

Title: Bayesian Analysis of WMAP3 Data

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Abstract:



Slides

Slide 1: Bayesian analysis of WMAP3 data

Slide 2: The WMAP3-year data and Cosmological Initial Conditions

- Why analyze the WMAP3 data?
- Why not use Bayesian analysis?
- Issues
- Low redshift foregrounds
- Implications for CMB maps

Slide 3: Why analyze the WMAP3 data?

Slide 4: WMAP3: Cosmological parameters

Slide 5: WMAP3: Cosmological parameters

Normal Outline Notes Handout Slide Sorter

Bayesian analysis of WMAP3 data

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University of Illinois at Urbana-Champaign

November 11, 2006

Budapest Workshop

Tasks

Master Pages

Layouts

Custom Animation

Slide Transition





Slides

Slide 1: Bayesian analysis of WMAP3 data

Slide 2: The WMAP 3-year data and Cosmological Initial Conditions

- Why analyze the WMAP 3-year?
- Why not use the 9-year combined?
- Issues:
 - Low SNR
 - Highly noisy
 - Implications for Cosmology

Slide 3: Why analyze the WMAP 3-year?

Slide 4: WMAP 3-year CMB anisotropy

The power spectrum is measured from 0.2% to 10% anisotropy.

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Bayesian analysis of WMAP3 data

$$r = T_c - T_{\text{occ}}$$

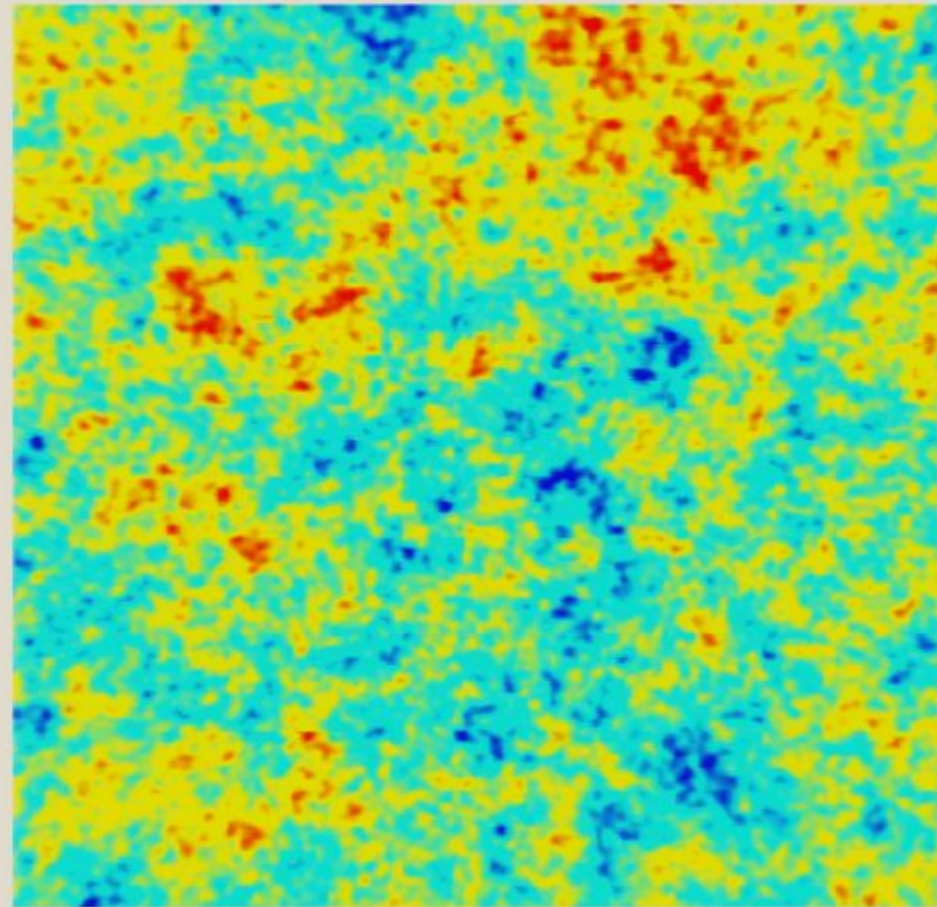
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The WMAP 3-year data and Cosmological Initial Conditions

- Why re-analyze the WMAP3 data?
- Why and how Bayesian analysis?
- Results
 - Low I results
 - High/All I results
- Implications for Cosmology



The WMAP 3-year data and Cosmological Initial Conditions

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Why re-analyze the WMAP3 data?



WMAP1: Bayesian Re-analysis reduces “low power on large scales” from 99.5% to 90% effect

Our analysis demonstrated that the power spectrum likelihoods at low ℓ have strong tails to high C_ℓ .

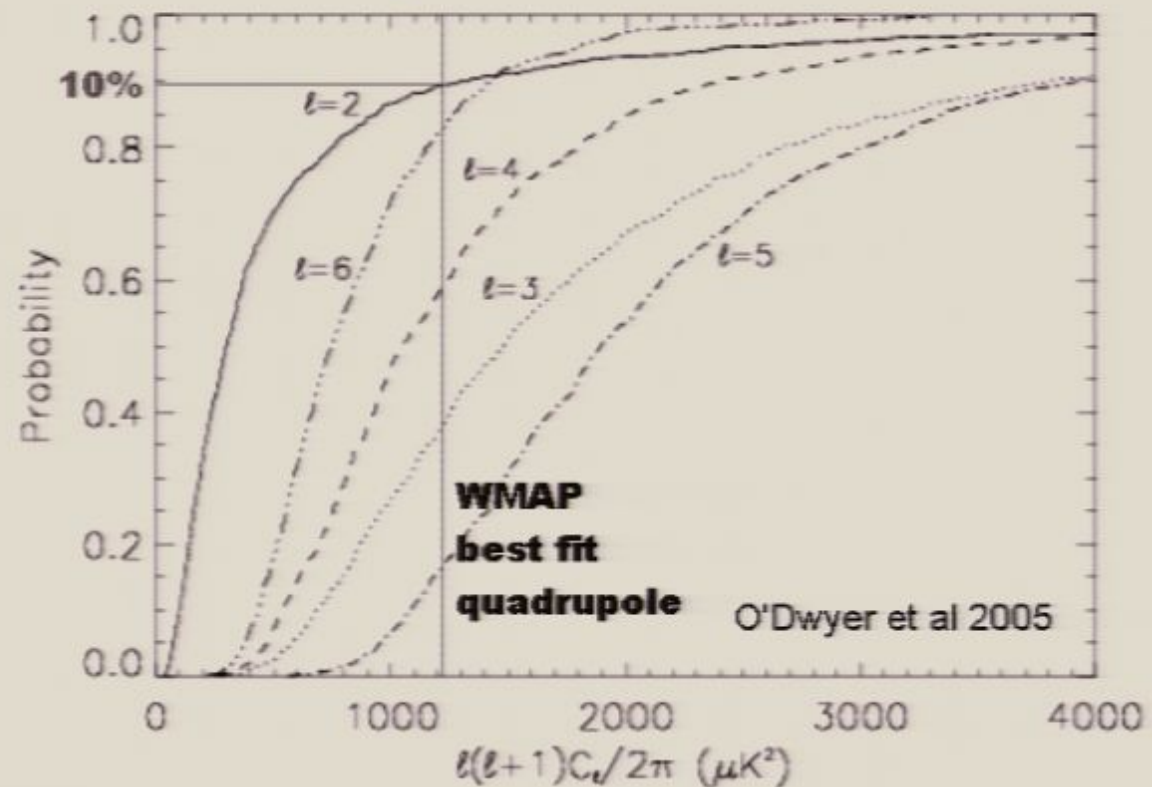
This leads to a probability in excess of 10% that the true C_2 is even larger than the WMAP best fit C_2 .

C_3 is unremarkable.

(Note: this is due to statistics, not Foreground marginalization, which adds ~5% to this effect)

Phys:0611096

$P(C_2 > x \mid \text{data})$



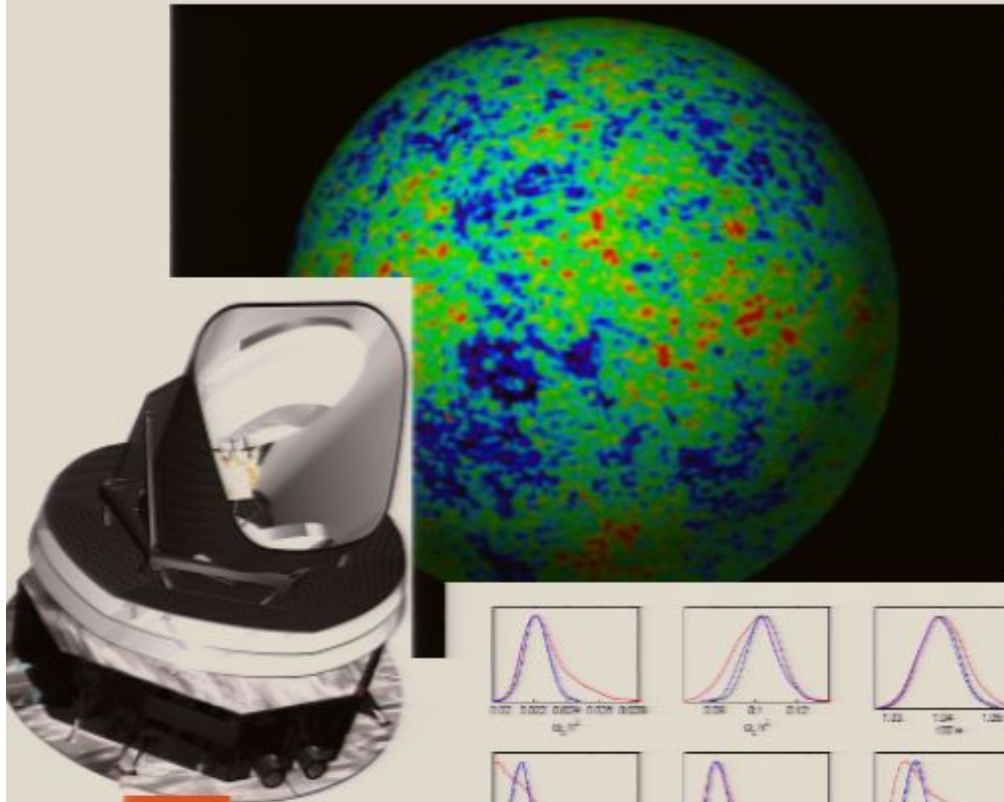
(O'Dwyer et al. 2005)

Why Bayesian analysis?



Bayesian Cosmological Data Analysis

Cosmological data analysis takes astronomical observations (D) and turns them into statistical statements about the parameters (θ) that define our Universe



- Conceptually straightforward:

$$P(\theta|D) \propto P(D|\theta)P(\theta)$$

- After COBE –for more than a decade– the field has had to cope with approximations that avoid the computational difficulty of evaluating the terms in this equation.



THE COSMOSTATISTICS PROBLEM



- Black bar: size of data set
- Red area: work required to evaluate

$$P(\theta|D) \propto P(D|\theta)P(\theta)$$



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THE CMB ANALYSIS PROBLEM

THE COSMOSTATISTICS PROBLEM

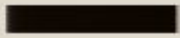


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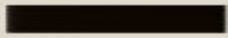


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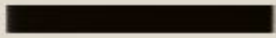


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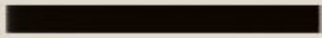


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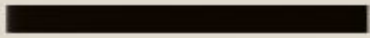


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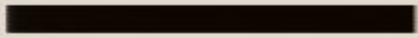


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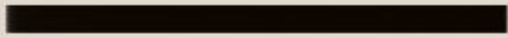


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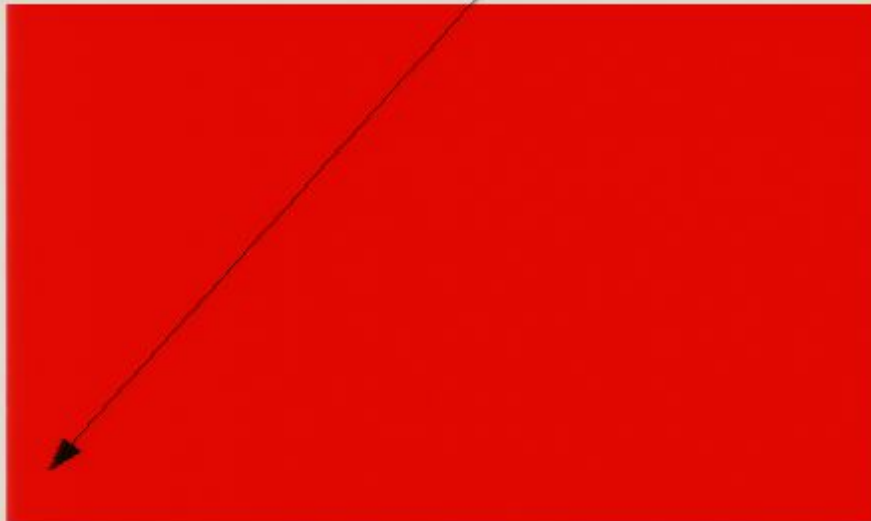


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10x COBE-DMR

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THE CMB ANALYSIS PROBLEM



Planck would take 4,000,000 years on a TeraFlop facility

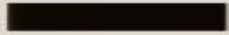
Computational Speed of Gibbs Sampling



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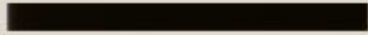
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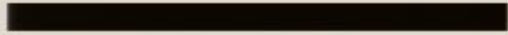
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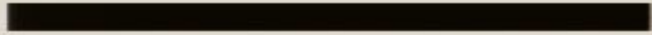
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Computational Speed of Gibbs Sampling

Feasible on existing facilities



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- The computational effort for each Gibbs sample is $O(N^{1.5})$ less than for the brute force techniques.

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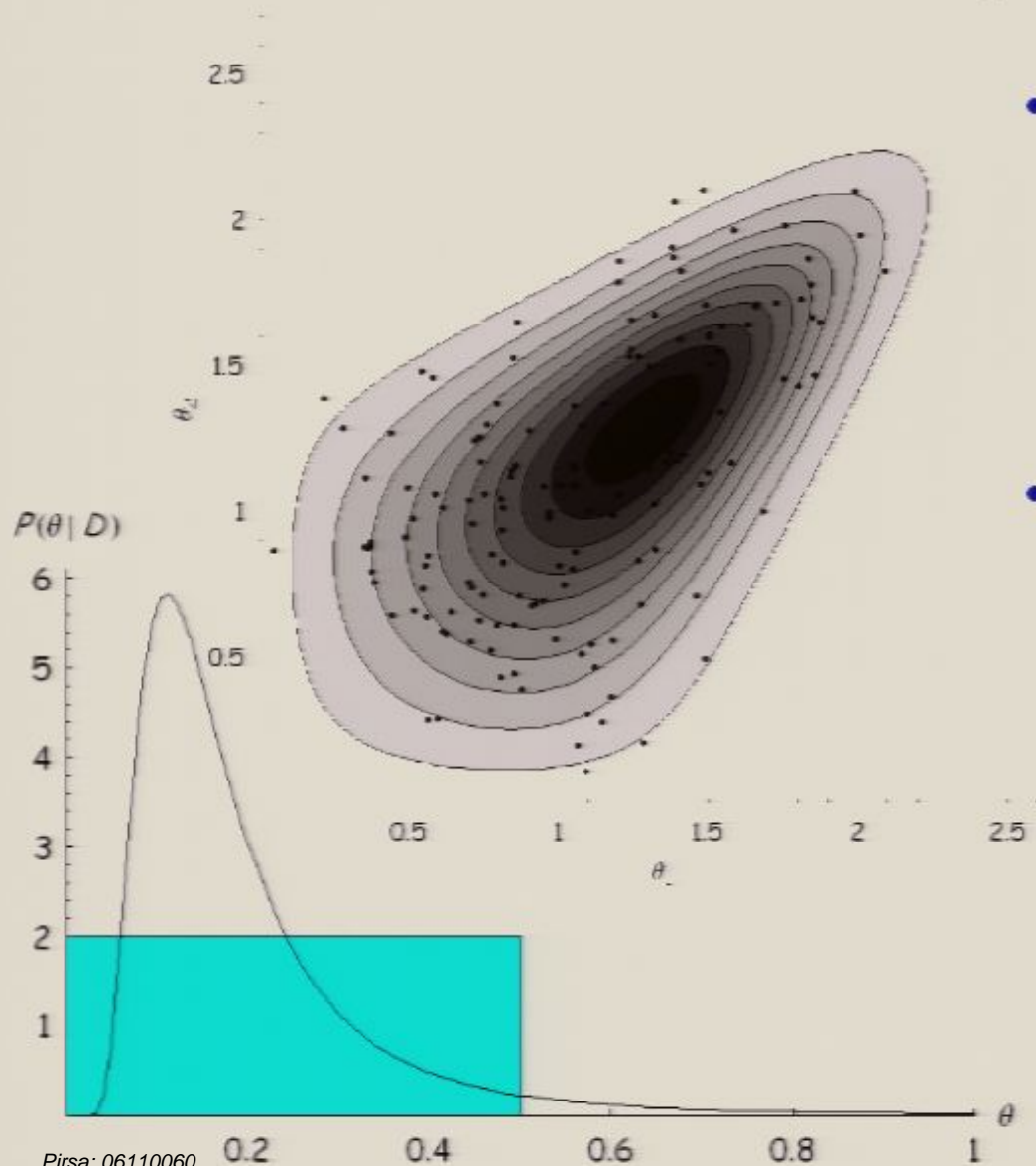


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- For WMAP and Planck $N \sim 10^7 \rightarrow N^{1.5} \sim 10^{10.5}$
- This speed-up is of the same order as the approximate (Pseudo- C_ℓ) techniques.

Gibbs Sampling: How?



