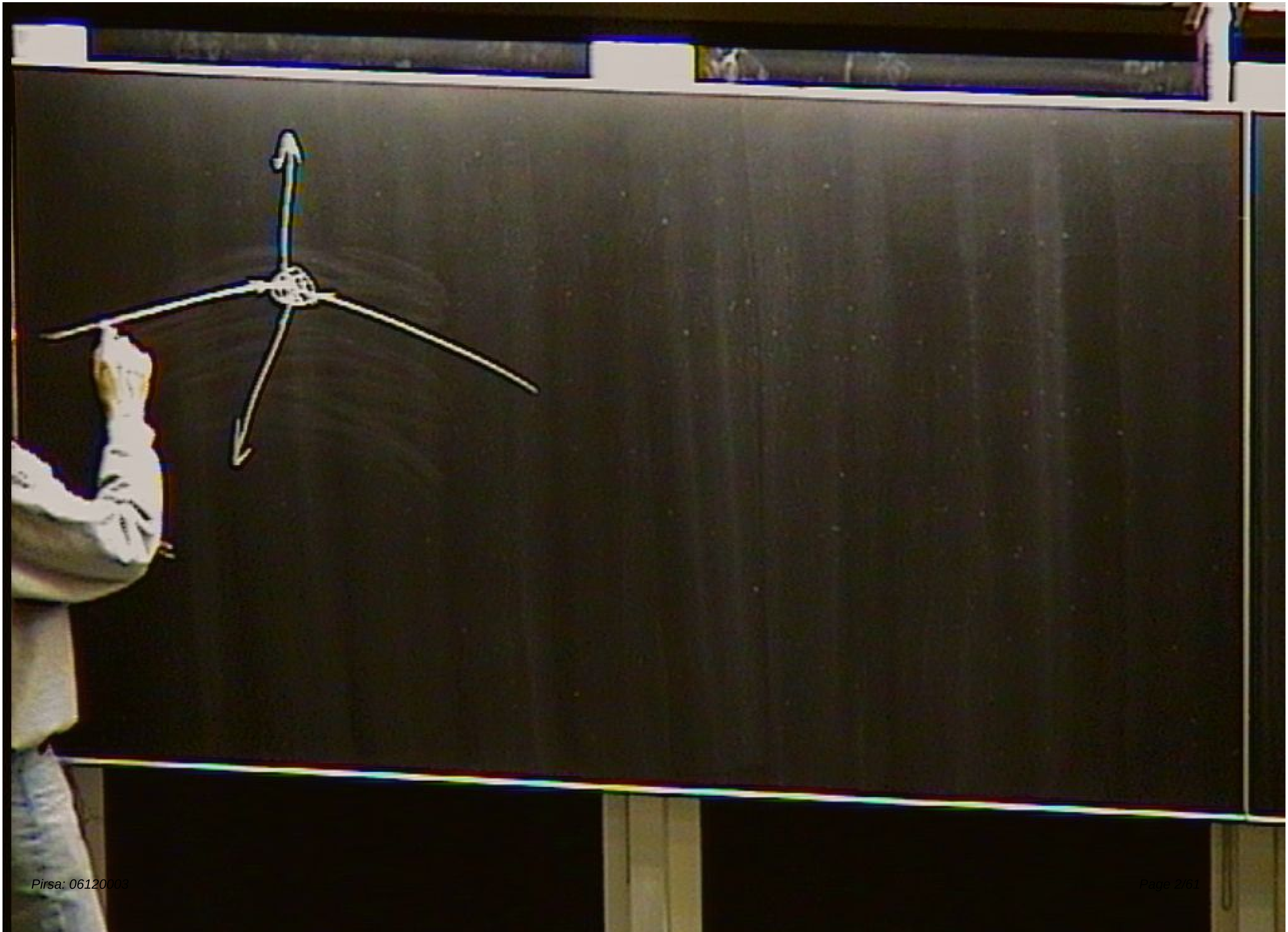
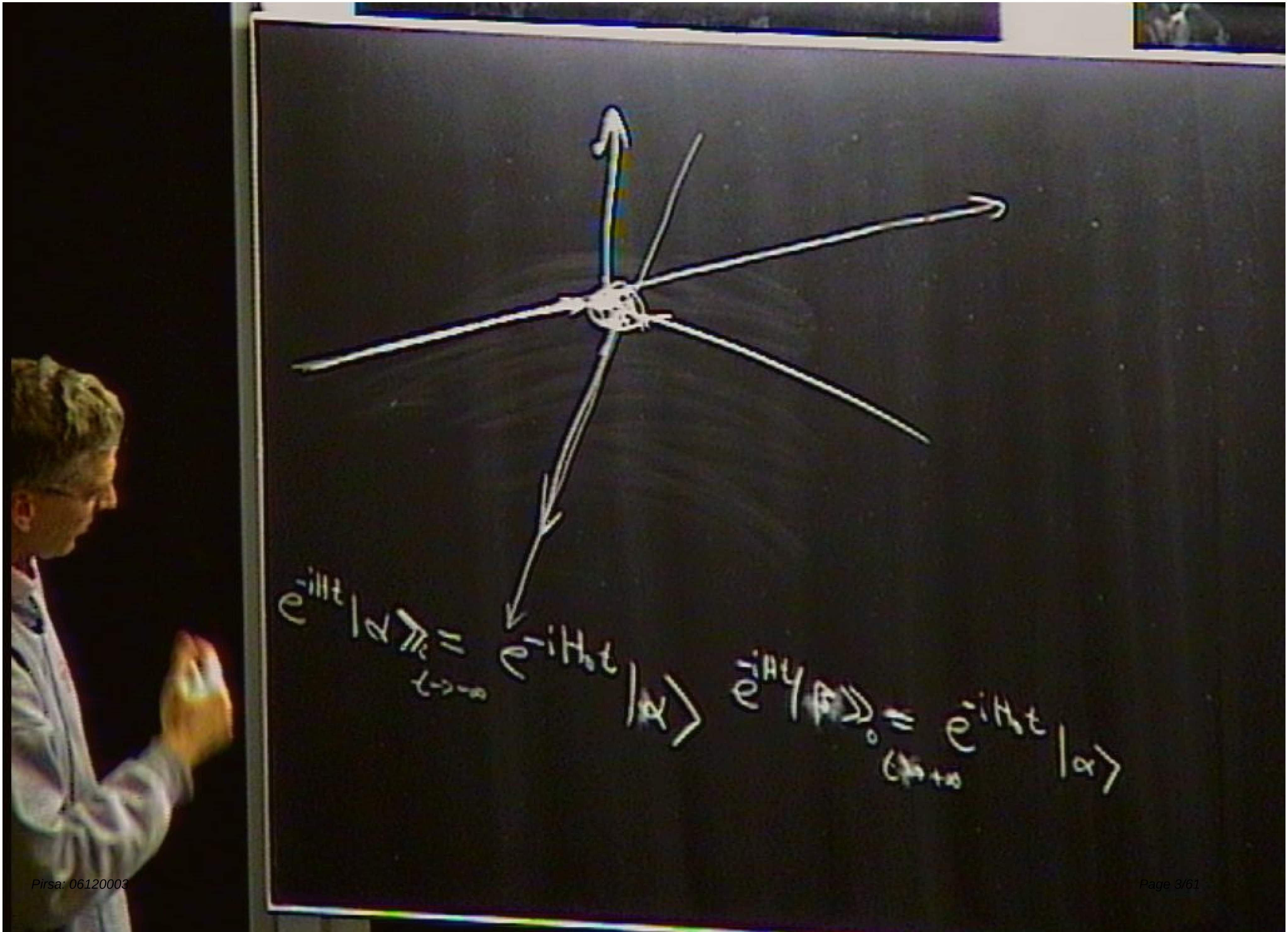
Title: Graduate Course on Standard Model & Quantum Field Theory - 8A

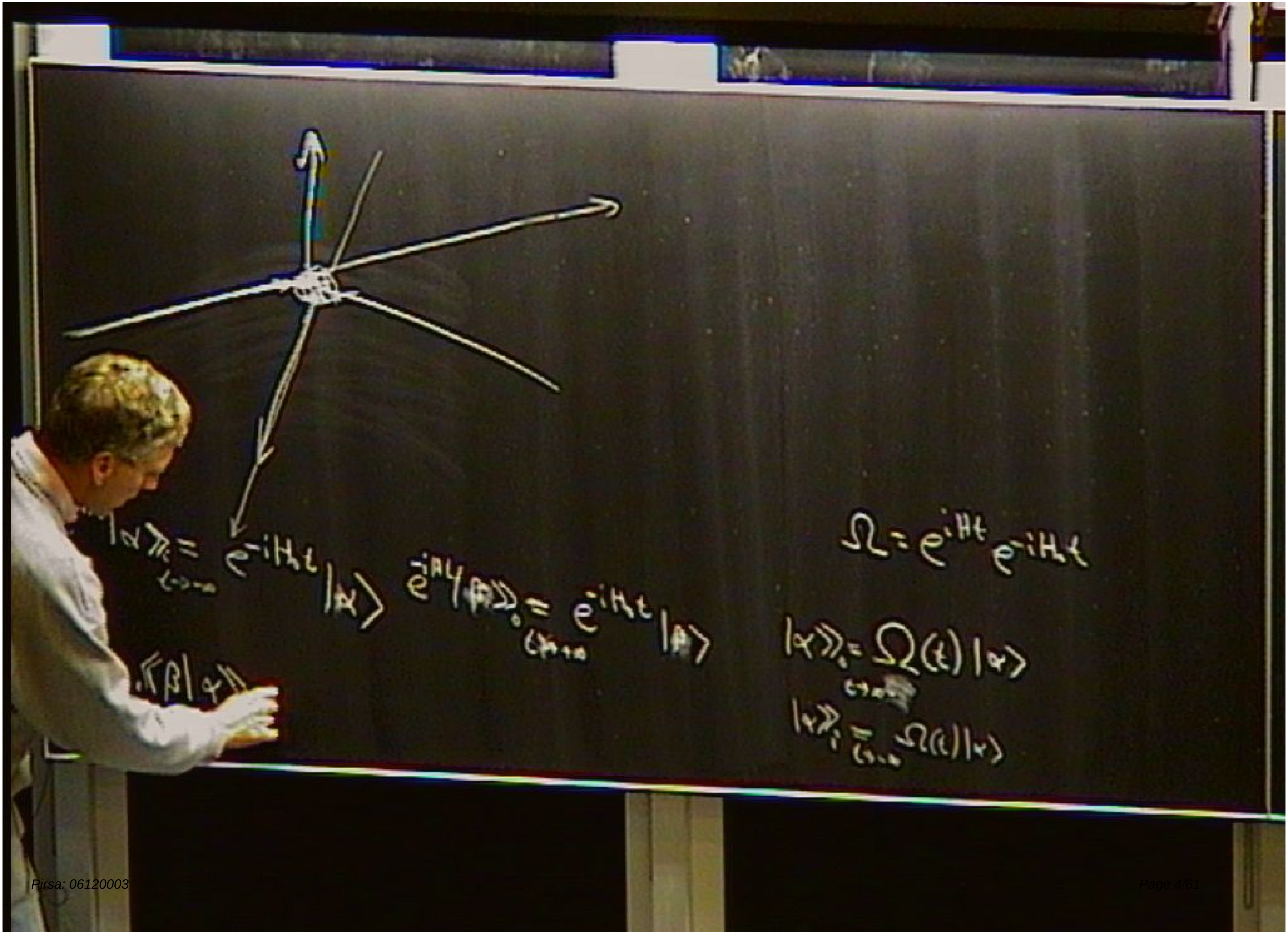
Date: Dec 06, 2006 11:00 AM

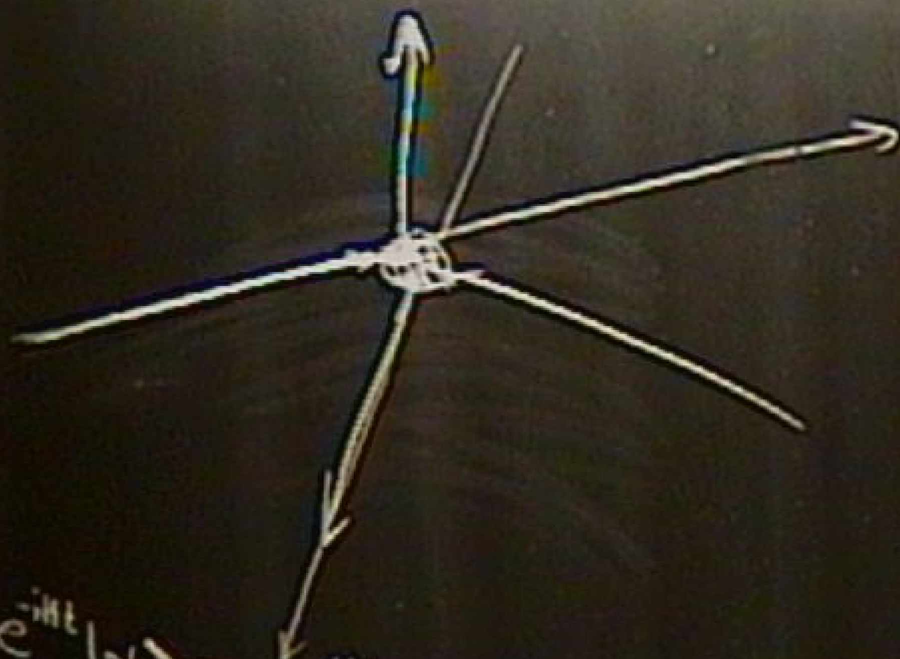
URL: http://pirsa.org/06120003

Abstract: Graduate Course on Standard Model & Quantum Field Theory




$e^{-iHt}|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} e^{-iH_0t}|\alpha\rangle$
$e^{-iHt}|\alpha\rangle\rangle_o \underset{t\to+\infty}{=} e^{-iH_0t}|\alpha\rangle$


Ω = e^{iHt} e^{-iH_0 t}



$$e^{-iHt}|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} e^{-iH_0 t}|\alpha\rangle$$

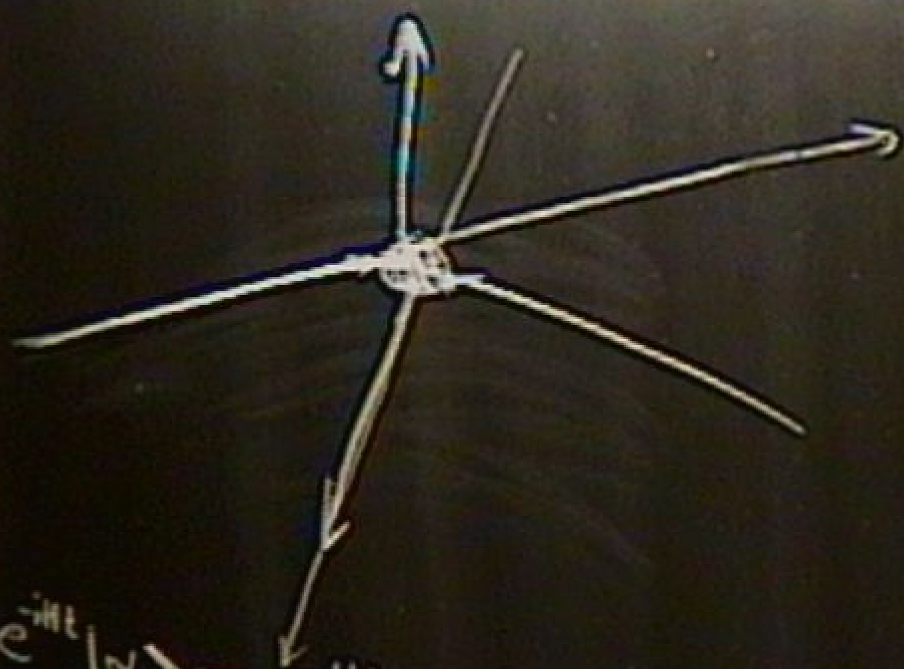
$$e^{-iHt}|\beta\rangle\rangle_0 \underset{t\to+\infty}{=} e^{-iH_0 t}|\beta\rangle$$

$$\Omega = e^{iHt}e^{-iH_0 t}$$

$$|\alpha\rangle\rangle_0 \underset{t\to\infty}{=} \Omega(t)|\alpha\rangle$$

$${}_0\langle\langle\beta|\alpha\rangle\rangle_i = \langle\beta|S|\alpha\rangle \qquad S = \Omega^{\dagger}(\infty)\Omega(-\infty)$$

$$|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} \Omega(t)|\alpha\rangle$$



