Title: Brane Gravity in Six Dimensional Flux Compactifications

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Abstract: I consider a six dimensional space-time, in which two of the dimensions are compactified by a flux. Matter can be localized on a codimension one brane coupled to the bulk gauge field and wrapped around an axis of symmetry of the internal space. By studying the linear perturbations around this background, I show that the gravitational interaction between sources on the brane is described by Einstein 4d gravity at large distances. This is one of the first complete study of gravity in a realistic brane model with two extra dimensions, in which the mechanism of stabilization of the extra space is fully taken into account.

Brane gravity in six dimensional flux compactifications

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Outline

- Motivations: why consider brane-worlds in six dimensions?
- A brane-world system: flux stabilisation and brane-worlds.
- Linear perturbations: interpretation of massless modes.
- Gravity at large distances: the role of flux stabilisation.
- Conclusions

General Motivations

Why to consider six dimensional models?

- They represented the first examples of supersymmetric flux compactification to 4D Minkowski space [Daemi et al., Salam-Sezgin]
  - Two dimensions compactified by magnetic flux, in a six dimensional space with positive cosmological constant
  - The radius of the sphere is fixed by the magnetic charge.

- Important for brane-world construction, in scenarios with flat large extra dimensions
  - Address the hierarchy problem [ADD]
  - Prediction: deviations from Newtonian law at short distances.

- Interesting for field theory/particle physics models
  - Break electroweak symmetry via Wilson-lines [Antoniadis; Csaki]
  - Predict number of chiral generations [Dobrescu, Poppitz]
  - Construct realistic GUT models [Asaka et al.]
  - Generate neutrino masses in BW with thickness [Dudas et al.]

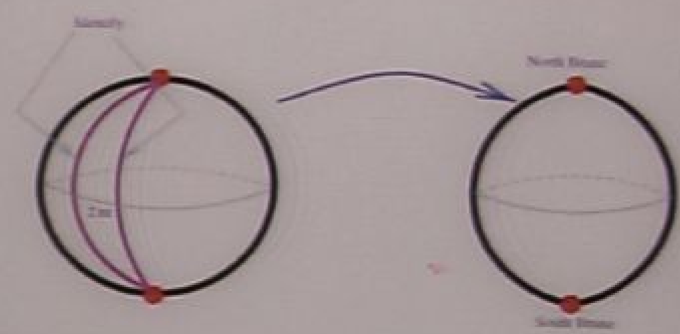
- They allow address the cosmological constant problem
  - The cosmological constant on the brane is adsorbed into bulk quantities via a self tuning mechanism [Carroll and Guica, Navarro, Aghababaie et al.]
  - The observed value of vacuum energy is due to susy breaking effects in a six dimensional sugra model [Burgess et al.]

- Gravitational aspects have been considered...
  - In AdS_6, models with gravity localized on a string core [Gherghetta, Shaposhnikov]
  - Bigravity models [Kogan et al.]
  - Various particular cosmological settings [Kanti et al.]

...although interesting issues have not been studied in detail.

Brane-worlds and flux compactifications

A recent approach combines in anatural way the flux compactification with the brane-world idea [Carroll et al.; Navarro; Aghababaie et al.]



The bulk action for this system is ($M_6 \simeq 1 - 10$ TeV)

$$S = \int d^6x \sqrt{-g_6} \left[M_6^4 \mathcal{R} - \Lambda - \frac{1}{4} F_{AB} F^{AB} \right] + \sum_{i=1}^{2} \int d^4x \sqrt{-\gamma_i} \lambda_i$$

The background solution has a magnetic monopole switched on:

$$\begin{aligned} ds_6^2 &= \eta_{\mu\nu} dx^\mu dx^\nu + R^2 \, d\theta^2 + R^2 \, \beta^2 \cos^2\theta \, d\phi^2 \\ F_{\theta\phi} &= \frac{\beta \, M_6^2}{R} \cos\theta \quad , \quad \Lambda = \frac{M_6^4}{2 \, R^2} \quad , \quad \lambda_i = 2\pi M_6^4 (1 - \beta) \end{aligned}$$

- The radius of the sphere is fixed by the monopole charge.
- The deficit angle is associated with the brane tension.

