Title: Non-Gaussianities from Multi-Field Inflation

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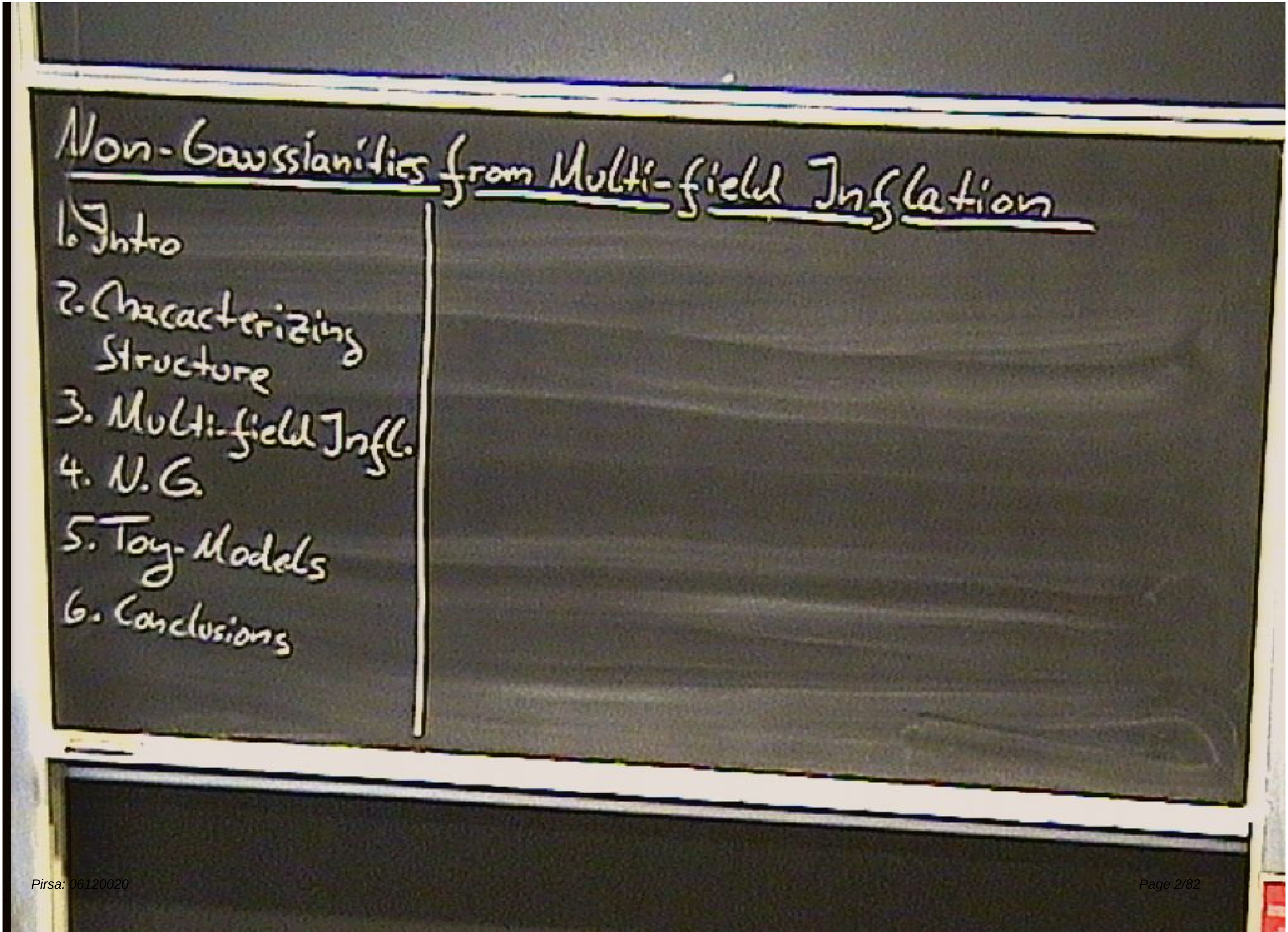
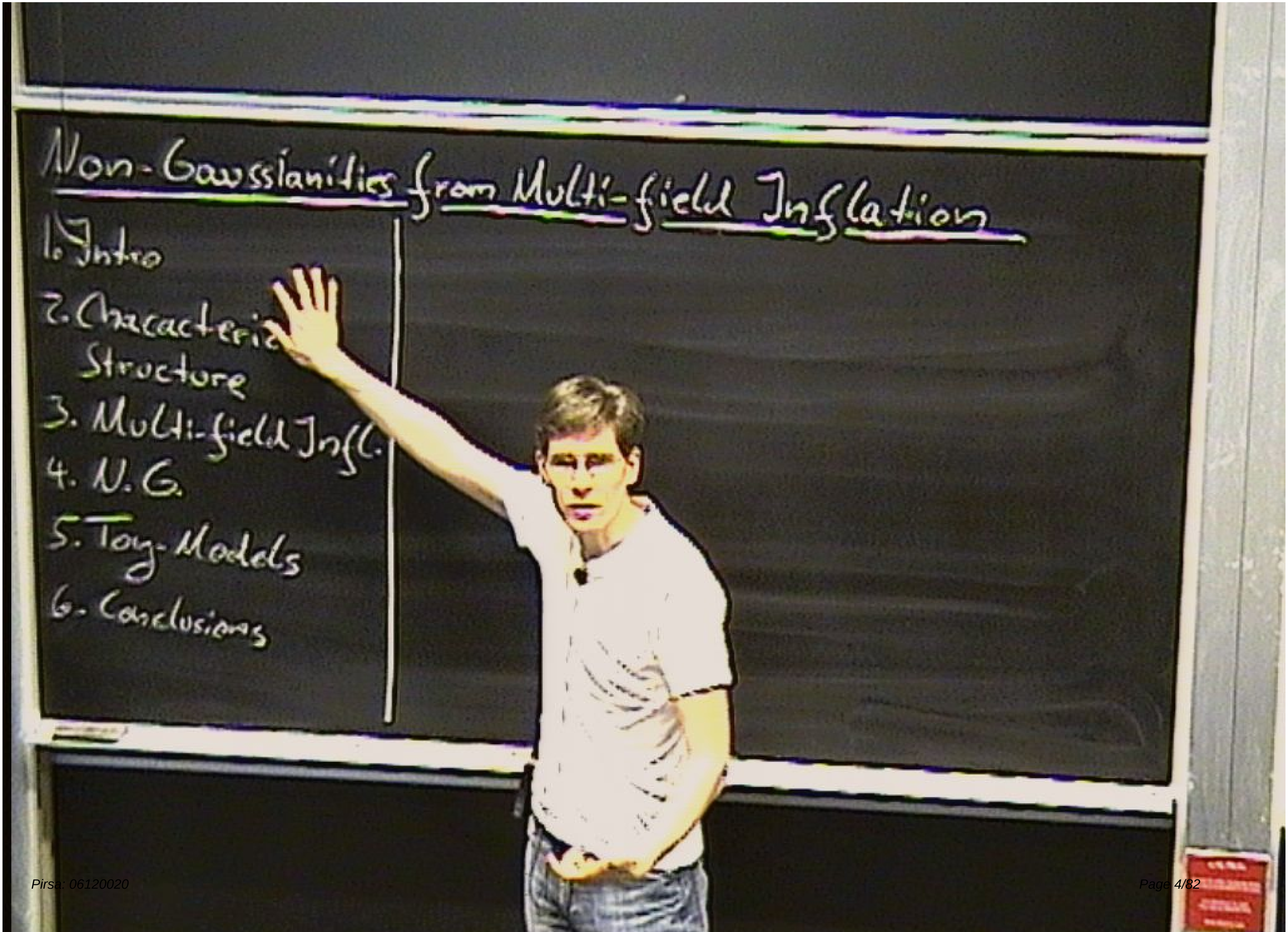
URL: http://pirsa.org/06120020

Abstract: Non-Gaussianities are a generic prediction of multi-field inflationary models and within reach of upcoming experiments. After reviewing current observational limits and the physical origin of a non-zero three point correlation function, I will discuss the possibility of detectable non-Gaussian signatures in a certain class of multi-field inflationary models, upon which assisted inflation/N-flation lies. Using the delta-N formalism within the slow roll approximation and focusing on N-flation (quadratic potentials without cross-coupling), we will see that said signatures are suppressed as the number of e-foldings grows, and that this suppression is increased in models with a broad spectrum of masses.
We thus conclude that the production of a large non-Gaussian signal in models of this type is very unlikely.


Non-Gaussianities from Multi-field Inflation
1. Intro
2. Characterizing Structure
3. Multi-field Infl.
4. N.G.
5. Toy-Models
6. Conclusions

references

- T.B., Easther astro-ph/0610296
- Vernizzi, Wands –"– /0603799
- Maldacena, –"– /0210603
- Seery, Lidsey, –"– /0506056
  –"– /0611034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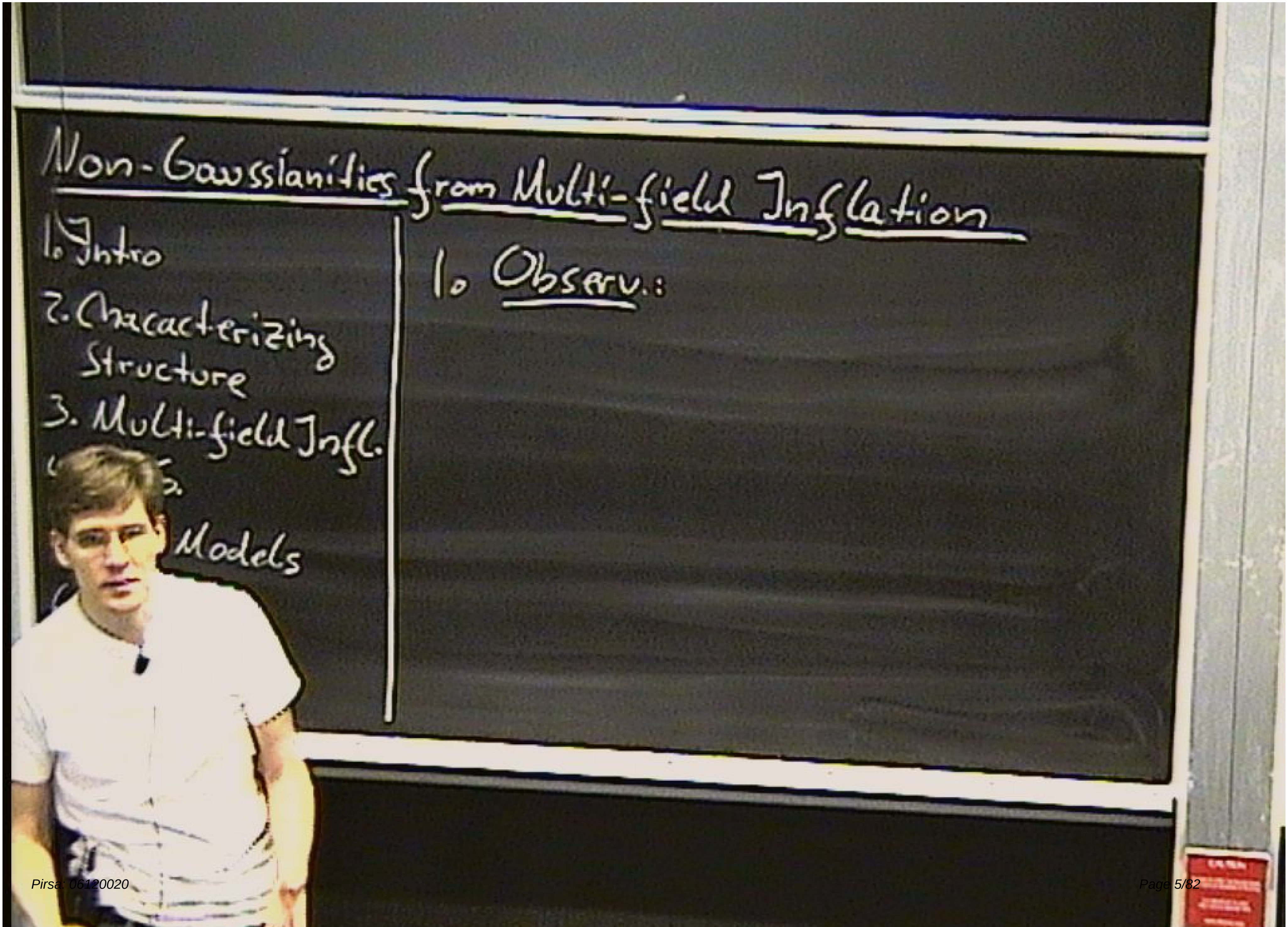
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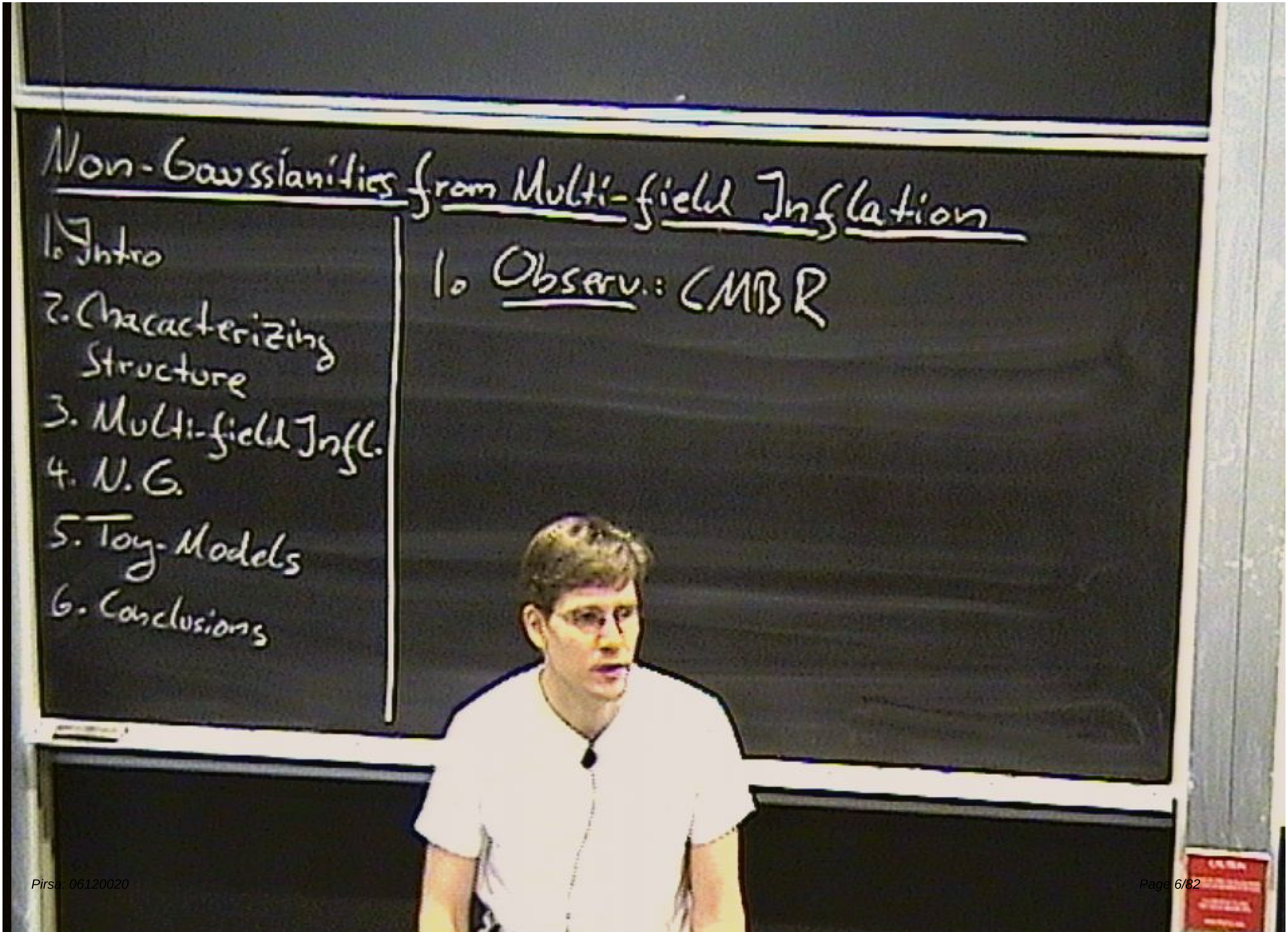
Models
1. Observ.: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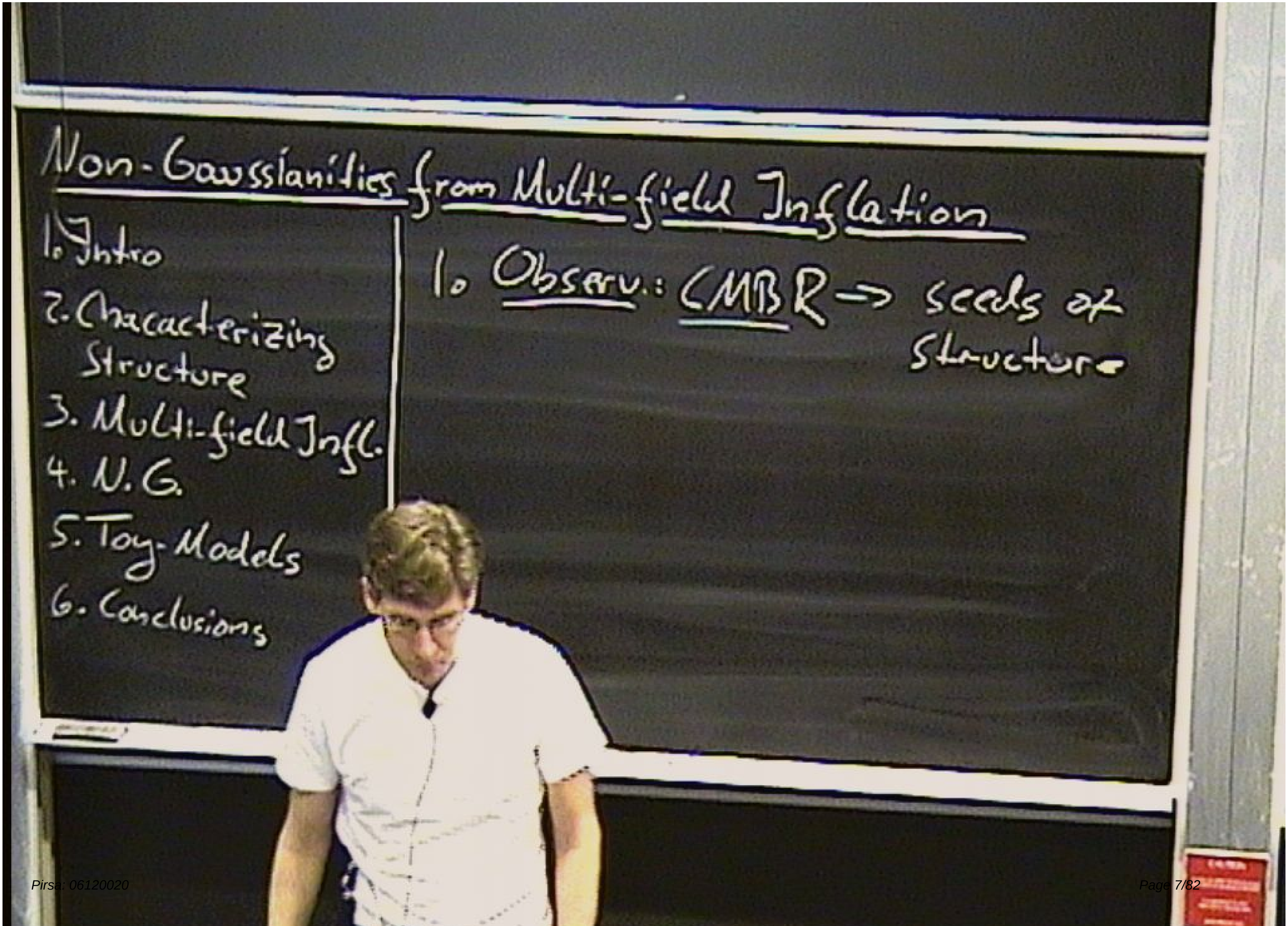
1. Observ.: CMBR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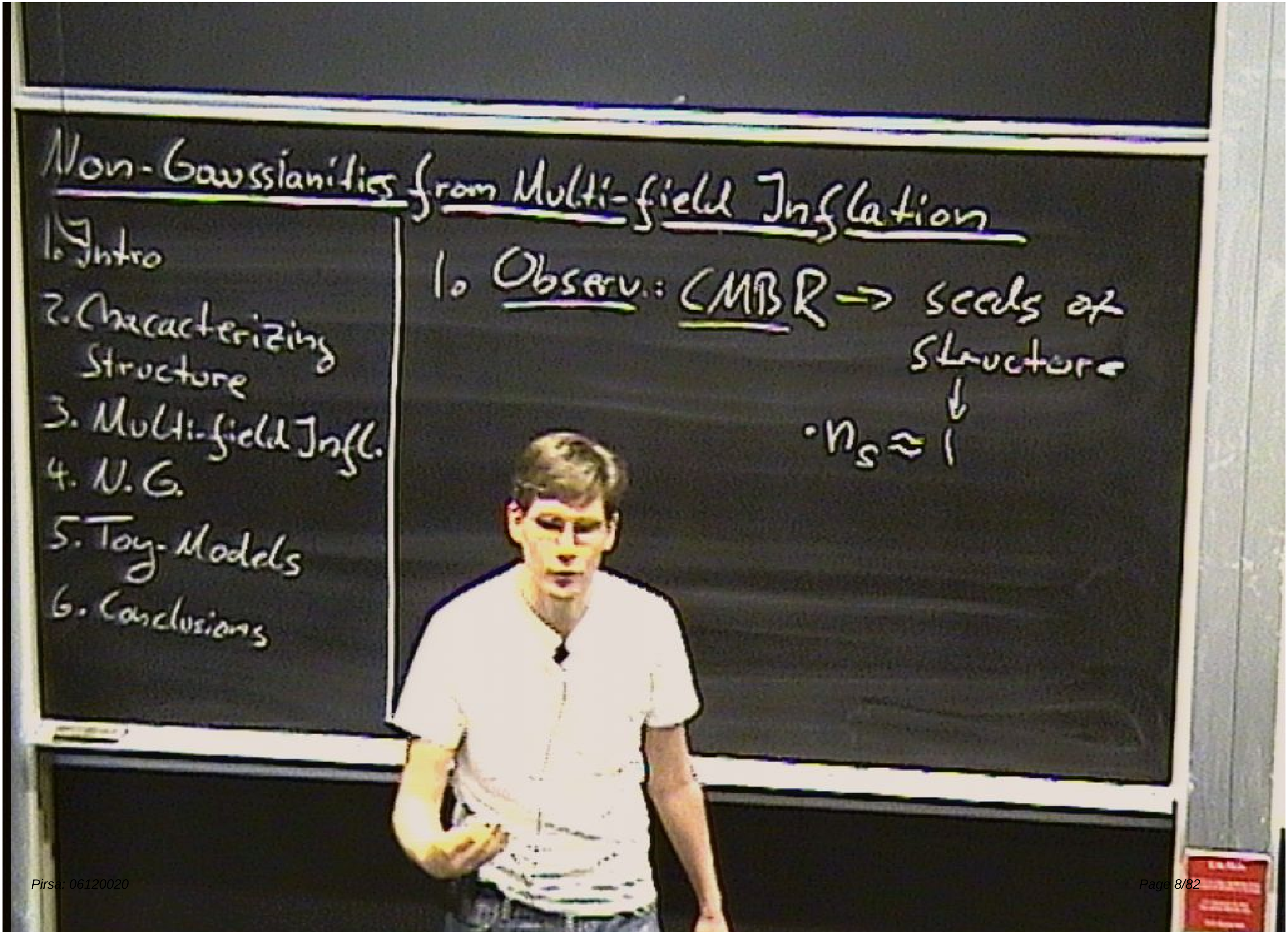
1. Observ.: CMBR → seeds of structure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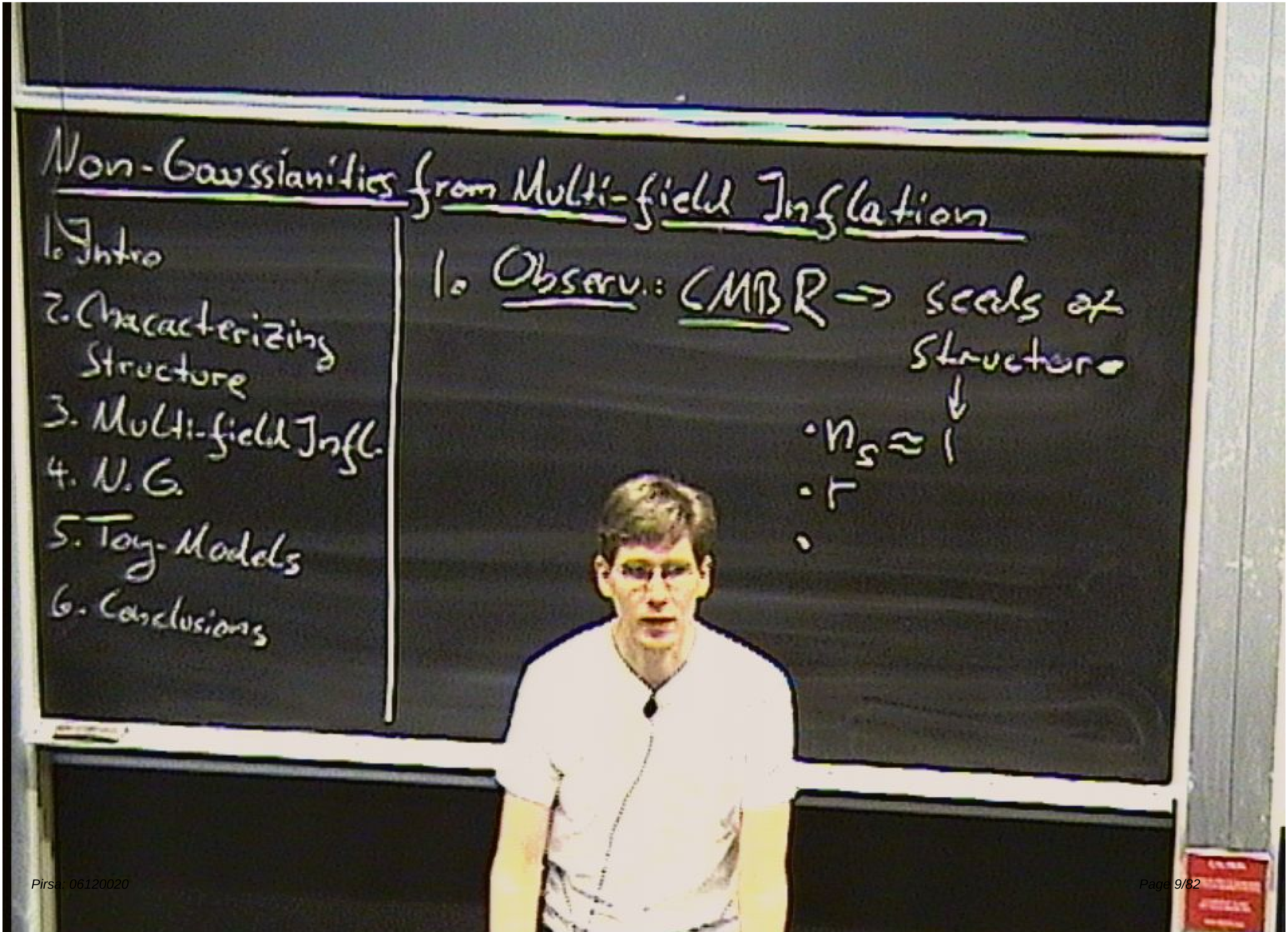
↓
• $n_s \approx 1$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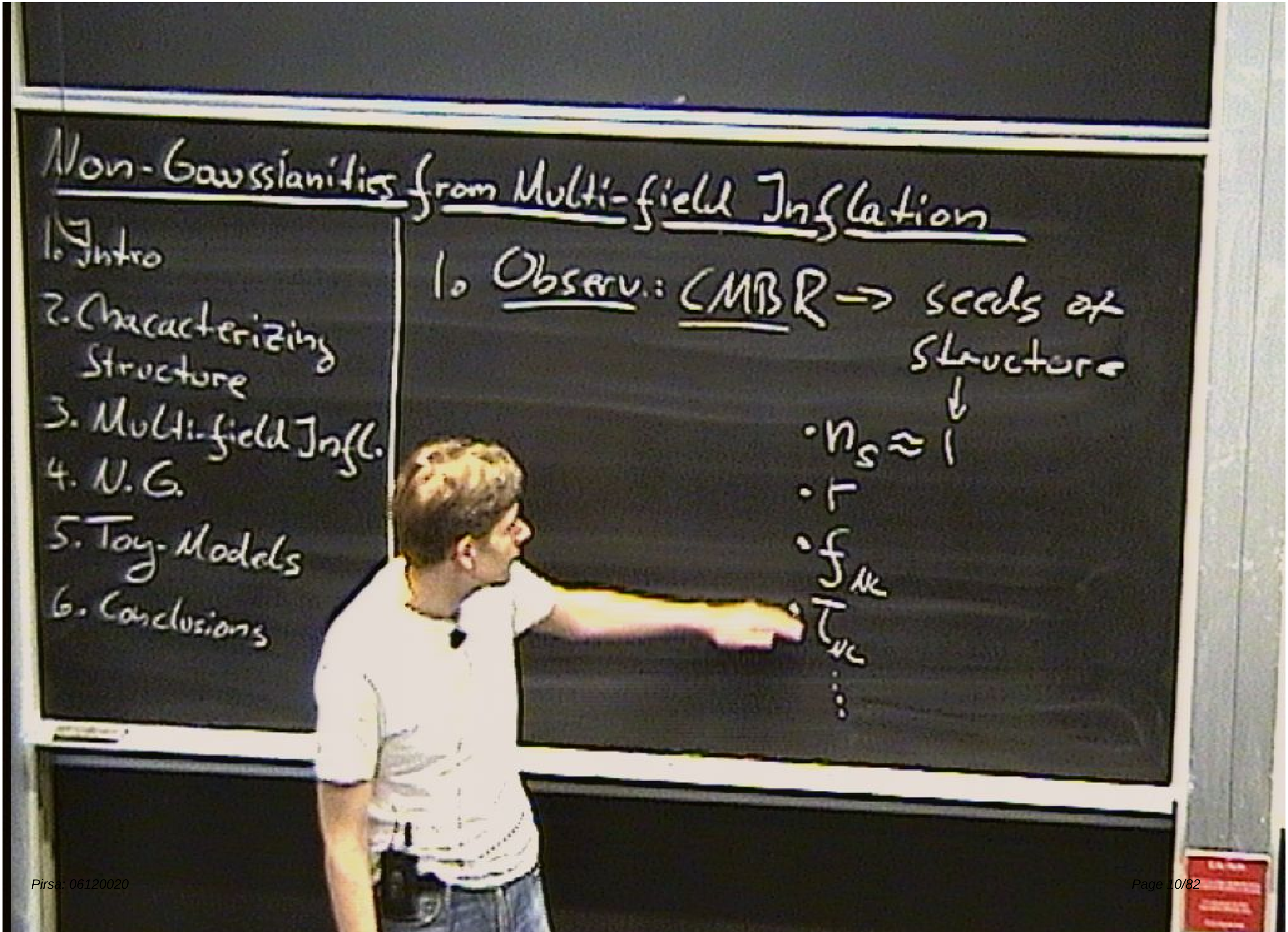
• r
•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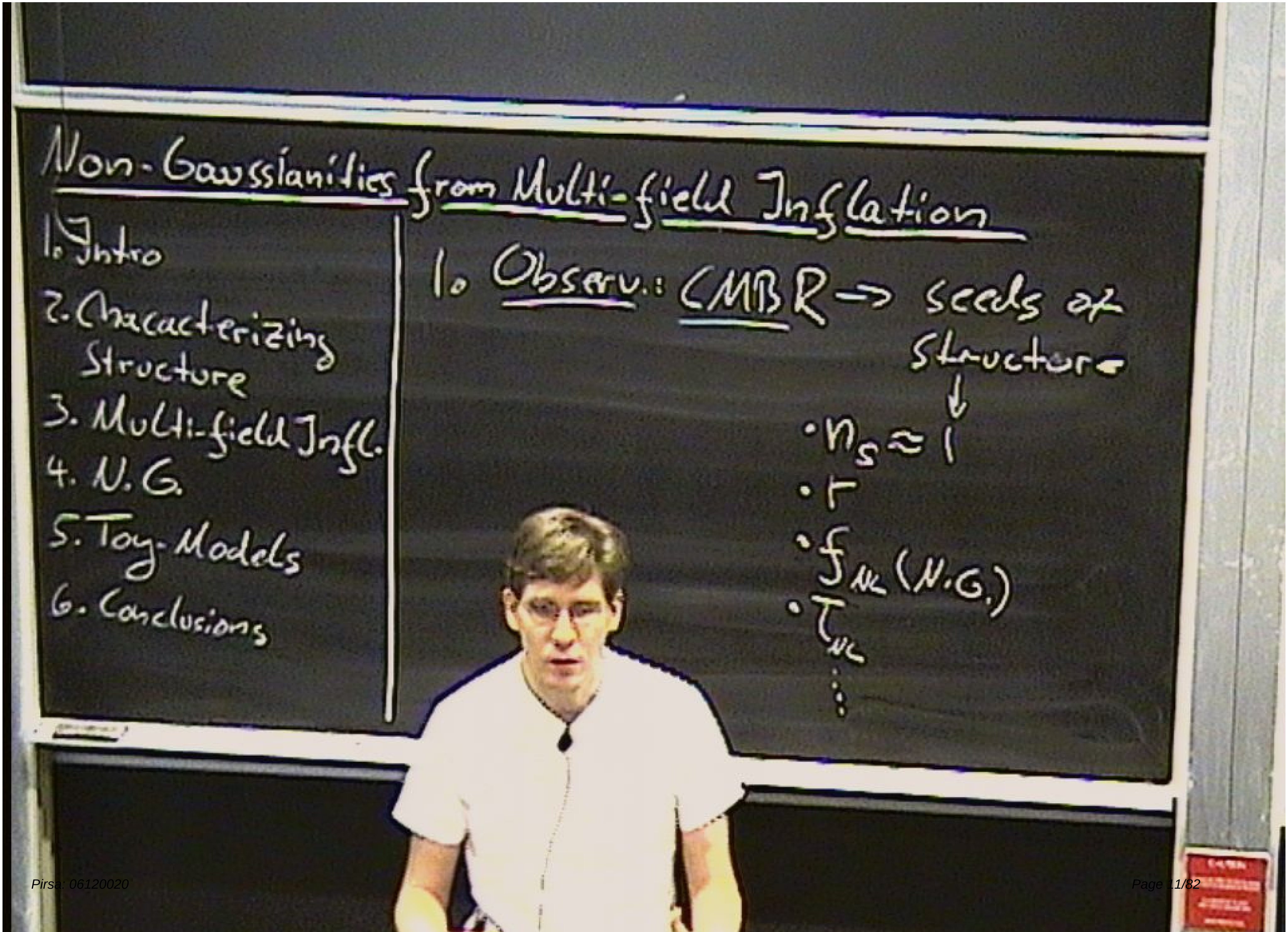
• $n_s \approx 1$
• r
• f_{NL}
• τ_{NL}
⋮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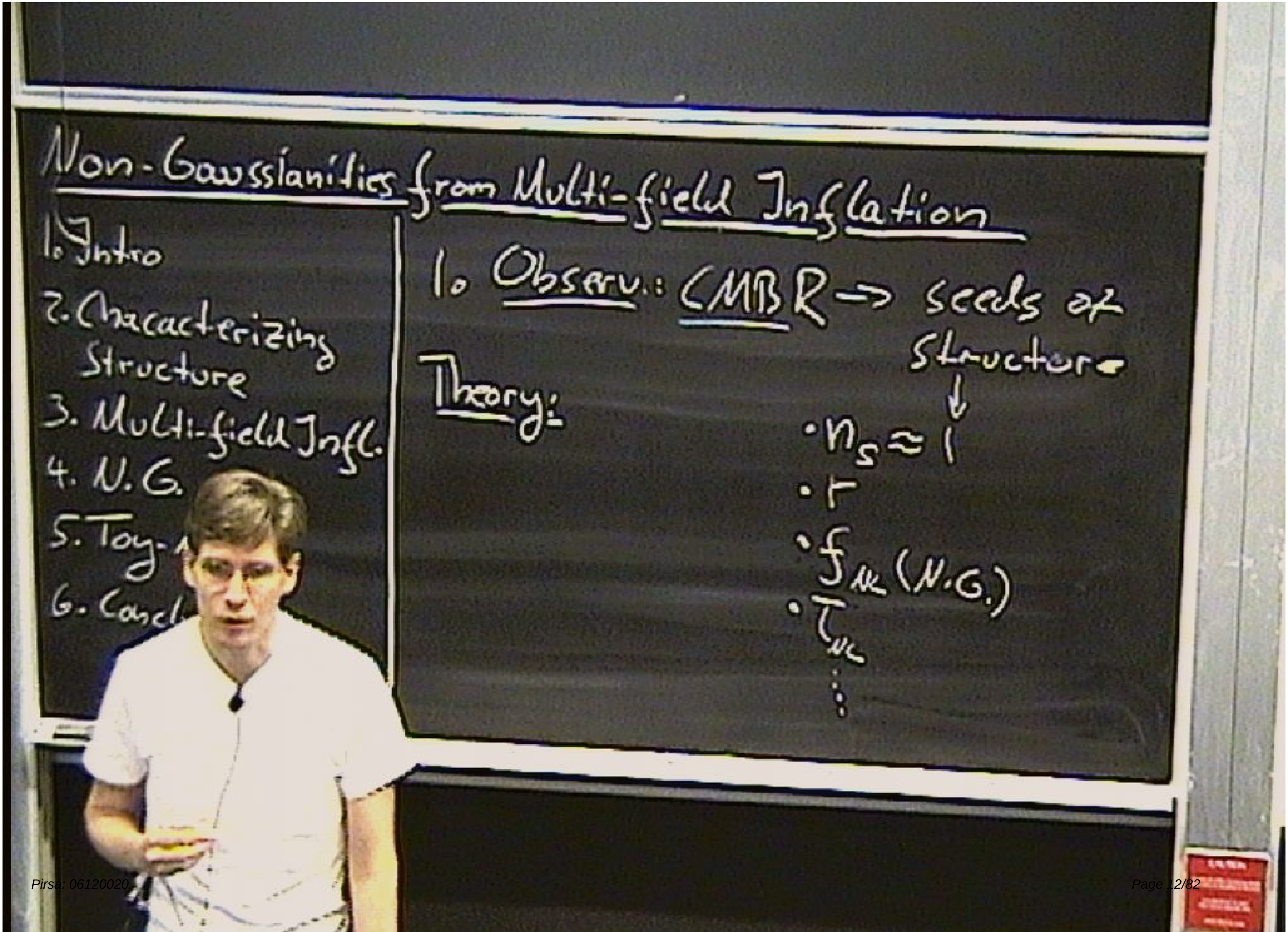
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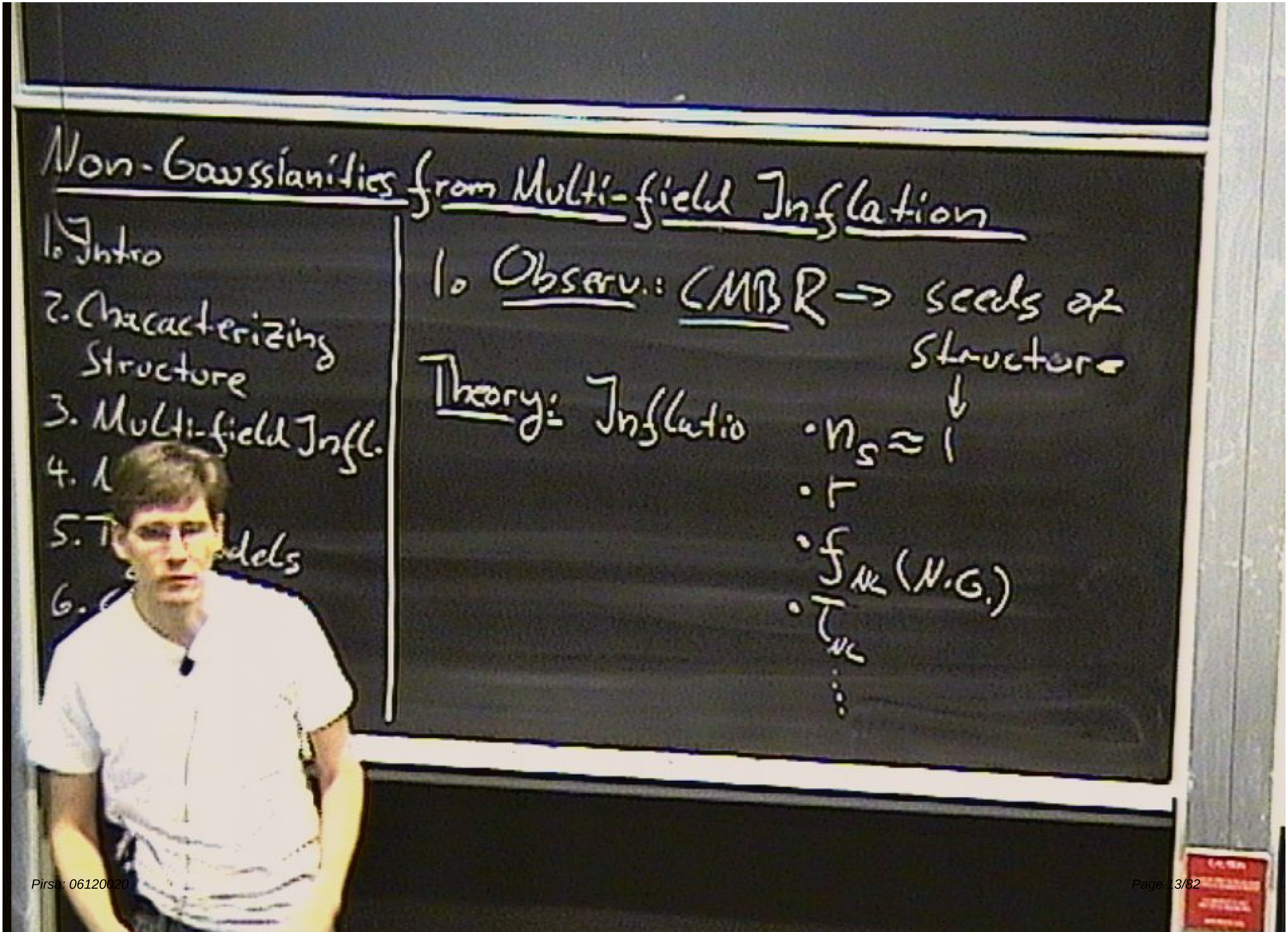
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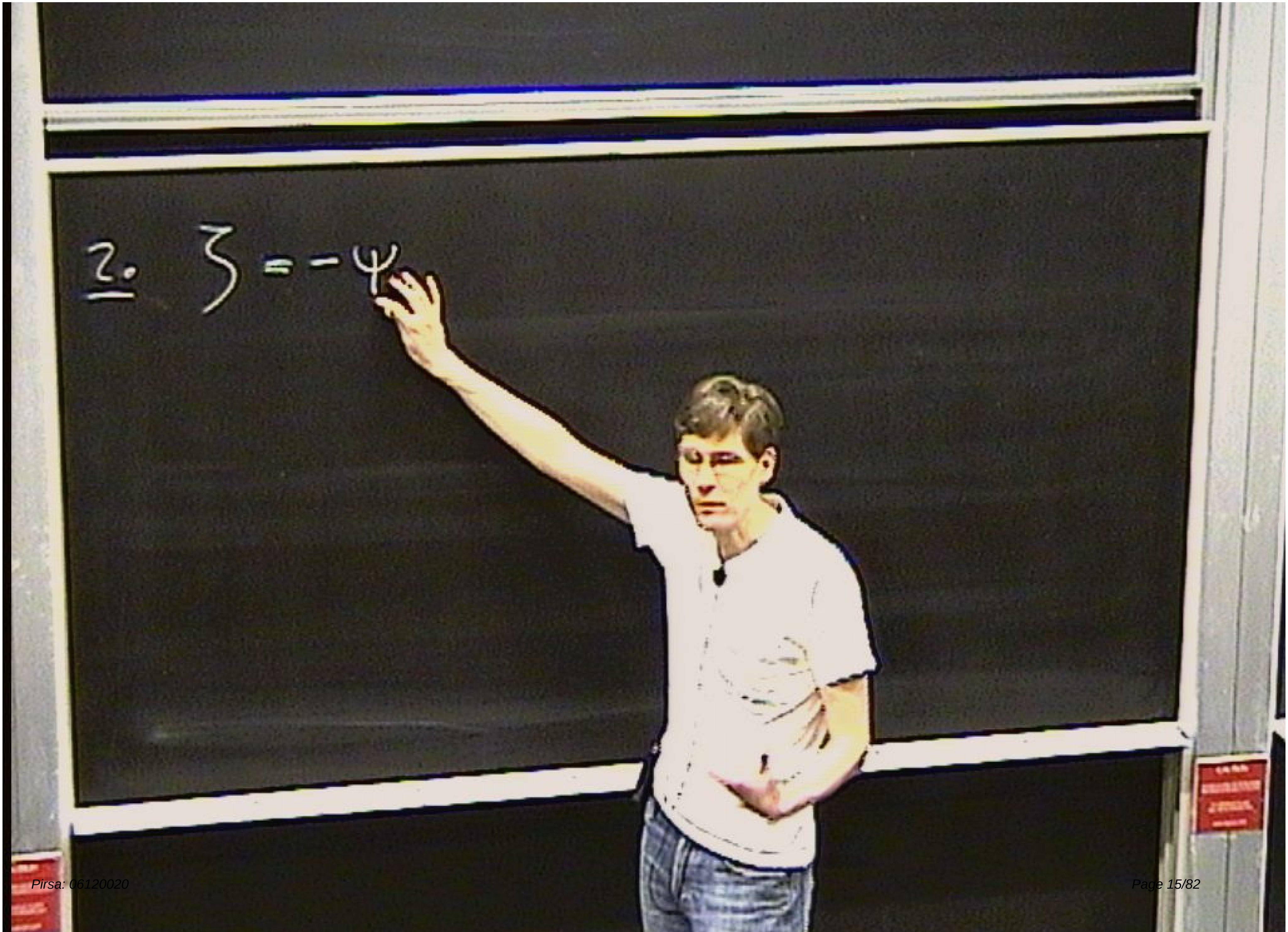
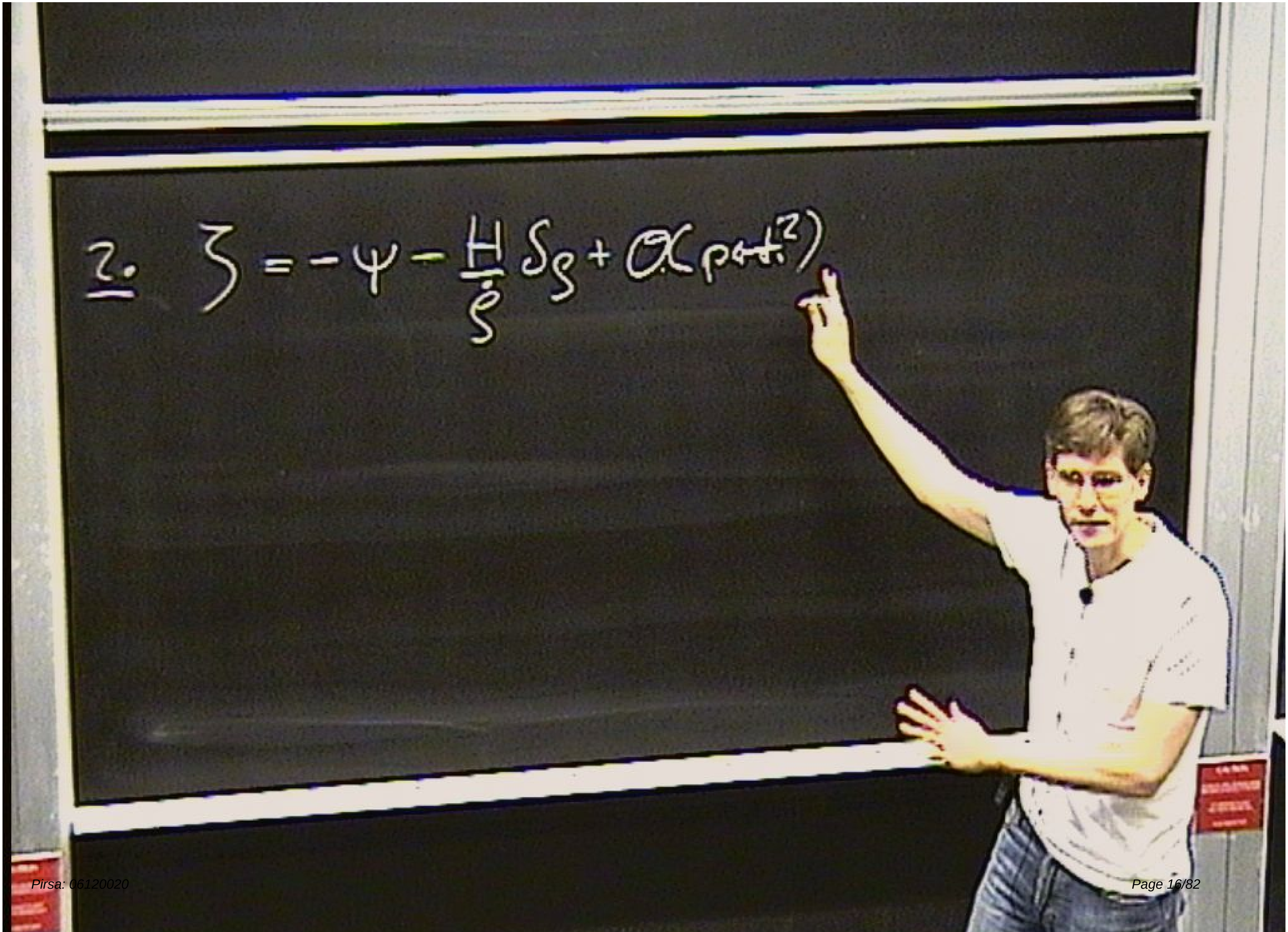
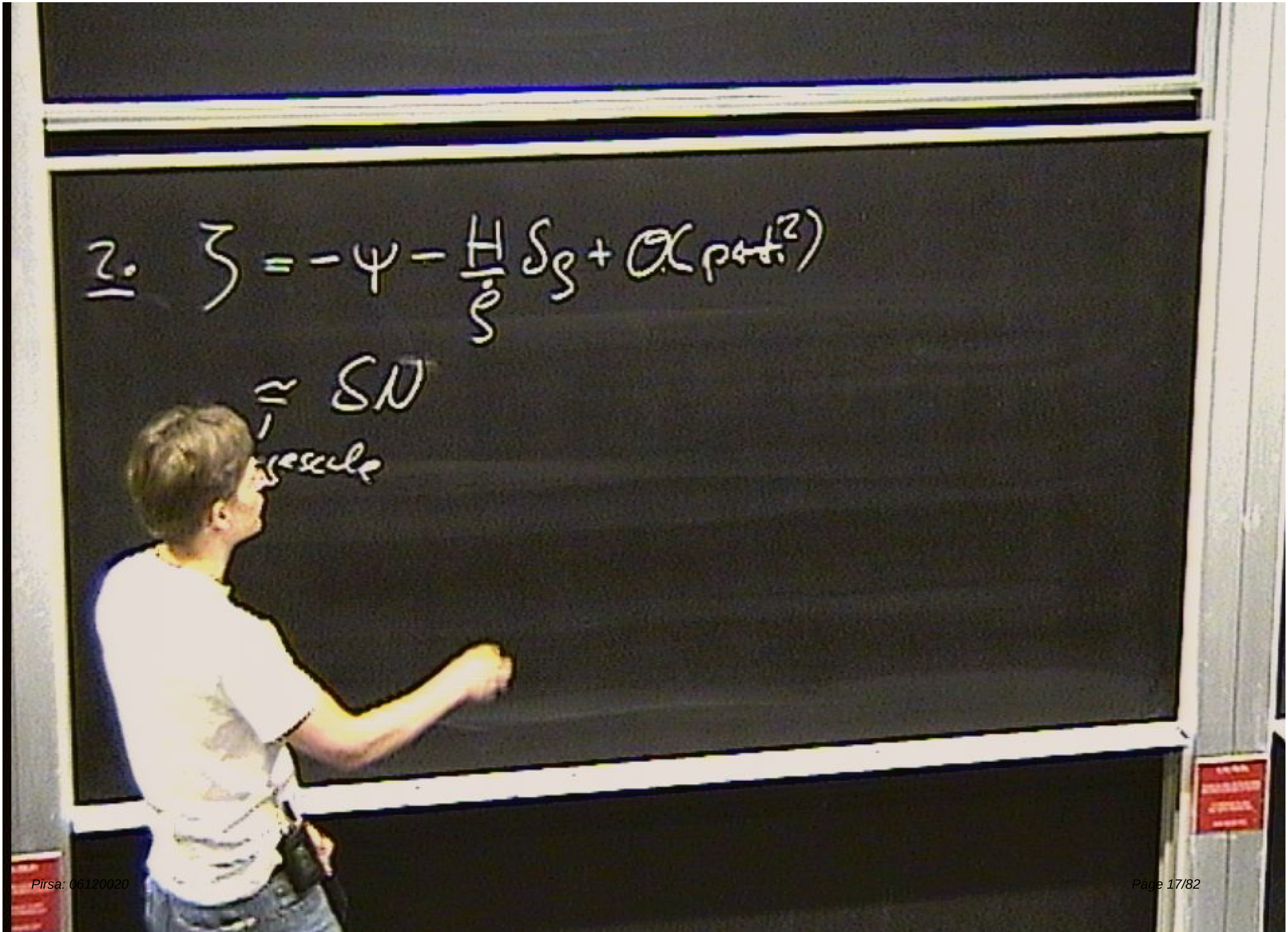
1. **Observ.:** CMBR → seeds of structure
   ↓