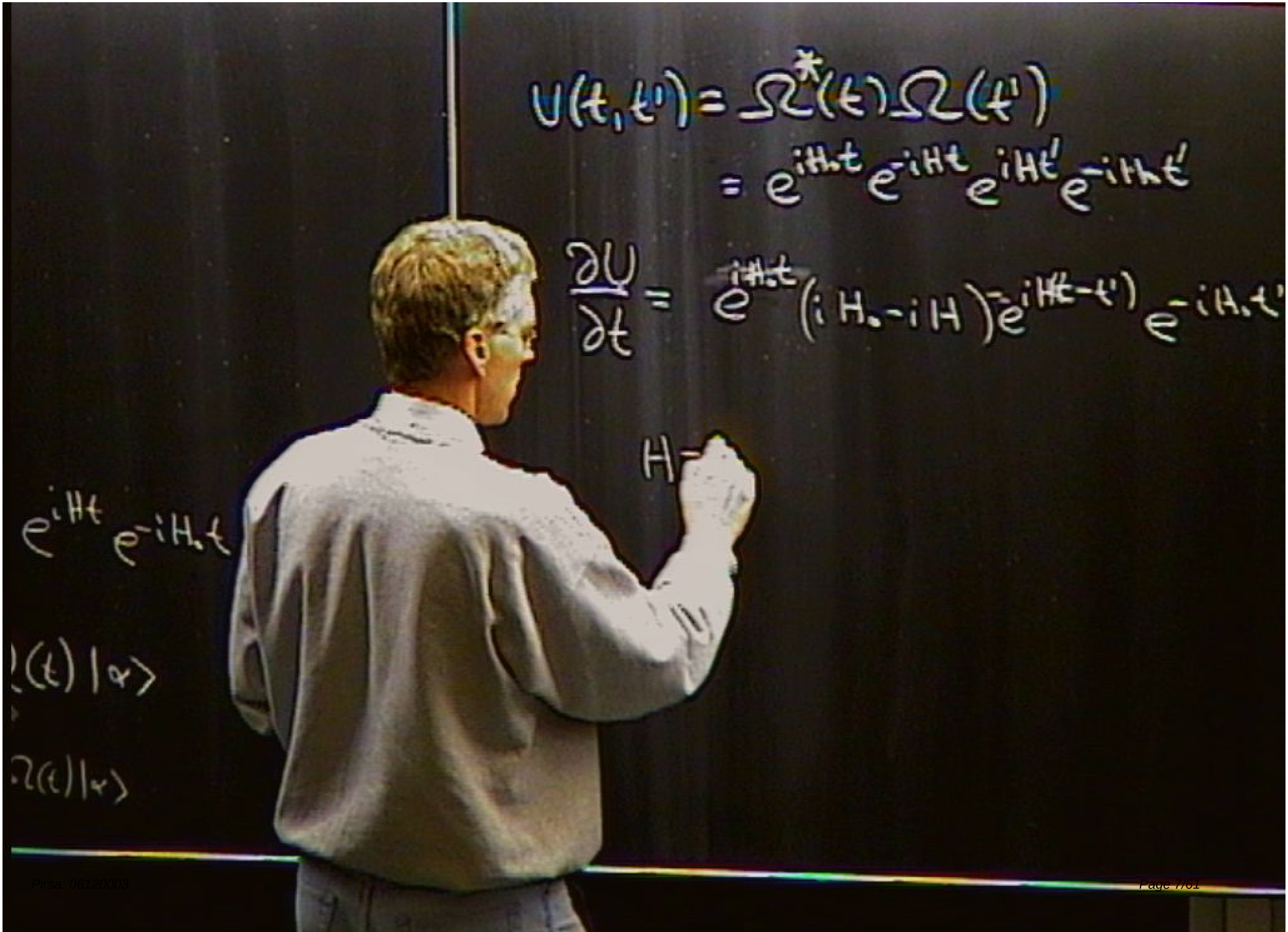
$$e^{-iHt}|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} e^{-iH_0 t}|\alpha\rangle \qquad e^{-iHt}|\beta\rangle\rangle_o \underset{t\to+\infty}{=} e^{-iH_0 t}|\beta\rangle$$

$$\Omega = e^{iHt}e^{-iH_0 t}$$

$$|\alpha\rangle\rangle_o \underset{t\to+\infty}{=} \Omega(t)|\alpha\rangle$$

$$_o\langle\langle\beta|\alpha\rangle\rangle_i = \langle\beta|S|\alpha\rangle \qquad S = \Omega^\dagger(\infty)\Omega(-\infty)$$

$$|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} \Omega(t)|\alpha\rangle$$


U(t,t') = Ω*(t) Ω(t')
= e^{iH₀t} e^{-iHt} e^{iHt'} e^{-iH₀t'}
∂U/∂t = e^{iH₀t} (iH₀ - iH) e^{-iH(t-t')} e^{-iH₀t'}
H
e^{iHt} e^{-iH₀t}
(t) |α>
(t) |α>

$$U(t,t') = \Omega^*(t)\,\Omega(t')$$
$$= e^{iH_0 t} e^{-iHt} e^{iHt'} e^{-iH_0 t'}$$

$$\frac{\partial U}{\partial t} = e^{iH_0 t} \underbrace{(iH_0 - iH)}_{-iV} e^{-iH(t-t')} e^{-iH_0 t'} = -i\,V(t)\,U(t,t')$$

$H = H_0 + V$ Define $V(t) = e^{iH_0 t} V e^{-iH_0 t}$ Interaction Rep'n of V

$$U(t,t') = \Omega^{*}(t)\,\Omega(t')$$
$$= e^{iH_0 t} e^{-iHt} e^{iHt'} e^{-iH_0 t'}$$

$$\frac{\partial U}{\partial t} = e^{iH_0 t}\underbrace{(iH_0 - iH)}_{-iV} e^{-iH(t-t')} e^{-iH_0 t'} = -i\,V(t)\,U(t,t')$$

$H = H_0 + V$ Define $V(t) = e^{iH_0 t} V e^{-iH_0 t}$ Interaction Rep^n of V.

Sol^n: $U(t,t') = 1 - i\int_{t'}^{t} d\tau\, V(\tau) U(\tau,t') + (-i)^2 \int_{t'}^{t} d\tau_1 \int_{t'}^{\tau_1} d\tau_2\, V(\tau_1) V(\tau_2) U(\tau_2,t')$

$U(t,t) = I$

$$U(t,t') = \Omega^{*}(t)\,\Omega(t')$$

$$= e^{iH_0 t} e^{-iHt} e^{iHt'} e^{-iH_0 t'}$$

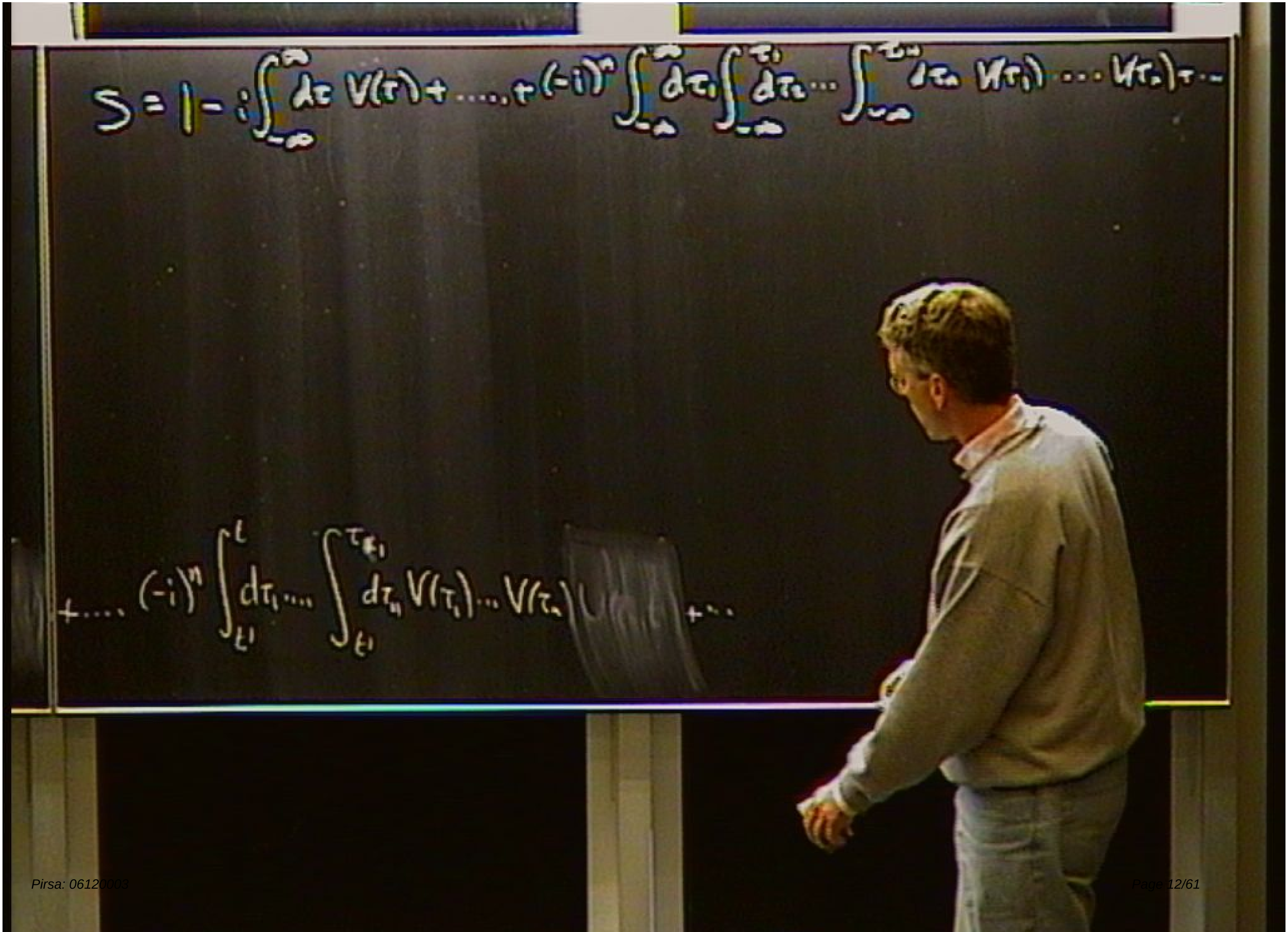
(1) $$\frac{\partial U}{\partial t} = e^{iH_0 t}\underbrace{(iH_0 - iH)}_{-iV} e^{-iH(t-t')} e^{-iH_0 t'} = -i\,V(t)\,U(t,t')$$

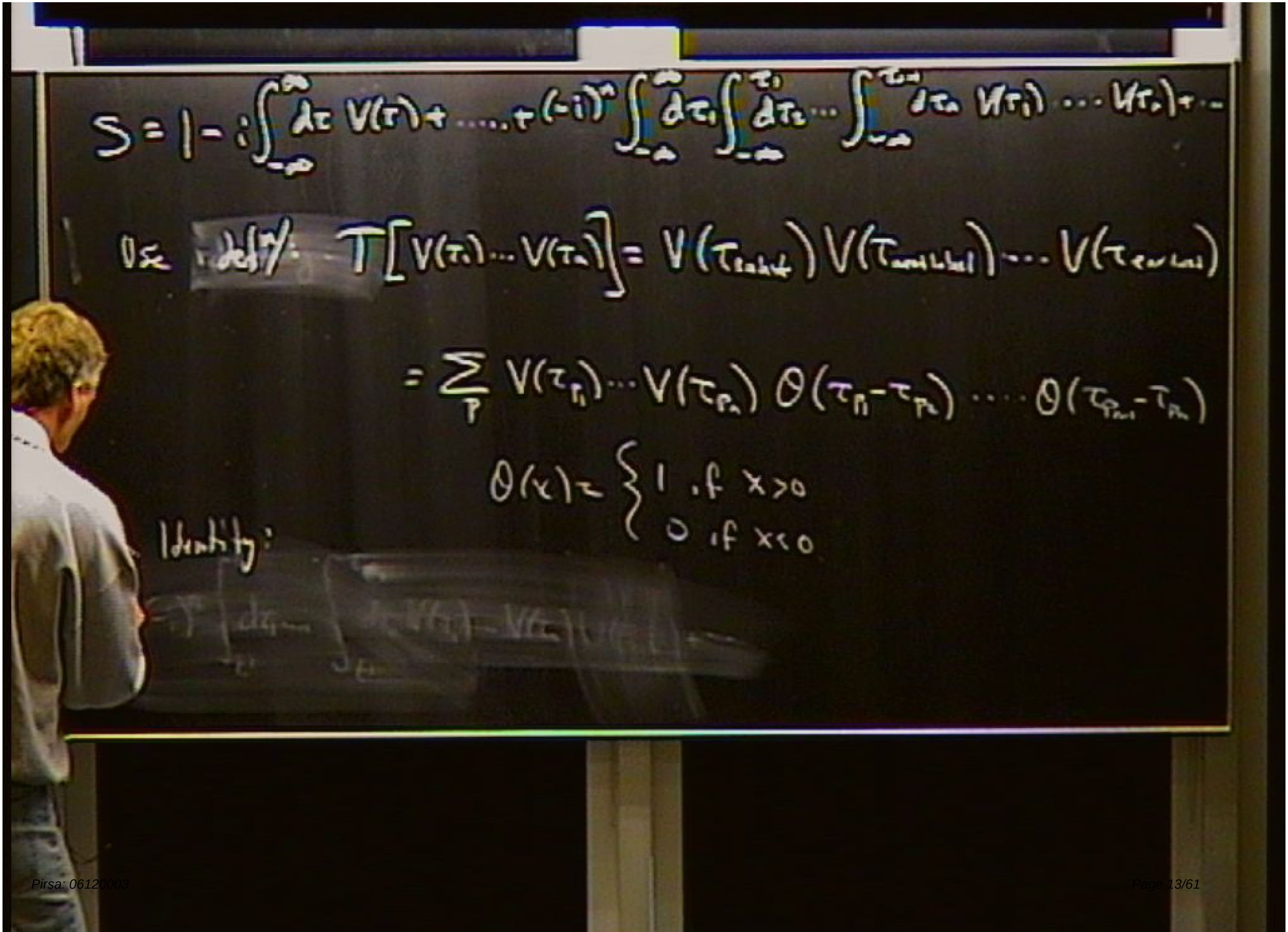
$H = H_0 + V$ Define $V(t) = e^{iH_0 t}\, V\, e^{-iH_0 t}$ Interaction Rep^{n} of V.

Sol^{n}: $$U(t,t') = 1 - i\int_{t'}^{t} d\tau\, V(\tau) + (-i)^2 \int_{t'}^{t} d\tau_1 \int_{t'}^{\tau_1} d\tau_2\, V(\tau_1)V(\tau_2) +$$

$U(t,t) = I$

$$+ \dots (-i)^n \int_{t'}^{t} d\tau_1 \dots \int_{t'}^{\tau_{n-1}} d\tau_n \, V(\tau_1) \dots V(\tau_n) \dots + \dots$$


$S = 1 - i\int_{-\infty}^{\infty} d\tau\, V(\tau) + \ldots + (-i)^n \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \cdots \int_{-\infty}^{\tau_{n-1}} d\tau_n\, V(\tau_1) \cdots V(\tau_n) + \ldots$
$+ \ldots (-i)^n \int_{t'}^{t} d\tau_1 \ldots \int_{t'}^{\tau_{n-1}} d\tau_n\, V(\tau_1) \ldots V(\tau_n) + \ldots$


$S = 1 - i\int_{-\infty}^{\infty} d\tau\, V(\tau) + \dots + (-i)^n \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \cdots \int_{-\infty}^{\tau_{n-1}} d\tau_n\, V(\tau_1) \cdots V(\tau_n) + \dots$
Use def'n: $T[V(\tau_1)\cdots V(\tau_n)] = V(\tau_{\text{latest}})\,V(\tau_{\text{next latest}}) \cdots V(\tau_{\text{earliest}})$
$= \sum_P V(\tau_{P_1}) \cdots V(\tau_{P_n})\, \theta(\tau_{P_1} - \tau_{P_2}) \cdots \theta(\tau_{P_{n-1}} - \tau_{P_n})$
$\theta(x) = \begin{cases} 1 & \text{if } x>0 \\ 0 & \text{if } x<0 \end{cases}$
Identity:

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d\tau_1 \dots d\tau_n \, T\left[V(\tau_1) \dots V(\tau_n)\right]$$

