Title: Twistor-Inspired Approach to QCD and Supergravity

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Abstract: We will discuss applications of the recently developed twistor-space methods in perturbative quantum field-theory. The two central topics will be 1) the unitarity bootstrap approach to hard scattering amplitudes in QCD and 2) the analysis of the UV structure of N=8 supergravity.

Introduction

Witten's 2003 proposal of a "weak-weak" duality between N=4 super Yang-Mills and string theory in twistor-space lead to major developments in perturbative quantum field-theory:

- ✓ 1-loop QCD amplitudes (6 gluon).
- ✓ Computation of up to 4-loop amplitudes in N=4 SYM.
- ✓ UV-structure of N=8 supergravity.

new methods:

- MHV rules (Cachazo, Svrcek and Witten)
- BCFW recursions (Britto, Cachazo, Feng and Witten)
- Unitarity and cut-constructibility (Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng)
- Recursions for loops (Bern, Dixon, Kosower, Berger, Forde, Bjerrum-Bohr, Dunbar, H.I.)
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Plan of Talk

- ✓ Unitarity Bootstrap Program for QCD
- ✓ UV-structure of N=8 supergravity

Unitarity Bootstrap Program for QCD

✓ Aim: Solve all-n scattering amplitudes of 1-loop QCD without explicit integration.

- Achieved already for N=4 SYM using generalised unitarity (Britto, Cachazo, Feng)

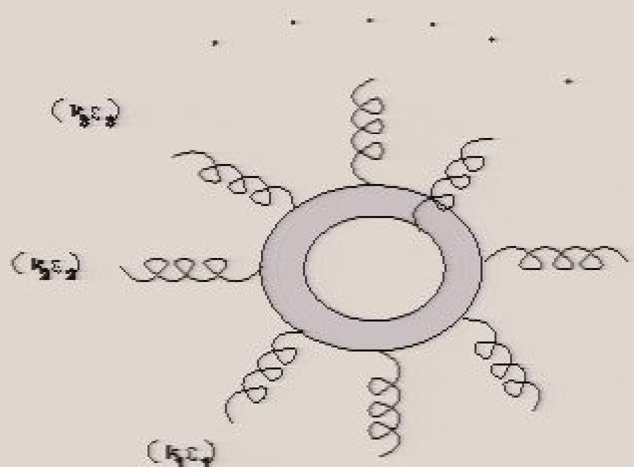
✓ Combine various structures:

- Spinor-helicity formalism
- Colour ordering
- Supersymmetric decomposition
- Unitarity methods
- Factorisation: physical + spurious
- BCFW recursions

⟶ Efficient bootstrap/recursion.

Scattering Amplitudes

- ✓ Hard n-parton amplitudes as function of momenta and polarisations (k,ε)
  - External gluons
  - Adjoint fermions
  - Massless fermions (good approximation for light flavours at LHC energies)



Conventions-Helicity

Xu, Zhang, Chang
Mangano, Parke

- ✓ Isomorphism $SO(3,1) \sim SL(2,C)$

$$K_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} + \mu_a \tilde{\mu}_{\dot{a}}$$

- ✓ Massless momenta:

$$P_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$$

- ✓ Polarisation vectors

$$\varepsilon^{+}_{a\dot{a}} = \frac{\rho_a \tilde{\lambda}_{\dot{a}}}{\langle \rho, \lambda \rangle}$$

⟹ Rational expressions in $\lambda_a, \tilde{\lambda}_{\dot{a}}$.

Colour Ordering

✓ Pull out colour factors at 1-loop,

$$\mathcal{A}_n^{\text{one-loop}}(\{k_i, a_i\}) = g^n \sum_{c=1}^{\lfloor n/2 \rfloor + 1} \sum_{\sigma \in S_n/S_{n;c}} \mathrm{Gr}_{n;c}(\sigma)\, A_{n;c}(\sigma),$$

i. Leading colour structure,

$$\mathrm{Gr}_{n;1}(1) = N_c \,\mathrm{Tr}\,(T^{a_1} \cdots T^{a_n})$$

ii. Sub-leading colour structure,

$$\mathrm{Gr}_{n;c}(1) = \mathrm{Tr}\,(T^{a_1} \cdots T^{a_{c-1}})\ \mathrm{Tr}\,(T^{a_c} \cdots T^{a_n})$$

