Title: Tests of Inflation from Precision Cosmology

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Abstract:

Tests of Inflation from Precision Cosmology

Richard Easther (Yale)

with: Peter Adshead, Xingang Chen, John ("Tom") Giblin, Eugene Lim and Hiranya Peiris

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Outline...

Now

Later

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- Constraining the inflationary parameter space
  - "Slow roll reconstruction"

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- Non-Gaussianity from single field inflation
    - Consistency checks for funky potentials

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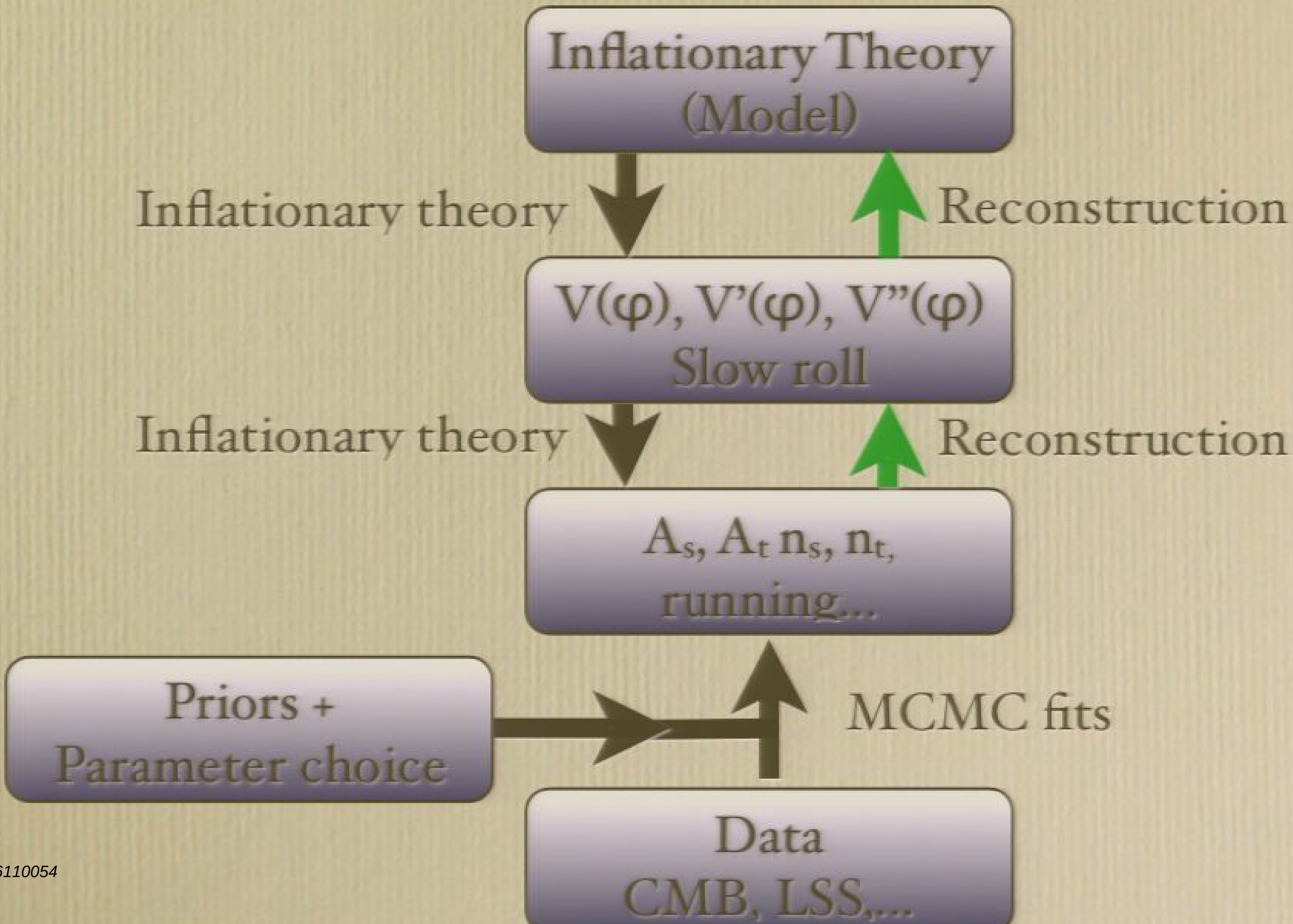
- Constraining the inflationary parameter space
  - "Slow roll reconstruction"
- Non-Gaussianity from single field inflation
  - Consistency checks for funky potentials
- Gravitational radiation from preheating
  - New window into inflationary physics
  - LIGO scales and *smaller*

Now

Later

Fit to Data: Standard Process



Reconstruction (mid 1990s)

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- Find values for n_s, n_t, $dn_s/d\ln k$, r etc from data
  - Not fundamental variables
  - Inflation: functions of slow roll parameters
  - Slow roll parameters ~ V'/V V''/V
    - Derivatives of the potential
    - Wind up with a Taylor expansion for $V(\phi)$

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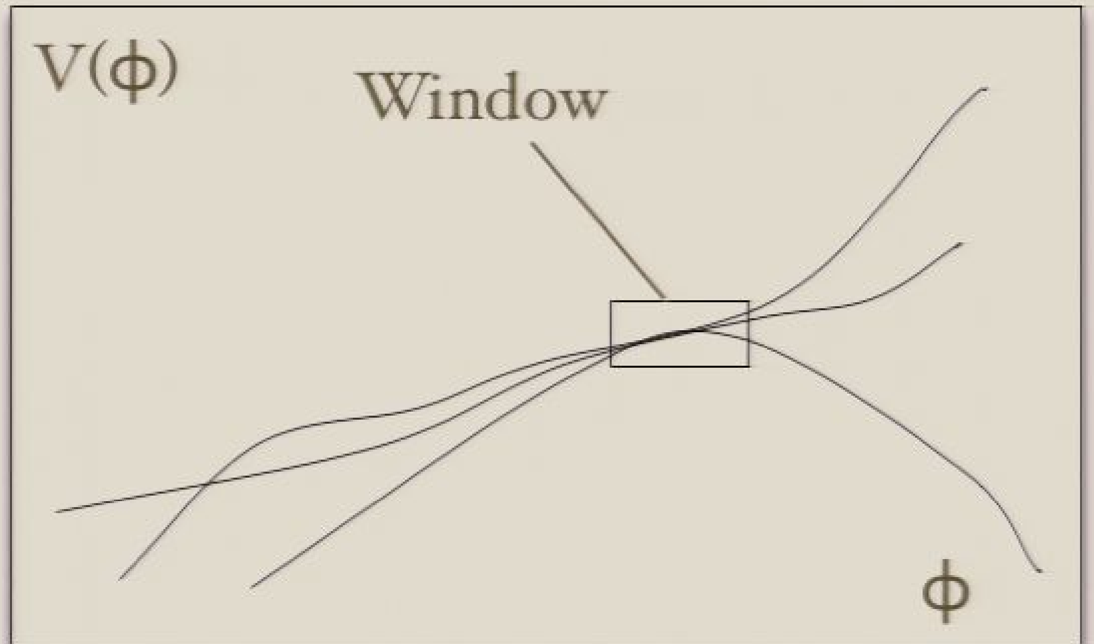
- Only good for a small window in k-space
  - Inflation lasts (at least) 50 e-folds
  - Cosmological perturbations probe ~ 10 e-folds
  - No control of potential beyond that.

Sketch...



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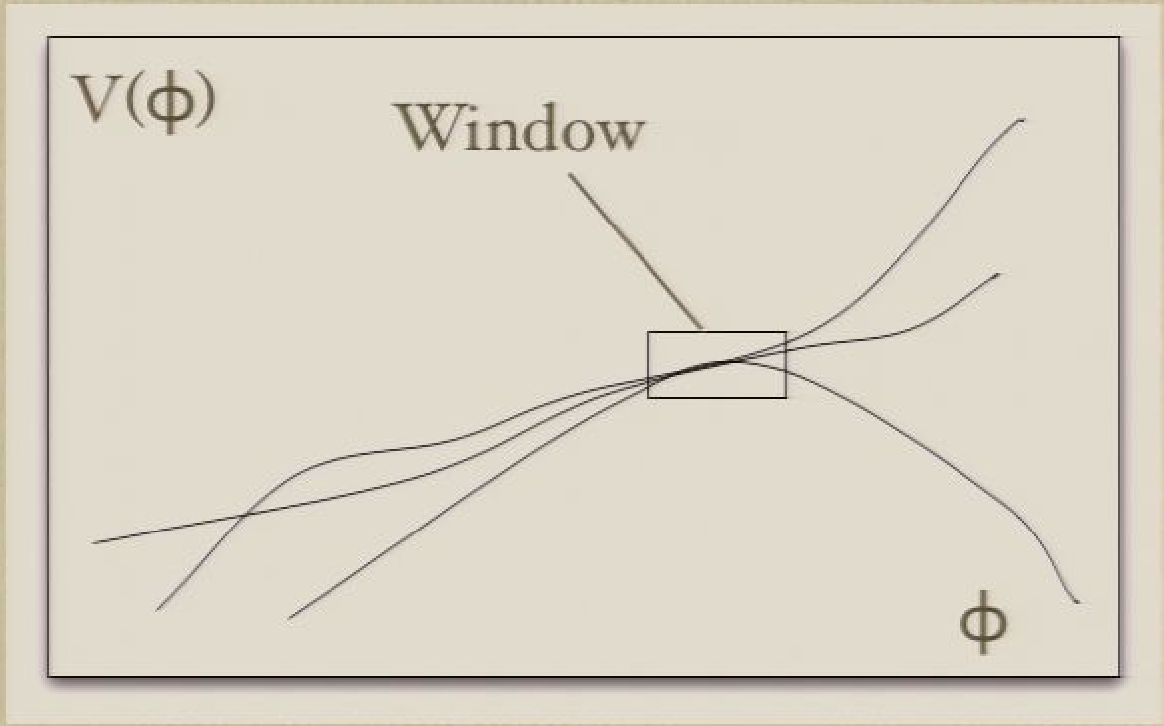

- Classical inverse problem: No unique solution

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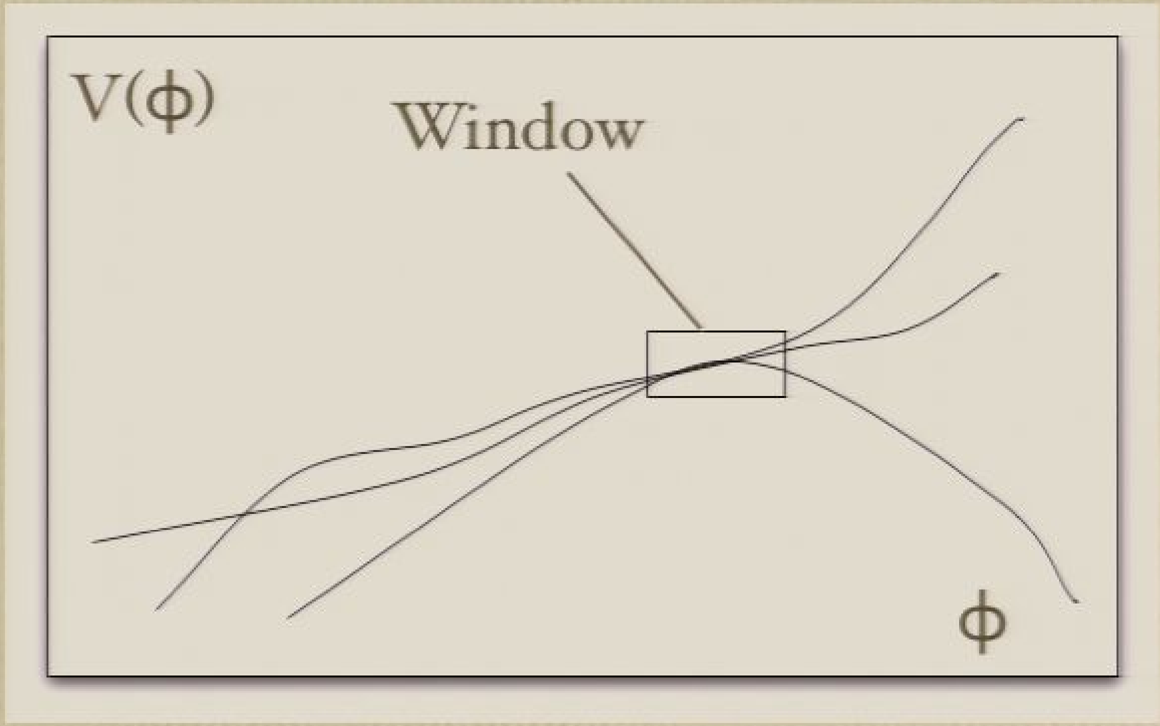

- Classical inverse problem: No unique solution
- Progress depends on how kind the universe is:
  - Can we measure tensor modes or running?

