Title: Emergence of a Background From Background Independent Quantum Gravity

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Abstract: TBA

Emergence of a background from background independent quantum gravity

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Phys. Lett. B641 (2006) 94-98
gr-qc/0607013

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Romuald Janik
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Emergence of geometry

- I will show an explicit example of an **exact analytical calculation** of a system where a **Semiclassical Geometry emerges** from **background independent** QG

- Emergence of geometry in 4d:
  EMERGENCE OF A 4-D WORLD FROM CAUSAL QUANTUM GRAVITY hep-th/0404156

Two dimensional Quantum Gravity

- No local degrees of freedom (no gravitons)
- 2d QG ⟷ conformal anomaly
- Exactly soluble

  ⬇
- Playground for diffeomorphism inv. Theories

Sum over topologies:

Taming the cosmological constant in 2D causal quantum gravity
with topology change
Nucl. Phys. B751 (2006) 419-435
hep-th/0507012
R.Loll,W.Westra,S.Zohren

- If one couples scalar fields -> string theory (work in progress)

2d QG: natural amplitudes

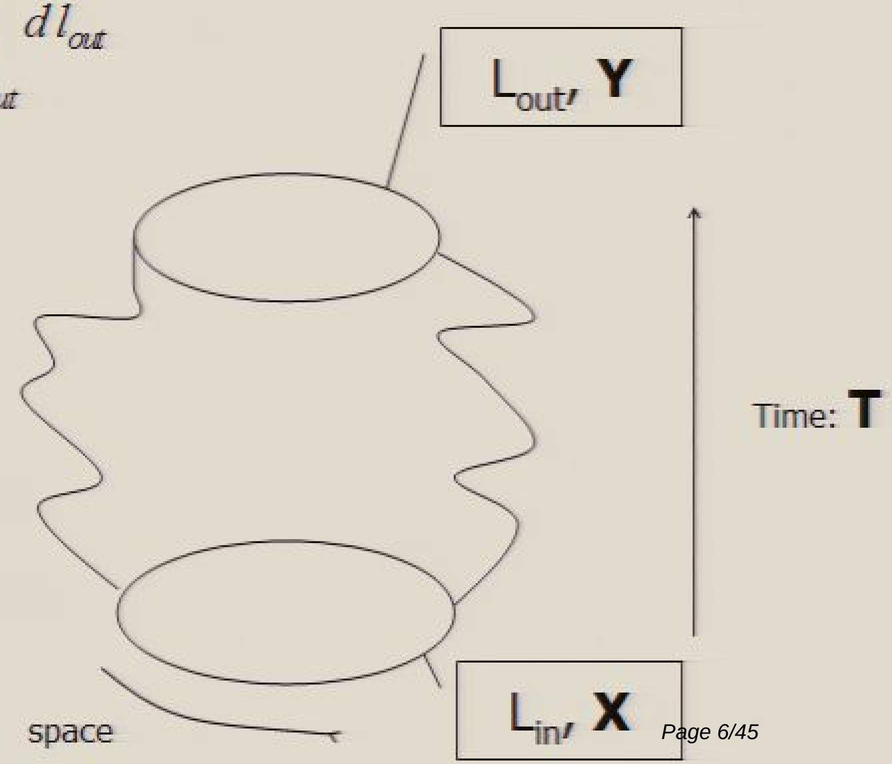
$$G_{\Lambda,\Gamma}(X,Y,T)=\int Dg_{\mu\nu}\, e^{-\int_M \sqrt{g}\,(-\Gamma R+\Lambda)\,-X\int_{\partial M_{in}} dl_{in}-Y\int_{\partial M_{out}} dl_{out}}$$

$$=\int Dg_{\mu\nu}\, e^{-\int_M \sqrt{g}\,\Lambda\,-X\int_{\partial M_{in}} dl_{in}-Y\int_{\partial M_{out}} dl_{out}}$$

- Cylinder amplitude
- Choice:

  - fix boundary geometry or,

  - fix "boundary cosmological constants" (X and Y)



2d QG: natural amplitudes

$$G_{\Lambda,\Gamma}(X,Y,T) = \int Dg_{\mu\nu}\, e^{-\int_M \sqrt{g}\,(-\Gamma R + \Lambda) - X \int_{\partial M_{in}} dl_{in} - Y \int_{\partial M_{out}} dl_{out}}$$

$$= \int Dg_{\mu\nu}\, e^{-\int_M \sqrt{g}\,\Lambda - X \int_{\partial M_{in}} dl_{in} - Y \int_{\partial M_{out}} dl_{out}}$$

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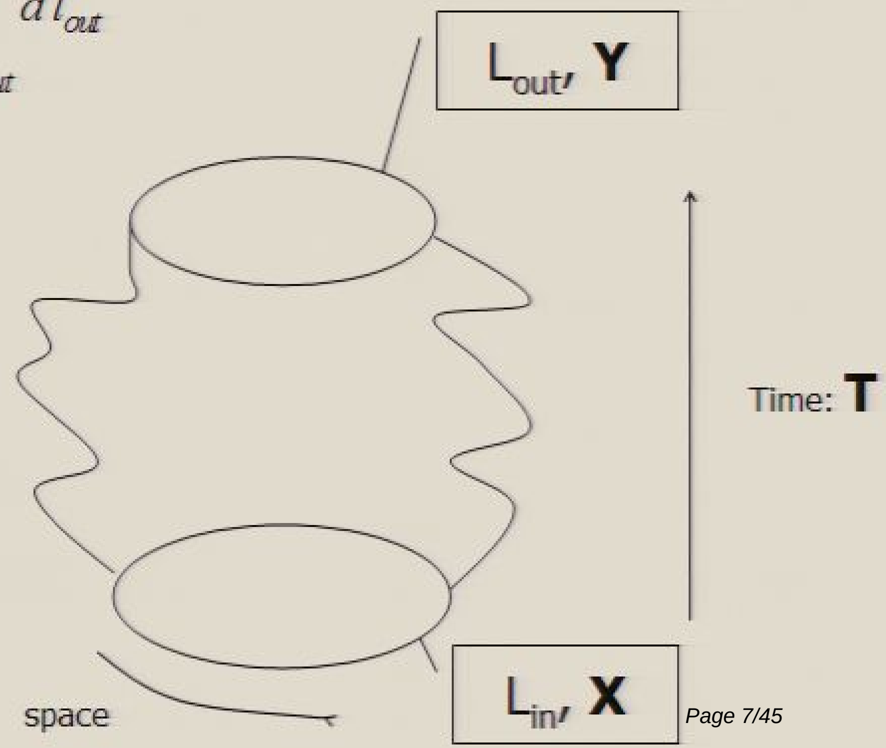


Calculating the amplitudes

- Continuum:

  R. Nakayama: proper-time gauge calculation of Polyakov's induced action (fixed boundary lengths)

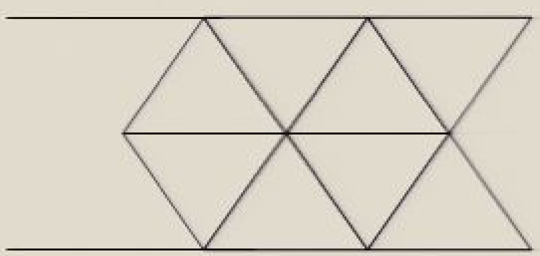
- CDT (Causal Dynamical Triangulations)

  Ambjørn & Loll: continuum limit of
  lattice regularization of
  sum over all **causal** geometries

- Nice agreement

CDT

- Path Integral ⟺ statistical mechanics
- 2D: Full analytical control over the sum over lattices
- Approximate surfaces → gluing identical building blocks (triangles in 2D)
- Flat: Curved:



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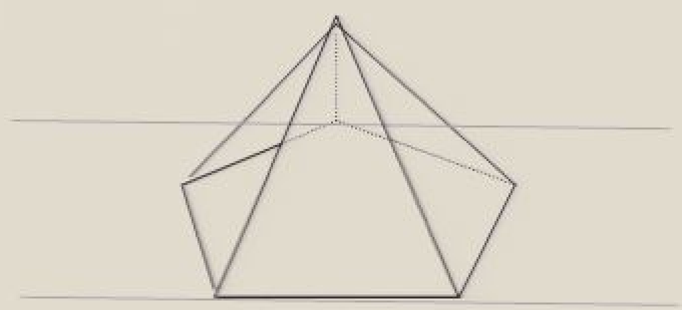

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□

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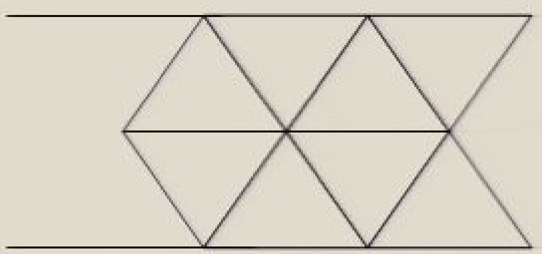

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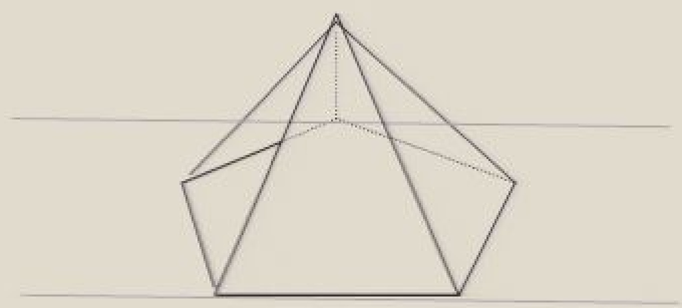

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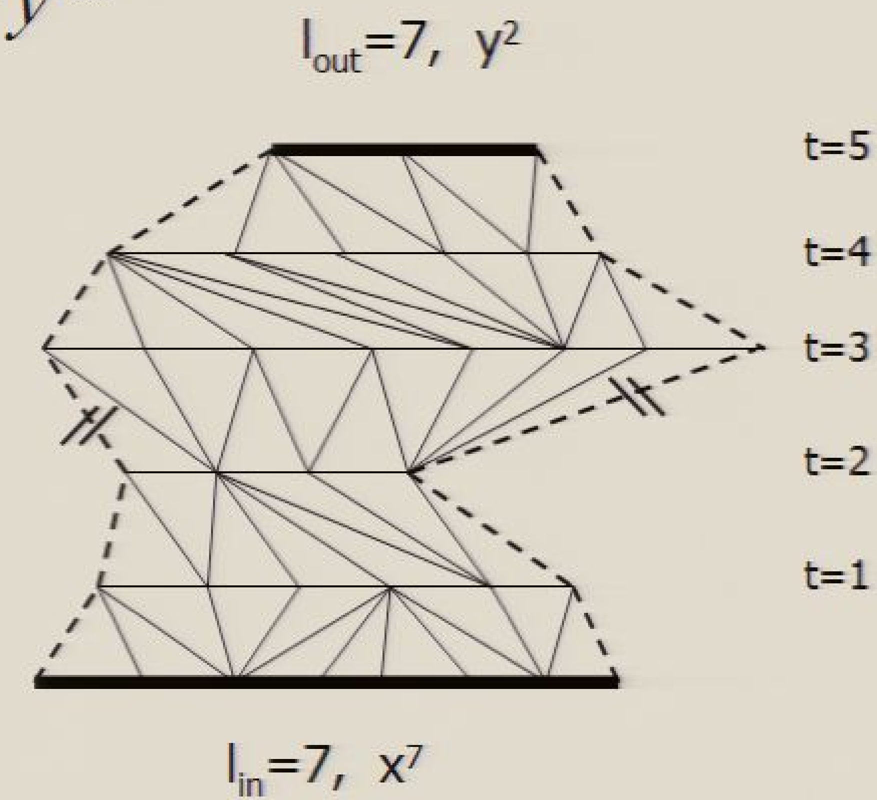
The regularized amplitude

