Title: Similarities Between Maximally Supersymmetric Gauge and Gravity Theories

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Abstract: Due to recent, as well as less recent, work on perturbative N=8 supergravity and N=4 super Yang-Mills in 4d, the two theories are appearing more and more closely related. These relations include similar \"MHV-rule\" constructions, one-loop structure and, perhaps, the same UV behavior, namely UV finiteness. This talk introduces some of the methods to study the relations.

About the Talk

- On the surface, it'll be about SYM/SUGRA duality.
- But it's also really about the methods underlying it. These methods are quite general.
- I'll focus particularly on how scaling behaviour of tree amplitudes and on-shell recursion lie behind most of the insight.

Contents

1. Preliminaries
2. MHV Constructions
3. One-Loop Structure
4. All-Loops, Conclusion, Outlook, *etc.*

Contents

$\mathcal{N} = 4$ Super-Yang–Mills

- The maximally supersymmetric gauge theory in 4d
- Contains one vector, four spin-$\frac{1}{2}$ fermions, and 3 complex scalars; all in the adjoint.
- Low-energy limit of the compactified open superstring.
- Has superconformal symmetry
- UV finite.

$\mathcal{N} = 8$ Supergravity

In general:

- The maximally supersymmetric gravity theory in 4d
- Contains one graviton, eight spin-$\frac{3}{2}$ fermions, 28 vectors, 56 spin-$\frac{1}{2}$ fermions, and 35 complex scalars.
- Dim. reduction of $\mathcal{N} = 1$ SUGRA in 11d, and low-energy limit of the compactified closed superstring.

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Perturbation theory:

- Feynman vertices go as $(\text{momentum})^2$, all order vertices.
- Dimensionful coupling constant.
- $\mathcal{N} < 8$ gravity is known to be non-renormalizable.
- Any Feynman diagram calculation is hideous.

Notation

Colour ordering:

- One can identify gaugle invariant sub-amplitudes with a cyclic ordering,

$$\begin{aligned} \mathcal{A}(1_a, 2_b, 3_c, 4_d) &= \mathrm{Tr}(t_a t_b t_c t_d) A(1,2,3,4) \\ &\quad + \mathrm{Tr}(t_a t_b t_d t_c) A(1,2,4,3) \\ &\quad + \mathrm{Tr}(t_a t_d t_b t_c) A(1,4,2,3) + \ldots \end{aligned}$$

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Spinor helicity notation:

- Massless momenta described by their Weyl spinors:

$$p_\mu \sigma^\mu_{a\dot{a}} = \lambda_a \widetilde{\lambda}_{\dot{a}} = |p\rangle [p|$$

- Polarizations chosen according to helicity

$$\epsilon^+_\mu(p) = \frac{\langle q|\sigma_\mu|p]}{\sqrt{2}\langle qp\rangle}, \qquad \epsilon^-_\mu(p) = \frac{[q|\sigma_\mu|p\rangle}{\sqrt{2}[qp]}$$

Previously Known Similarities

The KLT (Kawai, Lewellen, Tye) relations:

- Derived from string scattering amplitudes
- (gravity)=(gauge)^2.

$$
\begin{aligned}
M(1,2,3) &= A(1,2,3)\widetilde{A}(1,2,3) \\
M(1,2,3,4) &= s_{34}A(1,2,3,4)\widetilde{A}(1,2,4,3) \\
M(1,2,3,4,5) &= s_{\dots}s_{\dots}A(\dots)\widetilde{A}(\dots) + s_{\dots}s_{\dots}A(\dots)\widetilde{A}(\dots)
\end{aligned}
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Decomposition of helicities

- $2 = 1 + \widetilde{1}$
- $\frac{3}{2} = 1 + \widetilde{\frac{1}{2}}$ or $\frac{1}{2} + \widetilde{1}$
- $1 = 1 + \widetilde{0}$ or $\frac{1}{2} + \widetilde{\frac{1}{2}}$ or $0 + \widetilde{1}$.

On-shell Recursion

Basic idea (Britto, Cachazo, Feng, Witten):

- To calculate an amplitude A, choose two external particles (say 1 and 2) and make the analytic continuation

$$|\hat{1}] = |1] + z|2], \qquad |\hat{2}\rangle = |2\rangle - z|1\rangle$$

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$$|\hat{1}] = |1] + z|2], \qquad |\hat{2}\rangle = |2\rangle - z|1\rangle$$

- If $A(z) \longrightarrow 0$ as $z \longrightarrow \infty$, use Cauchy's Theorem

$$A(0) = \frac{1}{2\pi i}\oint dz \frac{A(z)}{z} = -\sum_i \frac{\text{Res}_i A(z)}{z_i} = \sum_i \frac{A_L(z)A_R(z)}{P_i^2}$$

- Expressions are more compact, but contain (apparent) unphysical poles.

