Title: Generalizing the Kodama State

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Introduction

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    - World appears to be asymptotically de-Sitter

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- Represents quantum (anti)de-Sitter space
- Cosmological data suggest we are in increasingly lambda dominated universe
  - World appears to be asymptotically de-Sitter
- Exact form in connection and spin network basis

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  - Not known to be under physical inner product
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  - Violates Lorentz invariance?
  - CPT inverted states have negative energy?

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- Reality constraint must be implemented

Outline

- Problems stem from complexification of phase space
  - Complexification comes from choice of Immirzi parameter: $\beta = \pm i$
- Need to extend state to real values of Immirzi parameter
- Can be done and answer is surprising:
  - Solves most of the problems associated with original
  - Opens up a large Hilbert space of states
- Existence of state stems from deep connection with underlying local de Sitter symmetry
  - Connection with Macdowell-Mansouri formalism
  - Freedom from gauge fixing?

Review of Kodama State

- Begin with left handed part of Lorentz group

$$so(3,1) \simeq su(2)_R \times su(2)_L$$

- Action is Einstein-Cartan of left handed curvature with cosmological constant
- Dynamical variables are complex su(2) connection and conjugate momentum

$$\left[A^{ij}_{(L)}|_P, \Sigma_{jk}|_Q\right] = 2ik\, \delta^{ij}_{jk}\, \tilde{\delta}(P,Q)$$

$$A = \Gamma + iK \qquad \hat{A} = A$$

$$\Sigma = E \wedge E \qquad \hat{\Sigma} = 2k\frac{\delta}{\delta A}$$

The Kodama State

- Hamiltonian constraint annihilates Chern-Simons state:

$$\int_\Sigma *\Sigma \wedge \left(F - \tfrac{\Lambda}{3}\Sigma\right) \Psi[A] = 0 \qquad \frac{\delta}{\delta A}\int_M Y_{CS}[A] = 2F$$

$$\boxed{\Psi[A] = N e^{\frac{3}{4k\Lambda}\int Y_{CS}[A]}}$$

- Solution is unique in Lorentzian signature
- Generally interpreted as de-Sitter space
- Reality conditions have yet to be implemented
  - Implemented through inner product
  - May change physical interpretation

Construction of Generalized State

- Start with arbitrary *imaginary* value of Immirzi parameter
- Imaginary values are measure of chiral asymmetry

$$S_H = \frac{1}{k}\int_M \star e \wedge e \wedge R + \frac{1}{\beta} e \wedge e \wedge R$$
$$= \frac{2}{k}\int_M \alpha_L \star \Sigma_L \wedge R_L + \alpha_R \star \Sigma_R \wedge R_R$$

$$\alpha_L + \alpha_R = 1$$

$$\beta = \frac{-i}{\alpha_L - \alpha_R}$$

- Begin quantization assuming left/right pieces are independent
- Impose primary constraint later: $\Sigma_L = \Sigma_R \quad (= \frac{1}{2} E \wedge E)$

Construction of Generalized State

- Canonical variables

$$\omega_{L/R} = \omega \pm iK$$

$$\begin{aligned}[\omega_R, \Sigma_L] &= [\omega_L, \Sigma_R] = 0 \\ [\omega_L, \Sigma_L] &= -2k/\alpha_L \, \tilde{\delta} \\ [\omega_R, \Sigma_R] &= +2k/\alpha_R \, \tilde{\delta}\end{aligned}$$

- Constraints split into left/right pieces

$$H = \alpha_L \int_M *\Sigma_L \wedge (F_L - \Lambda/3 \, \Sigma_L) + \{L \to R\}$$

- Chiral asymmetric Chern Simons state is immediate

$$\Psi[\omega_L, \omega_R] = N e^{\frac{3}{4k\Lambda} \int (\alpha_L Y_{CS}[\omega_L] - \alpha_R Y_{CS}[\omega_R])}$$

