Title: The Arrow of Time, Black Holes, and Quantum Mixing of Large N Yang-Mills Theories

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Abstract: Quantum gravity in an AdS space-time is described by an SU(N) Yang-Mills theory on a sphere, a bounded many-body system. We argue that in the high temperature phase the theory is intrinsically non-perturbative in the large N limit. At any nonzero value of the \\\'t Hooft coupling $lambda$, an exponentially large (in N^2) number of free theory states of wide energy range (of order N) mix under the interaction. As a result the planar perturbation theory breaks down. We argue that an arrow of time emerges and the dual string configuration should be interpreted as a stringy black hole

Spacelike singularities and time arrow

The equations of general relativity are time symmetric. One often finds an intrinsic time direction in solutions containing space-like singularities.

Examples are: FRW cosmologies and gravitational collapse.

The end result of gravitational collapse is a black hole. Since a black hole behaves like a thermodynamical system the direction of time appears to have a thermodynamical nature

In AdS spacetime a microscopic understanding of the emergence of thermodynamical behavior in a gravitational collapse can be achieved using the AdS/CFT correspondence.

The arrow of time in AdS-CFT

The AdS-CFT correspondence provides a nonperturbative framework to study the question

Quantum gravity on asymptotic AdS_5 spacetime	$\leftrightarrow$	$\mathcal{N}=4\ SU(N)$ SYM on S^3
Classical limit: $\frac{l_p}{R}\to 0$ and $\frac{l_s}{R}\to 0$	$\leftrightarrow$	$N\to\infty$ and $\lambda\to\infty$
Classical mass M distribution	$\leftrightarrow$	state of $E=O(N^2)$
$\downarrow$ collapse		$\downarrow$ thermalization
AdS_5 Black Hole	$\leftrightarrow$	Thermal density matrix at T_{BH}

Gravitational collapse is mapped to the thermalization of highly excited states in the SYM

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Importance of large N limit

The large N limit is essential

For finite N the gauge theory on S^3 is a bounded quantum mechanical system

$$\Downarrow$$

- The energy spectrum is discrete
- The theory is time reversible
- NO thermalization

We have to understand the large N limit in the gauge theory at high temperature.

Observables

We study real time two point functions of single trace scalar operators $O(t)$ of dimension $\Delta \sim O(N^0)$ at finite β

$$
\begin{aligned}
G_+(t) &= Z^{-1}\langle O(t)O(0)\rangle_\beta \\
G_R(t) &= iZ^{-1}\theta(t)\langle [O(t), O(0)]\rangle_\beta
\end{aligned}
$$

These correlators describe the linear response of the system to small perturbations. If the system thermalizes

$$
\begin{aligned}
G_+(t) &\to C \qquad t \to \infty \\
G_R(t) &\to 0 \qquad t \to \infty
\end{aligned}
$$

And an arrow of time is generated.

Frequently we will work in frequency space $G_+(\omega)$

Plan

- Properties of correlation functions for $N \to \infty$ and $\lambda \to \infty$
- Show that at each order in the planar perturbation expansion no time arrow is generated in the gauge theory
- Breakdown of the planar perturbation expansion at high temperatures
- Emergence of arrow of time for all $\lambda \neq 0$ in the large N limit at high temperature
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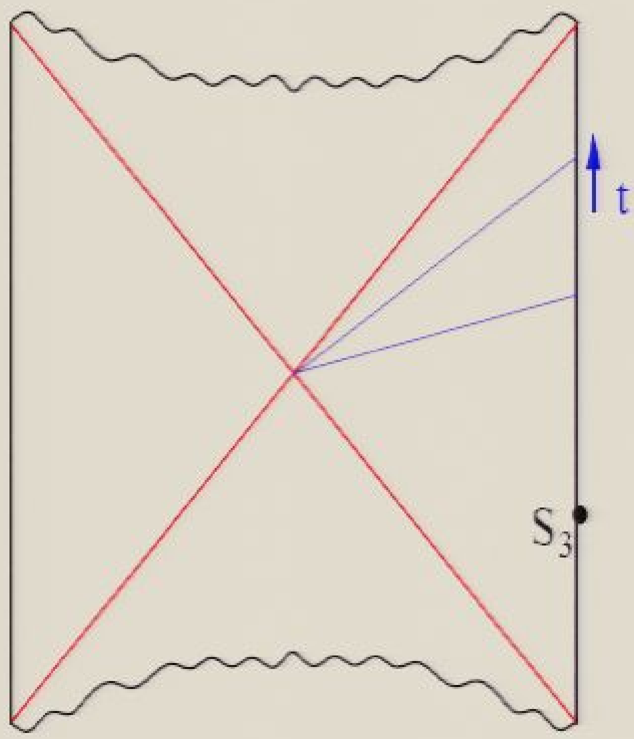
The $N \to \infty$, $\lambda \to \infty$ limit

