Title: Differences between quantum and generalised non-locality

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Abstract: TBA

Differences between quantum and generalised non-locality

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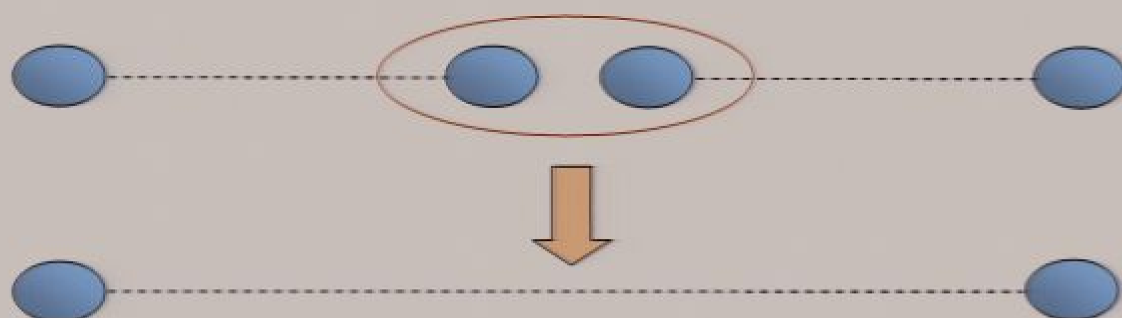
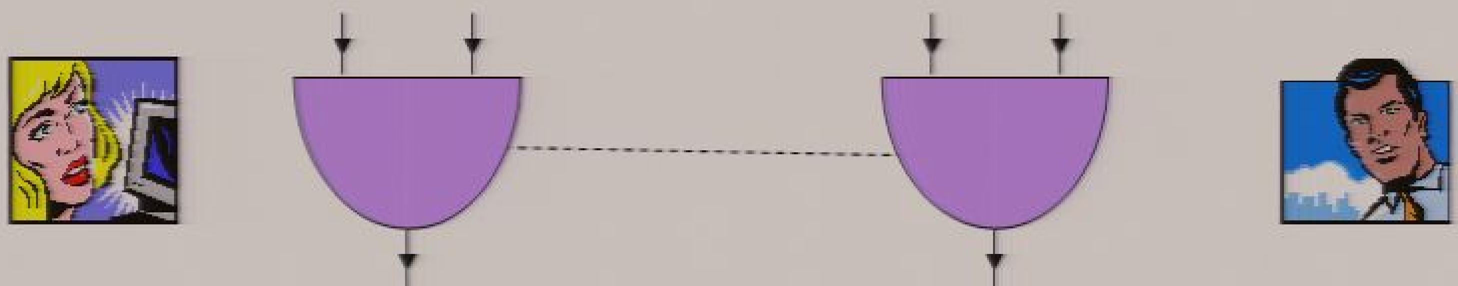
Contrasting quantum and generalised non-locality

- Non-local computation (with N.Linden, A.Winter, and S.Popescu)

- Joint measurements and non-locality swapping (with J.Barrett)



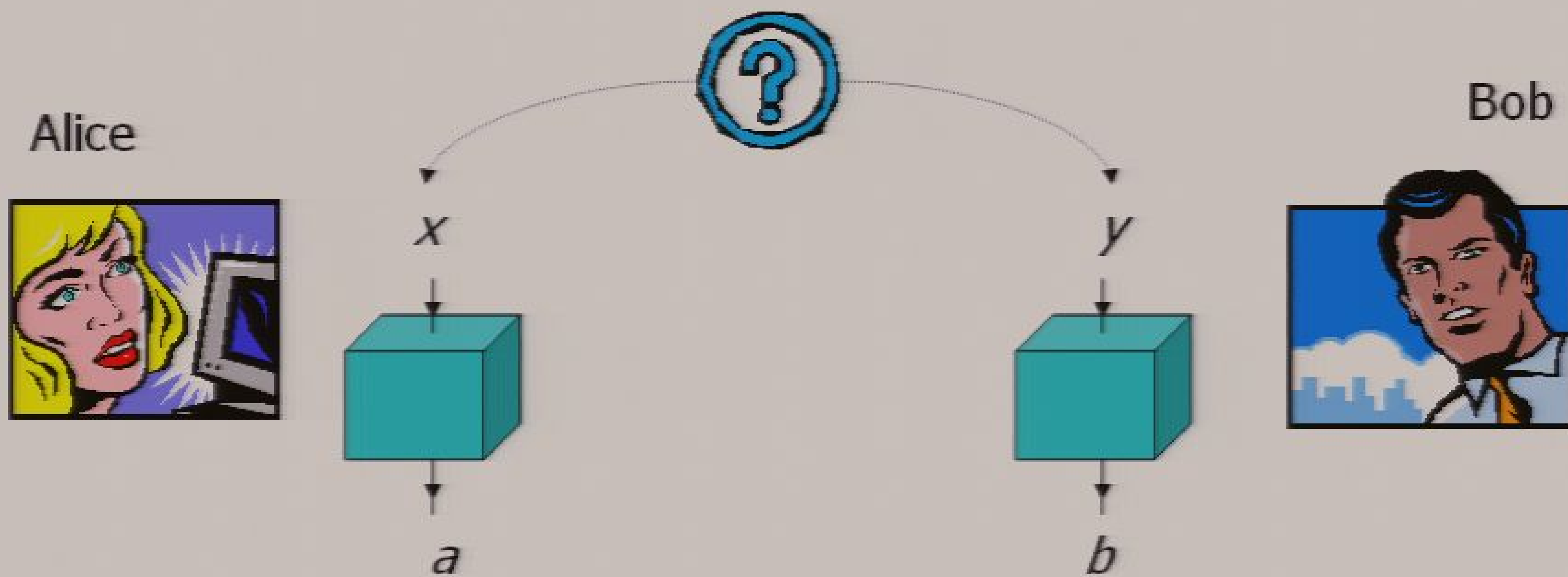
Non-local computation



Non-local computation: Overview

- An elementary non-local task
  - Success probability bounds:
    - Classical ($\Rightarrow$ CHSH inequality)
    - Quantum ($\Rightarrow$ Tsirelson inequality)
    - Generalised non-locality
- Non-local Computation
  - Success probability bounds
  - Example: nonlocal-AND
  - Extensions.
- Conclusions

An elementary non-local task.



Alice and Bob are set the following challenge: Given random input bits (x, y), they must generate output bits (a, b) such that

$$a \oplus b = xy$$

What is their maximum probability of success?

