Title: Kahler Moduli Inflation

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Kähler Moduli Inflation

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work in progress

Introduction

There are many suggested models of string theory inflation:

- D3/anti-D3 branes in a warped geometry
- D3/D7 branes
- axion/moduli fields ...

)ifferent inflationary scenarios can be realized in different regions of he string theory landscape.

low "general" are different inflationary scenarios?

s inflation a generic prediction of stringy motivated models?

Brane inflation models: highly fine-tuned, e.g. to avoid heavy inflaton problem ("η-problem") (D3/anti-D3 KKLMMT).
Similarly, most supergravity models suffer from the η-problem.
Alternatives: moduli fields lifted by non-perturbative effects.

We will focus on Kähler moduli of the type IIB string theory compactification on a Calabi-Yau (CY) manifold.

Any realistic stringy model of inflation, formulated in terms of an effective 4d supergravity, must

- have all moduli (axion-dilaton, complex structure and Kähler moduli, brane positions) stabilized
- have at least one modulus ("inflaton") with a potential flat enough to provide a slow-roll evolution

Most known examples of stabilization include:

- "No-scale" models — Giddings, Kachru, Polchinski, 2001, ...

  Both dilaton and complex structure moduli stabilized with fluxes in IIB string theory compactification on a CY manifold: effective 4d supergravity superpotential is

$$W_0 = \int_{CY} G_3 \wedge \Omega$$

where $G_3 = F_3 - iSH_3$ is the flux 3-form.
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- KKLT model Kachru, Kallosh, Linde, Trivedi, 2003

In addition to the above, the overall volume of CY is stabilized by non-perturbative effects: euclidean D3 brane instanton or gaugino condensate on D7 worldvolume. The superpotential is

$$W = W_0 + Ae^{-a\sigma}$$

with σ the modulus controlling the size of CY.
The model requires fine-tuning of the parameters,

$$W_0 \sim 10^{-4}$$

in order to have an AdS minimum and volume large enough.

- Heterotic flux compactifications on Non-Kähler manifolds

K.Becker, M.Becker, Dasgupta 2002, + P.Green 2003, + S.P. 2003, + ...

The tree level superpotential includes now the fundamental 3-form J:

$$W_0 = \int_{M_6} (H_3 + idJ) \wedge \Omega$$

The size of the manifold is stabilized by heterotic flux H_3 , using the relation following from above,

$$H_3 = \star dJ,$$

at values growing with the flux density, i.e. can be large.

Stabilization of the volume happens for a generic manifold supporting supersymmetric fluxes, without any fine-tuning.

- "Large volume" IIB flux compactifications on a CY

Balasubramanian, Berglund 2004, + Conlon, Quevedo 2005, + Suruliz 2005

Non-perturbatively corrected superpotential depending on all Kähler moduli T_i :

$$W = W_0 + \sum_{i=1}^{h_{11}} A_i e^{-a_i T_i}$$

α' - corrected Kähler potential:

$$K = -2 \ln \left(\mathcal{V}_s + \frac{\xi g_s^{\frac{3}{2}}}{2e^{\frac{3\phi}{2}}} \right)$$

K.Becker, M.Becker, Haack, Louis 2002

The minimum of the potential for T's exists for generic values of parameters (with the restriction $h_{1,2} > h_{1,1} > 1$) and gives a large value for the volume:

$$\mathcal{V}_s \sim \mathcal{O}(10^5 - 10^{15})$$

Why Kähler moduli?

There are several features of the Kähler moduli that make the study of inflationary scenarios based on them especially attractive:

- "Almost flat directions":
  Kähler moduli are generally left unfixed by tree level superpotentials, and can only be uplifted by quantum effects. These effects introduce negative exponential dependence on the moduli in the scalar potential, so that the potential is naturally flat, thus avoiding the η-problem.

- Both moduli stabilization and flatness of the potential are achieved by the same mechanism and, in the case of the "large volume" flux compactifications, are valid for a very large class of models.

