Title: OPE of Giant Wilson loops from D-branes and Matrix Models

Date: Dec 06, 2006 10:00 AM

URL: http://pirsa.org/06120028

Abstract: In the context of AdS/CFT, it has been recently proposed that Wilson loops in higher representation of the gauge group have a dual description in terms of D-branes in AdS_5xS^5. After reviewing this new dictionary, I will present a computation of correlators between chiral primaries of N=4 SYM and Wilson loops in large symmetric and antisymmetric representations. These correlators can be computed both in supergravity using D-branes and in gauge theory from a matrix model, with precise agreement between the two sides

OUTLINE

- MOTIVATION : AdS/CFT
- WILSON LOOPS IN $\mathcal{N}=4$ SYM
  - HALF-BPS LOOPS : LINE, CIRCLE
  - MATRIX MODEL FOR CIRCULAR LOOP
- STRING THEORY DUAL IN AdS
  - FUNDAMENTAL STRINGS
  - $\boxed{\text{D3 AND D5 BRANES : "GIANT WILSON LOOPS"}}$

    (Drukker-Fiol '05, Yamaguchi '06 Gomis-Passerini '06)
- CORRELATORS OF GIANT WILSON LOOPS AND CHIRAL PRIMARIES

  (S.G., Ricci, Trancanelli: hep-th/0608077)
  - O.P.E. OF WILSON LOOPS
  - SUPERGRAVITY CALCULATION FROM D-BRANES
  - GAUGE THEORY CALCULATION FROM MATRIX MODEL

  $\left.\begin{array}{l}\text{SUPERGRAVITY CALCULATION FROM D-BRANES}\\ \text{GAUGE THEORY CALCULATION FROM MATRIX MODEL}\end{array}\right\}$ PERFECT AGREEMENT

AdS/CFT DUALITY

$\mathcal{N}=4$ SYM $\longleftrightarrow$ IIB STRINGS ON $AdS_5 \times S^5$

WITH: $4\pi g_s = g_{YM}^2 \quad L^2 = \alpha' \sqrt{\lambda} \quad , \quad \lambda = g_{YM}^2 N$

- STRONG/WEAK DUALITY
  - SUPERGRAVITY LIMIT

    $N \to \infty \, , \, \lambda \gg 1$

- MANY OBSERVABLES:
  - EITHER PROTECTED BY SUPERSYMMETRY INDEPENDENT FROM λ, N
  - OR CAN BE COMPUTED ONLY AT WEAK COUPLING OR STRONG COUPLING

- SOME WILSON LOOPS ARE NON-TRIVIAL IN λ, N BUT MAY BE COMPUTED EXACTLY

WILSON LOOPS IN $\mathcal{N}=4$ SYM

$$W(C) = \frac{1}{N} \mathrm{Tr}\, P \exp \int_C \Big[i A_\mu(x(\tau))\dot{x}^\mu(\tau) + \underbrace{|\dot{x}(\tau)|\, \theta^I(\tau)\, \phi_I(x(\tau))}_{} \Big] d\tau$$

$C: (x^\mu(\tau), \theta^I(\tau))$

COUPLING TO $\mathcal{N}=4$ SCALARS
$I = 1, \dots, 6$

- $x^\mu(\tau)$ LOOP IN $\mathbb{R}^4$
- $\theta^I(\tau)$ UNIT VECTOR IN $\mathbb{R}^6$ $\quad \sum_{I=1}^{6} \theta^I \theta^I = 1$

SPECIAL HALF-BPS LOOPS

a) STRAIGHT LINE $\quad x^\mu(\tau) = (\tau, 0, 0, 0) \quad \theta^I = (1, 0, \dots, 0)$

- SYMMETRY GROUP $\quad SO(2,1) \times SO(3) \times SO(5)_R \subset SO(2,4) \times SO(6)_R$
- PRESERVES 8 Q's (POINCARÉ SUSY) AND 8 S's (SUPERCONFORMAL)

PROTECTED $\langle W_C \rangle = 1$ $\quad$ INDEPENDENT OF λ, N

b) CIRCULAR LOOP $\quad x^\mu(\tau) = a(\cos\tau, \sin\tau, 0, 0) \quad \theta^I = (1, 0, \dots, 0)$

- RELATED TO STRAIGHT LINE BY CONFORMAL TRANSFORMATION

- SAME SYMMETRY GROUP $\quad SO(2,1) \times SO(3) \times SO(5)_R$
- ALSO HALF-BPS $\quad$ (BUT NOW COMBINATIONS OF Q AND S)

NOT PROTECTED $\quad \langle W_C \rangle = f(\lambda, N)$

BUT CONJECTURED TO BE EXACTLY COMPUTABLE
IN GAUGE THEORY

CIRCULAR LOOP IN GAUGE THEORY: THE MATRIX MODEL

- THE EXPECTATION VALUE FOR A CIRCULAR WILSON LOOP IS CONJECTURED TO BE GIVEN EXACTLY FOR ALL λ, N BY THE HERMITEAN GAUSSIAN MATRIX MODEL

$$\boxed{\langle W_{\text{circle}} \rangle = \frac{1}{Z}\int DM \, \frac{1}{N}\operatorname{Tr} e^{M} \, e^{-\frac{2N}{\lambda}\operatorname{Tr} M^2}}$$

KEY OBSERVATION:

- THE COMBINED GAUGE–SCALAR PROPAGATOR IS A <u>CONSTANT</u>

$$-\langle A^a_\mu(x_1) A^b_\nu(x_2)\rangle \dot{x}_1^\mu \dot{x}_2^\nu + \langle \Phi^a_I(x_1)\Phi^b_J(x_2)\rangle \theta^I\theta^J |\dot{x}_1||\dot{x}_2|$$

$$= \frac{g^2}{4\pi^2}\,\frac{-\dot{x}^\mu(\tau_1)\dot{x}_\mu(\tau_2) + |\dot{x}(\tau_1)||\dot{x}(\tau_2)|}{(x(\tau_1)-x(\tau_2))^2}\,\delta^{ab} = \frac{g^2}{8\pi^2}\delta^{ab}$$

$= \frac{1}{2}$ FOR $x^\mu(\tau) = (\cos\tau, \sin\tau, 0, 0)$ $\quad$ $\left(= 0 \text{ FOR STRAIGHT LINE } x^\mu(\tau) = (\tau, 0, 0, 0)\right)$

→ THE SUM OF ALL LADDER DIAGRAMS (NO INTERNAL VERTICES) IS EQUAL TO A MATRIX INTEGRAL.

