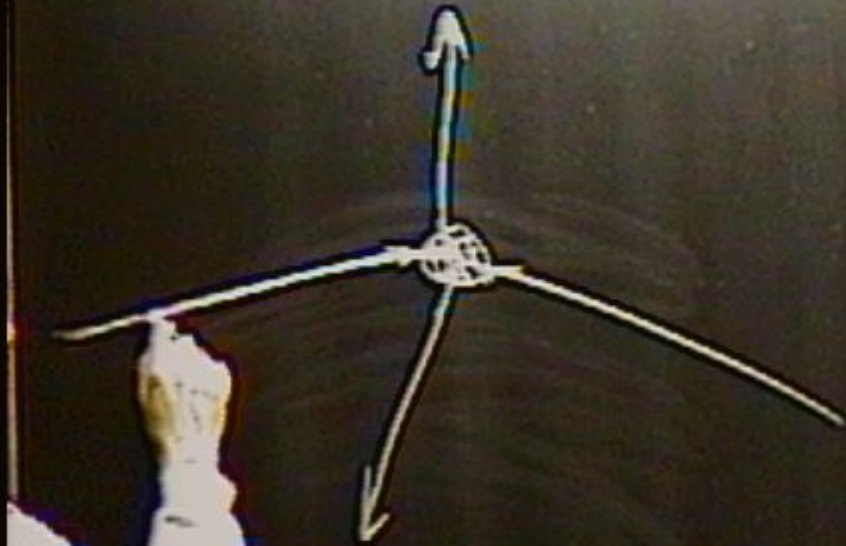


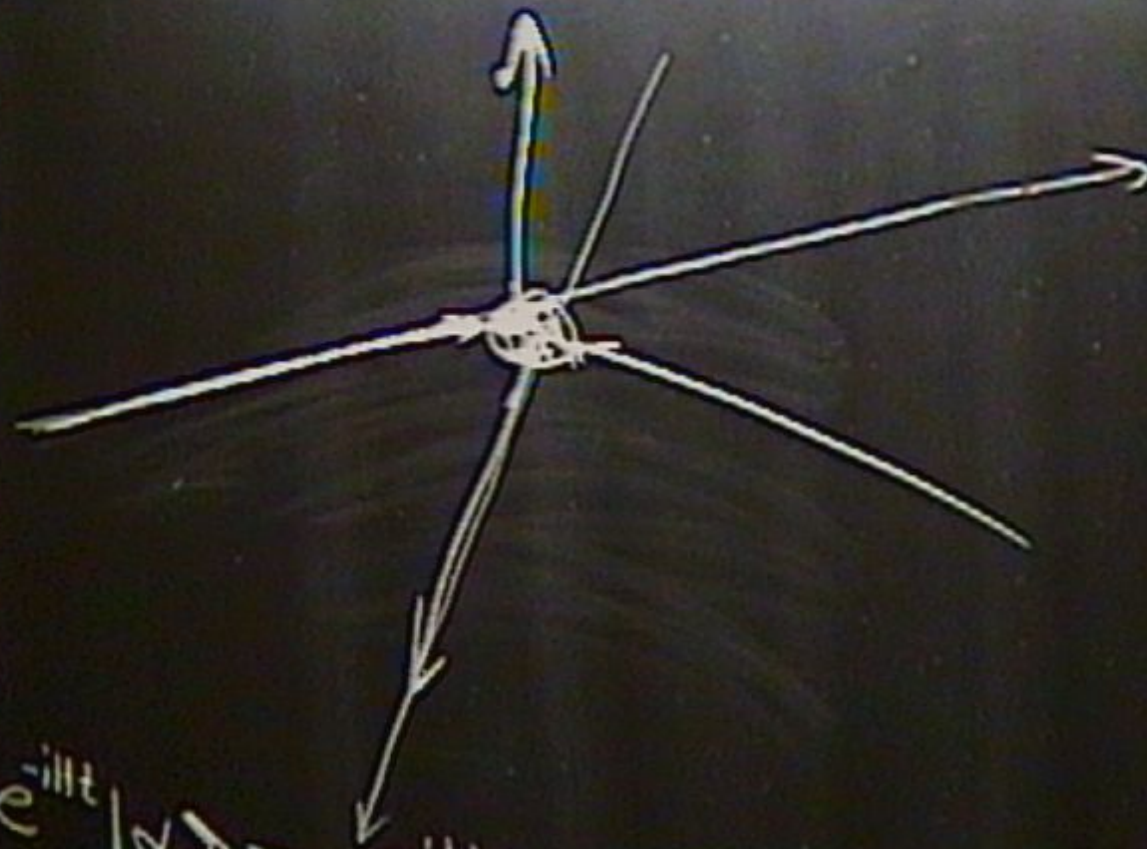
Title: Graduate Course on Standard Model & Quantum Field Theory - 8A

Date: Dec 06, 2006 11:00 AM

URL: <http://pirsa.org/06120003>

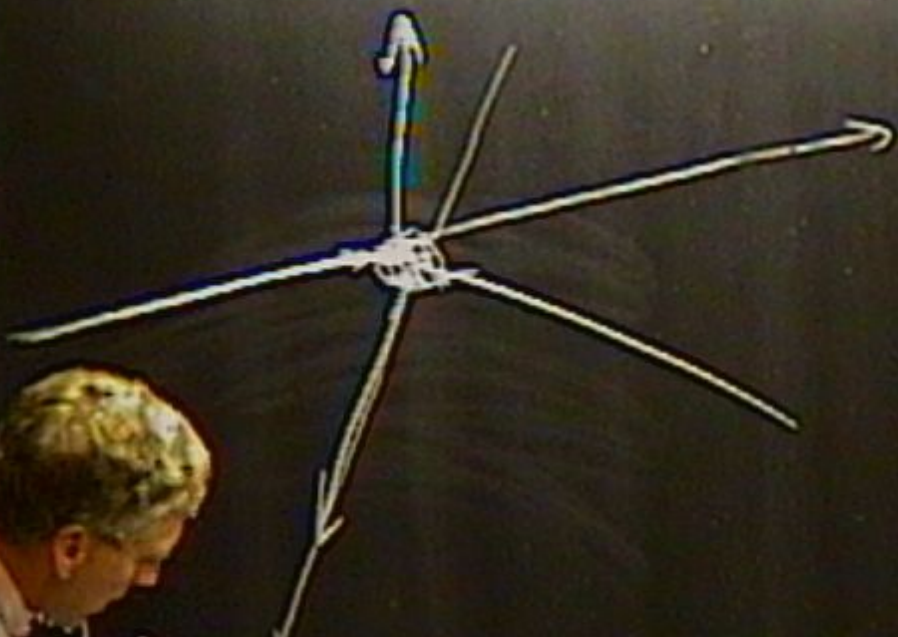
Abstract: Graduate Course on Standard Model & Quantum Field Theory





$$e^{-iHt} |\alpha\rangle = e^{-iHt} |\alpha\rangle \quad e^{-iHt} |\beta\rangle = e^{-iHt} |\beta\rangle$$