However,

- it should be checked if the volume is stable during the inflation,
- higher α' -corrections could lift the exponentially flat potential and restrict the region of validity of the model.

The Model

/e will consider a type IIB string theory compactified on a Calabi-Yau
ıanifold with

$$h_{1,2} > h_{1,1} > 2$$

ɪn effective 4d $\mathcal{N} = 1$ supergravity action is

$$S_{\mathcal{N}=1} = \int d^4x\sqrt{-G}\left[\frac{M_P^2}{2}\mathcal{R} - \mathcal{K}_{,i\bar{j}}D_\mu\varphi^i D^\mu\bar{\varphi}^j - V(\varphi,\bar{\varphi})\right],$$

where the scalar potential is

$$V(\varphi,\bar{\varphi}) = e^{\mathcal{K}/M_P^2}\left(\mathcal{K}^{i\bar{j}} D_i \hat{W} D_{\bar{j}} \bar{\hat{W}} - \frac{3}{M_P^2}\hat{W}\bar{\hat{W}}\right) + \text{ D-terms},$$

the α'^3- corrected Kähler potential is

$$\frac{\mathcal{K}}{M_P^2} = -2\ln\left(\mathcal{V}_s + \frac{\xi g_s^{\frac{3}{2}}}{2e^{\frac{3\phi}{2}}}\right) - \ln(S+\bar{S}) - \ln\left(-i\int_{CY}\Omega\wedge\bar{\Omega}\right),$$

where $\mathcal{V}_s$ is the volume of the Calabi-Yau manifold,

$\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3}$ represents α'- corrections,

$S = -iC_0 + e^{-\phi}$ is IIB axion-dilaton, and

Ω is the holomorphic 3-form of the CY manifold.

The superpotential including non-perturbative corrections is

$$\hat{W} = \frac{g_s^{\frac{3}{2}} M_P^3}{\sqrt{4\pi}} \left(W_0 + \sum A_i e^{-a_i T_i} \right), \quad W_0 = \frac{1}{l_s^2} \int_{CY} G_3 \wedge \Omega.$$

Here W_0 is the tree level flux-induced superpotential,

$G_3 = F_3 - iSH_3$ is the IIB flux 3-form, and

T_i are the Kähler moduli,

$$T_i = \tau_i + i\theta_i$$

with τ_i a 4-cycle volume and θ_i its axionic partner,

$a_i = \frac{2\pi}{g_s N}$ and A_i are some model dependent constants.

Perturbative Corrections

The leading perturbative (α') corrections appear in the model as the ξ - dependent term in the Kähler potential. They arise from the higher derivative terms in the ten dimensional IIB action,

$$S_{IIB} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g^{(10)}}e^{-2\phi}[R + 4(\partial\phi)^2 + \alpha'^3\frac{\zeta(3)}{3\cdot 2^{11}}J_0 - \alpha'^3\frac{(2\pi)^3\zeta(3)}{4}Q + ...]$$

where $J_0 \sim (R^{MN}{}_{PQ})^4$ and Q is a generalization of the six-dimensional Euler integrand, $\int_M d^6x\sqrt{g}Q = \chi$. Compactifying this action on a CY, we get an effective 4D SUGRA with the Kähler potential which after stabilizing the dilaton, takes the following form:

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right)$$

K.Becker, M.Becker, Haack, Louis, 2002

In the expression for the **Kähler potential**, the volume of the CY manifold is a function of all 4-cycle moduli τ_i . Indeed, the classical volume of the CY in terms of 2-cycle moduli is

$$\mathcal{V} = \frac{1}{6}\kappa_{ijk}t^i t^j t^k$$

4-cycle moduli can be expressed via 2-cycle moduli as $\tau_i = \frac{1}{2}\kappa_{ijk}t^j t^k t$, which gives the volume an implicit dependence on τ_i .