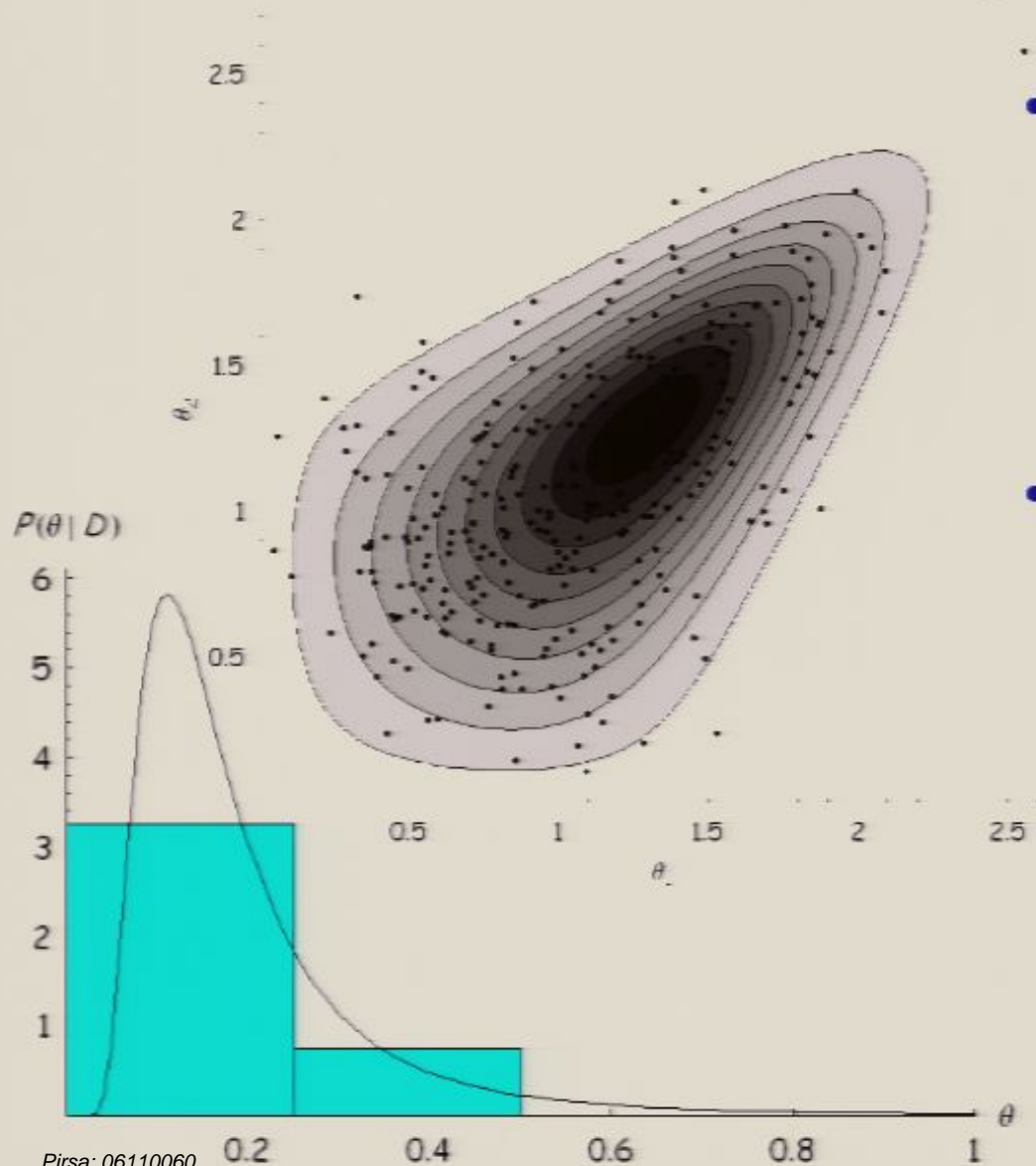
Pirsa: 06110060

- Gibbs sampling is a **Monte Carlo technique** for generating samples from the likelihood/posterior.
- It recovers the results of the full Bayesian approach without brute force evaluation of the likelihood.

(Jewell, Levin Anderson 2004,
Wandelt, Larson, Lakshmi.
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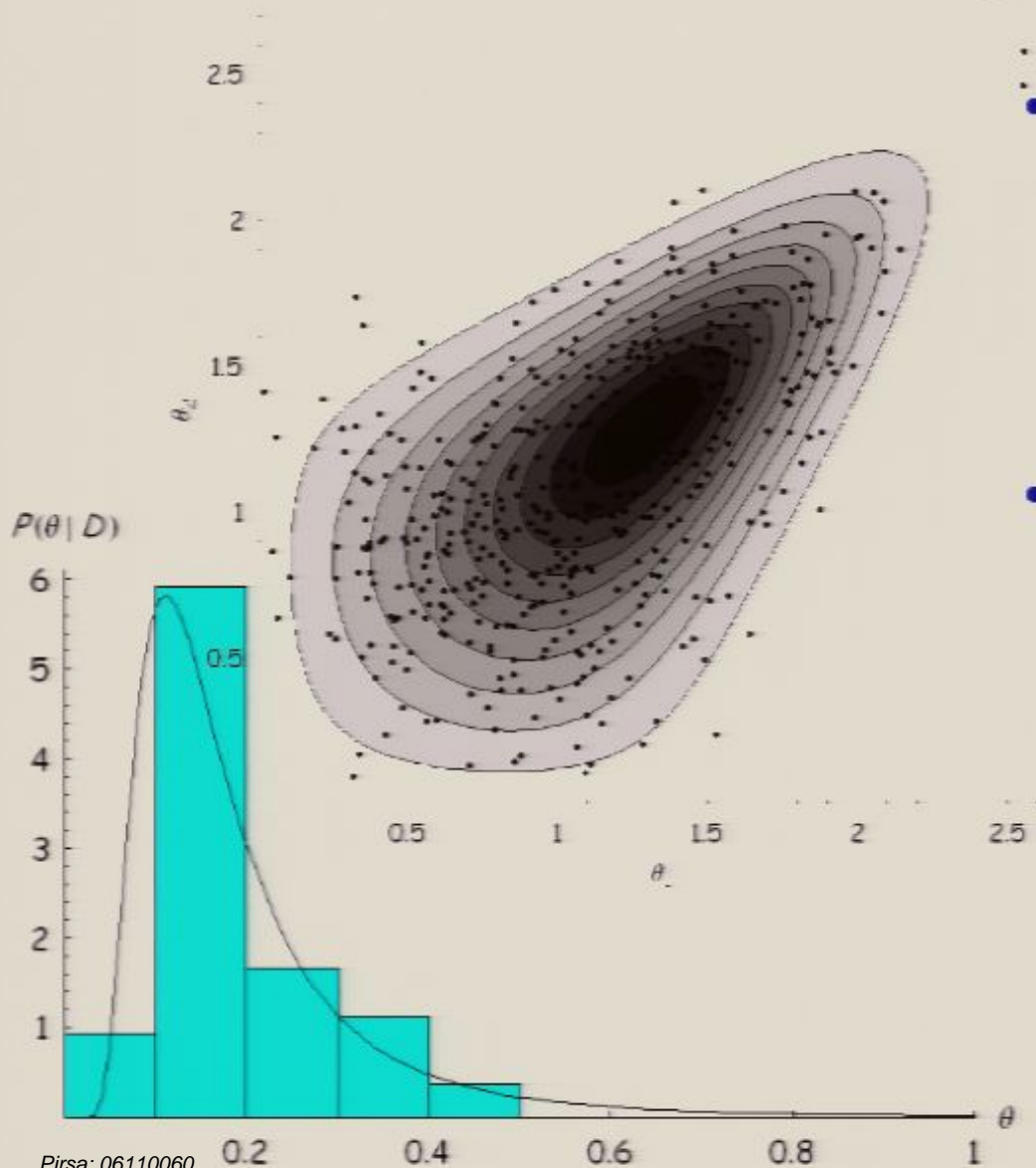
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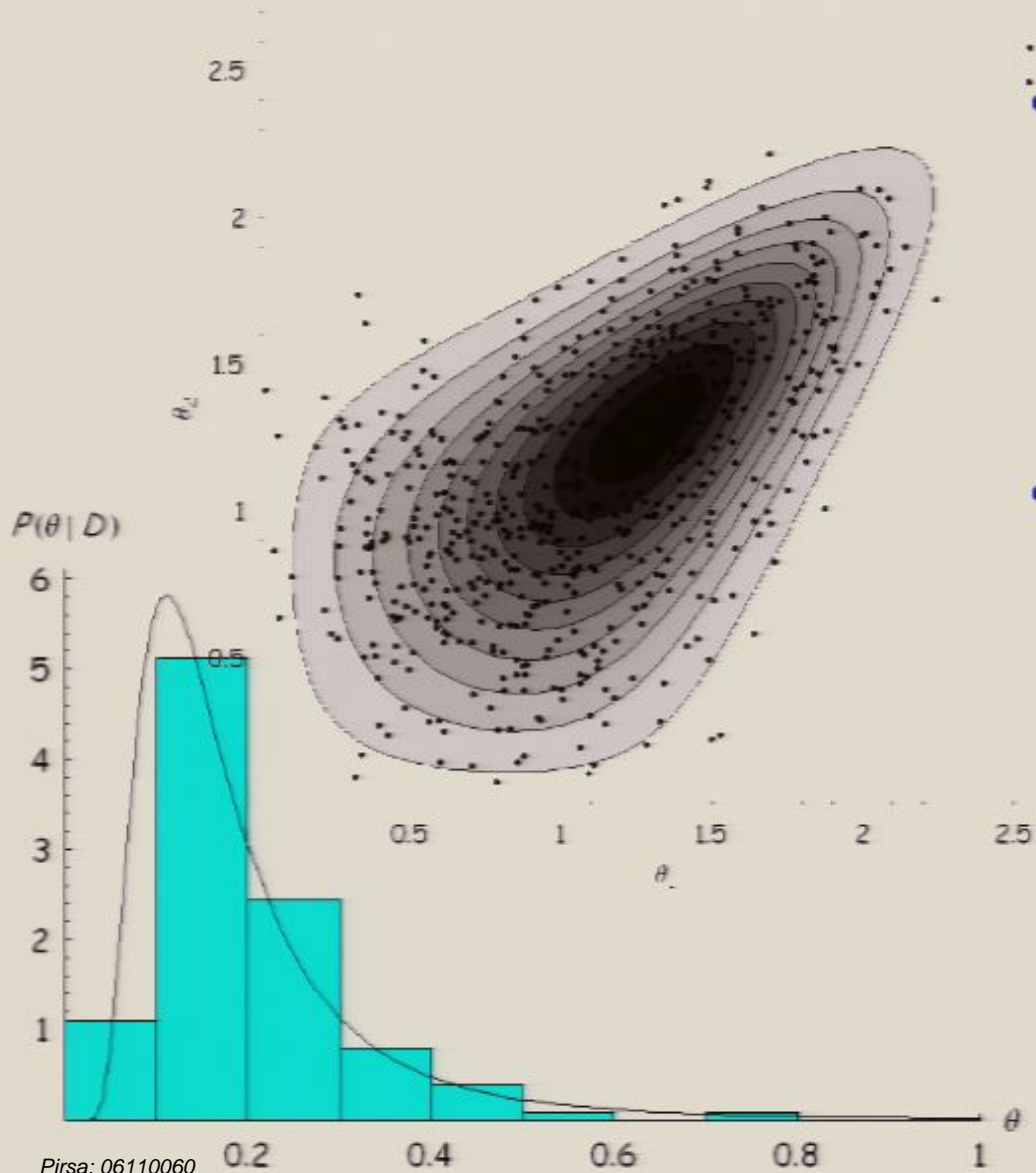
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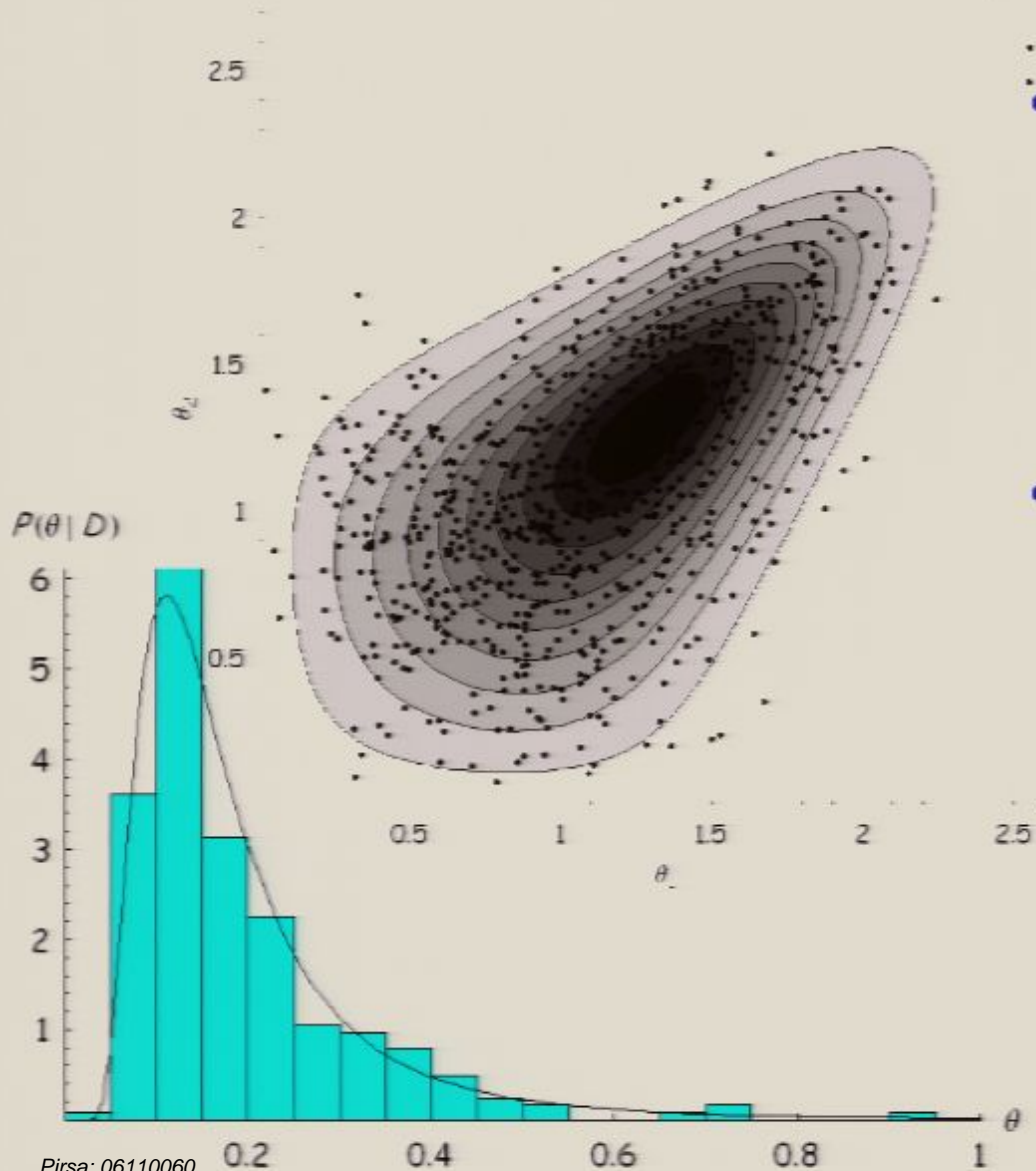


