Title: ï»¿ï»¿Cuscuton: Dark Energy meets Modified Gravity

Date: Nov 10, 2006 02:00 PM

URL: http://pirsa.org/06110056

Abstract:

Cuscuton: *Dark Energy meets Modified Gravity*

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Afshordi, Chung, & Geshnizjani; hep-th/0609150
Afshordi, Chung, Doran, & Geshnizjani; astro-ph/0610???

Outline

- What is *Cuscuton?* Dark Energy with infinite C_s
- Is *Cuscuton* causal?
  - *Cuscuton:* soap bubbles in Minkowski space
  - An underlying theory for *Cuscuton*
- *Cuscuton* Cosmology:
  - Dark Energy meets Modified Gravity
  - Quadratic *Cuscuton*: Early Dark Energy
  - Exponential *Cuscuton*: DGP-like cosmic history
- Why should we care?

What is *Cuscuton*?

- K-essence Action:

$$S_\varphi = \int d^4x \sqrt{-g}\left[\frac{1}{2}F(X,\varphi) - V(\varphi)\right]$$

$$X \equiv \partial_\mu \varphi \partial^\mu \varphi$$

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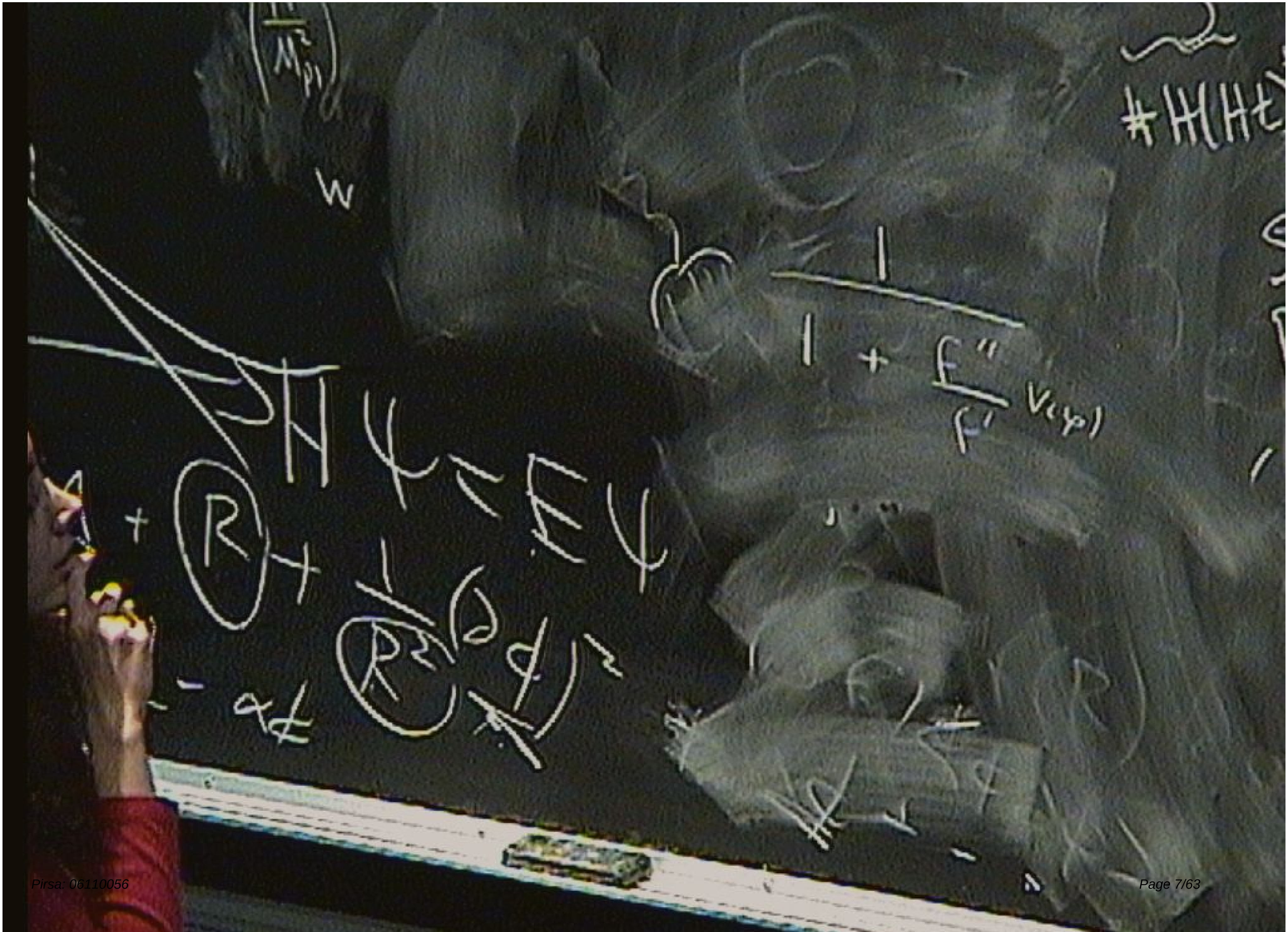
  $\delta\rho_\phi = 0$; **but** $\delta P_\phi \neq 0$ → $c_s = \infty$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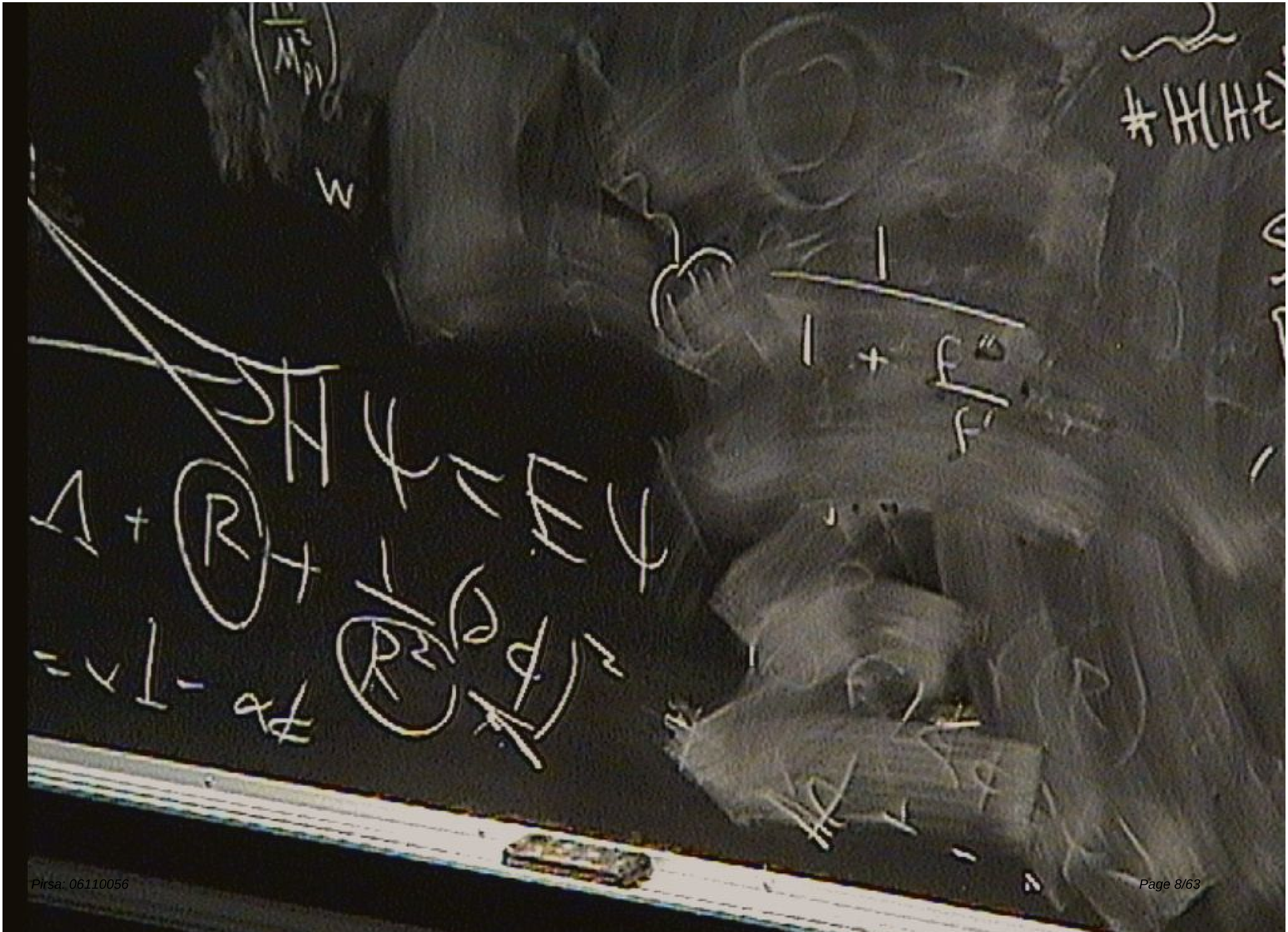
HΨ = EΨ



