Title: Probability in the Everett interpretation: How to live without uncertainty

Date: Dec 07, 2006 03:00 AM

URL: http://pirsa.org/06120038

Abstract: The Everett (many-worlds) interpretation has made great progress over the past 20-30 years, largely due to the role of decoherence in providing a solution to the preferred basis problem. This makes it a serious candidate for a realist solution to the measurement problem. A remaining objection to the Everett interpretation (and one that is often considered fatal) is that that interpretation cannot make adequate sense of quantum probabilities. Dvaid Deutsch and David Wallace have argued that, by applying decision theory to the case of a rational agent who believes in the many-worlds interpretation, we can prove that such agents _act as if_ the theory predicted objective probabilities in the sense of fundamental indeterminism, or ignorance of initial conditions. I raise the issue of whether or not this, if true, is all that the many-worlds theorist needs from \'probability\'. I first suggest a reason for thinking that the answer might be \'no\': the reason is that knowing how to act on the assumption that a given theory is true is prima facie irrelevant to the question of whether we have any reason to believe the theory in the first place. I then go on to offer a solution to this problem, drawing on resources from Bayesian confirmation theory. My conclusion is that the problem of probability in the Everett interpretation has been solved.

Probability in the Everett interpretation: How to live without uncertainty

Hilary Greaves
Rutgers University

Outline of the talk

1. The many-worlds interpretation & the problem of probability
2. The decision-theoretic program

   How to act, if you believe MW?

3. The epistemic problem

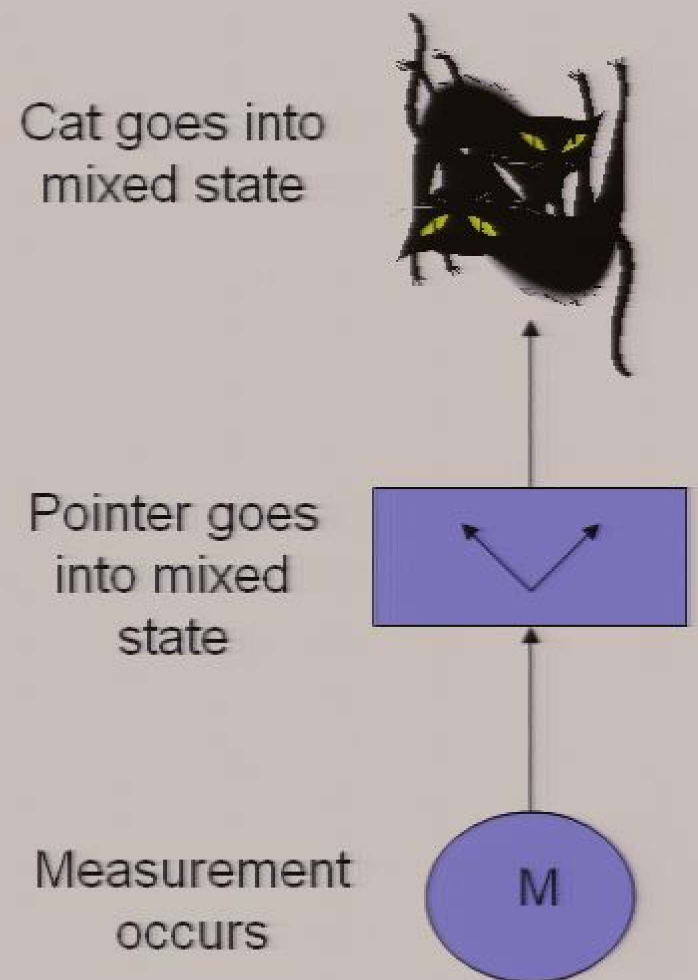
   Why believe MW in the first place?

4. Solution to the epistemic problem
5. Concluding remarks

Outline of the talk

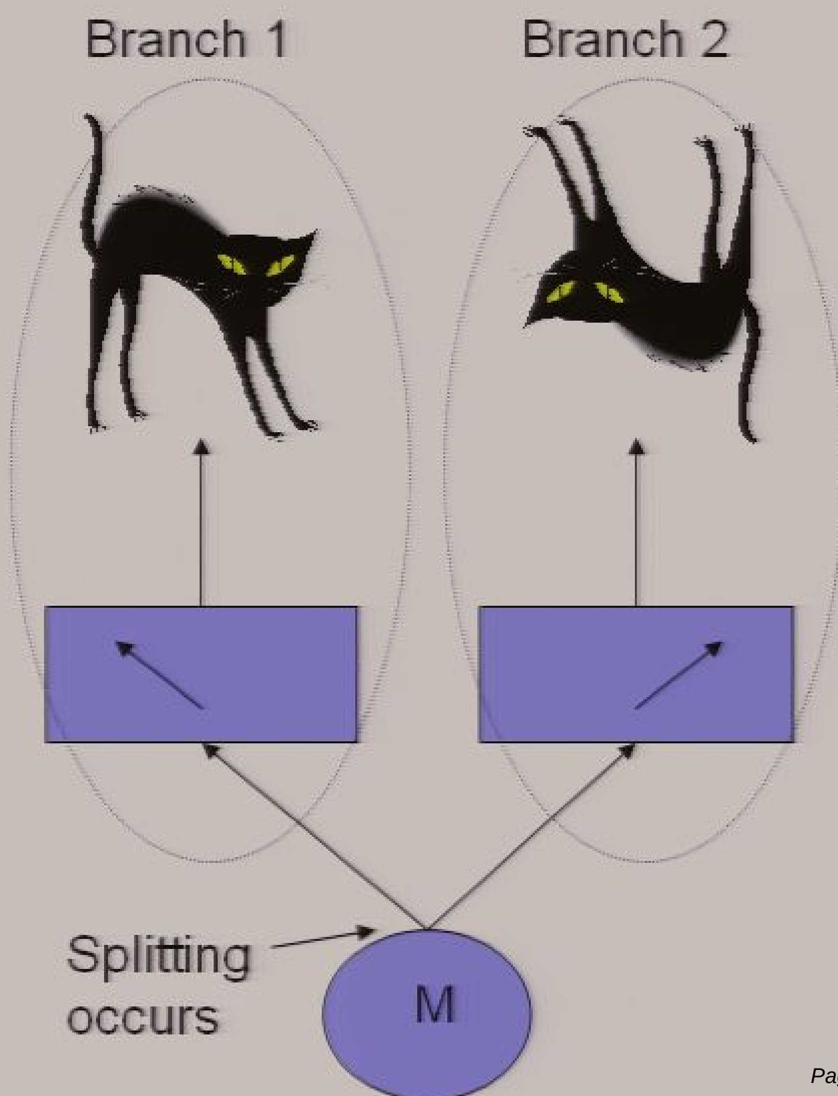
1. The many-worlds interpretation & the problem of probability
2. The decision-theoretic program
   - How to act, if you believe MW?
3. The epistemic problem
   - Why believe MW in the first place?
4. Solution to the epistemic problem
5. Concluding remarks

1.1 Many-worlds interpretations (MWI) introduced



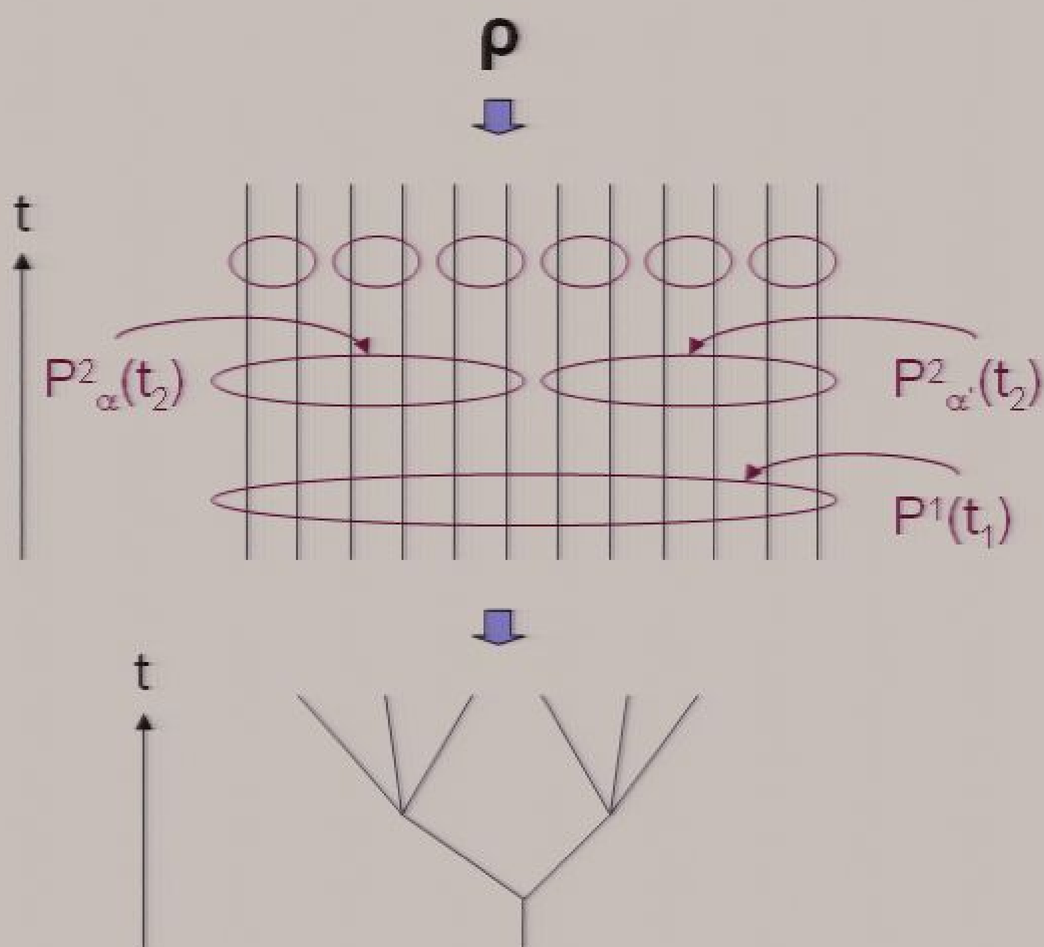
1.1 Many-worlds interpretations (MWI) introduced

- A first pass: "When a quantum measurement is performed, the world splits into multiple branches, and each 'possible' outcome is realized in some branch"



1.2 MWI via consistent histories

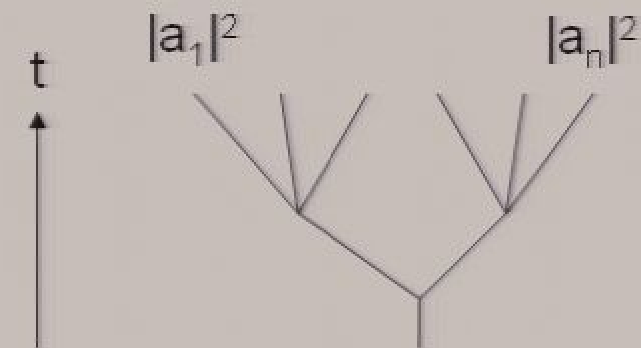
- **What there is:** ρ (≡|Ψ⟩⟨Ψ|), undergoing unitary evolution
- **How the macroworld supervenes on** ρ: via a decomposition into histories
- Preferred basis problem: *which* history set?
  - Use dynamical decoherence (Zurek, Zeh, Gell-Mann and Hartle, Saunders, Wallace)
- Emergent branching structure



1.3 The problem of probability

- "If one postulates that all of the histories… are realised … then ***no role has been assigned to the probabilities***, and ***there seems no obvious way of introducing further assumptions which would allow probabilistic statements to be deduced***."

