Title: Quantum Gravity from one-loop Super-Yang-Mills Theory

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Abstract: It has been conjectured that maximally supersymmetric SU(N) Yang-Mills theory is dual to a String Theory on asymptotically AdS_5 times S^5 backgrounds. This is known as the AdS/CFT correspondence. In this talk I will show how using one-loop calculations in the gauge theory, one can study the emergence of the dual String Theory. We will see, quite explicitly, the emergence of closed strings, D-branes, open strings and space-time itself. This is done in a reduced sector (SU(2) sector), where the gauge theory can be written as Matrix Quantum Mechanics. This simple sector provides a toy model of a non-perturbative quantum theory of gravity.

Quantum Gravity from one-loop $\mathcal{N} = 4$ SYM

Samuel E. Vázquez
Physics Department, University of California at Santa Barbara

Based on:

D. Berenstein, D. H. Correa S.E.V hep-th/0502172

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Motivation

- We want a non-perturbative definition of Quantum Gravity
- String Theory itself will be emergent from such a theory
- Example: AdS/CFT
  - Want to study how gravity and String Theory emerges from a large N, $SU(N)$ QFT

Plan of Talk

- SU(2) reduced sector of one-loop SYM
- Matrix Quantum Mechanics as a toy model of Quantum Gravity in 3 + 1 dimensions
- We will study the emergence of:
  - Closed Strings
  - D-branes
  - Open Strings
- Gravitational Back-reaction
  - Closed Strings on non-trivial backgrounds from the matrix model
- Conclusions

AdS/CFT:

SU(N) $\mathcal{N} = 4$ SYM on R x S^3 $\cong$ IIB String Theory in asymptotically $AdS^5 \times S^5$ with N units of 5-form flux

$$\frac{1}{N} = \left(\frac{l_P}{R}\right)^4 \longrightarrow 0$$ (Quantum gravity effects)

$$\frac{\alpha'}{R^2} = \frac{1}{\sqrt{\lambda}} = \text{fixed} \ll 1$$ (String theory effects)

$(\lambda = g^2_{YM} N)$

(R = radius of AdS_5 and S^5)

AdS/CFT:

Global symmetries of $\mathcal{N} = 4$ SYM $\cong$ Isometries of the asymptotic $AdS^5 \times S^5$

- In particular:

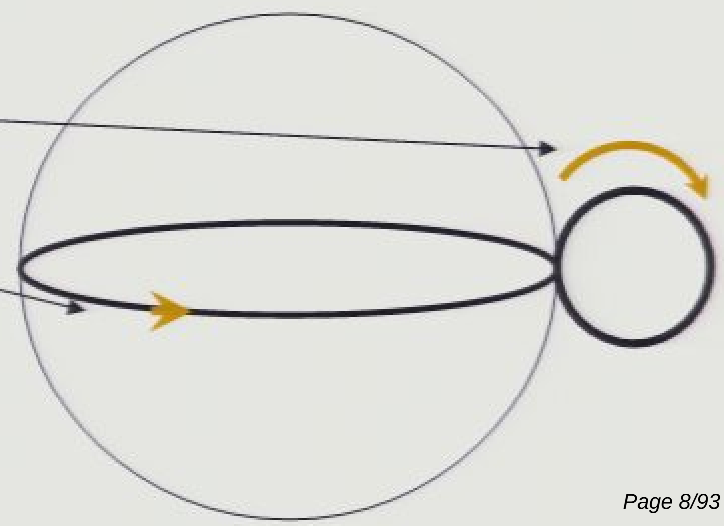
SU(4) ~ SO(6) global R-symmetry $\cong$ Isometries of asymptotic S^5

Reduced Sectors

- **SU(2) Scalar Sector of $\mathcal{N} = 4$ SYM :**

States with two $U_R(1) \subset SU(4)$ charges $\cong$ String States with two angular momenta on (asymp.) S^5

$$\mathrm{Tr}\left(\hat{A}^\dagger_Y \hat{A}^\dagger_Z \hat{A}^\dagger_Y \cdots\right) \mathrm{Tr}\left(\hat{A}^\dagger_Z \hat{A}^\dagger_Y \cdots\right) |0\rangle$$



$$[(\hat{A}_Z)^j_i, (\hat{A}^\dagger_Z)^l_k] = \delta^l_i \delta^l_k$$

$$[(\hat{A}_Y)^j_i, (\hat{A}^\dagger_Y)^l_k] = \delta^l_i \delta^j_k$$

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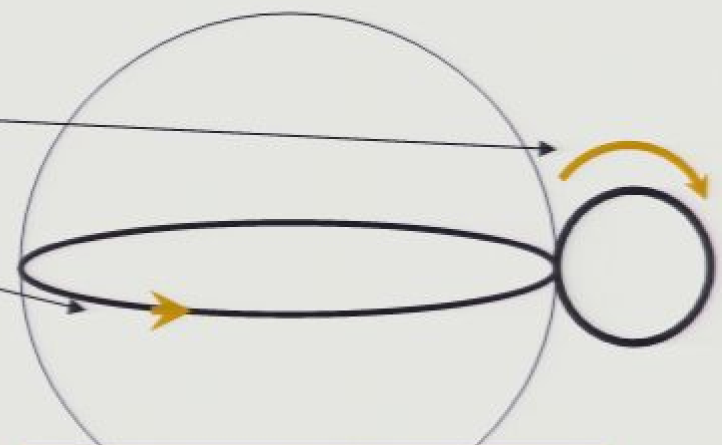

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$$[(\hat{A}_Y)_i^j, (\hat{A}_Y^\dagger)_k^l] = \delta_i^l \delta_k^j$$

String sees a reduced $R \times S^3$ geometry

Toy Model of 3+1 dimensional Quantum Gravity

- Hamiltonian in the SU(2) sector

$$\hat{H} - \hat{J}_Z - \hat{J}_Y = -\lambda \hbar \, \mathrm{Tr}[\hat{A}_Z^\dagger, \hat{A}_Y^\dagger][\hat{A}_Z, \hat{A}_Y] + \mathcal{O}(\lambda^2) + \dots$$

One loop truncation

$$\hat{J}_Z = \mathrm{Tr}(\hat{A}_Z^\dagger \hat{A}_Z) \qquad \hat{J}_Y = \mathrm{Tr}(\hat{A}_Y^\dagger \hat{A}_Y)$$

$$[\hat{H}, \hat{J}_Z] = [\hat{H}, \hat{J}_Y] = 0 \qquad \hbar = \frac{1}{N}$$

Toy Model of 3+1 dimensional Quantum Gravity

- New notation: **Coherent States**

$$(\hat{A}_Z)_i^j|Z\rangle = Z_i^j|Z\rangle \quad (\hat{A}_Y)_i^j|Y\rangle = Y_i^j|Y\rangle$$

$$|\psi\rangle = \psi(\hat{A}_Y^\dagger, \hat{A}_Z^\dagger)|0\rangle \to \psi(Y, Z)$$

$$\hat{H} - \hat{J}_Z - \hat{J}_Y \to -\lambda\hbar \,\mathrm{Tr}[Z, Y][\partial_Z, \partial_Y]$$

$$\langle\psi|\psi\rangle = \frac{\int [d^2Y d^2Z] e^{-\,\mathrm{Tr}(|Z|^2+|Y|^2)/\hbar} |\psi(Y, Z)|^2}{\int [d^2Y d^2Z] e^{-\,\mathrm{Tr}(|Z|^2+|Y|^2)/\hbar}} \equiv \langle|\psi(Y, Z)|^2\rangle$$

Random Matrix Model!

Closed Strings

- Single-trace states ~ Single string state
  - In the large N limit the Hamiltonian only acts on nearest neighbors.

$$-\lambda\hbar\ \mathrm{Tr}[Z,Y][\partial_Z,\partial_Y] \quad \mathrm{Tr}(YYZYZZ\cdots)$$

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- Bosonic Lattice

$$|n_1, n_2, \ldots, n_L\rangle = \mathrm{Tr}(YZ^{n_1}YZ^{n_2}Y\cdots YZ^{n_L})$$

Closed Strings

- The Hamiltonian is (periodic boundary cond.)

$$\hat{H} = \lambda \sum_{l=1}^{L} (\hat{a}_l^\dagger - \hat{a}_{l+1}^\dagger)(\hat{a}_l - \hat{a}_{l+1})$$

hopping

$$|n_1, n_2, n_3 \ldots, n_L\rangle$$

$$\hat{a}^\dagger |n\rangle = |n+1\rangle \qquad \hat{a}\hat{a}^\dagger = 1$$

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- Coherent States:

$$\hat{a}|z\rangle = z|z\rangle \qquad |z\rangle = \sqrt{1-|z|^2} \sum_{n=0}^{\infty} z^n |n\rangle$$

Emergent World Sheet

- Action for coherent states:

$$S = \int d\tau \left(i\langle CS|\partial_\tau|CS\rangle - \langle CS|\hat{H}|CS\rangle \right)$$

$$= \int d\tau \sum_{l=1}^{L} \left(i\langle z_l|\partial_\tau|z_l\rangle - \lambda|z_l - z_{l+1}|^2 \right)$$

$$V = \frac{z}{1-|z|^2}$$

$$\rightarrow \boxed{-L \int d\tau \int_0^1 d\sigma \left(\frac{i}{2} V \dot{\bar{z}} - \frac{i}{2} \bar{V} \dot{z} + \frac{\lambda}{L^2} |z'|^2 \right)}$$

$$|z| < 1$$

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$$\to \boxed{-L\int d\tau \int_0^1 d\sigma \left(\frac{i}{2}V\dot{\bar{z}} - \frac{i}{2}\bar{V}\dot{z} + \frac{\lambda}{L^2}|z'|^2 \right)} \qquad |z| < 1$$

Large L (λ/L^2 = fixed <1) gives classical limit

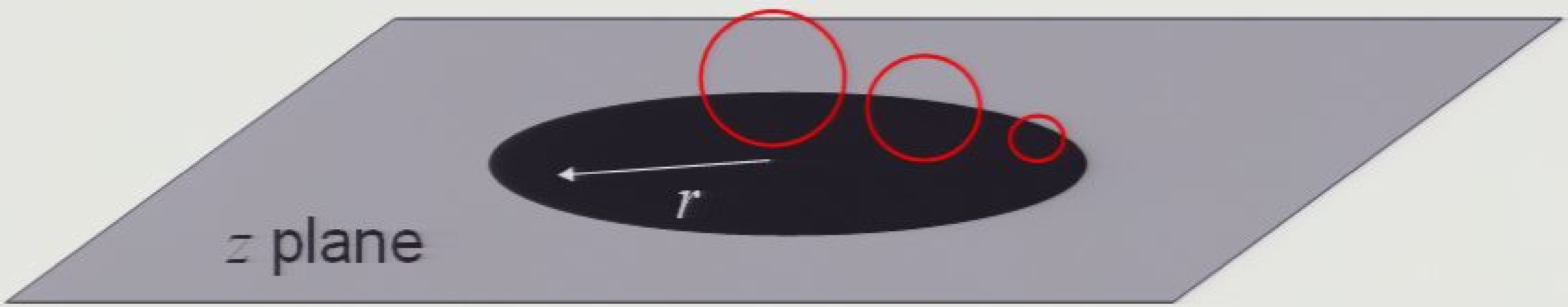
- States with large angular momentum become ***classical*** closed strings

String Theory Interpretation

- Consider a string on R x S^3 :

$$X = 0\,, \quad Z = re^{i(t-\phi)}\,, \quad Y = \pm\sqrt{1-r^2}e^{i\varphi}\,, \quad |X|^2 + |Y|^2 + |Z|^2 = 1$$

$$z \equiv re^{i\phi}$$



String Theory Interpretation

- Pick a gauge in the Polyakov action, where the angular momentum in φ is distributed uniformly along string.

$$t = \tau\,, \quad p_\varphi = \text{const.} \qquad L = \sqrt{\lambda_{YM}} \int_0^{2\pi} \frac{d\sigma}{2\pi} p_\varphi = \sqrt{\lambda_{YM}}\, p_\varphi$$