In the $N \to \infty$ and $\lambda \to \infty$ limit we can use classical gravity to compute SYM correlators



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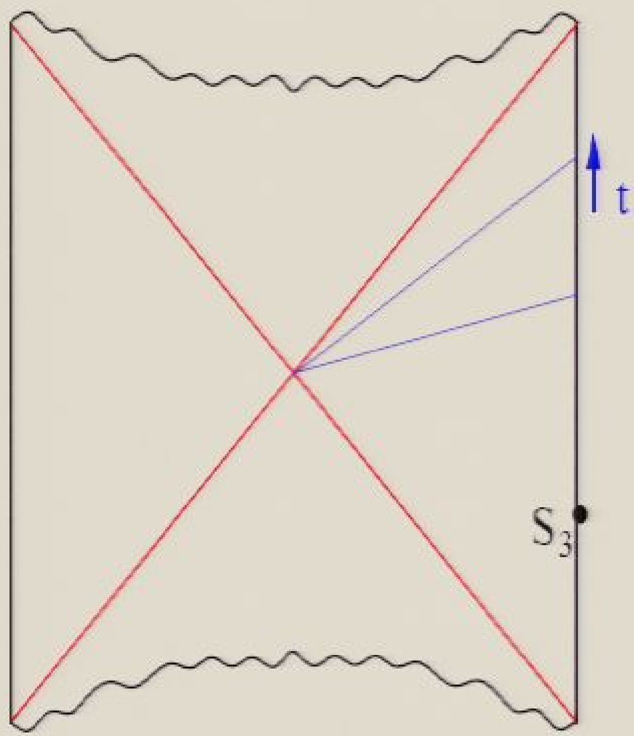
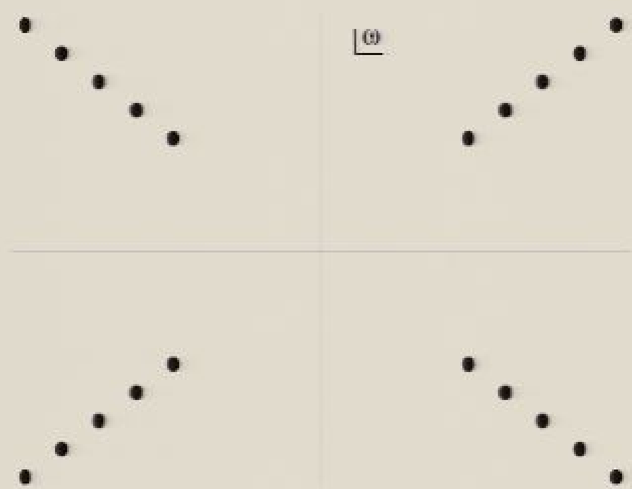


- Do correlators decay in time?
- Is the region behind the horizon encoded in the SYM correlation functions?
- What are the signatures of the singularity in the gauge theory in the $N \to \infty$, $\lambda \to \infty$ limit?
- Do this signatures go away at finite N or finite λ?