GRAPHS WITH INTERNAL VERTICES CONJECTURED TO VANISH (CHECKED UP TO THREE LOOPS).

- <u>DRUKKER–GROSS</u>: "CONFORMAL ANOMALY" ARGUMENT

  THE CIRCLE AND THE LINE ARE RELATED BY A SINGULAR CONFORMAL TRANSFORMATION

• FROM THE MATRIX MODEL, ONE CAN GET EXACT EXPRESSIONS INTERPOLATING BETWEEN WEAK AND STRONG COUPLING

– AT LARGE N

EIGENVALUE DENSITY (WIGNER'S LAW)

$$\langle W_C \rangle = \left\langle \frac{1}{N} \mathrm{Tr}\, e^{M} \right\rangle_{m.m.} \overset{N\to\infty}{=} \frac{2}{\pi\lambda} \int_{-\sqrt{\lambda}}^{\sqrt{\lambda}} dx \sqrt{\lambda - x^2}\, e^{x}$$

$$= \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \begin{cases} \simeq 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \cdots & \text{WEAK COUPLING} \\ \approx e^{\sqrt{\lambda}} & \text{STRONG COUPLING} \end{cases}$$

Bessel function

AGREES WITH STRING COMPUTATION IN AdS

– CAN EVEN SOLVE AT FINITE N (Drukker–Gross)

$$\langle W_C \rangle = \frac{1}{N} L^{1}_{N-1}\left(-\frac{\lambda}{4N}\right) e^{\lambda/8N}$$

Laguerre polynomial

$$= \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{48N^2} I_2(\sqrt{\lambda}) + \cdots$$

D-BRANES WILL CAPTURE A SET OF THESE NON-PLANAR CONTRIBUTIONS

STRING THEORY DUAL IN AdS

- STANDARD PRESCRIPTION TO COMPUTE $\langle W_C \rangle$ IN AdS|CFT: CONSIDER A FUNDAMENTAL STRING OF MINIMAL AREA LANDING ON $\mathcal{C}$ AT THE BOUNDARY OF AdS_5

$$ds^2_{AdS} = \frac{1}{z^2}\left(dz^2 + d\vec{x}^2\right)$$

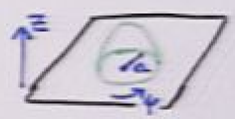
$$\langle W_C \rangle = Z_{string} \sim e^{-S_{F1}}$$

Maldacena '98
Rey-Yee '98

- CIRCULAR WILSON LOOP
  - TAKE METRIC $ds^2_{AdS} = \frac{1}{z^2}\left(dz^2 + dr_1^2 + r_1^2 d\psi^2 + dr_2^2 + r_2^2 d\phi^2\right)$

    LOOP IS AT $z=0$, $r_1 = a$, $r_2 = 0$, PARAMETRIZED BY ψ
  - MINIMAL STRING SURFACE IS

$$r_1^2 + z^2 = a^2$$



INDUCED METRIC AdS_2

- COMPUTING NAMBU-GOTO ACTION

$$S_{F1} = \frac{1}{2\pi\alpha'}\int dr\, d\psi\, \frac{r}{z^2}\sqrt{1+\partial_r z^2} = \underset{\frac{1}{\alpha'}}{\sqrt{\lambda}}\left(-1 + \frac{a}{z_{MIN}}\right)$$

CUTOFF

AFTER REGULARIZATION

$$S^{reg}_{F1} = -\sqrt{\lambda}$$

$$\rightarrow \langle W_C \rangle = e^{\sqrt{\lambda}}$$

AS THE STRONG COUPLING LIMIT OF MATRIX MODEL

- THIS WORKS FOR WILSON LOOP IN FUNDAMENTAL REPRESENTATION

  WHAT ABOUT HIGHER REPRESENTATIONS?

STRING THEORY DUAL IN AdS

- STANDARD PRESCRIPTION TO COMPUTE $\langle W_C \rangle$ IN AdS|CFT: CONSIDER A FUNDAMENTAL STRING OF MINIMAL AREA LANDING ON C AT THE BOUNDARY OF AdS_5

$$ds^2_{AdS} = \frac{1}{z^2}\left(dz^2 + d\vec{x}^2\right)$$

$$\langle W_C \rangle = Z_{string} \sim e^{-S_{F1}}$$

Maldacena '98
Rey-Yee '98

- CIRCULAR WILSON LOOP
  - TAKE METRIC $ds^2_{AdS} = \frac{1}{z^2}\left(dz^2 + dr_1^2 + r_1^2 d\psi^2 + dr_2^2 + r_2^2 d\phi^2\right)$

    LOOP IS AT $z=0$, $r_1 = a$, $r_2 = 0$, PARAMETRIZED BY ψ
  - MINIMAL STRING SURFACE IS

    $$r_1^2 + z^2 = a^2$$

INDUCED METRIC AdS_2

- COMPUTING NAMBU-GOTO ACTION

$$S_{F1} = \frac{1}{2\pi\alpha'}\int dr\, d\psi\, \frac{r}{z^2}\sqrt{1+\partial_r z^2} = \underset{\frac{1}{\alpha'}}{\sqrt{\lambda}}\left(-1 + \frac{a}{z_{MIN}}\right)$$

CUTOFF

AFTER REGULARIZATION

$$S_{F1}^{reg} = -\sqrt{\lambda}$$

$$\rightarrow \langle W_C \rangle = e^{\sqrt{\lambda}}$$

AS THE STRONG COUPLING LIMIT OF MATRIX MODEL

- THIS WORKS FOR WILSON LOOP IN FUNDAMENTAL REPRESENTATION

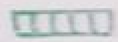
  WHAT ABOUT HIGHER - REPRESENTATIONS?

