Title: Quantum Computing with Delocalised Qubits

Date: Dec 08, 2006 10:20 AM

URL: http://pirsa.org/06120041

Abstract: TBA

Quantum Computing with Delocalised Qubits

Joe Fitzsimons

Department of Materials
Oxford University

Outline

- Part 1: Quantum State Transfer
  - Introduction
  - State Transfer
- Part 2: Quantum Computation
  - Single Qubit Gates
  - Two Qubit Gates
- Part 3: Optimization
  - Decreasing Overhead
  - Dense Packing
- Part 4: Fault Tolerance
- Conclusion & Acknowledgements

Outline

- Part 1: Quantum State Transfer
  - Introduction
  - State Transfer
- Part 2: Quantum Computation
  - Single Qubit Gates
  - Two Qubit Gates
- Part 3: Optimization
  - Decreasing Overhead
  - Dense Packing
- Part 4: Fault Tolerance
- Conclusion & Acknowledgements

Part 1: Quantum State Transfer

Introduction

Goal:

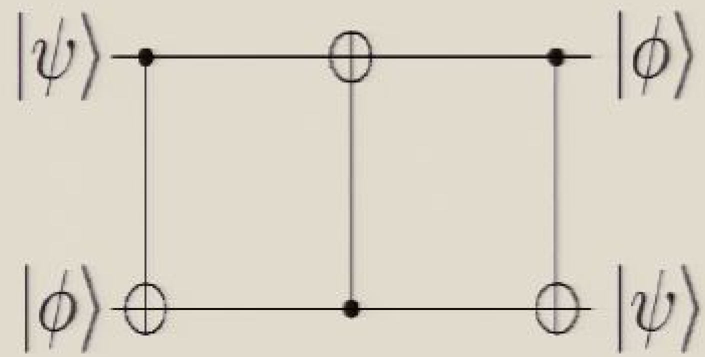
- Perfect quantum state transfer in a spin chain
- Only global addressing of intermediate qubits
- Fault-tolerance

Motivation:

- Spin chains can already be realised in many systems
- Using global control overcomes many addressability concerns
- Spins at ends of chain have a different energy level structure since they have only one neighbour.

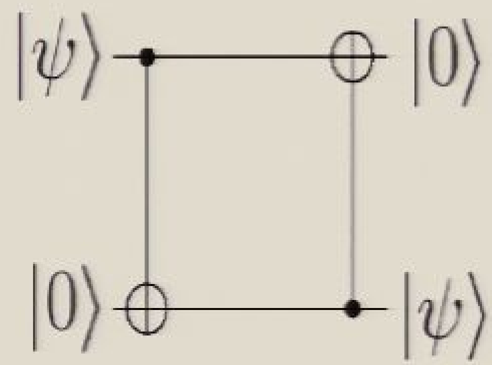
State Transfer

The Swap Circuit:



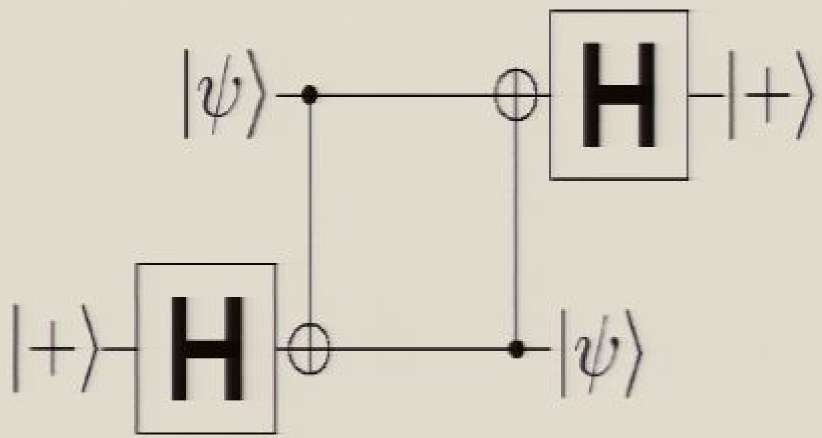
State Transfer

The Swap Circuit:



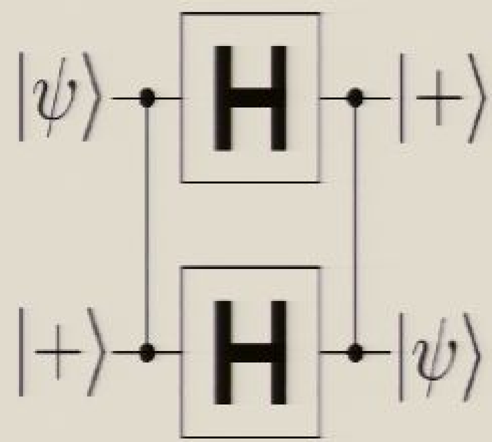
State Transfer

The Swap Circuit:



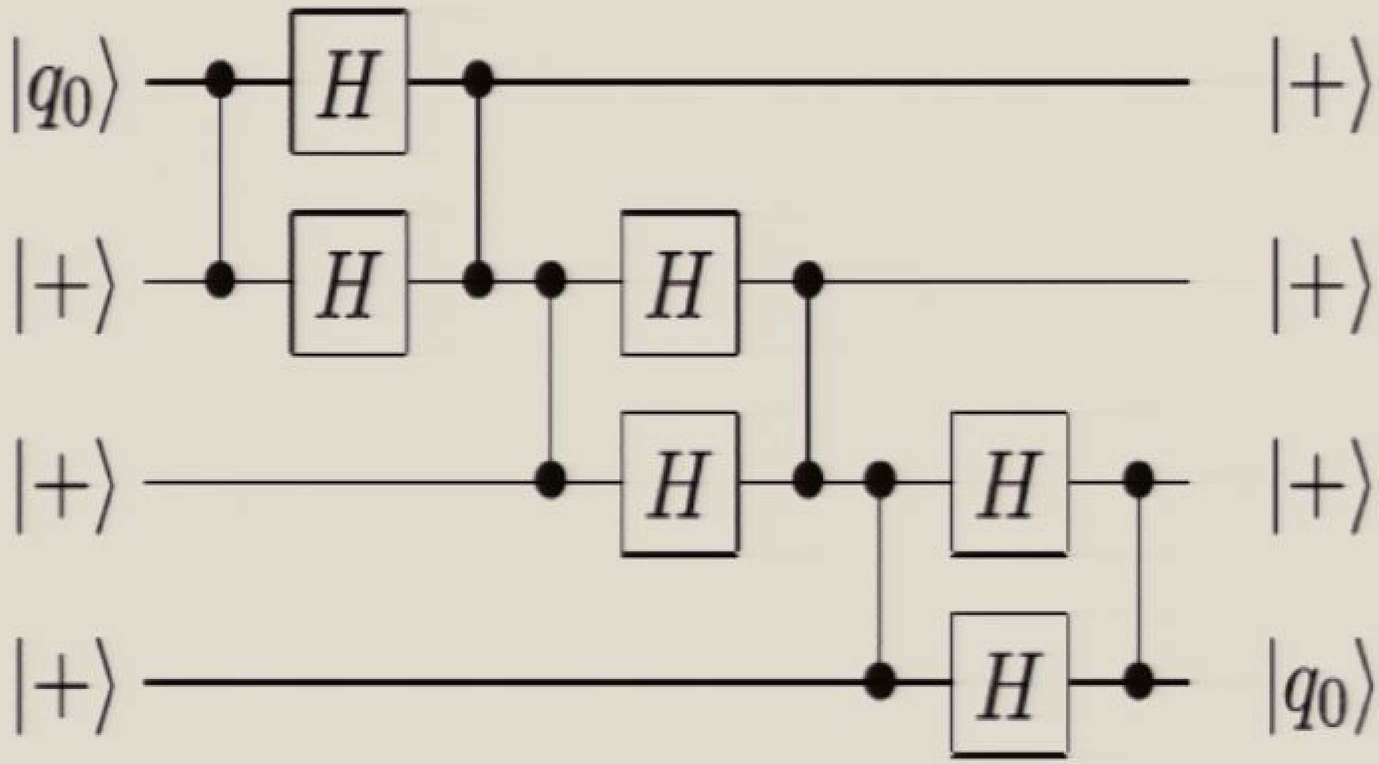
State Transfer

The Swap Circuit:



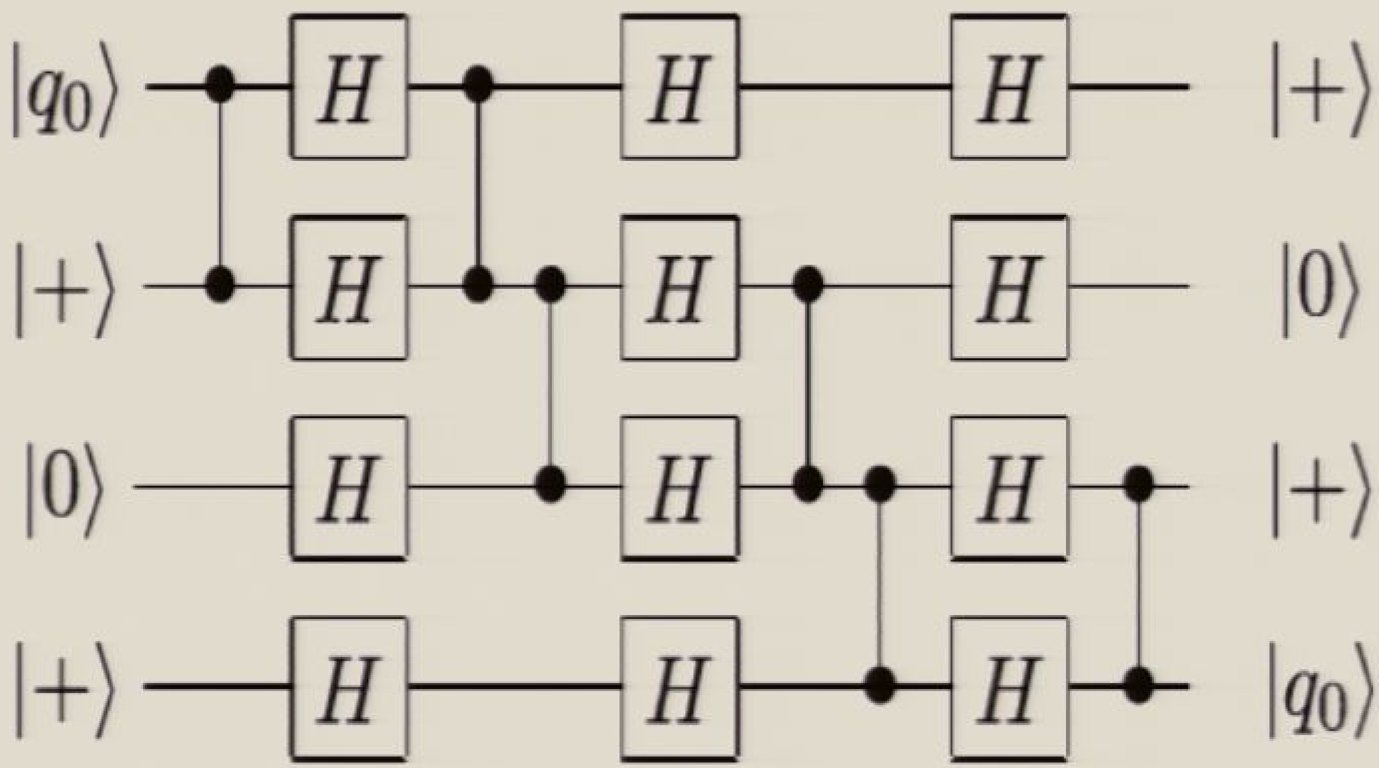
State Transfer

The Transfer Circuit: Concatenate Swaps



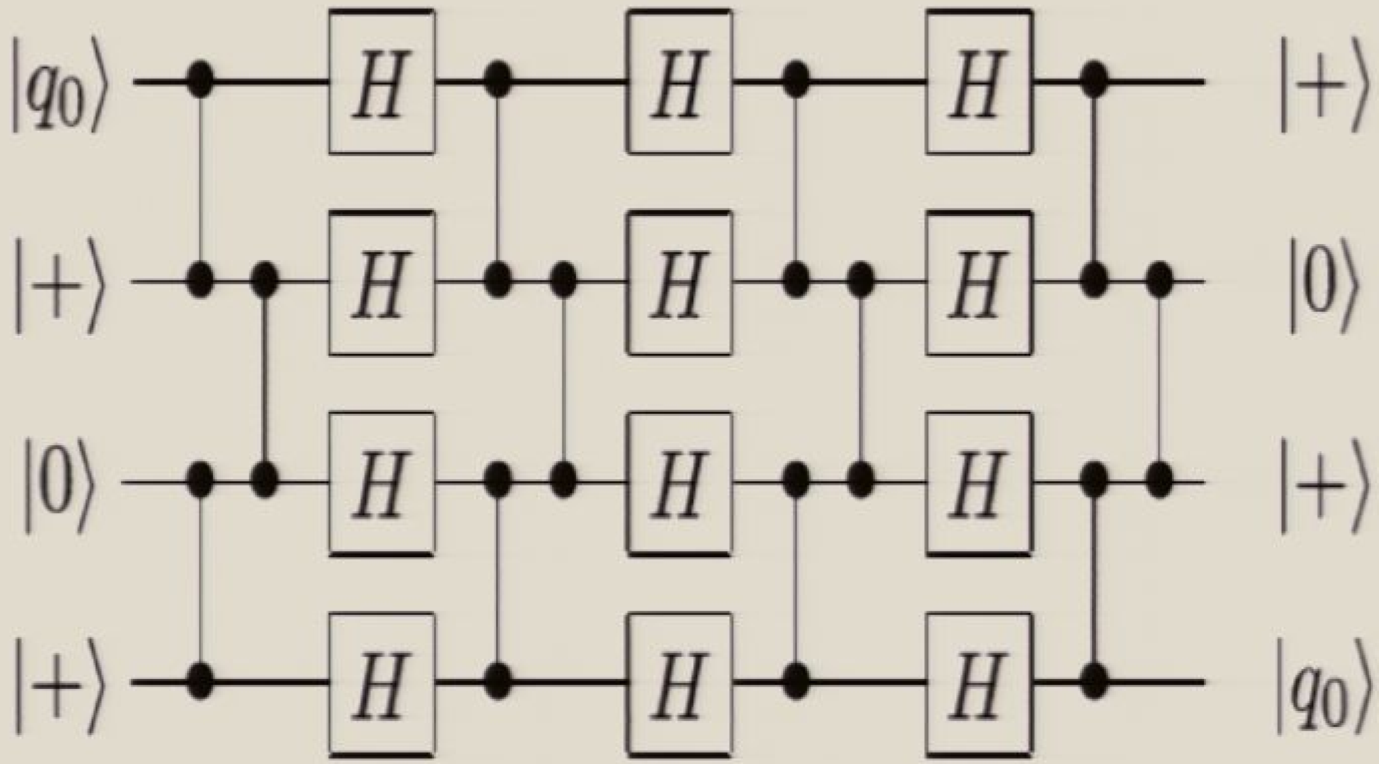
State Transfer

The Transfer Circuit: Concatenate Swaps



State Transfer

The Transfer Circuit: Concatenate Swaps[1]



[1] Robert Raussendorf, Phys. Rev. A 72, 052301 (2005)

State Transfer

The Controlled-Z gate between to qubits, *a* and *b*,can be written as

$$U_{CZ}^{(a,b)}=\frac{1}{2}\left(I+\sigma_z^{(a)}+\sigma_z^{(b)}-\sigma_z^{(a)}\sigma_z^{(b)}\right)$$

If S is the operator corresponding to performing a C-Z between all neighbouring sites in a chain, then