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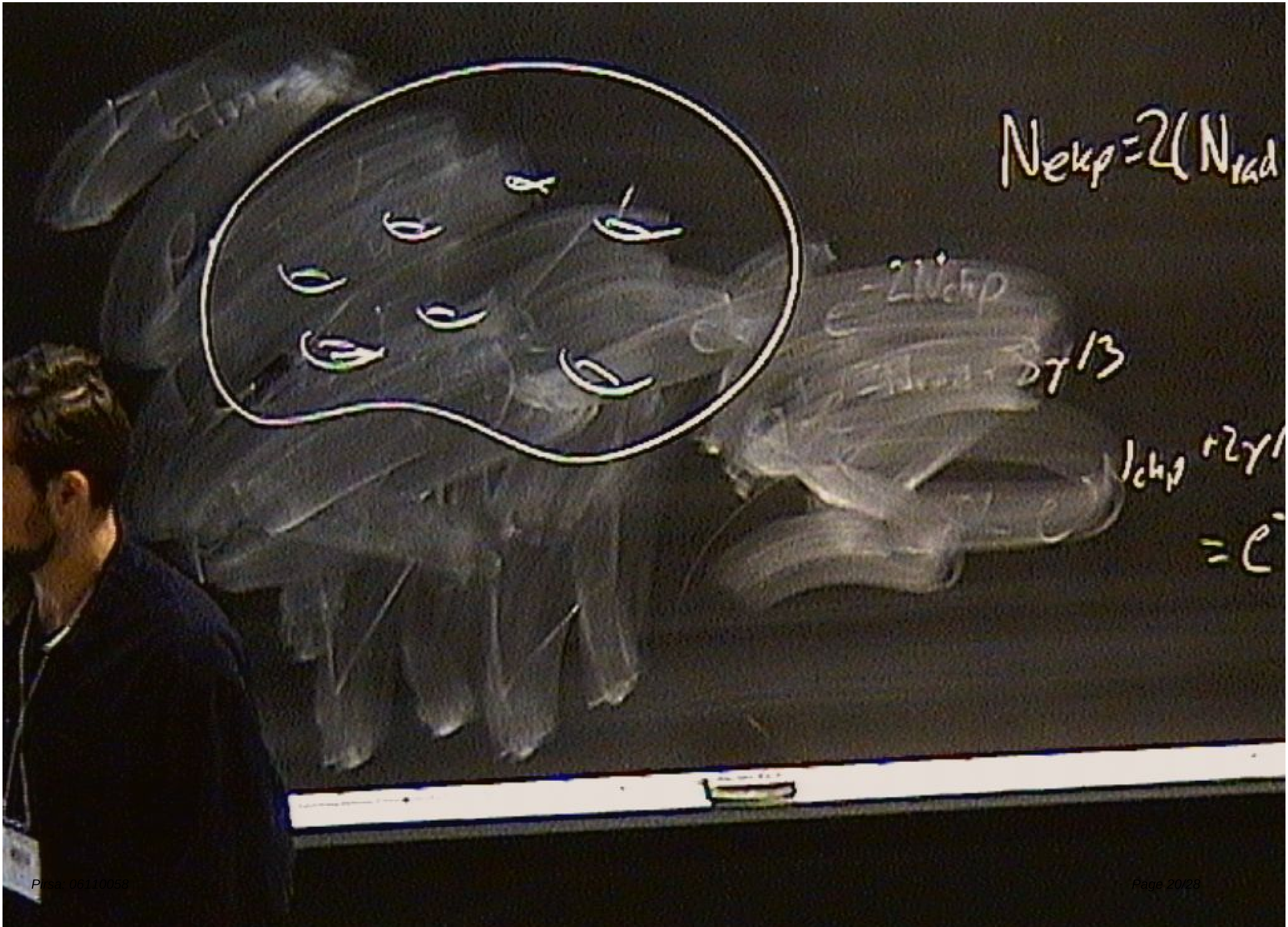
In the case where each of the 4-cycles has a non-vanishing triple intersection only with itself, the expression for the volume has a particularly simple form:

$$\mathcal{V} = \alpha \left(\tau_1^{3/2} - \sum_{i=2}^{n} \lambda_i \tau_i^{3/2} \right)$$