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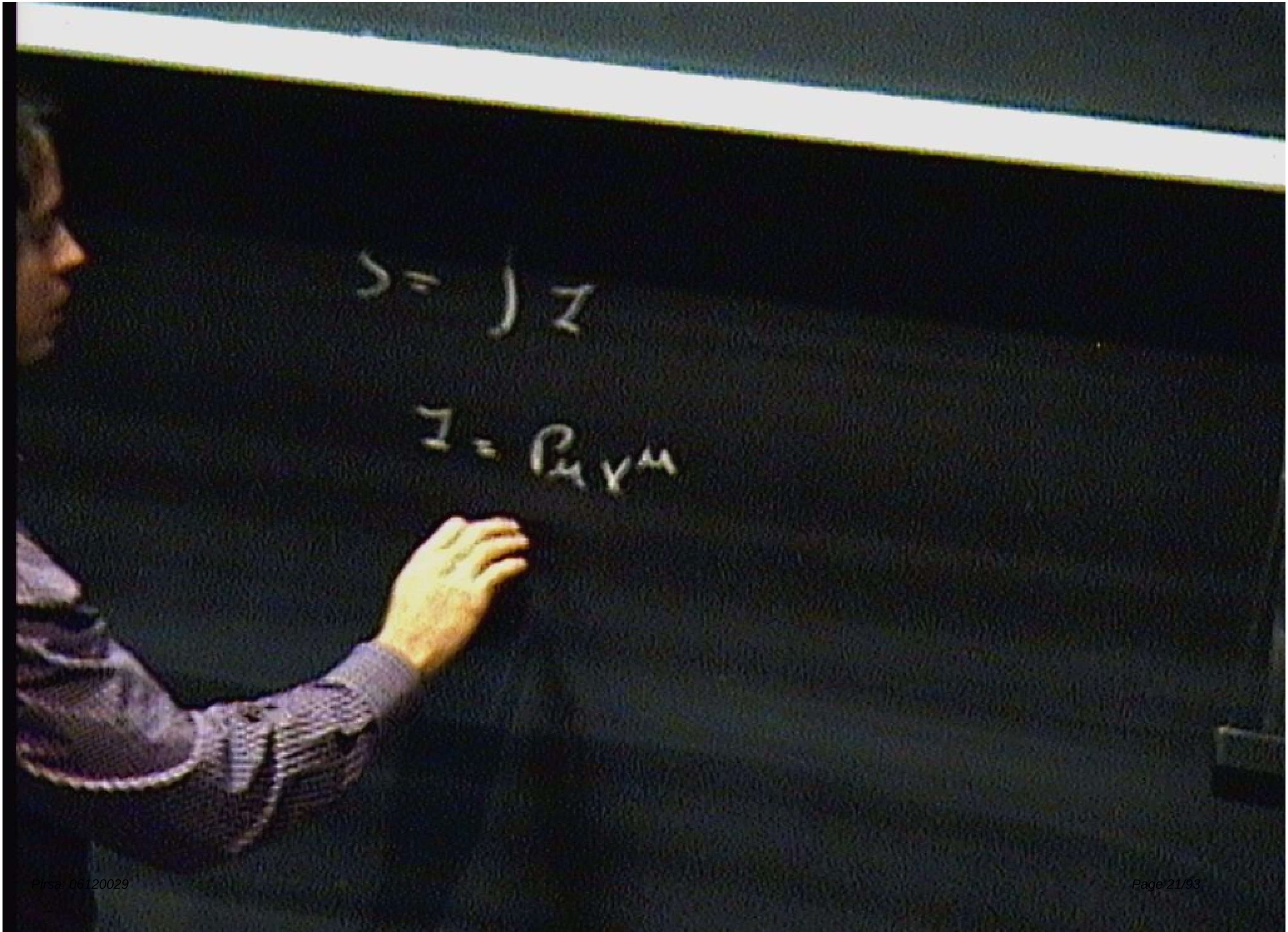
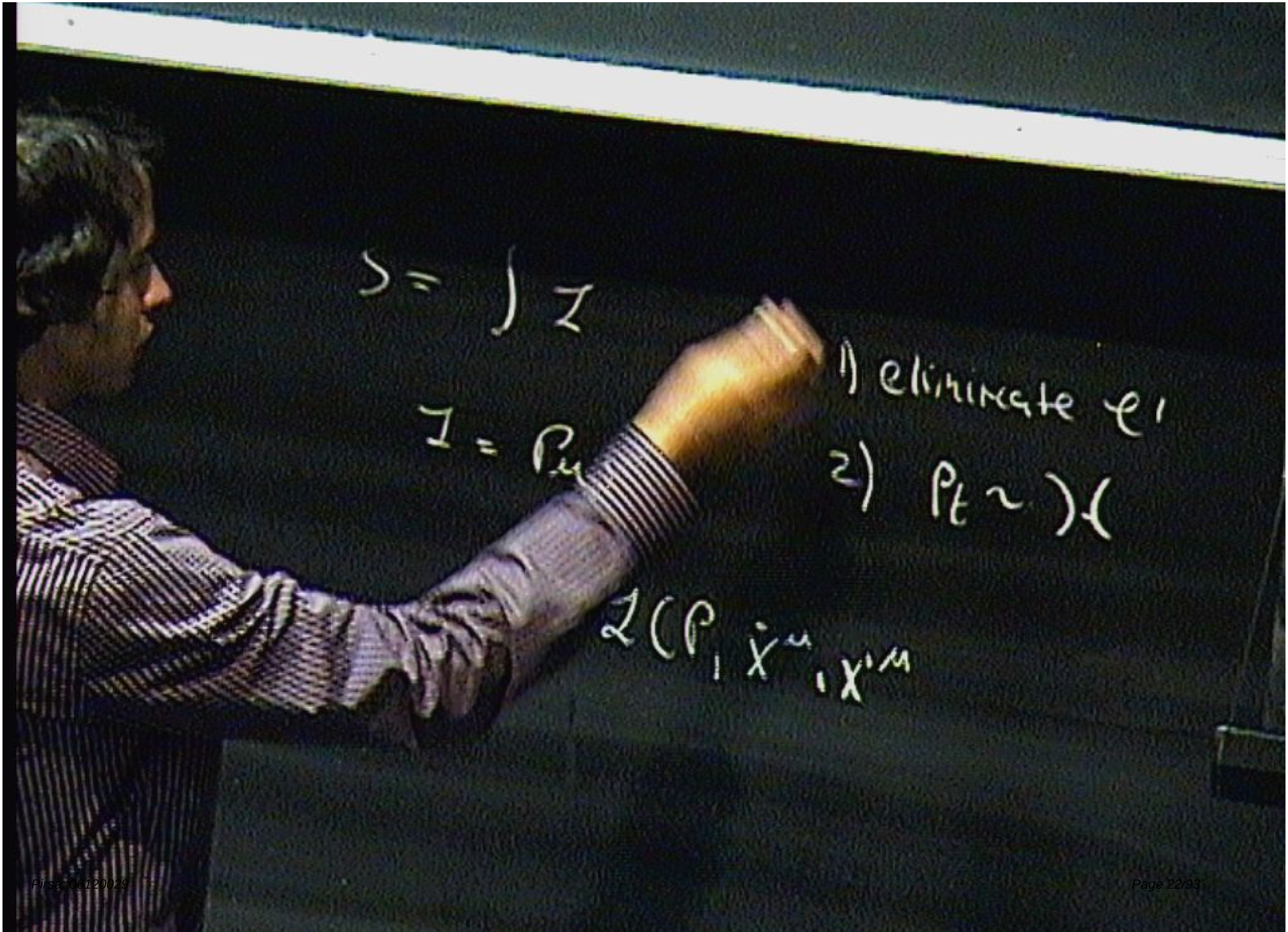
- Take the limit L $\longrightarrow\infty$ with λ/L^2 = fixed <<1. One can show that the Polyakov action becomes,

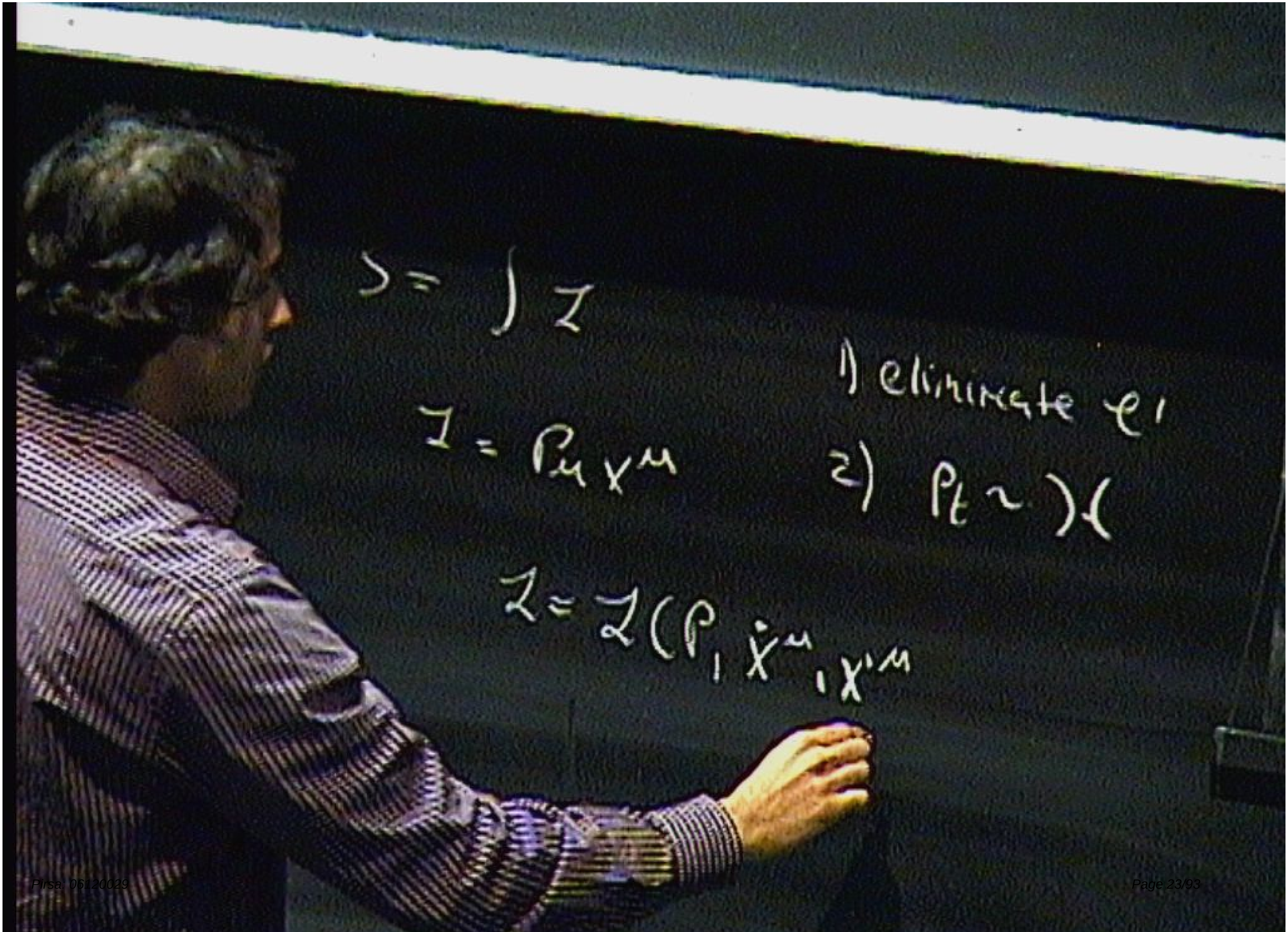
$$S_p \approx -L \int d\tau \int_0^1 d\sigma \left[\frac{i}{2} V \dot{\bar{z}} - \frac{i}{2} \bar{V} \dot{z} + \frac{\lambda}{L^2} |z'|^2 + \mathcal{O}\left(\frac{\lambda^2}{L^4} \right) \right]$$

$$V = \frac{z}{1 - |z|^2}$$

Just like Matrix Model!!!




S = ∫ L
L = P
1) eliminate e'
2) Pt ~ H
L(P, ẋ^μ, x'^μ


S = ∫ L
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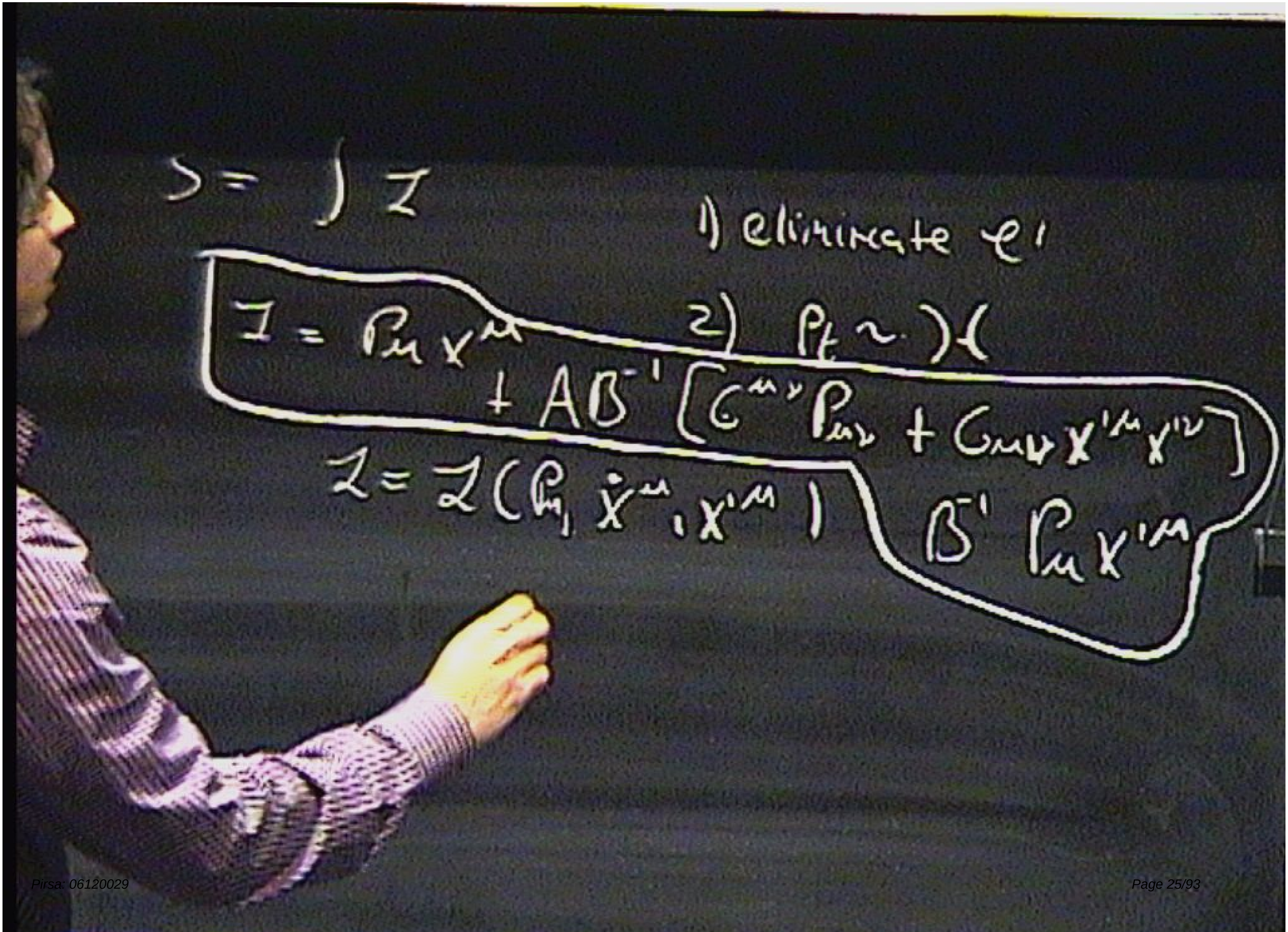
$$S = \int \mathcal{L}$$