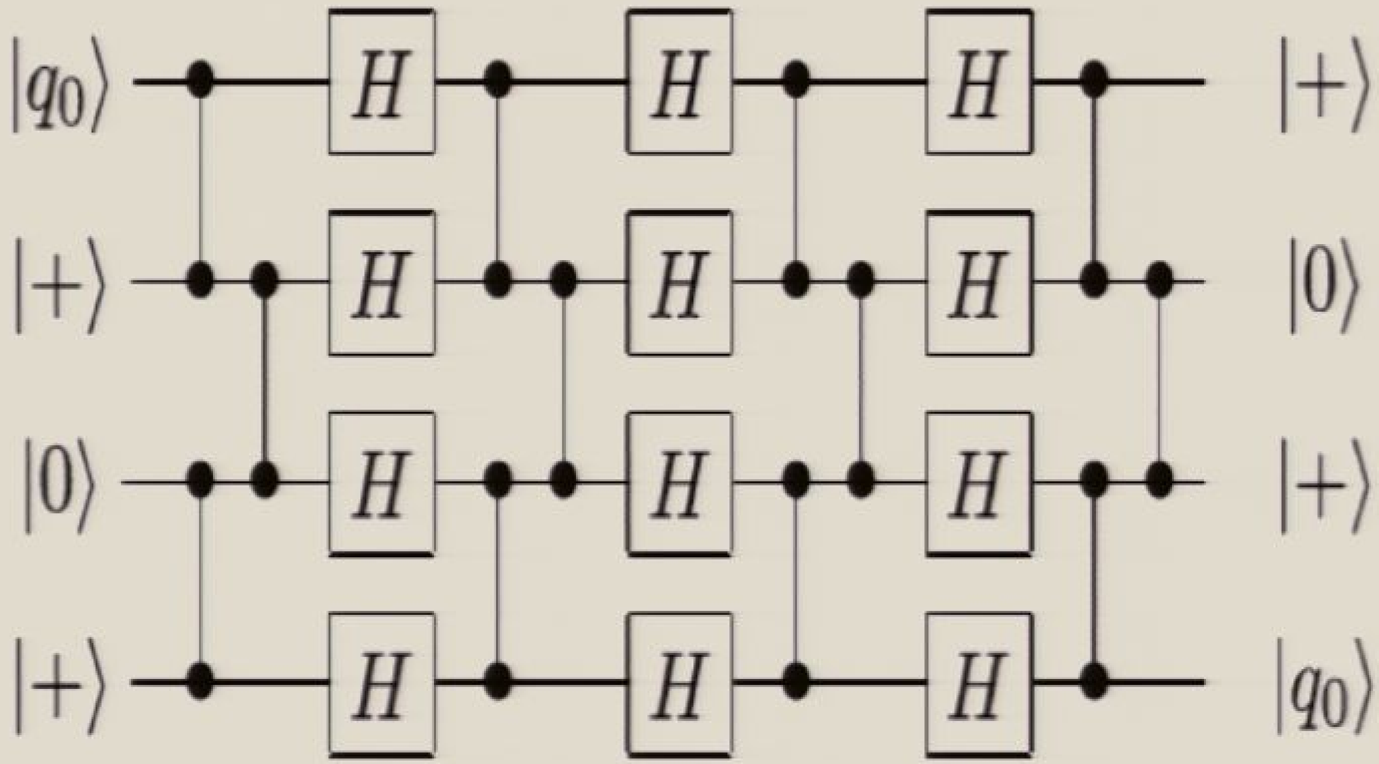
$$S=\prod_a U_{CZ}^{(a,a+1)}=\prod_a \frac{1}{2}\left(I+\sigma_z^{(a)}+\sigma_z^{(a+1)}-\sigma_z^{(a)}\sigma_z^{(a+1)}\right)$$

The generating Hamiltonian for this transformation is

$$H=\hbar g\sum_a \frac{1-\sigma_z^{(a)}}{2}\frac{1-\sigma_z^{(a+1)}}{2}$$

State Transfer

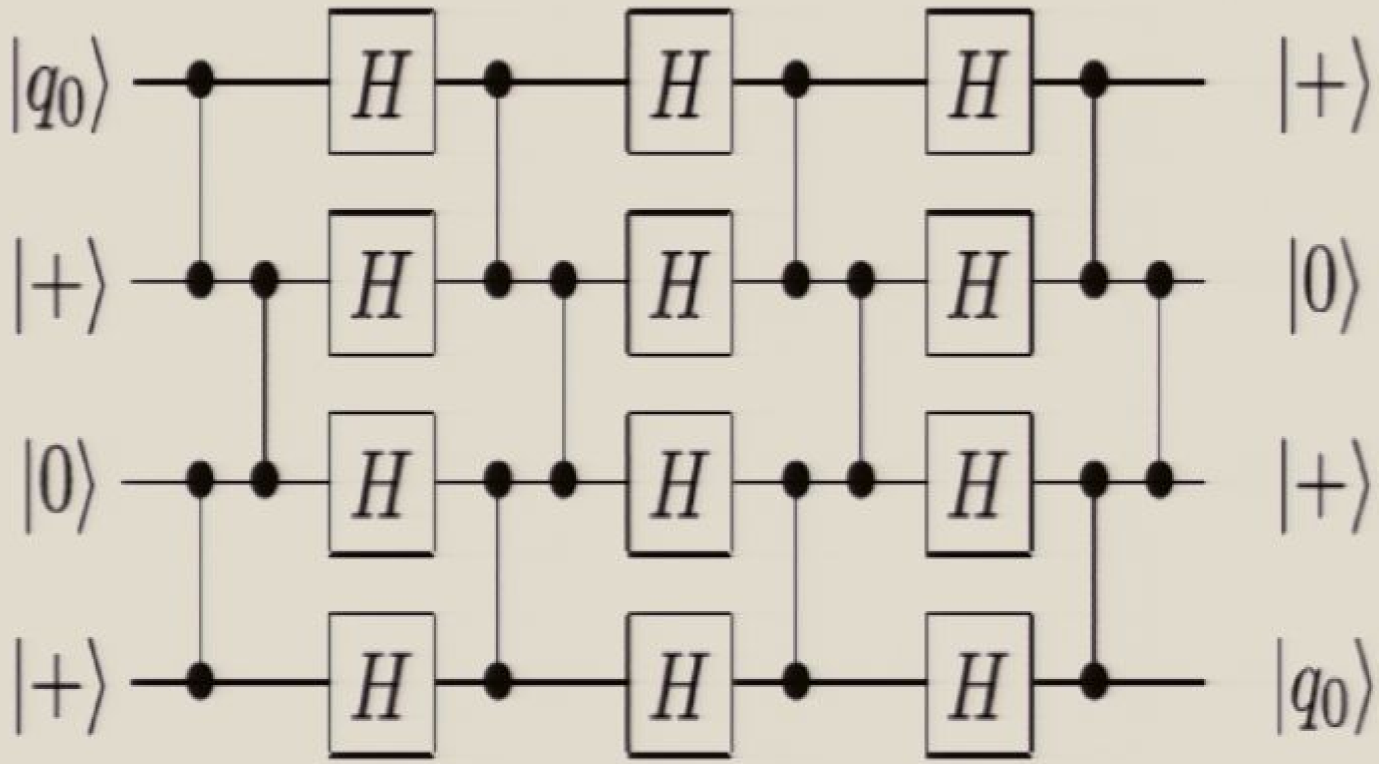
The Transfer Circuit: Concatenate Swaps[1]



[1] Robert Raussendorf, Phys. Rev. A 72, 052301 (2005)

State Transfer

The Transfer Circuit: Concatenate Swaps[1]



[1] Robert Raussendorf, Phys. Rev. A 72, 052301 (2005)

State Transfer

The Controlled-Z gate between to qubits, *a* and *b*,can be written as

$$U_{CZ}^{(a,b)}=\frac{1}{2}(I+\sigma_z^{(a)}+\sigma_z^{(b)}-\sigma_z^{(a)}\sigma_z^{(b)})$$

If S is the operator corresponding to performing a C-Z between all neighbouring sites in a chain, then

$$S=\prod_a U_{CZ}^{(a,a+1)}=\prod_a \frac{1}{2}(I+\sigma_z^{(a)}+\sigma_z^{(a+1)}-\sigma_z^{(a)}\sigma_z^{(a+1)})$$

The generating Hamiltonian for this transformation is

$$H=\hbar g\sum_a \frac{1-\sigma_z^{(a)}}{2}\frac{1-\sigma_z^{(a+1)}}{2}$$

State Transfer

The Controlled-Z gate between to qubits, *a* and *b*,can be written as

$$U_{CZ}^{(a,b)} = \frac{1}{2}\left(I + \sigma_z^{(a)} + \sigma_z^{(b)} - \sigma_z^{(a)}\sigma_z^{(b)}\right)$$

If S is the operator corresponding to performing a C-Z between all neighbouring sites in a chain, then

$$S = \prod_a U_{CZ}^{(a,a+1)} = \prod_a \frac{1}{2}\left(I + \sigma_z^{(a)} + \sigma_z^{(a+1)} - \sigma_z^{(a)}\sigma_z^{(a+1)}\right)$$

The generating Hamiltonian for this transformation is

$$H = \hbar g \sum_a \frac{1-\sigma_z^{(a)}}{2}\frac{1-\sigma_z^{(a+1)}}{2}$$

State Transfer

So S can be rewritten as

$$S = \exp\left(-i\pi\sum_a \frac{1-\sigma_z^{(a)}}{2}\frac{1-\sigma_z^{(a+1)}}{2}\right)$$

which can then be expanded, giving

$$S = \exp\left(-i\frac{\pi}{4}\sum_a \sigma_z^{(a)}\sigma_z^{(a+1)}\right)\times\prod_a \exp\left(-i\frac{\pi}{4}\right)\exp\left(-i\frac{\pi}{4}\sigma_z^{(a)}\right)\exp\left(-i\frac{\pi}{4}\sigma_z^{(a+1)}\right)$$

The first term of which corresponds to an Ising interaction, and the remainder correspond to a global phase and to local Z rotations.

State Transfer

We can again rewrite S, ignoring the global phase to get

$$S = \exp\left(+i\frac{\pi}{4}(\sigma_z^{(1)} + \sigma_z^{(N)})\right)\exp\left(-i\frac{\pi}{4}\sum_a \sigma_z^{(a)}\sigma_z^{(a+1)}\right) \times \prod_a \exp\left(-i\frac{\pi}{2}\sigma_z^{(a)}\right)$$

which contains terms corresponding to an Ising interaction between neighbours, a −π/2 Z rotation on all spins and a π/4 Z rotation on the spins at either end of the chain.

This allows us to perform controlled-z gates across all spin sites, without having to individually address them. The only sites which need to be addressed individually are the spins on either end of the chain.

State Transfer

Any pure state of a qubit *a* can be written as

$$|\Psi_a\rangle = \alpha_a|0\rangle + \beta_a|1\rangle = (\alpha_a + \beta_a\sigma_x^{(a)})|0\rangle$$

So, for any operator *M*,

$$M|\Psi_a\rangle = [M(\alpha_a + \beta_a\sigma_x^{(a)})]|0\rangle = \alpha_a M|0\rangle + \beta_a M\sigma_x^{(a)}|1\rangle$$

The initial state for the spin chain is:

$$|\phi\rangle = |0\rangle\otimes|+\rangle\otimes|0\rangle\otimes|+\rangle\otimes|0\rangle\otimes|+\rangle\otimes\cdots\otimes|0\rangle$$

This satisfies $S|\phi\rangle = |\phi\rangle$ and $SH|\phi\rangle = H|\phi\rangle$

So for any $|\psi_a\rangle = (\alpha_a + \beta_a\sigma_x^{(a)})|\phi\rangle$

$$(HS)^m|\psi\rangle = \alpha_a H^m|\phi\rangle + \beta_a(HS)^m\sigma_x^{(a)}|\phi\rangle$$

State Transfer

To see how the transport circuit actually performs a multi-qubit swap gate, it is important to note the following identities:

$$S\sigma_z^{(a)} = \sigma_z^{(a)} S$$

$$S\sigma_x^{(0)} = \sigma_x^{(0)} \sigma_z^{(1)} S$$

$$S\sigma_x^{(N)} = \sigma_z^{(N-1)} \sigma_x^{(N)} S$$

$$S\sigma_x^{(a)} = \sigma_z^{(a-1)} \sigma_x^{(a)} \sigma_z^{(a+1)} S$$

$$H\sigma_z^{(a)} = \sigma_x^{(a)} H$$

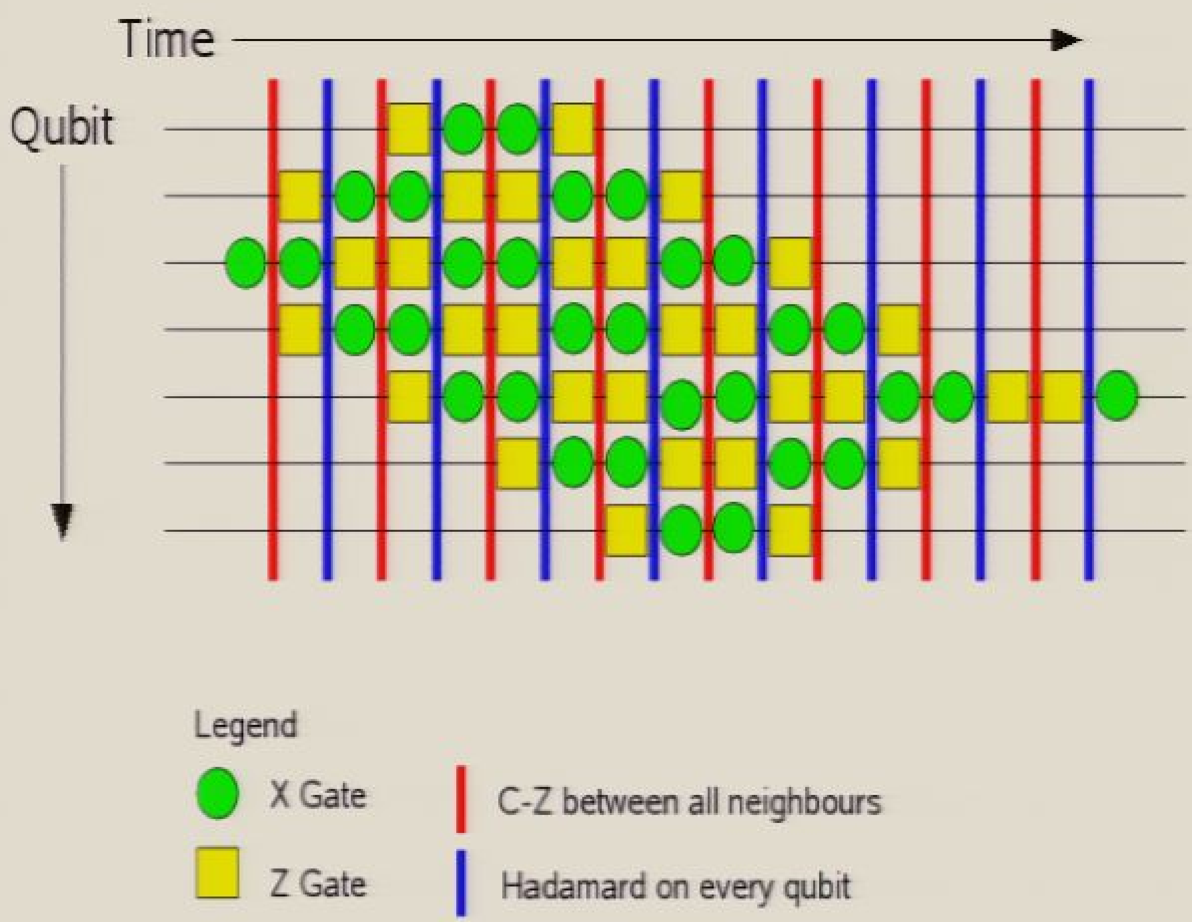
These allow us to produce equivalent operators for different times within the scheme. For example applying the operator

after the first two rounds of the Ising interaction and Hadamards has the same result as having applied $\sigma_x^{(5)}$ initially, since

$$(HS)^2 \sigma_x^{(5)} = \sigma_x^{(3)} \sigma_z^{(4)} \sigma_x^{(5)} \sigma_z^{(6)} \sigma_x^{(7)} (HS)^2$$

State Transfer

For example, the following operations are all equivalent:



QuNaT

State Transfer

For any qubit, *a*:

$$(HS)^{N+1}\sigma_x^{(a)}=\sigma_x^{(N-a)}(HS)^{N+1}$$ Eq. 1

$$(HS)^{N+1}\sigma_z^{(a)}=\sigma_z^{(N-a)}(HS)^{N+1}$$ Eq. 2

Using the Eq. 1,

$$(HS)^{N+1}|\psi_a\rangle=(\alpha_a+\beta\sigma_x^{(N-a)})|\psi\rangle$$

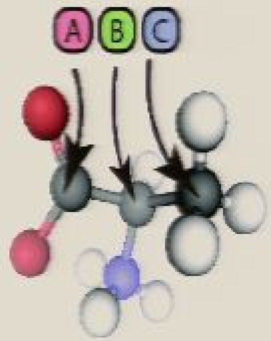
Similarly, by Eq. 2,

$$(HS)^{N+1}(\alpha_a+\beta_a\sigma_z^{(a)})|\phi\rangle=(\alpha_a+\beta\sigma_z^{(N-a)})|\psi\rangle$$

Since this is true for all *a*, the sites have been inverted, accomplishing a multi-qubit SWAP gate

