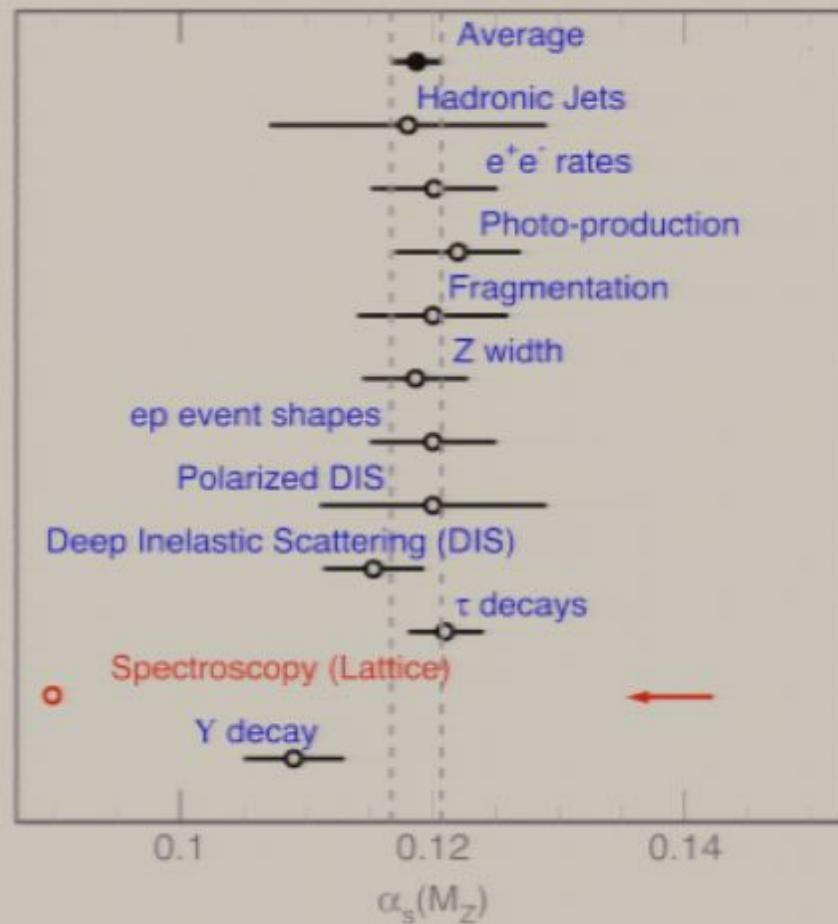
$$Y^{(i)} = \sum_{n=1}^{\infty} c_n^{(i)} \alpha_s^n (d^{(i)}/a)$$



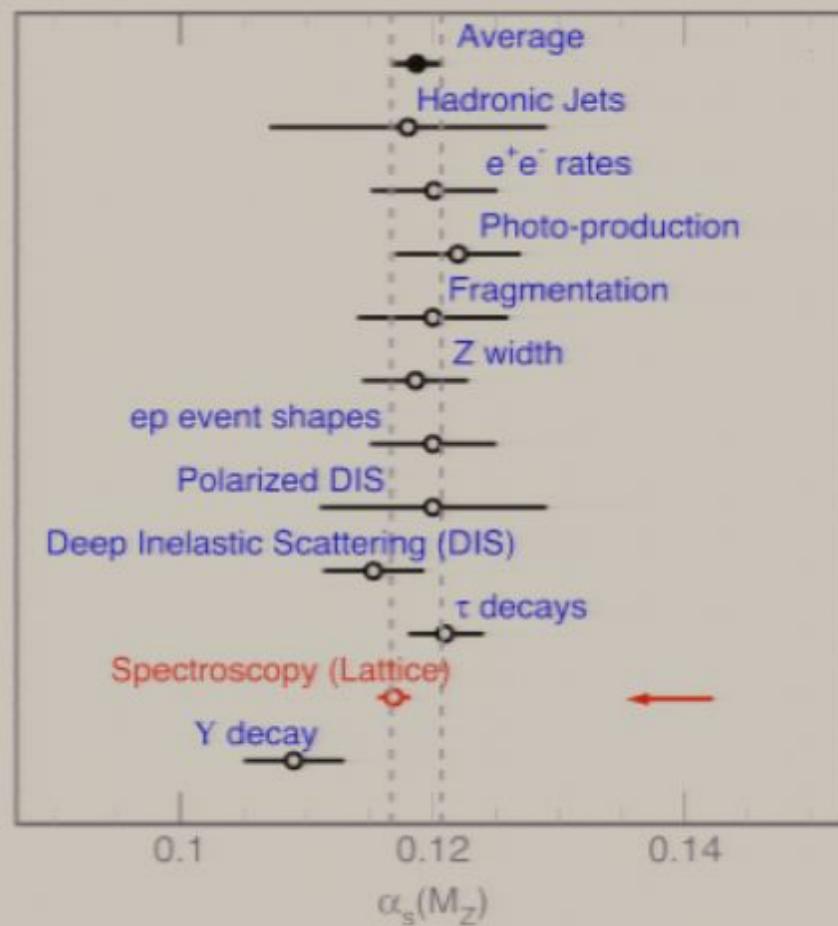
Context:



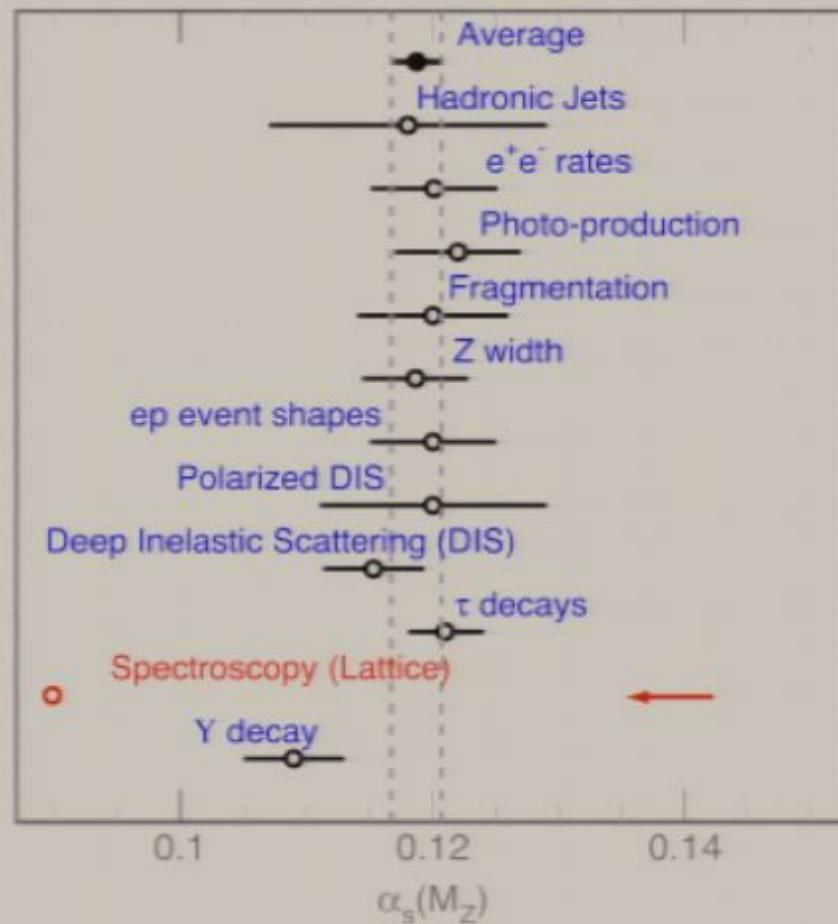
And without light-quark vacuum polarization:



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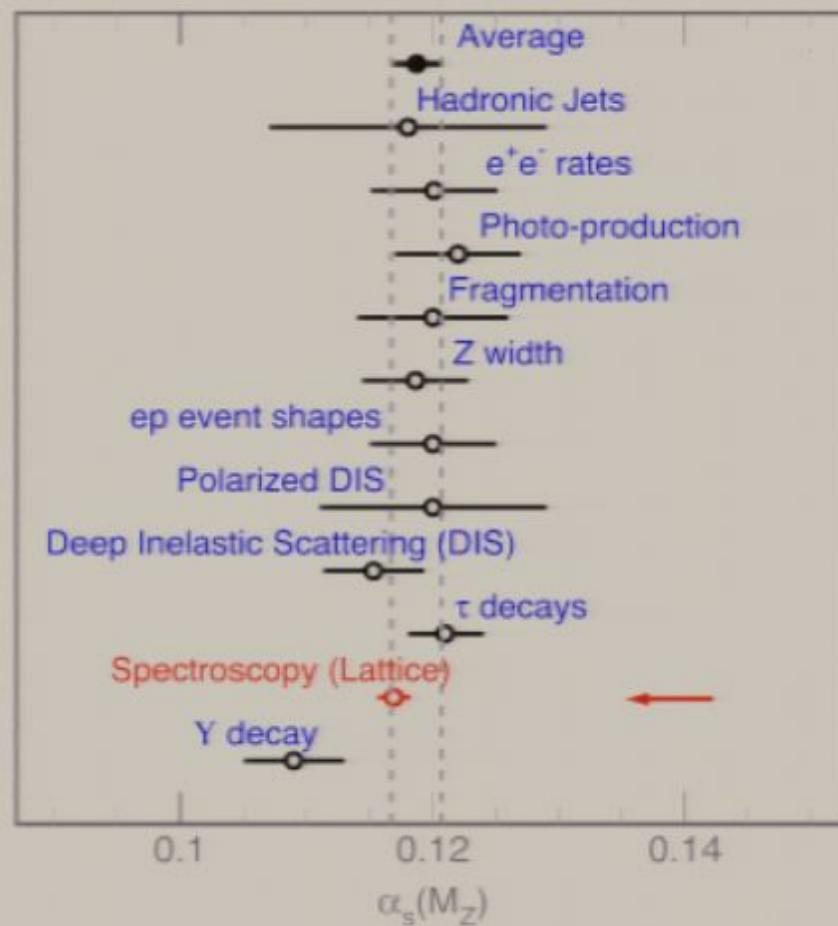