$$|\alpha\rangle_t = e^{-iHt} |\alpha\rangle$$

$$e^{iA} |\beta\rangle_0 = e^{-iHt} |\beta\rangle$$

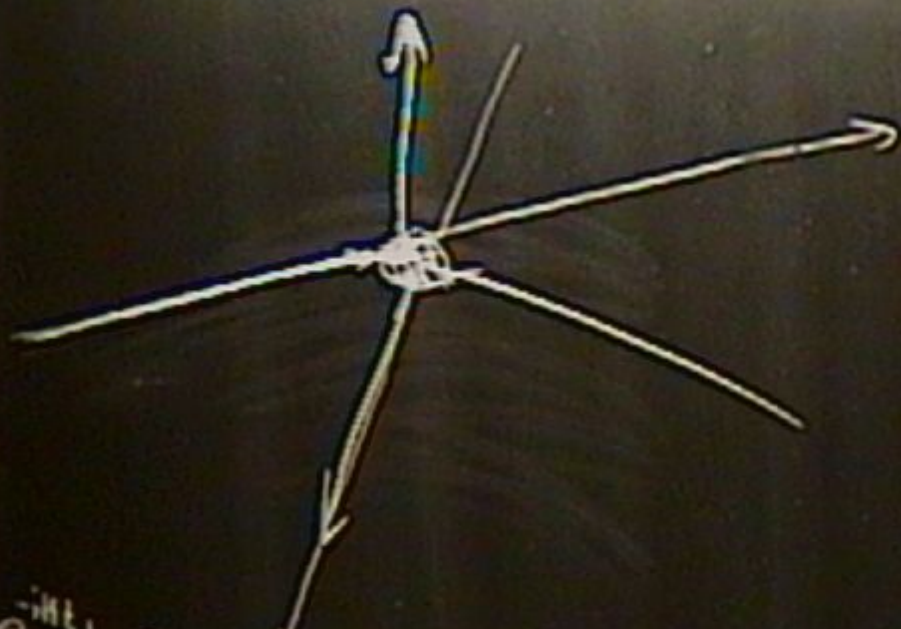
$$\langle \beta | \alpha \rangle$$

$$\Omega = e^{iHt} e^{-iHt}$$

$$|\alpha\rangle_t = \Omega(t) |\alpha\rangle$$

$$|\alpha\rangle_0 = \Omega(0) |\alpha\rangle$$





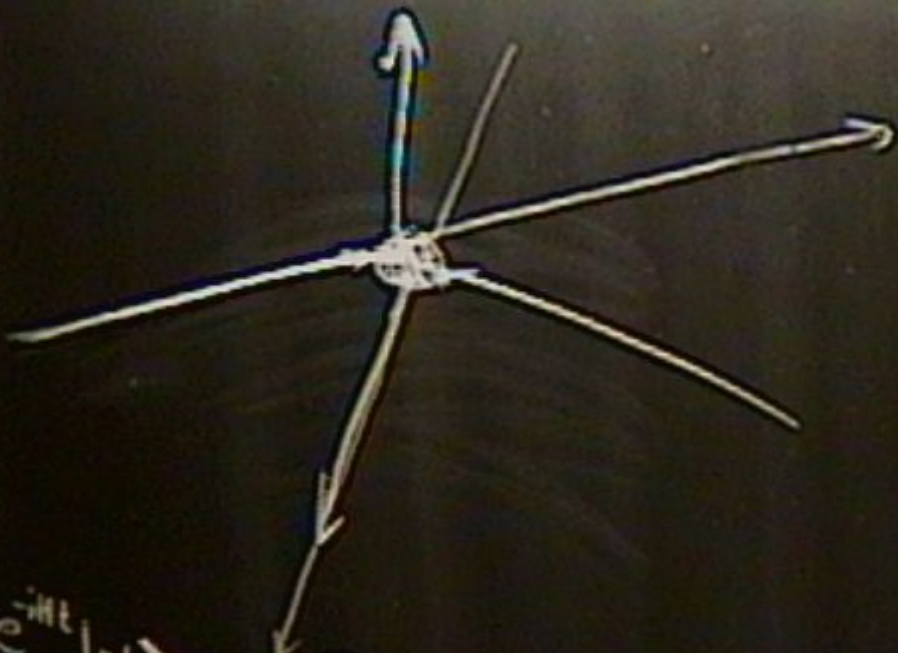
$$e^{-iHt} |\alpha\rangle_t = e^{-iHt} |\alpha\rangle \quad e^{iA\tau} |\beta\rangle_0 = e^{iH\tau} |\beta\rangle$$

$$\langle \beta | \alpha \rangle_t = \langle \beta | S | \alpha \rangle \quad S = \Omega^\dagger(\infty) \Omega(-\infty)$$

$$\Omega = e^{iHt} e^{-iHt}$$

$$|\alpha\rangle_t = \Omega(t) |\alpha\rangle$$

$$|\alpha\rangle_t = \Omega(t) |\alpha\rangle$$



$$e^{-iHt} |\alpha\rangle = e^{-iH_0 t} |\alpha\rangle \quad e^{iH_0 t} |\alpha\rangle = e^{-iH_0 t} |\alpha\rangle$$

$$\langle \beta | \psi \rangle = \langle \beta | S | \psi \rangle \quad S = \Omega^\dagger(\infty) \Omega(-\infty)$$

$$\Omega = e^{iH_0 t} e^{-iH t}$$

$$|\alpha\rangle = \Omega(t) |\psi\rangle$$

$$|\psi\rangle = \Omega(-\infty) |\alpha\rangle$$





$$U(t, t') = \Omega^*(t) \Omega(t')$$

$$= e^{iH_0 t} e^{-iH t} e^{iH t'} e^{-iH_0 t'}$$

$$\frac{\partial U}{\partial t} = e^{iH_0 t} (iH_0 - iH) e^{iH(t-t')} e^{-iH_0 t'}$$

$$H =$$

$$e^{iH_0 t} e^{-iH_0 t}$$

$$\Omega(t) |\psi\rangle$$

$$\Omega(t) |\psi\rangle$$

$$U(t, t') = \Omega^*(t) \Omega(t')$$

$$= e^{iH_0 t} e^{-iH_0 t} e^{iH_0 t'} e^{-iH_0 t'}$$

$$\frac{\partial U}{\partial t} = e^{iH_0 t} \underbrace{(iH_0 - iH)}_{-iV} e^{-iH_0 t'} e^{-iH_0 t'} = -iV(t) U(t, t')$$

$$H = H_0 + V$$

$$\text{Define } V(t) = e^{iH_0 t} V e^{-iH_0 t}$$

Interaction  
Rep<sup>n</sup> of V.



$$U(t, t') = \Omega^*(t) \Omega(t')$$

$$= e^{iH_0 t} e^{-iH_0 t} e^{iH_0 t'} e^{-iH_0 t'}$$

$$\textcircled{v} \frac{\partial U}{\partial t} = e^{iH_0 t} (iH_0 - iH) e^{-iH_0 t'} e^{-iH_0 t'} = -iV(t) U(t, t')$$

$-iV$

$$H = H_0 + V$$

Define  $V(t) = e^{iH_0 t} V e^{-iH_0 t}$

Interaction  
Rep<sup>n</sup> of V.

Sol<sup>n</sup>:  $U(t, t') = 1 - i \int_{t'}^{t'} d\tau V(\tau) U(\tau, t') + (-i)^2 \int_{t'}^{t'} d\tau_1 \int_{t'}^{\tau_1} d\tau_2 V(\tau_1) V(\tau_2) U(\tau_2, t')$

$U(t, t) = I$



$$U(t, t') = \Omega^*(t) \Omega(t')$$

$$= e^{iH_0 t} e^{-iH t} e^{iH t'} e^{-iH_0 t'}$$

$$\textcircled{2} \frac{\partial U}{\partial t} = e^{iH_0 t} (iH_0 - iH) e^{-iH(t-t')} e^{-iH_0 t'} = -i V(t) U(t, t')$$

$$H = H_0 + V$$

$$\text{Define } V(t) = e^{iH_0 t} V e^{-iH_0 t}$$

Interaction  
Rep<sup>n</sup> of V.