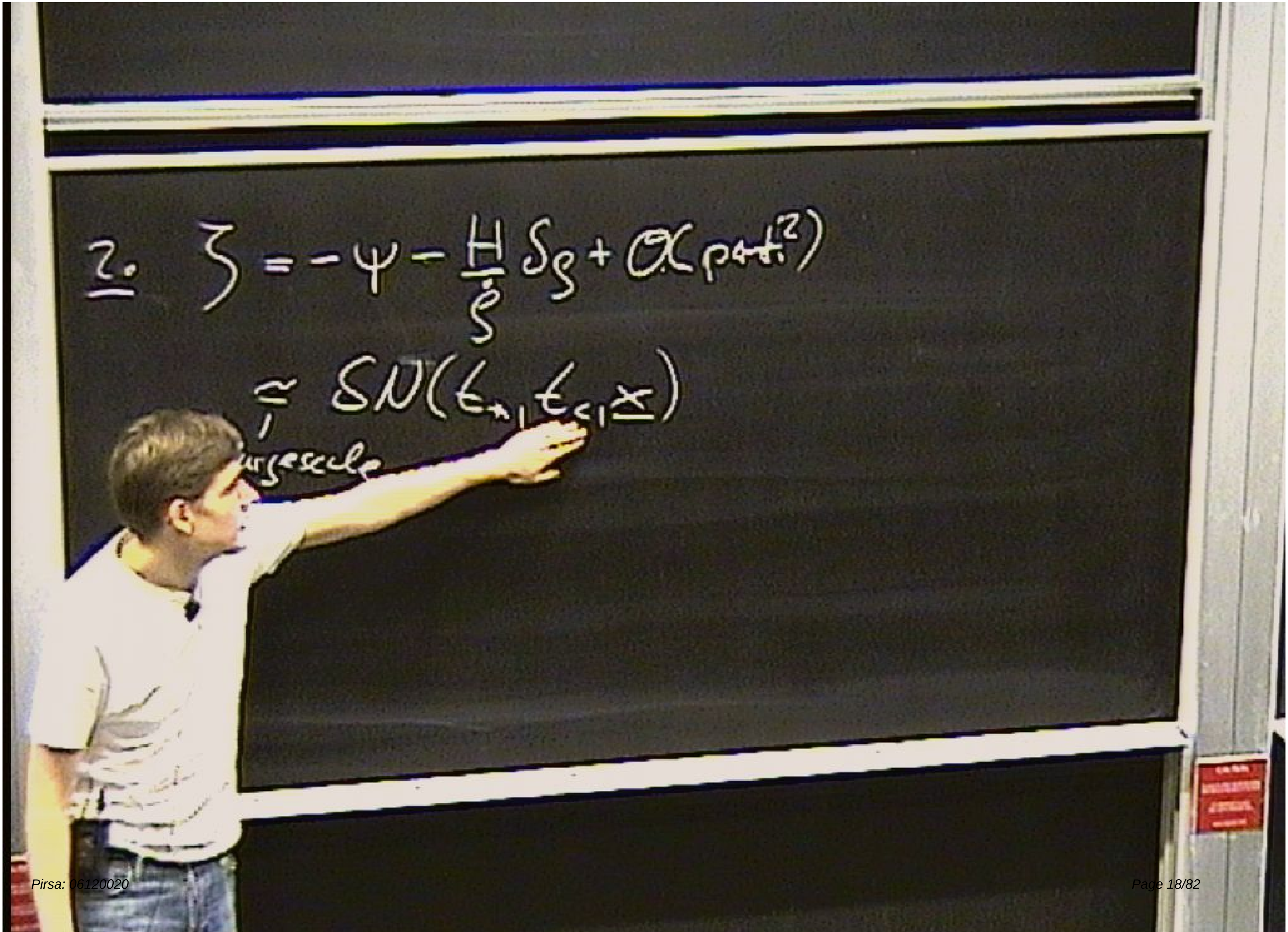
Theory: Inflation
- $n_s \approx 1$
- r
- f_{NL} (N.G.)
- τ_{NL}
- ⋮

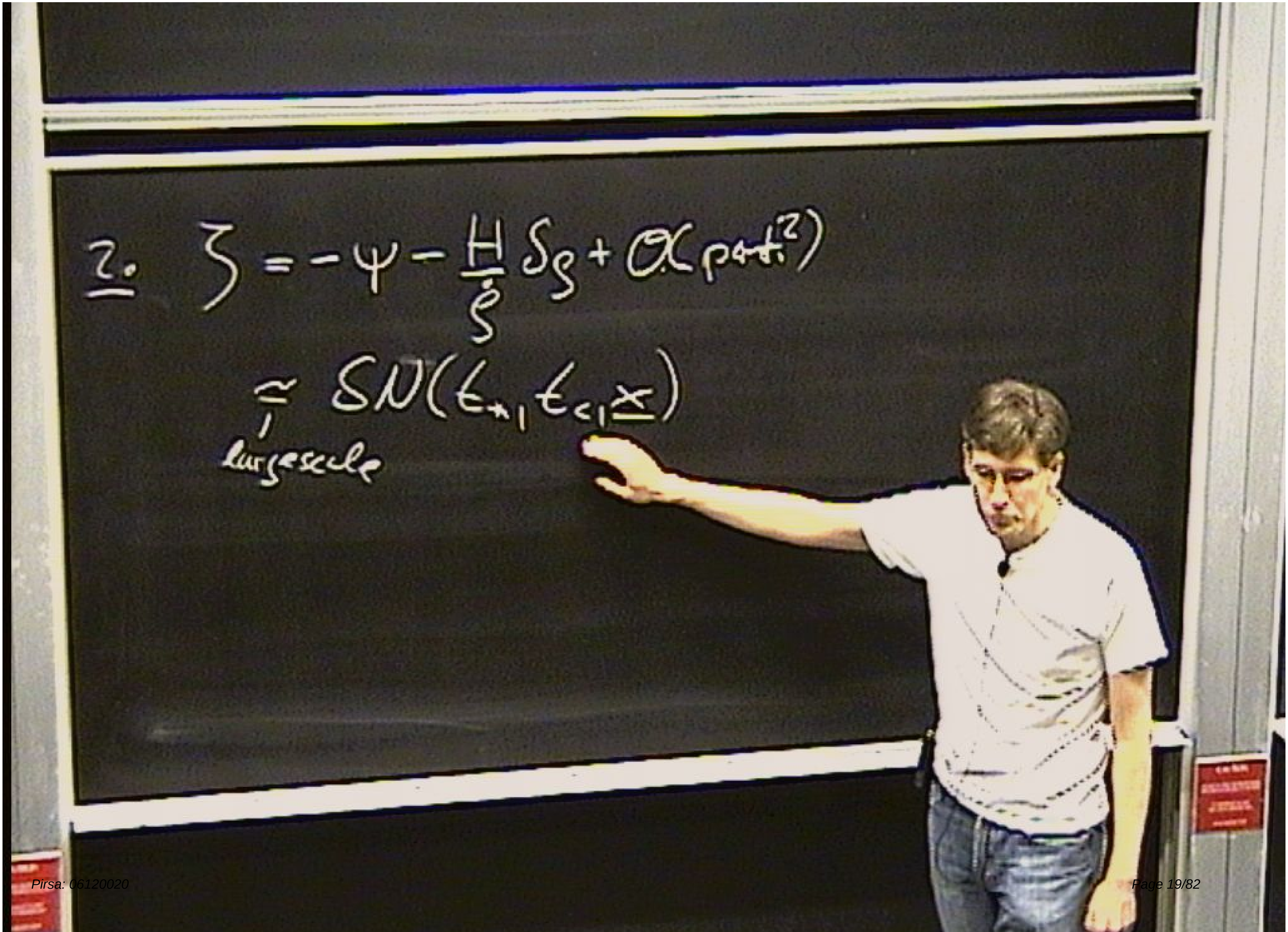
constr. →

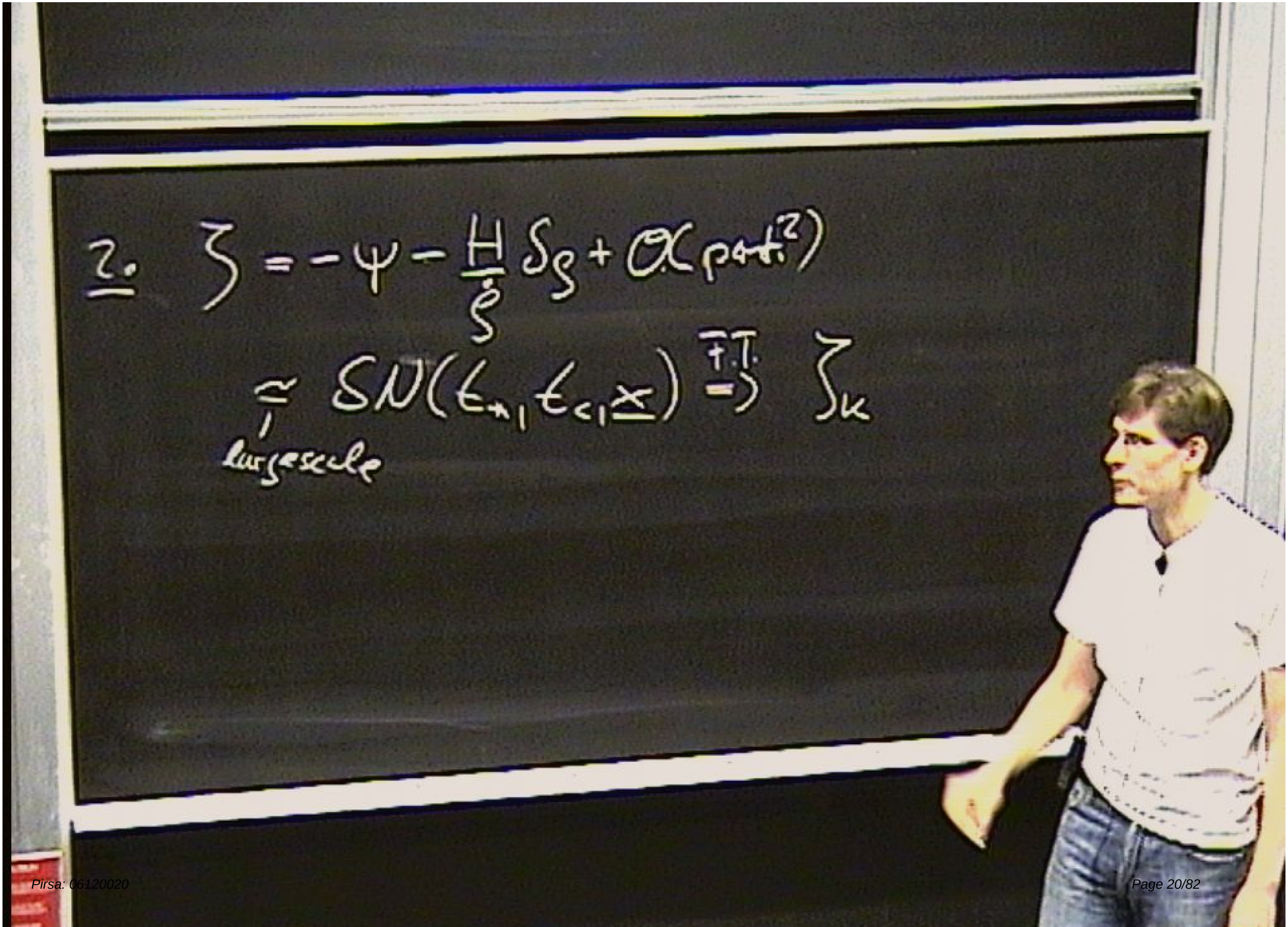

2. ζ = −ψ

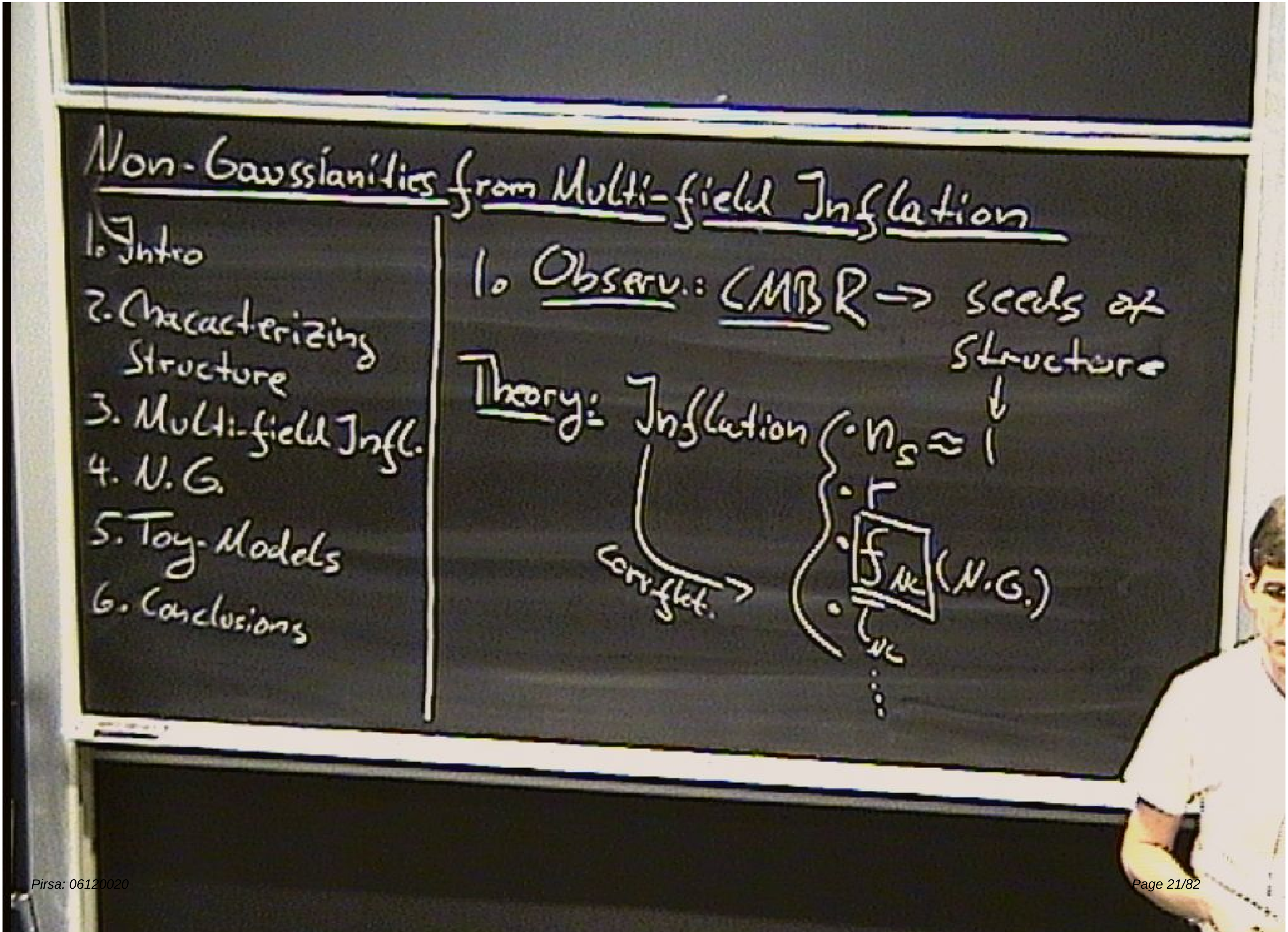

2. $\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho + \mathcal{O}(\text{pert.}^2)$


2. $\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho + \mathcal{O}(\text{pert.}^2)$
$\simeq \delta N$
scale


2. $\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho + \mathcal{O}(\text{pert}^2)$
$\simeq \delta N(t_*, t_c, \underline{x})$
rgescale


2. $\zeta = -\psi - \frac{H}{\dot{\rho}} \delta\rho + \mathcal{O}(\text{pert}^2)$
$\simeq \delta N(t_*, t_c, \underline{x})$
large scale


2. $\zeta = -\psi - \frac{H}{\dot\rho}\delta\rho + \mathcal{O}(\text{pert.}^2)$
$\simeq \delta N(t_*, t_c, \underline{x}) \xrightarrow{F.T.} \zeta_k$
large scale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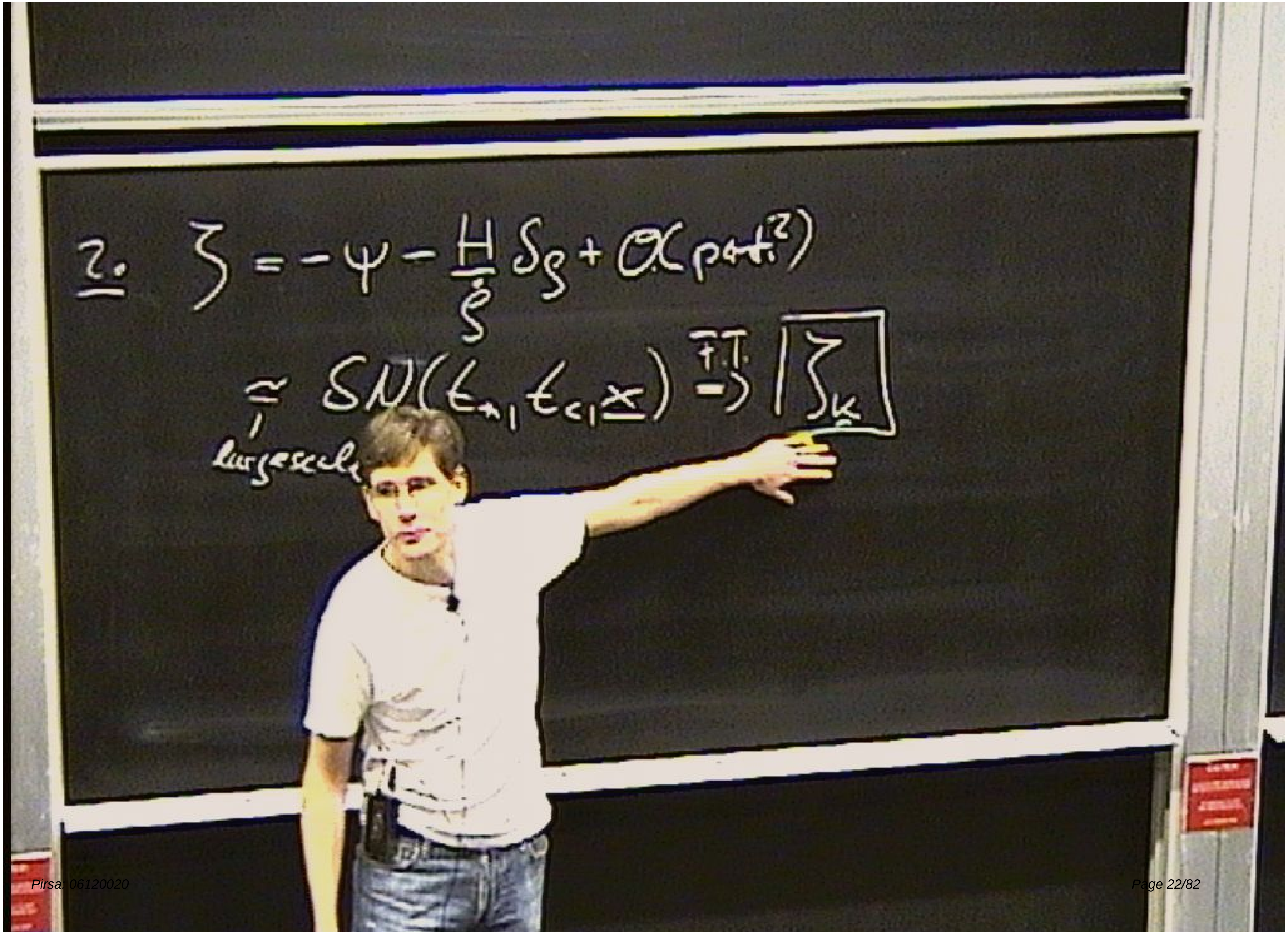
Theory: Inflation
• n_s ≈ 1
• r
• f_NL (N.G.)
• τ_NL
conflict.


