

Title: Non-Gaussianities from Multi-Field Inflation

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Abstract: Non-Gaussianities are a generic prediction of multi-field inflationary models and within reach of upcoming experiments. After reviewing current observational limits and the physical origin of a non-zero three point correlation function, I will discuss the possibility of detectable non-Gaussian signatures in a certain class of multi-field inflationary models, upon which assisted inflation/N-flation lies. Using the delta-N formalism within the slow roll approximation and focusing on N-flation (quadratic potentials without cross-coupling), we will see that said signatures are suppressed as the number of e-foldings grows, and that this suppression is increased in models with a broad spectrum of masses. We thus conclude that the production of a large non-Gaussian signal in models of this type is very unlikely.

Non-Gaussianities from Multi-field Inflation

1. Intro
2. Characterizing Structure
3. Multi-field Infl.
4. N.G.
5. Toy Models
6. Conclusions

references

- T.B. Easter astro-ph/0610296
- Vernizzi, Wands - " - /0603799
- Maldacena, - " - /0210603
- Seery, Lidsey, - " - /0506056
- " - /0611034

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Non-Gaussianities from Multi-field Inflation

1. Intro

2. Characterizing
Structure

3. Multi-field Infl.

4. Models

1. Observ.

Non-Gaussianities from Multi-field Inflation

1. Intro

1. Observ.: CMBR

2. Characterizing
Structure

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1. Observ.: CMBR $\rightarrow$ seeds of Structure

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1. Observ.: CMBR $\rightarrow$ seeds of structure

$\downarrow$
 $\bullet n_s \approx 1$



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1. Observ.: CMBR $\rightarrow$ seeds of structure

$\downarrow$
 $\bullet n_s \approx 1$

$\bullet r$

$\bullet$

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1. Observ.: CMBR $\rightarrow$ seeds of structure

$\downarrow$
 $\bullet n_s \approx 1$

$\bullet r$

$\bullet f_{\text{NL}}$

$\bullet T_{\text{NL}}$

$\vdots$

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1. Intro

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1. Observ.: CMBR $\rightarrow$ seeds of structure

$\downarrow$

- $n_s \approx 1$

- r

- $f_{NL}(N.G.)$

- τ_{NL}

- $\dots$



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1. Intro

2. Characterizing Structure

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4. N.G.

5. Toy-1

6. Concl.

1. Observ.: CMBR $\rightarrow$ seeds of structure

Theory:

$\cdot n_s \approx 1$

$\cdot r$

$\cdot f_{NL}(N.G.)$

$\cdot \tau_{NL}$
 $\vdots$

Non-Gaussianities from Multi-field Inflation

1. Intro

2. Characterizing Structure

3. Multi-field Infl.

4. 1

5. Models

6. 1

1. Observ.: CMBR $\rightarrow$ seeds of structure

Theory: Inflation

$\cdot n_s \approx 1$

$\cdot r$

$\cdot f_{NL} (N.G.)$

$\cdot T_{NL}$

$\vdots$

Non-Gaussianities from Multi-field Inflation

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6. Conclusions

1. Observ.: CMBR $\rightarrow$ seeds of structure

Theory: Inflation

$\left\{ \begin{array}{l} \bullet n_s \approx 1 \\ \bullet r \\ \bullet f_{NL}(N.G.) \\ \bullet \tau_{NL} \\ \vdots \end{array} \right.$

conflict $\rightarrow$

$$\underline{z.} \quad \zeta = -\psi$$

$$\underline{2.} \quad \mathcal{Z} = -\psi - \frac{H}{\dot{\xi}} \delta g + \mathcal{O}(\rho + t^2)$$

$$\underline{2.} \quad \underline{\zeta} = -\psi - \frac{H}{\dot{\xi}} \delta g + \mathcal{O}(p+1)^2)$$

$\approx SN$
 $\approx scale$

$$\underline{2.} \quad \zeta = -\psi - \frac{H}{\dot{\zeta}} \delta g + O(p+1^2)$$

$$\approx SN(\epsilon_n, \epsilon_c, x)$$

angeschle

$$\underline{2.} \quad \zeta = -\psi - \frac{H}{\xi} S_g + \mathcal{O}(p_+ t^2)$$

$$\underset{\text{Lageschle}}{\approx} SN(\epsilon_+, \epsilon_-, x)$$

$$\begin{aligned}
 \underline{2.} \quad \zeta &= -\psi - \frac{H}{\dot{\zeta}} \delta g + \mathcal{O}(\rho + t^2) \\
 &\approx \underset{\text{kurzgesch.}}{SN(\epsilon_n, \epsilon_c, x)} \frac{\overline{T.T.}}{\underline{\quad}} \sum_k
 \end{aligned}$$