➡
- Simpler gauge-invariant objects
- Planar factorisation properties

Super-symmetric Decomposition

- ✓ Regroup amplitudes into pieces with susy multiplets in the loop,

$$A_n^{\mathcal{N}=4} \equiv A_n^{[1]} + 4A_n^{[1/2]} + 3A_n^{[0]},$$
$$A_n^{\mathcal{N}=1\text{ vector}} \equiv A_n^{[1]} + A_n^{[1/2]},$$
$$A_n^{\mathcal{N}=1\text{ chiral}} \equiv A_n^{[1/2]} + A_n^{[0]}.$$

- ✓ Non-susy amplitudes in terms of susy amplitudes,

$$A_n^{[1]} = A_n^{\mathcal{N}=4} - 4A_n^{\mathcal{N}=1\text{ chiral}} + A_n^{[0]}$$

➡ Primitive amplitudes:

- Closed under factorisation
- Characteristic UV-behaviour

- ✓ Primitive amplitudes in basis of integral functions and rational coefficients,

$$A = \sum c_i I_i .$$

Constraints from Factorisation

Bern, Chalmers

✓ Universal factorisation properties,

$$A_n^{1-loop} \xrightarrow{K^2 \longrightarrow 0} A_{m+1}^{1-loop} \frac{i}{K^2} A_{n-m+1}^{tree} + A_{m+1}^{tree} \frac{i}{K^2} A_{n-m+1}^{1-loop} + A_{m+1}^{tree} \frac{i}{K^2} A_{n-m+1}^{tree} \mathcal{F}_n^{cor.} \,.$$

✓ Constraint for particular coefficient,

$$c_{i,n} \xrightarrow{K^2 \to 0} \sum_h A_{n-m+1}^h \frac{i}{K^2} c_{i,m+1}^{-h} \,.$$

- Can factorisation be inverted?
- Can we glue amplitudes from its parts?

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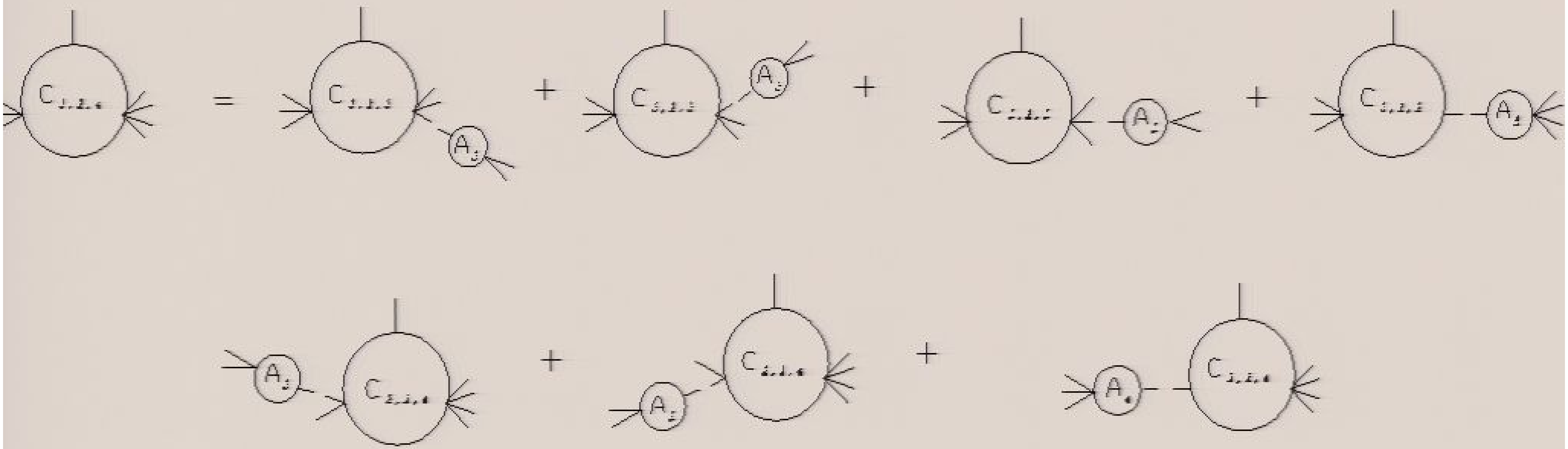
- Can factorisation be inverted?
- Can we glue amplitudes from its parts?

- Why not glue together factor amplitudes of all factorisations?