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→ Field equation becomes a **constraint equation** that uniquely determines φ as a function of metric

→ No internal dynamics; only follows what it couples to

→ *Cuscuton* (käs-kū̄-tän): derived from the Latin name for the parasitic plant of dodder, "Cuscuta"

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→Causality will constrain *Cuscuton* couplings

Cuscuton as Soap bubbles

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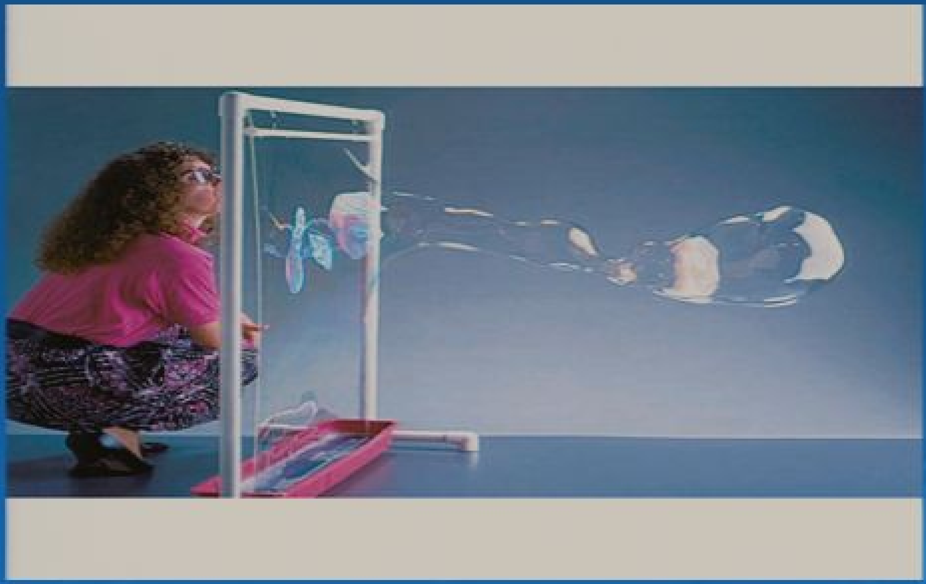

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&= \int d^4x\sqrt{-g}[\mu^2|u^\mu\partial_\mu\varphi| - V(\varphi)] \\
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Surface Tension — Pressure difference

→ **Similar to soap bubbles Energy**

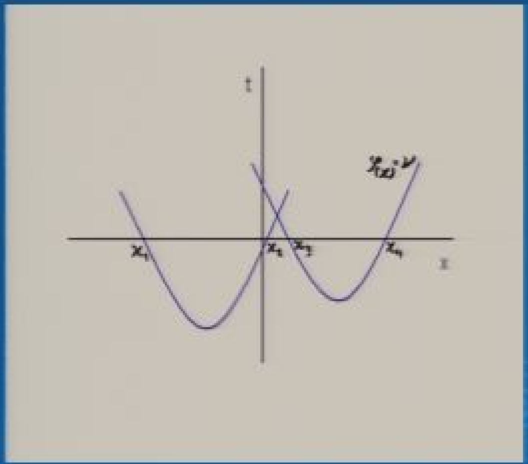
EXACT SOLUTIONS, UNIQUENESS AND SINGULARITIES??

- In general, the question of existence and uniqueness of CMC surfaces for a given boundary condition, is of significant subtlety.
- In 1+1 dimensions, field equation can be exactly solved.

$$[t - t_0(\varphi)]^2 - [x - x_0(\varphi)]^2 = \frac{\mu^4}{V'^2(\varphi)}$$

Hyperbola

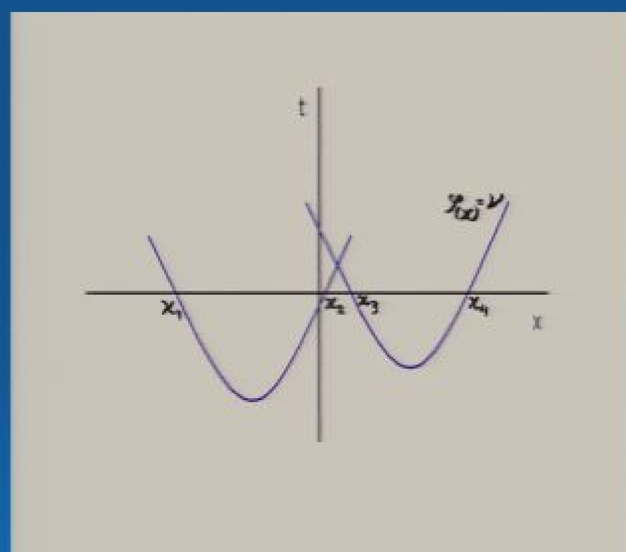
- Dirichlet data $\varphi(t=0,x)=\varphi_0(x)$, fixes the Minkowski curvature of hyperbolae at t=0 slices.



⇒ The sets of possible hyperbolae form a discrete set.

- After imposing just the Dirichlet conditions, general Cauchy initial condition, which fixes both $\varphi_0(x)$ and $\dot{\varphi}_0(x)$ typically overconstrains the system globally, resulting in no solution!

- The hyperbolae can intersect, in these cases, singularities or discontinuities will generically develop in the solutions within a finite time (in nature similar to development of shocks in fluid mechanics)!



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- Given Dirichlet initial/boundary conditions, admitting a discrete set of solutions is still a reasonable conjecture.
- Generic singularity of the solutions are also expected as in 1+1.

An underlying theory for *Cuscuton*

- Imagine a field theory with discrete degrees of freedom

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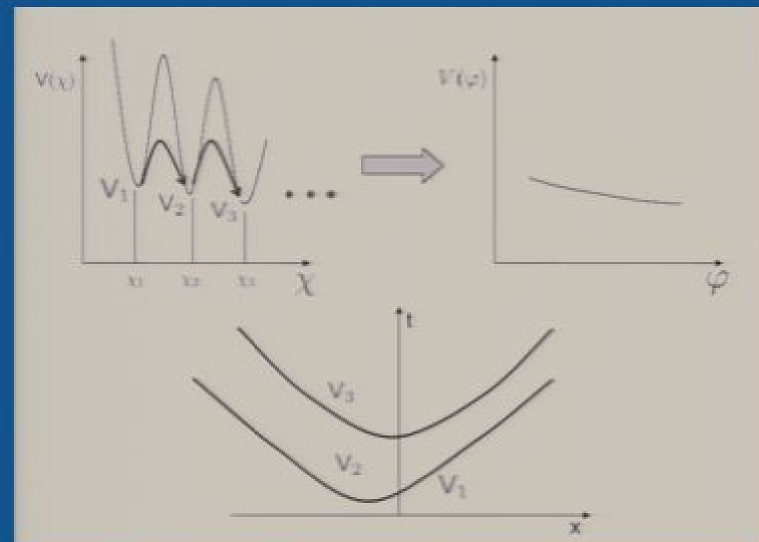
- The only local action in the low energy limit (in lieu of other couplings) is the discrete *Cuscuton* action

$$S_{\rm eff} \simeq \mu^2 \sum_i (\varphi_{i+1} - \varphi_i) \int d\Sigma_i - \int d^4x\, V(\varphi),$$