- Need to impose constraint $C = \Sigma_L - \Sigma_R \sim 0$

Construction of Generalized State

- Constraint has several implications
- Vanishing 3-torsion is second class constraint

$$[H,C] \sim D_\omega * \Sigma \rightarrow T = 0 \qquad (\omega = \Gamma[E])$$

- Can define new variables

$$A_{\frac{1}{\beta}} = \Gamma + \frac{1}{\beta} K$$
$$A_\beta = \Gamma - \beta K$$
$$\Sigma = \Sigma_L + \Sigma_R$$

$$[A_{1/\beta}, C] = -4k\,\tilde{\delta}$$
$$[A_\beta, \Sigma] = -i2k\beta\,\tilde{\delta}$$
$$[A_{1/\beta}, \Sigma] = 0$$
$$[A_\beta, C] = 0$$

- Constraint becomes

$$\frac{\delta}{\delta A_{1/\beta}} \Psi = 0 \rightarrow \Psi = \Psi[A_\beta]$$

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  - Analytically extend to real Immirzi
  - Reinterpret role of momentum dependence

Interpretation

- Generalized Kodama state is like NR momentum state

$$\langle x|p\rangle = \Psi_p[x] = \mathcal{N} exp[i\, x \cdot p - i\, E\, t]$$

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$$\langle p'|p\rangle \sim \int dx\, e^{-ix\cdot(p'-p)} = \delta(p'-p)$$

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$$\langle R'|R\rangle \sim \int \mathcal{D}A\, e^{-i\alpha \int A\wedge(R'-R)} = \delta(R'-R)$$

Gauge Invariant Inner Product

- Need to address gauge invariance of inner product
- Kodama States are *not* invariant under SU(2) or diffeos
  - Curvature acts as "background" structure
  - Familiar from spin network states

$$U_\phi \Psi_\Gamma[A] = \Psi_\Gamma[\phi(A)] = \Psi_{\phi^{-1}(\Gamma)}[A] \qquad \phi = \phi_{\bar{N}}$$

$$U_\phi \Psi_R[A] = \Psi_R[\phi(A)] = \Psi_{\phi^{-1}(R)}[A] \qquad \phi = \phi_{\{\bar{N},g\}}$$

- Allows us to define gauge invariant inner product:

$$\langle R'|R\rangle_{kin} = \int \mathcal{D}\phi \, \langle U_{\phi^{-1}} \Psi_{R'} | \Psi_R \rangle = \delta(\mathcal{R}' - \mathcal{R})$$

Progress Report

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Levi-Civita Curvature Operator

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- Momentum operator in momentum basis:

$$\hat{p} = \int dp \;\; p|\Psi_p\rangle\langle\Psi_p| \;\rightarrow\; \hat{p}|\Psi_p\rangle = p|\Psi_p\rangle$$

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  - Introduce test function lambda
  - Integrate over all curvatures and gauges

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$$\int_\Sigma \lambda \wedge \hat{R}_\Gamma = \int \mathcal{D}\phi\mathcal{D}\Gamma' \left[\left(\int_\Sigma \lambda \wedge \phi R'_{\Gamma'} \right) |\Psi_{\phi R'}\rangle\langle\Psi_{\phi R'}| \right]$$

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$$\boxed{\int_\Sigma \lambda \wedge \hat{R}_\Gamma \, |\Psi_R\rangle = \int_\Sigma \lambda \wedge R_\Gamma \, |\Psi_R\rangle}$$

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$$H = \int_{\Sigma} *\Sigma \wedge \left(F + (1+\beta^2)\left(\frac{1}{\beta} D_\Gamma K - K \wedge K\right) - \frac{\Lambda}{3}\Sigma \right)$$