The self tuning idea

Idea: If you change the brane tension, the four dimensional metric remains flat! Only the value of the deficit angle changes.

$$
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\end{aligned}
$$

However, in this simple case, the monopole charge is quantized (Dirac quantization condition): you cannot continuously change the parameter β.

One can hope to overcome the problem by considering the system in a supergravity set-up,

⇒ You have more fields to play with.

⇒ The breaking of SUSY due to the branes can allow to get the observed value of the cosmological constant (SLED proposal [Burgess et al.])

but the problem with the quantization condition remains.

Recent approaches look at the system from another point of view

⇒ Time-dependent configurations that connect vacua with different values of β [Burgess, Tolley et al.].

Here we are interested to a different issue: how does gravity behave for a brane observer?

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Question: How does the brane energy-momentum tensor backreact on the geometry and on the background flux?

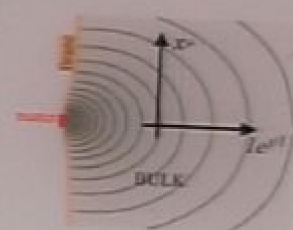
- In the codimension one, AdS_5 warped case, without flux, this is a central question to understand the behaviour of gravity on the brane
  - Start with a warped metric in AdS_5, and a brane is situated at $y = 0$, and has tension $\sigma = 3/4\pi l G_5$

$$ds^2 = dy^2 + e^{-2|y|/l}\eta_{\mu\nu}dx^\mu dx^\nu$$

  - Perturbing the metric and brane energy momentum, and calling ζ the perturbation of the brane position, the brane bends

$$\partial^2\zeta = \frac{\kappa}{6}T \qquad \text{[Garriga, Tanaka]}$$

this is essential in order to recover Einstein gravity at large distances.



- Is there an analog effect with more than one extra dimension?
- How does the brane energy momentum tensor interact with the stabilization mechanism?

Why this question is interesting?

- The answer to this question can allow to learn some features that are possibly in common with a more realistic string set-up.
- This system mimics, in a simple BW setting, properties of string theory compactifications with branes and background fluxes.

  ⇒ Analysis of perturbations is difficult to perform in the string case due to the large number of fields involved [Giddings, Maharana]
- The system offers a consistent framework to study brane-world gravity with flat extra dimensions

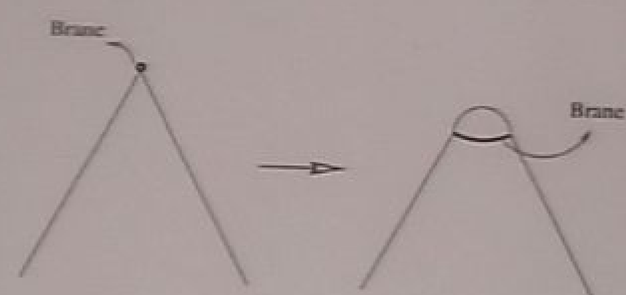
  ⇒ Maybe we can address issues that are difficult in warped geometries: exact solutions for **BHs on the brane**? [Kaloper, Kiley]