WILSON LOOPS IN HIGHER REPRESENTATIONS

- GENERAL WILSON LOOP OPERATOR

$$W_R(C) = \frac{1}{\dim R} \mathrm{Tr}_R\, P \exp \oint_C \left[i A_\mu(x) \dot{x}^\mu + |\dot{x}|\, \theta^I \Phi_I \right] d\tau$$

$R =$  FUNDAMENTAL → STRING

 SYMMETRIC

 ANTISYMMETRIC

RANK = NUMBER OF BOXES

- WHAT IS THE HOLOGRAPHIC DUAL FOR HIGHER REPRESENTATIONS?

INTUITIVELY, A RANK K REPRESENTATION WOULD CORRESPOND TO A SYSTEM OF K COINCIDENT FUNDAMENTAL STRINGS.

BETTER EFFECTIVE DESCRIPTION?

IF K IS LARGE ENOUGH $(K \sim N)$, STRINGS MAY "POLARIZE" INTO A SINGLE D-BRANE WITH K UNITS OF FUND. STRING CHARGE.

- THE D-BRANES DESCRIBING HIGHER RANK HALF-BPS LOOPS (LINE OR CIRCLE) SHOULD

  - PRESERVE HALF SUPERSYMMETRIES
  - PRESERVE SYMMETRY GROUP $\underset{AdS_2}{SO(2,1)} \times \underset{S^2}{SO(3)} \times \underset{S^4}{SO(5)}$
  - CARRY K UNITS OF ELECTRIC FLUX
  - PINCH OFF AT THE AdS BOUNDARY LANDING ON C

D-BRANES/WILSON LOOP DICTIONARY

Fiol-Drukker '05
Yamaguchi '06
Gomis-Passerini '06

LARGE RANK REPRESENTATIONS , $k \sim N$

a) SYMMETRIC REPR. $R = \underbrace{\square\square\square\square\square}_{k}$

$W_{\square\square\square}(e) \longleftrightarrow D3_k$ WRAPPING $AdS_2 \times S^2$

$\uparrow$ ELECTRIC FLUX

$S^2 \subset AdS_5$

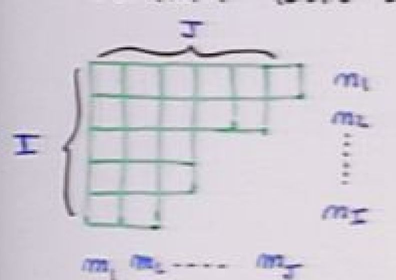
b) ANTISYMMETRIC REPR. $R = \left.\begin{array}{c}\square\\ \square\\ \square\\ \square\end{array}\right\} k$

$W_{\text{▯}}(e) \longleftrightarrow D5_k$ WRAPPING $AdS_2 \times S^4$

$S^4 \subset S^5$

- k UNITS OF ELECTRIC FLUX $\longleftrightarrow$ RANK k
- AdS_2 FACTOR $\approx$ STRING WORLDSHEET
- BLOWN UP SPHERES , RADIUS $\sim \frac{k}{N}$
  - $S^2 \subset AdS_5$ RADIUS ARBITRARILY LARGE $\longleftrightarrow R = \overbrace{\square\square\square\square\square}^{\text{ANY } k}$
  - $S^4 \subset S^5$ RADIUS BOUND BY S^5 $\longleftrightarrow R = \left.\begin{array}{c}\square\\ \square\\ \square\\ \square\end{array}\right\} k \leq N$

FOR ARBITRARY YOUNG DIAGRAMS



CAN BE DESCRIBED EITHER BY

1) A COLLECTION OF I D3 WITH FLUXES $(m_1, m_2, \ldots, m_I)$

2) A COLLECTION OF J D5 WITH FLUXES $(m_1, m_2, \ldots m_J)$

Gomis-Passerini

GIANT GRAVITONS vs "GIANT WILSON LOOPS"

• $\frac{1}{2}$-BPS CHIRAL PRIMARIES $\quad \mathcal{O}_J \sim \mathrm{Tr}\, Z^J \leftarrow \phi_1 + i\phi_2$

- $J \ll N$ SUPERGRAVITY FLUCTUATIONS
- $J \sim N$ SPHERICAL D3 BRANES ($R \times S^3$), GIANT GRAVITONS

$S^3 \subset S^5$, GIANT $\qquad\qquad S^3 \subset AdS_5$, DUAL GIANT

$\mathcal{O} \sim \mathrm{Tr}_{J} Z$  $\qquad\qquad \mathcal{O} \sim \mathrm{Tr}_{J}\, Z$ 

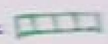
- $J \sim N^2$ NEW GEOMETRIES (BRANES BACK REACTION)
  (Lin, Lunin, Maldacena '04)

$\sim N^2$ BOXES

• $\frac{1}{2}$-BPS WILSON LOOPS $\quad W_R(\mathcal{C})$, $\mathrm{rank}(R) = k$

- $k \ll N$ FUNDAMENTAL STRING
- $k \sim N$ D-BRANES, "GIANT WILSON LOOPS"

$D5_k \quad AdS_2 \times S^4 \subset S^5$, GIANT $\qquad\qquad D3_k \quad AdS_2 \times S^2 \subset AdS_5$, DUAL GIANT

$R =$  $\qquad\qquad R =$ 

- $k \sim N^2$ NEW GEOMETRIES
  (Yamaguchi '06, Lunin '06)