Conditions of Use

But what about the condition $A(z) \to 0$ as $z \to \infty$?

- For gauge theory, you can consider "worst Feynman diagram"
- Behaviour is often better.

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Conclusion:

- We can do recursion on gravity amplitudes,
- but we cannot strictly prove we are right (in general).

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MHV Rules for Gauge Theory

The Parke–Taylor Amplitudes:

$$A(1^\pm, 2^+, 3^+, \ldots, n^+) = 0$$

$$A(1^-, \ldots, i^-, \ldots, n^+) = \frac{\langle 1i\rangle^4}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle} \quad \text{(MHV)}$$

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MHV Rules (Cachazo, Svrček, Witten):

- Compute amplitudes by stringing together MHV vertices and scalar propagators.
- For internal lines, subtract sufficient momentum to put it on-shell,

$$P_\mu \;\longrightarrow\; P^\flat_\mu = P_\mu - \frac{P^2}{2P\cdot\eta}\eta_\mu, \qquad |P^\flat\rangle = P|\eta] = P_\mu \sigma^\mu_{a\dot{a}}\tilde{\eta}^{\dot{a}}$$

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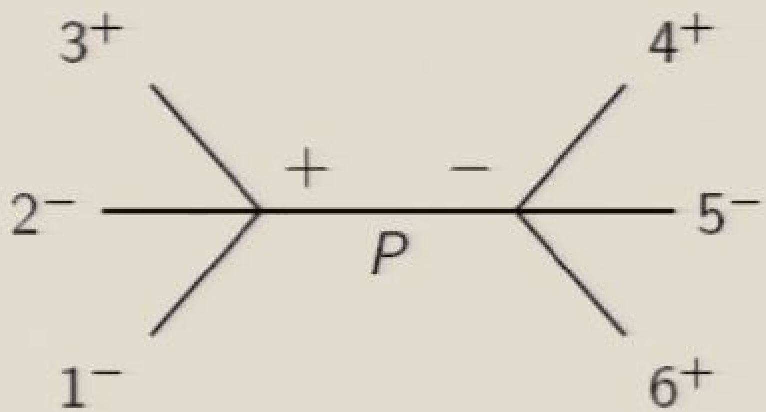
Deep connection to twistor string theory.

MHV Rules: An Example

Example diagram from $A(1^-, 2^-, 3^+, 4^+, 5^-, 6^+)$:



$$\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 3P^\flat\rangle\langle P^\flat 1\rangle}\frac{1}{P^2}\frac{\langle(-P^\flat)5\rangle^4}{\langle 45\rangle\langle 56\rangle\langle 6(-P^\flat)\rangle\langle(-P^\flat)4\rangle}$$

$$= \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 3(1+2)\eta]\langle 1(2+3)\eta]}\frac{1}{P^2}\frac{\langle 5(4+6)\eta]^4}{\langle 45\rangle\langle 56\rangle\langle 6(4+5)\eta]\langle 4(5+6)\eta]}$$

MHV Rules from Recursion

MHV rules can also be seen as coming from recursion (K.R.)

- To calculate an NMHV amplitude $A_{\text{NMHV}}(m_1^-, m_2^-, m_3^-)$, make the analytic continuation

$$|\widehat{m}_1] = |m_1] + z|\eta]\langle m_2 m_3\rangle, \qquad |\widehat{m}_2] = \dots$$

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- This splits the amplitude into

$$\sum_i A_{\text{MHV}}(\widehat{m}_1^-, \widehat{P}_i^-, \dots)\frac{1}{P_i^2}A_{\text{MHV}}(\widehat{m}_2^-, \widehat{m}_3^-, -\widehat{P}_i^+, \dots)$$

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- We must use $|\widehat{m}_j]$ instead of $|m_j]$ but it doesn't appear in the MHV expression.
- $\widehat{P}_i = P_i - z|m_1\rangle\langle m_2 m_3\rangle[\eta| \Rightarrow |\widehat{P}_i\rangle \propto |P_i\eta]$

MHV Rules from Recursion, continued

What if there are more than three negative helicity gluons (say, four)?

- Choose some analytic continuation

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What is required for this to work in general?