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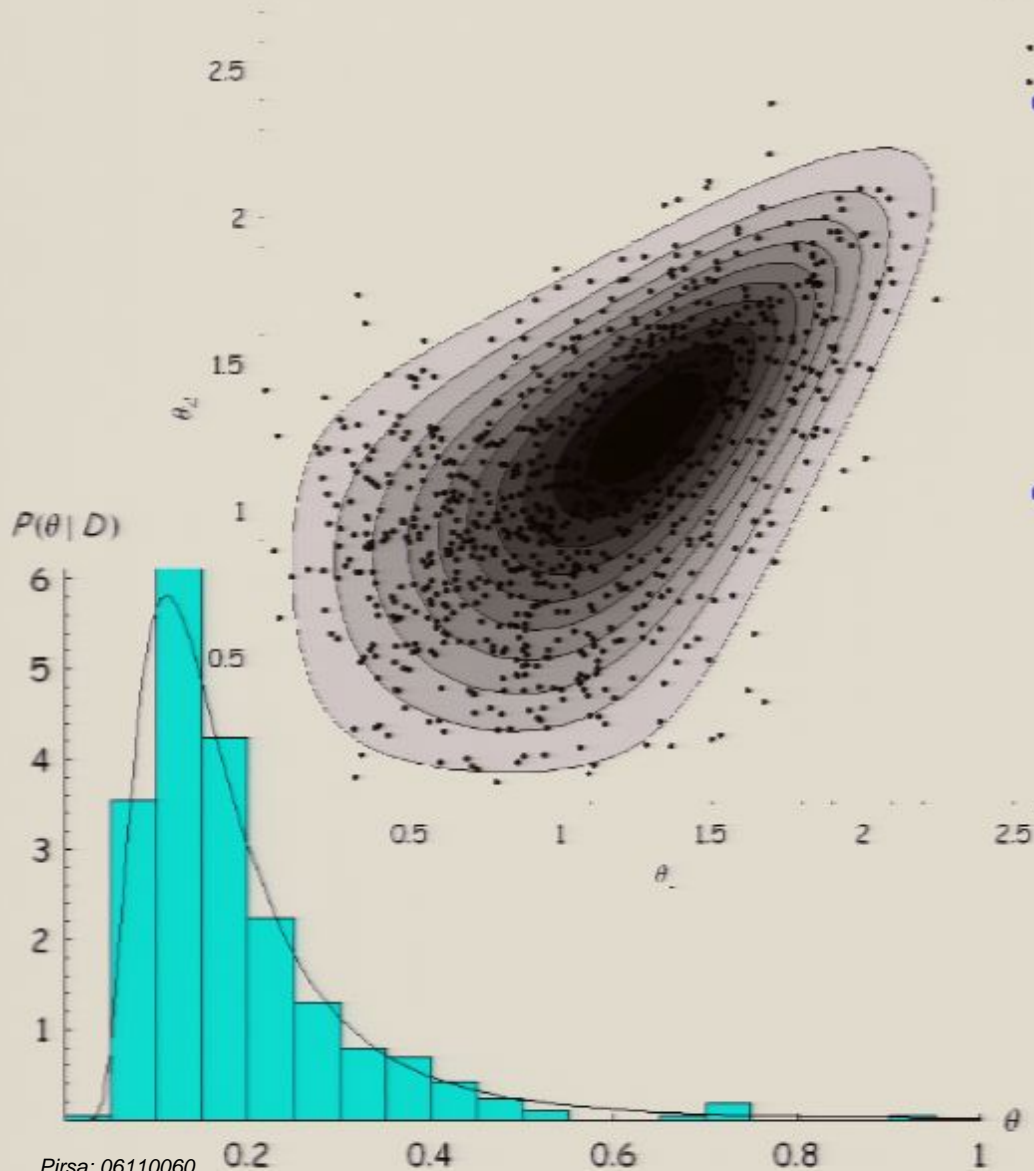
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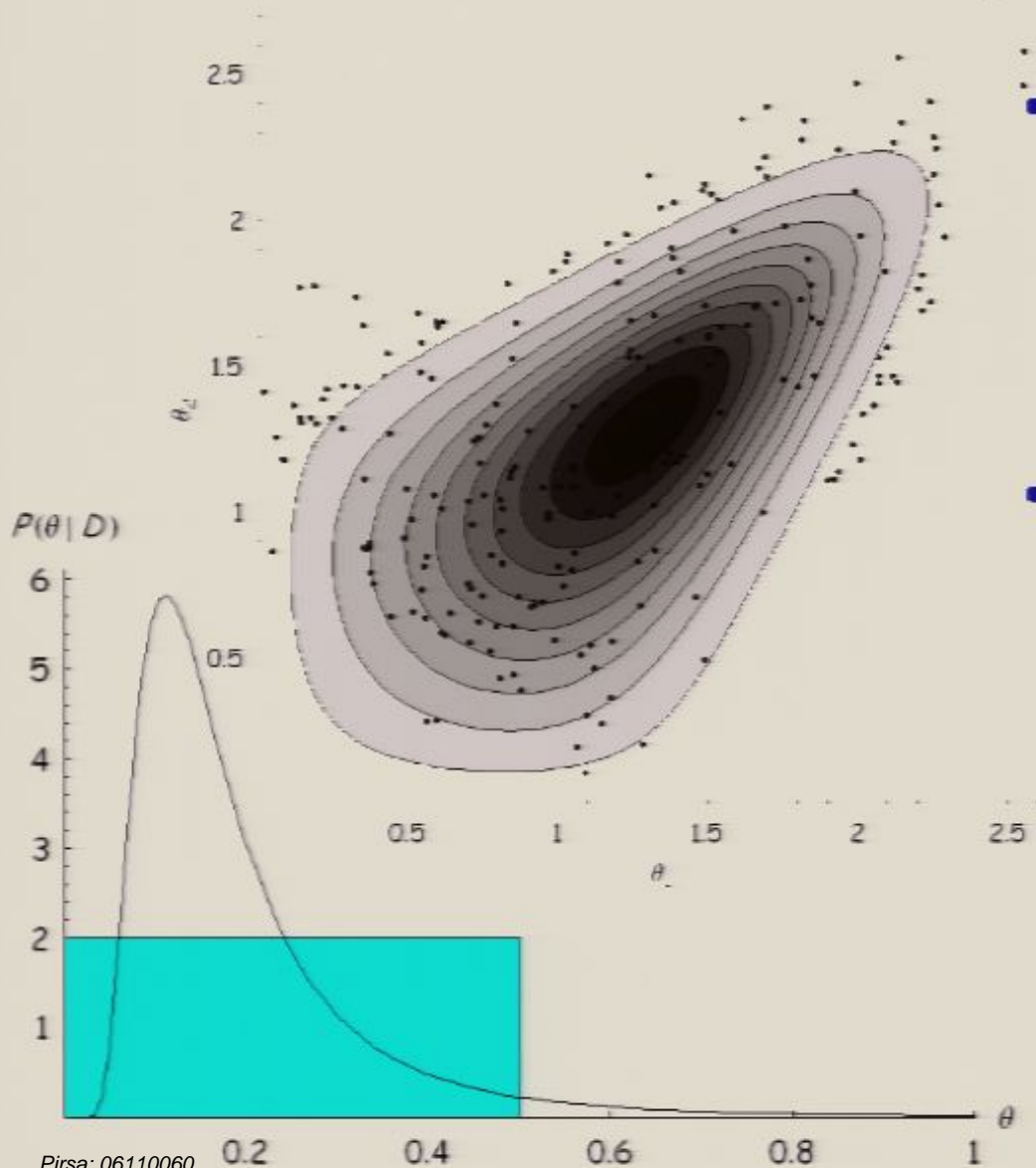
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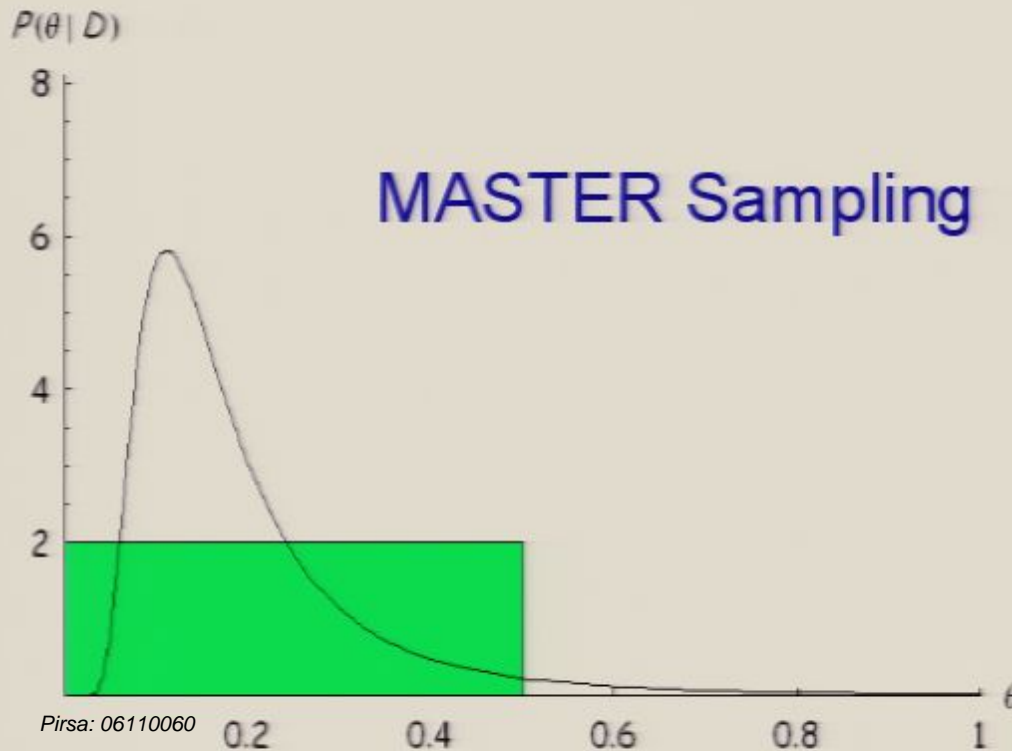
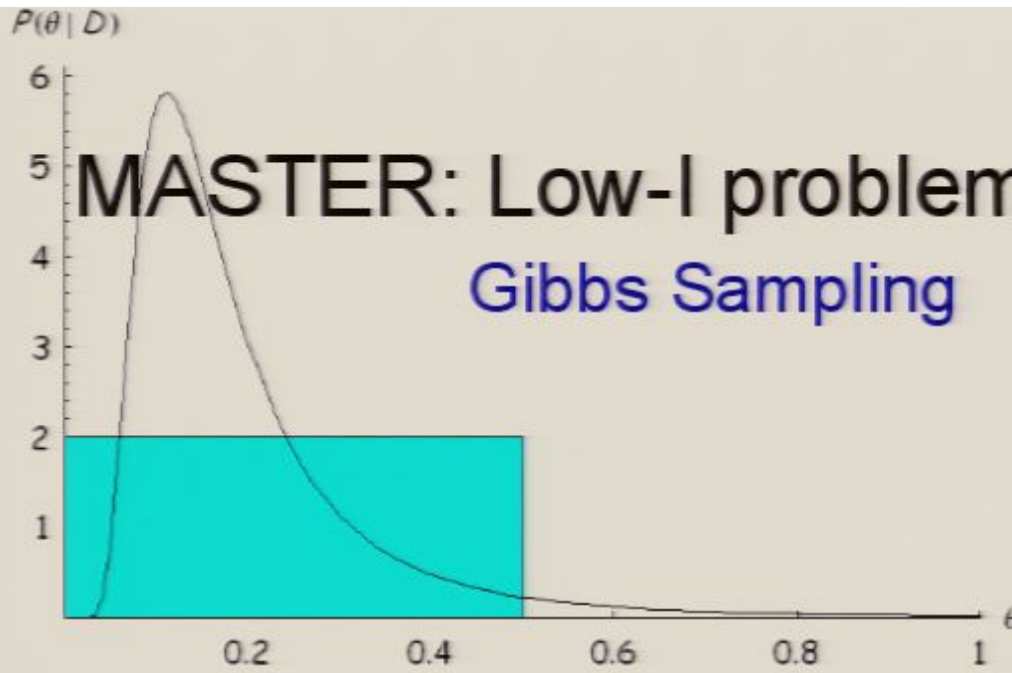


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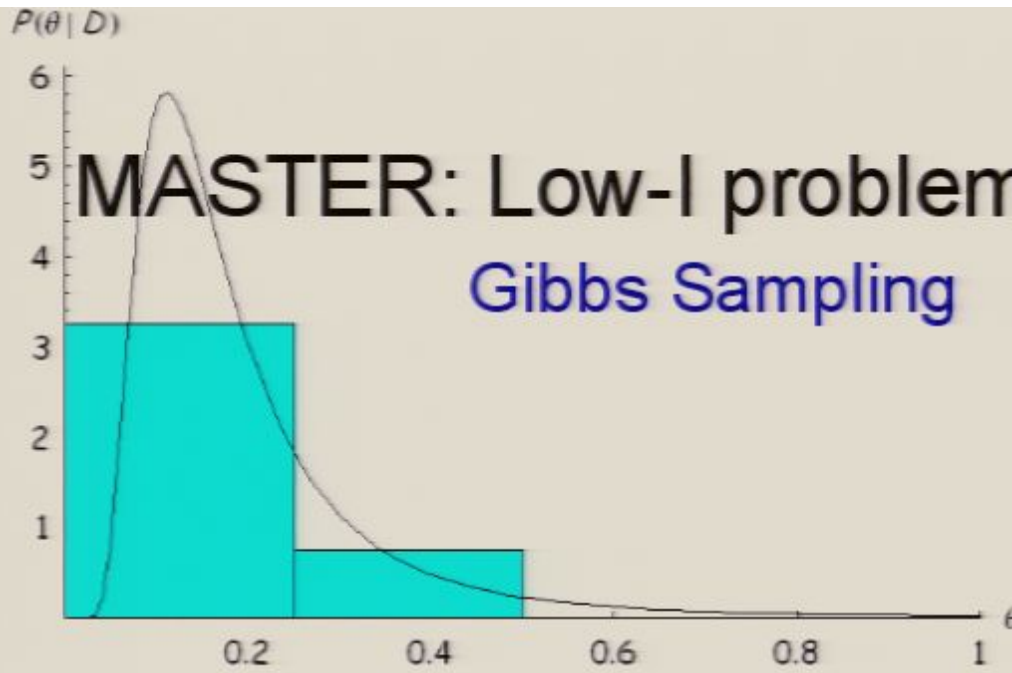
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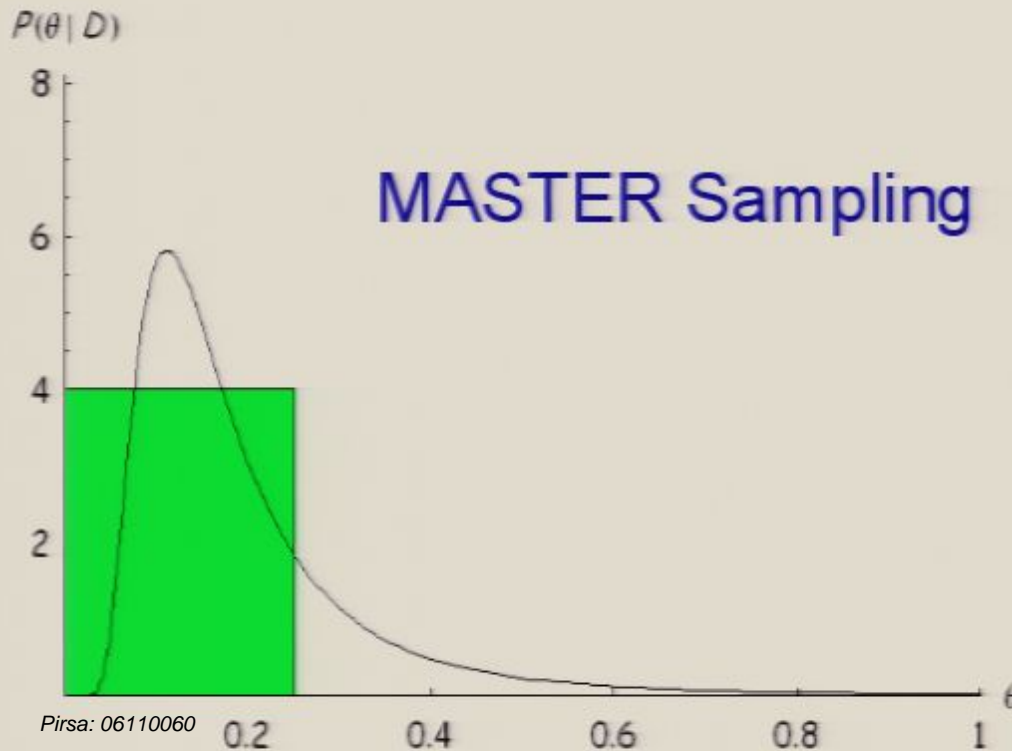
- MASTER: Low-l problems even for Perfect Data**
- Gibbs Sampling**
- What do the error bars from a MASTER-like approx. technique mean?
 - Here is an example using perfect, all sky data without any noise.
 - We focus on $\theta = C_4$.
 - In both cases, the solid line shows the actual likelihood for C_ℓ at $\ell=4$.
 - The samples are obtained using Gibbs sampling and by “regular” Monte Carlo, like MASTER.
 - MASTER is *neither frequentist*



