Title: Children's Drawings From Seiberg-Witten Curves

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Abstract:

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Eleonora Dell'Aquila
Rutgers University

based on hep-th/0611082, with Sujay Ashok and Freddy Cachazo

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Outline

- A physics problem: phases of $\mathcal{N}=1$ vacua
- A math problem: classification of children's drawings into Galois orbits
- Dictionary
- Conjectures
- Questions and future directions

Classification of SUSY vacua

[Cachazo, Seiberg, Witten - hep-th/0301006] consider $\mathcal{N}=2$ $U(N)$ Super-Yang-Mills consider $\mathcal{N}=2$ $U(N)$ pure Super-Yang-Mills

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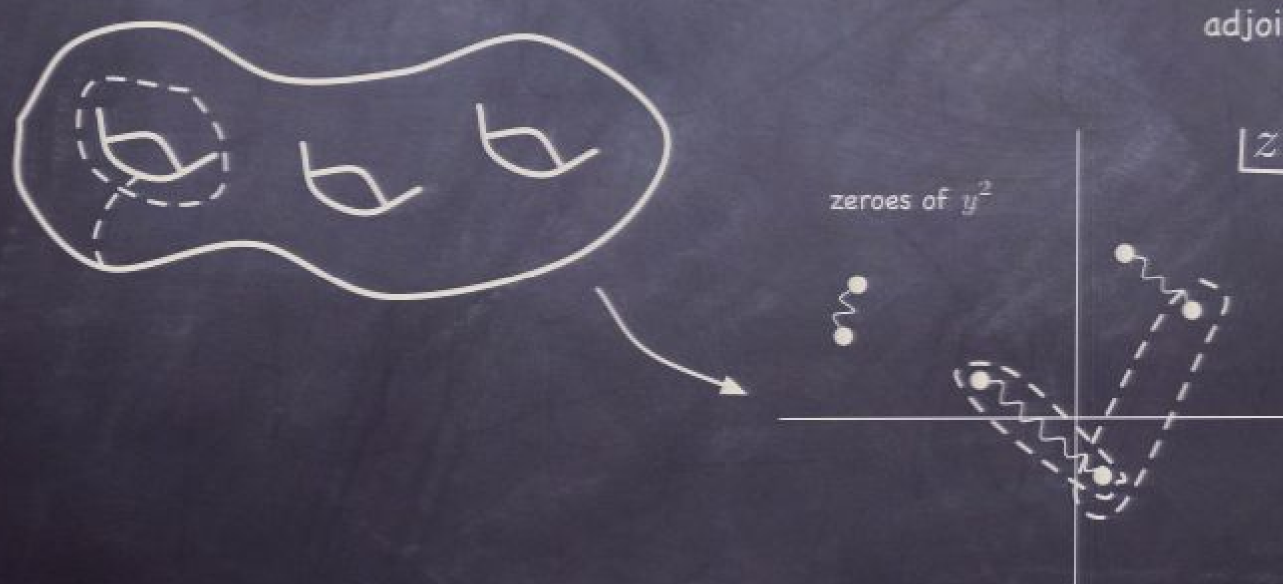
adjoint scalar



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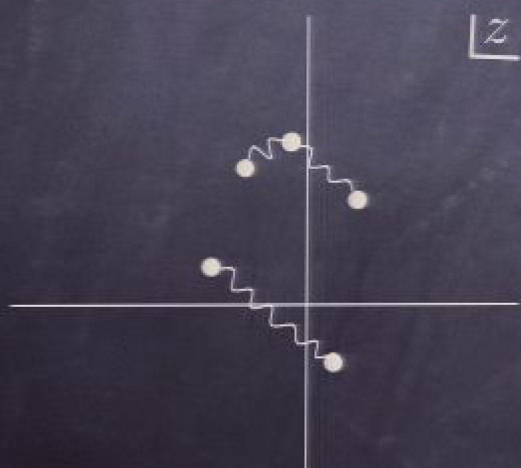

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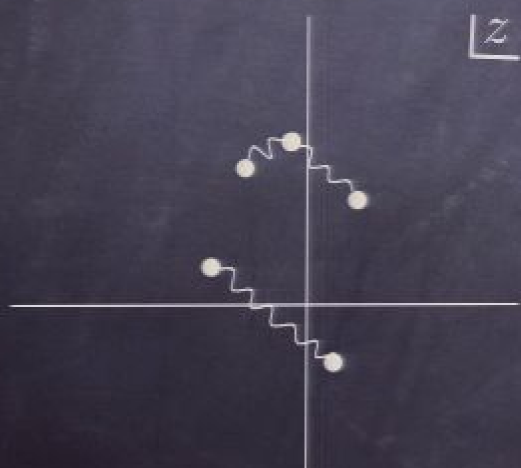

- cycles shrink to zero size
- charged particles becomes massless

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× points where an additional monopole becomes massless

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we can tune the parameters to obtain rigid factorizations at special points

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semiclassically $U(N) \to U(N_1) \times \ldots \times U(N_n)$

relative theta angle of $U(N_i)$ and $U(N_{n+1})$

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More order parameters? Probably, yes.