$$G(x,y,t)=\sum_{Triangulations} g^{N} x^{l_{in}} y^{l_{out}}$$

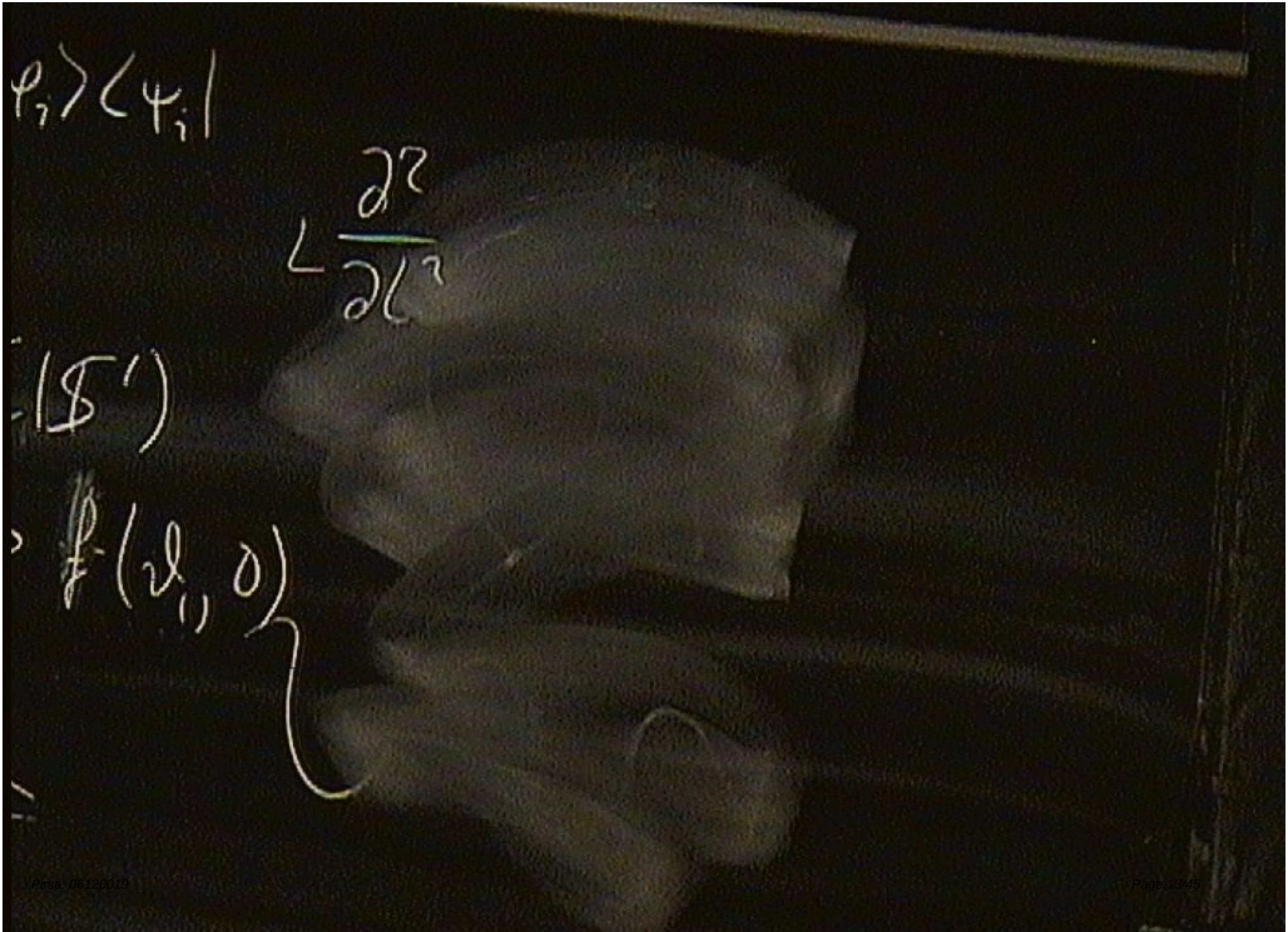


N = # triangles

$g=e^{-\Lambda \mathrm{Vol}\Delta}$

$x=e^{-X\, \Delta \mathrm{length}}$

$y=e^{-Y\, \nabla \mathrm{length}}$



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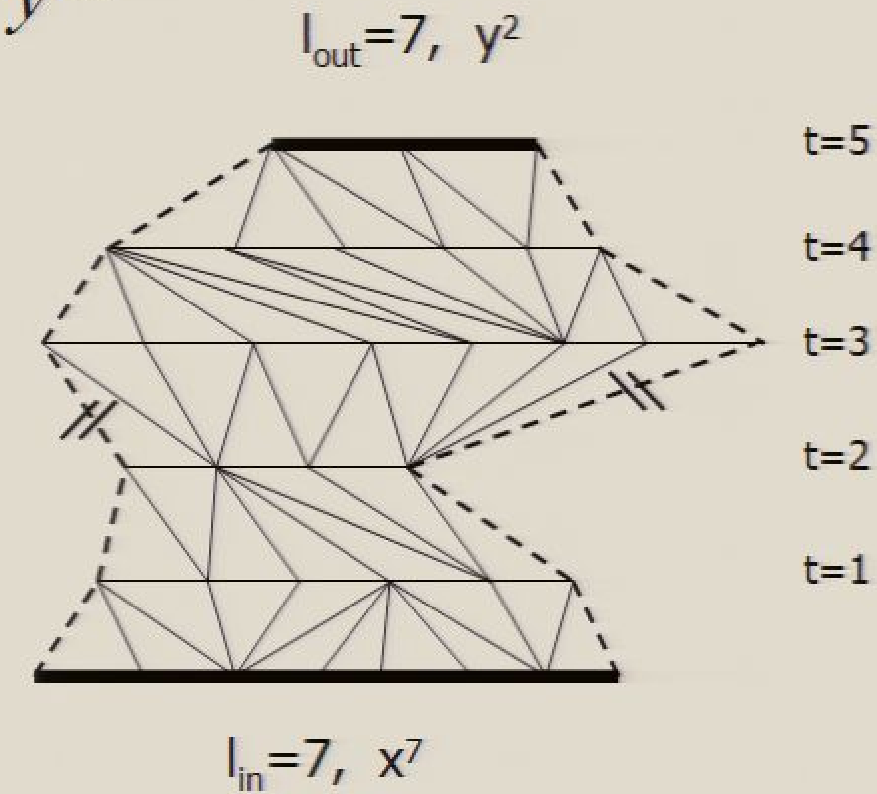

The continuum limit

- Step1:

  $g \to 1/2 : N \to$ infinity

- Step2:

  scale size of triangles such that the total volume and boundary lenghts are finite

  $Vol\Delta \sim a^2$

  $\Delta_{length} \sim a$

N = # triangles

$g \to \frac{1}{2}\exp(-\Lambda a^2)$

$X \to e^{-\mathbf{X}\, a}$

$Y \to e^{-\mathbf{Y}\, a}$

The amplitude

$$G(X,Y,T) = -\log\left(1 - \frac{\sqrt{\Lambda}}{X\sinh\left(\sqrt{\Lambda}T\right) + \sqrt{\Lambda}\cosh\left(\sqrt{\Lambda}T\right)}\frac{\sqrt{\Lambda}}{Y\sinh\left(\sqrt{\Lambda}T\right) + \sqrt{\Lambda}\cosh\left(\sqrt{\Lambda}T\right)}\right)$$

$$G(L_1, L_2, T) = \frac{e^{-\sqrt{\Lambda}\coth\left(\sqrt{\Lambda}T\right)(L_1+L_2)}}{\sinh\left(\sqrt{\Lambda}T\right)}\frac{\sqrt{\Lambda}}{\sqrt{L_1 L_2}} I_1\left(\frac{2\sqrt{\Lambda L_1 L_2}}{\sinh\left(\sqrt{\Lambda}T\right)}\right)$$