THE D3-BRANE SOLUTION

Fiol - Drukker

- CONVENIENT TO CHANGE COORDINATES

$$ds^2_{AdS} = \frac{1}{z^2}\left(dz^2 + dr_1^2 + r_1^2 d\psi^2 + dr_2^2 + r_2^2 d\phi^2\right)$$

$$\to \frac{1}{\sin^2\eta}\left(d\eta^2 + \cos^2\eta \, d\psi^2 + d\rho^2 + \mathrm{sh}^2\rho\,\big(\underbrace{d\theta^2 + \sin^2\theta \, d\phi^2}_{S^2}\big)\right)$$

CIRCULAR WILSON LOOP AT BOUNDARY $\quad z=0,\ r_1=a,\ r_2=0 \ \to\ \eta=0,\ \rho=0 \quad \psi$ PARAMETRIZES THE CIRCLE

- WORLDVOLUME COORDINATES : ψ, ρ, θ, ϕ (POINT ON S^5)

ANSATZ : $\eta(\rho)$, $\underbrace{F_{\psi\rho}(\rho)}_{\text{ELECTRIC FIELD}}$

- D3-BRANE ACTION

RR 4-FORM ALONG AdS

$$S_{D3} = T_{D3}\int\sqrt{\det(\gamma + 2\pi\alpha' F)} - T_{D3}\int C_{(4)} \qquad T_{D3} = \frac{N}{2\pi^2}$$

CONSTRAINT $\quad k = -i\,\frac{\delta S_{D3}}{\delta F_{\psi\rho}} \quad$ k UNITS OF FLUX

$\to$ SOLUTION : $\quad \sin\eta = \frac{1}{\kappa}\,\mathrm{sh}\rho \qquad F_{\psi\rho} = \frac{i k \lambda}{8\pi N \,\mathrm{sh}^2\rho} \qquad \kappa = \frac{k\sqrt{\lambda}}{4N}$

FIXED

INDUCED METRIC TURNS OUT TO BE

$$ds^2_{D3} = \sqrt{1+\kappa^2}\, dH_2 + \kappa\, d\Omega_2 \qquad AdS_2 \times S^2$$

FOR $\frac{k}{N} \to 0 \quad S^2$ SHRINKS AND $D3 \to F1$

- ON-SHELL D3 ACTION
  - AS FOR STRING, THERE ARE DIVERGENCES. AFTER REGULARIZATION

$$S_{D3} = S_{DBI} + S_{WZ} + S_{bdy} = -2N\left(\kappa\sqrt{1+\kappa^2} + \sinh^{-1}\kappa\right)$$

$$\kappa = \frac{k\sqrt{\lambda}}{4N}$$

SO THE AdS/CFT PREDICTION FOR CIRCULAR LOOP IN SYMM. REPRESENTATION IS

$$\langle W_{S_k}\rangle = \exp\left[2N\left(\kappa\sqrt{1+\kappa^2} + \sinh^{-1}\kappa\right)\right]$$

- AGREES WITH MATRIX MODEL

$$\langle W_{S_k}\rangle = \frac{1}{Z}\int DM\, \frac{1}{\dim S_k}\, \mathrm{Tr}_{S_k} e^{M}\, e^{-\frac{2N}{\lambda}\mathrm{Tr}M^2} \xrightarrow[\kappa \text{ FIXED}]{N\to\infty} e^{-S_{D3}}$$

(Fiol-Drukker '05, Hartnoll-Kumar '06)

- WHEN $\frac{k}{N}\to 0$ REPRODUCES STRING RESULT

$$\langle W_{S_k}\rangle \xrightarrow[k/N\to 0]{} \exp\left(k\sqrt{\lambda}\right)$$

- S_{D3} INCLUDES AN INFINITE SUM OF NON-PLANAR $\frac{1}{N}$ CORRECTIONS (HIGHER GENUS)

$$S_{D3} = -k\sqrt{\lambda} - \frac{k^3\lambda^{3/2}}{96N^2} + \frac{k^5\lambda^{5/2}}{10240N^4} + \cdots$$

THE D5-BRANE SOLUTION

Yamaguchi '06

- CONVENIENT COORDINATES

$$ds^2 = ch^2u\,(dy^2 + sh^2y\,d\psi^2) + du^2 + sh^2u\,(d\vartheta^2 + \sin^2\vartheta\,d\phi^2) \qquad AdS_5$$
$$+ \; d\theta^2 + \sin^2\theta\,d\Omega_4^2 \qquad S^5$$

WILSON LOOP $\quad y \to \infty\,,\; u = 0 \quad$ PARAMETRIZED BY ψ

- WORLDVOLUME OF D5 : $\quad y, \psi, \Omega_4$

ANSATZ : $\quad u = 0 \qquad \theta = \text{const.} \equiv \theta_k\,, \quad F_{\psi y}(y)$

- ACTION $\qquad \to$ INDUCED METRIC $AdS_2 \times S^4$

$$S_{D5} = T_{D5}\int\sqrt{\det(g + 2\pi\alpha' F)} - 2\pi\alpha' i\,T_{D5}\int F \wedge C_{(4)} \qquad T_{D5} = \frac{N\sqrt{\lambda}}{8\pi^4}$$

SOLUTION IF $\quad \underline{\theta_k - \sin\theta_k\cos\theta_k = \frac{\pi k}{N}}$

S^5 $\quad \leftarrow S^4$ AT θ_k

$0 \leq \theta_k \leq \pi$

$0 \leq k \leq N$

AND $\quad F_{\psi y} = \frac{i\sqrt{\lambda}}{2\pi}\, sh\,y\,\cos\theta_k$

- TOTAL (REGULARIZED) ACTION

$$S_{D5} = -\frac{2N}{3\pi}\sqrt{\lambda}\,\sin^3\theta_k$$

$$\to \langle W_{A_k}\rangle = \exp\left(\frac{2N}{3\pi}\sqrt{\lambda}\,\sin^3\theta_k\right)$$

– AGREES WITH MATRIX MODEL $\left\langle \frac{1}{\dim A_k}\,\mathrm{Tr}_{A_k} e^{M}\right\rangle_{m.m.}$
(Yamaguchi '06, Hartnoll-Kumar '06)