Inflationary Dynamics

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- Inflaton field becomes the clock

Slow Roll Hierarchy

$$\begin{aligned} \epsilon(\phi) &\equiv \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{H'(\phi)}{H(\phi)}\right]^2 \\ {}^{\ell}\lambda_H &\equiv \left(\frac{m_{\rm Pl}^2}{4\pi}\right)^{\ell}\frac{(H')^{\ell-1}}{H^{\ell}}\frac{d^{(\ell+1)}H}{d\phi^{(\ell+1)}};\ \ell \geq 1. \\ \eta &= {}^{1}\lambda_H \\ \xi &= {}^{2}\lambda_H \end{aligned}$$

Definition

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- Describing dynamics in terms of H (energy density)
  - Potential can be derived from H.

Flow Equations

$$\frac{d\epsilon}{dN} = 2\epsilon(\eta - \epsilon)$$

$$\frac{d\eta}{dN} = -\epsilon\eta + \xi$$

$$\frac{d\xi}{dN} = \xi(\eta - 2\epsilon) + {}^3\lambda_H$$

$$\frac{d\left({}^\ell\lambda_{\rm H}\right)}{dN} = \left[\frac{\ell-1}{2}\eta + (\ell-2)\,\epsilon\right]\left({}^\ell\lambda_{\rm H}\right) + {}^{\ell+1}\lambda_{\rm H}$$

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  - Truncated hierarchy is closed
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Summary

$$
\begin{aligned}
\frac{d^{(M+2)}H}{d\phi^{(M+2)}} &= 0 \\
H(\phi) &= H_0\left[1 + B_1\left(\frac{\phi}{m_{Pl}}\right) + \cdots + B_{M+1}\right] \\
\epsilon(\phi) &= \frac{m_{\rm Pl}^2}{4\pi}\left[\frac{\left(\frac{B_1}{m_{Pl}}\right) + \cdots + (M+1)\left(\frac{B_{M+1}}{m_{Pl}}\right)\left(\frac{\phi}{m_{Pl}}\right)^M}{1 + B_1\left(\frac{\phi}{m_{Pl}}\right) + \cdots + B_{M+1}\left(\frac{\phi}{m_{Pl}}\right)^{M+1}}\right]^2 \\
B_1 &= \sqrt{4\pi\epsilon_0} \qquad B_{\ell+1} = \frac{(4\pi)^\ell}{(\ell+1)!B_1^{\ell-1}}{}^\ell\lambda_{H,0};\ \ell \geq 1.
\end{aligned}
$$

Summary

- HSR hierarchy captures full inflationary dynamics
  - Truncate it, get an approximate potential
  - Truncated hierarchy has an exact solution

$$
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Inflation Meets Data

Easther and Peiris

astro-ph/0603587, 0604214 & 0609003

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ASTRO-PH/0603587, 0604214 & 0609003

- We fit to the (Hubble) slow roll parameters
  - From these we can rebuild the potential
  - "Thorough" inflationary prior.
  - Do not explicitly drop smoothness

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- We fit to the (Hubble) slow roll parameters
  - From these we can rebuild the potential
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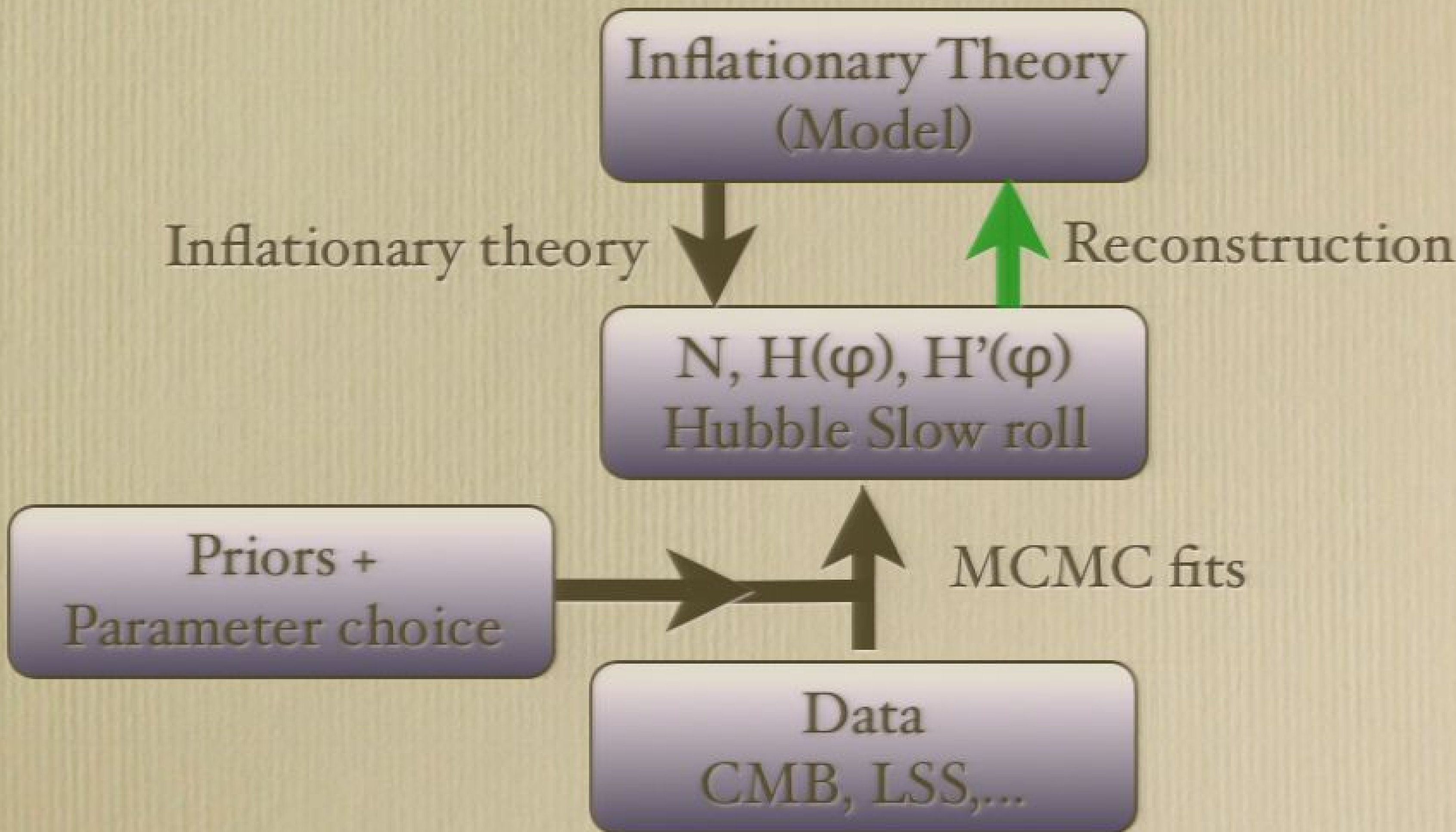
- Fundamental physics in cosmological parameter set
  - May not give "unique" answer
  - But will give a weighted ensemble of potentials
  - Simultaneously fit to late universe variables

Fit to Data:
Slow Roll Reconstruction



Inflationary Observables

$$P_{\mathcal{R}} = \frac{[1-(2C+1)\epsilon + C\eta]^2}{\pi\epsilon}\left(\frac{H}{m_{\rm Pl}}\right)^2\Bigg|_{k=aH},$$

$$P_h = [1-(C+1)\epsilon]^2\,\frac{16}{\pi}\left(\frac{H}{m_{\rm Pl}}\right)^2\Bigg|_{k=aH},$$

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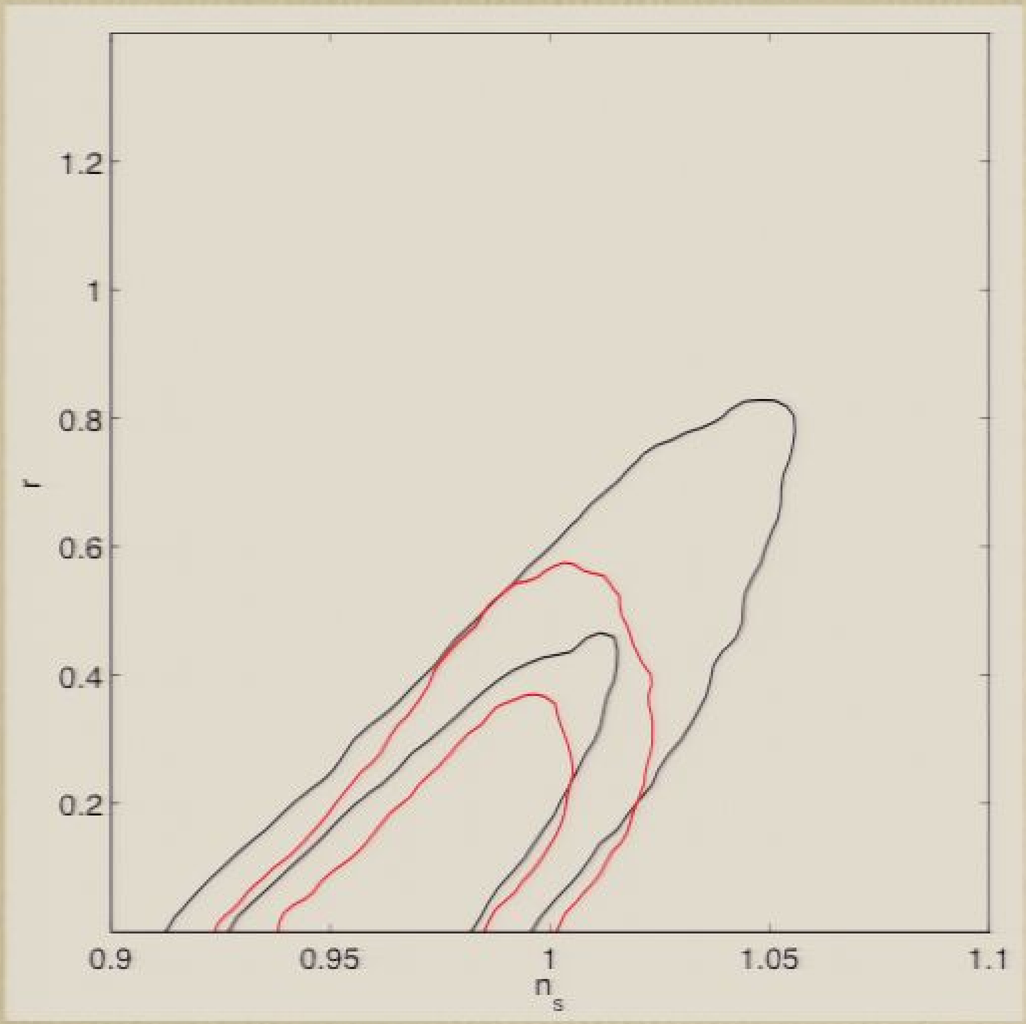
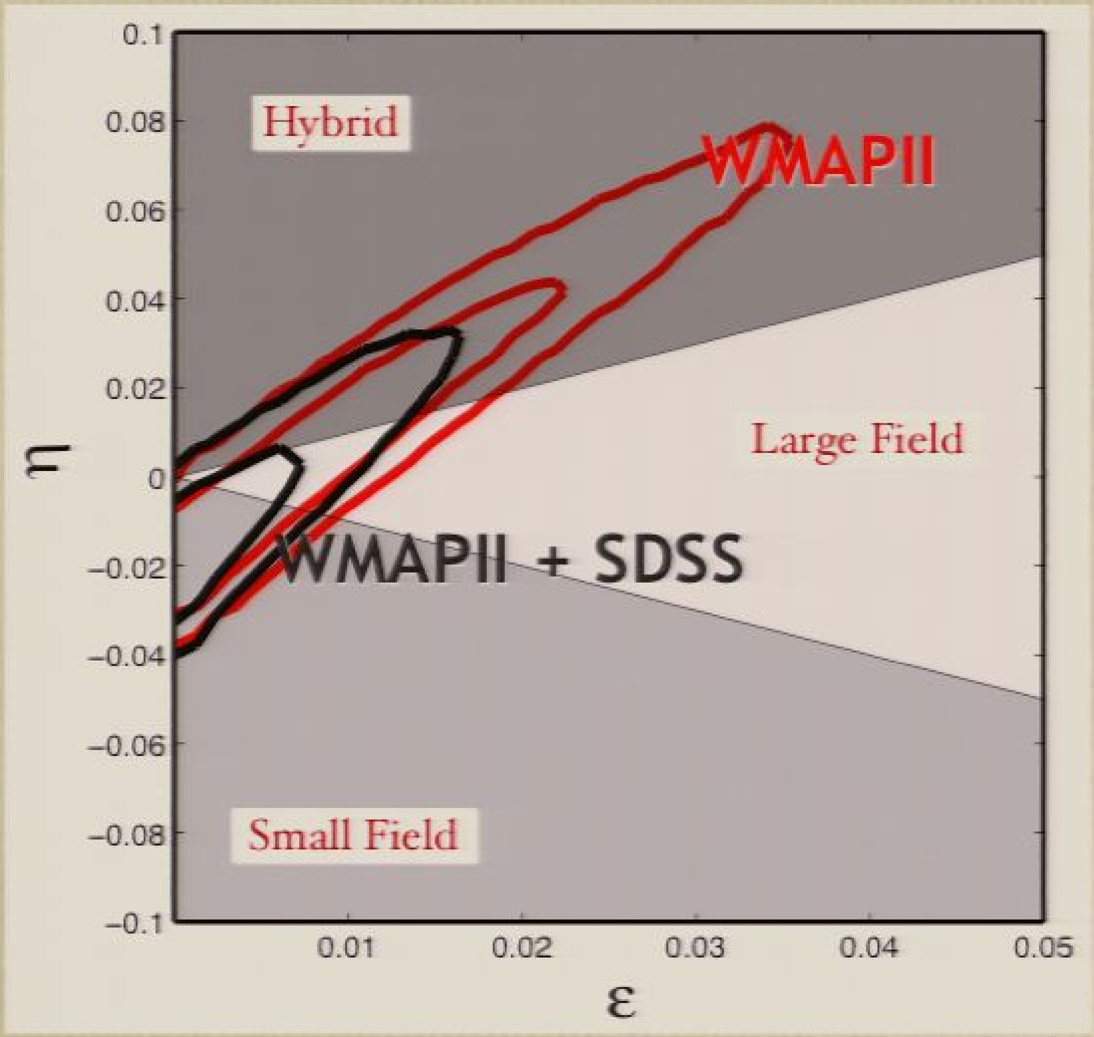
- Avoids the use of a pivot
- Could compute the spectrum exactly if needed
- Can include constraints on N
  - What about models where inflation never ends?