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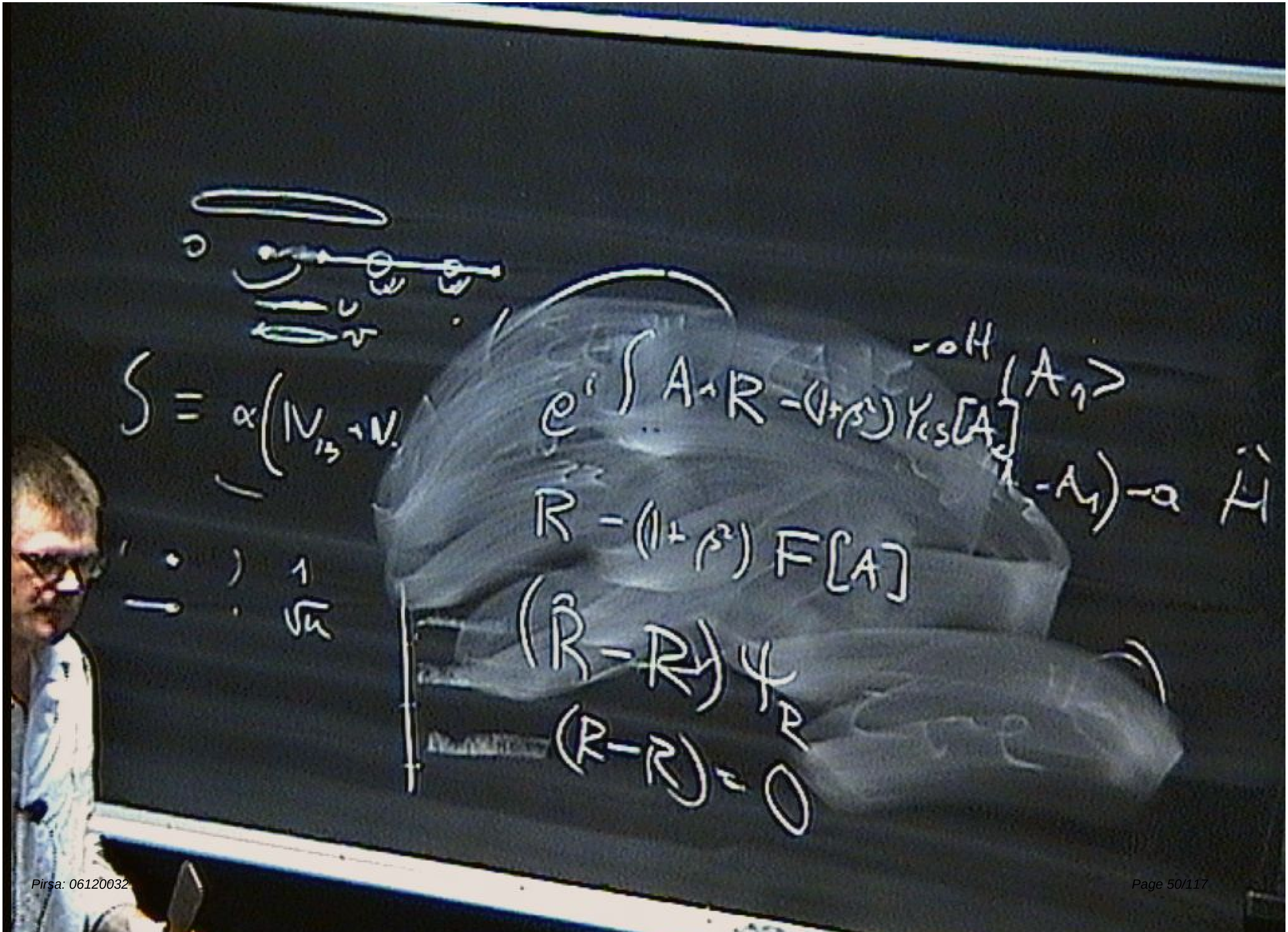
$$\boxed{\hat{H}\, \Psi_R[A] = 0 \quad !!!!}$$


R − (1+β²) F[A]
(R̂ − R) Ψ_R
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Progress Report