– REPRODUCES STRING LIMIT FOR $\frac{k}{N} \to 0$

$$\langle W_{A_k}\rangle \xrightarrow[\frac{k}{N}\to 0]{} \exp(k\sqrt{\lambda})$$

O.P.E. OF GIANT WILSON LOOPS

(S.G., R. Ricci, D. Trancanelli)
0608077

- WHAT CAN WE CHECK BEYOND $\langle W_R(C)\rangle$?
  - CORRELATORS WITH LOCAL OPERATORS $\langle W_R(C)\, \mathcal{O}(x)\rangle$
- WHEN PROBED AT DISTANCES MUCH LARGER THAN ITS SIZE a, A WILSON LOOP CAN BE EXPANDED IN LOCAL OPERATORS

$$W_R(C) = \langle W_R(C)\rangle \left(1 + \sum_n c_{R,n}\, a^{\Delta_n}\, \mathcal{O}_n(0)\right)$$

DIMENSION Δ_n

- O.P.E. COEFFICIENTS $c_{R,n} \sim \dfrac{\langle W_R(C)\,\mathcal{O}_n\rangle}{\langle W_R(C)\rangle}$

- WE ARE INTERESTED IN $\frac{1}{2}$-BPS CHIRAL PRIMARIES

$$\mathcal{O}_\Delta(x) = C_{I_1 I_2 \cdots I_\Delta}\, \mathrm{Tr}\left(\phi^{I_1}\cdots\phi^{I_\Delta}\right) \qquad \text{SYMM. TRACELESS}$$

2 POINT FUNCTION $\langle \mathcal{O}_\Delta(x)\,\mathcal{O}_{\Delta'}(x')\rangle = \dfrac{\delta_{\Delta\Delta'}}{|x-x'|^{2\Delta}}$ UNIT NORMALIZED

THE O.P.E. COEFFICIENT CAN THEN BE EXTRACTED FROM

$$\frac{\langle W_R(C)\,\mathcal{O}_\Delta(L)\rangle}{\langle W_R(C)\rangle} = \underline{c_{R,\Delta}}\, \frac{a^\Delta}{L^{2\Delta}} + \cdots$$

$L \gg a$

- $c_{R,\Delta}$ CAN BE COMPUTED BOTH IN SUGRA AND FROM MATRIX MODEL AND ARE NON-TRIVIAL FUNCTIONS OF λ, N.
  - $R = \square$ — Berenstein et al '98 — SUGRA; Semenoff-Zarembo '01 — MATRIX MODEL
  - $R = \square\square\square\square$, $\begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}$ — S.G., Ricci, Trancanelli 0608077 — SUGRA AND MATRIX MODEL

CHIRAL PRIMARIES FROM SUPERGRAVITY

- WE CONSIDER $\mathcal{O}_\Delta$ WITH $\Delta \ll N$ $\longrightarrow$ SUPERGRAVITY FLUCTUATIONS

$$\begin{cases} G_{mn} = g_{mn} + h_{mn} \\ F_{(5)ijklm} = \bar{F}_{(5)ijklm} + 5\nabla_{[i}a_{jklm]} \end{cases}$$

$i, j \ldots = 1, \ldots, 10$

FLUCTUATIONS AROUND $AdS_5 \times S^5$

- CHIRAL PRIMARIES ARE DUAL TO THE LIGHTEST SCALARS $s(x)$, $M_s^2 = \Delta(\Delta-4)$ ARISING FROM K.K. COMPACTIFICATION

$$s(x,y) = \sum_I s^I(x) Y^I(y)$$

$x \in AdS$ $y \in S$

$$\nabla^\mu \nabla_\mu s^I = \Delta(\Delta-4) s^I$$

SCALAR HARMONICS ON S^5

$$\nabla^\alpha \nabla_\alpha Y^I = -\Delta(\Delta+4) Y^I$$

s^I ARE LINEAR COMBINATIONS OF KK MODES OF METRIC AND 4-FORM ALONG S^5, h^α_α $a_{\alpha\beta\gamma\delta}$ Kim et al. '85

- TURNING ON s EXCITES AT FIRST ORDER

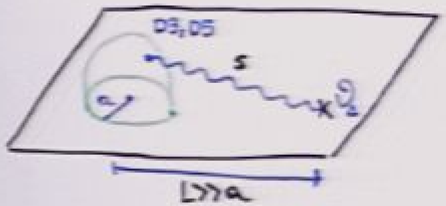
$$AdS_5 \begin{cases} h_{\mu\nu} = -\frac{6}{5}\Delta s\, g_{\mu\nu} + \frac{4}{\Delta+1}\nabla_{(\mu}\nabla_{\nu)} s \\ a_{\mu_1\mu_2\mu_3\mu_4} = -4\,\epsilon_{\mu_1\mu_2\mu_3\mu_4}\nabla^\mu s \end{cases}$$

$$s = \sum_I s^I Y^I$$

$$S^5 \begin{cases} h_{\alpha\beta} = 2\Delta s\, g_{\alpha\beta} \\ a_{\alpha_1\alpha_2\alpha_3\alpha_4} = 4\,\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha}\, s^I(x)\nabla^\alpha Y^I(y) \end{cases}$$

(Lee et al. '98)

SETUP OF SUGRA CALCULATION



EXCHANGE OF SUGRA MODES BETWEEN BOUNDARY AND D-BRANE WORLDVOLUME

STRATEGY:

- COMPUTE THE COUPLING OF s TO THE WORLDVOLUME (EXPAND AROUND $AdS_5 \times S^5$ AND USE PREVIOUS EXPRESSIONS)

$$S_{BRANE}[g+h, C_{(4)}+a_{(4)}] \longrightarrow S^{(0)}_{BRANE} + S^{(1)}_{BRANE}$$

LINEAR IN s

- WRITE FLUCTUATION AS

$$s^I(\vec{x}, z) = \int d^4 x' \, G_\Delta(\vec{x}'; \underbrace{\vec{x}, z}_{\text{ON D-BRANE}}) \, s_0^I(\vec{x}')$$