Two parameter chains...



Peiris & Easther (2006)

"Zoo Plots"

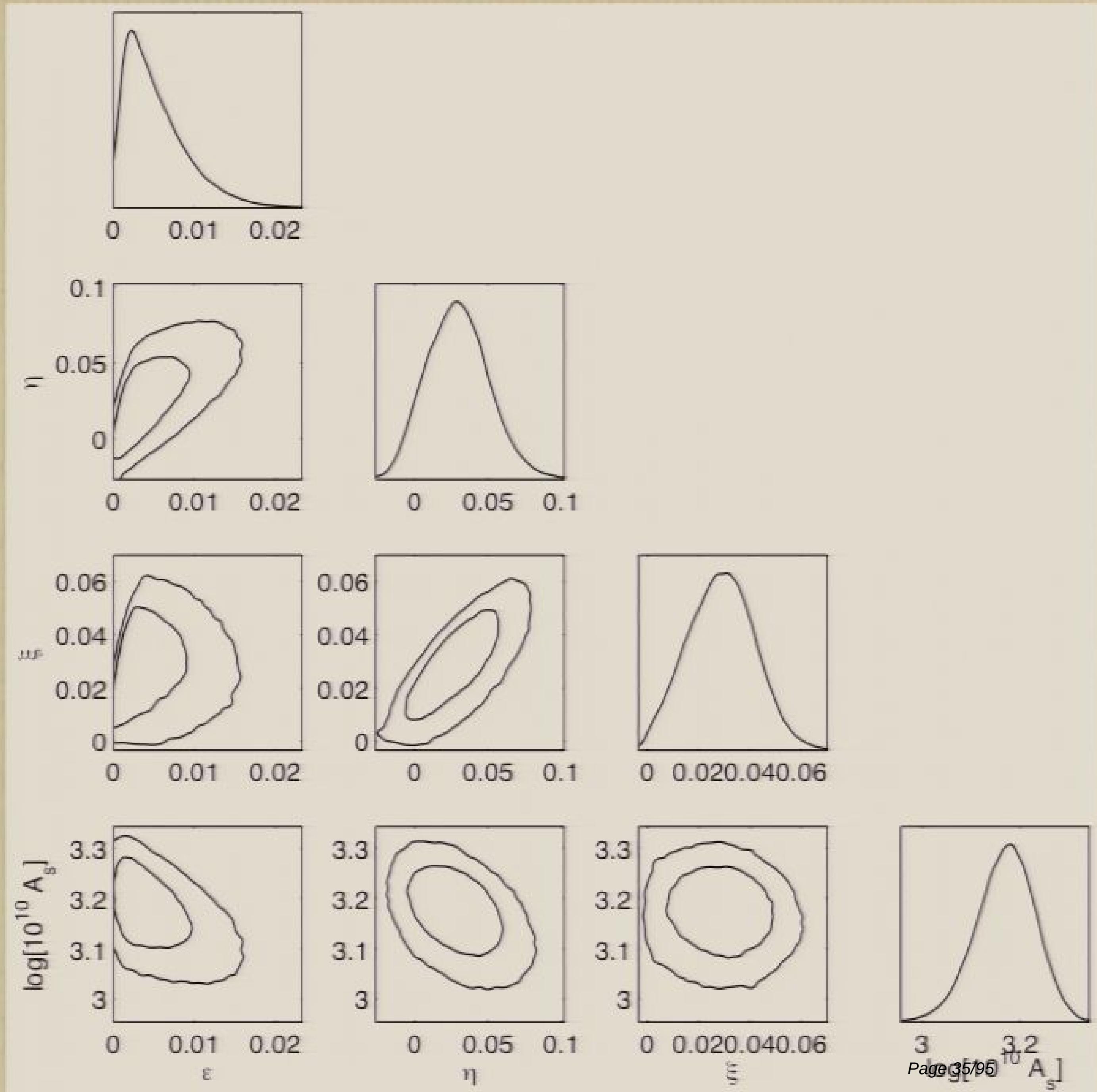


Peiris & Easther (2006)

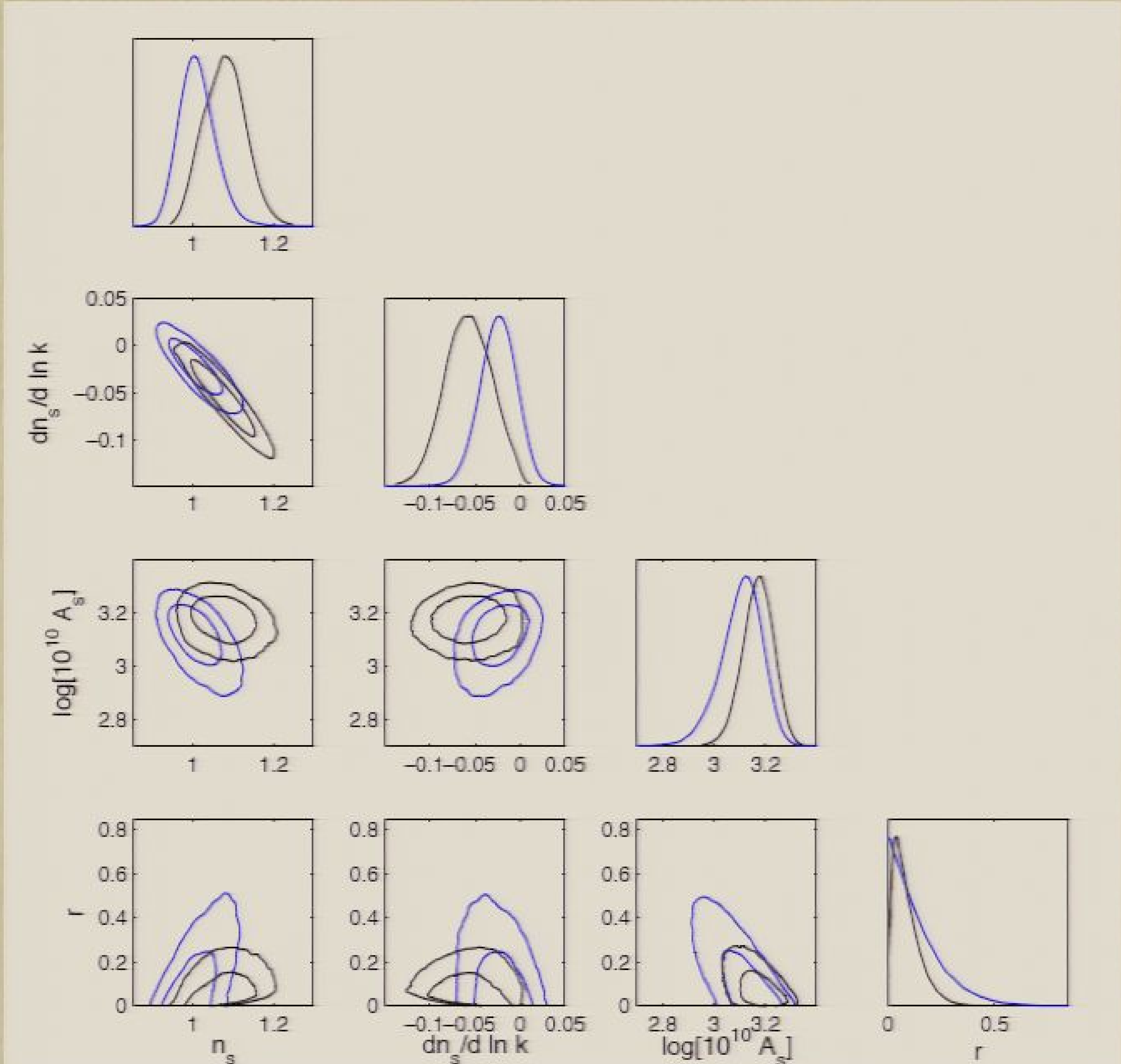
Constraints on first two HSR parameters at k=0.002 Mpc^{-1}

3 Slow Roll Parameters

WMAPII+SDSS



Comparison with WMAP analysis



NB: Not expected to overlap...

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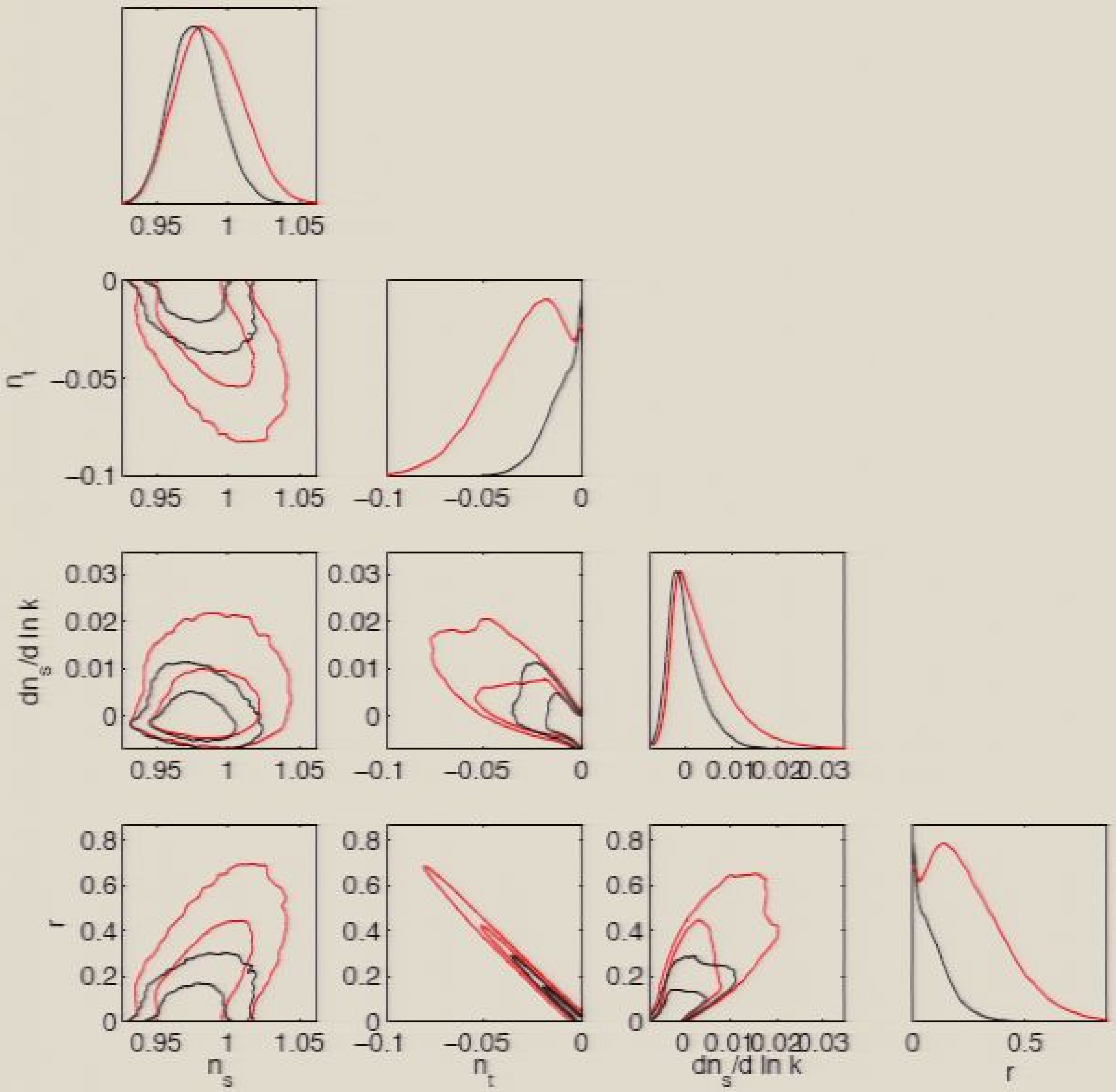
Standard Parameters N>30



