

Title: TBA

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Abstract: TBA

Outline

- Quantum state discrimination
 - Measurements in Quantum Mechanics
 - State discrimination strategies (theory & expt.)
- Maximum confidence measurements -Theory
Stephen Barnett, John Jeffers, Erika Andersson,
Claire Gilson
- Experiment
Peter Mosley, Ian Walmsley
- Conclusions and Future Work

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Quantum State Discrimination

- Non-orthogonal quantum states cannot be perfectly distinguished
 - Outcome of measurements probabilistic
 - Measurement disturbs state
- Advantages in using non-orthogonal states for e.g. quantum communication
- Would like to know ‘how well’ it is possible to discriminate between members of a given set

Measurements in Quantum Mechanics

- States represented as normalised state kets in a complex, linear vector space; $|\Psi\rangle$
- Von Neumann measurement along orthogonal directions $\{|i\rangle\}$
- Probability of obtaining result i :

$$P(i|\Psi) = |\langle\Psi|i\rangle|^2$$

- More general measurements also possible, outcomes ω_i associated with operators $\hat{\Pi}_i$, can define probabilities

$$P(\omega_i|\Psi) = \langle\Psi|\hat{\Pi}_i|\Psi\rangle = \text{Tr}(\hat{\rho}\hat{\Pi}_i)$$

Generalised Measurements

- Described by operators satisfying:

$$\sum_i \hat{\Pi}_i = \hat{I}$$

$$\hat{\Pi}_i^\dagger = \hat{\Pi}_i$$

$$\hat{\Pi}_i \geq 0$$

- Probability of obtaining result ω_i :

$$P(\omega_i | \Psi) = \langle \Psi | \hat{\Pi}_i | \Psi \rangle = \text{Tr}(\hat{\rho} \hat{\Pi}_i)$$

- Probability Operator Measure (POM) or Positive Operator Valued Measure (POVM)

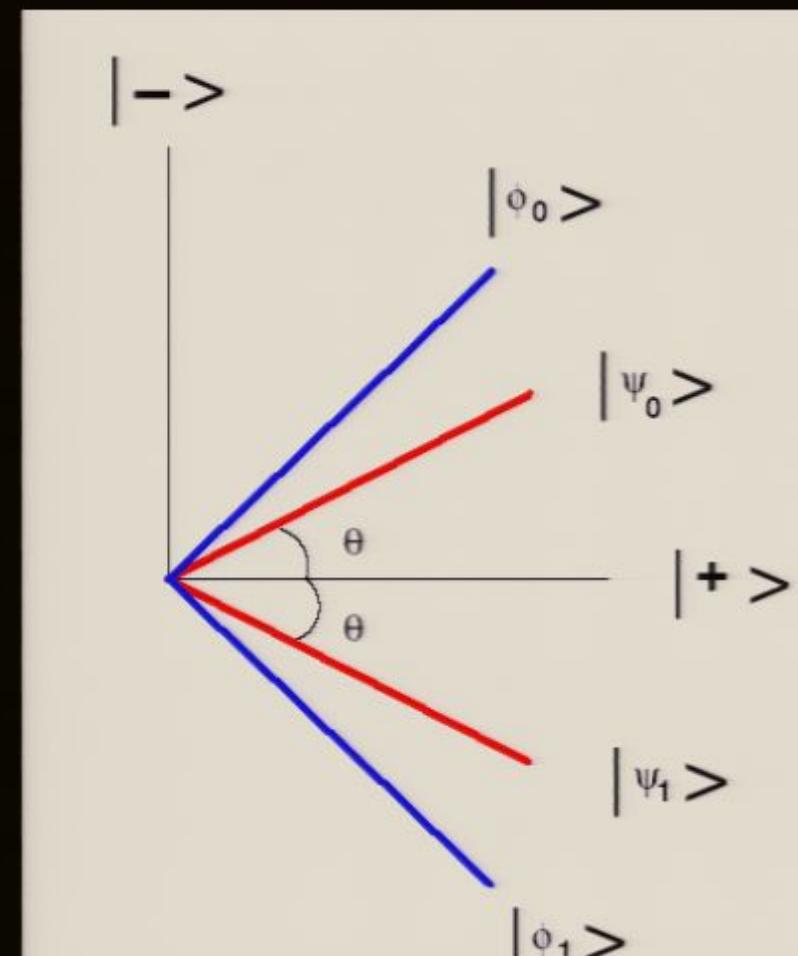
Minimum Error Measurement

- Minimises the probability of making an error

$$P_E = 1 - \sum_j P(\psi_j) P(\omega_j | \psi_j)$$
$$= 1 - \sum_j p_j \text{Tr}(\hat{\rho}_j \hat{H}_j)$$

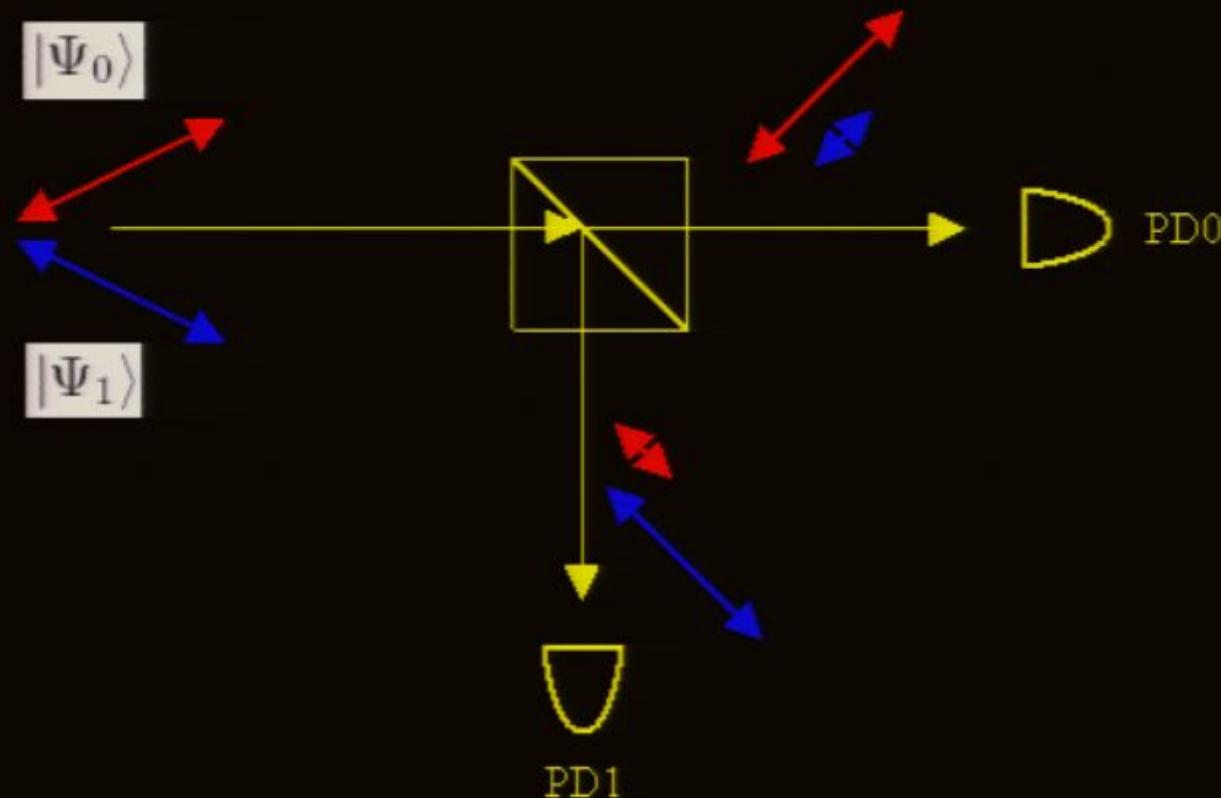
- Measurement known in certain cases, e.g. 2 states in a 2-D space

Helstrom, 1976; Holevo 1973; Yuen et al, 1973



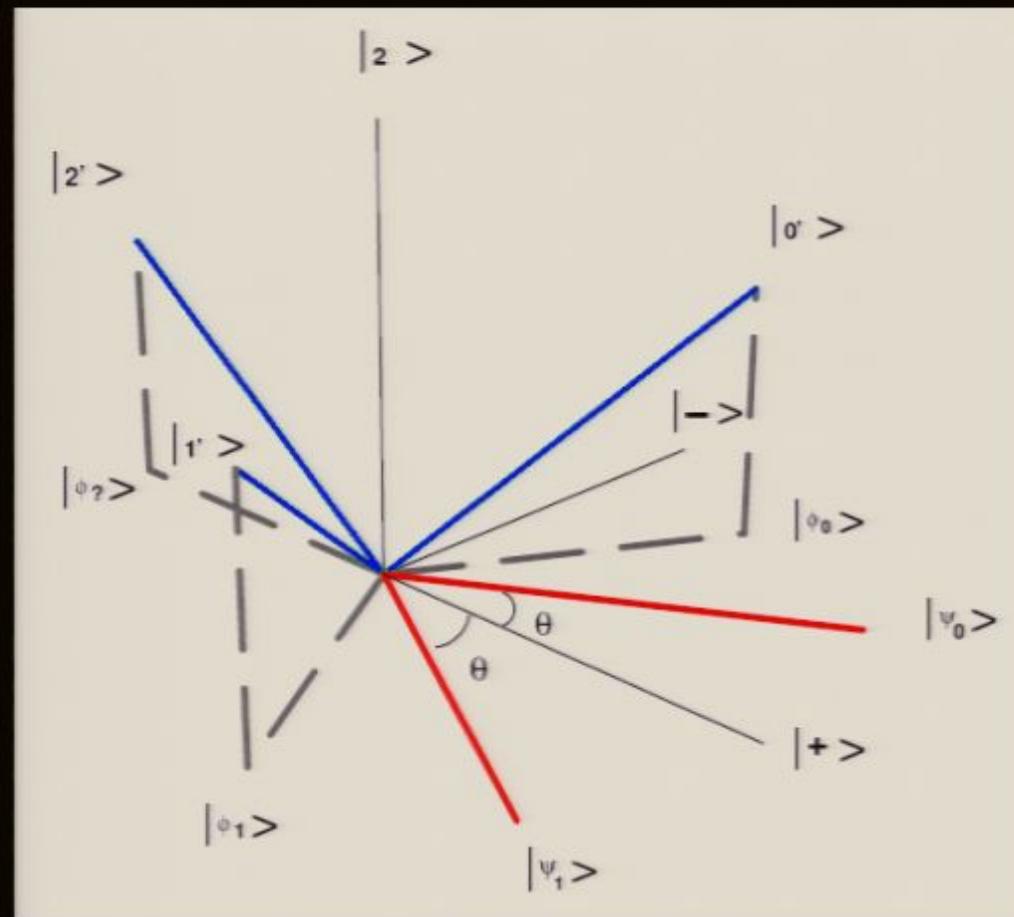
Optical Implementation

Barnett and Riis, 1997

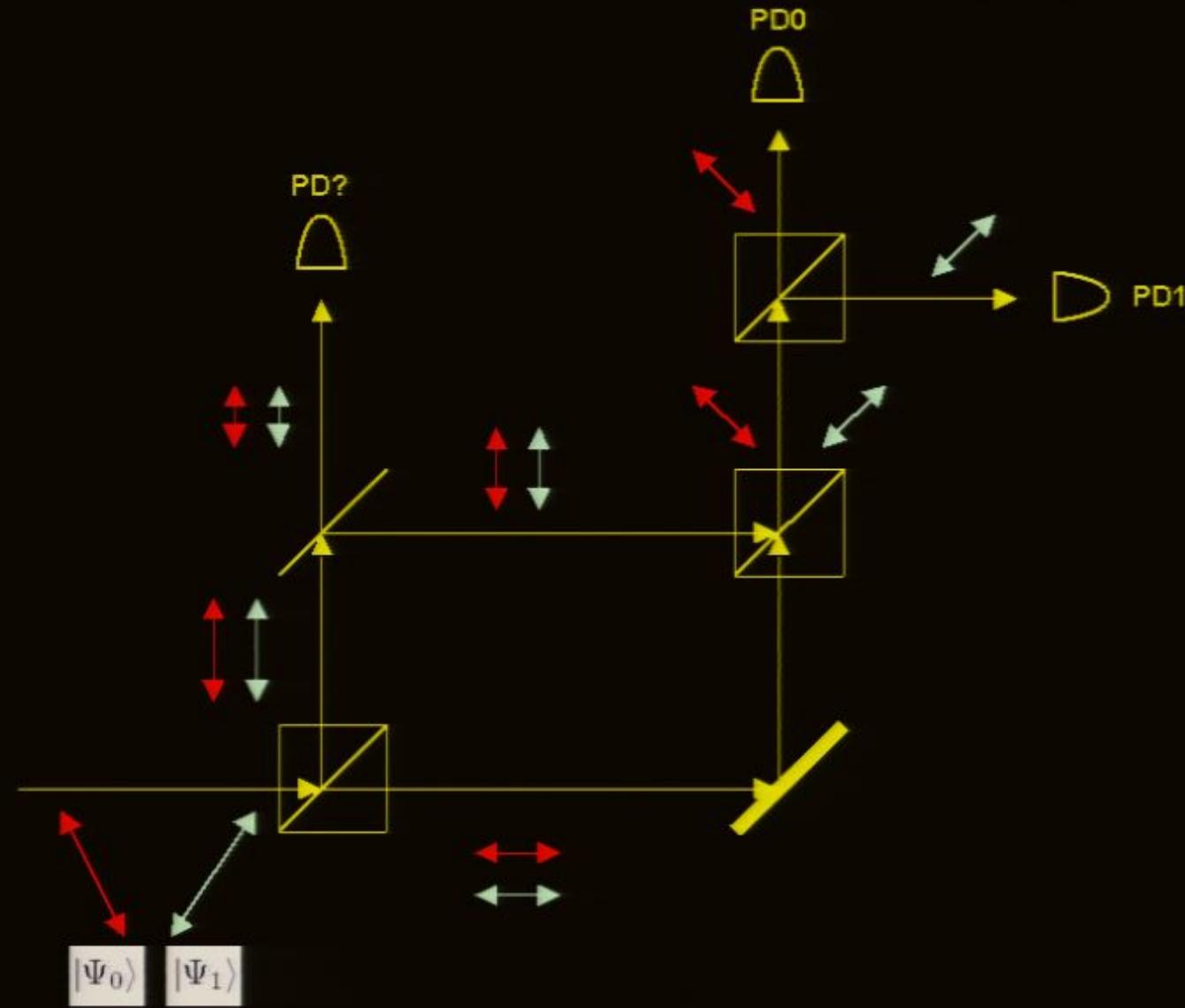


Unambiguous Discrimination

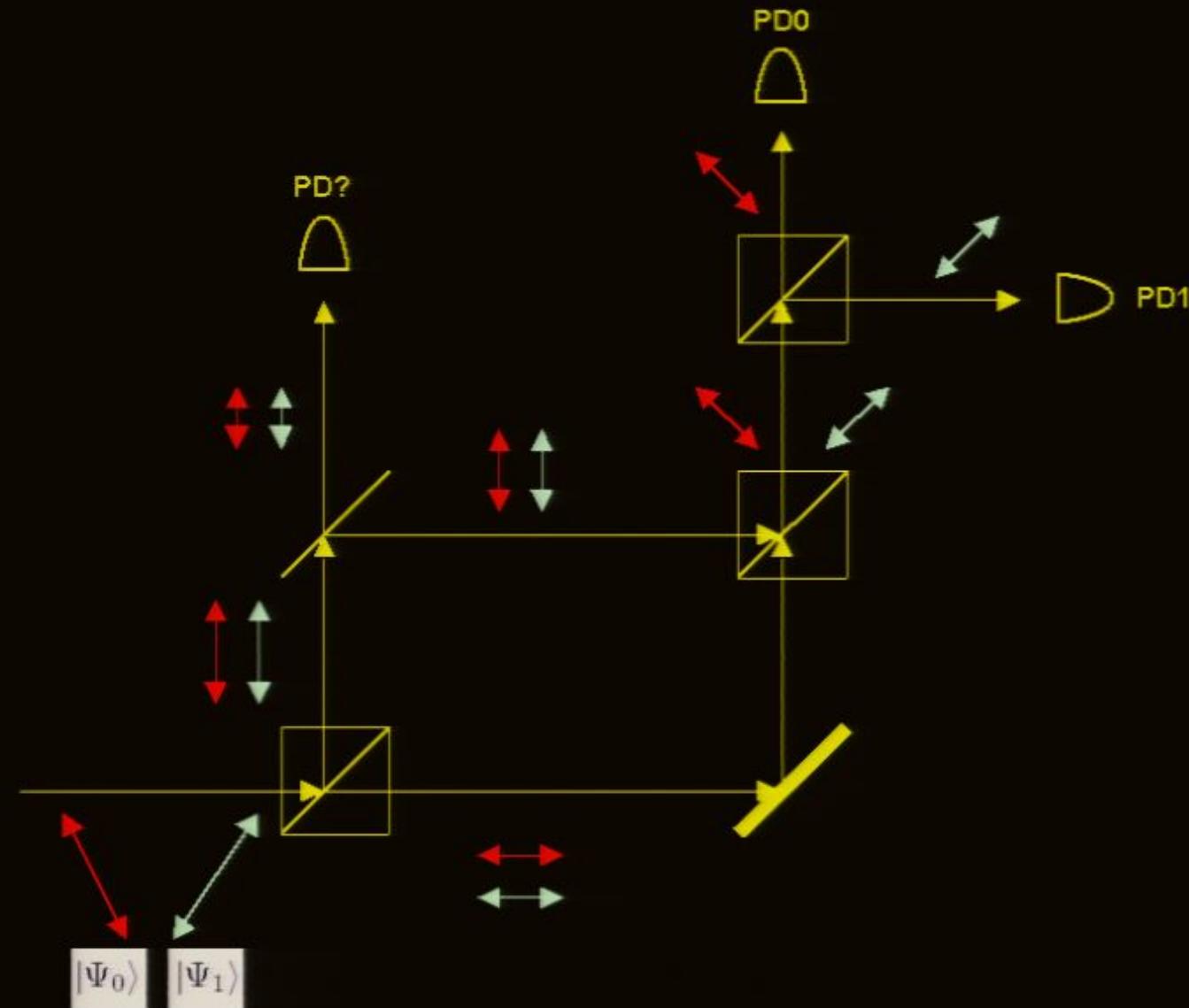
- If $P(\omega_j|\psi_i) = 0$,
 $\forall i \neq j$ then
unambiguous
discrimination is
possible. (IDP, 1987,
1988)
- Inconclusive
result needed
- Only possible for
linearly
independent sets
(Chefles, 1998)