Sol<sup>n</sup>:  $U(t, t') = 1 - i \int_{t'}^{t} d\tau V(\tau) U(t, \tau) + (-i)^2 \int_{t'}^{t} d\tau_1 \int_{t'}^{\tau_1} d\tau_2 V(\tau_1) V(\tau_2) U(t, \tau_2) + \dots$

$$U(t, t) = I$$



$$+ \dots (-i)^n \int_{t_1}^t dt_1 \dots \int_{t_1}^{\tau_{n-1}} d\tau_{n-1} \dots V(\tau_1) \dots V(\tau_n) \dots + \dots$$

$$S = 1 - i \int_{-\infty}^{\infty} dt V(t) + \dots + (-i)^n \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \dots \int_{-\infty}^{t_{n-1}} dt_n V(t_1) \dots V(t_n) + \dots$$

$$+ \dots + (-i)^n \int_{t_1}^L dt_1 \dots \int_{t_1}^{t_{n-1}} dt_n V(t_1) \dots V(t_n) + \dots$$



$$S = 1 - i \int_{-\infty}^{\infty} dt V(t) + \dots + (-i)^n \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\tau_1} dt_2 \dots \int_{-\infty}^{\tau_{n-1}} dt_n V(t_1) \dots V(t_n) + \dots$$

Use identity:  $T[V(\tau_1) \dots V(\tau_n)] = V(\tau_{n+1}) V(\tau_{n+2}) \dots V(\tau_{n+1})$

$$= \sum_{\mathcal{P}} V(\tau_{\mathcal{P}_1}) \dots V(\tau_{\mathcal{P}_n}) \theta(\tau_{\mathcal{P}_1} - \tau_{\mathcal{P}_2}) \dots \theta(\tau_{\mathcal{P}_{n-1}} - \tau_{\mathcal{P}_n})$$

$$\theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Identity:

~~$$T \int_{-\infty}^{\infty} dt V(t) = \int_{-\infty}^{\infty} dt V(t) = V(t) V(t) = \dots$$~~

$$S = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_{-a}^a \int_{-a}^{\tau_1} \dots \int_{-a}^{\tau_{n-1}} d\tau_n \dots d\tau_1 T[V(\tau_1) \dots V(\tau_n)]$$

Use identity:  $T[V(\tau_1) \dots V(\tau_n)] = V(\tau_{(1)}) V(\tau_{(2)}) \dots V(\tau_{(n)})$

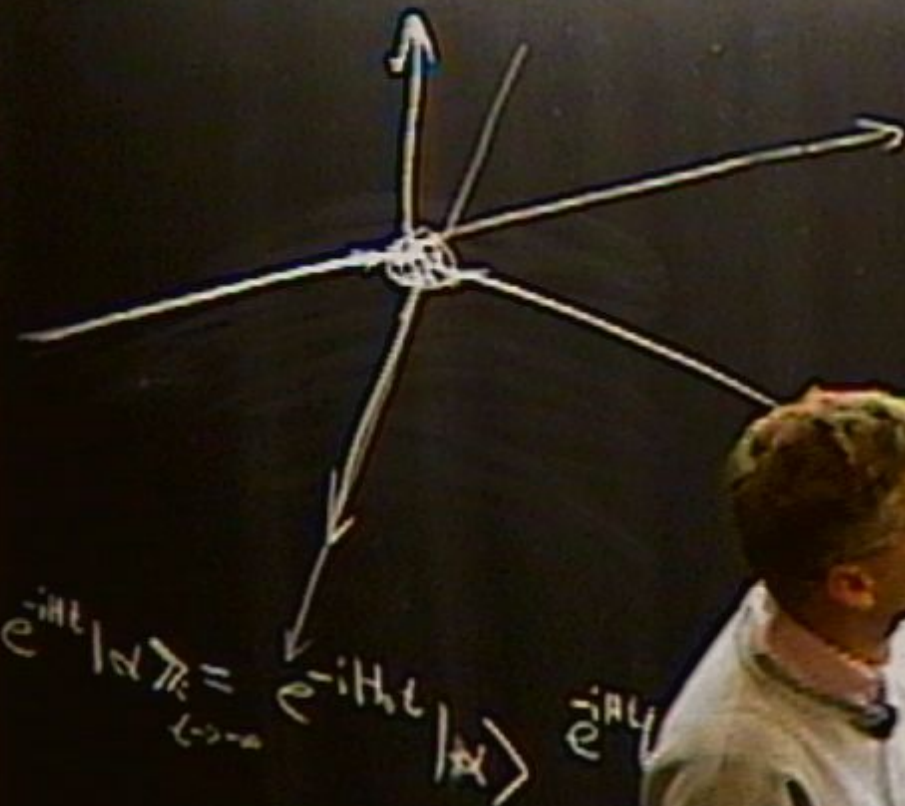
$$= \sum_{\mathcal{P}} V(\tau_{n_1}) \dots V(\tau_{n_r}) \theta(\tau_{n_1} - \tau_{n_2}) \dots \theta(\tau_{n_{r-1}} - \tau_{n_r})$$

$$\theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Ren Identity:

$$\int_{-a}^a d\tau_1 \int_{-a}^{\tau_1} d\tau_2 \dots \int_{-a}^{\tau_{n-1}} d\tau_n = \frac{1}{n!} \int_{-a}^a d\tau_1 \dots \int_{-a}^a d\tau_n T(V(\tau_1) \dots V(\tau_n))$$





$$\text{if } V = \int d^3x \mathcal{H}_{int}(x, t)$$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \dots d^4x_n T \left[ \mathcal{H}_{int}(x_1) \dots \mathcal{H}_{int}(x_n) \right]$$

$$\Omega = e^{iHt} e^{-iHt}$$

$$|\alpha\rangle_t = \Omega(t) |\alpha\rangle$$

$$|\alpha\rangle_i = \Omega(i) |\alpha\rangle$$

$$e^{-iHt} |\alpha\rangle_t = e^{-iHt} |\alpha\rangle e^{iHt}$$

$$\langle \beta | \alpha \rangle_t = \langle \beta | S | \alpha \rangle$$

$$U(t,0) = \Omega(t) \Omega(0)$$

If  $H_{int}(x) = e^{i p x} H_{int}(0) e^{-i p x}$

(3)  
[8]

$$\textcircled{x} \frac{\partial U}{\partial t} = \frac{\partial}{\partial t} (H_0 - (H_0 - V) e^{-i p x}) e^{i p x} U(t,0) = -i V(t) U(t,0)$$