- The **Euclidian instanton** action for the successive

$$S_E = \int d^4x [\frac{1}{2}\partial_a\chi\partial_a\chi + V(\chi)]$$



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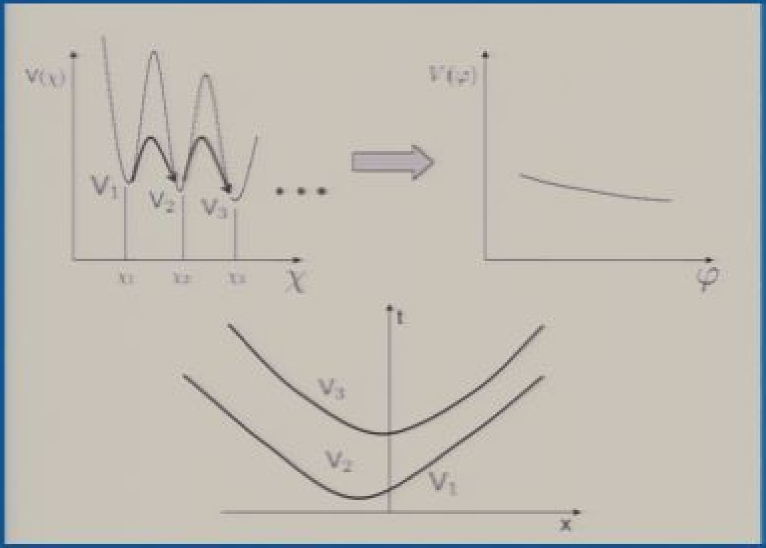

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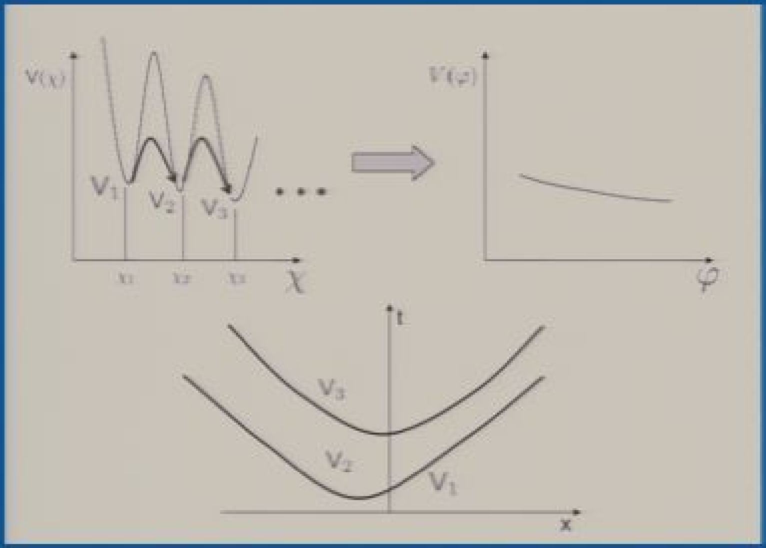

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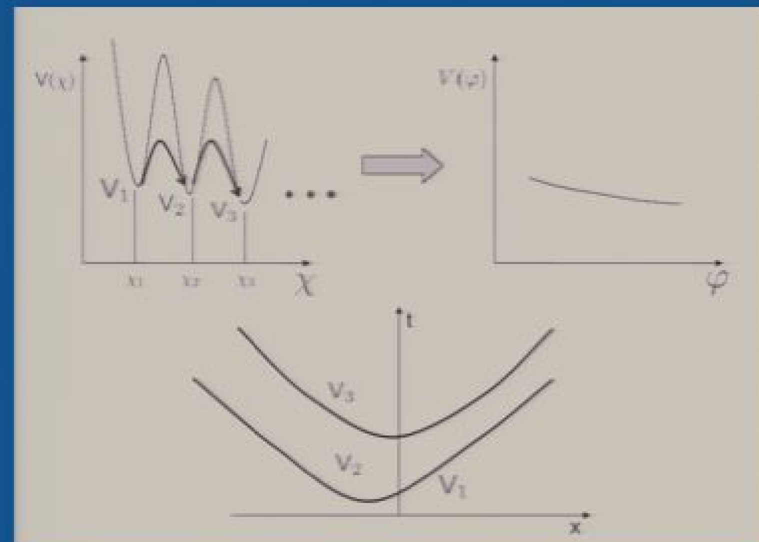

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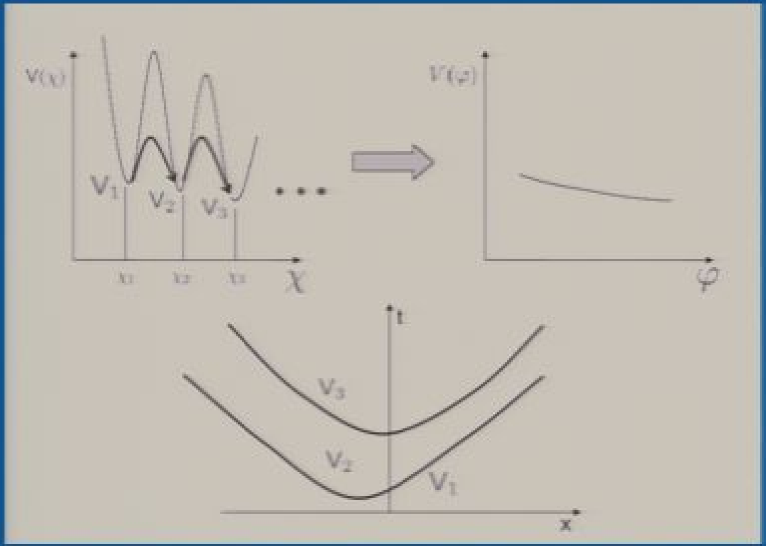

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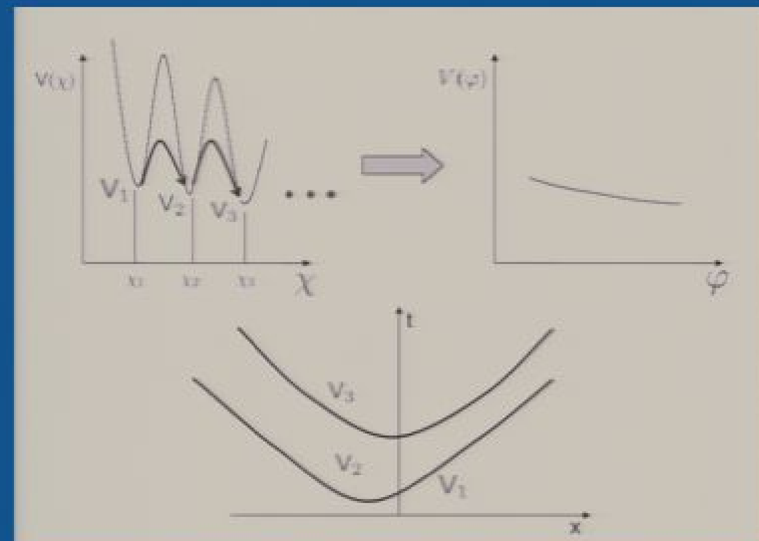

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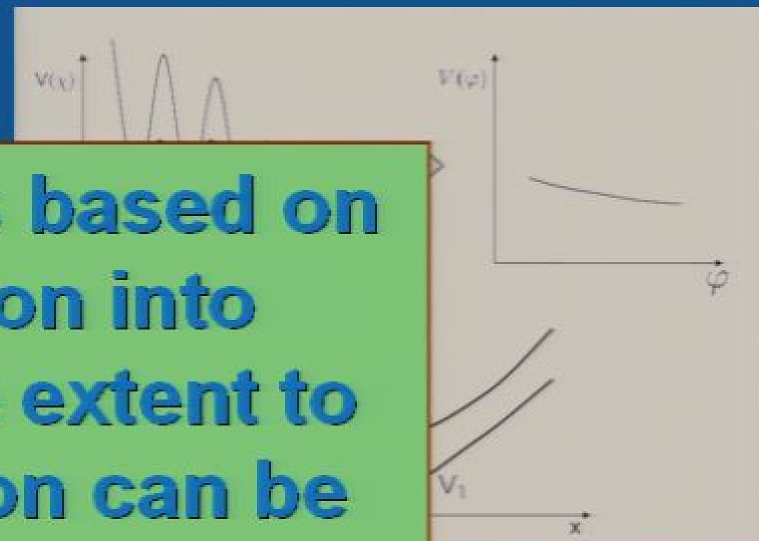