Optical Implementation –Huttner *et al*, 1996, Clarke *et al*. 2001



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Maximum Confidence Measurements

- Bayes' Rule:

$$P(\psi_i|\omega_j) = \frac{P(\psi_i)P(\omega_j|\psi_i)}{P(\omega_j)} = \frac{p_i \langle \psi_i | \hat{\Pi}_j | \psi_i \rangle}{\sum_k p_k \langle \psi_k | \hat{\Pi}_j | \psi_k \rangle}$$

- If result ω_j is taken to imply that the initial state was $|\psi_j\rangle$, the probability that this is correct is $P(\psi_j|\omega_j)$
- This figure of merit is maximised by the Maximum Confidence measurement strategy
 - S Croke, E Andersson, S M Barnett, C R Gilson, J Jeffers, PRL 96, 070401 (2006)

Optimisation:

$$P(\hat{\rho}_j | \omega_j) = \frac{p_j \text{Tr}(\hat{\rho}_j \hat{H}_j)}{\text{Tr}(\hat{\rho} \hat{H}_j)},$$

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$$= \text{Tr} (\zeta_j \rho \rho^{-1/2} Q_j \rho^{-1/2})$$

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Example

- 3 states in a 2-dimensional space

$$\begin{aligned} |\Psi_0\rangle &= \cos\theta|0\rangle + \sin\theta|1\rangle \\ |\Psi_1\rangle &= \cos\theta|0\rangle + e^{2\pi i/3}\sin\theta|1\rangle \\ |\Psi_2\rangle &= \cos\theta|0\rangle + e^{-2\pi i/3}\sin\theta|1\rangle \end{aligned}$$

- Maximum Confidence Measurement:

$$\hat{\Pi}_j = a_j |\phi_j\rangle\langle\phi_j|$$

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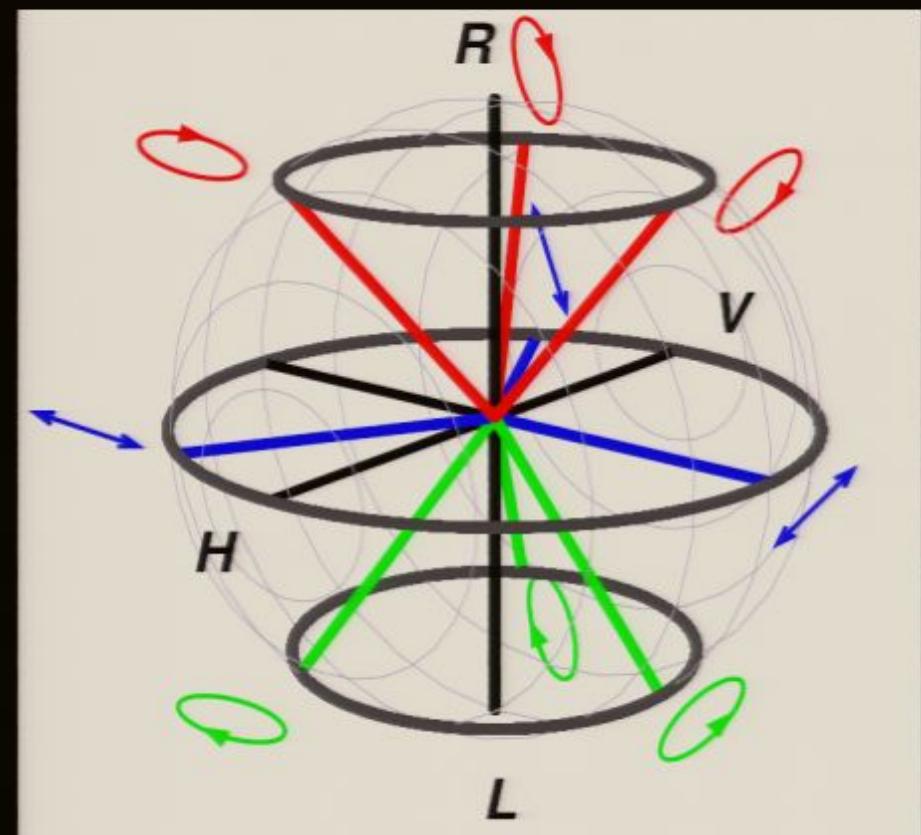
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$$\hat{\Pi}_j^{ME} = \frac{2}{3} |\phi_j^{ME}\rangle\langle\phi_j^{ME}|$$



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$$\rho = \cos^2 \theta | \alpha \times \beta | + \sin^2 \theta | |\alpha| |\beta|$$



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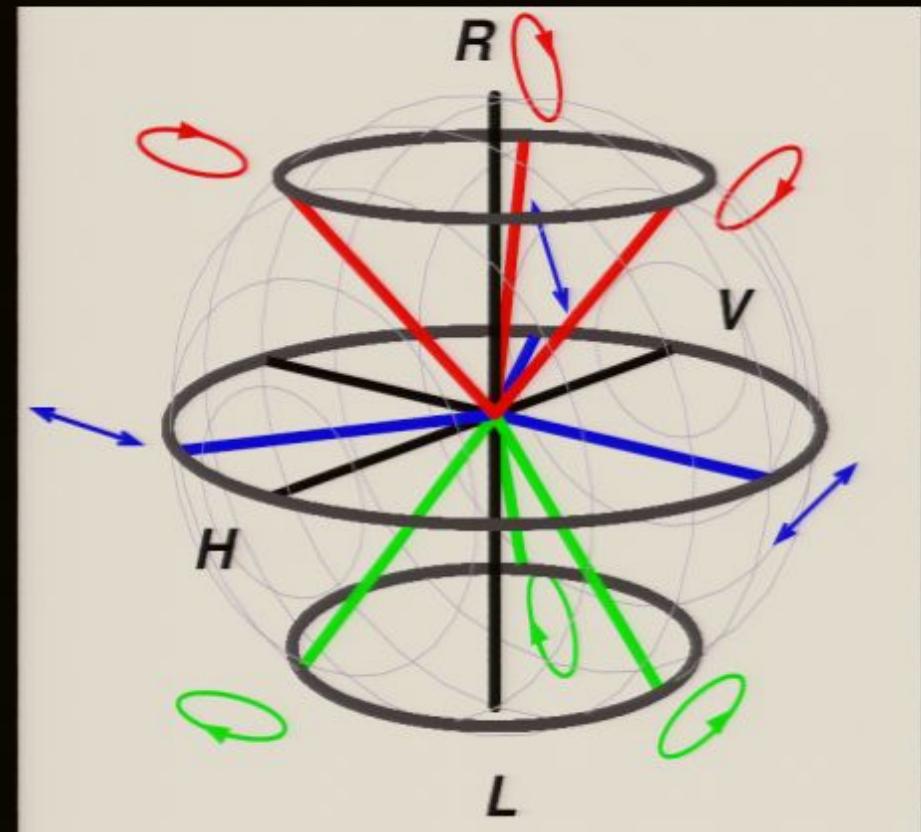
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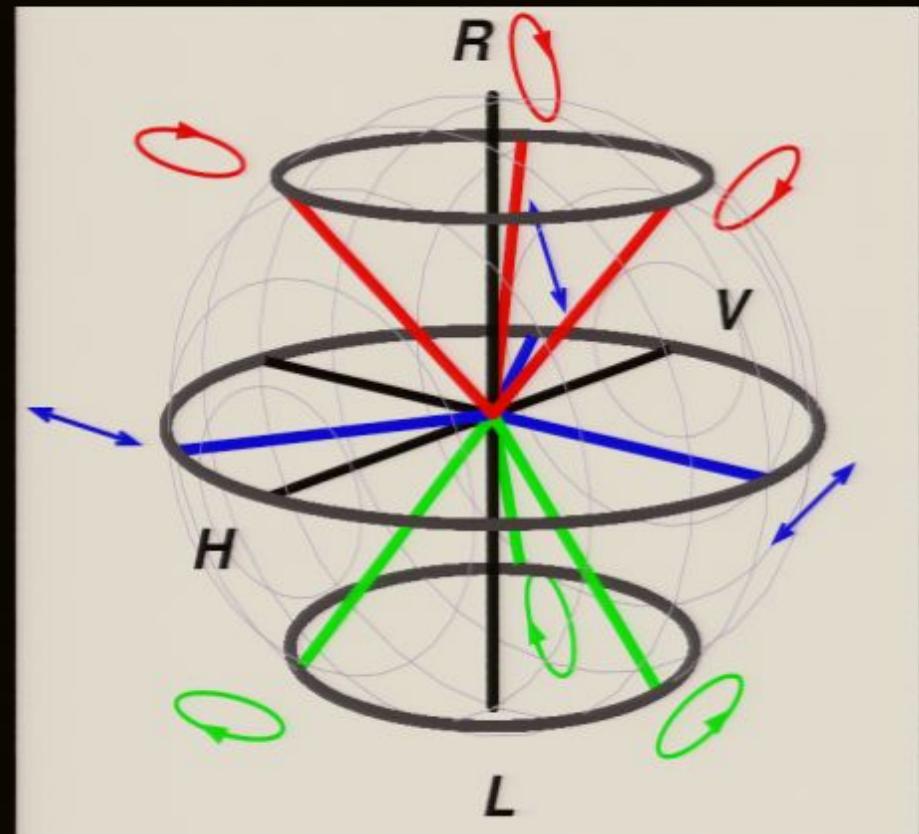


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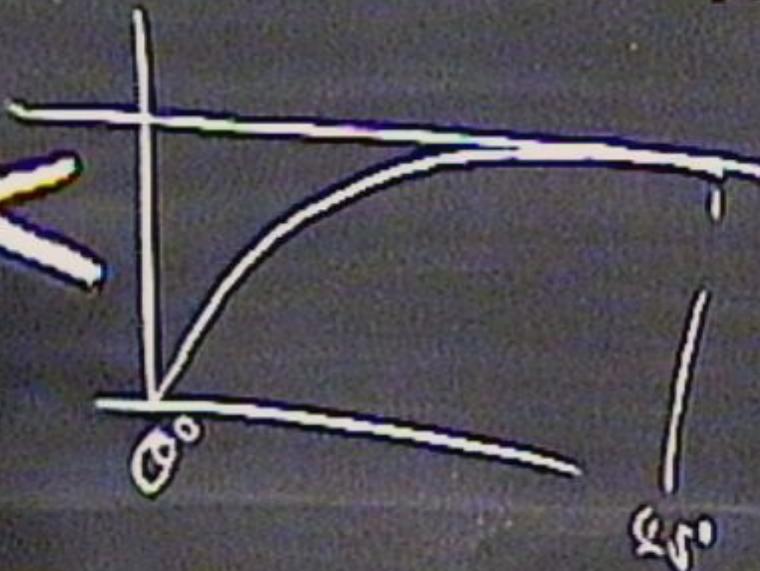
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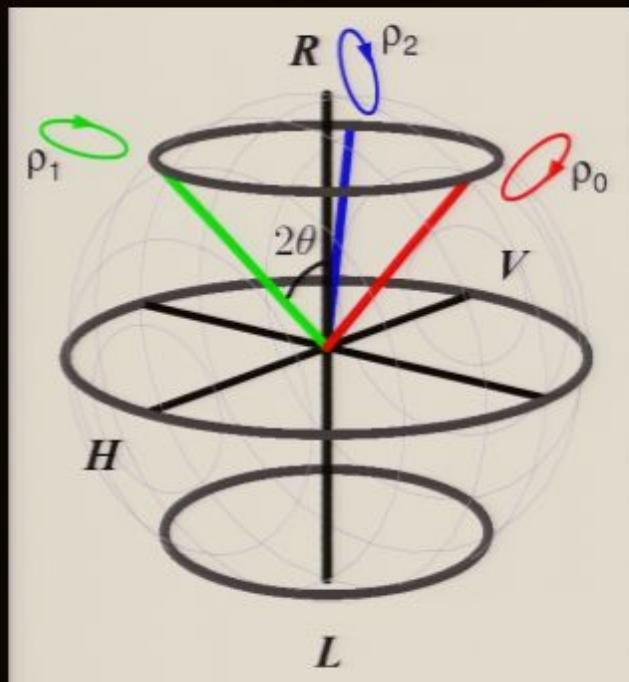
$$\begin{aligned}|\phi_0^{ME}\rangle &= \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \\ |\phi_1^{ME}\rangle &= \frac{1}{\sqrt{2}}(|+\rangle + e^{2\pi i/3}|-\rangle) \\ |\phi_2^{ME}\rangle &= \frac{1}{\sqrt{2}}(|+\rangle + e^{-2\pi i/3}|-\rangle)\end{aligned}$$