  (Dowker & Kent (1994))

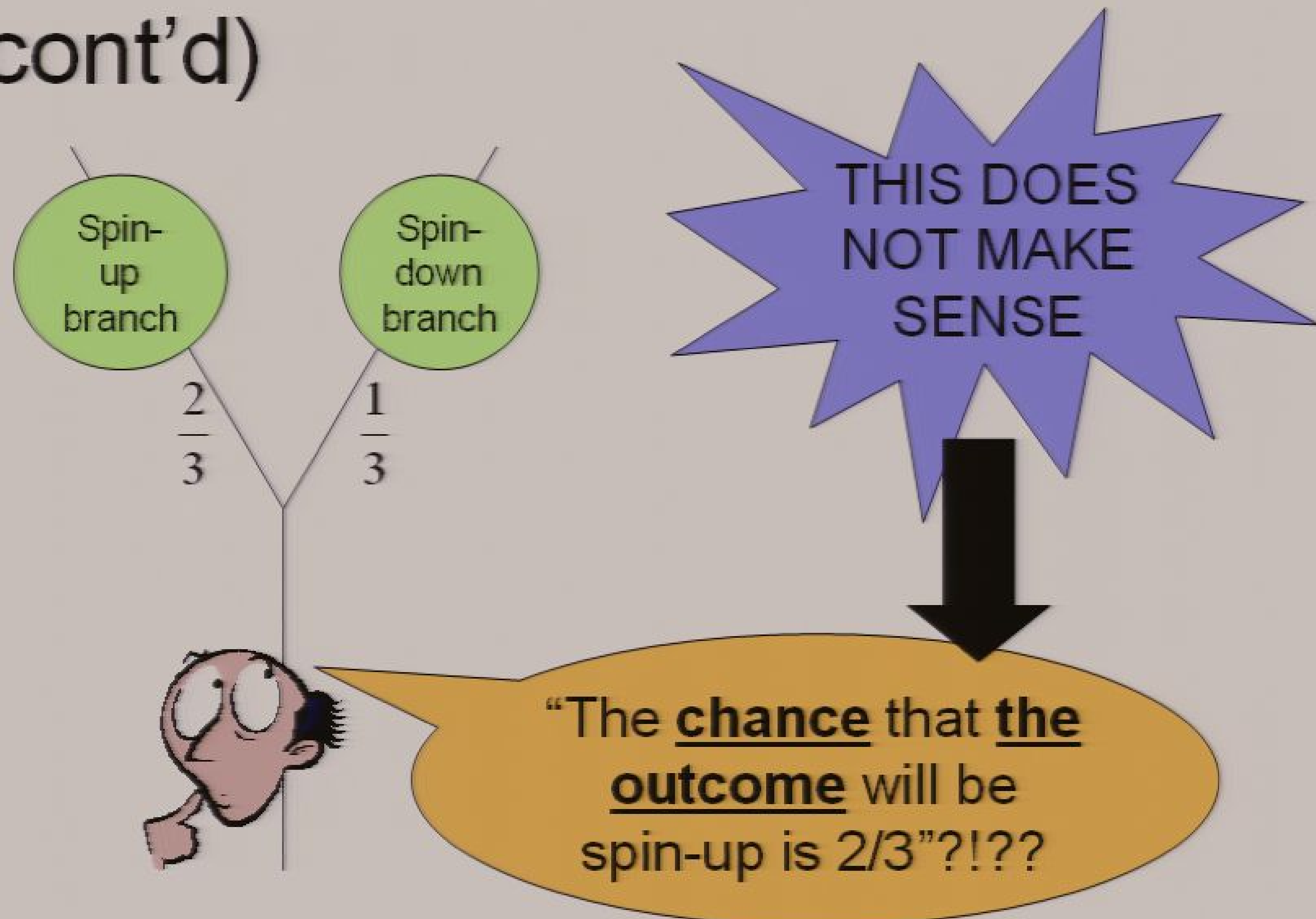


Quantum weight of *i*th branch,

$|a_i|^2 := || C_{\alpha_i} |\Psi\rangle ||^2$

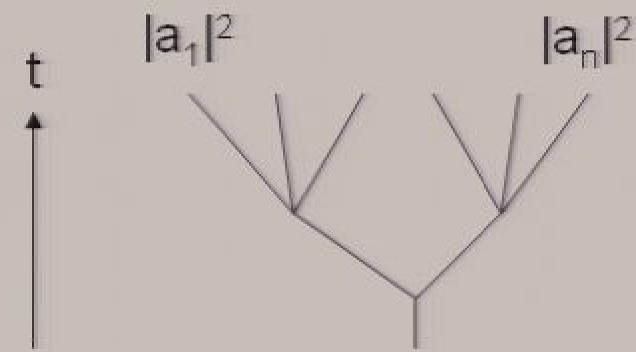
- The quantum weights:
  - satisfy the axioms of probability…
  - …but mean……??
- Talking about relative frequencies won't help

1.4 The problem of probability (cont'd)



1.3 The problem of probability

- "If one postulates that all of the histories… are realised … then ***no role has been assigned to the probabilities***, and ***there seems no obvious way of introducing further assumptions which would allow probabilistic statements to be deduced***."
  (Dowker & Kent (1994))

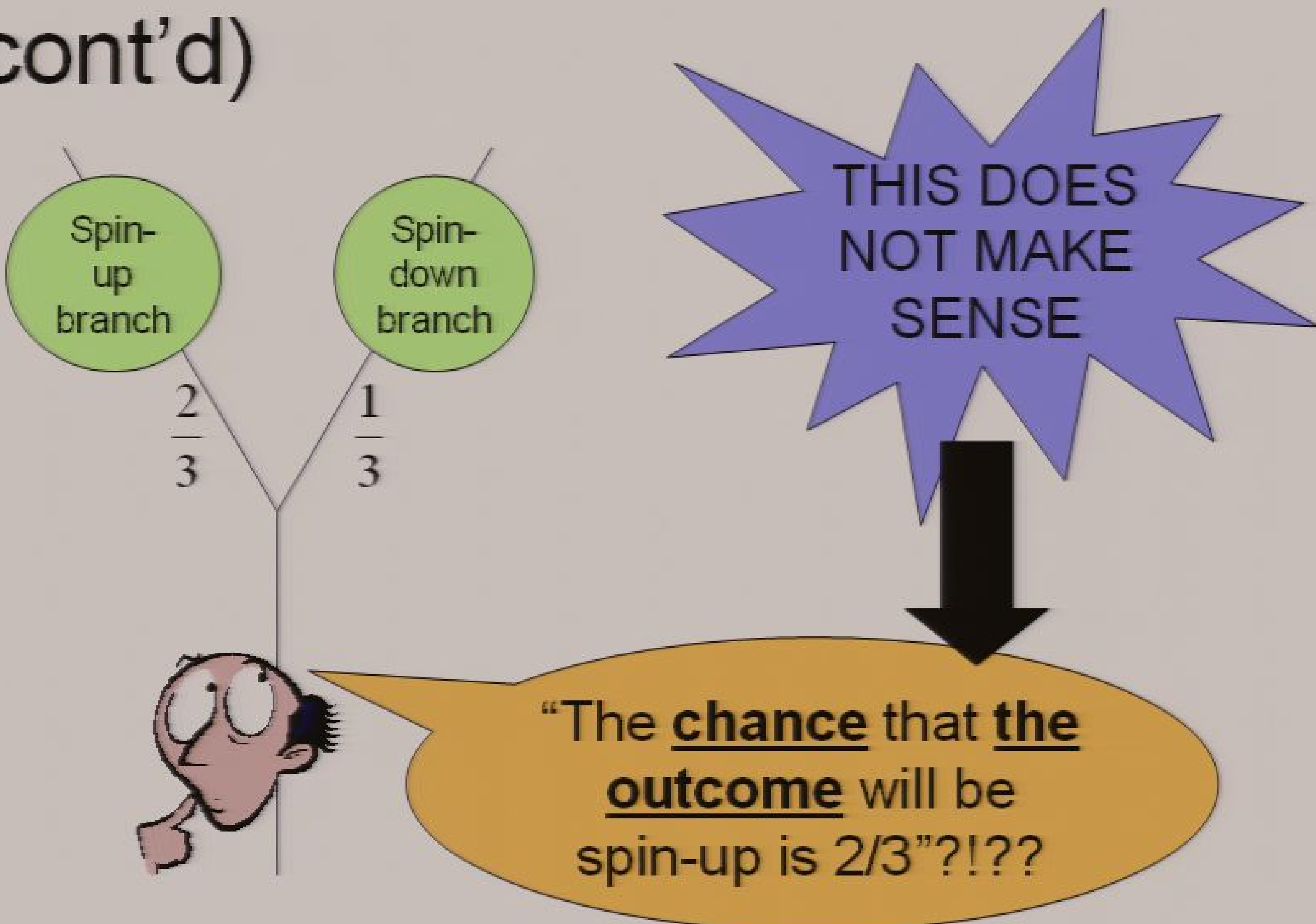


Quantum weight of *i*th branch,

$|a_i|^2 := || C_{\alpha_i} |\Psi\rangle ||^2$

- The quantum weights:
  - satisfy the axioms of probability…
  - …but mean……??
- Talking about relative frequencies won't help

1.4 The problem of probability (cont'd)



1.5 Claim 1:

Worries about *probability*
are not a reason to reject
the many-worlds interpretation.

Outline of the talk

1. The many-worlds interpretation & the problem of probability
2. The decision-theoretic program

   How to act, if you believe MW?

3. The epistemic problem

   Why believe MW in the first place?