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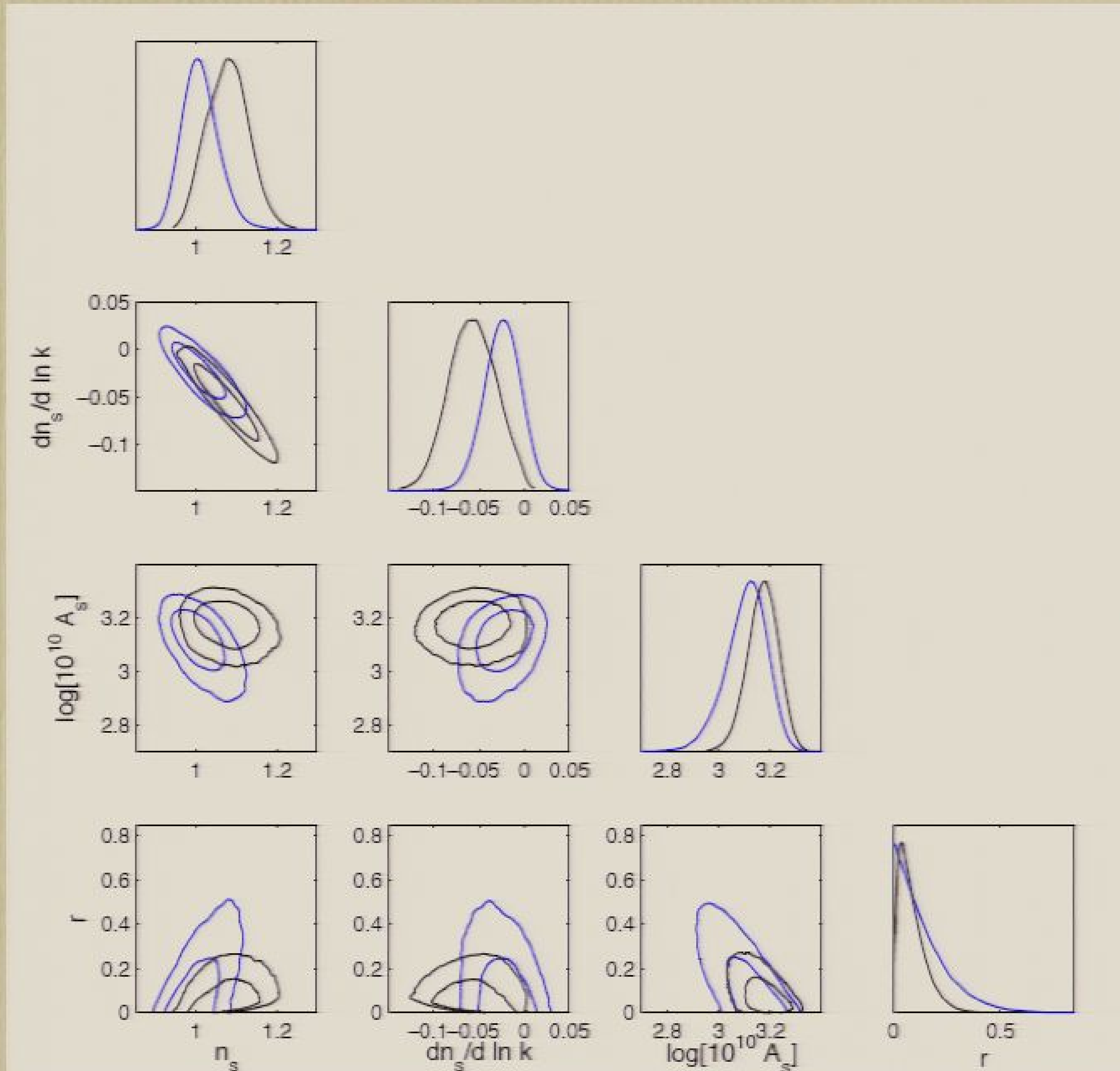

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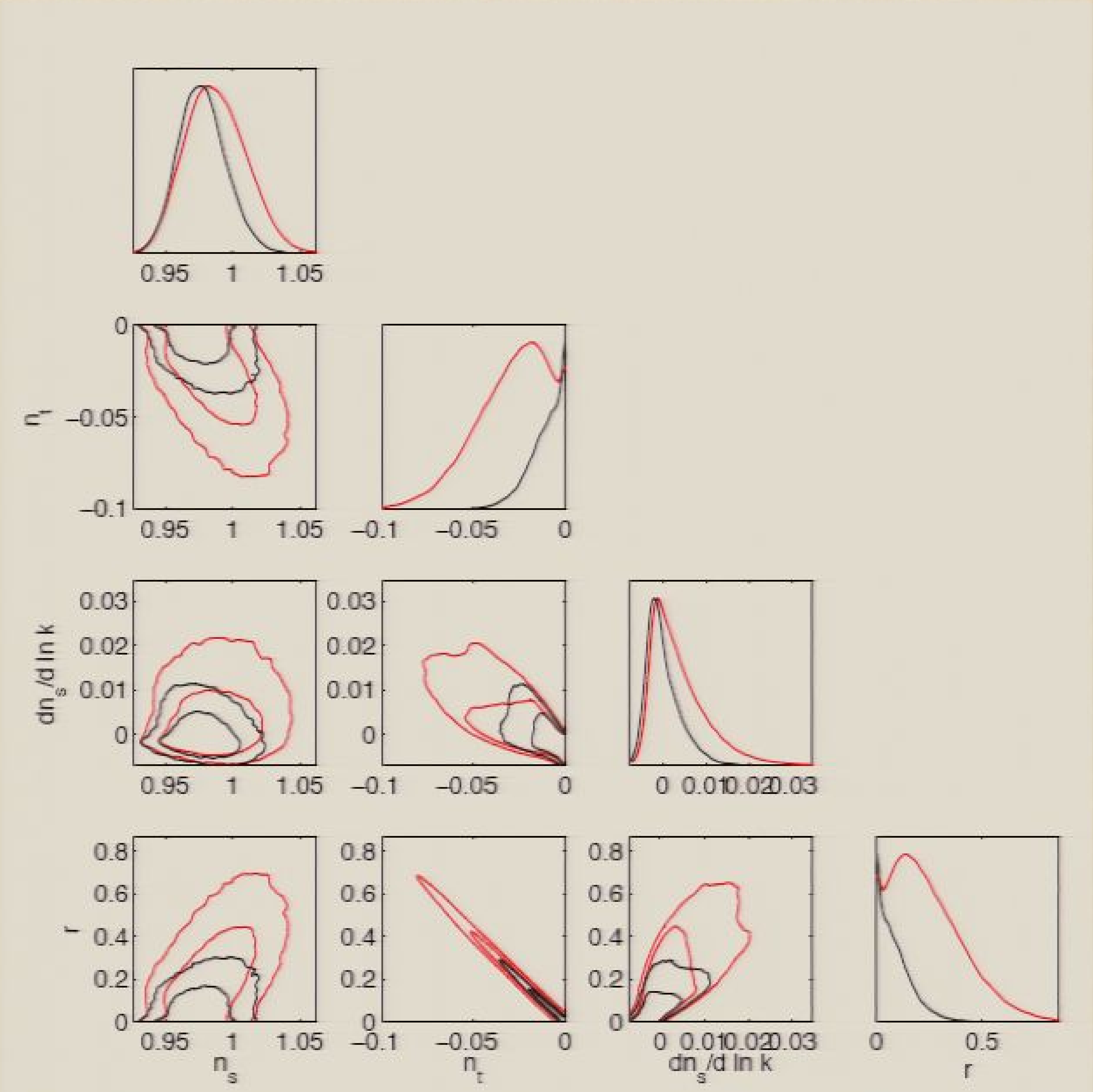
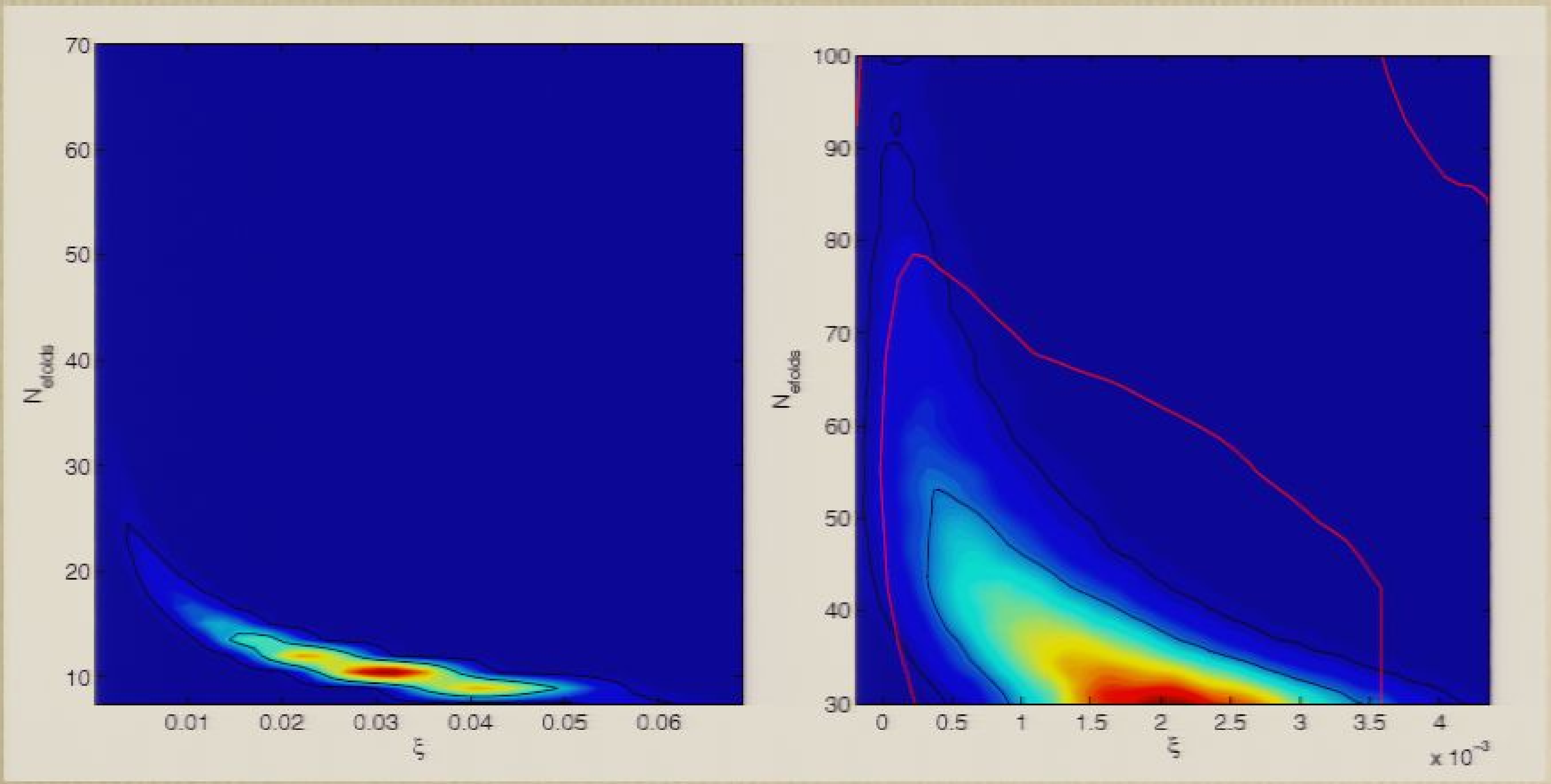


Peiris & Easther (2006)