And without light-quark vacuum polarization:



The Future

Context:



High-Precision Now

Few % accuracy for dozens of “gold-plated” calculations:

- Masses, form factors, decay constants, mixing amplitudes for π, K, p, n (but **not $\rho, \phi, \Delta\dots$**).
- Masses, decay constants, semileptonic form factors, and mixing for D, D_s, B, B_s (but **not $D^*\dots$**).
- Masses, leptonic widths, electromagnetic form factors, and mixing for any meson in ψ and Υ families well below D/B threshold.

High-precision \Rightarrow masses and amplitudes with at most one hadron in the initial and/or final state, for stable or nearly stable hadrons.

Physics Focus

1) Heavy quark physics.

- Major experimental program to measure weak-interaction decays of c and b quarks to few % (BaBar, Belle, CLEO-c).
⇒ Standard Model pushed to point of failure (supersymmetry, extra dimensions...?).
⇒ Lattice QCD essential (for high-precision):

$$\text{quark decay} = \text{weak-interaction} \times \text{QCD}.$$

- Lattice QCD *predictions* confirmed: $m(B_c), f_D, f_{B_s}, \dots$

- Gold-plated quantities for almost every CKM matrix elements (and K - \bar{K} mixing):

$$\left(\begin{array}{ccc} V_{ud} & V_{us} & V_{ub} \\ \pi \rightarrow l\nu & K \rightarrow l\nu & B \rightarrow \pi l\nu \\ & K \rightarrow \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \rightarrow l\nu & D_s \rightarrow l\nu & B \rightarrow D l\nu \\ D \rightarrow \pi l\nu & D \rightarrow K l\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle & \end{array} \right)$$

- Extensive cross-checks for error calibration: $\Upsilon, B, \psi, D, \dots$

2) Hadronic spectrum, structure...

- Major experimental programs at DESY, JLab ...
 - ⇒ Structure functions, form factors ...
 - ⇒ Low-energy nuclear physics from scattering + χ PTh.
 - ⇒ Exotic/hybrid mesons.

3) QCD at finite temperature and density. (RHIC)

4) Strong coupling beyond QCD. (LHC?, ILC?)

- 2 of 3 known interactions strongly coupled (QCD, gravity).
- Generic at low energies in non-abelian gauge theories ...
- ... unless gauge symmetry spontaneously broken (\Rightarrow strong coupling!).

Conclusion

Few percent precision \Rightarrow superb opportunity for lattice QCD to have an impact on particle physics.

- LQCD essential to high-precision B/D physics at BaBar, Belle, CLEO-c, Fermilab...
- *Predicting* CLEO-c, BaBar/Belle results \Rightarrow much needed credibility for LQCD.
- Critical to focus on gold-plated quantities; **low-mass $u/d/s$ vacuum polarization essential for high-precision**.
- Landmark in history quantum field theory: quantitative verification of nonperturbative technology (c.f., 1950s).
- Ready for beyond the Standard Model, strong coupling beyond QCD?

Problems that remain: for example,

- Hadronization of quark and gluon jets.
 - Thermodynamics of quark matter.
 - Axial gauge symmetry, supersymmetry....
 - ...
- ⇒ Non-Monte Carlo techniques (e.g., strong-coupling expansions), domain-wall fermions...?

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