4. Solution to the epistemic problem
5. Concluding remarks

2.1 Using QM as a guide to life

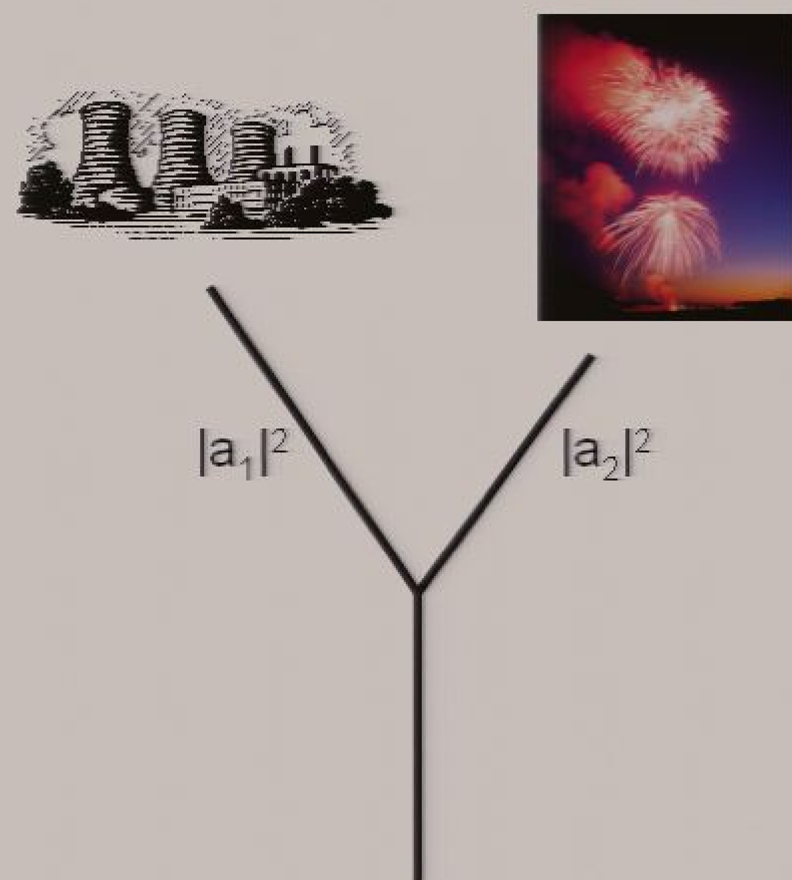
- Nuclear power plant design A:

  p(disaster) = 0.0000…..07.

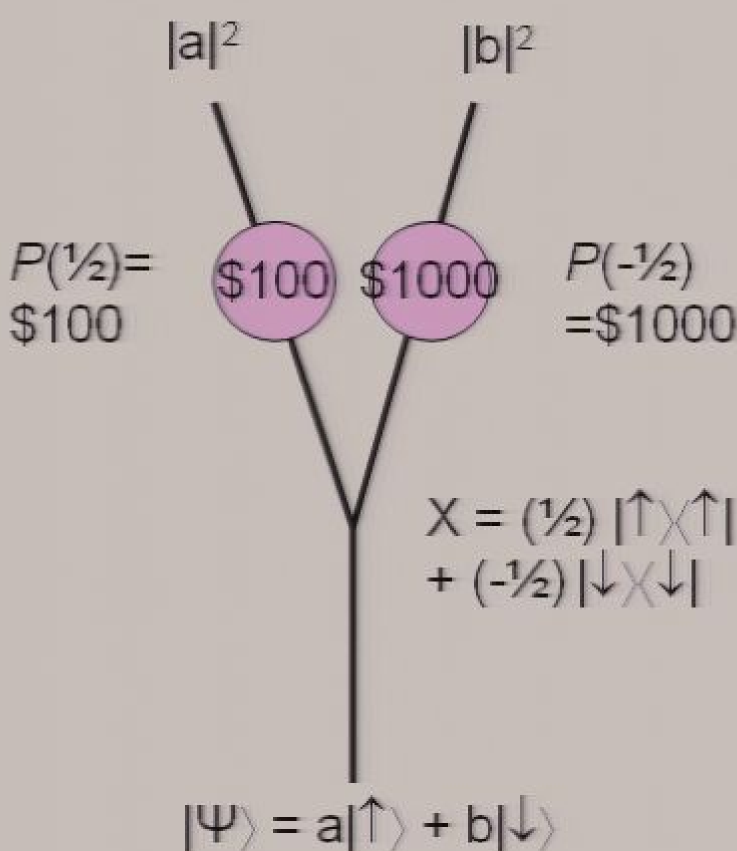
- Nuclear power plant design B:

  p(disaster) = 0.9999…..93.

- What to do?



2.2 The decision-theoretic program (Deutsch, Wallace)



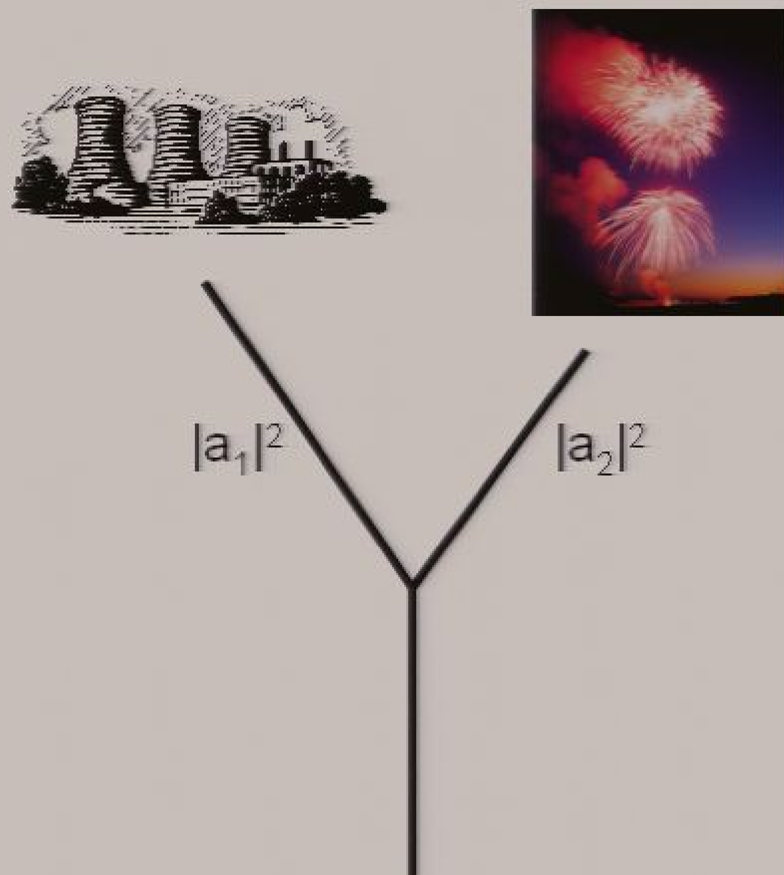
- Quantum games: $(|\Psi\rangle, X, P)$
- Utility function, U
- Probability function, p: 'decision-theoretic branch weights'
- Structural claim: Maximization of expected utility (MEU)
- Quantitative claim: decision-theoretic branch weight = quantum branch weight
- So: "The rational agent ***acts as if*** the Born rule were true." (Deutsch (1999))