2.
ζ = −ψ − H/ρ̇ δρ + O(pert²)
≃ δN(t_m, t_c, x) F.T. → ζ_k
large scale

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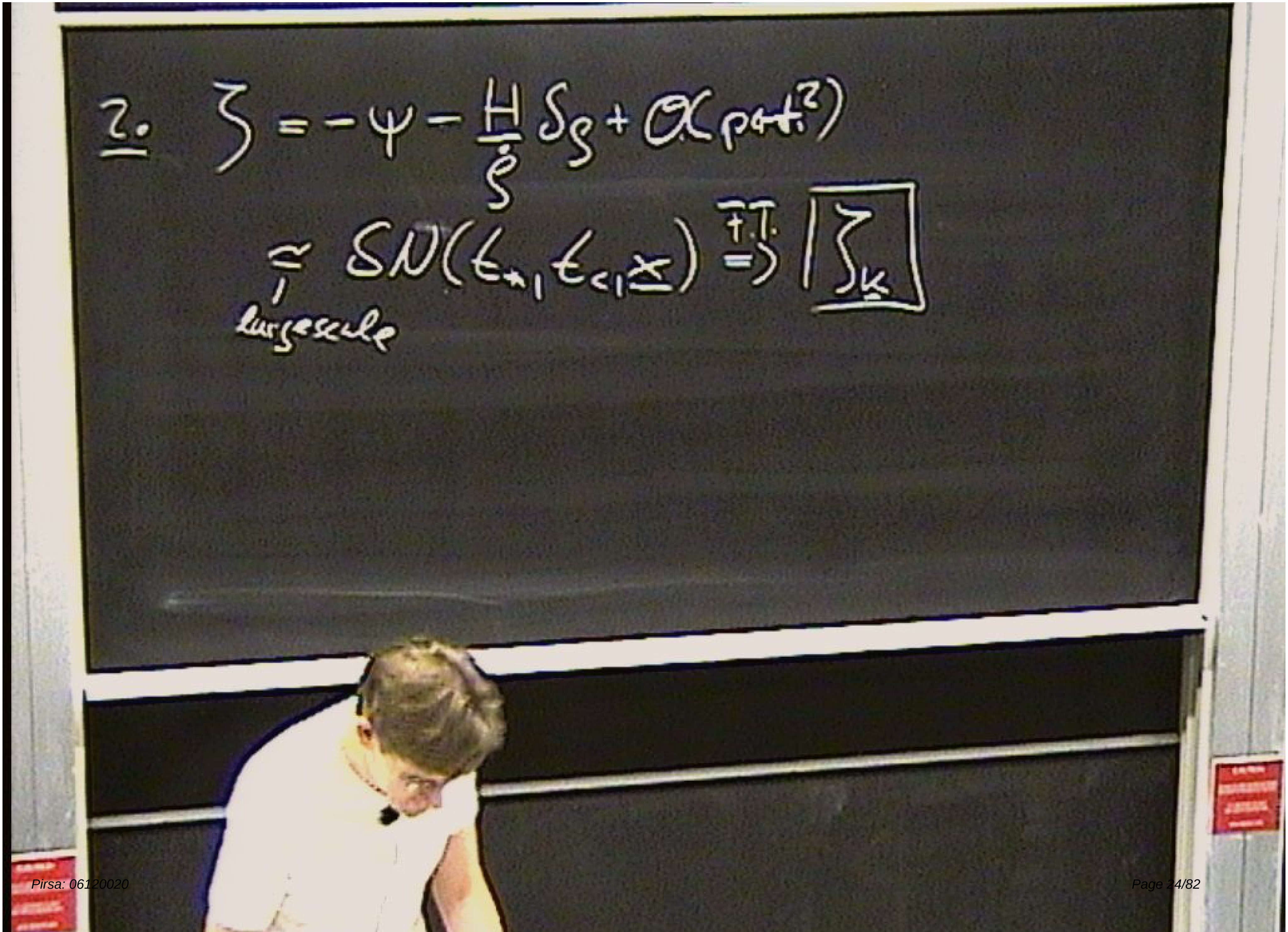
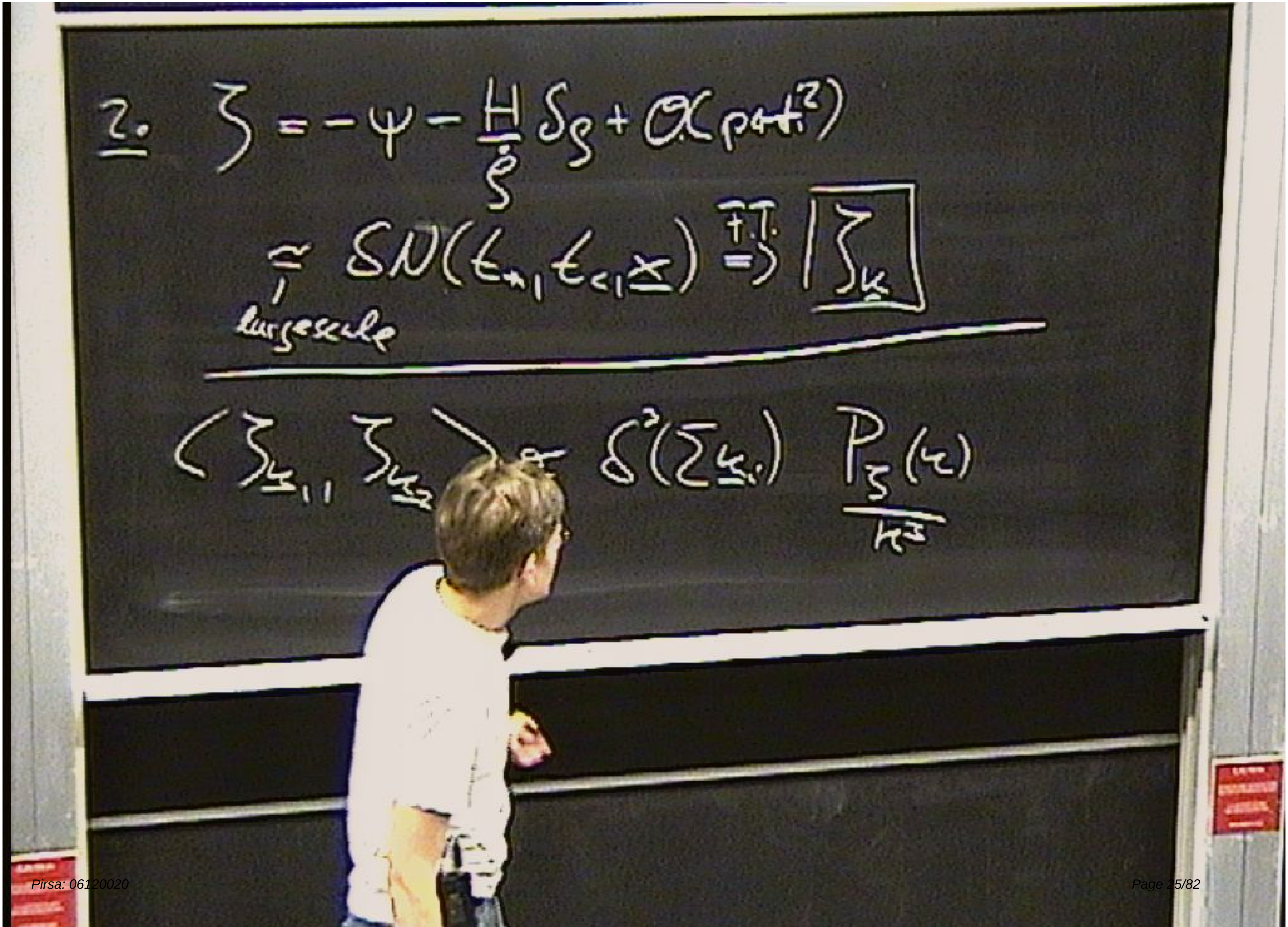
1. Observ.: CMBR → seeds of structure

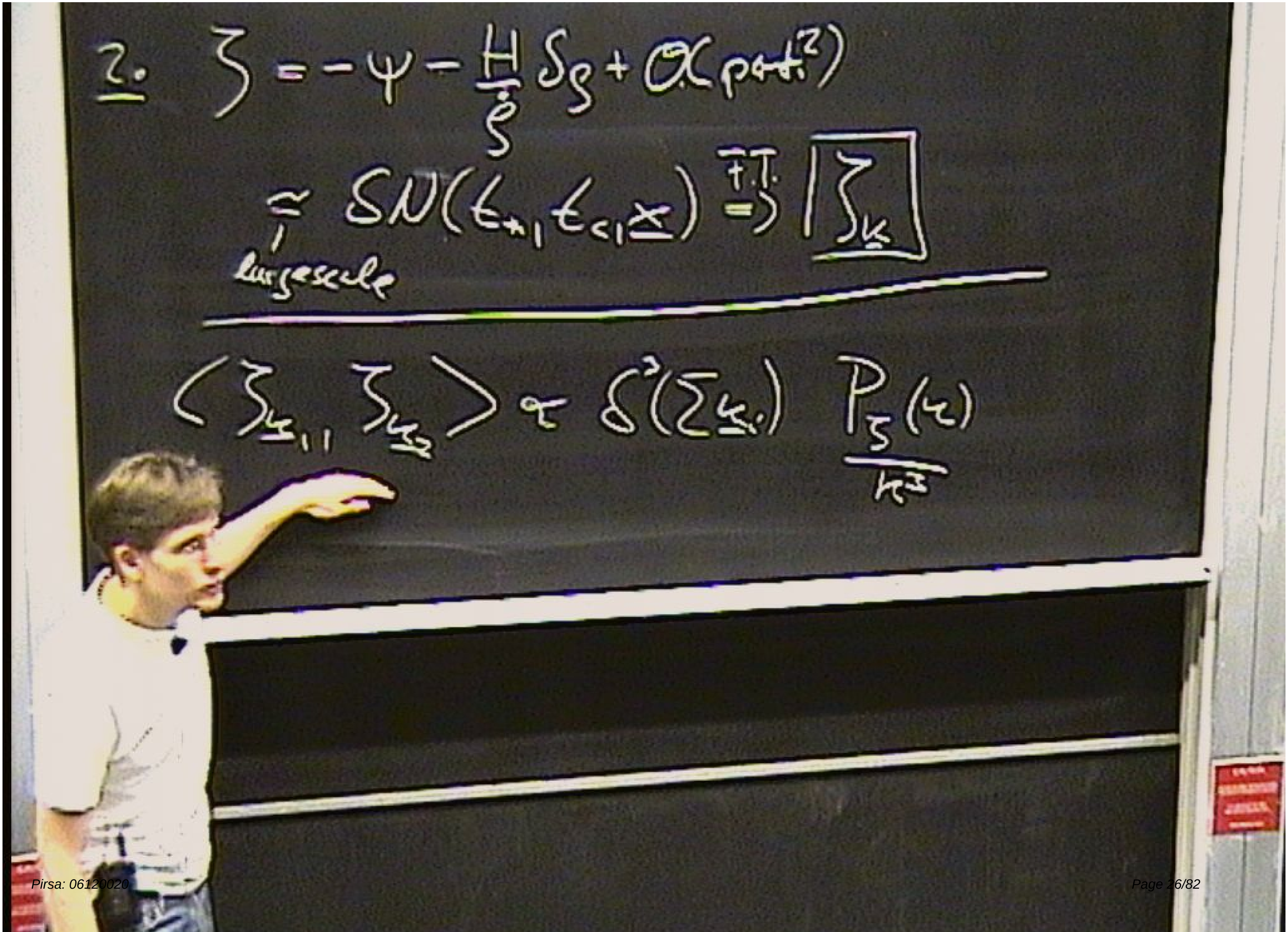
↓

Theory: Inflation — constraints →

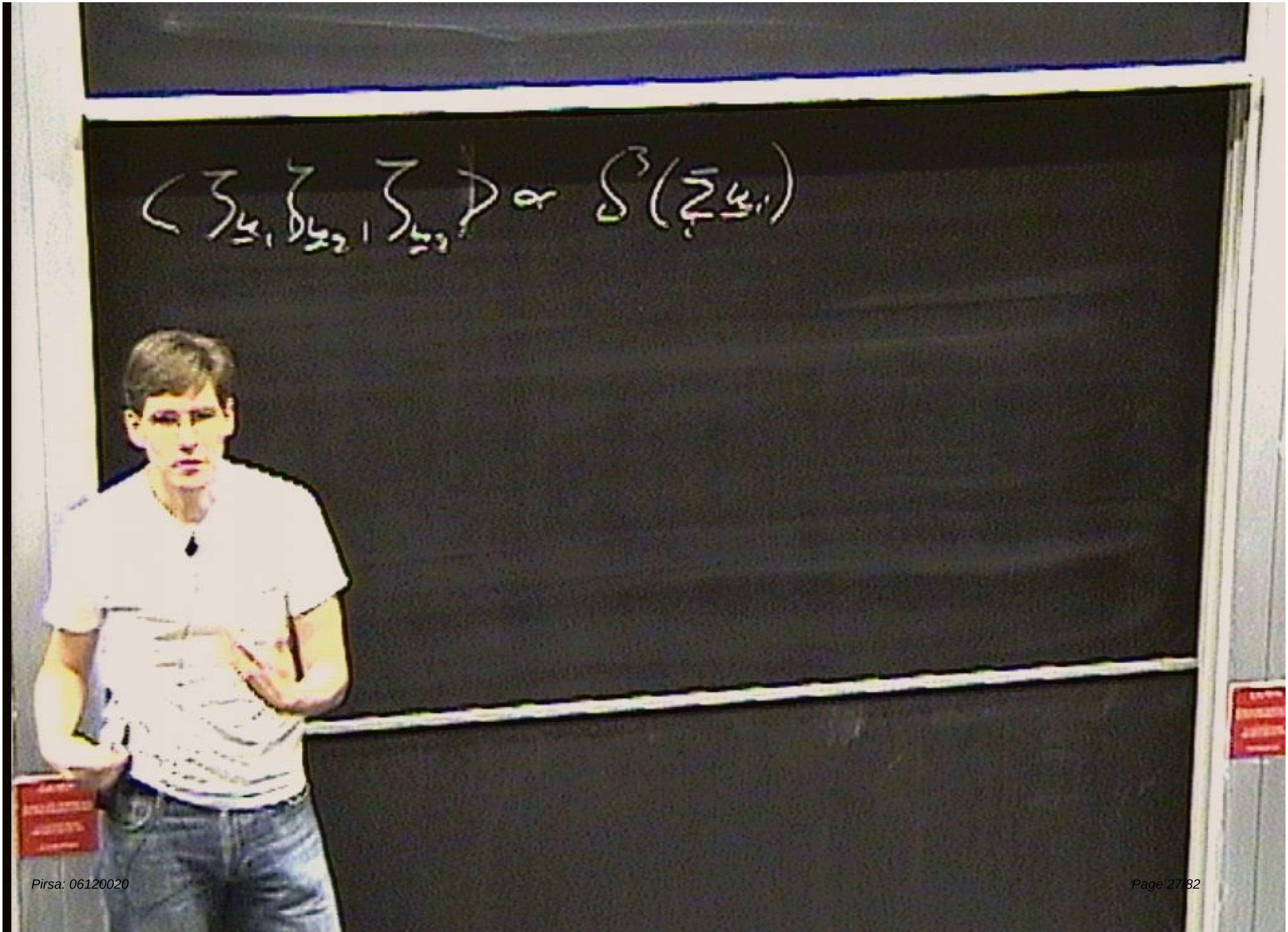
- $n_s \simeq 1$
- r
- f_{NL} (N.G.)
- τ_{NL}
- ⋮

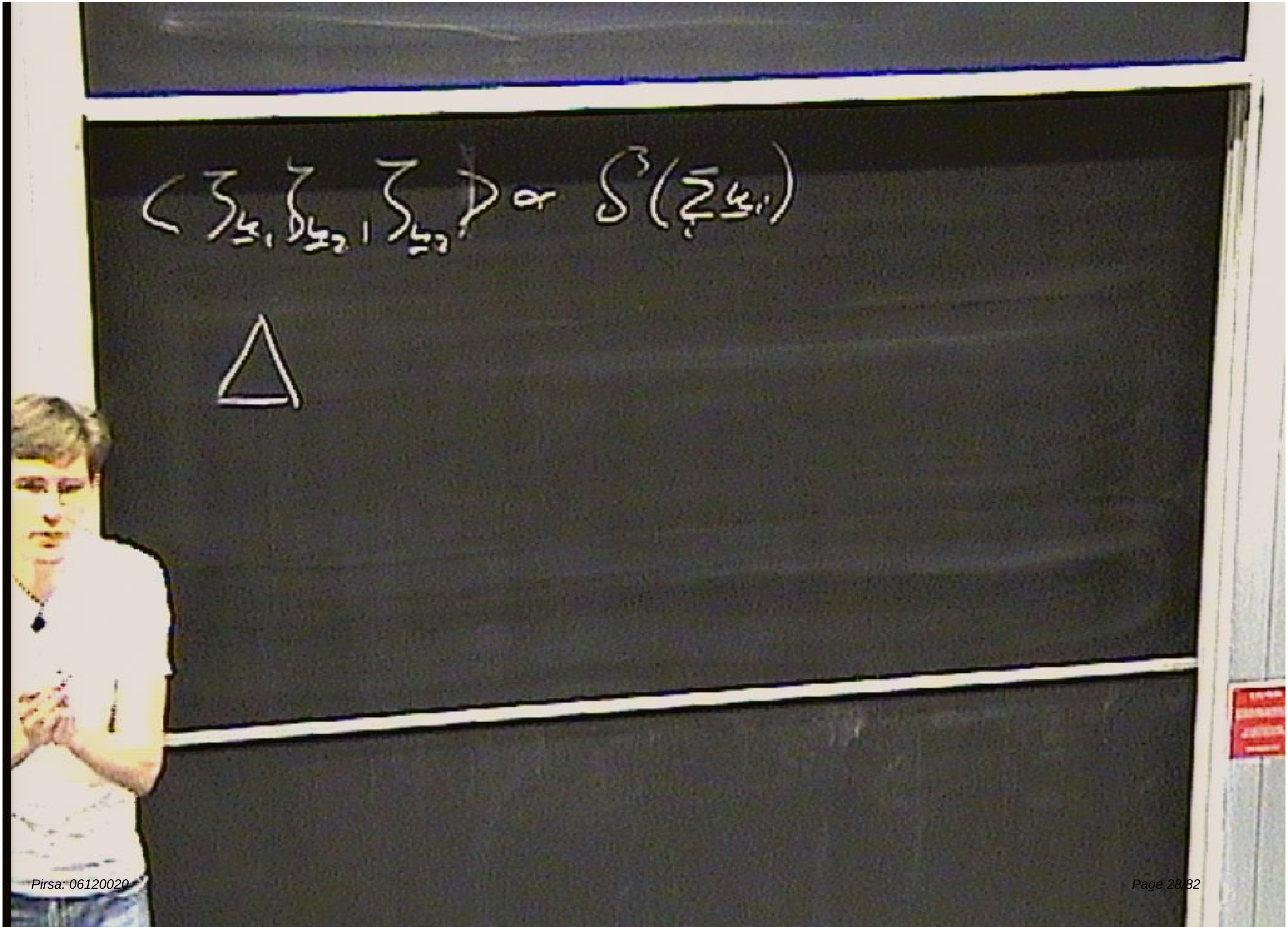

2. $\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho + \mathcal{O}(\text{pert.}^2)$
$\simeq \delta N(t_*, t_c, \underline{x}) \overset{F.T.}{\Longrightarrow} \boxed{\zeta_{\underline{k}}}$
large scale

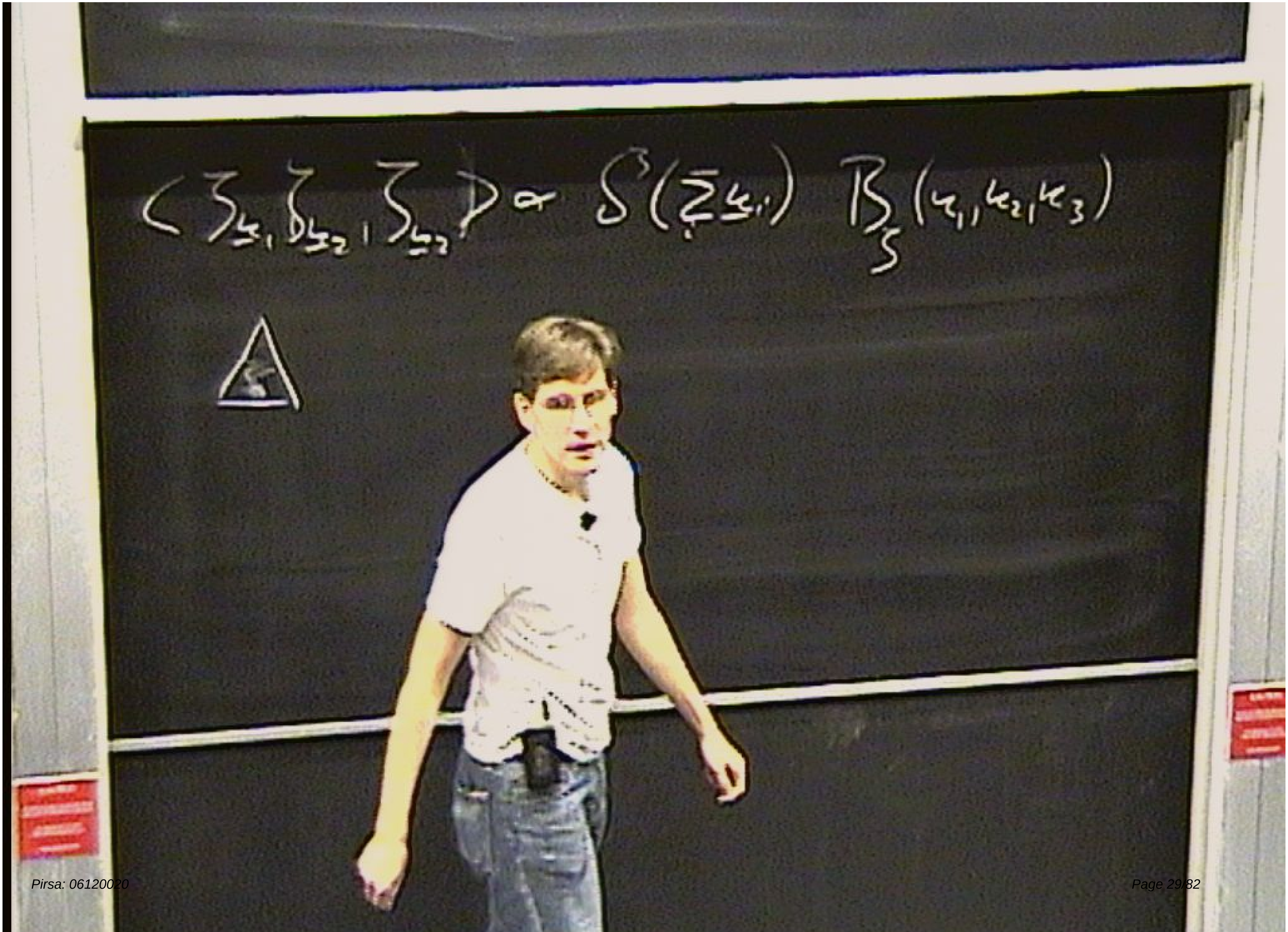

2. ζ = −ψ − H/ρ̇ δρ + O(pert.³)
≃ δN(t_*, t_c, x) → ζ_k
large scale
⟨ζ_k1, ζ_k2⟩ = δ³(Σk_i) P_ζ(k)/k³



2. $\zeta = -\psi - \frac{H}{\dot{\rho}}\delta\rho + O(\text{pert.}^2)$
$\simeq \delta N(t_*, t_c, \underline{x}) \overset{F.T.}{\Longrightarrow} \zeta_{\underline{k}}$
largescale
$\langle \zeta_{\underline{k}_1} \zeta_{\underline{k}_2} \rangle \propto \delta^3(\sum \underline{k}_i) \frac{P_\zeta(k)}{k^3}$

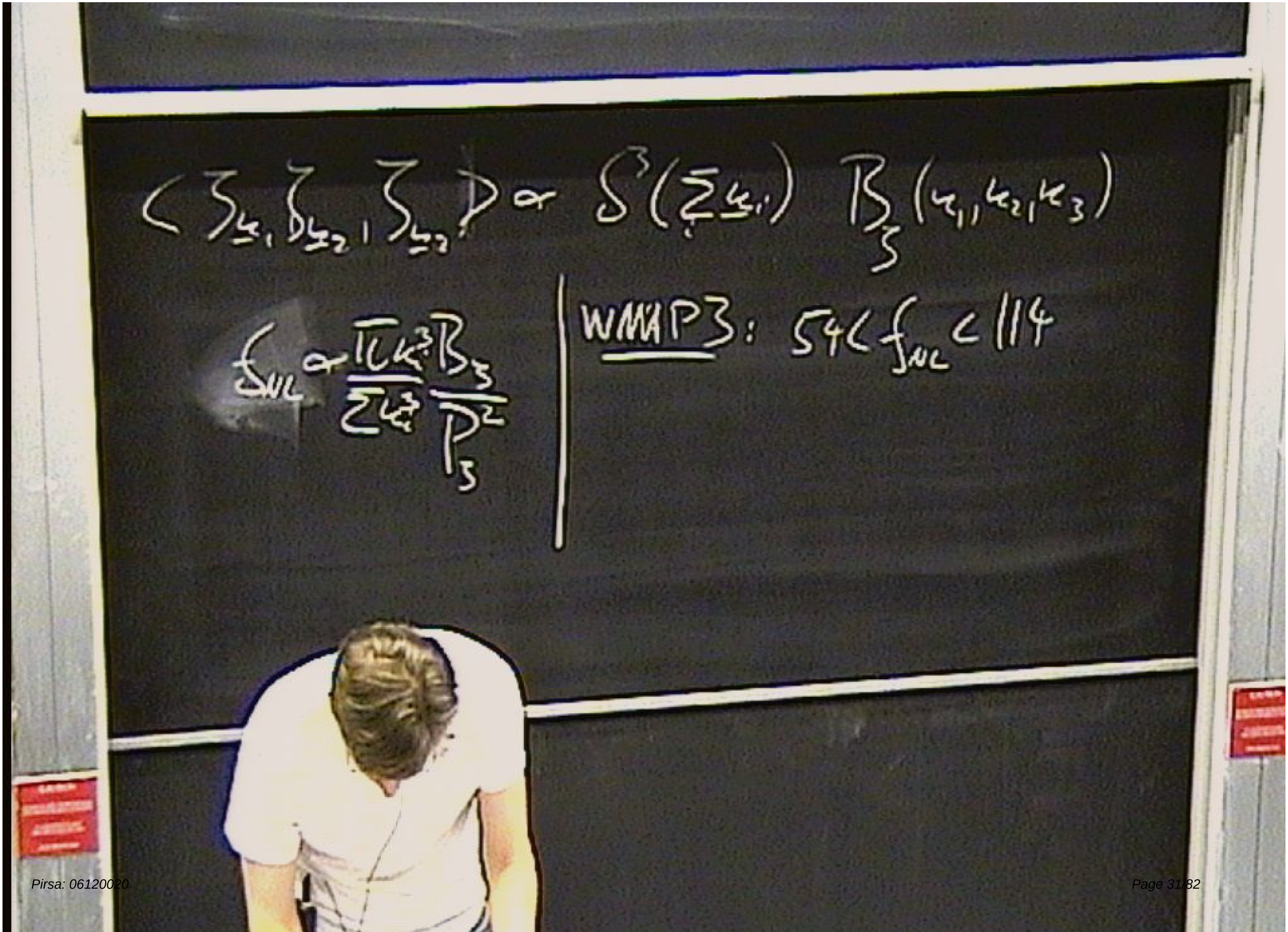

$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \propto \delta^3(\sum k_i)$

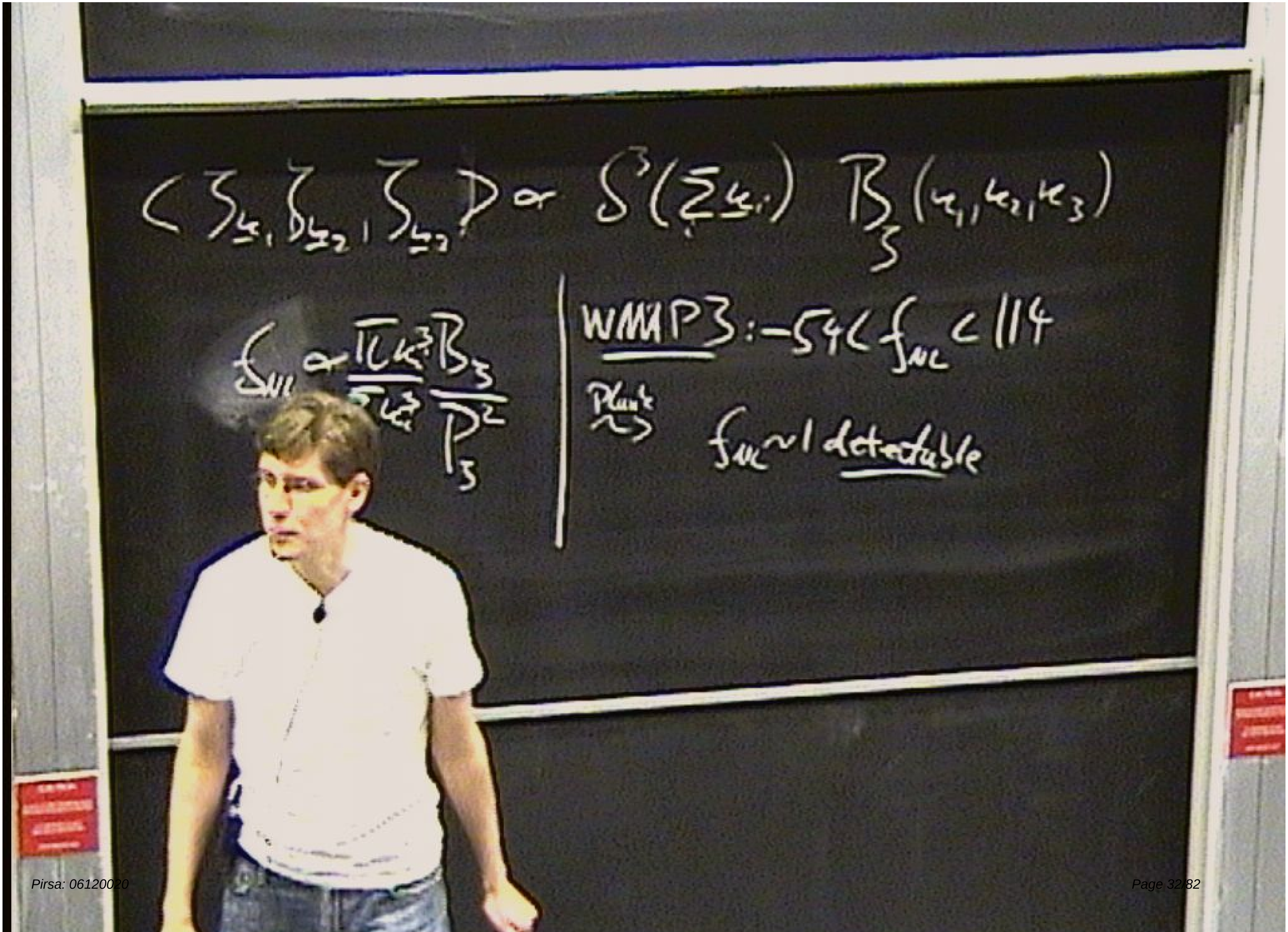

$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \propto \delta^3(\sum k_i)$


$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \propto \delta^3(\sum k_i) \, B_\zeta(k_1, k_2, k_3)$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \propto \delta^3\left(\sum_i k_i\right) B_\zeta(k_1, k_2, k_3)$$

$$f_{NL} \propto \frac{\Pi k_i^3}{\Sigma k_i^3} \frac{B_\zeta}{P_\zeta^2}$$


$\langle \zeta_{\underline{k}_1} \zeta_{\underline{k}_2} \zeta_{\underline{k}_3} \rangle \propto \delta^3(\sum_i \underline{k}_i)\, B_\zeta(k_1, k_2, k_3)$
$f_{NL} \propto \frac{\prod k_i^3}{\sum k_i^3} \frac{B_\zeta}{P_\zeta^2}$
WMAP3: $54 < f_{NL} < 114$