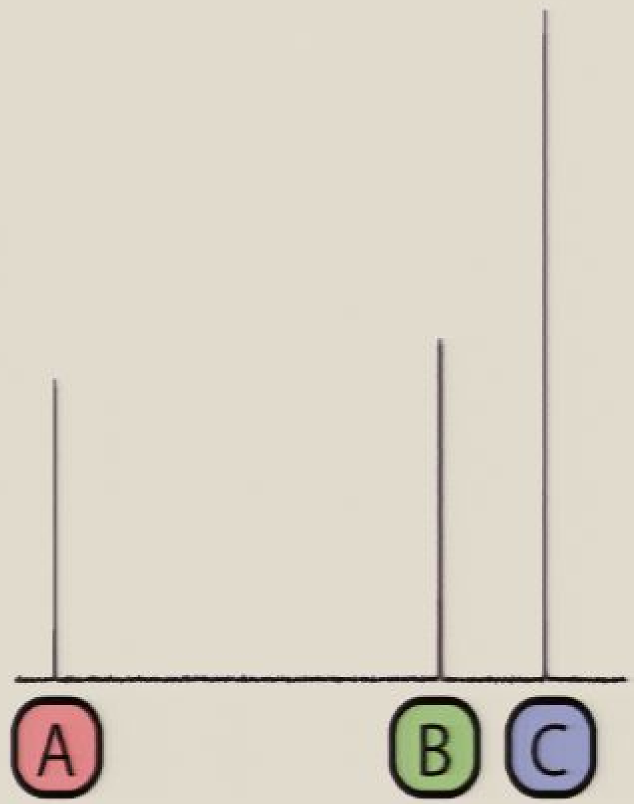
State Transfer

Experimental results



$\nu_A = +13034.5$ Hz
$\nu_B = -5869.7$ Hz
$\nu_C = -11504.2$ Hz

$J_{AB} = +54.1$ Hz
$J_{BC} = +35.0$ Hz
$J_{AC} = -1.3$ Hz



State Transfer

For any qubit, *a*:

$$(HS)^{N+1}\sigma_x^{(a)}=\sigma_x^{(N-a)}(HS)^{N+1}$$ Eq. 1

$$(HS)^{N+1}\sigma_z^{(a)}=\sigma_z^{(N-a)}(HS)^{N+1}$$ Eq. 2

Using the Eq. 1,

$$(HS)^{N+1}|\psi_a\rangle=(\alpha_a+\beta\sigma_x^{(N-a)})|\psi\rangle$$

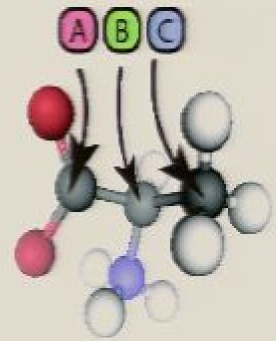
Similarly, by Eq. 2,

$$(HS)^{N+1}(\alpha_a+\beta_a\sigma_z^{(a)})|\phi\rangle=(\alpha_a+\beta\sigma_z^{(N-a)})|\psi\rangle$$

Since this is true for all *a*, the sites have been inverted, accomplishing a multi-qubit SWAP gate