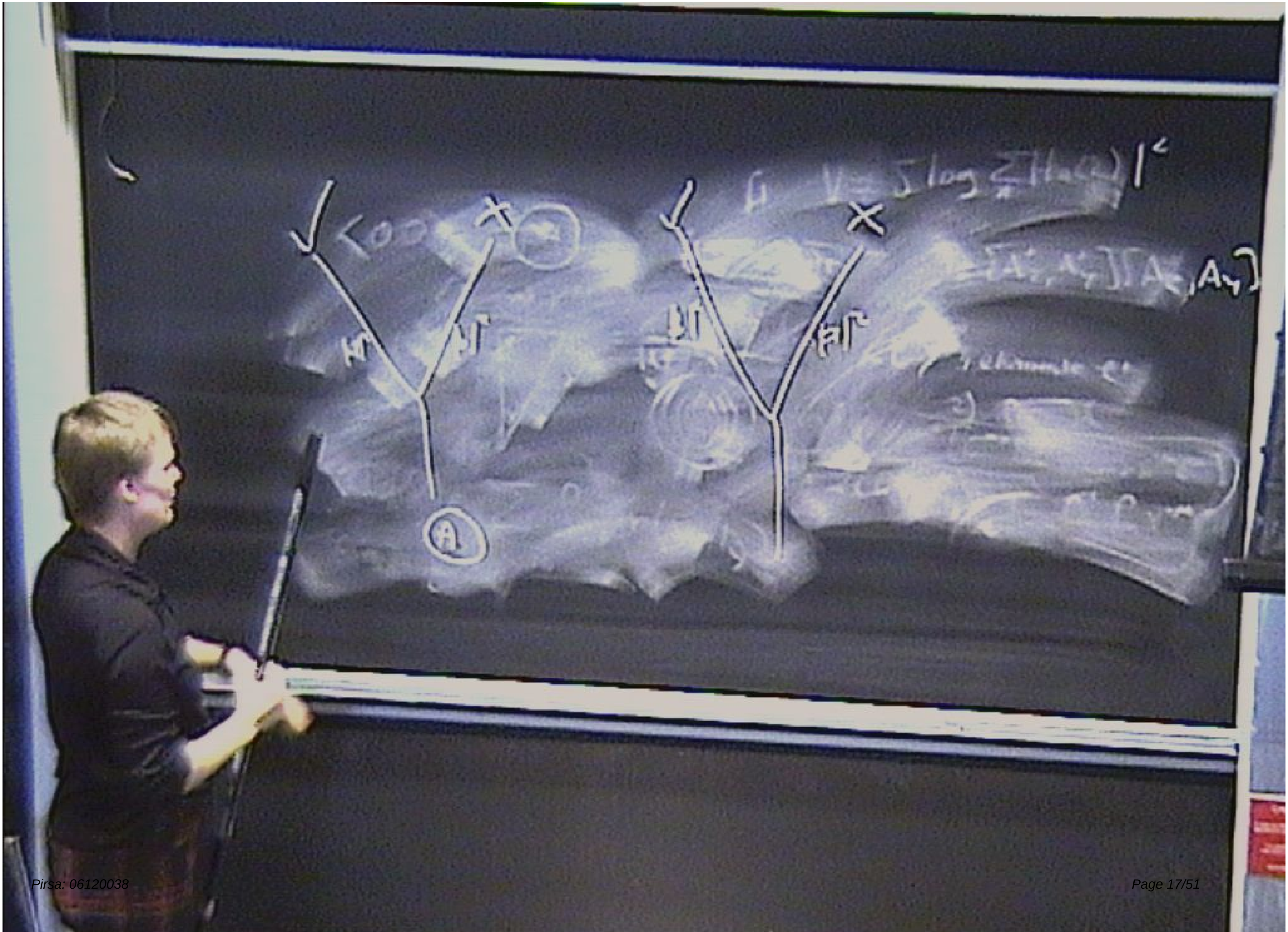
2.1 Using QM as a guide to life

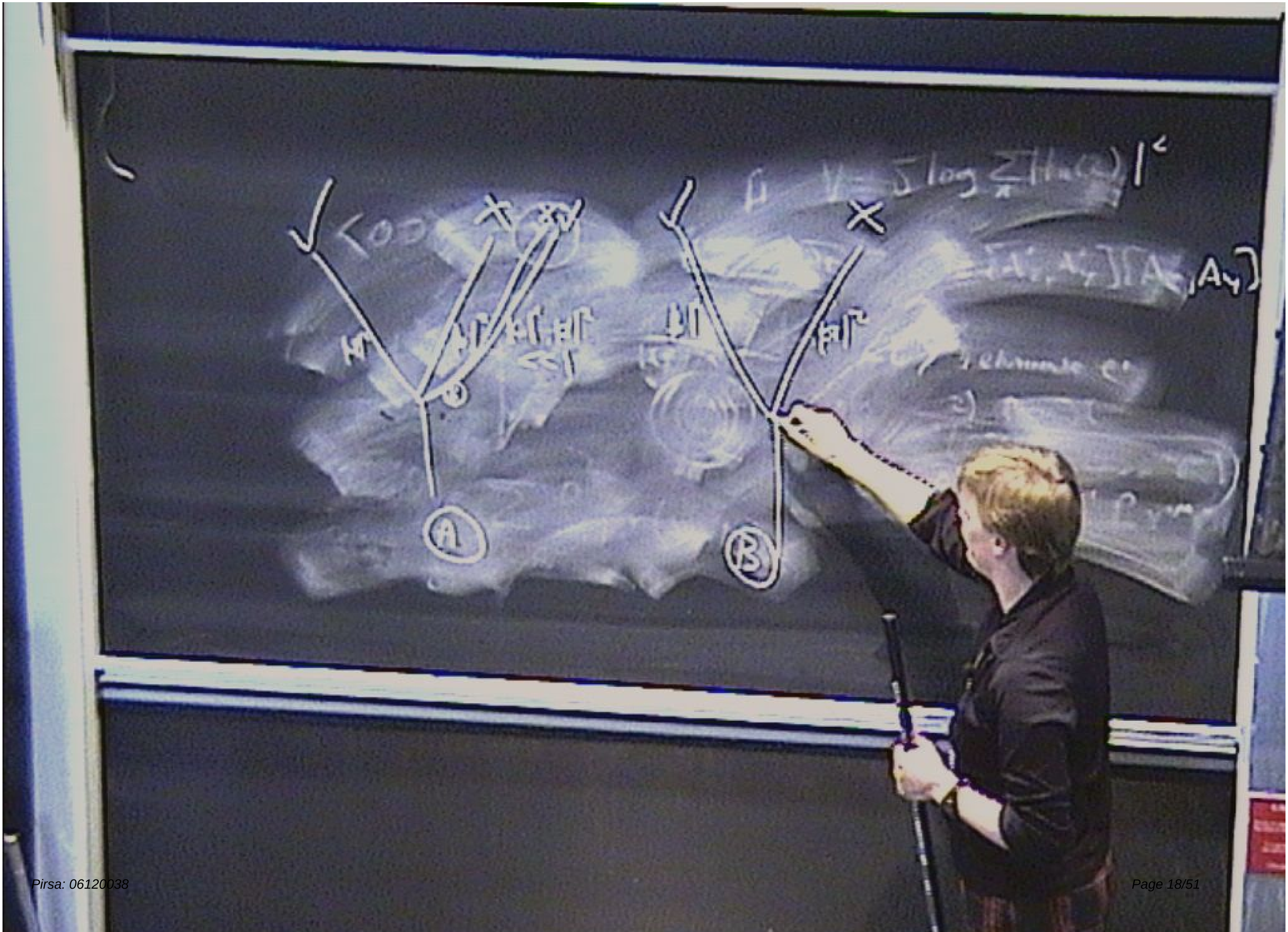
- Nuclear power plant design A:

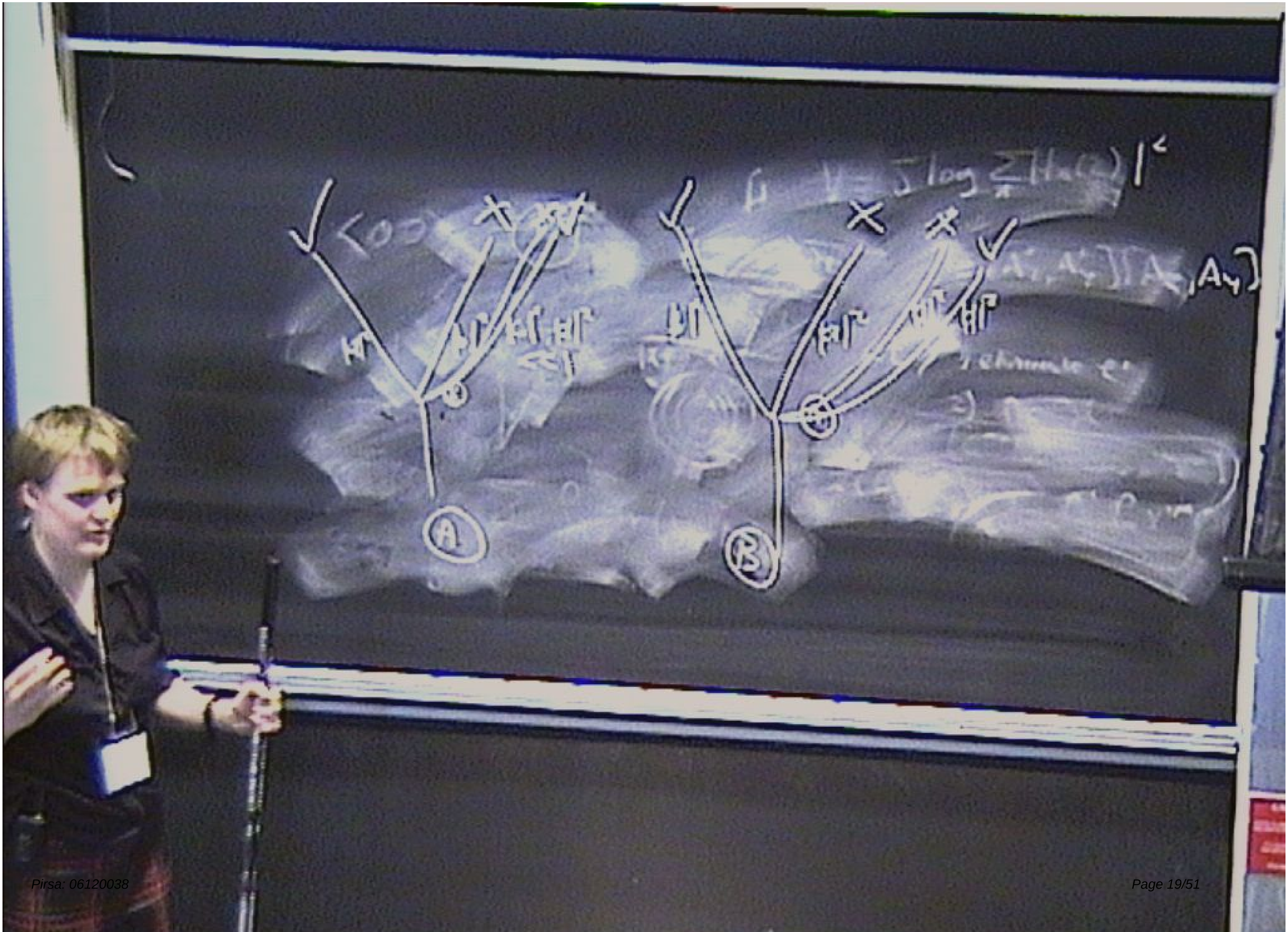
  p(disaster) = 0.0000…..07.
- Nuclear power plant design B:

  p(disaster) = 0.9999…..93.
- What to do?







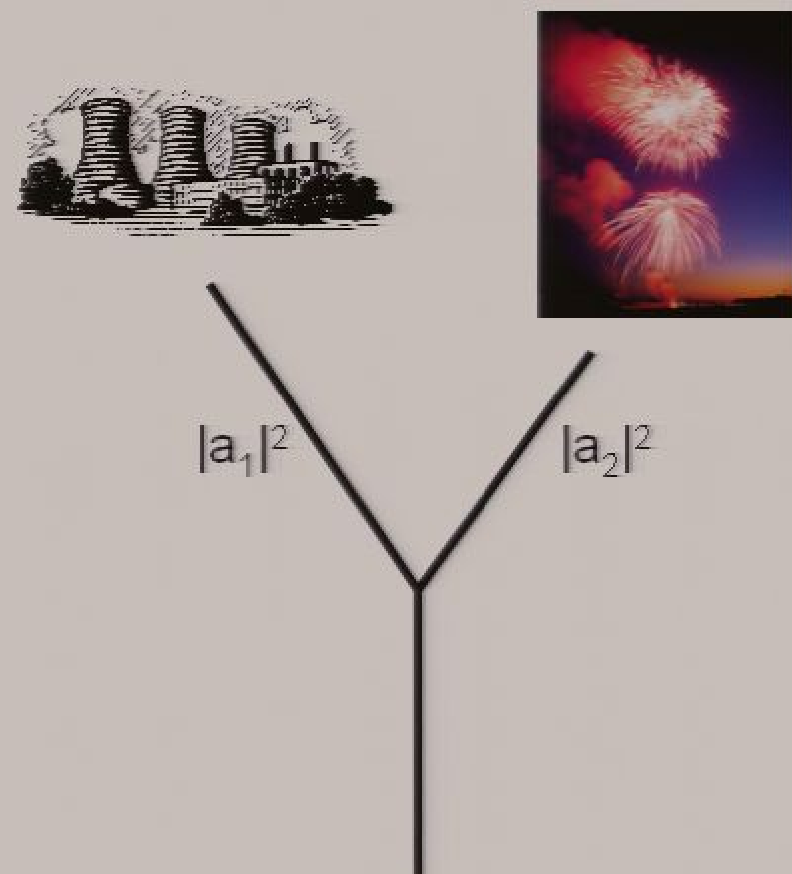

A
B

2.1 Using QM as a guide to life

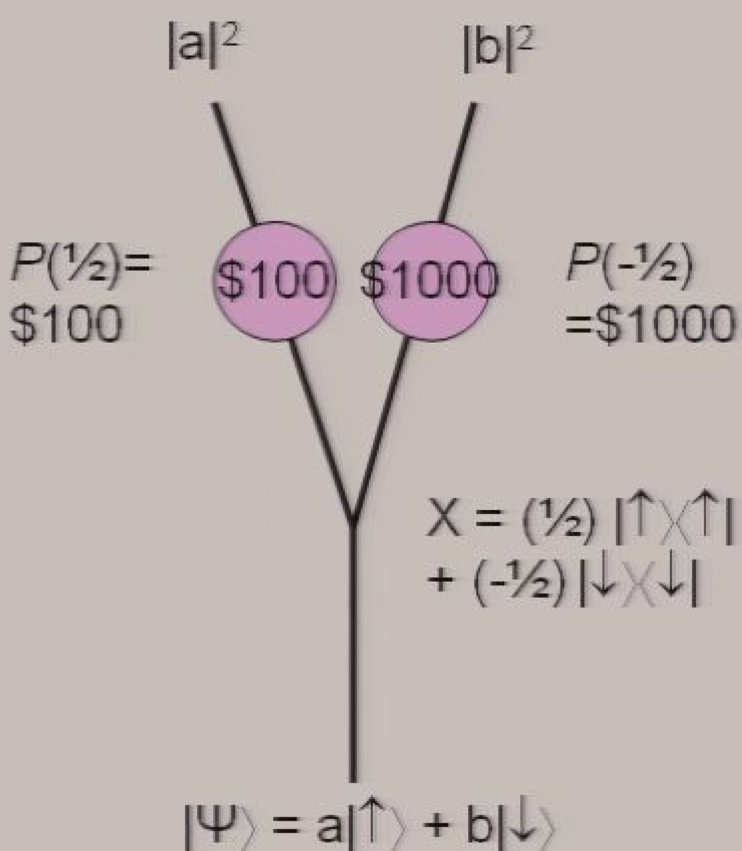
- Nuclear power plant design A:

  p(disaster) = 0.0000…..07.
- Nuclear power plant design B:

  p(disaster) = 0.9999…..93.
- What to do?