1) eliminate e'

$$\mathcal{L} = p_\mu \dot{x}^\mu$$

2) $p_t \sim x$

$$\mathcal{L} = \mathcal{L}(p, \dot{x}^\mu, x'^\mu)$$


S = ∫ L
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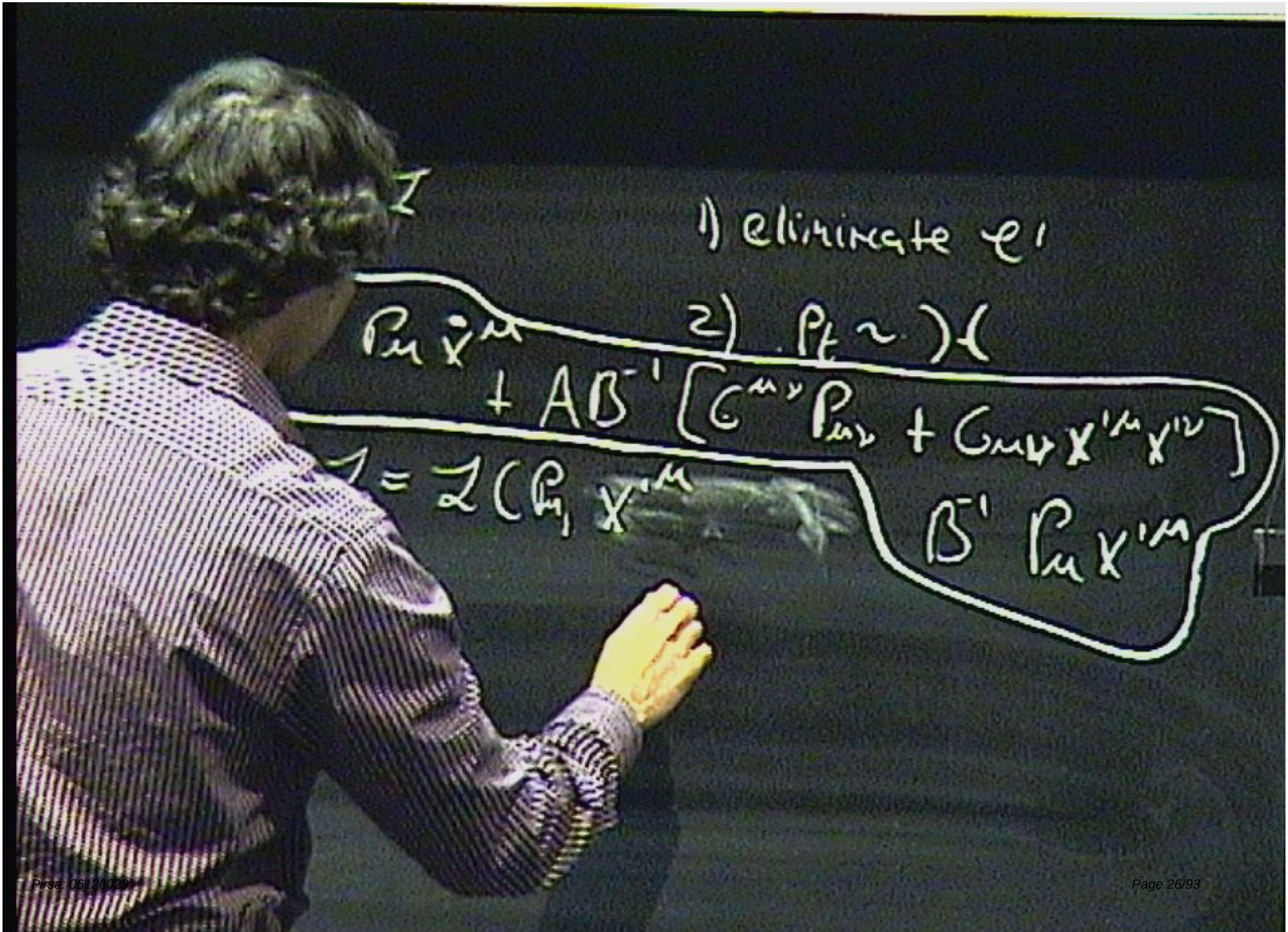
L = P_μ ẋ^μ + AB^{-1} [G^{μν} P_μ P_ν + G_{μν} x'^μ x'^ν]
L = L(P_μ, ẋ^μ, x'^μ)
B' P_μ x'^μ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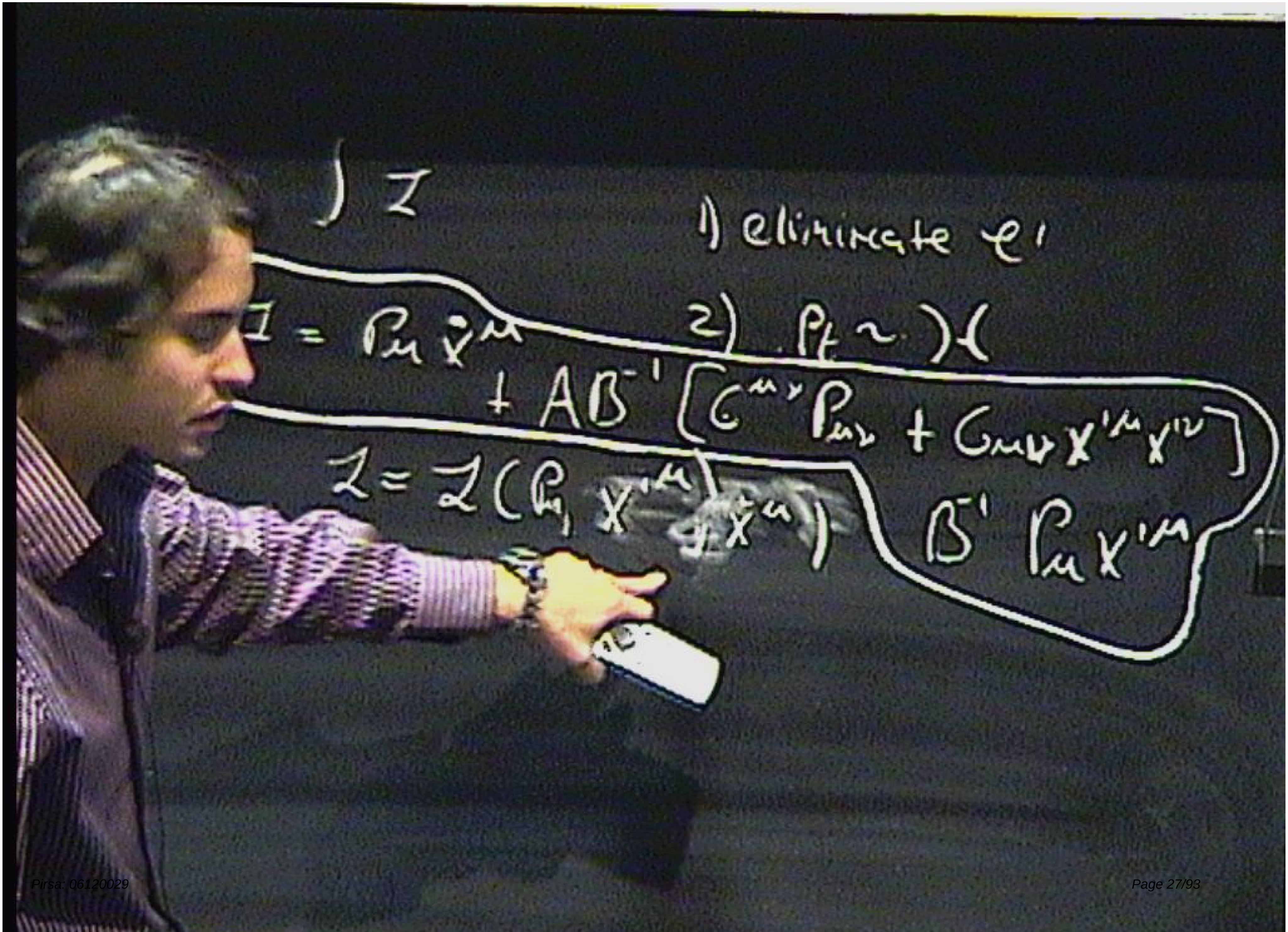
B' Pμ x'^μ
= L(Pμ, x'^μ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D-branes

- Consider "Heavy" states with $E = J_Z \equiv p \lesssim N$

$$\psi_p = \epsilon^{j_1 \cdots j_p}_{i_1 \cdots i_p} Z^{i_1}_{j_1} \cdots Z^{i_p}_{j_p}$$

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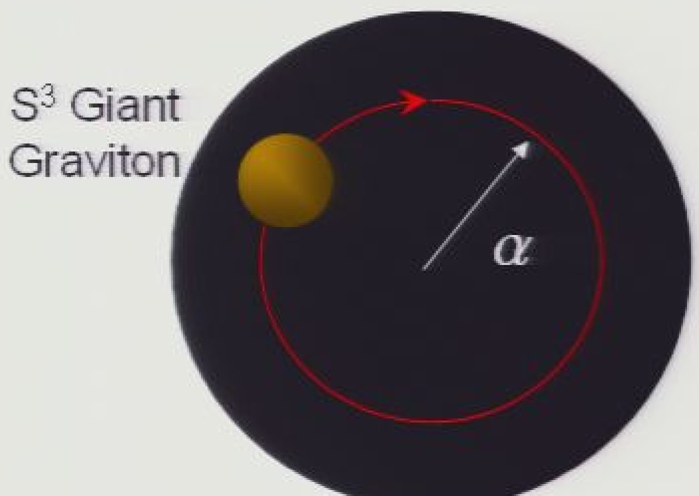
- Claim: This is dual to a Giant Graviton (spherical D3-brane of type IIB string theory on $AdS^5 \times S^5$)



$$\alpha = \sqrt{1 - \frac{p}{N}}$$

$$E = J_Z \equiv p$$

Open Strings on D-branes

- Add a word to the state

$$|n_1, n_2, \ldots, n_L\rangle = \epsilon^{j_1 \cdots j_p}_{i_1 \cdots i_p} Z^{i_1}_{j_1} \cdots Z^{i_{p-1}}_{j_{p-1}} (Y Z^{n_1} Y \cdots)^{i_p}_{j_p}$$

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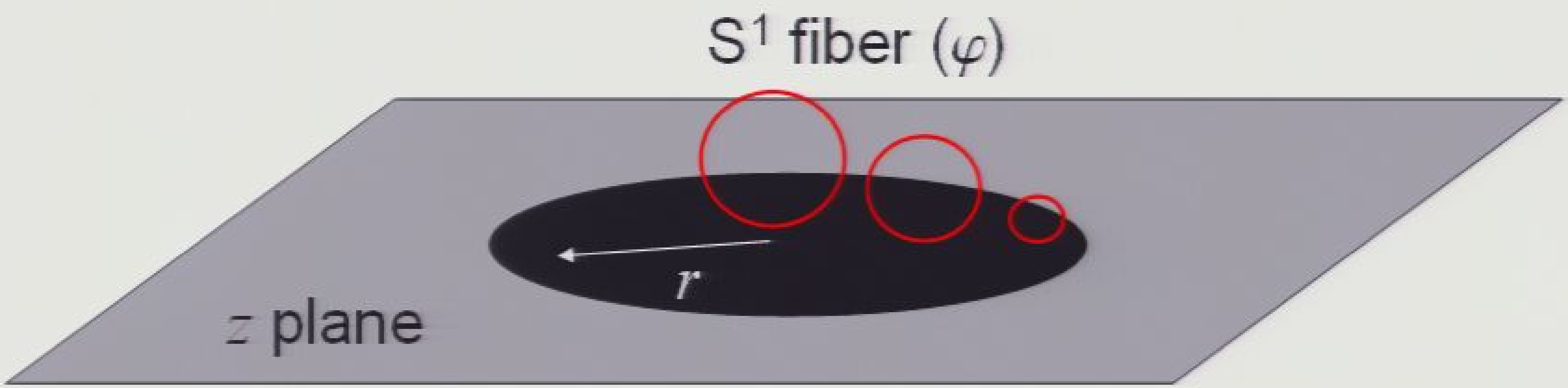

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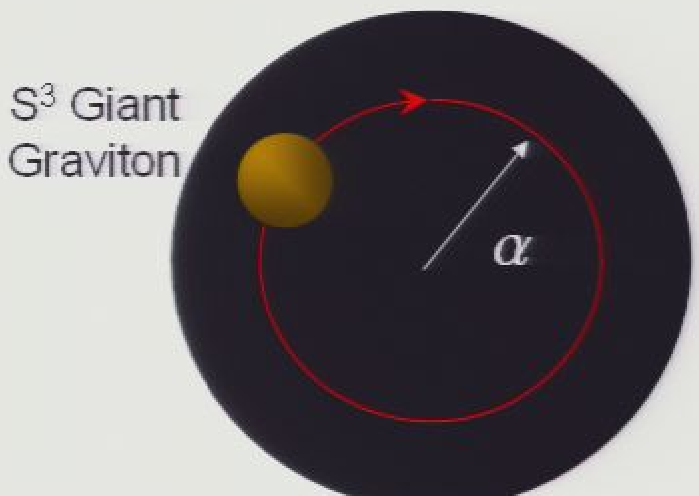