State Transfer

Experimental results



ν_A = +13034.5 Hz

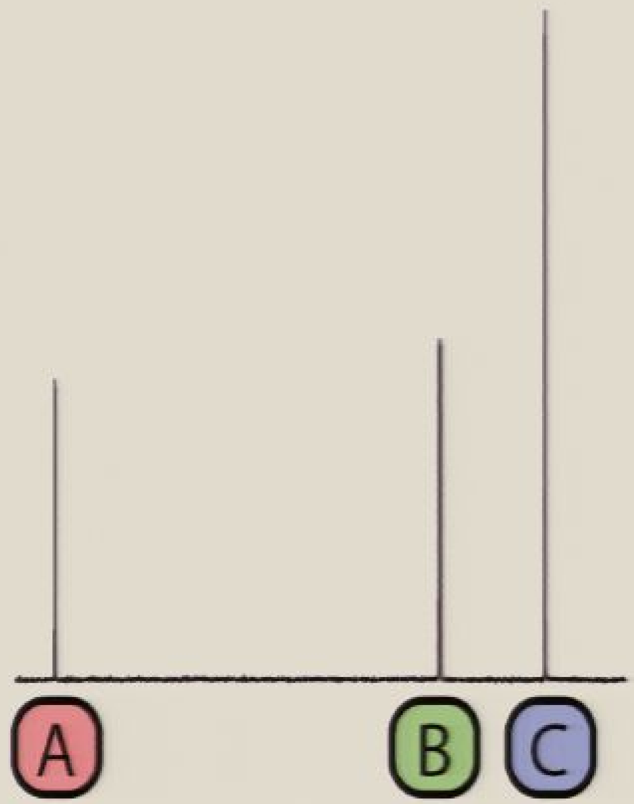
ν_B = -5869.7 Hz

ν_C = -11504.2 Hz

J_{AB} =+54.1 Hz

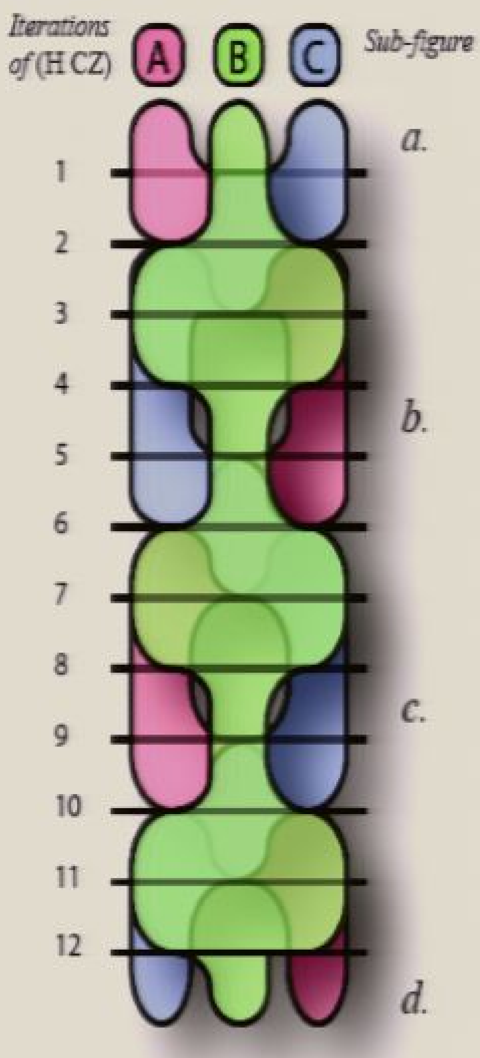
J_{BC} =+35.0 Hz

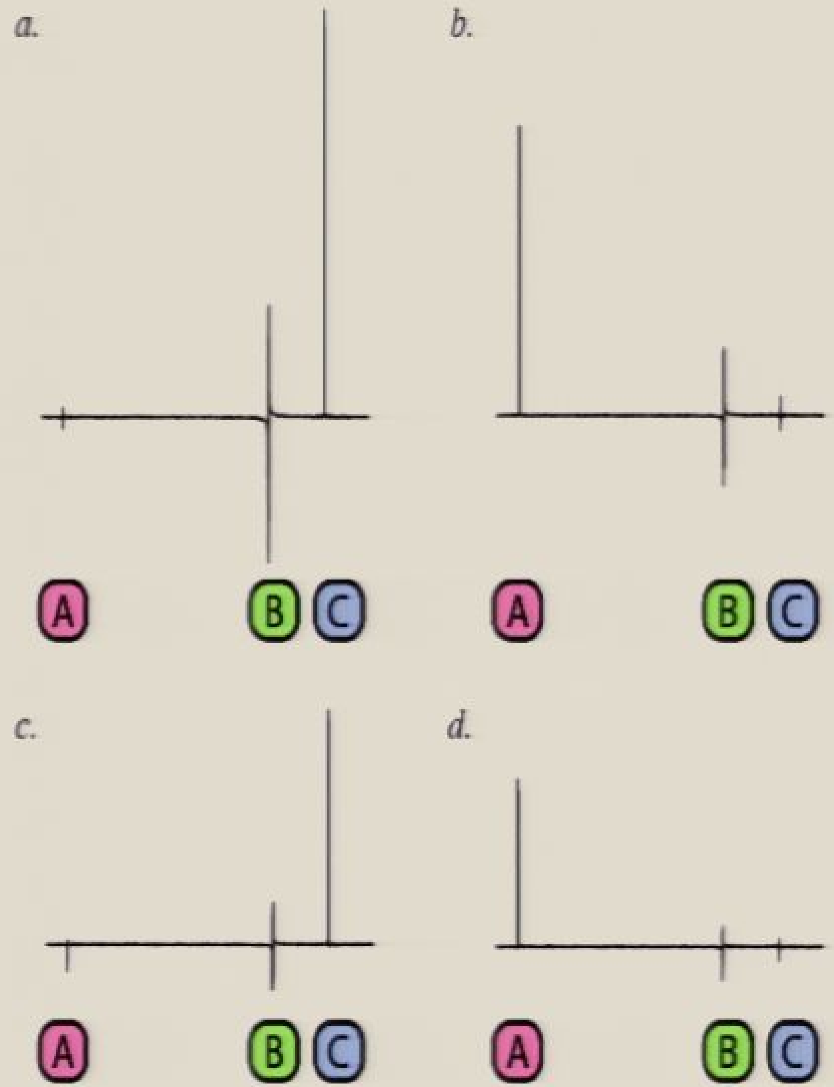
J_{AC} =-1.3 Hz



State Transfer

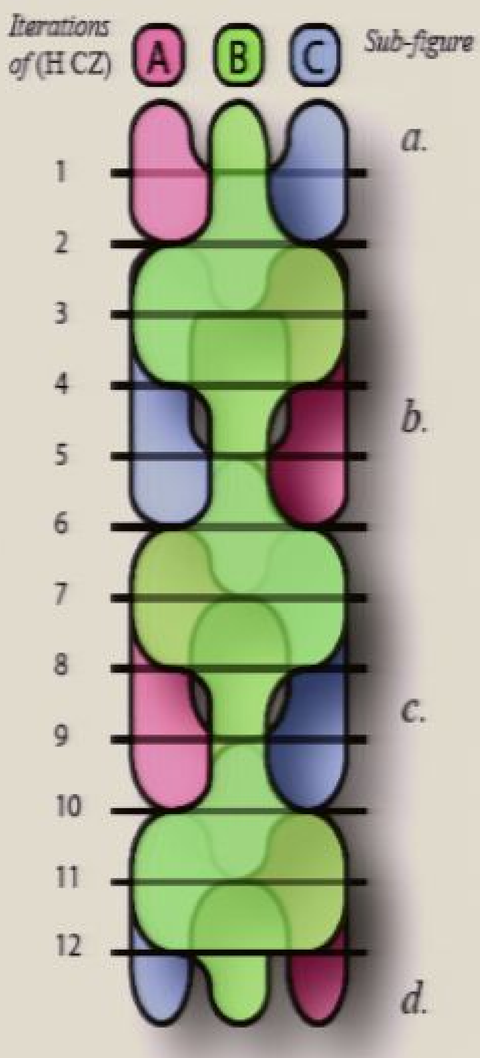
Experimental results

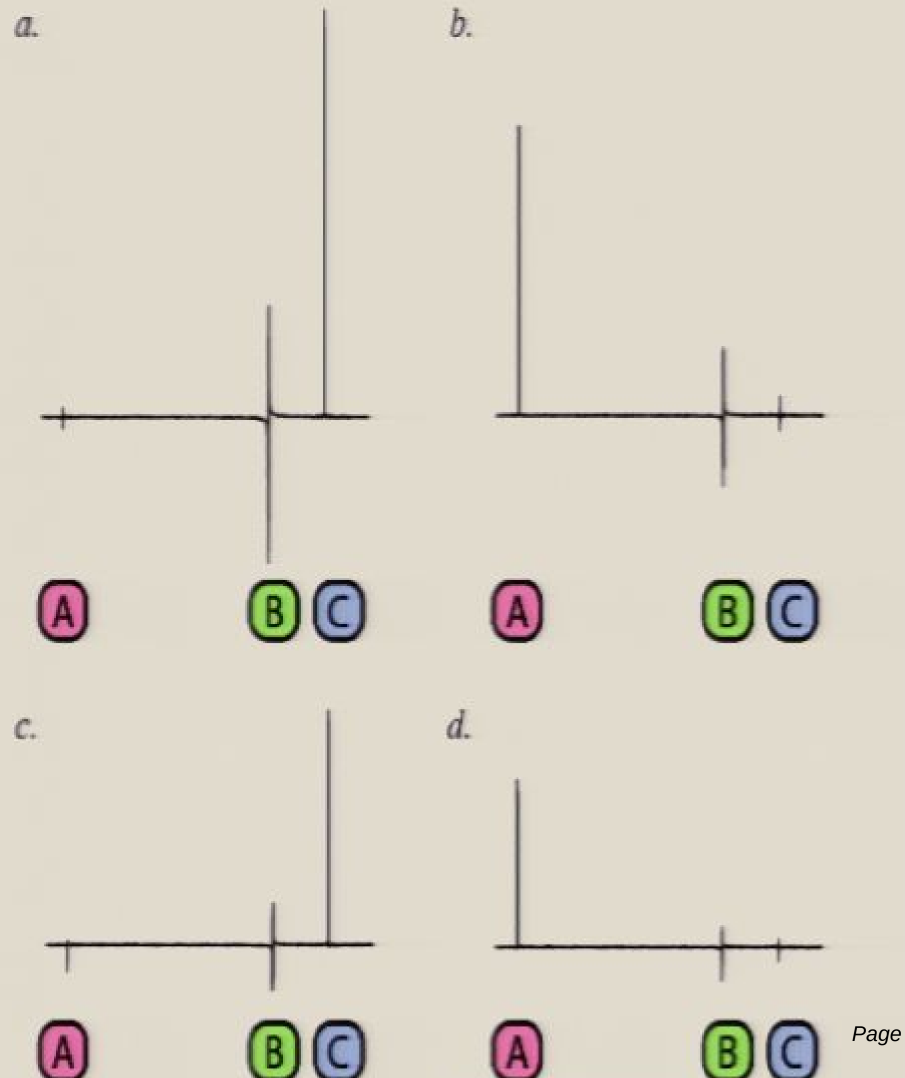




State Transfer

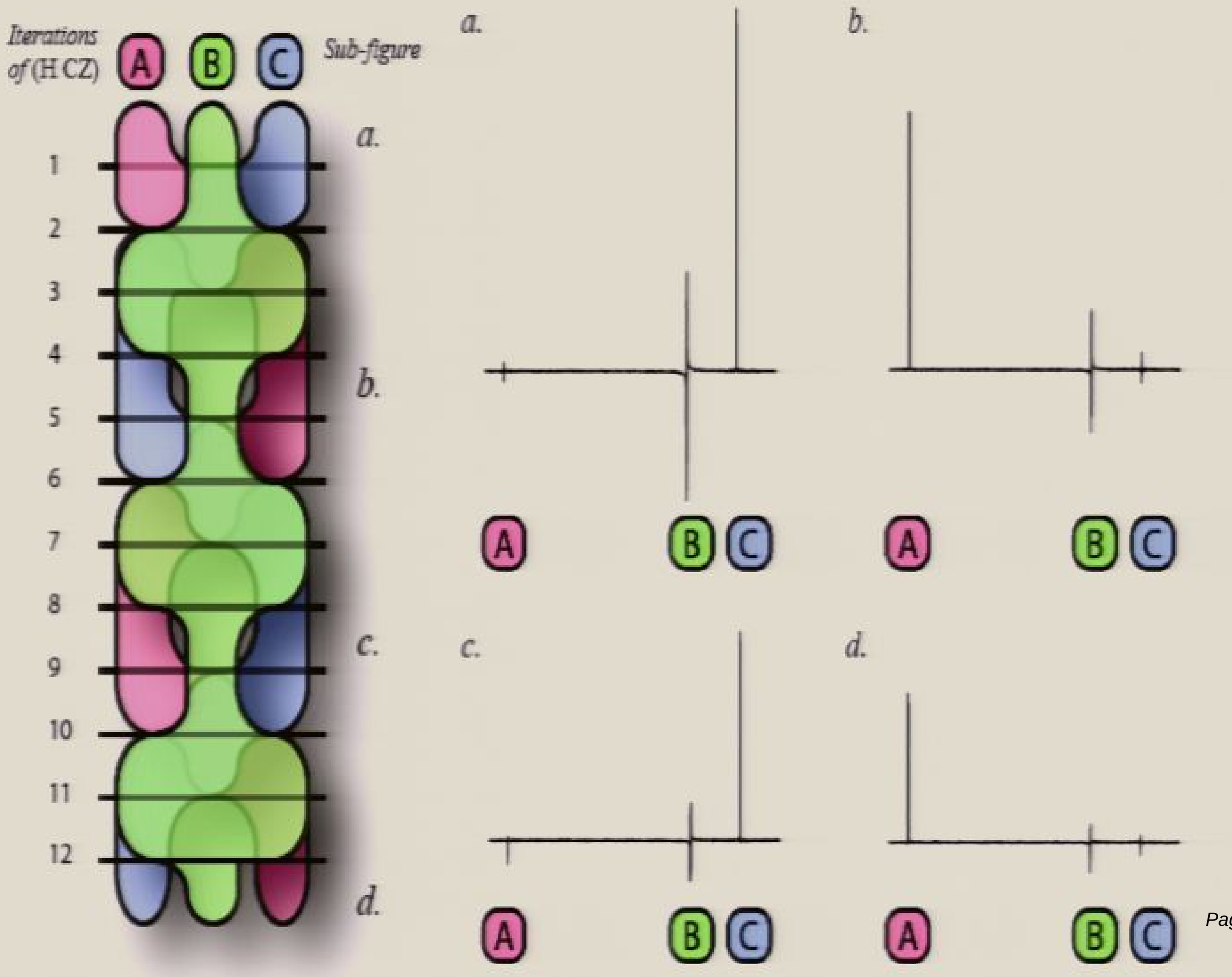
Experimental results





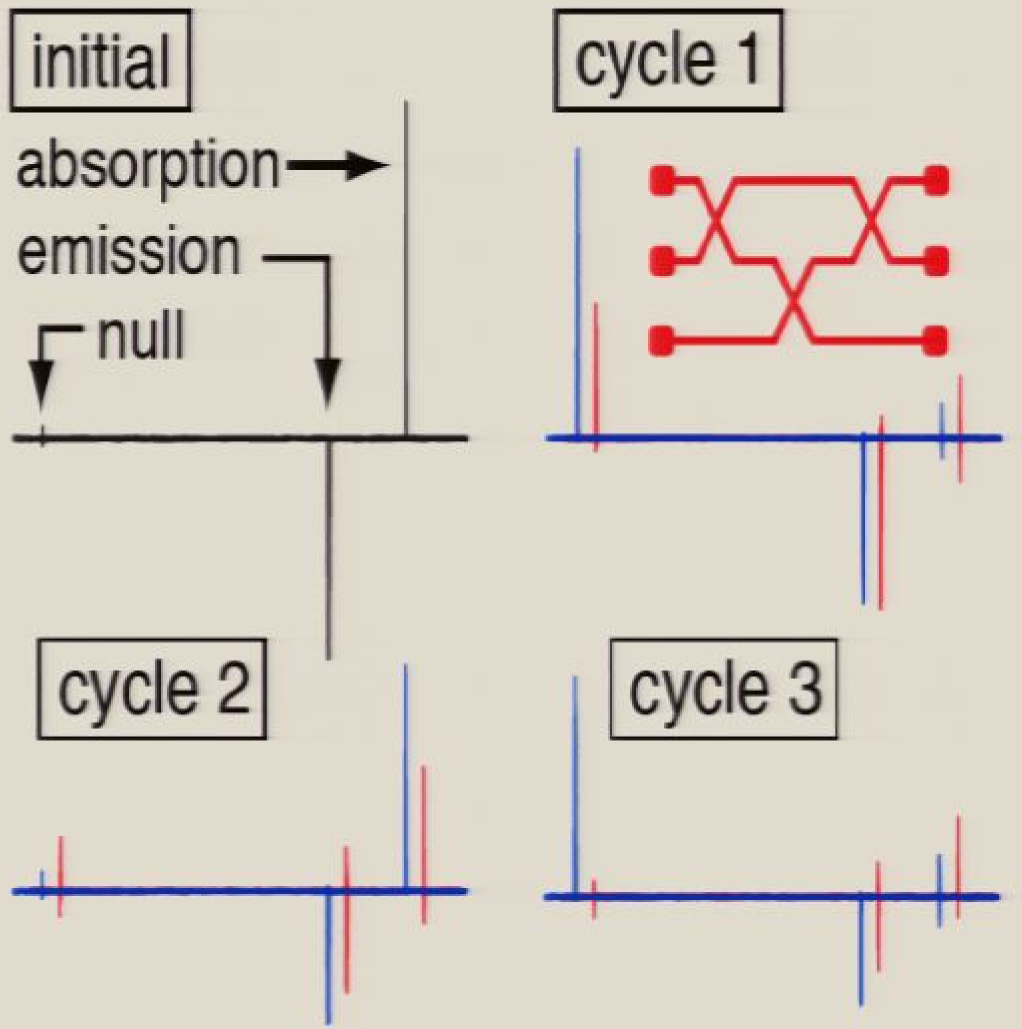
State Transfer

Experimental results

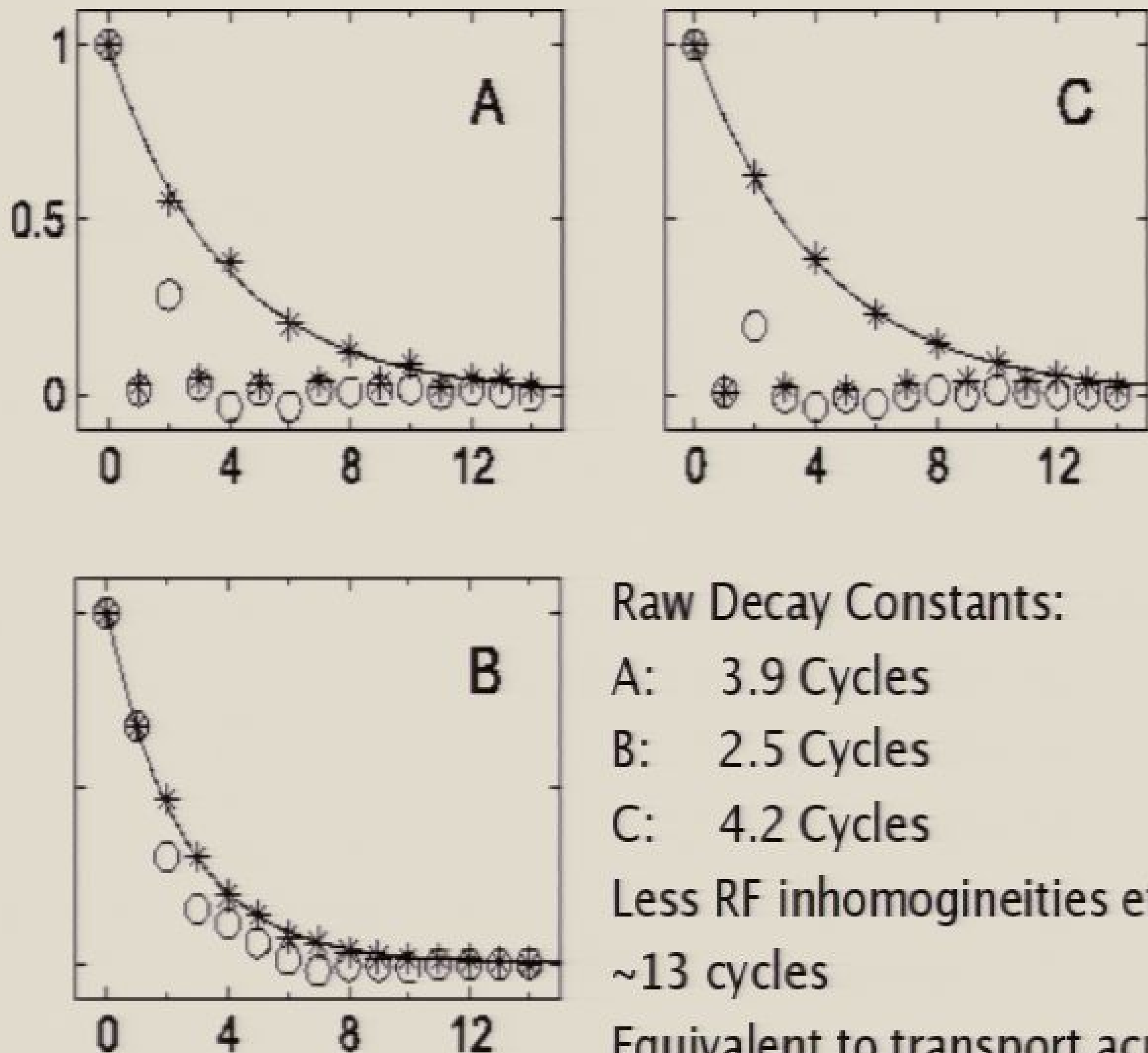


State Transfer

quantum mirror versus SWAP network



State Transfer



Raw Decay Constants:

A: 3.9 Cycles

B: 2.5 Cycles

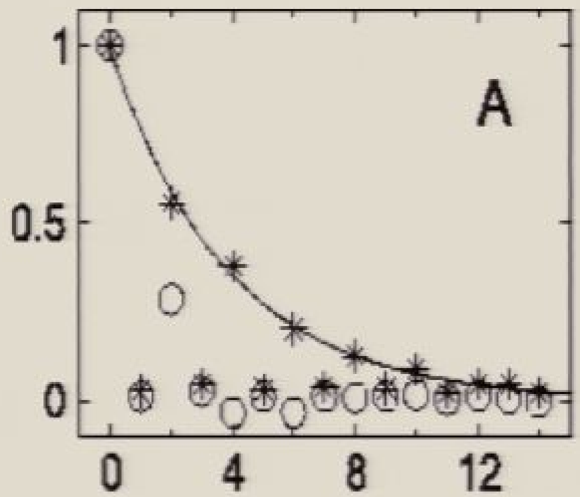
C: 4.2 Cycles

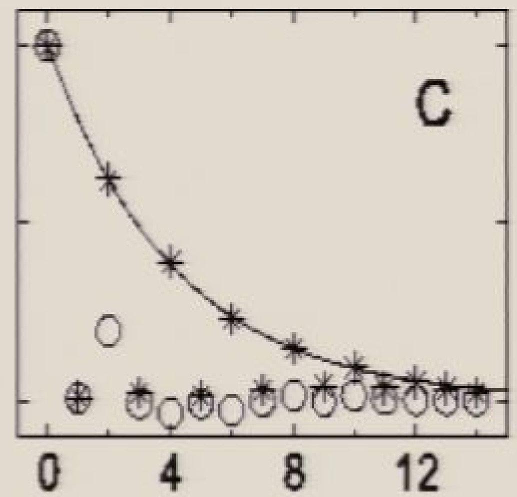
Less RF inhomogineities etc.:

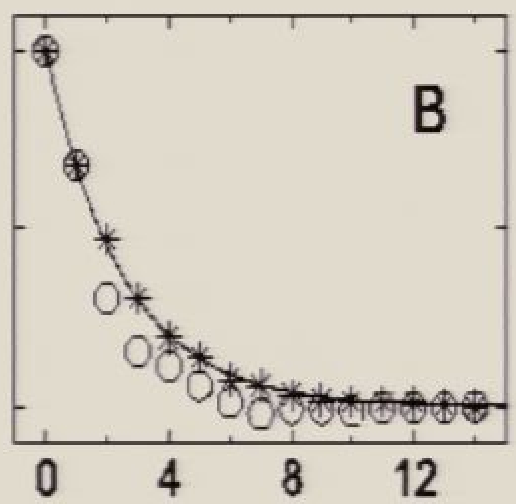
~13 cycles

Equivalent to transport across approximately 52 spins

State Transfer







Raw Decay Constants:

A: 3.9 Cycles

B: 2.5 Cycles

C: 4.2 Cycles

Less RF inhomogineities etc.:

~13 cycles

Equivalent to transport across approximately 52 spins

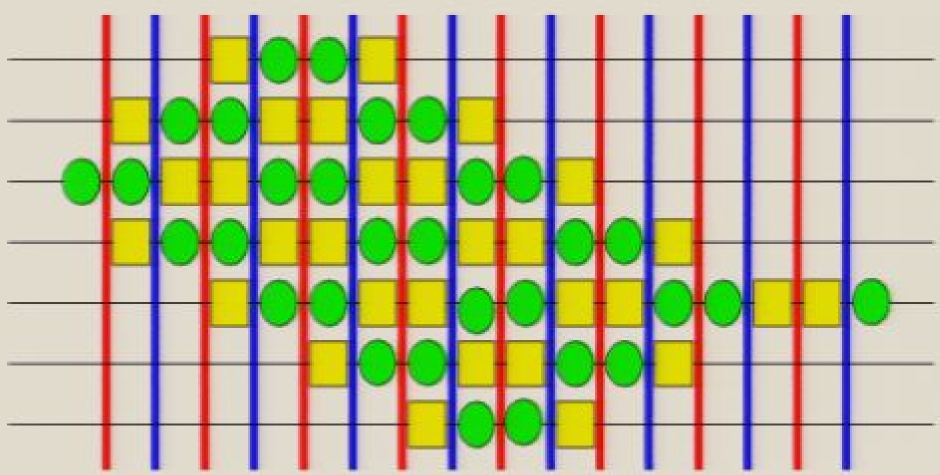
Part 2: Quantum Computation

Single Qubit Gates

To build upon this scheme, to allow single qubit operations to be performed, it is necessary to separate the logical qubits, adding a $|+\rangle$ state between each. So

$$|\psi\rangle = |\psi_0\rangle \otimes |+\rangle \otimes |\psi_1\rangle \otimes |+\rangle \otimes |\psi_2\rangle \otimes |+\rangle \otimes \cdots \otimes |\psi_N\rangle$$

This ensures that the state of the physical qubit, at a given end, is only affected by one logical qubit.

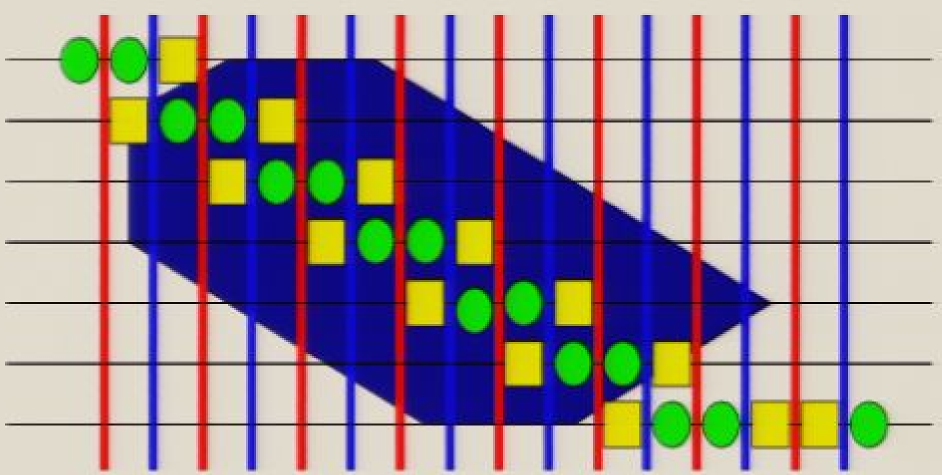


Single Qubit Gates

To build upon this scheme, to allow single qubit operations to be performed, it is necessary to separate the logical qubits, adding a $|+\rangle$ state between each. So

$$|\psi\rangle = |\psi_0\rangle \otimes |+\rangle \otimes |\psi_1\rangle \otimes |+\rangle \otimes |\psi_2\rangle \otimes |+\rangle \otimes \cdots \otimes |\psi_N\rangle$$

This ensures that the state of the physical qubit, at a given end, is only affected by one logical qubit.

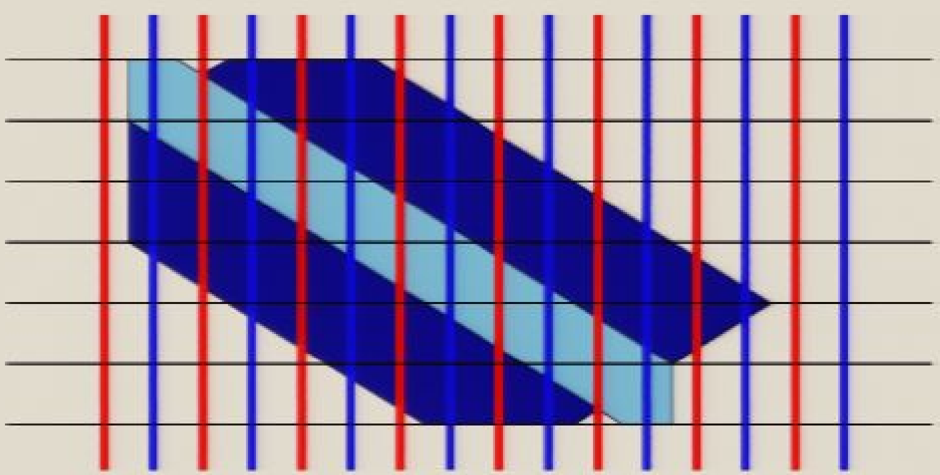


Single Qubit Gates

To build upon this scheme, to allow single qubit operations to be performed, it is necessary to separate the logical qubits, adding a $|+\rangle$ state between each. So

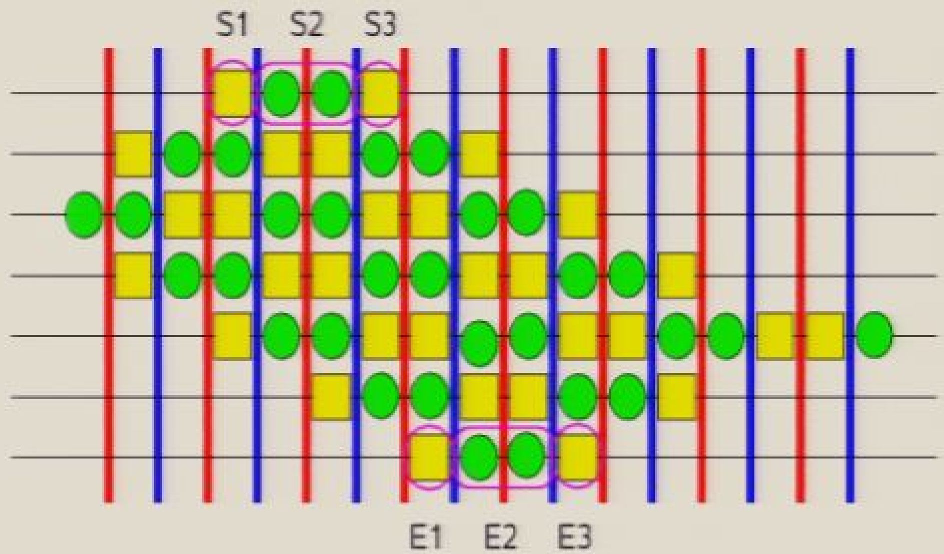
$$|\psi\rangle = |\psi_0\rangle \otimes |+\rangle \otimes |\psi_1\rangle \otimes |+\rangle \otimes |\psi_2\rangle \otimes |+\rangle \otimes \cdots \otimes |\psi_N\rangle$$

This ensures that the state of the physical qubit, at a given end, is only affected by one logical qubit.



