Title: Constraining the Statistical Isotropy of Primordial Perturbations

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Abstract:

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Cosmology on the Great Lakes Conference
November 2006

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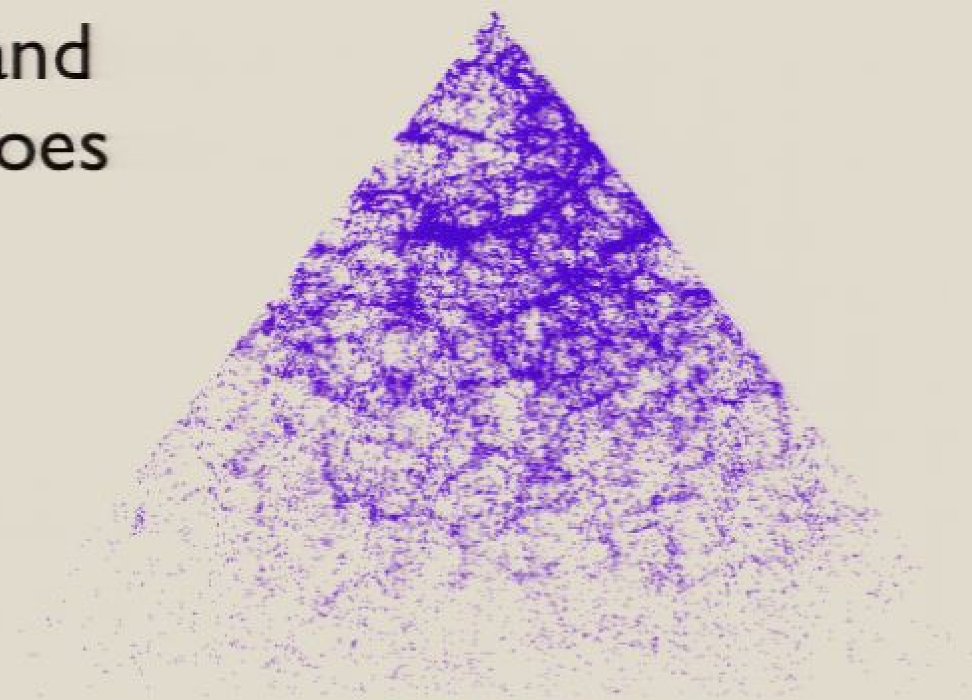
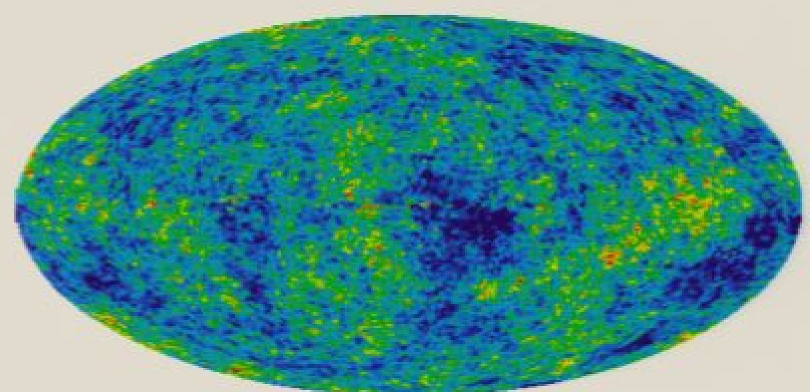
Primordial Perturbations

• In an exactly homogeneous and isotropic universe, structure does not form.

• The universe must have contained initial "primordial" perturbations, the seeds of structure.

• Primordial perturbations grow under the action of gravity to form the structures that we observe.





* 2dFGRS * WMAP

Primordial Perturbations

- Simplifying assumptions:

Assumption	Measure	Status
Adiabaticity	$r_{CDM}<0.13$	✓
Gaussianity	$-54<f_{NL}<114$	✓
Scale Invariance	$n_S=0.96$	(✓)
Statistical Isotropy	???	?

* R. Bean, J. Dunkley, E. Pierpaoli, astro-ph/0606685
* D. Spergel et. al., astro-ph/0603449
* H. Eriksen et. al., astro-ph/0606088

Related Works (CMB)

* A. Oliveira-Costa, M. Tegmark, M. Zaldarriaga, A. Hamilton, "The Signifcance of the largest scale CMB fluctuations in WMAP."