Computing the success probability

- The average probability of success in this task is given by

$$P_{success} = \sum_{xy} P(x,y) P(a_x \oplus b_y = xy)$$

$$= \sum_{xy} \frac{1}{4} \left\langle \frac{1+(-1)^{a_x+b_y+xy}}{2} \right\rangle$$

Writing $A_x=(-1)^{a_x}$ and $B_y=(-1)^{b_y}$,

$$P_{success} = \frac{1}{2} + \frac{1}{4}\left(\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \right)$$

Maximal success probability: Classical

- The success probability for classical strategies is bounded by the Clauser-Horne-Shimony-Holt (CHSH) inequality:

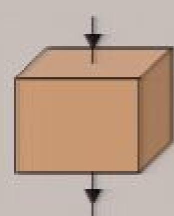
$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

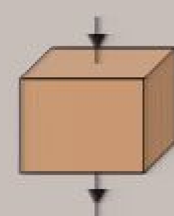
Which gives

$$\boxed{\max P^{C}_{success} = \frac{3}{4}}$$

e.g.

x	a
0	0
1	0





y	b
0	0
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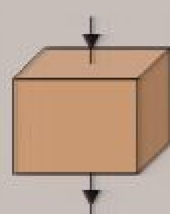
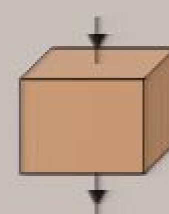




y	b
0	0
1	0

Maximal success probability: Quantum

- If Alice and Bob share an entangled state, they can use it to generate *non-local correlations*:

$$P(a,b\,|\,x,y) \neq \sum_i P(i)P(a\,|\,x,i)P(b\,|\,y,i)$$

Their success probability is bounded by the Tsirelson inequality:

$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2\sqrt{2}$$

Which gives

$$\boxed{\max P^{Q}_{success} = \frac{2+\sqrt{2}}{4}}$$

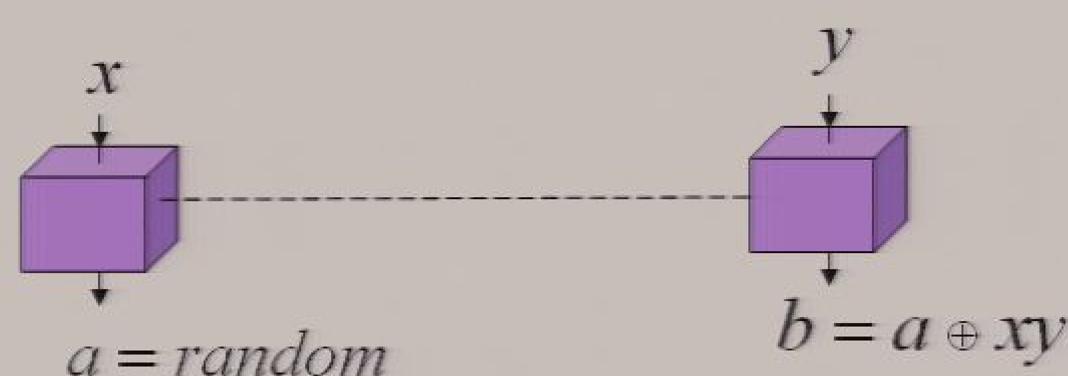
Maximal success probability: Generalised non-locality

- Now consider generalised non-local correlations, where any P(a,b|x,y) is allowed that does not allow signalling between Alice and Bob.

With such super-strong non-local correlations

$$\boxed{\max P^{G}_{success} = 1}$$

e.g.



A hierarchy of success probabilities

- Bell and Tsirelson inequalities can be understood as bounds on the maximal success probability in non-local tasks.

- In this particular non-local task, the maximal success probability increases with the amount of attainable non-locality:

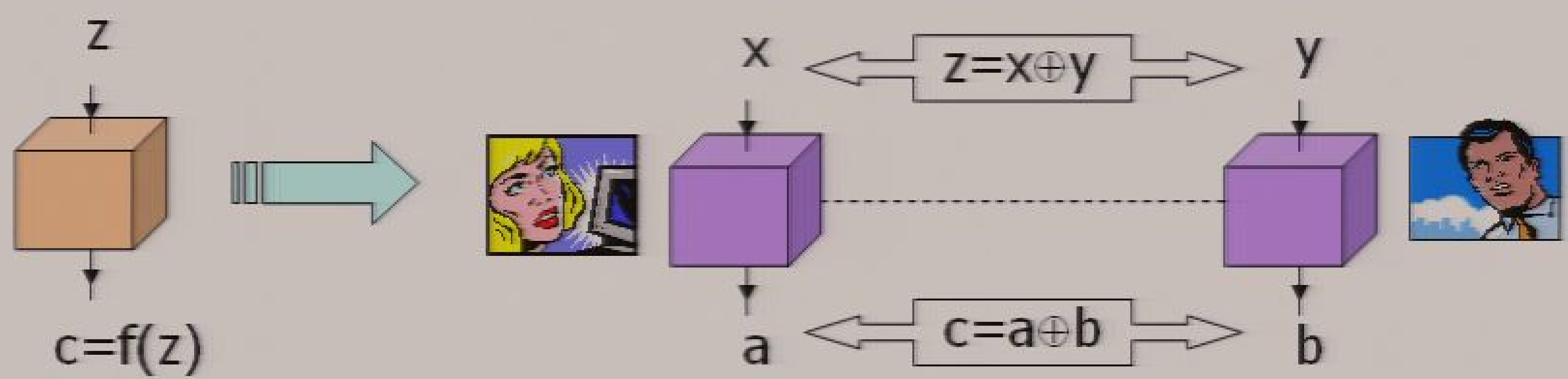
Greater non-locality $\Rightarrow$ *Greater success probability*

$$\boxed{\max P^{C}_{success} < \max P^{Q}_{success} < \max P^{G}_{success}}$$

- Is this a feature of *all* non-local tasks?

Non-local Computation

- Consider the non-local computation of a Boolean function *c=f(z)* from n bits ($z=z_1z_2...z_n$) to 1 bit, in which each party individually learns nothing about c or z.



Given random input bit strings (*x, y*), Alice and Bob must generate output bits (*a, b*) such that

$$a \oplus b = f(x \oplus y)$$

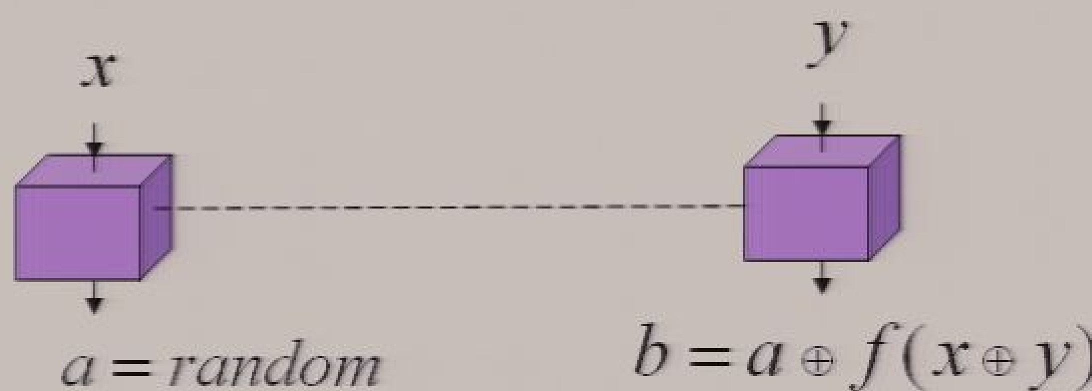
Maximal success probabilities: Generalised non-locality.