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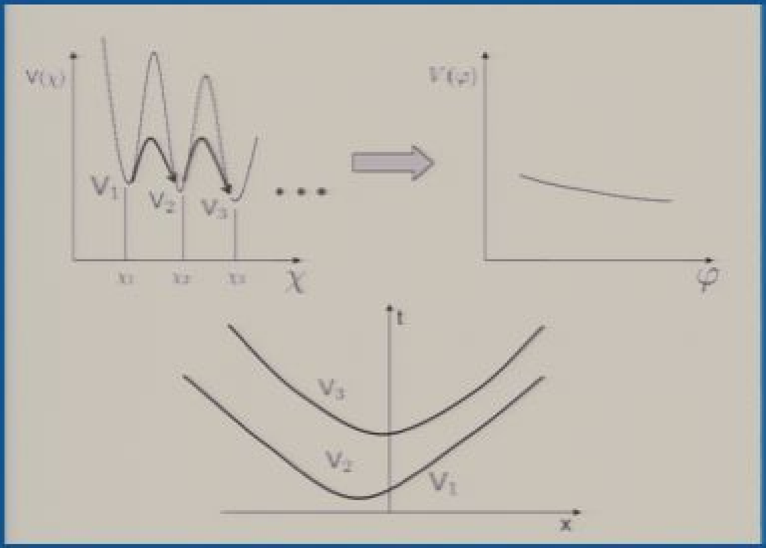

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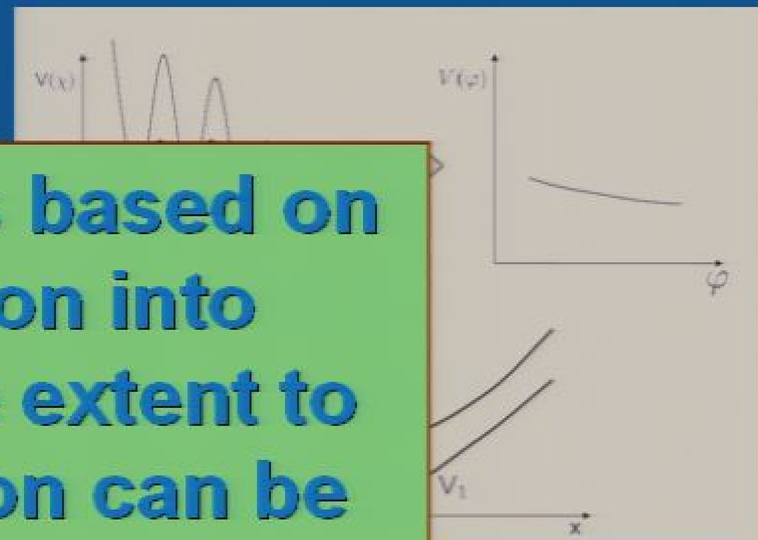

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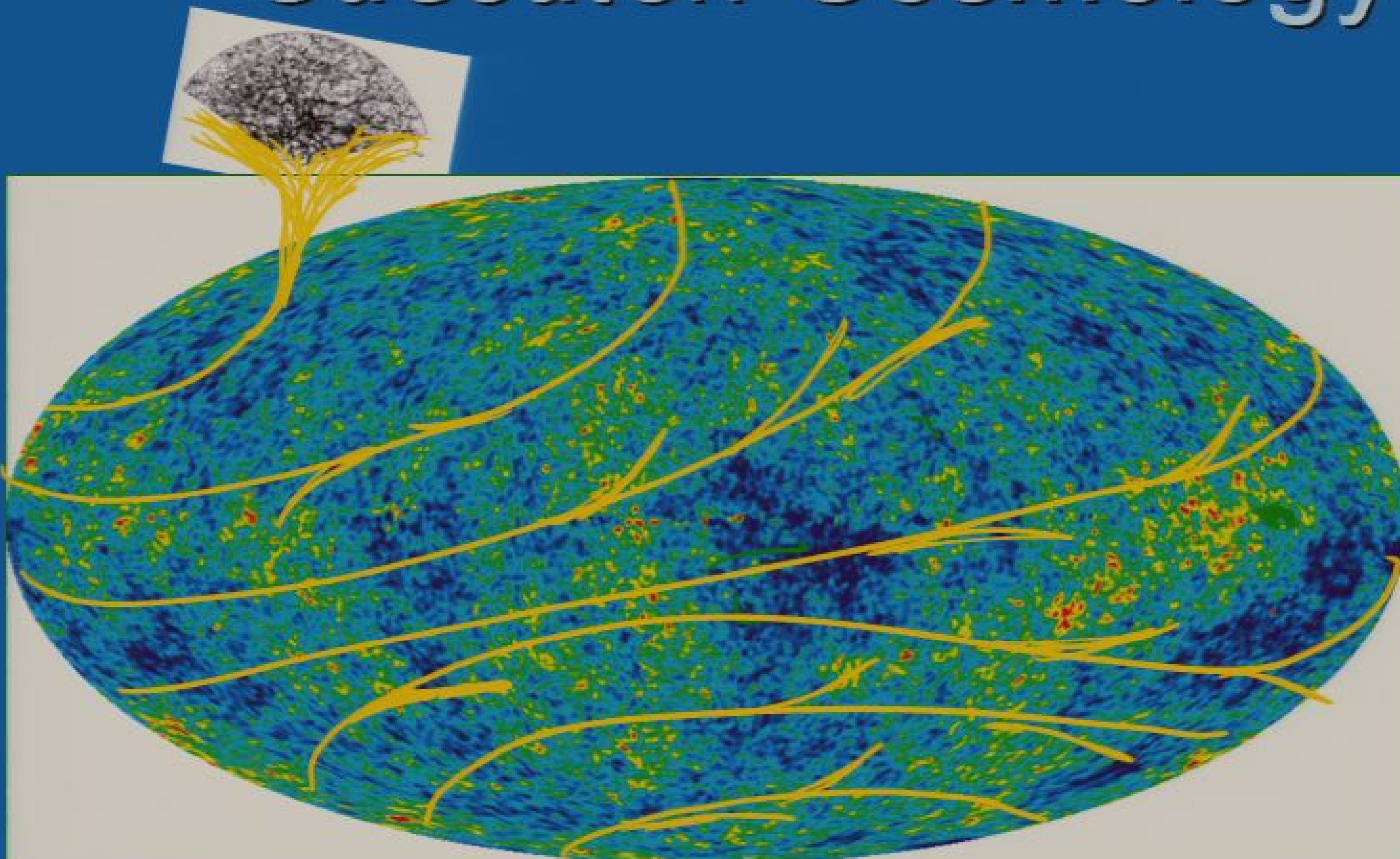
Cuscuton Cosmology



Artist's Conception

Friedmann *Cuscuton* Cosmology

- Field Equation (remember bubbles):
- → Friedmann equation:

$$(3\mu^2 H)\ \mathrm{sgn}(\dot{\varphi}) + V'(\varphi) = 0.$$

- → (in a flat universe)

$$\left(\frac{M_p^2}{3\mu^4}\right) V'^2(\varphi) - V(\varphi) = \rho_m$$

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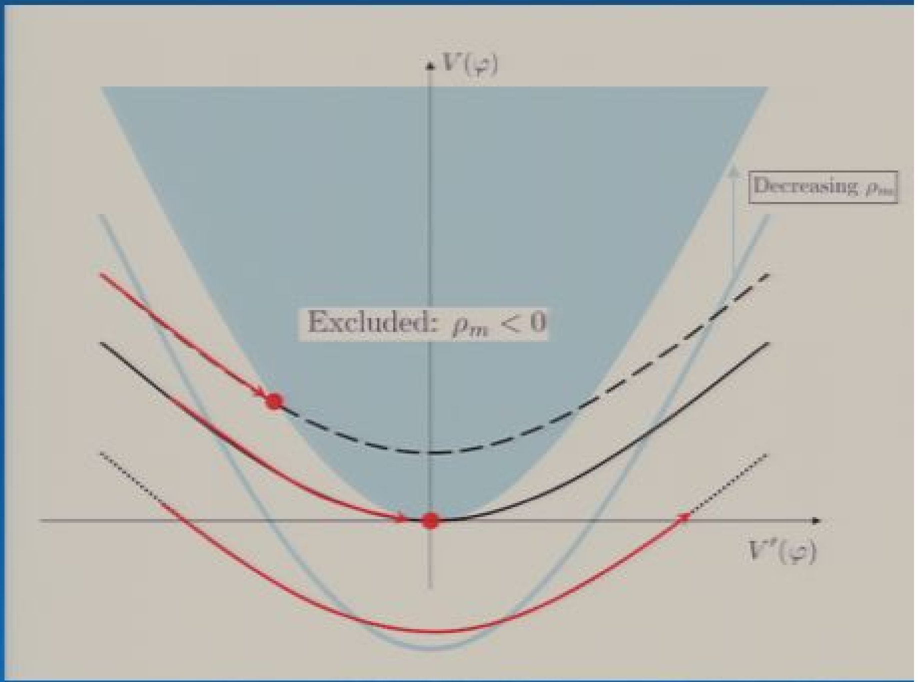