Analytic structure of $G_+(\omega)$

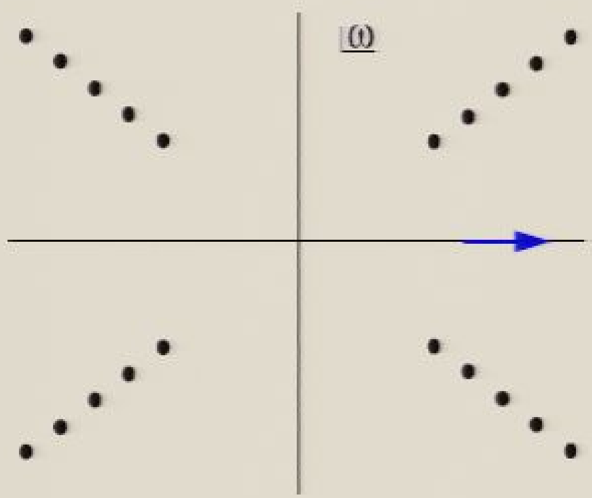


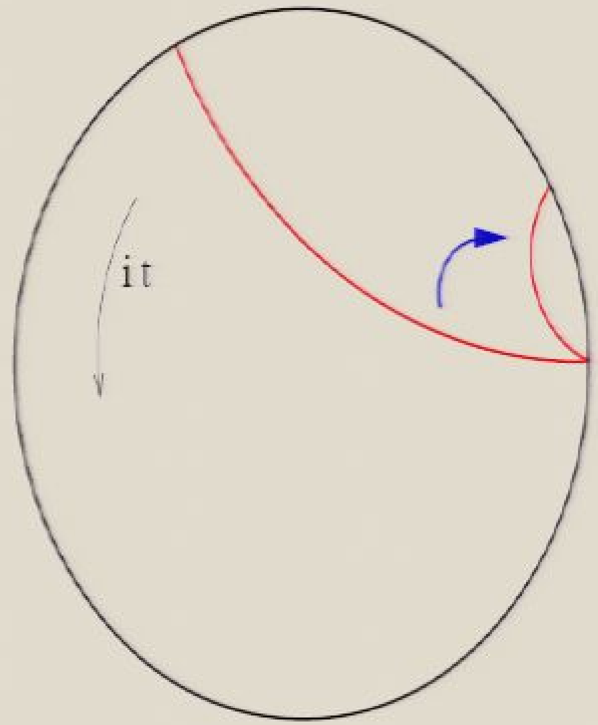
- The presence of the horizon implies a continuous spectrum for $G_+(\omega)$
- The complex ω plane is divided in sectors by lines of quasinormal poles [Starinets, Nuñez,Cardoso...]
- The poles are away from the real axis implying exponential decay in time for $G_+(t)$
- An arrow of time is generated.

Geodesics and Correlators

In the large Δ limit $G_+(\omega)$ has a direct relation with spacelike geodesics in the bulk: [Hong Liu,G.F.]

For $\omega \in \mathbb{R}$ the geodesic probes the Euclidean section of the Black Hole, UV-IR connection, a spacelike coordinate is generated holographically

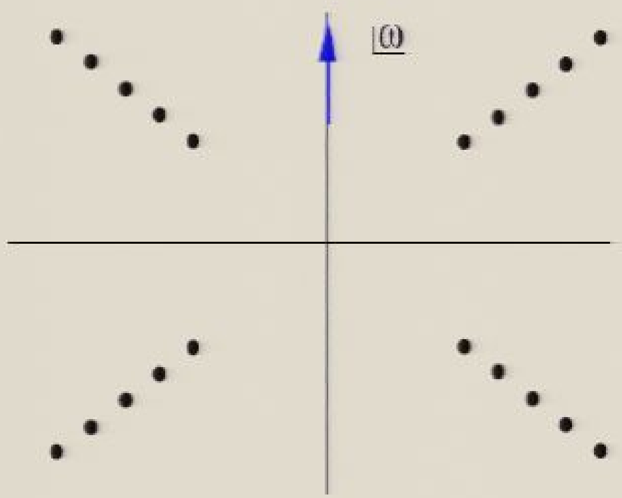


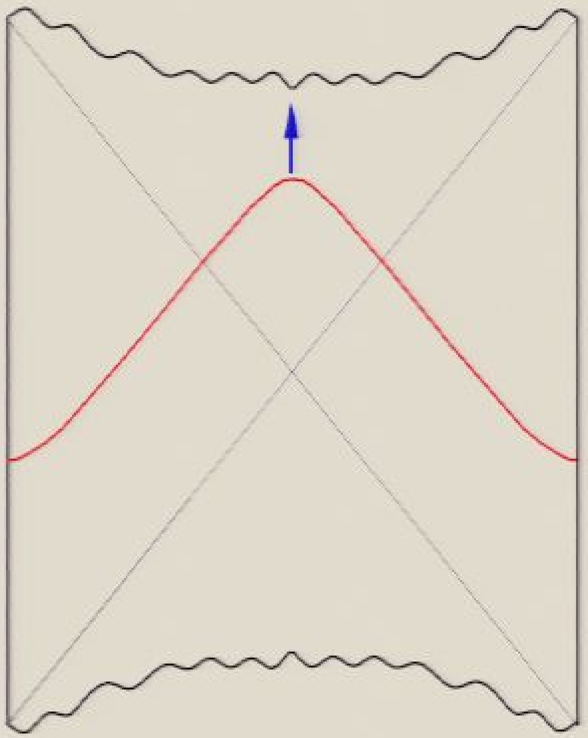


Geodesics and Correlators

In the large Δ limit $G_+(\omega)$ has a direct relation with spacelike geodesics in the bulk: [Hong Liu,G.F.]