- We allow an arbitrary probability distribution $P_{in}(z)$ of logical inputs $z=x \oplus y$, although x and y individually remain maximally random so that Alice and Bob cannot learn z. Hence

$$P(x,y)=\frac{P_{in}(x \oplus y)}{2^n}$$

- As before, generalised non-locality allows perfect success:

$$\max P^{G}_{success} = 1$$



Maximal success probabilities: Quantum

- When Alice and Bob share a quantum state, their success probability is given by

$$P_{success} = \frac{1}{2}\left(1 + \sum_{xy} P(x,y)(-1)^{\hat{a}_x + \hat{b}_y + f(x\oplus y)}\right)$$
$$= \frac{1}{2}\left(1 + \langle \alpha | \mathbf{1} \otimes \Phi | \beta \rangle\right)$$

where

$$|\alpha\rangle = 2^{-n/2} \sum_x (-1)^{\hat{a}_x} |\psi\rangle \otimes |x\rangle$$
$$|\beta\rangle = 2^{-n/2} \sum_y (-1)^{\hat{b}_y} |\psi\rangle \otimes |y\rangle$$
$$\Phi = \sum_{xy} (-1)^{f(x\oplus y)} P_{in}(x\oplus y) |x\rangle\langle y|$$

Maximal success probabilities: Quantum

This means that the quantum success probability is bounded by

$$P^{Q}_{success} \leq \frac{1}{2}\left(1 + \|\Phi\|\right)$$

Φ has eigenstates $\quad |u\rangle = \sum_x (-1)^{u.x} |x\rangle$

with eigenvalues $\quad \varphi_u = \sum_z (-1)^{f(z)+u.z} P_{in}(z) \quad$ hence

$$\boxed{\max P^{Q}_{success} = \frac{1}{2}\left(1 + \max_u \left| \sum_z (-1)^{f(z)+u.z} P_{in}(z) \right| \right)}$$

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Maximal success probabilities: Classical

- Surprisingly, the same maximal success probability can be attained by adopting a classical strategy:

$$a = u.x \qquad\qquad b = u.y \oplus \delta$$

giving

$$\boxed{\max P^{C}_{success} = \max_{u,\delta} \frac{1}{2}\left(1 + \sum_{z} (-1)^{f(z)+u.z+\delta} P_{in}(z)\right) = \max P^{Q}_{success}}$$

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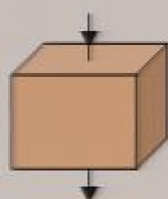
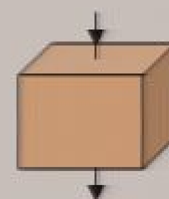
x

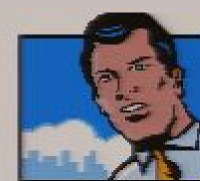


$$a = u.x$$

y



$$b = u.y \oplus \delta$$



giving

$$\max P_{success}^{C} = \max_{u,\delta} \frac{1}{2}\left(1 + \sum_{z} (-1)^{f(z)+u.z+\delta} P_{in}(z)\right) = \max P_{success}^{Q}$$

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Non-local Computation: Summary

- For all non-local computations with a single output bit, where Alice and Bob must jointly compute $c=f(z_1,z_2 \ldots z_n)$ without individually learning c or z, quantum non-locality is useless:

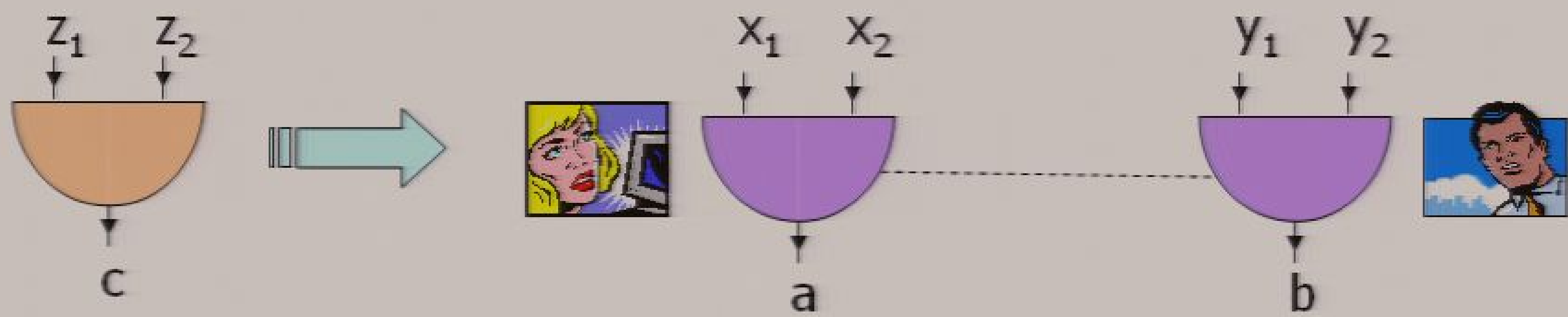
Greater non-locality $\not\Rightarrow$ *Greater success probability*

$$\boxed{\max P^{C}_{success} = \max P^{Q}_{success} \leq \max P^{G}_{success}}$$

- Note that each choice of f(z) and $P_{in}(z)$ also corresponds to a pair of identical Bell and Tsirelson inequalities.

Example: Nonlocal-AND

As a simple example, consider the non-local version of AND



$$c = z_1 z_2 \quad \Rightarrow \quad (a \oplus b) = (x_1 \oplus y_1)(x_2 \oplus y_2)$$

For maximally random inputs ($P_{in}(z)=1/4$), we obtain:

$$\max P^{C}_{success} = \frac{3}{4} \quad = \quad \max P^{Q}_{success} = \frac{3}{4} \quad < \quad \max P^{G}_{success} = 1$$

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Maximal success probabilities: Classical

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x $\qquad\qquad\qquad\qquad$ y

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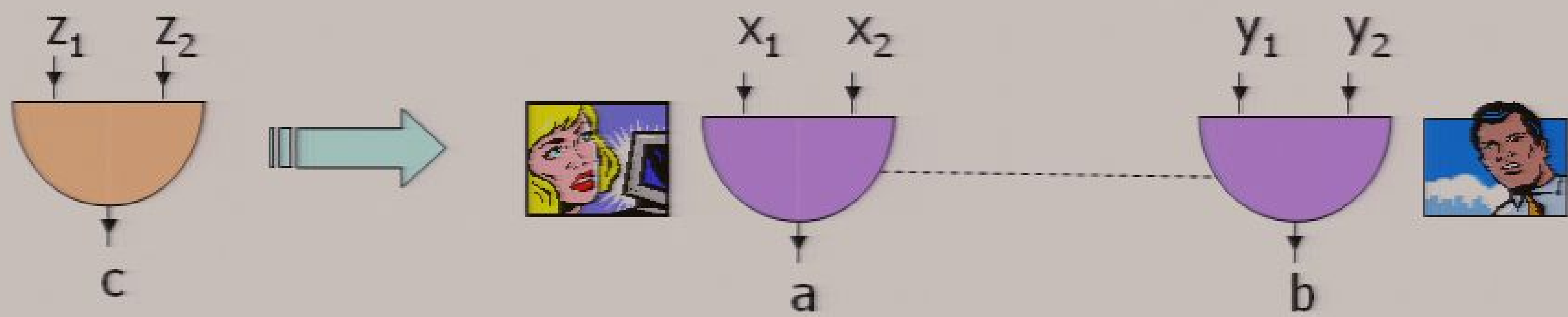