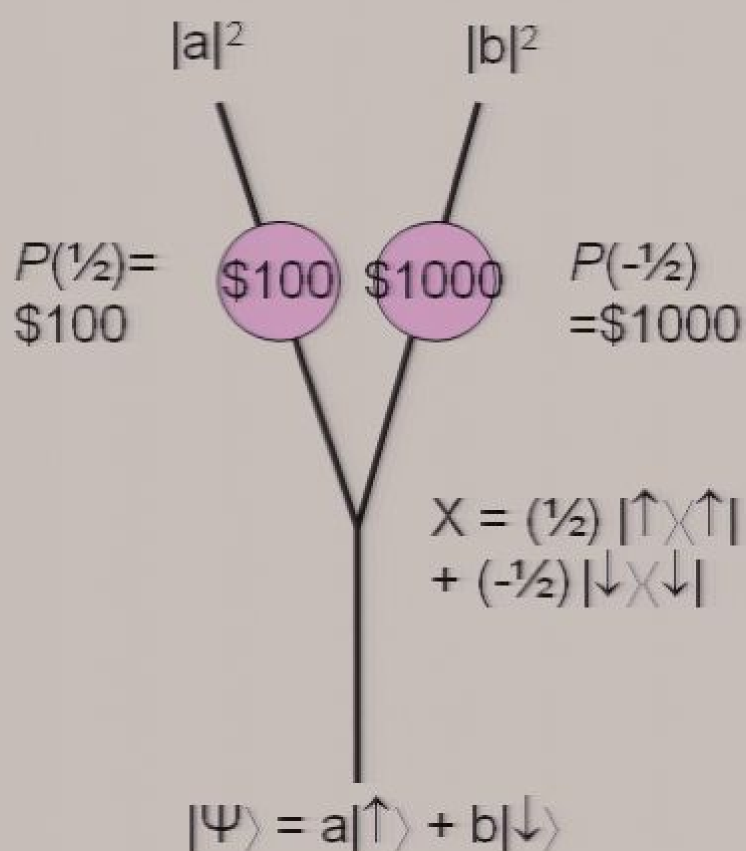


2.2 The decision-theoretic program (Deutsch, Wallace)



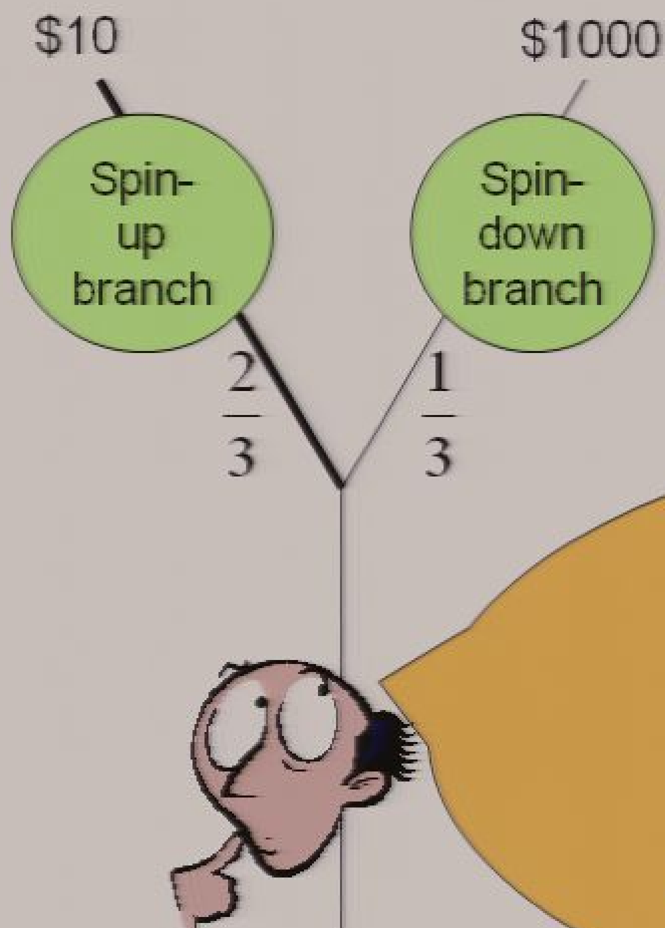
- Quantum games: $(|\Psi\rangle, X, P)$
- Utility function, U
- Probability function, p: 'decision-theoretic branch weights'
- Structural claim: Maximization of expected utility (MEU)
- Quantitative claim: decision-theoretic branch weight = quantum branch weight
- So: "The rational agent ***acts as if*** the Born rule were true." (Deutsch (1999))

2.2 The decision-theoretic program (Deutsch, Wallace)

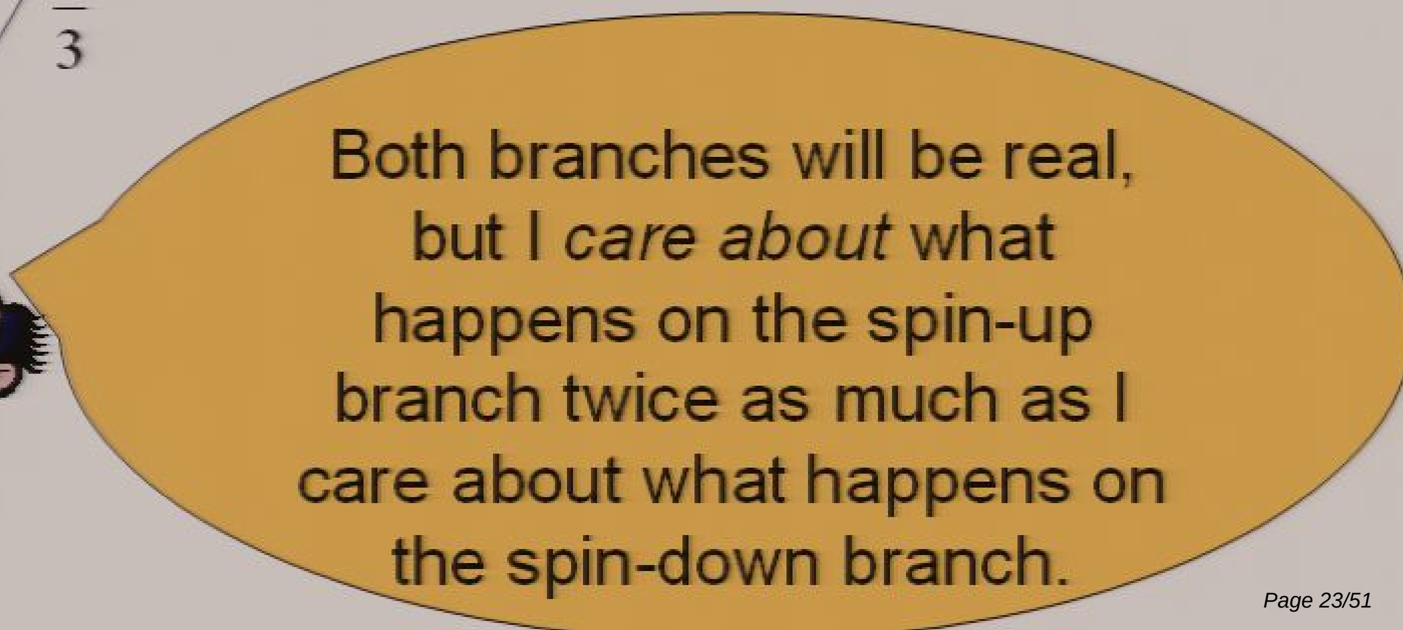


- Quantum games: $(|\Psi\rangle, X, P)$
- Utility function, U
- Probability function, p: 'decision-theoretic branch weights'
- Structural claim: Maximization of expected utility (MEU)
- Quantitative claim: decision-theoretic branch weight = quantum branch weight
- So: "The rational agent ***acts as if*** the Born rule were true." (Deutsch (1999))

2.3 How to think about decision-making in the face of branching (a suggestion)



Think of the decision-theoretic branch weights as a '**caring measure**': they quantify the degree to which the agent *cares about* what happens on the branch in question.



Outline of the talk

1. The many-worlds interpretation & the problem of probability
2. The decision-theoretic program

   How to act, if you believe MW?

3. The epistemic problem

   Why believe MW in the first place?

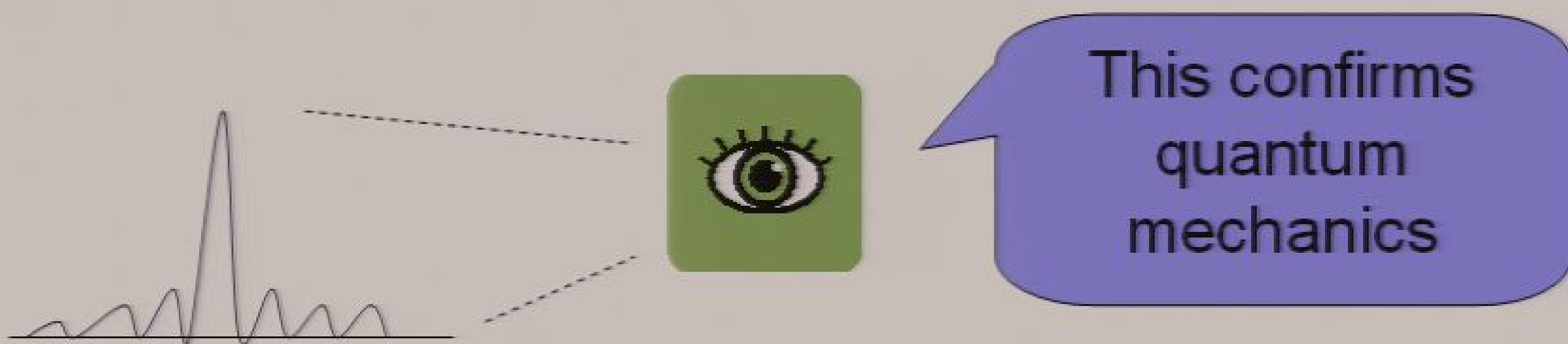
4. Solution to the epistemic problem
5. Concluding remarks

3.1 Two problems of probability

- The practical problem: How should I act, if I have (somehow) come to believe that MWQM is true?
- The epistemic problem: Do I have any reason to think that MWQM ***is*** true? (or approximately true in a certain domain, or on the path to truth...)