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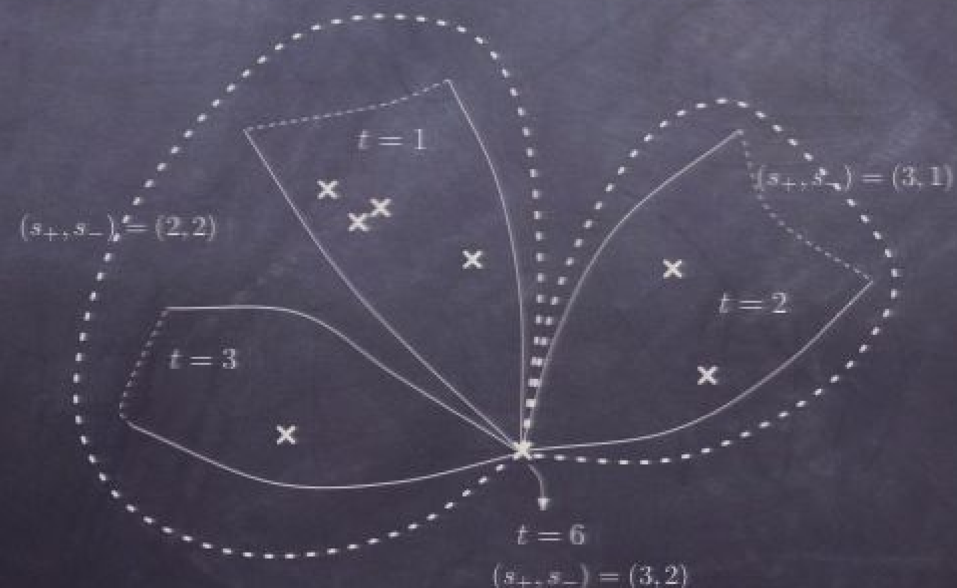

Summary so far:

- consider a $U(N)$ $\mathcal{N}=2$ theory without matter
- break to $\mathcal{N}=1$ adding a superpotential $W_{\rm tree}$
- the $\mathcal{N}=1$ vacua correspond to particular factorizations of the S-W curve, which depend on the parameters of $W_{\rm tree}$
  $\longrightarrow$ moduli space of vacua
- the modulispace is composed of different branches, distinguished by order parameters
- a complete list of order parameters is not known

Dessins d’Enfants

Galois groups

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This is the traditional approach to Galois theory, based on permutation groups. The modern approach is based on the theory of fields.

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the corresponding Galois group is $\mathrm{Gal}(\mathbf{Q}(2^{1/4}, \mathrm{i})/\mathbf{Q}) = \mathrm{D}_4$

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In 1983 Grothendieck introduced the "dessins d'enfants" ("children's drawings") as a set on which the absolute Galois group acts faithfully. The goal of the programme proposed by Grothendieck is to understand how such "dessins" are organized in orbits under the action of the Galois group.

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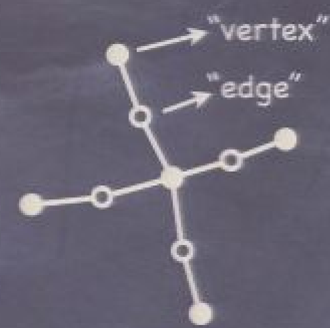

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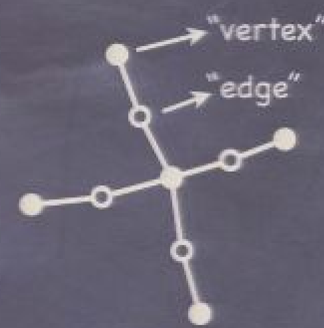

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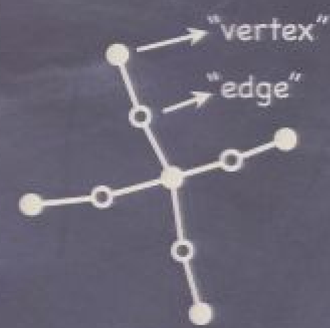

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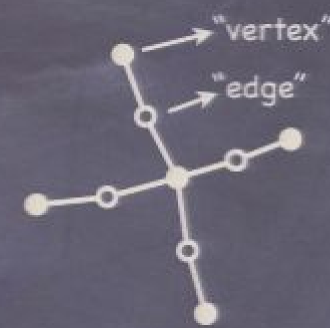

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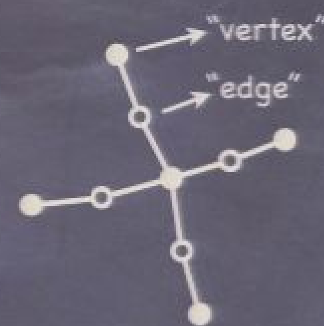