Cuscuton as Modified Gravity

- Since *Cuscuton* has no internal dynamics, it can be viewed as a (non-local) modification to gravity

$$\left(\frac{k^2}{a^2}\right)\phi + \left[3H + \frac{9H(2\dot{H} + 3H^2\Omega_m)}{2\left(\frac{k^2}{a^2} - 3\dot{H}\right)}\right](\dot{\phi} + H\phi) + (2M_p^2)^{-1}\delta\rho_m = 0,$$

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- Non-locality is exponentially suppressed beyond the Hubble radius, i.e. for k << H, the evolution is local

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  - **Exponential potential:** $V(\varphi) = V_0 \exp(-r_c\varphi)$
    - Geometry evolves exactly as DGP cosmology

$$H = \frac{1}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{\rho_m}{3M_p^2}}$$

Quadratic *Cuscuton* Cosmology

- $V(\varphi) = V_0 + \frac{1}{2}m^2\varphi^2$ + Friedmann eq. →

$$\Omega_Q = \frac{\frac{1}{2}m^2\varphi^2}{\rho_{tot}} = \frac{3\mu^4}{2M_p^2 m^2} = \text{const.}$$

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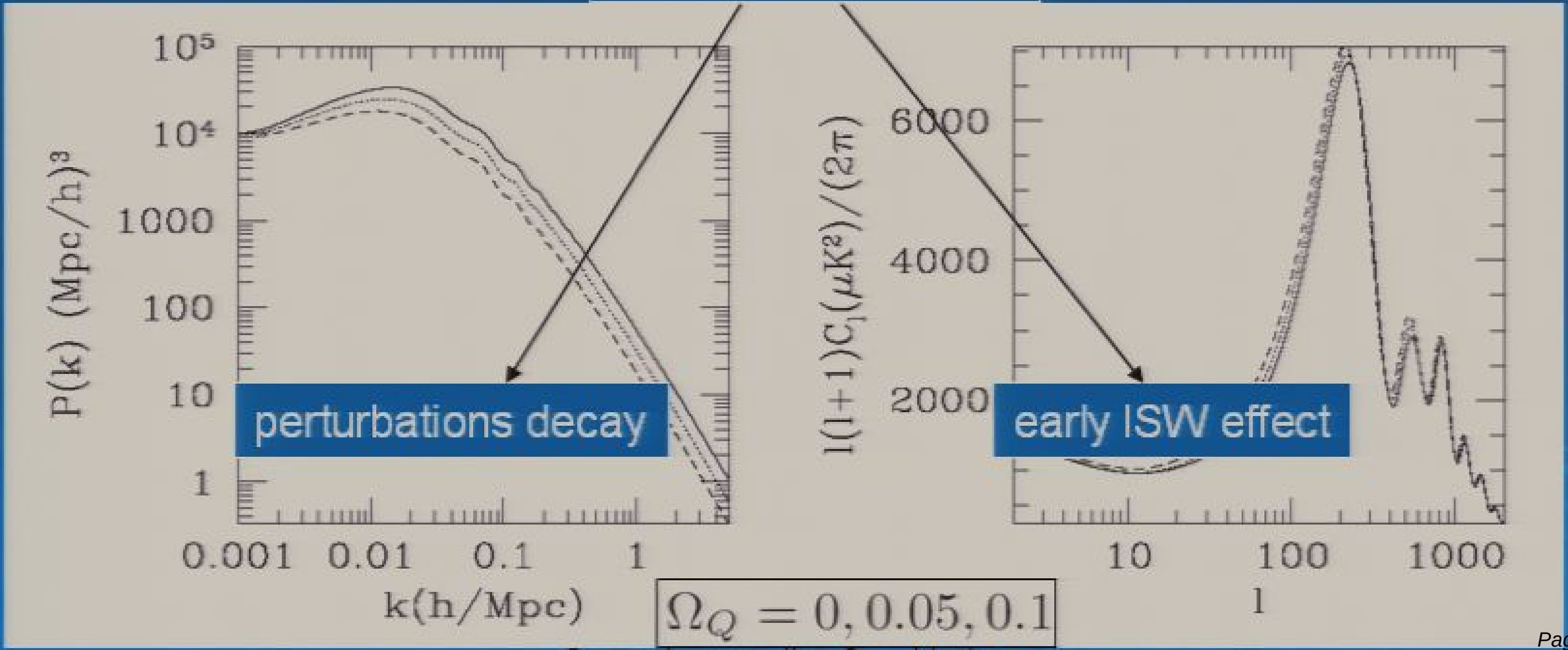
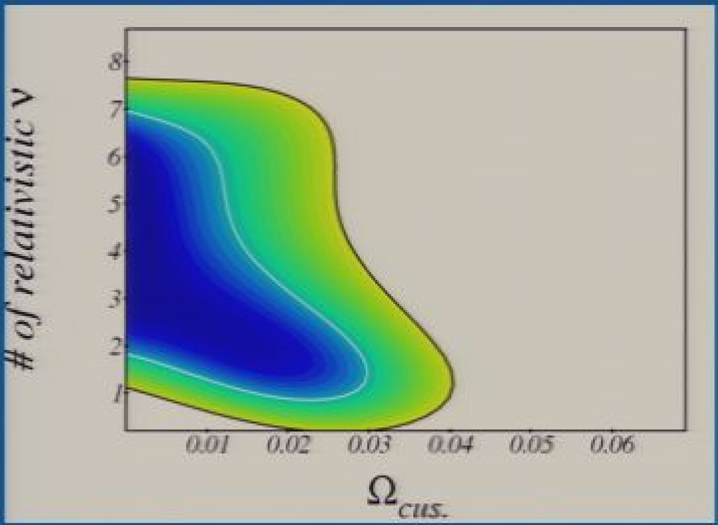
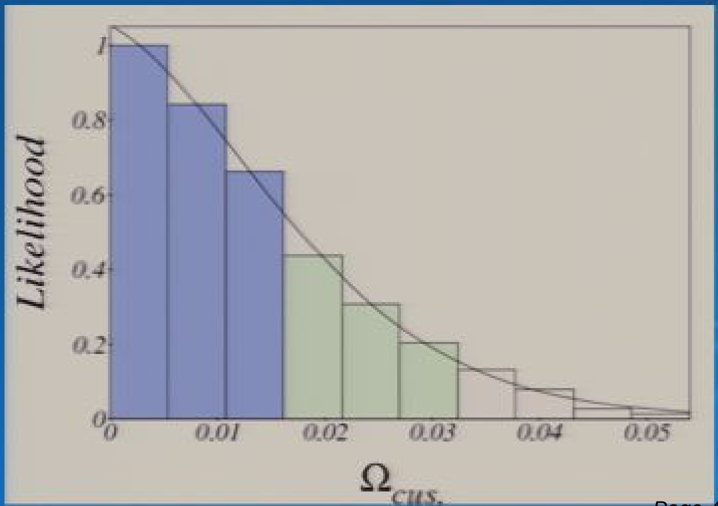