$H_0 = H_0 + V$       Define  $V(t) = e^{i p x} V e^{-i p x}$

Interaction  
Rep V, At V

$$U(t,0) = T \left[ e^{-i \int_0^t V(t') dt'} \right] U(0,0)$$



$$\text{if } V = \int d^3x \mathcal{H}_{int}(x, t)$$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \dots d^4x_n T[\mathcal{H}_{int}(x_1) \dots \mathcal{H}_{int}(x_n)]$$

$$\Omega = e^{iH_0 t} e^{-iH t}$$

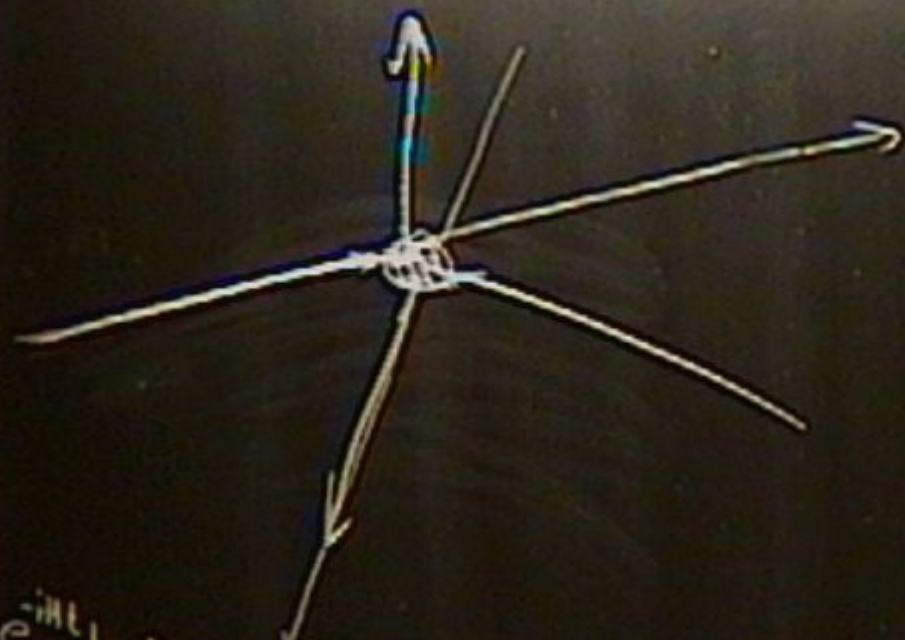
$$|\psi\rangle = \Omega(t) |\psi\rangle$$

$$|\psi\rangle = \Omega(t) |\psi\rangle$$

$$U(t, t') = \Omega^\dagger(t) \Omega(t')$$

$$\text{if } \mathcal{H}_{int}(x) = e^{i p x} \mathcal{H}_{int}(0) e^{-i p x}$$

*(Faint, mostly illegible handwritten notes on the right chalkboard, including some equations and diagrams.)*



$$\text{if } V = \int d^3x \mathcal{H}_{int}(x, t)$$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \dots d^4x_n T[\mathcal{H}_{int}(x_1) \dots \mathcal{H}_{int}(x_n)]$$

$$\Omega = e^{iHt} e^{-iHt}$$

$$e^{iHt} |\alpha\rangle_t = e^{-iHt} |\alpha\rangle \quad e^{iHt} |\beta\rangle_0 = e^{-iHt} |\beta\rangle$$

$$\langle \beta | \alpha \rangle_t = \langle \beta | S | \alpha \rangle \quad S = \Omega^\dagger(\infty) \Omega(-\infty)$$

$$|\alpha\rangle_t = \Omega(t) |\alpha\rangle$$

$$|\alpha\rangle_i = \Omega(i) |\alpha\rangle$$







of  $V = \text{span}\{v_1, v_2, \dots, v_n\}$

is a subspace of  $V$  if and only if it is closed under addition and scalar multiplication.

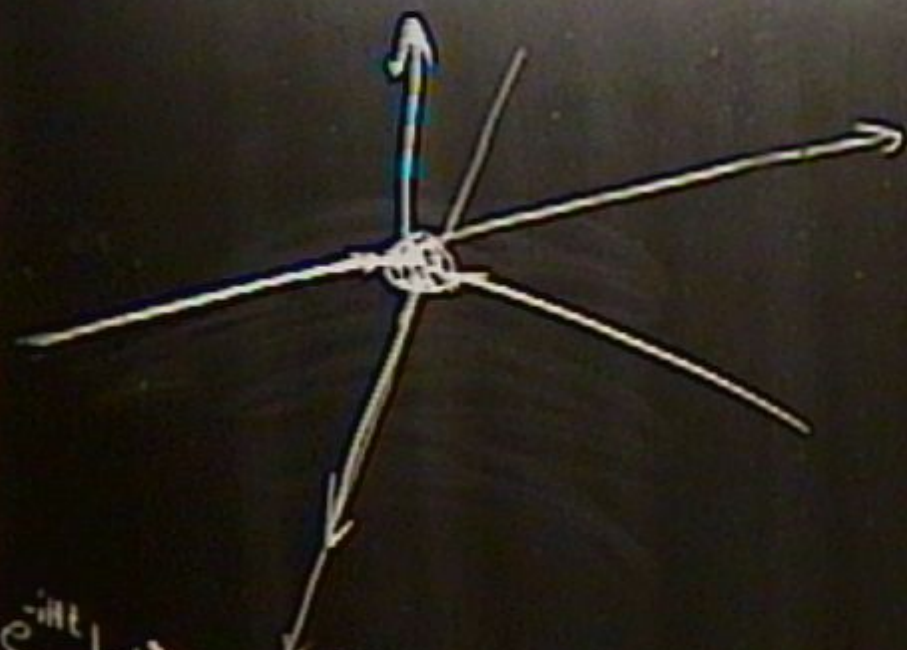
Let  $W$  be a subspace of  $V$ . Then  $W$  is a vector space in its own right, with the same operations as  $V$ .

Example:

$W = \{0\}$

$W = V$





if  $V = \int d^3x \mathcal{H}_{int}(x, t)$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \dots d^4x_n T[\mathcal{H}_{int}(x_1) \dots \mathcal{H}_{int}(x_n)]$$

$$\Omega = e^{iHt} e^{-iHt}$$

$$e^{-iHt} |\alpha\rangle_t = e^{-iHt} |\alpha\rangle \quad e^{iHt} |\beta\rangle_0 = e^{-iHt} |\beta\rangle$$

$$|\alpha\rangle_t = \Omega(t) |\alpha\rangle$$

$$|\beta\rangle_0 = \Omega(-\infty) |\beta\rangle$$

$$\langle \beta | \alpha \rangle_t = \langle \beta | S | \alpha \rangle \quad S = \Omega^\dagger(\infty) \Omega(-\infty)$$

$$\langle \beta | S | \alpha \rangle = -\delta^4(\mathbf{P}_\beta - \mathbf{P}_\alpha) (2\pi)^4 M_{\beta\alpha} + \delta(\alpha - \beta)$$

if  $|\alpha\rangle$  is a one-particle state then, its decay rate



$$\langle \beta | S | \alpha \rangle = -\delta^4(P_\beta - P_\alpha) (2\pi)^4 M_{\beta\alpha} + \delta(\alpha - \beta)$$

if  $|\alpha\rangle$  is a one-particle state then, its decay rate

$$\Gamma(\alpha \rightarrow \beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4 \delta^4(P_\alpha - P_\beta) \frac{|M_{\beta\alpha}|^2}{2E_\alpha} d\beta$$

$$d\beta = \pi \frac{d^3 p_i}{2E_i}$$

$$\langle \beta | S | \alpha \rangle = -\delta^4(P_\beta - P_\alpha) (2\pi)^4 i M_{\beta\alpha} + \delta(\alpha - \beta)$$

if  $|\alpha\rangle$  is a one-particle state then, its decay rate

$$\Gamma(\alpha \rightarrow \beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4 \delta^4(P_\alpha - P_\beta) \frac{|M_{\beta\alpha}|^2}{2E_\alpha} d\beta$$

$$d\beta = \frac{\pi^3 d^3 p_i}{2E_i (2\pi)^3}$$



$$\langle \beta | S | \alpha \rangle = -\delta^4(P_\beta - P_\alpha) (2\pi)^4 i M_{\beta\alpha} + \delta(\alpha - \beta)$$

if  $|\alpha\rangle$  is a one-particle state then, its decay rate

$$\Gamma(\alpha \rightarrow \beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4 \delta^4(P_\alpha - P_\beta) \frac{|M_{\beta\alpha}|^2}{2E_\alpha} d\beta$$

$$d\beta = \prod_i \frac{d^3 p_i}{2E_i (2\pi)^3}$$

$$S = \sum_{n=0}^{\infty} \frac{(t_0)^n}{n!} \int_{-a}^{t_0} \int_{-a}^{\tau_1} \dots \int_{-a}^{\tau_{n-1}} T[V(\tau_1) \dots V(\tau_n)] d\tau_1 \dots d\tau_n$$