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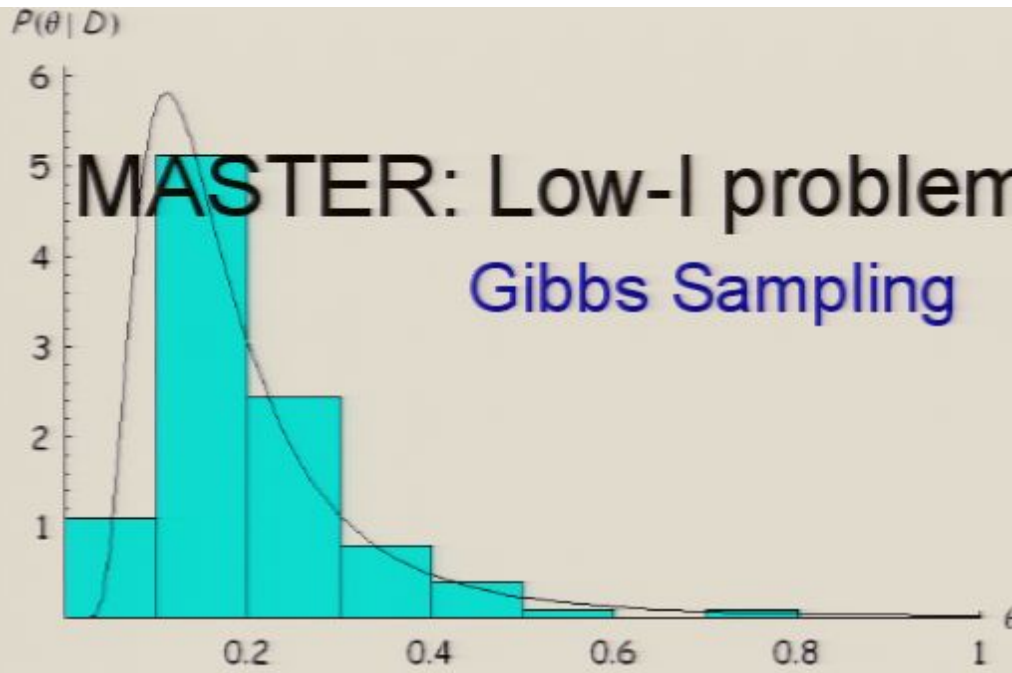
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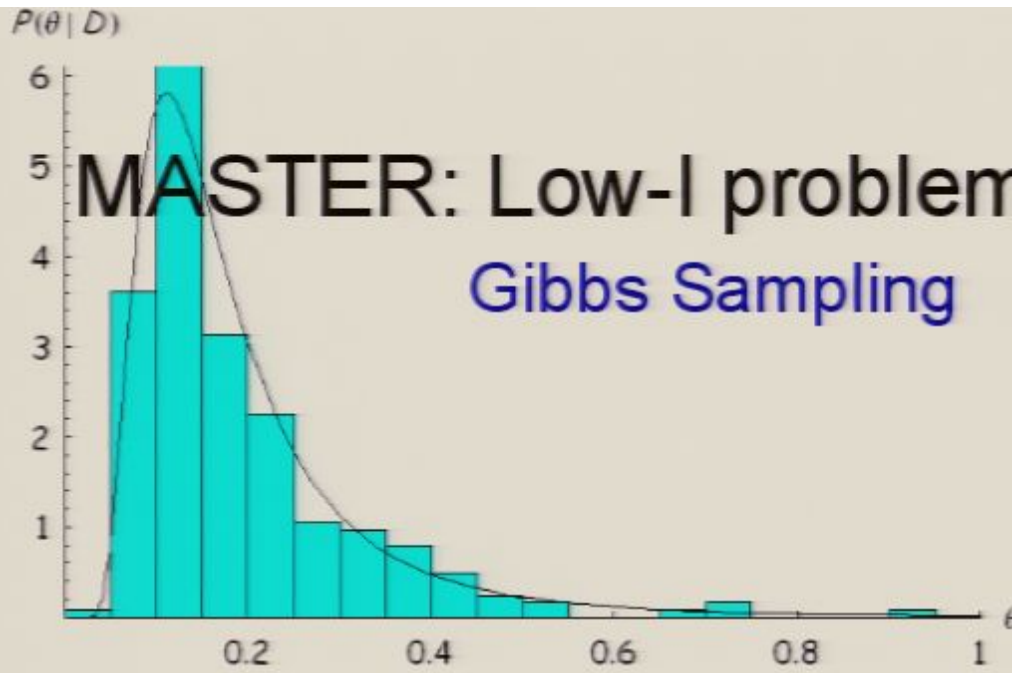
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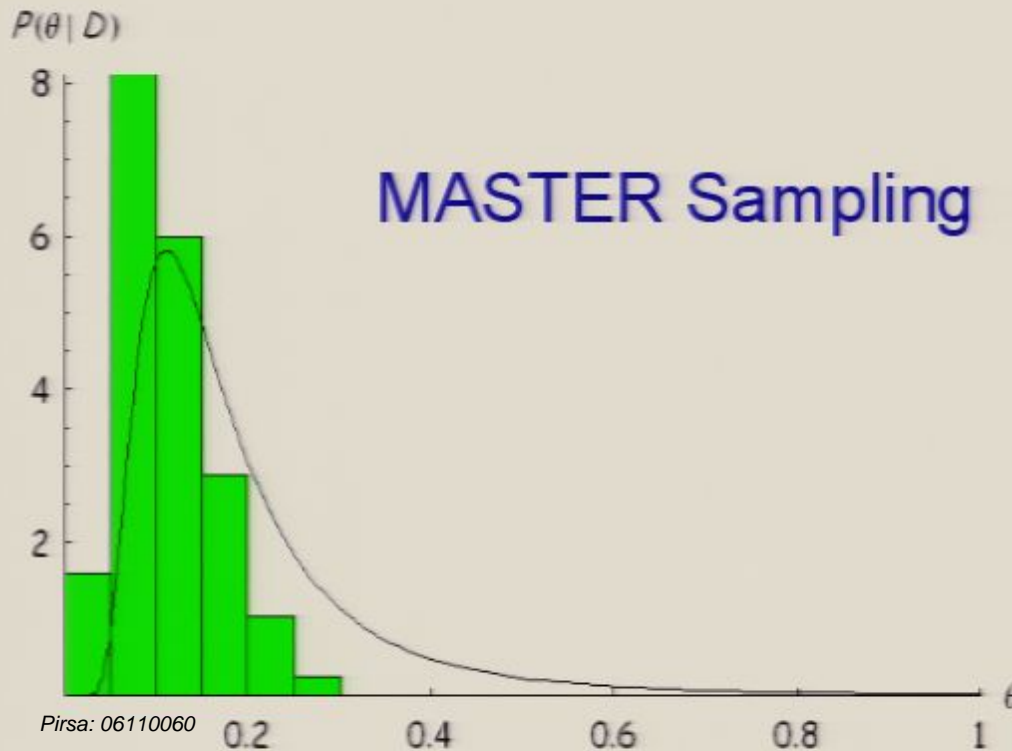
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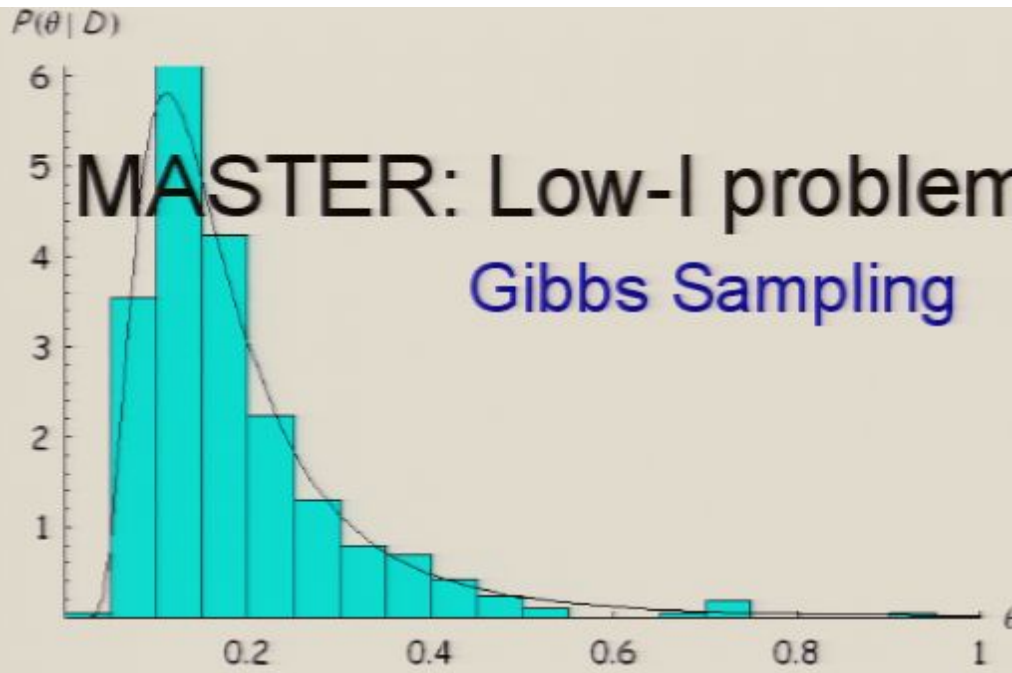
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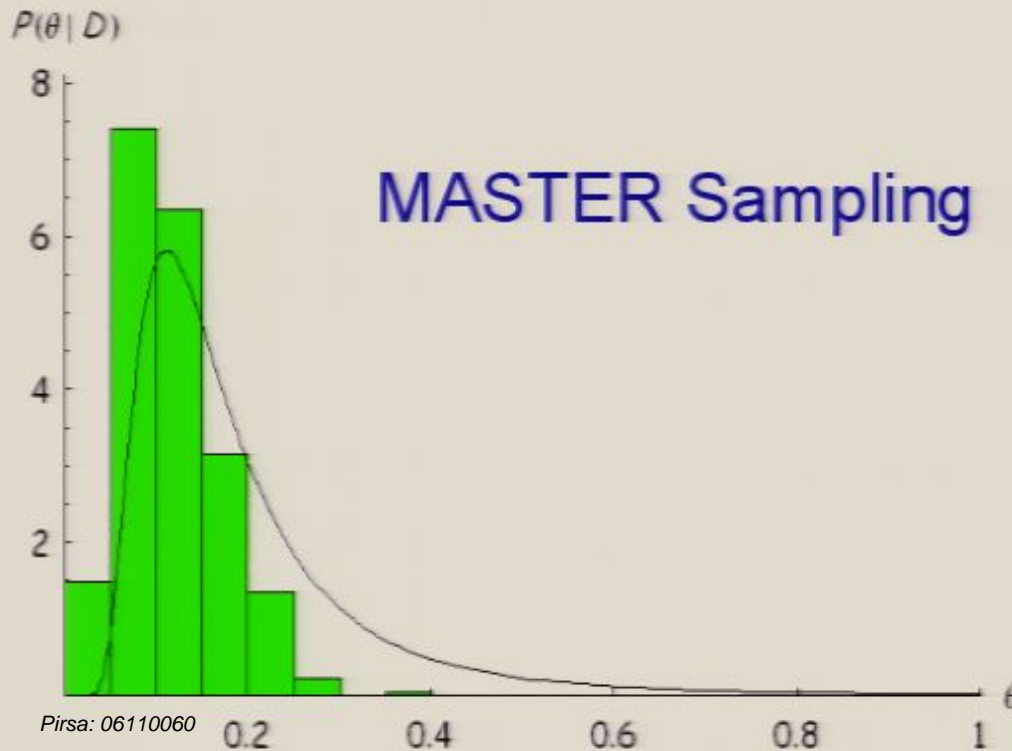
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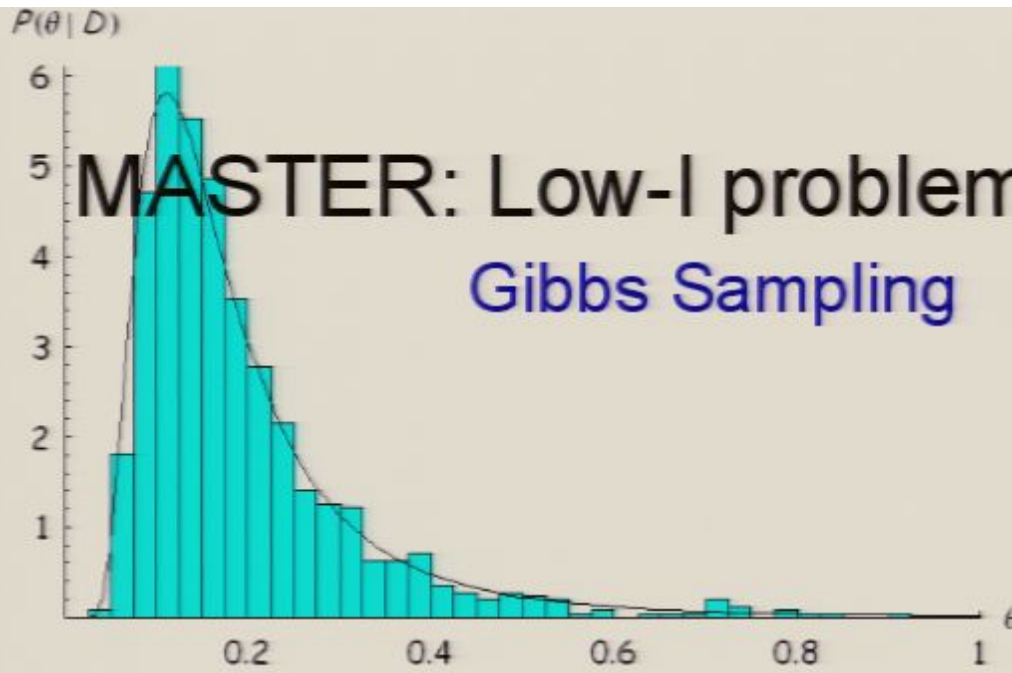
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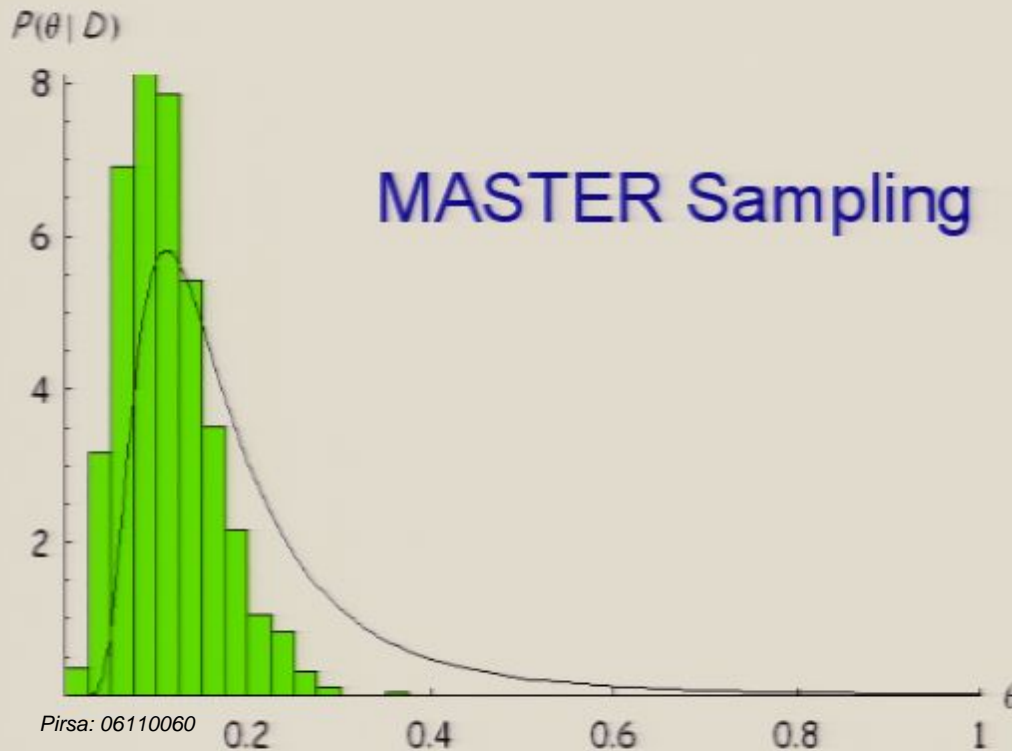
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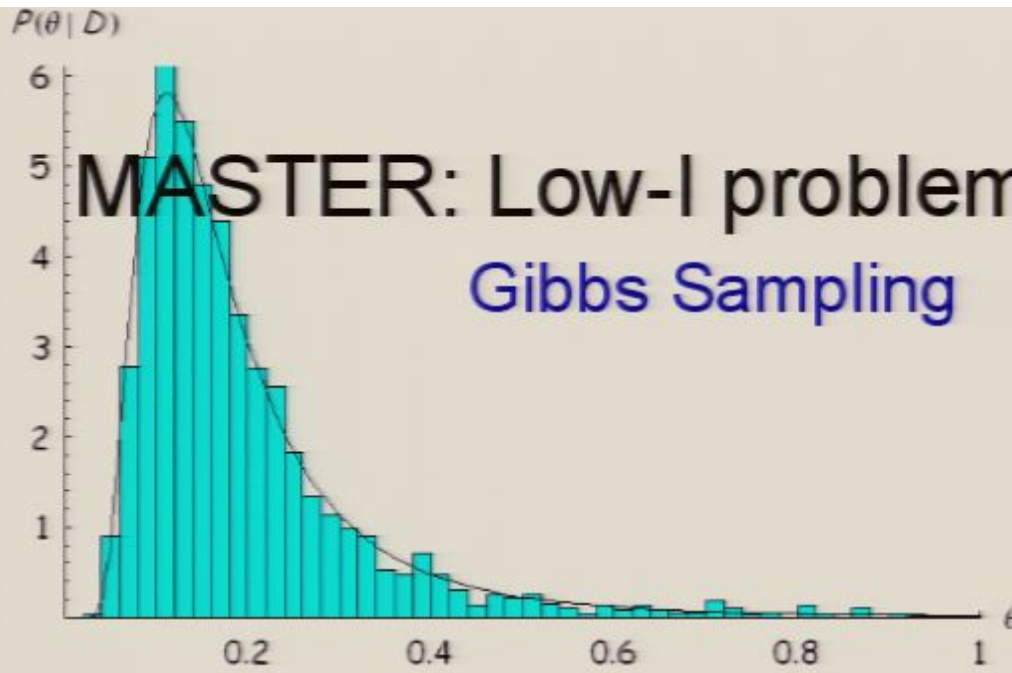
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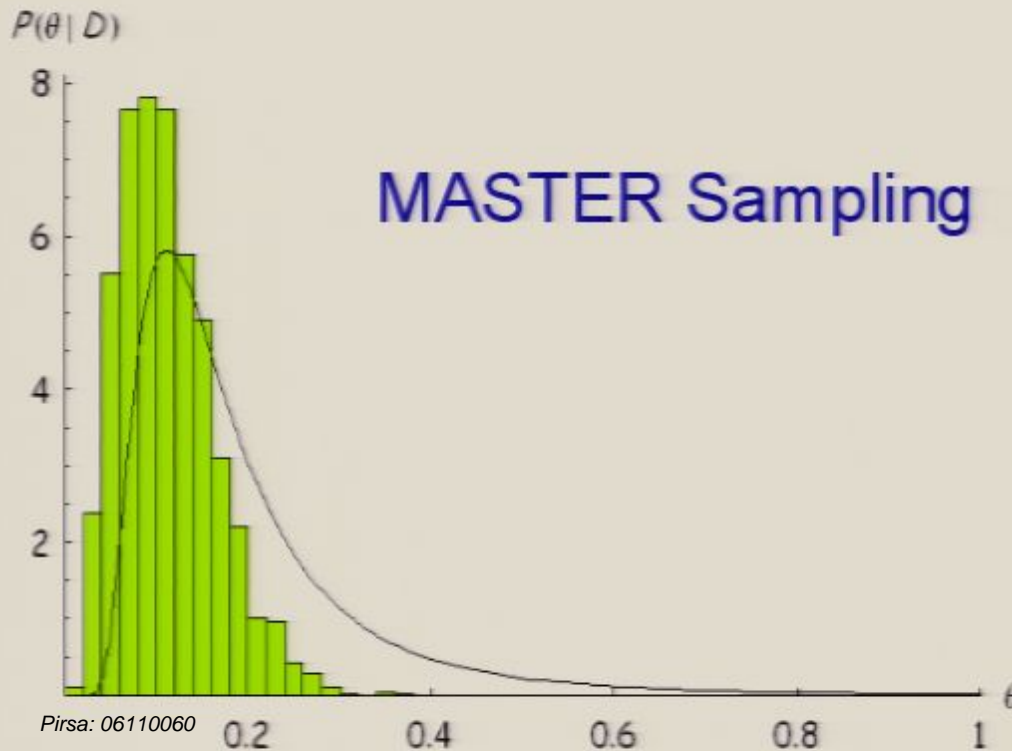
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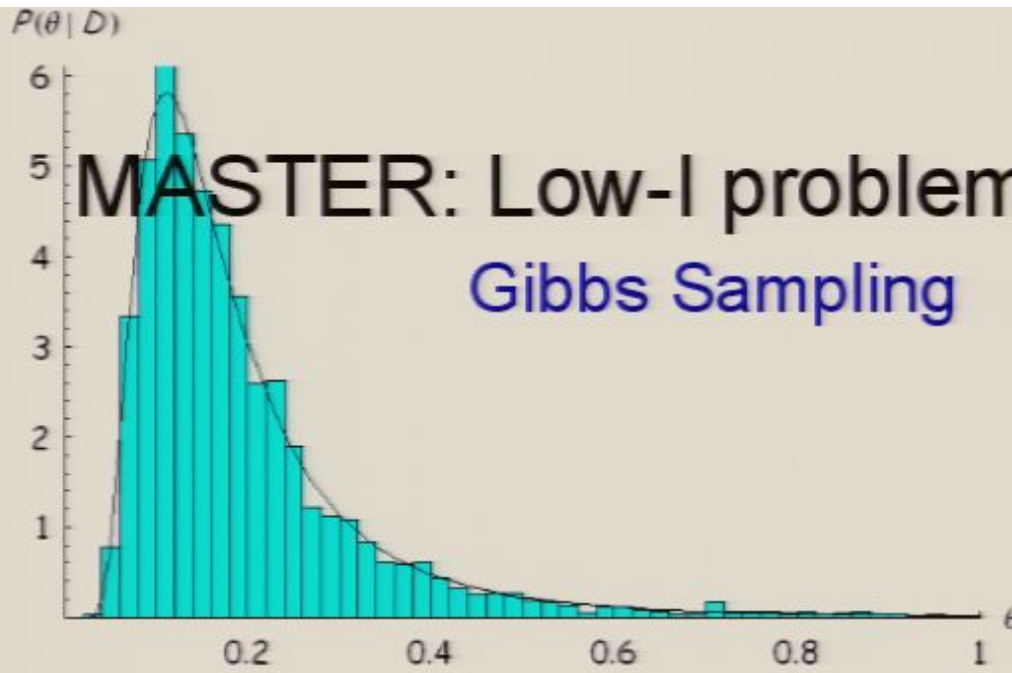
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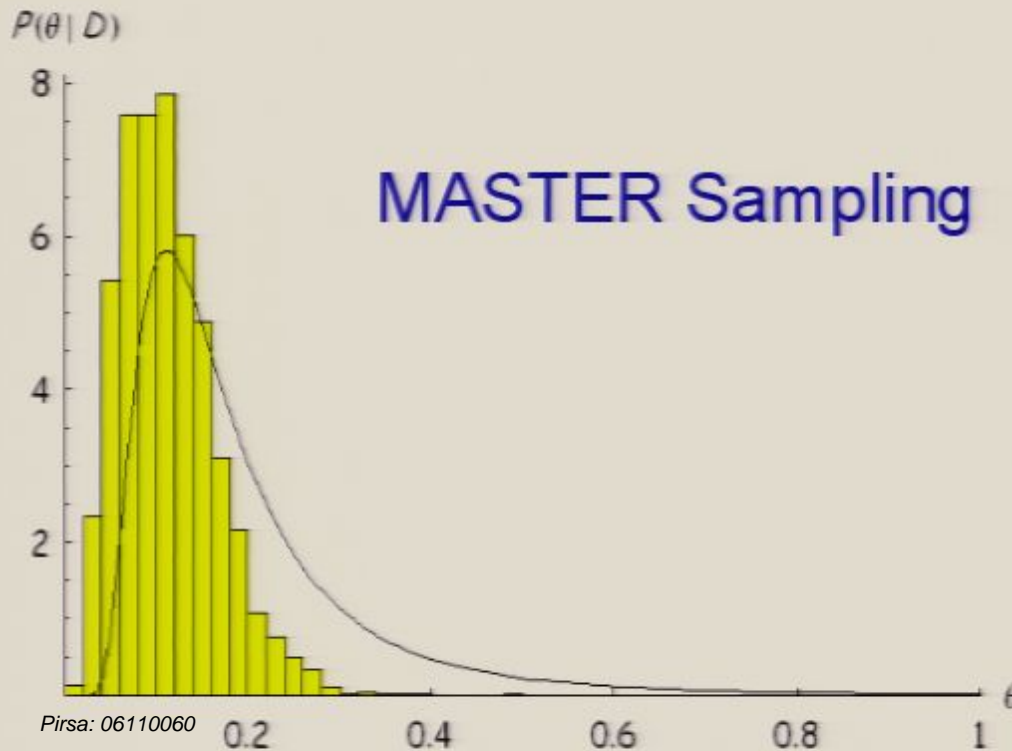
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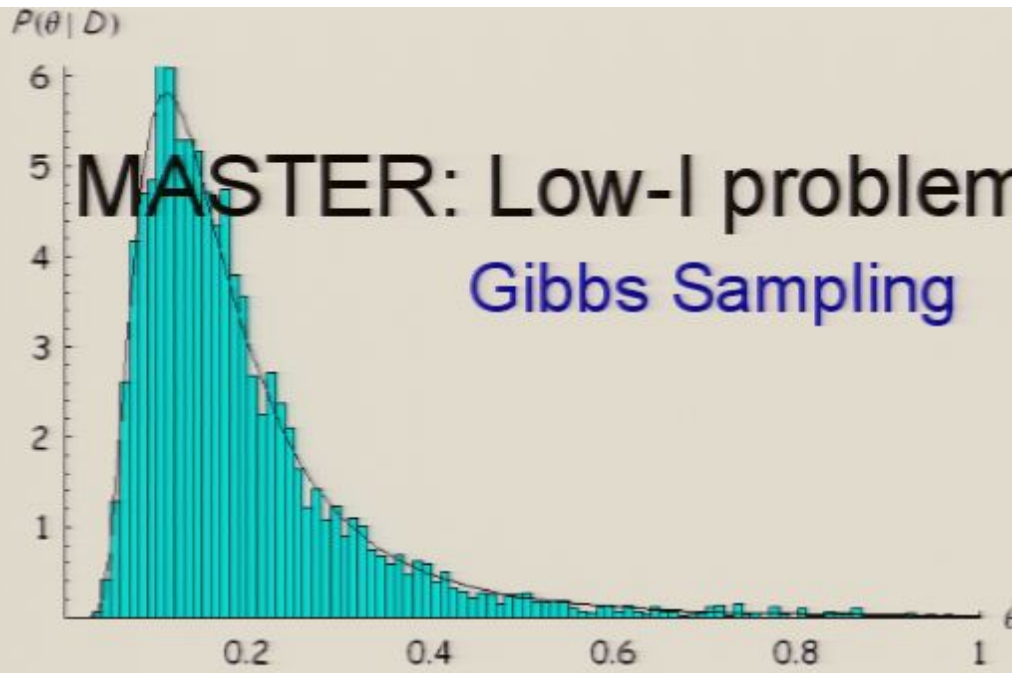
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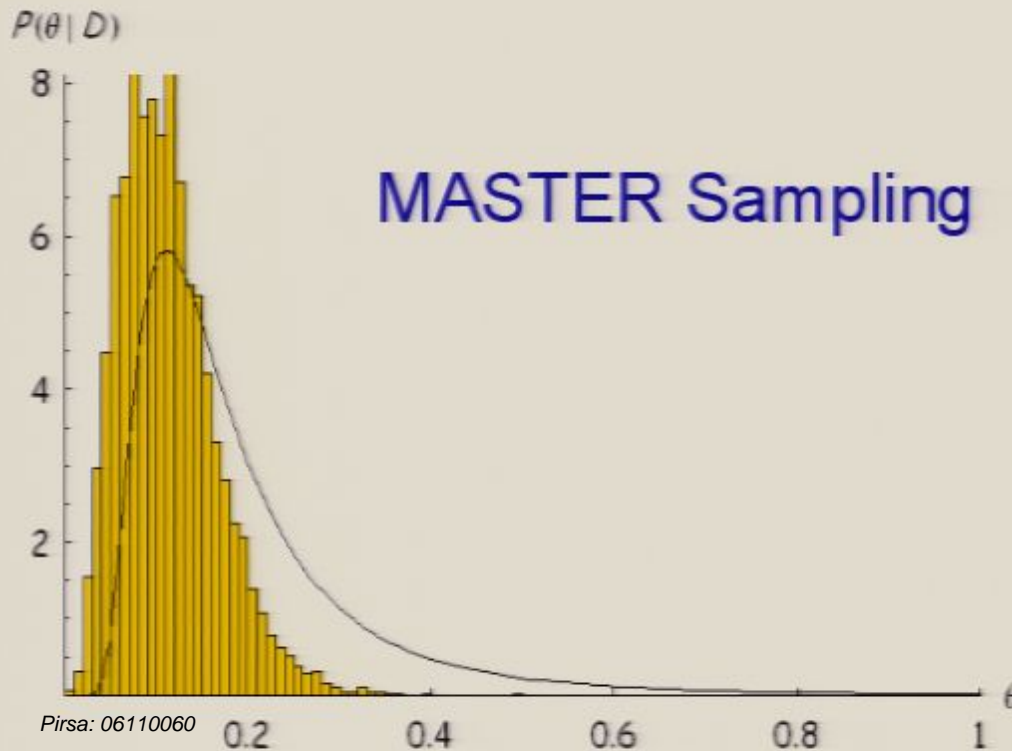
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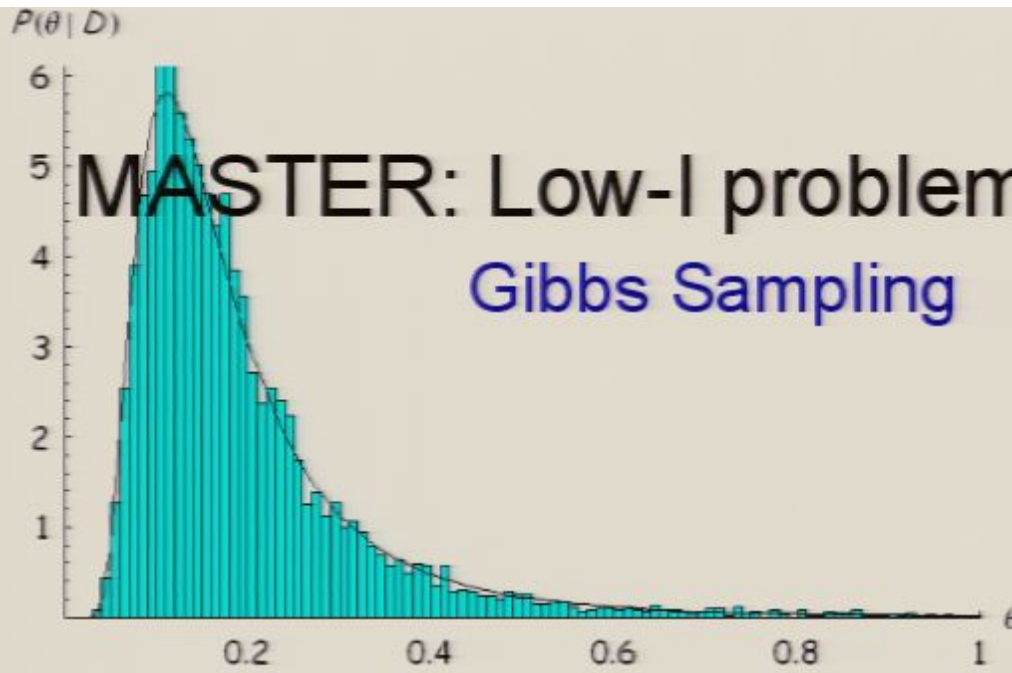
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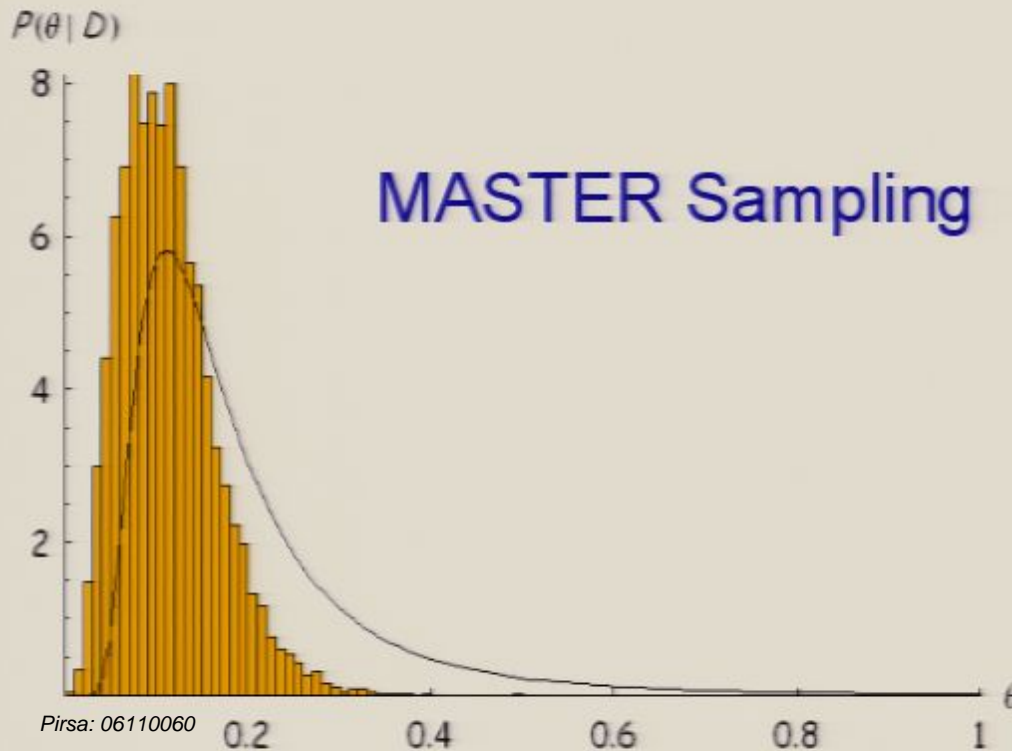
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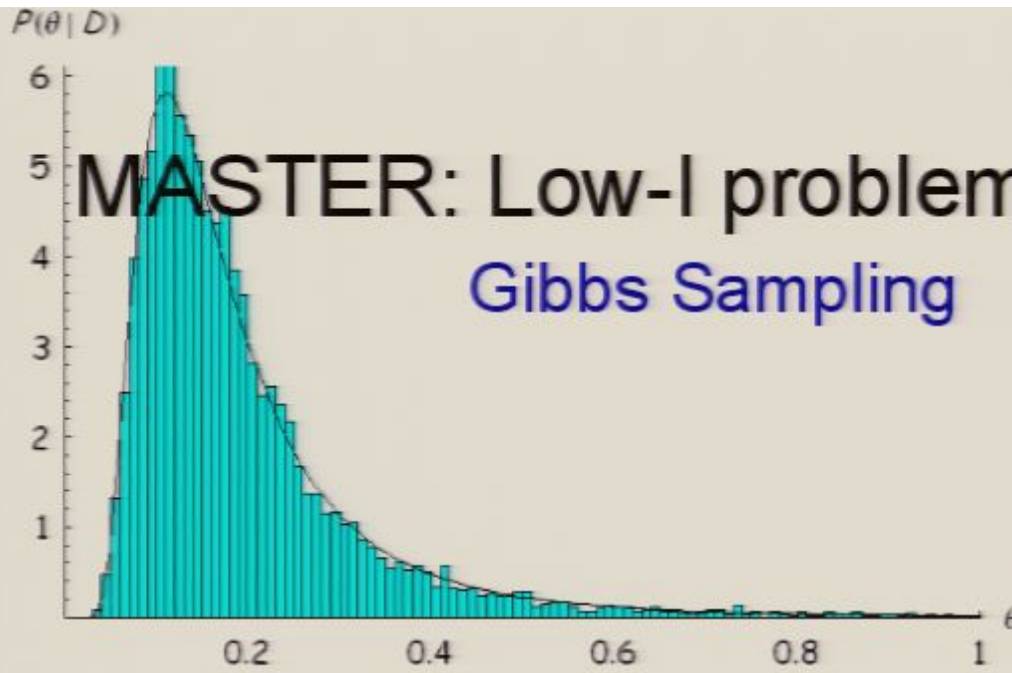
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MASTER Sampling