BOUNDARY SOURCE

"BULK-TO-BOUNDARY" PROPAGATOR

$$G_\Delta(\vec{x}'; \vec{x}, z) = \alpha \left(\frac{z}{z^2 + |\vec{x} - \vec{x}'|^2} \right)^\Delta$$

α: NORMALIZATION CONSTANT

- GET CORRELATION FUNCTION AS

$$\frac{\langle W_R(C) \mathcal{O}_\Delta(L) \rangle}{\langle W_R(C) \rangle} = - \left. \frac{\delta S^{(1)}_{BRANE}}{\delta s_0^I(L)} \right|_{s_0 = 0} = \underline{C_{R,\Delta}} \frac{a^\Delta}{L^{2\Delta}} + \cdots$$

FOR $L \gg a$, WE CONSISTENTLY USE APPROXIMATIONS

$$G_\Delta(L; \vec{x}, z) \approx \alpha \frac{z^\Delta}{L^{2\Delta}}$$

$$\partial_z s^I \approx \frac{\Delta}{z} s^I \qquad \partial_z^2 s^I \approx \frac{\Delta(\Delta-1)}{z^2} s^I$$

D3 BRANE CALCULATION

$R=$ ☐☐☐☐☐

- DBI FLUCTUATION (PLUG $G_{\mu\nu} = \overset{AdS_5}{g_{\mu\nu}} + h_{\mu\nu}$)

$$S_{DBI} = T_{D3}\int \sqrt{\det\left(g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + 2\pi\alpha' F_{ab}\right)} \cdot \qquad a,b=1,\dots,4$$

$$\cdot \left(1 + \frac{1}{2}\left(g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + 2\pi\alpha' F_{ab}\right)^{-1} h_{\rho\sigma}\partial_a X^\rho \partial_b X^\sigma + \dots\right)$$

LINEAR IN s^I

- W.Z FLUCTUATION (PLUG $C_{(4)} \to C_{(4)} + a_{(4)}$)

$$S^{(1)}_{WZ} = -T_{D3}\int a_{(4)} = 4T_{D3}\Delta\int C_{(4)}\, s$$

- USING THE EXPLICIT D3 SOLUTION, THE SUM OF DBI AND WZ GIVES

$$S^{(1)}_{D3} = S^{(1)}_{DBI} + S^{(1)}_{WZ} = -4N\Delta\int_0^{\mathrm{sh}^{-1}\kappa} d\rho \int_0^{\pi} d\theta\, \frac{\sin\theta}{(\mathrm{ch}\rho - \mathrm{sh}\rho\cos\theta)^2}\cdot s$$

DIFFERENTIATING W.R.T. SOURCE $s_0(L)$ AND INTEGRATING WE GET

$$\boxed{C_{S_k,\Delta} = \frac{2^{\Delta/2+1}}{\sqrt{\Delta}}\,\mathrm{sh}\left(\Delta\,\mathrm{sh}^{-1}\kappa\right)} \qquad \kappa = \frac{k\sqrt{\lambda}}{4N}$$

- FOR $\frac{k}{N}\to 0$ ($\kappa\to 0$) REPRODUCES THE KNOWN RESULT FOR FUNDAMENTAL REPRESENTATION

$$C_{S_k,\Delta} \xrightarrow[k\to 0]{} 2^{\Delta/2-1}\frac{\sqrt{\Delta\lambda}}{N}k$$

Berenstein et al '98

D5 BRANE CALCULATION

R = 

- PROCEED SIMILARLY AS D3, BUT WE ALSO NEED FLUCTUATIONS ALONG S^5
WE GET

$$S^{(1)}_{DBI} = \frac{N\sqrt{\lambda}}{3\pi}\int dj\, \mathrm{sh}j\, \sin^5\theta_k \left(\frac{-4\Delta}{\mathrm{ch}^2 j \sin^2\theta_k} + 8\Delta\right) s^\Delta Y^{\Delta,0}(\theta_k)$$

$$S^{(1)}_{WZ} = \frac{8N\sqrt{\lambda}}{3\pi}\int dj\, \mathrm{sh}j\, \sin^4\theta_k \cos\theta_k\, s^\Delta \partial_{\theta_k} Y^{\Delta,0}(\theta_k)$$

RECALL: θ_k LATITUDE AT WHICH $S^4 \subset S^5$ SITS

$$\theta_k - \sin\theta_k \cos\theta_k = \frac{\pi k}{N}$$

AND $\quad Y^{\Delta,0}(\theta_k) = \int_{S^4} \mu(\Omega_4) Y^I \quad$ SO(5) INVARIANT HARMONIC

$$\underline{Y^{\Delta,0}(\theta_k) = \mathcal{N}_\Delta C^{(2)}_\Delta(\cos\theta_k)}$$ GEGENBAUER POLYNOMIALS

$$\left[(1-x^2)\partial_x^2 - 5x\partial_x\right] C^{(2)}_\Delta(x) = -\Delta(\Delta+4) C^{(2)}_\Delta(x)$$

- TAKING $\frac{\delta}{\delta S_0}$, INTEGRATING OVER J AND USING RECURRENCE RELATIONS FOR GEGENBAUER $C^{(2)}_\Delta(x)$, THE FINAL RESULT IS

$$\boxed{C_{A_k,\Delta} = \frac{2^{\Delta/2}}{3\pi}\sqrt{\Delta\lambda}\, \sin^3\theta_k \frac{6(\Delta-2)!}{(\Delta+1)!} C^{(2)}_{\Delta-2}(\cos\theta_k)}$$

– CAN CHECK THAT FOR $\frac{k}{N}\to 0$ REDUCES TO STRING RESULT

$$C_{A_k,\Delta} \xrightarrow[\frac{k}{N}\to 0]{} 2^{\Delta/2-1}\frac{\sqrt{\Delta\lambda}}{N} k$$