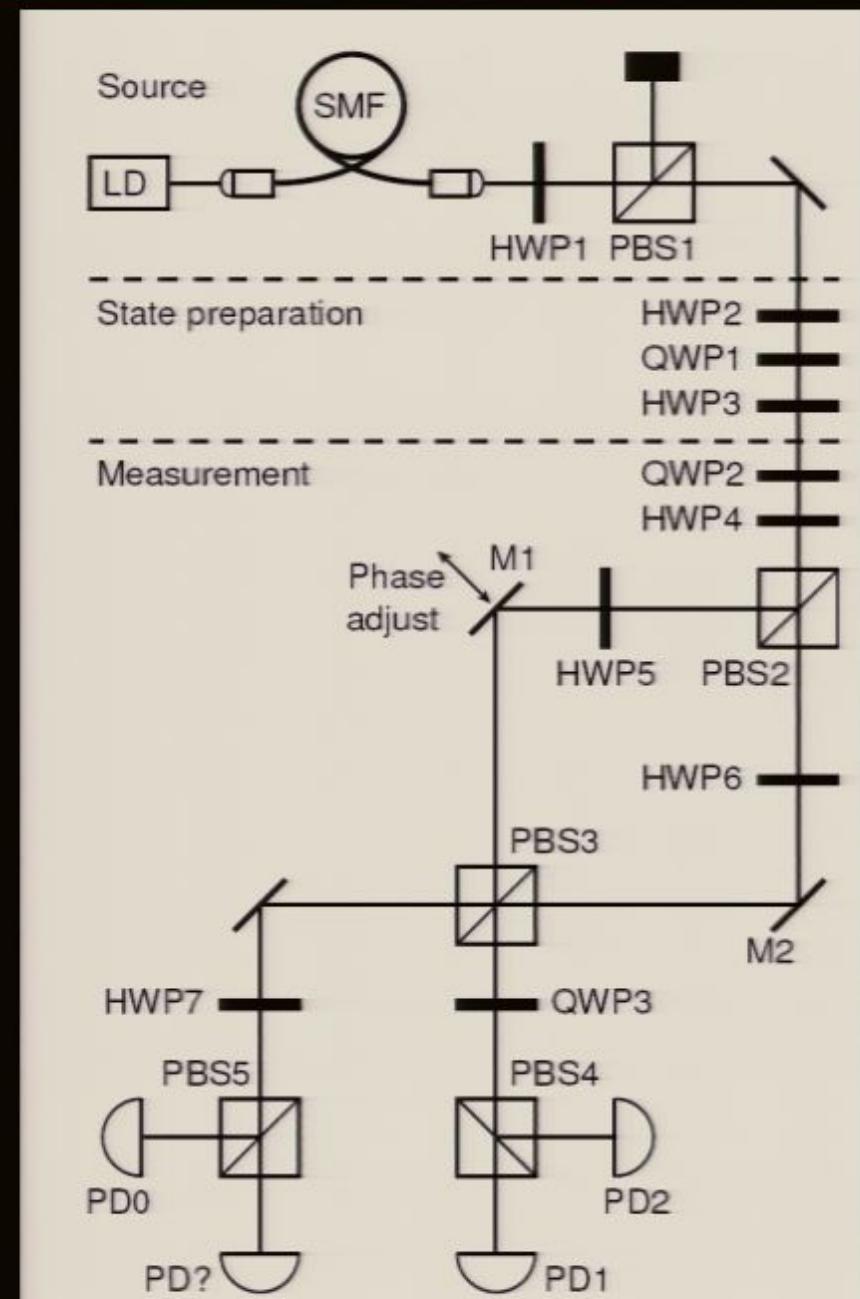
$$\rho = \cos^2 \theta |0\rangle\langle 0| + \sin^2 \theta |1\rangle\langle 1|$$



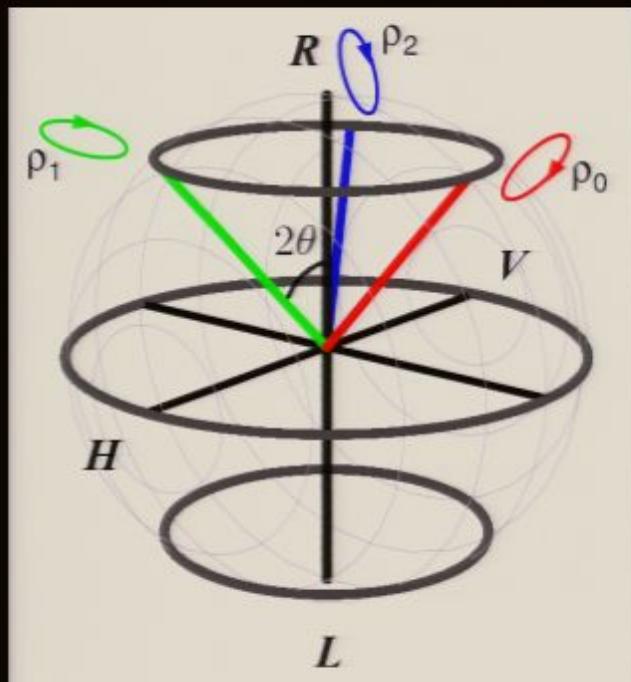
Experimental Implementation



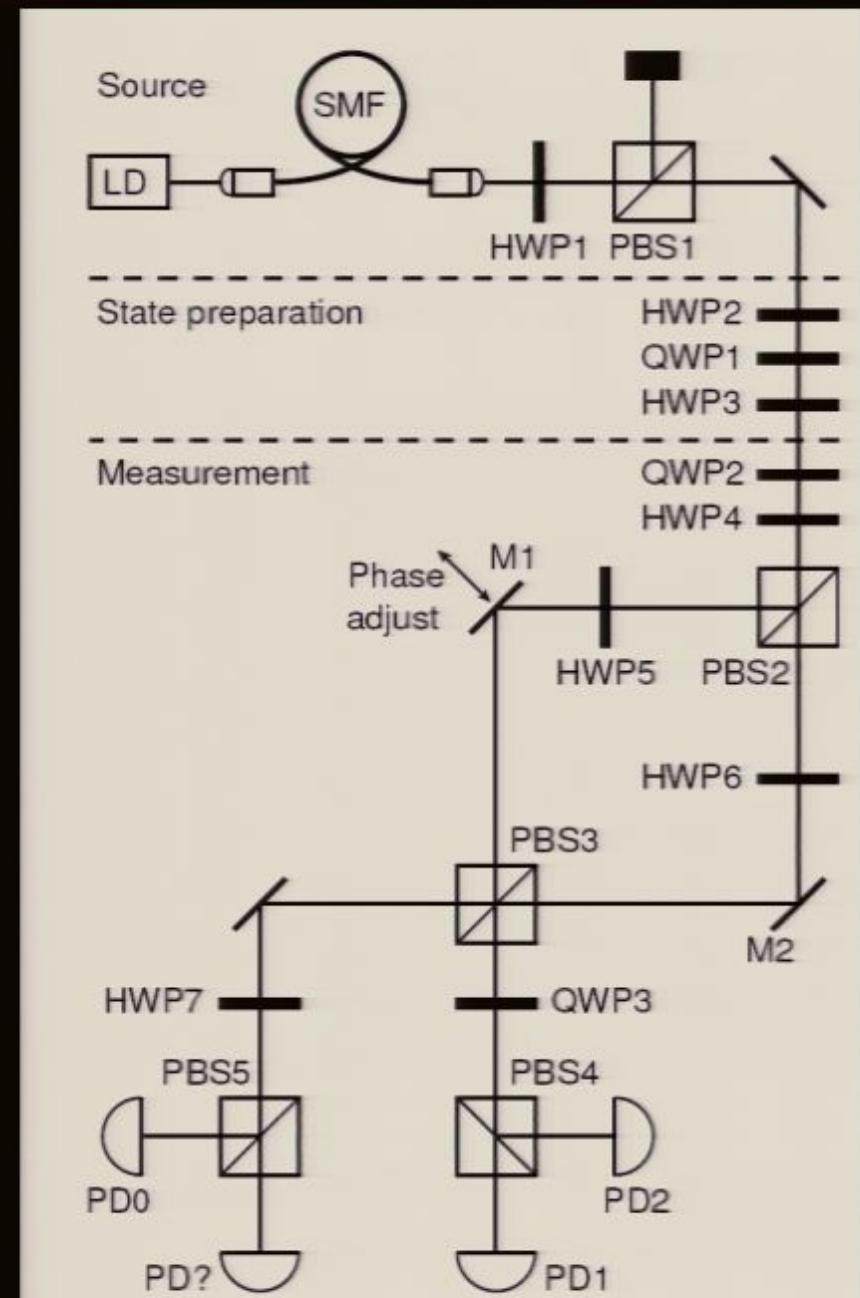
P J Mosley, S Croke, I A Walmsley, S M Barnett, PRL 97, 193601 (2006)