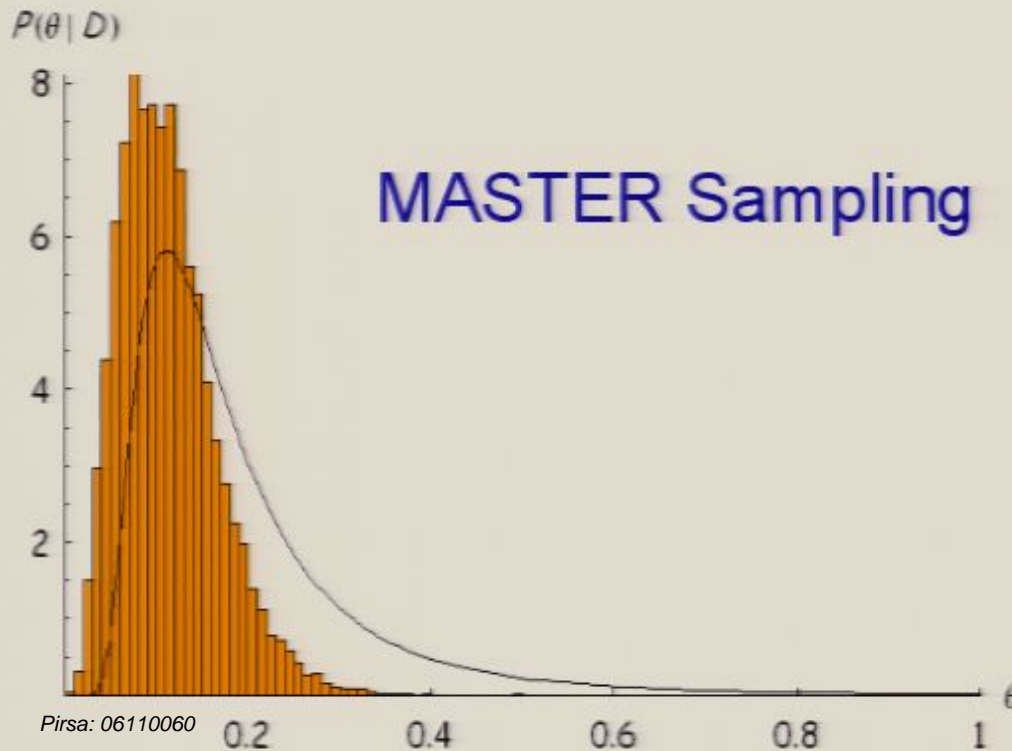
- We focus on $\theta = C_4$.
- In both cases, the solid line shows the actual likelihood for C_ℓ at $\ell=4$.
- The samples are obtained using Gibbs sampling and by “regular” Monte Carlo, like MASTER.
- MASTER is *neither frequentist*



MASTER: Low- ℓ problems even for Perfect Data

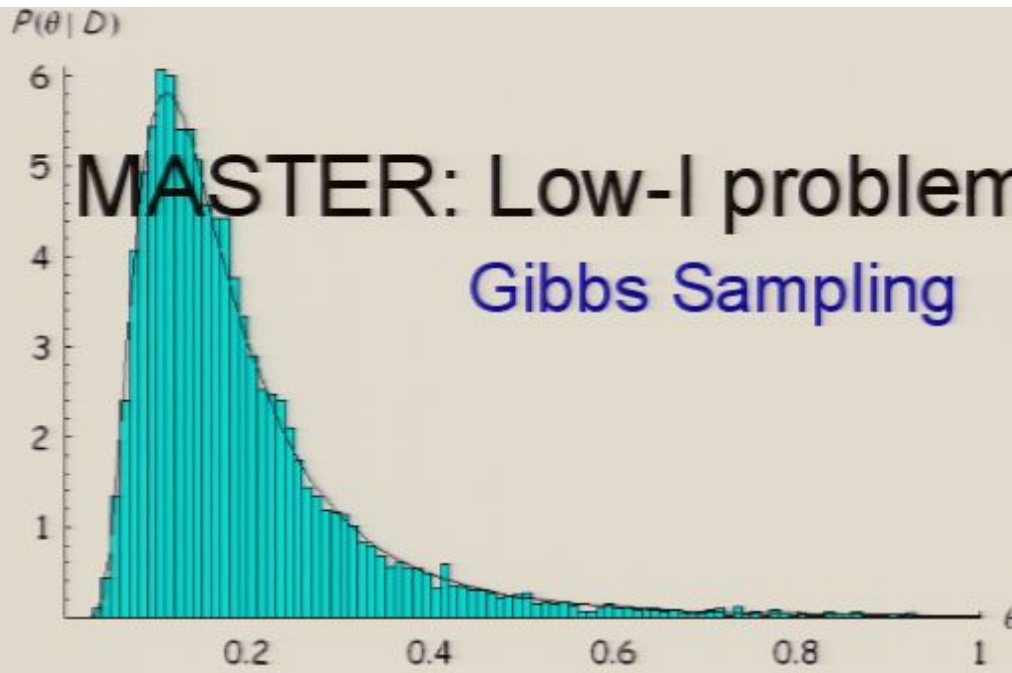
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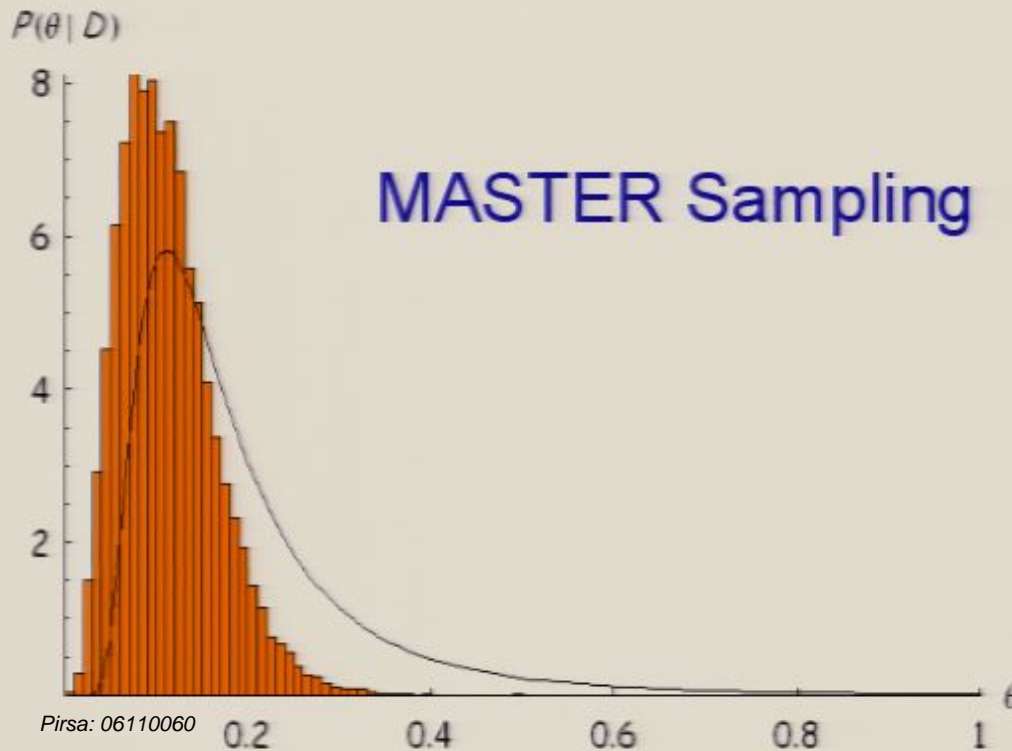
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MASTER: Low-I problems even for Perfect Data