MATRIX MODEL CALCULATION

- RECALL HERMITEAN MATRIX MODEL

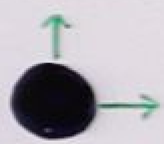
$$\langle W_R(e)\rangle = \frac{1}{Z}\int DM \, \frac{1}{\dim R}\mathrm{Tr}_R e^{M} \, e^{-\frac{2N}{\lambda}\mathrm{Tr} M^2}$$

- AN EQUIVALENT COMPLEX MATRIX MODEL WAS PROPOSED BY OKUYAMA AND SEMENOFF (0604209)

$$\langle W_R(e)\rangle = \frac{1}{Z_n}\int_{[Z,\bar Z]=0} [d^2 Z] \, e^{-\frac{2N}{\lambda}\mathrm{Tr} Z\bar Z} \, \frac{1}{\dim R}\mathrm{Tr}_R e^{\frac{1}{\sqrt{2}}(Z+\bar Z)-\frac{\lambda}{8N}}$$

"NORMAL MATRIX MODEL"

- AT LARGE N, EIGENVALUES ARE UNIFORMLY DISTRIBUTED IN INCOMPRESSIBLE DROPLETS IN THE COMPLEX PLANE



FUNDAMENTAL REPRESENTATION :
GAUSSIAN POTENTIAL
$\rightarrow$ CIRCULAR DROPLET

- FREE FERMION INTERPRETATION
- ANALOGY WITH MATRIX MODEL DESCRIPTION OF CHIRAL PRIMARIES AND $\frac{1}{2}$-BPS GEOMETRIES

- SUITABLE TO COMPUTE CORRELATORS WITH CHIRAL PRIMARIES

$$\langle W_\square(e)\, \mathcal{O}_\Delta\rangle = \frac{2^{\Delta/2}}{Z_n}\int_{[Z,\bar Z]=0} [d^2 Z] \, e^{-\mathrm{Tr} Z\bar Z} \, \frac{1}{N}\mathrm{Tr}\, e^{\frac{1}{2}\sqrt{\frac{\lambda}{N}}(Z+\bar Z)-\frac{\lambda}{8N}} \, \underbrace{\frac{\mathrm{Tr} Z^\Delta}{\sqrt{\Delta N^\Delta}}}_{\text{CHIRAL PRIMARY}}$$

SYMMETRIC REPRESENTATION

- IT TURNS OUT THAT AT LARGE N, λ (AND $k \sim N$), SYMMETRIC REPR. IS EQUIVALENT TO MULTIPLE K-WINDING (Hartnoll-Kumar)

$$\langle W_{\square\square\square}(C) \rangle \simeq \langle W_{\square}^{(k)}(C) \rangle$$

SO D3 BRANE DESCRIBES BOTH

- MULTIPLY WOUND LOOP IS EASIER IN MATRIX MODEL. SIMPLY TAKE $R = \square$ WITH $\lambda \to k^2 \lambda$

$$\langle W_{\square}^{(k)} \mathcal{O}_\Delta \rangle = \frac{2^{\Delta/2}}{Z_m} \int_{[Z,\bar{Z}]=0} [d^2 Z]\, e^{-\mathrm{Tr}\, Z\bar{Z}}\, \frac{1}{N} \mathrm{Tr}\, e^{\frac{k}{2}\sqrt{\frac{\lambda}{N}}(Z+\bar{Z}) - \frac{\lambda k^2}{8N}}\, \frac{\mathrm{Tr}\, Z^\Delta}{\sqrt{\Delta N^\Delta}}$$

- THIS CAN BE SOLVED EXACTLY TO GIVE

$$\langle W_{\square}^{(k)} \mathcal{O}_\Delta \rangle = \frac{2^{\Delta/2}}{N\kappa\sqrt{\Delta}}\, e^{2N\kappa^2} \oint \frac{dw}{2\pi i}\, e^{2N\kappa w} \left(1 + \frac{2\kappa}{w}\right)^N \left[(2\kappa + w)^\Delta - w^\Delta\right]$$

$$\kappa = \frac{k\sqrt{\lambda}}{4N}$$

- SADDLE POINT AT LARGE N, $\frac{k\sqrt{\lambda}}{4N}$ FIXED

$$\partial_w \left(2\kappa w + \ln\left(1 + \frac{2\kappa}{w}\right)\right) = 0 \;\to\; w_* = \sqrt{1+\kappa^2} - \kappa$$

- EVALUATING AT w_*

$$\frac{\langle W_{\square}^{(k)} \mathcal{O}_\Delta \rangle}{\langle W_{\square}^{(k)} \rangle} = \frac{2^{\Delta/2+1}}{\sqrt{\Delta}}\, \mathrm{sh}\left(\Delta\, \mathrm{sh}^{-1} \kappa\right)$$

$\exp\left[2N\left(\kappa\sqrt{1+\kappa^2} + \mathrm{sh}^{-1}\kappa\right)\right]$

EXACT AGREEMENT WITH D3 BRANE!

ANTISYMMETRIC REPRESENTATION

- NEED TO EVALUATE THE CORRELATOR

$$\langle W_{A_k} \mathcal{O}_\Delta \rangle = \frac{2^{\Delta/2} e^{k\lambda/8N}}{Z_m} \int_{[Z,\bar{Z}]=0} [d^2 Z]\, e^{-\mathrm{Tr}\, Z\bar{Z}} \frac{1}{\dim A_k} \mathrm{Tr}_{A_k} e^{\frac{1}{2}\sqrt{\frac{\lambda}{N}}(Z+\bar{Z})}\, \mathrm{Tr}\, Z^\Delta$$

– USE GENERATING FUNCTION FOR Tr_{A_k}

$$\mathrm{Tr}_{A_k} e^{\frac{1}{2}\sqrt{\frac{\lambda}{N}}(Z+\bar{Z})} = \oint \frac{dt}{2\pi i} t^{k-1} \underbrace{\exp\left[\mathrm{Tr}\log\left(1+\frac{1}{t} e^{\frac{1}{2}\sqrt{\frac{\lambda}{N}}(Z+\bar{Z})}\right)\right]}_{\equiv F_A(t,Z)}$$