For $\omega \in i\mathbb{R}$ the geodesic probes the Lorentzian section of the Black Hole and the region beyond the horizon. As $\omega \to i\infty$ it approaches the singularity. A time-like coordinate is generated holographically.





Summary

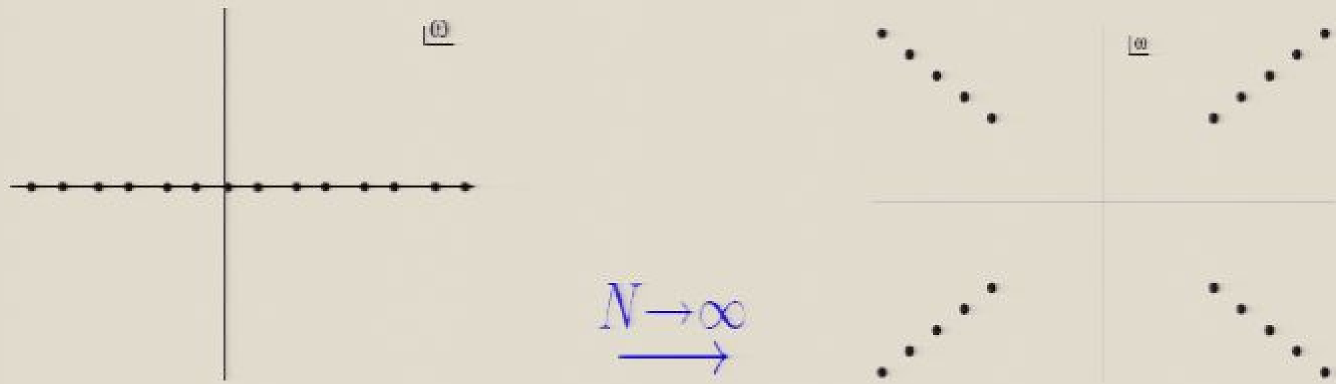
We studied the signatures of the presence of the black hole singularity in the gauge theory correlation functions in the $N \to \infty, \lambda \to \infty$ limit:

- New asymptotic regions for $G_+(\omega)$ open up because of the presence of lines of quasi-normal poles.
- The presence of the black hole singularity is encoded in the behaviour of $G_+(\omega)$ for $\omega \to i\infty$
- $G_+(\omega)$ has a continuous spectrum.
- All poles are away from the real axis $\Rightarrow G_+(t)$ decays as $t \to \infty$

Finite N

At finite N the SYM theory on S^3 is a bounded quantum mechanical system therefore $G_+(\omega)$ has a discrete spectrum

$$G_+(\omega) = \sum_n a_n \delta(\omega - \omega_n)$$



- All signatures of the singularity disappear! No time arrow.
- Quantum gravity effects (finite N) should resolve the singularity.
- What about α' corrections (finite λ)?

Many questions arise

- Do the features of $G_+(\omega)$ found at $\lambda \to \infty$ and $N \to \infty$ survive at finite λ?

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- Do the features of $G_+(\omega)$ found at $\lambda \to \infty$ and $N \to \infty$ survive at finite λ?
- What is the physical mechanism for the emergence of an arrow of time in the large N limit of the gauge theory?
- Is there a qualitative change at a finite value of λ in the properties of real time correlation functions in the $N \to \infty$ limit?

Plan

- Properties of correlation functions for $N \to \infty$ and $\lambda \to \infty$
- **Show that at each order in the planar perturbation expansion no time arrow is generated in the gauge theory**
- Breakdown of the planar perturbation expansion at high temperatures
- Emergence of arrow of time for all $\lambda \neq 0$ in the large N limit at high temperature
- Speculations

The model

We will consider matrix models like:

$$S = \frac{N}{2}\mathrm{tr}\left[\int dt \sum_{\alpha} \left((D_t M_\alpha)^2 - \omega_\alpha^2 M_\alpha^2\right)\right] - N\lambda \int dt\, V(M_\alpha)$$

- $U(N)$ gauge symmetry
- $\omega_\alpha \neq 0 \Rightarrow$ the theory has a mass gap
- $V(M_\alpha)$ is a sum of single traces
- There are at least two matrices

$\mathcal{N} = 4$ SYM on S^3 is of this form.