- Problems:
  i. Off-shell continuation needed
  ii. Over-counting of gluing

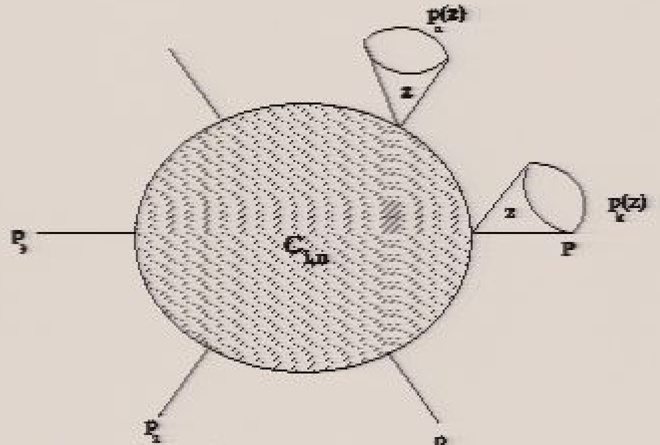
On-shell Recursion Relations

- ✓ Tree recursion relations found by Britto, Cachazo, Feng and Witten can be applied at loops.

→ elegant fix of problems:

Bern, Bjerrum-Bohr, Dunbar, H.I.

- Two momenta vary over complexified light-cone, the rest fixed,



$$p_a(z) = \lambda_a\tilde{\lambda}_a + z\lambda_a\tilde{\lambda}_b\,,$$
$$p_b(z) = \lambda_b\tilde{\lambda}_b - z\lambda_a\tilde{\lambda}_b\,.$$

- Total momentum conserved.

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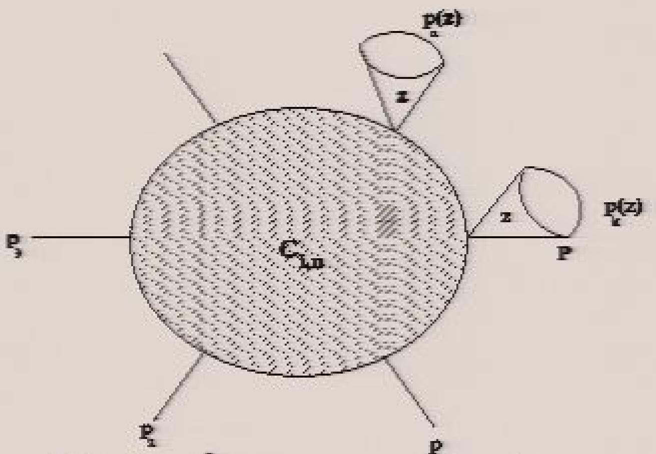

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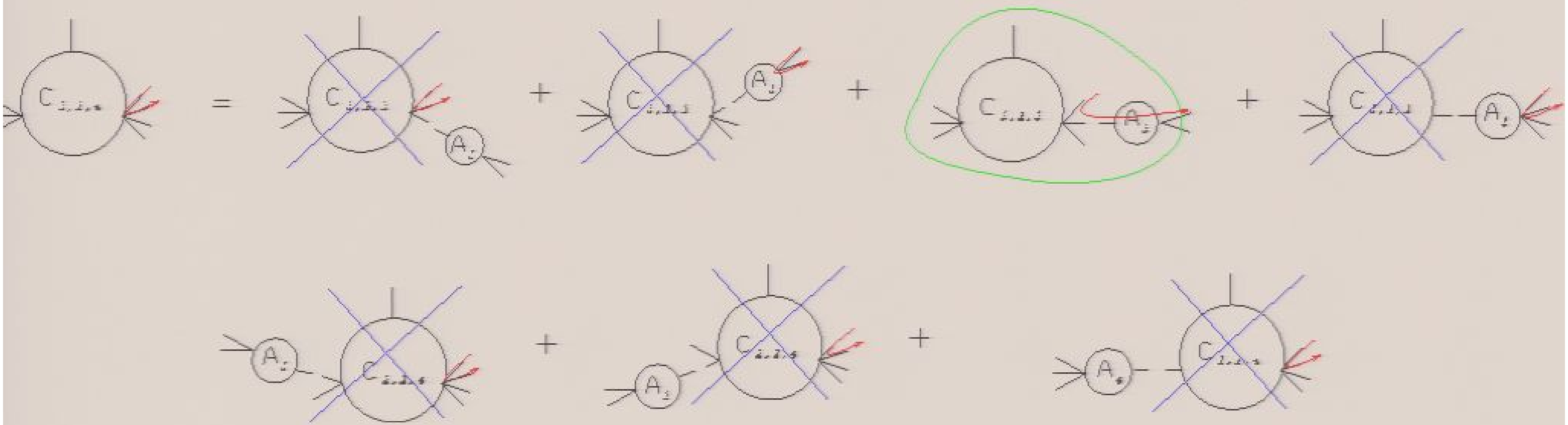
- Total momentum conserved.