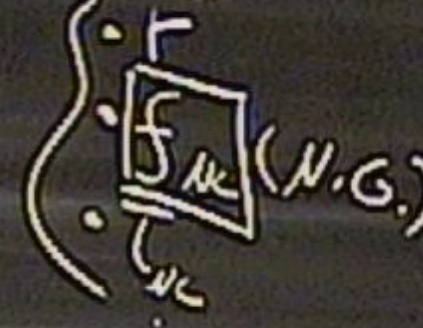
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1. Observ.: CMBR $\rightarrow$ seeds of structure

Theory: Inflation $\left\{ \begin{array}{l} \bullet n_s \approx 1 \\ \bullet f_{NL} \text{ (N.G.)} \end{array} \right.$

Corr. fct. $\rightarrow$



$$\underline{2.} \quad \mathcal{Z} = -\psi - \frac{H}{\xi} \delta g + \mathcal{O}(p+q^2)$$

$$\underset{\text{kurzgesch.}}{\approx} SN(\epsilon_n, \epsilon_c, x) \overset{\text{F.T.}}{\rightarrow} \boxed{\sum_k}$$

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1. Observ.: CMBR $\rightarrow$ seeds of structure

Theory: Inflation $\left\{ \begin{array}{l} \bullet n_s \approx 1 \\ \bullet f \\ \bullet \end{array} \right.$

curv. $\rightarrow$ $\left[\begin{array}{l} f \\ f_{NL} \end{array} \right] (N.G.)$

$\downarrow$

$$\underline{2.} \quad \zeta = -\psi - \frac{H}{\xi} \delta g + \mathcal{O}(p+1)^2)$$

$$\underset{\text{large scale}}{\approx} SN(t_*, t_c, x) \stackrel{\text{T.T.}}{=} \boxed{\sum_k}$$

$$\underline{2.} \quad \zeta = -\psi - \frac{H}{\xi} \delta g + \mathcal{O}(p+1)^2$$

$$\underset{\text{large scale}}{\zeta} \approx \delta N(t_n, t_{c, \pm}) \overset{\text{F.T.}}{=} \boxed{\zeta_k}$$

$$\langle \zeta_{k_1}, \zeta_{k_2} \rangle \propto \delta^2(\sum k_i) \frac{P_\zeta(k)}{k^3}$$

$$\underline{2.} \quad \mathcal{Z} = -\psi - \frac{H}{\mathcal{Z}} \delta g + \mathcal{O}(p+q^2)$$

$$\underset{\text{Langescale}}{\mathcal{Z}} \approx SN(\epsilon_{+1}, \epsilon_{-1}, x) \overset{\text{F.T.}}{=} \boxed{\mathcal{Z}_k}$$

$$\langle \mathcal{Z}_{k_1}, \mathcal{Z}_{k_2} \rangle \propto \delta^2(\sum \epsilon_i) \frac{P_{\mathcal{Z}}(k)}{k^3}$$

$$\langle \vec{z}_1, \vec{z}_2, \vec{z}_3 \rangle \propto \delta^3(\vec{z}_1)$$



$$\langle \zeta_{z_1}, \zeta_{z_2}, \zeta_{z_3} \rangle \propto \delta^3(\bar{z}_{z_1})$$



$$\langle \bar{z}_1, \bar{z}_2, \bar{z}_3 \rangle \propto \delta(\bar{z}_1) \mathcal{B}_3(k_1, k_2, k_3)$$



$$\langle \zeta_{k_1}, \zeta_{k_2}, \zeta_{k_3} \rangle \propto \delta(\sum \zeta_{k_i}) \mathcal{P}_3(k_1, k_2, k_3)$$

$$\zeta_{\text{NL}} \propto \frac{\overline{16 k^3 \mathcal{P}_3}}{\sum k_i^3 \overline{P^2}} \Big|_3$$

$$\langle \vec{k}_1, \vec{k}_2, \vec{k}_3 \rangle \propto \delta^3(\vec{\Sigma k}_i) \mathcal{P}_3(k_1, k_2, k_3)$$