$\Omega_Q = 0, 0.05, 0.1$

Preliminary constraints on Quadratic *Cuscuton*

- Main constraints from ISW effect, i.e. WMAP3





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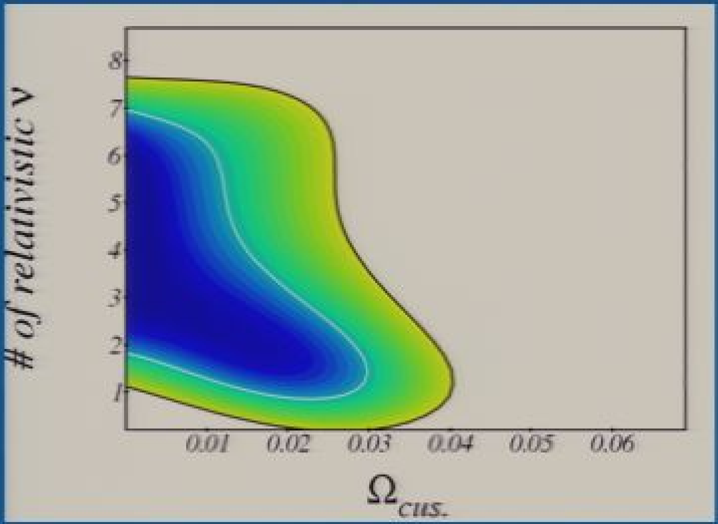
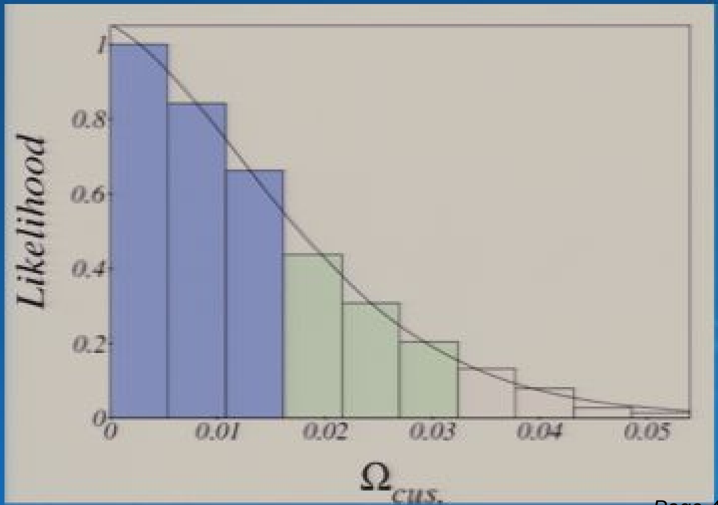
- Main constraints from ISW effect, i.e. WMAP3
- $\Omega_Q < 0.035$ (95%)

→ or $\mu < 0.4\ m^{1/2}$

(in Planck units) for the action:

$$S = \int d^4x\sqrt{-g}[\mu^2\sqrt{|g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi|} - \frac{1}{2}m^2\varphi^2]$$





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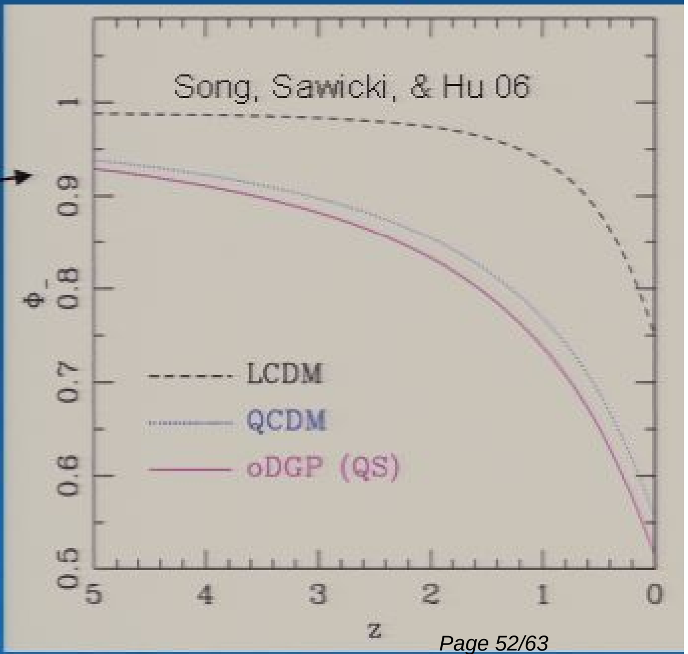

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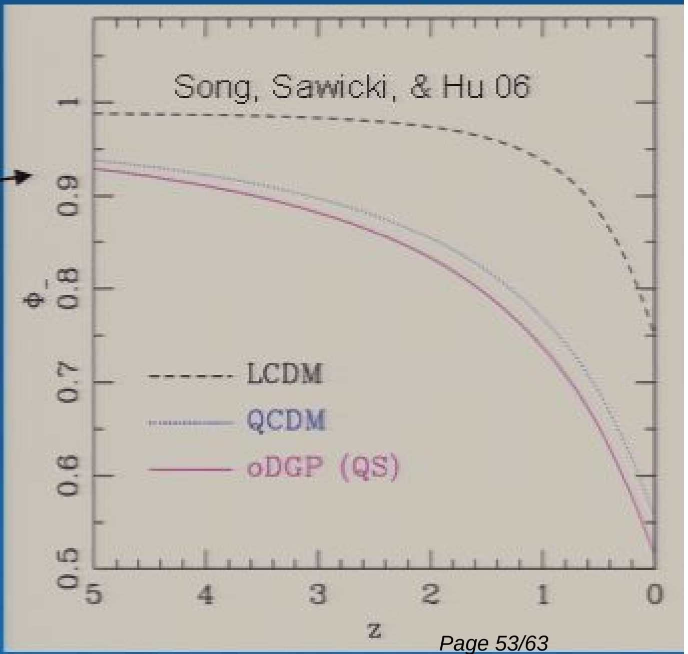
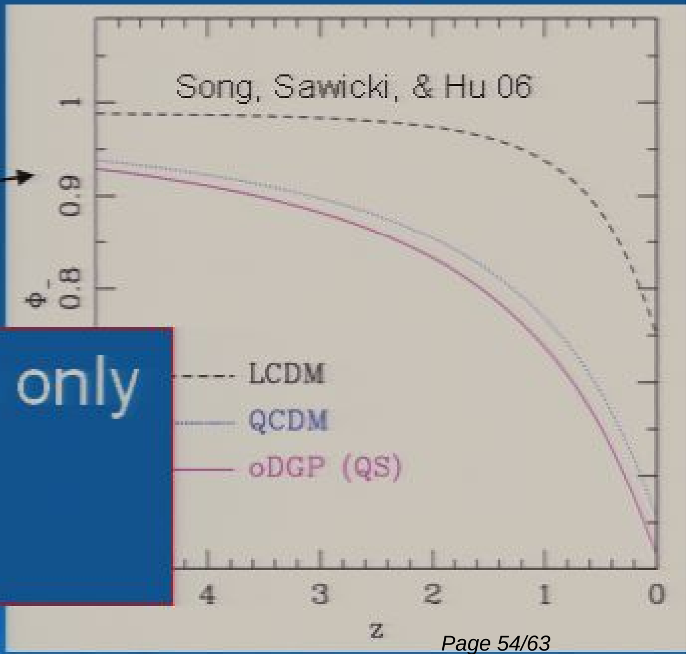
→ detectable at < 3σ with weak lensing+ext (Huterer & Linder 06)



Exponential (DGP-like) *Cuscuton*

- Exact same expansion history as flat DGP self-accelerating model (no difference in geometric tests)
- But, different sub-horizon perturbation theory
  - Like quintessence with the same expansion history
- 5-10% difference with DGP



→ DGP and exponential *Cuscuton* can only be marginally distinguished at low z

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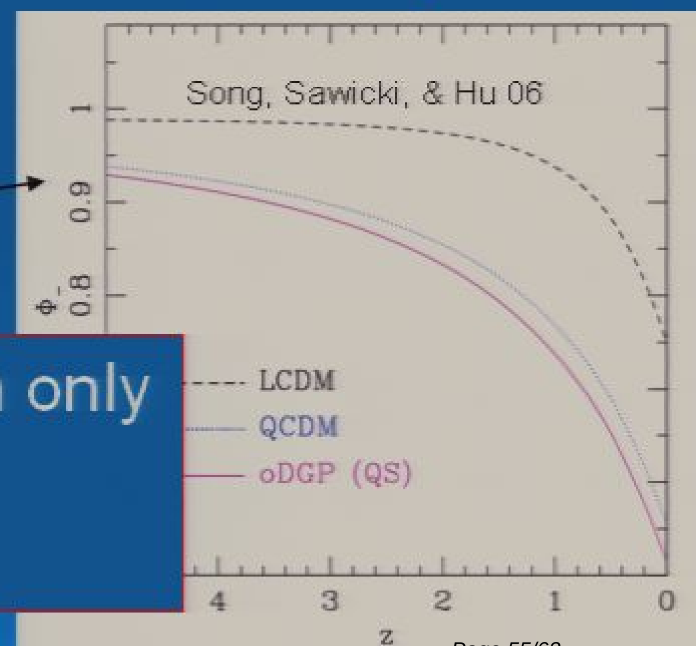