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Physical Interpretation

- Do states represent something like de-Sitter spacetime?
  - Yes!
  - States are WKB for De-Sitter spacetime and beyond
- Started with Holst action:

$$S_H = \frac{1}{k}\int_M \star e\wedge e\wedge R + \frac{1}{\beta} e\wedge e\wedge R - \frac{\lambda}{6}\star e\wedge e\wedge e\wedge e$$

- All solutions to vacuum field equations look like this:

$$R = \frac{\lambda}{3} e\wedge e + C \qquad T^I = 0$$

- Plug this back into action to get WKB state

$$\Psi_{WKB} \simeq e^{iS_0}$$

Physical Interpretation

- Assume Weyl is small (keep only first order terms)
- Action becomes topological

$$S_0 \simeq \frac{3}{2k\lambda} \int_M \star R \wedge R + \frac{1}{\beta} R \wedge R$$

- Take boundary of manifold to be spacelike hypersurface
- Set torsion to zero, rewrite in terms of A

$$S_0 \simeq -\frac{3}{4k\lambda} \int_{\partial M = \Sigma} Y_{CS}[A] - (1+\beta^2)(Y_{CS}[\Gamma] - 2\beta K \wedge R)$$

- We have an exact WKB state: $\Psi_R[A] = N e^{iS_0[A]}$

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Regaining de-Sitter spacetime

- Have shown set of states contains small perturbations to de-Sitter
- Which one is de-Sitter?
- There is a slicing of de-Sitter spacetime in which three-curvature is flat and spatial topology is R^3
- Kodama State takes special form when R=0: take this to be de-Sitter solution

$$\langle A|\Psi_{dS}\rangle = \Psi_{R=0} = \mathcal{P}\exp\left[-\frac{3i}{4k\lambda\beta^3}\int_{\mathbb{R}^3} Y_{CS}[A]\right]$$

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Evidence of Cosmological Horizon

- q-deformed loop expansion of state is well known

$$\langle\Gamma|\Psi_{dS}\rangle = \sum_{\Gamma} K_{\Gamma}(q)\,|\Gamma\rangle$$

- Edges labeled by representations of $SU_q(2)$ with

$$q = e^{\frac{2\pi i}{\kappa+2}} \qquad \kappa = \frac{3}{2G\lambda\beta^3}$$

- At roots of unity, spectrum of area operator is bounded

$$A = 8\pi G\beta\sqrt{j(j+1)} \longrightarrow A_{max} \simeq 2\pi\left(\frac{r_0}{\beta}\right)^2$$

- Immirzi Parameter appears to be significant at very large scales as well as very small scales

$$small\ areas \sim \beta \qquad large\ areas \sim \frac{1}{\beta^2}$$

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CPT Invariance

- C, P, and T have action in both base manifold and SO(3,1) representation space

- Work with mixture of 3D and 4D forms:

$$\Psi_R[A] = N \exp\left[\frac{3i}{2k\lambda}\int_M \star R\wedge R + \frac{1}{\beta}R\wedge R\right]$$

$$= N \exp\left[\frac{3i}{2k\lambda}\int_\Sigma Y[\omega] + \frac{1}{\beta}\star Y[\omega]\right]$$

$$Y[\omega] = \omega\wedge d\omega + \frac{\lambda}{3}\omega\wedge\omega\wedge\omega \qquad \text{(No Trace)}$$

- State uses both inner products:

$$\langle A,B\rangle = Tr(AB) \qquad \langle A,B\rangle_\star = Tr(\star AB) \qquad \star = -i\gamma_5$$

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CPT

- States are CPT invariant for real values of the Immirzi parameter

$$\Psi_\beta \xrightarrow{\mathcal{C}} \Psi_\beta \xrightarrow{\mathcal{P}} \Psi_{-\beta} \xrightarrow{\mathcal{T}} \Psi_\beta$$

$$\mathcal{CPT}(\Psi_\beta) = \Psi_\beta$$

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True Inner Product: MM-Gravity