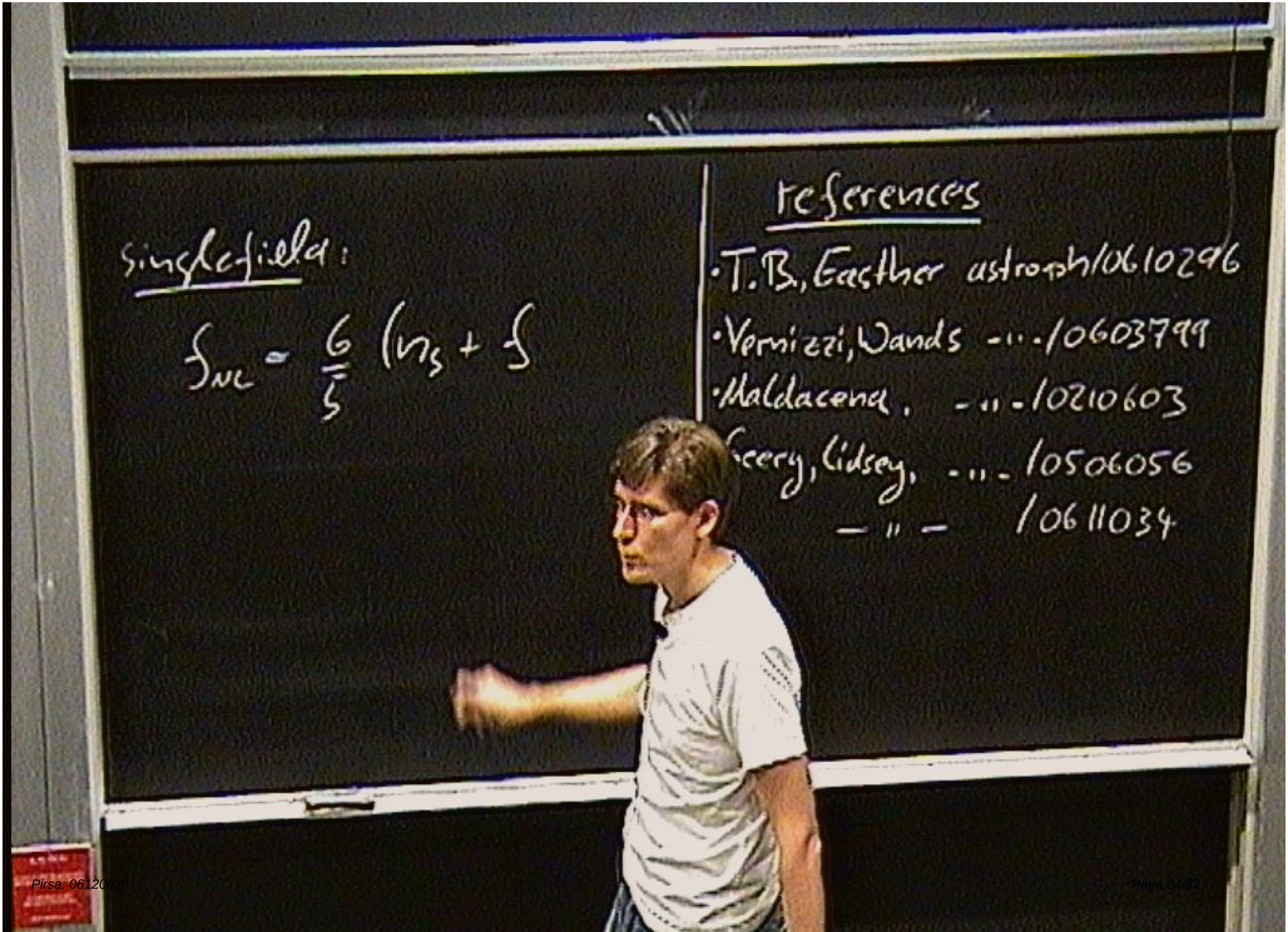

WMAP3: −54 < f_NL < 114
Planck
f_NL ~ 1 detectable

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \propto \delta^3\left(\sum_i k_i\right) B_\zeta(k_1, k_2, k_3)$$

$$f_{NL} \propto \frac{\prod k_i^3}{\sum k_i^3} \frac{B_\zeta}{P_\zeta^2}$$

WMAP3: $-54 < f_{NL} < 114$

Planck $\leadsto$ $f_{NL} \sim 1$ detectable


single field:
f_NL = 6/5 (n_s + f
references
• T. B., Easther astro-ph/0610296
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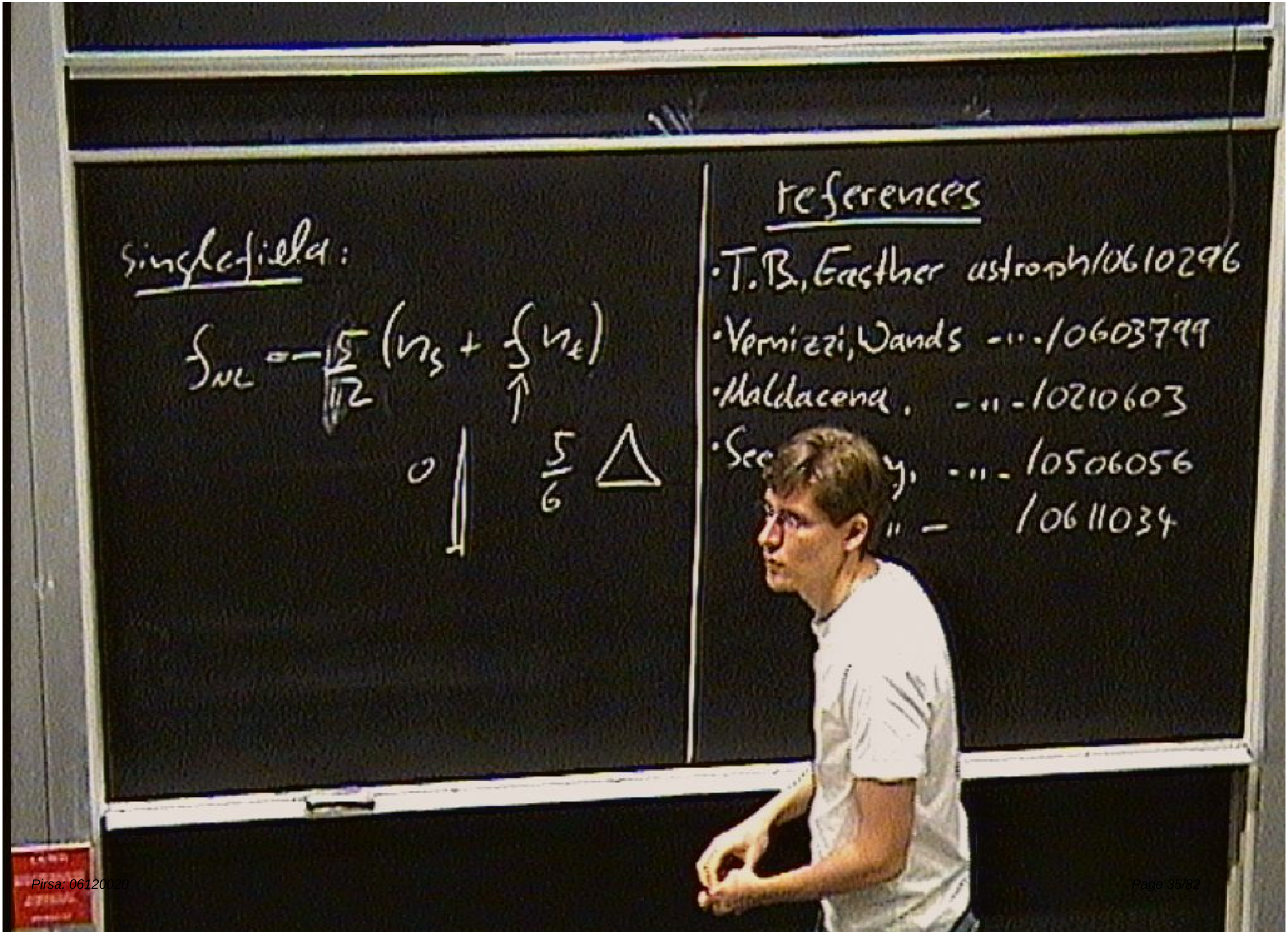
• Maldacena, – " – /0210603
• Seery, Lidsey, – " – /0506056
– " – /0611034


single field:
f_NL = -5/12 (n_s + f n_t)
0
5/6
references
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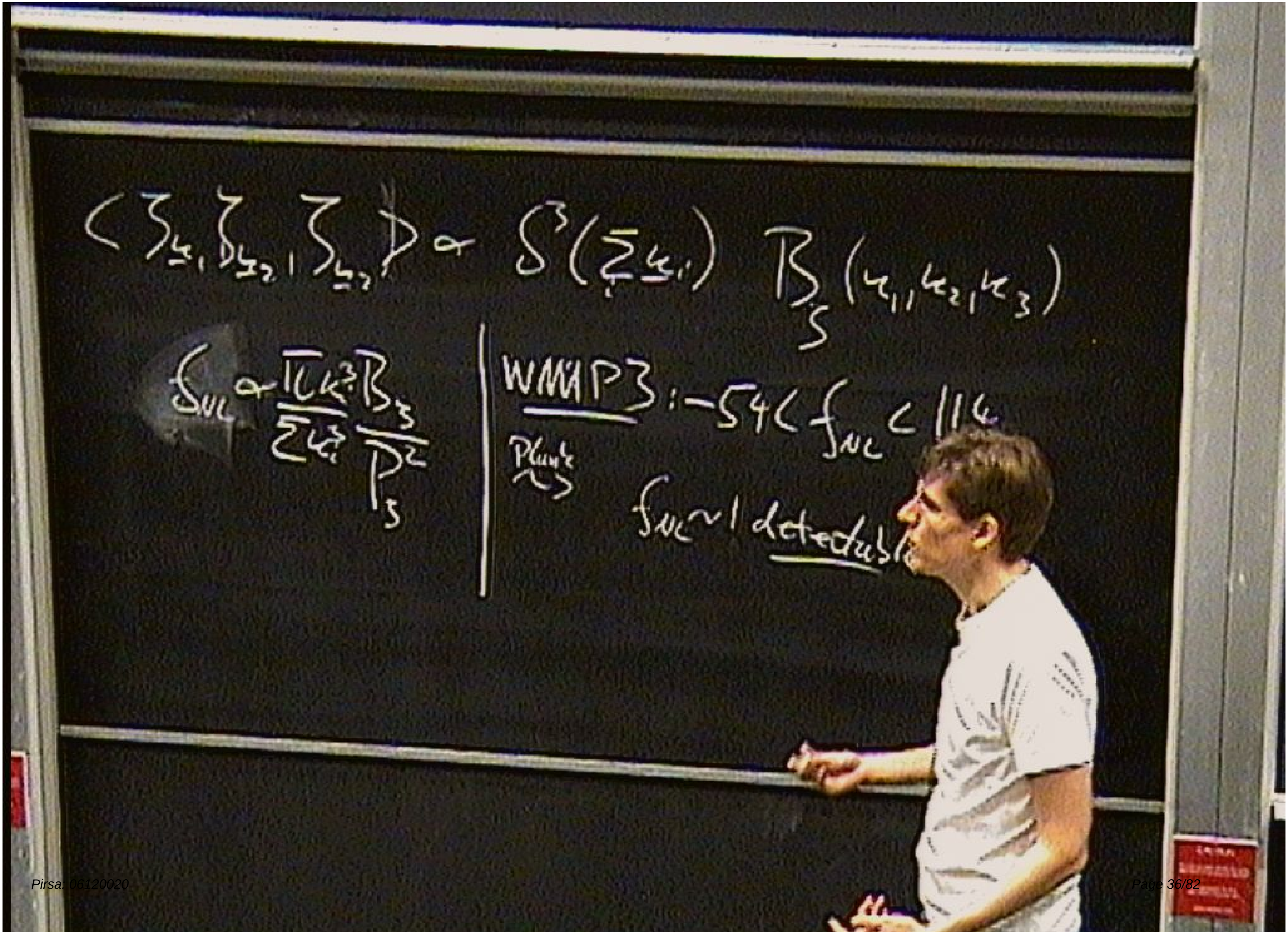
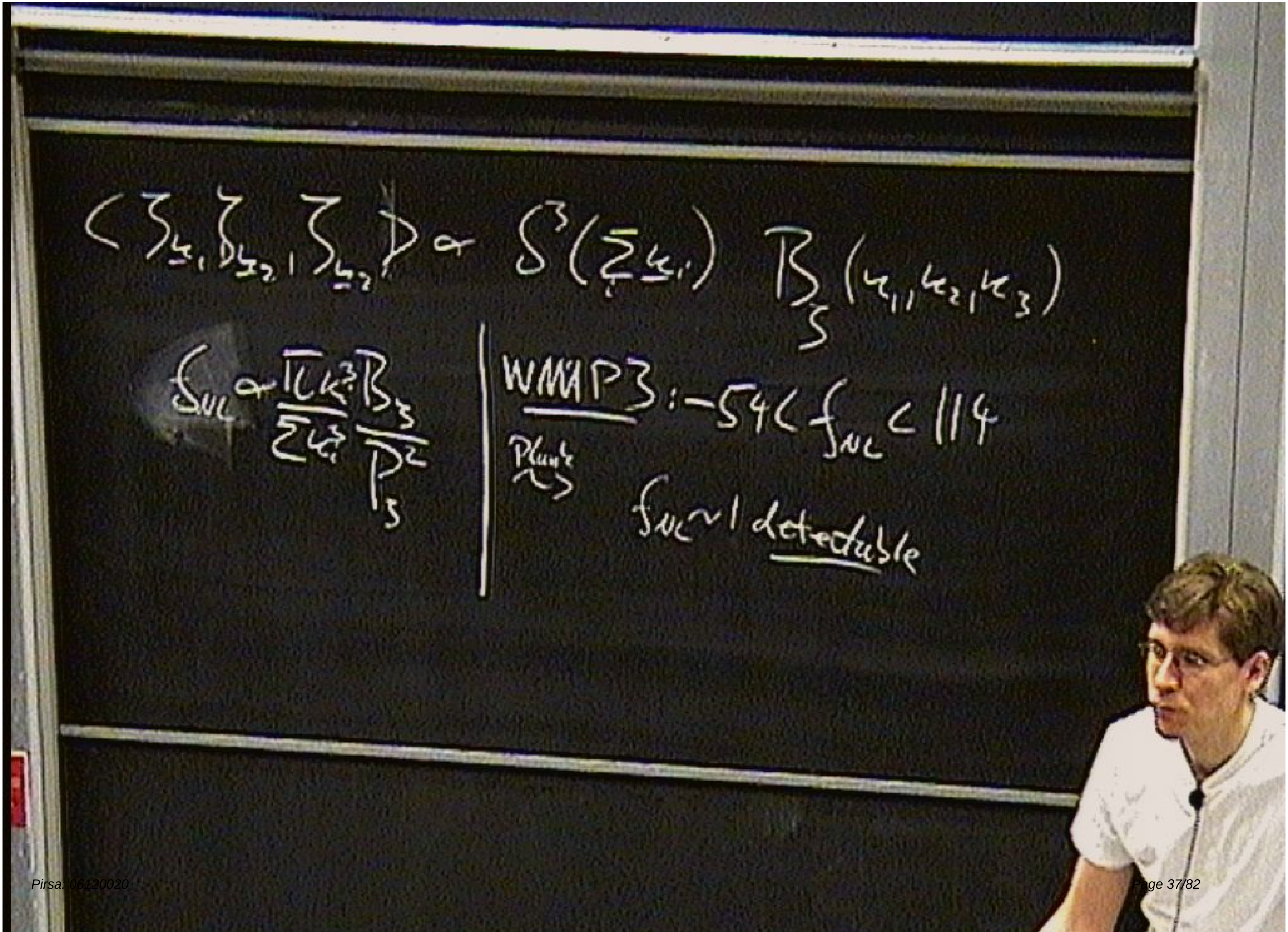
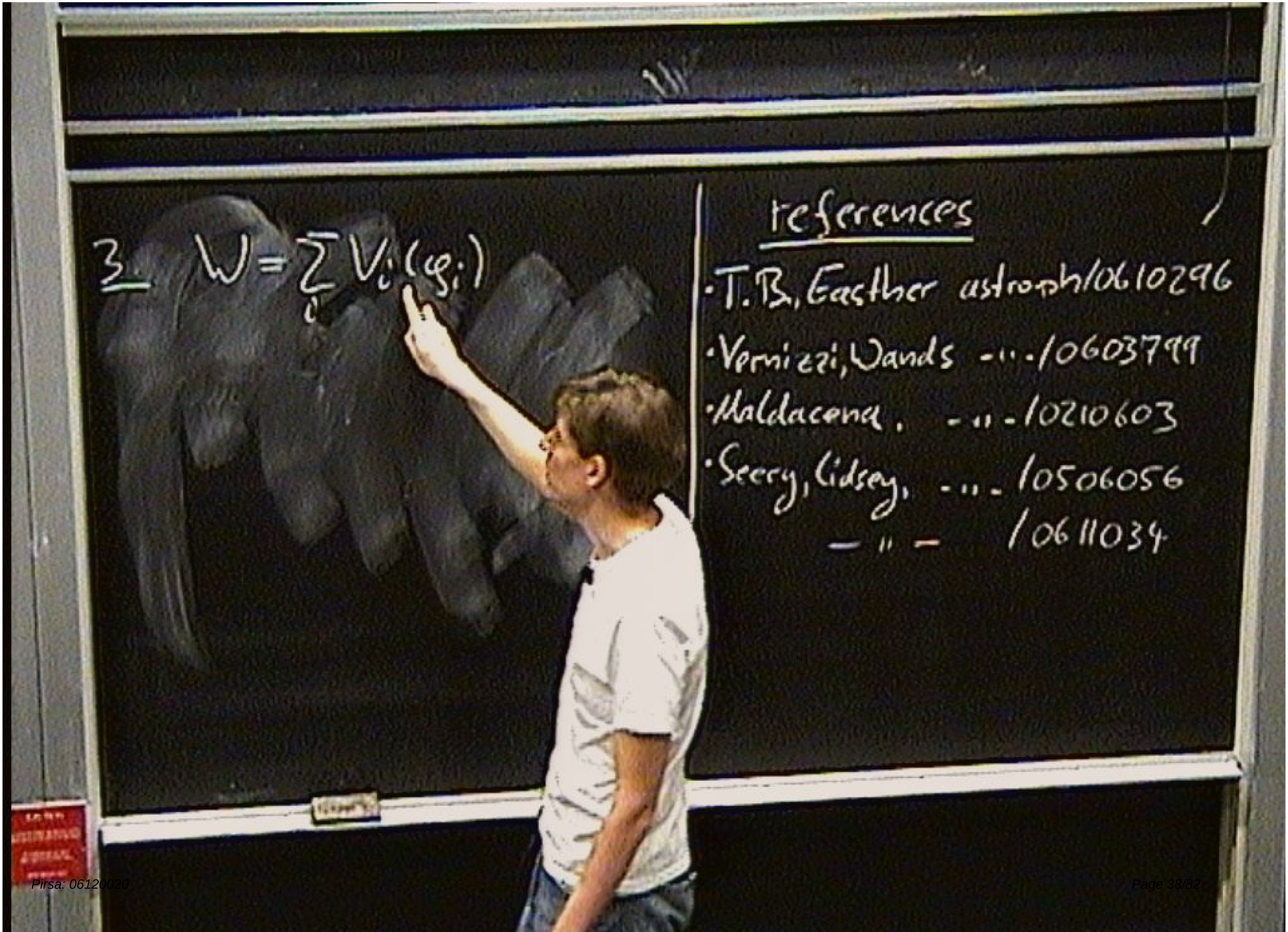
• Vernizzi, Wands -"-/0603799
• Maldacena -"-/0210603
-"-/0506056
/0611034



WMAP3: −54 < f_NL < 114
Planck
f_NL ~ 1 detectable


$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \propto \delta^3(\sum_i k_i)\, B_\zeta(k_1, k_2, k_3)$
$f_{NL} \propto \frac{\prod k_i^3 B_\zeta}{\sum k_i^3 P_\zeta^2}$
WMAP3: $-54 < f_{NL} < 114$
Planck
$f_{NL} \sim 1$ detectable


3. W = ∑ V_i(φ_i)
references
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·Maldacena. -"- /0210603
·Seery, Lidsey. -"- /0506056
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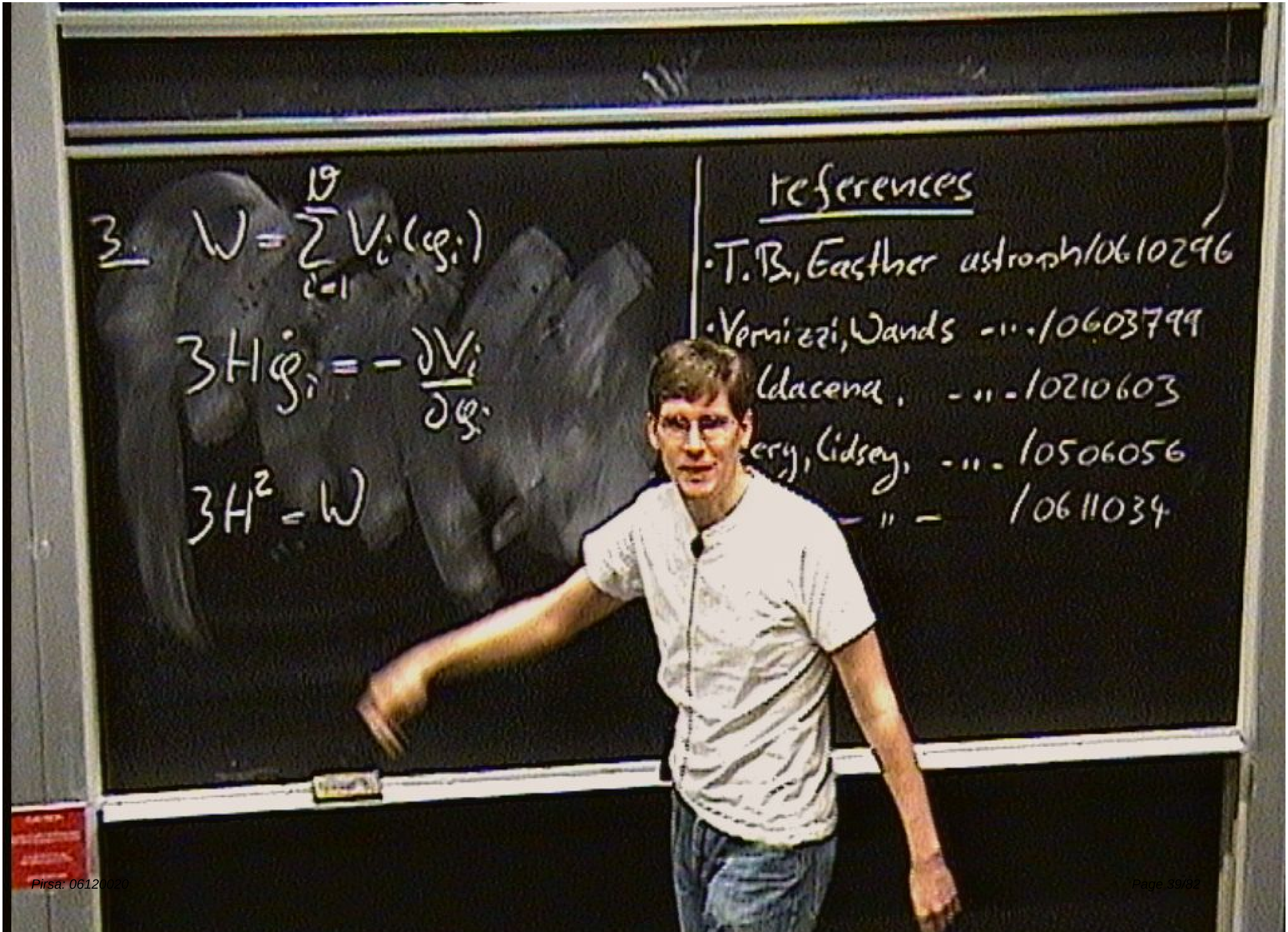
3.
$W = \sum_{i=1}^{10} V_i(\varphi_i)$
$3H\dot{\varphi}_i = -\frac{\partial V_i}{\partial \varphi_i}$
$3H^2 = W$
references
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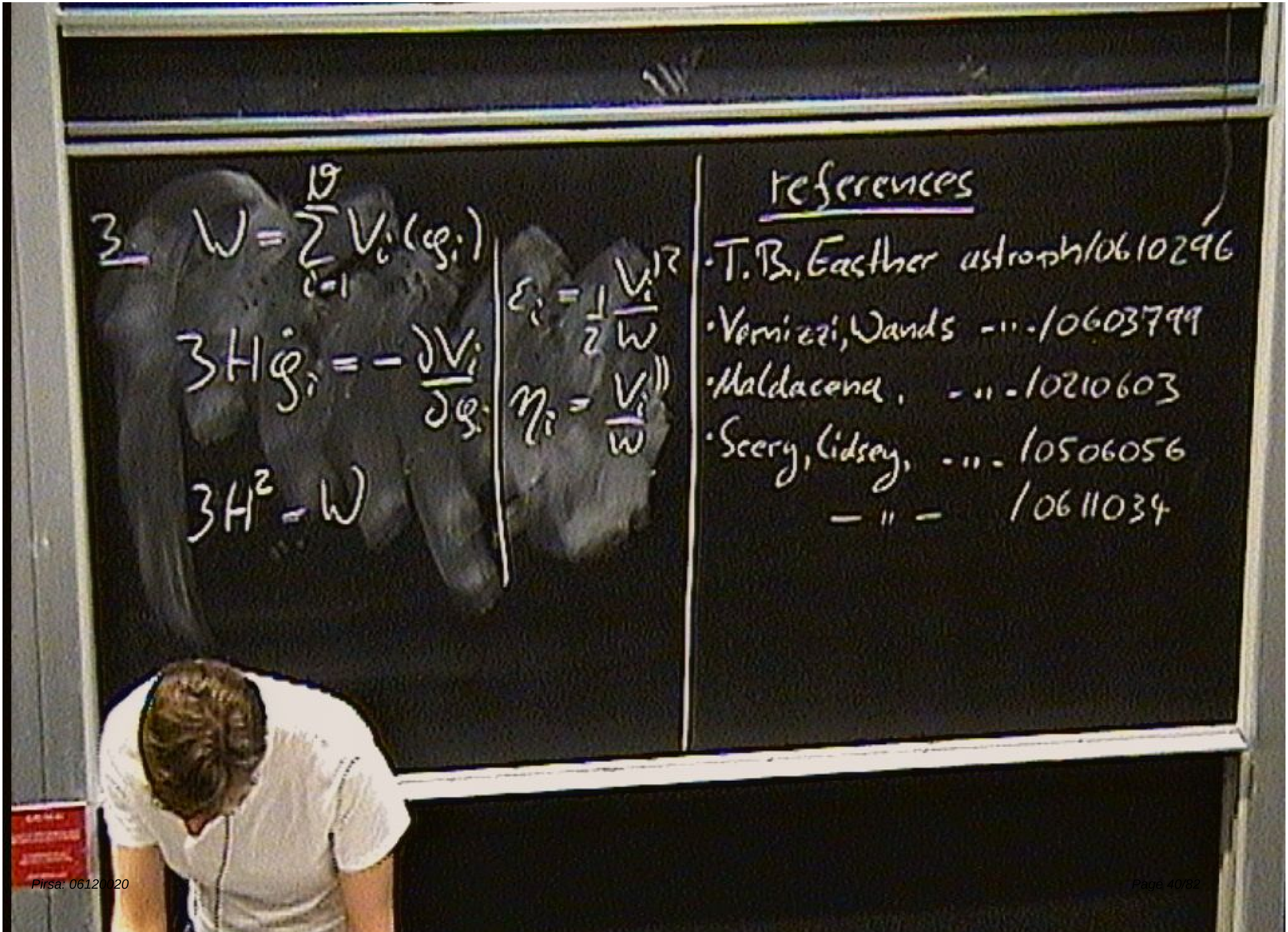
•Vernizzi, Wands -"- /0603799
ldacena, -"- /0210603
ery, Lidsey, -"- /0506056
-"- /0611034


3. $W = \sum_{i=1}^{N} V_i(\varphi_i)$
$3H\dot{\varphi}_i = -\frac{\partial V_i}{\partial \varphi_i}$
$3H^2 = W$
$\epsilon_i = \frac{1}{2}\frac{V_i'^2}{W}$
$\eta_i = \frac{V_i''}{W}$
references
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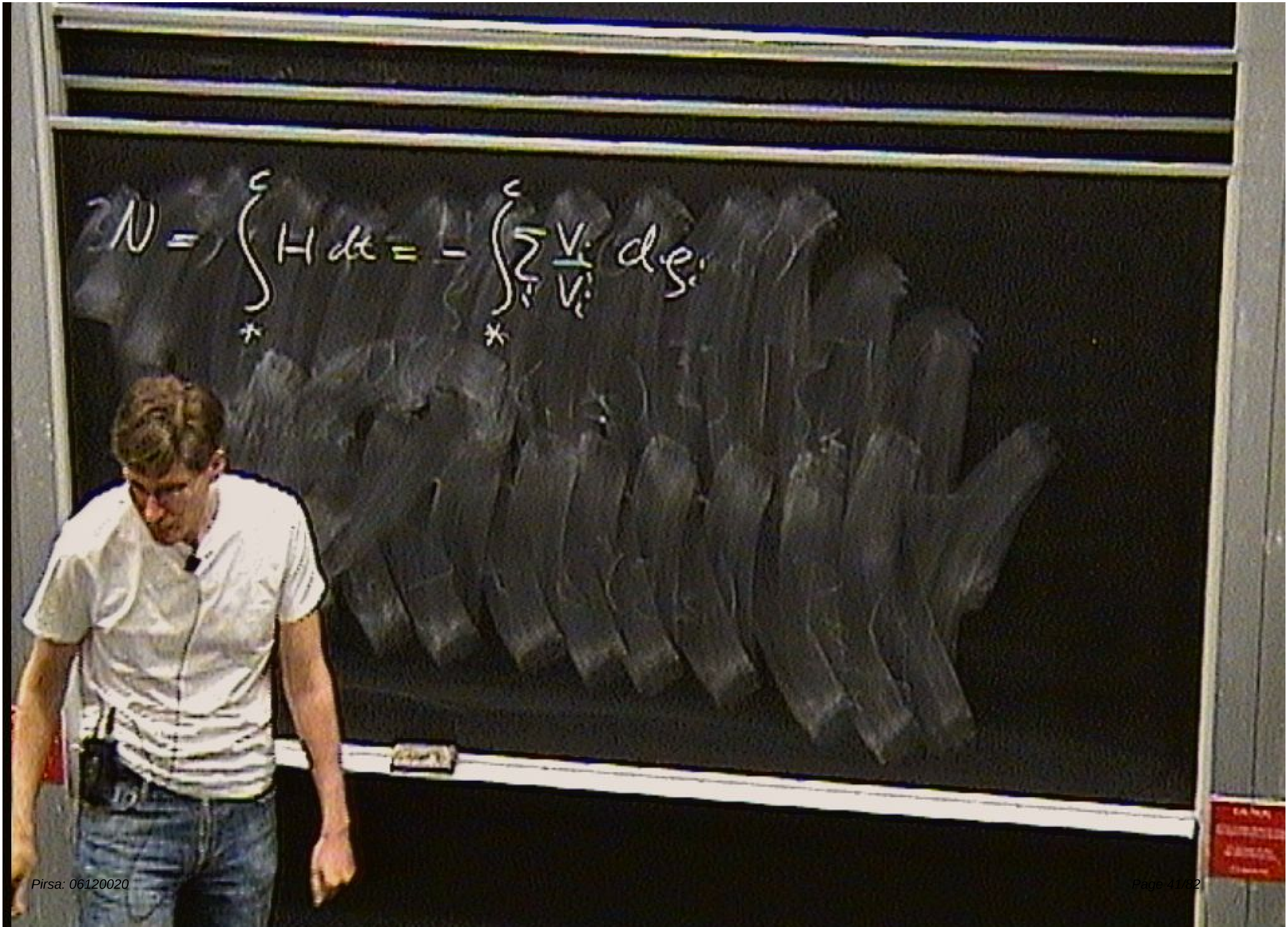
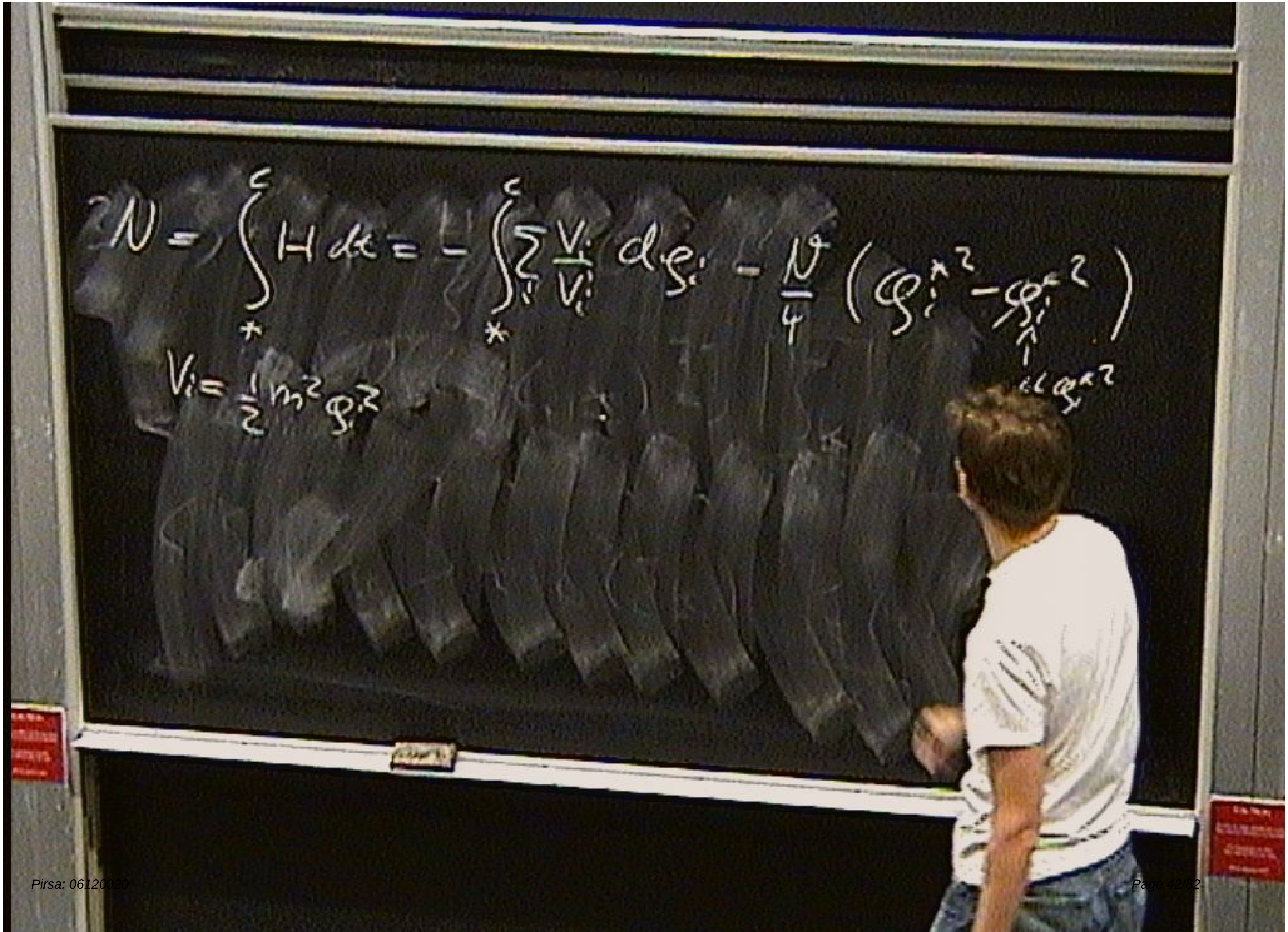
·Maldacena, -"-/0210603
·Seery, Lidsey, -"-/0506056
-"- /0611034