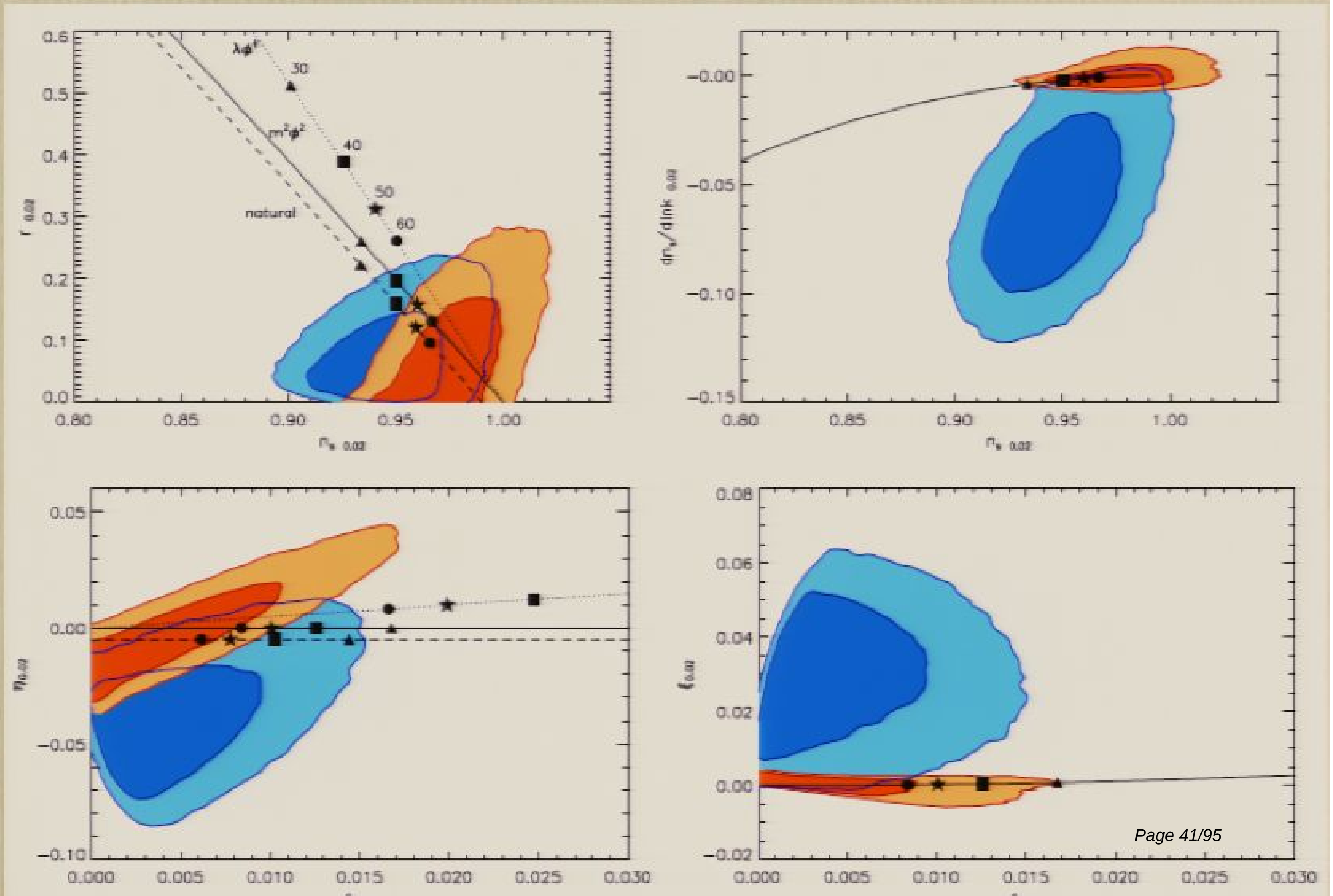
Number of e-folds correlated with ξ



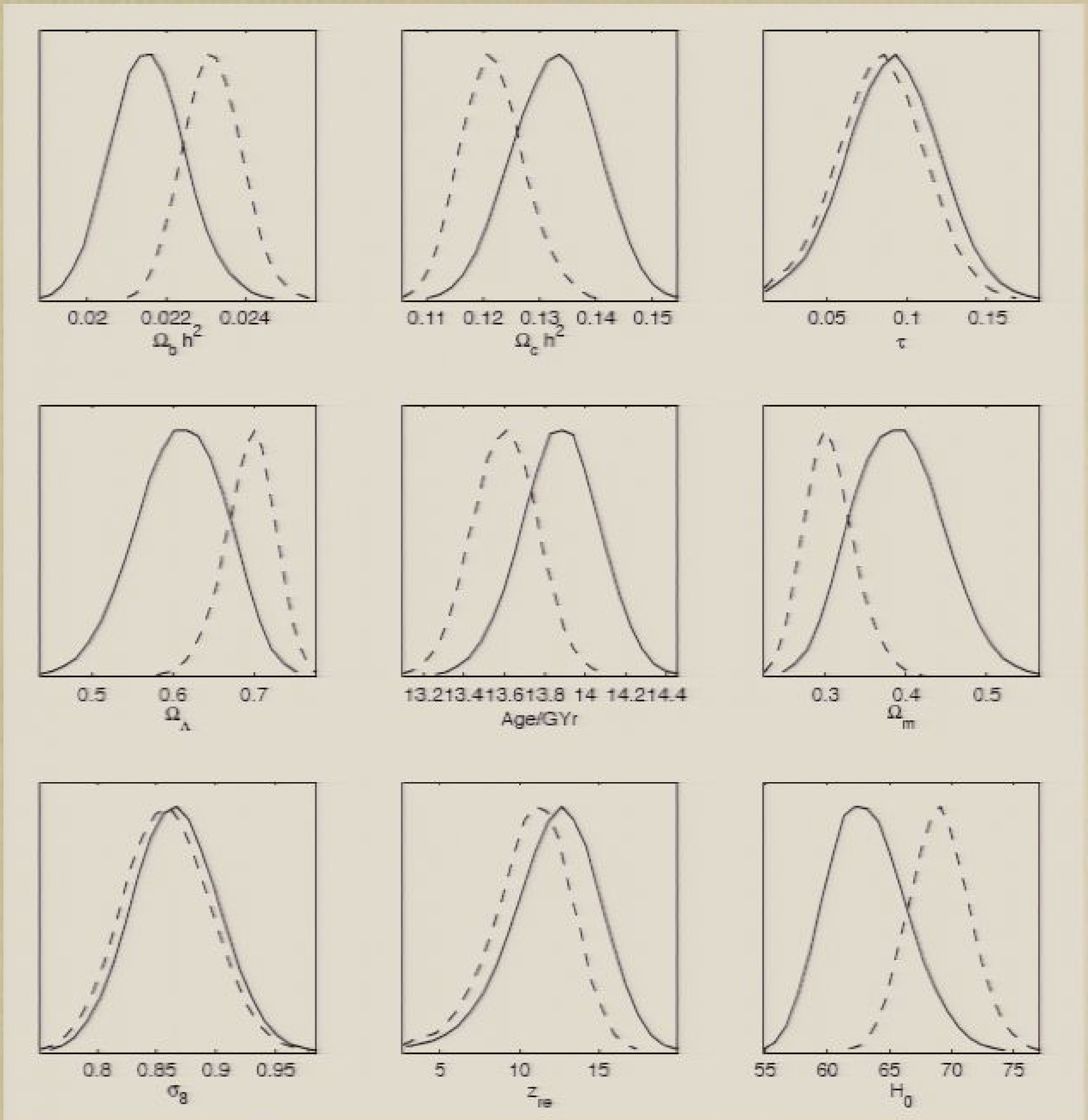
Peiris & Easther (2006)

Inflationary Trajectories WMAP+SDSS

N>30
N free



"Late" parameters and the N-prior



What Do We Learn?

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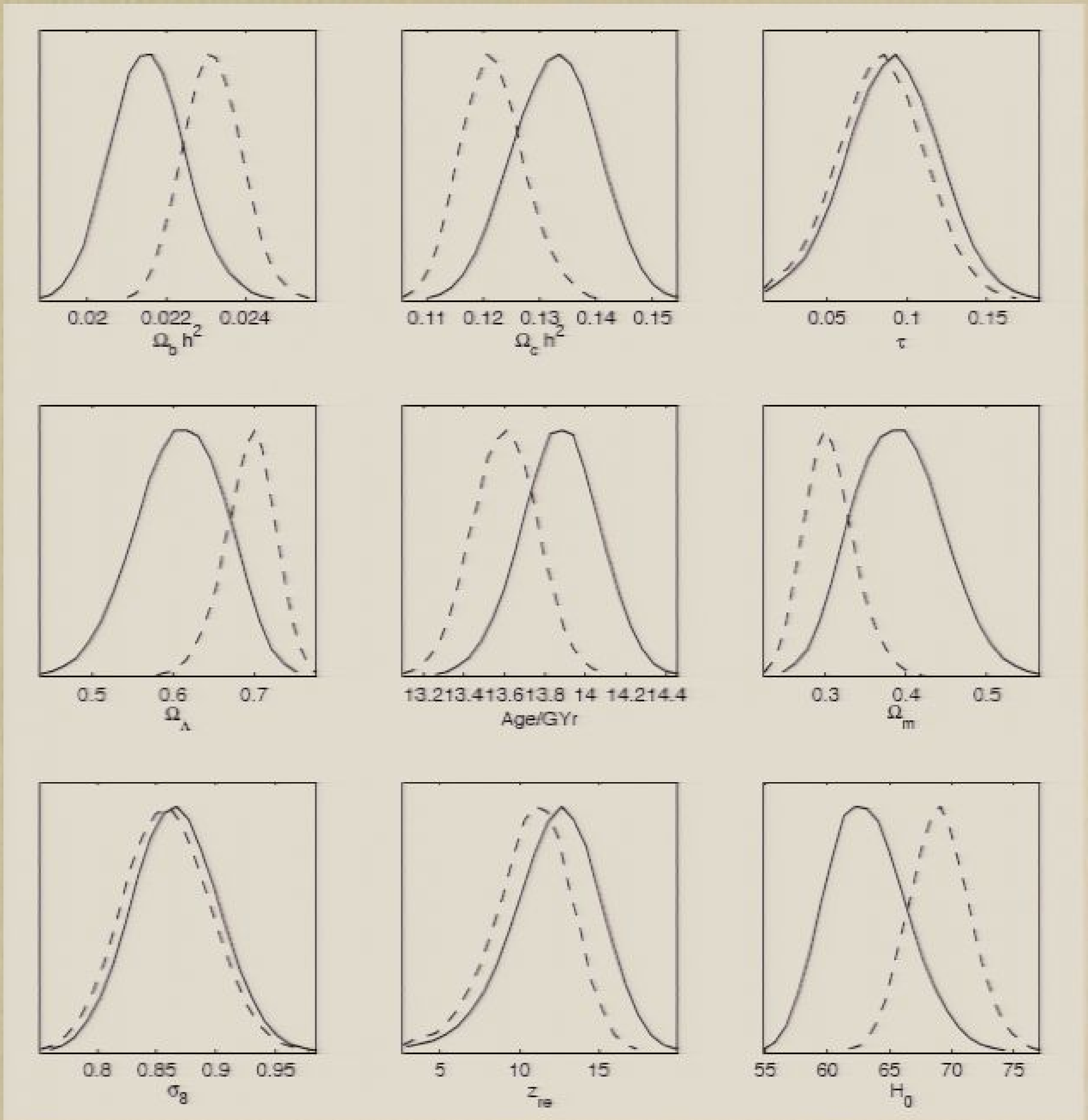

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- Three ways out:
  - Higher order terms non-trivial
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- Confirming the current WMAP centroid would cause serious problems for (minimal) inflation.
  - Seek single experiment / high-l probes of CMB
  - Better constraints on Ω_m etc to use as priors.
  - Evidence for running present at same level as in WMAP analysis

Future Work...

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- Look at all sources of information
  - CMB: WMAP-N, ACT, Planck, CMBPol
  - Large scale structure (SDSS, LSST, ...)
  - Ly-α forest
  - High z 21 centimeter

  Large scale

  - Direct detection of primordial GWs (BBO)

  Solar system scales

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Non-Gaussianities

ASTRO-PH/0611???

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ASTRO-PH/0611???

- Power spectrum is only one statistic
  - 2 point correlation function
  - But there can be more information in the sky

- Inflation "predicts" perturbations are Gaussian
  - Know 2-point function, can calculate n-point
  - But there are exceptions...

State of Play...