i. Sum only over factorisations with shifted momentum in propagator,



ii. Gluing with intermediate momentum on-shell on complexified light-cone,

$$c_n(0) = \sum_{\alpha,h} A^h_{n-m_\alpha+1}(z_\alpha)\,\frac{i}{K_\alpha^2}\,c^{-h}_{m_\alpha+1}(z_\alpha)$$

$K_\alpha^2(z_\alpha)=0$, only propagator i/K_α^2 off-shell.

Conclusions I

✓ Physical factorisation can be inverted also at loop-level

✓ Important issues:

- $C_{i,n}$ only part of amplitude $\rightarrow$ unphysical factorisations
- Un-real poles

✓ In most general case use unitarity method, physical and unphysical factorisation:

"bootstrap" = loops from trees

UV-structure of N=8 Supergravity

't Hooft, Veltman; Goroff, Sagnotti

✓ S-matrix of pure gravity UV-finite at 1-loop, diverges at 2-loop

Grisaru, Nieuwenhuizen, Vermaseren

✓ Supergravity theories finite at least up to two loops

✓ Susy is believed to tame UV-divergences:

- Renormalisable $\rightarrow$ finite (N=4 SYM)
- Rare: non-renormalisable $\rightarrow$ Renormalisable

? Might N=8 supergravity be just that special?

How to Consider Finiteness?

- ✓ Compare perturbative expansion of N=8 supergravity and N=4 SYM.
- ✓ Map N=8 supergravity to a finite string theory. Without string-tower. Achieved by twistor string-theory? (Abou-Zeid, Hull, Mason)
- ✓ Non-renormalisation from duality web of string- and M-theory. (Chalmers; Green, Russo, Vanhove)
- ✓ Non-renormalisation from multi-loop Berkovits-string.

Perturbative Quantum Gravity

✓ Quantum field-theory defined by Einstein-Hibert action,

$$L = \frac{2}{\kappa^2} \int \sqrt{g} R, \quad g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}.$$

✓ Perturbative expansion in $\kappa = \sqrt{32\pi G_n}$.

✓ Consider n-graviton scattering:

$$\mathcal{M}_n(1, 2, \ldots n)$$

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N=8 Supergravity in 4D

- ✓ Matter content: $g_{\mu\nu} + 8\,\psi_{\mu,\alpha} + 28\,v_\mu + 56\,\psi_\alpha + 70\,s_r$
- ✓ Scalars parametrise coset: $E_7/SU(8)$
- ✓ Un-gauged version! Flat space not minimum of SO(8)-gauged supergravity with N=8 preserved.
- ✓ Phenomenology:
  - SU(3)× SU(2)× U(1) too big for SO(8)
  - Fermions in non-chiral (real) representations

Counter-terms

- ✓ Famous 7-loop counter-term (Howe, Lindstrom) based on assuming existence of superspace in N=8 supergravity
- ✓ Combination of Berkovits string and Green, Russo, Vanhove: not before 9-loops.
  - N=4 SYM tests of formalism?
  - F-terms right place to look at?