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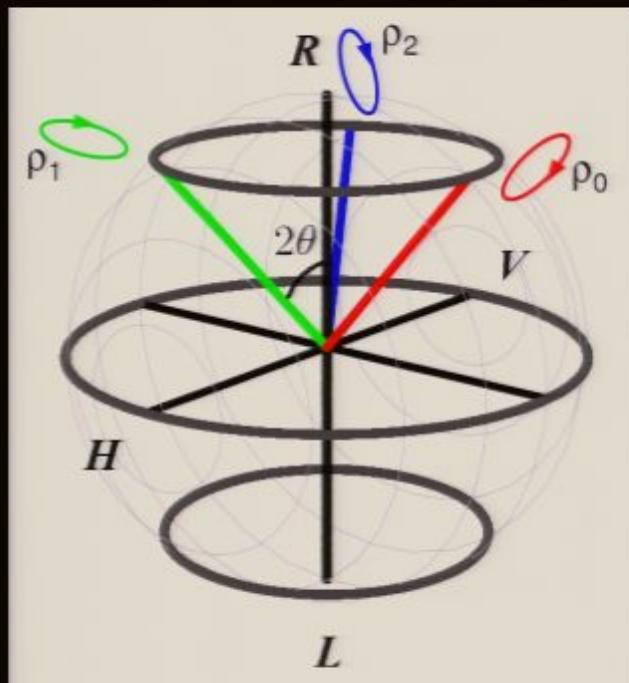
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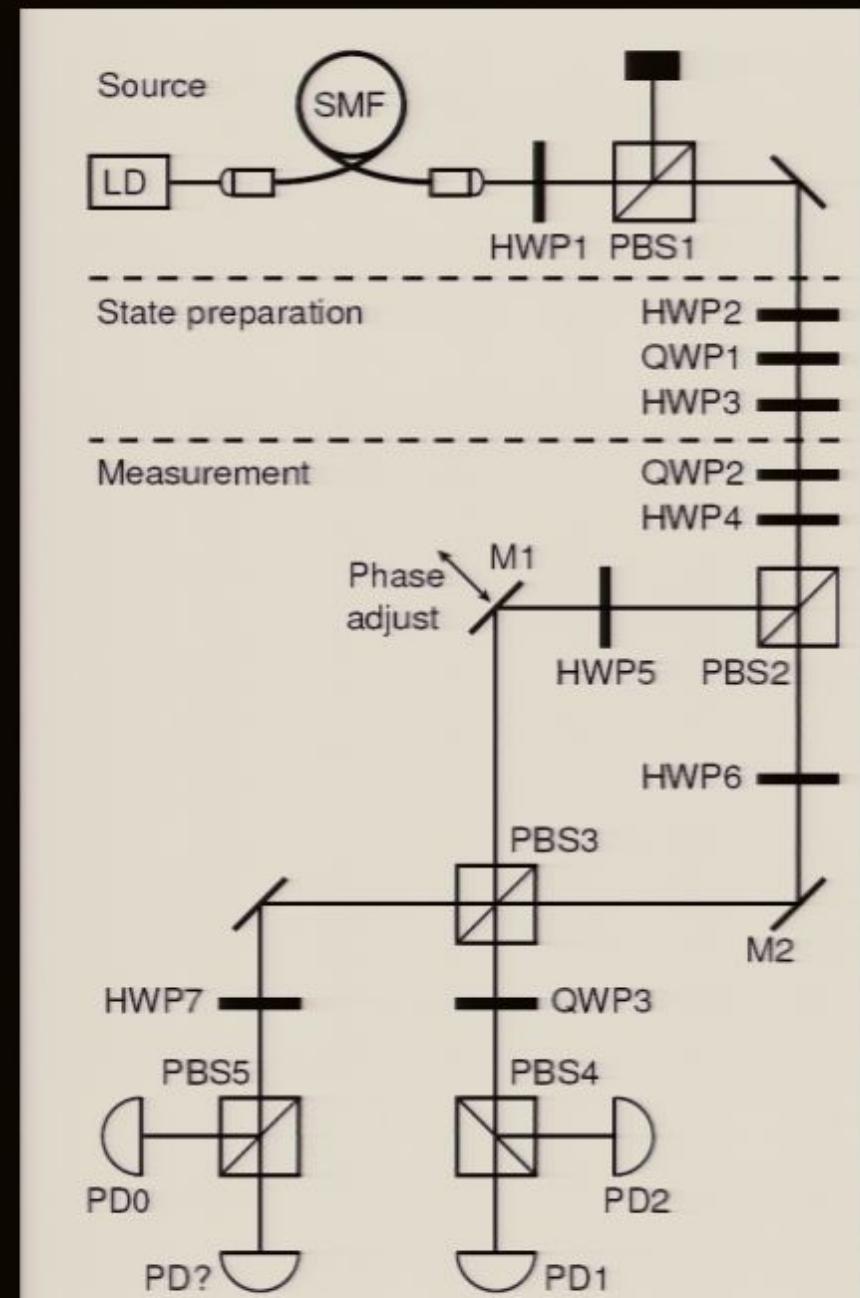
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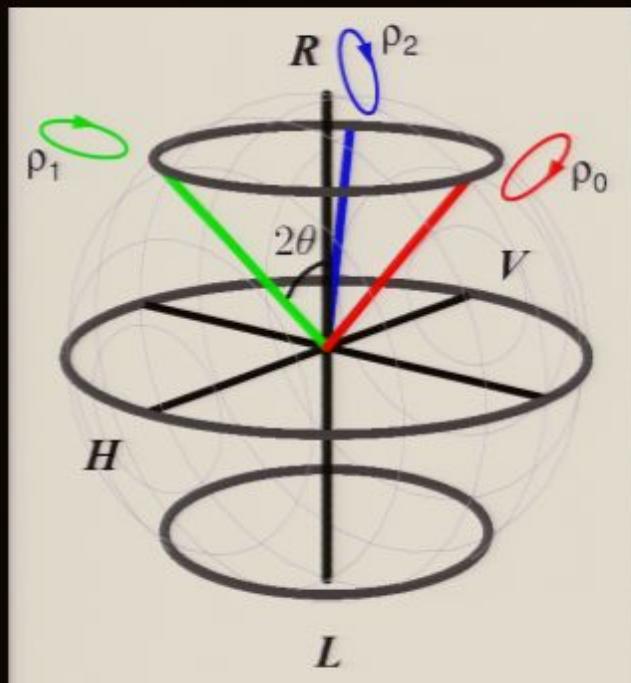
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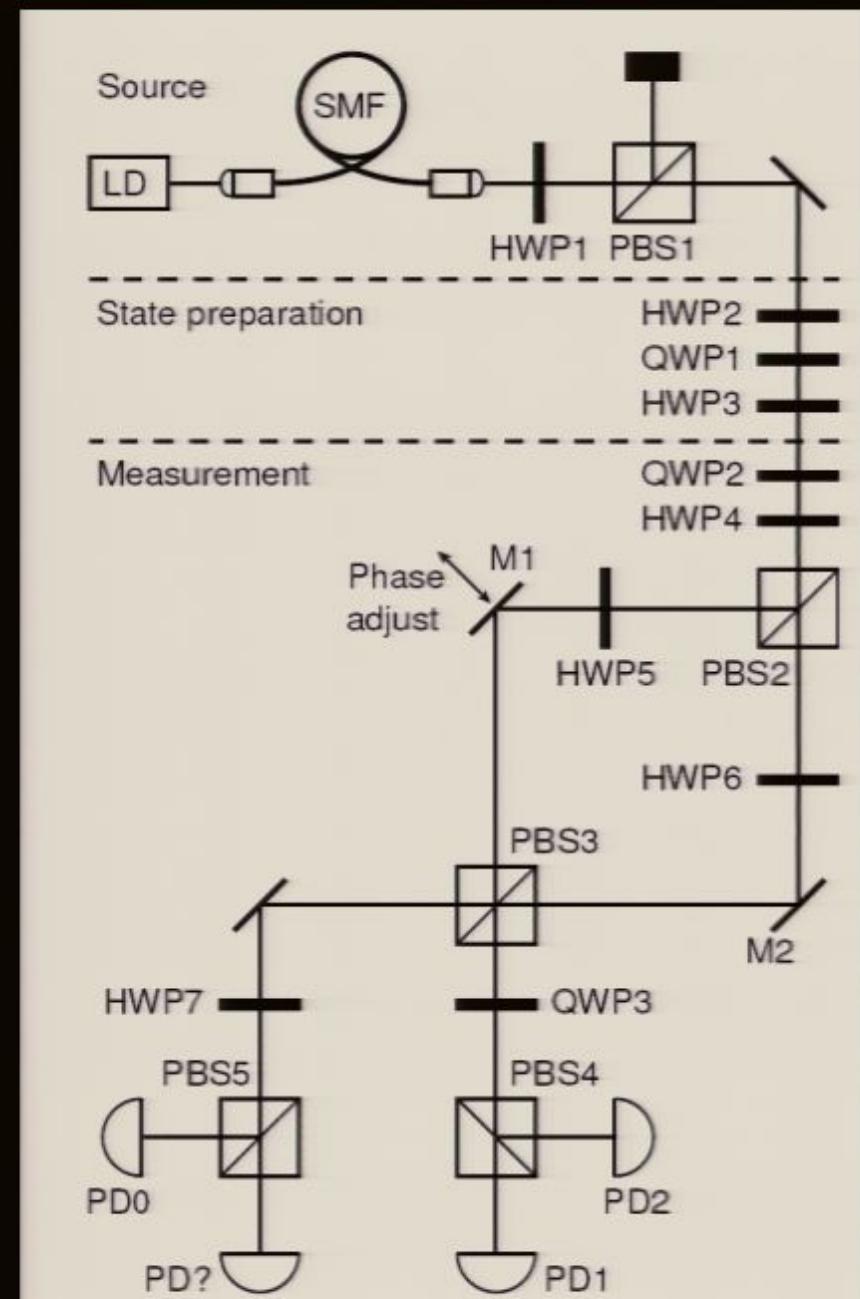
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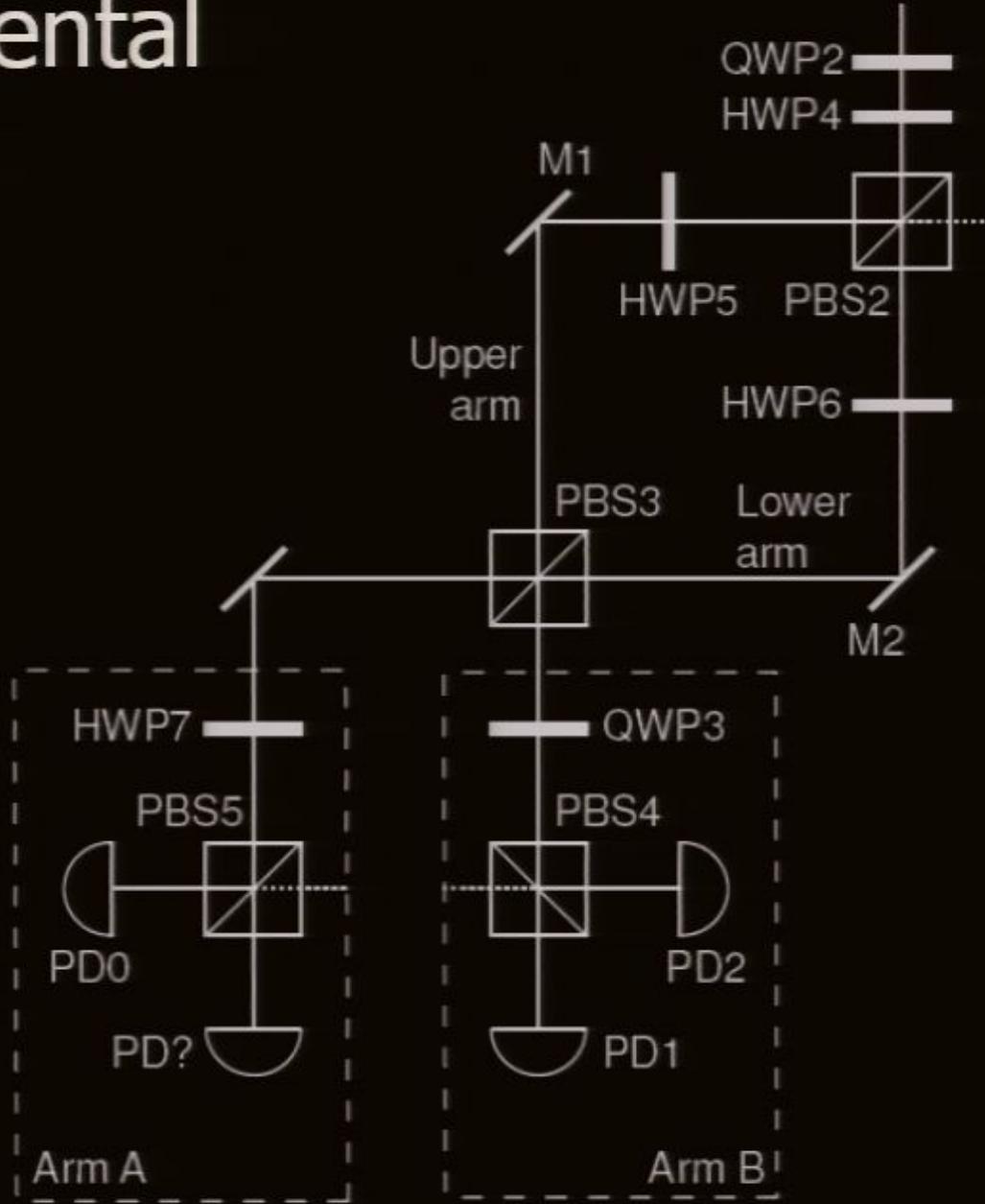
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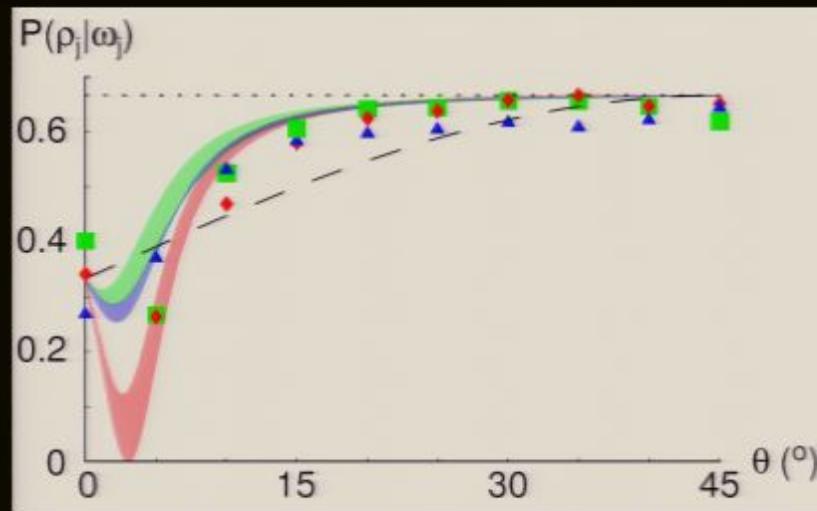
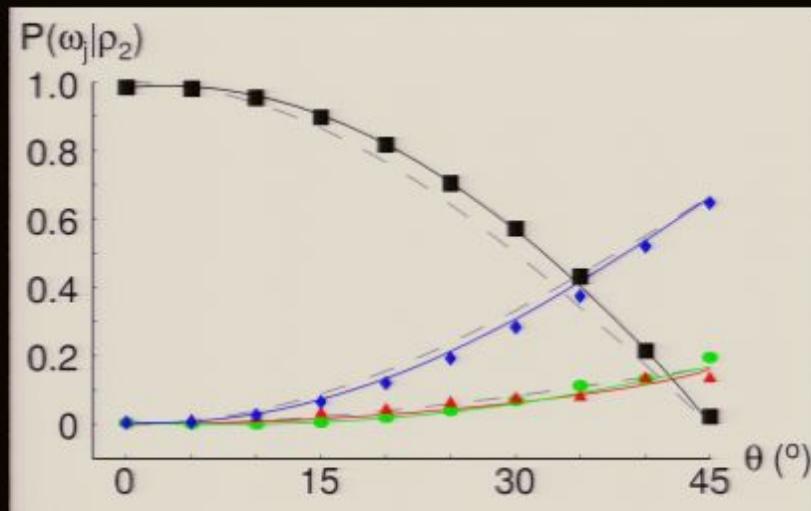
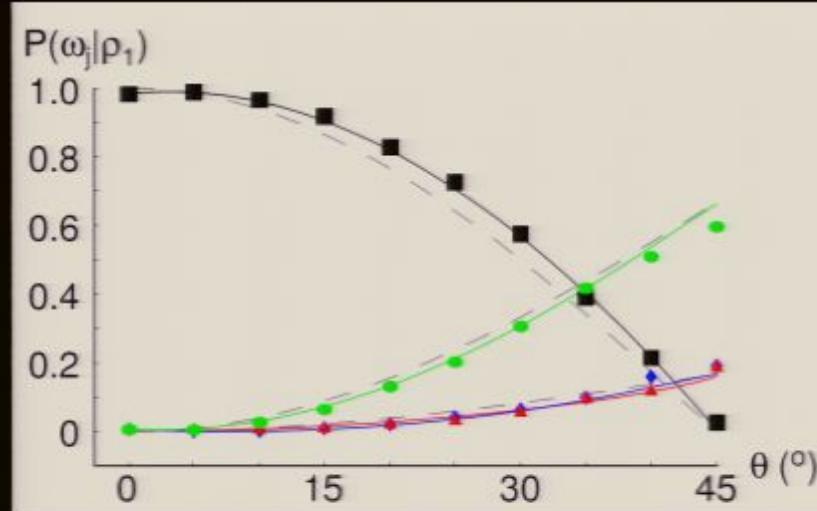
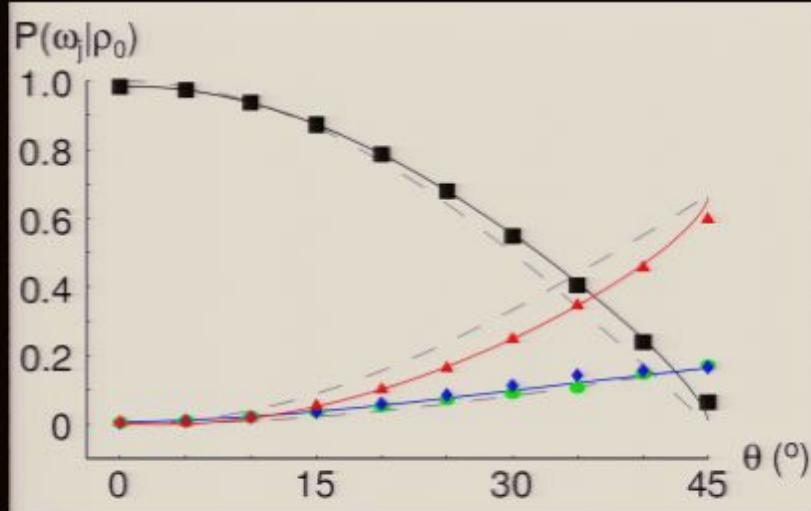
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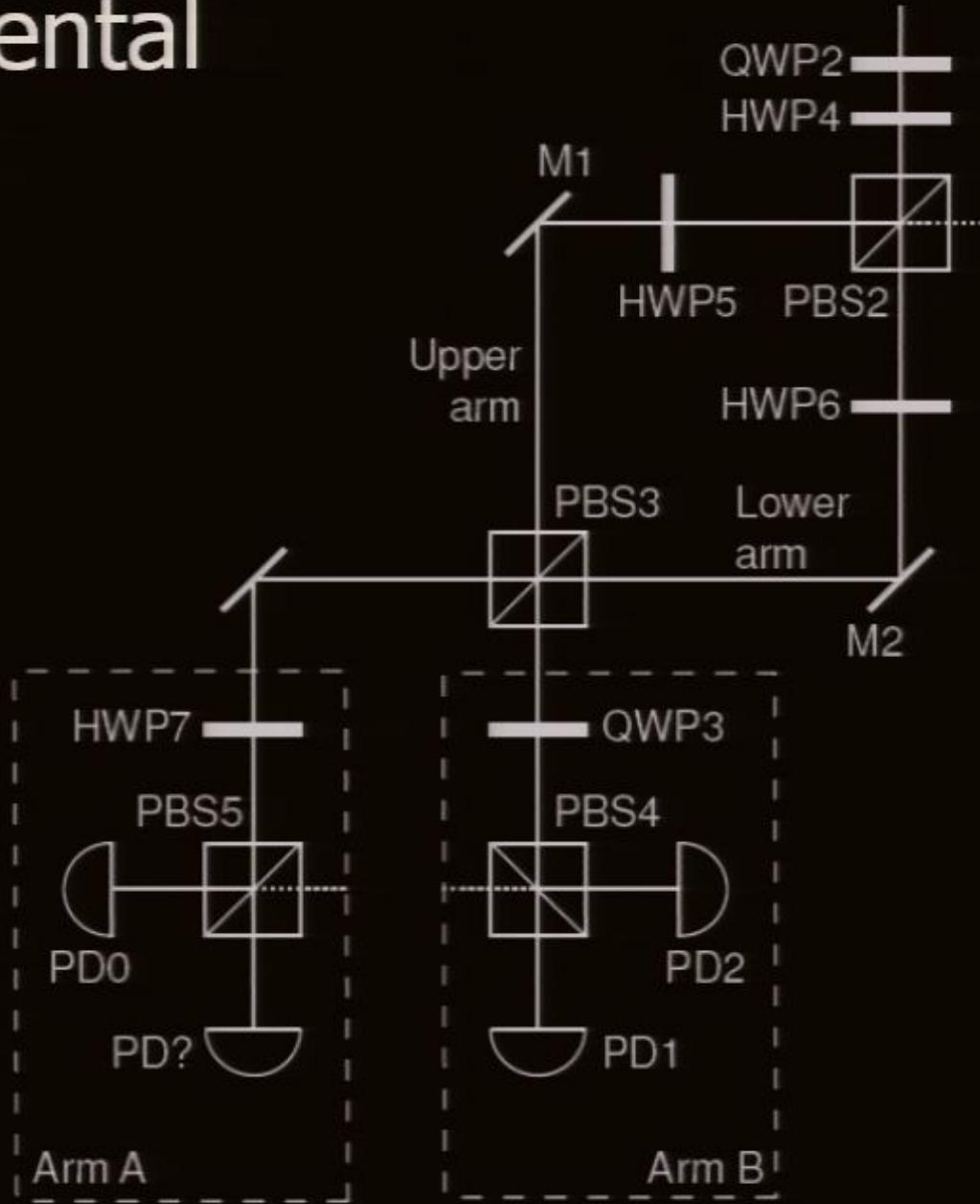
Experimental Design:



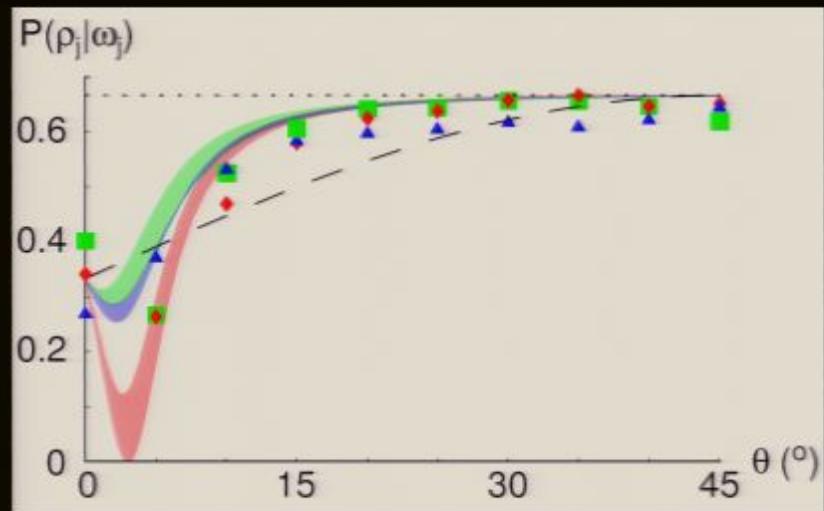
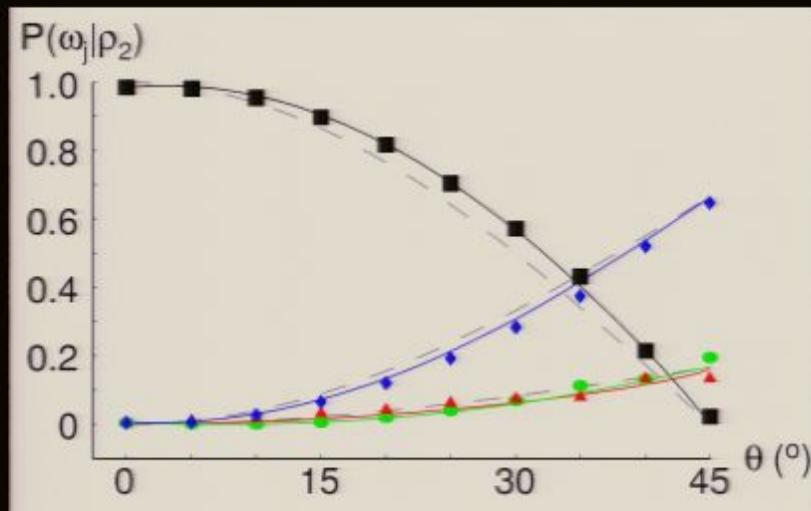
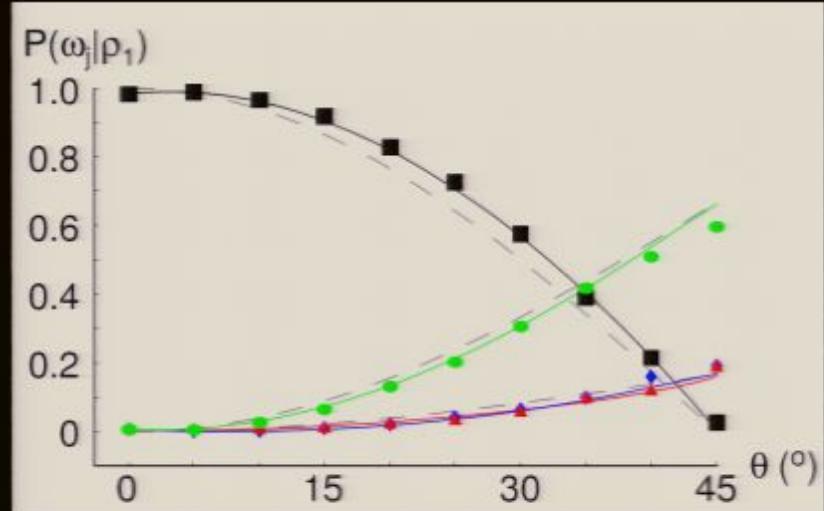
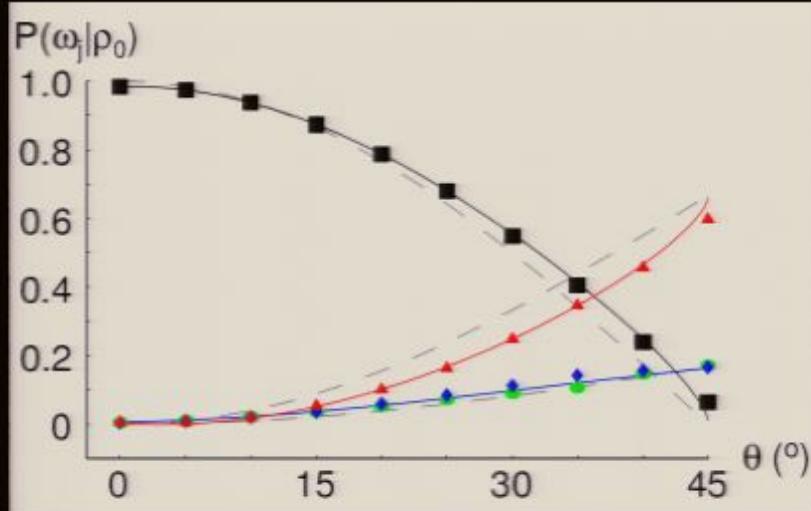
Results:



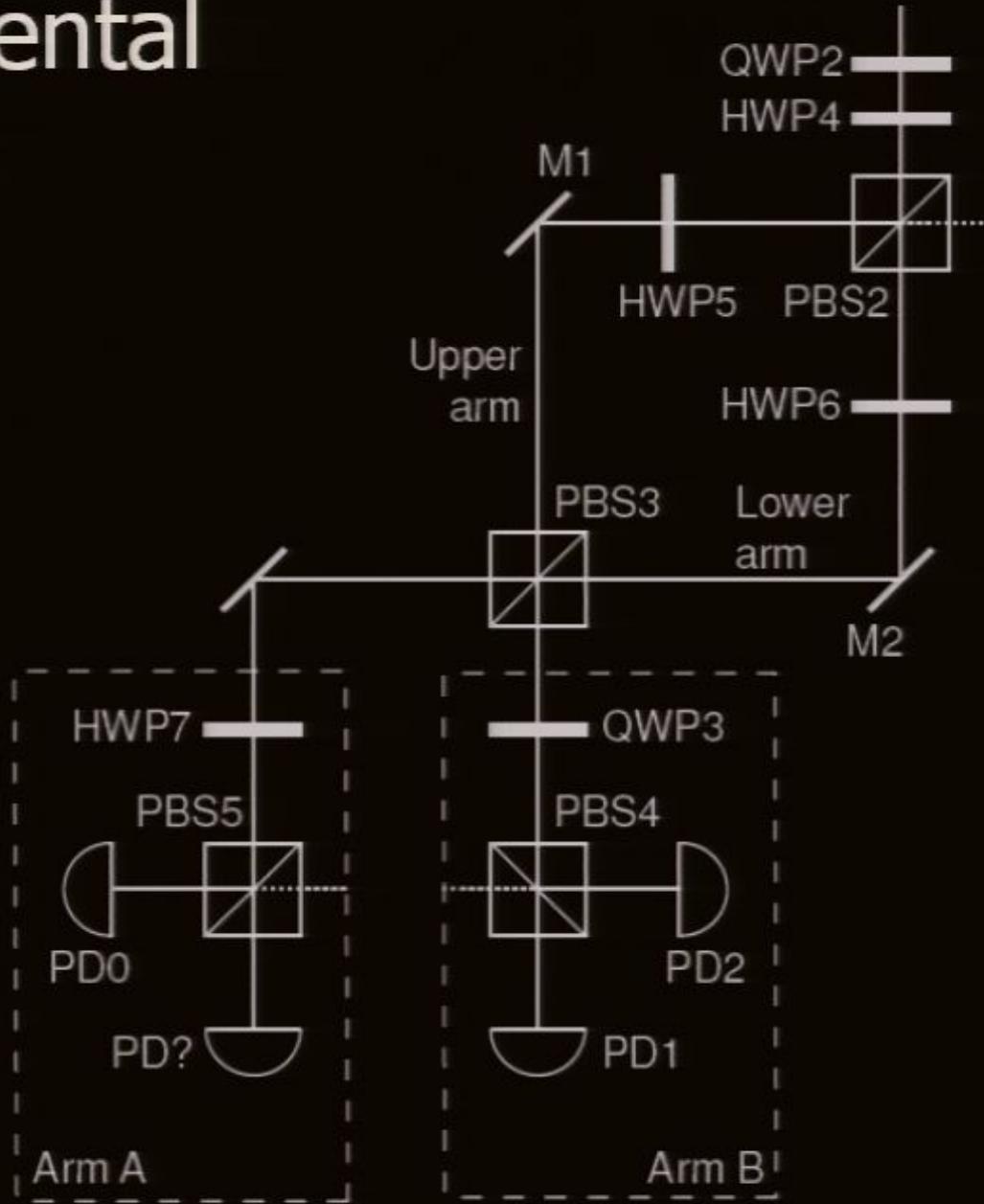
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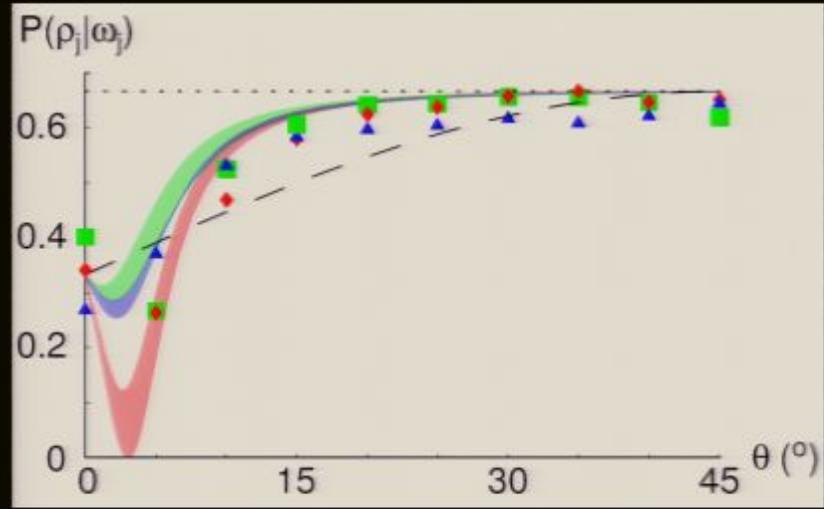
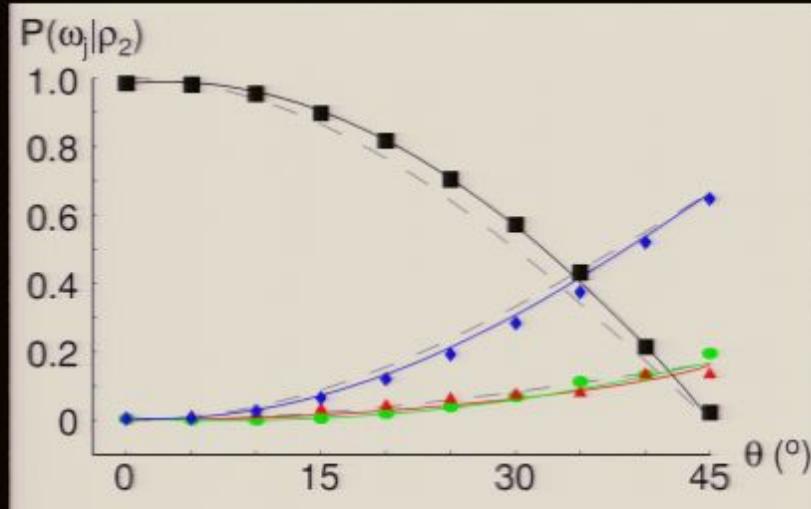
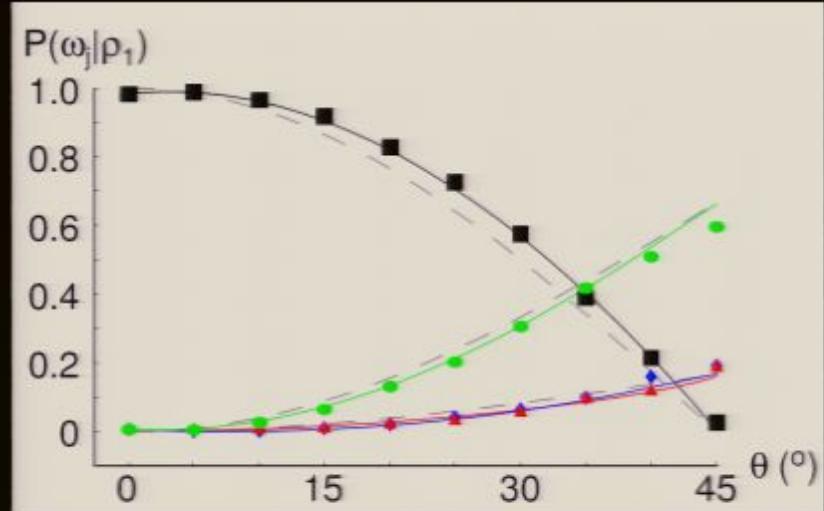
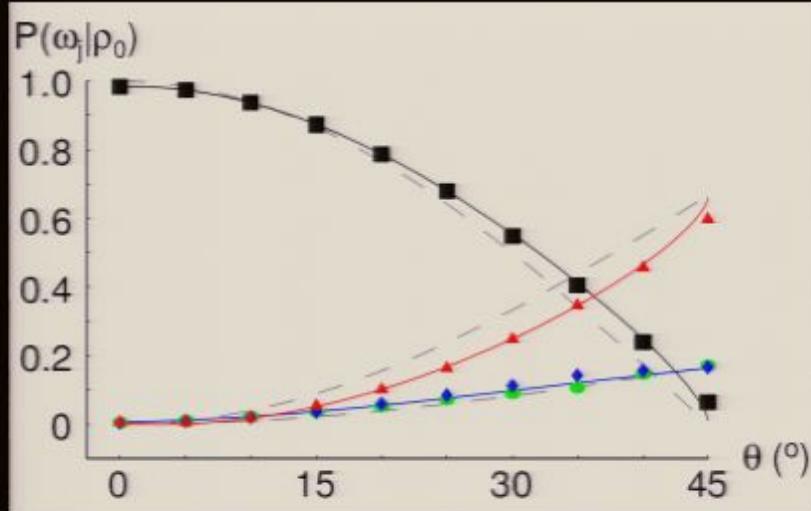
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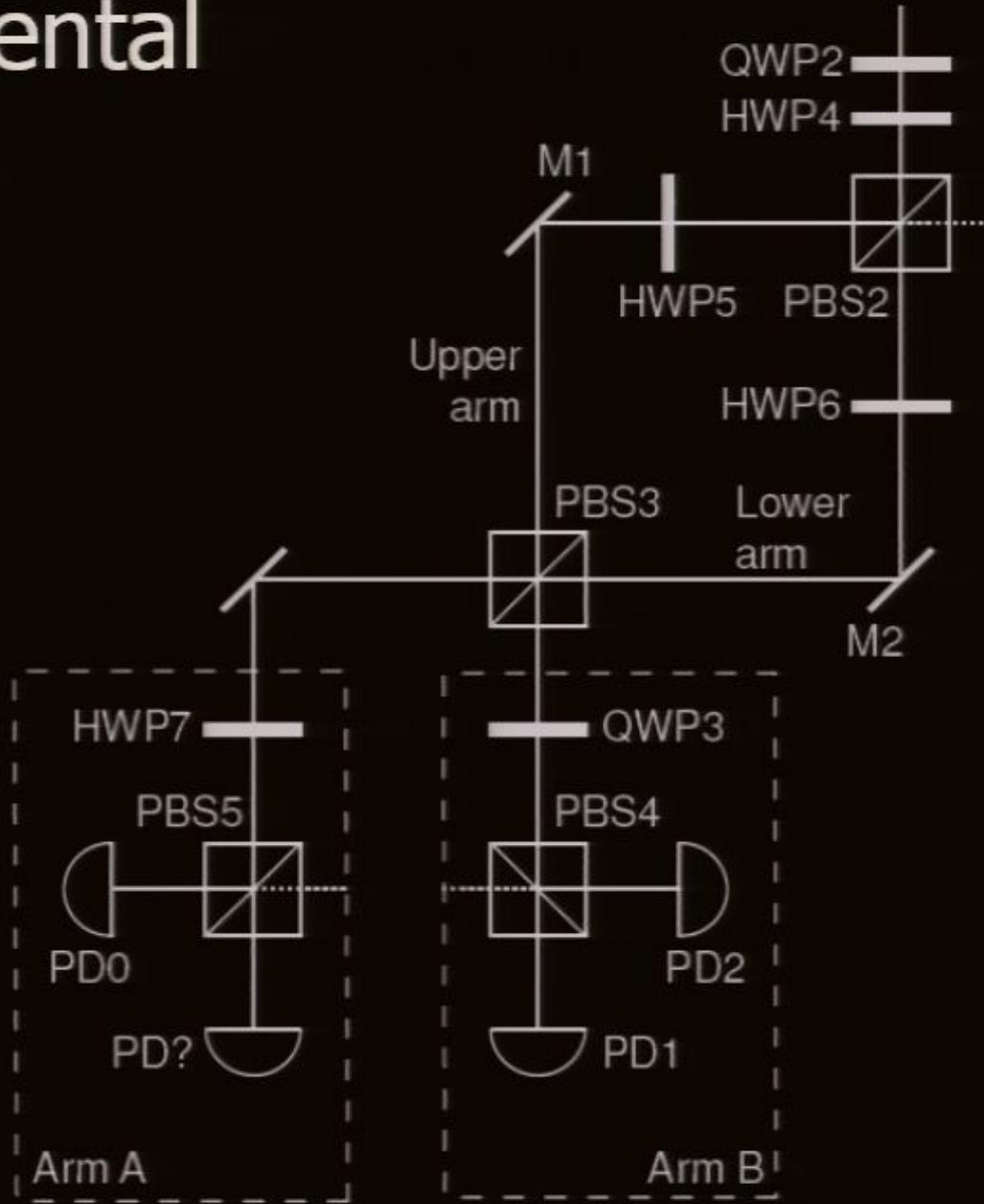
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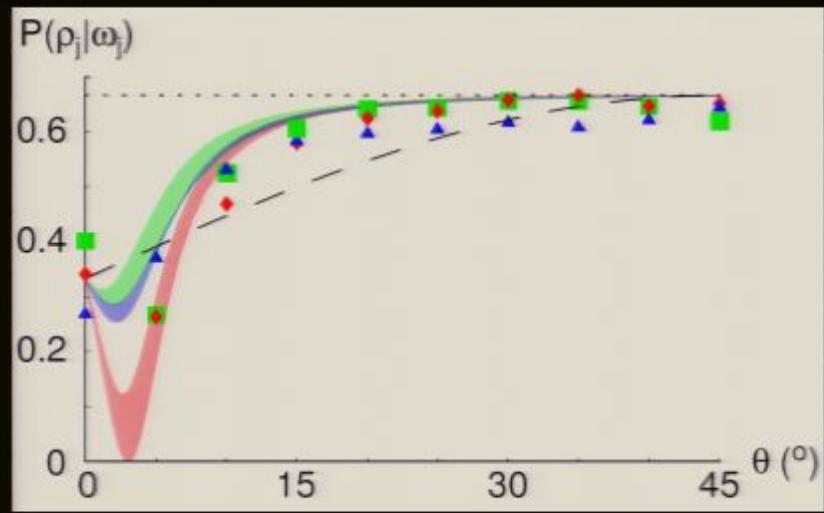
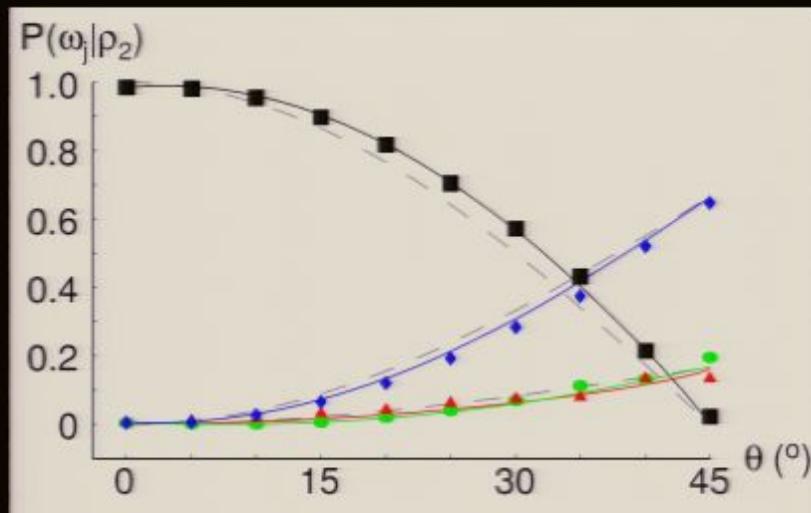