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  - f_{NL}: $\Phi(x) = \Phi_L(x) + f_{NL}(\Phi_L(x)^2 - \langle \Phi_L(x)^2 \rangle)$

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- Single field, slow roll $f_{NL} \sim 0.01$
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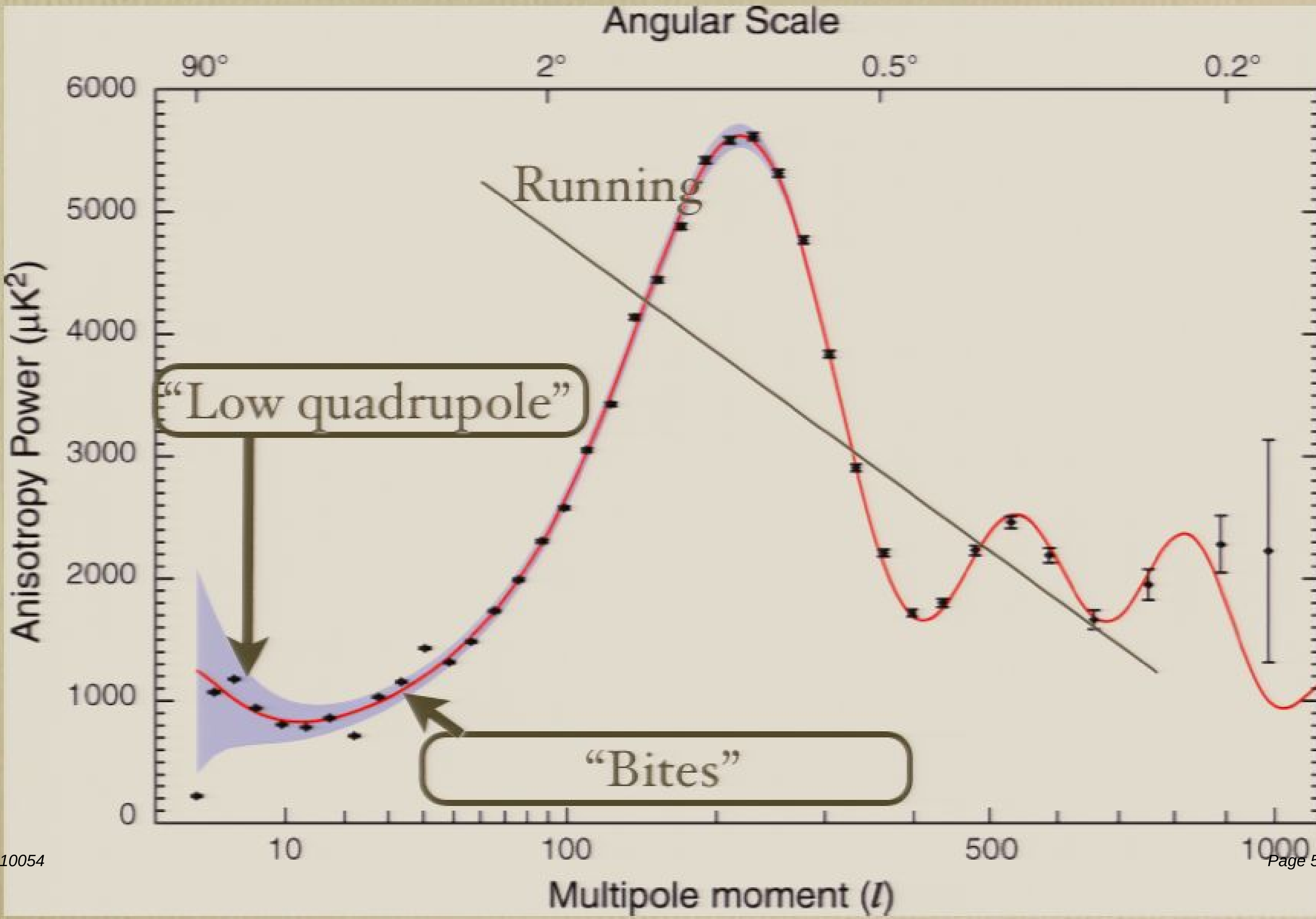
- DBI in the sky, brane inflation: $f_{NL} \sim 100$
  - Sound speed << c

WMAP-II (2006)



Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
Anisotropy Power (μK²)
Running
"Low quadrupole"
"Bites"
10
100
500
1000
Multipole moment (l)

Local Violations of Slow Roll

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- Explain "bites" via features in the potential
  - Analysis: Adams, Cresswell Easther ('01)

$$V(\phi) = \frac{1}{2}m^2\phi^2\left[1 + c\tanh\left(\frac{\phi - \phi_{\rm step}}{d}\right)\right]$$

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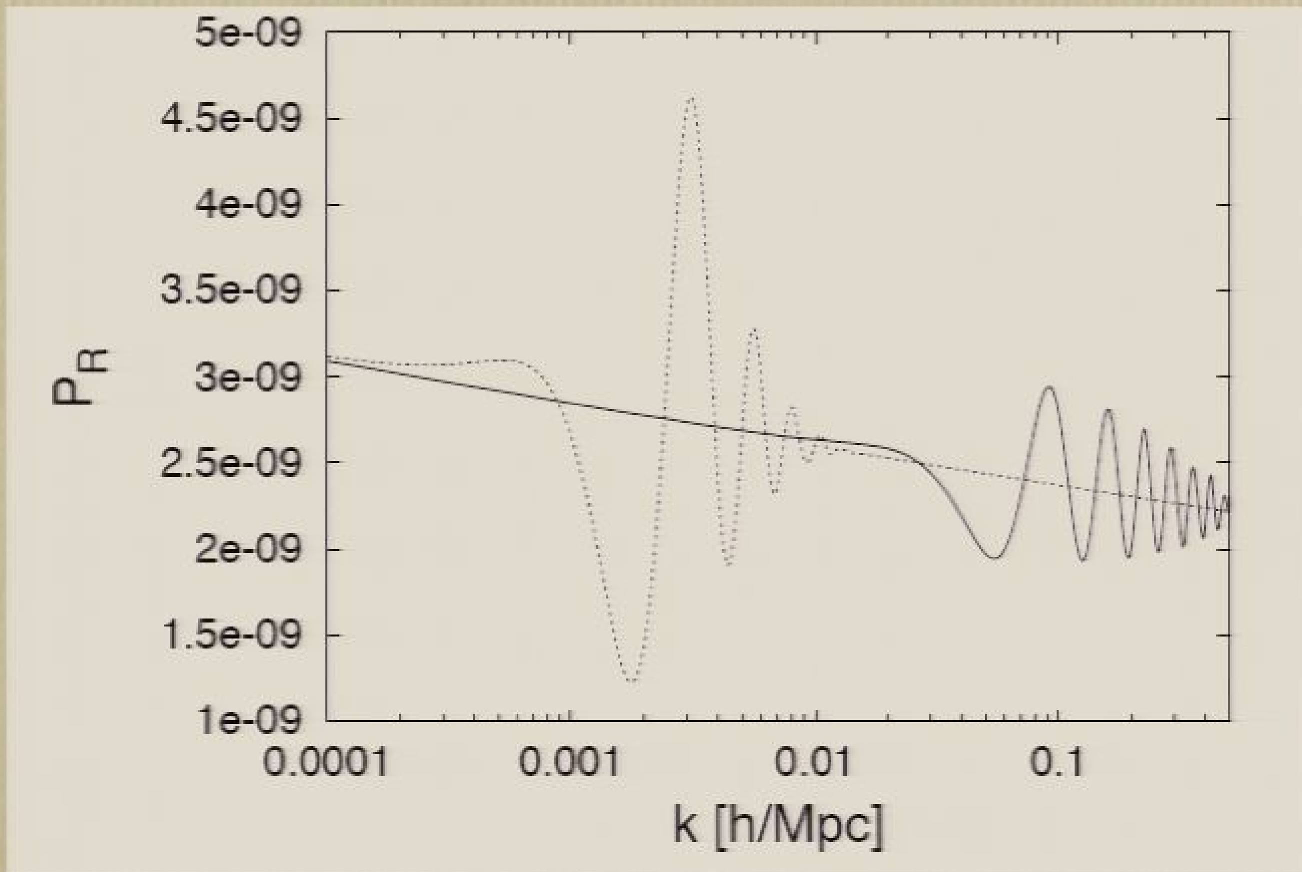
- Comparison with observations:
  - First fit to data: Peiris et al (2003) (WMAPI)
  - WMAPII: Covi et al. ASTRO-PH/0606452
  - Some support for a feature at l~30.
    - c ~ 0.002 (small step)

2 Point Function



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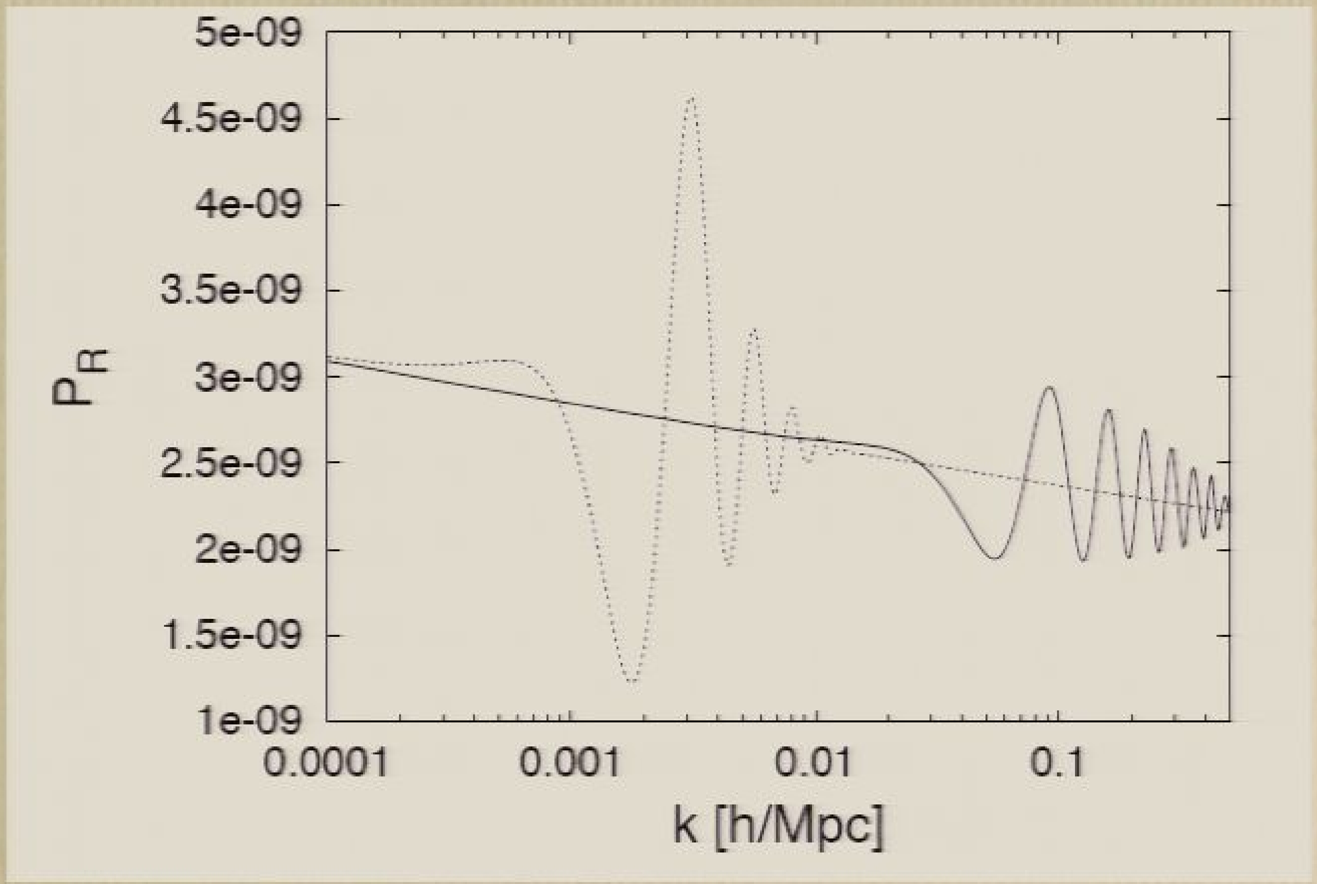

What About 3 Point Function?