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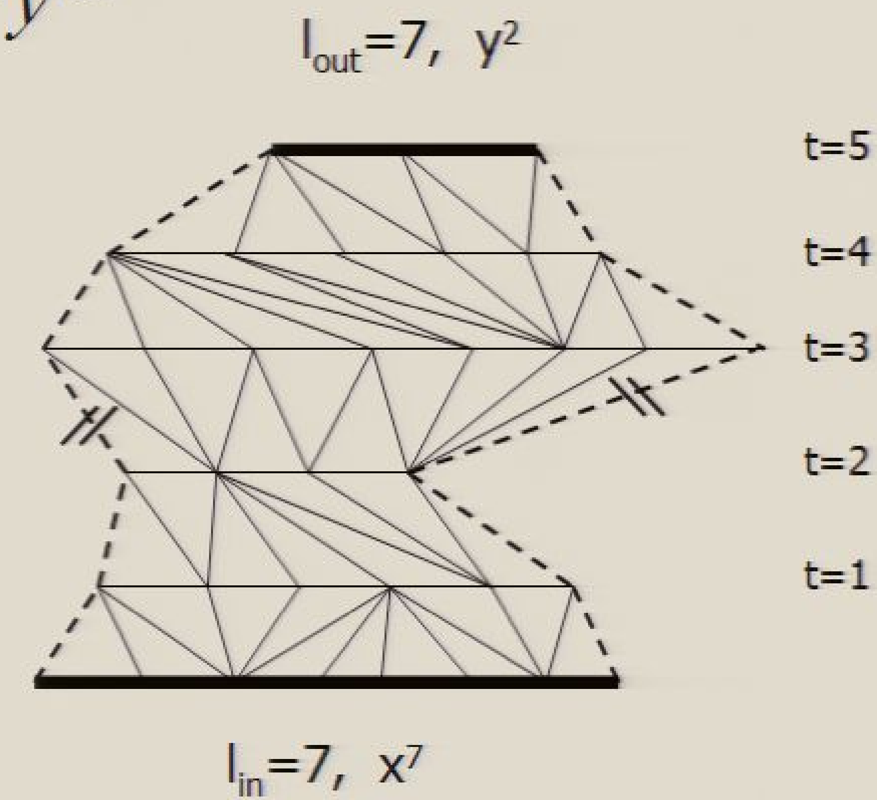

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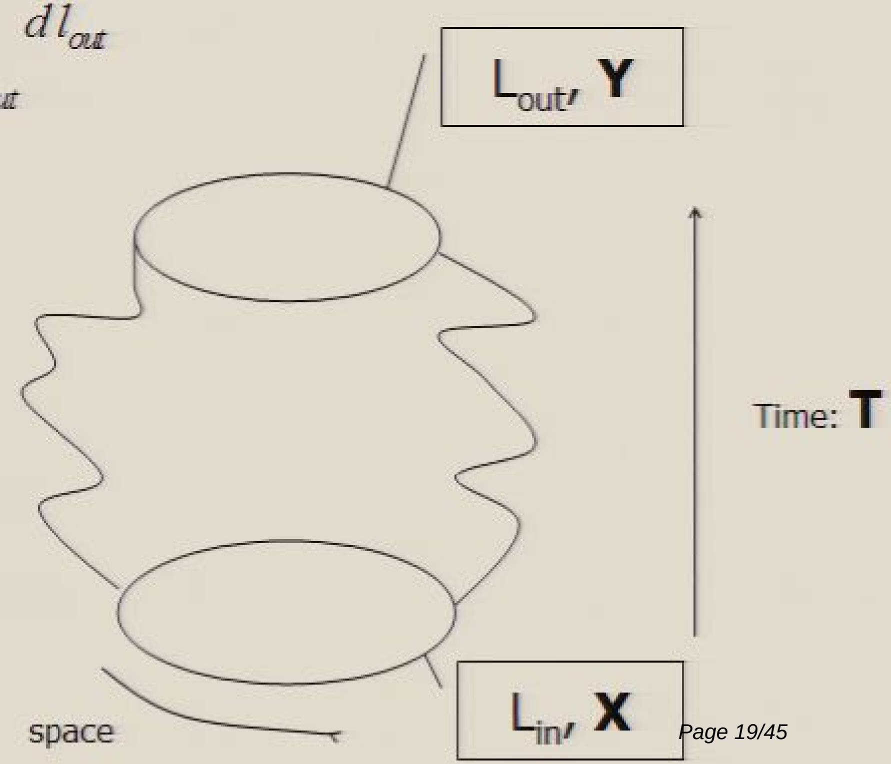

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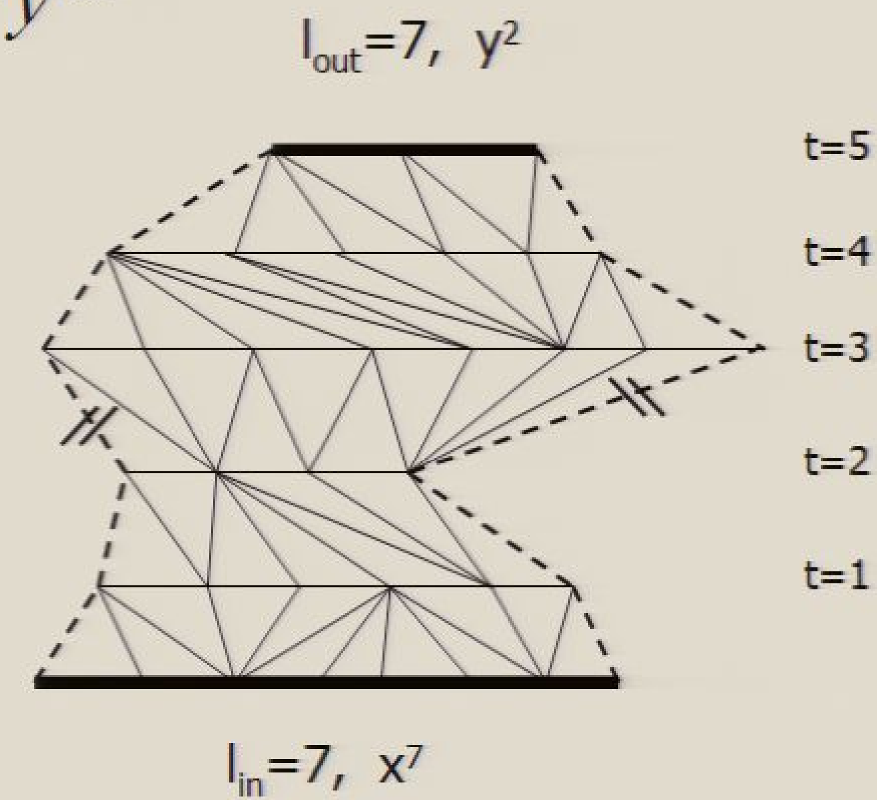