Use identity:  $T[V(\tau_1) \dots V(\tau_n)] = V(\tau_{(n)}) V(\tau_{(n-1)}) \dots V(\tau_{(1)})$

$$= \sum_P V(\tau_{p_1}) \dots V(\tau_{p_n}) \theta(\tau_{p_1} - \tau_{p_2}) \dots \theta(\tau_{p_{n-1}} - \tau_{p_n})$$

$$\theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then Identity:

$$\int_{-a}^{t_0} \int_{-a}^{\tau_1} \int_{-a}^{\tau_2} \dots \int_{-a}^{\tau_{n-1}} d\tau_1 \dots d\tau_n = \frac{1}{n!} \int_{-a}^{t_0} \int_{-a}^{\tau_1} \dots \int_{-a}^{\tau_{n-1}} T(V(\tau_1) \dots V(\tau_n))$$



If  $|\alpha\rangle$  is a 2-particle state:

It is convenient to divide the reaction rate by a factor which expresses the probability of having the reactants coming together, to get a result which is:

- 1) independent of how the reactants are combined
- 2) Lorentz invariant

$$* \langle \beta | S | \alpha \rangle = -\delta^4(P_\beta - P_\alpha) (2\pi)^4 M_{\beta\alpha} + \delta(\alpha - \beta)$$

if  $|\alpha\rangle$  is a one-particle state then, its decay rate

$$* \Gamma(\alpha \rightarrow \beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4 \delta^4(P_\alpha - P_\beta) \frac{|M_{\beta\alpha}|^2}{2E_\alpha} d\beta$$

$$d\beta = \frac{V_0}{(2\pi)^3} \frac{d^3 P_\beta}{2E_\beta} = N_\beta = \# \text{ particles in } |\beta\rangle$$



$$d\sigma(\alpha \rightarrow \beta) = \frac{d\Gamma(\alpha \rightarrow \beta)}{F}$$

$$H = V = \int d^3x \mathcal{L}_{int}(\psi, \psi^\dagger)$$

$$= (2\pi)^4 \delta^4(P_\alpha - P_\beta) \frac{|M_{\alpha\beta}|^2}{f}$$

$$f \propto |U_{cd}|^2$$

$$d\beta$$

$$\hookrightarrow \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

$$f = 4 \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}$$

$$\langle \alpha | \hat{p} | \beta \rangle = \langle \alpha | S | \beta \rangle$$

$$J = Q^\dagger \alpha Q$$

$$d\sigma(\alpha \rightarrow \beta) = \frac{d\Gamma(\alpha \rightarrow \beta)}{F}$$

$$= (2\pi)^4 \delta^4(P_\alpha - P_\beta) \frac{|M_{\alpha\beta}|^2}{f}$$

$$f = V \int d^3x \dots$$

$d\beta$

$$\int \prod_{i=1}^{n_f} \frac{d^3p_i}{(2\pi)^3 2E_i}$$

$$f = 4 \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}$$

↑↑  
2 initial particles

$\propto \sqrt{s}$

$\alpha \propto \sqrt{s}$

$p_z = -m_1 E_2$  (in rest frame of 1)

$-m_1 m_2$

$\frac{1}{1 - \beta^2}$



$$d\sigma(\alpha \rightarrow \beta) = \frac{d\Gamma(\alpha \rightarrow \beta)}{F}$$

$$F = \int d^3x d^3p_i d^3p_f \delta^4(x_f - x_i)$$

$$= (2\pi)^4 \delta^4(p_f - p_i) \frac{|M_{\beta\alpha}|^2}{f}$$

$$d\beta \rightarrow \prod_{i=1}^{N_f} \frac{d^3p_i}{(2\pi)^3 2E_i}$$

$f \propto v_{rel}$

$\underline{p}_1 \cdot \underline{p}_2 = -m_1 E_2$  (in rest frame of 1)

$= -m_1 m_2 \sqrt{1 - v_{rel}^2}$

$$f = 4 \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}$$

$\uparrow \uparrow$   
2 initial particles

$$\propto v_{rel}$$

$$d\sigma(\alpha \rightarrow \beta) = \frac{d\Gamma(\alpha \rightarrow \beta)}{F}$$

$$F = \int d^3x d^3y \delta^4(x-y)$$

$$= (2\pi)^4 \delta^4(p_\alpha - p_\beta) \frac{|M_{\beta\alpha}|^2}{f}$$

$d\beta$

$$\int \prod_{i=1}^{N_\beta} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

$\propto v_{rel}$

$$f = 4 \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}$$

↑↑  
2 initial particles

$v \propto v_{rel}$

$\underline{p_1 \cdot p_2} = -m_1 E_2$  (in rest frame of 1)

$= -m_1 m_2 \gamma$

$\gamma = \frac{1}{\sqrt{1 - v_{rel}^2}}$



$$U(t) = e^{-iH_0 t} U^{(1)}(t)$$

$$* \langle \beta | S | \alpha \rangle = -\delta^4(P_\beta - P_\alpha) (2\pi)^4 i \underline{M_{\beta\alpha}} + \delta(\alpha - \beta)$$

if  $|\alpha\rangle$  is a one-particle state then, its decay rate

$$* \Gamma(\alpha \rightarrow \beta) = \frac{\text{probability}}{\text{per unit time}} = (2\pi)^4 \delta^4(P_\alpha - P_\beta) \frac{|M_{\beta\alpha}|^2}{2E_\alpha} d\beta$$

$$d\beta = \frac{N_\beta}{(2\pi)^3} \frac{d^3 p_i}{2E_i} \quad N_\beta = \# \text{ particles in } |\beta\rangle$$

$$d\sigma(\alpha \rightarrow \beta) = \frac{d\Gamma(\alpha \rightarrow \beta)}{F}$$

$$F = V \int \mathcal{L}_{int}(\alpha, t)$$



$$= (2\pi)^4 \delta^4(P_{\alpha} - P_{\beta}) \frac{|M_{\alpha\beta}|^2}{f}$$

$d\beta$

$$\int \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

$f \propto V_{rel}$

$$P_1 \cdot P_2 = -m_1 E_2 \quad (\text{in rest frame of } 1)$$

$$= -m_1 m_2 \frac{1}{\sqrt{1 - v_{rel}^2}}$$

$E_1 E_2$

$$f = 4 \sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2}$$

$$= 4(P_1 \cdot P_2) V_{rel}$$

↑  
2 initial particles

$$\prod_{i=1}^n P_{2,i}$$

$$P_1 \cdot P_2 = (P_1 \cdot P_2)$$

$$J = Q'(\alpha)$$



Calculate  $\mathbb{Z} \rightarrow$  anything  $|\alpha\rangle = |\mathbb{Z}(p, \mathbb{S})\rangle = a_{p, \mathbb{S}}^* |0\rangle$

$$S = 1 - i \int d^4x \mathcal{H}_{int} + \frac{(-i)^2}{2} \int d^4x \mathcal{T} [\mathcal{H}_{int}(x) \mathcal{H}_{int}(y)]$$

Calculate  $\mathbb{Z} \rightarrow$  anything  $|\alpha\rangle = |Z(p, \beta)\rangle = a_{p, \beta}^* |0\rangle$

$$S = 1 - i \int d^4x \mathcal{H}_{int} + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)]$$

+ ...