$N = \int_{*} H\,dt = -\int_{*} \sum_i \frac{\dot V_i}{V_i}\,dq_i$

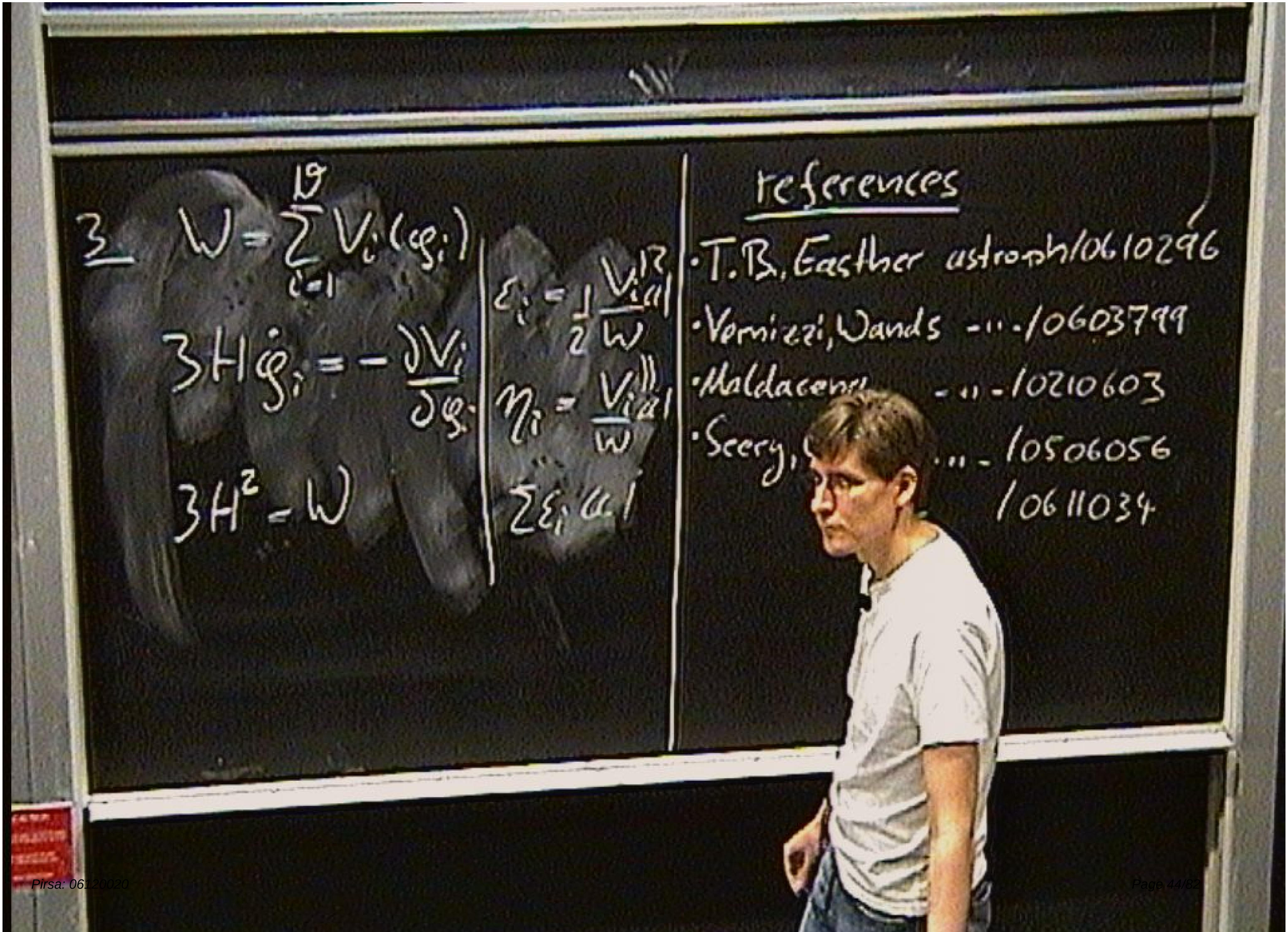

$N = \int_{*} H\,dt = -\int_{*} \sum_i \frac{V_i'}{V_i}\, d\varphi_i - \frac{N}{4}\left(\varphi_i^{*2} - \varphi_i^{2}\right)$
$V_i = \frac{1}{2} m^2 \varphi_i^2$

$$N = \int_*^c H\,dt = -\int_*^c \sum_i \frac{V_i'}{V_i}\,d\varphi_i - \frac{N}{4}\left(\varphi_i^{*2} - \varphi_i^{c\,2}\right)$$

$$\varphi_i^{c\,2} \ll \varphi_i^{*2}$$

$$V_i = \frac{1}{2} m^2 \varphi_i^2$$

$$\rightarrow \varphi_i^{*2} = \frac{240}{N}$$


3
W = Σ_{i=1}^{N} V_i(φ_i)
3H φ̇_i = − ∂V_i/∂φ_i
3H² = W
ε_i = ½ V'²_{i,i}/W
η_i = V''_{i,i}/W
Σε_i ≪ 1
references
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• Maldacena -"- /0210603
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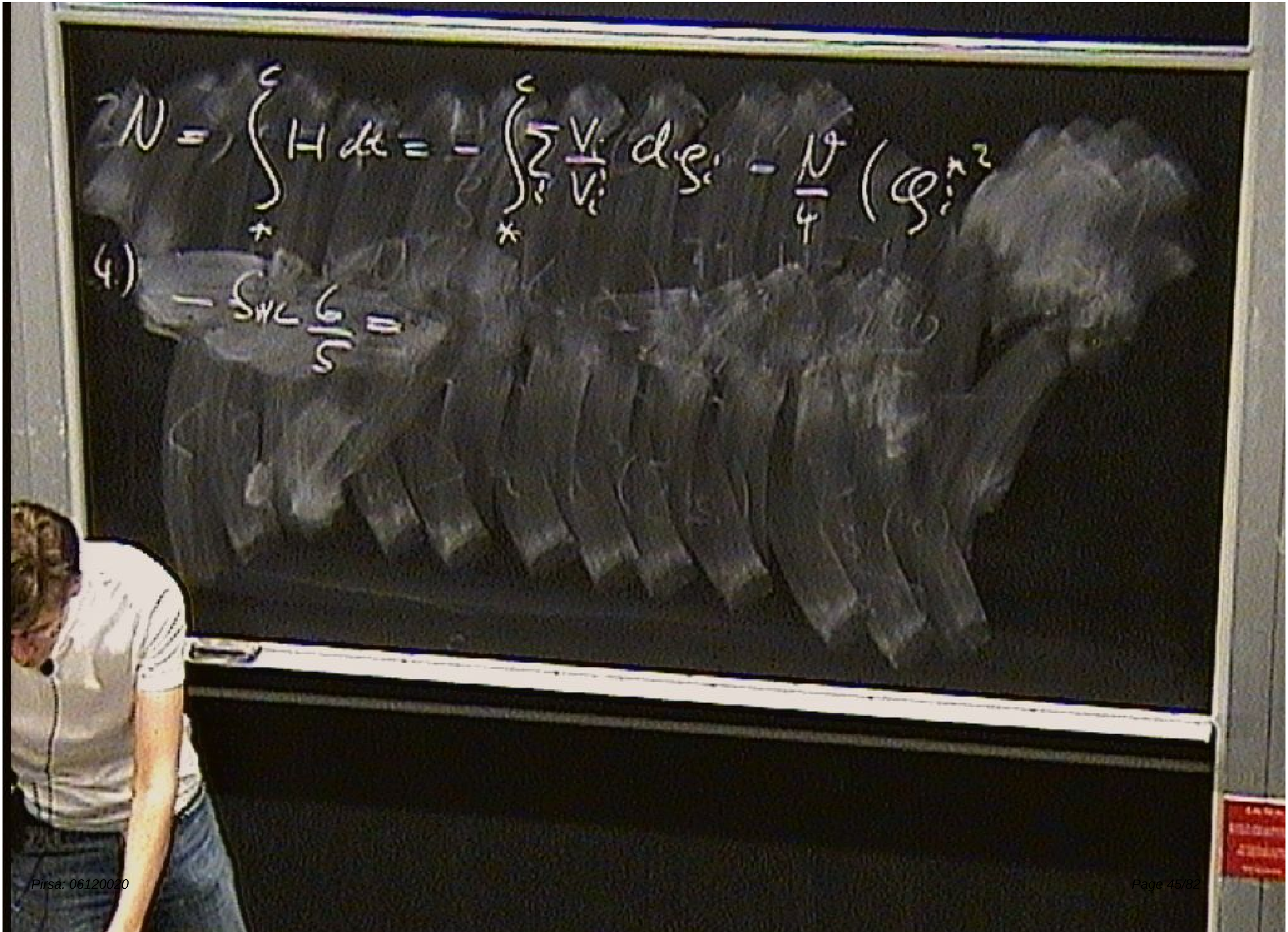
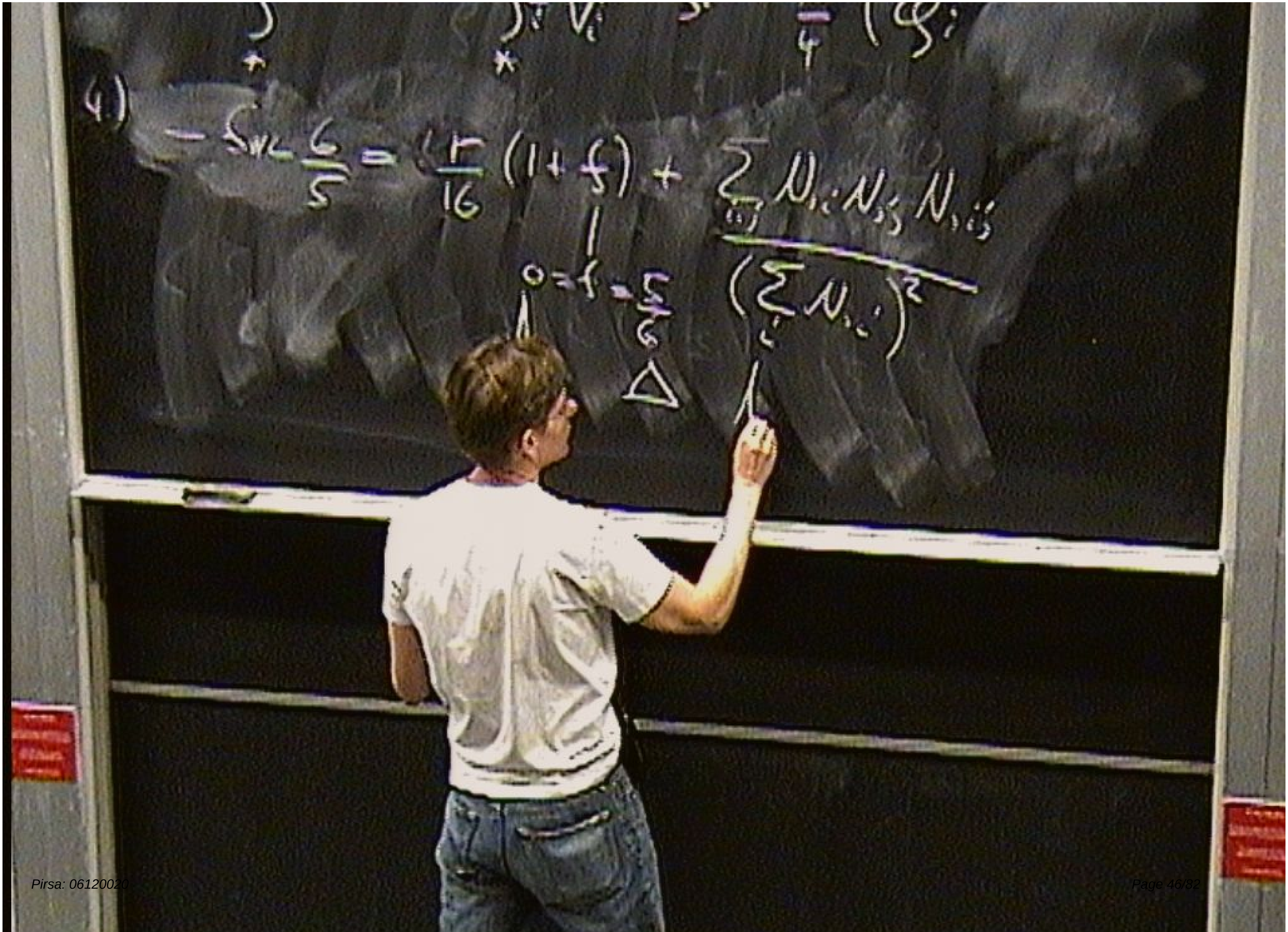
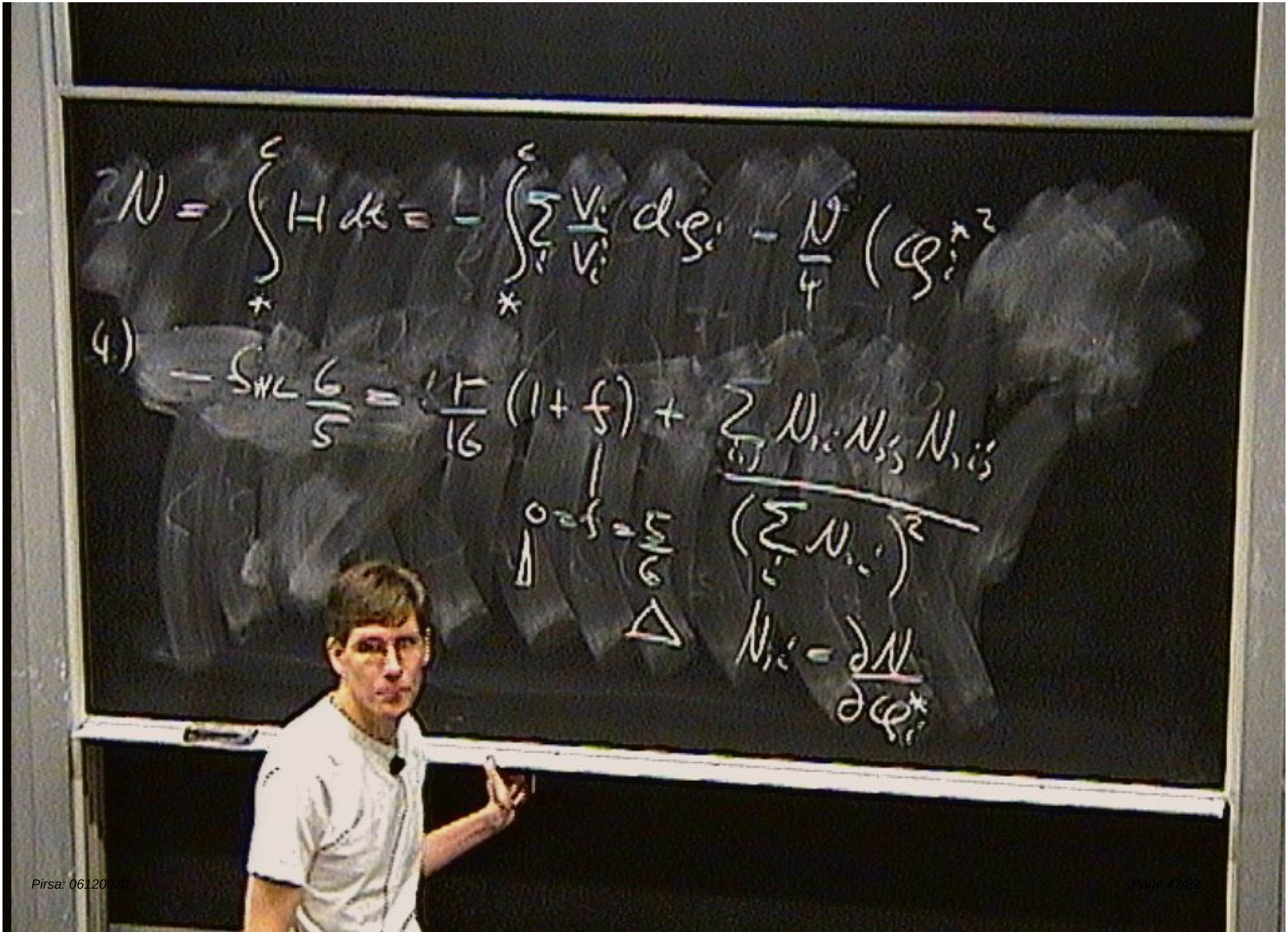
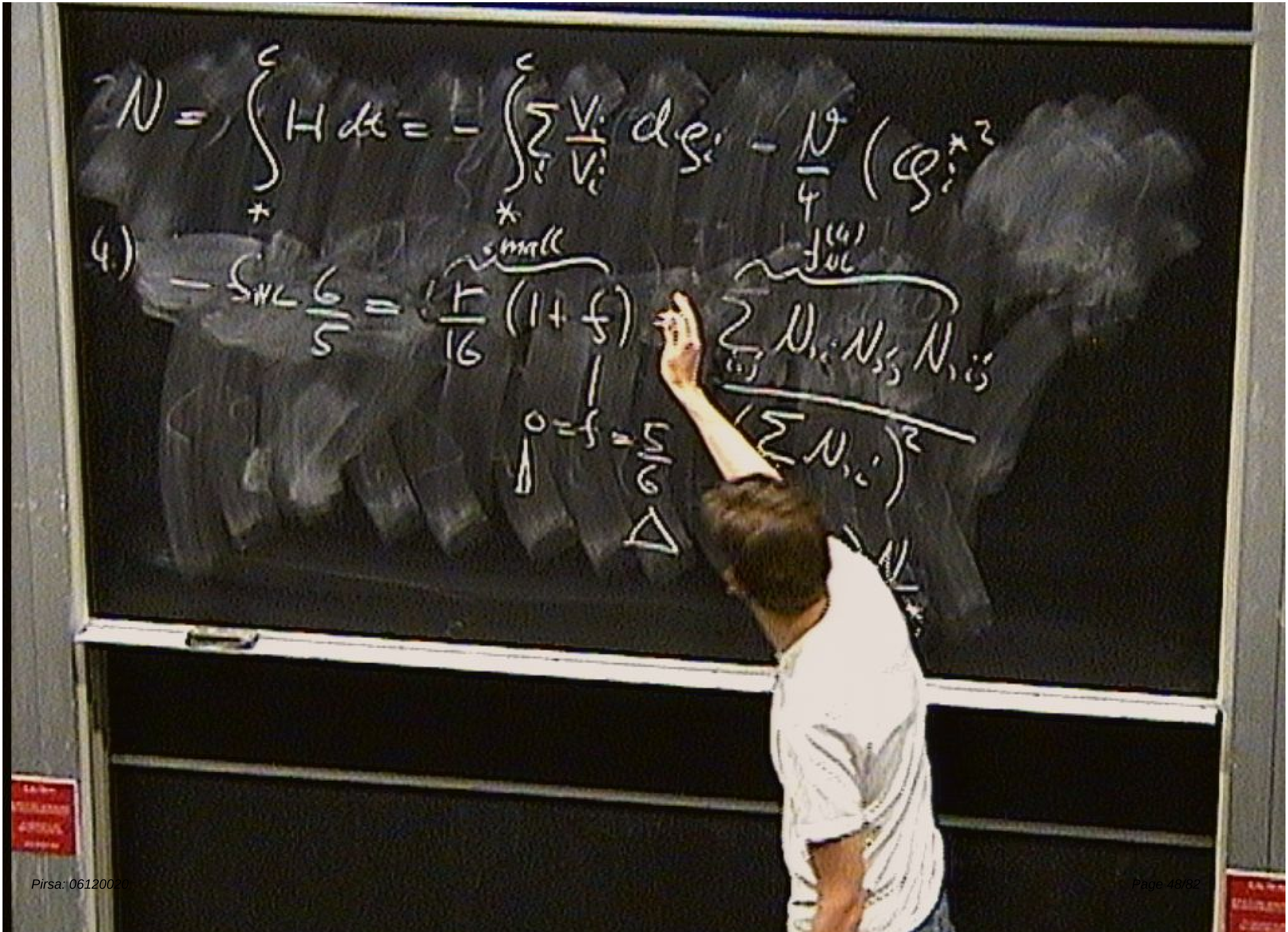
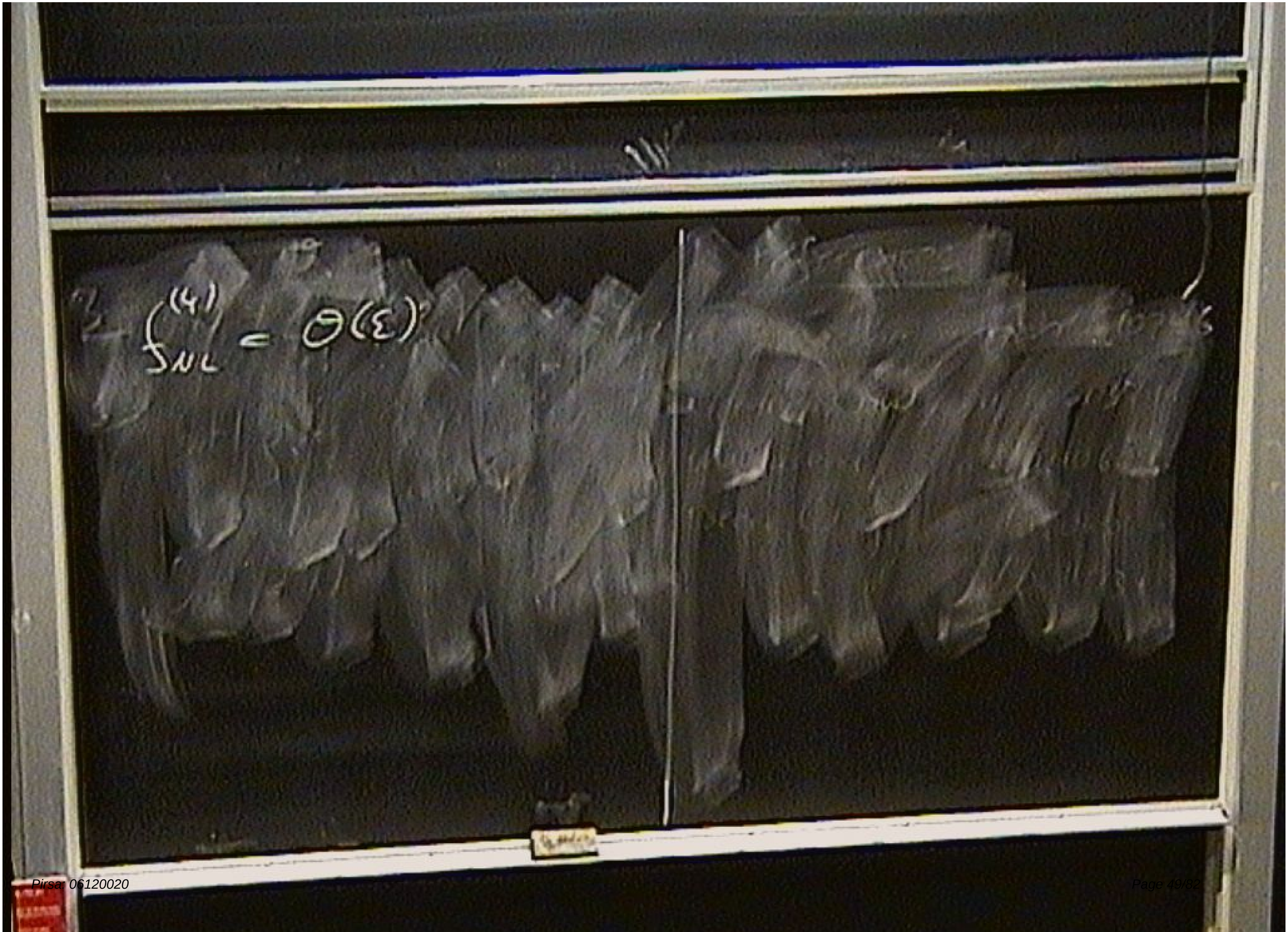
/0611034

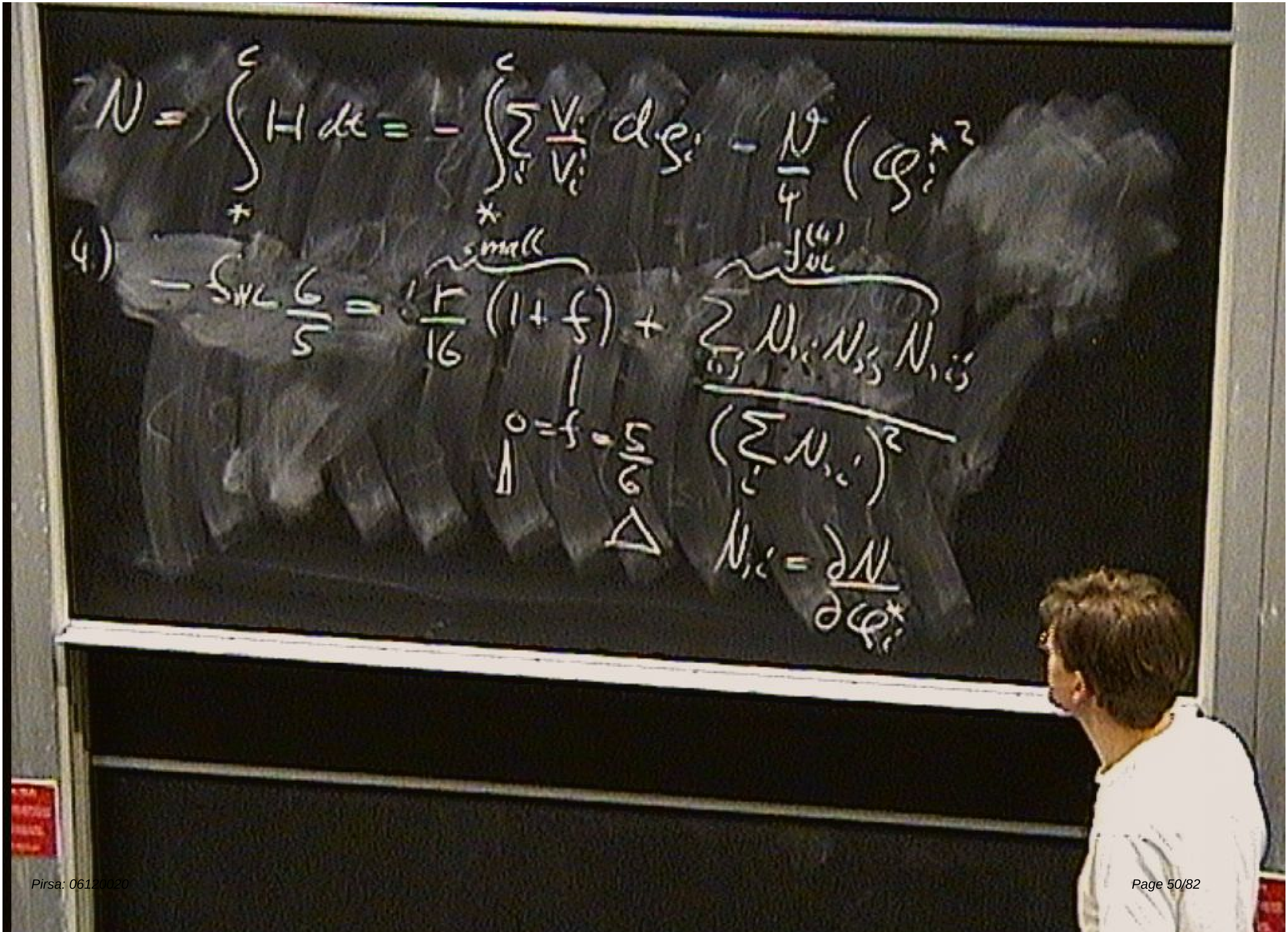

2N = ∫ H dt = − ∫ Σ Vi/Vi dρi − N/4 (ρi²
4.)
− Swl 6/5 =



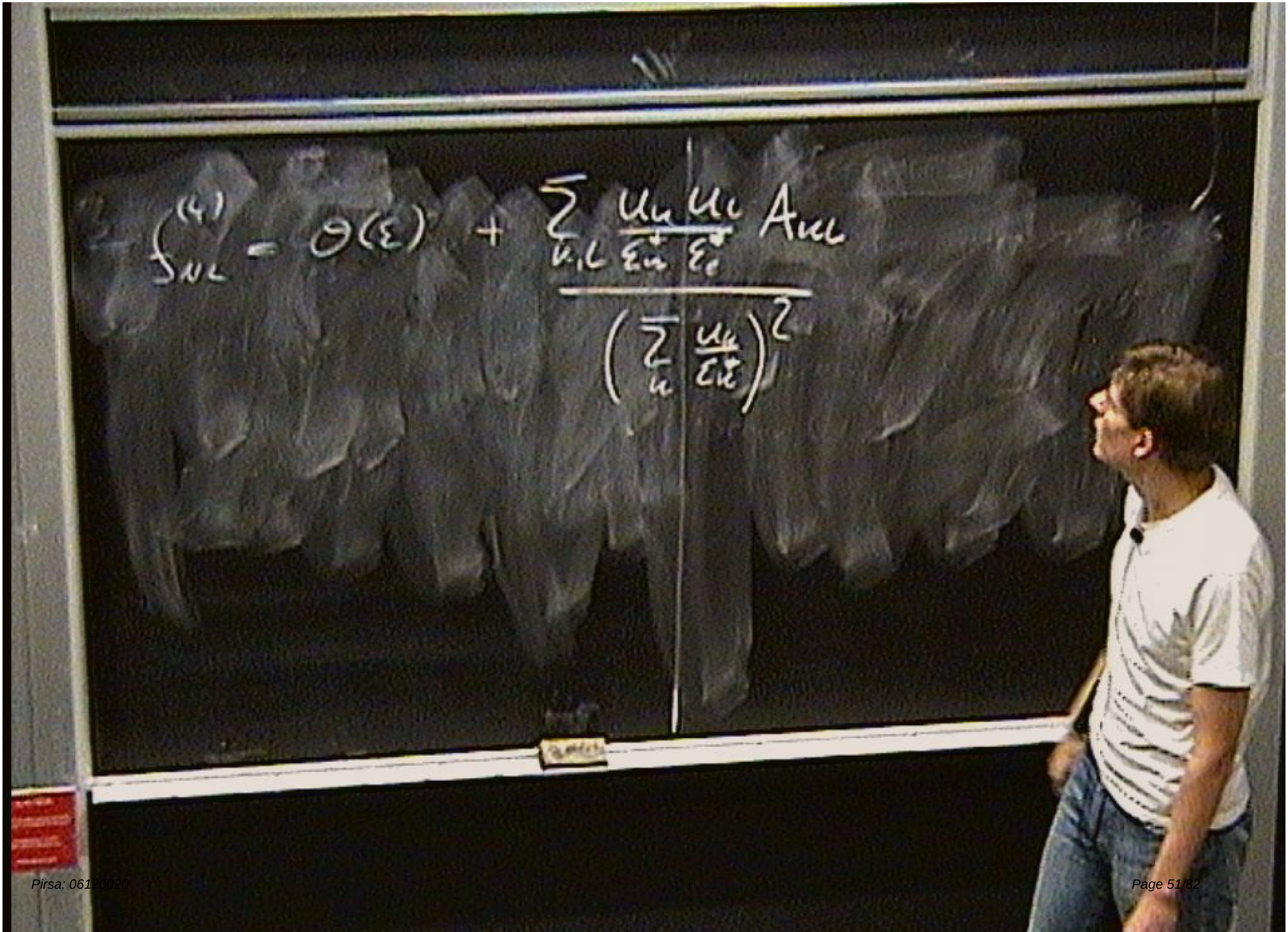

3) N = ∫ H dt = - ∫ Σ V_i / V_i' dφ_i - N/4 (φ_i*²
4) - S_NL 6/5 = 5/16 (1 + f) + Σ N_,i N_,j N_,ij / (Σ N_,i)²
0 < f < 5/6
Δ
N_,i = ∂N/∂φ_i*