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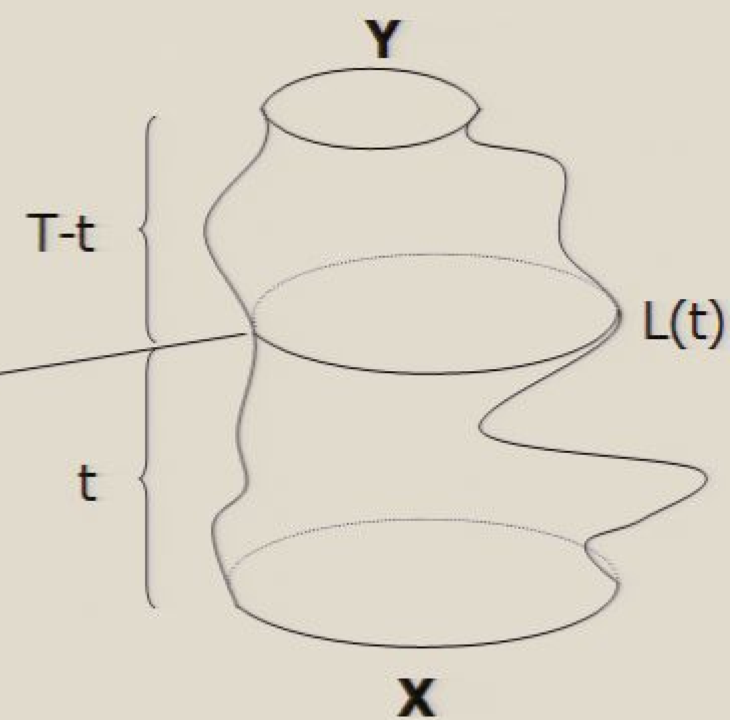
Observables

$$\left\langle L(t)^m \right\rangle_{X,Y,T} = \frac{\int dL\, G(X,L,t) L^m G(L,Y,T-t)}{G(X,Y,T)}$$

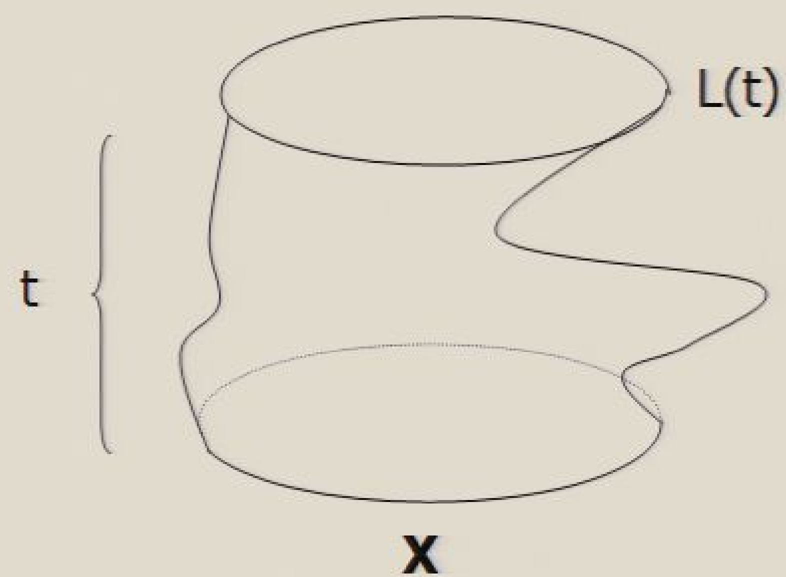
$$\Delta^2 = \left\langle L(t)^2 \right\rangle - \left\langle L(t) \right\rangle^2$$

spatial slice ->
"One dimensional universe"



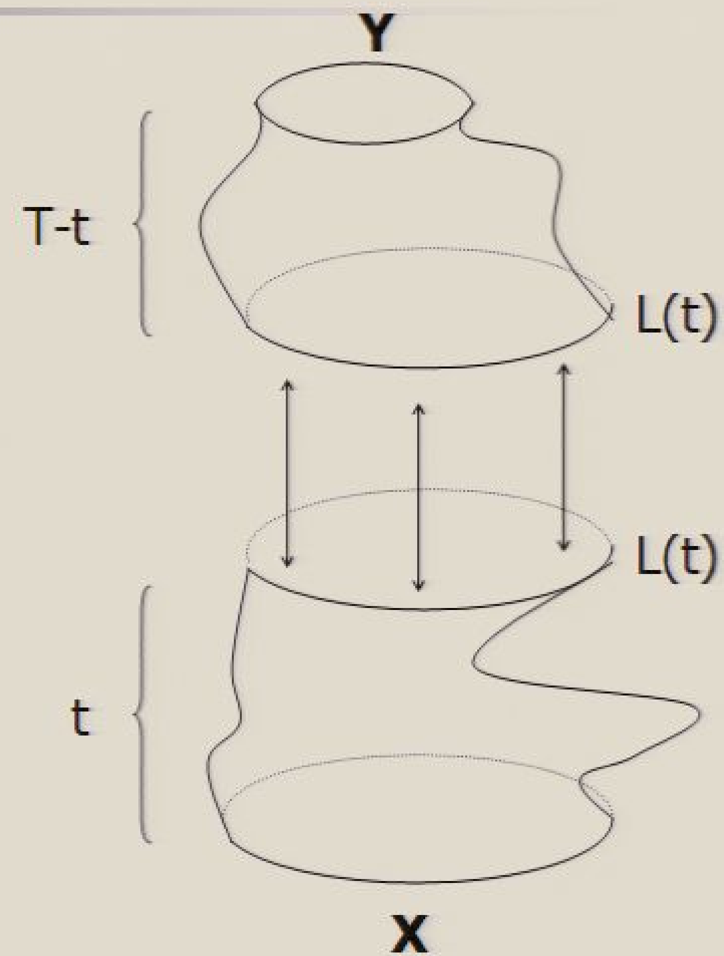
Step 1

- Calculate amplitude G(X,L(t),t)



Step 2

- Calculate amplitude
- G(X, L(t), t)
  x
  G(L(t),Y,T-t)

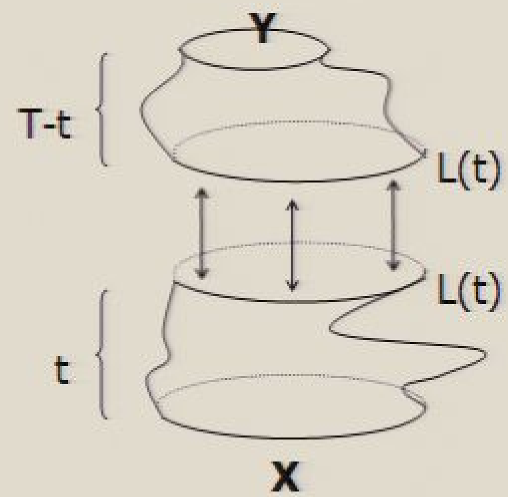


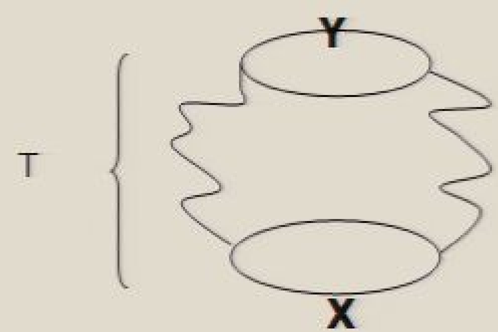
$$\langle L(t)\rangle_{X,Y,T} = \frac{\int dL G(X,L,t) L G(L,Y,T-t)}{G(X,Y,T)}$$