- Kinematical inner product unlikely to be true inner product
- True inner product defined by path integral methods:

$$\langle \Psi_{R'}, \Sigma_2 | \Psi_R, \Sigma_1 \rangle_{true} = \langle \Psi_{R'} | U(\Sigma_2, \Sigma_1) | \Psi_R \rangle$$

$$= \int \mathcal{D}A_2 \mathcal{D}A_1 \, \Psi^*_{R'}[A_2] \Psi_R[A_1] \int_{A_1}^{A_2} \mathcal{D}\omega \mathcal{D}e \; e^{iS_{EC}}$$

$$= \int_{E_1}^{E_2} \mathcal{D}\omega \mathcal{D}e \; e^{-i\frac{3}{2k\lambda}\int_M \star R\wedge R + \frac{1}{\beta} R\wedge R} \, e^{iS_{EC+\beta}}$$

- Claim this is related to Macdowell-Mansouri Gravity

Adding Immirzi to MM-Gravity

- Macdowell-Mansouri begins with de-Sitter connection

$$\Lambda = \omega + \frac{i}{r_0} e \qquad F = d\Lambda + \Lambda \wedge \Lambda = R + \frac{i}{r_0} T - \frac{\lambda}{3} e \wedge e$$

$$r_0 = \sqrt{\lambda/3}$$

- MM action is

$$\boxed{S_{MM} = -\frac{3}{2k\lambda} \int_M \star F \wedge F = S_{EC+\lambda} - \frac{3}{2k\lambda} \int_M \star R \wedge R}$$

- Adding Immirzi to Einstein Cartan means perturbing curvature by its dual: $R \to R - \frac{1}{\beta} \star R \qquad S_{EC} \to S_{EC+\beta}$

- Try this on MM action: $F \to F - \theta \star F \qquad S_{MM} \to ??$

Adding Immirzi to MM Action

- Action becomes:

$$S_{MM} \to S_{MM+\beta} = \alpha' \int_M \star(F - \theta \star F) \wedge (F - \theta \star F)$$

- For appropriate choice of constants we have

$$S_{MM+\beta} = S_{EC+\lambda+\beta} - \frac{3}{2k\lambda} \int_M \star R \wedge R + \frac{1}{\beta} R \wedge R$$

- This has following implication for inner product:

$$\langle \Psi_{R'}, \Sigma_2 | \Psi_R, \Sigma_1 \rangle = \int_{E_2}^{E_1} \mathcal{D}\omega \mathcal{D}e \; e^{i\frac{-3}{2k\lambda}\int_M \star R \wedge R + \frac{1}{\beta} R \wedge R} \; e^{iS_{EC+\lambda+\beta}}$$

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Conclusions on MM Relation

- Inner product of two Kodama states equivalent to Hawking sum over histories in MM gravity
  - Sum over histories fixes two geometries on end caps
  - Geometries fix R' and R

- Difference between EC gravity and MM gravity: MM gravity has Kodama states "built in"

- This is similar to two formulations of CP problem in Yang-Mills

States have theta-ambiguity

or

Action has theta-ambiguity

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Freedom from Time Gauge?

- Freedom from gauge fixing would be nice for a variety of reasons
- The WKB state prior to gauge fixing is suggestive

$$\Psi[\omega] = \mathcal{P}\ \exp\left[\frac{3i}{2k\lambda}\int_{\Sigma}\star Y[\omega] + \frac{1}{\beta}Y[\omega]\right]$$

- Can one obtain this state from canonical construction without gauge fixing?
- We can obtain a slightly modified version

Canonical construction without gauge fixing

- Begin with a modified Holst action

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- Dynamical variables are connection and frame

Position	Momentum	Primary Constraint
ω	$\Pi_\omega = \frac{1}{k}\Sigma$	$\Sigma = \star e \wedge e$
e	$\Pi_e = -\frac{1}{k\beta} T$	$T = De$

Canonical Constraints

- Symplectic structure defines Poisson bracket:

$$\{A,B\} = k\int_{\Sigma} \delta_{\omega} A \wedge \delta_{\Sigma} B - \beta\, \delta_{e} A \wedge \delta_{T} B \;-(A \leftrightarrow B)$$