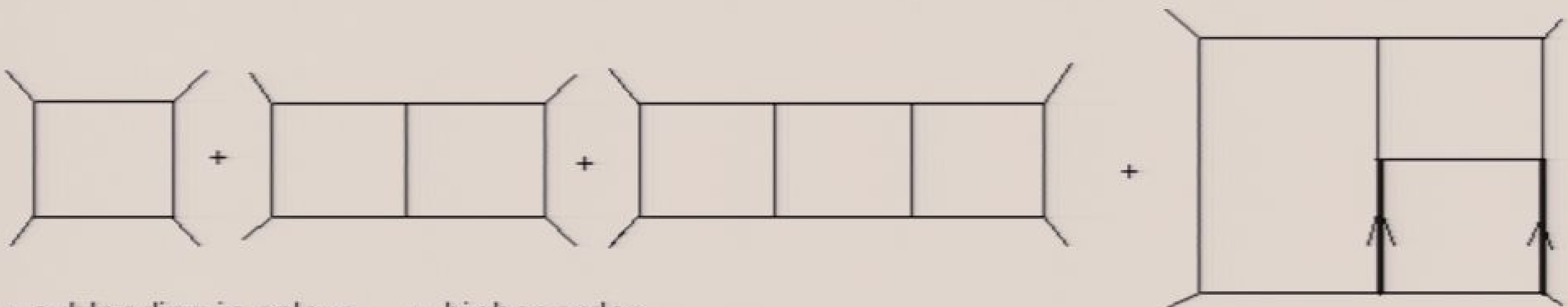
?Can perturbative approach keep the pace?

Measure of Finiteness - Integral Functions

- ✓ Idea: keep the pace with more sensitive observable.

Bern et al.

- ✓ 4-point N=4 SYM amplitudes given in terms of scalar integrals with momentum factors.



+ subleading in colour +..higher order

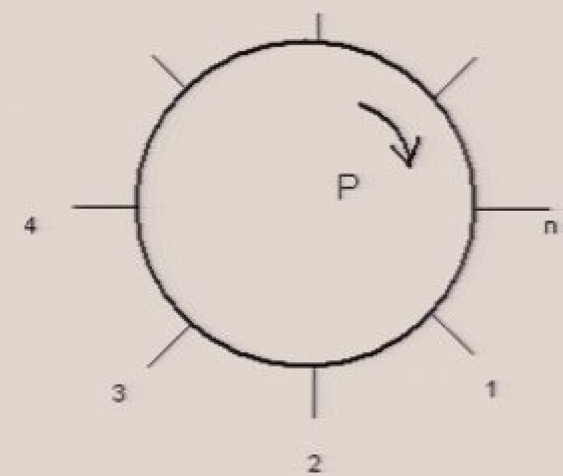
? Can N=8 be written in terms of the very same integral functions?

1-loop: No-triangle Hypothesis

Bern, Bjerrum-Bohr, Dunbar; Dunbar, Bjerrum-Bohr, H.I.
Bjerrum-Bohr, Dunbar, H.I., Perkins, Risager

✓ N=4 SYM 1-loop power counting:

- $\int dp^4 \, F(p)/(p^2)^n$
- F(p) polynomial order (n-4) .



✓ Passarino-Veltman Reduction:

$$2\, p.k = p^2 - (p-k)^2$$

super-symmetries

- Gluons:

$$\int dp^4 \;\; 1/(p^2)^4$$

- Naively for Gravitons: F(p) order (2n-8)

$$\int dp^4 \;\; 1/(p^2)^{(2n-8)}$$

spin

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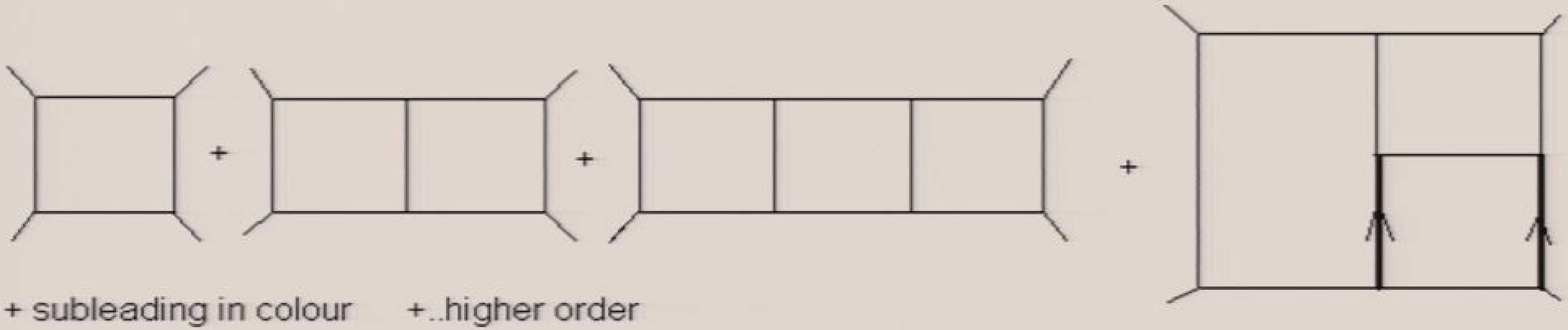