A closer look to the brane

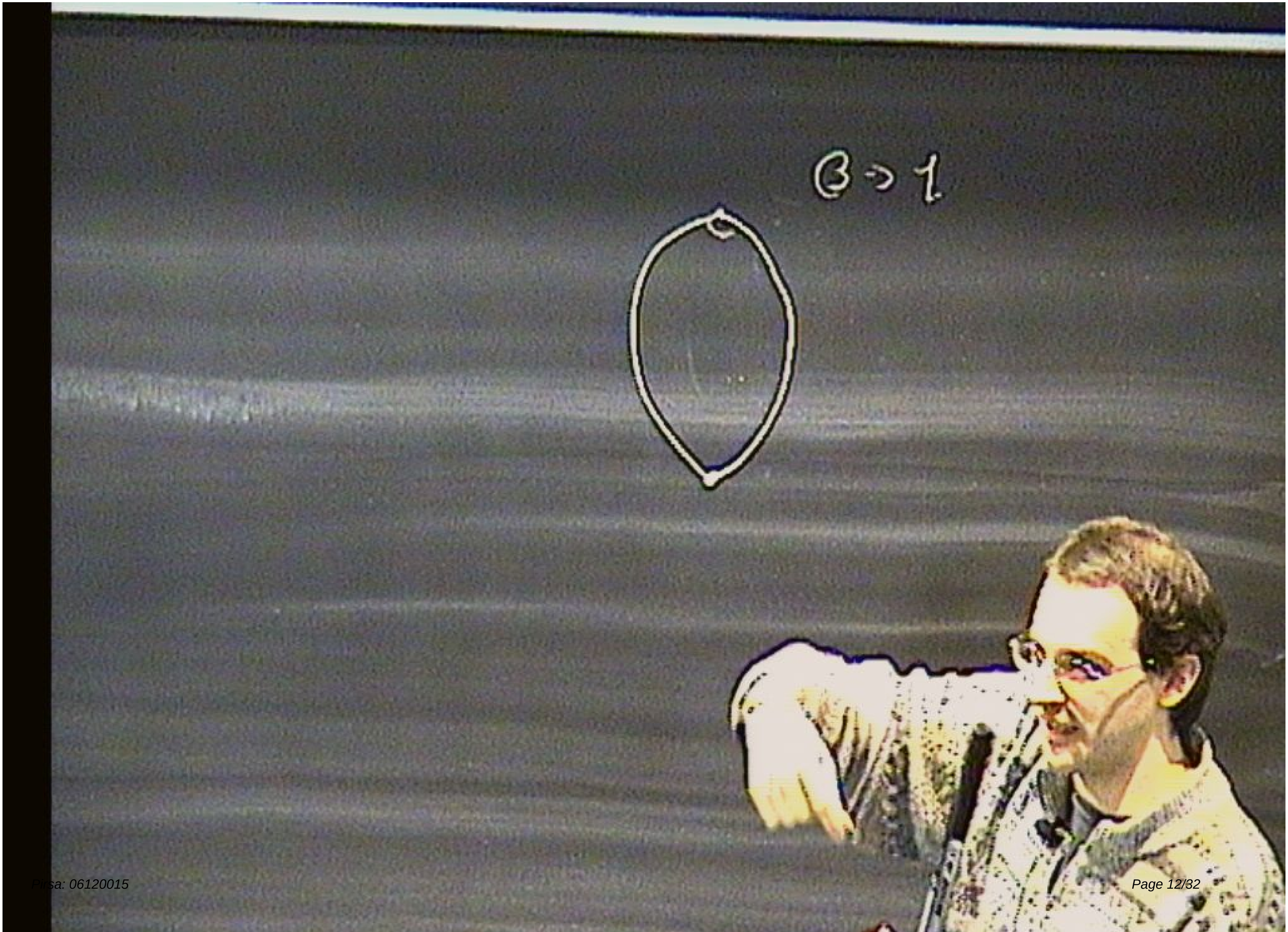
- The situation is more subtle with respect to the codimension one case: one finds that only tension can be placed on a strict codimension two defect! [Cline et al., Bostock et al.]

$$T^{sing}_{\mu\nu} \propto \gamma_{\mu\nu}$$

- There are two options to overcome the problem
  1. Add higher order curvature terms (Gauss-Bonnet) in the bulk
  2. Regularize the brane, promoting it to a thick defect
- We choose the second option.



- The brane becomes a codimension one, 1+4-dimensional object, in which one of the dimensions is compactified on a small circle.
  - ⇒ The outer side of the space corresponds to the conical geometry
  - ⇒ The inner side smooths the tip to the cone, substituting it with a spherical cap.
- **Question:** what does fix the position of the brane?
  - ⇒ A coupling with the magnetic field.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The background system

We consider **two** different actions (and background solutions), one inside, one outside the brane position.

$$S_{o,i} = \int d^6x\sqrt{-g_6}\left[M^4\mathcal{R} - \Lambda_{o,i} - \frac{1}{4}F_{AB}F^{AB}\right]$$

$$S_{br} = -\int d^5x\sqrt{-\gamma}\left[\lambda_+ + \frac{q^2}{2}A_M A^M\right]$$

The brane action breaks the bulk gauge symmetry, with a mass term $\propto q^2$ at the brane position.