- The concept of an MHV amplitude.
- An N^nMHV amplitude must $\to z^{-n}$ as $z \to \infty$.

MHV Rules for Gravity

Amazingly, this seems to work for (super)gravity also (Bjerrum-Bohr, Dunbar, Ita, Perkins, K.R.):

- Gravity has the concept of MHV amplitudes (get them from *e.g.* the KLT relations).
- Gravity MHV amplitudes depend on $|\cdot]$'s too, so things are not as simple as for gauge theory.
- Is there a relation to a twistor formulation of (super)gravity?

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There's a problem, of course:

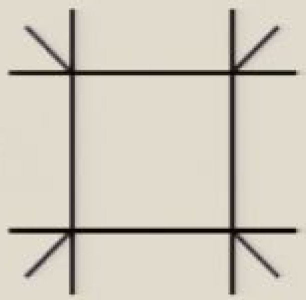
- We can't prove the asymptotic behaviour as $z \to \infty$.
- Actual behaviour is way better than the naïve expectation.
- Understanding of (super)gravity depends on validity of certain recursion relations.

Contents

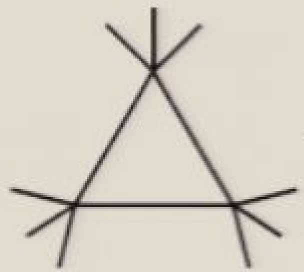
Structure of One-Loop Amplitudes

By using various reduction methods, any massless one-loop amplitude can be written as a linear combination of scalar integrals:

$$\sum_i c_i$$



$$+\sum_j d_j$$



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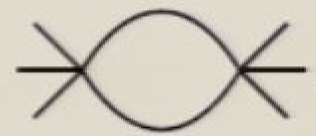

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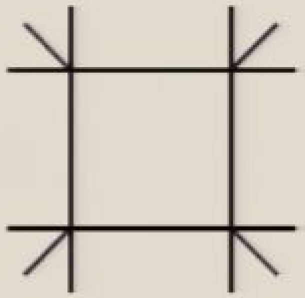
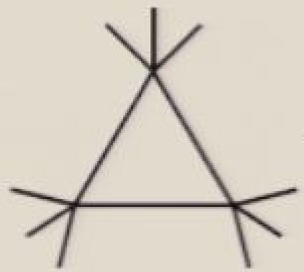
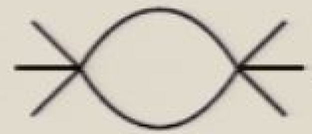
$\sum_i c_i$  $+ \sum_j d_j$  $+ \sum_k e_k$  $+ R$

In $\mathcal{N} = 4$ SYM, a cancellation reduces this to boxes only:

- Naïvely, an n-point one-loop diagram can have up to n powers of loop momentum in the numerator (and n propagators).
- $\mathcal{N} = 4$ SUSY reduces this to $n - 4$.
- Each power of loop momentum in the numerator can remove a propagator, so there are four left.

The No-Triangle Hypothesis

What about $\mathcal{N} = 8$ SUGRA?

- Naïvely, a one-loop n-point amplitude can have up to $2n$ powers of loop momentum in the numerator.
- $\mathcal{N} = 8$ SUSY reduces this to $2n - 8$.
- Lots of loop momenta left in the numerator to cancel propagators.

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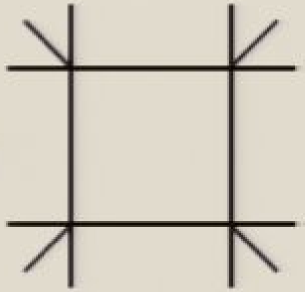
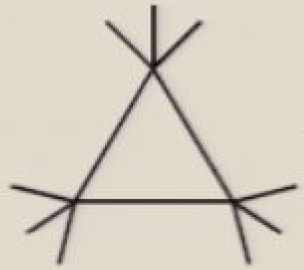
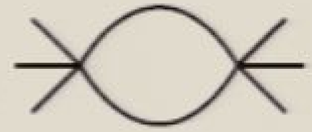
Experience shows that it might be better, though:

- At tree level gravity is "better behaved" than *e.g.* the KLT relations suggest.
- There could be cancellations across diagrams.

No general methods exhibit this better behaviour, so we have to calculate. . .

Quadruple Cuts

$\sum_i c_i$  $+\sum_j d_j$  $+\sum_k e_k$  $+R$

How to compute a box coefficient (Britto, Cachazo, Feng):

- In the (complex) space of loop momentum, there is a place where the four internal propagators go on shell.