$$\xi_{NL} \propto \frac{16 k^3 \mathcal{P}_3}{\Sigma k_i^3 \mathcal{P}_3^2}$$

$$\text{WMAP3: } 54 < f_{NL} < 114$$

$$\langle \zeta_1, \zeta_2, \zeta_3 \rangle \propto \delta^3(\vec{\zeta}_i) \mathcal{P}_\zeta(k_1, k_2, k_3)$$

$$\zeta_{\text{lin}} \propto \frac{10 k^3 \mathcal{P}_\zeta}{5 k^3 \mathcal{P}_\zeta^2}$$

$$\text{WMAP 3: } -54 < f_{\text{NL}} < 114$$

$\xrightarrow{\text{Planck}}$

$f_{\text{NL}} \sim 1$ detectable

$$\langle \zeta_{k_1}, \zeta_{k_2}, \zeta_{k_3} \rangle \propto \delta^3(\vec{\zeta}_{k_1}) B_{\zeta}(k_1, k_2, k_3)$$

$$f_{NL} \propto \frac{16k^3 B_{\zeta}}{\sum k_i^3 P_{\zeta}^2}$$

$$\text{WMAP3: } -54 < f_{NL} < 114$$

P_{NL}^{int}

$f_{NL} \sim 1$ detectable

singlefield:

$$S_{\text{sc}} = \frac{6}{5} (m_s + \xi)$$

References

- T.B. Easther astro-ph/0610296
- Vernizzi, Wands - " - 10603799
- Maldacena. - " - 10210603
- Geary, Lidsey, - " - 10506056
- " - 10611034

single field:

$$S_{NL} = -\frac{\sqrt{5}}{\sqrt{2}} \left(n_s + \underset{\substack{\uparrow \\ 0 \downarrow}}{\frac{5}{6} \triangle}} n_t \right)$$

references

- T. B. Easther astro-ph/0610296
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- Maldacena, - " - 10210603
- See "y, - " - 10506056
- " - 10611034

$$\langle \vec{z}_1, \vec{z}_2, \vec{z}_3 \rangle \propto \delta^3(\vec{z}_1) B_S(k_1, k_2, k_3)$$

$$f_{NL} \propto \frac{\overline{k^3} B_3}{\overline{k^2} \overline{P_2}} \Big|_S$$

$$\underline{WMAP3}: -54 < f_{NL} < 114$$

Planck
→

$f_{NL} \sim 1$ detectable

$$\langle \zeta_{k_1}, \zeta_{k_2}, \zeta_{k_3} \rangle \propto \delta^3(\vec{\zeta}_{k_1}) P_{\zeta}(k_1, k_2, k_3)$$

$$f_{NL} \propto \frac{\overline{U} k^3 P_{\zeta}}{\sum k_i^3 \overline{P}_{\zeta}^2}$$

$$\text{WMAP3: } -54 < f_{NL} < 114$$

Planck

$f_{NL} \sim 1$ detectable

$$\underline{3.} \quad W = \sum_i V_i(\varphi_i)$$

references

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- Vernieri, Wands - " - *10603799*
- Maldacena, - " - *10210603*
- Seery, Lidsey, - " - *10506056*
- " - *10611034*

$$3. \quad W = \sum_{i=1}^N V_i(q_i)$$

$$3H\dot{q}_i = -\frac{\partial V_i}{\partial q_i}$$

$$3H^2 = W$$

References

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- " - *10611034*

$$3. \quad W = \sum_{i=1}^N V_i(\varphi_i)$$

$$3H\dot{\varphi}_i = -\frac{\partial V_i}{\partial \varphi_i}$$

$$3H^2 = W$$

$$\epsilon_i = \frac{1}{2} \frac{V_i^2}{W}$$

$$\eta_i = \frac{V_i''}{W}$$

References

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- Seery, Lidsey, - " - 10506056
- " - 10611034

$$N = \int_{\Sigma} H dt = - \int_{\Sigma} \sum_i \frac{v_i}{v_i} d\xi_i$$

$$N = \int_{\star}^{\epsilon} H dx = - \int_{\star}^{\epsilon} \sum_i \frac{V_i}{V_i} d\varphi_i = - \frac{N}{4} (\varphi_i^{\star 2} - \varphi_i^{\epsilon 2})$$