Single Qubit Gates

At times S2 and E2 a z-rotation on the physical qubit at the indicated end of the chain will result in the same z-rotation on the corresponding logical qubit.



Single Qubit Gates

At the end of each cycle, a ±π/2 X rotation is performed on all qubits.

This will leave the buffer qubits unchanged, but rotate the logical qubits.

$$R_x(\theta)|+\rangle = |+\rangle$$

By repeating the procedure for a single qubit Z rotation, we can now also perform Y rotations.

$$R_x(\frac{\pi}{4}) R_z(\theta) R_x(-\frac{\pi}{4})|\phi\rangle = R_y(-\theta)|\phi\rangle$$

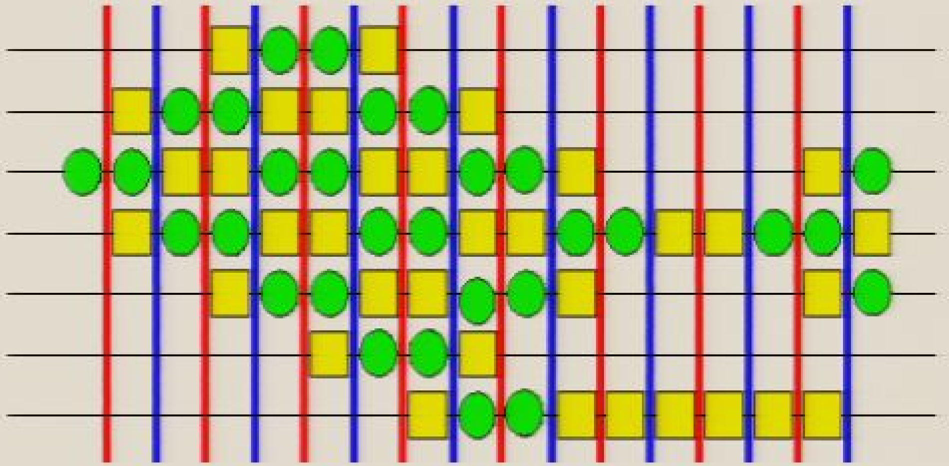
Since an arbitrary rotation can be written as $R_Z(a)R_Y(b)R_Z(c)$, arbitrary single qubit unitaries can be performed.

Two Qubit Gates

For universal quantum computation, it is also necessary to be able to perform a two qubit gate, such as a C-Z or a CNOT.

To accomplish this, we need to decouple one of the end spins. This can be done by applying stroboscopicly an X gate half way between the Hadamard gates.

When the state of the spin, at a chosen end of the chain, is only affected by the state of the CONTROL qubit, it is decoupled as shown.

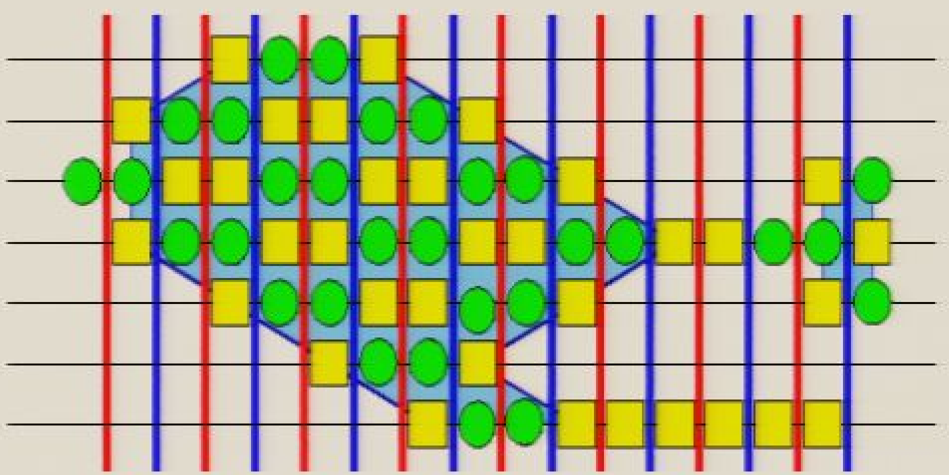


Two Qubit Gates

For universal quantum computation, it is also necessary to be able to perform a two qubit gate, such as a C-Z or a CNOT.

To accomplish this, we need to decouple one of the end spins. This can be done by applying stroboscopicly an X gate half way between the Hadamard gates.

When the state of the spin, at a chosen end of the chain, is only affected by the state of the CONTROL qubit, it is decoupled as shown.

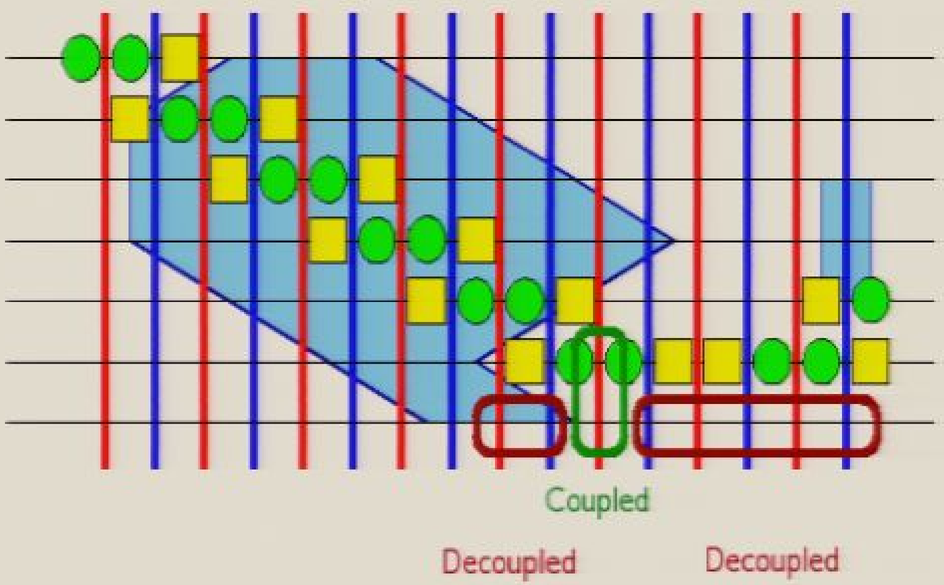


Two Qubit Gates

To interact the TARGET qubit with the CONTROL, we stop decoupling the end spin for one round of the Ising interaction, allowing a C-Z to be performed.

The state of the neighbouring qubit is $|q_{N-1}\rangle = |0\rangle$ if $|\psi_a\rangle = |0\rangle$

and $|q_{N-1}\rangle = |1\rangle$ if $|\psi_a\rangle = |1\rangle$

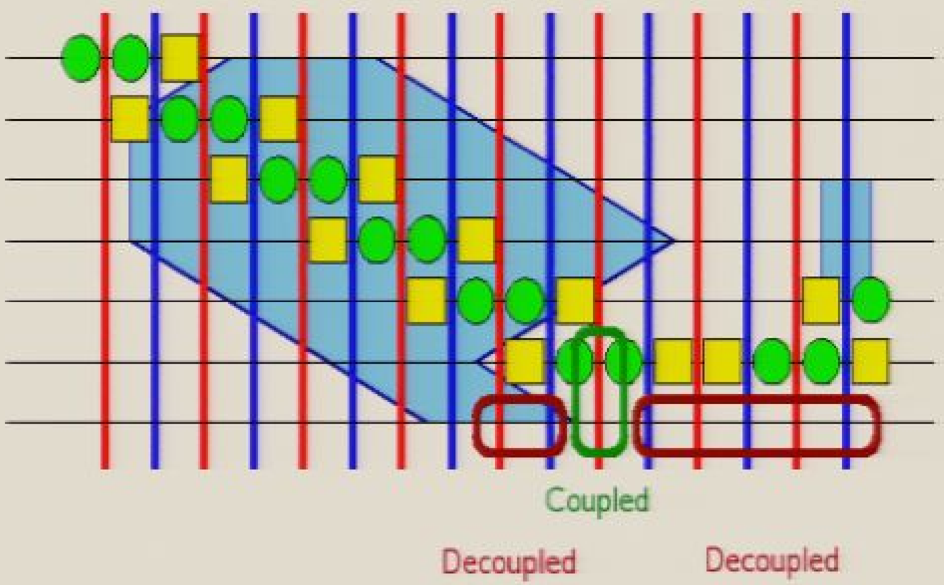


Two Qubit Gates

The state of the end qubit is $|q_N\rangle = |0\rangle$ if $|\psi_b\rangle = |0\rangle$

and $|q_N\rangle = |1\rangle$ if $|\psi_b\rangle = |1\rangle$

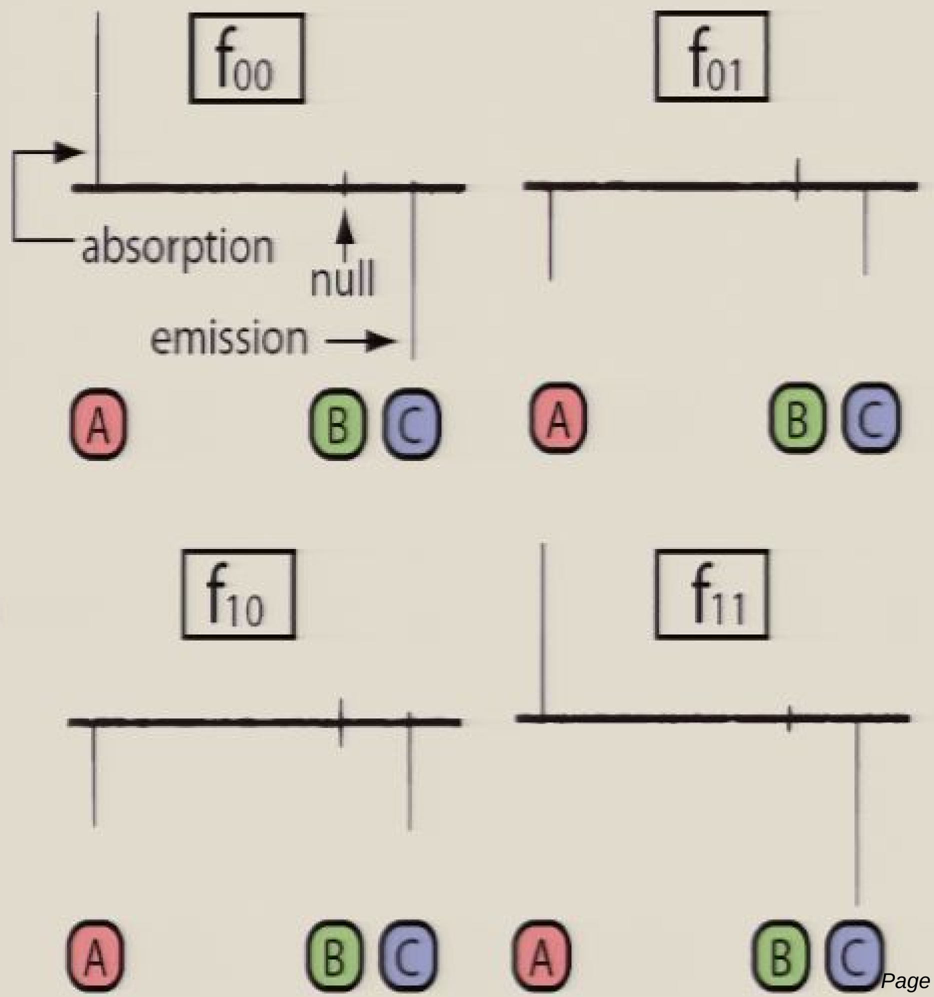
By allowing a CZ between these two physical qubits we obtain a CZ between the logical qubits. To localise the logical qubits again, we skip a set of Hadamard gates, and run the process backwards, this time skipping the isolated coupling.

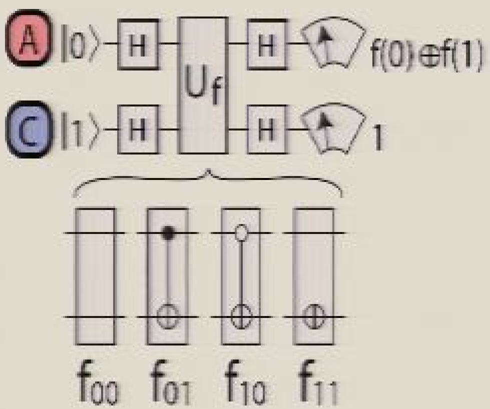


QuNaT

Two Qubit Gates

Deutsch Algorithm





Part 3: Optimization

Decreasing Overhead

This scheme has an overhead on all operations which scales linearly with N.

This can be reduced, however, since

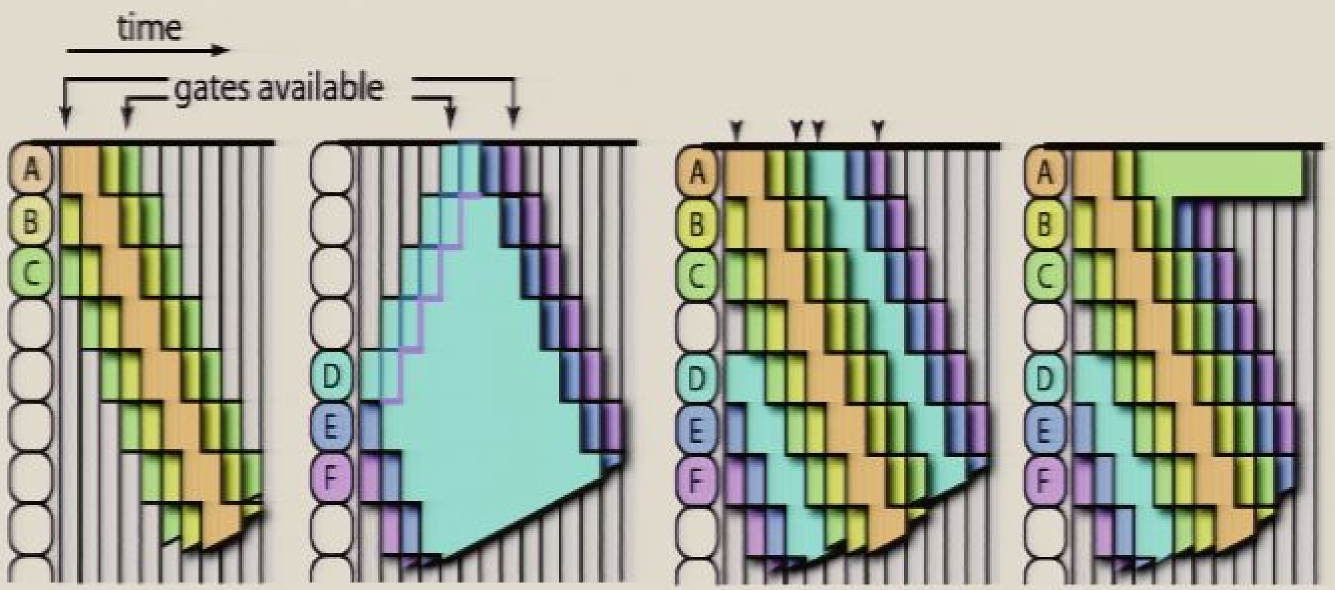
- All qubits can be rotated in a single mirror inversion cycle
- Any number of CZs, or Controlled Phase operations with the same controlling qubit can be performed in a single mirror inversion cycle.