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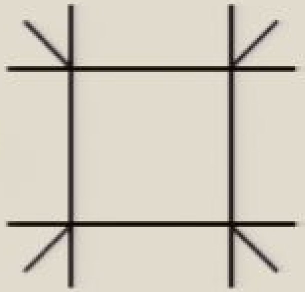
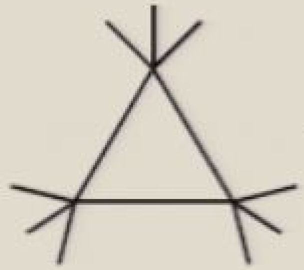
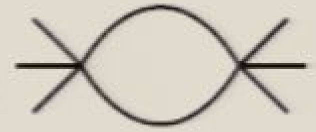
$\sum_i c_i$  $+ \sum_j d_j$  $+ \sum_k e_k$  $+ R$

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- This gives four overlapping "poles" whose residue can be computed.

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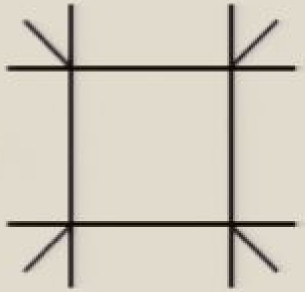
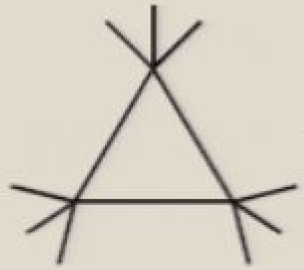
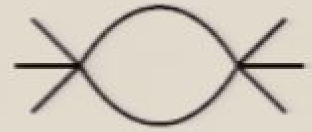
$\sum_i c_i$  $+\sum_j d_j$  $+\sum_k e_k$  $+R$

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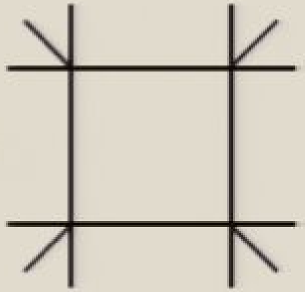
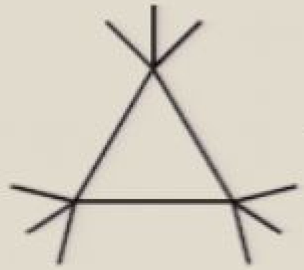
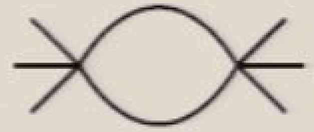
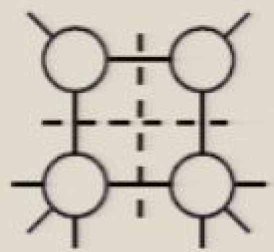
$\sum_i c_i$  $+\sum_j d_j$  $+\sum_k e_k$  $+ R$

How to compute a box coefficient (Britto, Cachazo, Feng):

- In the (complex) space of loop momentum, there is a place where the four internal propagators go on shell.
- This gives four overlapping "poles" whose residue can be computed.
- Only the box in question contributes to this residue, so it is essentially the coefficient.
- The residue is just the product of the four 'corner amplitudes' on the condition that internal momenta be on-shell.

$c = \frac{1}{2}$ 

SUGRA at One-Loop

(Bern, Bjerrum-Bohr, Dunbar, Ita, Perkins, K.R.)

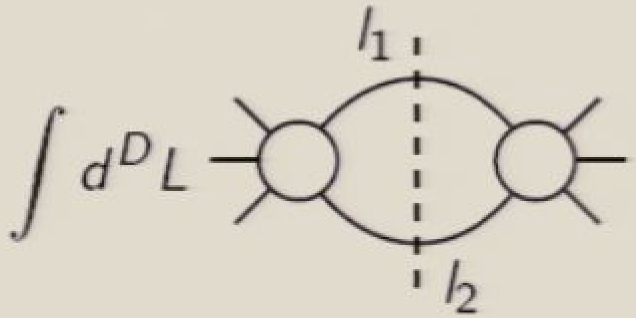
Step 1, boxes:

- Box coefficeients reduce to products of trees. This is taken care of by the KLT relation or recursion relations.
- General trend: Boxes account for all IR divergences.
- This can probably be proven using recursion relations (but we still don't have a failsafe proof of those).