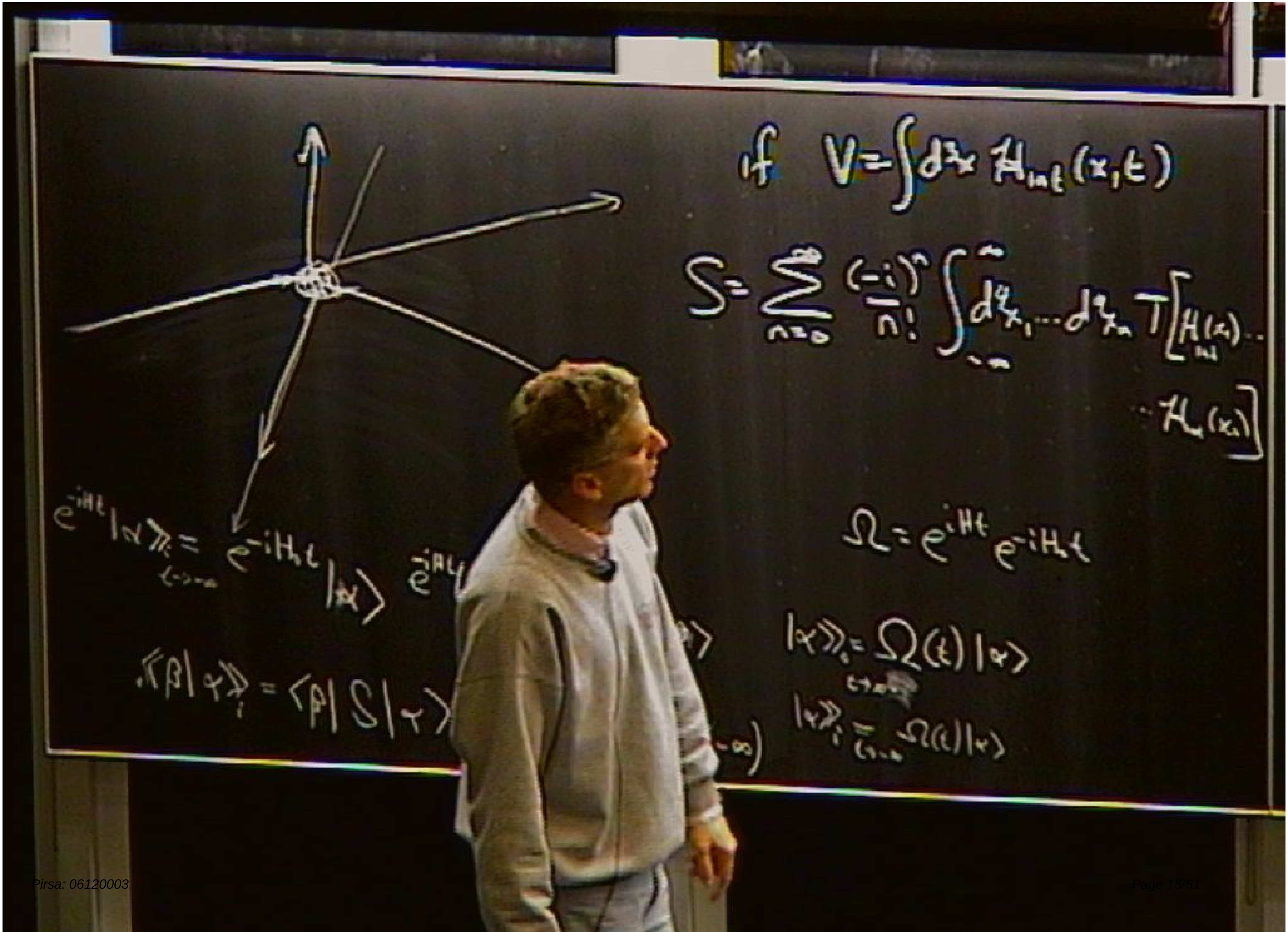
Use defn. $T\left[V(\tau_1)\dots V(\tau_n)\right] = V(\tau_{\text{latest}})\,V(\tau_{\text{next latest}}) \cdots V(\tau_{\text{earliest}})$

$$= \sum_{P} V(\tau_{P_1}) \dots V(\tau_{P_n})\,\theta(\tau_{P_1} - \tau_{P_2}) \cdots \theta(\tau_{P_{n-1}} - \tau_{P_n})$$

$$\theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

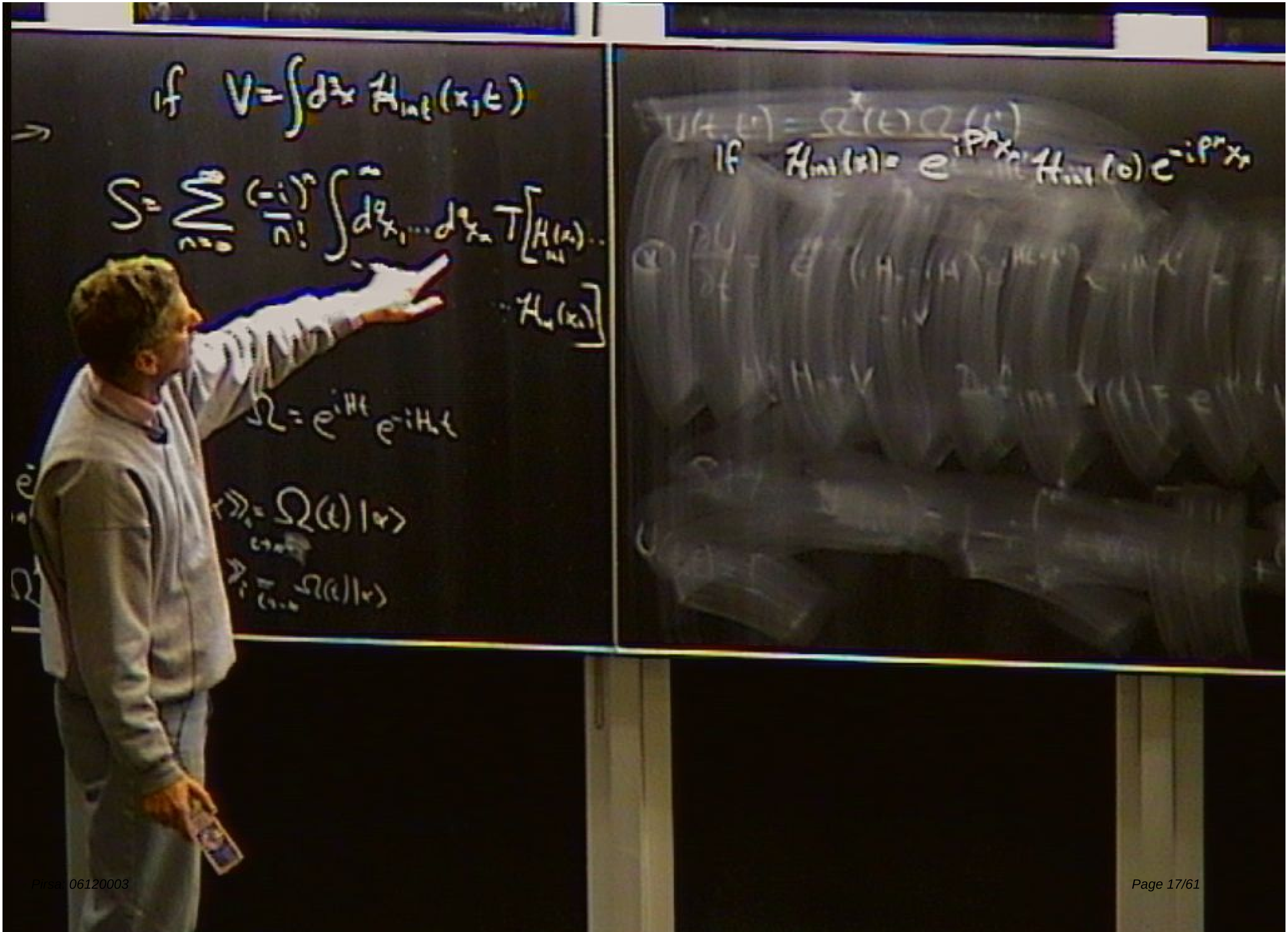
Then Identity:

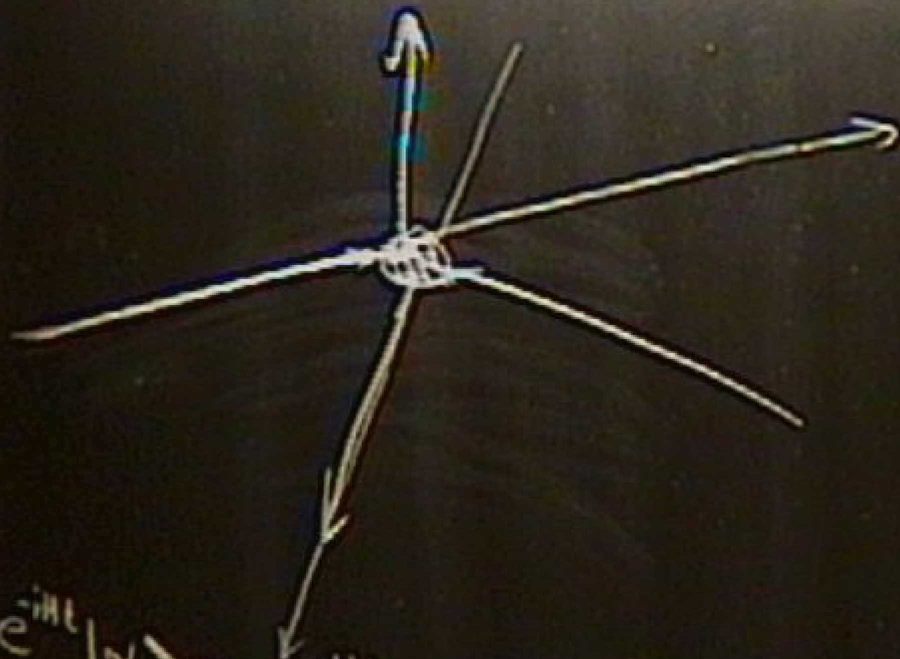
$$\int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \cdots \int_{-\infty}^{\tau_{n-1}} d\tau_n = \frac{1}{n!} \int_{-\infty}^{\infty} d\tau_1 \cdots \int_{-\infty}^{\infty} d\tau_n \, T\left(V(\tau_1) \dots V(\tau_n)\right)$$



if $V = \int d^3x\, \mathcal{H}_{int}(x,t)$
$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots d^4x_n\, T[\mathcal{H}_{int}(x_1) \cdots \mathcal{H}_{int}(x_n)]$
$e^{-iHt}|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} e^{-iH_0 t}|\alpha\rangle$
$\Omega = e^{iHt} e^{-iH_0 t}$
$|\alpha\rangle\rangle_i = \Omega(t)|\alpha\rangle$
$|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} \Omega(t)|\alpha\rangle$
$\langle\langle\beta|\alpha\rangle\rangle_i = \langle\beta|S|\alpha\rangle$

If $\mathcal{H}_{int}(x) = e^{iP^\mu x_\mu}\,\mathcal{H}_{int}(0)\,e^{-iP^\mu x_\mu}$


if $V = \int d^3x\, \mathcal{H}_{int}(x,t)$
$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \cdots d^4x_n\, T[\mathcal{H}_{int}(x_1) \cdots \mathcal{H}_{int}(x_n)]$
$\Omega = e^{iHt} e^{-iH_0 t}$
$|\alpha\rangle\rangle_{\pm} = \Omega(t)|\alpha\rangle$ $t \to \mp\infty$
If $\mathcal{H}_{int}(x) = e^{iP^\mu x_\mu} \mathcal{H}_{int}(0) e^{-iP^\mu x_\mu}$



if $V = \int d^3x\, \mathcal{H}_{int}(x,t)$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots d^4x_n\, T\left[\mathcal{H}_{int}(x_1) \cdots \mathcal{H}_{int}(x_n)\right]$$

$$\Omega = e^{iHt} e^{-iH_0 t}$$

$$e^{-iHt}|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} e^{-iH_0 t}|\alpha\rangle$$

$$e^{-iHt}|\beta\rangle\rangle_0 \underset{t\to+\infty}{=} e^{-iH_0 t}|\beta\rangle$$

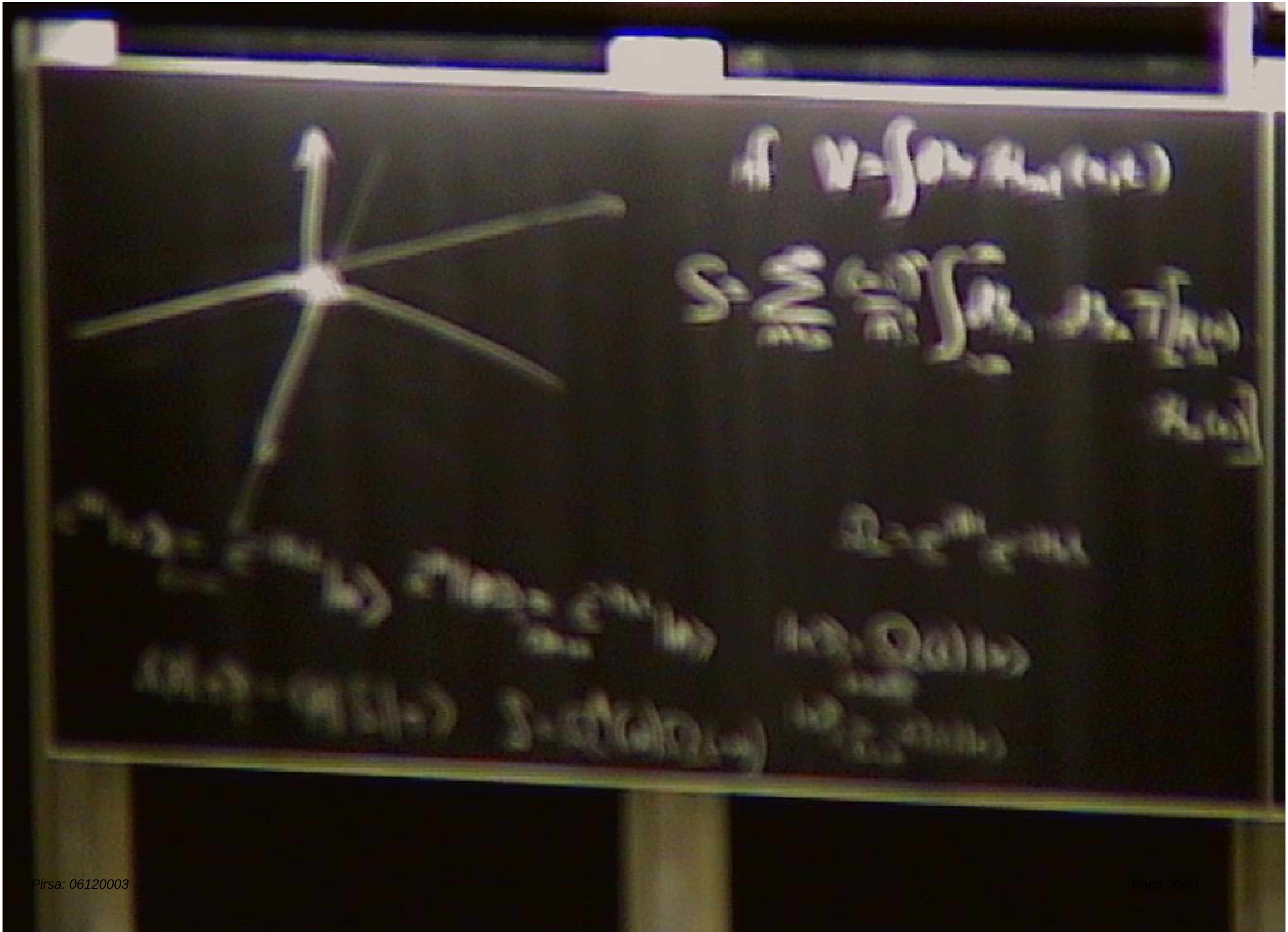
$$|\alpha\rangle\rangle_0 \underset{t\to\infty}{=} \Omega(t)|\alpha\rangle$$