- VIEW THE INSERTION OF CHIRAL PRIMARY AS A SMALL PERTURBATION OF GAUSSIAN POTENTIAL

$$\int_{[Z,\bar{Z}]=0} [d^2 Z]\, e^{-\mathrm{Tr}\, Z\bar{Z}}\, \mathrm{Tr}\, Z^\Delta \xrightarrow{} \tfrac{1}{2}(\mathrm{Tr}\, Z^\Delta + \mathrm{Tr}\, \bar{Z}^\Delta)\; F_A(t,Z) = \frac{\partial}{\partial\alpha} \int_{[Z,\bar{Z}]=0} [d^2 Z]\, e^{-\mathrm{Tr}\, Z\bar{Z} + \frac{\alpha}{2}\mathrm{Tr}(Z^\Delta + \bar{Z}^\Delta)} F_A(t,Z) \Big|_{\alpha=0}$$

$$= \frac{\partial}{\partial\alpha} \langle F_A(t,Z) \rangle_\alpha \Big|_{\alpha=0}$$

EVALUATE CORRELATOR IN A DEFORMED MATRIX MODEL WITH POTENTIAL

$$V(Z,\bar{Z}) = -\mathrm{Tr}\, Z\bar{Z} + \frac{\alpha}{2}\mathrm{Tr}(Z^\Delta + \bar{Z}^\Delta)$$

- AT LARGE N, EIGENVALUE DENSITY GIVEN BY SADDLE POINT EQ.

$$z - \frac{\Delta\alpha}{2}\bar{z}^{\Delta-1} = N\int d^2z' \frac{\rho_\alpha(z',\bar{z}')}{\bar{z}-\bar{z}'}$$

FROM VANDERMONDE FACTOR

ρ_α IS CONSTANT INSIDE A DEFORMED DROPLET AND ZERO OUTSIDE

$\alpha=0$. GAUSSIAN RADIUS$=\sqrt{N}$

DEFORMED DROPLETS

WE WANT TO FIND THE CURVE WHICH BOUNDS THE DROPLET.
AT FIRST ORDER IN α

$$r(\phi) = \sqrt{N}\left(1+\alpha f(\phi)\right) \qquad z = re^{i\phi}$$

PERIODICITY, $z \leftrightarrow \bar{z}$ SYMMETRY, AND AREA CONSERVATION RESTRICT

$$f(\phi) = \sum_{m=1}^{\infty} a_m \cos m\phi$$

PLUGGING INTO SADDLE POINT EQUATION, WE FIND

$$\underline{r(\phi) = \sqrt{N}\left(1+\frac{\alpha}{2}\Delta N^{\Delta/2-1}\cos\Delta\phi\right)}$$

• THE CORRELATOR

$$\langle F_A(t,z)\rangle_\alpha \to \exp\left[N\int d^2z\, \underbrace{\rho_\alpha(z)}_{\text{DENSITY FOR DEFORMED DROPLET}} \log\left(1+\frac{1}{t}e^{\frac{1}{2}\sqrt{\frac{\lambda}{N}}(z+\bar z)}\right)\right]$$

$$\langle W_{A_k}\mathcal{O}_\Delta\rangle \sim \oint \frac{dt}{2\pi i}\, t^{k-1}\,\frac{\partial}{\partial\alpha}\langle F_A(t,z)\rangle_\alpha\Big|_{\alpha=0}$$

- t INTEGRAL CAN BE EVALUATED BY SADDLE POINT AT LARGE λ

$$t = e^{\sqrt{\lambda}w} \qquad w_* = \cos\theta_k \qquad \left(\theta_k - \sin\theta_k\cos\theta_k = \frac{\pi k}{N}\right)$$

AS D5

FINAL RESULT

$$\frac{\langle W_{A_k}\mathcal{O}_\Delta\rangle}{\langle W_{A_k}\rangle} = \frac{2^{\Delta/2}}{\sqrt{\Delta}N^{\Delta/2}}\frac{\partial}{\partial\alpha}\left[N\int d^2z\,\rho_\alpha \log\left(1+e^{\frac{1}{2}\sqrt{\lambda}\left(\frac{z+\bar z}{\sqrt{N}}-2\cos\theta_k\right)}\right)\right]_{\alpha=0}$$

$\frac{r}{\sqrt{N}}\cos\phi \geq \cos\theta_k$

$$= \frac{2^{\Delta/2}\sqrt{\Delta\lambda}}{\pi}\int_0^{\theta_k} d\phi\,\cos\Delta\phi\,(\cos\phi-\cos\theta_k)$$

$$= \frac{2^{\Delta/2}}{3\pi}\sqrt{\Delta\lambda}\,\sin^3\theta_k\,\frac{6(\Delta-2)!}{(\Delta+1)!}\,C^{(2)}_{\Delta-2}(\cos\theta_k)$$

EXACTLY AS THE D5 CALCULATION !

CONCLUSIONS

- NEW D-BRANE / WILSON LOOP DICTIONARY
  "GIANT WILSON LOOPS"
- COMPUTED WILSON LOOP / CHIRAL PRIMARY CORRELATORS IN BOTH SUGRA AND GAUGE THEORY.
  NON-TRIVIAL TESTS OF
  - GIANT WILSON LOOP DICTIONARY
  - MATRIX MODEL CONJECTURE FOR CIRCULAR LOOPS
  - AdS / CFT
- FUTURE DIRECTIONS
  - CORRELATORS WITH GIANT GRAVITONS ?
  - LESS SUPERSYMMETRIC WILSON LOOPS
    e.g. $\frac{1}{4}$-BPS CIRCULAR LOOPS (Drukker '06)
    D3 / D5 BRANES FOR THESE LOOPS ?
    (WORK IN PROGRESS WITH DRUKKER, RICCI, TRANCANELLI)
    GENERAL CLASSIFICATIONS FOR $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$-BPS LOOPS ?
    (AS Mikhailov '00 FOR GIANT GRAVITONS)