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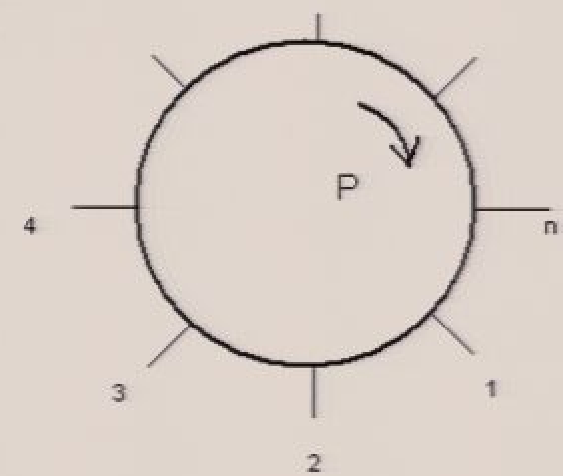

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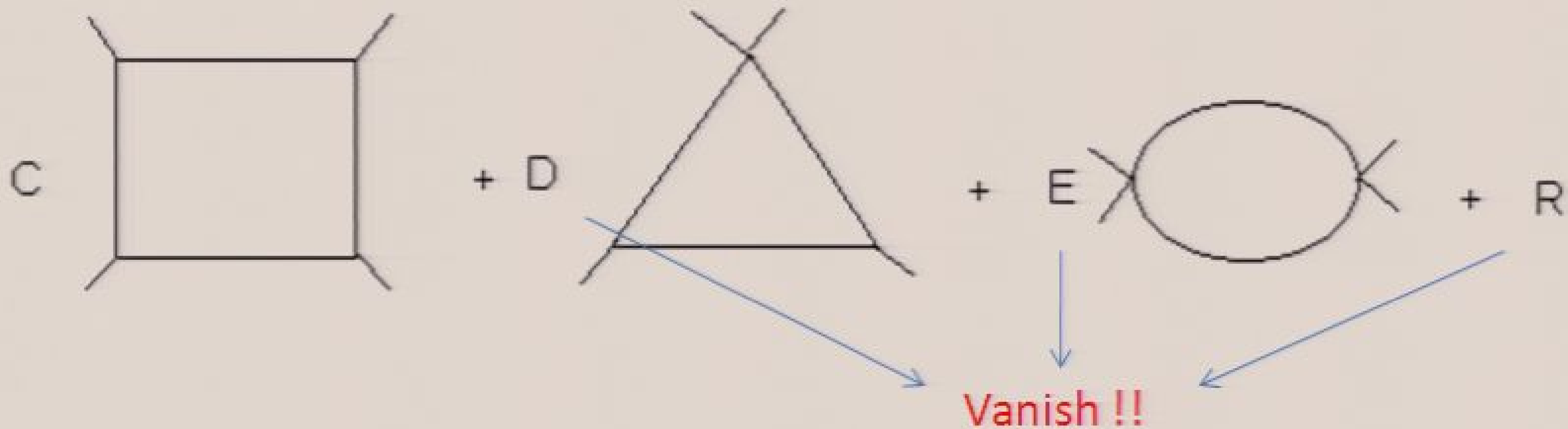
✓ N=4 SYM contains at most box functions

✓ N=8: UV-finite combination of integrals,



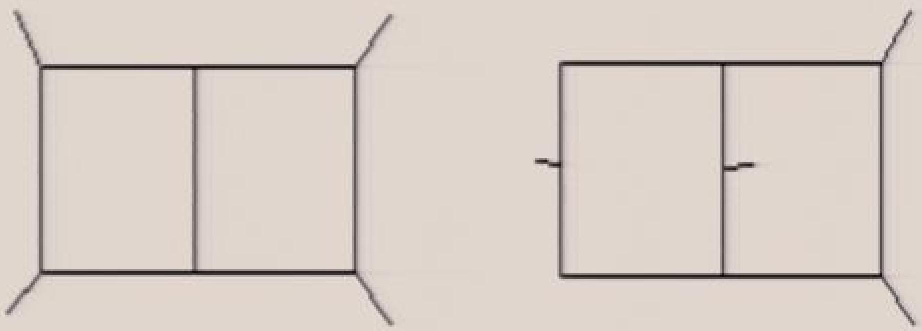
- Four point (Green, Schwarz, Brink)
- n-point MHV (Bern, Dixon, Perelstein, Rozowsky)
- 6-point and parts of 7-point (Bjerrum-Bohr, Dunbar, H.I; Bjerrum-Bohr, Dunbar, H.I., Perkins, Risager)