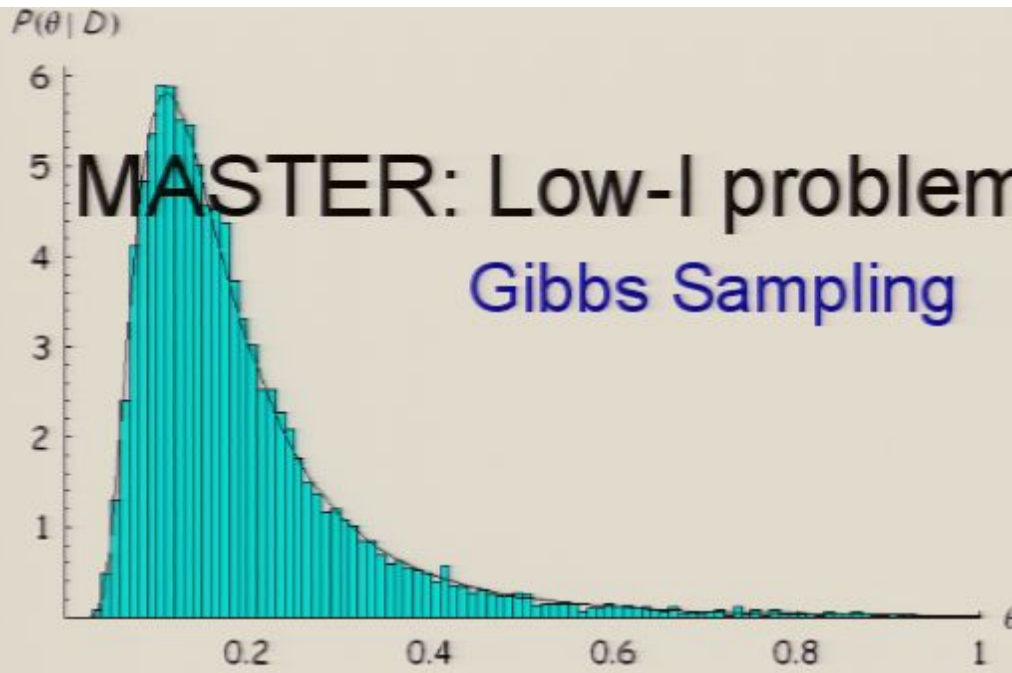
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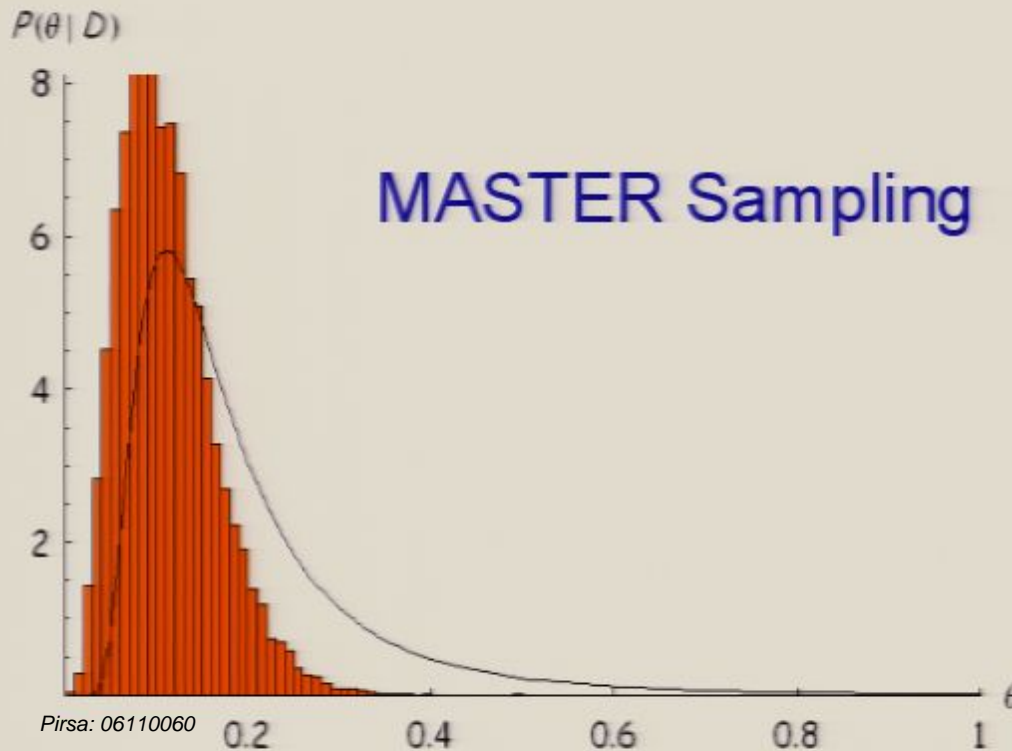
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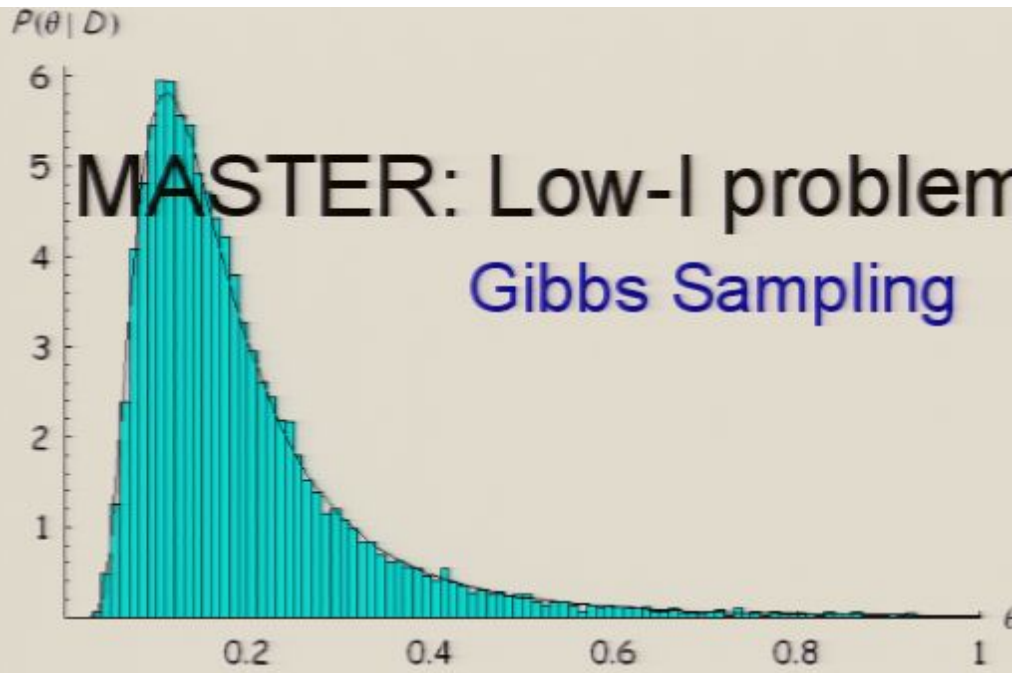
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MASTER Sampling

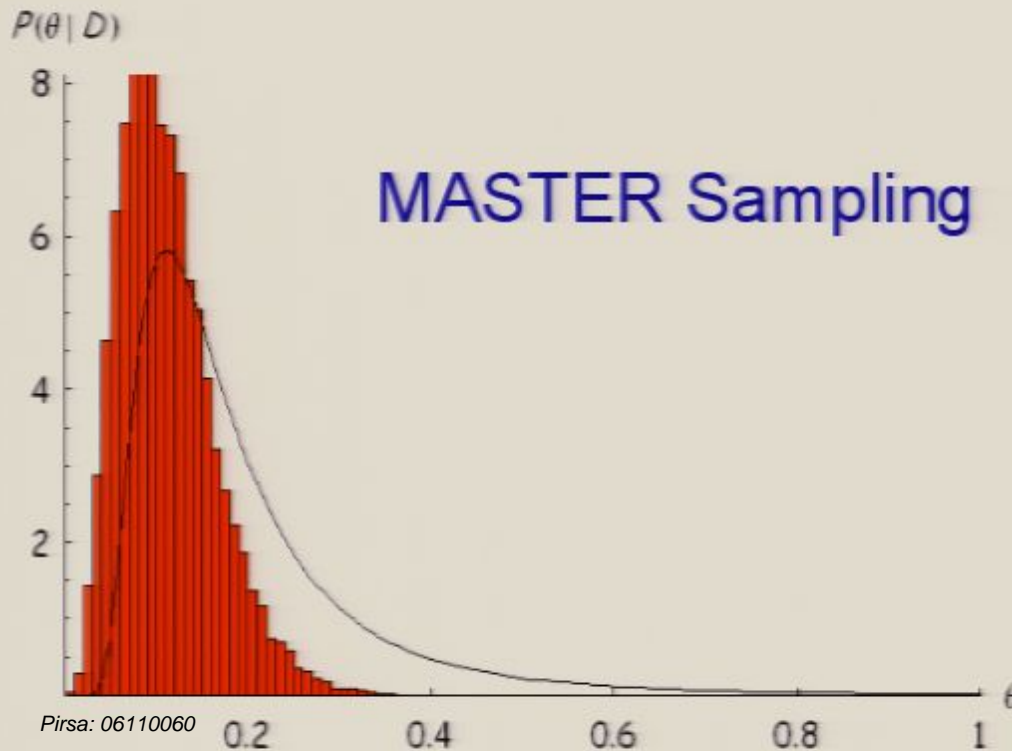
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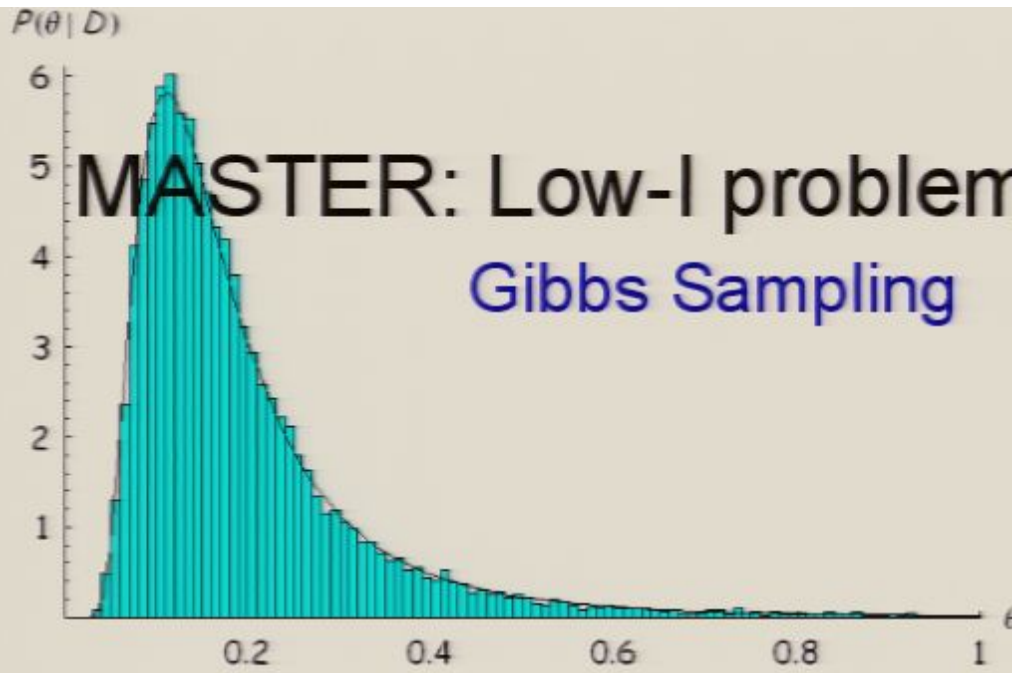
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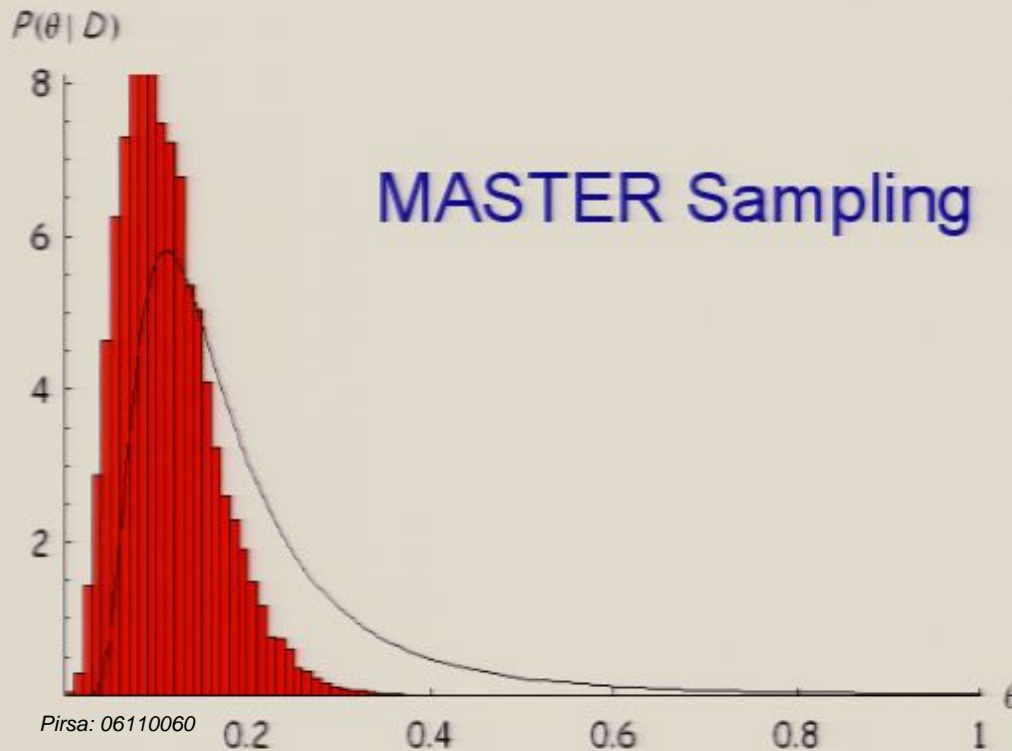
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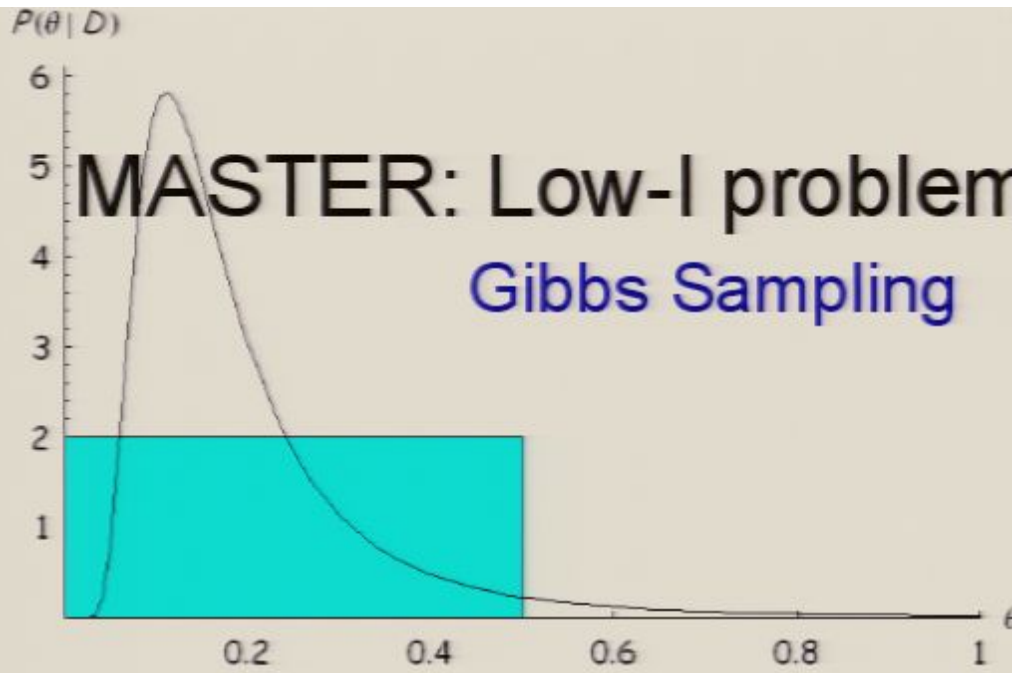
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MASTER Sampling

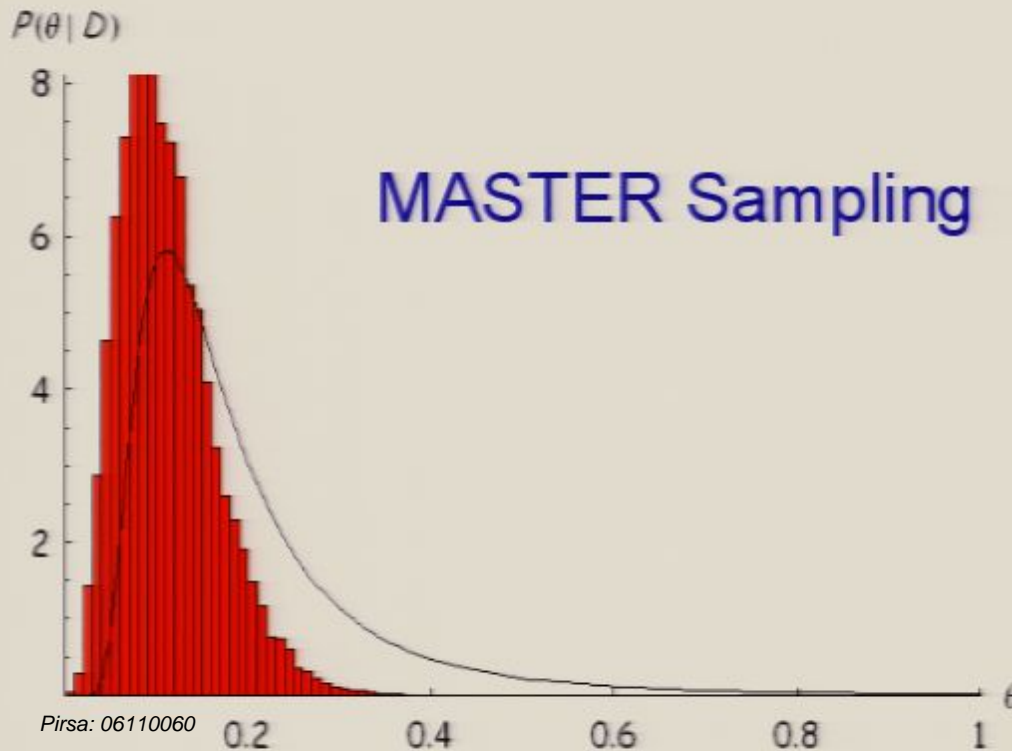
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MASTER: Low-l problems even for Perfect Data