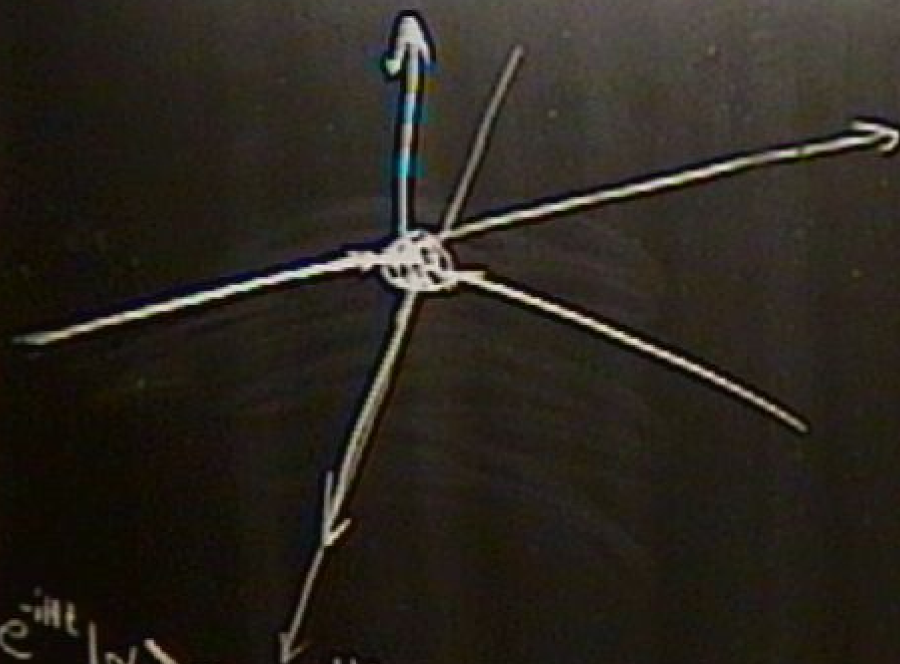
$$|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} \Omega(t)|\alpha\rangle$$

$${}_0\langle\langle\beta|\alpha\rangle\rangle_i = \langle\beta|S|\alpha\rangle$$

$$S = \Omega^\dagger(\infty)\,\Omega(-\infty)$$

$$\langle \beta | S | \alpha \rangle = \delta^4(p_\beta^\mu - p_\alpha^\mu)(2\pi)^4 i\, M_{\beta\alpha}$$





if $V = \int d^3x \, \mathcal{H}_{int}(x,t)$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots d^4x_n \, T\left[\mathcal{H}_{int}(x_1) \cdots \mathcal{H}_{int}(x_n)\right]$$

$$\Omega = e^{iHt} e^{-iH_0 t}$$

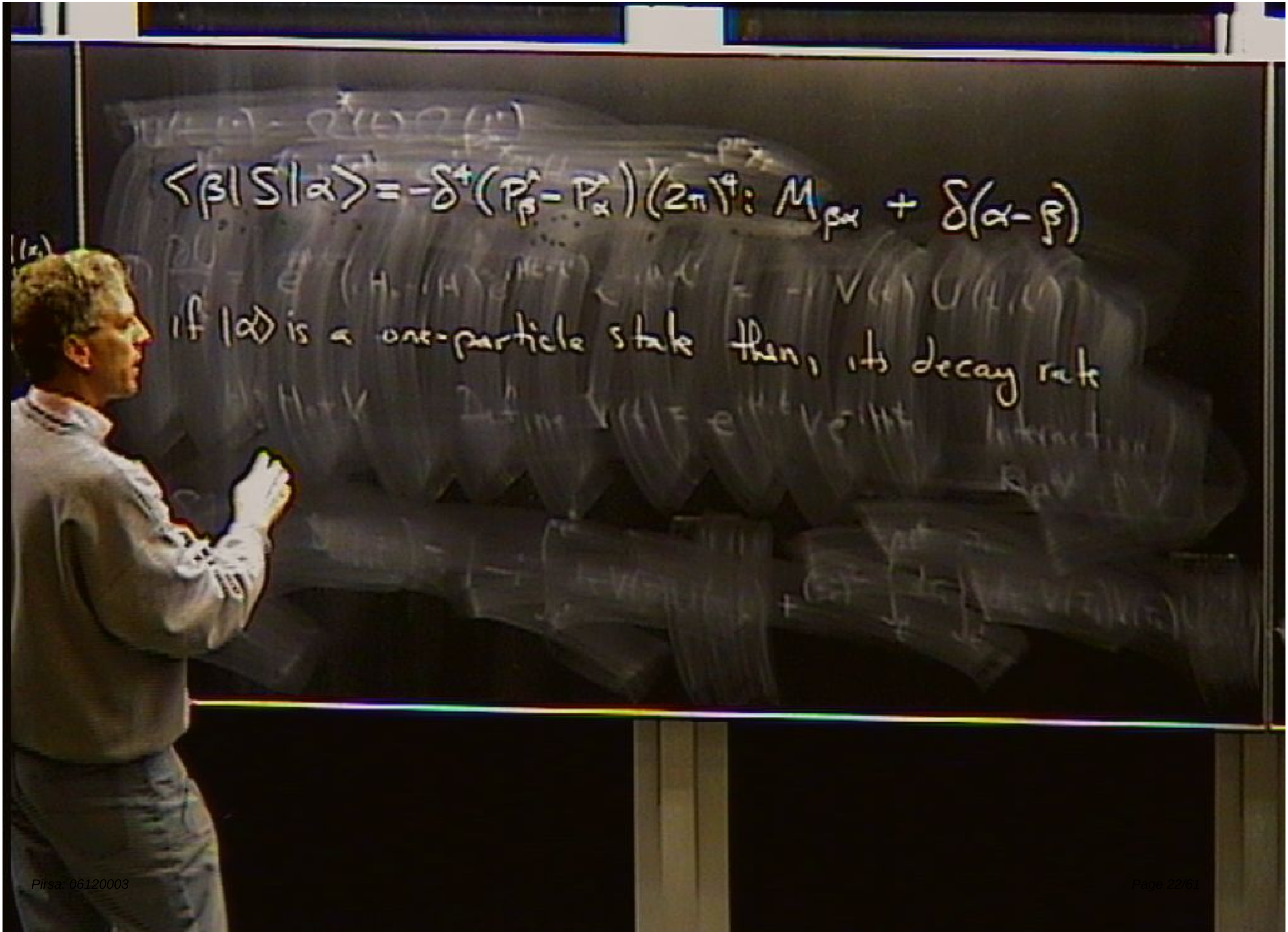
$$e^{-iHt}|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} e^{-iH_0 t}|\alpha\rangle$$

$$e^{-iHt}|\beta\rangle\rangle_0 \underset{t\to+\infty}{=} e^{-iH_0 t}|\beta\rangle$$

$$|\alpha\rangle\rangle_0 \underset{t\to+\infty}{=} \Omega(t)|\alpha\rangle$$

$$|\alpha\rangle\rangle_i \underset{t\to-\infty}{=} \Omega(t)|\alpha\rangle$$

$${}_0\langle\langle\beta|\alpha\rangle\rangle_i = \langle\beta|S|\alpha\rangle \qquad S = \Omega^\dagger(\infty)\,\Omega(-\infty)$$

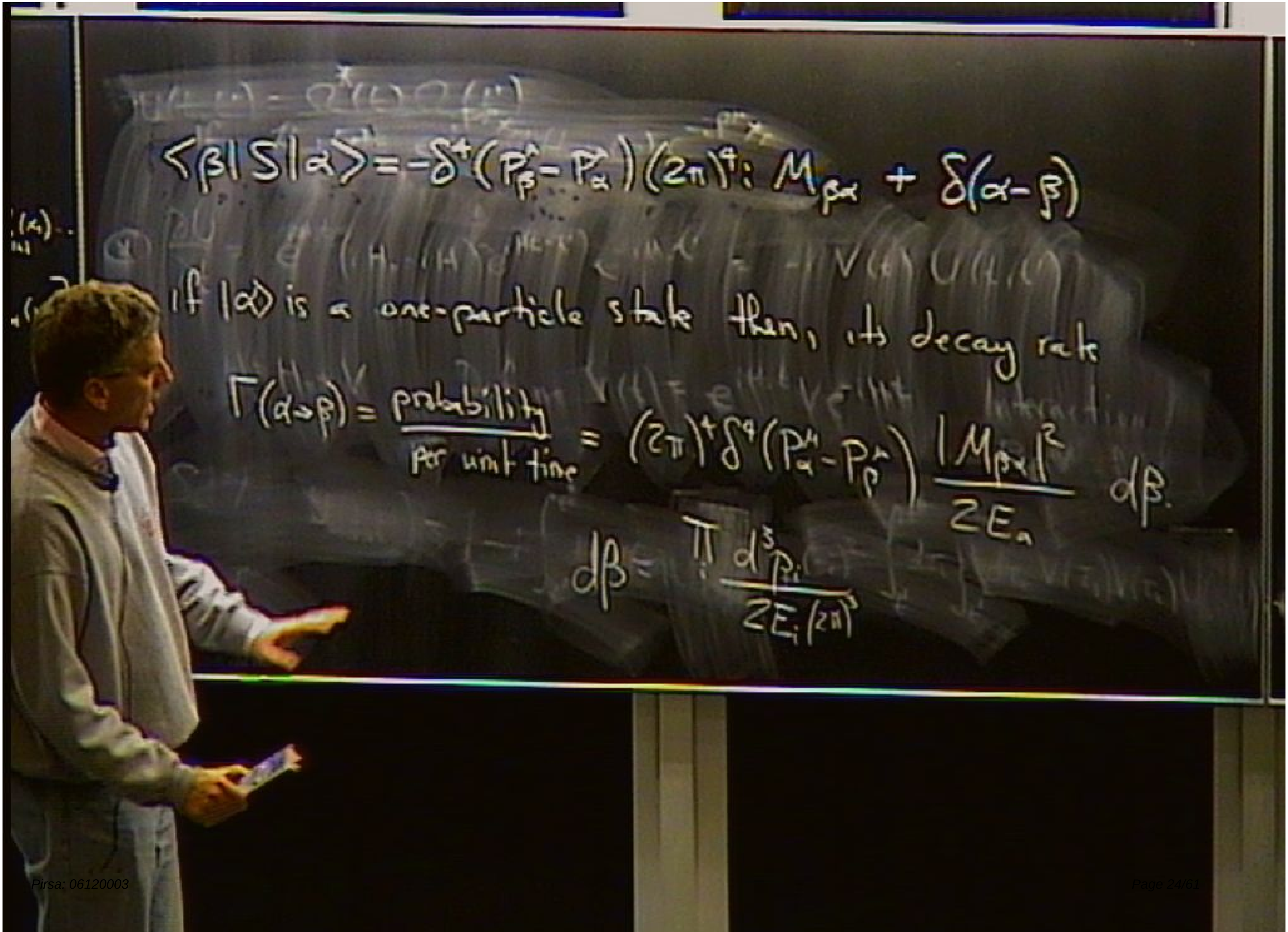

$\langle\beta|S|\alpha\rangle = -\delta^4(p'_\beta - p_\alpha)(2\pi)^4 i\, M_{\beta\alpha} + \delta(\alpha-\beta)$
if $|\alpha\rangle$ is a one-particle state then, its decay rate

$$\langle\beta|S|\alpha\rangle = -\delta^4(p_\beta^\mu - p_\alpha^\mu)(2\pi)^4 i\, M_{\beta\alpha} + \delta(\alpha-\beta)$$

if $|\alpha\rangle$ is a one-particle state then, its decay rate

$$\Gamma(\alpha\to\beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4\delta^4(p_\alpha^\mu - p_\beta^\mu)\frac{|M_{\beta\alpha}|^2}{2E_\alpha}\, d\beta.$$

$$d\beta = \prod_i \frac{d^3p_i}{2E_i}$$


⟨β|S|α⟩ = −δ⁴(p_β − p_α)(2π)⁴ i M_βα + δ(α−β)
if |α⟩ is a one-particle state then, its decay rate
Γ(α→β) = probability per unit time = (2π)⁴ δ⁴(P_α − P_β) |M_βα|² / 2E_α dβ.
dβ = Π d³p_i / 2E_i (2π)³

$$\langle\beta|S|\alpha\rangle = -\delta^4(p_\beta^\mu - p_\alpha^\mu)(2\pi)^4 i\, M_{\beta\alpha} + \delta(\alpha-\beta)$$

if $|\alpha\rangle$ is a one-particle state then, its decay rate

$$\Gamma(\alpha\to\beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4\delta^4(p_\alpha^\mu - p_\beta^\mu)\,\frac{|M_{\beta\alpha}|^2}{2E_\alpha}\, d\beta.$$

$$d\beta = \prod_i \frac{d^3p_i}{2E_i(2\pi)^3}$$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d\tau_1 \cdots d\tau_n \, T\left[V(\tau_1) \cdots V(\tau_n)\right]$$

Use defn: $T\left[V(\tau_1)\cdots V(\tau_n)\right] = V(\tau_{\text{latest}})\, V(\tau_{\text{next latest}}) \cdots V(\tau_{\text{earliest}})$

$$= \sum_{P} V(\tau_{P_1}) \cdots V(\tau_{P_n})\, \theta(\tau_{P_1} - \tau_{P_2}) \cdots \theta(\tau_{P_{n-1}} - \tau_{P_n})$$

$$\theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then Identity:

$$\int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \cdots \int_{-\infty}^{\tau_{n-1}} d\tau_n = \frac{1}{n!} \int_{-\infty}^{\infty} d\tau_1 \cdots \int_{-\infty}^{\infty} d\tau_n \, T\left(V(\tau_1) \cdots V(\tau_n)\right)$$

If $|\alpha\rangle$ is a 2-particle state:

It is convenient to divide the reaction rate by a factor which expresses the probability of having the reactants coming together, to get a result which is 1) independent of how the reactants are combined

2) Lorentz invariant

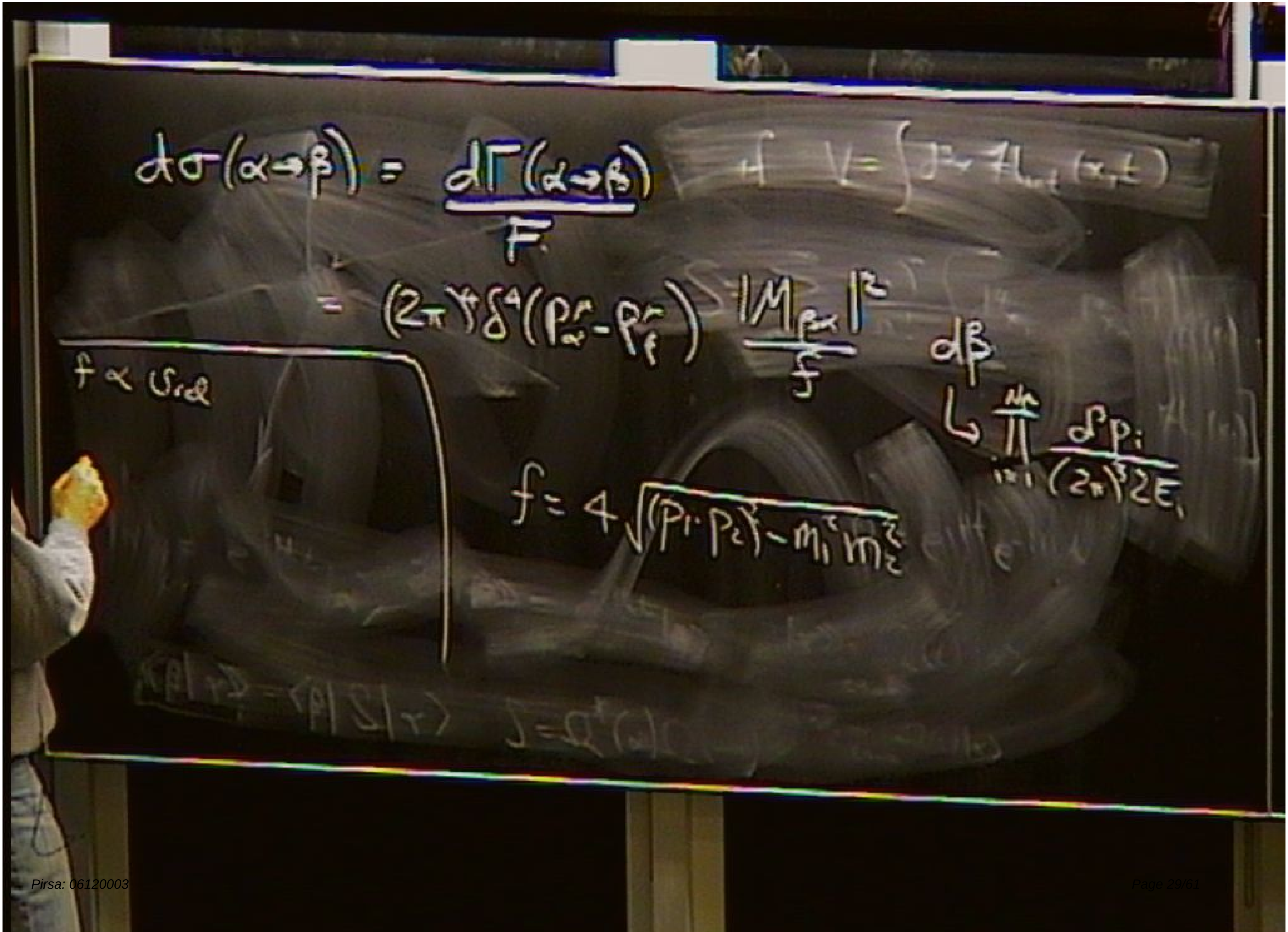
$$* \langle \beta | S | \alpha \rangle = -\delta^4(P_\beta^\mu - P_\alpha^\mu)(2\pi)^4 i\, M_{\beta\alpha} + \delta(\alpha - \beta)$$