Dense Packing

- Qubits can be packed into blocks

- Blocks are separated by a buffer state

- Qubits at the edge of blocks can undergo gates as described earlier

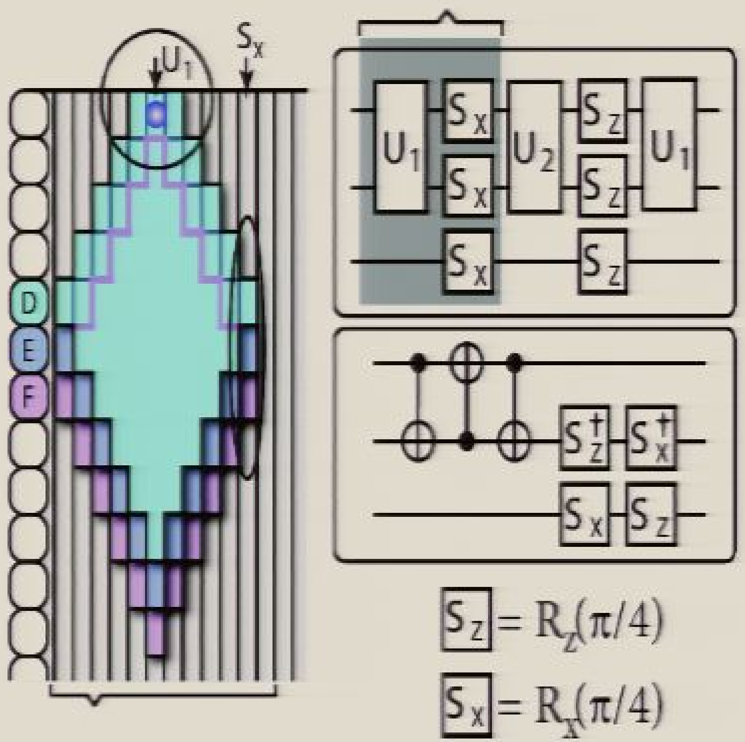


Block 1 | Block 2 | combined blocks - single qubit gates | inter-block logic two-qubit gates

Dense Packing

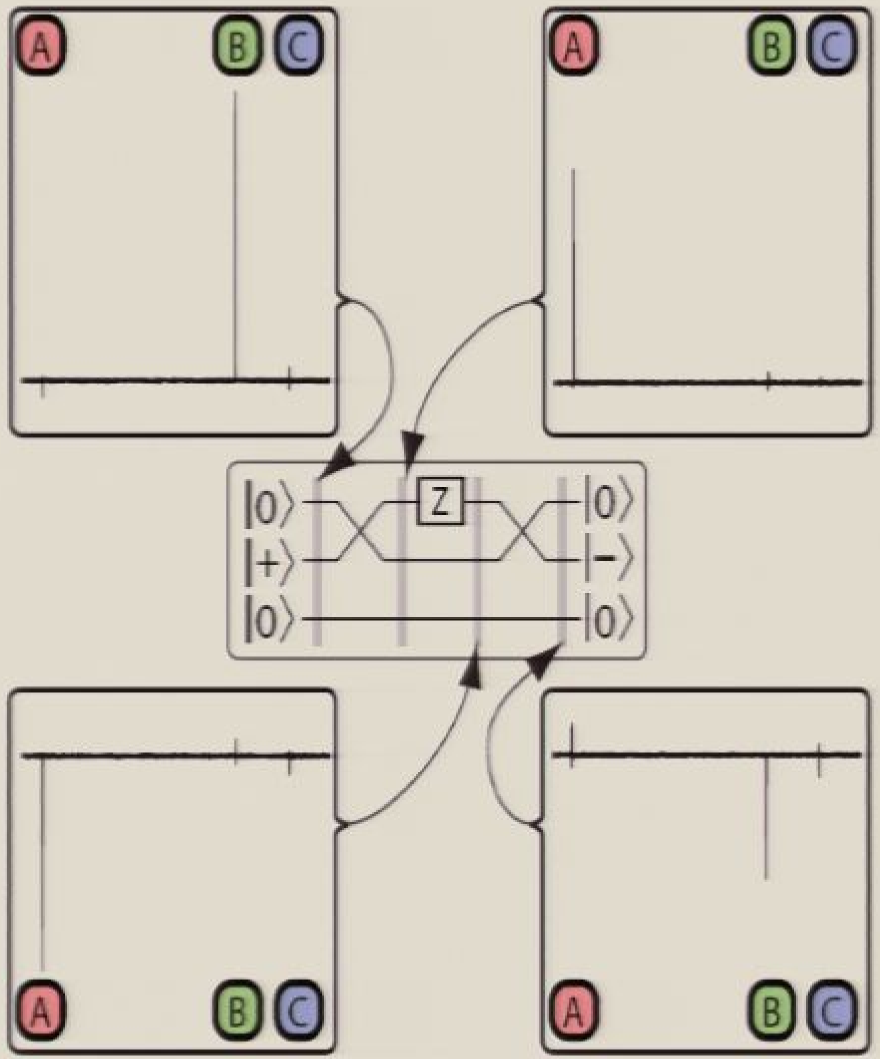
- Qubits on the interior of a block must be swapped to the edge before logic gates can be performed on them

- Trade off between time and space: Gates take O(Nxm) for a block size of m → chain length = (N + N/m–1)