Step 2, triangles:

- One- and two-mass triangles ruled out by IR divergences.
- Three-mass triangles can be shown to vanish at $n \leq 7$ by looking at triple cuts.

"Old" Unitarity Cuts



- Two internal momenta are put on-shell. This makes the integrand simple.
- Remember to sum over SUSY multiplet.
- Try to guess/compute what expression the cut came from, preferably without doing the integration.

One method is to write the integrand as

$$\sum_{i,j} c_{ij} \frac{1}{L_i^2 L_j^2} + \sum_k d_k \frac{1}{L_k^2} + e.$$

SUGRA at One-Loop, continued

Step 3, bubbles:

- Do the recursive shift

$$|\widehat{l_1}] = |l_1] + z|l_2], \qquad |\widehat{l_2}\rangle = |l_2\rangle + z|l_1\rangle$$

on the integrand.

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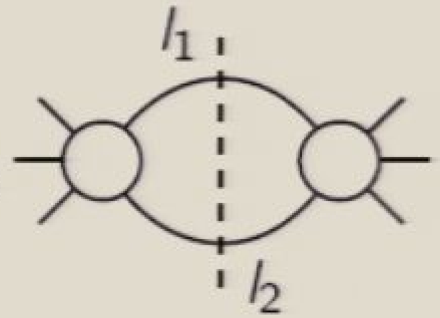
  on the integrand.
- If the integrand goes as z^{-1} or better, there must be at least one more propagator, and thus no bubbles.
- If there is no SUSY multiplet summation, recursion should work on both amplitudes, giving z^{-1} from each.
- If there is SUSY multiplet summation, recursion ought fail, but the summation always saves us by providing z^{-8}.

"Old" Unitarity Cuts

$\int d^D L$ 

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- Try to guess/compute what expression the cut came from, preferably without doing the integration.

One method is to write the integrand as

$$\sum_{i,j} c_{ij} \frac{1}{L_i^2 L_j^2} + \sum_k d_k \frac{1}{L_k^2} + e.$$

SUGRA at One-Loop, continued

Step 3, bubbles:

- Do the recursive shift

$$|\widehat{l_1}] = |l_1] + z|l_2], \qquad |\widehat{l_2}\rangle = |l_2\rangle + z|l_1\rangle$$

  on the integrand.
- If the integrand goes as z^{-1} or better, there must be at least one more propagator, and thus no bubbles.
- If there is no SUSY multiplet summation, recursion should work on both amplitudes, giving z^{-1} from each.
- If there is SUSY multiplet summation, recursion ought fail, but the summation always saves us by providing z^{-8}.

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Step 4, rationals:

- Would be sort of freaky, now that the triangles and bubbles aren't there.

The Body of Evidence

- Loads of circumstancial evidence.
- Proof for $n \leq 6$
- Limits and factorization: Let two momenta go collinear in an n-point amplitude; that gives you the $n-1$-point.
- If triangles, bubbles and rationals appear at high n, how could they disappear at low n?

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Conclusion: The No-Triangle Hypothesis is now a firm conjecture.

Caveat: Our arguments always end up using recursion relations in some form. Those are not strictly proven.

Contents

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- The methods used here primarily run on tree level input.
- If the No-Triangle Conjecture is true, one-loop looks similar in $N = 4$ SYM and $N = 8$ SUGRA.
- Why should this stop at one-loop? Apparently it doesn't: Two- and three-loop confirm the general picture of similarity (Bern, Dixon, Roiban, Kosower, Perelstein, Rozowsky).
- Input from other directions (Green, Risso, Vanhove)
- Next week there's even a conference about it!

A New Symmetry?

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- The uexplained cancellations may be due to an unknown symmetry of $\mathcal{N} = 8$ SUGRA
- Supersymmetry doesn't seem to be the whole answer.
- $\mathcal{N} = 4$ SYM has super-*conformal* symmetry.

$$\begin{aligned} M(1,2,3,4) &= s_{34} A(1,2,3,4)\widetilde{A}(1,2,4,3) \\ (\mathcal{N} = 8 \ + \ ??) &= (\mathcal{N} = 4 \ + \ \text{conf})^2 \end{aligned}$$

Where does the conformal symmetry go? No simple answer.

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Conclusion

- Recent methods are a leap forward in understanding.
- Tree-level feeds into loop-level more than expected.
- Perturbative $\mathcal{N} = 8$ supergravity seems closely related to $\mathcal{N} = 4$ super-Yang–Mills, particularly wrt. UV behaviour.
- There's something out there waiting to be discovered …