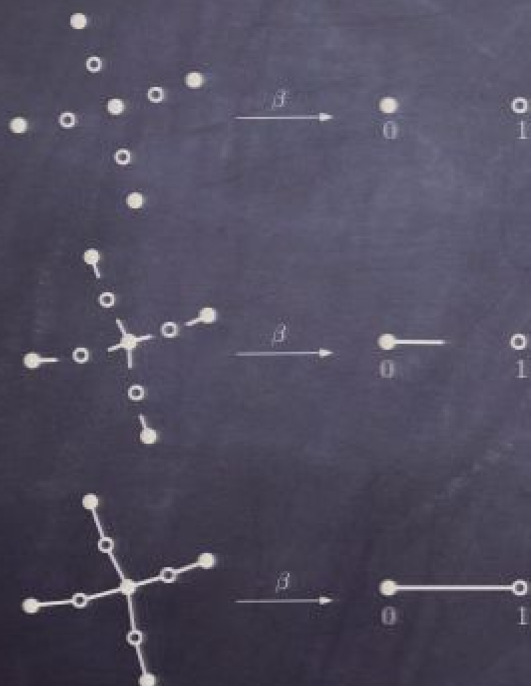

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The hope is to be able to characterize the action of the Galois group by combinatorial data associated to the dessins.

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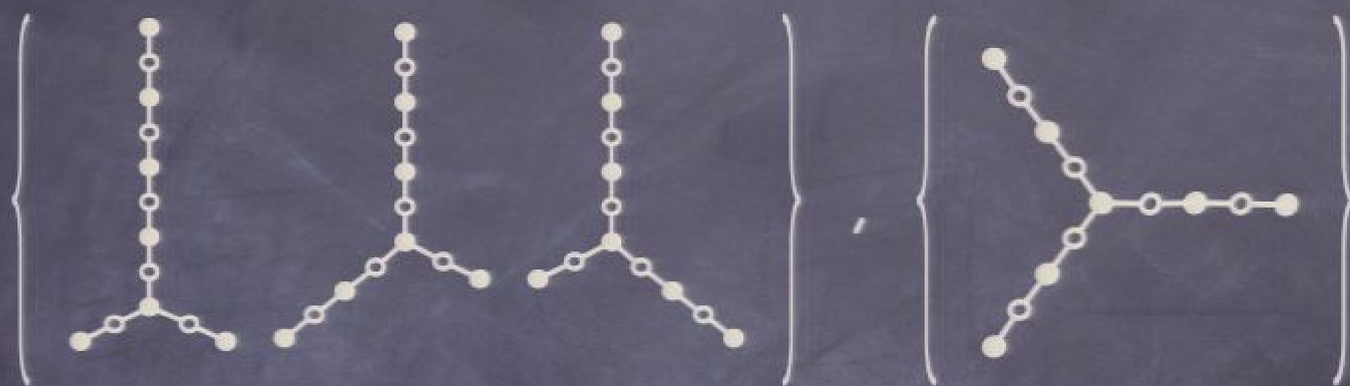

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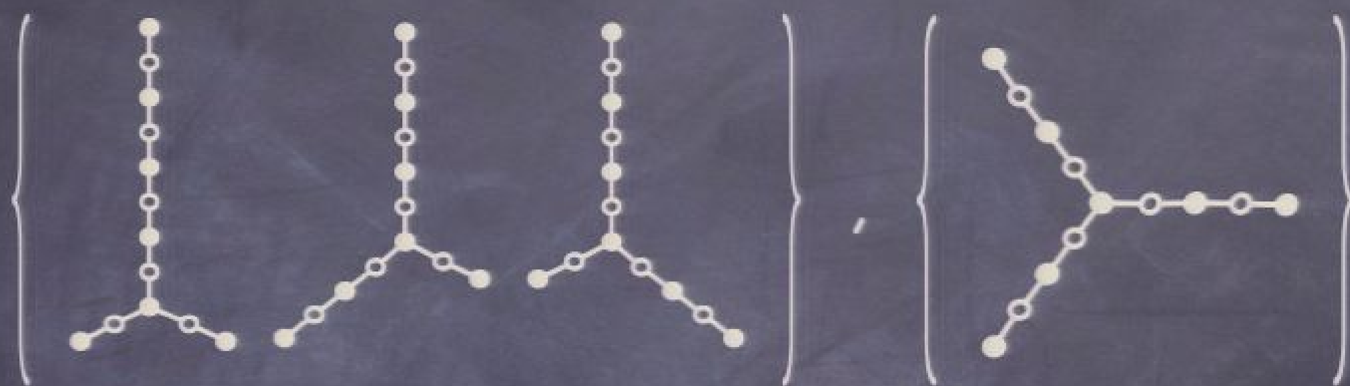

How? ideally one would need a complete set of Galois invariants, such that if two dessins differ in the value of any of the invariants then the dessins are in different orbits.