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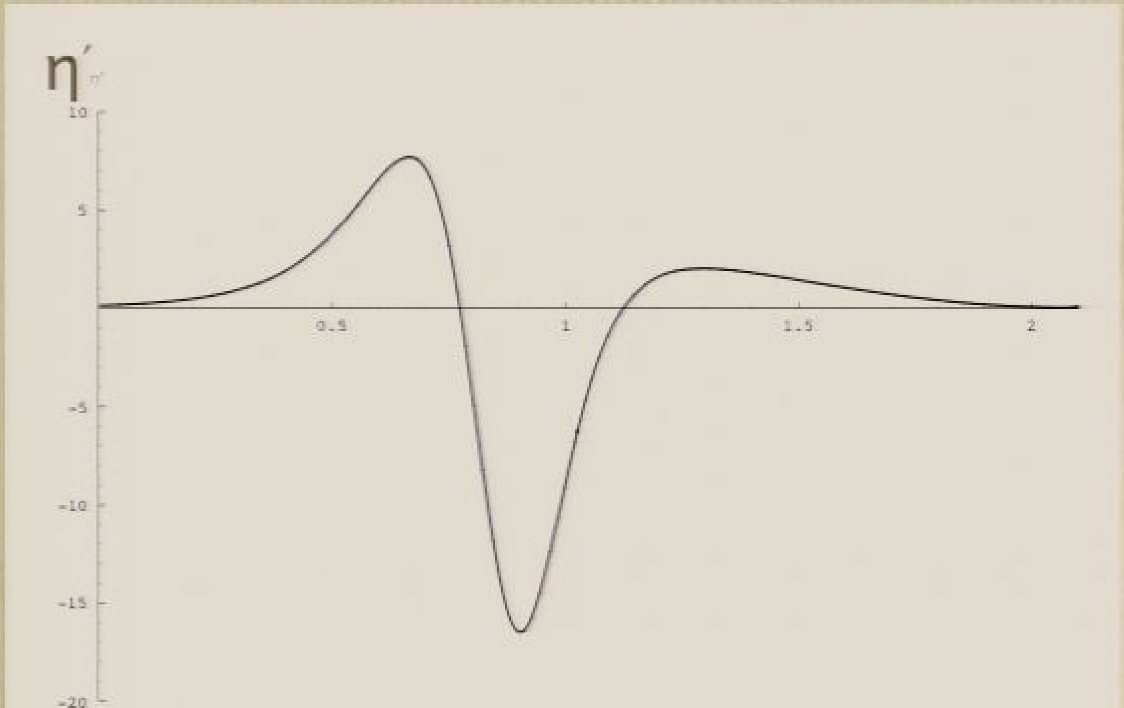
- "f_{NL}" ~ slow roll parameters.
  - ϵ, η and η' - η' enhanced ~ $O(10^3)$ at step
  - Large non-Gaussian signal



Compute 3-point function

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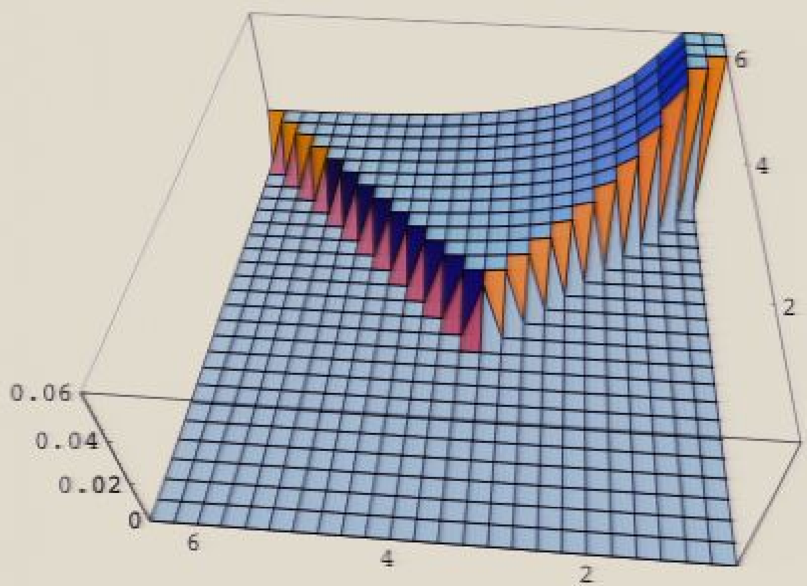
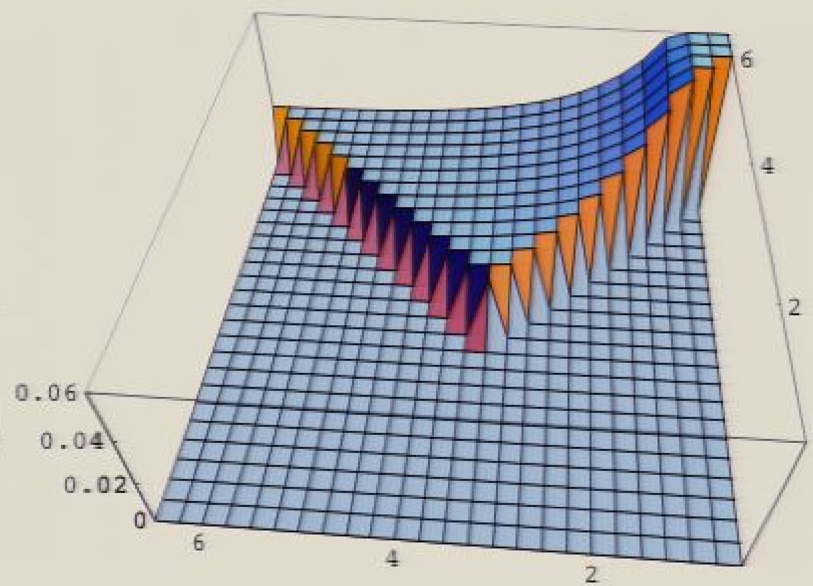
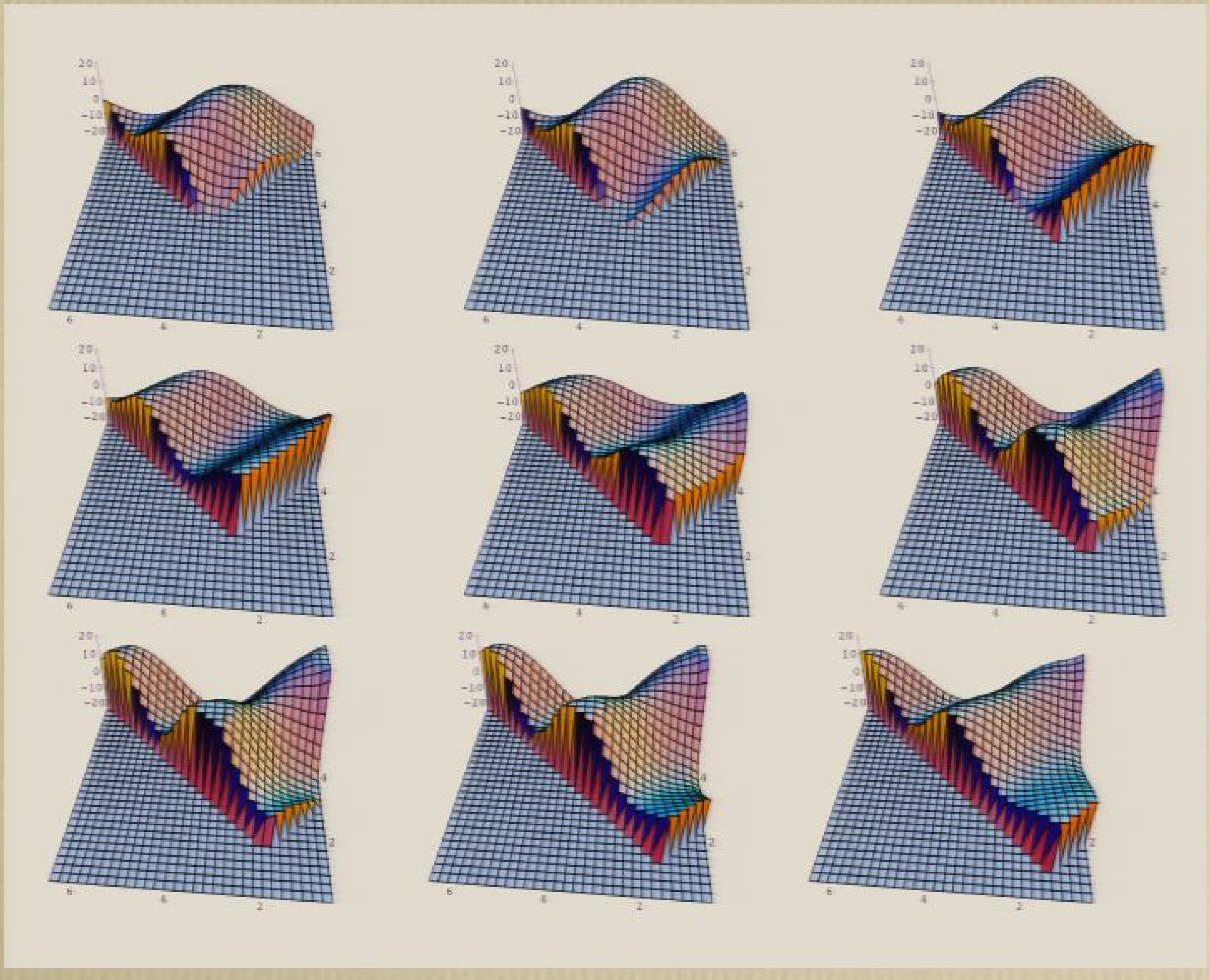
- Find strong shape and scale dependence
  - Have to think carefully about best statistic

No Step Case...





Signal...



Conclusions II

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- Can improve fit to C_l by assuming a funky potential
  - But: 3-point function now strongly enhanced
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  - Working to understand this now...
  - Powerful consistency test
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- Generalize this to multi-field models.
  - Looked at slow roll with uncoupled fields.
  - Do other cases numerically.

Reheating and Preheating

ASTRO-PH/0601617 & 0611???

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ASTRO-PH/0601617 & 0611???

- Inflationary universe cold and empty
- Must "reheat" the universe
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- Originally, this was assumed to be done by coupling of the inflation to other fields (reheating)
  - Couplings necessarily weak

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- But can also get parametric resonance / preheating
  - Non-perturbative effect
  - Akin to stimulated emission

Parametric Resonance: Math (Continued Again)

- A > 2q
- A and q depend weakly on time.

move in and out

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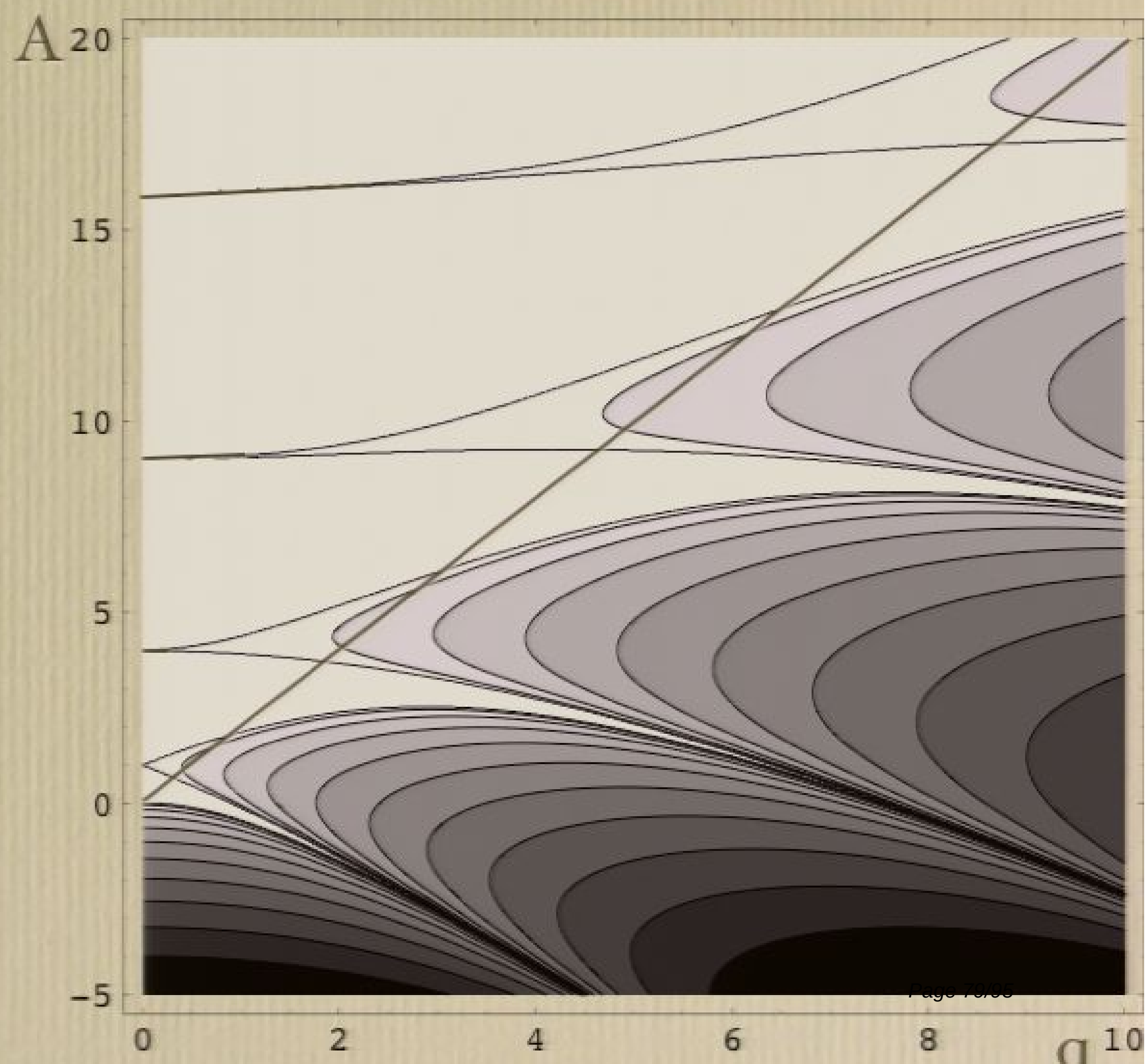