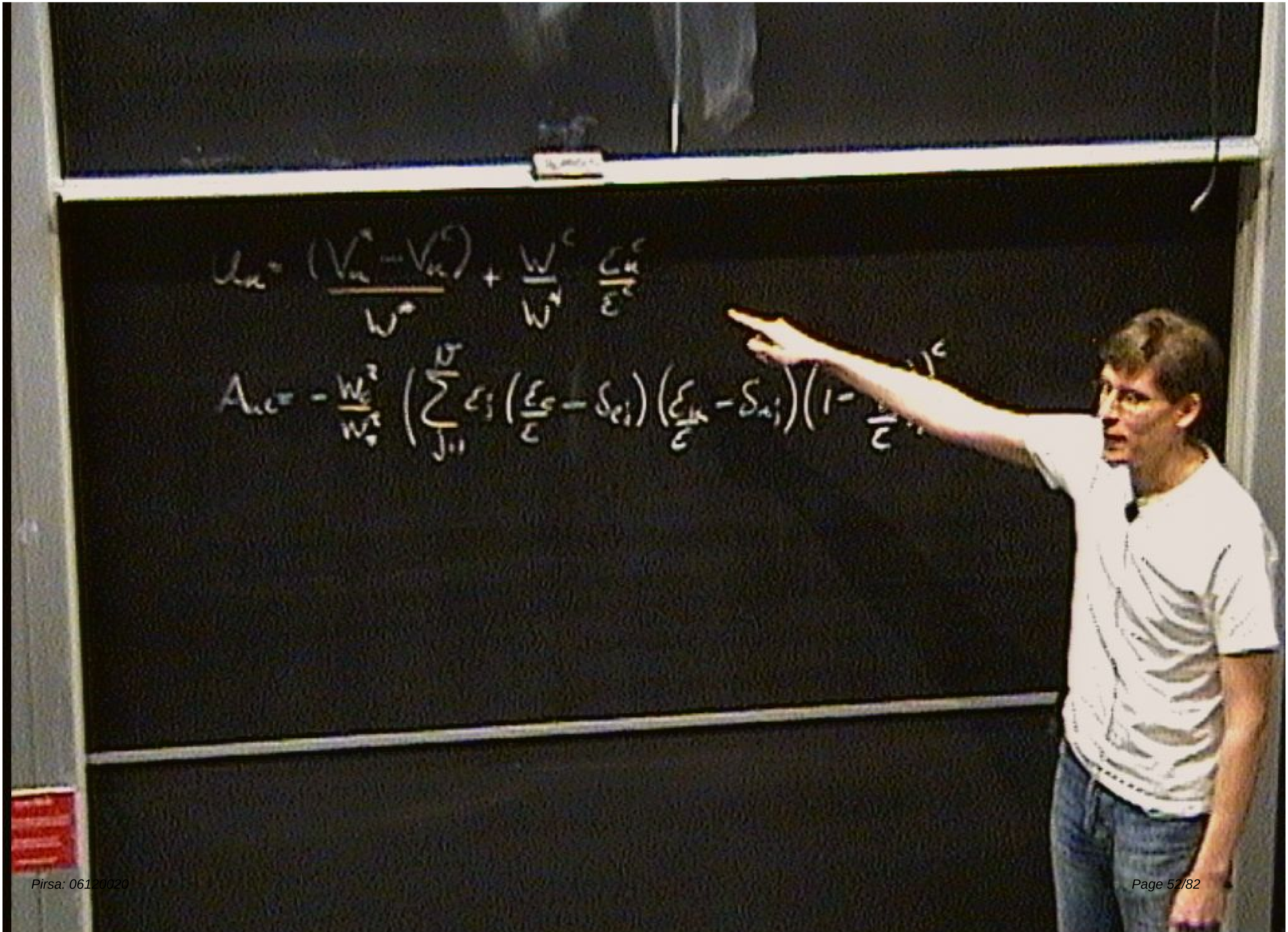

N = ∫ H dt = − ∫ Σ V̇i/Vi dφi − N/4 (φi*²
4.)
small
(1+f)
0=f=5/6
Δ

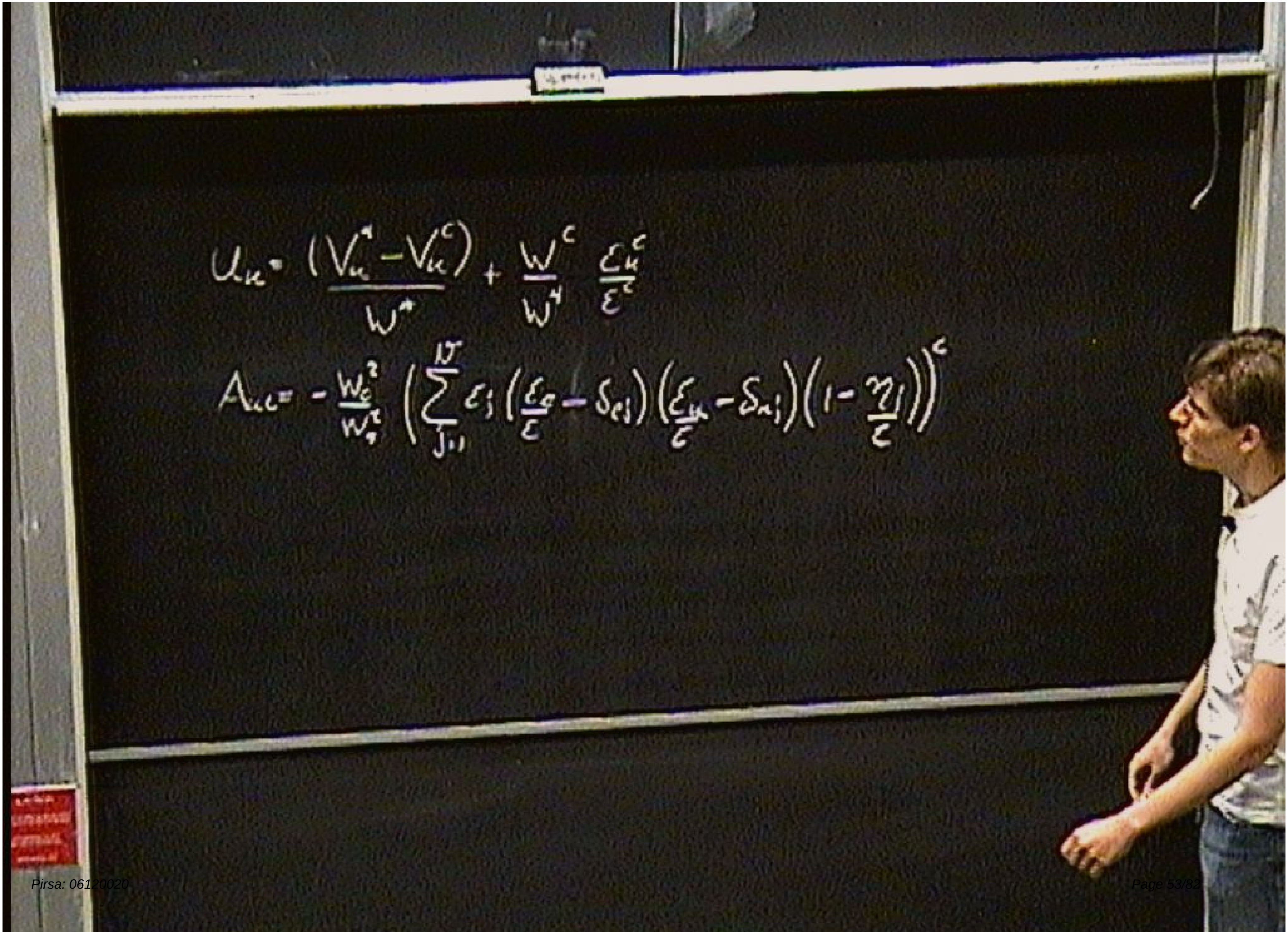

$f_{NL}^{(4)} = O(\varepsilon)$

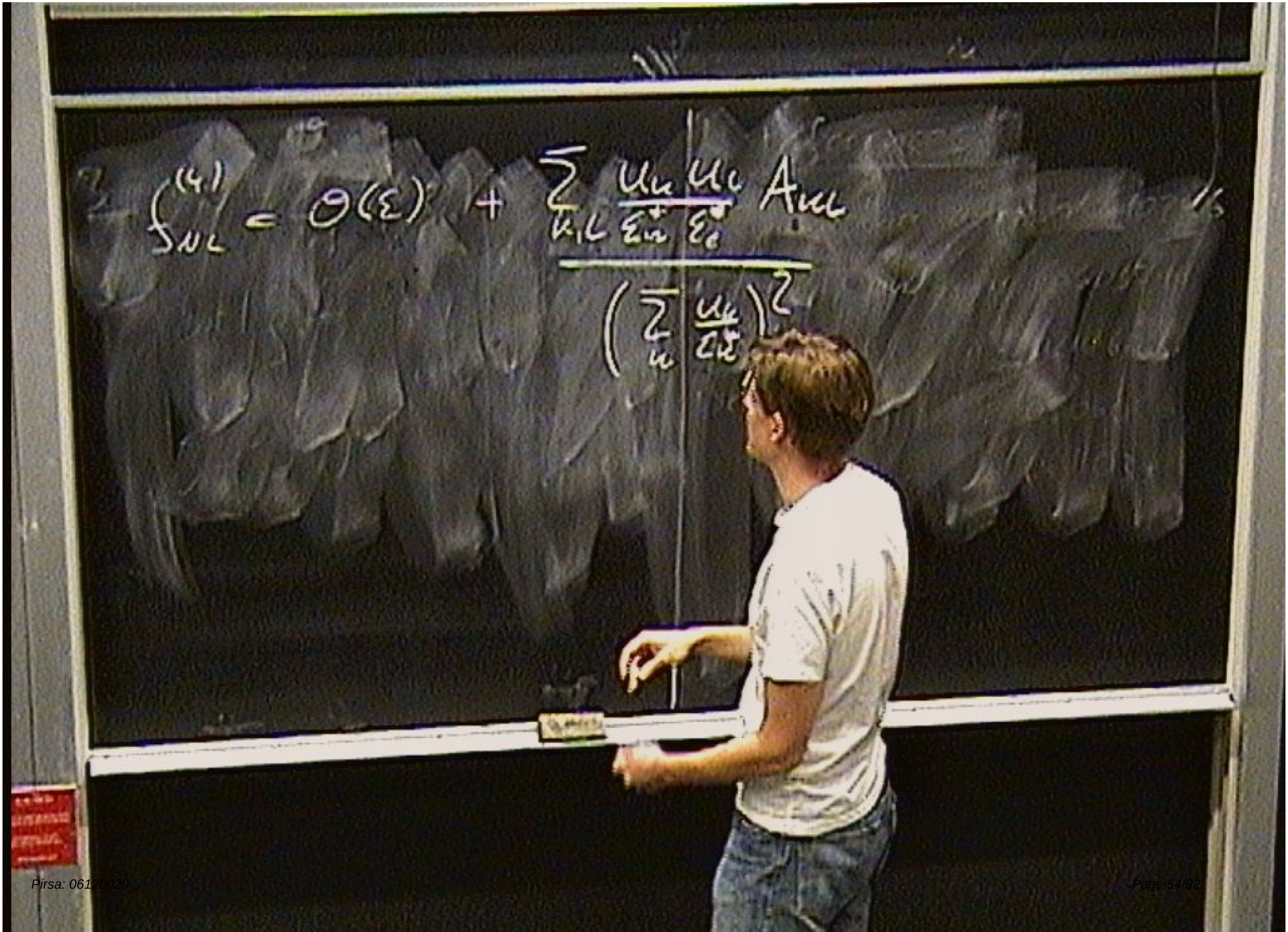


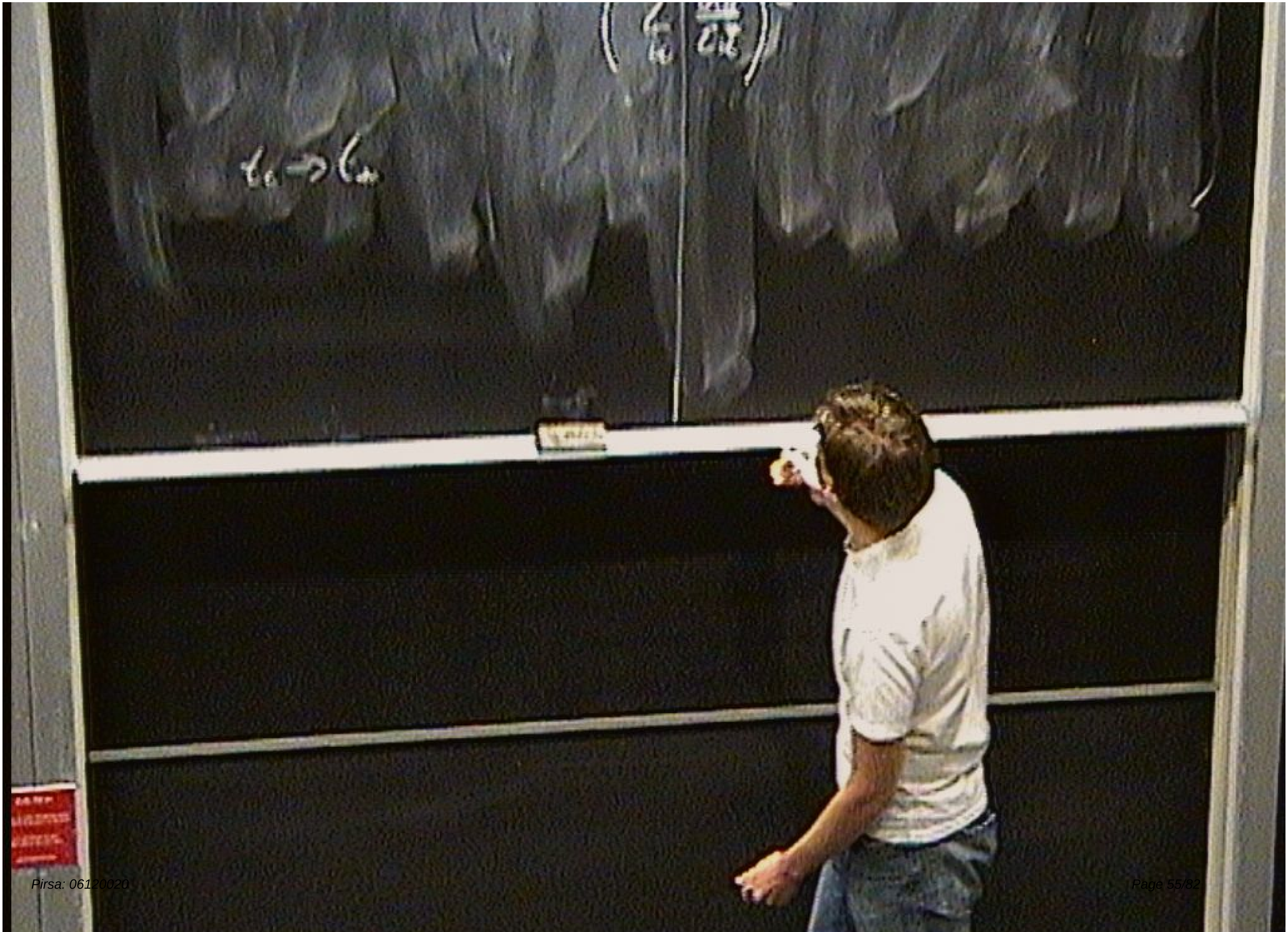
$N = \int_{*}^{c} H\, dt = -\int_{*}^{c} \sum_i \frac{V_i'}{V_i}\, d\varphi_i - \frac{N}{4}\left(\varphi_i^{*2}\right.$
4) $-f_{NL}\frac{6}{5} = \frac{5}{16}(1+f) + \frac{\sum_{ij} N_{,i} N_{,j} N_{,ij}}{\left(\sum_i N_{,i}^2\right)^2}$
$0 = f = \frac{5}{6}$
Δ
$N_{,i} = \frac{\partial N}{\partial \varphi_i^*}$

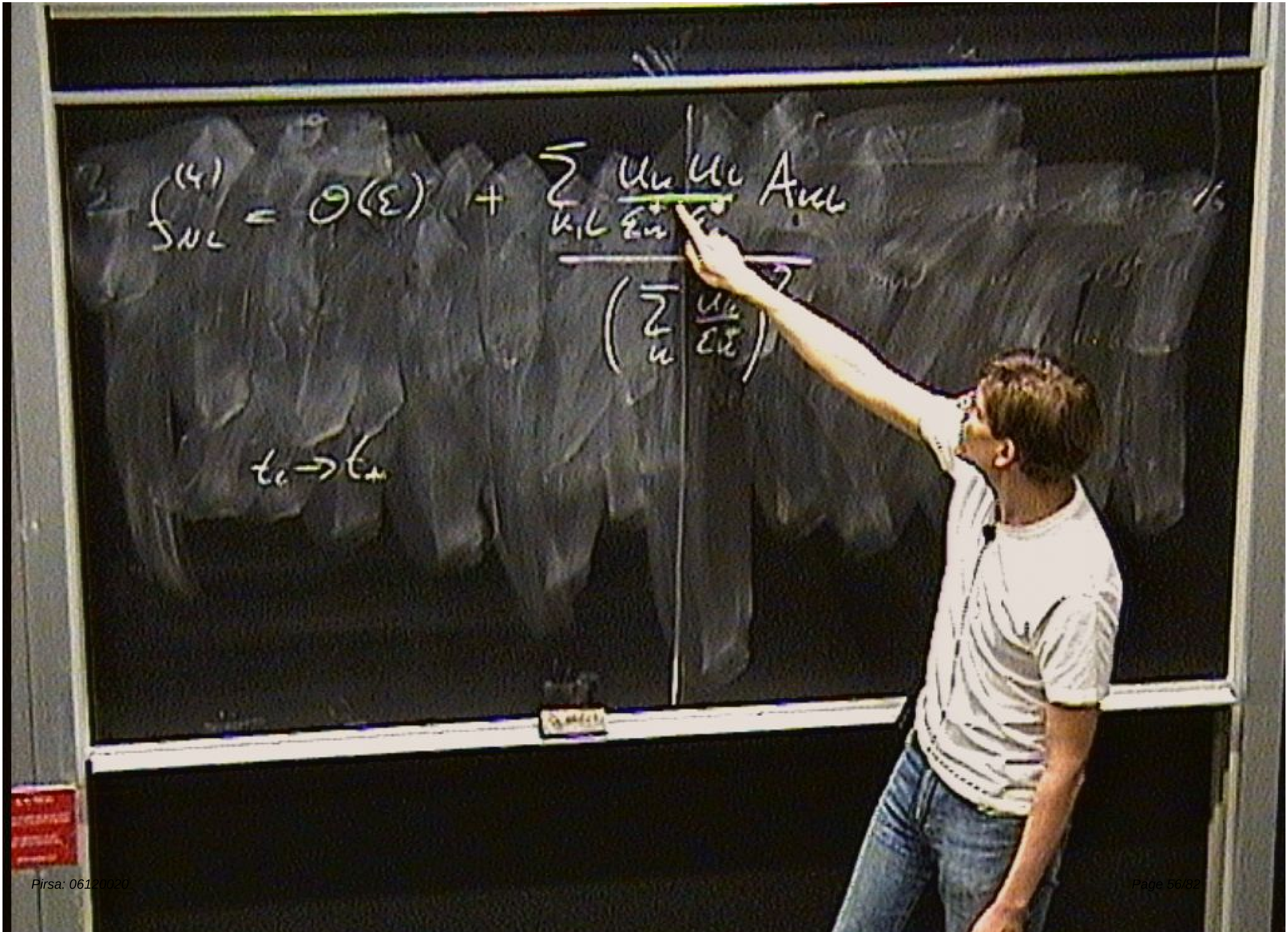

$f_{NL}^{(4)} = O(\varepsilon) + \frac{\sum_{k,l} \frac{u_k u_l}{\varepsilon_k^* \varepsilon_l^*} A_{kl}}{\left(\sum_k \frac{u_k}{\varepsilon_k^*}\right)^2}$

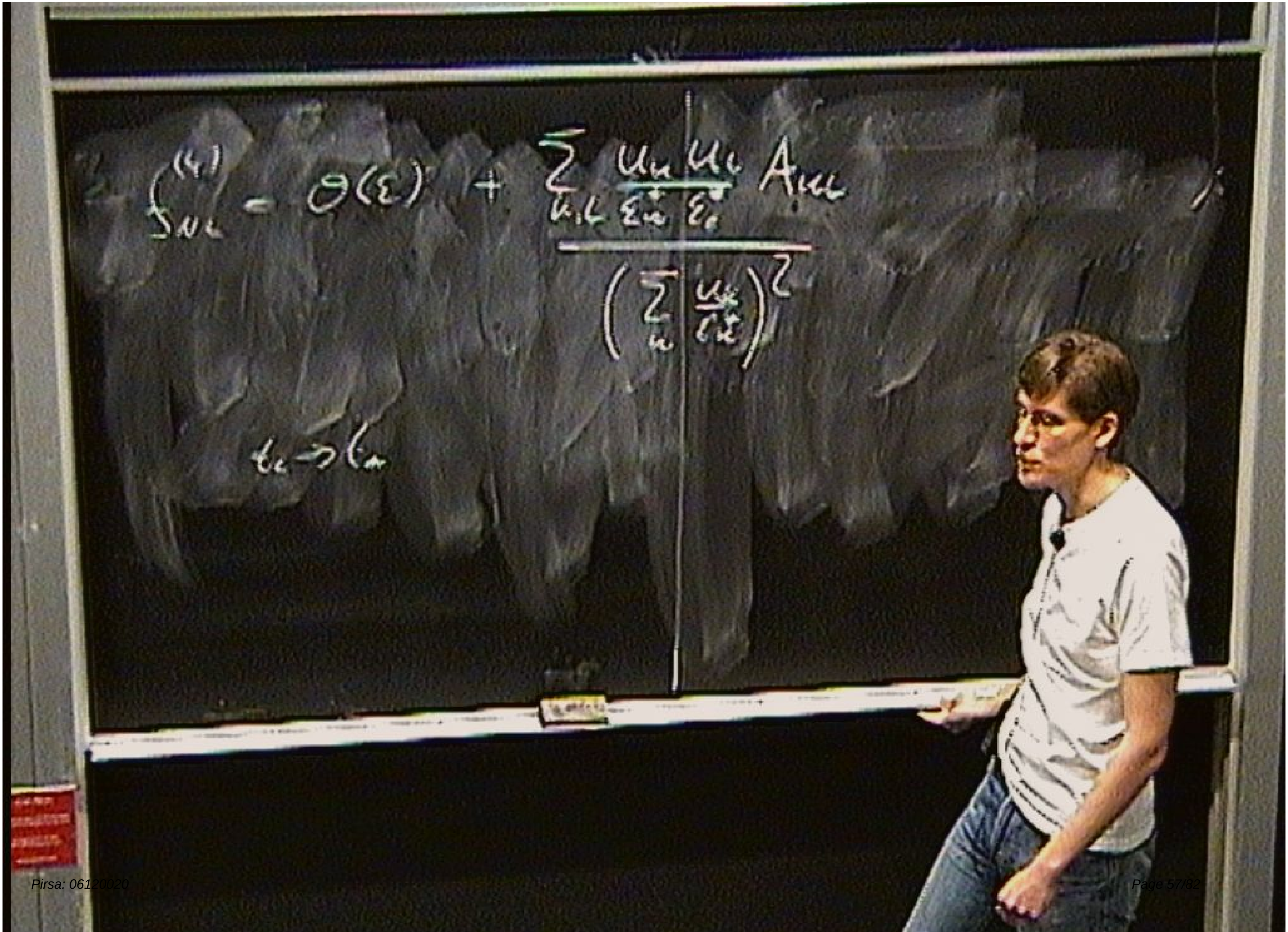


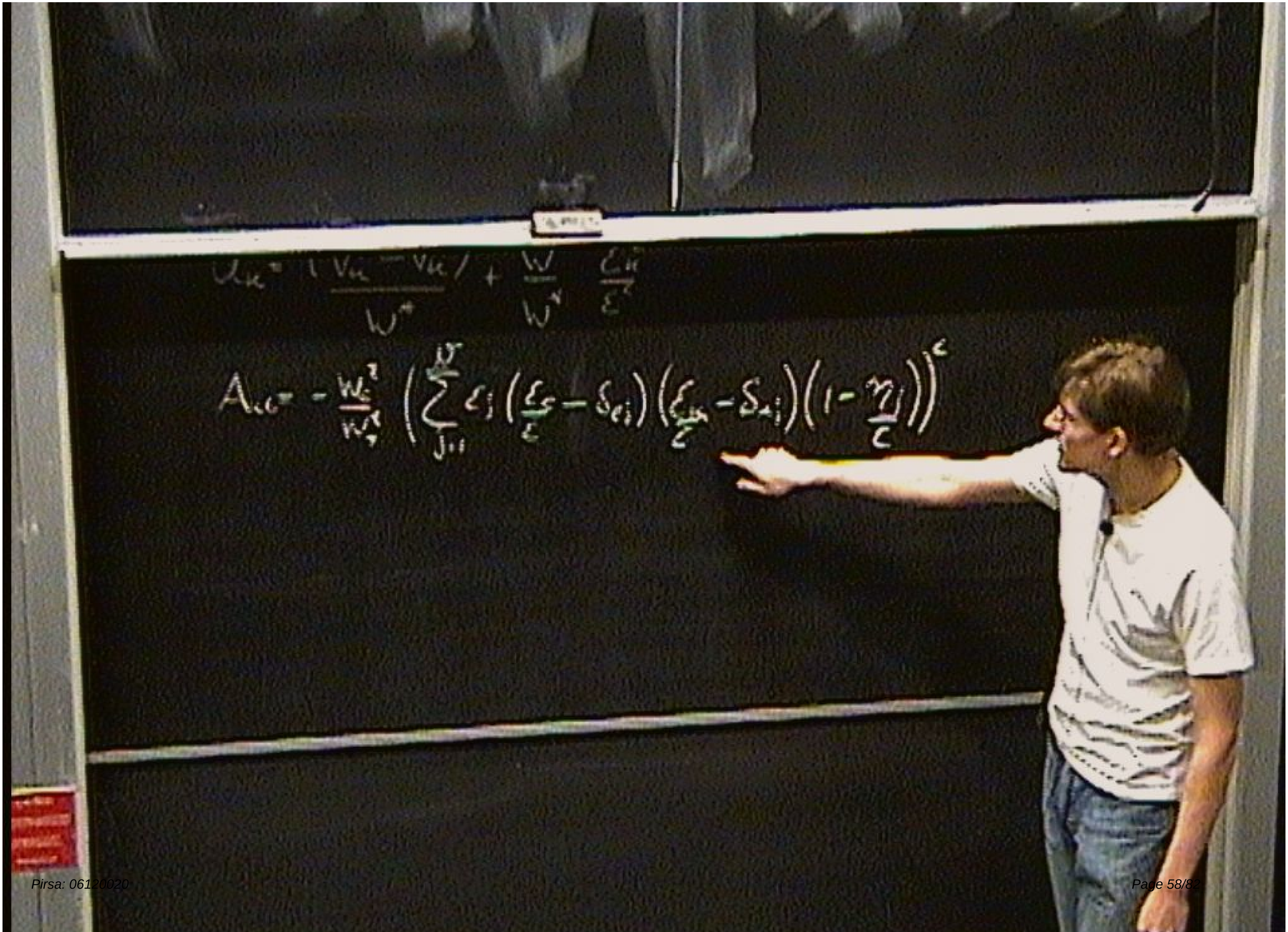

$U_k = \frac{(V_k^A - V_k^c)}{W^A} + \frac{W^c}{W^A}\frac{\varepsilon_k^c}{\varepsilon^c}$
$A_{kl} = -\frac{W_c^2}{W_A^2}\left(\sum_{j=1}^{N}\varepsilon_j\left(\frac{\varepsilon_l}{\varepsilon} - \delta_{lj}\right)\left(\frac{\varepsilon_k}{\varepsilon} - \delta_{kj}\right)\left(1 - \frac{\eta_j}{\varepsilon}\right)\right)^c$

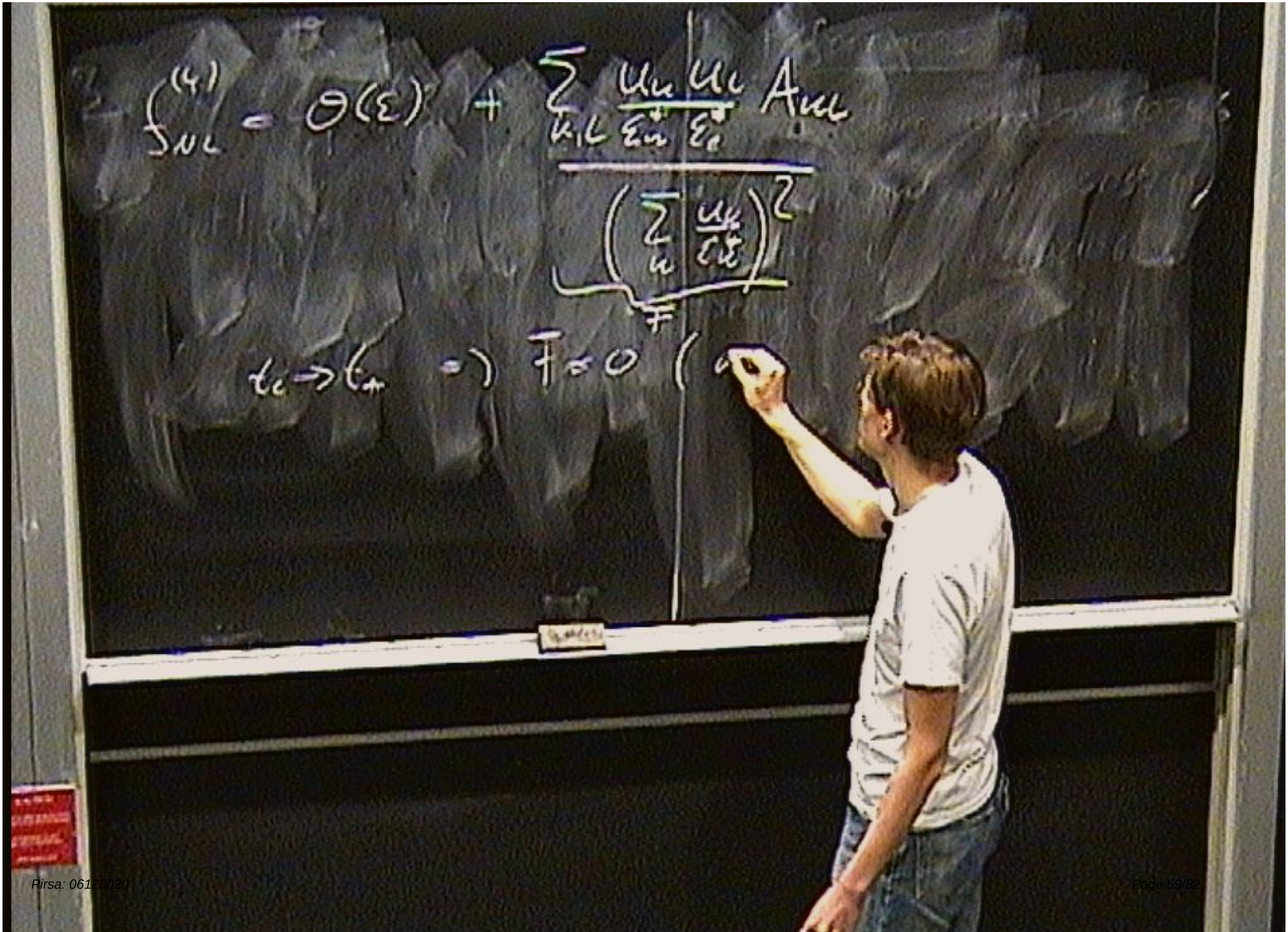

$f_{NL}^{(4)} = \mathcal{O}(\varepsilon) + \frac{\sum_{k,l} \frac{u_k u_l}{\varepsilon_k \varepsilon_l} A_{kl}}{\left(\sum_k \frac{u_k}{\varepsilon_k}\right)^2}$

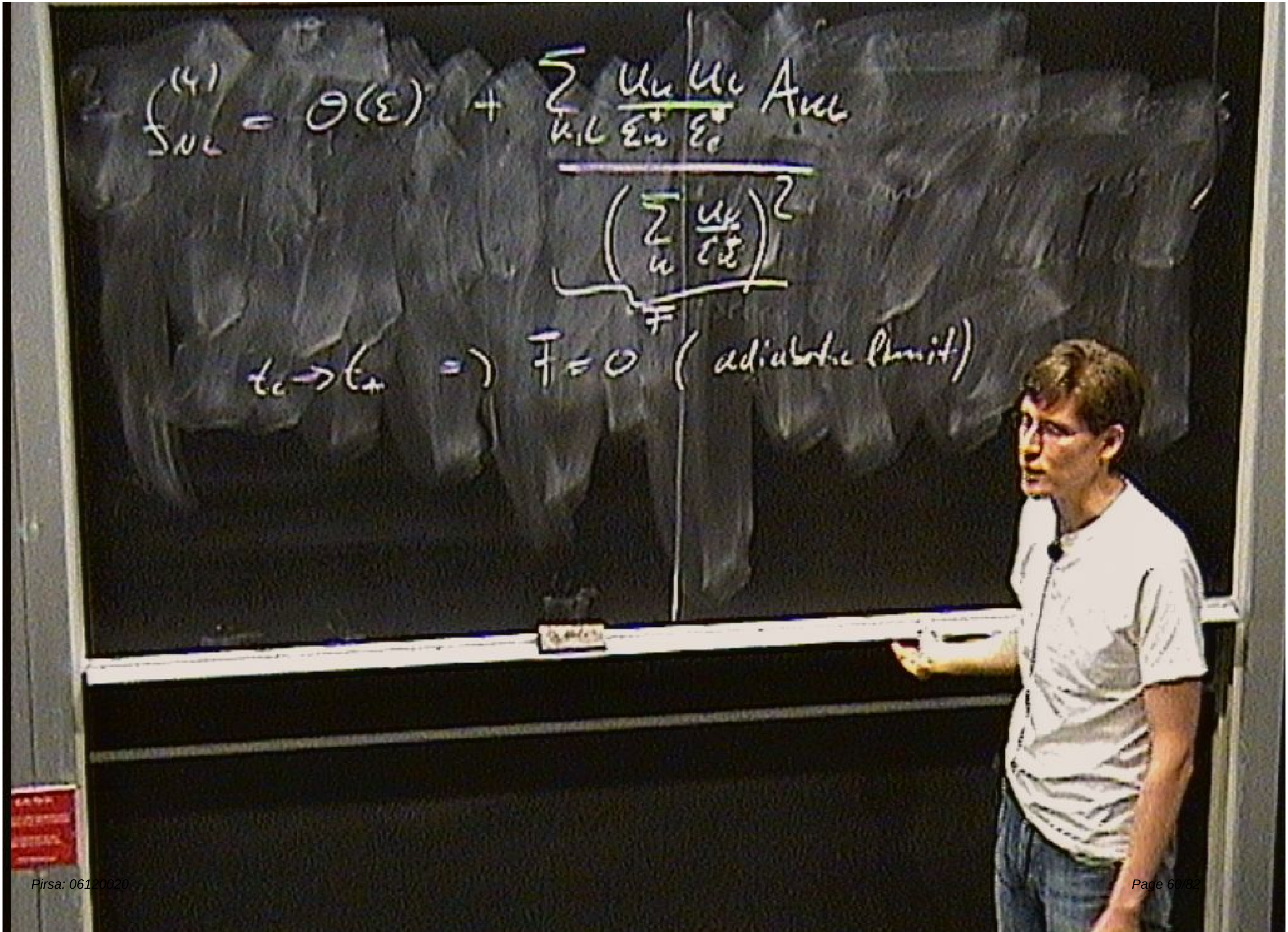


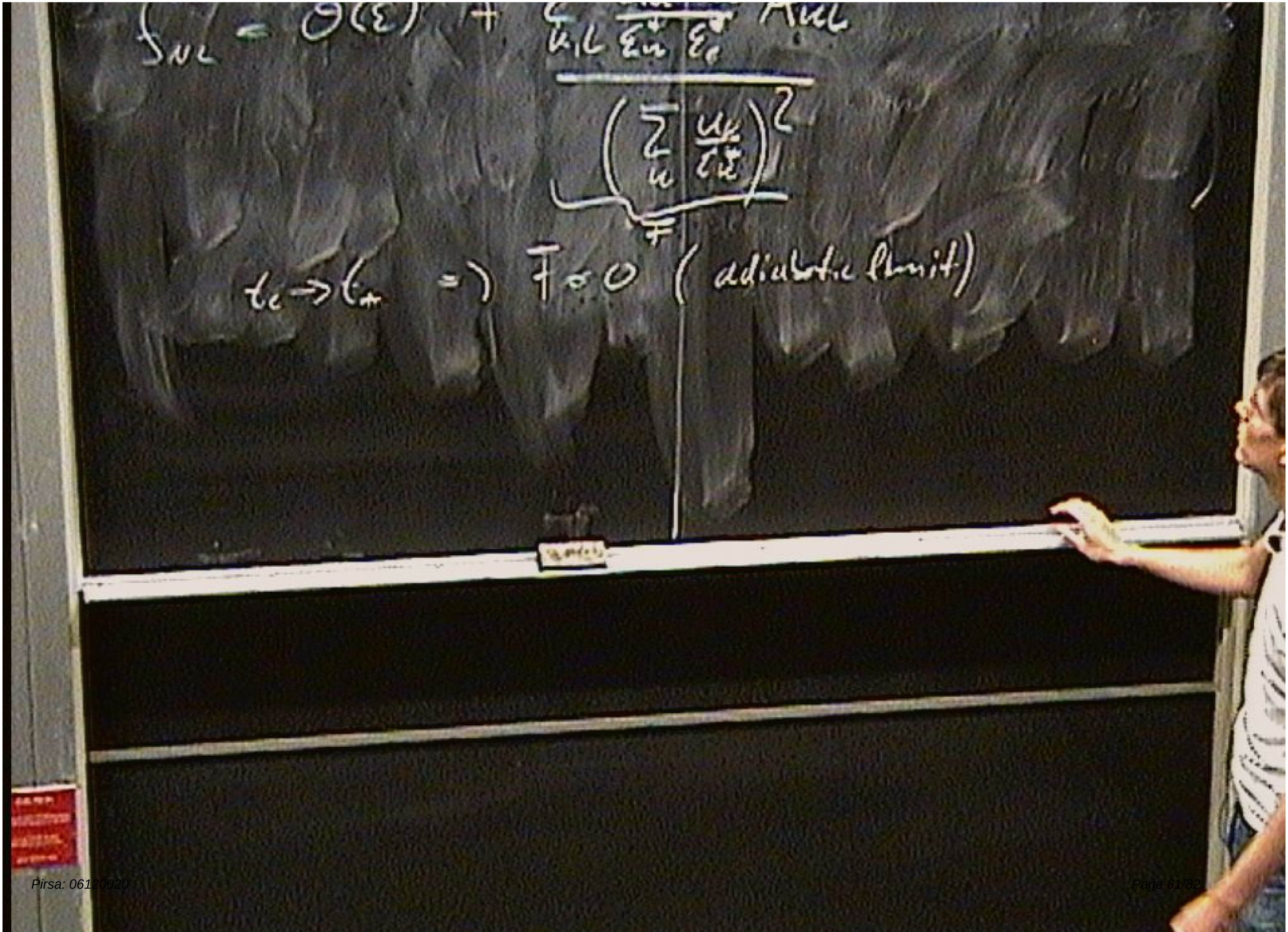

$f_{NL}^{(4)} = O(\epsilon) + \dfrac{\sum_{k,l} \frac{u_k}{\epsilon_k} \frac{u_l}{\epsilon_l} A_{kl}}{\left(\sum_k \frac{u_k}{\epsilon_k}\right)^2}$
$t_i \to t_*$


$\xi_{NL}^{(4)} = \Theta(\varepsilon) + \dfrac{\sum_{k,l} \frac{u_k u_l}{\varepsilon_k^* \varepsilon_l^*} A_{kl}}{\left(\sum_k \frac{u_k}{\varepsilon_k^*}\right)^2}$
$\varepsilon_k \to \varepsilon_m$



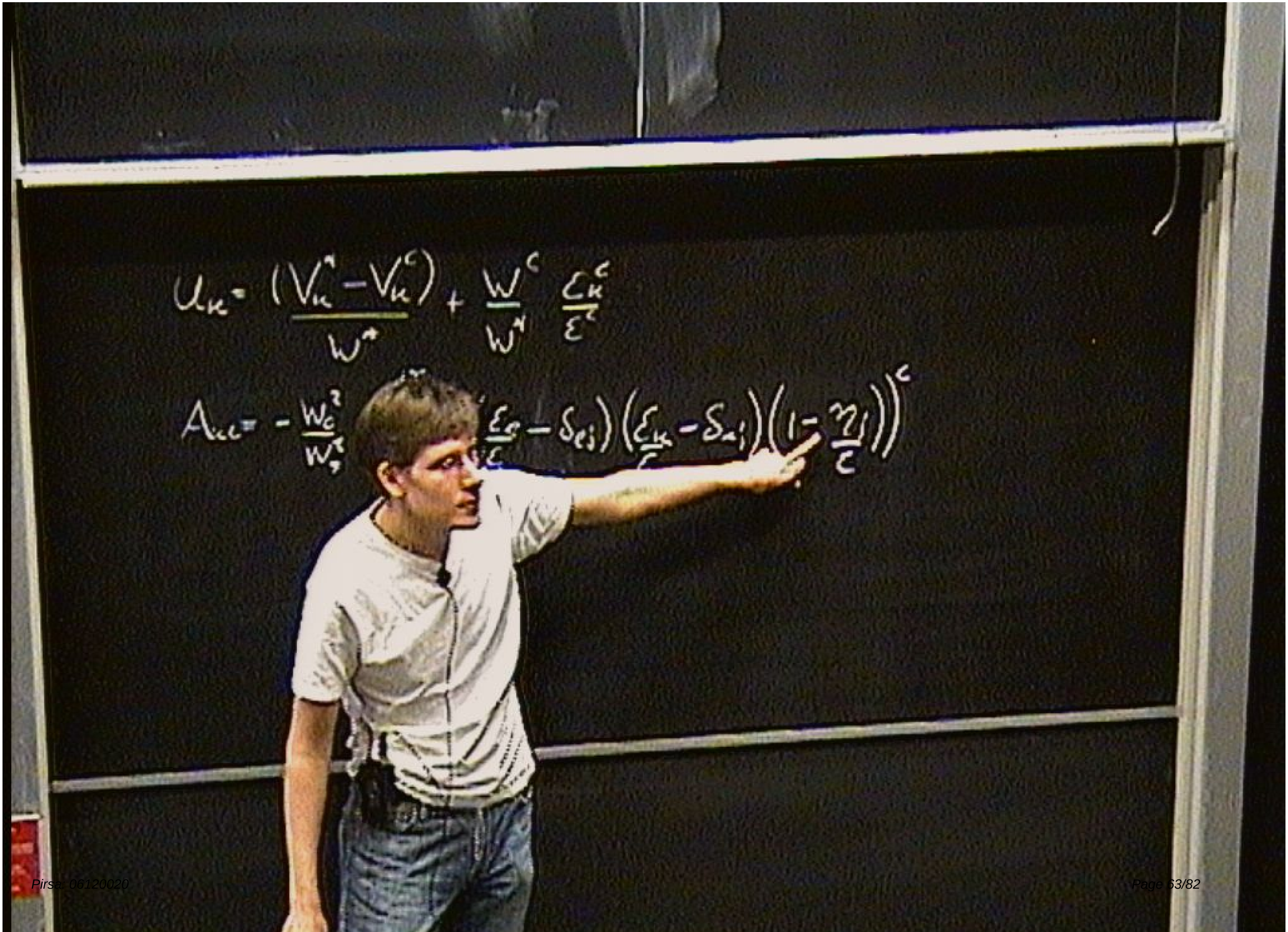

ζ_NL^(4) = O(ε) + Σ_{k,l} (u_k/ε_k)(u_l/ε_l) A_kl / (Σ_k u_k/ε_k)^2
F
ε_l → ε_m ⇒ F ≈ O(