3.1 The epistemic problem: Why believe QM in the first place?

- e.g. A 2-slit experiment:



3.2 The epistemic problem: Why believe QM in the first place?

- Compare and contrast:
  - "Quantum mechanics predicted that the relative frequency would approximately equal *R* **with very high probability**. We observed relative frequency *R*. This gives us a reason to regard QM as empirically confirmed."
    - Seems fine
  - "MWQM predicted that the relative frequency would approximately equal *R* **on the majority of branches** [according to the 'caring measure']. We observed relative frequency *R*. This gives us a reason to regard MWQM as empirically confirmed."
    - ???
- 'Empirical incoherence': Coming to believe the theory would undermine our reason for believing anything like it (cf. Boltzmann)

3.3 Claim 2:

The epistemic problem
(not only the practical problem)
can be solved in a way
favorable to MWQM.

3.2 The epistemic problem: Why believe QM in the first place?

- Compare and contrast:
  - "Quantum mechanics predicted that the relative frequency would approximately equal *R* **with very high probability**. We observed relative frequency *R*. This gives us a reason to regard QM as empirically confirmed."
    - Seems fine
  - "MWQM predicted that the relative frequency would approximately equal *R* **on the majority of branches** [according to the 'caring measure']. We observed relative frequency *R*. This gives us a reason to regard MWQM as empirically confirmed."
    - ???
- 'Empirical incoherence': Coming to believe the theory would undermine our reason for believing anything like it (cf. Boltzmann)

3.3 Claim 2:

The epistemic problem
(not only the practical problem)
can be solved in a way
favorable to MWQM.

Outline of the talk

1. The many-worlds interpretation & the problem of probability
2. The decision-theoretic program

   How to act, if you believe MW?

3. The epistemic problem

   Why believe MW in the first place?

4. Solution to the epistemic problem
5. Concluding remarks

4.1 Strategy for solving the epistemic problem

- Ask: how *exactly* do we deal with the epistemic issue in the *non*-MW case?
  - Dynamics of rational belief: A Bayesian model of common-or-garden empirical confirmation
  - Illustrate how 2-slit experiments (etc) confirm QM
- Argue that: the same solution *(mutatis mutandis)* works for *MW*QM
  - Work out the dynamics of rational belief *for an agent who has non-zero credence in MWQM*
  - Deduce that 2-slit experiments (etc) confirm MWQM

4.2 A naïve theory of confirmation

- "If your theory predicted that X would probably occur, and you see X, you've gained evidence for your theory"
  - This will not work! (e.g. let X = 'something')

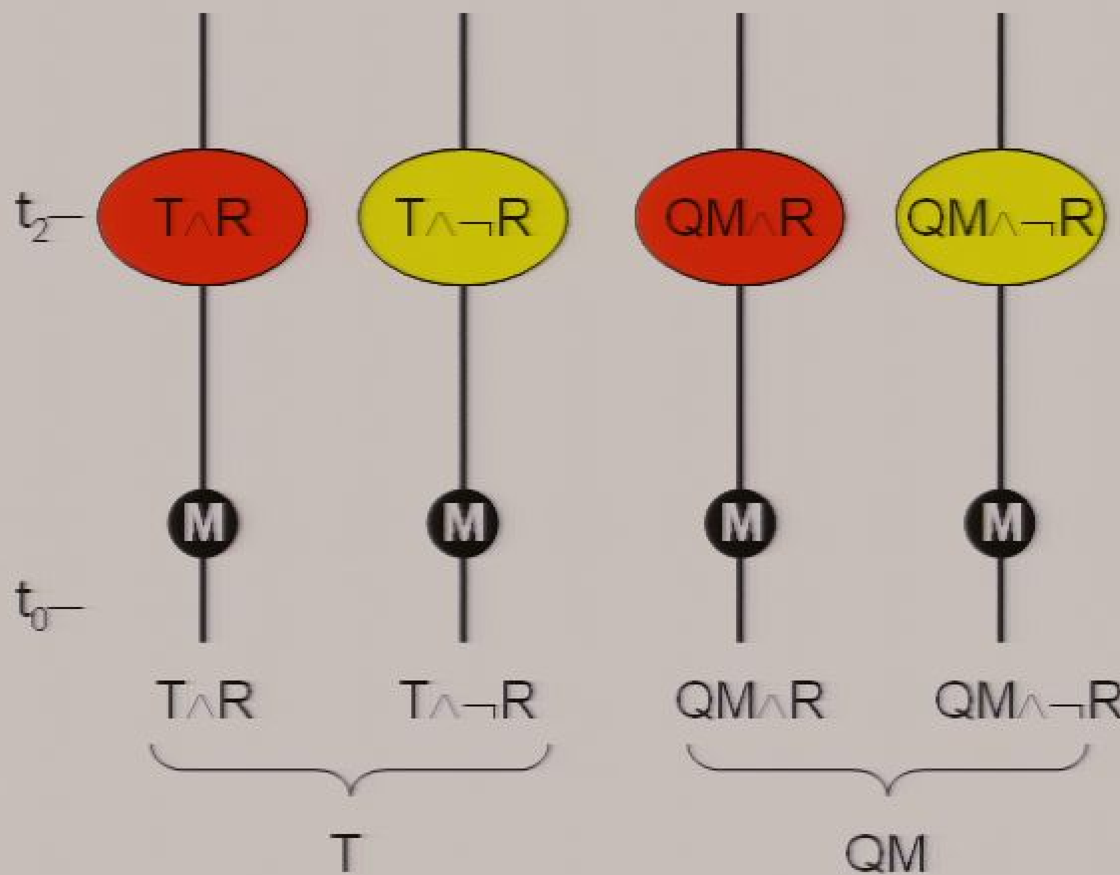
    → Need to move to Bayesian model

4.3 Bayesian confirmation theory (non-branching case)

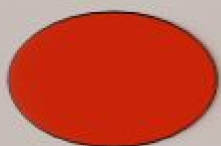
- Suppose I have two theories, QM and T
- Suppose I perform an experiment with two possible outcomes, R and ¬R
- Four 'possible worlds':

  W={T∧R, T∧¬R, QM∧R, QM∧¬R}
- Credence function Cr_0 at time t_0, prior to experiment
  - Cr_0 obeys the 'Principal Principle', i.e.:

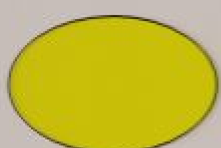
  $Cr_0(\cdot|QM) = Ch_{QM}(\cdot)$



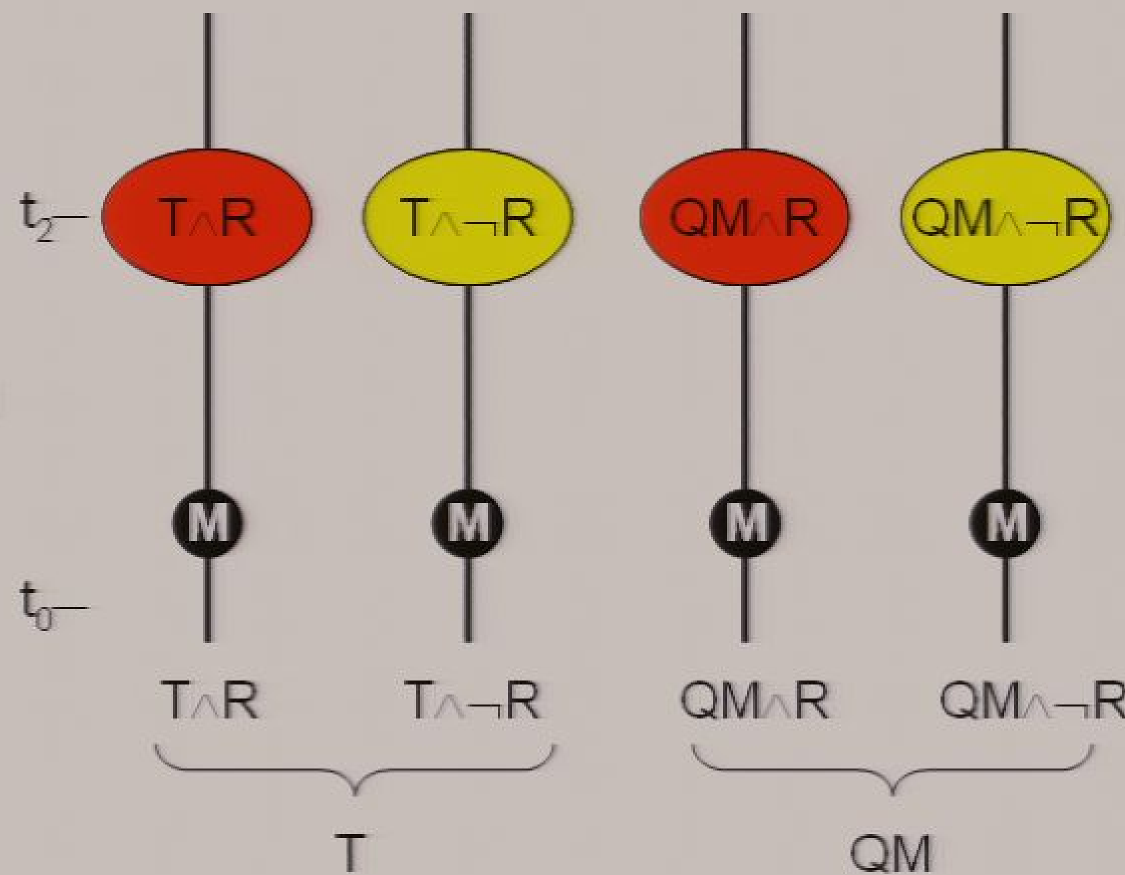
4.4 Bayesian confirmation theory (non-branching case)



Centered world in which the agent adopts credence function Cr_2^R over W



Centered world in which the agent adopts credence function $Cr_2^{\neg R}$ over W



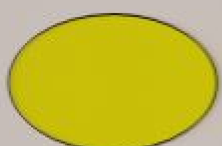
4.5 How to update beliefs: choosing Cr_2^R and $Cr_2^{\neg R}$

- Conditionalization on observed outcome: use posterior credence functions $Cr_2^R = Cr_0(\cdot|R)$, $Cr_2^{\neg R} = Cr_0(\cdot|\neg R)$
- IF
  - Cr_0 obeys the Principal Principle, and
  - the agent updates by conditionalization