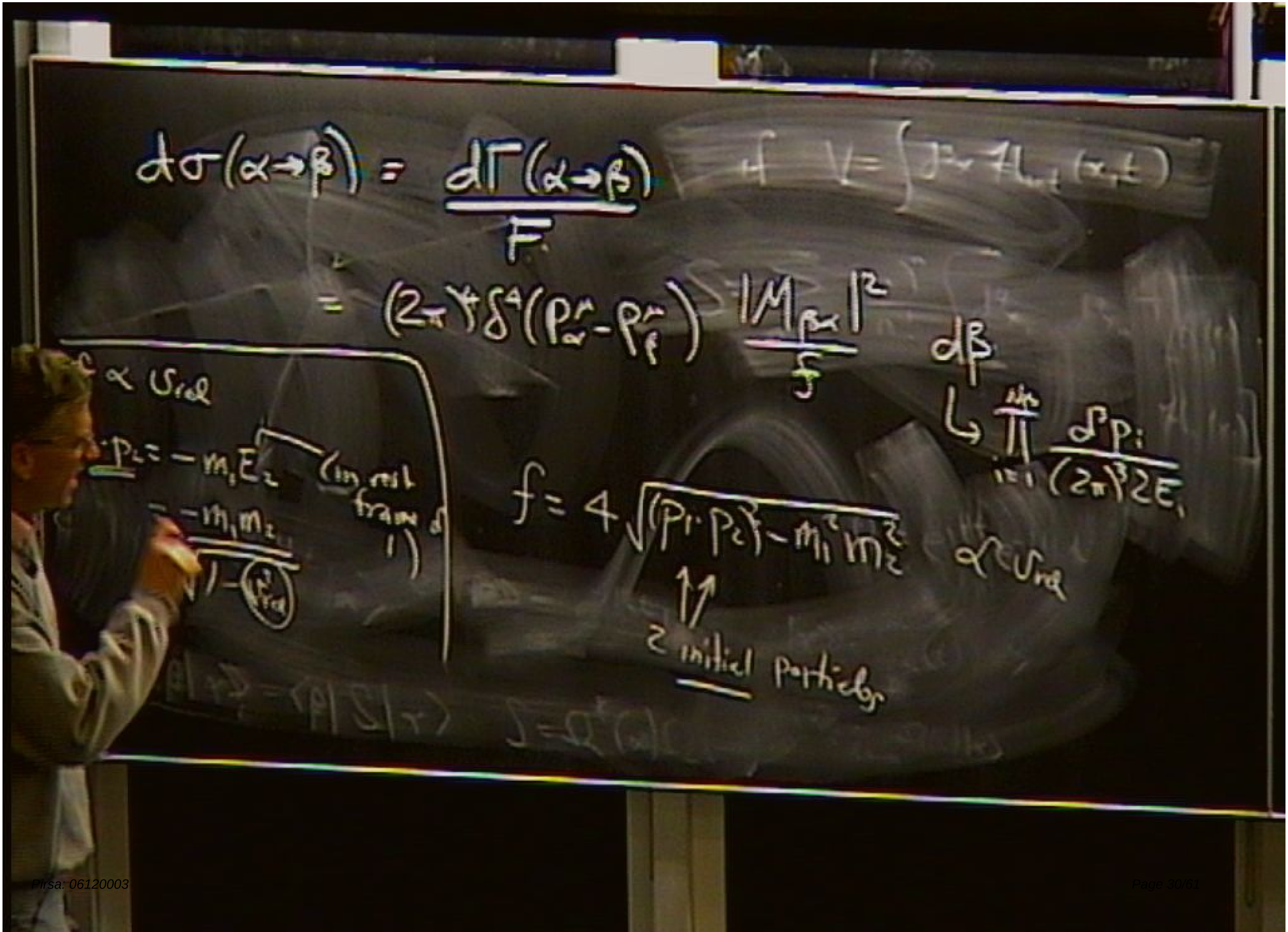
if $|\alpha\rangle$ is a one-particle state then, its decay rate

$$* \quad \Gamma(\alpha \to \beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4 \delta^4(P_\alpha^\mu - P_\beta^\mu) \frac{|M_{\beta\alpha}|^2}{2E_\alpha} \, d\beta.$$

$$d\beta = \prod_i^{N_\beta} \frac{d^3 p_i}{2E_i (2\pi)^3}$$

$N_\beta = \#$ particles in $|\beta\rangle$


$d\sigma(\alpha\to\beta) = \frac{d\Gamma(\alpha\to\beta)}{F}$
$= (2\pi)^4 \delta^4(p_\alpha^\mu - p_f^\mu) \frac{|M_{\beta\alpha}|^2}{f} d\beta$
$d\beta \to \prod_{i=1}^{n_f} \frac{d^3p_i}{(2\pi)^3 2E_i}$
$f \propto v_{rel}$
$f = 4\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2}$


$d\sigma(\alpha\to\beta) = \frac{d\Gamma(\alpha\to\beta)}{F}$
$= (2\pi)^4\delta^4(p_\alpha^\mu - p_\beta^\mu)\frac{|M_{\beta\alpha}|^2}{f}\, d\beta$
$d\beta \hookrightarrow \prod_{i=1}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i}$
$f = 4\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2} \propto v_{rel}$
2 initial particles
$\propto v_{rel}$
$p_1\cdot p_2 = -m_1E_2$
(in rest frame of 1)
$= \frac{-m_1m_2}{\sqrt{1-v_{rel}^2}}$

$$d\sigma(\alpha \to \beta) = \frac{d\Gamma(\alpha \to \beta)}{F}$$

$$= (2\pi)^4 \delta^4(p_\alpha^\mu - p_\beta^\mu) \frac{|M_{\beta\alpha}|^2}{f} \, d\beta$$

$$d\beta \hookrightarrow \prod_{i=1}^{N_\beta} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

$f \propto v_{rel}$

$p_1 \cdot p_2 = -m_1 E_2$ (in rest frame of 1)

$$= \frac{-m_1 m_2}{\sqrt{1 - v_{rel}^2}}$$

$$f = 4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} \propto v_{rel}$$

2 initial particles

$$d\sigma(\alpha\to\beta) = \frac{d\Gamma(\alpha\to\beta)}{F}$$

$$= (2\pi)^4\delta^4(p_\alpha^\mu - p_\beta^\mu)\,\frac{|M_{\beta\alpha}|^2}{f}\,d\beta$$

$$d\beta \hookrightarrow \prod_{i=1}^{N_\beta}\frac{d^3p_i}{(2\pi)^3 2E_i}$$

$f \propto v_{rel}$

$p_1\cdot p_2 = -m_1 E_2$ (in rest frame of 1)

$= \dfrac{-m_1 m_2}{\sqrt{1-v_{rel}^2}}$

$$f = 4\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2} \propto v_{rel}$$

2 initial particles

$$* \langle \beta | S | \alpha \rangle = -\delta^4(P_\beta^\mu - P_\alpha^\mu)(2\pi)^4 i \, \underline{M_{\beta\alpha}} + \delta(\alpha - \beta)$$

if $|\alpha\rangle$ is a one-particle state then, its decay rate

$$* \quad \Gamma(\alpha \to \beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4 \delta^4(P_\alpha^\mu - P_\beta^\mu) \frac{|M_{\beta\alpha}|^2}{2E_\alpha} \, d\beta.$$

$$d\beta = \prod_{i=1}^{N_\beta} \frac{d^3 p_i}{2E_i (2\pi)^3} \qquad N_\beta = \# \text{ particles in } |\beta\rangle$$

$$d\sigma(\alpha\to\beta) = \frac{d\Gamma(\alpha\to\beta)}{F}$$

$$= (2\pi)^4\delta^4(p_\alpha^\mu - p_\beta^\mu)\,\frac{|M_{\beta\alpha}|^2}{F}\,d\beta$$

$$d\beta \hookrightarrow \prod_{i=1}^{n_\beta}\frac{d^3p_i}{(2\pi)^3 2E_i}$$

$F \propto v_{rel}$

$p_1\cdot p_2 = -m_1E_2$ (in rest frame of 1)

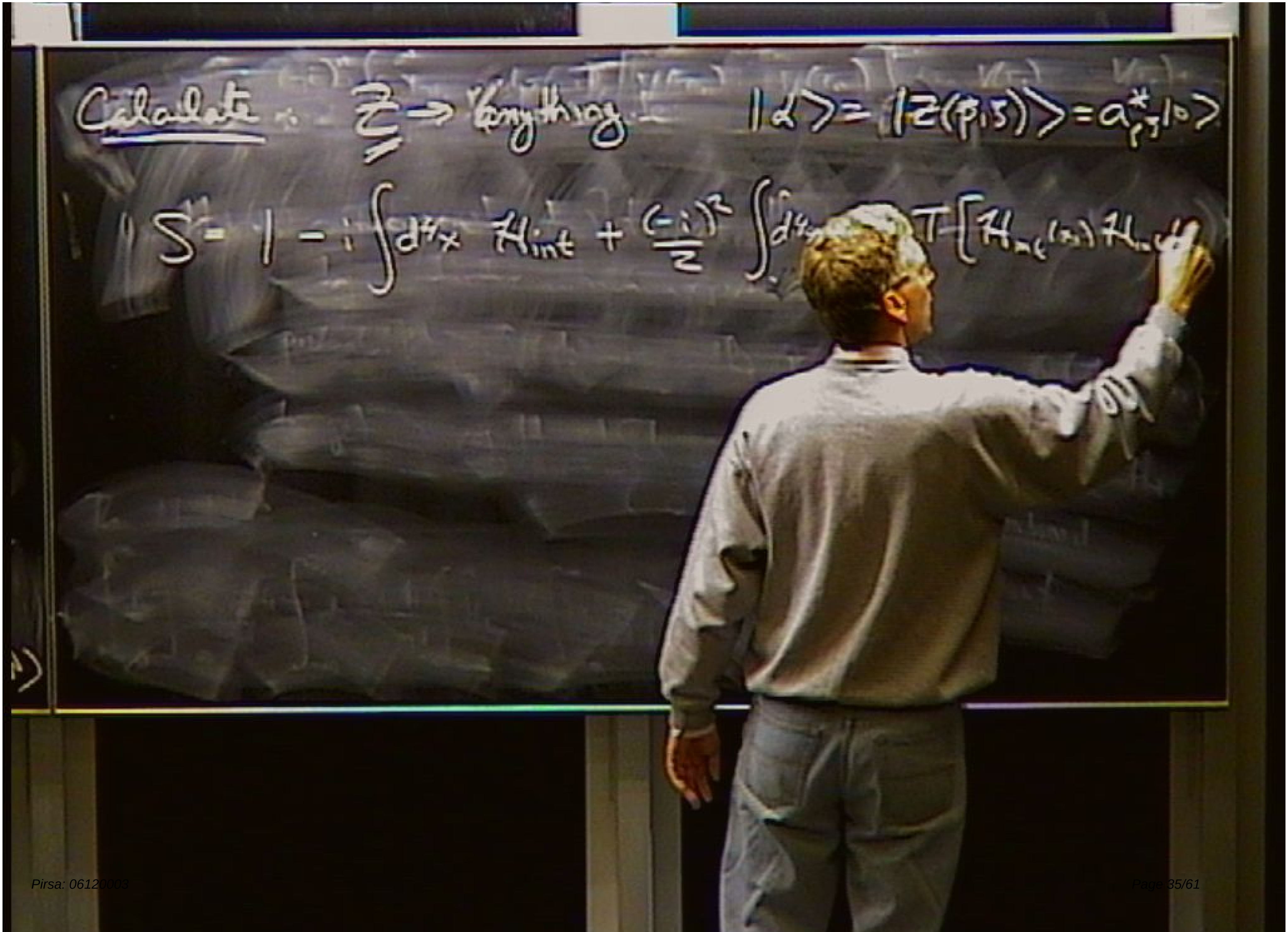
$$= \frac{-m_1 m_2}{\sqrt{1-v_{rel}^2}}$$

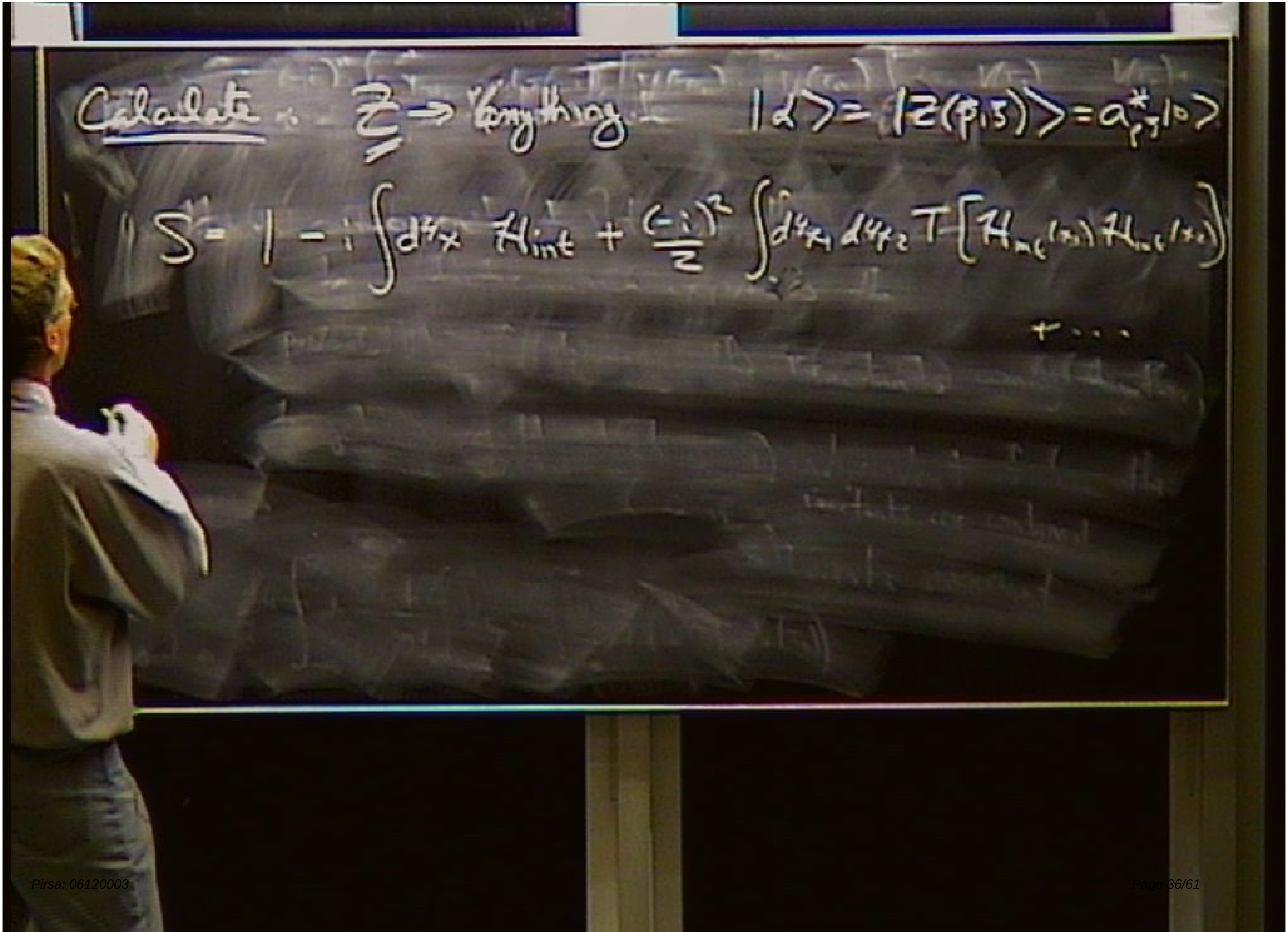
E_1E_2

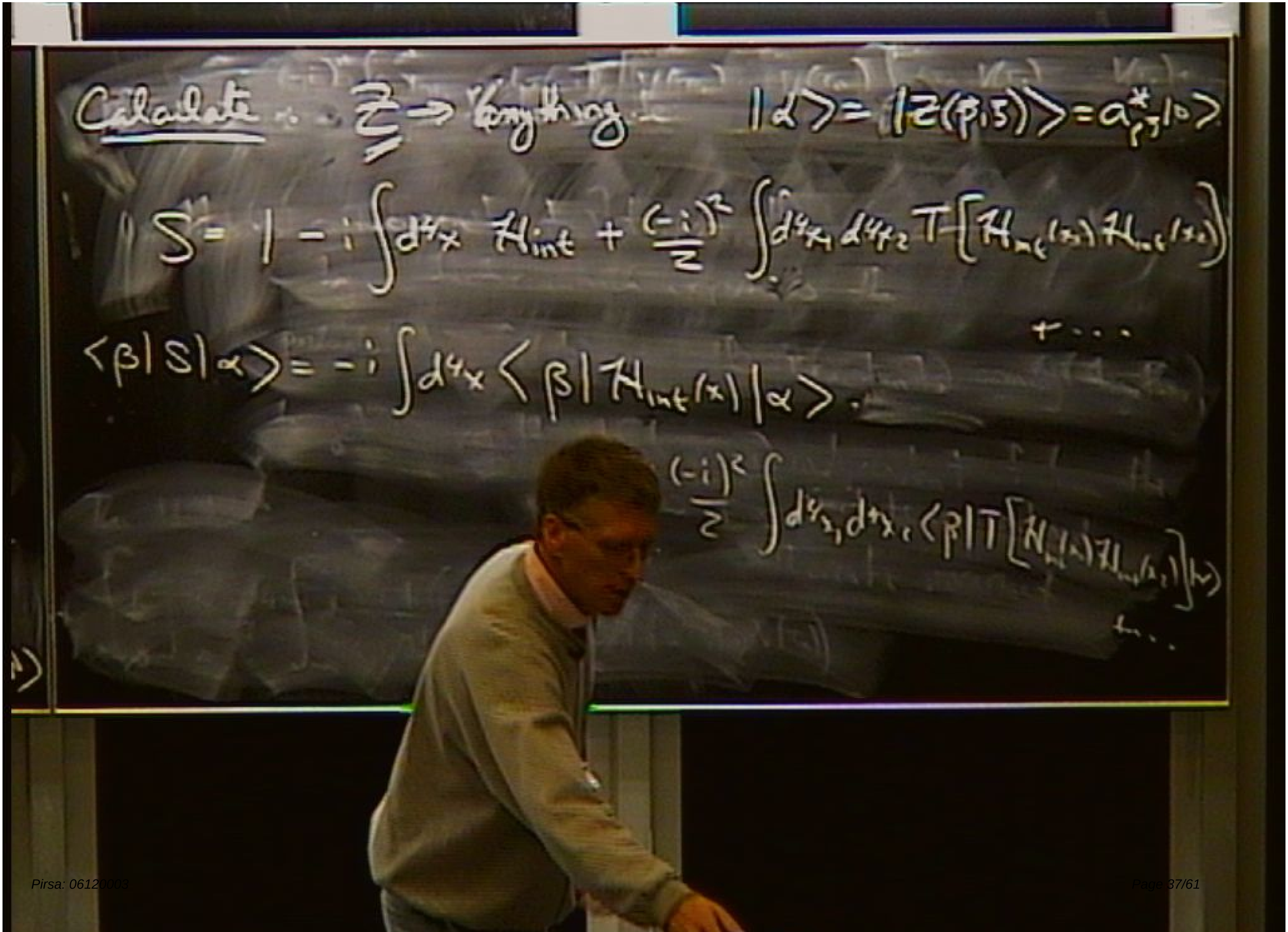
$$F = 4\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2} = -4(p_1\cdot p_2)\,v_{rel}$$