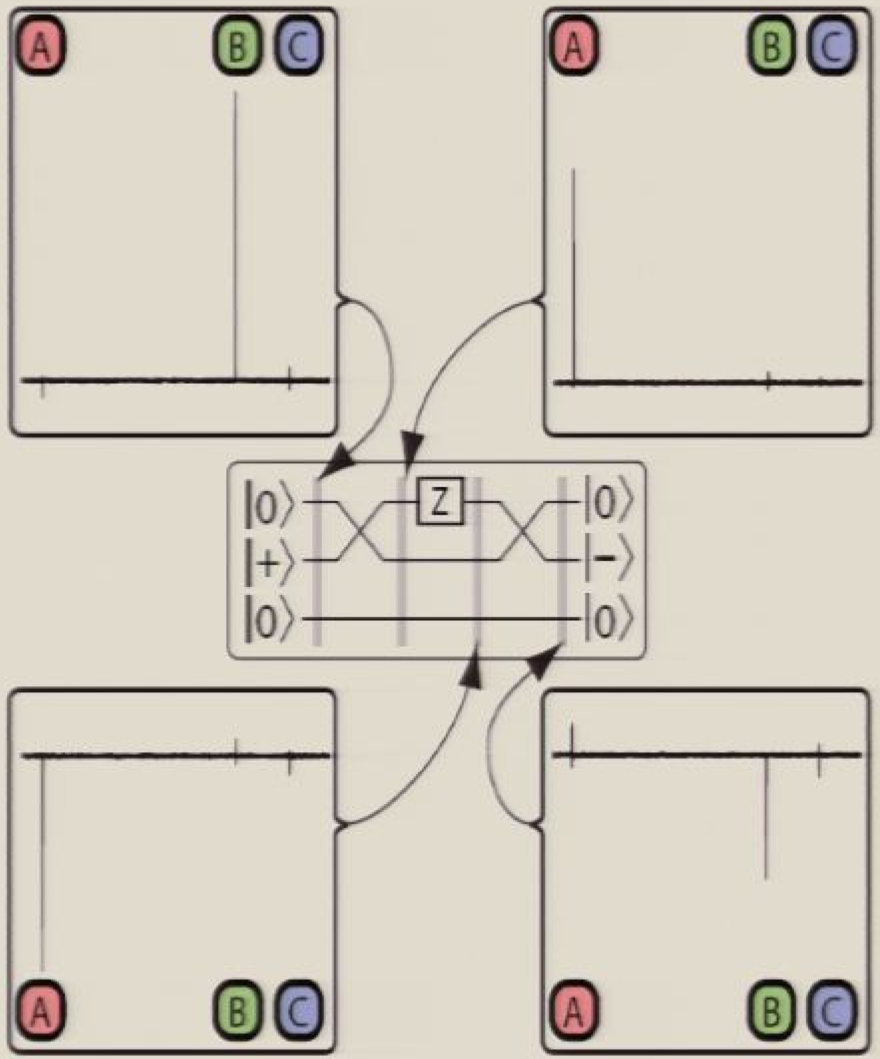
Dense Packing

Single Qubit Rotation



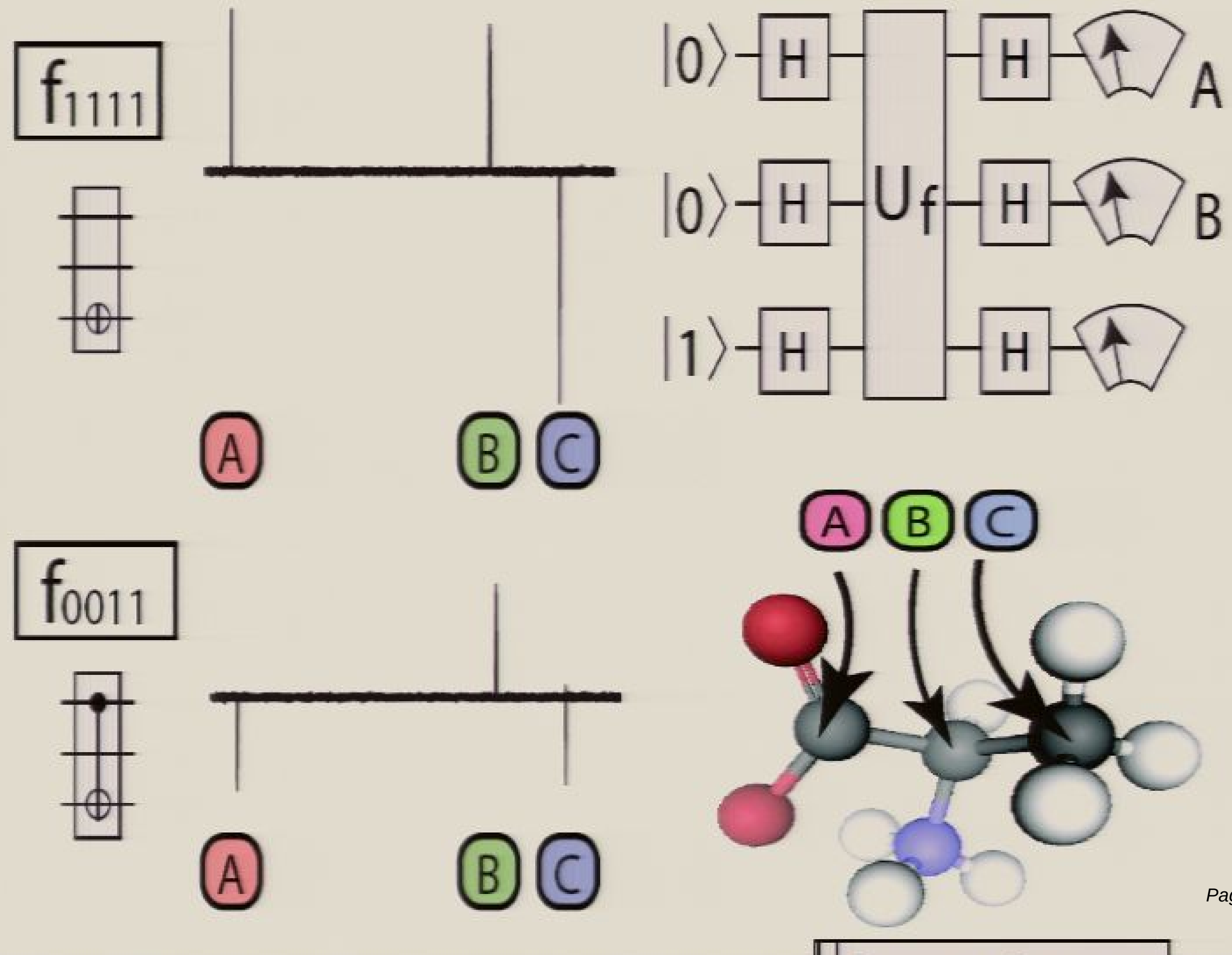
Dense Packing

Single Qubit Rotation



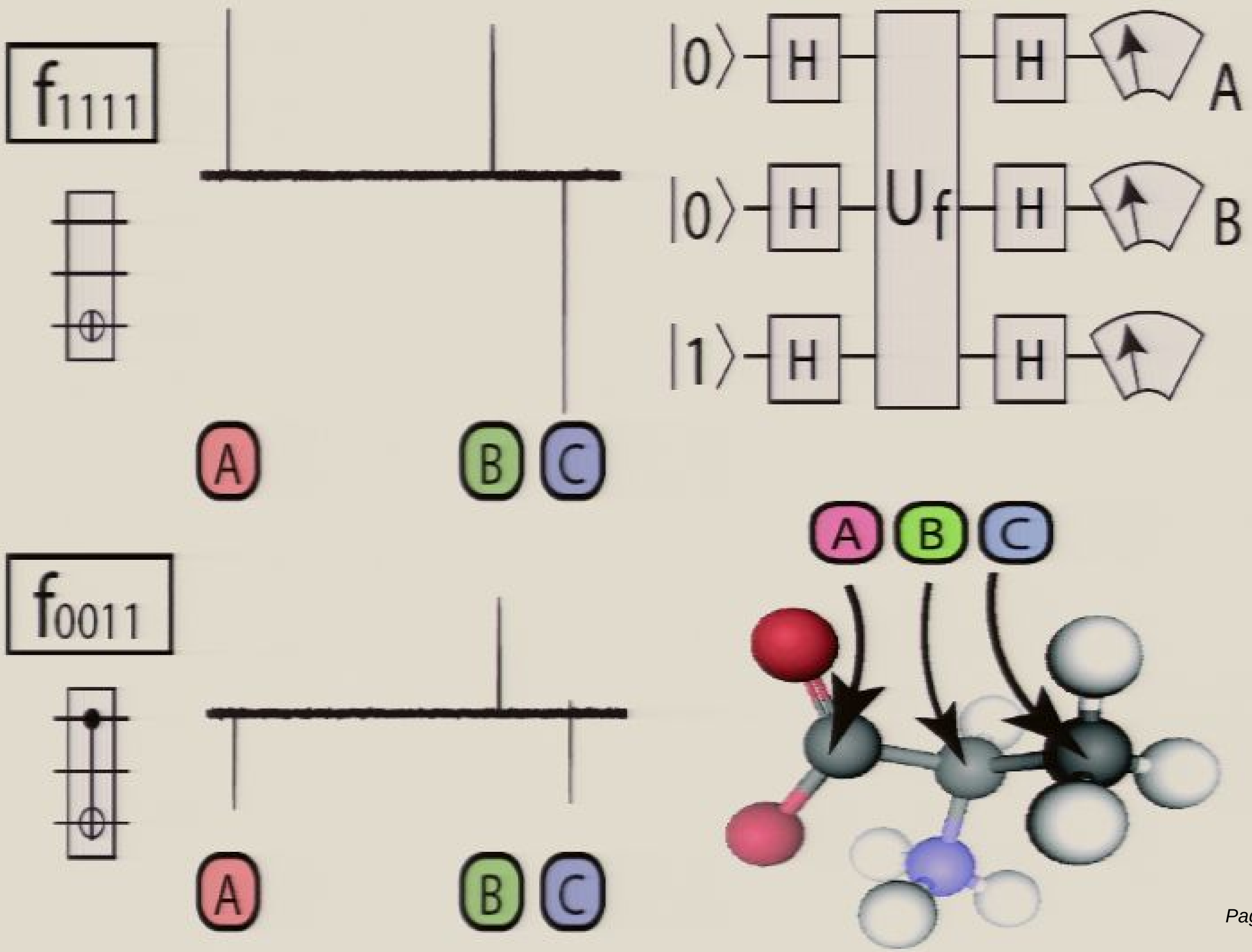
Dense Packing

Deutsch-Jozsa Algorithm



Dense Packing

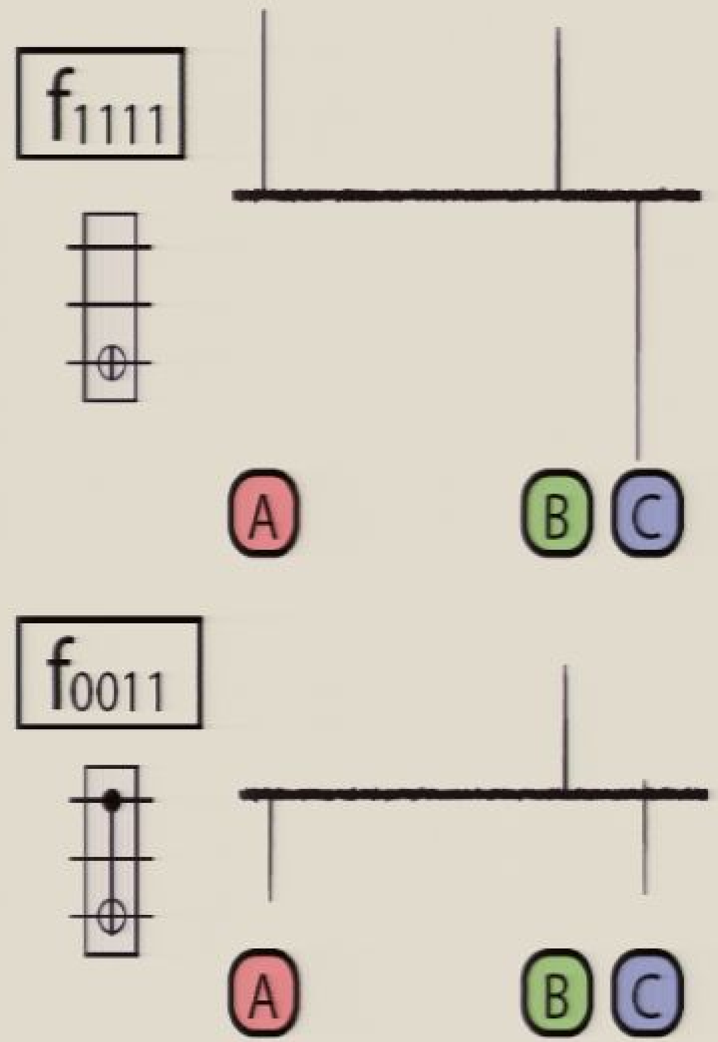
Deutsch-Jozsa Algorithm

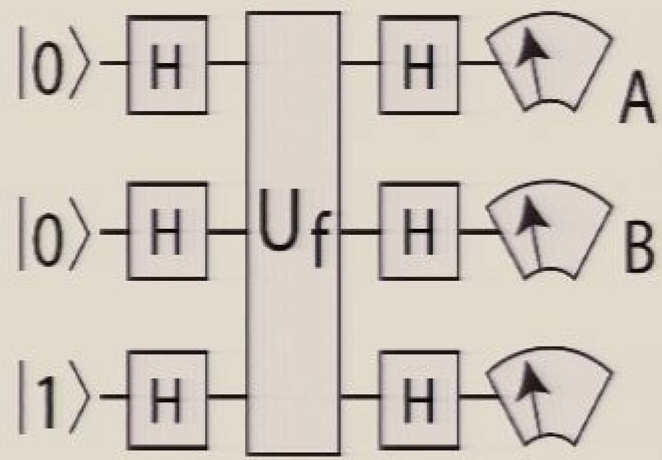


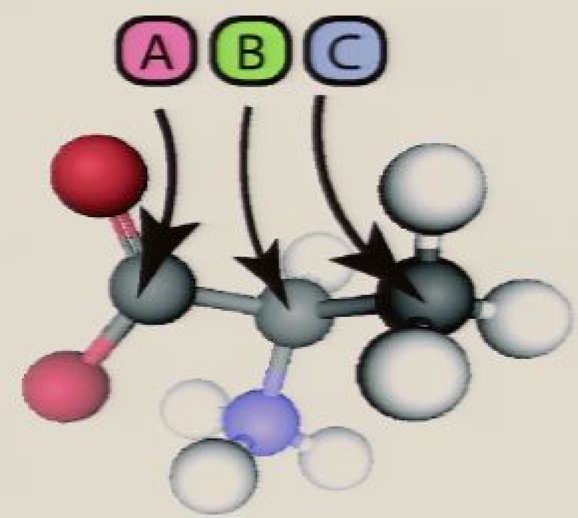
Part 4: Fault Tolerance

Dense Packing

Deutsch-Jozsa Algorithm

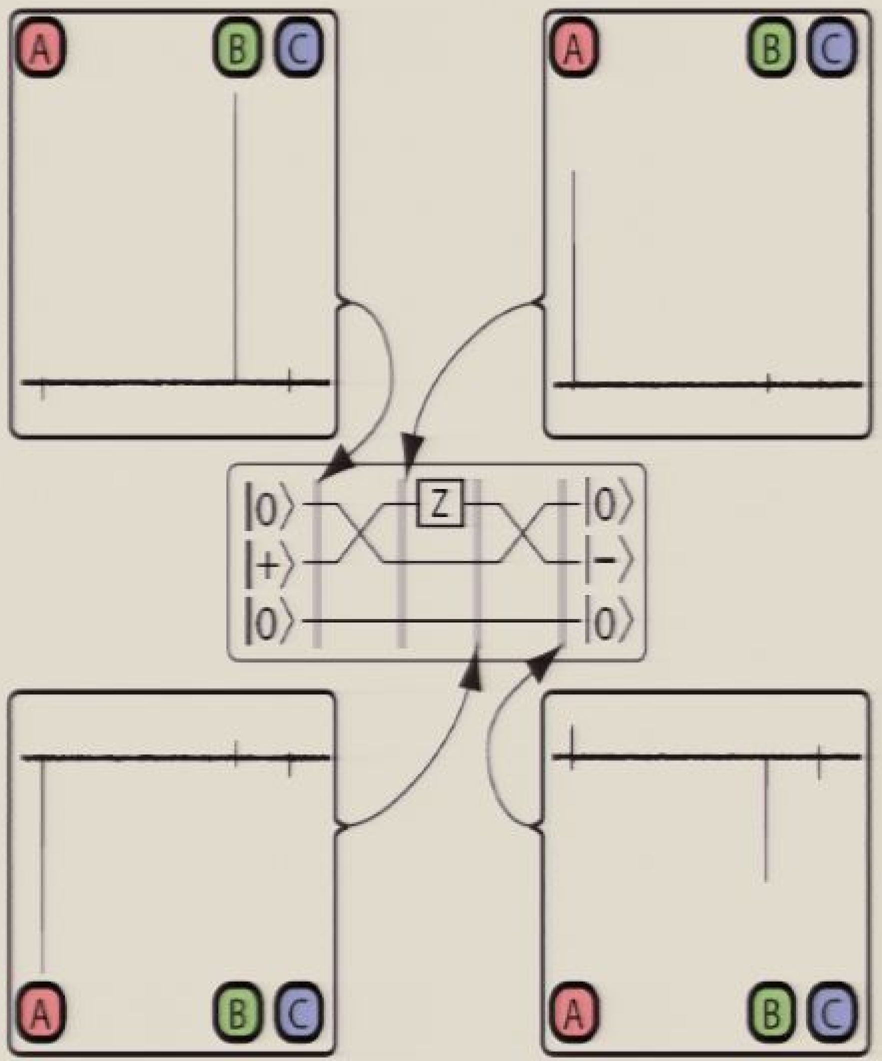






Dense Packing

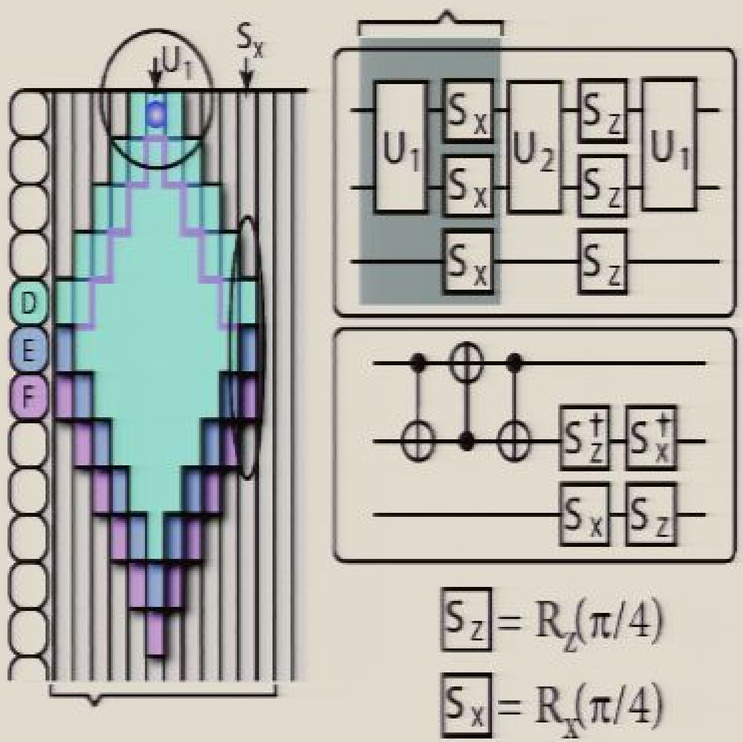
Single Qubit Rotation



Dense Packing

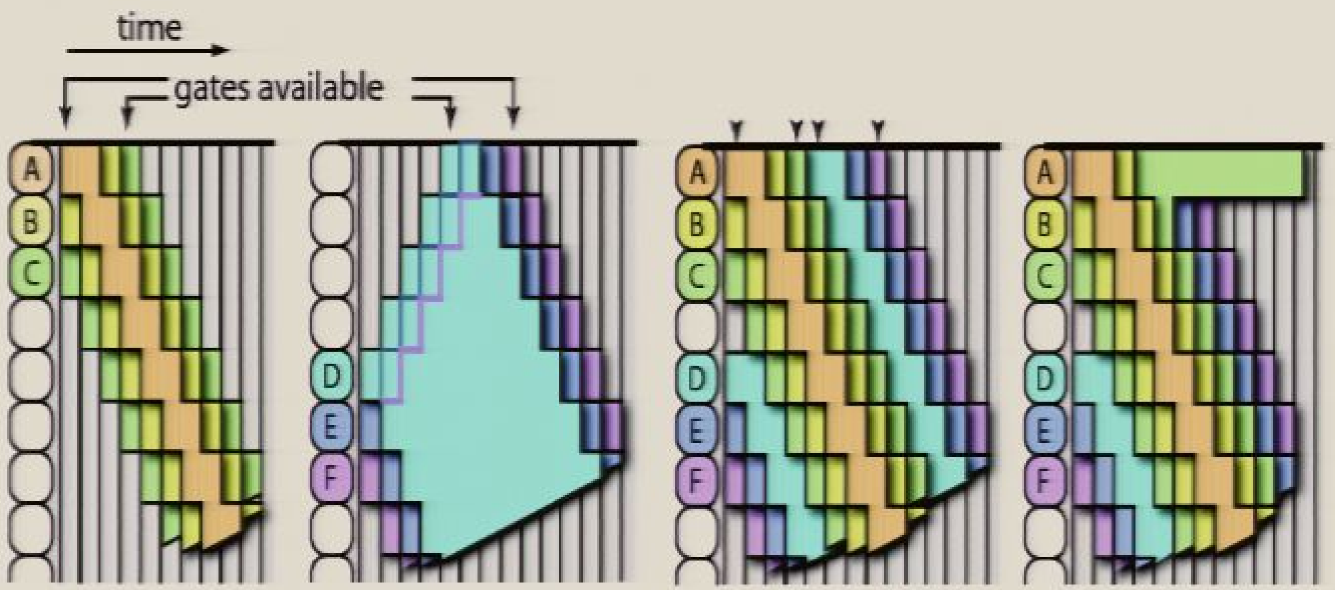
- Qubits on the interior of a block must be swapped to the edge before logic gates can be performed on them

- Trade off between time and space: Gates take O(Nxm) for a block size of m → chain length = (N + N/m−1)



Dense Packing

- Qubits can be packed into blocks
- Blocks are separated by a buffer state
- Qubits at the edge of blocks can undergo gates as described earlier



Block 1

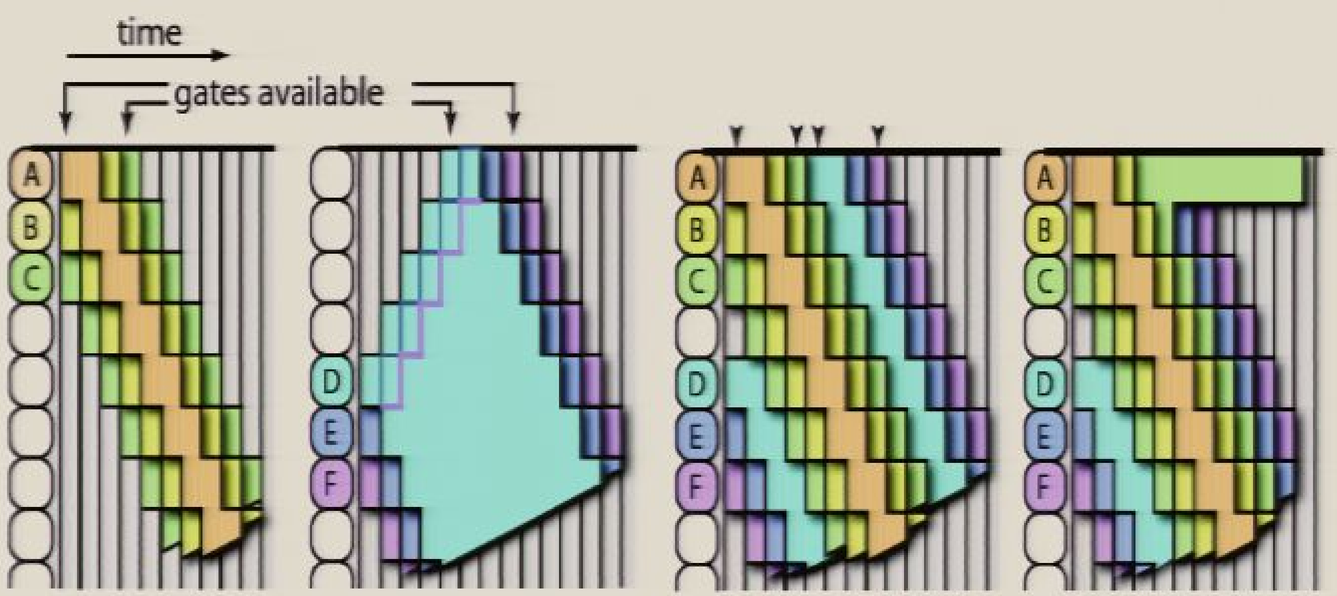
Block 2

combined blocks - single qubit gates

inter-block logic two-qubit gates

Dense Packing

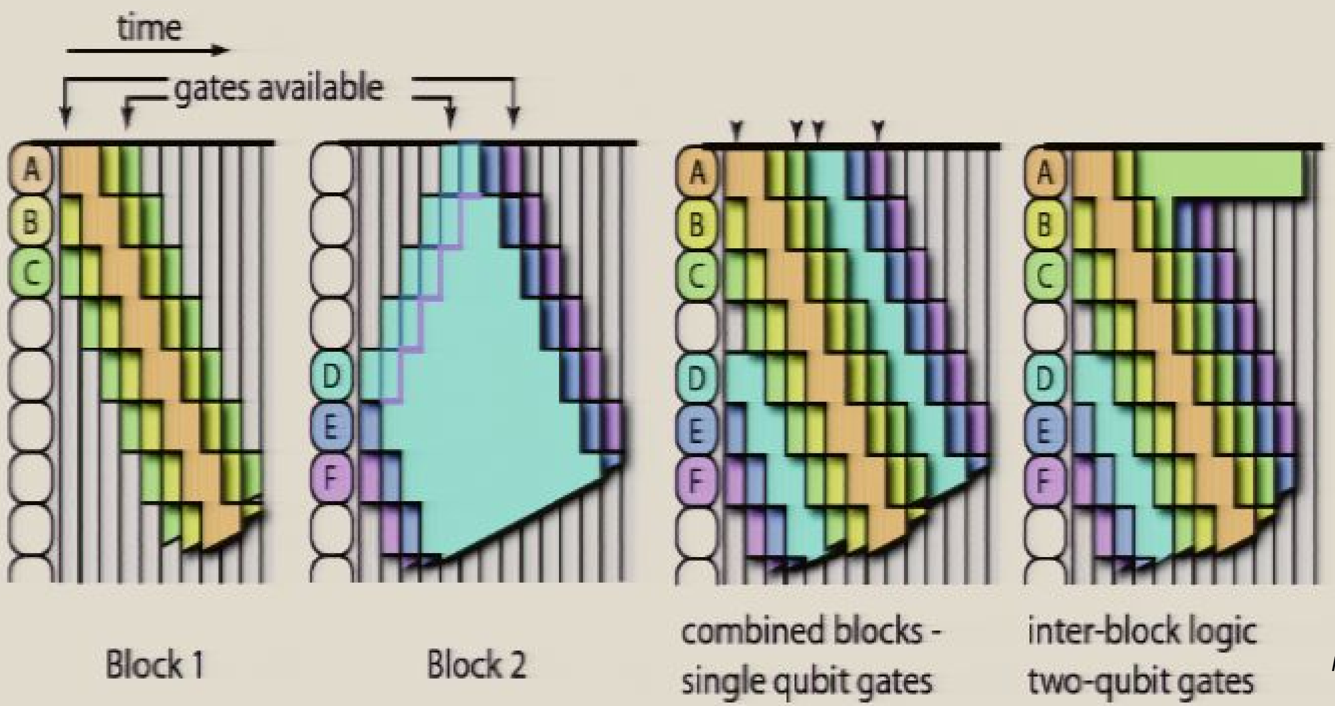
- Qubits can be packed into blocks
- Blocks are separated by a buffer state
- Qubits at the edge of blocks can undergo gates as described earlier