$\Rightarrow$ This term gives the coupling with the gauge field. It is necessary in order to render the system consistent, and will be useful for the stabilization procedure.

$\Rightarrow$ **Continuity** conditions require to choose

$$\Lambda_i = \frac{M^4}{2R^2} \quad , \quad \Lambda_o = \frac{M^4}{2\beta^2 R^2}$$

explaining why we take two values for the bulk cosmological constants.

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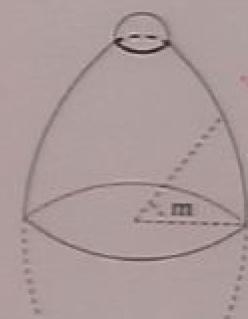

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The solution in the bulk

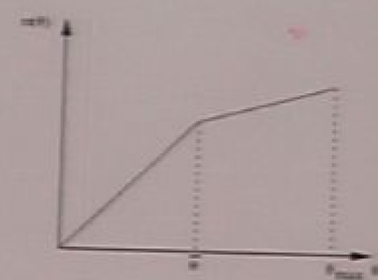
The solution can be expressed in a quite **compact form**

$$ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu + d\theta^2 + \cos^2\left[m\left(\theta\right)\right]\, d\phi^2$$

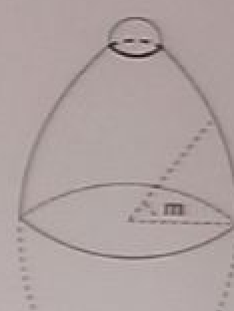
$$F_{\theta\phi} = M^2\, m'\left(\theta\right)\, \cos\left[m\left(\theta\right)\right]$$

where the function $m\left(\theta\right)$ is (Θ corresponds to the Heaviside step function)

$$m\left(\theta\right) = \bar{\theta} + \frac{1}{R}\left(\theta - R\bar{\theta}\right)\left[\Theta\left(R\bar{\theta} - \theta\right) + \frac{1}{\beta}\Theta\left(\theta - R\bar{\theta}\right)\right] \,.$$



Here $m(\theta)$ is the latitudinal coordinate: the value $m(\bar{\theta}) = \bar{\theta}$ corresponds to the **brane position**, while $m = \frac{\pi}{2}$ to the position of the **pole**. The coordinate ϕ corresponds instead to the azimuthal coordinate.



Junction conditions

The brane action is $S_{br} = -\int d^5x \sqrt{-\gamma}\left[\lambda_* + \frac{q^2}{2} A_M A^M\right]$
with associated anisotropic EMT

$$S_{\mu\nu} = -\left[\lambda_* + \frac{q^2}{2} A_\phi A^\phi\right] \eta_{\mu\nu}$$
$$S_{\phi\phi} = -\left[\lambda_* - \frac{q^2}{2} A_\phi A^\phi\right] \gamma_{\phi\phi}$$

Conditions to solve

- **Israel** junction conditions:

$$\left[\tilde{K}_{AB}\right]_J = -\frac{S_{AB}}{M^4}.$$

where $\tilde{K}_{MN} = K_{MN} - \gamma_{MN} K$, $K_{MN} = \nabla_M n_M$

Inserting the background solution, one finds

$$\left[\tilde{K}_{\mu\nu}\right]_J = \left(1 - \frac{1}{\beta}\right) \tan\bar{\theta}\, \eta_{\mu\nu} \quad , \quad \tilde{K}_{\phi\phi} = 0$$

- From the **Maxwell** equation, one gets the condition

$$[(\sqrt{g_{\theta\theta}} F^{\theta\phi})]_J = -q^2 A^\phi|_{brane}$$

from which

$$\left(1 - \frac{1}{\beta}\right) \cos\bar{\theta} = -q^2 \sin\bar{\theta}$$

We have **three** independent conditions for the **three** available parameters: λ_* (the brane tension), q^2 (the mass term) and $\bar{\theta}$ (the brane position). The relations that connect them are

$$q^2 = -\left(1 - \frac{1}{\beta}\right) \frac{1}{\tan \theta} = \frac{2\lambda_*}{\tan^2 \theta}$$

As a byproduct, the brane position is **stabilized**, once you choose the parameters λ_* and q^2!