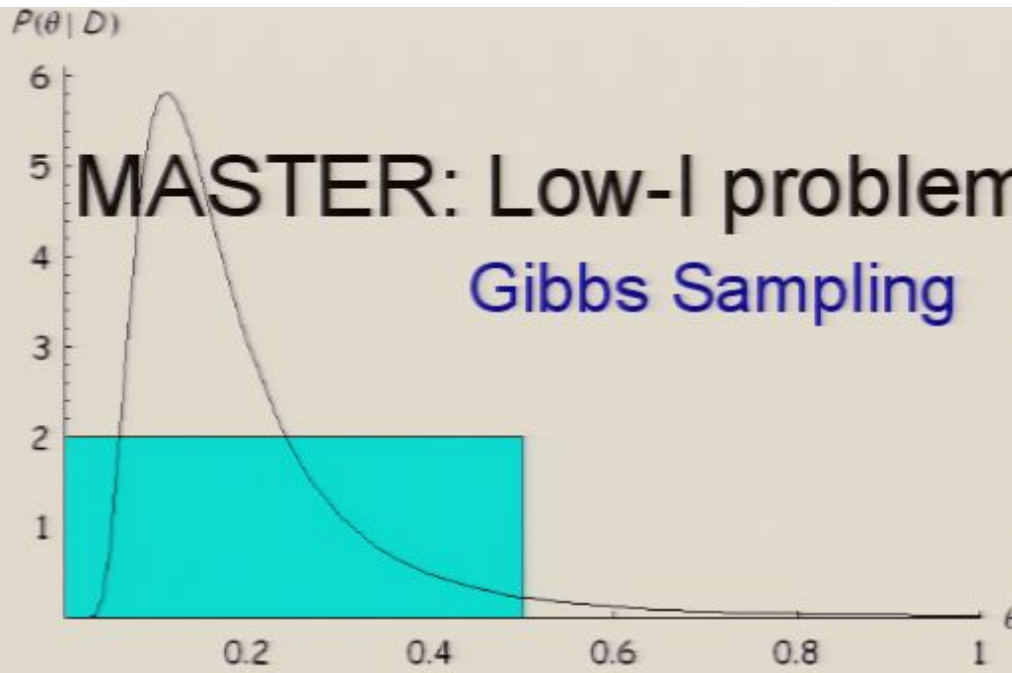
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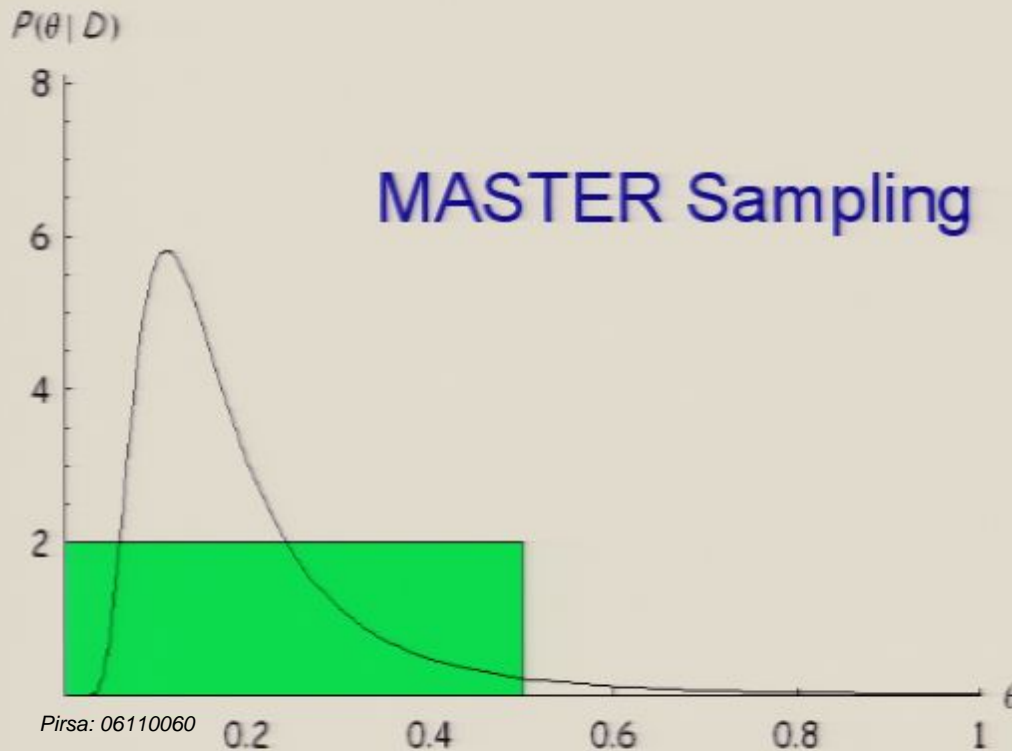
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The Team

(alphabetical by institution)

- IUCAA, IIT Kanpur
 - Tarun Souradeep and students (IUCAA, IIT Kanpur)
- JPL/Caltech
 - Jeff Jewell
 - Ian O'Dwyer
 - Krzysztof Górski
- Max Planck Institut für Astrophysik
 - Anthony Banday
- University of California at Davis
 - Lloyd Knox
 - J. Dick
- University of Illinois at Urbana-Champaign
 - Ben Wandelt
 - Greg Huey
 - David Larson
- University of Oslo
 - Hans-Kristian Eriksen
 - Frode Hansen

Eriksen et al., ApJ in press,
astro-ph/0606088

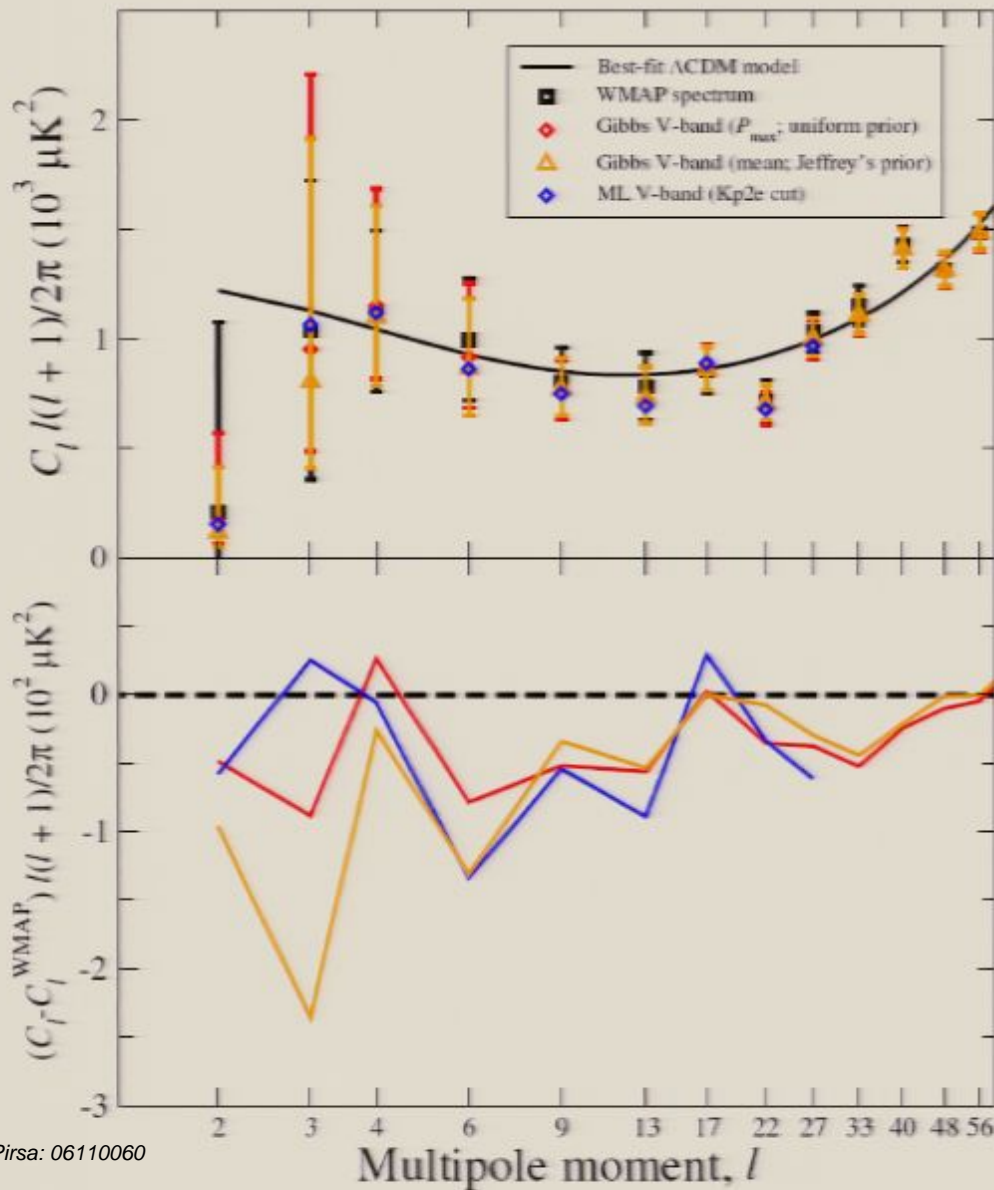


WMAP re-analysis

- Cross-checking approach adopted throughout
- 5 different research groups (JPL, Illinois, Oslo, “India,” Davis)
- 4 different analysis methods
 - Gibbs sampling (2 versions of priors)
 - Maximum likelihood
 - Metropolis Hastings exploration of exact low-l likelihood
 - MASTER (two different foreground treatments)
- This approach allows us to check for not just for systematic differences in the analysis but also various other errors (data handling etc...)



Results of low l analysis

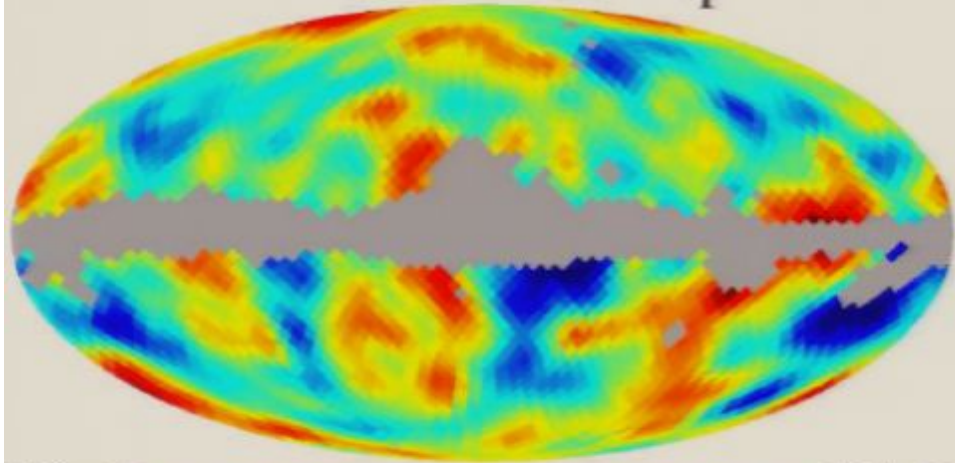


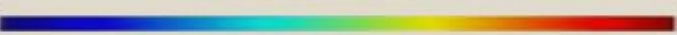
- First result: very good agreement l by l for all methods.
- Second look: the small offset compared to WMAP spectrum is correlated across l .
- These correlated deviations can int