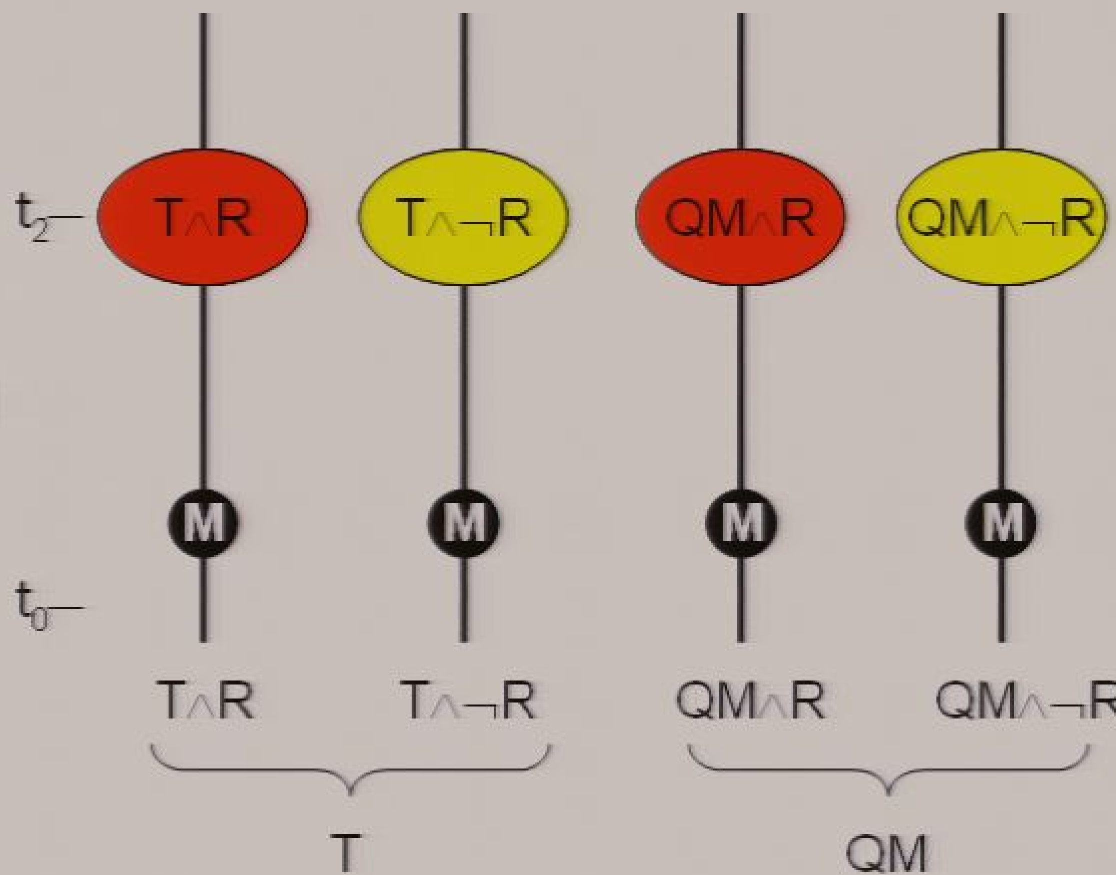
  THEN *observing R increases credence in QM at the expense of credence in T*
- This is why observing R counts as confirmatory of QM

4.4 Bayesian confirmation theory (non-branching case)

Centered world in which the agent adopts credence function Cr_2^R over W



Centered world in which the agent adopts credence function $Cr_2^{\neg R}$ over W



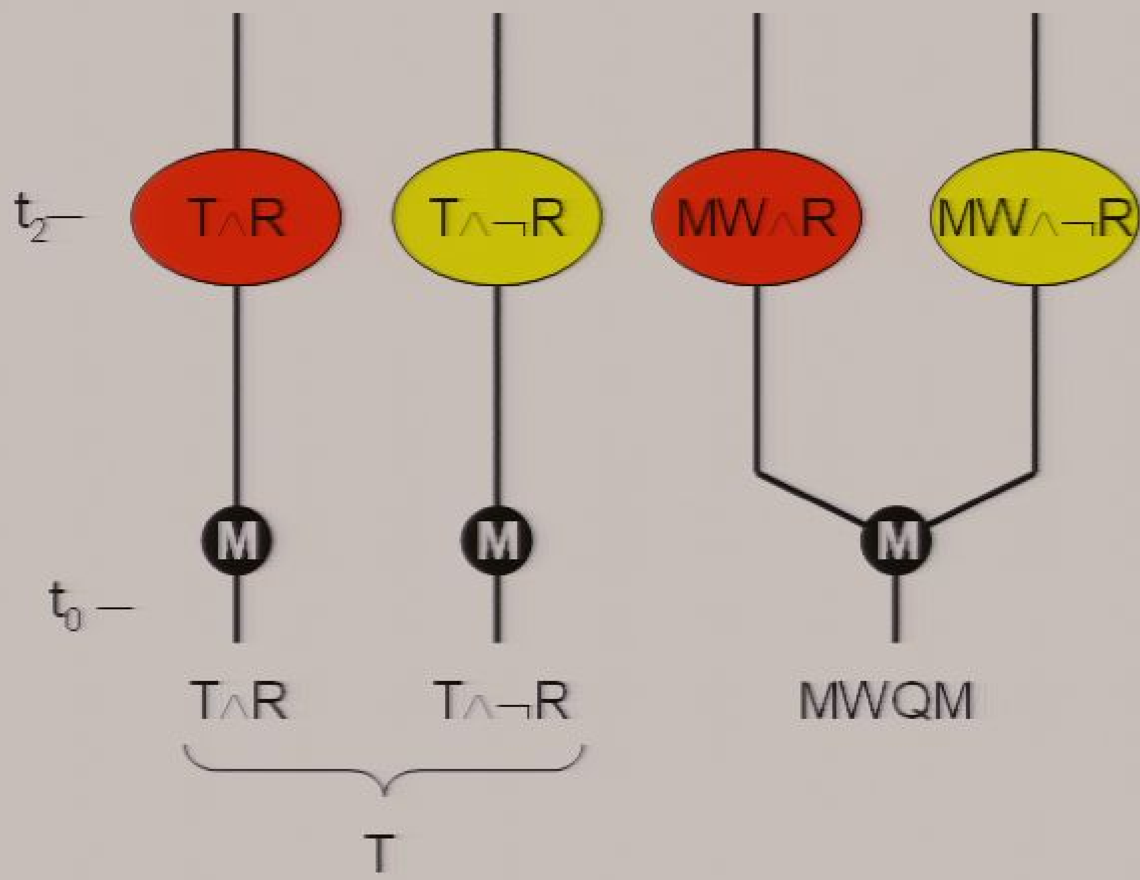
4.5 How to update beliefs: choosing Cr_2^R and $Cr_2^{\neg R}$

- Conditionalization on observed outcome: use posterior credence functions $Cr_2^R = Cr_0(\cdot|R)$, $Cr_2^{\neg R} = Cr_0(\cdot|\neg R)$
- IF
  - Cr_0 obeys the Principal Principle, and
  - the agent updates by conditionalization

  THEN *observing R increases credence in QM at the expense of credence in T*
- This is why observing R counts as confirmatory of QM

4.6 Generalized Bayesian confirmation theory ('branching case')

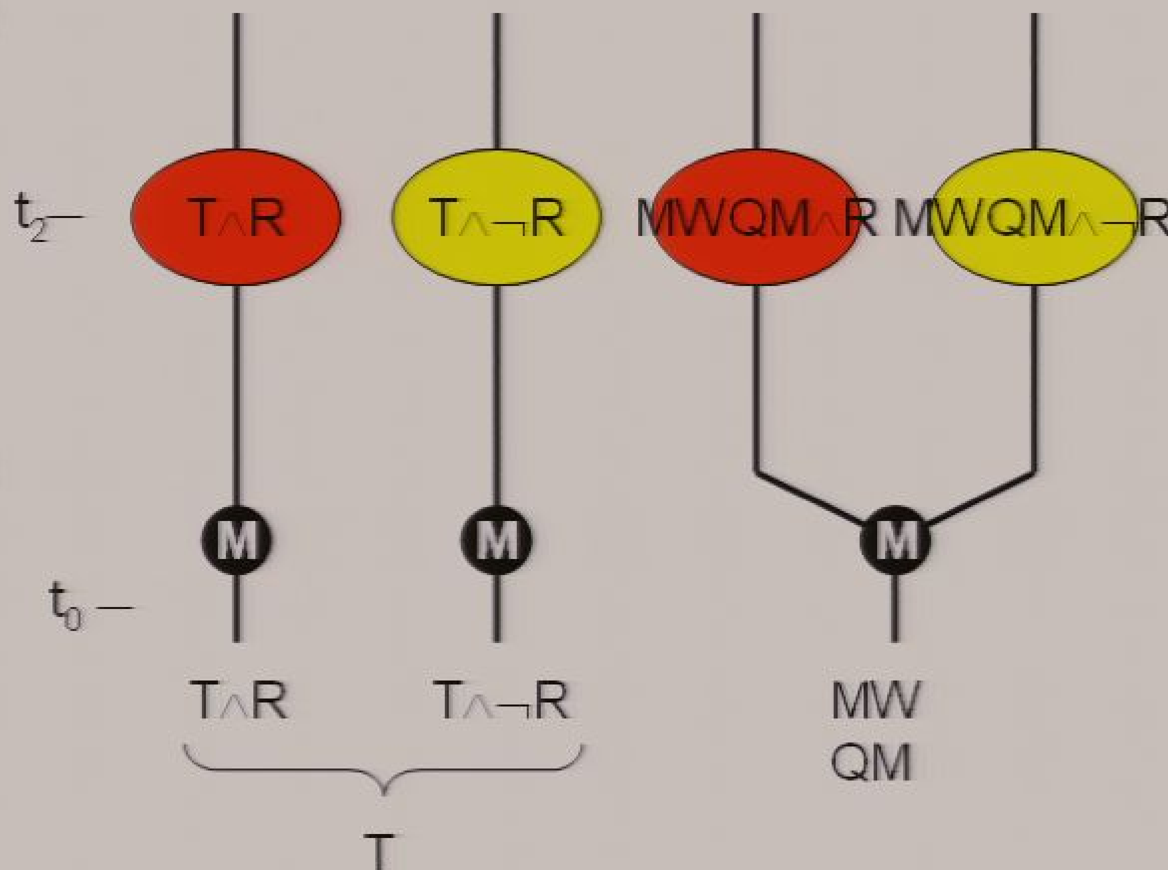
- Candidate theories: MWQM, T
- Possible worlds:
  $W = \{T \wedge R, T \wedge \neg R, MWQM\}$
- Centered possible worlds at time t_2:
  $W^c = \{T \wedge R, T \wedge \neg R, MWQM \wedge R, MWQM \wedge \neg R\}$



4.7 Generalized Bayesian confirmation theory ('branching case')

Centered world in which the agent adopts credence function Cr_2^R over W

Centered world in which the agent adopts credence function $Cr_2^{\neg R}$ over W