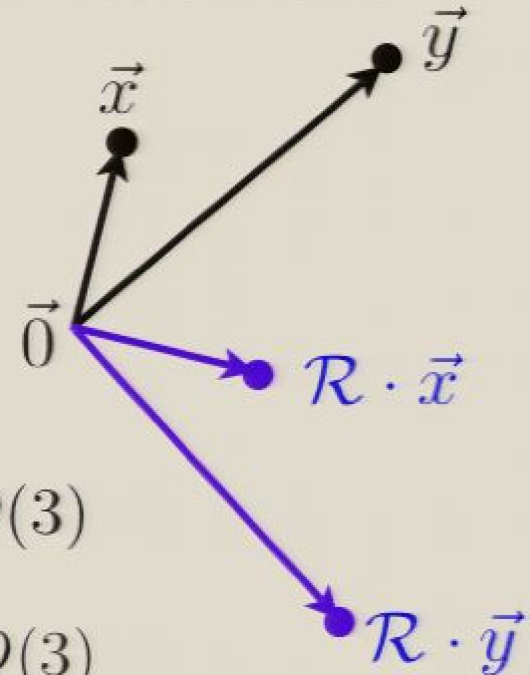
* C. Copi, D. Huterer, G. Starkman, "Multipole Vectors."

* H. Eriksen, F. Hansen, A. Banday, K. Gorski, P. Lilje, "Assymetries in the CMB anisotropy field."

* K. Land, J. Magueijo, "The axis of evil."

* A. Hajian, T. Souradeep, "The cosmic microwave background bipolar spectrum."

Statistical Isotropy



- In a statistically isotropic universe, correlation functions of perturbations are invariant under rotations:

$$\langle \Phi^*(\vec{x})\,\Phi(\vec{y})\rangle = \langle \Phi^*(\mathcal{R}\cdot\vec{x})\,\Phi(\mathcal{R}\cdot\vec{y})\rangle \quad \forall\mathcal{R}\in SO(3)$$

- In Fourier space, $\mathcal{P}(\mathcal{R}\vec{k}) = \mathcal{P}(\vec{k}) \quad \forall\mathcal{R}\in SO(3)$

$$\langle \Phi^*(\vec{k})\Phi(\vec{k}')\rangle = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')\frac{\pi}{2k^3}\mathcal{P}(\vec{k})$$

- In a statistically isotropic universe, the power spectrum only depends on the magnitude of the wave vector

Parameterizing Statistical Isotropy

- Because power spectrum is positive, write it as a square

$$\mathcal{P}(\vec{k}) = k^3 \bar{\Phi}^*(\vec{k})\bar{\Phi}(\vec{k})$$

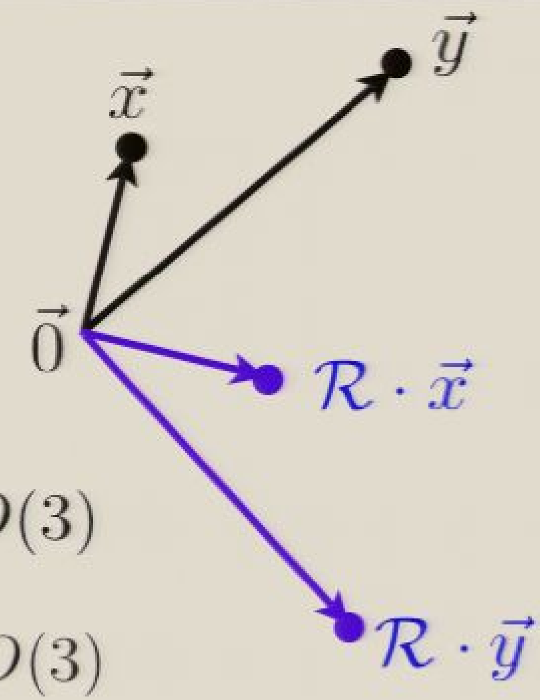
- Decompose the square root in spherical harmonics,

$$\bar{\Phi}(\vec{k}) = \sqrt{4\pi} \sum_{lm} \bar{\Phi}_{lm}(k) Y_{lm}(\hat{k})$$

- Substitute back,

$$\begin{aligned} \mathcal{P}(\vec{k}) &= \sqrt{4\pi} \sum_{lm} \mathcal{P}_{lm}(k) Y_{lm}(\hat{k}), \quad \text{where} \\ \mathcal{P}_{lm}(k) &= \sum_{l_1 m_1, l_2 m_2} (-1)^{m_1} k^3 \, \bar{\Phi}^*_{l_1 m_1}(k) \bar{\Phi}_{l_2 m_2}(k) D(l_1 - m_1; l_2, m_2 | l, m) \end{aligned}$$

Statistical Isotropy



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- The functions $\bar{\Phi}_{lm}(k)$ characterize the primordial power

- For statistically isotropic primordial perturbations

$$\bar{\Phi}_{lm} = 0 \Rightarrow \mathcal{P}_{lm} = 0 \quad \text{for} \quad l \geq 1.$$

- The spherical harmonics of the power spectrum are constrained by angular momentum addition.