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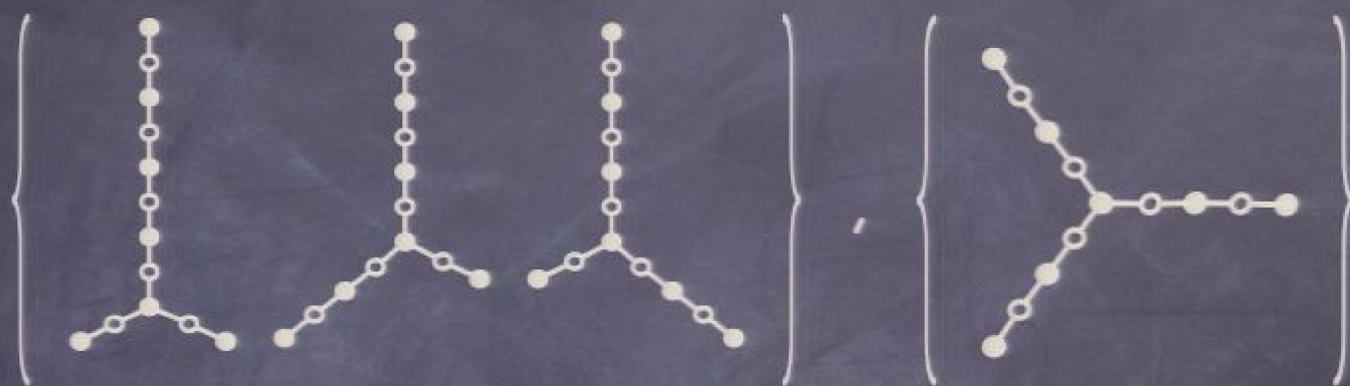

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A full classification of dessins in orbits would provide information about the representations of the absolute Galois group $\mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q})$, but a complete list of invariants is not known.

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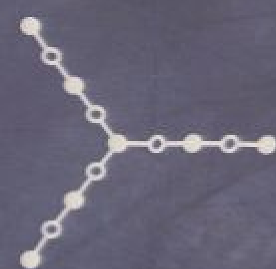

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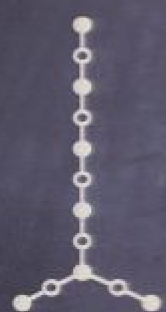

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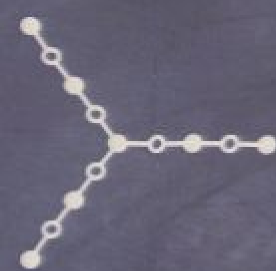

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⟶ We can't tell from the valency list that these dessins belong to different Galois orbits.

Galois invariants:

valency list: $V = \{u_1, \ldots, u_k\}$ $u_i = \#$ vertices with valence i

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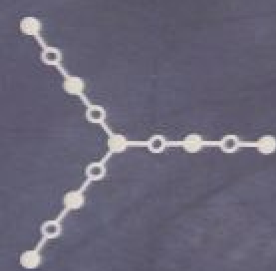

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We need a more refined invariant

refined valency list: (only well defined for trees)

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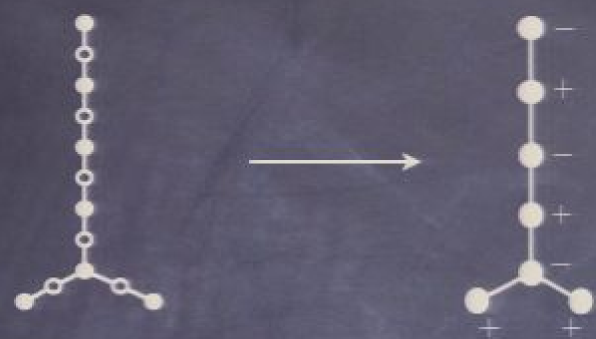

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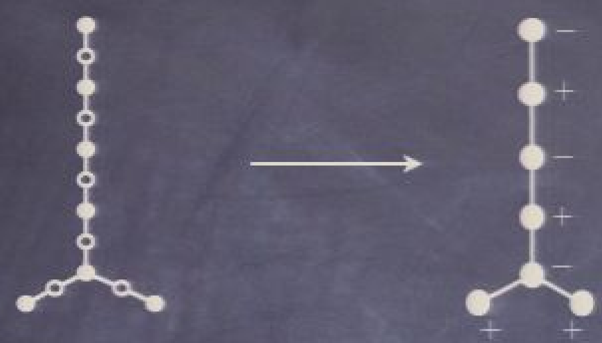
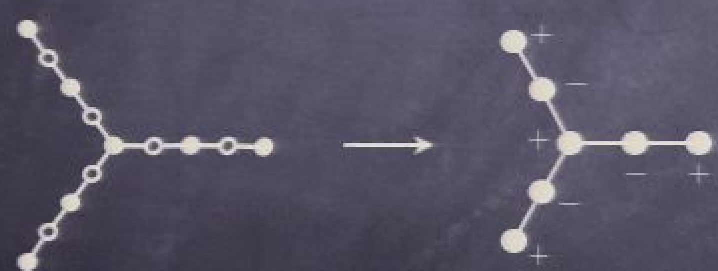