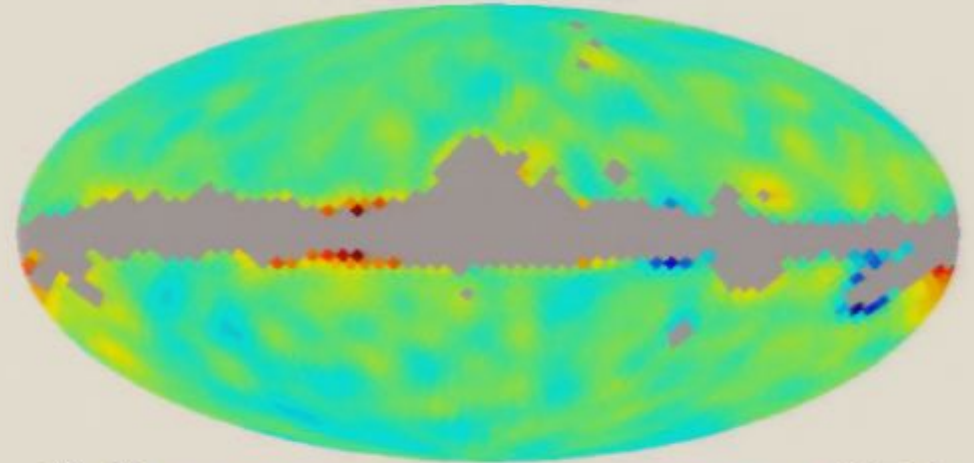
Masking issues in the WMAP3 analysis

De-biased ILC map



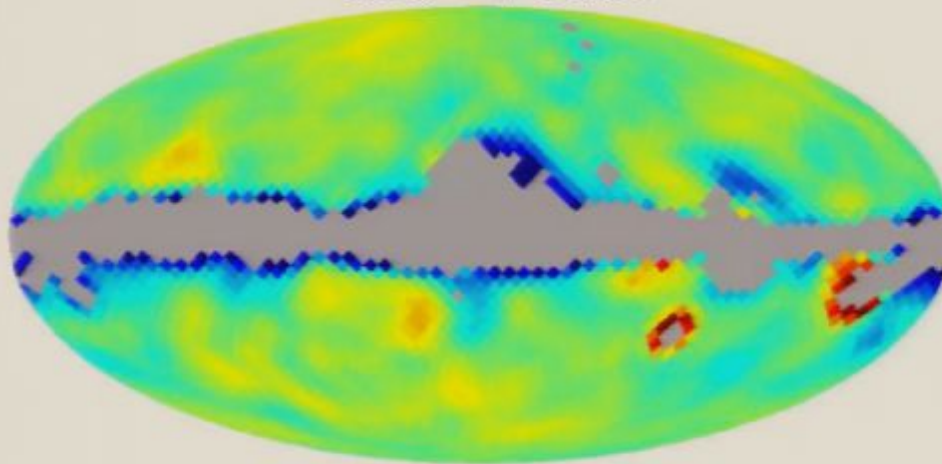
-100 μK  100 μK

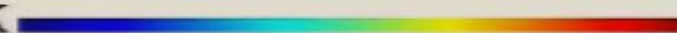
V-band - W-band



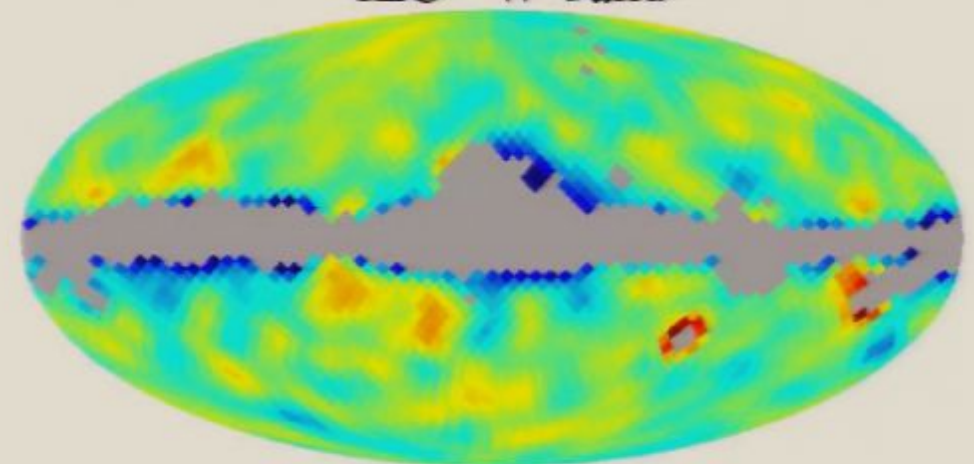
-20 μK  20 μK

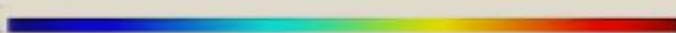
ILC - V-band



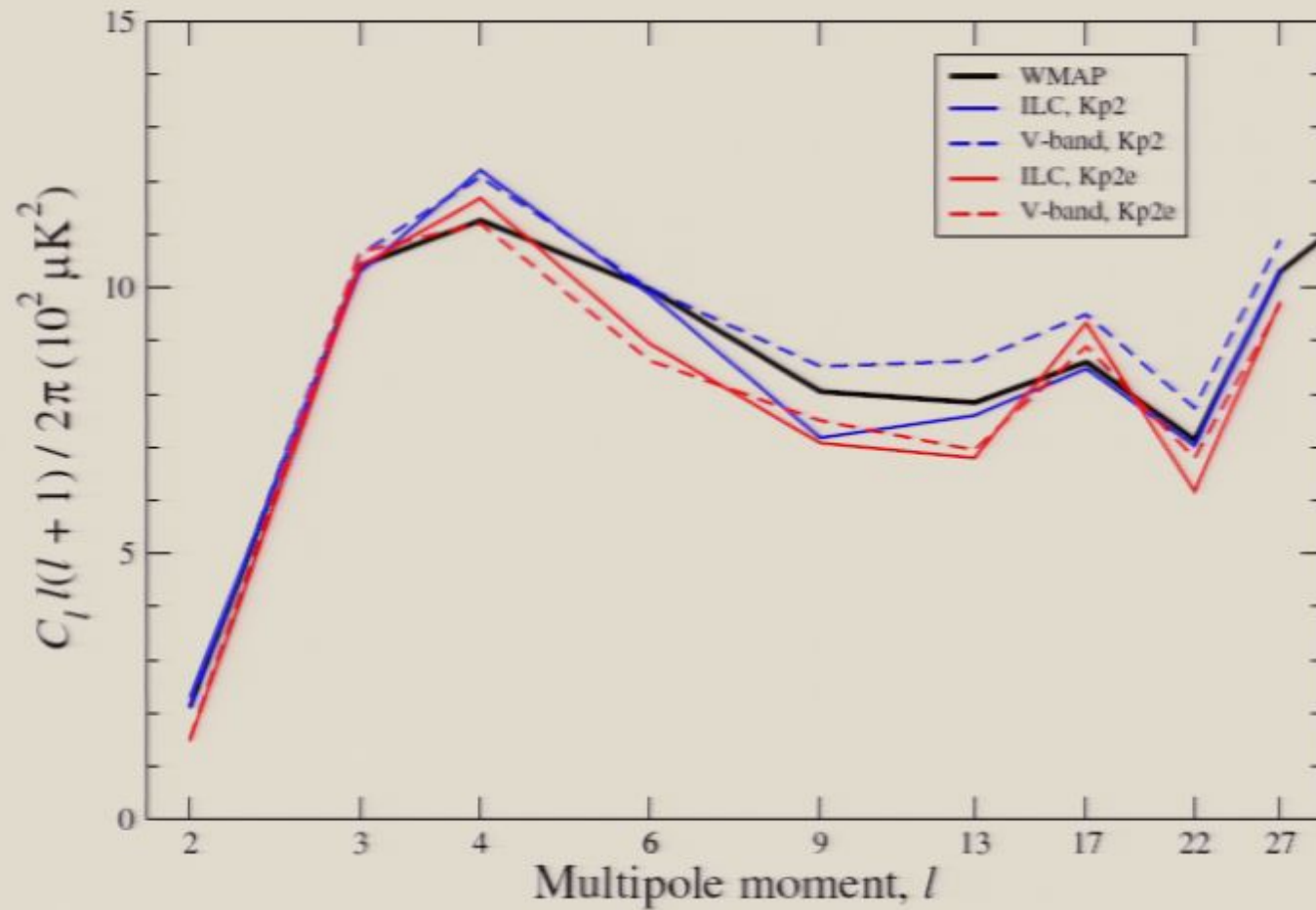
-20 μK  20 μK

ILC - W-band

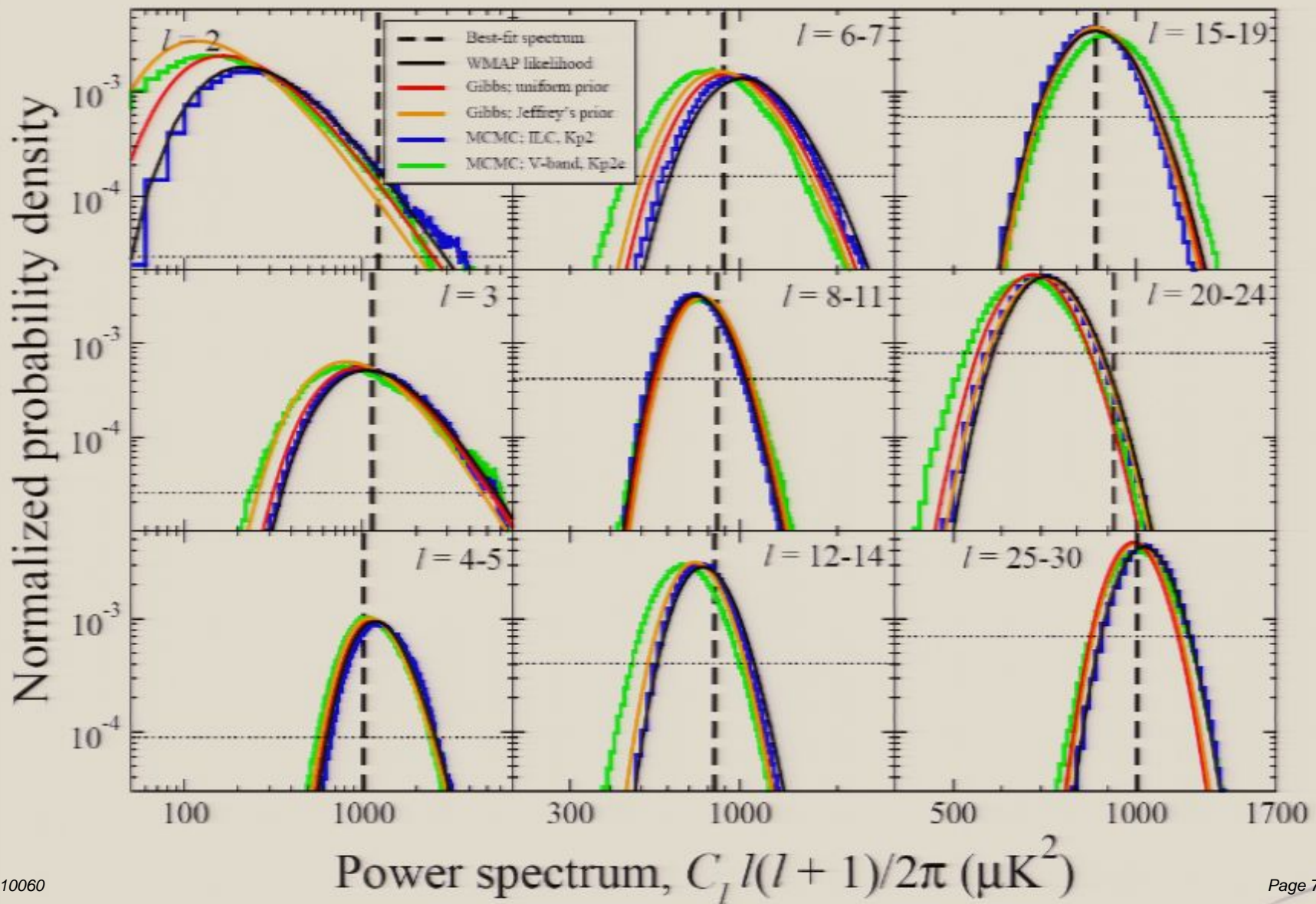


-20 μK  20 μK

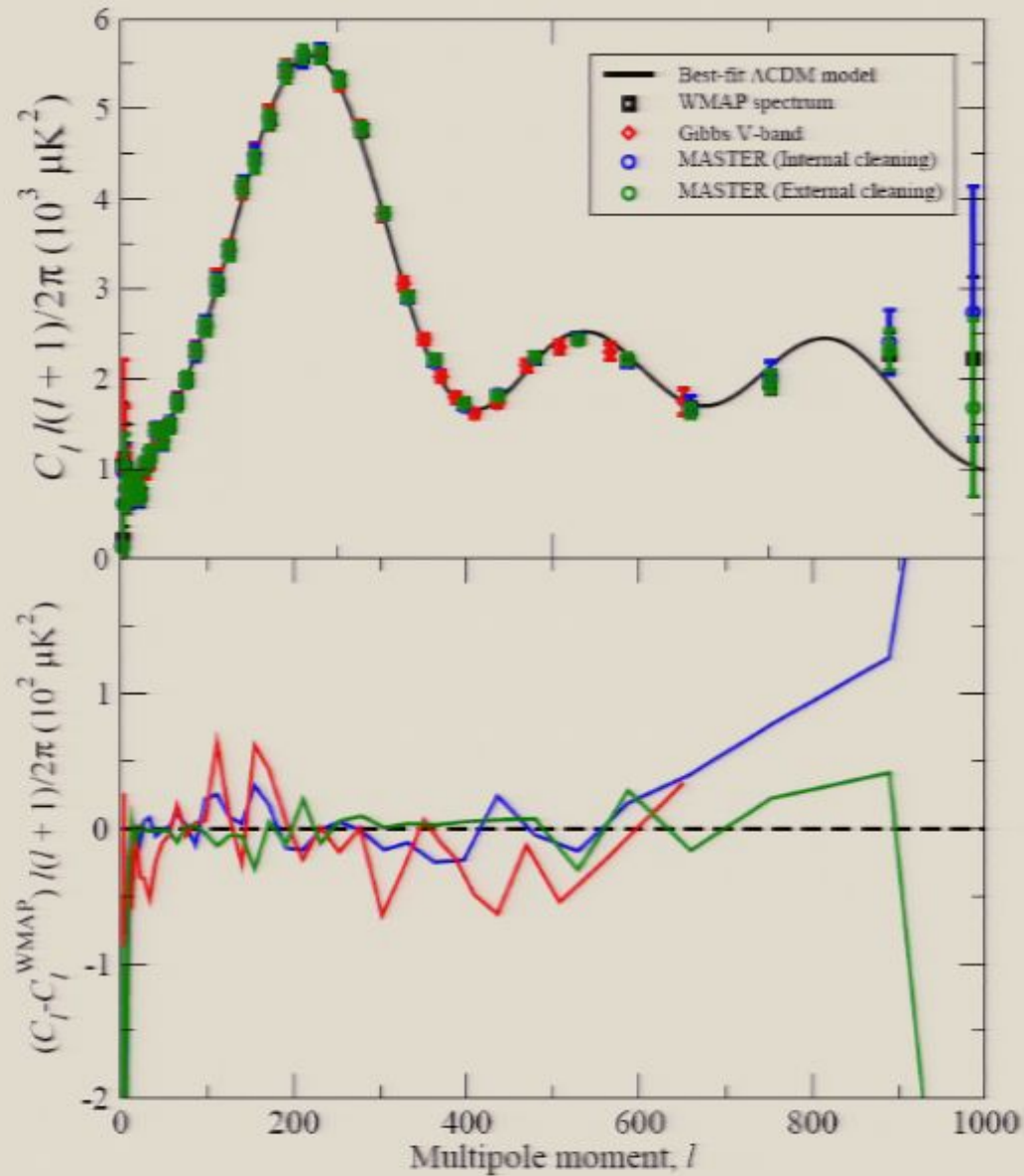
Masking effects on low l power spectrum estimates



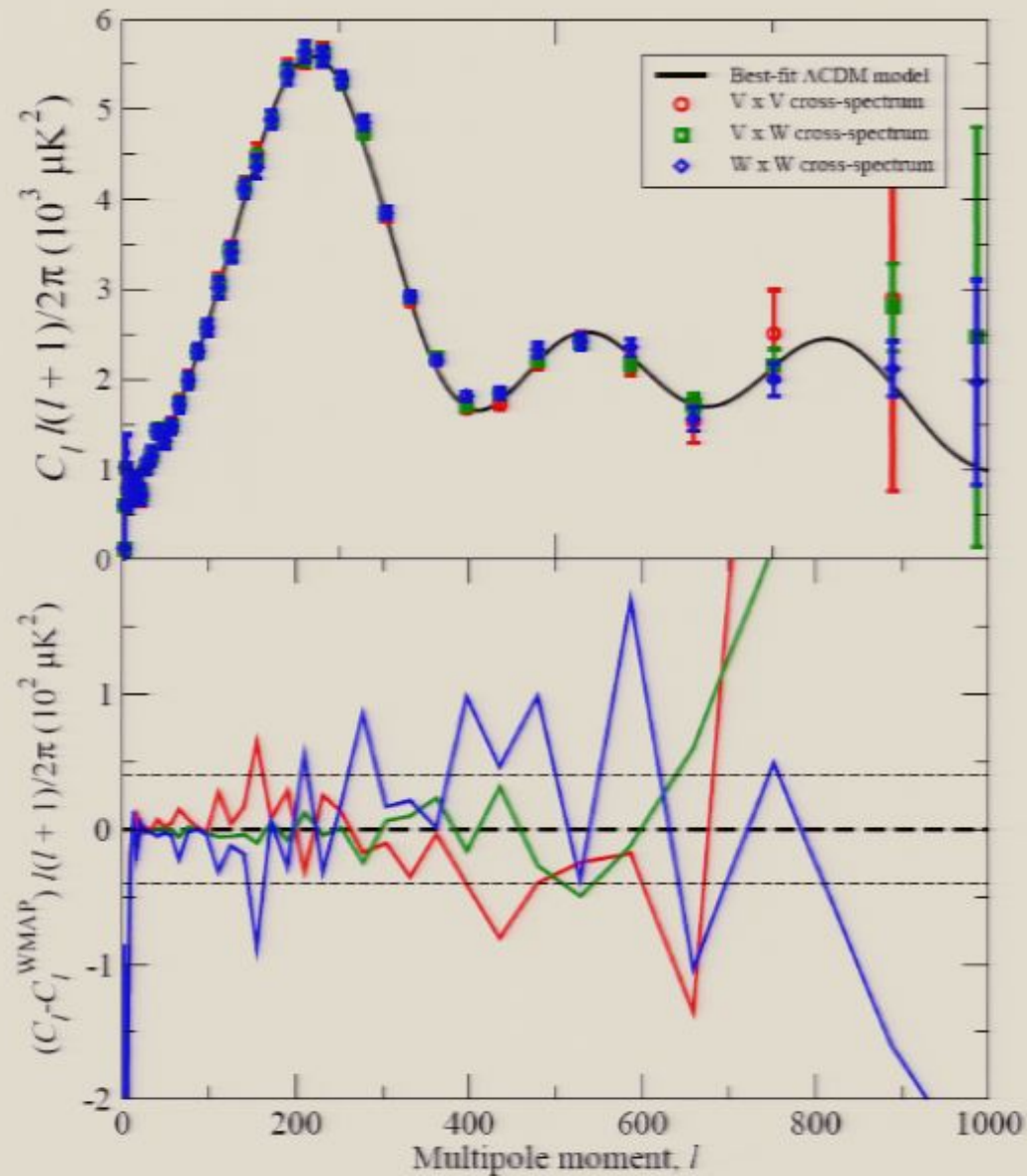
Low l likelihoods and posteriors for C_l



High resolution (all-scale) analysis



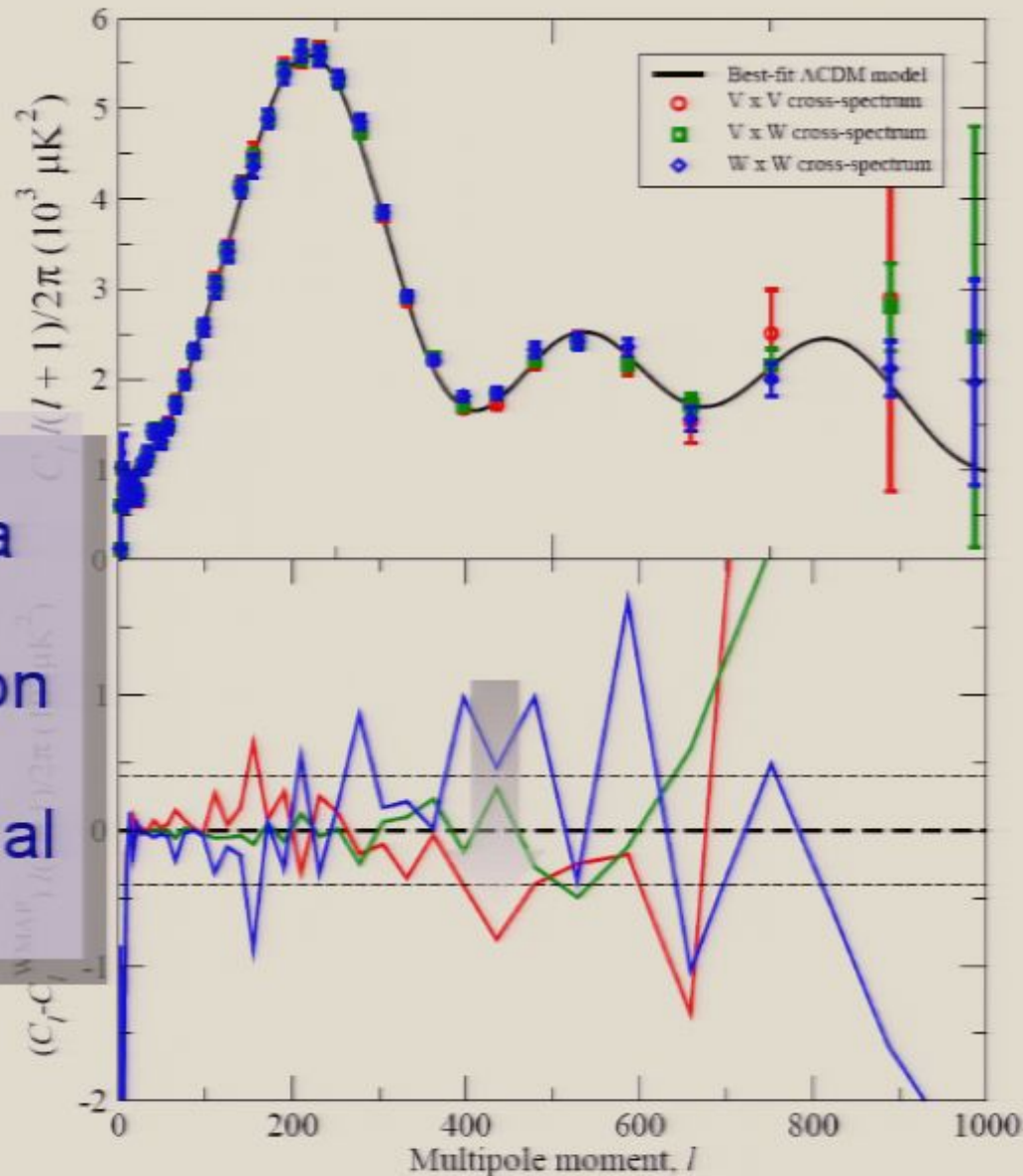
MASTER for Individual Frequency Combinations



Band shows error estimate due to beam asymmetries



MASTER for Individual Frequency Combinations



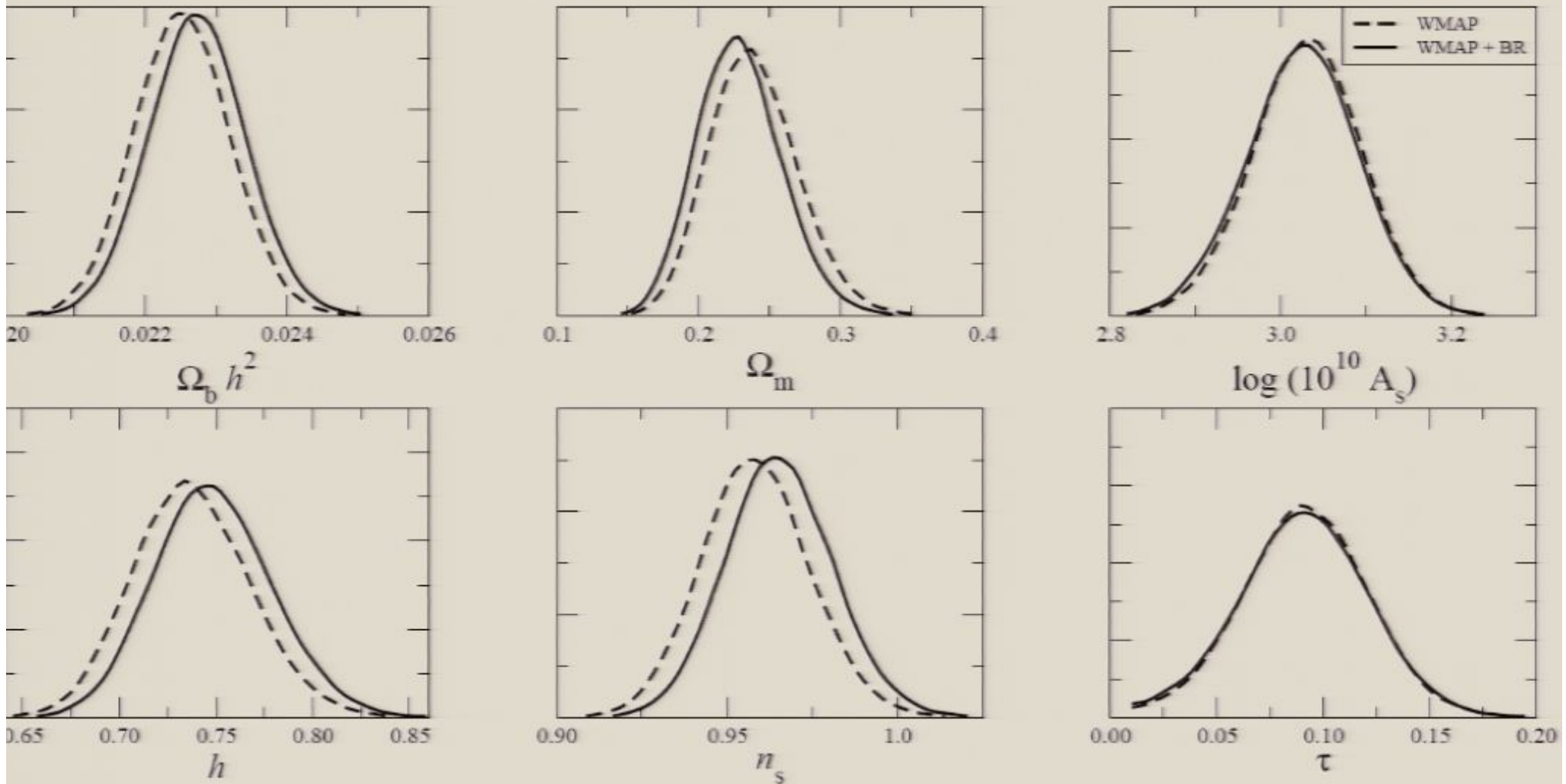
This effect is consistent with a different point source population

Huffenberger et al 2006

Band shows error estimate due to beam asymmetries



Cosmological implications



Conclusions

- Statistically rigorous analysis of the CMB is now feasible using Bayesian sampling techniques (Gibbs sampling).
- Pseudo- C_ℓ techniques are very convenient, but the error bars are “special,” especially at low l . This is dangerous when S/N is ~ 1 .
- The WMAP 3-year power spectrum contains a low- l bias, at l around 30
- The WMAP 3-year data power spectrum also contains a bias at high l (400-600) which is consistent with overcorrection for point sources
- The net result of these biases is reduced evidence for $n_s < 1$:
 - The exact low- l likelihood reduces significance from 2.7σ to 2.3σ
 - A new high l point source correction (Huffenberger et al) further reduces the significance to 2σ .
- The new version of the WMAP3 likelihood code on LAMBDA gives results consistent with these conclusions.