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- The one-loop Hamiltonian (large N) becomes an ***open*** bosonic lattice (hep-th/**0502172, 0604123)**

$$H = 2\lambda \sum_{l=1}^{L} \hat{a}_l^\dagger \hat{a}_l - \lambda \sum_{l=1}^{L-1} (\hat{a}_l^\dagger \hat{a}_{l+1} + \hat{a}_l \hat{a}_{l+1}^\dagger)$$
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source/sink

$$|n_1, n_2, n_3 \ldots, n_L\rangle$$

source/sink

Open Strings on D-branes

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Z exchange

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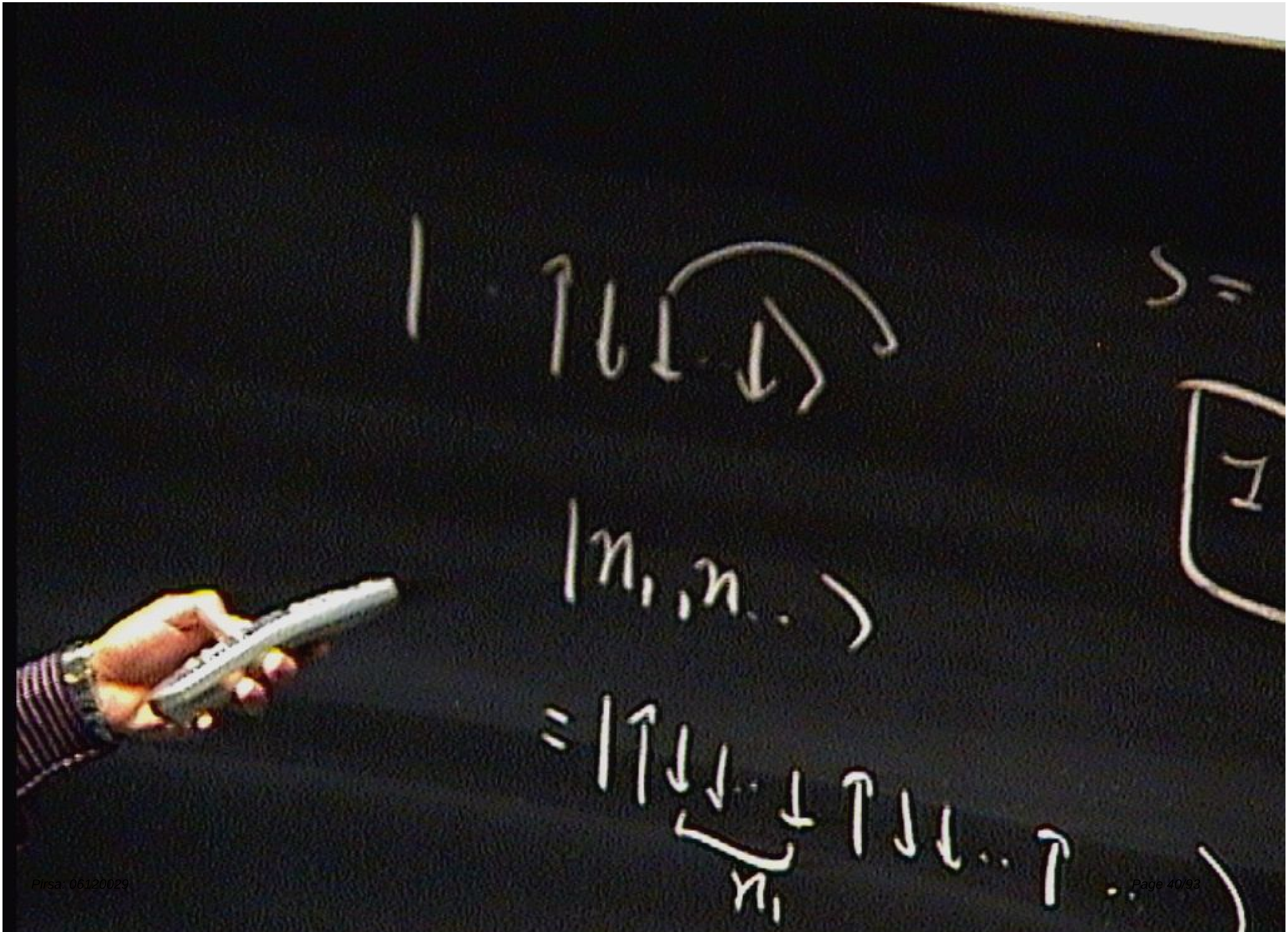
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|n₁, n₂ … ⟩
= |↑↓↓…↓↑↓↓..↑ … ⟩
n₁
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Open Strings on D-branes

- In coherent states, and taking the L $\longrightarrow\infty$ limit

$$\begin{aligned} \langle H \rangle &= \frac{\lambda}{L}\int_0^1 d\sigma(r'^2 + r^2\phi'^2) \\ &+ \lambda\left[\alpha^2\sin^2\phi + (\alpha\cos\phi + r)^2\right]\Big|_{\sigma=0} \\ &+ \lambda\left[\alpha^2\sin^2\phi + (\alpha\cos\phi + r)^2\right]\Big|_{\sigma=1} \end{aligned}$$

- At large λ, these terms localize the ends of the open string on the Giant Graviton ***just like in String Theory!***

$$\begin{aligned} r|_{\sigma=0,1} &= \sqrt{1-p/N} \\ \phi|_{\sigma=0,1} &= \pi \end{aligned}$$

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Gravitational Back-reaction

- The Hamiltonian

$$\hat{H} = -\lambda\hbar \, \mathrm{Tr}[Z,Y][\partial_Z, \partial_Y]$$

has a very degenerate ground state: any function of (say) Z only

- Some “heavy" ground states behave classically in the large N limit
  - Example:

$$\psi = e^{\,\mathrm{Tr}\,\Omega(Z)/\hbar}$$

Gravitational Back-reaction

- Normalization in terms of eigenvalues of Z:

$$\langle\psi|\psi\rangle \propto \prod_{i=1}^{N}\int d^2 z_i e^{\sum_j W(z_j,\bar{z}_j)/\hbar+\sum_{i<j}\log|z_i-z_j|^2}$$

$$W(z,\bar{z}) = -|z|^2+\Omega(z)+\overline{\Omega(z)}$$

- At large N, use Saddle Point approximation

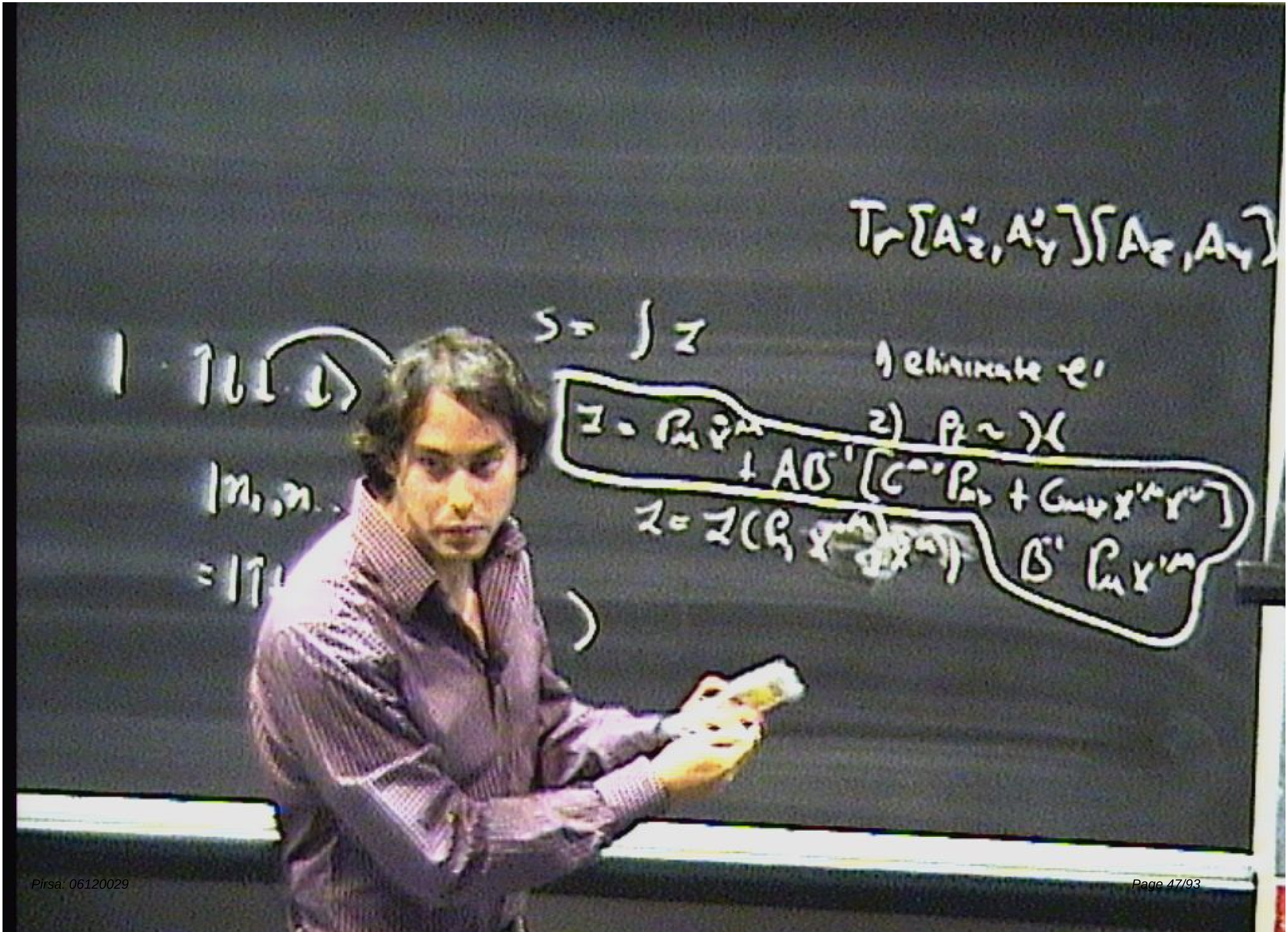
$$E[\rho] = \int d^2 z\rho(z,\bar{z})W(z,\bar{z})/\hbar+\frac{1}{2}\int d^2 z d^2 z'\rho(z,\bar{z})\rho(z',\bar{z}')\log|z-z'|^2$$