- In the limit $\bar{\theta} \to \pi/2$, the coupling q^2 vanish.
- One recovers the brane action for a codimension two brane.

$\Downarrow$

Analysis of linear perturbations allows to understand

- How the **stabilized moduli** behave when we add perturbations on the energy momentum tensor on the brane?
- How **gravity** behaves at large distances on the brane? Which kind of corrections to Einstein gravity one does find for this system?
- How do the answers to the previous questions depend on the brane position $\bar{\theta}$?

Perturbations

The degrees of freedom

- We impose azimuthal symmetry, look at zero modes with $\partial_\varphi^2 = 0$.
- We impose a Z_2 symmetry at the equator, while the brane position does not enjoy a particular status.
- We split the metric perturbations into scalar, (vector), and tensor components from a four dimensional point of view

$$\begin{aligned} ds^2 &= (1+2\Phi)\, d\theta^2 + 2\, A\, d\theta\, d\phi + (1+2C)\, \cos^2 m\, d\phi^2 \\ &+ 2\partial_\mu T d\theta dx^\mu + 2\partial_\mu V\, d\phi\, dx^\mu \\ &+ \{\eta_{\mu\nu}\,(1+2\Psi) + 2\, E_{,\mu\nu} + h_{\mu\nu}\}\, dx^\mu\, dx^\nu \end{aligned}$$

  and we demand $\partial^\mu h_{\mu\nu} = 0 = h^\mu_\mu$

- The perturbations of the gauge field read

$$\delta A_\phi \equiv a_\phi,\ \delta A_\theta \equiv a_\theta,\ \delta A_\mu \equiv \partial_\mu a$$

- An additional mode plays a crucial role: the brane **bending**

$$\bar{\theta} + \zeta(x^\mu)$$

- We then consider perturbations on brane energy momentum tensor

$$S_{\mu\nu} \to S_{\mu\nu} + T_{\mu\nu} \quad , \quad S_{\phi\phi} \to S_{\phi\phi} + T_{\phi\phi}$$

A partial gauge fixing

- The perturbations transform under infinitesimal coordinate transformations $x^A \to x^A + \xi^A$. After fixing $T = V = 0$, one can define gauge invariant variables

$$\hat{\Phi} \equiv \Phi + E'' \qquad \hat{C} \equiv C - m' \mathrm{tg}\, m\, E'$$
$$\hat{\zeta} \equiv \zeta - E' \qquad \hat{a}_\phi \equiv a_\phi + m' M^2 \cos m E'$$

Some results

- The equations for the gauge invariant scalar perturbations for the metric can be solved exactly, in both sides of the brane

  ⇒ They depend only on **two modes**, C_Ψ and D_Ψ.

- The three scalar modes have a well definite geometrical interpretation

  ⇒ The mode ζ corresponds to the brane position

  ⇒ The perturbation of the volume of the internal space, and of the length of the circumference, depend only on the mode D_Ψ

$$\Delta V \propto D_\Psi$$
$$\Delta L_c \propto D_\Psi$$

Junction conditions

- We perturb energy-momentum tensor , with components T_{MN}.
- One finds a junction condition from the Maxwell equation

$$\left[4\tan^2\bar{\theta}\,\Psi' - m'\left(\tan\bar{\theta}\,\hat{C} - \frac{\hat{a}_\phi}{M^2\cos\bar{\theta}} - \frac{m'\hat{\zeta}}{\cos^2\bar{\theta}}\right)\right]_J = 0$$

it provides a constraint independent on the presence of brane EMT

$\Rightarrow$ It **fixes** one of the three scalar modes.

- The Israel junction conditions give

$$\left[-\hat{\zeta}_{\mu\nu} + \frac{1}{2}h'_{\mu\nu}\right]_J = \frac{1}{M^4}\left(T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}\right)$$
$$\left[\partial^2\hat{\zeta} - \frac{4\Psi'}{\cos^2\theta}\right]_J = \frac{T^\phi_\phi}{M^4}$$

  - Matter bends the brane: $\left[\partial^2\hat{\zeta}\right]_J = \frac{T}{3M^4}$
    it acts as a source for the bending mode.
  - The $\phi\phi$ component gives the relation

$$D_\Phi = -\frac{\cos\bar{\theta}}{8\pi M_6^4}\left[\frac{T}{3} - T^\phi_\phi\right]$$

Matter on the brane fixes the scalar that controls the volume: the mode D_Φ is frozed to a certain value.