2 initial particles

$p_1^\mu p_{2\mu}$


Calculate: Z → "anything"
|α> = |Z(p,s)> = a†p,s|0>
S = 1 − i ∫d4x Hint + (−i)2/2 ∫d4x


Calculate: $Z \to$ "anything"
$|\alpha\rangle = |Z(p,s)\rangle = a^{*}_{p,s}|0\rangle$
$S = 1 - i\int d^4x\, \mathcal{H}_{int} + \frac{(-i)^2}{2}\int d^4x_1\, d^4x_2\, T\left[\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)\right]$
$+ \ldots$



Calculate ... $Z \rightarrow$ everything
$|\alpha\rangle = |Z(p,s)\rangle = a^{*}_{p,s}|0\rangle$
$S = 1 - i\int d^4x\, \mathcal{H}_{int} + \frac{(-i)^2}{2}\int d^4x_1\, d^4x_2\, T\left[\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)\right] + \ldots$
$\langle\beta|S|\alpha\rangle = -i\int d^4x \langle\beta|\mathcal{H}_{int}(x)|\alpha\rangle$.
$\frac{(-i)^2}{2}\int d^4x_1\, d^4x_2 \langle\beta|T\left[\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)\right]|\alpha\rangle$
$+ \ldots$

Calculate ... $\Sigma \Rightarrow$ "anything" $\quad |\alpha\rangle = |Z(p,s)\rangle = a^*_{p,s}|0\rangle$

$$S = 1 - i\int d^4x\, \mathcal{H}_{int} + \frac{(-i)^2}{2}\int d^4x_1\, d^4x_2\, T\left[\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)\right] + \dots$$

$$\langle\beta|S|\alpha\rangle = -i\int d^4x \langle\beta|\mathcal{H}_{int}(x)|\alpha\rangle + \frac{(-i)^2}{2}\int d^4x_1\, d^4x_2 \langle\beta|T\left[\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)\right]|\alpha\rangle + \dots$$

The only interaction which can destroy a Z + leave only particles lighter than M_Z is

$$\mathcal{L}_{NC} = \frac{ie}{s_W c_W} Z_\mu \sum_f \left[\bar{f} \gamma^\mu \left[T_3 \gamma_L - Q s_W^2 \right] f \right]$$

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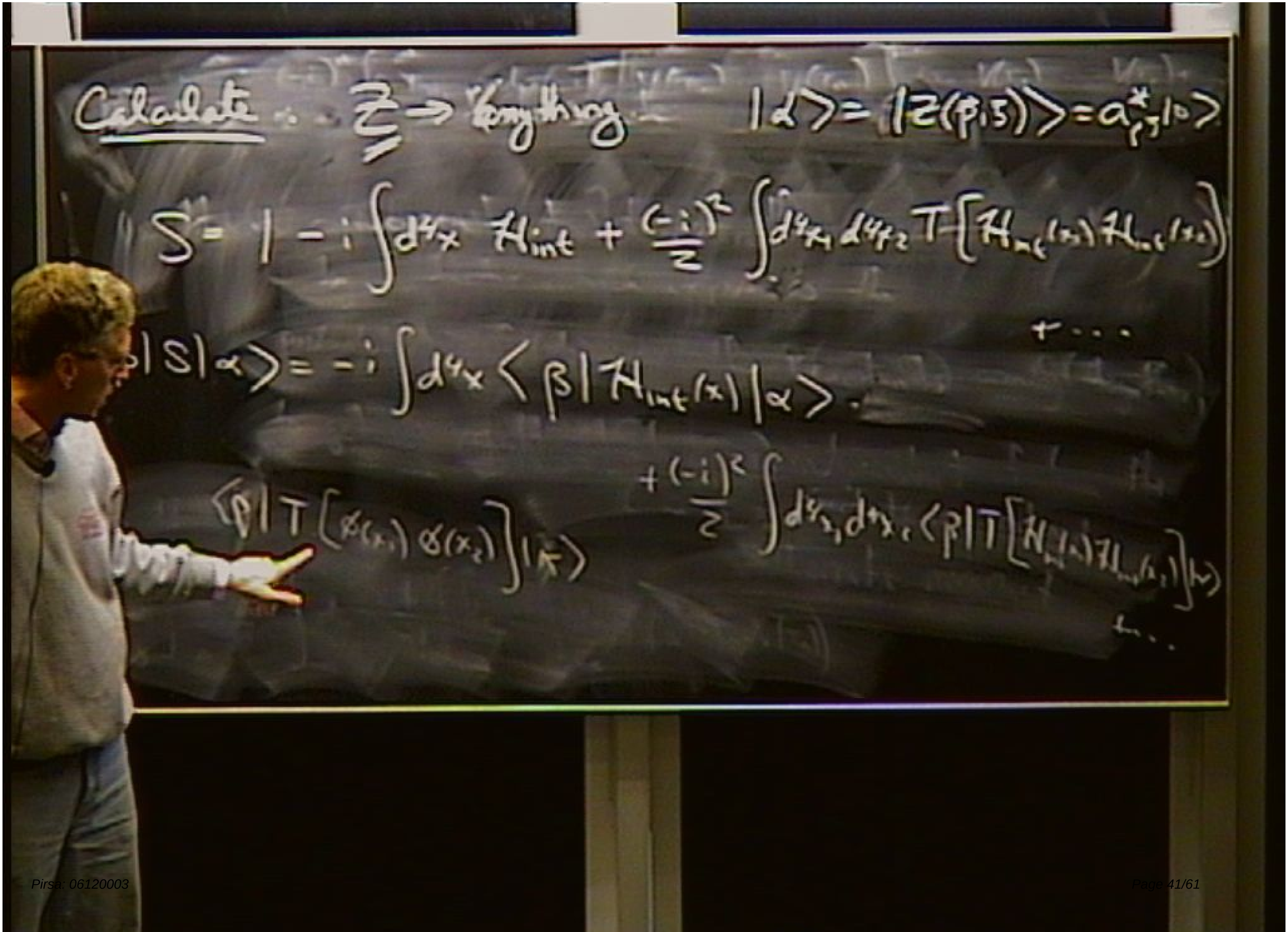
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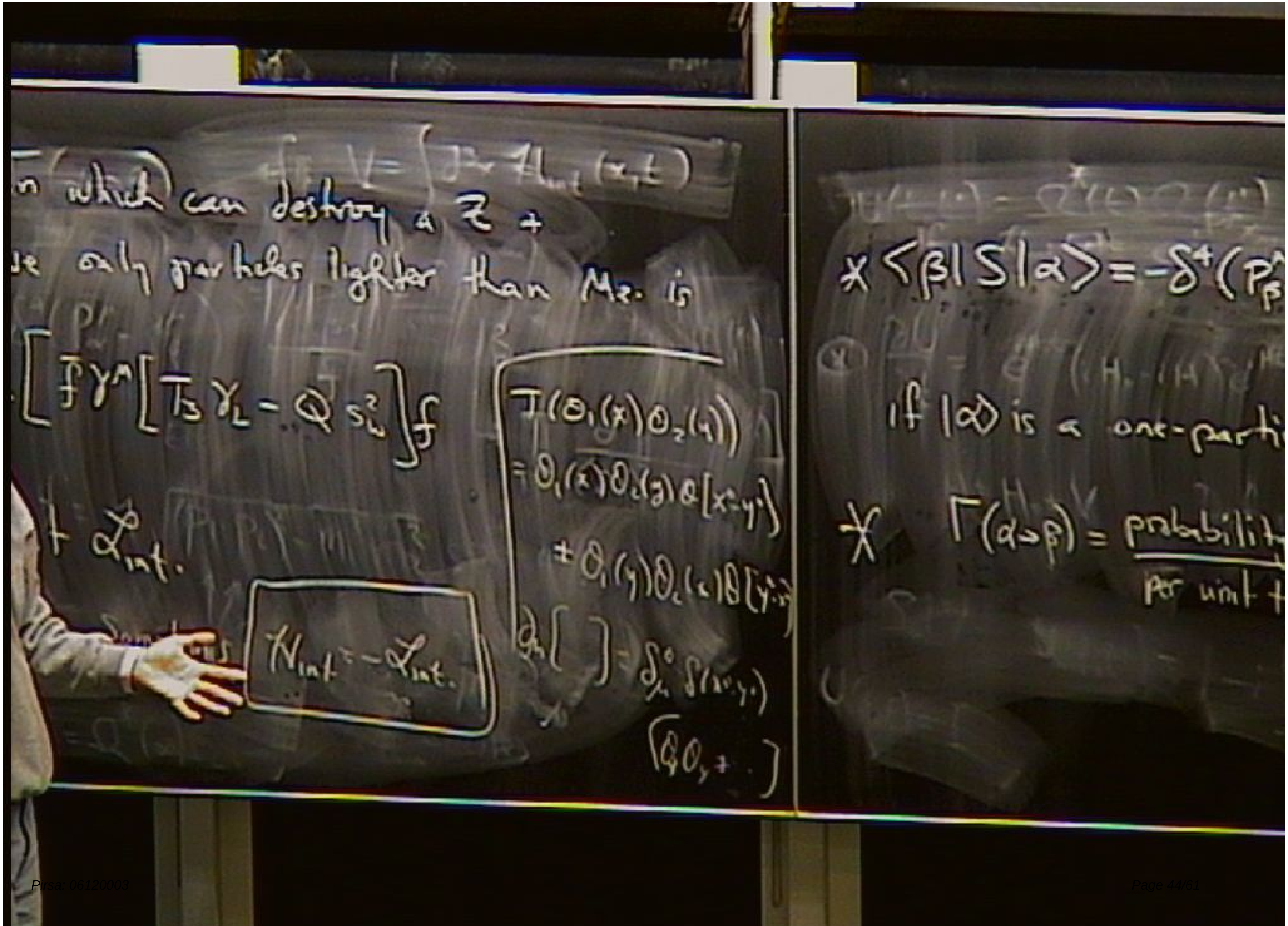
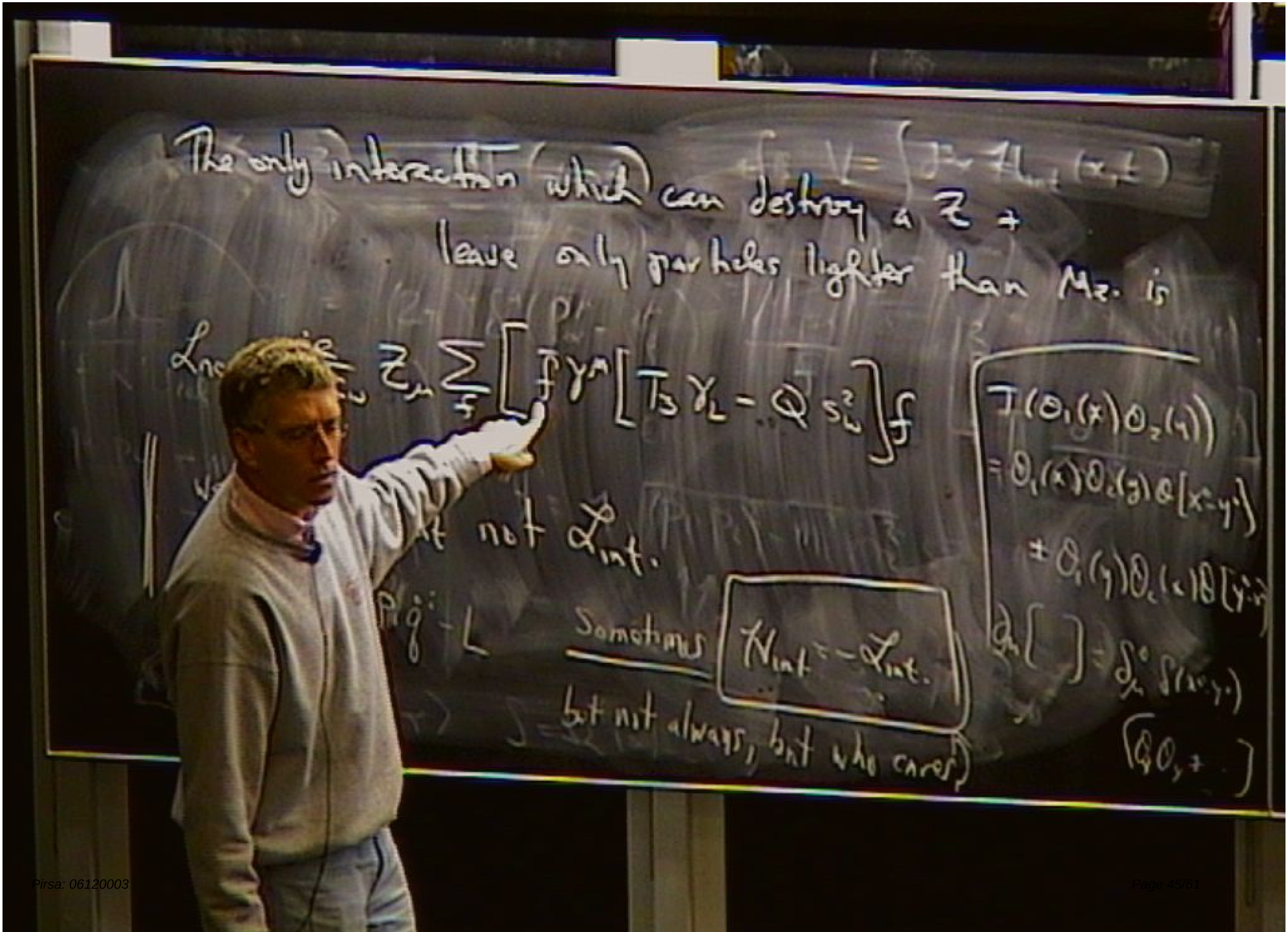
$$\partial_\mu \langle 0|T\left[\phi(x_1)\phi(x_2)\right]|0\rangle$$