Block 1 | Block 2 | combined blocks - single qubit gates | inter-block logic two-qubit gates

Dense Packing

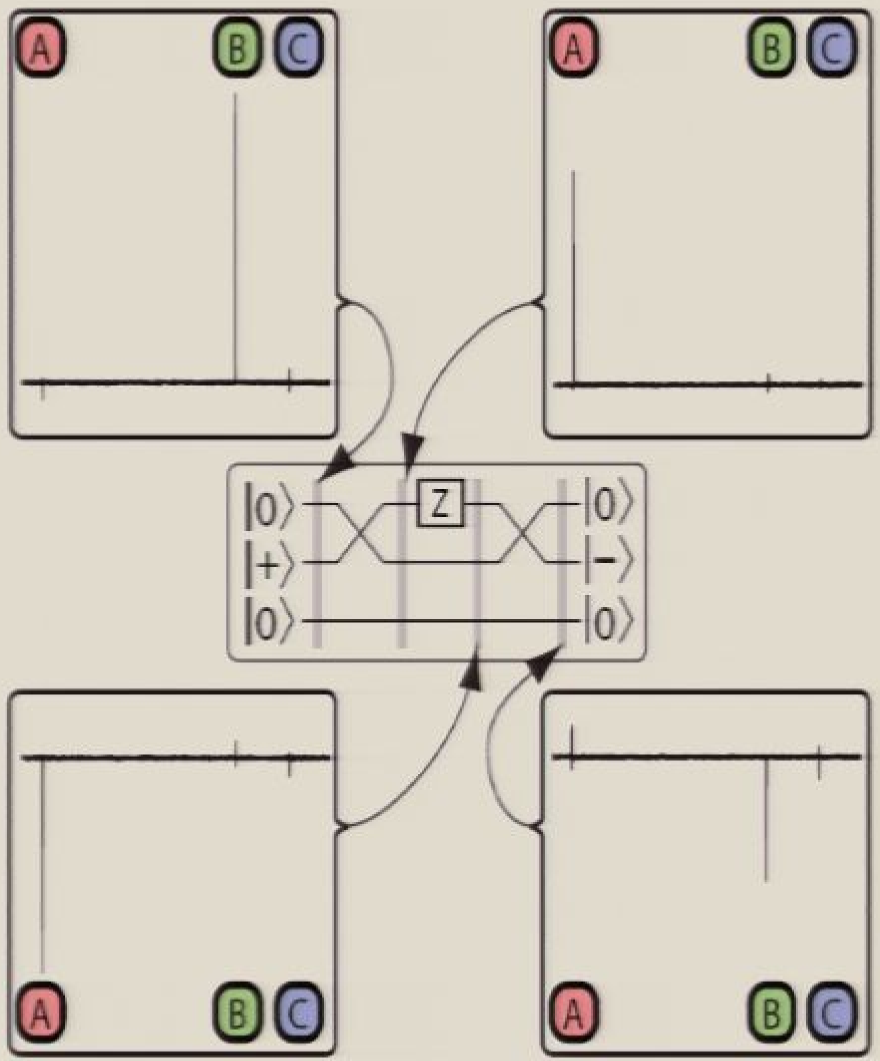
- Qubits can be packed into blocks
- Blocks are separated by a buffer state
- Qubits at the edge of blocks can undergo gates as described earlier



QuNaT

Dense Packing

Single Qubit Rotation



Part 4: Fault Tolerance

QuNaT

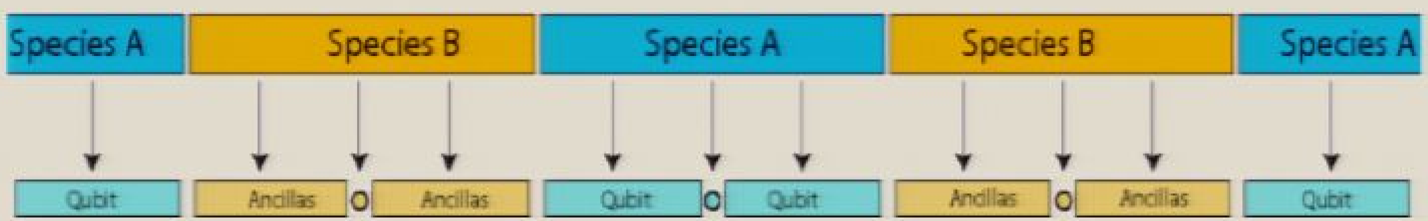
Fault Tolerance

Spin chain layout

- Need at least two species so that ancillas can be reset without destroying the states of the logical qubits.

- Subchains can be decoupled by applying X operator to one species. This will commute with the interaction everywhere except at the boundaries between species.

- Need to be able to reset ancillas without knowing their state
  → Need a lambda level structure for species B



Fault Tolerance

Universal QC on subchains of species A

Procedure:

- Set species B to $|0\rangle$
- Interaction between species becomes a local z-rotation on the last spin of species A.

$$e^{i\theta z_A z_B}|0\rangle_B = e^{i\theta z_A}$$

- Decoupling can be used to control angle of rotation
- Single spin rotations and entangling gates between spins are then accomplished as in the case of a chain of a single species spin chain.

We can now do all mirror symmetric operations on the subchains of species A.

Fault Tolerance

QuNaT

Error correction on subchains of species A

Let each sub-chain of species A be constructed as

$$|Ancilla_1\rangle \otimes |Encoded\ qubit_1\rangle \otimes |Encoded\ qubit_2\rangle \otimes |Ancilla_2\rangle$$

With this layout, error correction is a mirror symmetric operation, and so can be achieved using our original proposal.

Controlled phase operations between spins is also a symmetric operation and so can be achieved within the same framework.

All qubits undergo the same error correction procedure, so it we need only accomplish the same operations on each sub-chain of species A.

Fault Tolerance

Flushing ancillas

The neighbouring qubits in species A and B can be interacted as follows.

The sub-chains are decoupled while controlled phase gates are applied between neighbours in all sub-chains. The sub-chains are then recoupled, allowing controlled phase operations between all neighbouring spins in the chain.

These cancel with the previous controlled phase operations, except at the boundary between species. This allows controlled phase gates to be applied between the end spins of a neighbouring chains.

These gates can be used to swap qubits between chains. The ancillas are swapped species B chain and then flushed.

Fault Tolerance

QuNaT

Flushing ancillas

The neighbouring qubits in species A and B can be interacted as follows.

The sub-chains are decoupled while controlled phase gates are applied between neighbours in all sub-chains. The sub-chains are then recoupled, allowing controlled phase operations between all neighbouring spins in the chain.

These cancel with the previous controlled phase operations, except at the boundary between species. This allows controlled phase gates to be applied between the end spins of a neighbouring chains.

These gates can be used to swap qubits between chains. The ancillas are swapped species B chain and then flushed.

Fault Tolerance

Flushing ancillas

The neighbouring qubits in species A and B can be interacted as follows.

The sub-chains are decoupled while controlled phase gates are applied between neighbours in all sub-chains. The sub-chains are then recoupled, allowing controlled phase operations between all neighbouring spins in the chain.

These cancel with the previous controlled phase operations, except at the boundary between species. This allows controlled phase gates to be applied between the end spins of a neighbouring chains.

These gates can be used to swap qubits between chains. The ancillas are swapped species B chain and then flushed.

Fault Tolerance

Gates between neighbouring blocks of species A

Procedure:

- Target qubits CNOTed with the ends of the ancilla chain

$$(a_1|0\rangle_B + b_1|1\rangle_B) \otimes |0\ldots0\rangle_B \otimes (a_1|0\rangle_B + b_1|1\rangle_B)$$
$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|00\ldots11\rangle + b_1 a_2|11\ldots00\rangle + b_1 b_2|11\ldots11\rangle$$

- Ancilla chain is mirrored

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|01\ldots01\rangle + b_1 a_2|10\ldots10\rangle + b_1 b_2|11\ldots11\rangle$$

- A conditional /8 phase gate is then applied between species as described in the previous slide

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|01\ldots01\rangle + b_1 a_2|10\ldots10\rangle - b_1 b_2|11\ldots11\rangle$$

- The mirroring and CNOTs are then repeated to yield

$$\rightarrow CZ_{1,2}(a_1|0\rangle_B + b_1|1\rangle_B) \otimes |0\ldots0\rangle_B \otimes (a_1|0\rangle_B + b_1|1\rangle_B)$$

Fault Tolerance

Gates between neighbouring blocks of species A

Procedure:

- Target qubits CNOTed with the ends of the ancilla chain

$$(a_1|0\rangle_B + b_1|1\rangle_B) \otimes |0\ldots0\rangle_B \otimes (a_1|0\rangle_B + b_1|1\rangle_B)$$

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|00\ldots11\rangle + b_1 a_2|11\ldots00\rangle + b_1 b_2|11\ldots11\rangle$$

- Ancilla chain is mirrored

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|01\ldots01\rangle + b_1 a_2|10\ldots10\rangle + b_1 b_2|11\ldots11\rangle$$

- A conditional /8 phase gate is then applied between species as described in the previous slide

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|01\ldots01\rangle + b_1 a_2|10\ldots10\rangle - b_1 b_2|11\ldots11\rangle$$

- The mirroring and CNOTs are then repeated to yield

$$\rightarrow CZ_{1,2}(a_1|0\rangle_B + b_1|1\rangle_B) \otimes |0\ldots0\rangle_B \otimes (a_1|0\rangle_B + b_1|1\rangle_B)$$

Fault Tolerance

QuNaT

Gates between neighbouring blocks of species A

Procedure:

- Target qubits CNOTed with the ends of the ancilla chain

$$(a_1|0\rangle_B + b_1|1\rangle_B)\otimes|0\ldots0\rangle_B\otimes(a_1|0\rangle_B + b_1|1\rangle_B)$$
$$\rightarrow a_1a_2|00\ldots00\rangle + a_1b_2|00\ldots11\rangle + b_1a_2|11\ldots00\rangle + b_1b_2|11\ldots11\rangle$$

- Ancilla chain is mirrored

$$\rightarrow a_1a_2|00\ldots00\rangle + a_1b_2|01\ldots01\rangle + b_1a_2|10\ldots10\rangle + b_1b_2|11\ldots11\rangle$$

- A conditional /8 phase gate is then applied between species as described in the previous slide

$$\rightarrow a_1a_2|00\ldots00\rangle + a_1b_2|01\ldots01\rangle + b_1a_2|10\ldots10\rangle - b_1b_2|11\ldots11\rangle$$

- The mirroring and CNOTs are then repeated to yield

$$\rightarrow CZ_{1,2}(a_1|0\rangle_B + b_1|1\rangle_B)\otimes|0\ldots0\rangle_B\otimes(a_1|0\rangle_B + b_1|1\rangle_B)$$

Fault Tolerance

Gates between neighbouring blocks of species A

Procedure:

- Target qubits CNOTed with the ends of the ancilla chain

$$(a_1|0\rangle_B + b_1|1\rangle_B) \otimes |0\ldots0\rangle_B \otimes (a_1|0\rangle_B + b_1|1\rangle_B)$$
$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|00\ldots11\rangle + b_1 a_2|11\ldots00\rangle + b_1 b_2|11\ldots11\rangle$$

- Ancilla chain is mirrored

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|01\ldots01\rangle + b_1 a_2|10\ldots10\rangle + b_1 b_2|11\ldots11\rangle$$

- A conditional /8 phase gate is then applied between species as described in the previous slide

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|01\ldots01\rangle + b_1 a_2|10\ldots10\rangle - b_1 b_2|11\ldots11\rangle$$

- The mirroring and CNOTs are then repeated to yield

$$\rightarrow CZ_{1,2}(a_1|0\rangle_B + b_1|1\rangle_B) \otimes |0\ldots0\rangle_B \otimes (a_1|0\rangle_B + b_1|1\rangle_B)$$

QuNaT

Fault Tolerance

Gates between neighbouring blocks of species A

Procedure:

- Target qubits CNOTed with the ends of the ancilla chain

$$\left(a_1|0\rangle_B + b_1|1\rangle_B\right) \otimes |0\ldots0\rangle_B \otimes \left(a_1|0\rangle_B + b_1|1\rangle_B\right)$$
$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|00\ldots11\rangle + b_1 a_2|11\ldots00\rangle + b_1 b_2|11\ldots11\rangle$$

- Ancilla chain is mirrored

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|01\ldots01\rangle + b_1 a_2|10\ldots10\rangle + b_1 b_2|11\ldots11\rangle$$

- A conditional /8 phase gate is then applied between species as described in the previous slide

$$\rightarrow a_1 a_2|00\ldots00\rangle + a_1 b_2|01\ldots01\rangle + b_1 a_2|10\ldots10\rangle - b_1 b_2|11\ldots11\rangle$$

- The mirroring and CNOTs are then repeated to yield

$$\rightarrow CZ_{1,2}\left(a_1|0\rangle_B + b_1|1\rangle_B\right) \otimes |0\ldots0\rangle_B \otimes \left(a_1|0\rangle_B + b_1|1\rangle_B\right)$$

Fault Tolerance

We can now :

- Perform all operations required to perform error correction
- Perform arbitrary rotations on each qubit, as long as each logical qubit has the same rotation applied
- Interact spins between logical qubits, and hence perform logic gates between qubits in a fault-tolerant manner.

Back at the start! We now have a meta-Ising spin chain where each of the species A chains takes the place of a spin in the original chain.

This meta-chain is self-correcting, and we can apply the original scheme on top of it, without having to worry about error correction or fault tolerance.

Conclusion & Acknowledgements

Conclusion

- Full QC under global control with low overhead
- Experimentally implemented in NMR
- Can be made fault tolerant with the addition of a second species

Acknowledgements

Theory: Jason Twamley, Simon Benjamin

Experiments: Li Xiao, Jonathan Jones

References

- Theory: quant-ph/0601120, *Phys. Rev. Lett.* 97, 090502
  - *Joseph Fitzsimons and Jason Twamley*
- Experiment: quant-ph/0606188
  - *Joseph Fitzsimons, Li Xiao, Simon C. Benjamin, Jonathan A. Jones*

Fault Tolerance

We can now :

- Perform all operations required to perform error correction
- Perform arbitrary rotations on each qubit, as long as each logical qubit has the same rotation applied
- Interact spins between logical qubits, and hence perform logic gates between qubits in a fault-tolerant manner.

Back at the start! We now have a meta-Ising spin chain where each of the species A chains takes the place of a spin in the original chain.

This meta-chain is self-correcting, and we can apply the original scheme on top of it, without having to worry about error correction or fault tolerance.

QuNaT

Conclusion & Acknowledgements

Conclusion

- Full QC under global control with low overhead
- Experimentally implemented in NMR
- Can be made fault tolerant with the addition of a second species

Acknowledgements

Theory: Jason Twamley, Simon Benjamin

Experiments: Li Xiao, Jonathan Jones

References

- Theory: quant-ph/0601120, *Phys. Rev. Lett.* 97, 090502
  - *Joseph Fitzsimons and Jason Twamley*
- Experiment: quant-ph/0606188
  - *Joseph Fitzsimons, Li Xiao, Simon C. Benjamin, Jonathan A. Jones*

QuNaT

Conclusion & Acknowledgements

Conclusion

- Full QC under global control with low overhead
- Experimentally implemented in NMR
- Can be made fault tolerant with the addition of a second species

Acknowledgements

Theory: Jason Twamley, Simon Benjamin

Experiments: Li Xiao, Jonathan Jones

References

- Theory: quant-ph/0601120, *Phys. Rev. Lett.* 97, 090502
  - *Joseph Fitzsimons and Jason Twamley*
- Experiment: quant-ph/0606188
  - *Joseph Fitzsimons, Li Xiao, Simon C. Benjamin, Jonathan A. Jones*

Conclusion & Acknowledgements

Conclusion

- Full QC under global control with low overhead
- Experimentally implemented in NMR
- Can be made fault tolerant with the addition of a second species

Acknowledgements

Theory: Jason Twamley, Simon Benjamin

Experiments: Li Xiao, Jonathan Jones

References

- Theory: quant-ph/0601120, *Phys. Rev. Lett.* 97, 090502
  - *Joseph Fitzsimons and Jason Twamley*
- Experiment: quant-ph/0606188
  - *Joseph Fitzsimons, Li Xiao, Simon C. Benjamin, Jonathan A. Jones*