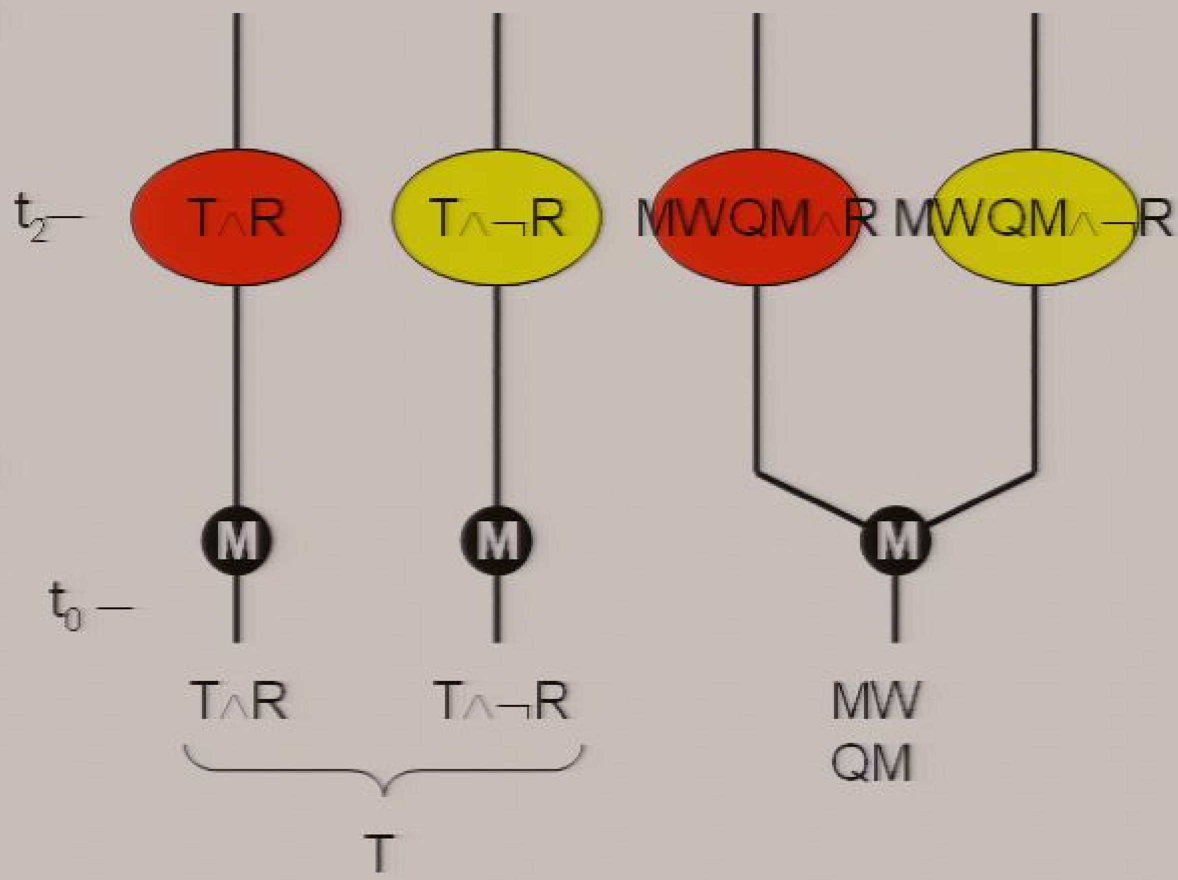
4.8 Choosing Cr_2^R and $Cr_2^{\neg R}$ in the branching case

- Two prima facie plausible updating policies:
  - Naïve conditionalization
  - Extended conditionalization
- Both of these are generalizations of ordinary conditionalization

4.7 Generalized Bayesian confirmation theory ('branching case')

Centered world in which the agent adopts credence function Cr_2^R over W

Centered world in which the agent adopts credence function $Cr_2^{\neg R}$ over W



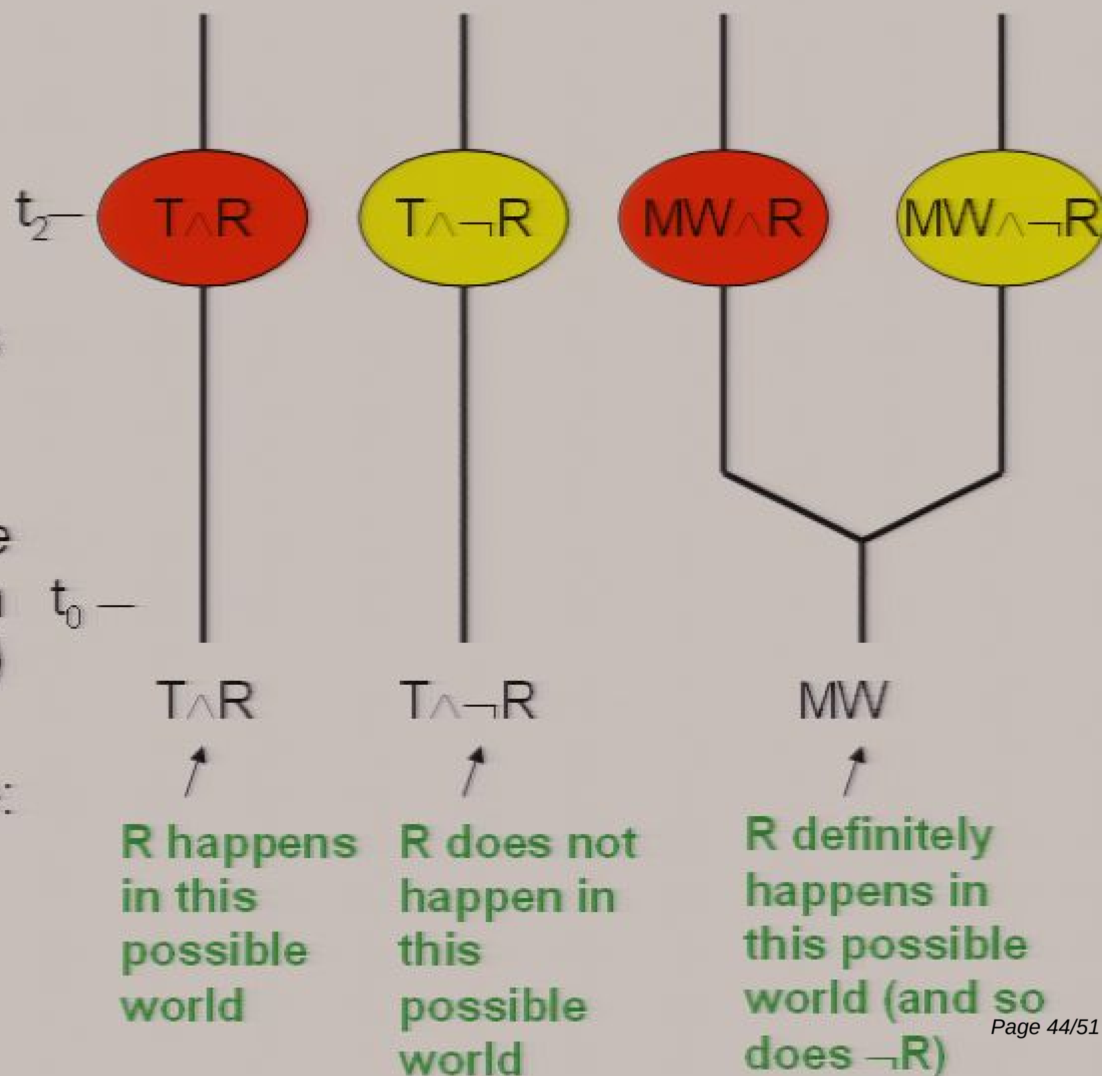
4.8 Choosing Cr_2^R and $Cr_2^{\neg R}$ in the branching case

- Two prima facie plausible updating policies:
  - Naïve conditionalization
  - Extended conditionalization
- Both of these are generalizations of ordinary conditionalization

4.9 Naïve conditionalization

- Some very natural, but pernicious intuitions:
  - ‘Caring measure’ has nothing to do with *credence*
  - The agent’s *credence that R occurs* is given by: $Cr_0(R) = Cr_0(T \wedge R) + Cr_0(MWQM)$
  - How to conditionalize:

$$Cr_2^R(\cdot) = Cr_0(\cdot|R) \equiv Cr_0(\cdot \wedge R)/Cr_0(R)$$



4.10 Naïve conditionalization is bizarre

- Observation: Naïve conditionalization has the consequence that: credence in MW increases at the expense of credence in T, regardless of whether R or ¬R occurs

  i.e. $Cr_2^R(MW) > Cr_0(MW)$

  and $Cr_2^{\neg R}(MW) > Cr_0(MW)$

  - This is not surprising
- Auxiliary premise: No rational updating policy can allow any theory to enjoy this sort of 'free ticket to confirmation'
- Conclusion: Naïve conditionalization is *not* the rational updating policy for an agent who has nonzero credence in a branching-universe theory

4.11 Defining Extended Conditionalization

- Construct an 'effective credence function', Cr'_0 (defined on W^C), from Cr_0 and Car_0
  - $Cr'_0(T \wedge R) = Cr_0(T \wedge R)$
  - $Cr'_0(T \wedge \neg R) = Cr_0(T \wedge \neg R)$
  - $Cr'_0(MW \wedge R) = Cr_0(MW) \times Car_0(R)$
  - $Cr'_0(MW \wedge \neg R) = Cr_0(MW) \times Car_0(\neg R)$
- Updating policy: obtained by conditionalizing the *effective* credence function on R and on ¬R
  - $Cr_2^R(.) = Cr'_0(.|R)$
  - $Cr_2^{\neg R}(.) = Cr'_0(.|\neg R)$
- Note: This policy would have the effect that credence in MW responds to evidence in just the same way that credence in QM responds to evidence

4.12 Defending Extended Conditionalization

- *Is* Extended Conditionalization the rational updating policy for an agent who thinks the universe might be branching?
- Yes:
  - All the arguments we have in favour of conditionalization in the ordinary case apply just as well in the branching case, and favour Extended Conditionalization over Naïve Conditionalization

4.13 Defending (ordinary) conditionalization: The (diachronic) Dutch Book argument

- Assume that degrees of belief give betting quotients
  - This holds because the agent is an expected utility maximizer
  - A fair bet is a bet with zero net expected utility
- *If the agent updates other than by conditionalization*, a Dutch Book can be made against her

4.14 Defending Extended Conditionalization: diachronic) Dutch Book argument

- If the agent is an expected-utility maximizer in Deutsch's/Wallace's sense (+…), her betting quotients are given by her *effective* credence function, Cr'_0
- *If the agent updates other than by Extended Conditionalization*, a Dutch Book can be made against her
- (Other arguments for conditionalization can be generalized in the same sort of way)

4.15 On black magic

- How these arguments manage to connect a ‘caring measure’ to credences:
  - Cast the confirmation question in terms of rational belief-updating
  - Choosing an updating policy is an epistemic action
  - *Epistemic action is a species of action*
  - The caring measure is relevant to *all* choices of actions, including epistemic ones

5 Concluding remarks

- There exists a natural measure over Everett branches, given by the Born rule (we knew this already)
- The measure governs:
  - rational action (Deutsch/Wallace have argued); so we know how to use the theory as a guide to life
  - rational belief (I have argued); so **we are justified in believing the theory on the basis of our empirical data**, just as in the non-MW case
  - What more could we want?
- **Worries about probability are not a reason to reject the many-worlds interpretation.**