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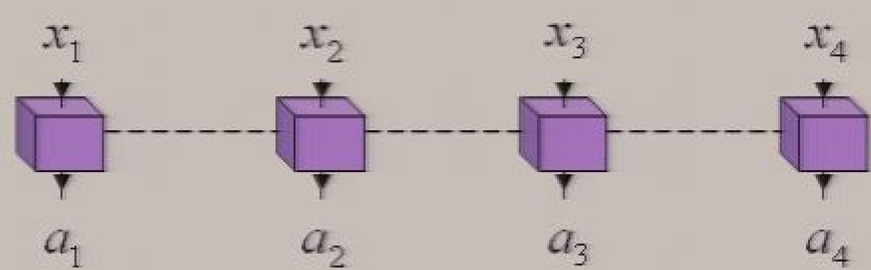
Non-local computation: Extensions

- These results also extend to further cases:
  1. Non-local computations by any number of parties:



$$z = x_1 \oplus x_2 \oplus x_3 \oplus x_4$$
$$c = a_1 \oplus a_2 \oplus a_3 \oplus a_4$$

  2. Non-local computations with multiple output bits where strategies are scored according to the number of correct bits.
  3. Other non-local tasks requiring $a \oplus b = f(x,y)$, for which

$$\Phi' = \sum_{xy} (-1)^{f(x,y)} P(x,y) |x\rangle\langle y|$$

has a maximal-eigenvalue eigenstate $|u\rangle = \sum_x (-1)^{u.x} |x\rangle$

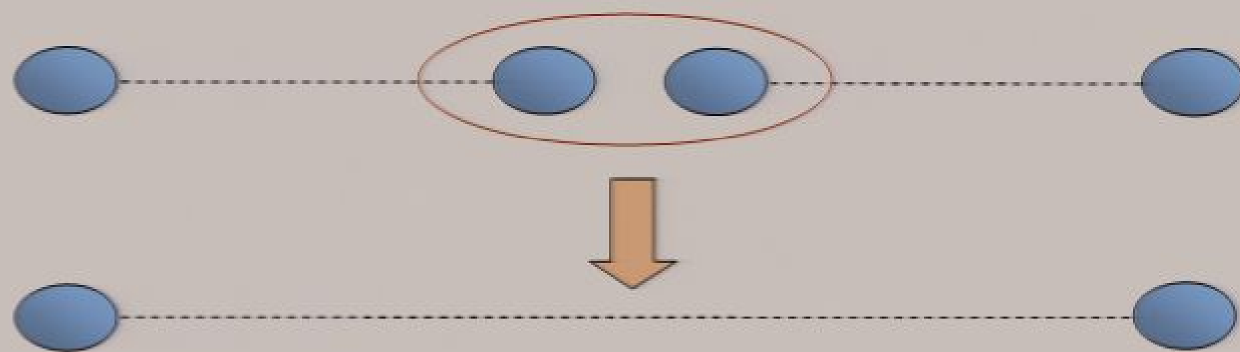
Distributed Computation: Conclusions

- Non-local computation provides a natural class of tasks in which generalised non-local correlations allow perfect success, yet quantum non-locality is useless.

$$\boxed{\max P^{C}_{success} = \max P^{Q}_{success} < \max P^{G}_{success}}$$

- Do all non-quantum non-local correlations help in some non-local computation?

Joint measurements and non-locality swapping

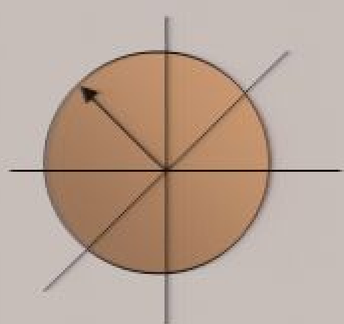


Joint measurements and non-locality swapping: Overview

- A general framework for probabilistic theories.
  - Representing states
  - The no-signalling condition
  - Generalised Non-Signalling Mechanics (GNSM)
  - Representing measurements
- Measurements in GNSM
  - Limitation to post-selected fiducial measurements
  - Impossibility of `swapping' non-locality
- Conclusions

Representing quantum states as probability vectors

- Instead of representing quantum states as density matrices, we take a more operational approach (Hardy, Barrett):
  - A state is completely represented by a vector P(a|x) of outcome probabilities (a) for some set of *fiducial* measurements (x).
  - E.g. For a single qubit, we might choose σ_x, σ_y, σ_z as fiducial measurements



$$\underline{P} = \begin{pmatrix} \begin{pmatrix} P(+1 \mid \sigma_x) \\ P(-1 \mid \sigma_x) \end{pmatrix} \\ \begin{pmatrix} P(+1 \mid \sigma_y) \\ P(-1 \mid \sigma_y) \end{pmatrix} \\ \begin{pmatrix} P(+1 \mid \sigma_z) \\ P(-1 \mid \sigma_z) \end{pmatrix} \end{pmatrix}$$

  - This framework can be used to express quantum, classical and more general theories, allowing comparisons between them.

Multipartite systems

- The state of a multipartite system can be given by specifying the output probabilities for every combination of fiducial measurements on the subsystems (I.e. P(**a**|**x**)= P(a_1…a_n| x_1…x_n))

e.g. The singlet state

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$P_{singlet} = \left.\overbrace{\begin{pmatrix} \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} & \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} \\ \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} & \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} \\ \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} & \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix} & \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \end{pmatrix}}^{a_2 | x_2}\right\} a_1 | x_1$$

$$P(a_1 = +1, a_2 = -1 \,|\, x_1 = \sigma_z, x_2 = \sigma_z)$$

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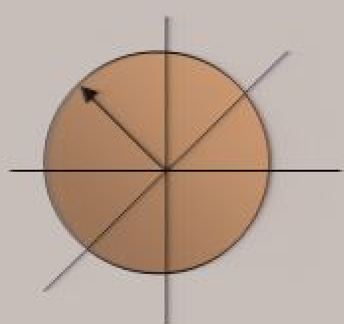

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$P(a_1 = +1, a_2 = -1 \,|\, x_1 = \sigma_z, x_2 = \sigma_z)$

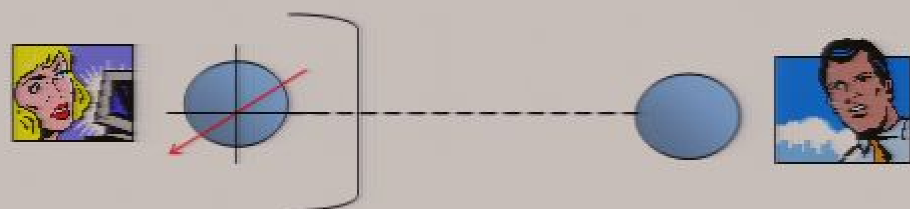
The no-signalling condition

- All P(**a**|**x**) representing allowed states must satisfy:

  I. Positivity: $P(\mathbf{a}\,|\,\mathbf{x}) \geq 0$

  II. Normalisation: $\sum_{\mathbf{a}} P(\mathbf{a}\,|\,\mathbf{x}) = 1$

  III. No-signalling: $\sum_{a_n} P(\mathbf{a}\,|\,\mathbf{x})$ is independent of x_n



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Generalised non-signalling mechanics (GNSM)

- However, there exist distributions P(**a**|**x**) satisfying the positivity, normalisation, and no-signalling constraints that do not correspond to any quantum system.

e.g.