✓ Finite for D<8 like N=4 SYM

✓ Feeds into higher-loop via unitarity

2-loop

- ✓ Very same integral functions (Bern, Dixon, Dunbar, Perelstein, Rozowsky)

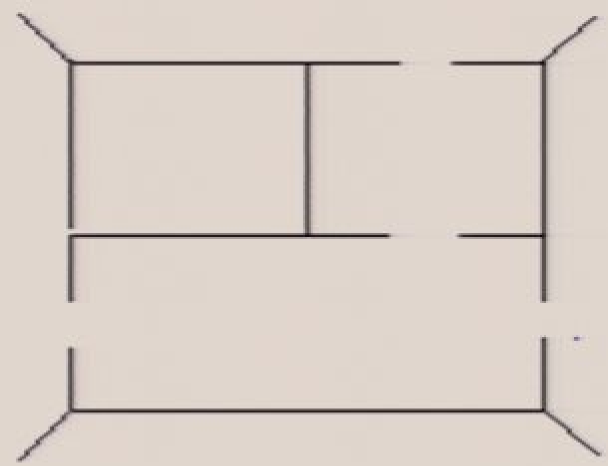


- ✓ Finite for D<7 like N=4 SYM

3-loop

Bern, Dixon, Roiban

- ✓ No complete 3-loop computation done
- ✓ Guess from part of contributions: iterated two-particle cuts (rung "rule")



It turns out that integrals differ!

? Can UV structure of theories still be the same?

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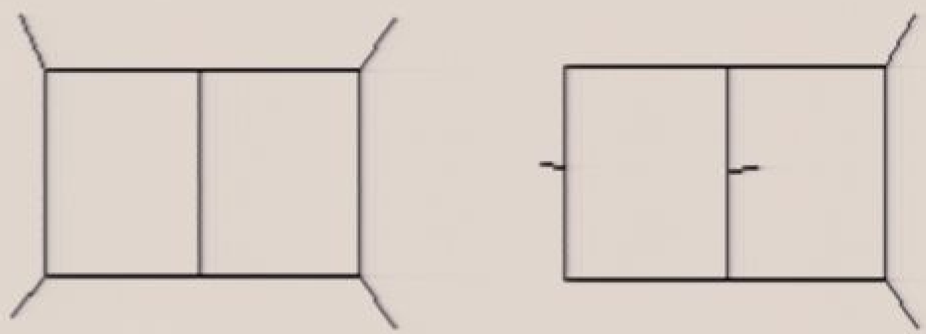

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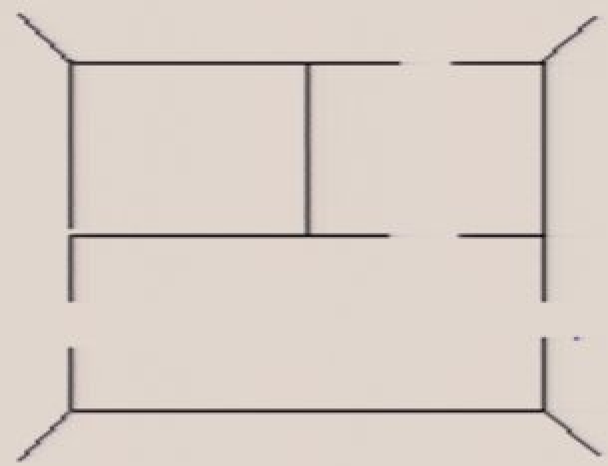

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✓ Degree of divergence for L>2 differs!

- Supergravity: D<10/L+2
- SYM: D< 6/L+4

No finiteness? Too early to conclude!

✓ Only parts of the full amplitude

✓ Consistency with no-triangle hypothesis indicates cancellations:

planar vs. non-planar (which are not in iteration)

Conclusions II

- ✓ Surprising cancelations in N=8 supergravity.
- ✓ No full computation shows deviations of UV structure from N=4 SYM, which is finite.
- ✓ Non-trivial cancellations at 3-loop? 3-loop amplitude needed.