High temperature phase

In the large N limit these theories have a phase transition at $T = T_c$ such that

$$F(T) \sim O(N^0), \quad S(T) \sim O(N^0) \quad \text{for} \quad T < T_c$$
$$F(T) \sim O(N^2), \quad S(T) \sim O(N^2) \quad \text{for} \quad T > T_c$$

- The high temperature phase has been displayed both in the weakly interacting regime [Aharony et Al] and in the strongly interacting one [Witten]

Planar perturbation theory I

At $\lambda = 0$ the spectrum is discrete. O is a single trace of $O(1)$ length and mediates the exchange of a finite number of excitations $\Rightarrow$ Correlators are quasi-periodic.

In the case $\omega_\alpha = \omega_0 \quad \forall \alpha$ we obtain:

$$G_+(\omega) = \sum g_{k,l}^{(n)} \lambda^n \delta^{(l)}(\omega - k\omega_0)$$

where $l = 0, 1, ..., n \qquad k = n - 2\Delta, ..., n + 2\Delta$

- At each order in perturbation theory only a finite number of frequencies appear
- Quasi-periodic behavior; no thermalization at weak coupling?

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Perturbation theory II

What are the convergence properties of the small λ expansion?

In the planar limit at $T = 0$ the perturbation theory converges for small λ; what happens in the hight temperature phase ?

We will consider a simple two matrix model for illustration:

$$S = \frac{N}{2} \int dt \, \mathrm{tr} \left[(D_t M_1)^2 + (D_t M_2)^2 - \omega_0^2 (M_\alpha^2 + M_\alpha^2) + \lambda M_1 M_2 M_1 M_2 \right]$$

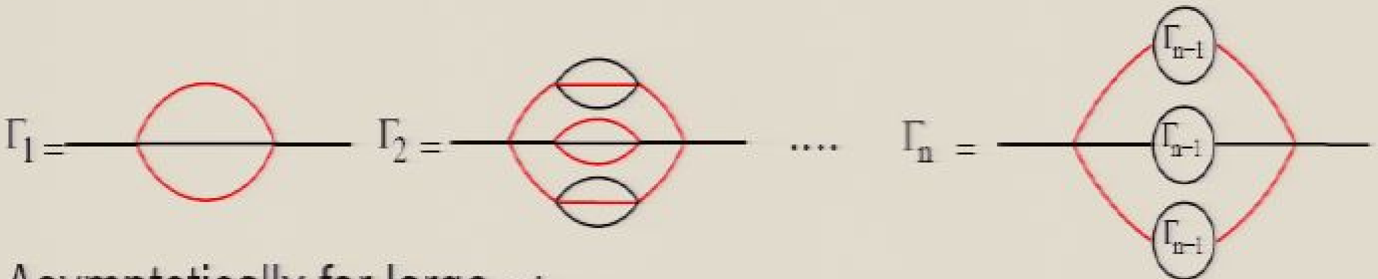
And extract the following contribution to the propagator for M_1

$$D_F(t) \;=\; Z^{-1} \langle \mathrm{T}[M_1(t) M_1(0)] \rangle_\beta = \sum_n \lambda^n D_F^{(n)}(t)$$

$$= \; D_F^{(0)}(t) \sum_n a_n (\lambda \, t)^n + \ldots$$

A subset of diagrams

We identify a subset of planar diagrams at order $d_n = 3^n - 1$ that leads to a divergent contribution



Asymptotically for large n:

$$\sum_n \Gamma_n \approx D_F^{(0)}(t) \sum_n (c(\beta)\lambda t)^{d_n} + ...$$

The radius of convergence of the perturbation series decreases with time $\lambda_c \sim \frac{1}{tc(\beta)}$

This results in the breakdown of the planar expansion for real time correlation functions in frequency space:

$$G_F(\omega) \approx \sum_n (c(\beta)\lambda)^{d_n}\, d_n! \left(\frac{i}{\omega^2 - \omega_0^2 + i\epsilon}\right)^{d_n}$$