Here α, λ_i are positive constants depending on a particular model. This formula suggests a "Swiss-cheese" picture of a CY where τ_1 describes the 4-cycle of maximal size and $\tau_2, \ldots, \tau_n$ the blow-up cycles. The modulus τ_1 controls the overall volume of the CY and can take an arbitrary large value, whereas $\tau_2, \ldots, \tau_n$ describe the holes in the CY and cannot be larger then the overall size of the manifold.

One of τ_i-s, that is the last to attain its minimum plays the role of the inflaton in the model. In these models inflation occurs almost inevitably, and the most important results do not depend on the initial conditions and details of microphysics such as the model dependent parameters W_0, A_i, a_i (Conlon, Quevedo, 2006).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The Potential

After stabilizing axion-dilaton, complex structure moduli, internal volume, and all other Kähler moduli except $T_2 = \{\tau, \theta\}$, the resulting scalar potential has the following asymptotic form at $\mathcal{V} \sim \exp(-a\tau)$, $\mathcal{V} \gg 1$:

$$V(\tau, \theta) = \frac{8(a_2 A_2)^2 \sqrt{\tau} e^{-2a_2\tau}}{3\alpha\lambda_2 \mathcal{V}_m} - \frac{4W_0 a_2 A_2 \tau e^{-a_2\tau} \cos(a_2\theta)}{\mathcal{V}_m^2} + \Delta V$$

where the constant term ΔV contains the contributions of other stabilized moduli, as well as an uplift necessary to obtain dS or Minkowski space (e.g. D-term potential from the fluxes on D7).

An important feature of the model: large volume at the minimum of the potential. $\mathcal{V}_m \sim W_0 e^{a\tau}$

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The potential is exponentially flat at large values of τ :
first stringy realization of the self-reproduction regime ?