Imprint on the CMB

- The temperature fluctuations seen by an observer are

$$\Delta_T(\vec{x}_0,\hat{n}) \equiv \sum_{lm} a_{lm} Y_{lm}(\hat{n}) = \int \frac{d^3k}{(2\pi)^3}\, \Phi(\vec{k})\, \Delta(\vec{k},\hat{n})\, e^{i\vec{k}\cdot x_0}$$

- Describes the amplitude of primordial perturbations ($\Phi(\vec{k})$)

- Describes the evolution of the perturbations ($\Delta(\vec{k},\hat{n})$)

- In a homogeneous and isotropic universe

$$\Delta(\vec{k},\hat{n}) = \sum_l (2l+1)(-i)^l \Delta_l(k) P_l(\hat{k}\cdot\hat{n}).$$

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- The two-point function of the CMB temperature anisotropies is

$$\langle a^*_{l_1 m_1} a_{l_2 m_2} \rangle = (-i)^{l_2 - l_1} \sum_{lm} D(l_1, m_1; l, m | l_2, m_2) K(l_1, l_2; l, m), \quad \text{where}$$

$$K(l_1, l_2; l, m) = \int \frac{dk}{k} \Delta^*_{l_1}(k) \Delta_{l_2}(k) \mathcal{P}_{lm}(k),$$

- Angular power spectrum $C_l \equiv \langle \hat{C}_l \rangle \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} \langle a^*_{lm} a_{lm} \rangle$

$$C_l = \int \frac{dk}{k} \left| \Delta_l(k) \right|^2 \mathcal{P}_{00}(k) \quad \textbf{where} \quad \mathcal{P}_{00} = \sum_{l,m} k^3 \, \bar{\Phi}^*_{lm} \bar{\Phi}_{lm}$$

- Angular power spectrum is insensitive to statistical anisotropy

Estimating Primordial Anisotropy

- Construct an estimator of $K(l_1, l_2; l, m)$:

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- For L=0, M=0 the estimator reduces to $\hat{C}_l$

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- Look for a dipole $\bar{\Phi}_{1m}$ in the primordial perturbations

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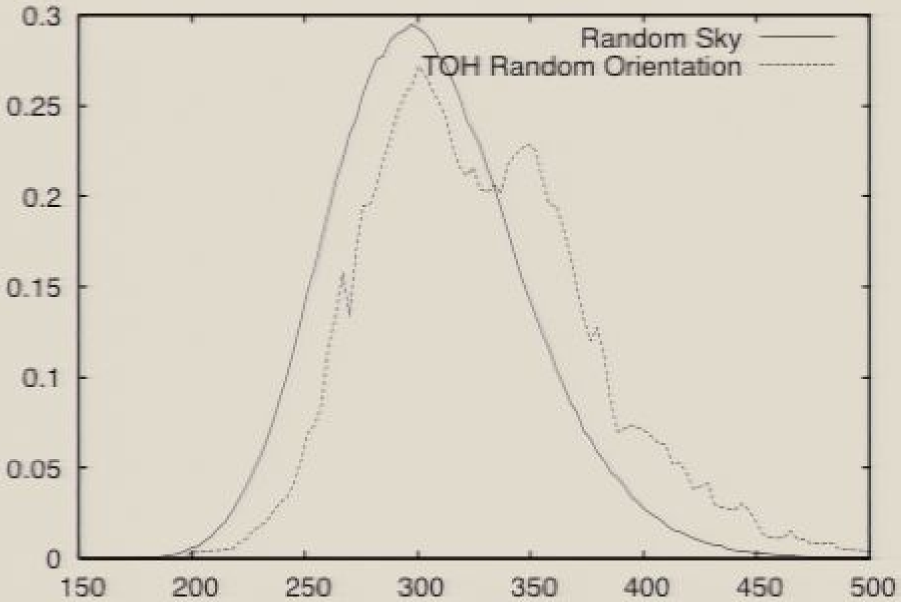