Calculate  $\mathbb{Z} \rightarrow$  anything  $|\alpha\rangle = |\mathbb{Z}(\beta, S)\rangle = a_{\beta, S}^* |10\rangle$

$$S = 1 - i \int d^4x \mathcal{H}_{int} + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)]$$

$$\langle \beta | S | \alpha \rangle = -i \int d^4x \langle \beta | \mathcal{H}_{int}(x) | \alpha \rangle + \dots$$

$$\frac{(-i)^2}{2} \int d^4x_1 d^4x_2 \langle \beta | T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)] | \alpha \rangle + \dots$$

Calculate  $\mathbb{Z} \rightarrow$  anything  $|d\rangle = |\mathbb{Z}(\beta, \beta)\rangle = a_{\beta, \beta}^* |0\rangle$

$$S = 1 - i \int d^4x \mathcal{H}_{int} + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)] + \dots$$

$$\langle \beta | S | \alpha \rangle = -i \int d^4x \langle \beta | \mathcal{H}_{int}(x) | \alpha \rangle + \dots$$

$$+ \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 \langle \beta | T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)] | \alpha \rangle + \dots$$



The only interaction which can destroy a  $Z$  +  
 leave only particles lighter than  $M_Z$  is

$$\mathcal{L}_{nc} = \frac{ie}{s_W c_W} Z_\mu \sum_f \left[ \bar{f} \gamma^\mu \left[ T_3 \gamma_L - Q s_W^2 \right] f \right]$$

The only interaction which can destroy a  $Z$  +  
 leave only particles lighter than  $M_Z$  is

$$\mathcal{L}_{nc} = \frac{ie}{s_W c_W} Z_\mu \sum_f \left[ \bar{f} \gamma^\mu \left[ T_3 \gamma_L - Q s_W^2 \right] f \right]$$

we want  $\mathcal{H}_{int}$  not  $\mathcal{L}_{int}$ .

$$H = P \dot{q}^i - L$$

Sometimes  $\mathcal{H}_{int} = -\mathcal{L}_{int}$ .

$$\mathcal{H}_{int} = \mathcal{H}(S) - \mathcal{L}(S)$$



Calculate ...  $\Xi \rightarrow$  "anything" ...  $|d\rangle = |Z(\beta, S)\rangle = a_{\beta, S}^* |0\rangle$

$$S = 1 - i \int d^4x \mathcal{H}_{int} + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)] + \dots$$

$$\langle \beta | S | \alpha \rangle = -i \int d^4x \langle \beta | \mathcal{H}_{int}(x) | \alpha \rangle + \dots$$

$$\langle \beta | T[\phi(x_1) \phi(x_2)] | \alpha \rangle + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 \langle \beta | T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)] | \alpha \rangle + \dots$$

Calculate  $\mathbb{Z} \rightarrow$  "anything"  $|\alpha\rangle = |\mathbb{Z}(\beta, \mathbb{S})\rangle = a_{\beta, \mathbb{S}}^* |10\rangle$

$$S = 1 - i \int d^4x \mathcal{H}_{\text{int}} + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 T[\mathcal{H}_{\text{int}}(x_1) \mathcal{H}_{\text{int}}(x_2)]$$

$$\langle \beta | S | \alpha \rangle = -i \int d^4x \langle \beta | \mathcal{H}_{\text{int}}(x) | \alpha \rangle + \dots$$

$$\langle \beta | T[\phi(x_1) \phi(x_2)] | \alpha \rangle + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 \langle \beta | T[\mathcal{H}_{\text{int}}(x_1) \mathcal{H}_{\text{int}}(x_2)] | \alpha \rangle + \dots$$



Calculate  $\mathbb{Z} \rightarrow$  anything  $|\alpha\rangle = |\mathbb{Z}(\beta, S)\rangle = a_{\beta}^* |0\rangle$

$$S = 1 - i \int d^4x \mathcal{H}_{int} + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)]$$

$$\langle \beta | S | \alpha \rangle = -i \int d^4x \langle \beta | \mathcal{H}_{int}(x) | \alpha \rangle + \dots$$

$$\partial_\mu \langle 0 | T[\phi(x_1) \phi(x_2)] | 0 \rangle + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 \langle \beta | T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)] | \alpha \rangle$$

$$\langle 0 | T[\partial_\mu \phi(x_1) \phi(x_2)] | 0 \rangle$$



in which can destroy a  $\bar{z} +$   
 the only particles lighter than  $M_z$  is

$$\int \bar{f} \gamma^\mu [T_3 \gamma_L - Q s_w^2] f$$

$+ \mathcal{L}_{int.}$

$$\mathcal{H}_{int.} = -\mathcal{L}_{int.}$$

$$\begin{aligned} & T(\theta_1(x) \theta_2(y)) \\ &= \theta_1(x) \theta_2(y) \theta(x-y) \\ & \quad + \theta_1(y) \theta_2(x) \theta(y-x) \\ & \mathcal{G}_n[\ ] = \int \delta^n \delta(x-y) \\ & \quad [\theta_{0,+}] \end{aligned}$$

$$* \langle \beta | S | \alpha \rangle = -\delta^+(P_\beta^\mu)$$

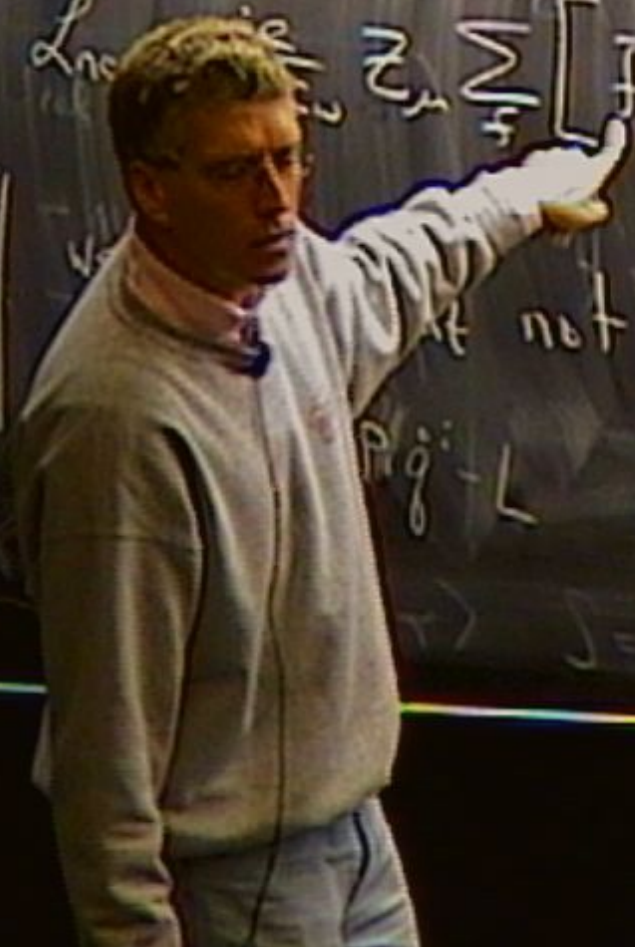
if  $|\alpha\rangle$  is a one-particle

$$* \Gamma(\alpha \rightarrow \beta) = \frac{\text{probability}}{\text{per unit } t}$$



The only interaction which can destroy a  $Z$  +  
 leave only particles lighter than  $M_Z$  is

$$L_{\text{int}} = \sum_f \bar{\psi}_f \gamma^\mu [T_3 \gamma_L - Q_s s_w^2] \psi_f$$



not  $\mathcal{L}_{\text{int}}$ .