$$V_i = \frac{1}{2} m^2 \varphi_i^2$$

$$\varphi_i^{\epsilon 2}$$

$$2N = \int_{\star}^c H dx = - \int_{\star}^c \sum_i \frac{V_i}{V_i} d\varphi_i - \frac{N}{4} (\varphi_i^{\star 2} - \varphi_i^c{}^2)$$

$$V_i = \frac{1}{2} m^2 \varphi_i^2$$

$$\rightarrow \varphi_i^{\star 2} = \frac{240}{N}$$

" $\varphi_i^c{}^2$ "

$$3 \quad W = \sum_{i=1}^{19} V_i(\varphi_i)$$

$$3H\dot{\varphi}_i = -\frac{\partial V_i}{\partial \varphi_i}$$

$$3H^2 = W$$

$$\varepsilon_i = \frac{1}{2} \frac{V_i^2}{W}$$

$$\eta_i = \frac{V_i}{W}$$

$$\sum \varepsilon_i \ll 1$$

references

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- Maldacena - " - *10210603*
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10611034

$$N = \int_{\star}^{\epsilon} H dx = - \int_{\star}^{\epsilon} \sum_i \frac{V_i}{V_i} d\zeta_i - \frac{N}{4} (\zeta_i^{\star})^2$$

$$4) - \sum_{\star} \frac{G}{S} =$$

$$4) - \sum_{i,j} \frac{C_{ij}}{S} = \left(\frac{1}{16} (1 + \frac{f}{S}) \right) + \frac{\sum_{i,j} N_{i,j} N_{j,i}}{(\sum_i N_{i,i})^2}$$

$0 = f = \frac{S}{6}$
 Δ



$$N = \int_{\Sigma} H d\epsilon = - \int_{\Sigma} \sum_i \frac{V_i}{V_i} d\epsilon_i - \frac{N}{4} (\varphi_i^*)^2$$

$$4) -S_{\text{H.L.}} \frac{G}{5} = \frac{1}{16} (1+f) + \frac{\sum_i N_i N_{i3} N_{i5}}{(\sum_i N_i)^2}$$

$0 = f = \frac{5}{6}$
 Δ

$N_{i3} = \frac{\partial N}{\partial \varphi_i^*}$

$$N = \int_{\star}^{\epsilon} H dx = - \int_{\star}^{\epsilon} \sum_i \frac{V_i}{V_i} d\varphi_i - \frac{N^2}{4} (\varphi_i^{\star})^2$$

$$(4) -S_{\text{H.C.}} \frac{6}{5} = \underbrace{\left(\frac{1}{16} (1+f) \right)}_{\text{small}} + \underbrace{\sum_{i,j} N_i N_j N_{i,j}}_{\text{vol}} \\ \underbrace{0=f=\frac{5}{6}}_{\Delta} \quad \frac{(\sum N_i)^2}{2}$$

$$\sum_{NL}^{(4)} = \Theta(\varepsilon)$$

$$N = \int_{\star}^{\epsilon} H dt = - \int_{\star}^{\epsilon} \sum_i \frac{V_i}{V_i} d\varphi_i - \frac{N^2}{4} (\varphi_i^{\star})^2$$

$$4) -5\pi L \frac{6}{5} = \underbrace{\frac{1}{16} (1+f)}_{\text{small}} + \underbrace{\sum_{i,j} N_{i,j} N_{j,i} N_{i,i}}_{\substack{0=f=\frac{5}{6} \\ \Delta}} \frac{(\sum_i N_{i,i})^2}{N_{i,i} = \frac{\partial N}{\partial \varphi_i^{\star}}}$$

$$S_{NL}^{(4)} = \mathcal{O}(\varepsilon) + \frac{\sum_{k,l} \frac{u_k u_l}{\varepsilon_k \varepsilon_l} A_{kll}}{\left(\sum_k \frac{u_k}{\varepsilon_k} \right)^2}$$

$$J_{in} = \frac{(V_{in}^* - V_{in})}{W^*} + \frac{W^c}{W^*} \frac{\epsilon_{in}^c}{\epsilon^c}$$

$$A_{in} = -\frac{W_c^2}{W^*} \left(\sum_{j=1}^N \epsilon_j \left(\frac{\epsilon_j}{\epsilon} - S_{ej} \right) \left(\frac{\epsilon_{in}^c}{\epsilon} - S_{in,j} \right) \left(1 - \frac{\epsilon_j}{\epsilon} \right) \right)^c$$