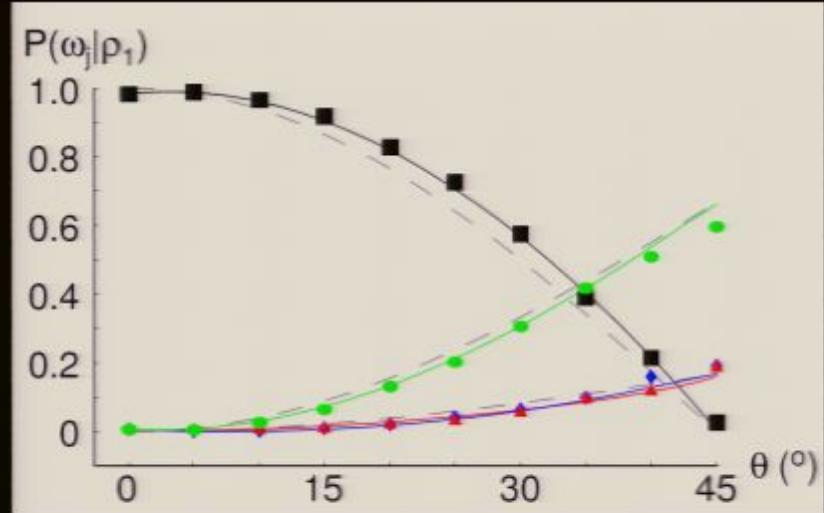
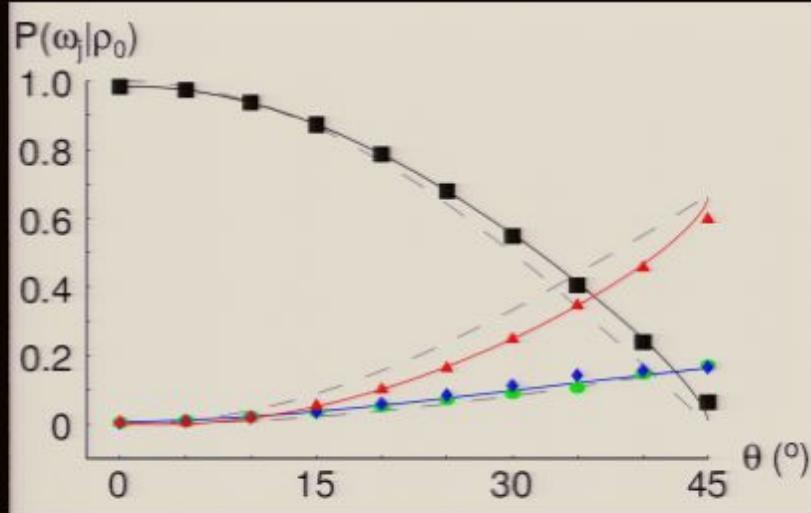
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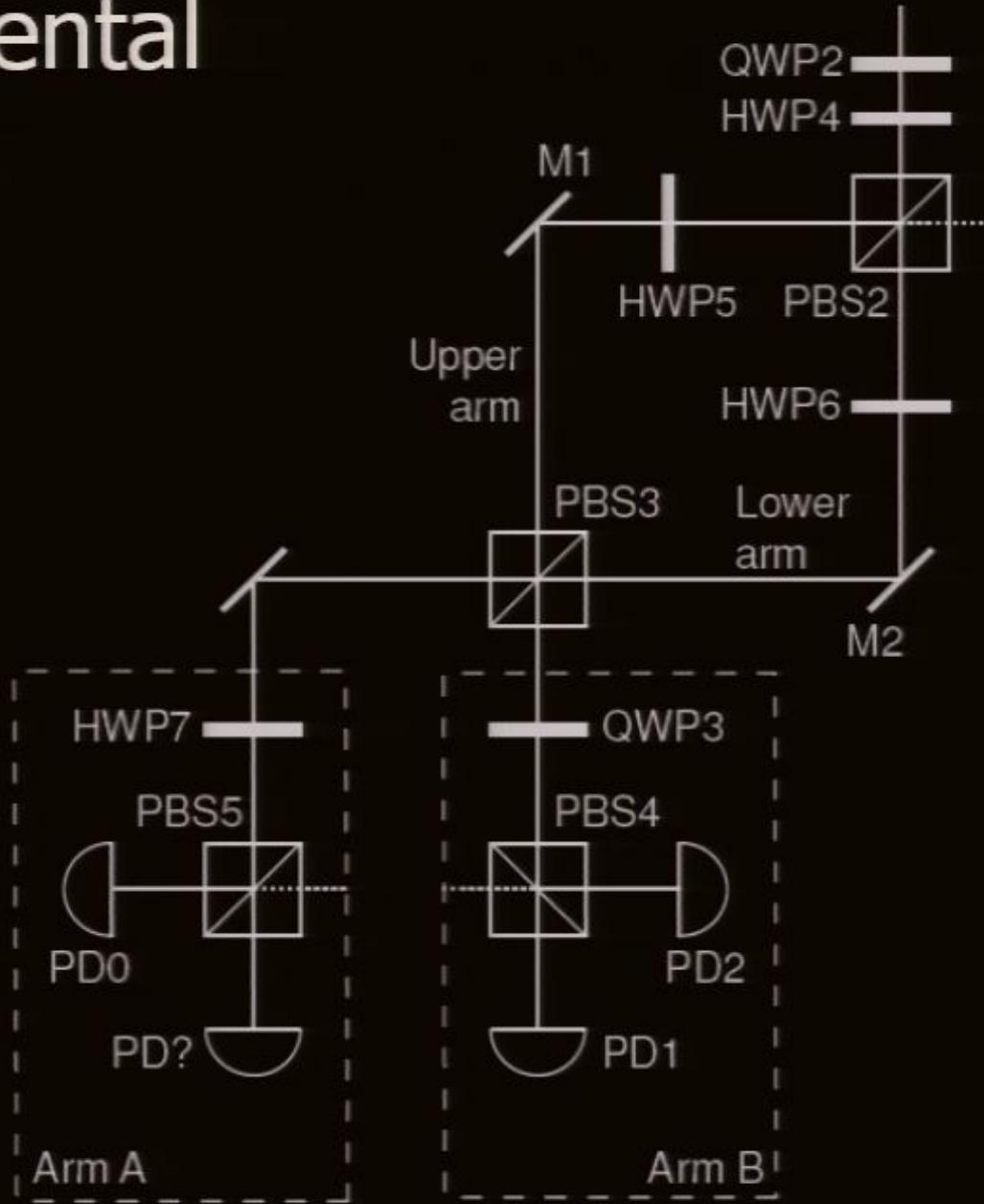
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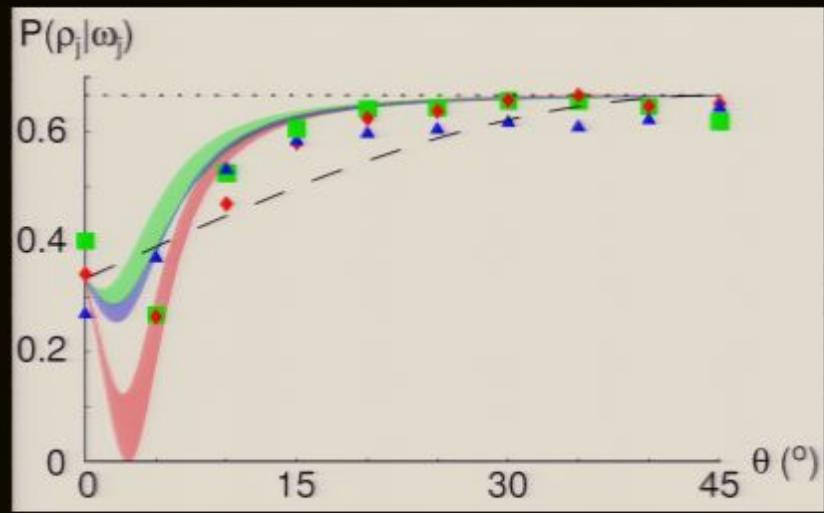
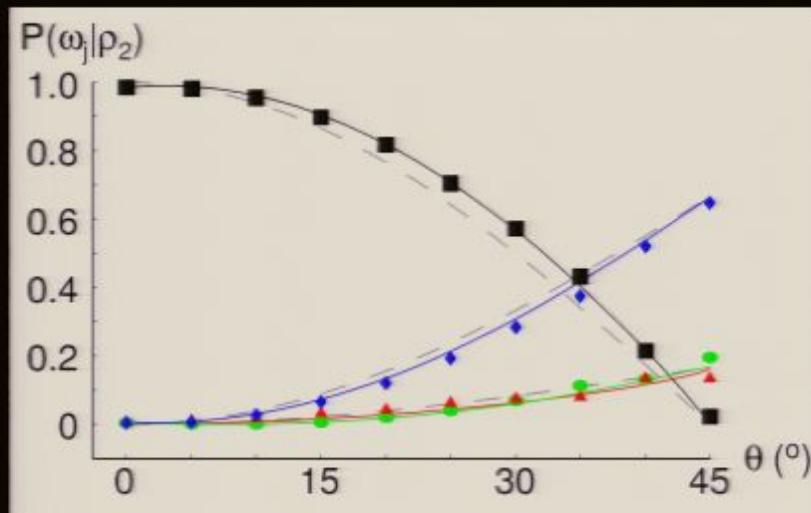
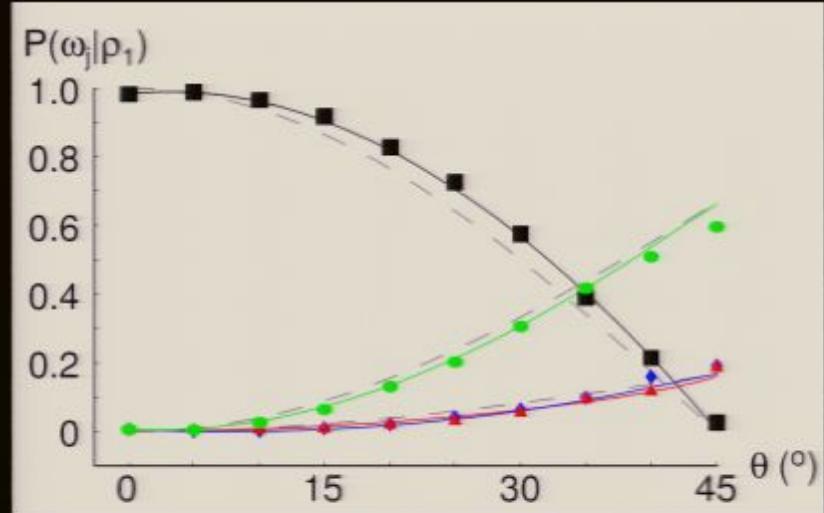
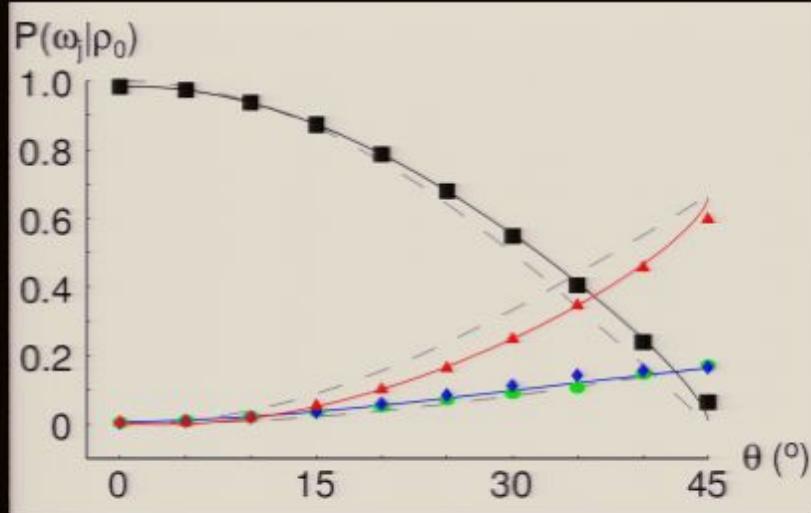
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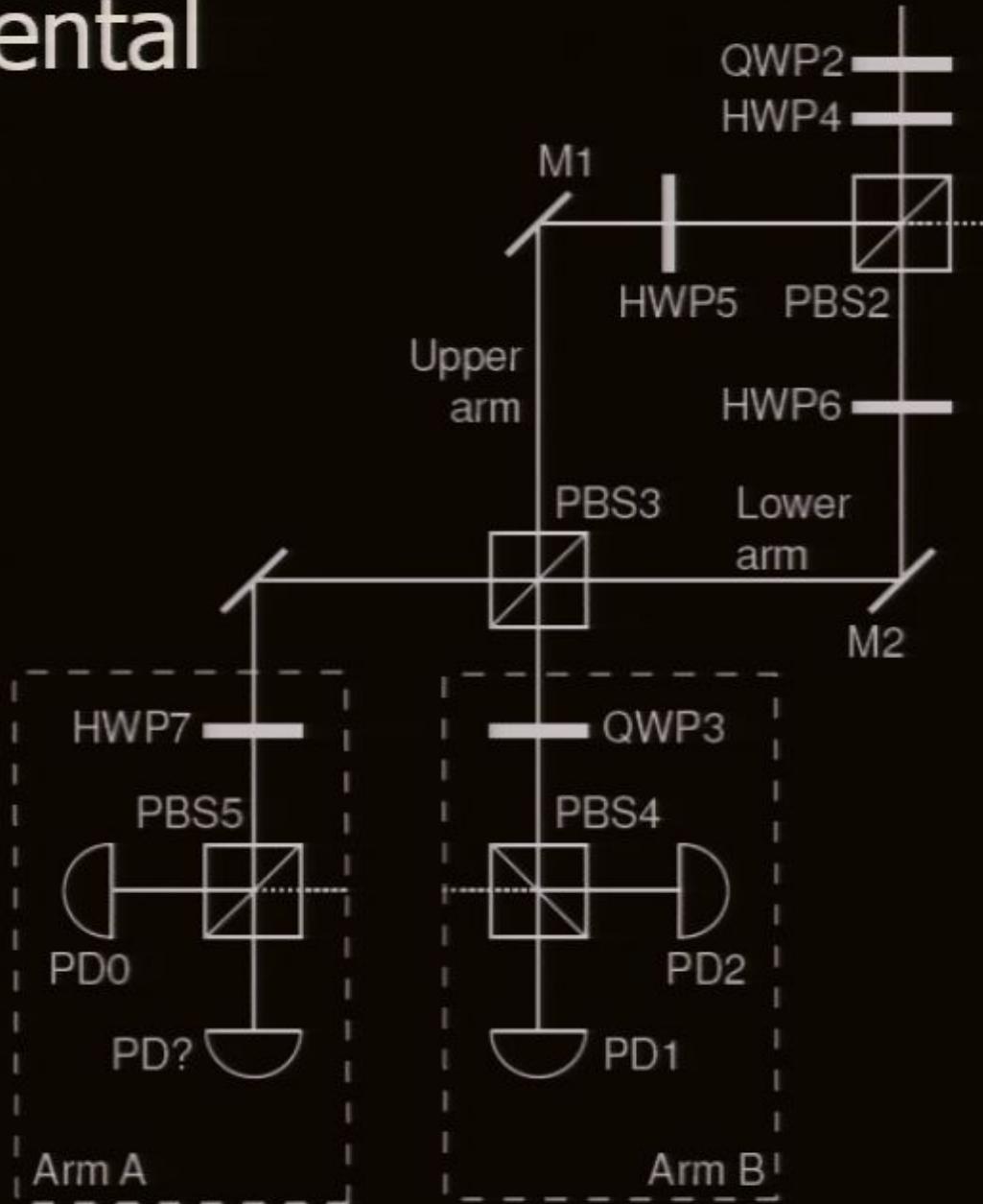
Errors:

- Largest errors due to leakage of ‘wrong’ polarisation at PBS2-3
- Errors modelled by:

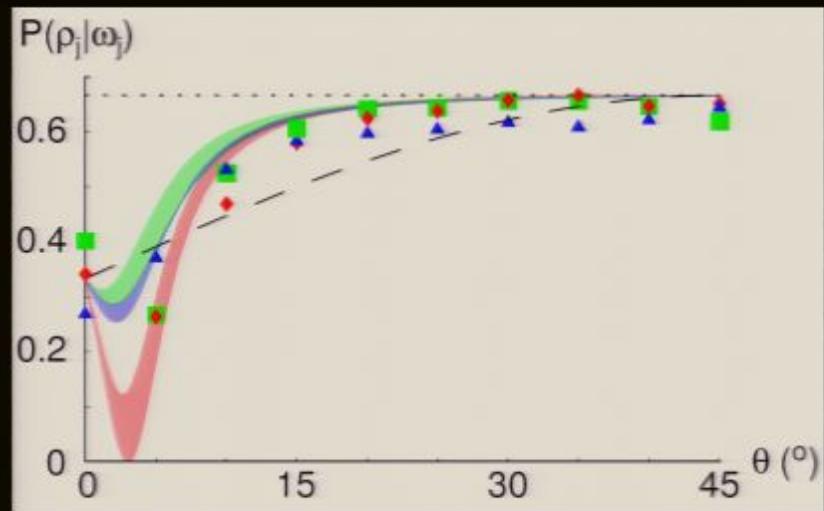
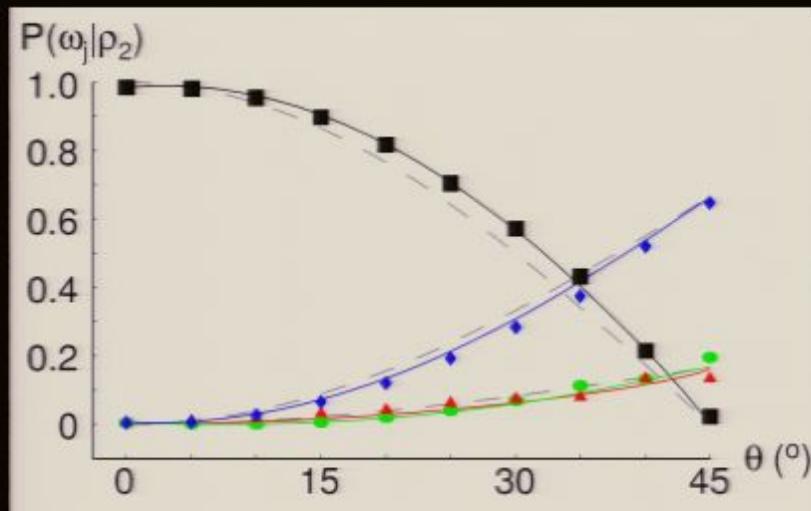
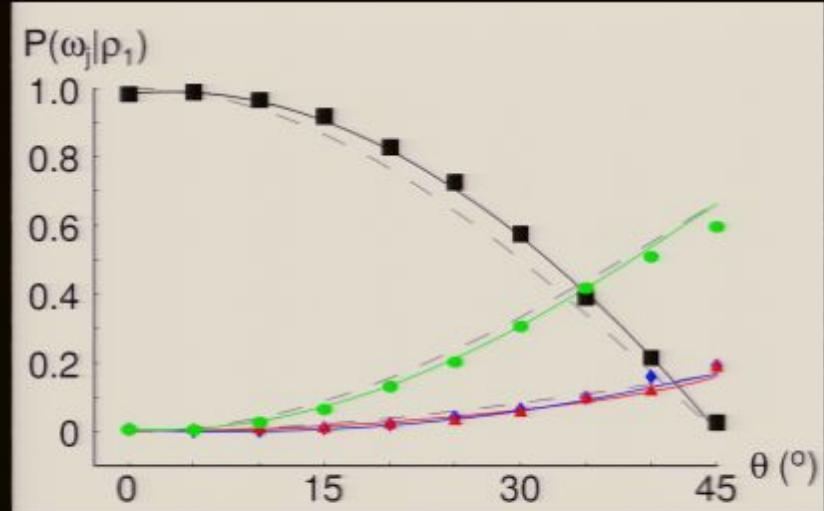
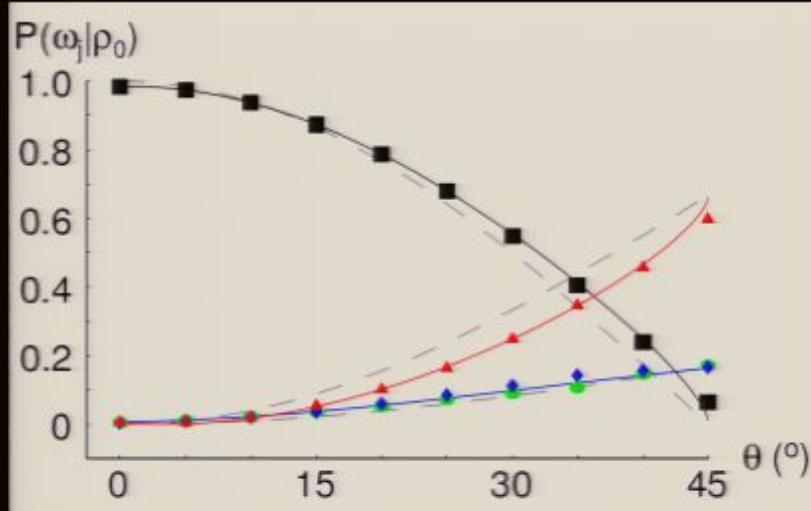
$$\begin{aligned} PBS = & |H_L\rangle(i e^{-i\chi}\sqrt{0.005}\langle H_L| + \sqrt{0.995}\langle H_U|) \\ & + |V_L\rangle(\sqrt{0.995}\langle V_L| + i e^{i\chi}\sqrt{0.005}\langle V_U|) \\ & + |H_U\rangle(\sqrt{0.995}\langle H_L| + i e^{i\chi}\sqrt{0.995}\langle H_U|) \\ & + |V_U\rangle(i e^{-i\chi}\sqrt{0.005}\langle V_L| + \sqrt{0.995}\langle V_U|) \end{aligned}$$

consistent with measured properties of the polarising beamsplitters

Experimental Design:



Results:



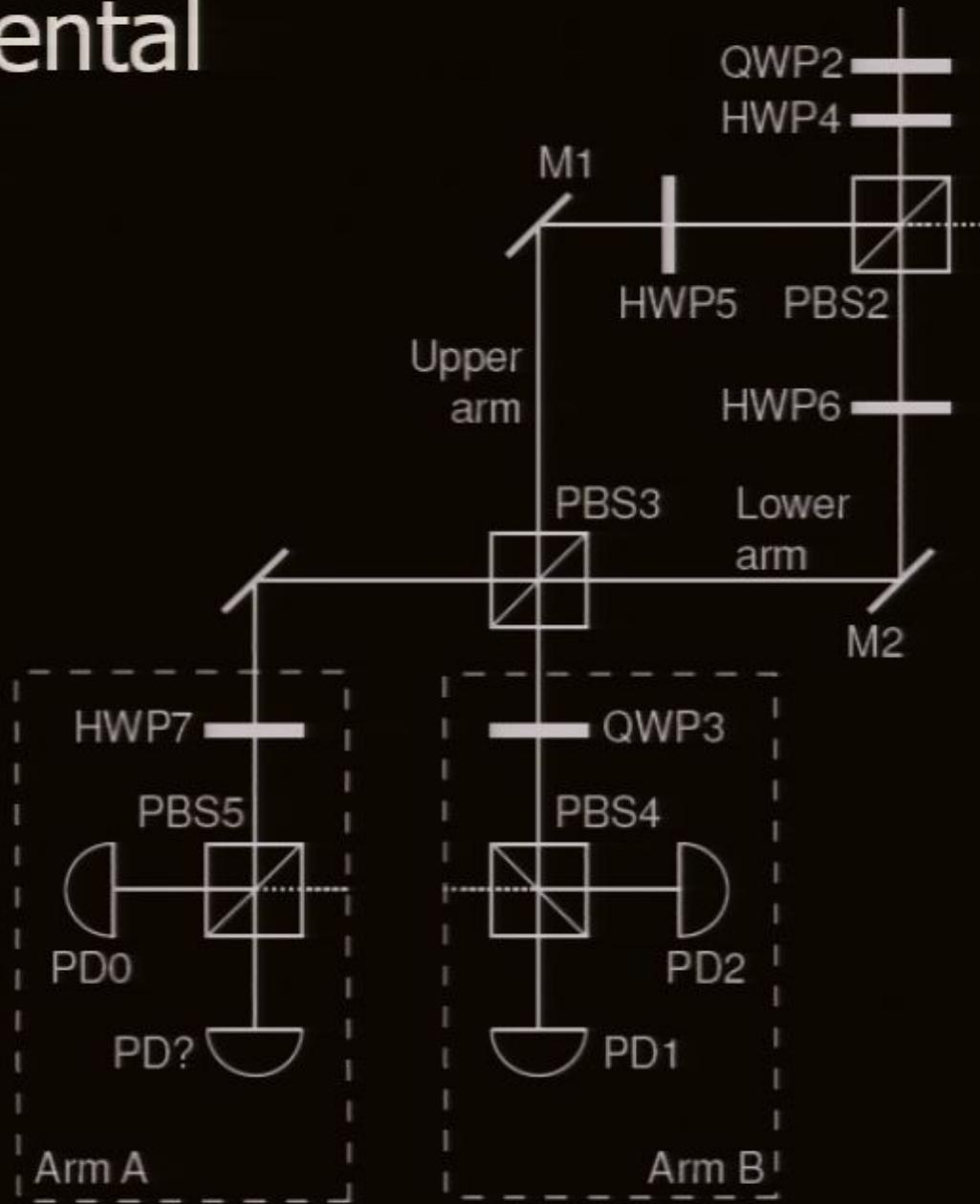
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consistent with measured properties of the polarising beamsplitters

Experimental Design:



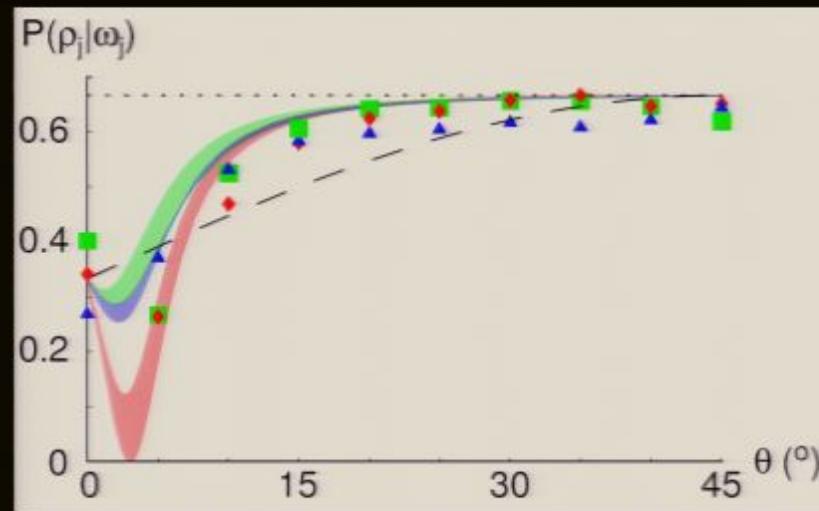
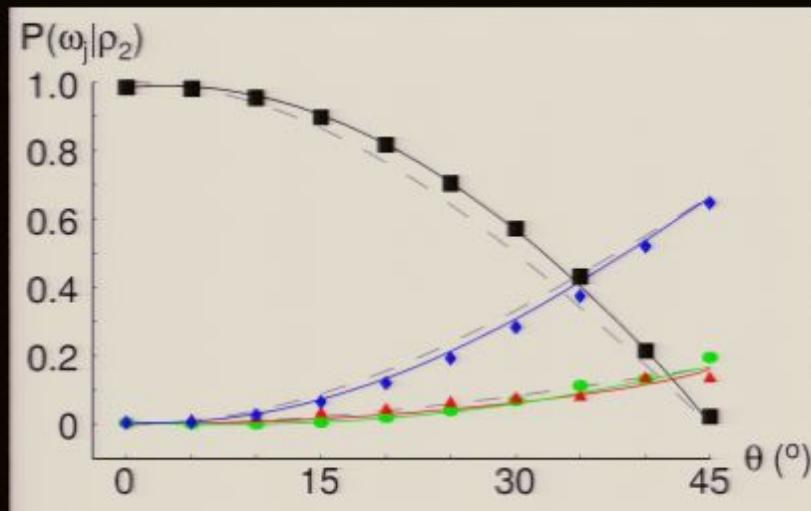
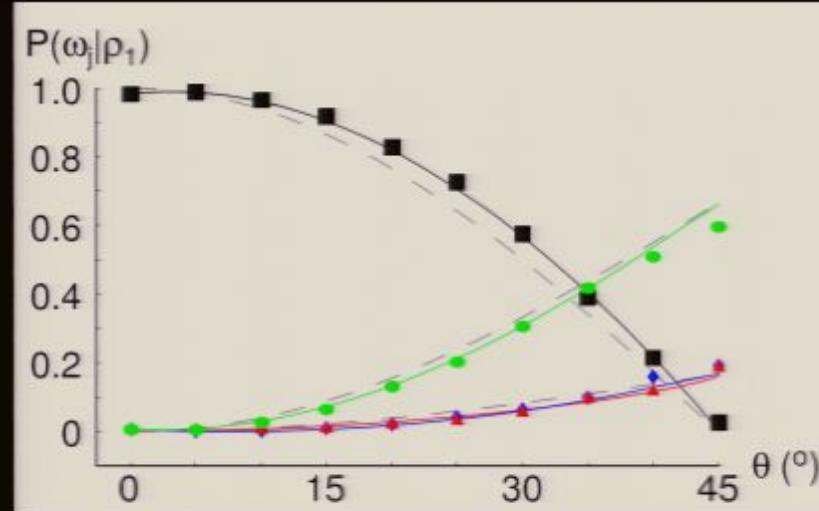
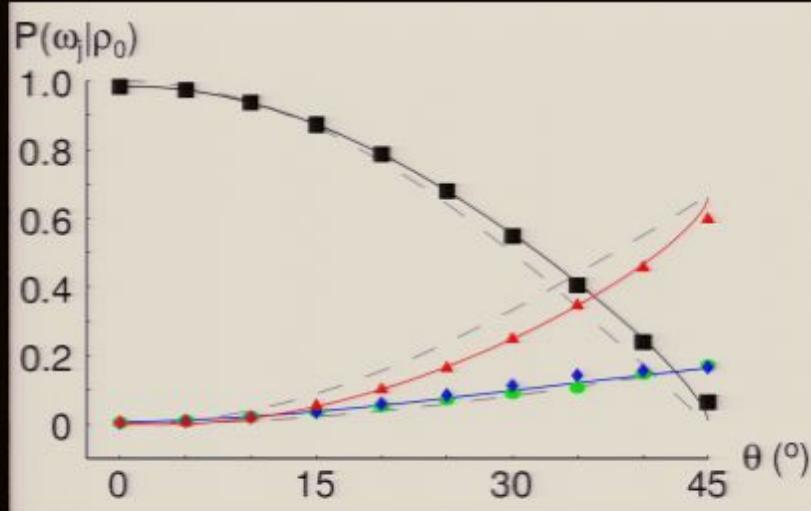
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consistent with measured properties of the polarising beamsplitters

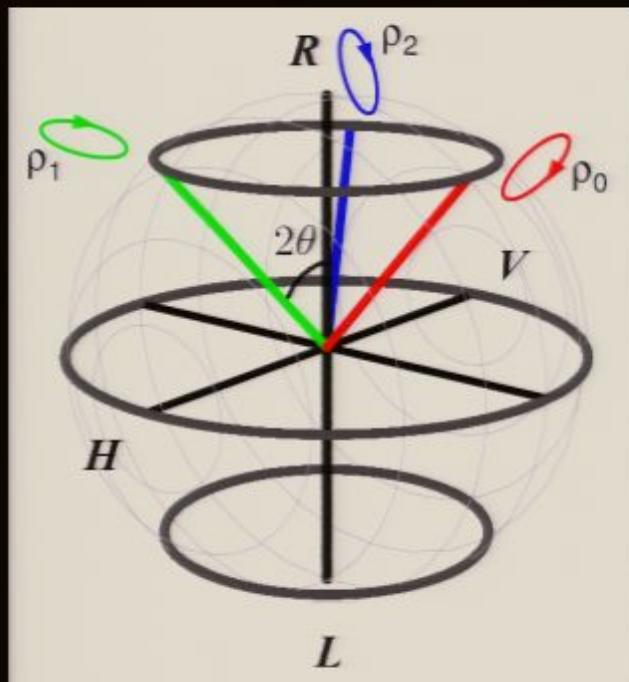
Conclusions

- Constructed a measurement which maximises the confidence in identifying a state from a given set.
- First explicit experimental demonstration that an improvement in this figure of merit is possible for linearly dependent states.