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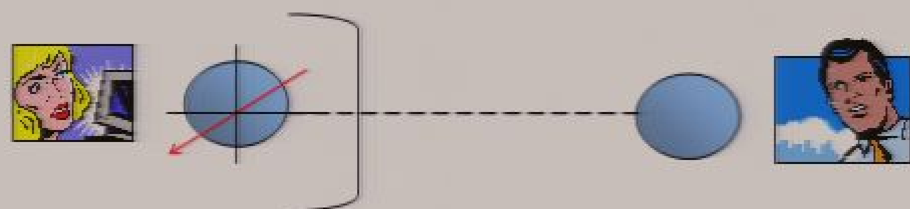

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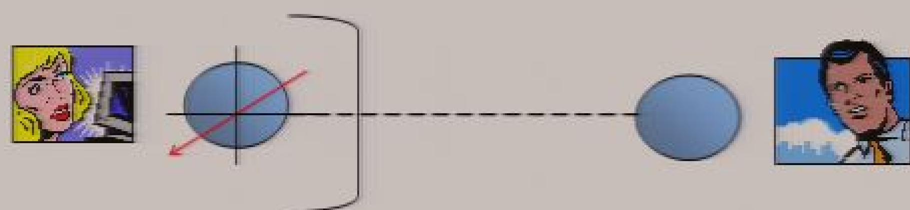

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GNSM contains stronger than quantum non-locality

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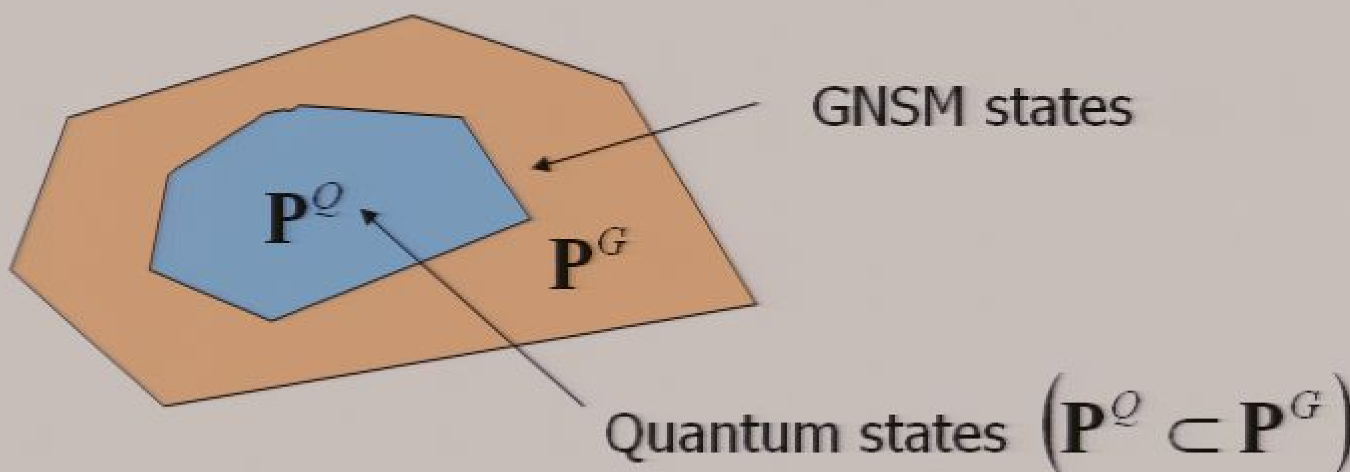

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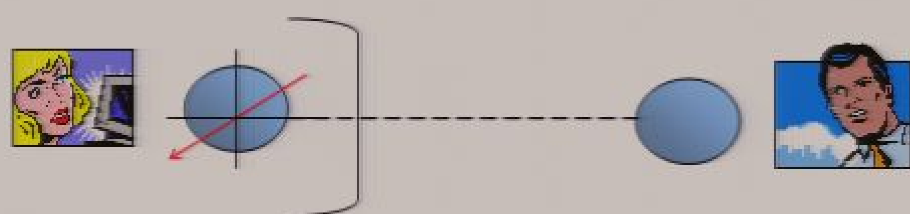

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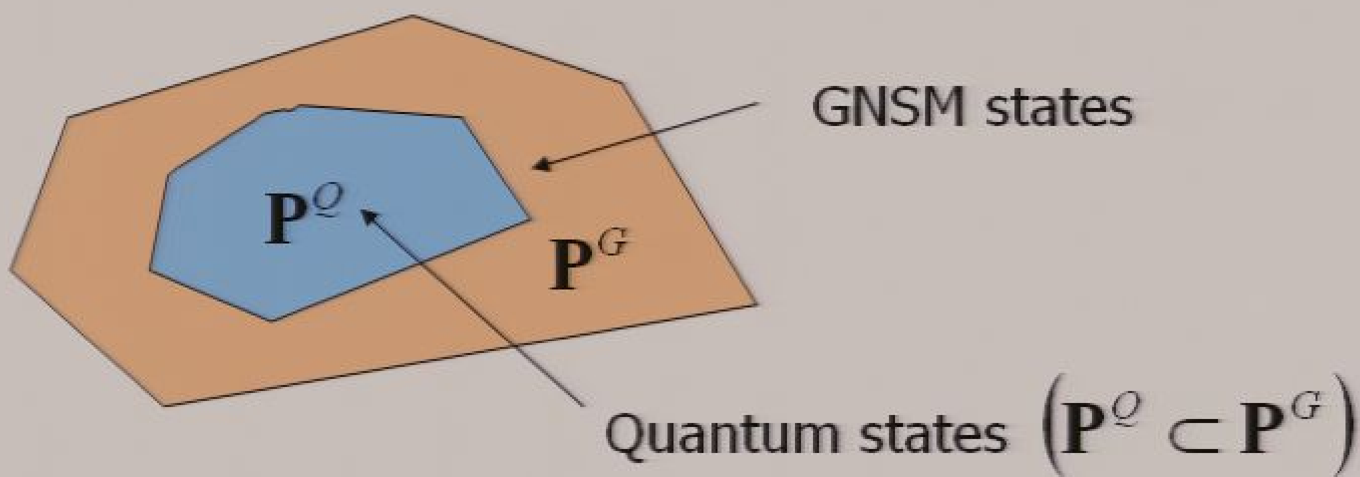