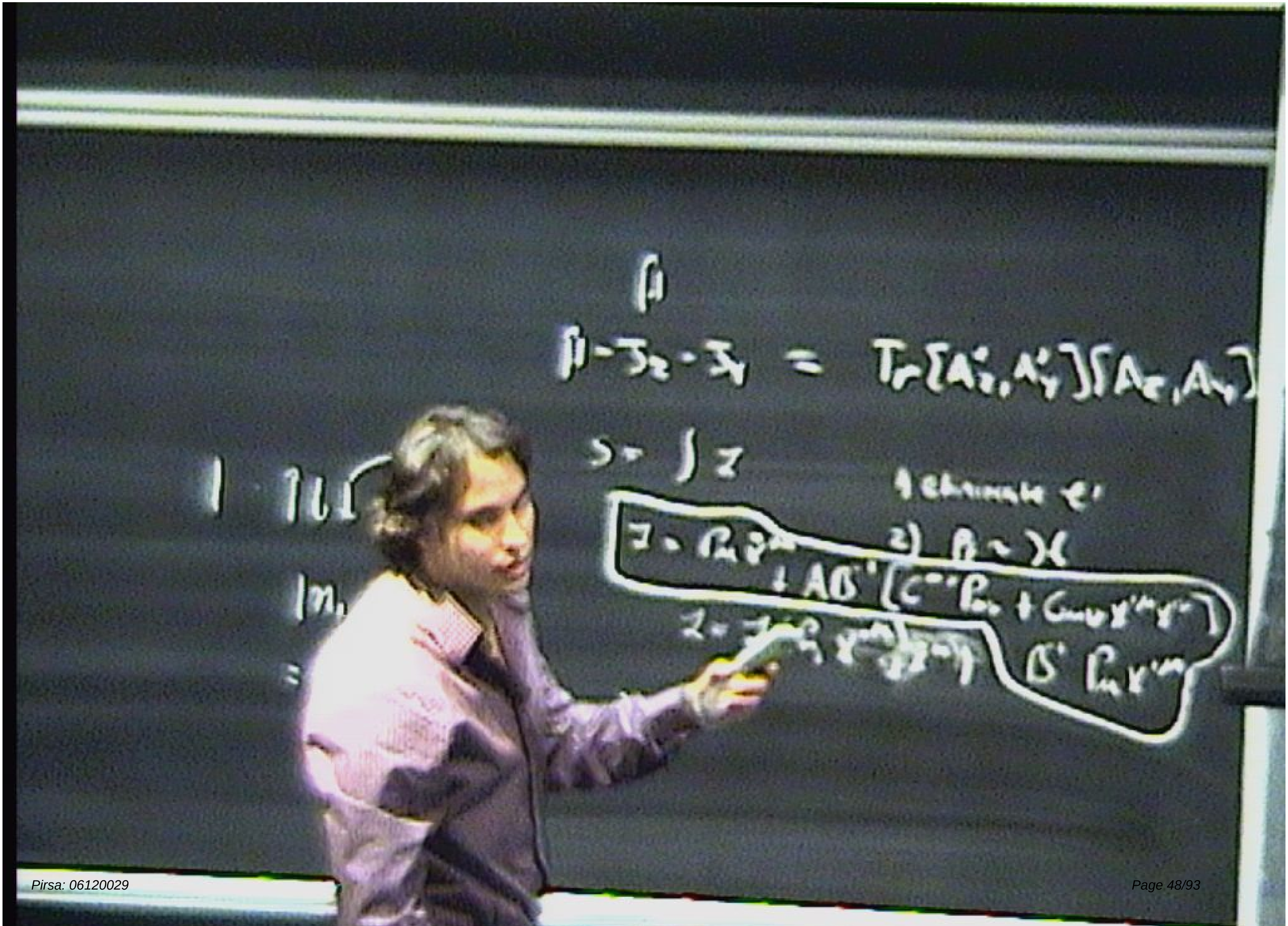
$$\delta E[\rho] = 0$$

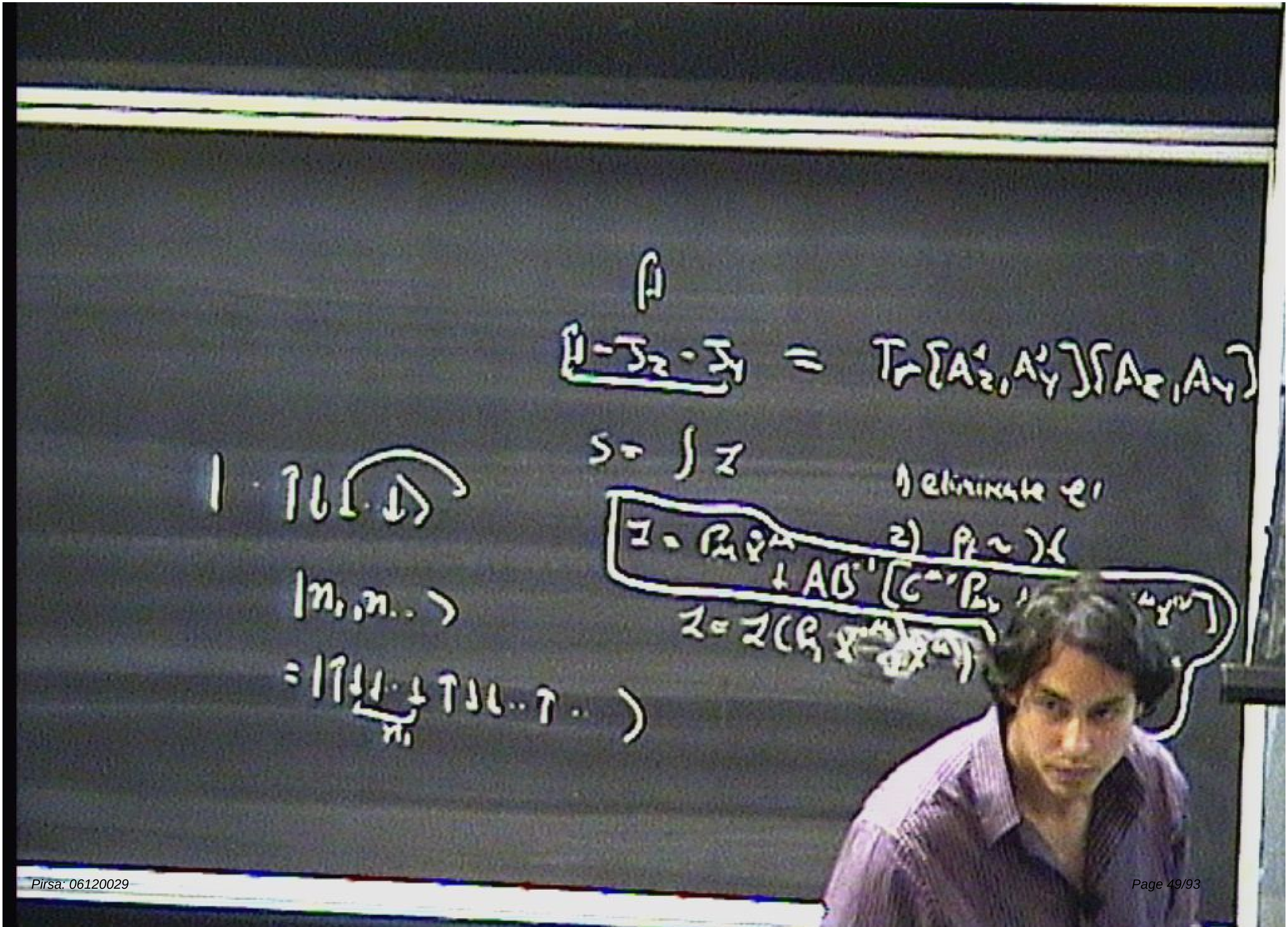
$$N = \int d^2 z\rho(z,\bar{z})$$

Gravitational Back-reaction

- At large N, we have a "Landscape" of constant density droplets on the complex plane:







Gravitational Back-reaction

- At large N, we have a "Landscape" of constant density droplets on the complex plane:

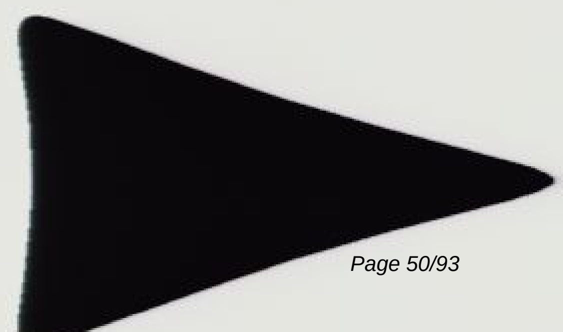
- Examples:

$\Omega(z) = 0$
(unit disk)

$= |0\rangle$

$\Omega(z) = t_2 z^2$
(elliptical droplet)

$\Omega(z) = t_3 z^3$
(Hypotrochoid)



(Closed) Probe String

- Probe string state:

$$|n_1, \ldots, n_L\rangle = \ \mathrm{Tr}(Y\psi_{n_1}(Z)Y \cdots Y\psi_{n_L}(Z))e^{\ \mathrm{Tr}\ \Omega(Z)/\hbar}$$

where, in the large N limit

$$\langle n_1, \ldots, n_L | n'_1, \ldots, n'_L\rangle \approx \prod_{l=1}^{L} \langle \hbar\ \mathrm{Tr}(\overline{\psi_{n_l}(Z)}\psi_{n'_l}(Z))\rangle$$

$$\langle \mathcal{O} \rangle \ = \ \frac{\int [d^2 Z] e^{\ \mathrm{Tr} W/\hbar} \mathcal{O}}{\int [d^2 Z] e^{\ \mathrm{Tr} W/\hbar}}$$

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Gravitational Back-reaction

- Normalization in terms of eigenvalues of Z:

$$\langle\psi|\psi\rangle \propto \prod_{i=1}^{N} \int d^2 z_i e^{\sum_j W(z_j,\bar{z}_j)/\hbar + \sum_{i<j} \log|z_i - z_j|^2}$$

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- Assume orthonormal basis:

$$\langle\hbar\,\mathrm{Tr}(\overline{\psi_n(Z)}\psi_m(Z))\rangle = \delta_{nm}$$

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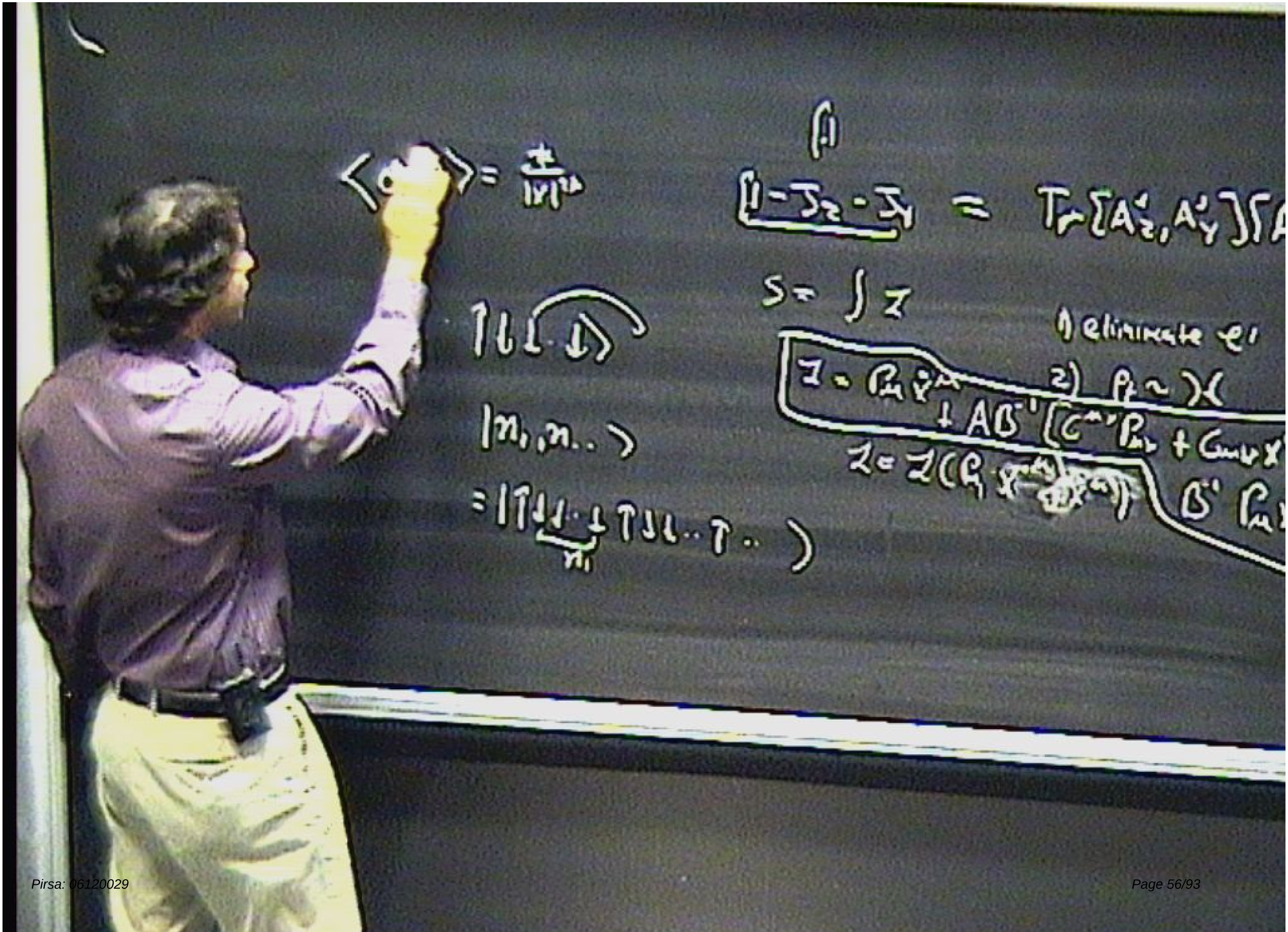
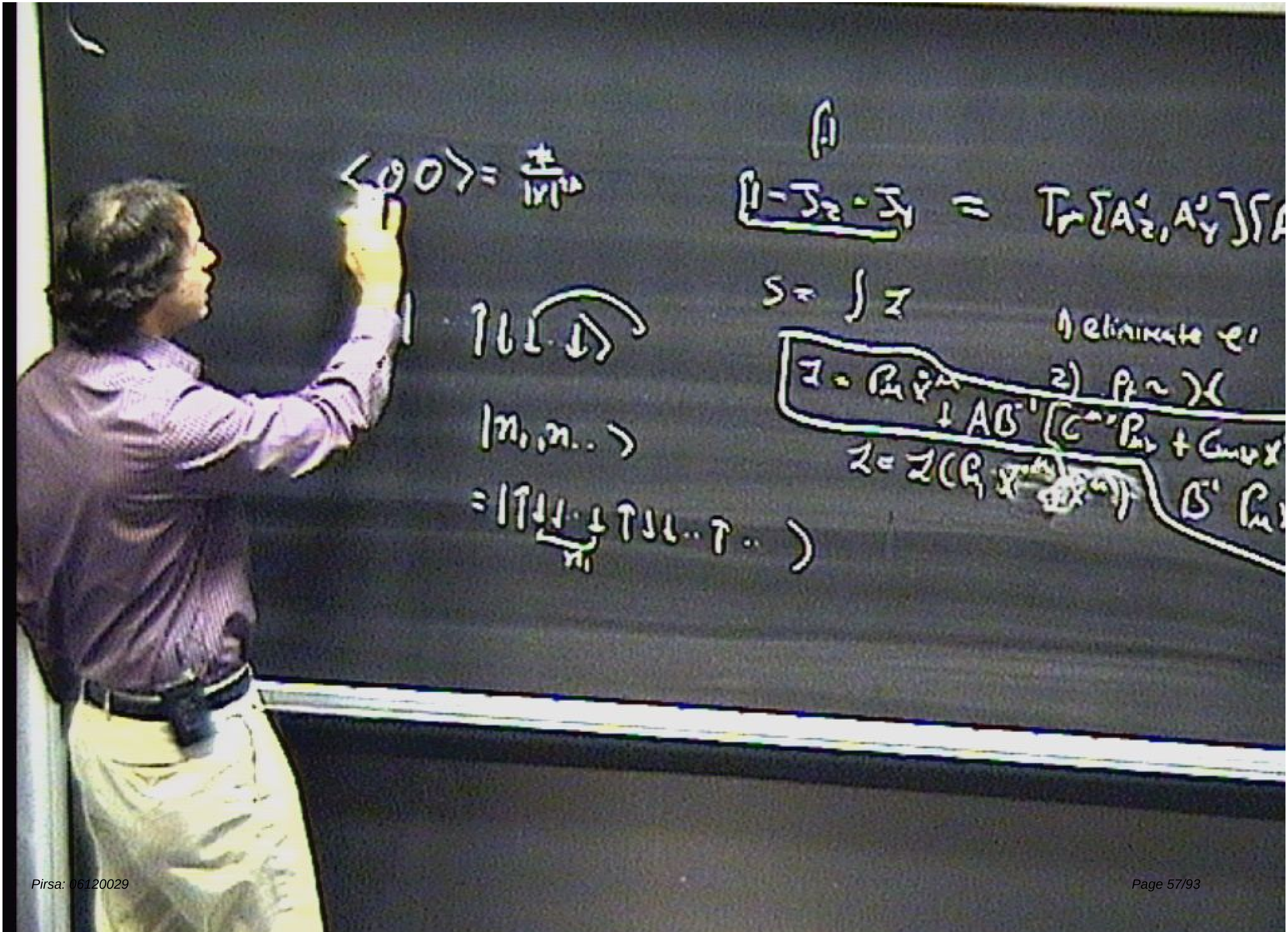