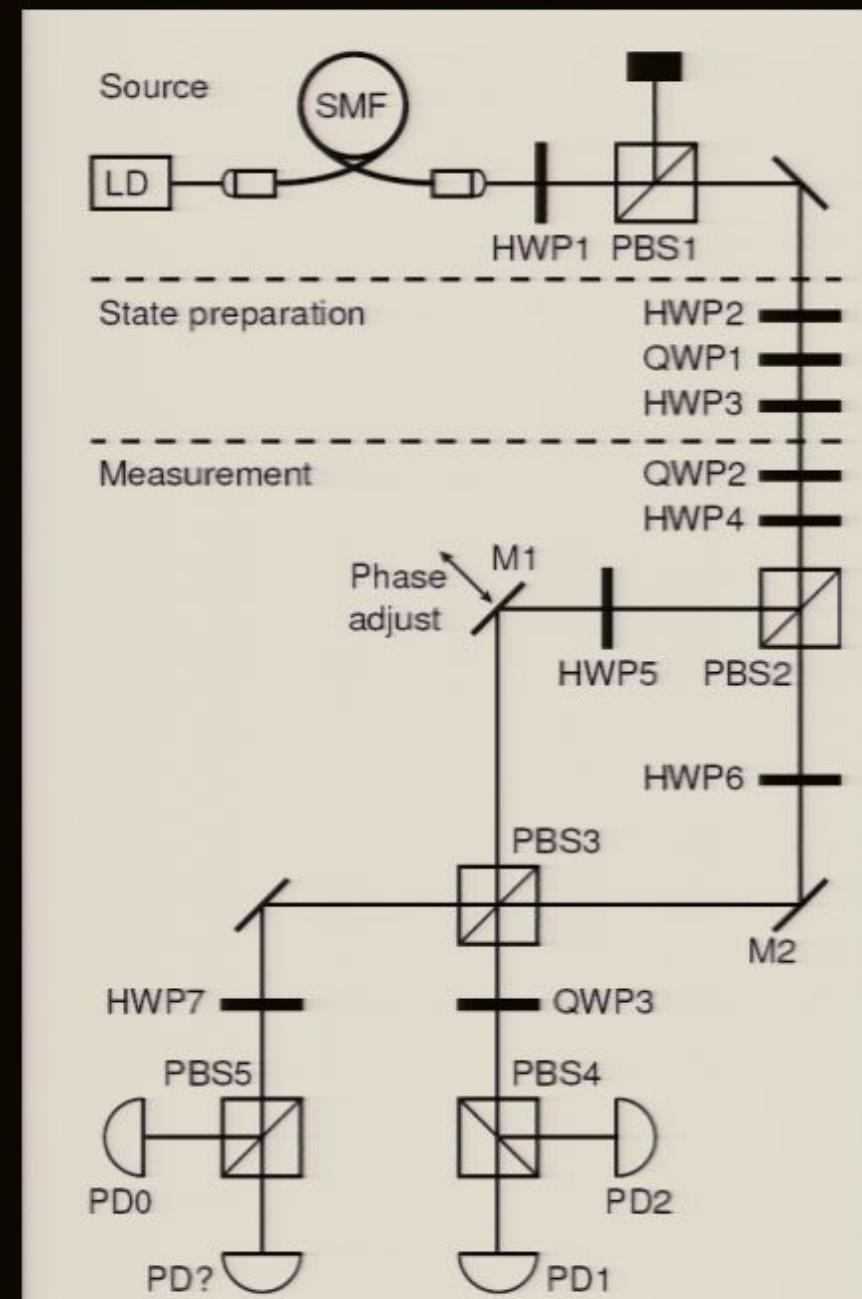
Future Work:

- No signalling arguments and connection to entanglement concentration.
- Maximum confidence measurement in the limit of continuous distributions of states.

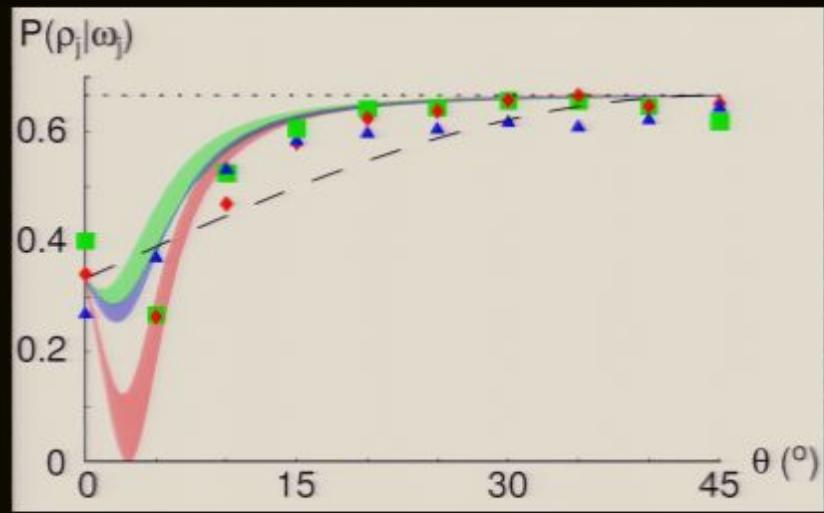
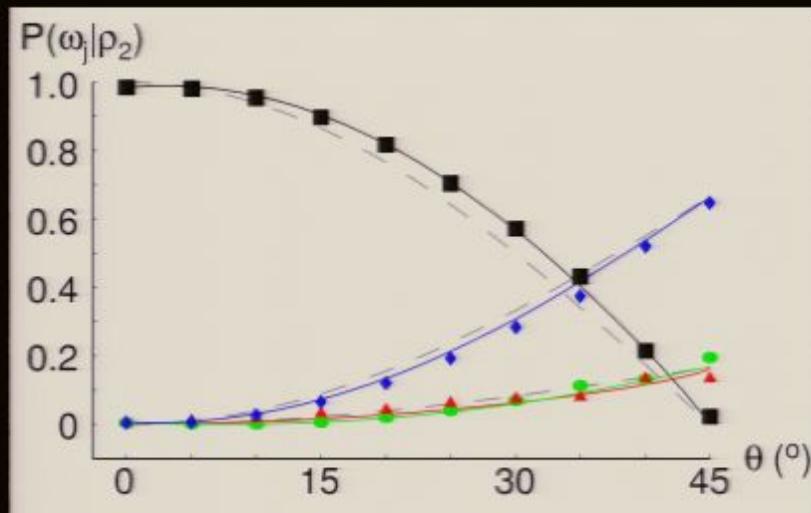
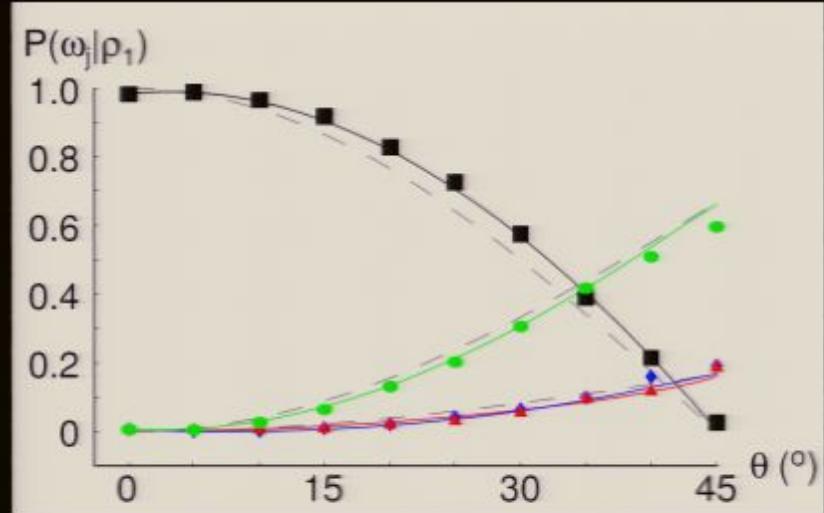
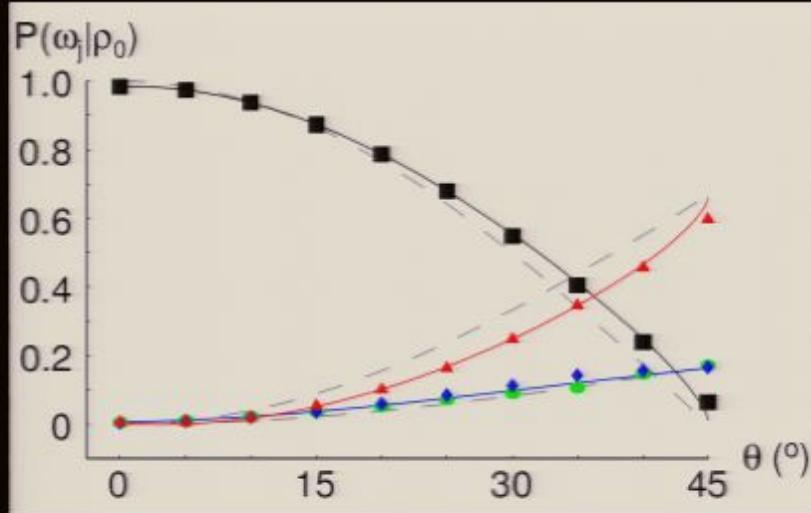
Experimental Implementation



P J Mosley, S Croke, I A Walmsley, S M Barnett, PRL 97, 193601 (2006)



Results:



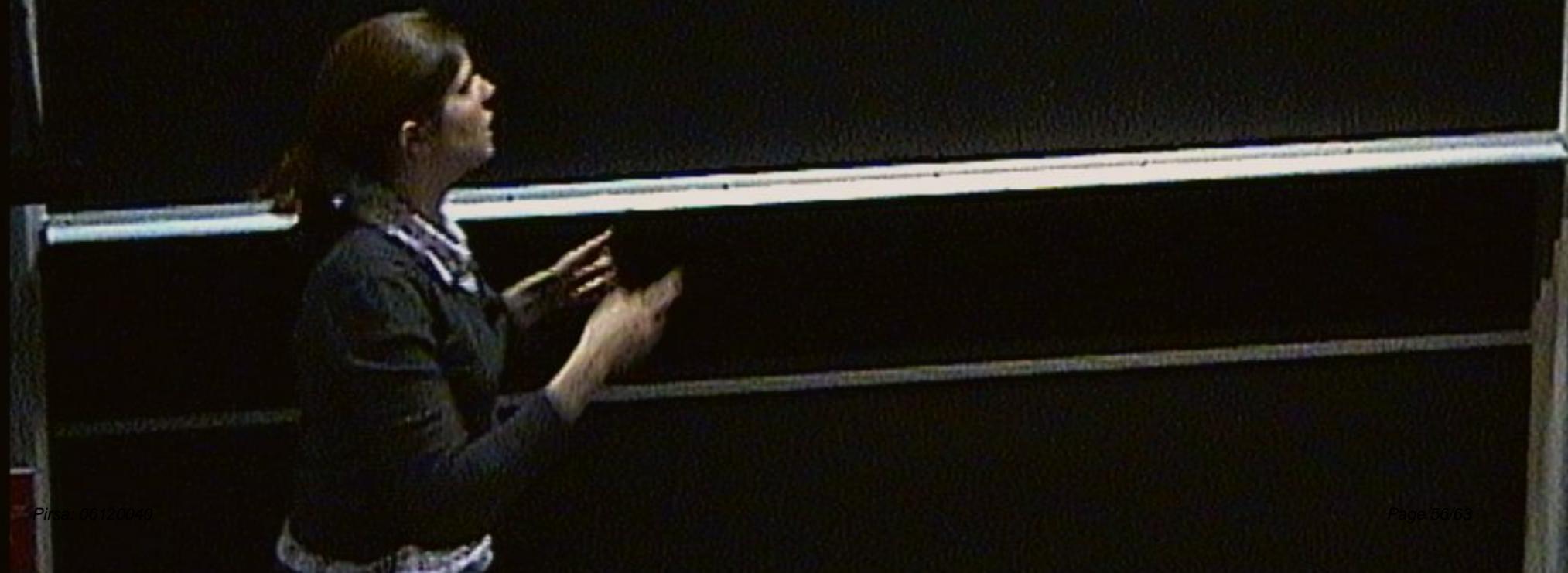
$$|\Psi\rangle = |0\rangle_{\text{L}}|\psi_0\rangle + |1\rangle_{\text{L}}|\psi_1\rangle + |2\rangle_{\text{L}}|\psi_2\rangle$$



$$|\Psi\rangle = |0\rangle_{\text{L}}|\Psi_0\rangle + |1\rangle_{\text{L}}|\Psi_1\rangle + |2\rangle_{\text{L}}|\Psi_2\rangle$$

\uparrow

$$\langle 0| \rho_{\text{L}} | 0 \rangle$$



Optimisation:

$$P(\hat{\rho}_j | \omega_j) = \frac{p_j \text{Tr}(\hat{\rho}_j \hat{H}_j)}{\text{Tr}(\hat{\rho} \hat{H}_j)},$$

$$\hat{H}_j = c_j \hat{\rho}^{-1/2} \hat{Q}_j \hat{\rho}^{-1/2},$$

$$\begin{aligned} P(\hat{\rho}_j | \omega_j) &= p_j \text{Tr}(\hat{\rho}^{-1/2} \hat{\rho}_j \hat{\rho}^{-1/2} \hat{Q}_j) \\ &= p_j \text{Tr}(\hat{\rho}_j \hat{\rho}^{-1}) \text{Tr}(\hat{\rho}'_j \hat{Q}_j), \end{aligned}$$

$$\hat{\rho}'_j = \hat{\rho}^{-1/2} \hat{\rho}_j \hat{\rho}^{-1/2} / \text{Tr}(\hat{\rho}_j \hat{\rho}^{-1})$$

$$\hat{Q}_j = \hat{\rho}'_j$$

$$\hat{H}_j \propto \hat{\rho}^{-1} \hat{\rho}_j \hat{\rho}^{-1}$$

$$|\Psi\rangle = |0\rangle \underbrace{|\psi_0\rangle}_{\uparrow} + |1\rangle |\psi_1\rangle + |2\rangle |\psi_2\rangle$$

$\langle 0 | \underline{p_{\psi}} | 0 \rangle$

$$\sum_k p_{\psi_k} |\psi_k\rangle \langle \psi_k|$$
$$\sum_k p_{\psi_k}$$



$$|\Psi\rangle = |0\rangle_{\text{L}} |\gamma_0\rangle_{\text{R}} + |1\rangle_{\text{L}} |\gamma_1\rangle_{\text{R}} + |2\rangle_{\text{L}} |\gamma_2\rangle_{\text{R}}$$

$$\langle 0| \rho_{\text{L}} |0\rangle$$

$$\sum_k \gamma_{kk} |\gamma_k\rangle_{\text{L}} |\gamma_k\rangle_{\text{R}}$$

$$\sum_k \gamma_k^{-1} |$$



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