$$V_+ = \{2, 2, 0\}$$
$$V_- = \{1, 1, 1\}$$



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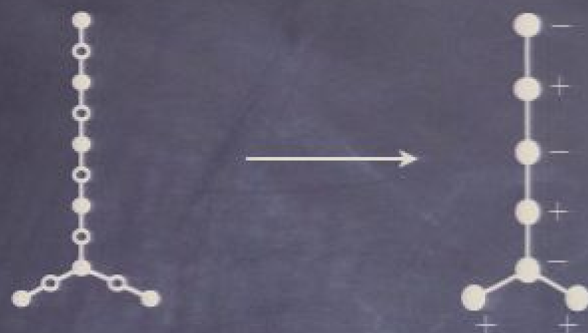

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The refined valency list are different and this is enaough to decide that the dessins belong to different orbits.

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monodromy group:

permutation group generated by

$\sigma_+ \longrightarrow$ cyclic permutations of edges around + vertices

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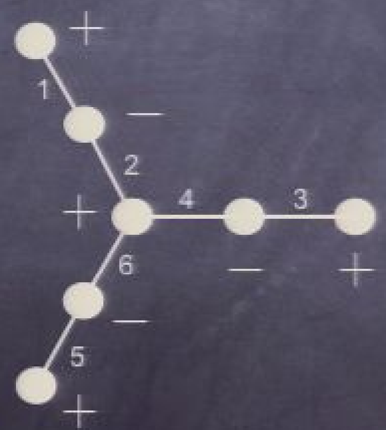
ex:



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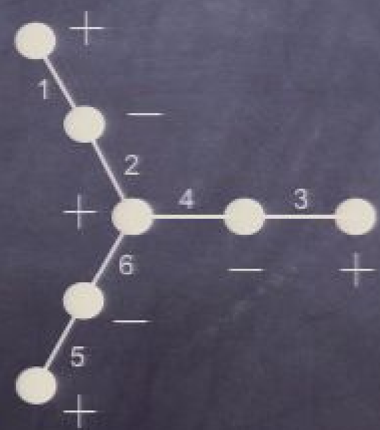

$$\sigma_+ = (6,4,2)$$

$$\sigma_- = (1,2)(3,4)(5,6)$$

$$M = \mathbf{Z}_3 \times S_3$$

and more invariants...

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- some example of invariants: valency lists, monodromy group, etc...

Dictionary

Dictionary

Classification of branches of $\mathcal{N} = 1$ vacua ⟷ Organization of dessins into Galois orbits

Rigid S-W curves / Belyi maps

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let's go back to the $U(6)$ example

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Rigid S-W curves / Belyi maps

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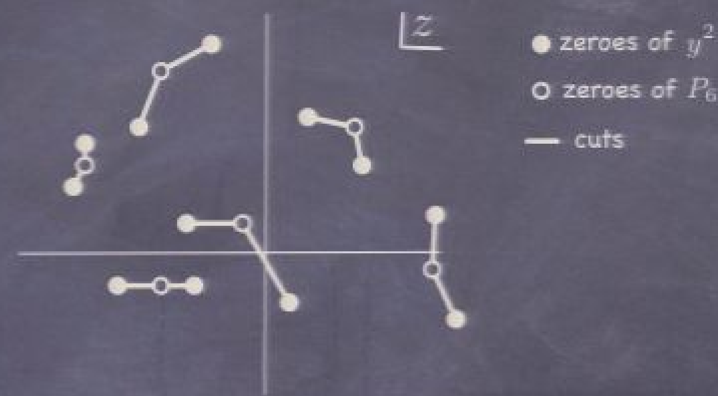

Rigid S-W curves / Belyi maps

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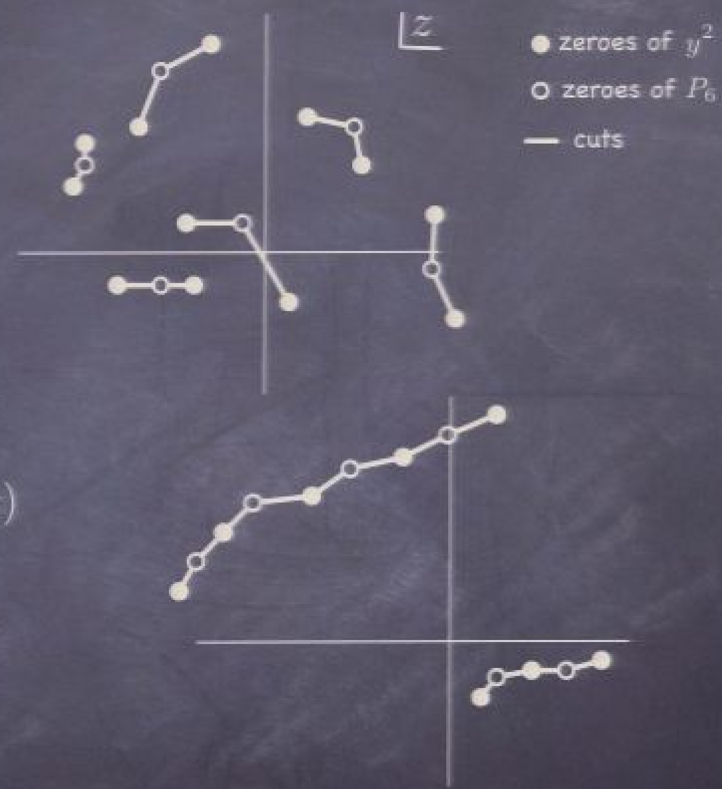