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- In connection-tetrad representation define the operators

$$\hat{\omega} = \omega \qquad \hat{\Sigma} = -ik\frac{\delta}{\delta\omega}$$
$$\hat{e} = e \qquad \hat{T} = ik\beta\,\frac{\delta}{\delta e} \qquad \Psi = \Psi[\omega, e]$$

- Can solve all of the naïve quantum constraints by a version of the Kodama state

$$\Psi[\omega, e] = \exp\left[\frac{3i}{2k\lambda}\int_{\Sigma} \star Y[\omega] + \frac{1}{\beta}Y[\omega] - \frac{\lambda}{3\beta}e\wedge De\right]$$

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Primary Constraints

- Primary constraint says that for any functional, f, of position and momentum,

$$f[e, \omega, \Sigma, T] = f[e, \omega, \star e \wedge e\,, De]$$

- If this holds on constraint manifold, it will hold for any function on constraint manifold. So we need

$$C'_{\{H,G,D\}}[\omega, e] \equiv C_{\{H,G,D\}}[\omega, e, \star e \wedge e\,, De]$$

$$C'_{\{H,G,D\}}[\omega, e] \sim 0$$

- Since this is a constraint on position variables, it can be implemented through inner product

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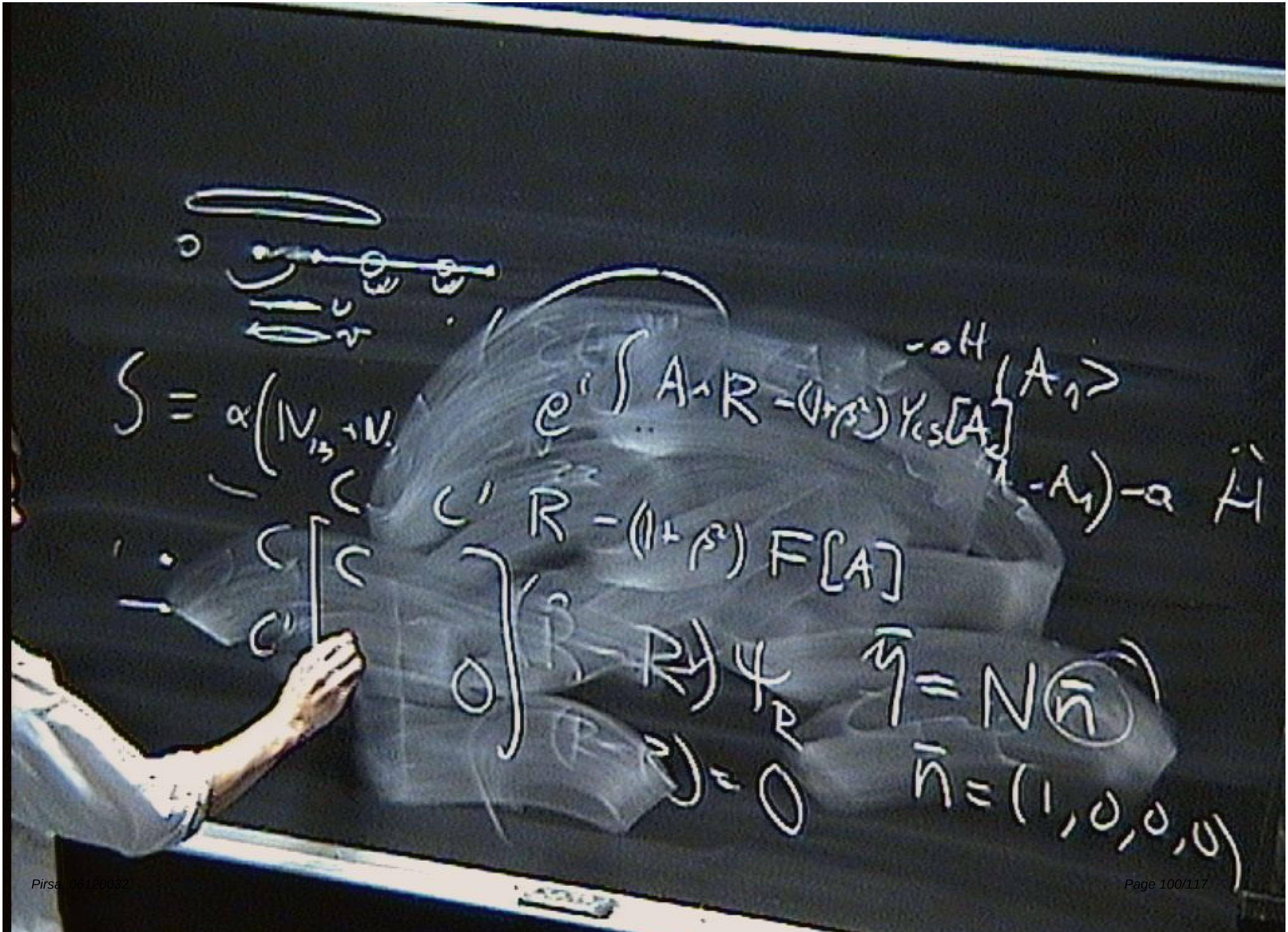
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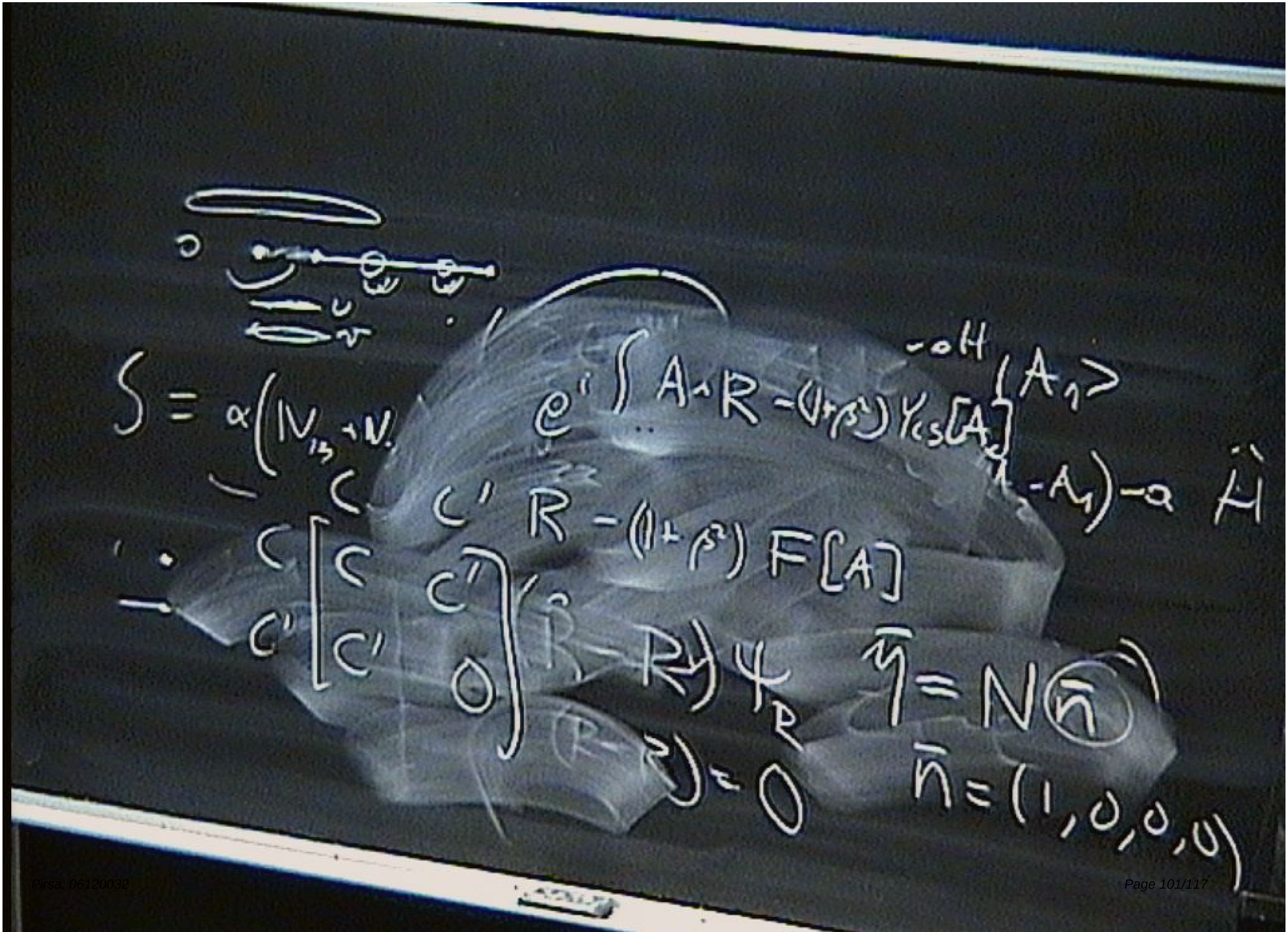
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- Then one can show that the inner product of the Kodama states once again reproduces a version of the Macdowell-Mansouri path integral:

$$\langle\Psi,\Sigma_2|\Psi,\Sigma_1\rangle = \int \mathcal{D}\omega\mathcal{D}e\ \delta(C'_{\{H,G,D\}})\,\Psi^*_{\Sigma_2}[\omega_2,e_2]\Psi_{\Sigma_1}[\omega_1,e_1]$$

$$= \int \mathcal{D}\omega\,\mathcal{D}e\ \exp\left[\frac{3i}{2k\lambda}\int_M \star F\wedge F + \frac{1}{\beta}F\wedge F\right]$$

Canonical Constraints

- Symplectic structure defines Poisson bracket:

$$\{A,B\} = k\int_\Sigma \delta_\omega A \wedge \delta_\Sigma B - \beta\, \delta_e A \wedge \delta_T B \; - (A \leftrightarrow B)$$

- Naïve Constraints (prior to primary constraints) are

$$C_D = \tfrac{1}{k}\int_\Sigma \mathcal{L}_{\bar N}\omega \wedge \Sigma - \tfrac{1}{\beta}\mathcal{L}_{\bar N} e \wedge T \qquad \bar t = \bar\eta + \bar N$$

$$C_G = -\tfrac{1}{k}\int_\Sigma D\alpha \wedge \Sigma + \tfrac{1}{\beta}[\alpha, e] \wedge T \qquad \alpha \in so(3,1)$$

$$C_H = \tfrac{1}{k}\int_\Sigma [\eta, e] \wedge (\star R - \tfrac{\lambda}{3}\Sigma) + \tfrac{1}{\beta}\, D\eta \wedge T \qquad \eta \equiv e(\bar\eta)$$

Canonical construction without gauge fixing

- Begin with a modified Holst action

$$S = \frac{1}{k}\int_M \star e \wedge e \wedge R - \frac{1}{2\beta} T \wedge T$$

$$\frac{1}{\beta} e \wedge e \wedge R - \frac{1}{2\beta} d(e \wedge T)$$

- Dynamical variables are connection and frame

Position	Momentum	Primary Constraint
ω	$\Pi_\omega = \frac{1}{k}\Sigma$	$\Sigma = \star e \wedge e$
e	$\Pi_e = -\frac{1}{k\beta} T$	$T = De$

Canonical Constraints

- Symplectic structure defines Poisson bracket:

$$\{A,B\} = k\int_{\Sigma} \delta_{\omega} A \wedge \delta_{\Sigma} B - \beta\, \delta_{e} A \wedge \delta_{T} B \; - (A \leftrightarrow B)$$