Step 3

- Normalize amplitude to obtain average

$$\langle L(t)\rangle_{X,Y,T} =$$



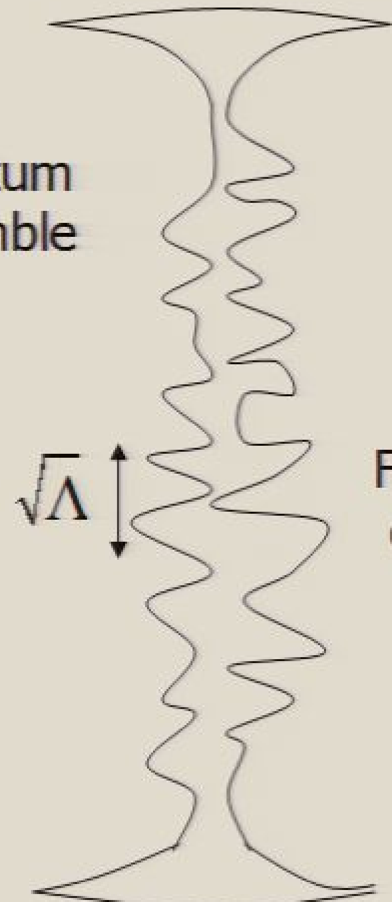


Quantum geometry for T $\geq \sqrt{\Lambda}$

- No semiclassical background → only fluctuations

"Classical" minimal surface solution

Quantum ensemble



Fluctuations in L(t) of the same order as it's expectation value

$$\langle L(t)^2\rangle - \langle L(t)\rangle^2 \approx \langle L(t)\rangle^2$$

Emergent geometry??

The new trick:

Emergence of backgrounds
From
Background independent QG

Emergence of backgrounds
From
Background independent QG

Emergence of backgrounds
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Emergence of backgrounds
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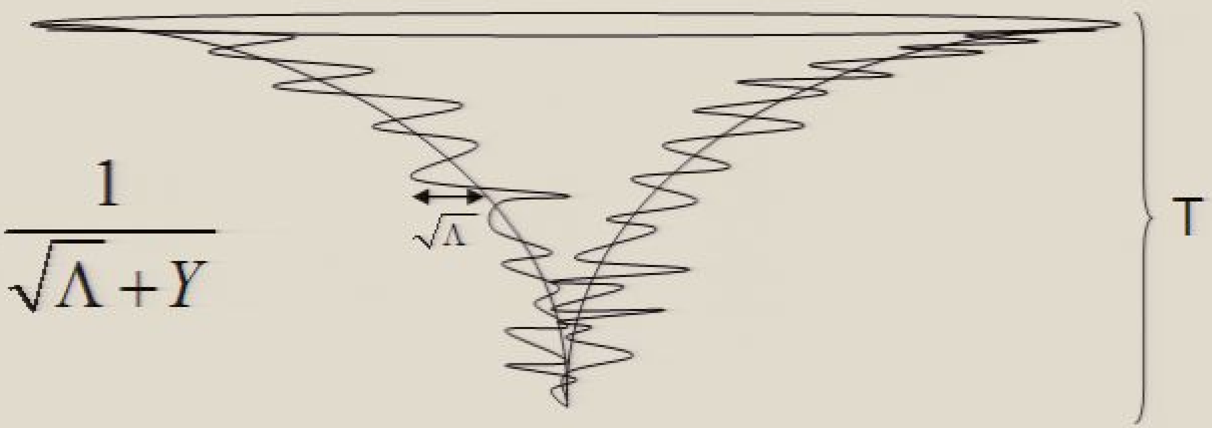
from compact to non-compact

- Boundary length -> infinity for Y -> $-\sqrt{\Lambda}$

$$\left\langle L(T) \right\rangle_{L_1=0,Y,T\to\infty} = \frac{1}{\sqrt{\Lambda}+Y}$$



- This mechanism is similar to non critical string theory:
- ZZ branes appear for critical value boundary cosmological constant -> compact to non compact transition Ambjørn et al. hep-th/0406108