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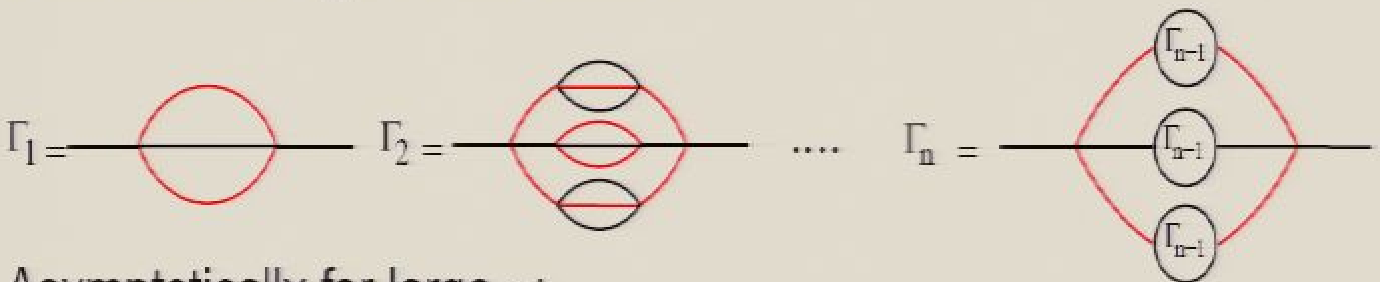

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Perturbation theory summary

- We found an essentialsingularity for real time correlation functions in frequency space at $\lambda = 0$
- The argument can be carried over to more complicated interactions
- However there could be unforeseen cancelations (SUSY at finite T is not enough)
- Non perturbative methods to deal with the large N limit of the theory at $T > T_c$ are needed
- A qualitative explanation for the breakdown.

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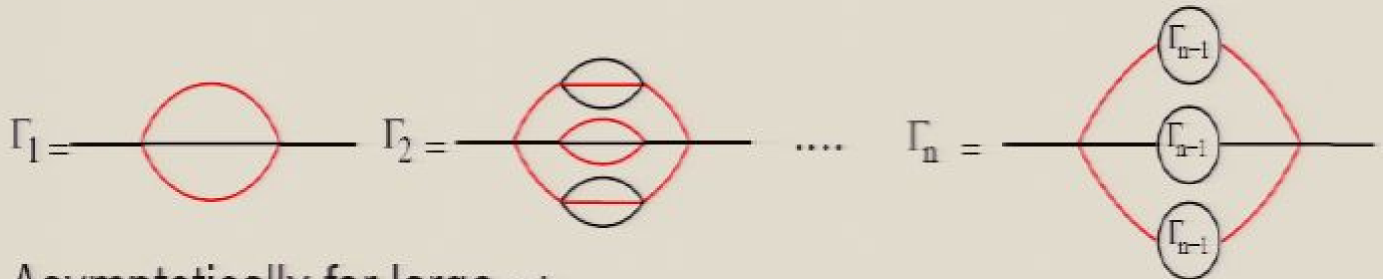

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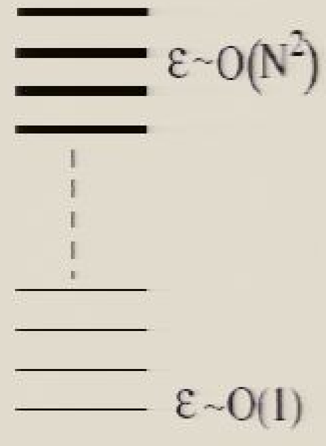
Interpretation

At $\lambda = 0$ the spectrum of $\mathcal{N} = 4$ SYM has the following properties:

- The energy levels are equally spaced
- The degeneracy of states of energy $\epsilon \sim O(N^0)$ is $O(N^0)$
- The degeneracy of states of energy $\epsilon \sim O(N^2)$ is $\exp(O(N^2))$



In the high temperature phase we probe the states with energy $O(N^2)$ above the ground state. At first order in time independent perturbation theory we have to diagonalize V in a degenerate subspace.

- The exponential degeneracy in the high energy sector is generically lifted, the level spacing is of order $\exp(-O(N^2))$
- The eigenvalues have a spread of order λN greater than the free theory level spacing. Perturbation theory breaks down.

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Statistical approach

Treat the interaction as a random matrix with the following structure

- Banded: only states with finite energy difference are connected.
- Sparse (in appropriate basis) but connectivity of $O(N^4)$
- Density of states in the free theory increases with energy

Then we get for any $\lambda \neq 0$ in the large N limit:

- The spectrum of $G_+(\omega)$ is continuous.
- $G_+(t) \to C$ as $t \to \infty$. A direction of time is generated!

Conclusions

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Speculations

The high temperature phase of a weakly coupled YM in the large N limit is dual to a stringy black hole

Black hole singularities may survive α' corrections.

The behavior we found for $|\langle i|O|j\rangle|^2$ has been conjectured as a hallmark of Quantum Chaos. Is there a connection with BKL behaviour near the singularity ?

Improving the nonperturbative statistical analysis could help in understanding these issues

Thank You

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