Sometimes  $\mathcal{N}_{\text{int}} = -\mathcal{L}_{\text{int}}$ .

but not always, but who cares?

$$\begin{aligned} T(\theta_1(x)\theta_2(y)) &= \theta_1(x)\theta_2(y)\delta(x-y) \\ &\neq \theta_1(y)\theta_2(x)\delta(y-x) \\ \mathcal{D}_\mu[\ ] &= \int_{x_0}^0 \mathcal{D}(x,y) \\ &[\mathcal{Q}_0, +] \end{aligned}$$



Calculate  $\mathbb{Z} \rightarrow$  anything  $|\alpha\rangle = |\mathbb{Z}(\beta|\mathbb{Z})\rangle = a_{\beta}^* |0\rangle$

$$S = 1 - i \int d^4x \mathcal{H}_{int} + \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)] + \dots$$

$$\langle \beta | S | \alpha \rangle = -i \int d^4x \langle \beta | \mathcal{H}_{int}(x) | \alpha \rangle + \dots$$

$$\frac{\partial}{\partial \phi} \langle 0 | T[\phi(x_1) \phi(x_2)] | 0 \rangle$$

$$\langle 0 | T^*[\partial_\mu \phi(x_1) \phi(x_2)] | 0 \rangle$$

$$+ \frac{(-i)^2}{2} \int d^4x_1 d^4x_2 \langle \beta | T[\mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2)] | \alpha \rangle + \dots$$







The only nonzero matrix element of the form:

$$\langle \beta | \alpha_{nc} | \alpha \rangle = \delta(\alpha - \beta)$$

is for  $|\beta\rangle = |f(\mathbf{p}, \sigma)\rangle$

$$\langle f(\mathbf{p}, \sigma) | \alpha_{nc} | \alpha_{nc} \rangle$$



The only nonzero matrix element of the form:

$$\langle \beta | \alpha_{nc} | \alpha \rangle = \delta(\alpha - \beta)$$

is for  $|\beta\rangle = |\mathcal{F}(p, \sigma) \mathcal{F}(p, \sigma)\rangle$

$$\langle \mathcal{F}(p, \sigma) \mathcal{F}(p, \sigma) | \alpha_{nc}(\alpha) | \mathcal{Z}(k, \xi) \rangle$$

The only nonzero matrix element of the form:

$$\langle \beta | \alpha_{nc} | \alpha \rangle = \delta(\alpha - \beta)$$

is for  $|\beta\rangle = |f(\vec{p}, \sigma) \bar{f}(\vec{p}, \bar{\sigma})\rangle$

$$\langle f(\vec{p}, \sigma) \bar{f}(\vec{p}, \bar{\sigma}) | \alpha_{nc}(x) | \alpha(k, \lambda) \rangle$$

$$\langle f(\vec{p}, \sigma) \bar{f}(\vec{p}, \bar{\sigma}) | S | \alpha(k, \lambda) \rangle = +i \int d^4x \langle f(\vec{p}, \sigma) \bar{f}(\vec{p}, \bar{\sigma}) | \alpha_{nc}(x) | \alpha(k, \lambda) \rangle$$



The only nonzero matrix element of the form:

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$$\langle f(\vec{p}, \sigma) \bar{f}(\vec{p}, \bar{\sigma}) | \alpha_{nc}(x) | \mathcal{Z}(k, \zeta) \rangle$$

$$\langle f(\vec{p}, \sigma) \bar{f}(\vec{p}, \bar{\sigma}) | S | \mathcal{Z}(k, \zeta) \rangle = +i \int d^4x \langle f(\vec{p}, \sigma) \bar{f}(\vec{p}, \bar{\sigma}) | \alpha_{nc}(x) | \mathcal{Z}(k, \zeta) \rangle$$

$$\bar{\psi}_\mu(x) = \frac{1}{\sqrt{L}} \int \frac{dk}{2\pi} \left[ \epsilon_\mu(k, \tau) e^{ikx} a_{k\tau} + \epsilon_\mu^*(k, \tau) e^{-ikx} a_{k\tau}^\dagger \right]$$





$$\psi(x) = \sum_n \int \frac{d^3k}{(2\pi)^3 2E_k} \left[ \epsilon_n(k, \vec{x}) e^{ikx} a_{n\vec{k}} + \epsilon_n^*(k, \vec{x}) e^{-ikx} a_{n\vec{k}}^* \right]$$

$$f(x) = \sum_\lambda \int d^3p \left[ u(p, \lambda) e^{ipx} c_{p\lambda} + v(p, \lambda) e^{-ipx} \bar{c}_{p\lambda}^* \right]$$

$\bar{f}$

$\bar{u}$

$e^{-ipx}$

$c_{p\lambda}^*$

$\bar{v}$

$e^{ipx}$

$\bar{c}_{p\lambda}$

Check:

$$\langle f(p=) \bar{f}(p, \vec{x}) | \alpha_{n\vec{k}}(y) | Z(t, \vec{x}) \rangle = \frac{ie}{8\pi\omega} \frac{\epsilon_n(k, \vec{x})}{((2\pi)^3 2E_n)^2}$$

$$\tilde{z}_n(x) = \sum_s \int \frac{d^3k}{(2\pi)^3 2E_k} \left[ \epsilon_n(k, s) e^{ikx} a_{ks} + \epsilon_n^*(k, s) e^{-ikx} a_{ks}^* \right]$$

$$f(x) = \sum_\lambda \int \frac{d^3p}{(2\pi)^3} \left[ u(p, \lambda) e^{ipx} c_{p\lambda} + \bar{v}(p, \lambda) e^{-ipx} \bar{c}_{p\lambda} \right]$$

$$\bar{f} = \int \bar{u} \frac{e^{-ipx} c_{p\lambda}^*}{(2\pi)^3} + \bar{v} e^{ipx} \bar{c}_{p\lambda}$$

check:

$$\langle f(p=) \bar{f}(p=) | \alpha_{nc}(x) | Z(t, S) \rangle = \frac{ie}{S_0 c_n} \frac{\epsilon_n(k, s) e^{ikx}}{(2\pi)^3 2E_k} \bar{u}(p, \lambda) e^{-ipx}$$



$$\bar{\psi}(x) = \sum_s \int \frac{d^3k}{(2\pi)^3 2E_k} \left[ \bar{u}_s(k) e^{ikx} a_{ks} + \bar{v}_s(k) e^{-ikx} a_{ks}^\dagger \right]$$

$$\psi(x) = \sum_s \int \frac{d^3p}{(2\pi)^3 2E_p} \left[ u_s(p) e^{-ipx} c_{ps} + v_s(p) e^{ipx} c_{ps}^\dagger \right]$$

$$\bar{\psi} \quad \bar{u} \quad e^{-ipx} \quad c_{ps}^\dagger \quad \bar{v} \quad e^{ipx} \quad c_{ps}$$

check:

$$\langle \bar{\psi}(p) \bar{\psi}(p') | \alpha_{nc}(x) | \psi(t, \vec{y}) \rangle = \frac{ie}{4V\omega} \frac{\bar{u}_s(k) e^{ikx}}{(2\pi)^3 2E_k} \bar{u}(p) e^{-ipx} \gamma^\mu [T_x \gamma_\mu - Q_1 \gamma_0] v(p')$$

$$\bar{z}_\mu(x) = \sum_s \int \frac{d^3k}{(2\pi)^3 2E_k} \left[ \epsilon_\mu(k, s) e^{ikx} a_{ks} + \epsilon_\mu^*(k, s) e^{-ikx} a_{ks}^* \right]$$

$$f(x) = \sum_\lambda \int \frac{d^3p}{(2\pi)^3} \left[ u(p, \lambda) e^{ipx} c_{p\lambda} + \bar{v}(p, \lambda) e^{-ipx} \bar{c}_{p\lambda}^* \right]$$