$$\langle 0|T\left[\partial_\mu\phi(x_1)\phi(x_2)\right]|0\rangle$$



which can destroy a Z +
only particles lighter than M_Z is
$[\bar{f}\gamma^\mu[T_3\gamma_L - Q s_W^2]f$
$+ \mathcal{L}_{int.}$
$\mathcal{H}_{int} = -\mathcal{L}_{int.}$
$T(O_1(x)O_2(y)) = O_1(x)O_2(y)\theta[x^0-y^0] \pm O_1(y)O_2(x)\theta[y^0-x^0]$
$\partial_\mu[\ ] = \delta_\mu^0\,\delta(x^0-y^0)$
$[Q, O_y + \ ]$
$\langle\beta|S|\alpha\rangle = -\delta^4(P_\beta$
if $|\alpha\rangle$ is a one-parti
$\Gamma(\alpha\to\beta) = $ probability
per unit t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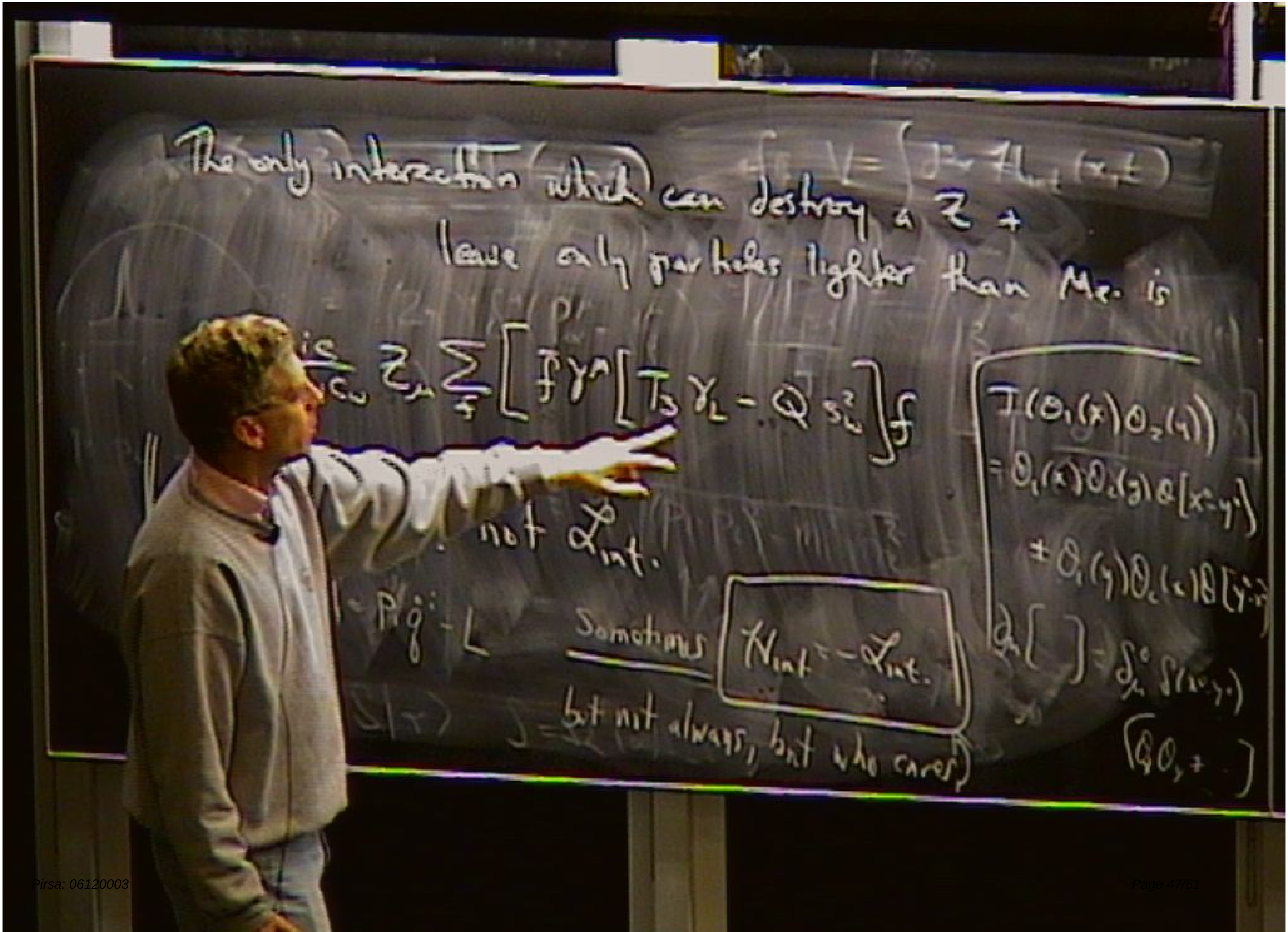
$$\parallel$$
$$\langle 0|T^{*}\left[\partial_\mu\phi(x_1)\,\phi(x_2)\right]|0\rangle$$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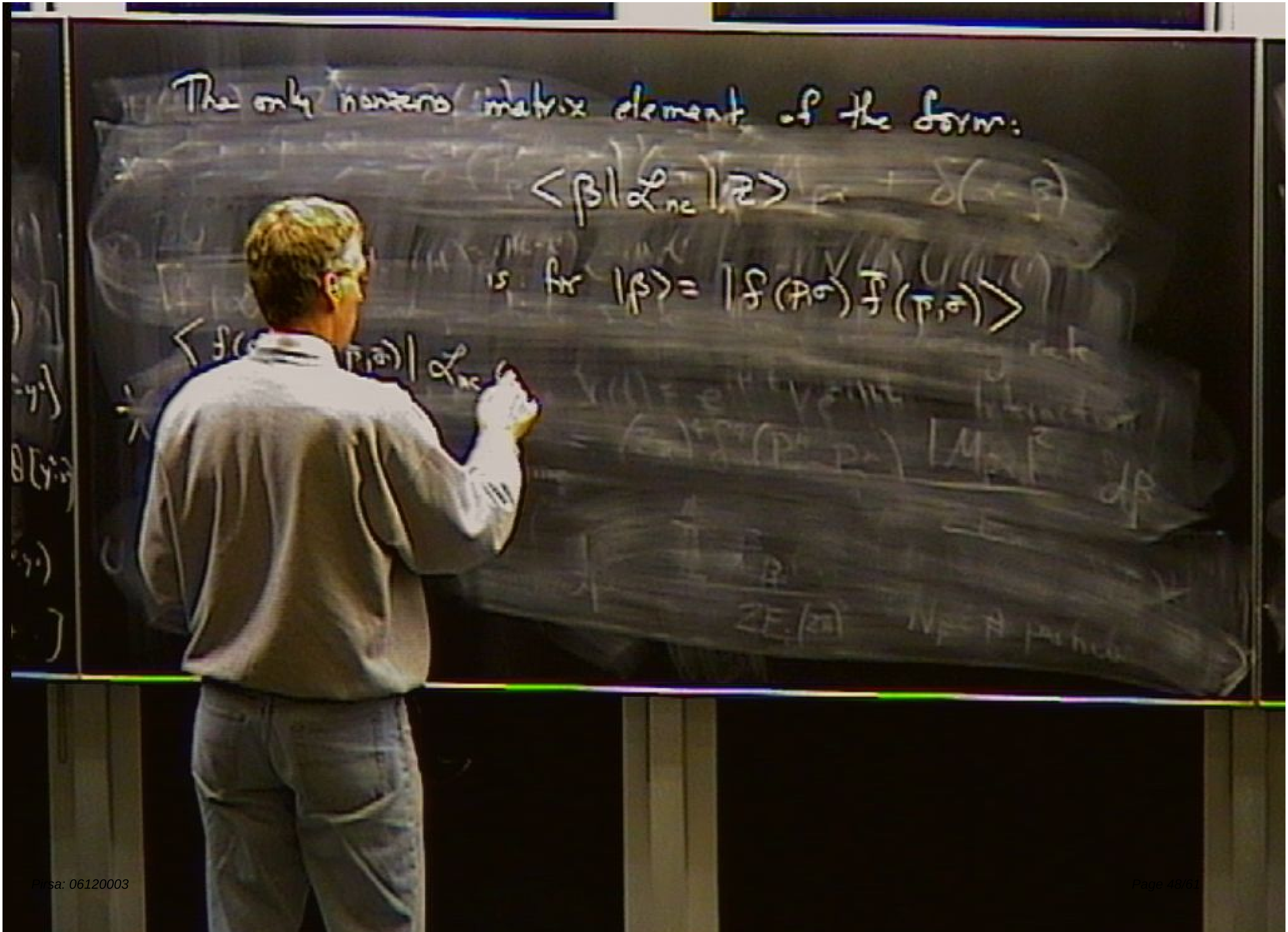
The only nonzero matrix element of the form:
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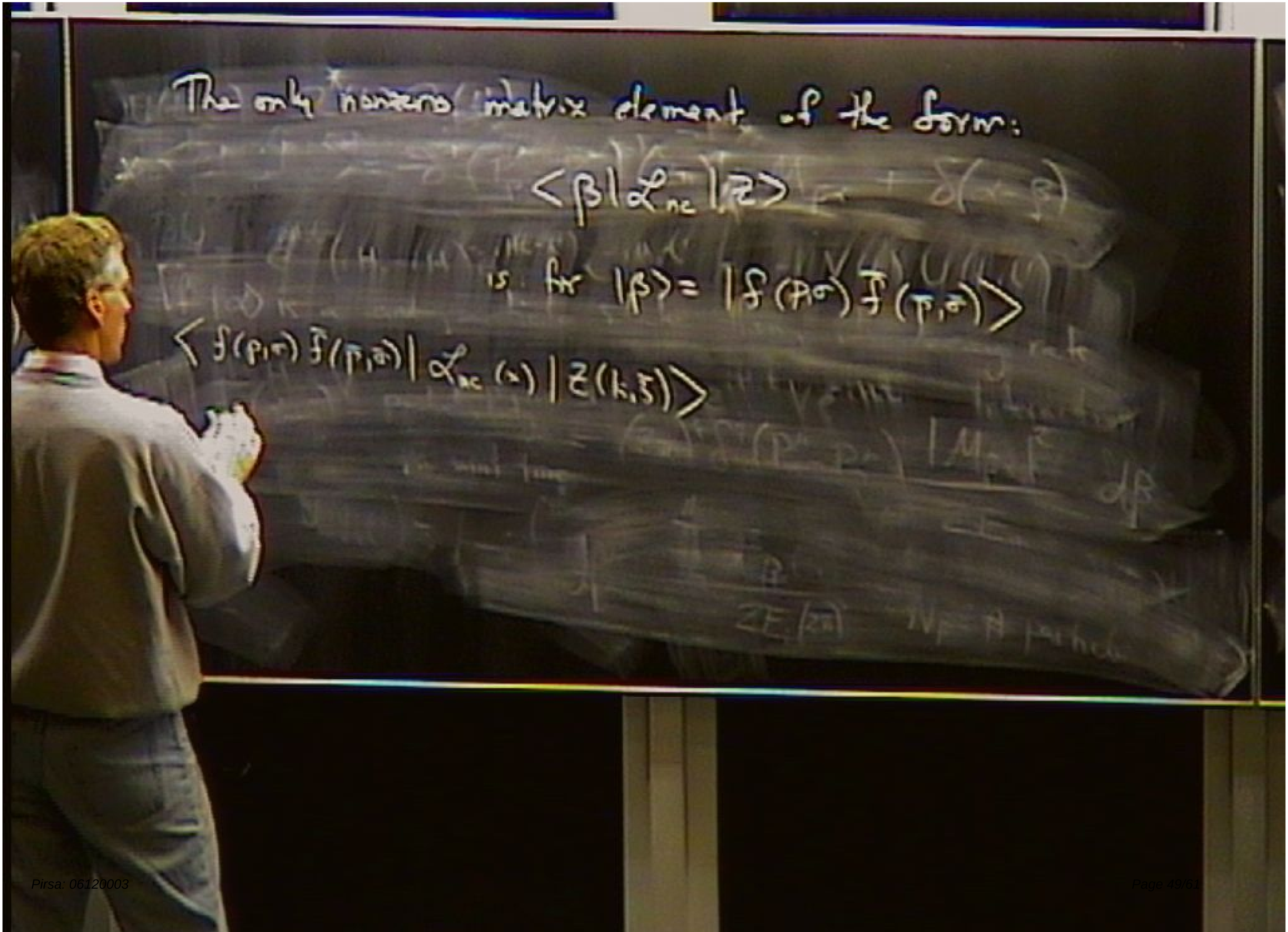
is for $|\beta\rangle = |f(\vec{p},\sigma)\,\bar{f}(\vec{p},\sigma)\rangle$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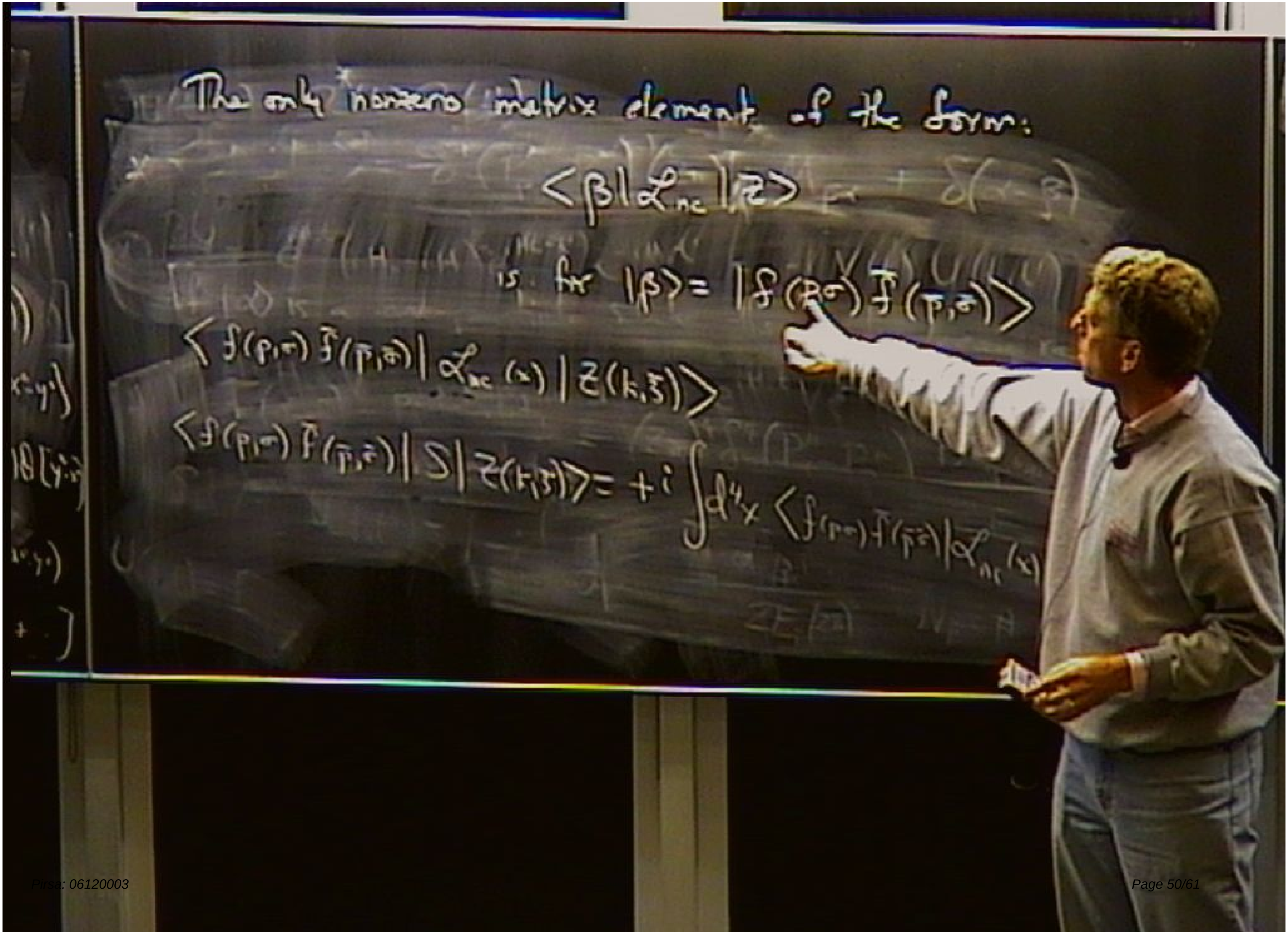
<f(p,σ) f̄(p̄,σ̄)| L_nc(x) |Z(k,ξ)>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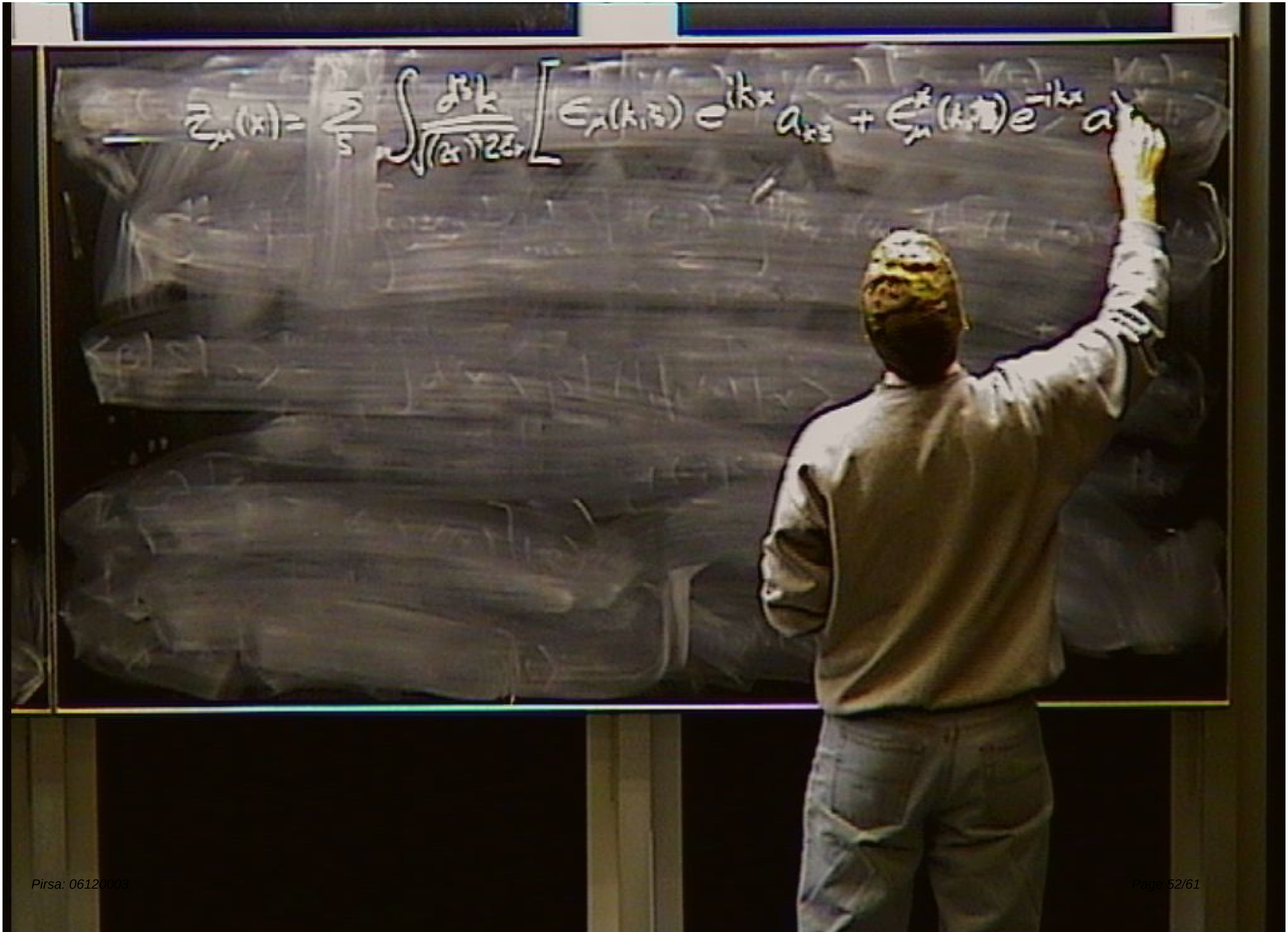
$$\langle f(\vec{p},\sigma)\,\bar{f}(\bar{\vec{p}},\bar{\sigma})|\,S\,|Z(\vec{k},\xi)\rangle = +i\int d^4x\,\langle f(\vec{p},\sigma)\,\bar{f}(\bar{\vec{p}},\bar{\sigma})|\,\mathcal{L}_{nc}(x)\,|Z(\vec{k},\xi)\rangle$$