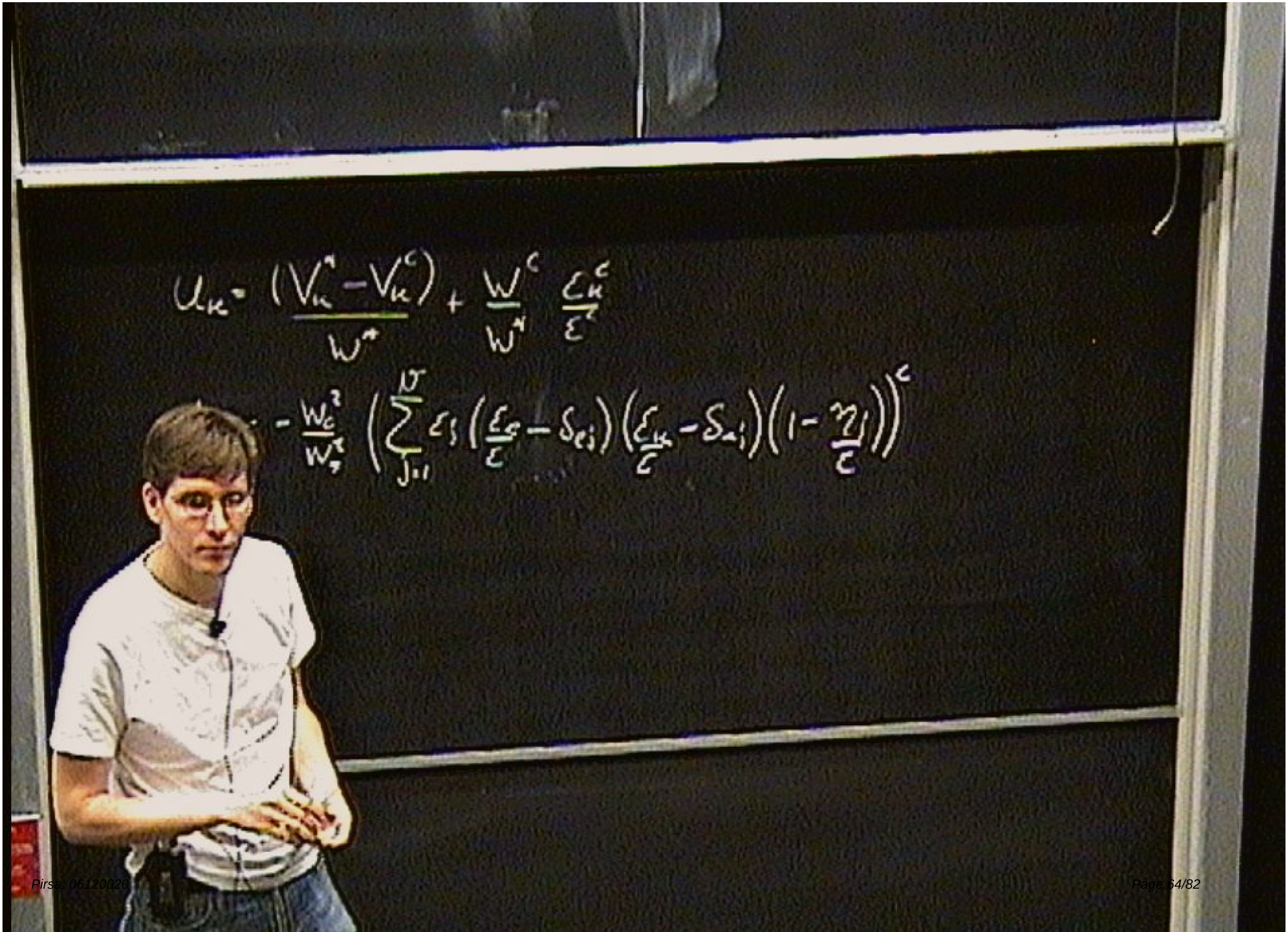

adiabatic limit

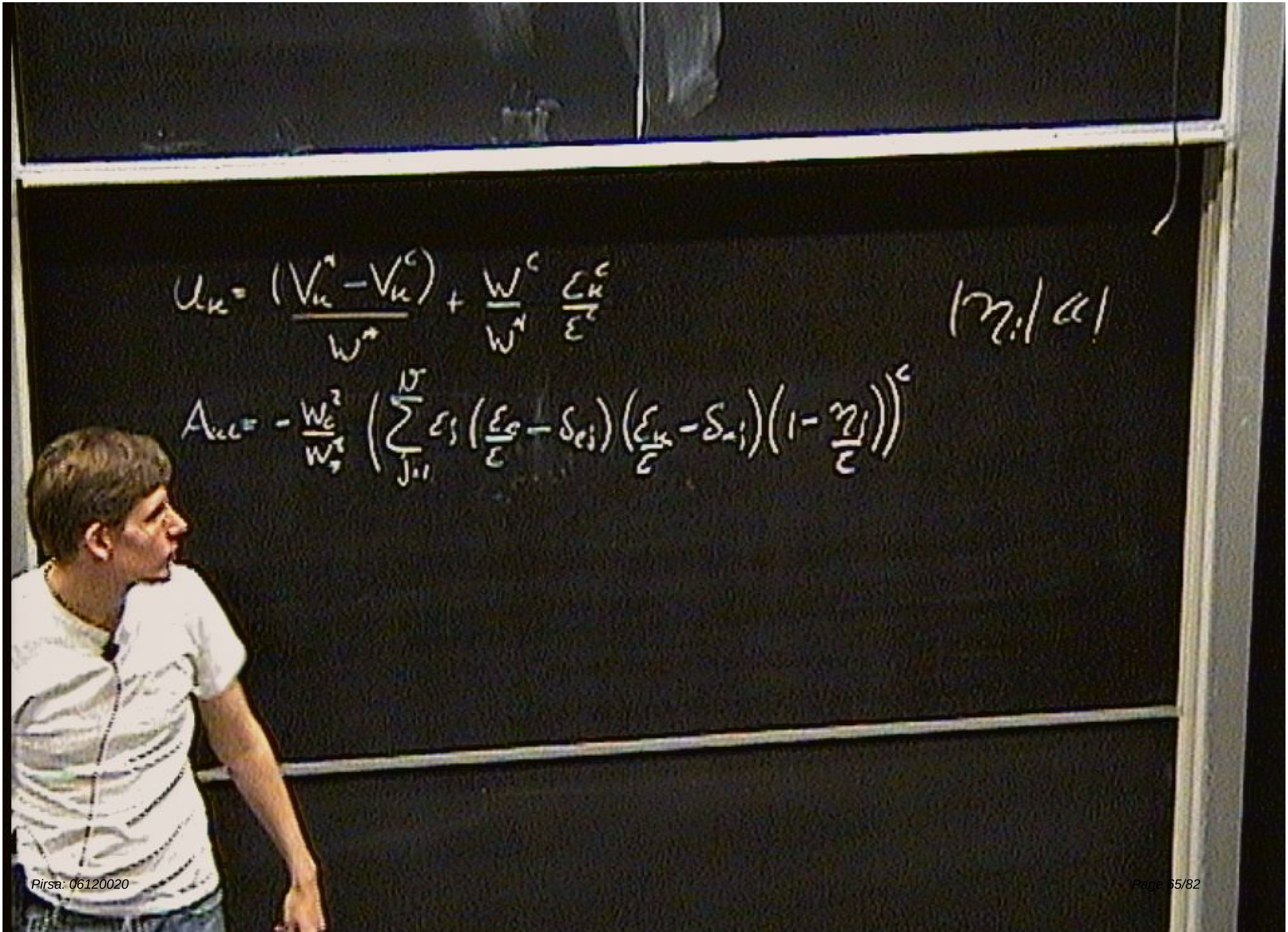

(adiabatic limit)

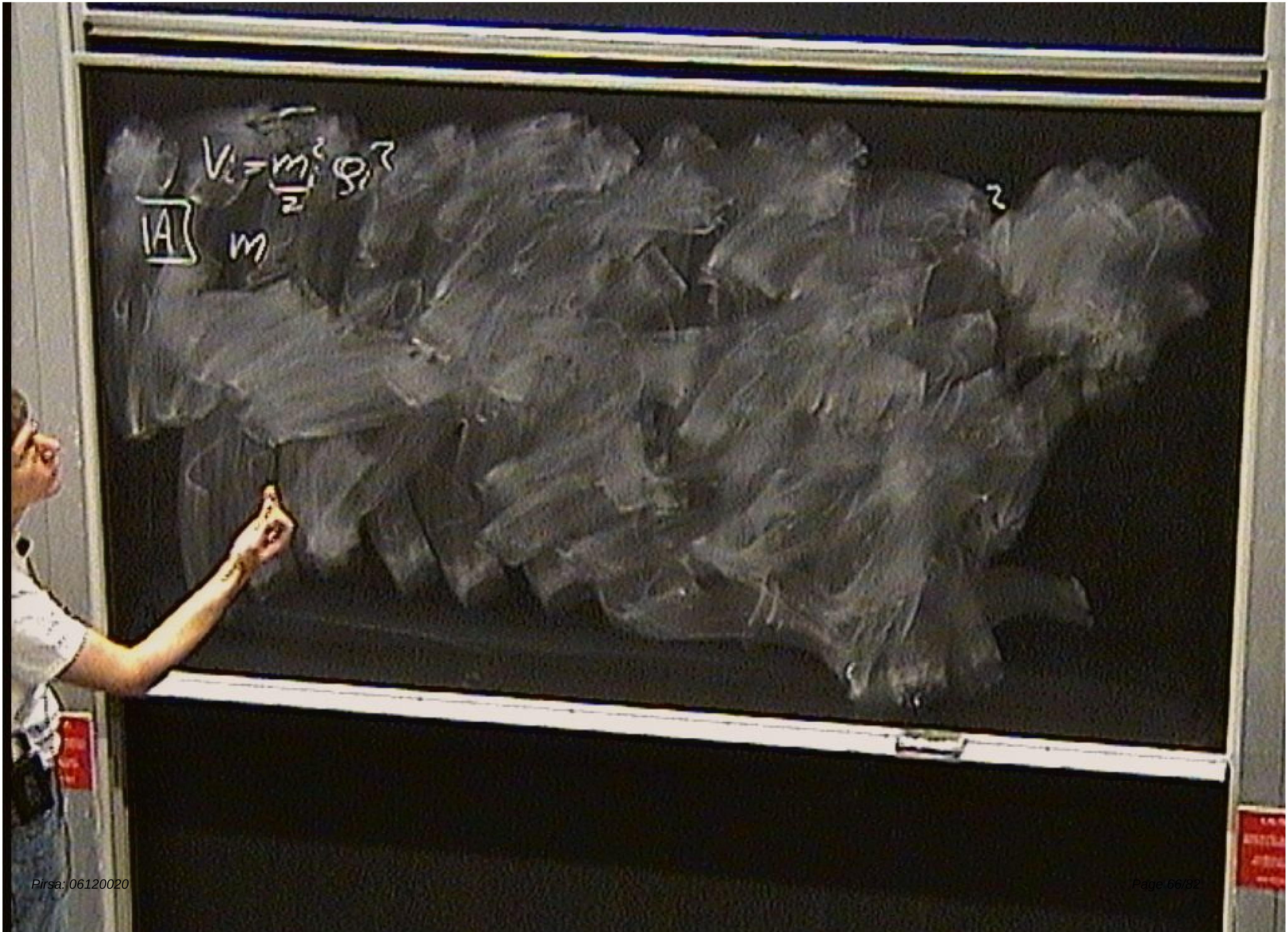
$$f_{NL}^{(4)} = O(\epsilon) + \underbrace{\frac{\sum_{k,L} \frac{u_k u_L}{\epsilon_k^* \epsilon_L^*} A_{kL}}{\left(\sum_k \frac{u_k^2}{\epsilon_k^*}\right)^2}}_{F}$$

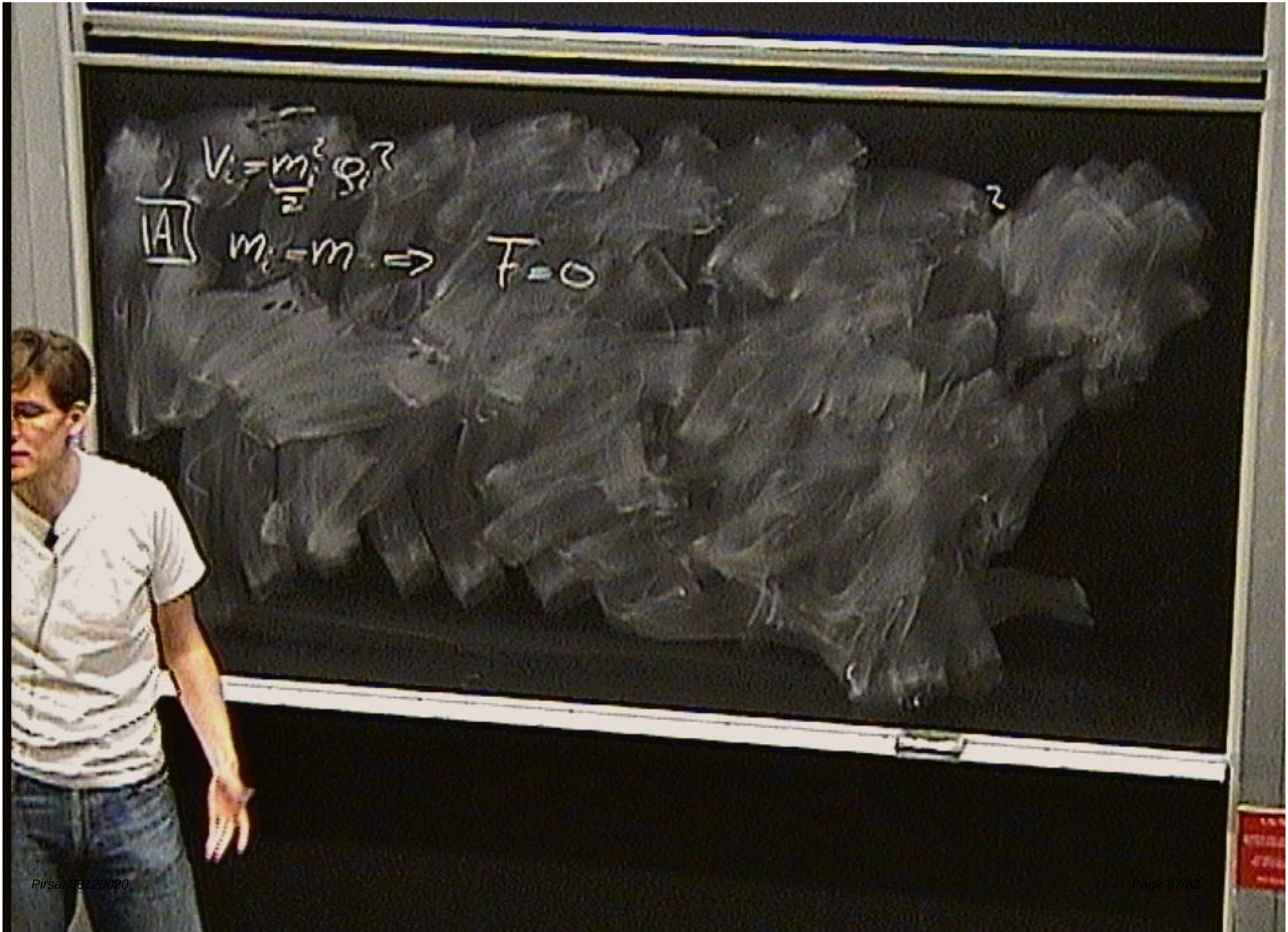
$$t_c \to t_* \Rightarrow F = 0 \quad (\text{adiabatic limit})$$

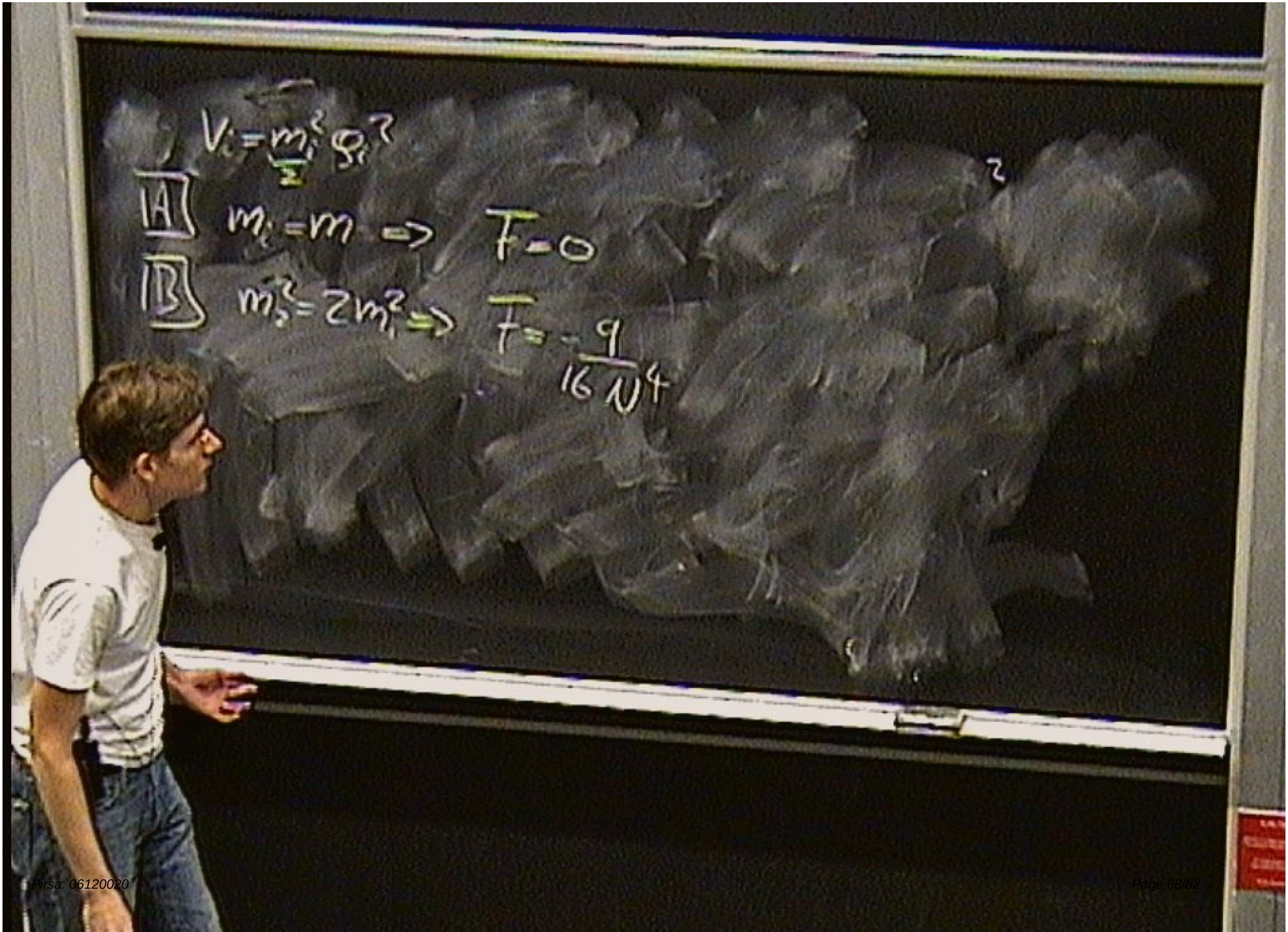

$U_K = \frac{(V_K^a - V_K^c)}{W^a} + \frac{W^c}{W^a} \frac{\varepsilon_K^c}{\varepsilon^c}$

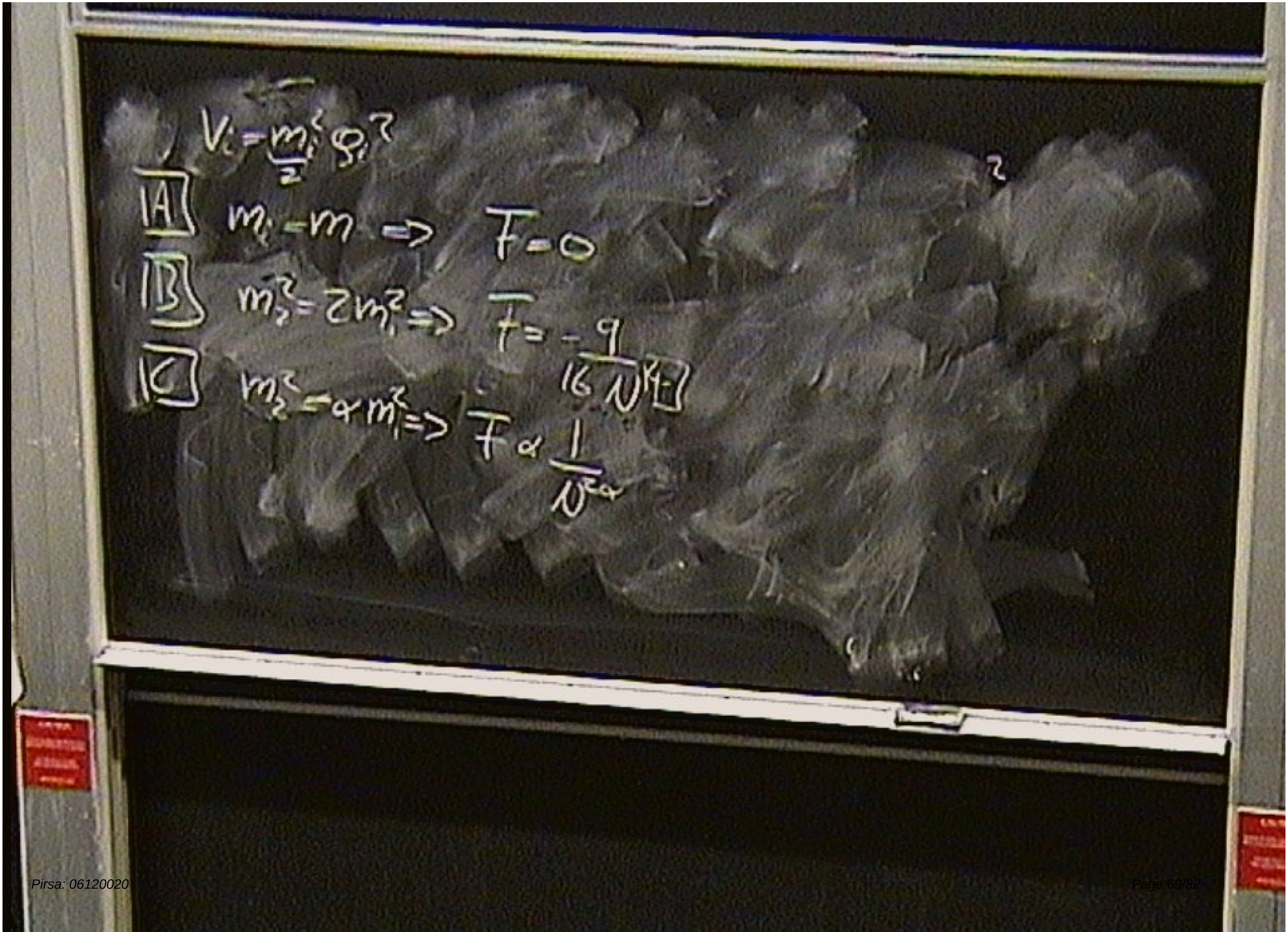

U_k = (V_k^a − V_k^c)/W^a + (W^c/W^a) (ε_k^c/ε^c)
− (W_c^2/W_a^3) (Σ_{j=1}^{N} ε_j (ε_a/ε − δ_{aj}) (ε_k/ε − δ_{kj}) (1 − η_j/ε))^c

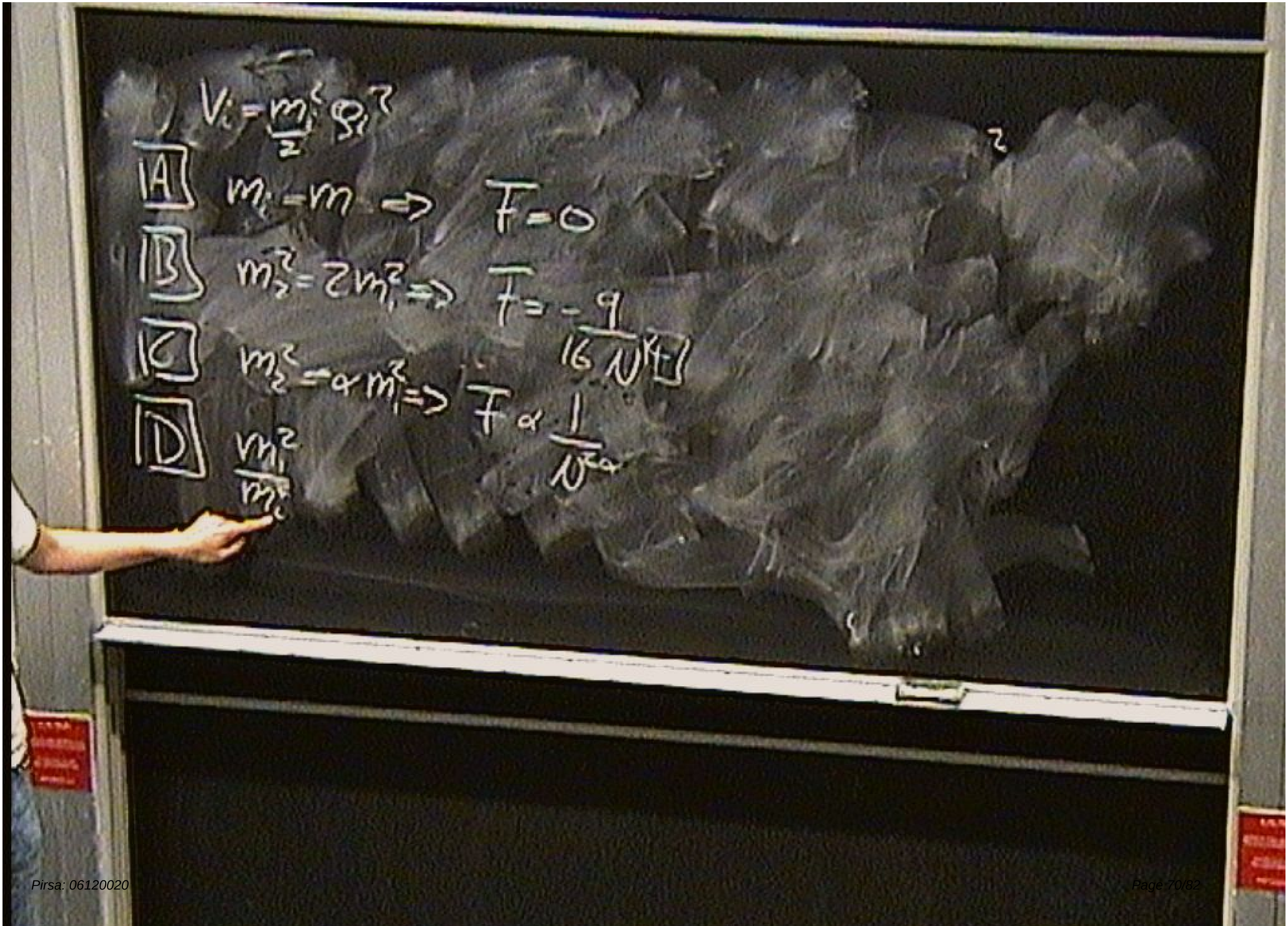

$U_{kk} = \frac{(V_k^a - V_k^c)}{W^a} + \frac{W^c}{W^a}\frac{\varepsilon_k^c}{\varepsilon^c}$
$|\eta_i| \ll 1$
$A_{kk} = -\frac{W_c^2}{W_a^2}\left(\sum_{j=1}^{N} \varepsilon_j \left(\frac{\varepsilon_e}{\varepsilon} - \delta_{ej}\right)\left(\frac{\varepsilon_k}{\varepsilon} - \delta_{kj}\right)\left(1 - \frac{\eta_j}{\varepsilon}\right)\right)^c$



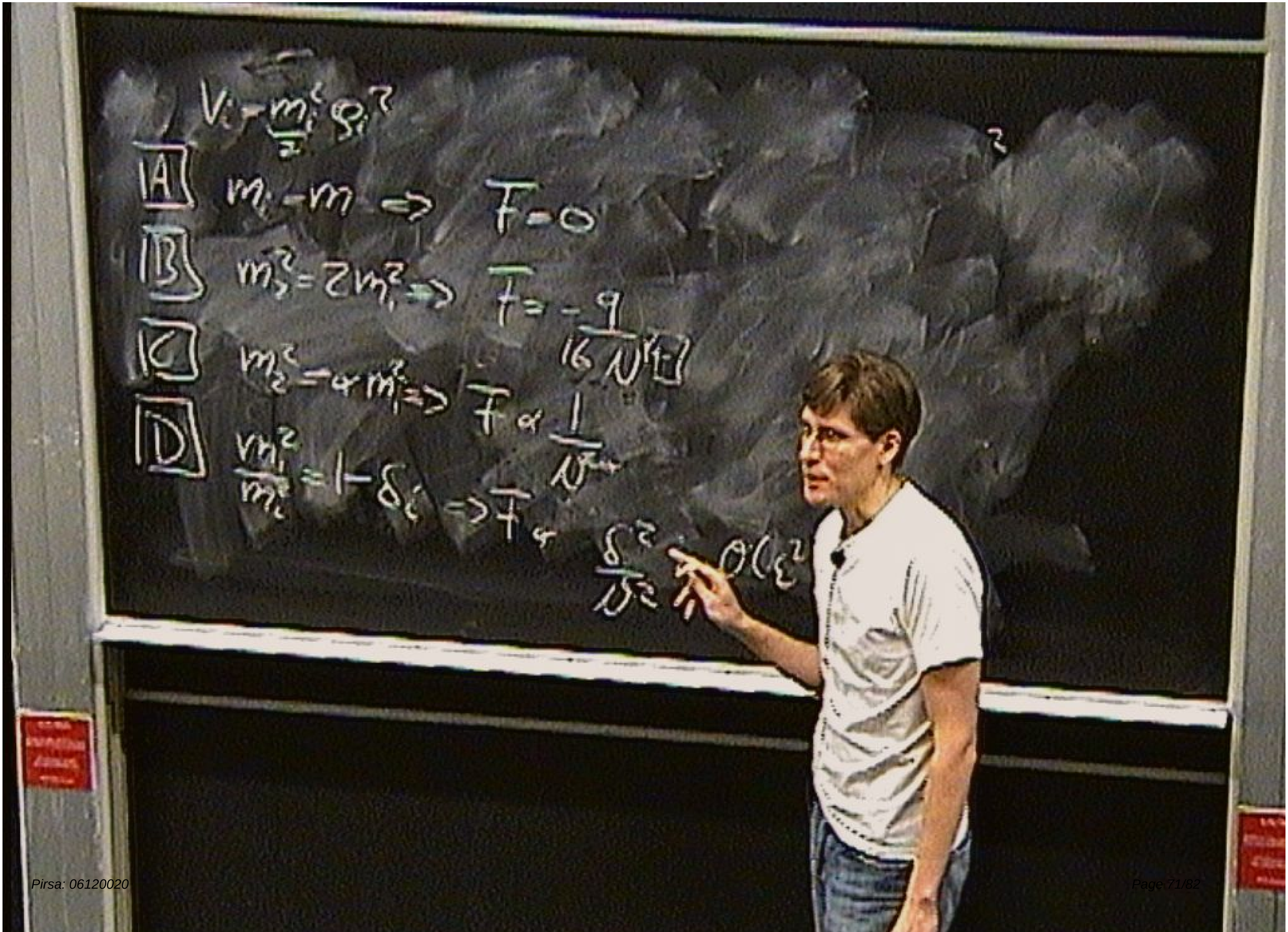

V_i = m_i^2/2 φ_i^2
A m_i = m → F = 0

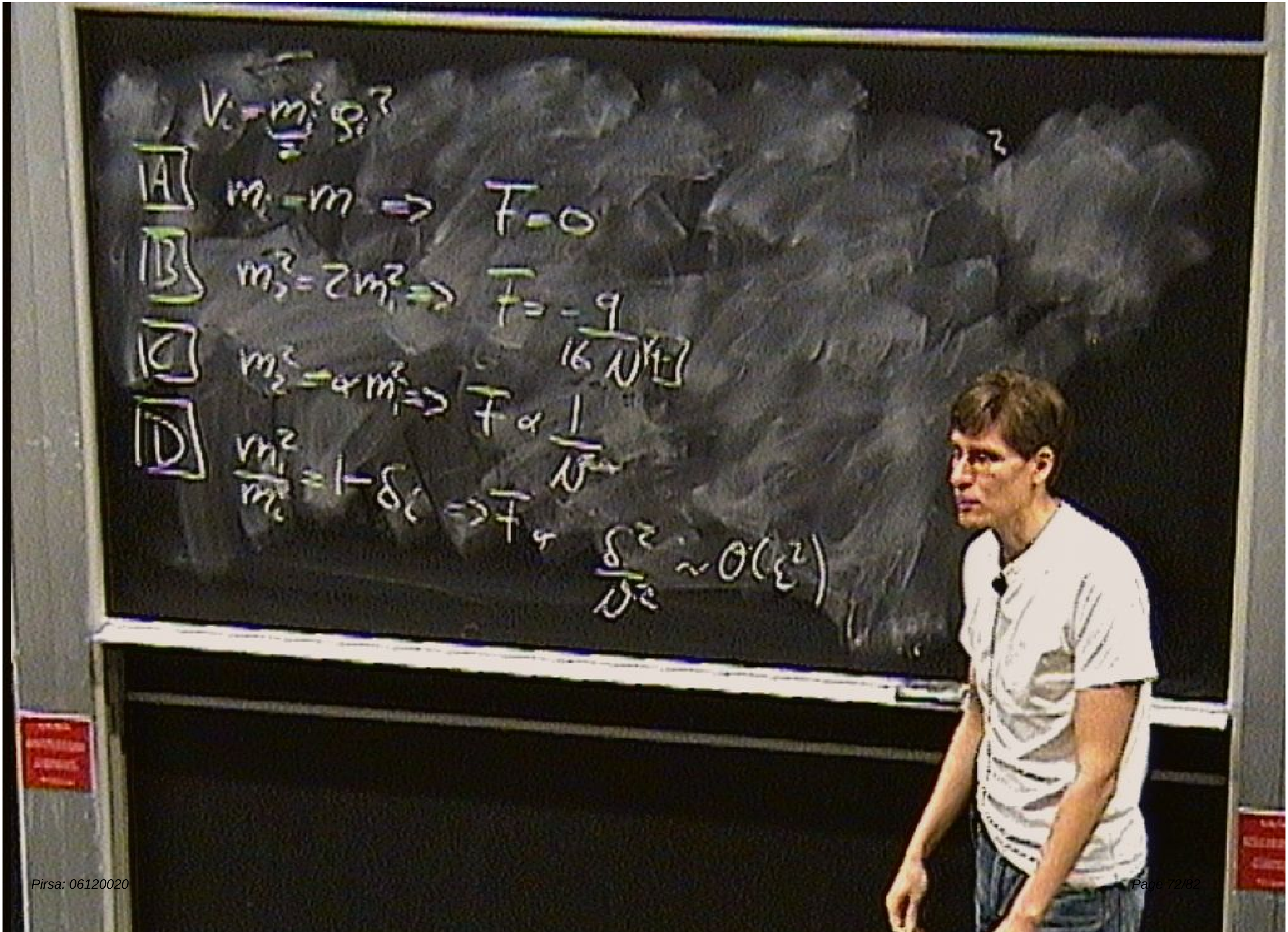

$V_i = \frac{m_i^2}{2} \varphi_i^2$
A $m_i = m \Rightarrow F = 0$
B $m_2^2 = 2m_1^2 \Rightarrow F = -\frac{9}{16 N^4}$

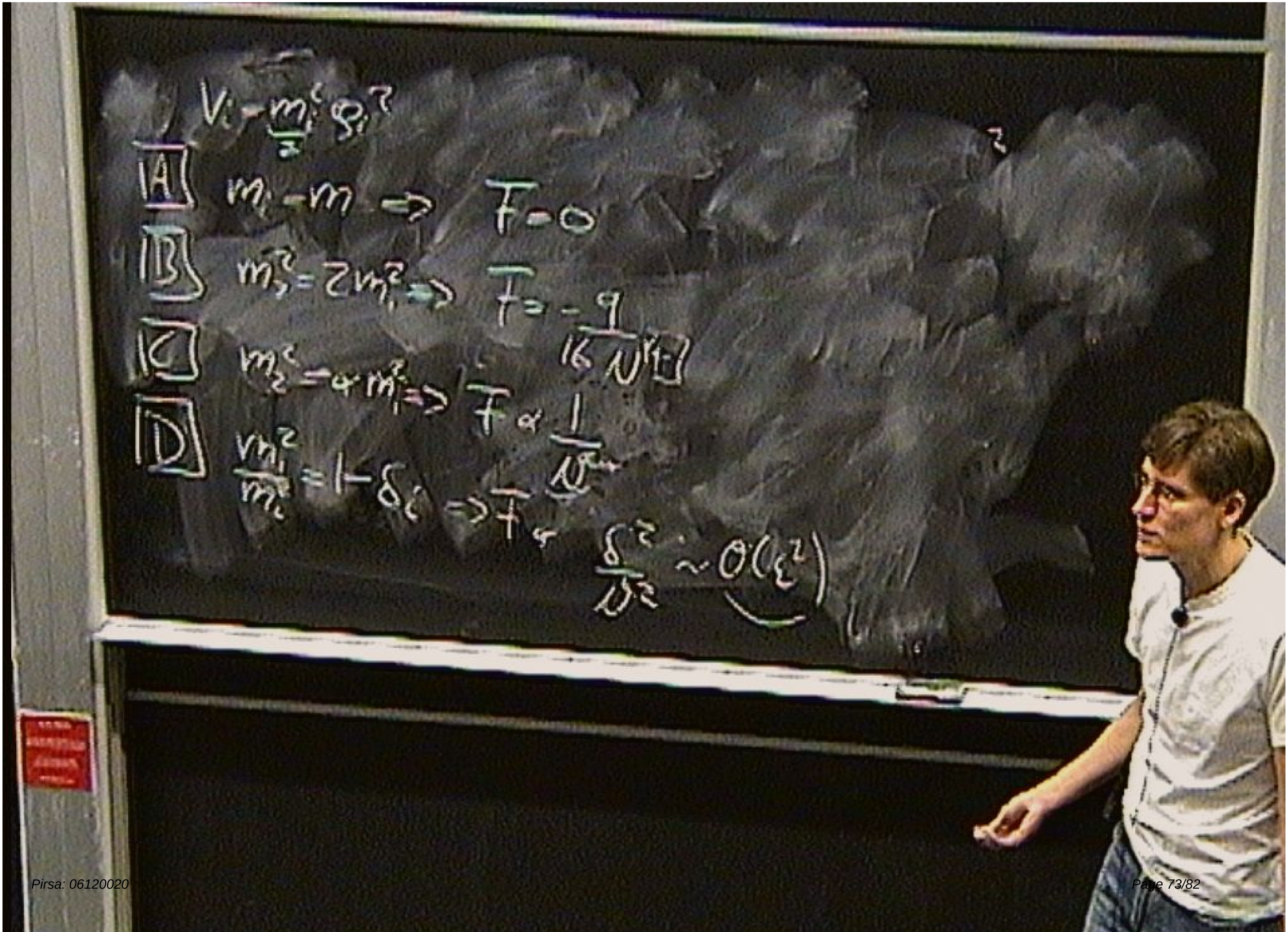

V_i = m_i^2/2 φ_i^2
A m_i = m => F = 0
B m_2^2 = 2m_1^2 => F = -9/16N
C m_2^2 = α m_1^2 => F α 1/N^2α