Rigid S-W curves / Belyi maps

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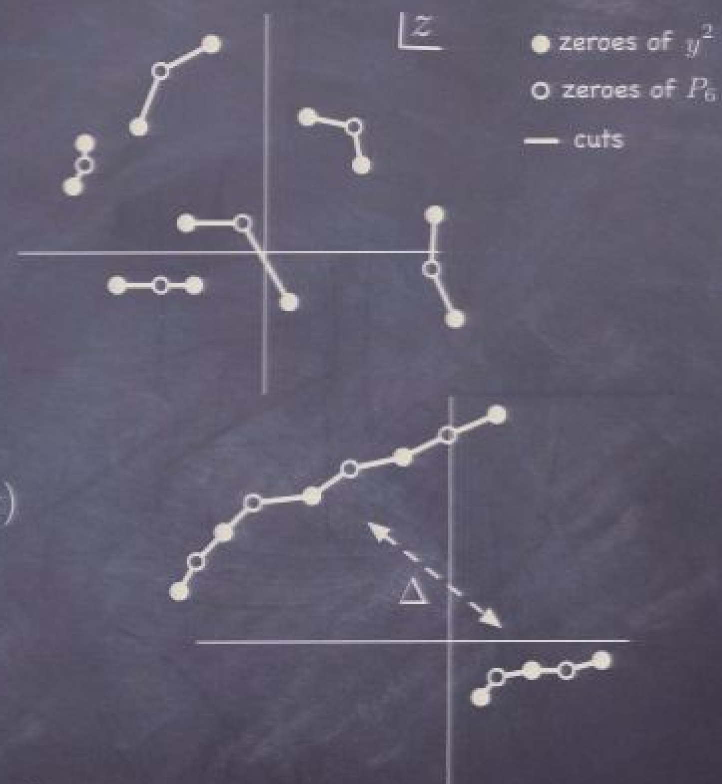

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(1 free parameter)

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$$P_6^2(z) - 4\Lambda^{12} = F_3(z) Q_1(z)^3 H_3^2(z)$$

(rigid)

Rigid S-W curves / Belyi maps

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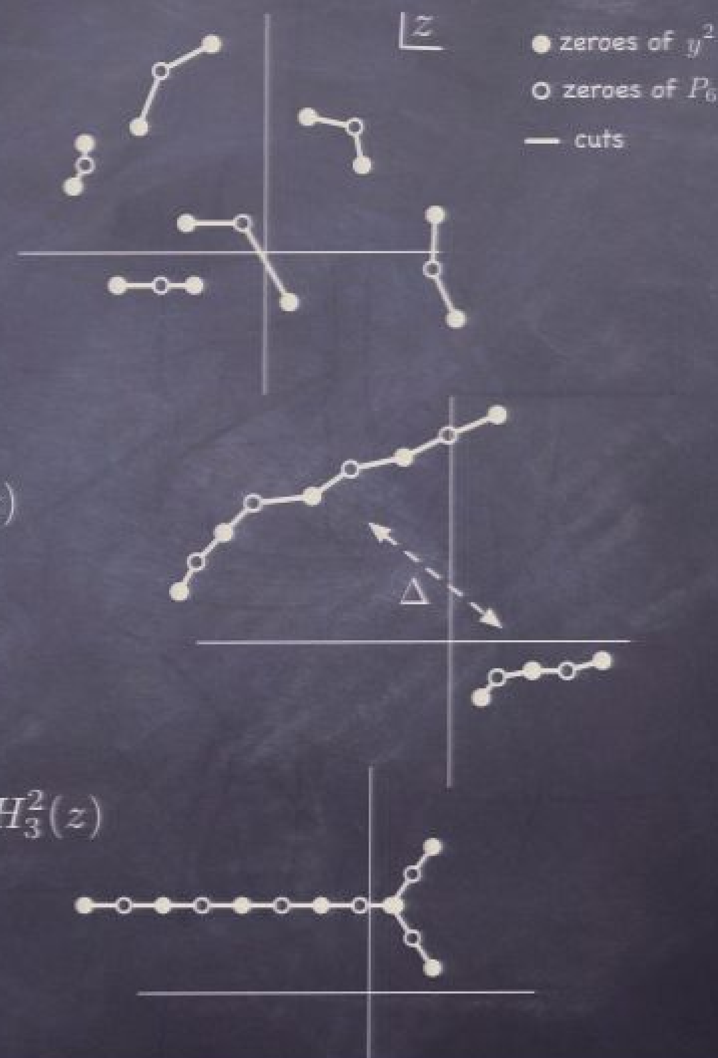