To the critical situation

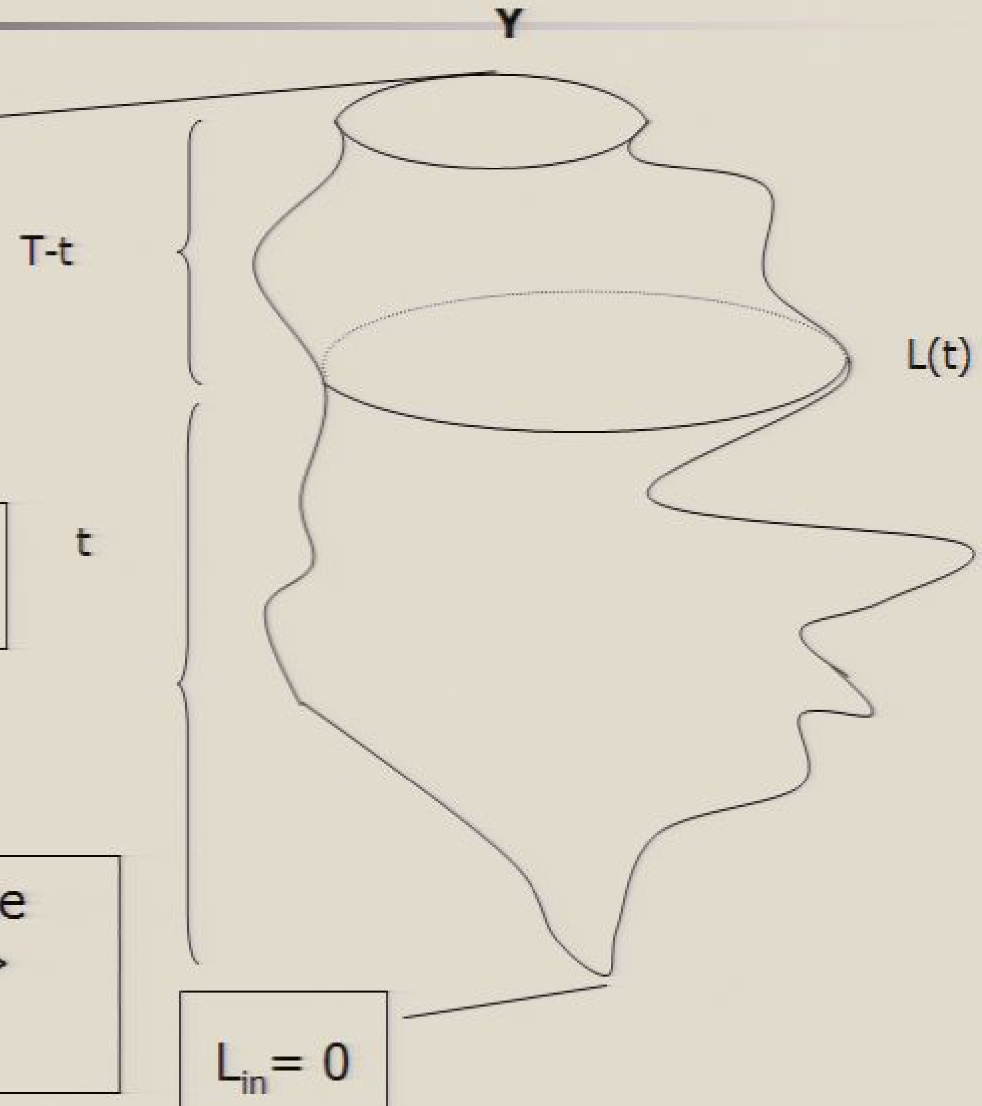
Tune Y to its critical value

⇓

Boundary length diverges

⇓

Typical geometry in the quantum ensemble -> non compact

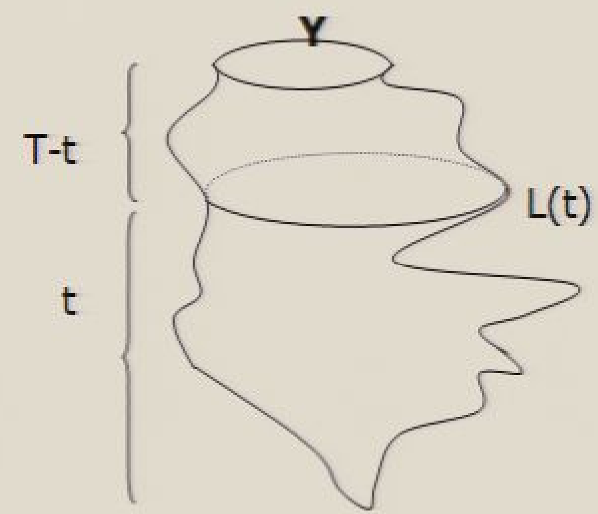


Boundary conditions at T = ∞

$$L_{crit.} = \left\langle L(t) \right\rangle_{L_1=0, Y=-\sqrt{\Lambda}, T}$$

$$\lim_{T\to\infty} \left\langle L(t) \right\rangle_{L_1=0, Y=-\sqrt{\Lambda}, T} = \lim_{T\to\infty} \left\langle L(t) \right\rangle_{L_1=0, L_{crit}, T}$$

$$\lim_{T\to\infty} \left\langle L(t)^2 \right\rangle_{L_1=0, Y=-\sqrt{\Lambda}, T} \neq \lim_{T\to\infty} \left\langle L(t)^2 \right\rangle_{L_1=0, L_{crit}, T}$$



Emergent hyperbolic plane

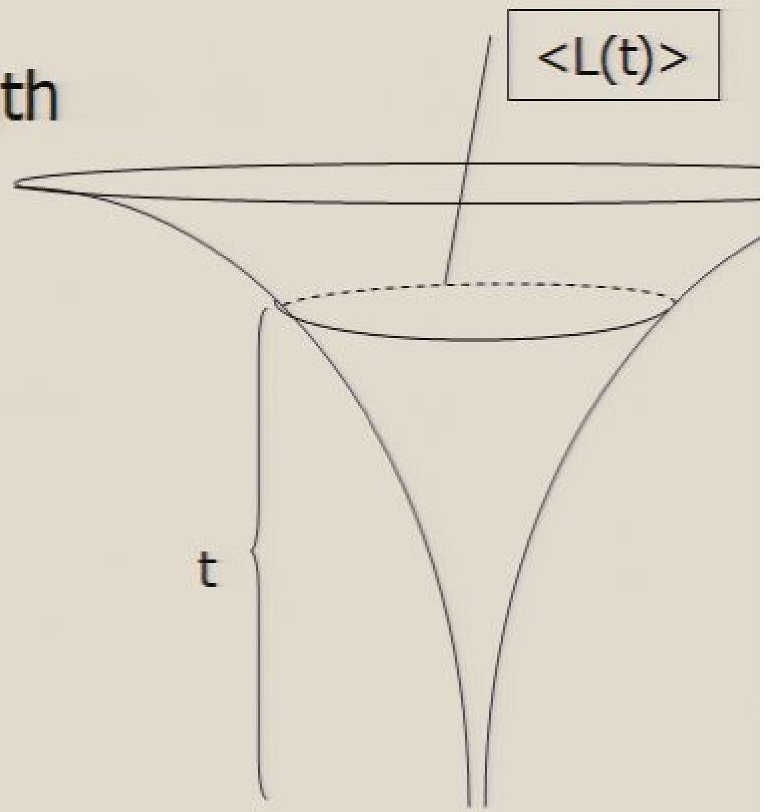
- Quantum Average of the spatial length

$$\langle L(t)\rangle = \frac{\pi}{\sqrt{\Lambda}}\sinh(\sqrt{\Lambda}t)$$

- The metric in proper-time gauge

$$ds^2 = dt^2 + \frac{\langle L(t)\rangle^2}{4\pi^2}d\theta^2$$

$$\theta \in [0, 2\pi]$$



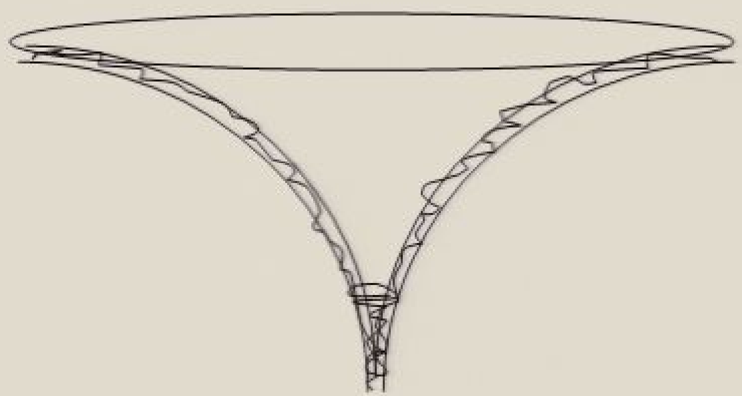
Small relative fluctuations!

- Fluctuations

$$\Delta L(t)^2 = \left\langle L^2(t) \right\rangle - \left\langle L(t) \right\rangle^2 \approx \frac{1}{\sqrt{\Lambda}} \left\langle L(t) \right\rangle$$

- For t $\approx \sqrt{\Lambda}$ relative fluctuations exponentially small!

$$\frac{\Delta L(t)}{L(t)} \approx e^{-\sqrt{\Lambda} t}$$



Emergent geometry??