1) eliminate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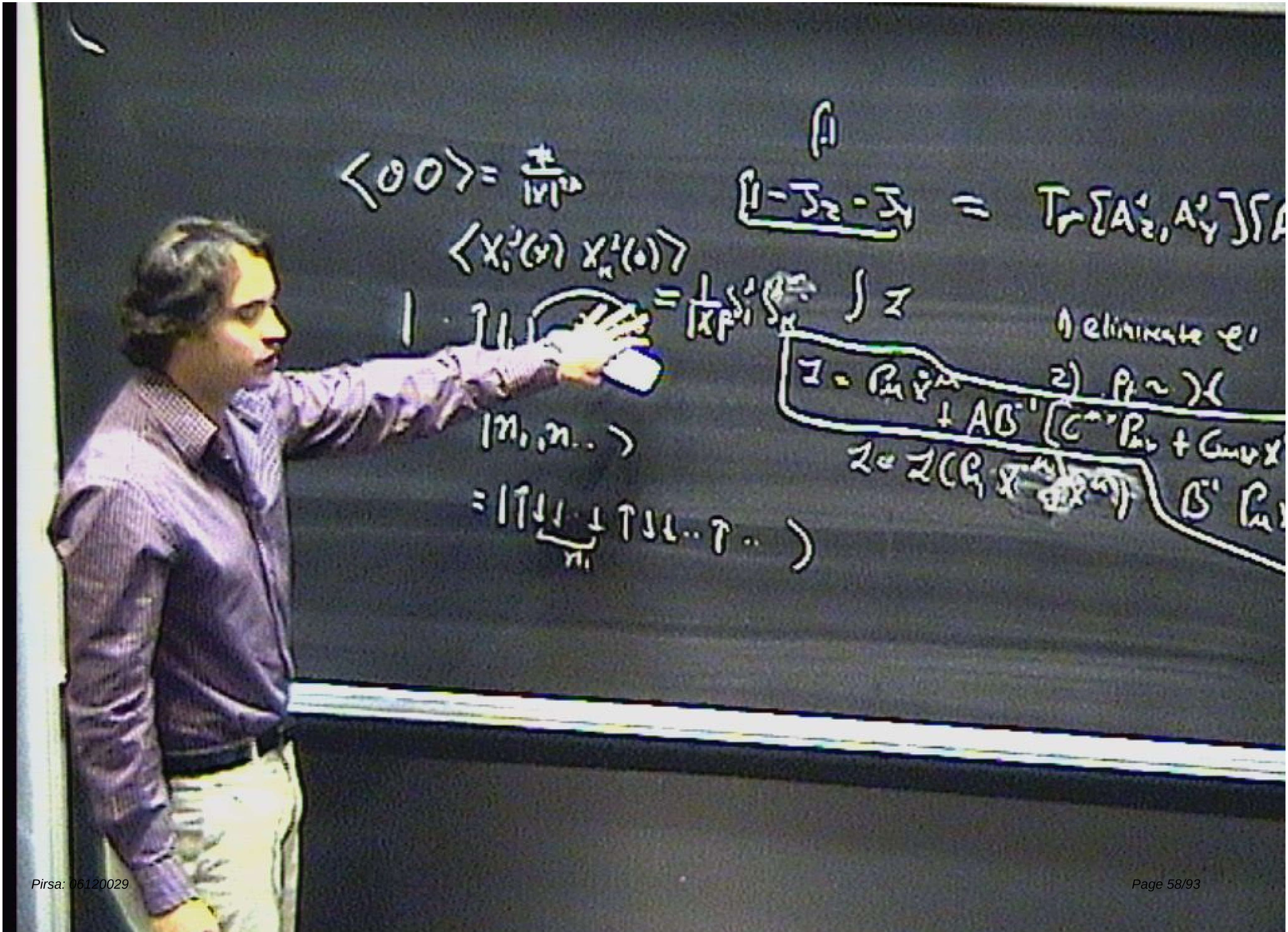

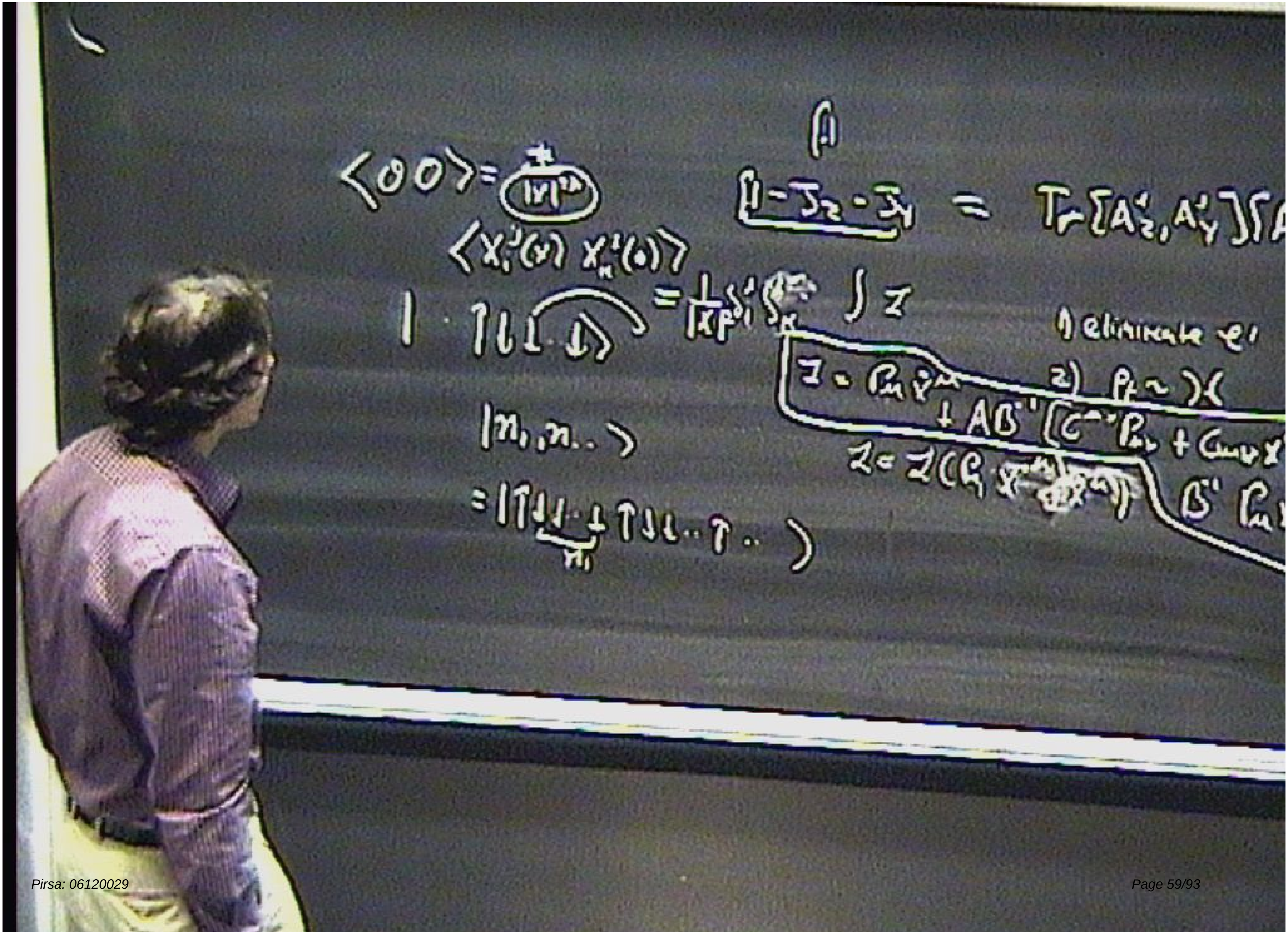
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- One can show that, in the large N limit, the Hamiltonian in this basis becomes ($|n_1, n_2, \ldots, n_L\rangle$), (hep-th/**0612014)**

$$\hat{H} = \lambda \sum_{l=1}^{L} (\hat{L}_l^\dagger - \hat{L}_{l+1}^\dagger)(\hat{L}_l - \hat{L}_{l+1})$$

$$\psi_n(Z) \simeq |n\rangle$$

$$|0\rangle = \psi_0(Z) \equiv \mathbf{1}$$

- One finds an interesting algebraic structure

$$[\hat{L}, \hat{L}^\dagger] = \hat{P}_0 \ ,$$

$$\hat{L} = \Omega'(\hat{L}^\dagger) + (\hat{L}^\dagger)^{-1} + \sum_{k>1} v^{(k)} (\hat{L}^\dagger)^{-k} \ ,$$

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Classical Limit

- In this representation, the *coherent states* are

$$\hat{L}|z\rangle = z|z\rangle \qquad |z\rangle \propto \sum_{n=0}^{\infty} \psi_n^*(z)|n\rangle$$

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- Action (in the large angular momentum limit):

$$S = L\int d\tau \int_0^1 d\sigma \left(\frac{i}{2}V\dot{\bar{z}} - \frac{i}{2}\bar{V}\dot{z} - \frac{\lambda}{L^2}|z'|^2\right) \qquad V = \bar{\partial}\log\left(\sum_{n=0}^{\infty}|\psi_n(\bar{z})|^2\right)$$

Examples

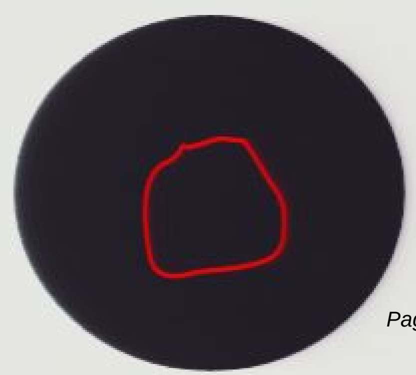
- Circular Droplet ($\Omega(z) = 0$): recover usual bosonic lattice

$$\hat{L}\hat{L}^\dagger = 1 \qquad [\hat{L}, \hat{L}^\dagger] = \hat{P}_0$$

$$\psi_n(z) = z^n$$

Normalized only inside unit disk

$$\langle z|z\rangle = \sum_{n=0}^{\infty} |\psi_n(\bar{z})|^2 = \frac{1}{1-|z|^2}$$



$$V = \frac{z}{1-|z|^2}$$

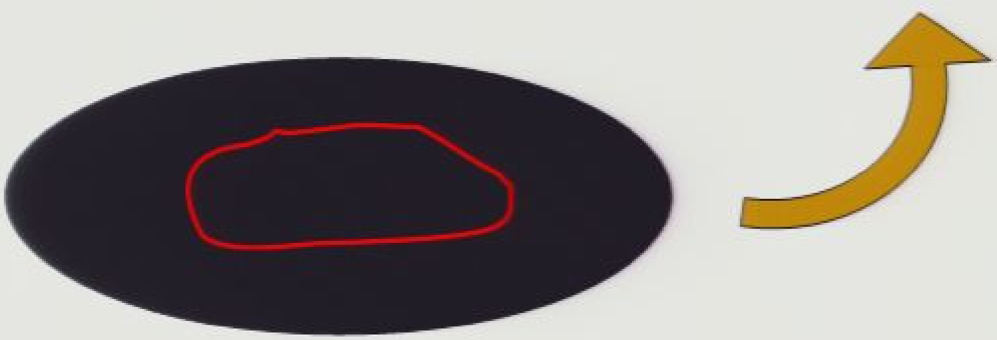
Examples

- Elliptical Droplet ($\Omega(z) = t_2 z^2 = \epsilon z^2/2$)

$$\hat{L}^\dagger = \frac{1}{\sqrt{1-\epsilon^2}}\left(\hat{a}^\dagger + \epsilon \hat{a}\right)$$

$\psi_n(z) =$ Chebyshev Polynomials of Second Kind

$$\langle z|z\rangle = \sum_{n=0}^{\infty} |\psi_n(\bar{z})|^2 = \frac{1}{1-\epsilon^2+\epsilon(z^2+\bar{z}^2)-|z|^2(1+\epsilon^2)}$$



Normalizable only inside ellipse

String Theory Interpretation

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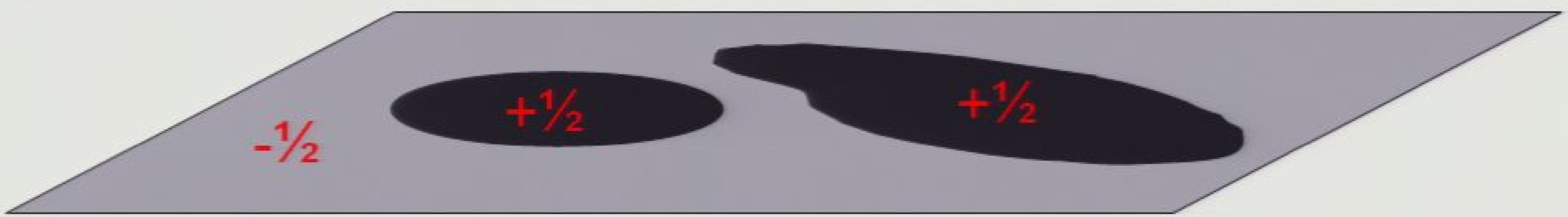