$$U_k = \frac{(V_k^* - V_k^c)}{W^*} + \frac{W^c}{W^*} \frac{\varepsilon_k^c}{\varepsilon^c}$$

$$A_{cc} = -\frac{W_c^2}{W_g^2} \left(\sum_{j=1}^{17} \varepsilon_j \left(\frac{\varepsilon_j}{\varepsilon} - \delta_{ej} \right) \left(\frac{\varepsilon_k}{\varepsilon} - \delta_{ek} \right) \left(1 - \frac{\eta_1}{\varepsilon} \right) \right)^c$$

$$S_{NL}^{(4)} = \mathcal{O}(\varepsilon) + \frac{\sum_{k,l} \frac{u_k u_l}{\varepsilon_k \varepsilon_l} A_{kll}}{\left(\sum_k \frac{u_k}{\varepsilon_k} \right)^2}$$

$$t_0 \rightarrow t_m$$

$$\left(\frac{1}{t_0} \frac{dt}{dt} \right)$$

$$\sum_{N,L}^{(4)} = O(\varepsilon) + \sum_{k,L} \frac{u_k u_L}{\varepsilon_k \varepsilon_L} A_{kLL}$$

$$\left(\sum_k \frac{u_k}{\varepsilon_k} \right)^2$$

$$t_c \rightarrow t_+$$

$$S_{NL}^{(4)} = O(\varepsilon) + \frac{\sum_{k,l} \frac{u_k u_l}{\varepsilon_k \varepsilon_l} A_{kll}}{\left(\sum_k \frac{u_k}{\varepsilon_k} \right)^2}$$

$$t_c \rightarrow t_m$$

$$U_{ke} = \frac{V_{ke} - V_{ke}}{W^*} + \frac{W}{W^*} \frac{\epsilon_{ke}}{\epsilon^*}$$

$$A_{ke} = - \frac{W_{ke}^2}{W_{st}^2} \left(\sum_{j=1}^N \epsilon_j \left(\frac{\epsilon_j}{\epsilon} - \delta_{ej} \right) \left(\frac{\epsilon_{jk}}{\epsilon} - \delta_{kj} \right) \left(1 - \frac{\eta_j}{\epsilon} \right) \right)^{\epsilon}$$

$$\sum_{n \in L}^{(4)} = O(\varepsilon) + \sum_{k, l \in \mathbb{Z}} \frac{u_k u_l}{\varepsilon_k \varepsilon_l} A_{kl}$$

$$\underbrace{\left(\sum_{k \in \mathbb{Z}} \frac{u_k}{\varepsilon_k} \right)^2}_{\neq 0}$$

$$t_c \rightarrow t_m \Rightarrow \bar{f} = 0$$

$$\sum_{n \neq l}^{(4)} = \mathcal{O}(\varepsilon) + \frac{\sum_{k, l} \frac{u_k u_l}{\varepsilon_k \varepsilon_l} A_{kl}}{\left(\sum_k \frac{u_k}{\varepsilon_k} \right)^2}$$

$$t_c \rightarrow t_+ \Rightarrow \overline{F} = 0 \quad (\text{adiabatic limit})$$

$$S_{NL} = \mathcal{O}(\epsilon) + \frac{\sum_k \frac{u_k}{\epsilon_k}}{\epsilon_0} \text{AveL}$$

$$\left(\sum_k \frac{u_k}{\epsilon_k} \right)^2$$

$$t_c \rightarrow t_+ \Rightarrow \bar{T} \rightarrow 0 \text{ (adiabatic limit)}$$

$$S_{NL}^{(4)} = \mathcal{O}(\varepsilon) + \sum_{k, l} \frac{u_k u_l}{\varepsilon_k \varepsilon_l} A_{k, l}$$

$$\underbrace{\left(\sum_k \frac{u_k}{\varepsilon_k} \right)^2}_{\overline{F}}$$

$$t_c \rightarrow t_+ \Rightarrow \overline{F} = 0 \text{ (adiabatic limit)}$$

$$U_k = \frac{(V_k^* - V_k^c)}{W^*} + \frac{W^c}{W^*} \frac{\varepsilon_k^c}{\varepsilon^c}$$