- Yes!
- Hyperbolic plane
- **Small fluctuations !**

Emergence of backgrounds
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Local structure

- Small fluctuations locally?
- fluctuating regions?

$$N(t)_{line-element} = \sqrt{\Lambda} L(t)$$

- "spatial universes" ->
  number of fundamental lengths

Regions fluctuate independently

- Spatial regions really separate since they fluctuate independently

$$\frac{\Delta N(t)}{\left\langle N\left(t\right)\right\rangle} \approx \frac{\Delta L(t)}{\left\langle L(t)\right\rangle} \approx \frac{1}{\sqrt{\sqrt{\Lambda}\left\langle L(t)\right\rangle}} \approx \frac{1}{\sqrt{N}}$$

- Independently fluctuating **spacetime** regions? ->
- Study temporal correlations

Local structure

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- fluctuating regions?

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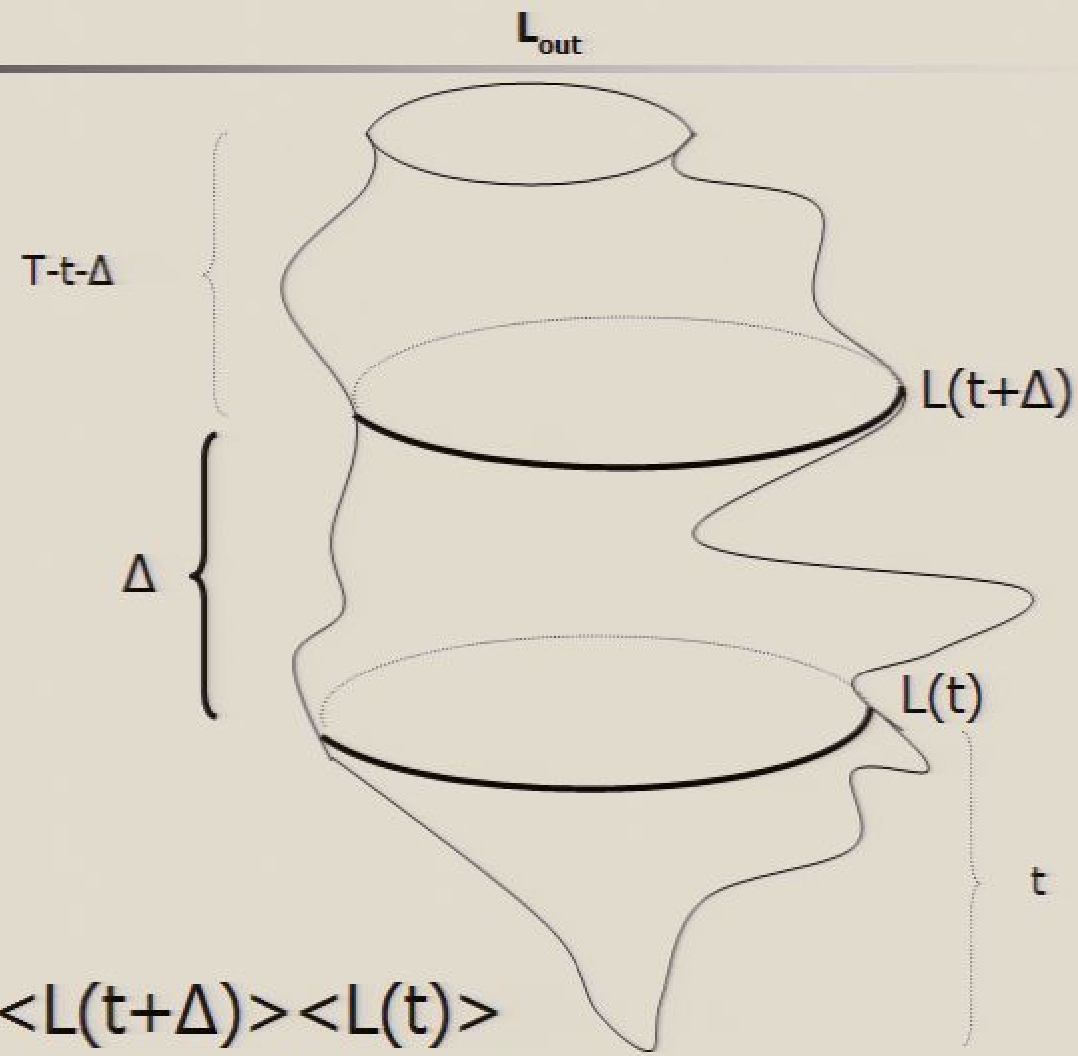
- Study temporal correlations

Temporal correlations I



$$\langle L(t+\Delta)L(t)\rangle - \langle L(t+\Delta)\rangle\langle L(t)\rangle$$

Temporal correlations II

- **Non critical case** ->
  complicated dependence on correlation time Δ

$$\langle L(t+\Delta)L(t)\rangle - \langle L(t+\Delta)\rangle\langle L(t)\rangle = \frac{2}{\Lambda}\frac{\sinh(\sqrt{\Lambda}t)^2\sinh(\sqrt{\Lambda}(T-t-\Delta))^2}{\sinh(\sqrt{\Lambda}T)^2} +$$

$$\frac{2L_{out}}{\sqrt{\Lambda}}\frac{\sinh(\sqrt{\Lambda}t)^2\sinh(\sqrt{\Lambda}(t+\Delta))\sinh(\sqrt{\Lambda}(T-t-\Delta))}{\sinh(\sqrt{\Lambda}T)^3}$$

Temporal correlations III

- **Critical case:** correlations for L(T) -> L_{crit} :

$$\left\langle L(t+\Delta)L(t)\right\rangle_{crit} - \left\langle L(t+\Delta)\right\rangle\left\langle L(t)\right\rangle_{crit} = \frac{2}{\Lambda}\sinh(\sqrt{\Lambda}t)^2$$

- **No dependence on** correlation time **Δ!** ->
- lengths at different time intervals seem to be perfectly correlated!?

Independently fluctuating spacetime patches

- Number of regions increases exponentially

$$N(t)_{line-element} = \sqrt{\Lambda} L(t) \approx e^{2\sqrt{\Lambda}t}$$

- independence on Δ of the correlations of the regions implies that the temporal correlation between the line elements decreases exponentially

- Number of spatial regions increases exponentially
- Their mutual correlations decrease exponentially
- Exponential decrease of correlations ->

regions independent also in time!

Summary: Emergence of geometry

- **Fully analytical** calculation where

  The hyperbolic plane dressed with

  small fluctuations

  emerges from an **exact** and

  background independent evaluation of the 2D gravity path integral

- CDT is an excellent tool to study the

Emergence of backgrounds From Background independent QG

Numerical:

Emergence of geometry in 4d:
EMERGENCE OF A 4-D WORLD FROM CAUSAL QUANTUM GRAVITY
hep-th/0404156

Analytical:

Emergence of geometry in 2d:
EMERGENCE OF AdS2 FROM QUANTUM FLUCTUATIONS
gr-qc/0607013 Phys. Lett. B641 (2006) 94-98

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