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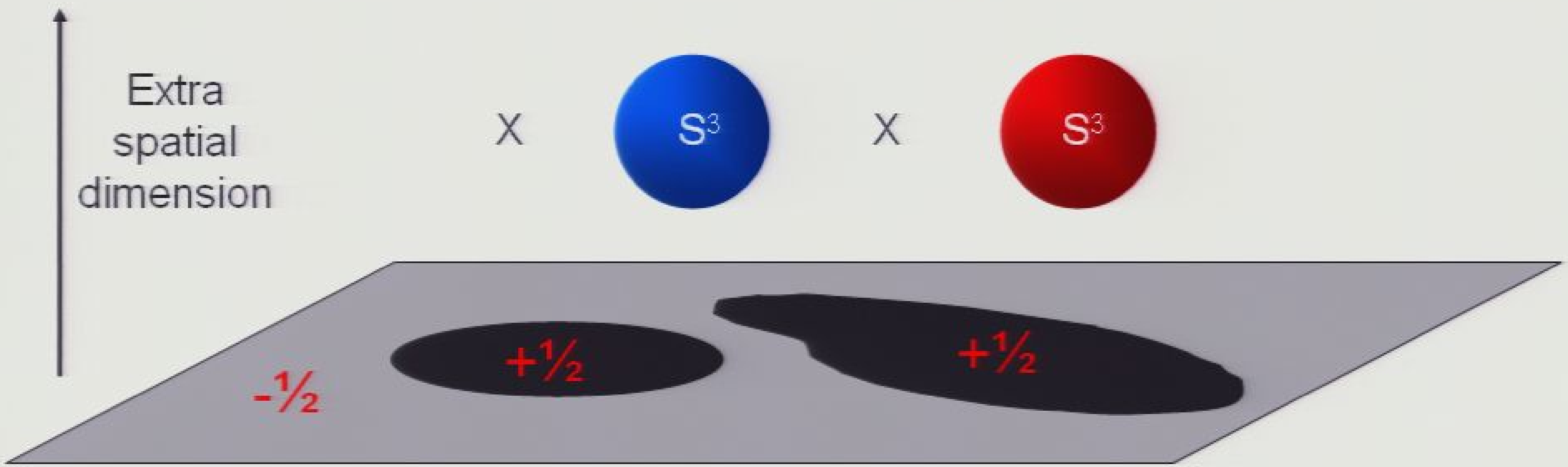
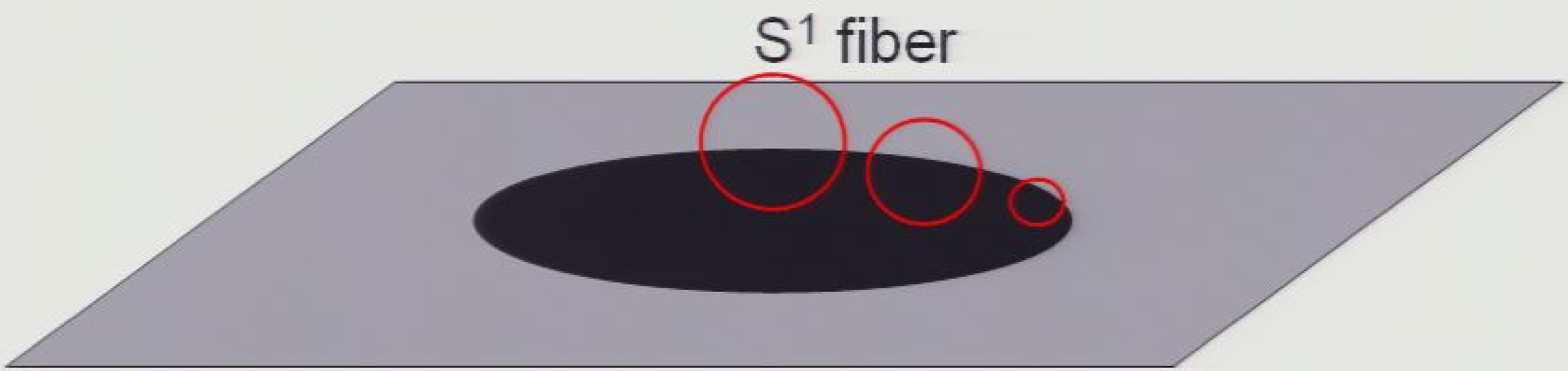


String Theory Interpretation

- The SU(2) sector is dual to strings that live inside the droplets and rotate on a $S^1 \subset S^3$ fiber.



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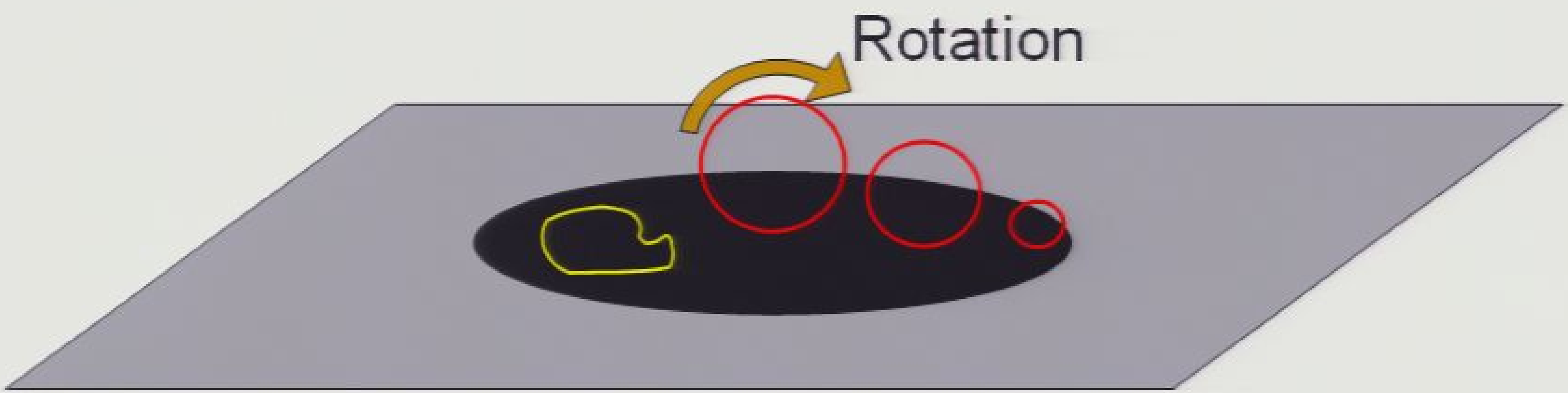

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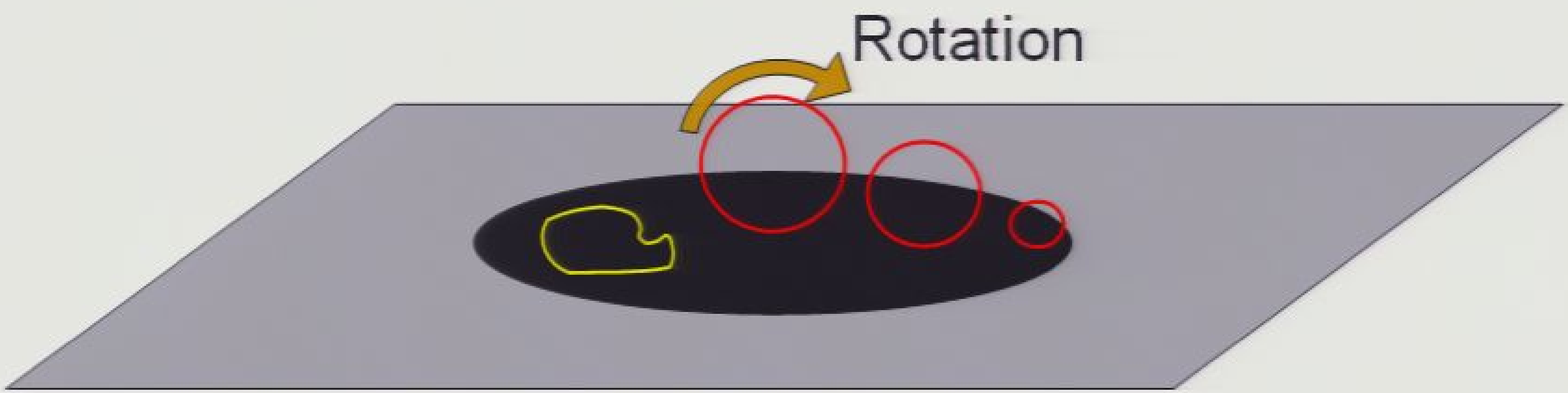

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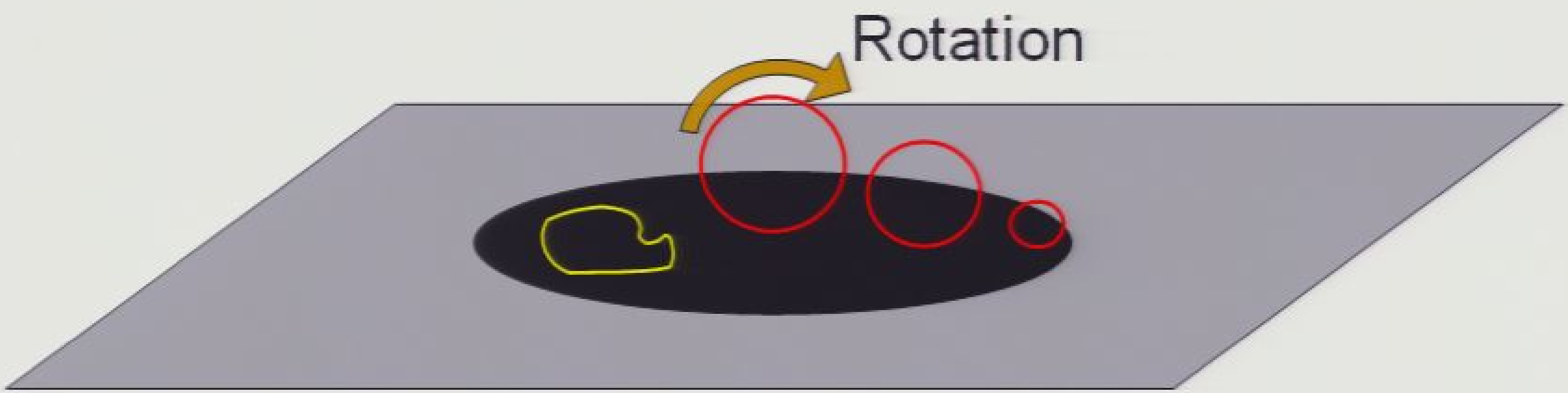

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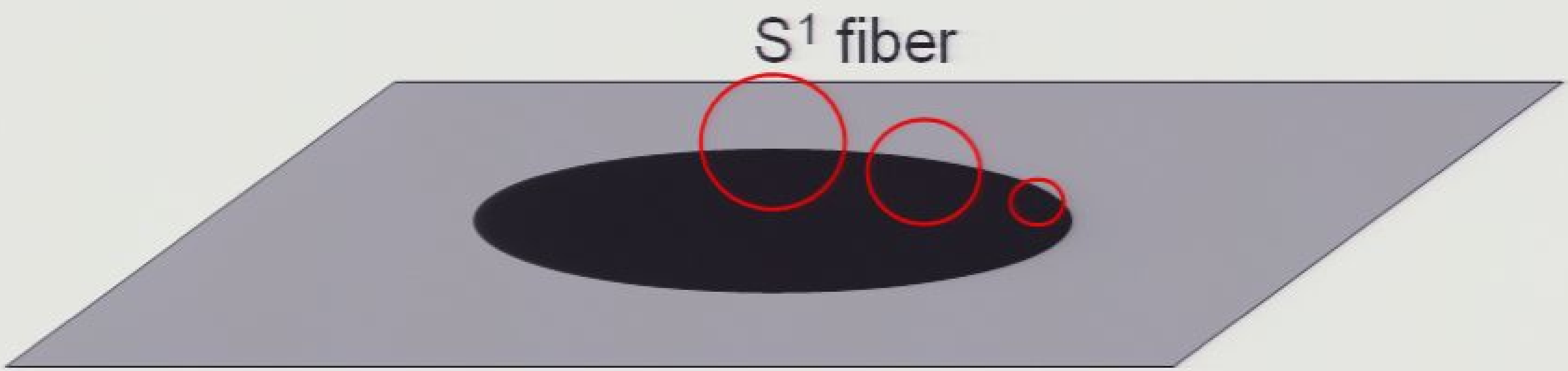