Gravity at large distances on the brane

- The dynamics of linear perturbations gives the behavior of gravity at large distances.
- The equation for the tensor mode, with brane contribution, reads

$$\partial_\theta \left(\cos m\, \partial_\theta h_{\mu\nu}\right) + \cos m\, \partial^2 h_{\mu\nu} =$$
$$= -2\delta\left(\theta - \bar\theta\right) \frac{\cos\bar\theta}{M^4} \left(T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu} + M^4 \left[\zeta_{\mu\nu}\right]_J\right)$$

- At the zero mode level, one can solve using (retarded) Green functions

$$h^{(0)}_{\mu\nu} \equiv h^{(m)}_{\mu\nu} + h^{(\zeta)}_{\mu\nu}$$
$$h^{(m)}_{\mu\nu} \equiv -\frac{4\pi\beta}{M^4 V_2} \cos\bar\theta \left(\partial^2\right)^{-1} \left[T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}\right]$$
$$h^{(\zeta)} \equiv -\frac{4\pi\beta}{V_2} \cos\bar\theta \left(\partial^2\right)^{-1} \left[\hat\zeta\right]_J$$

where V_2 is the volume of the internal space

- The metric in four dimension is obtained averaging the projected metric on the brane along the azimuthal direction.

$$g^{(4)}_{\mu\nu} = \frac{1}{2\pi\cos\bar\theta} \int_0^{2\pi\beta} d\phi \sqrt{\gamma_{\phi\phi}}\, \gamma_{\mu\nu}$$

where γ_{MN} is the projected metric on the brane.

- The corresponding Ricci tensor, using the previous information, reads

$$R^{(4)}_{\mu\nu} = \frac{2\pi\beta}{M^4 V_2}\cos\bar{\theta}\left[T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu} - \frac{T}{6}\eta_{\mu\nu}\right] + \left(\frac{1}{2}\partial^2\Upsilon\,\eta_{\mu\nu} + \Upsilon_{,\mu\nu}\right)$$

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with

$$\Upsilon \equiv \bar{\Phi} + \tan\bar{\theta} m'\bar{\zeta} - \frac{1}{2}\partial^2 h^{(\zeta)}$$

- **Two** scalar combinations contribute
  - **The first** depends on $\left[\bar{\zeta}\right]_J$, giving the right numerical factor $\frac{1}{2}$. (Exactly what happens in the codimension one, warped case)
  - **The second**, Υ, behaves as a massless scalar coupled to gravity

Results

- The **effective Planck mass** is given by the **ADD** formula

$$M_p^2 = M^4 V_2$$

- But the theory contains and additional **scalar contribution**.
- The scalar Υ is given in terms of the mode D_φ:

$$\Upsilon = \left[\frac{1 - \sin\bar{\theta}}{\sin\bar{\theta} + \beta(1 - \sin\bar{\theta})} F(\beta, \bar{\theta}) \right] D_\varphi$$

- The junction conditions provide a relation between the scalar Υ and the components of the brane energy momentum tensor, so

$$R^{(4)}_{\mu\nu} = \frac{1}{M_p^2} \left[T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{2} \eta_{\mu\nu} \right] -$$
$$- \frac{R^2 \beta}{4 M_{Pl}^2} \left(1 - \sin\bar{\theta}\right) F\left(\beta, \bar{\theta}\right) \left(\frac{1}{2} \eta_{\mu\nu} \partial^2 + \partial_\mu \partial_\nu \right) \left[\frac{T^{(4)}}{3} - \left(T^\phi_\phi\right)^{(4)} \right]$$

the second line is **subleading** in respect to the first line. It becomes sizable only when probing **short distances**, of order R.

- Notice that the factor

$$\left(1 - \sin\bar{\theta}\right)$$

suppresses even further this term: taking the singular limit, the second line becomes more and more **negligible** (if $T \simeq T^\phi_\phi$)

⇒ The corrections due to the frozen modulus are typically subdominant in respect to KK modes.

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The limit $\bar{\theta} \to \frac{\pi}{2}$

- In this limit, the correction term in the RHS of the Einstein equation decouples.