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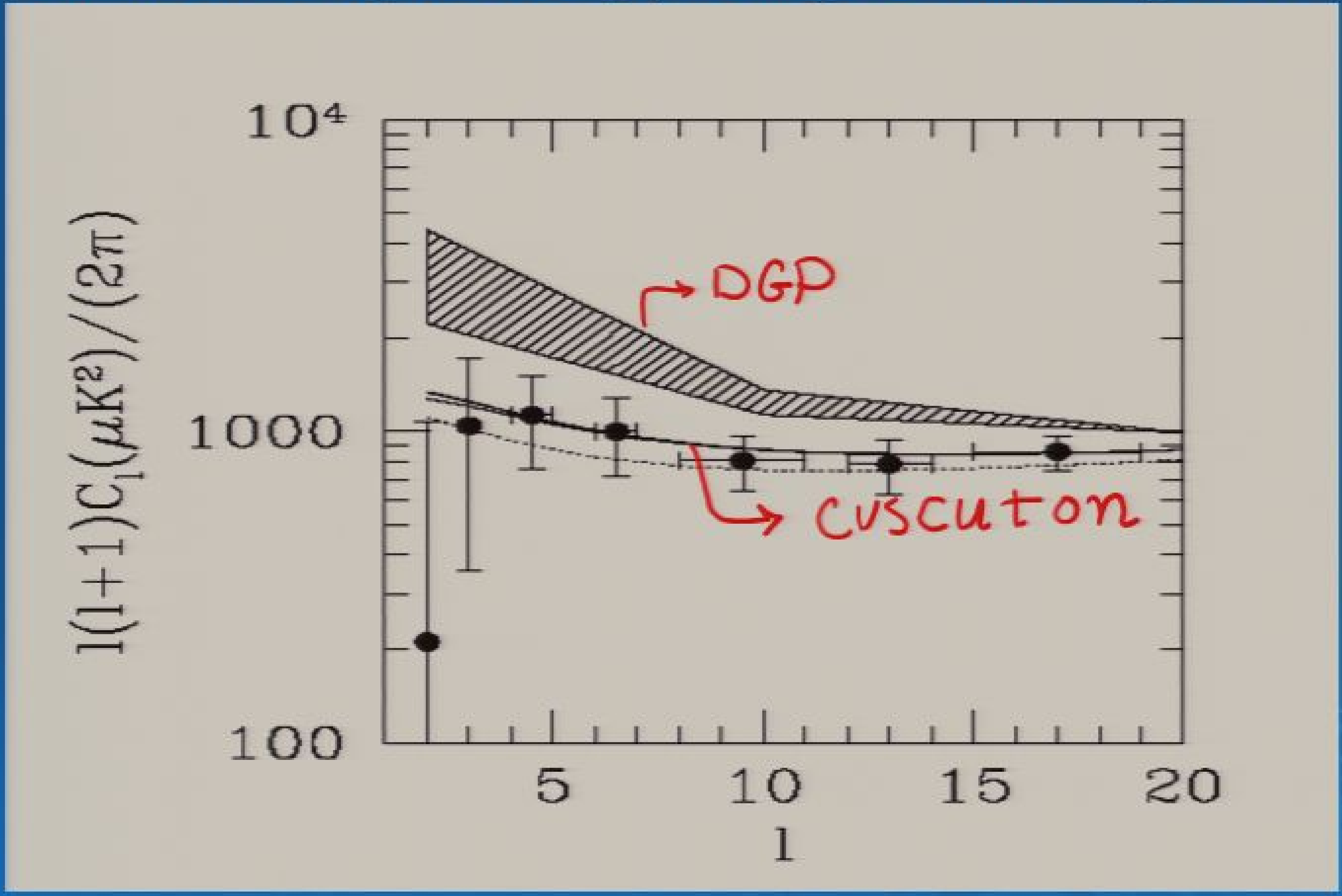
ISW effect: DGP-like *Cuscuton* vs. DGP

Song, Sawicki, & Hu 06

Due to anisotropic stress (?) DGP gives much larger ISW



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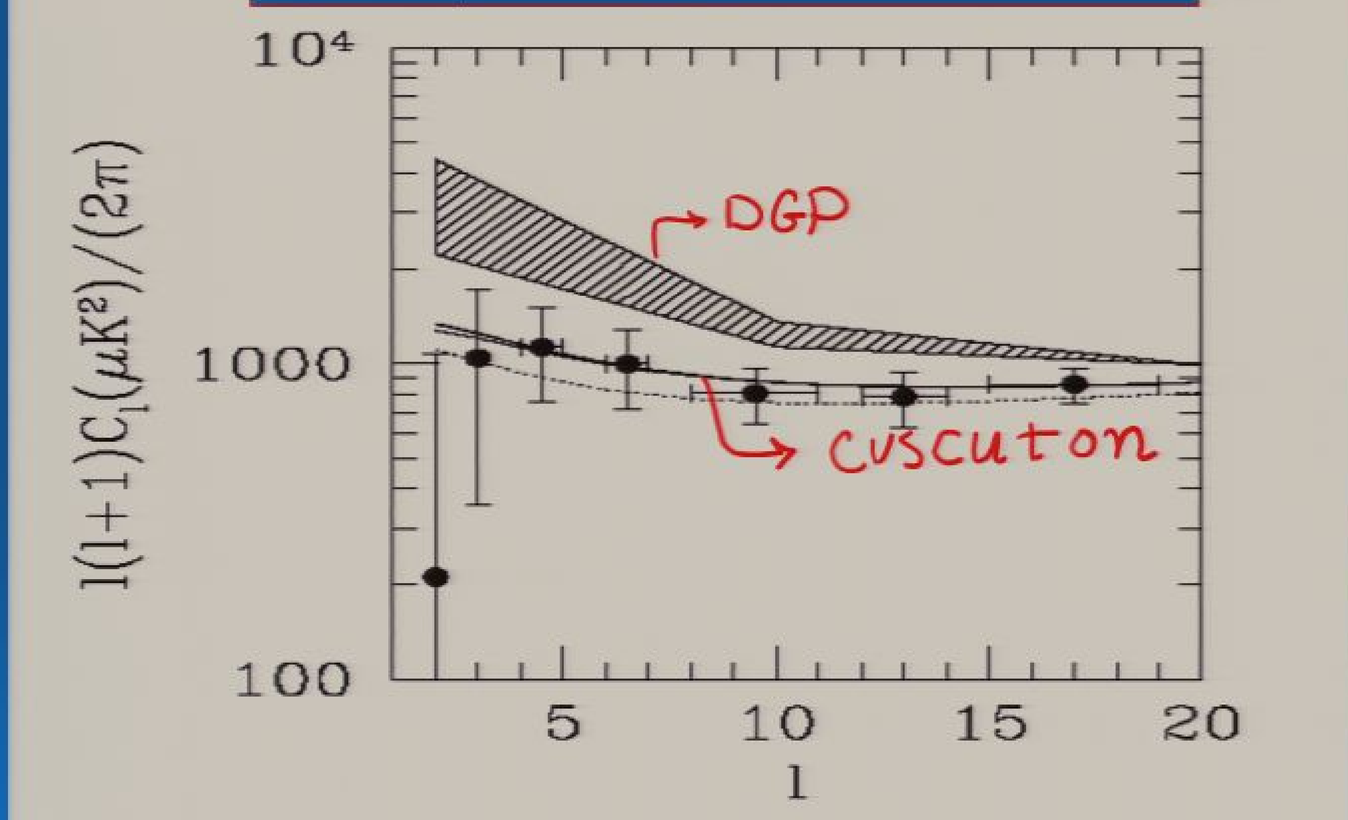
Due to an ... er ISW