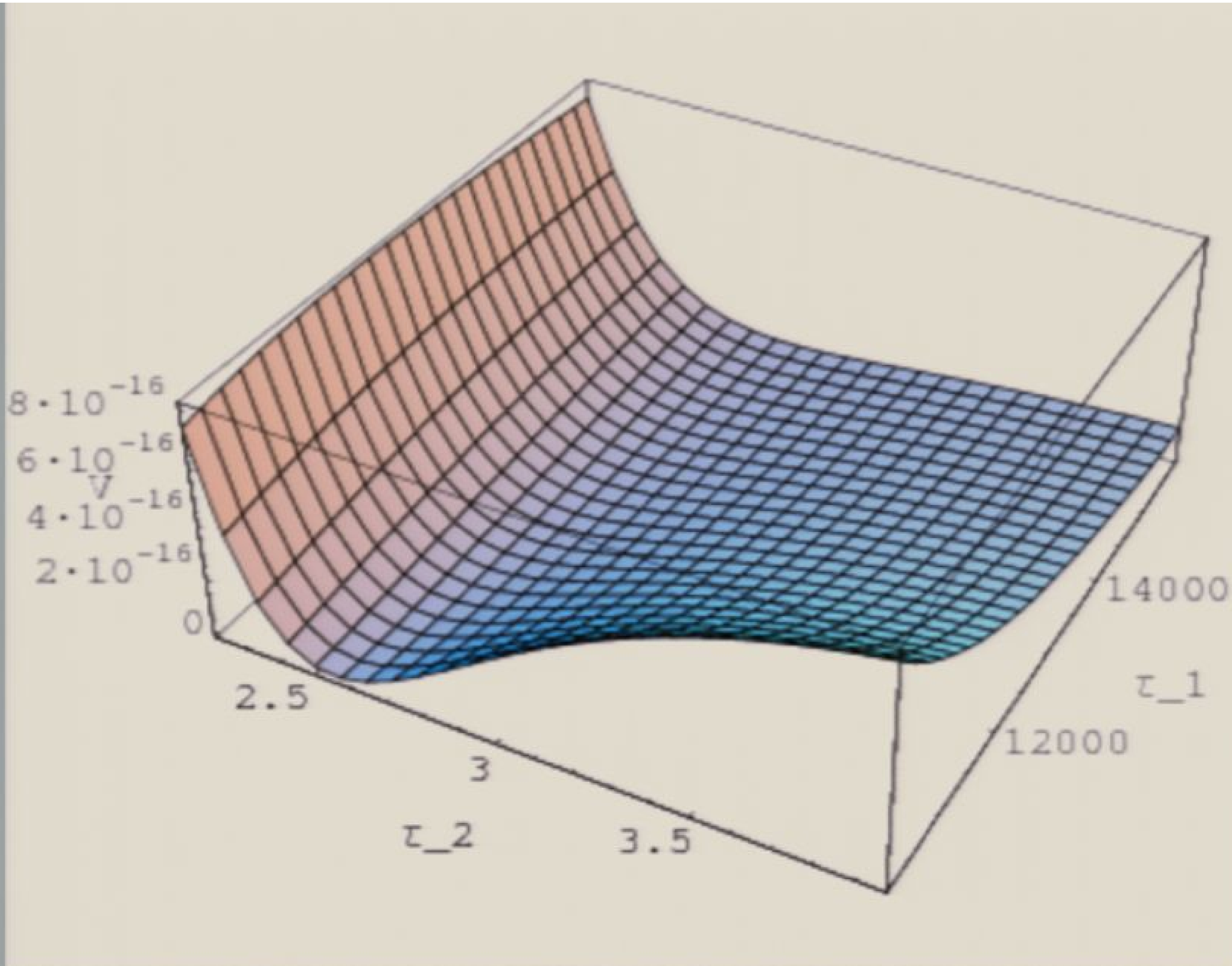
All settled moduli, including the overall volume, should remain stable during the evolution of the inflaton field. To obtain a criterion of stability, put all moduli at their minimum and get the potential for the volume:

$$V = -\frac{3W_0^2}{2\mathcal{V}^3}\left(\alpha\sum_{i=2}^{n}\left[\frac{\lambda_i}{a_i^{3/2}}\right](\ln\mathcal{V})^{3/2} - \frac{\xi}{2}\right) + \frac{\beta}{\mathcal{V}^2}$$