$$A_{cc} = - \frac{W_c^2}{W_s^2} \left(\frac{\varepsilon_s}{\varepsilon} - \delta_{sj} \right) \left(\frac{\varepsilon_k}{\varepsilon} - \delta_{sj} \right) \left(1 - \frac{\eta_j}{\varepsilon} \right)^c$$

$$U_k = \frac{(V_k^* - V_k^c)}{W^*} + \frac{W^c}{W^*} \frac{\varepsilon_k^c}{\varepsilon^c}$$

$$- \frac{W_c^2}{W^*} \left(\sum_{j=1}^N \varepsilon_j \left(\frac{\varepsilon_j}{\varepsilon} - \delta_{ej} \right) \left(\frac{\varepsilon_k}{\varepsilon} - \delta_{ek} \right) \left(1 - \frac{\eta_j}{\varepsilon} \right) \right)^c$$

$$U_k = \frac{(V_k^a - V_k^c)}{W^a} + \frac{W^c}{W^a} \frac{\epsilon_k^c}{\epsilon^c}$$

$$|\eta_i| \ll 1$$

$$A_{acc} = -\frac{W_c^2}{W_a^2} \left(\sum_{j=1}^N \epsilon_j \left(\frac{\epsilon_j}{\epsilon} - \delta_{ej} \right) \left(\frac{\epsilon_k}{\epsilon} - \delta_{aj} \right) \left(1 - \frac{\eta_j}{\epsilon} \right) \right)^c$$

$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$\boxed{IA}$ m

$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$$\boxed{IA} \quad m_i = m \Rightarrow F = 0$$

$$V_i = \frac{m_i^2}{2} \Omega_i^2$$

$$\boxed{IA} \quad m_1 = m \Rightarrow F = 0$$

$$\boxed{IB} \quad m_2 = 2m_1 \Rightarrow F = -\frac{9}{16 N^4}$$

$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$$\boxed{A} \quad m_1 = m \Rightarrow F = 0$$

$$\boxed{B} \quad m_2^2 = 2m_1^2 \Rightarrow F = -\frac{9}{16} \frac{N^4}{\epsilon^4}$$

$$\boxed{C} \quad m_2^2 = \alpha m_1^2 \Rightarrow F \propto \frac{1}{N^{2\alpha}}$$

$$V_i = \frac{m_i^2}{2} \Omega_i^2$$

A

$$m_2 = m \Rightarrow F = 0$$

B

$$m_2^2 = 2m_1^2 \Rightarrow F = -\frac{9}{16} \frac{N^2}{4}$$

C

$$m_2^2 = \alpha m_1^2 \Rightarrow F \propto \frac{1}{N^2 \alpha}$$

D

$$\frac{m_2^2}{m_1^2}$$

$$V_i = \frac{m_i^2}{2} \phi_i^2$$

[A] $m_1 = m \Rightarrow F = 0$

[B] $m_2 = 2m_1 \Rightarrow F = -\frac{9}{16} N^2$

[C] $m_2 = 4m_1 \Rightarrow F \propto \frac{1}{N^2}$

[D] $\frac{m_2^2}{m_1^2} = 1 - \delta \epsilon \Rightarrow F \propto \frac{\delta^2}{N^2} \propto \mathcal{O}(\epsilon^2)$

$$V_i = \frac{m_i^2}{2} \phi_i^2$$

A

$$m_1 = m \Rightarrow F = 0$$

B

$$m_2^2 = 2m_1^2 \Rightarrow F = -9$$

C

$$m_2^2 = 4m_1^2 \Rightarrow F \propto \frac{1}{N^2}$$

D

$$\frac{m_1^2}{m_2^2} = 1 - \delta \epsilon \Rightarrow F \propto \frac{\delta^2}{\delta \epsilon} \sim \mathcal{O}(\epsilon^2)$$



$$V_i = \frac{m_i^c}{2} \phi_i^2$$

A

$$m_1 = m \Rightarrow F = 0$$

B

$$m_2 = 2m_1 \Rightarrow F = -\frac{9}{16} \frac{1}{N^2}$$

C

$$m_2^c = 4m_1^c \Rightarrow F \propto \frac{1}{N^2}$$

D

$$\frac{m_1^c}{m_2^c} = 1 - \delta\epsilon \Rightarrow F \propto \frac{\delta^2}{\partial^2} \sim \mathcal{O}(\epsilon^2)$$

$$V_i = m_i^2 \phi_i^2$$

$$\alpha \geq 1$$

[A]

$$m_i = m \Rightarrow F = 0$$

[B]

$$m_2^2 = 2m_1^2 \Rightarrow F = -\frac{9}{16} \frac{1}{N^4}$$

[C]

$$m_2^2 = \alpha m_1^2 \Rightarrow F \propto \frac{1}{N^4}$$

[D]

$$\frac{m_1^2}{m_2^2} = 1 - \delta_i \Rightarrow F \propto \frac{\delta^2}{N^2} \sim O(\epsilon^2)$$