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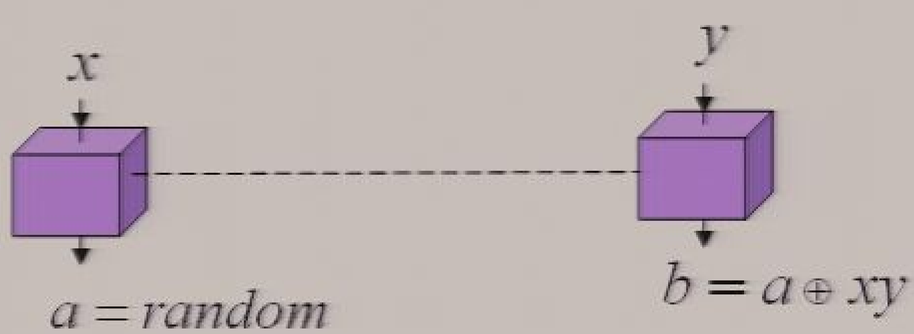
Non-local correlations

- Some states yield *non-local* correlations, for which

$$P(a_1 a_2 \mid x_1 x_2) \neq \sum_i p(k) P_k(a_1 \mid x_1) P_k(a_2 \mid x_2)$$

- GNSM states can produce stronger non-local correlations than quantum theory. E.g. $\underline{P}_{\text{nonlocal}} \in \mathbf{P}^G$ that allows perfect success in the non-local task introduced earlier (based on the CHSH inequality):

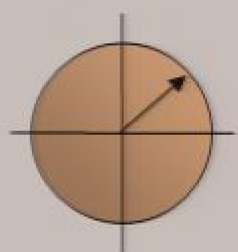


$$\underline{P}_{\text{nonlocal}} = \begin{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} & \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \\ \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} & \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \end{pmatrix}$$

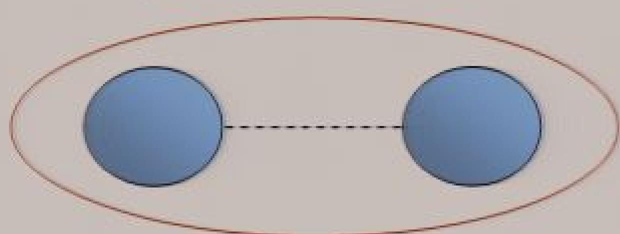
Why doesn't nature allow the full state-space/non-locality of GNSM?

Introducing non-fiducial measurements

- In addition to the fiducial measurements used to characterise the state, a theory may admit many other measurements.
- E.g. in quantum theory



$$\sigma^{45^\circ} = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$$ on a single qubit



A joint Bell measurement on two qubits

- What are the allowed measurements in GNSM?

Representing generalised measurements

- The probability p_i of obtaining a measurement output i with a mixed state must equal the mixture of output probabilities for the constituent states. It follows that measurements act linearly:

$$p_i = \underline{R}_i \cdot \underline{P} = \sum_{\mathbf{ax}} R_i(\mathbf{a} \mid \mathbf{x}) P(\mathbf{a} \mid \mathbf{x})$$

- Allowed measurements are represented by a set of vectors $\{ R_i(\mathbf{a}|\mathbf{x}) \}$ which satisfy:

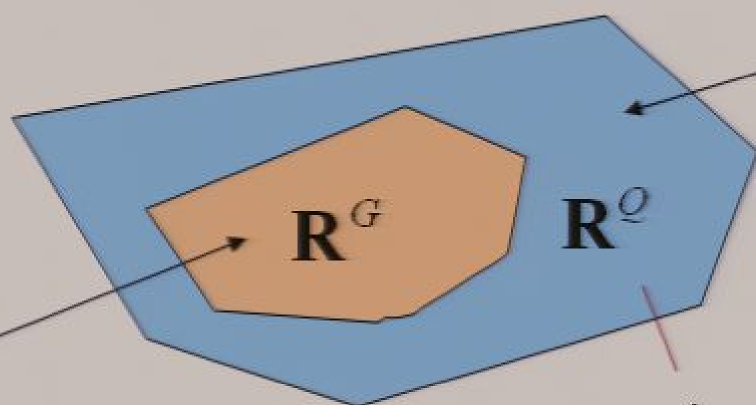
I. Positivity: $\underline{R}_i \cdot \underline{P} \geq 0 \quad \text{for all allowed } \underline{P}$

II. Normalisation: $\sum_i \underline{R}_i \cdot \underline{P} = 1 \quad \text{for all allowed } \underline{P}$

GNSM allows less measurements than quantum theory

- Like states, the allowed measurements form a convex set. However, as measurements in GNSM are constrained to give positive/normalised results for *more states*, the allowed measurement set is smaller.

Quantum measurements



GNSM measurements
$\left(\mathbf{R}^G \subset \mathbf{R}^Q\right)$

E.g. σ^{45°

$$\left\{\underline{R}_{+1}^{45^\circ}, \underline{R}_{-1}^{45^\circ}\right\} = \left\{ \begin{pmatrix} \begin{pmatrix} 2^{-3/2} \\ -2^{-3/2} \end{pmatrix} \\ \begin{pmatrix} 2^{-1} \\ 2^{-1} \end{pmatrix} \\ \begin{pmatrix} 2^{-3/2} \\ -2^{-3/2} \end{pmatrix} \end{pmatrix}, \begin{pmatrix} \begin{pmatrix} -2^{-3/2} \\ 2^{-3/2} \end{pmatrix} \\ \begin{pmatrix} 2^{-1} \\ 2^{-1} \end{pmatrix} \\ \begin{pmatrix} -2^{-3/2} \\ 2^{-3/2} \end{pmatrix} \end{pmatrix} \right\}$$

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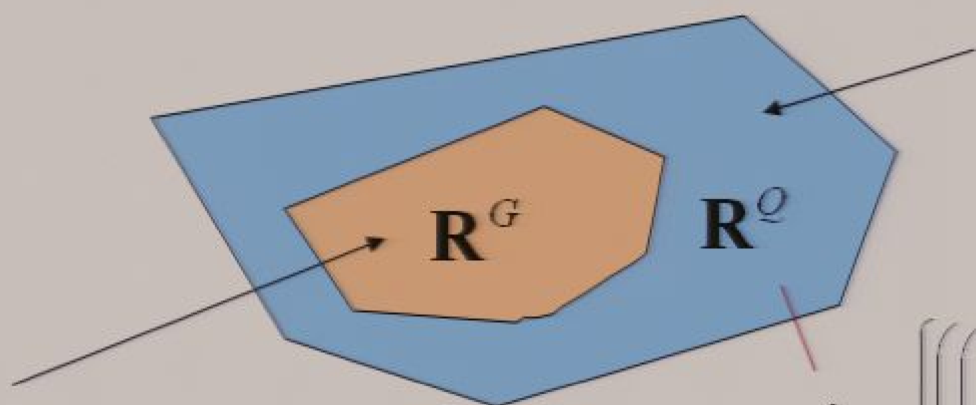