DGP-like *Cuscuton* yields only . 5% larger C_l's than DGP-like quintessence



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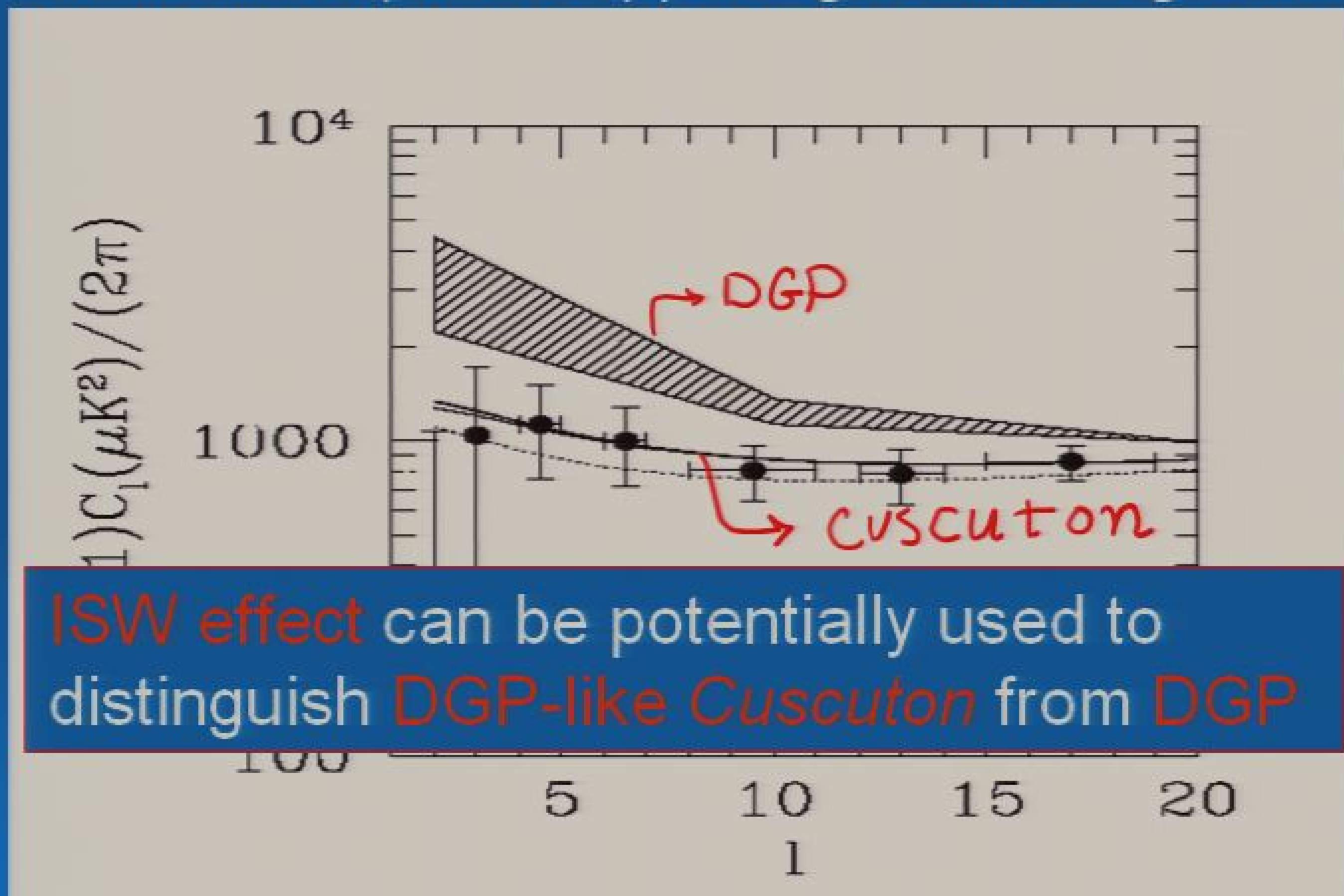
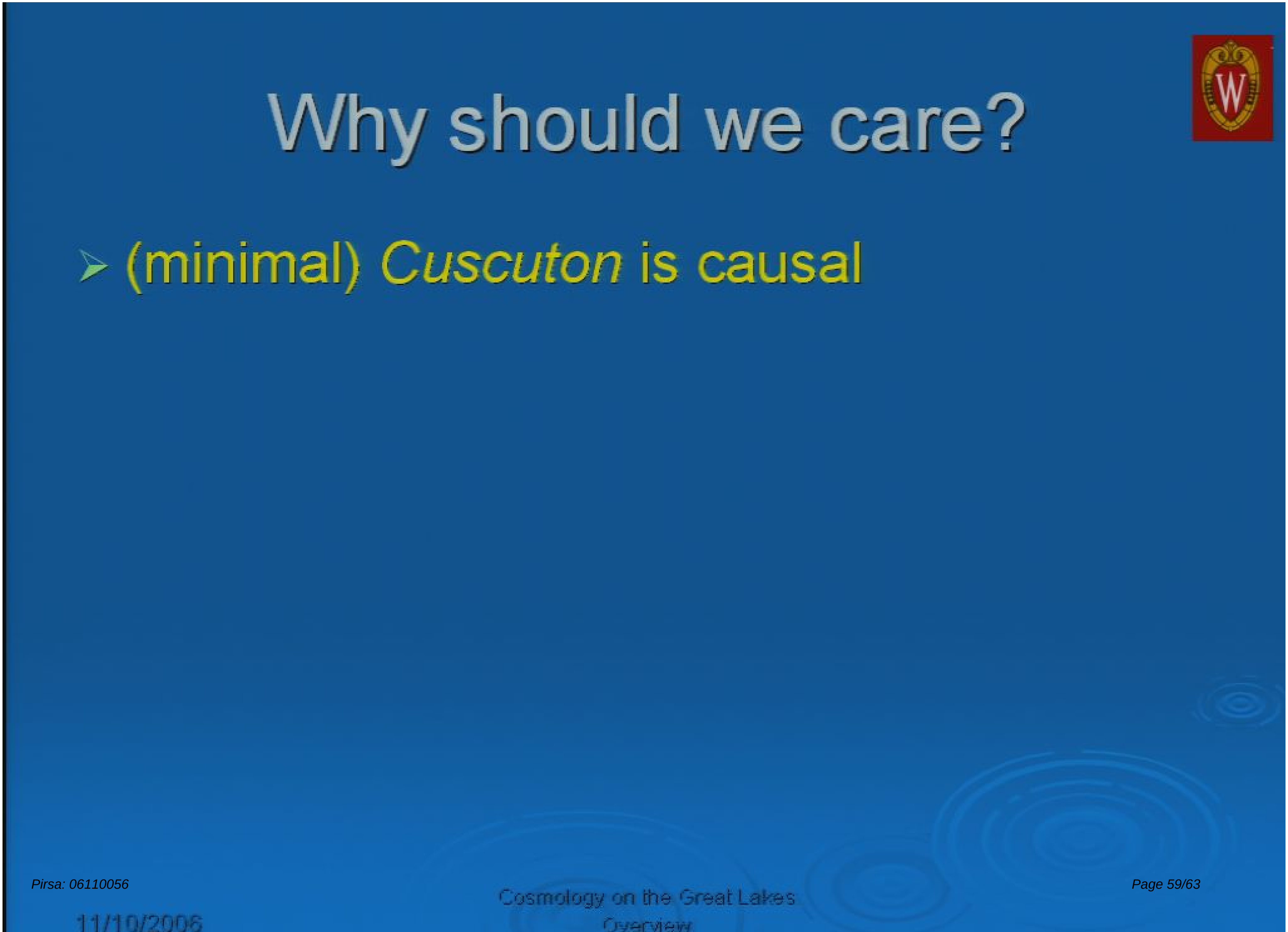


ISW effect can be potentially used to distinguish DGP-like *Cuscuton* from DGP


Why should we care?
(minimal) Cuscuton is causal
11/10/2006
Cosmology on the Great Lakes
Overview

Why should we care?

- (minimal) *Cuscuton* is causal
- is a minimal theory of evolving DE
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- (minimal) *Cuscuton* is causal
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  (no internal dynamics)
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- blurs the observational distinctions between modified gravity and dark energy models

End of slide show, click to exit.