$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$$\alpha \geq 1$$

$$m_i > m_j, i < j$$

A $m_2 = m_1 \Rightarrow F = 0$

B $m_2^2 = 2m_1^2 \Rightarrow F = -\frac{q}{16\pi^2}$

C $m_2^2 = \alpha m_1^2 \Rightarrow F \propto \frac{1}{N^{\frac{1}{2}}}$

D $\frac{m_2^2}{m_1^2} = 1 - \delta_i \Rightarrow F \propto \frac{\delta_i^2}{N^2} \sim O(\epsilon^2)$



$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$$\alpha \geq 1$$

$$m_i > m_j, |\xi_i| > 1$$

A $m_2 = m \Rightarrow F = 0$

B $m_2^2 = 2m_1^2 \Rightarrow F = -9$

C $m_2^2 = \alpha m_1^2 \Rightarrow F \propto \frac{1}{16 N^{\frac{1}{2}}}$

D $\frac{m_2^2}{m_1^2} = 1 - \delta \Rightarrow F \propto \frac{1}{N^{\frac{1}{2}}}$
 $\frac{\delta^2}{N^2} \sim O(\xi^2)$

$$(14) \quad S_{NL} = \mathcal{O}(\epsilon) + \sum_{k,l} \frac{u_k u_l}{\epsilon_k^+ \epsilon_l^+} A_{kl}$$

$\epsilon \rightarrow 0 \Rightarrow \overline{T} = 0$ (asymptotic limit)

$$\langle 4 \rangle_{NL} = \mathcal{O}(\epsilon) + \sum_{k,l} \frac{u_k u_l}{\epsilon_k \epsilon_l} A_{kl}$$

$$\left(\sum_k \frac{u_k}{\epsilon_k} \right)^2$$

(adiabatic limit)

$t_c \rightarrow \infty$

$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$$\alpha \geq 1$$

$$m_i > m_j, |i| > j$$

A $m_i = m \Rightarrow F = 0$

B $m_i^2 = 2m^2 \Rightarrow F = -\frac{9}{16 N^4}$

C $m_i^2 = \alpha m^2 \Rightarrow F \propto \frac{1}{N^2 \alpha}$

D $\frac{m_i^2}{m^2} = 1 - \delta_i \Rightarrow F \propto \left(\frac{\delta^2}{N^2} \sim O(\epsilon^2) \right)$

$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$$\alpha \geq 1$$

$$m_i > m_j, |i| > j$$

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D $\frac{m_i^2}{m_j^2} = 1 - \delta_i \Rightarrow F \propto \frac{\delta_i^2}{N^2} \sim \mathcal{O}(\epsilon^2)$



$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$$\alpha \geq 1$$

$$m_i > m_j, |\delta_i| > |\delta_j|$$

A $m_i = m \Rightarrow F = 0$

B $m_i^2 = 2m_j^2 \Rightarrow F = -\frac{9}{16 N^2}$

C $m_i^2 = \alpha m_j^2 \Rightarrow F \propto \frac{1}{N^{2\alpha}}$

D $\frac{m_i^2}{m_j^2} = 1 - \delta_i \Rightarrow F \propto \left(\frac{\delta_i^2}{N^2} \right) \sim \mathcal{O}(\epsilon^2)$



$$V_i = \frac{m_i^2}{2} \phi_i^2$$

$$\alpha \geq 1$$

$$m_i > m_j, |\xi_i| > |\xi_j|$$

A $m_i = m \Rightarrow F = 0$

B $m_i^2 = 2m_j^2 \Rightarrow F = -\frac{9}{16 N^4}$

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D $\frac{m_i^2}{m_j^2} = 1 - \delta_i \Rightarrow F \propto \frac{\delta_i^2}{N^2} \sim \mathcal{O}(\xi^2)$