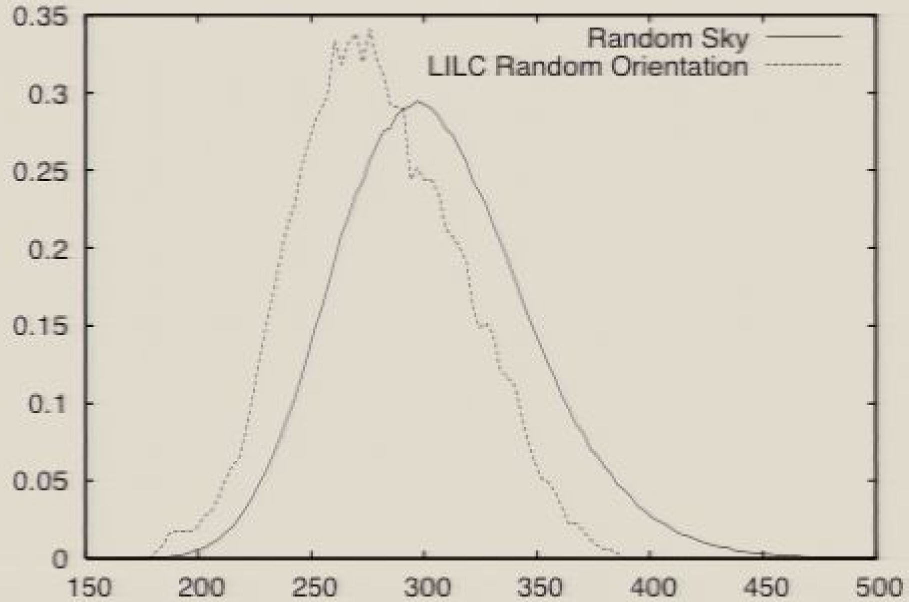

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No significance evidence for a quadrupole in the primordial perturbations

* M. Tegmark, A. Oliveira, A. Hamilton * H. Eriksen, A. Banday, K. Gorski, P. Lilje

Summary

i) We often assume that primordial perturbations are statistically isotropic.

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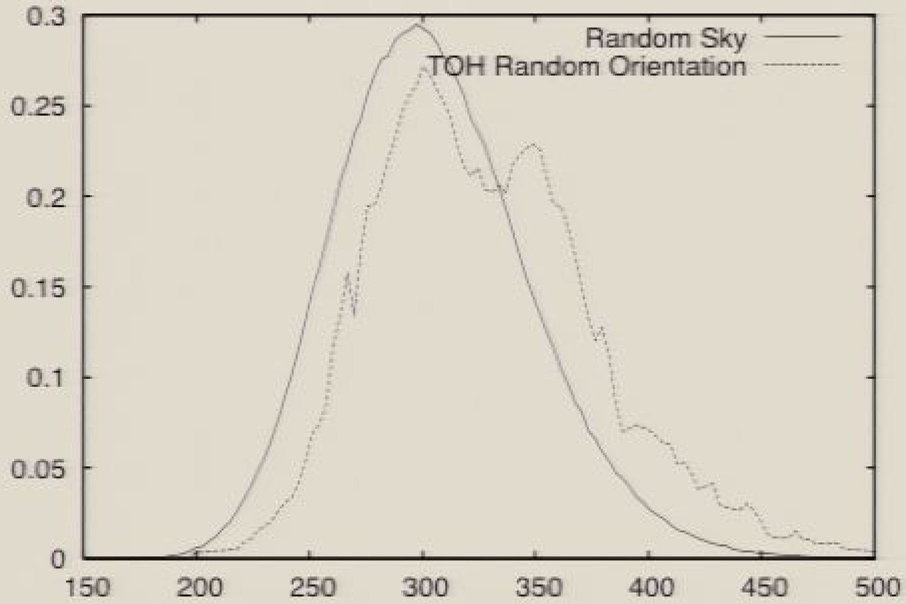