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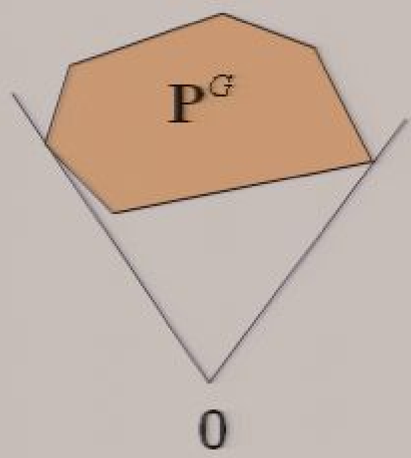
Results concerning GNSM measurements: I

I. All GNSM measurements can be represented by non-negative vectors $\mathbf{R_i(a|x) \geq 0}$



The proof follows from applying Farkas Lemma to the convex cone of un-normalised states.

Note that measurements in quantum theory do not have this property:

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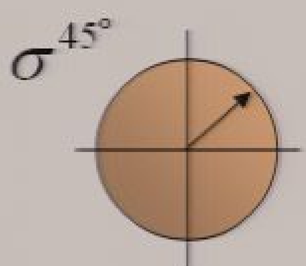

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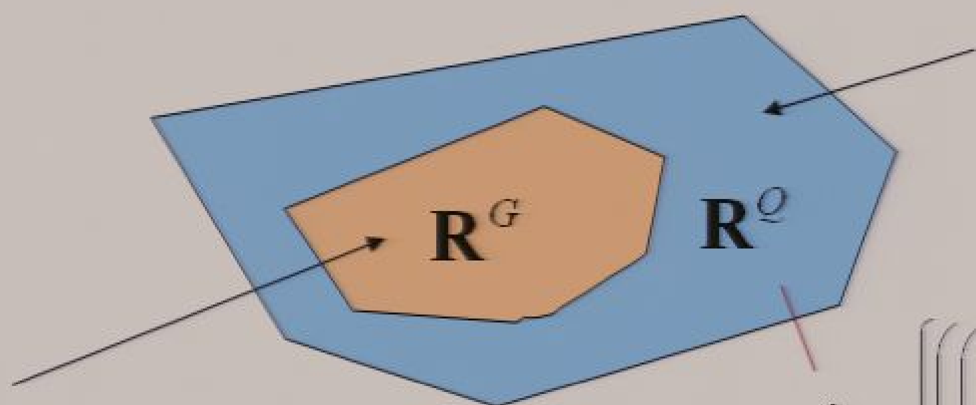

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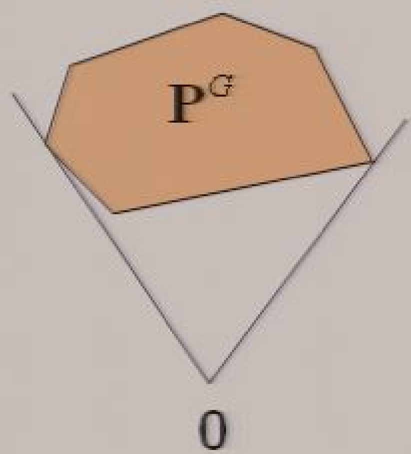

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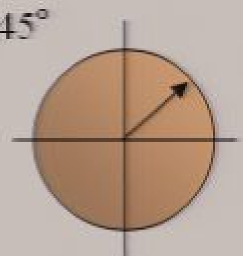

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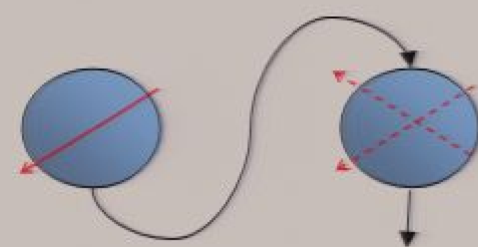
Results concerning GNSM measurements: II

II. All GNSM measurements on single and bi-partite systems can be performed using only fiducial measurements on the individual systems

This includes conditional sequences of measurements

e.g. $x_1 = 0,\ x_2 = a_1,\ \dot{\mathrm{i}} = \mathrm{a}_2$



There is therefore no analogue of a Bell measurement in GNSM.

However, note that for tri-partite (or larger) measurements fiducial measurements alone are *not* sufficient. However...

Results concerning GNSM measurements: III

III. All GNSM measurements can be simulated using fiducial measurements on individual systems and post-selection

A protocol to obtain any particular { $\underline{R}_i$ } is as follows:

a. Perform a maximally random set of fiducial measurements $\mathbf{x}=x_1 \ldots x_n$, obtaining outputs **a**

b. Give measurement output *i* or *fail* with probabilities:

$$q_i = \frac{R_i(\mathbf{a}\,|\,\mathbf{x})}{\max_{\mathbf{ax}} \sum_i R_i(\mathbf{a}\,|\,\mathbf{x})} \qquad q_{fail} = 1 - \frac{\sum_i R_i(\mathbf{a}\,|\,\mathbf{x})}{\max_{\mathbf{ax}} \sum_i R_i(\mathbf{a}\,|\,\mathbf{x})}$$

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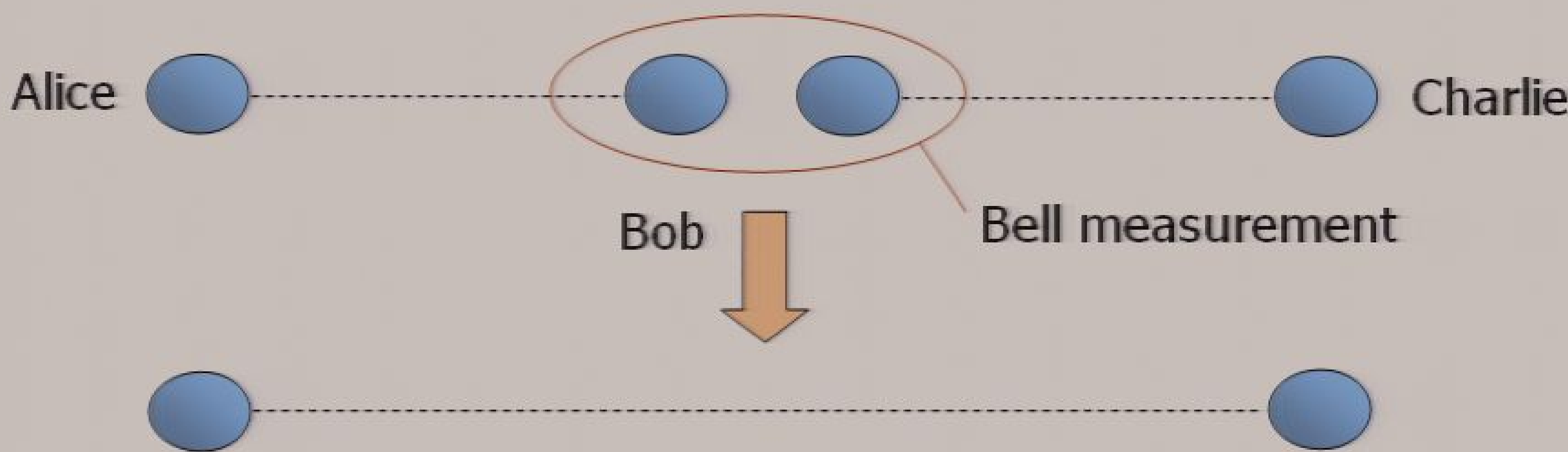
No 'swapping' of non-locality in GNSM

- In quantum theory, non-local correlations can be `swapped' between parties.