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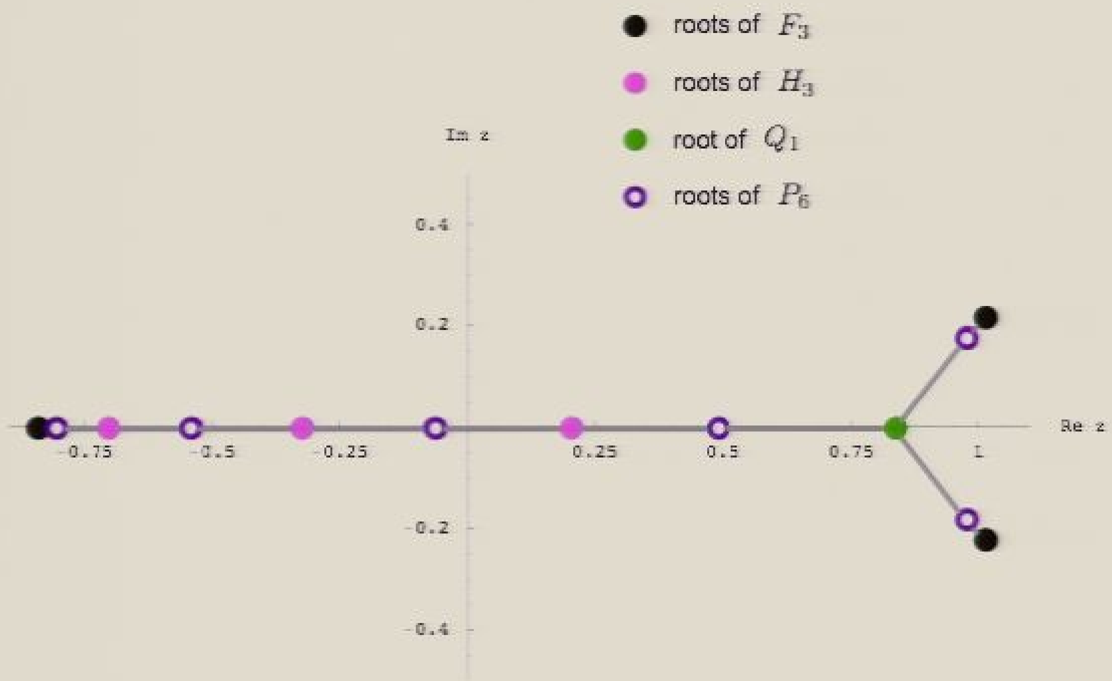
$$P_6^2(z) - 4\Lambda^{12} = F_3(z)\, H_3^2(z)\, Q_1^3(z)$$



Claim: $\beta \equiv -\frac{1}{4\Lambda^{12}}(P_6^2 - 4\Lambda^{12})$ with the factorization condition

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- it is a holomorphic map from the complex sphere to itself
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More generally:

There is a one-to-one correspondence between rigid S-W curves and Belyi maps, so it is possible to associate a dessin to every point in the moduli space where a rigid curve appears

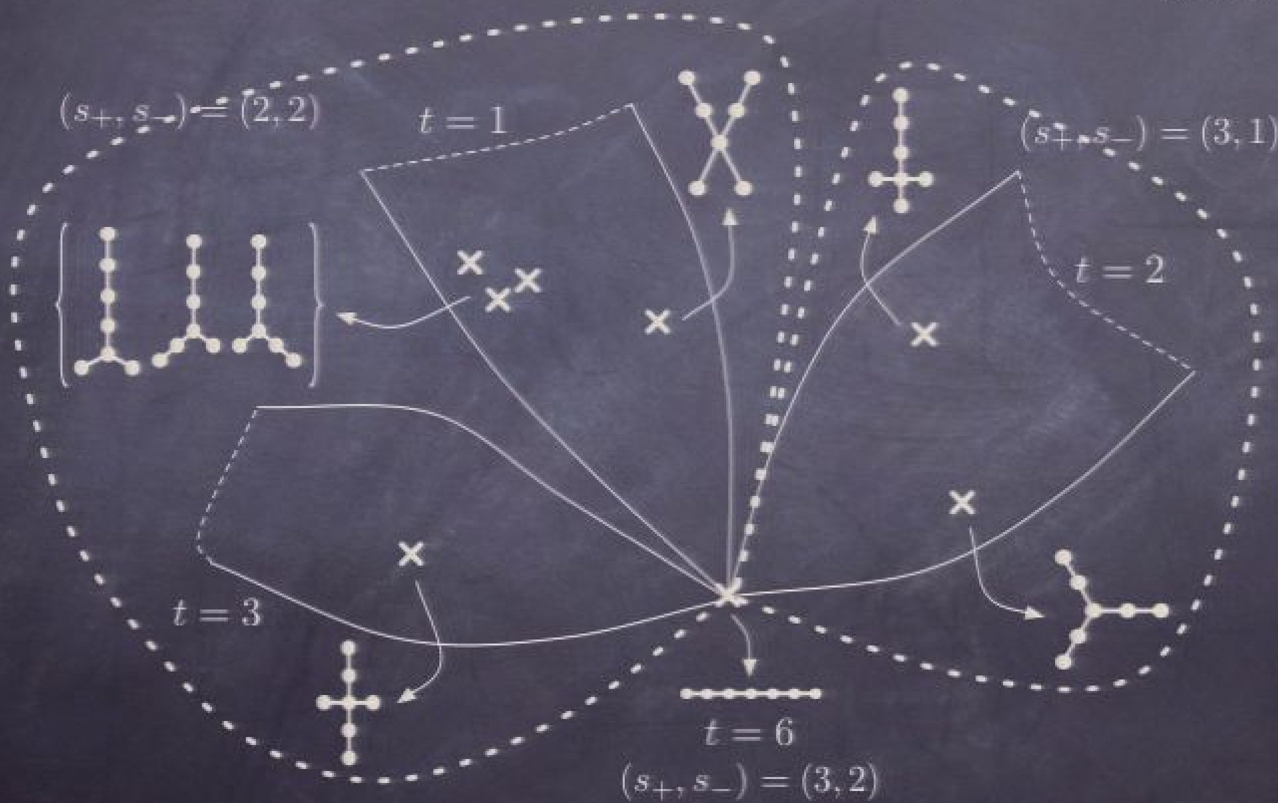
$U(6)$ example:

$$y^2 = P_6^2(z) - 4\Lambda^{12} = F_4(z) H_4^2(z)$$

$$\to P_6^2(z) - 4\Lambda^{12} = F_3(z) Q_1^3(z) H_3^2(z)$$

$$\to P_6^2(z) - 4\Lambda^{12} = F_2(z) H_5^2(z)$$

$$\to P_6^2(z) - 4\Lambda^{12} = F_4(z) Q_1^4(z) H_2^2(z)$$



Dictionary

gauge theory | children's drawings

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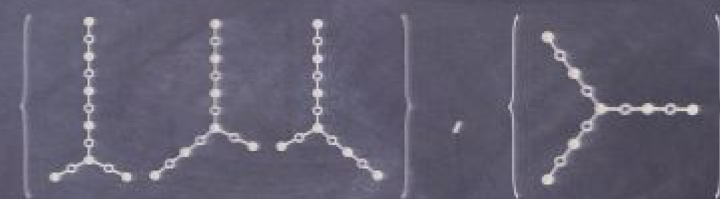
form of the rigid factorization

ex: $P_6^2(z) - 4\Lambda^{12} = F_3(z)Q_1^3(z)H_3^2(z)$ → $V = \{3, 3, 1\}$

valency list



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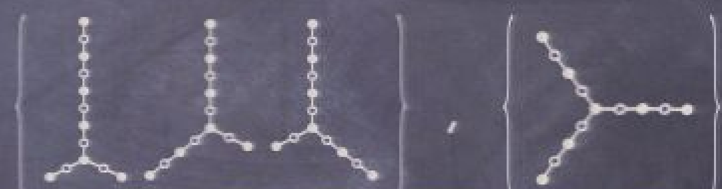

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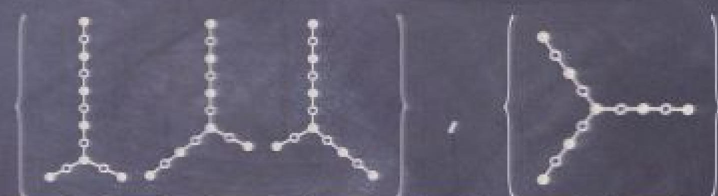

holomorphic invariants	refined valency list

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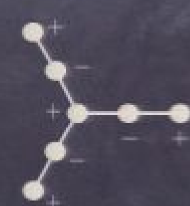
$$\begin{cases} P_6(z) - 2\Lambda^6 = \tilde{R}_{6-2s_-}(z)\tilde{H}^2_{s_-} \\ P_6(z) + 2\Lambda^6 = \tilde{R}_{6-2s_+}(z)\tilde{H}^2_{s_+} \end{cases}$$

←



$V_+ = \{3,0,1\}$

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gauge theory | children's drawings

gauge theory

confinement index

children's drawings

(new?) Galois invariant

$t = 1$

$t = 2$

gauge theory	children's drawings
confinement index	(new?) Galois invariant
multiplication map	Belyi extending maps
?	monodromy group

Conjectures

(and conclusion)

conjecture (strong form): Every Galois invariant is a physical order parameter

consequences:

- the monodromy group and other known Galois invariants would be order parameters
- especially useful for theories with matter, for which no order parameters are known

conjecture (weak form): if two dessins appear at points that belong to different branches, then they belong to different Galois orbits.