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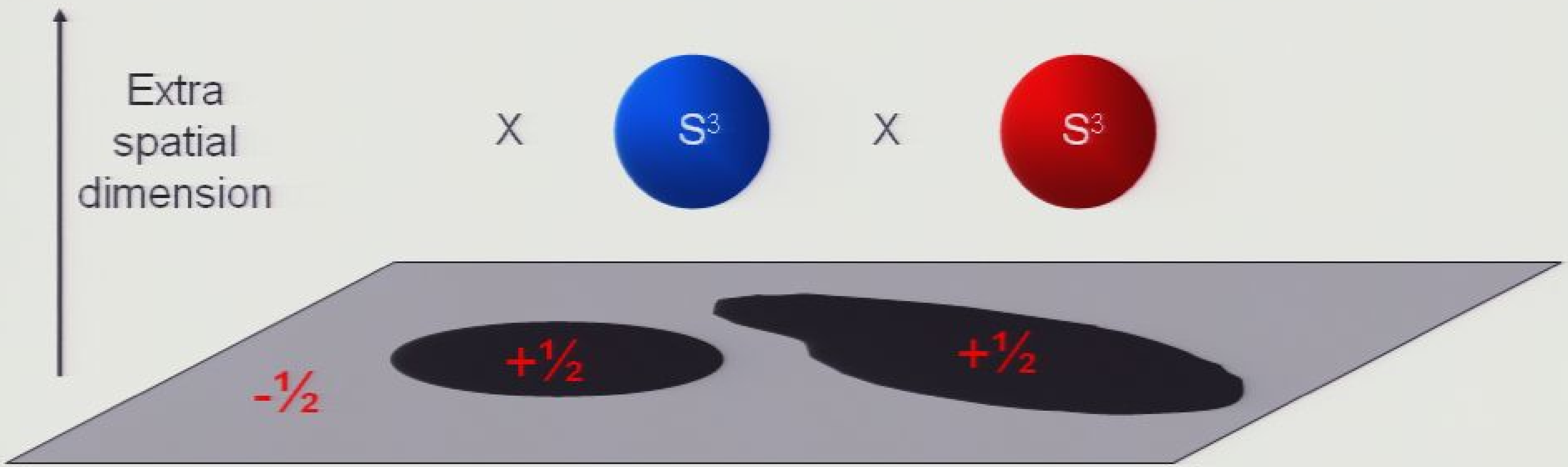

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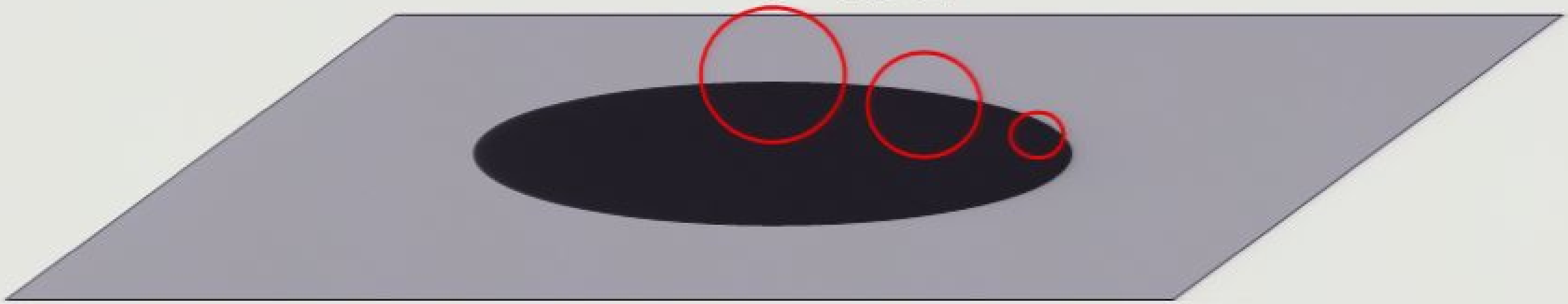

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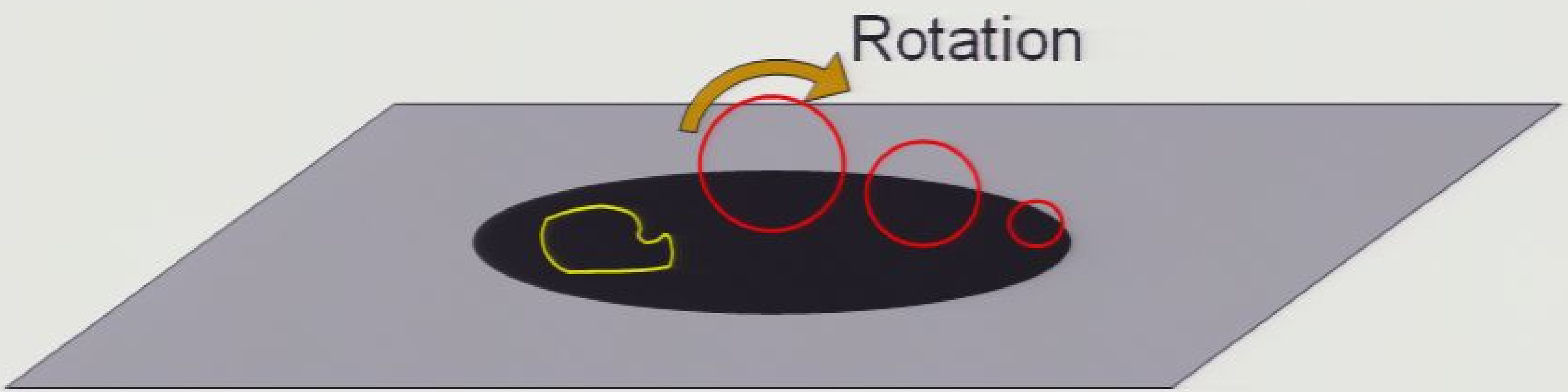

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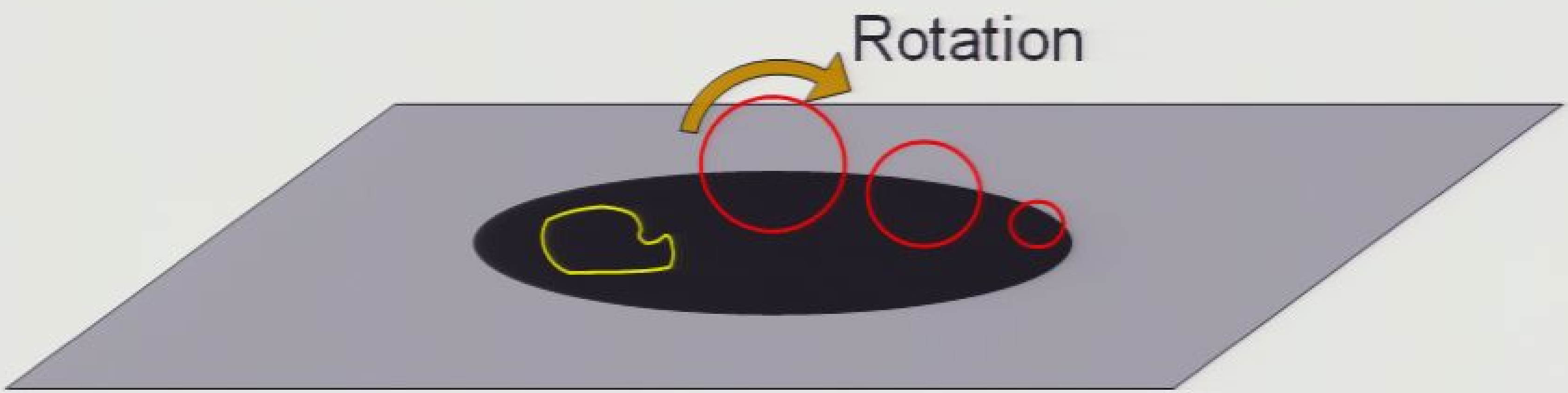

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- Metric inside the droplet

S[1] fiber ~ Y field in matrix model

$$ds^2 = -h^{-2}\left(dt - \frac{i}{2}V\,d\bar{z} + \frac{i}{2}\bar{V}\,dz\right)^2 + h^2 dz\,d\bar{z} + \boxed{h^{-2}d\varphi^2}$$

$$V = \bar{\partial}\log K\,, \quad h^4 = \partial\bar{\partial}\log K \qquad \log K \equiv \sum_i \oint_{\partial D_i} \frac{dz'}{2\pi i}\frac{1}{z-z'}\log(\bar{z}-\bar{z}')$$

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- One can show that strings rotating fast along circle are described by the effective sigma-model (L>>1, λ/L^2 = fixed <<1).

$$\boxed{S = L\int d\tau \int_0^1 d\sigma\left(\frac{i}{2}V\dot{\bar{z}} - \frac{i}{2}\bar{V}\dot{z} - \frac{\lambda}{L^2}|z'|^2\right)}$$

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- We discover that the full metric on the droplet can be reconstructed from the orthogonal polynomials:

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- One can explicitly verify this for all cases studied above, including the complicated "Hypotrochoid":

$$V = \bar{\partial} \log K = \frac{z}{1-|z|^2} - \frac{\left(z^4 + \bar{z}^2\,(3 - 2\,z\,\bar{z})\right)a}{(1-|z|^2)^2} + \frac{\left(2\,z + z^7 + 3\,\bar{z}^5 - 2\,z\,\bar{z}^6\right)a^2}{(1-|z|^2)^3} - \frac{\left(z^4\,(3+z^6) + 3\,\bar{z}^2 + 3\,\bar{z}^8 - 2\,z\,\bar{z}^9\right)a^3}{(1-|z|^2)^4} + \mathcal{O}(a^4)$$

$\Omega(z) = a\,z^3/3$

Summary and Conclusions

- The one-loop SU(2) sector of $\mathcal{N} = 4$ SYM can be regarded as a toy model of Quantum Gravity on some reduced 3+1 dimensional space-times

- The classical limit is reached at large N and for states with large quantum numbers (angular momenta).

- The classical limit agrees with String Theory
  - Closed Strings
  - D-branes
  - Open Strings
  - ½ BPS geometries (and closed strings on them)

Summary and Conclusions

Quantum

Matrix Model at $O(\lambda)$

Large N

Discrete lattice systems

Large L

reduced sigma-model

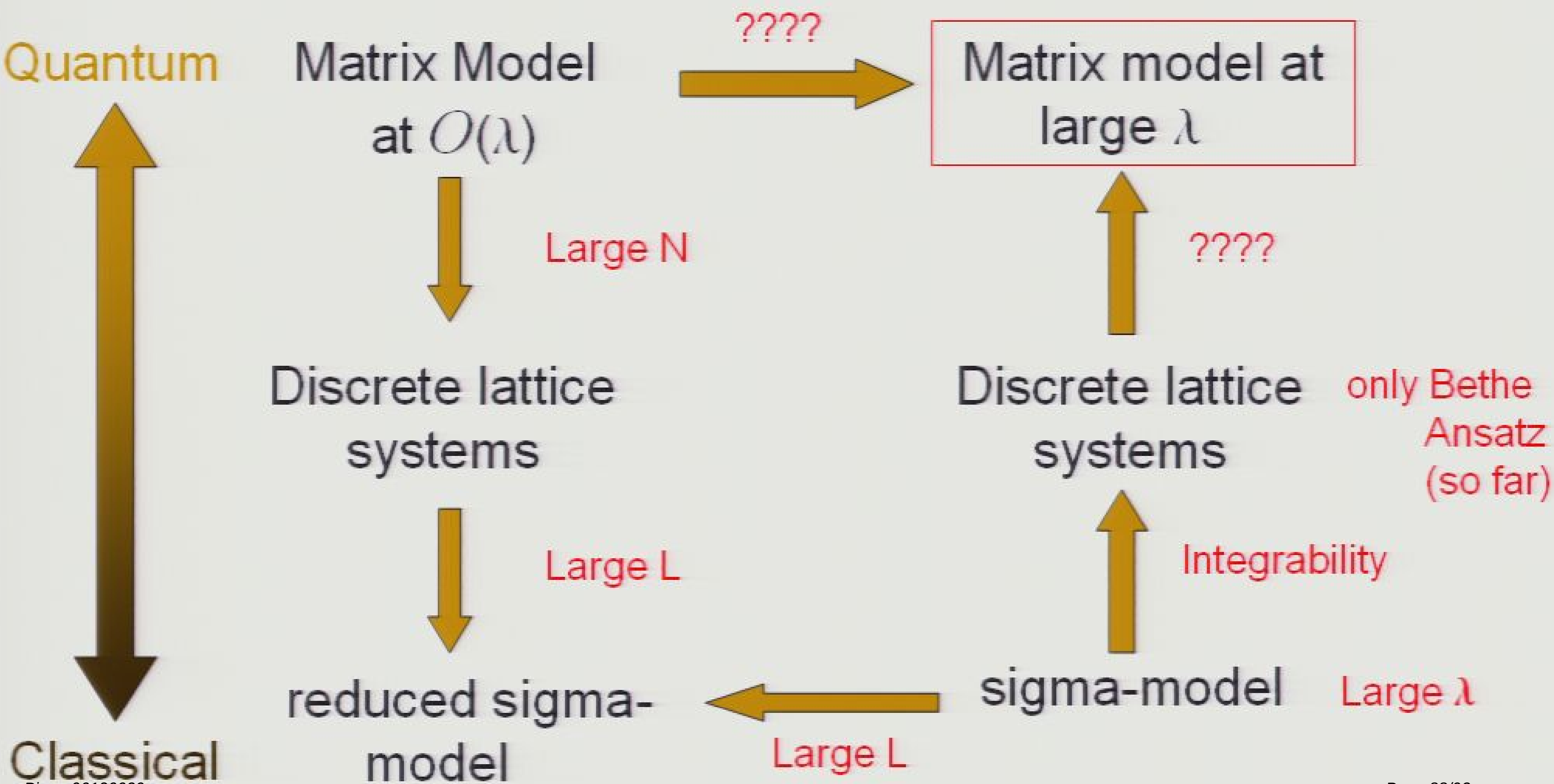
Large L

sigma-model

Large λ

Classical

Summary and Conclusions



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????

Matrix model at large λ

Some partial results in

D. Berenstein

hep-th/0507203

D. Berenstein, D. H. Correa, S.E.V.

hep-th/0509015

S.E.V.

hep-th/0607204

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