$Z_\mu(x) = \sum_s \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} \Big[\epsilon_\mu(k,s)\, e^{ikx} a_{ks} + \epsilon^*_\mu(k,s)\, e^{-ikx} a$

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$$f(x) = \sum_\lambda \int d^3q \left[u(q,\lambda)\, e^{iqx} c_{q\lambda} + v(q,\lambda)\, e^{-iqx} \bar{c}^*_{q\lambda} \right]$$

$$\bar{f} \qquad \bar{u} \quad e^{-iqx} c^*_{q\lambda} \qquad \bar{v} \quad e^{iqx} \bar{c}_{q\lambda}$$

Check:

$$\langle f(p,r)\, \bar{f}(p',r') | \mathcal{L}_{NC}(x) | Z(k,s) \rangle = \frac{ie}{s_W c_W} \frac{\epsilon_\mu(k,s)}{\left((2\pi)^3 2E_k\right)^{1/2}}$$

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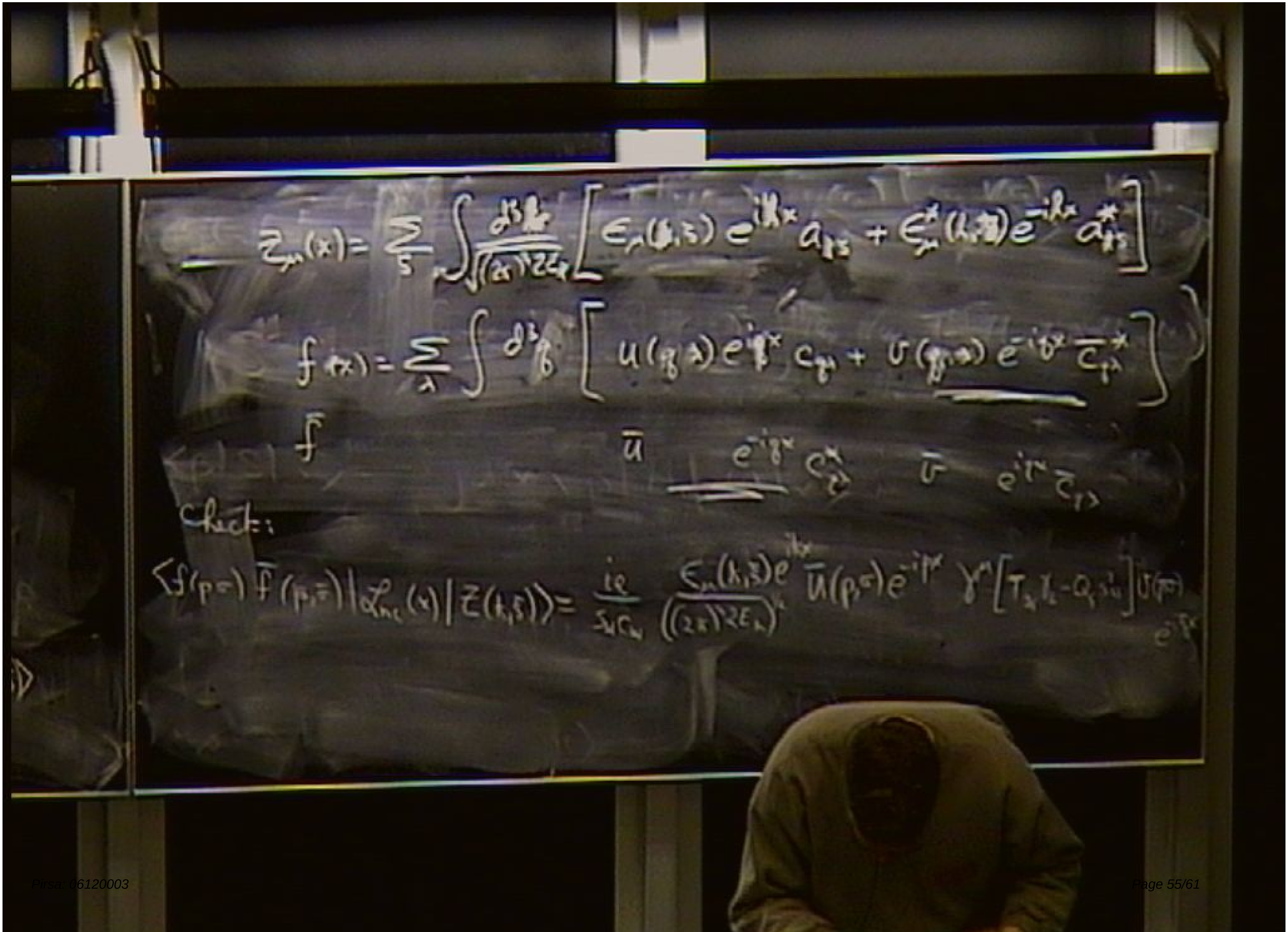
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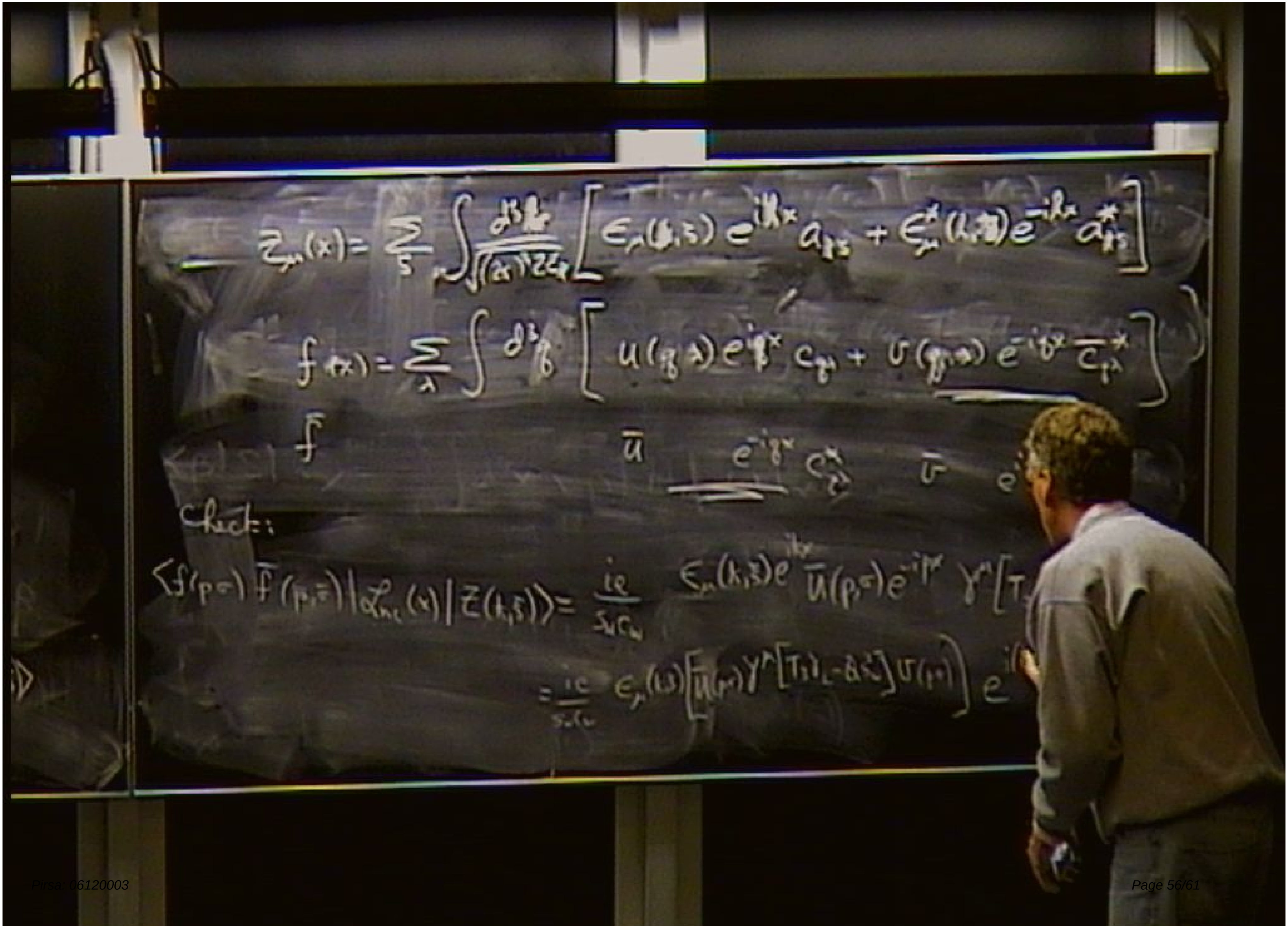
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$\bar{f}$ $\bar{u}$ $e^{-iqx} c^*_{q\lambda}$ $\bar{v}$ e
Check:
$\langle f(p,\sigma) \bar{f}(p,\sigma) | \mathcal{L}_{nc}(x) | Z(k,\xi) \rangle = \frac{ie}{s_W c_W} \epsilon_\mu(k,\xi) e^{ikx} \bar{u}(p,\sigma) e^{-ipx} \gamma^\mu [T$
$= \frac{ie}{s_W c_W} \epsilon_\mu(k,\xi) \left[\bar{u}(p) \gamma^\mu [T_3 \gamma_L - Q s_W^2] v(p) \right] e^{i}$

$$Z_\mu(x) = \sum_s \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} \left[\epsilon_\mu(k,s)\, e^{ikx}\, a_{ks} + \epsilon_\mu^*(k,s)\, e^{-ikx}\, a_{ks}^* \right]$$

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$$\int d^4x\, e^{i(k-p-p')x} = (2\pi)^4 \delta^4(k-p-p') \qquad = \frac{ie}{s_W c_W}\, \epsilon_\mu(k,s) \left[\bar{u}(p)\, \gamma^\mu \left[T_3 \gamma_L - Q s_W^2 \right] v(p') \right] e^{i(k-p-p')x}$$

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$$= -i(2\pi)^4 \, \delta^4(k-p-\bar{p})\, \mathcal{M}(Z \to f\bar{f})$$

S_0: $\mathcal{M} = -\frac{e}{s_W c_W} \epsilon_\mu(k,r) [\bar{u}(p,\sigma)$

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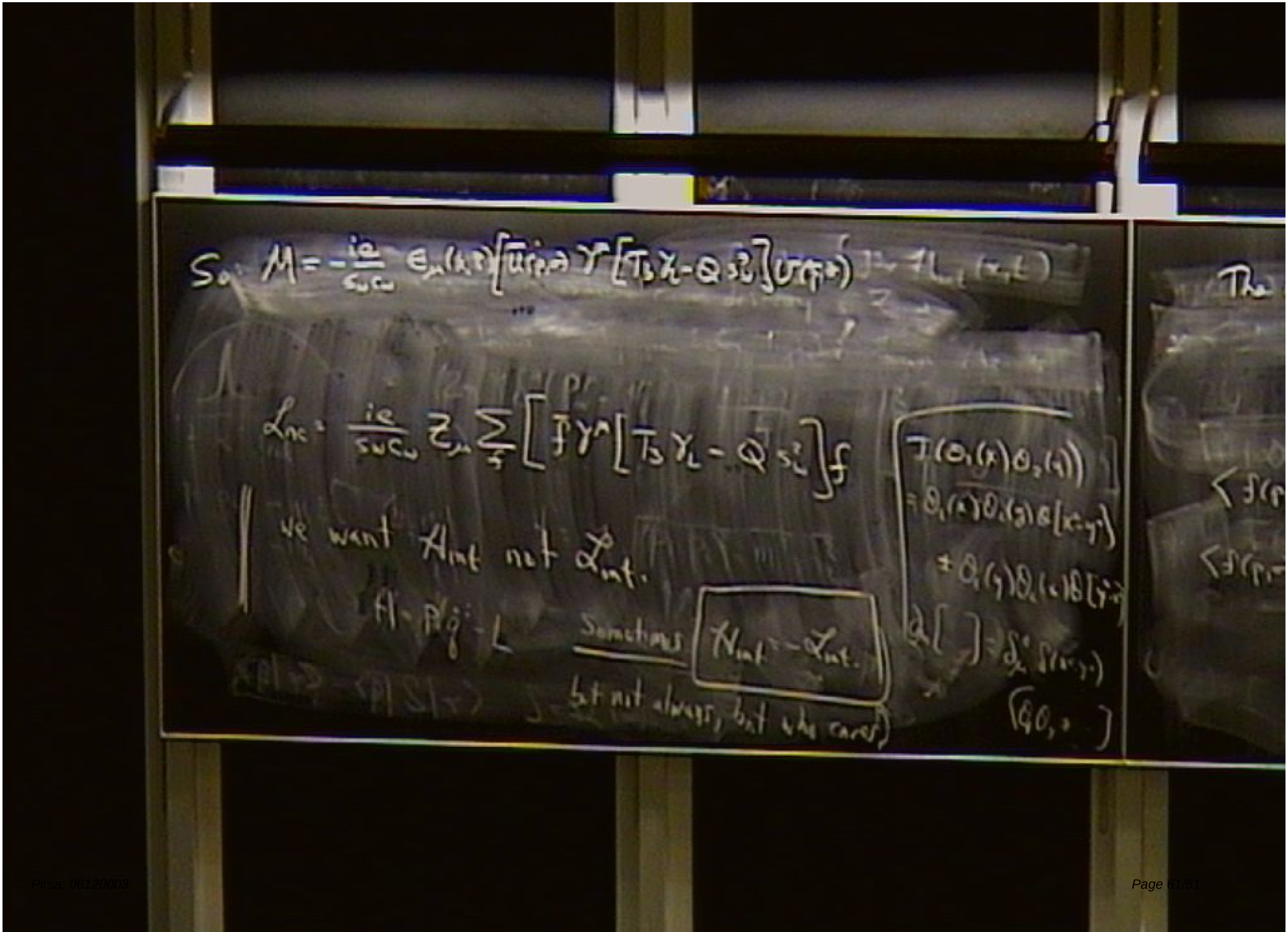
$[\partial O_1 +\ \ ]$



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