  $\Rightarrow$ The scalar just discussed decouples from the theory.

- Recall that one of the junction conditions is

$$\left[\hat{\zeta}_{\mu\nu}\right]_J = -\frac{1}{M^4 \cos\bar{\theta}}\left(T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{3}\eta_{\mu\nu}\right)$$

  In the limit $\bar{\theta} \to \frac{\pi}{2}$, the RHS diverges, unless $T^{(4)}_{\mu\nu} \to 0$: the brane cannot afford the presence of matter on it!

  - If $T^{(4)}_{\mu\nu} \neq 0$, the linear approximation breaks down, the results are no more reliable.
  - If $T^{(4)}_{\mu\nu} = 0$, it means we do not have any matter on the brane.

Result

Taking the limit in this way, analysis suggests that $R^{(4)}_{\mu\nu} = 0$.
This is essentially due to the behaviour of the bending mode $\hat{\zeta}$.

The limit $\bar{\theta} \to \frac{\pi}{2}$

- In this limit, the correction term in the RHS of the Einstein equation decouples.

  $\Rightarrow$ The scalar just discussed decouples from the theory.
- Recall that one of the junction conditions is

$$\left[\bar{\zeta}_{\mu\nu}\right]_J = -\frac{1}{M^4 \cos\bar{\theta}} \left(T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{3} \eta_{\mu\nu} \right)$$

In the limit $\bar{\theta} \to \frac{\pi}{2}$, the RHS diverges, unless $T^{(4)}_{\mu\nu} \to 0$: the brane cannot afford the presence of matter on it!

- If $T^{(4)}_{\mu\nu} \neq 0$, the linear approximation breaks down, the results are no more reliable.
- If $T^{(4)}_{\mu\nu} = 0$, it means we do not have any matter on the brane.

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Results

- The **effective Planck mass** is given by the **ADD** formula

$$M_p^2 \equiv M^4 V_2$$

- But the theory contains and additional **scalar contribution**.
- The scalar Υ is given in terms of the mode D_ψ:

$$\Upsilon = \left[\frac{1-\sin\bar{\theta}}{\sin\bar{\theta}+\beta(1-\sin\bar{\theta})} F(\beta,\bar{\theta})\right] D_\psi$$

- The junction conditions provide a relation between the scalar Υ and the components of the brane energy momentum tensor, so

$$R^{(4)}_{\mu\nu} = \frac{1}{M_p^2}\left[T^{(4)}_{\mu\nu} - \frac{T^{(4)}}{2}\eta_{\mu\nu}\right] -$$
$$- \frac{R^2\beta}{4M_{Pl}^2}\left(1-\sin\bar{\theta}\right) F\left(\beta,\bar{\theta}\right)\left(\frac{1}{2}\eta_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu\right)\left[\frac{T^{(4)}}{3} - \left(T^\phi_\phi\right)^{(4)}\right]$$

the second line is subleading in respect to the first line. It becomes sizable only when probing **short distances**, of order R.

- Notice that the factor

$$\left(1-\sin\bar{\theta}\right)$$

suppresses even further this term: taking the singular limit, the second line becomes more and more **negligible** (if $T \simeq T^\phi_\phi$)

$\Rightarrow$ The corrections due to the frozen modulus are typically subdominant in respect to KK modes.

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Conclusions

- We presented a regularized version of a **codimension two BW model** in a bulk compactified by fluxes.
- We shown that, at the background level, the brane position is **stabilized**.
- We studied linearized perturbations around this background, to investigate the behavior of gravity on the brane.
- We shown that (as expected) **GR is recovered at large distances**. We could quantify the corrections to GR directly in the effective Einstein equations.
- The limit of brane thickness $\to 0$ is still problematic. It seems to suggest that brane matter must disappear otherwise the bending mode diverges.

Outlook

- How does gravity behaves at the non-linear level? can we find exact solutions that describe **BHs on the brane**? (notoriously difficult question in AdS geometries.)
- Study cosmological models in flux stabilized BWs.

  $\Rightarrow$ The pure de Sitter case is a straightforward generalization of this.
- Develop a deeper understanding of the limit in which brane thickness vanishes.