- However, as all measurements in GNSM can be simulated by single-system measurements and post-selection (result III), there are no truly joint measurements in GNSN. Hence

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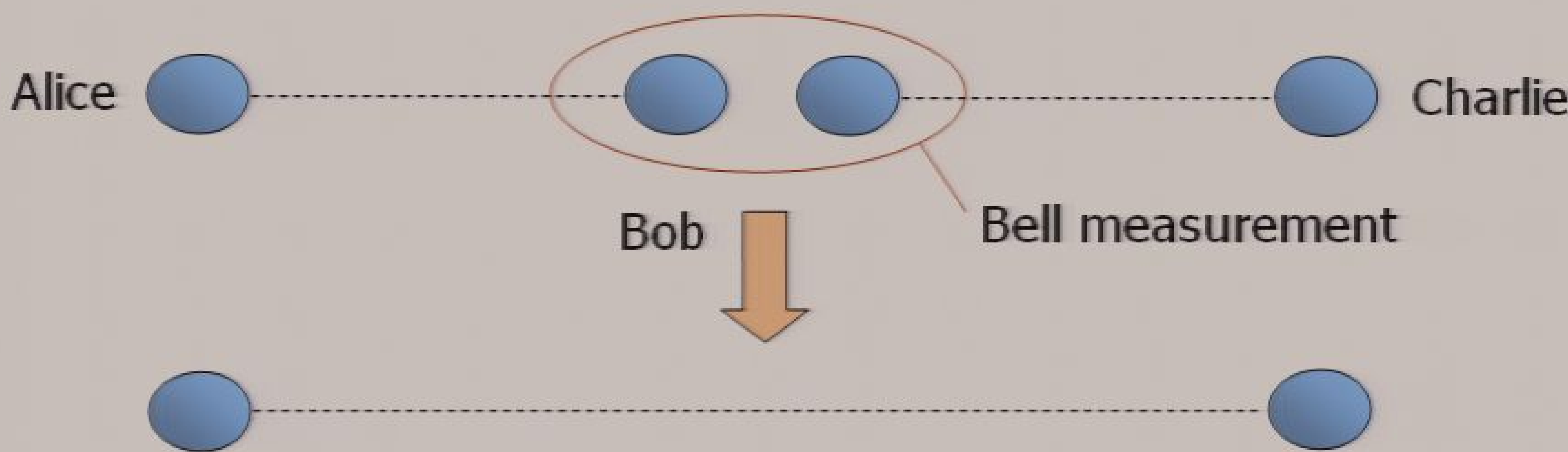

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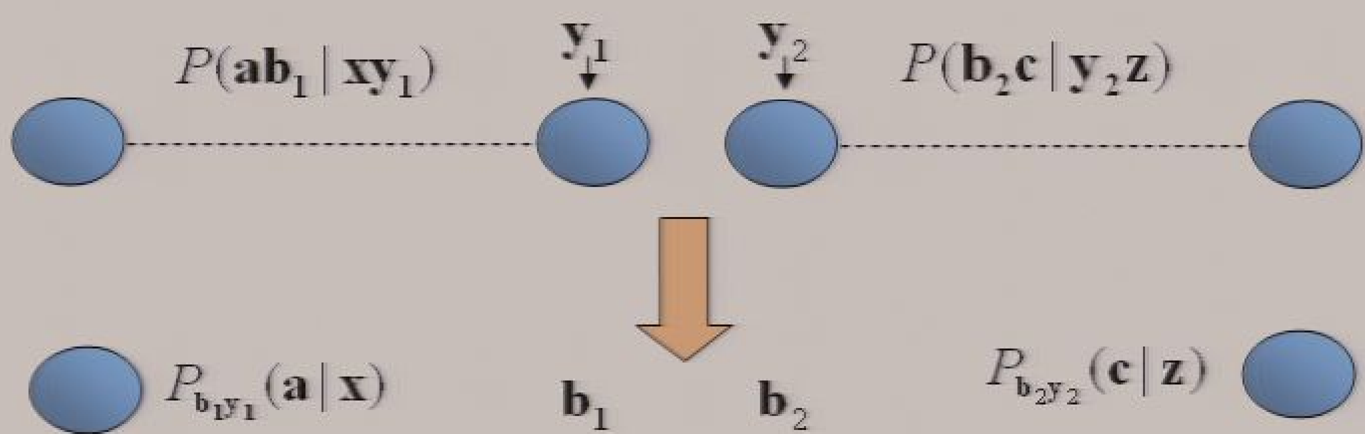
No 'swapping' of non-locality in GNSM

- Performing a set of fiducial measurements *collapses* the state



Hence using fiducial measurements and post-selection we can only obtain separable (i.e. local) final states

$$P(\mathbf{ac} | \mathbf{xz}) = \sum_{\mathbf{b}_1\mathbf{y}_1\mathbf{b}_2\mathbf{y}_2} P(\mathbf{b}_1\mathbf{y}_1\mathbf{b}_2\mathbf{y}_2 | \mathit{success}) P_{\mathbf{b}_1\mathbf{y}_1}(\mathbf{a} | \mathbf{x}) P_{\mathbf{b}_2\mathbf{y}_2}(\mathbf{c} | \mathbf{z})$$

Joint measurements and non-locality swapping: Conclusions

- We can construct a theory admitting generalised non-local correlations and quantum theory within a common framework.

- Generalised non-signalling mechanics allows any non-local correlations, but much less versatility in terms of measurements on a given state:
  - There are no truly joint measurements on separate subsystems, analogous to a Bell measurement.
  - There is no analogue of entanglement-swapping for generalised non-local correlations.
  - All measurements can be implemented using only fiducial measurements and (for >2 systems) post-selection.

Summary

Differences between quantum and generalised non-locality:

GNSM	**Quantum**
More allowed states	*More measurements and dynamics*
Stronger non-locality	*More versatile ('swappable') non-locality*
Allows perfect non-local computation	*No advantage in non-local computation*
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By viewing it within a broader framework, can we better understand the particular properties of quantum theory?

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miktex
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perimeter_t...
phil 103 grading scale
phil 225 grading scale
EditPlus 2
47G-B8-6-C...
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