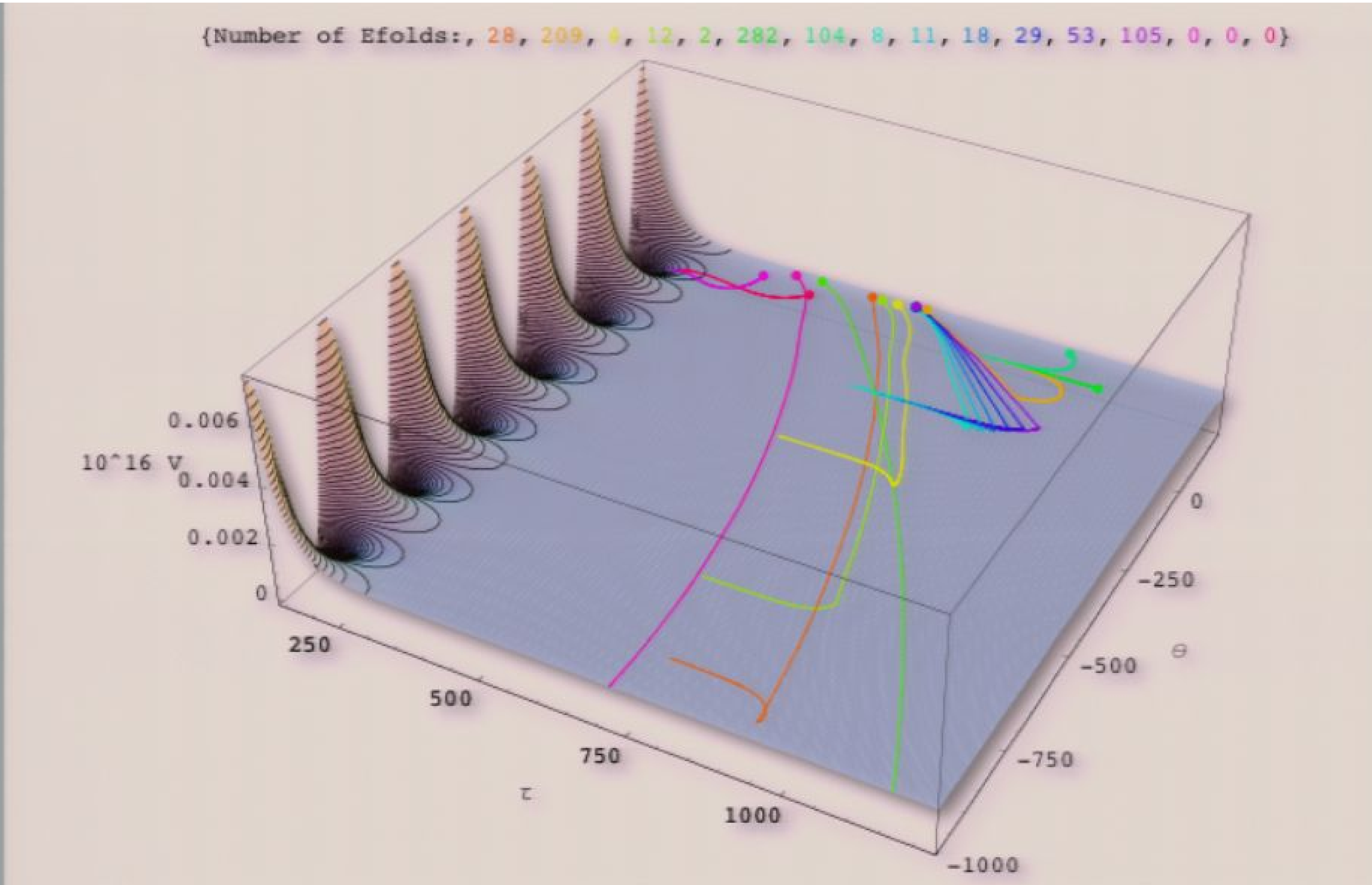
The functional form of the potential will not alter much if

$$\frac{\lambda_2}{a_2^{3/2}} \ll \sum_{i=2}^{n}\frac{\lambda_i}{a_i^{3/2}}$$

Note that in the models with many Kähler moduli, $n \gg 1$, this condition is satisfied for generic values of the parameters $\lambda_i,\ a_i$.



The potential $V(\tau_1, \tau_2)$ in a model with just two Kähler moduli.
It is difficult to have both inflation and stabilized volume.



"Ensemble" of inflationary trajectories in the model with many Kähler moduli, and the values of parameters:

$W_0 = 4 \times 10^5$, $A_2 = 1$, $a_2 = \pi/100$, $\lambda_2 = 1$, $\alpha = 1/9\sqrt{2}$, $\xi = 0.5$, $g_s = 0.1$, $\mathcal{V} \sim 10^9$

Conclusion

- "Large volume" IIB flux compactification scheme provides the possibility for realization of inflationary regimes with the Kähler moduli as the inflaton fields in a large class of models
- Axion partners of the Kähler moduli can play a non-trivial role in the inflationary dynamics, due to special features of the scalar potential
- The regime of self-reproduction can possibly be realized due to the exponential flatness of the potential at large values of the moduli
- However, it should be checked if higher α'- corrections don't break exponential flatness of the potential for the values of the inflaton fields relevant during the last sixty e-folds of inflation.