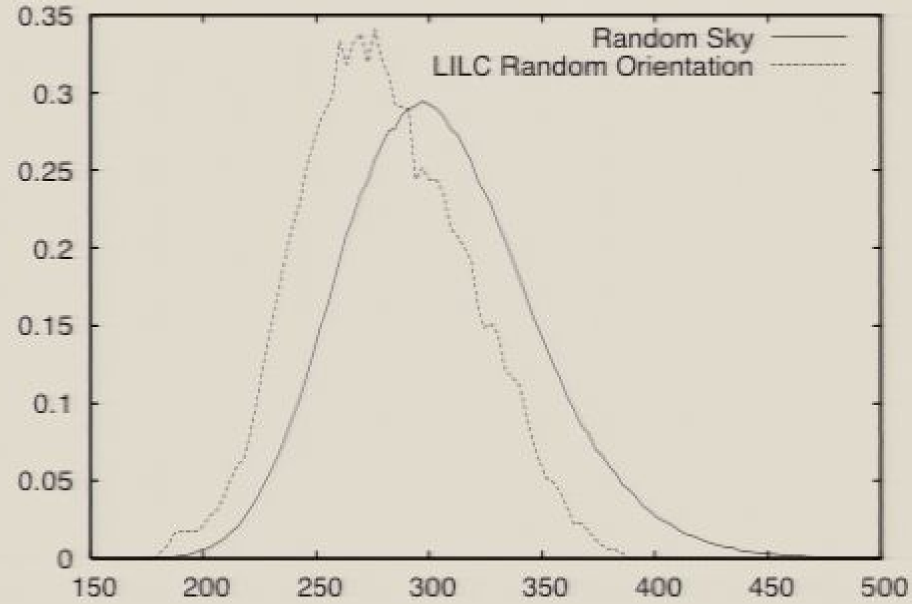

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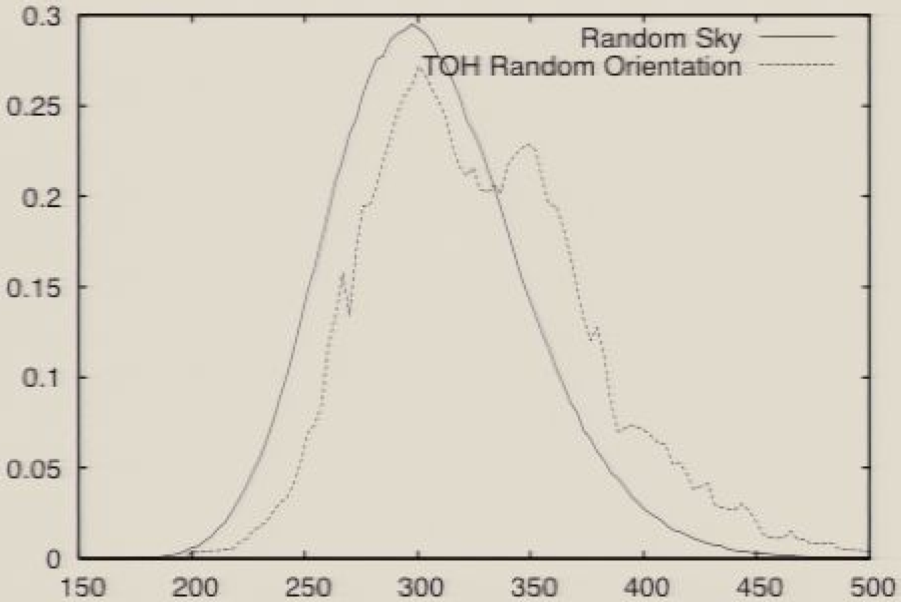

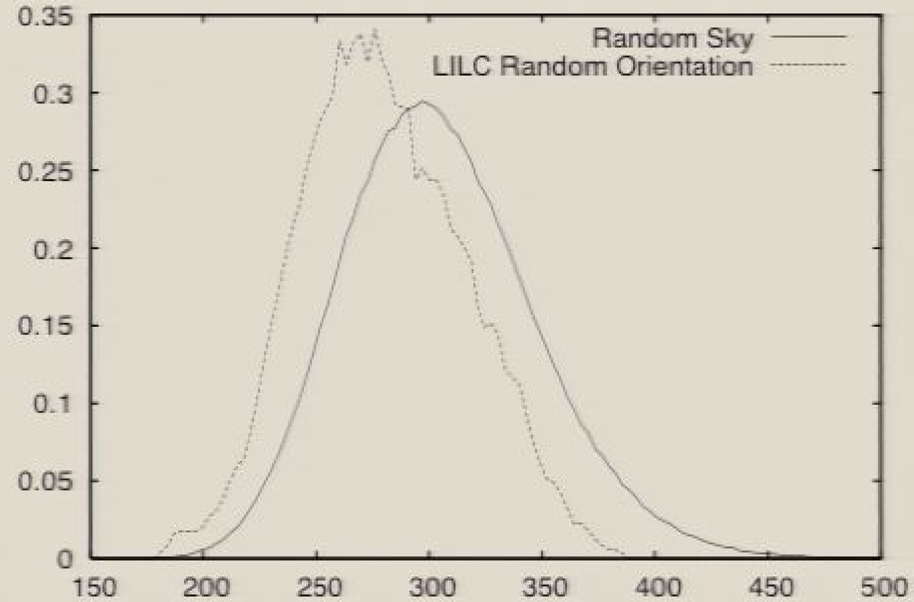

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