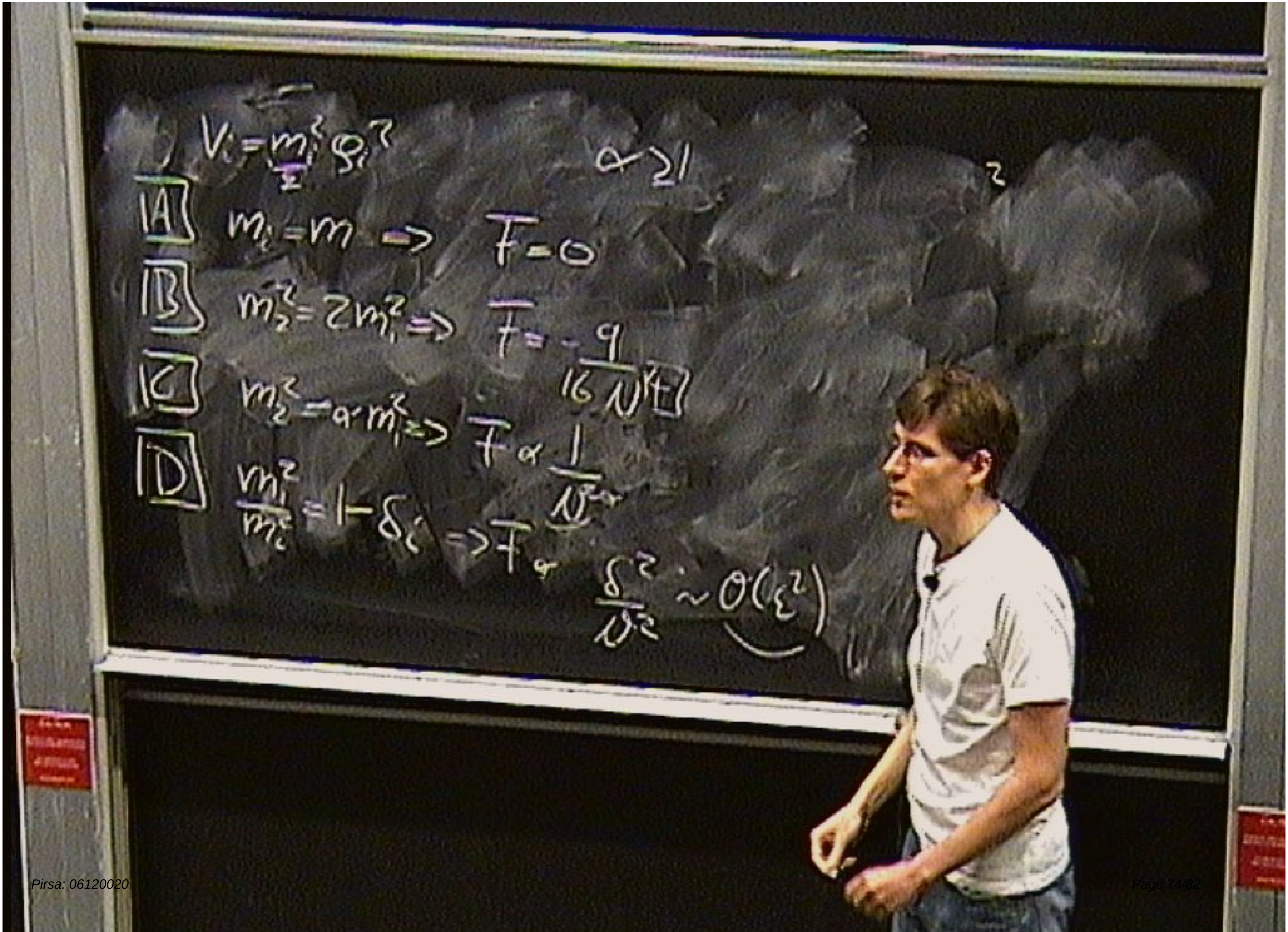


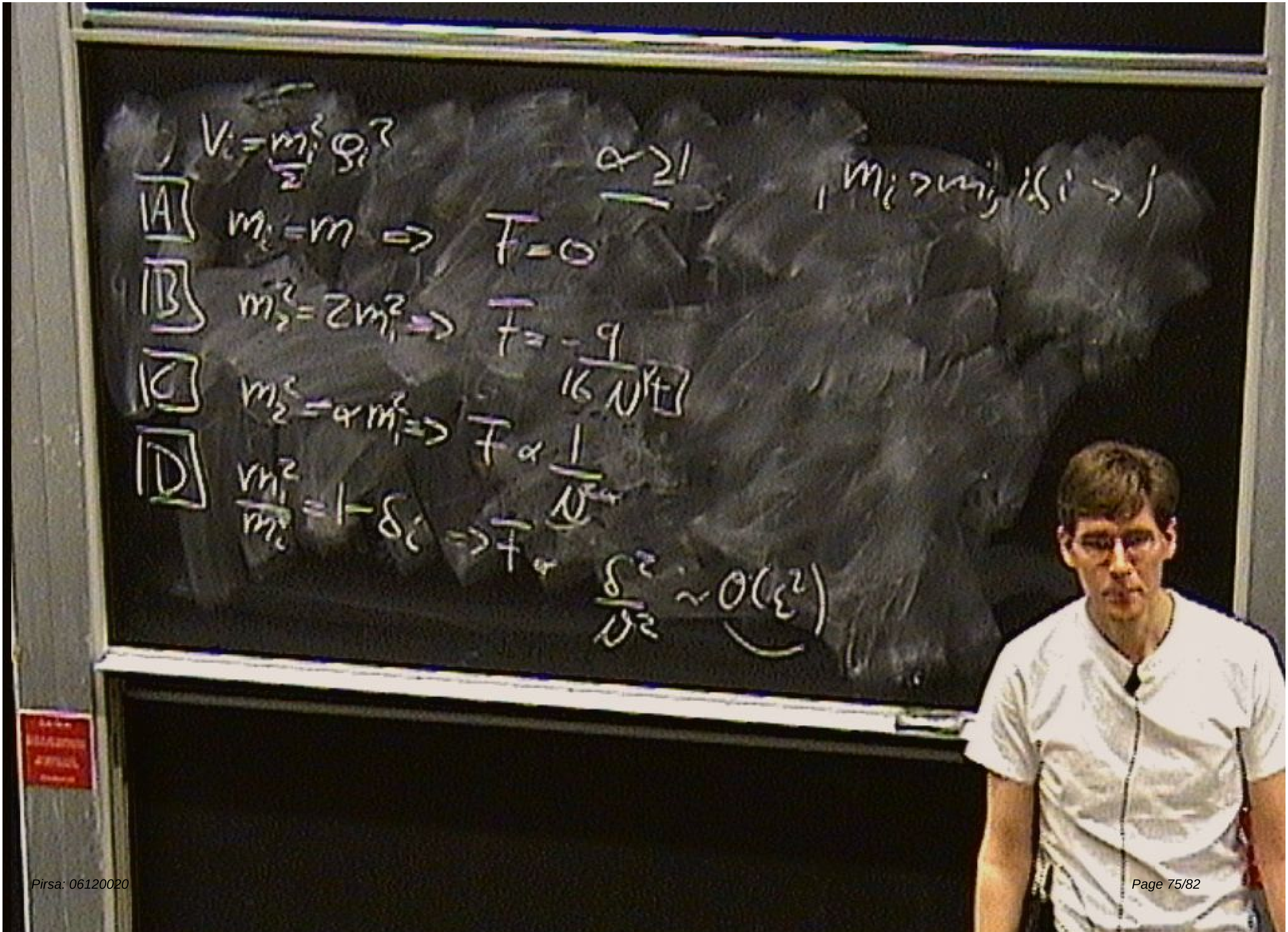
V_i = m_i^2/2 φ_i^2
A m_1 = m ⇒ F = 0
B m_2^2 = 2m_1^2 ⇒ F = -9/16 N
C m_2^2 = α m_1^2 ⇒ F ∝ 1/N^{2α}
D m_1^2/m_2^2

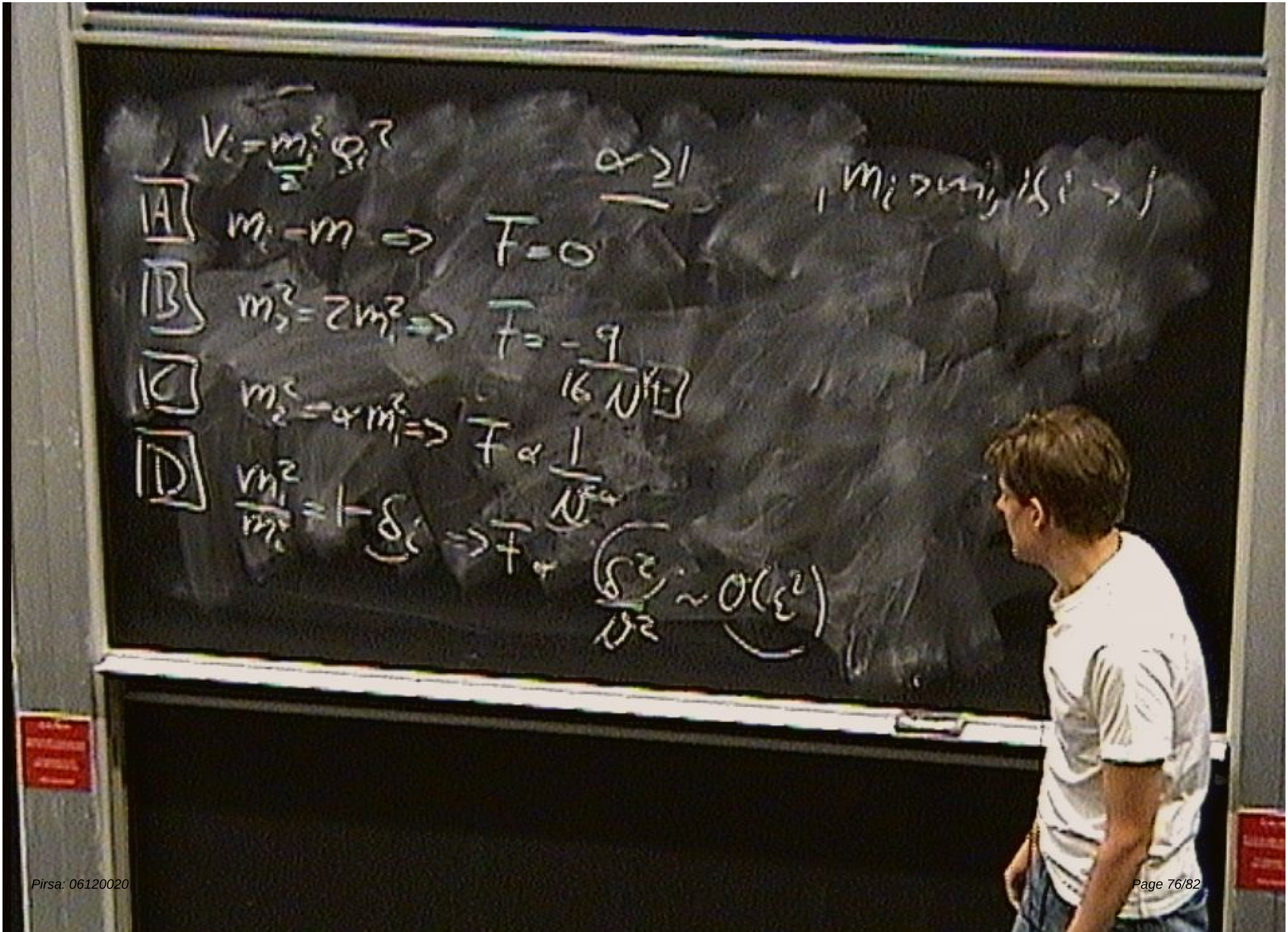

A
B
C
D
F=0

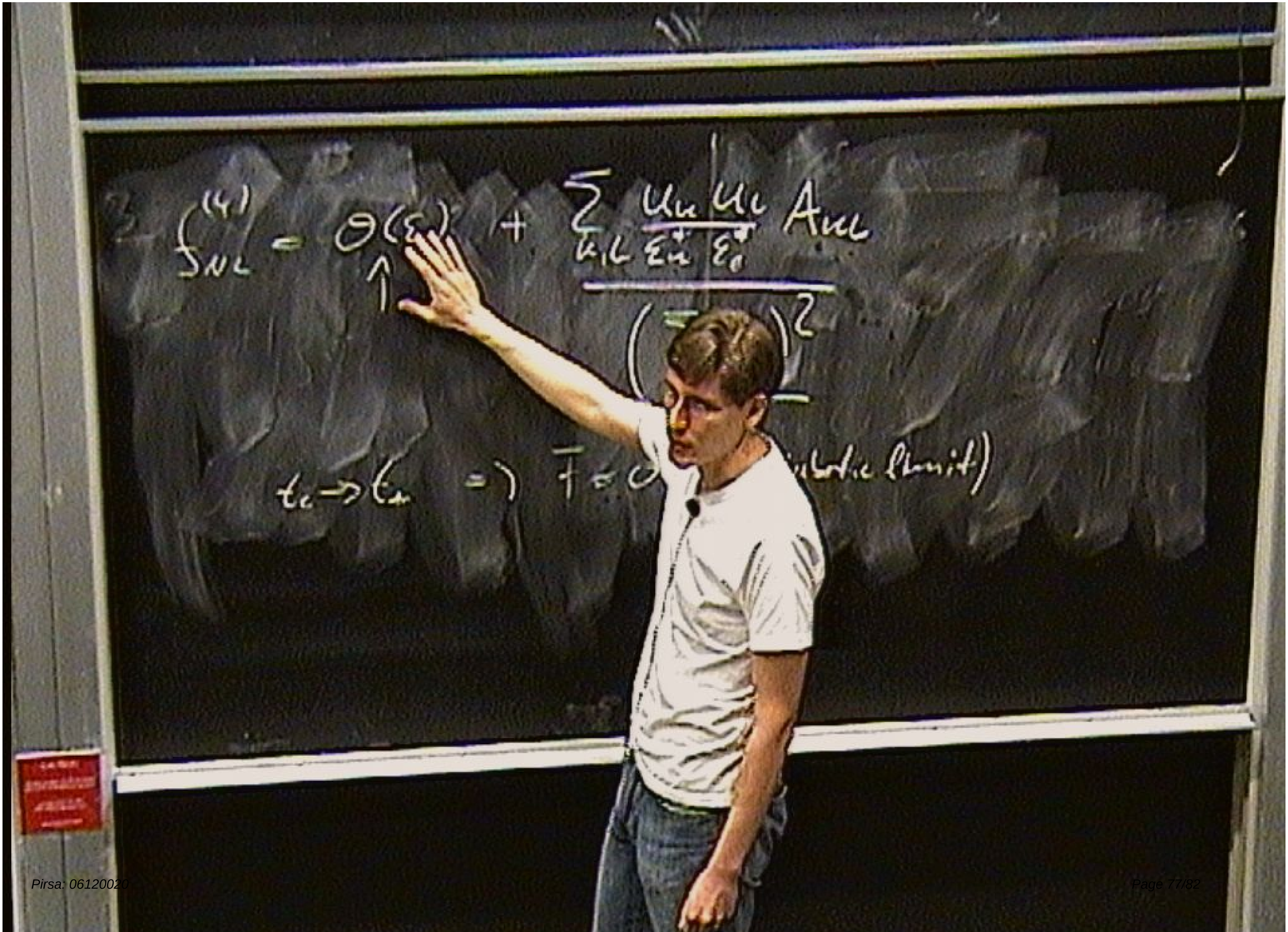

A
B
C
D
F = 0
F ∝
~ O(ε²)

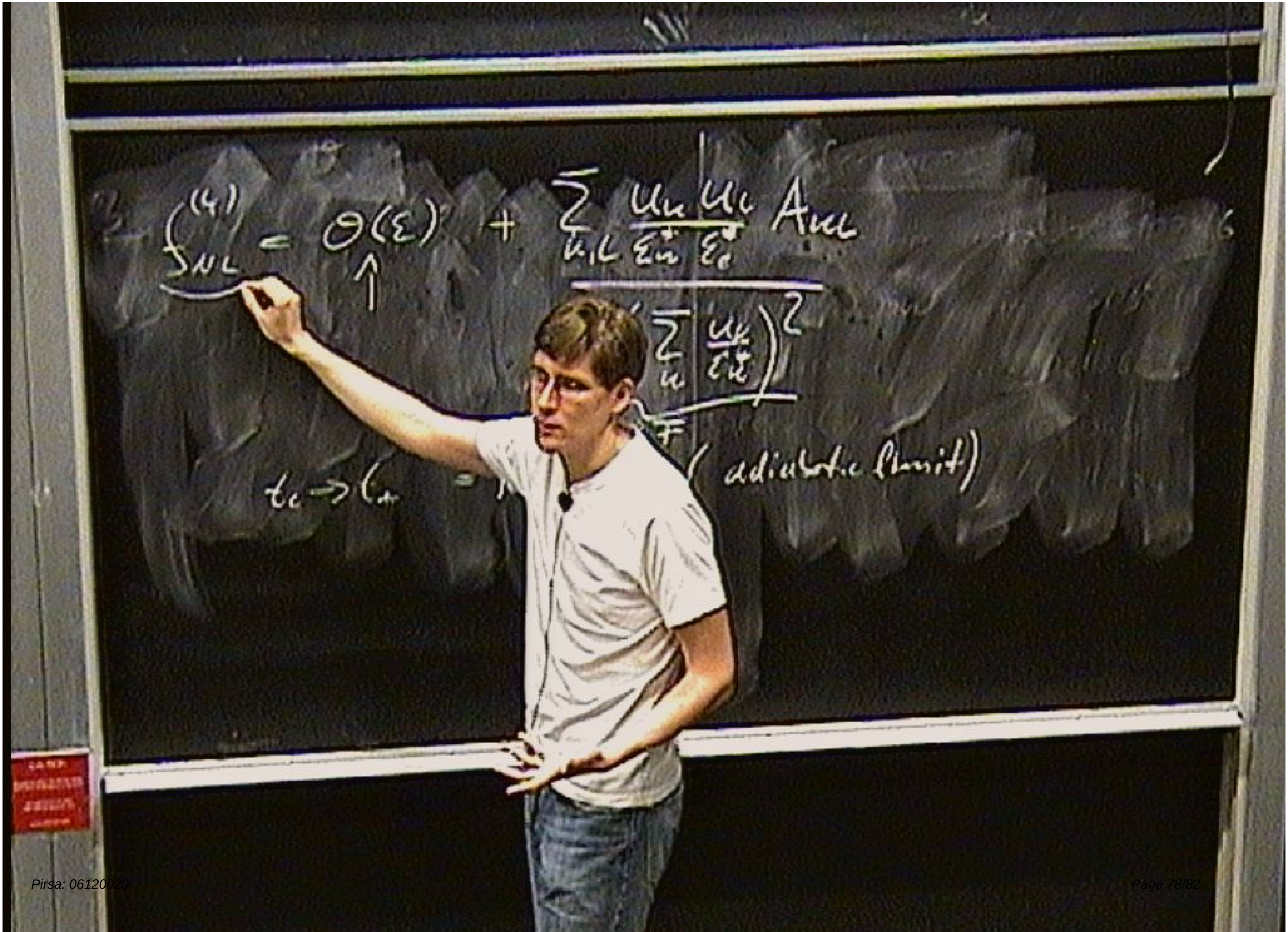

A
B
C
D
F = 0

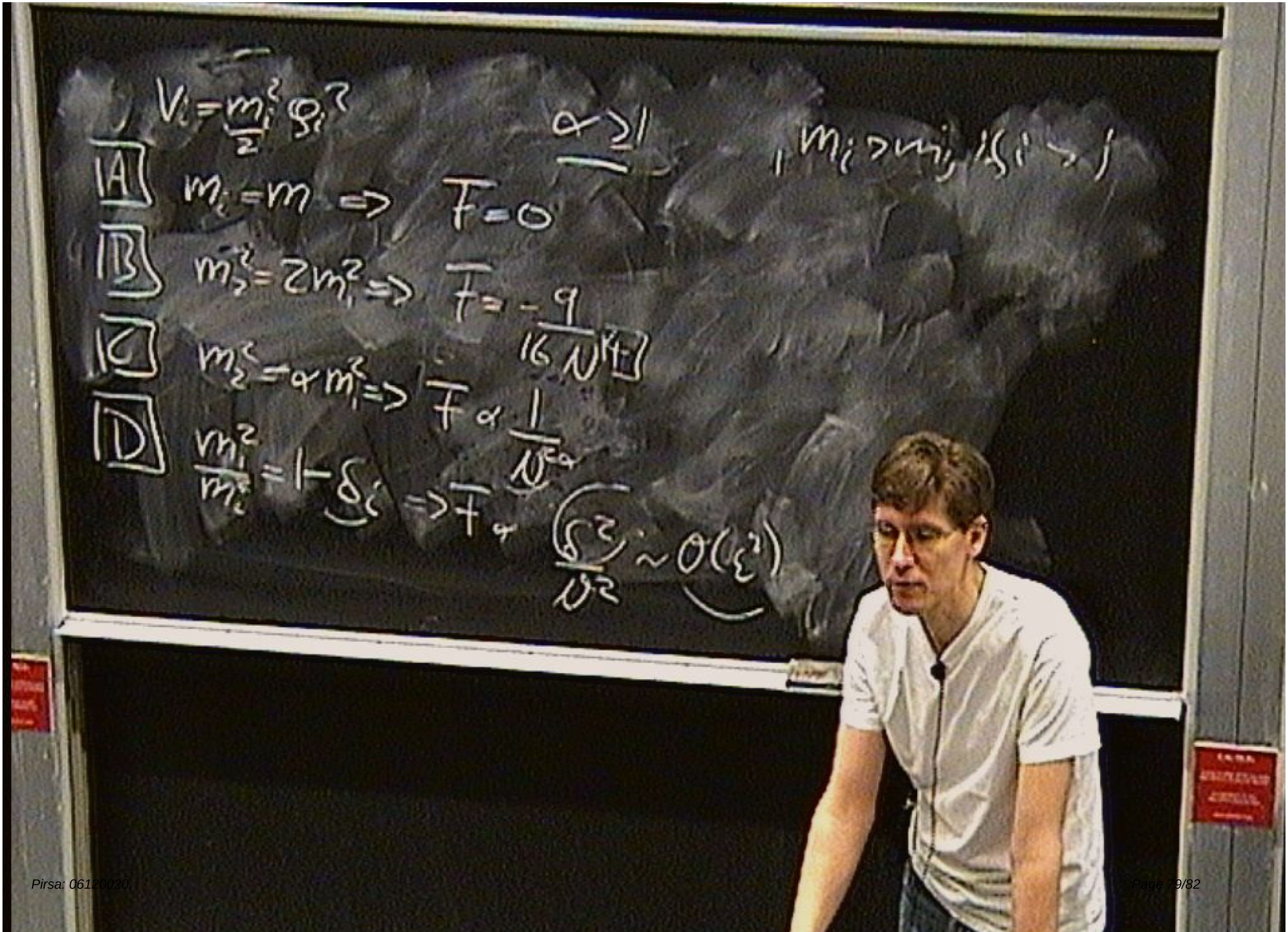

A
$m_i = m \Rightarrow F = 0$
B
C
D
$\sim O(\epsilon^2)$

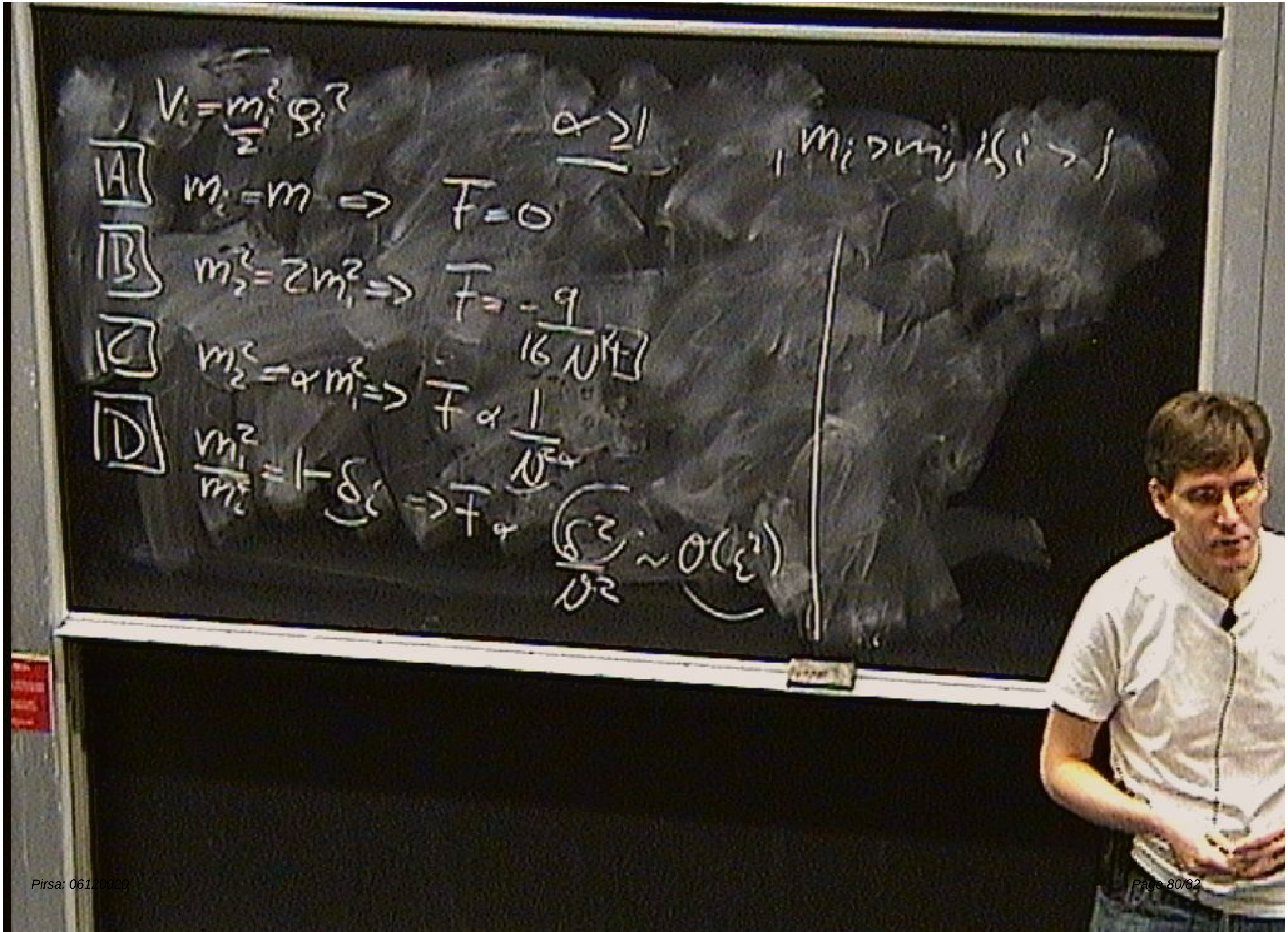

$V_i = \frac{m_i^2}{2} \varphi_i^2$
$\alpha \geq 1$
$m_i > m_j$ if $i > j$
A $m_i = m \Rightarrow \bar{F} = 0$
B $m_2^2 = 2m_1^2 \Rightarrow \bar{F} = -\frac{9}{16 N^{(t)}}$
C $m_2^2 = 4m_1^2 \Rightarrow \bar{F} \propto \frac{1}{N}$
D $\frac{m_i^2}{m_i^2} = 1 - \delta_i \Rightarrow \bar{F} \propto \frac{\delta^2}{N^2} \sim O(\xi^2)$

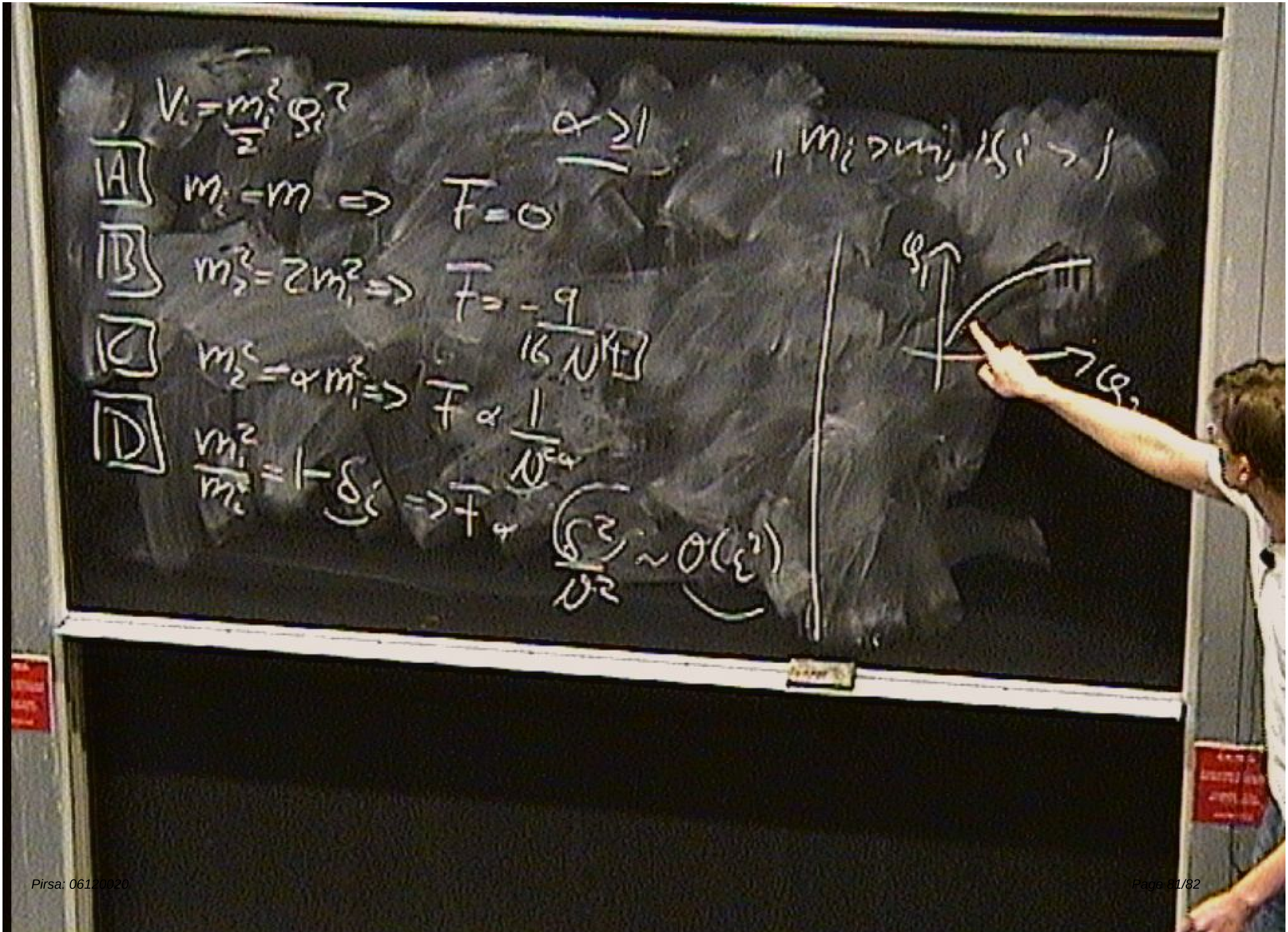


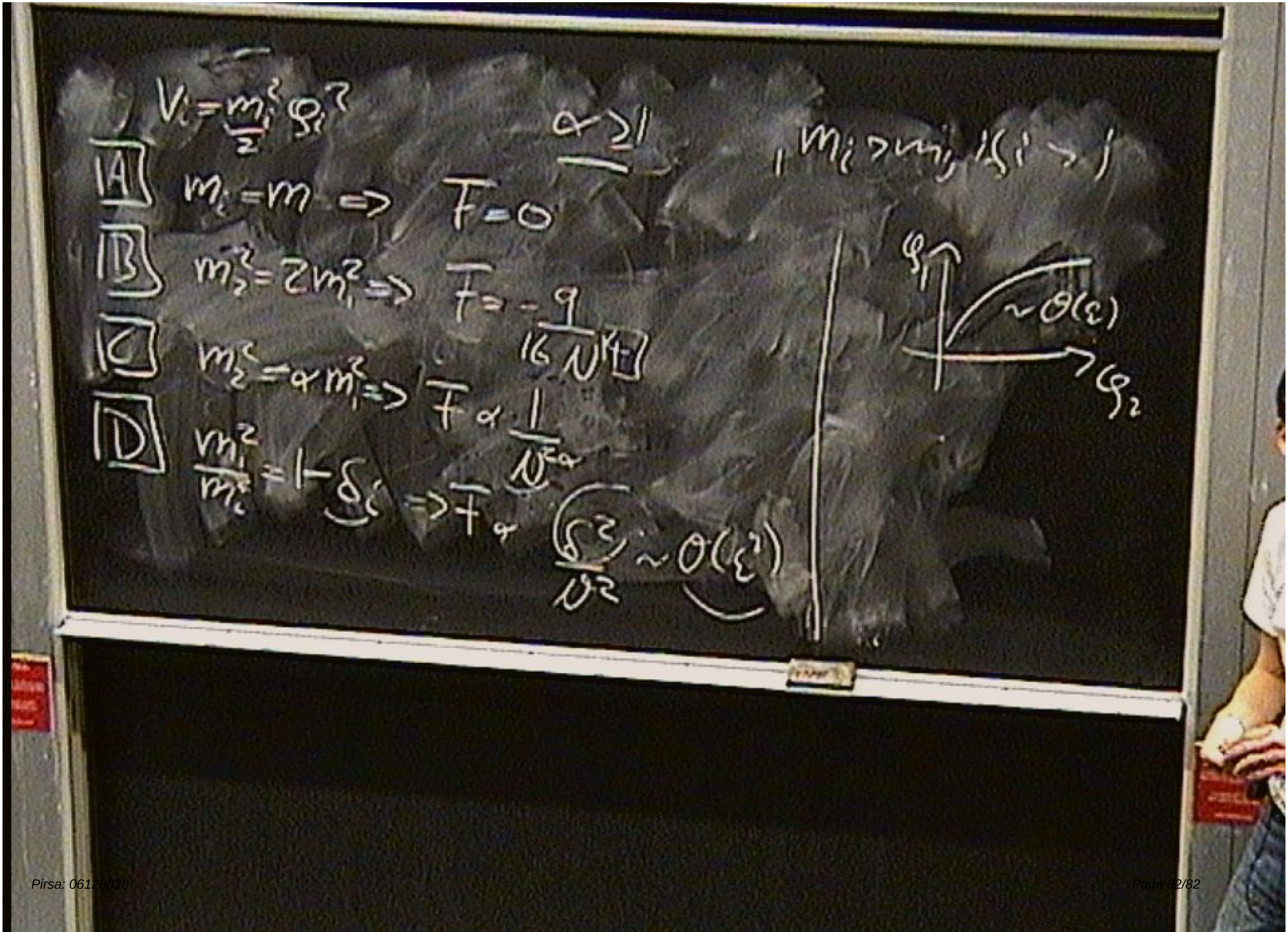



(adiabatic limit)


V_i = m_i^2/2 φ_i^2
α ≥ 1
A m_i = m => F = 0
B m_2^2 = 2m_1^2 => F = -9/16N
C m_2^2 = α m_1^2 => F ∝ 1/N
D m_i^2/m_i^2 = 1 - δ_i => F ∝ δ^2/N^2 ~ O(δ^2)


V_i = m_i^2/2 φ_i^2
α ≥ 1
m_i > m_j if i > j
A m_i = m
B m_2^2 = 2m_1^2
C m_2^2 = α m_1^2
D m_1^2/m_2^2 = 1 − δ_i
~ O(δ^2)


A
B
C
D
F=0


V_i = m_i^2/2 φ_i^2
α ≥ 1
m_i > m_j, i > j
A m_i = m => F = 0
B m_2^2 = 2m_1^2 => F = -9/16N
C m_2^2 = α m_1^2 => F α 1/N^2α
D m_i^2/m_c^2 = 1 - δ_i => F α δ^3/N^2 ~ O(ε^3)
φ_1
φ_2
~O(ε)