$\bar{f}$

$\bar{u}$

$e^{-ipx}$

$\bar{c}_{p\lambda}^*$

$\bar{v}$

$e^{ipx}$

check:

$$\langle f(p=) \bar{f}(p,=) | \alpha_{\mu\nu}(x) | \bar{z}(k, s) \rangle = \frac{ie}{s v c v} \epsilon_\mu(k, s) e^{ikx} \bar{u}(p, \lambda) e^{-ipx} \gamma^\mu [T]$$

$$= \frac{ie}{s v c v} \epsilon_\mu(k, s) \left[ \bar{u}(p, \lambda) \gamma^\mu [T] v(k, s) \right] e^{i(k-p)x}$$



$$\bar{\psi}(x) = \sum_s \int \frac{d^3k}{(2\pi)^3 2E_k} \left[ \bar{u}_s(k) e^{ikx} a_{ks} + \bar{v}_s(k) e^{-ikx} a_{ks}^\dagger \right]$$

$$\psi(x) = \sum_s \int \frac{d^3p}{(2\pi)^3} \left[ u_s(p) e^{ipx} c_{ps} + \bar{v}_s(p) e^{-ipx} c_{ps}^\dagger \right]$$

$$\bar{\psi} = \bar{u} e^{ipx} c_{ps}^\dagger + \bar{v} e^{ipx} c_{ps}$$

check:

$$\langle \bar{\psi}(p) \bar{\psi}(p') | \alpha_{ps}^\dagger(x) | \psi(k, s) \rangle = \frac{ie}{s v c} \bar{u}_s(k) e^{ikx} \bar{u}(p') e^{-ip'x} \gamma^0 [\gamma^0 \gamma^i - \delta_{ij}] u(p)$$

$$= \frac{ie}{s v c} \bar{u}_s(k) \left[ \bar{u}(p') \gamma^0 [\gamma^0 \gamma^i - \delta_{ij}] u(p) \right] e^{i(k-p)x}$$

$$\psi_{\mu}(x) = \sum_{\alpha} \int \frac{d^3 k}{(2\pi)^3 2E_k} \left[ \epsilon_{\mu}(k, \alpha) e^{ikx} a_{\alpha} + \epsilon_{\mu}^*(k, \alpha) e^{-ikx} a_{\alpha}^* \right]$$

$$f(x) = \sum_{\lambda} \int d^3 p \left[ u(p, \lambda) e^{ipx} c_{\lambda} + \bar{v}(p, \lambda) e^{-ipx} \bar{c}_{\lambda}^* \right]$$

$$\bar{f} = \bar{u} \frac{e^{-ipx} c_{\lambda}^*}{2} + \bar{v} e^{ipx} \bar{c}_{\lambda}$$

check:

$$\langle f(p=) \bar{f}(p=) | \alpha_{\mu}(x) | \psi(k, \alpha) \rangle = \frac{ic}{2E_k} \epsilon_{\mu}(k, \alpha) e^{ikx} \bar{u}(p, \alpha) e^{-ipx} \gamma^{\mu} [\tau_2 \tau_3 - \alpha \tau_1] u(p, \alpha)$$

$$\int d^3 x e^{i(k-p-p)x} = (2\pi)^3 \delta^3(k-p-p) = \frac{ic}{2E_k} \epsilon_{\mu}(k, \alpha) \left[ \bar{u}(p, \alpha) \gamma^{\mu} [\tau_2 \tau_3 - \alpha \tau_1] u(p, \alpha) \right] e^{i(k-p-p)x}$$



The only nonzero matrix element of the form:

$$\langle \beta | \alpha_{nc} | Z \rangle = \delta(\beta - \beta')$$

is for  $|\beta\rangle = |S(p, \sigma) \bar{S}(\bar{p}, \bar{\sigma})\rangle$

$$\langle S(p, \sigma) \bar{S}(\bar{p}, \bar{\sigma}) | \alpha_{nc}(x) | Z(k, \lambda) \rangle$$

$$\langle S(p, \sigma) \bar{S}(\bar{p}, \bar{\sigma}) | S | Z(k, \lambda) \rangle = +i \int d^4x \langle S(p, \sigma) \bar{S}(\bar{p}, \bar{\sigma}) | \alpha_{nc}(x) | Z(k, \lambda) \rangle$$

$$= -i (2\pi)^4 \delta^4(k - p - \bar{p}) M(Z \rightarrow H)$$

$$\text{So: } M = -\frac{e}{s\omega c_0} \mathbf{E}_\mu(k, \omega) \int \mathbf{u}(r, t) dr$$

$$\int_{-\infty}^{\infty} V = \int_{-\infty}^{\infty} \mathcal{L}_{int}(x, t)$$

$$\mathcal{L}_{int} = \frac{ie}{s\omega c_0} \sum_{\mu} \sum_f \left[ \bar{f} \gamma^{\mu} \left[ T_{3\mu} \gamma_L - Q s_{\omega}^2 \right] f \right]$$

we want  $\mathcal{H}_{int}$  not  $\mathcal{L}_{int}$ .

$$\mathcal{H} = P \dot{q} - L$$

Sometimes

$$\mathcal{H}_{int} = -\mathcal{L}_{int}$$

but not always, but who cares?

$$\begin{aligned} & \int (\theta_1(x) \theta_2(y)) \\ &= \theta_1(x) \theta_2(y) \delta(x-y) \\ & \neq \theta_1(y) \theta_2(x) \delta(y-x) \\ & \partial_x [\ ] = \delta_x^0 \delta(x-y) \\ & (0, +) \end{aligned}$$



$$\text{So: } M = -\frac{ie}{\epsilon_0 \omega} \mathbf{e}_\mu(x) \int \mathbf{u}(\mathbf{r}) \gamma \cdot [\mathbf{T}_3 \chi - Q \mathbf{s}] \mathbf{u}(\mathbf{r}) d\mathbf{r}$$

$$\mathcal{L}_{int} = \frac{ie}{\epsilon_0 \omega} \sum_{\mu} \int \mathbf{f} \gamma^\mu \cdot [\mathbf{T}_3 \chi_L - Q \mathbf{s}_L] \mathbf{f}$$

we want  $\mathcal{H}_{int}$  not  $\mathcal{L}_{int}$ .

$$\mathcal{H} = \mathcal{P} \dot{q} + L$$

Sometimes

$$\mathcal{H}_{int} = -\mathcal{L}_{int}$$

but not always, but who cares?

$$\begin{aligned} \mathcal{H}(\mathbf{0}_1(x), \mathbf{0}_2(y)) &= \mathbf{0}_1(x) \mathbf{0}_2(y) \mathcal{H}(x, y) \\ &= \mathbf{0}_1(x) \mathbf{0}_2(y) \mathcal{H}(x, y) \\ &= \mathbf{0}_1(x) \mathbf{0}_2(y) \mathcal{H}(x, y) \end{aligned}$$