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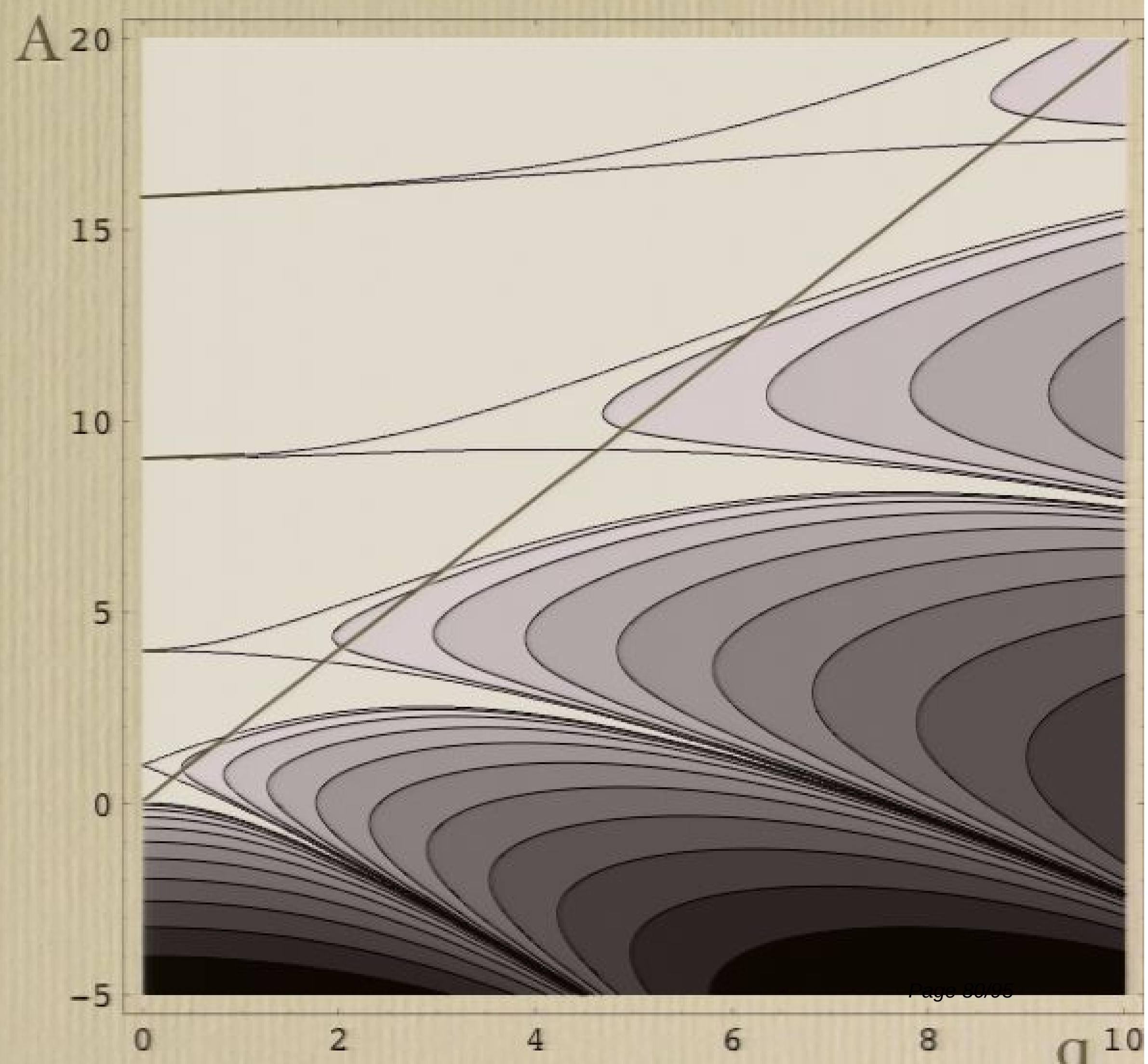
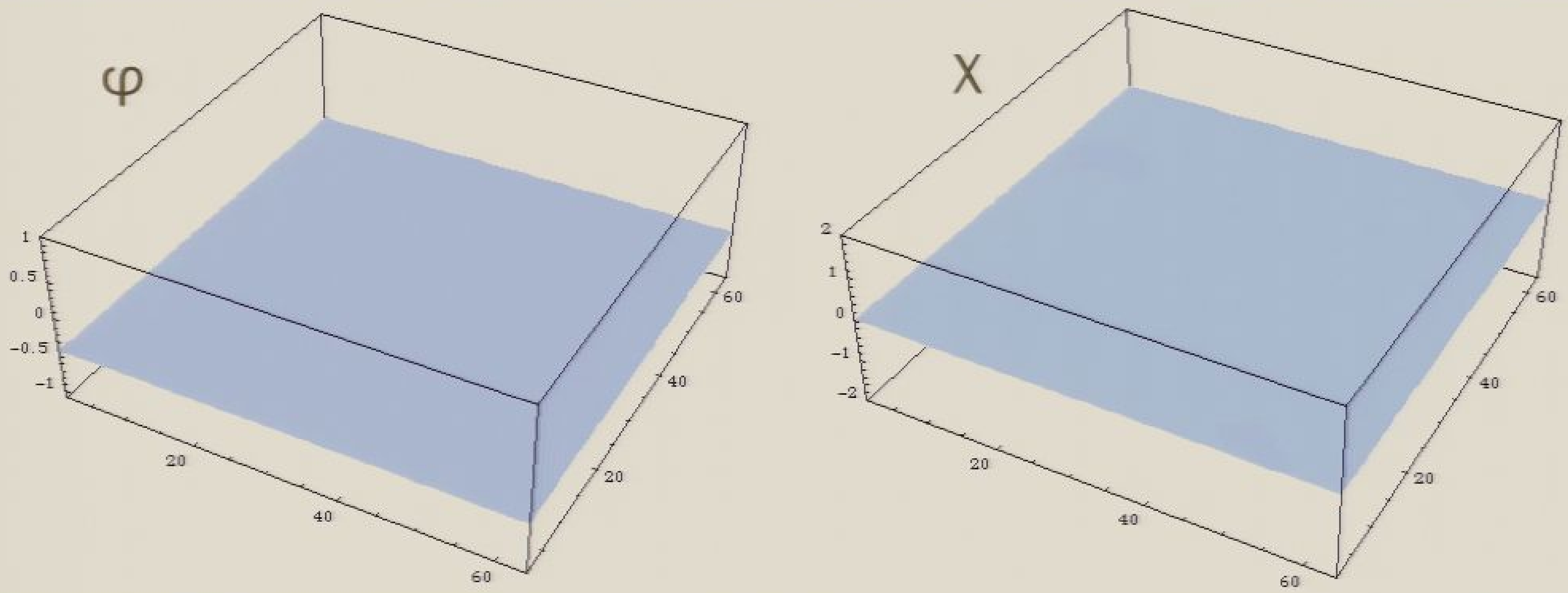
- A and q depend weakly on time.
- Modes move in and out of resonance

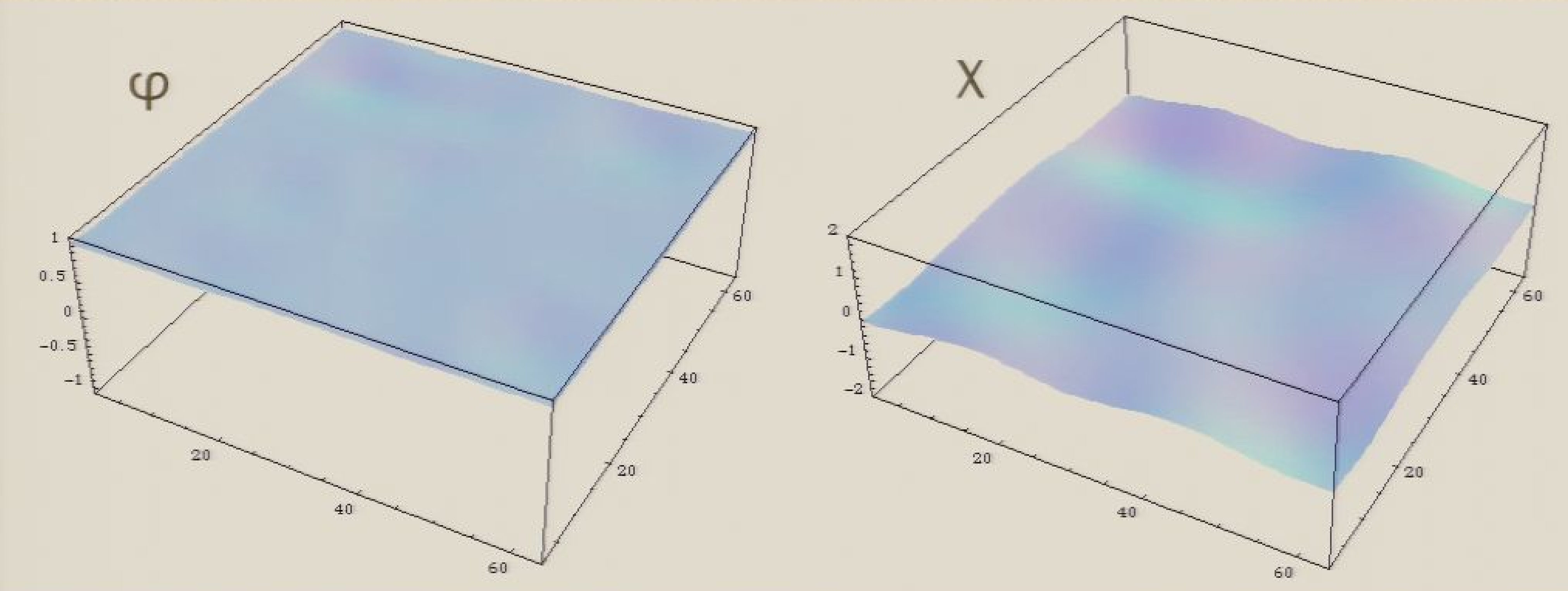


Parametric Resonance: For Beginners

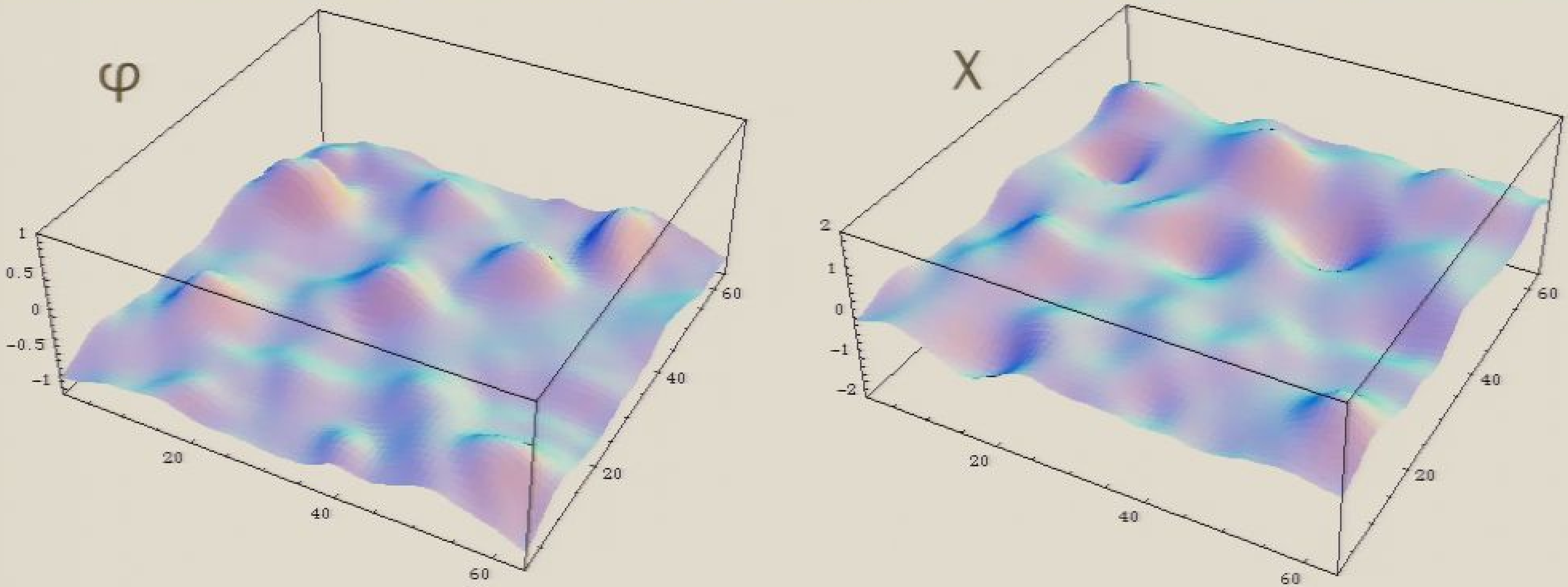
Simulations



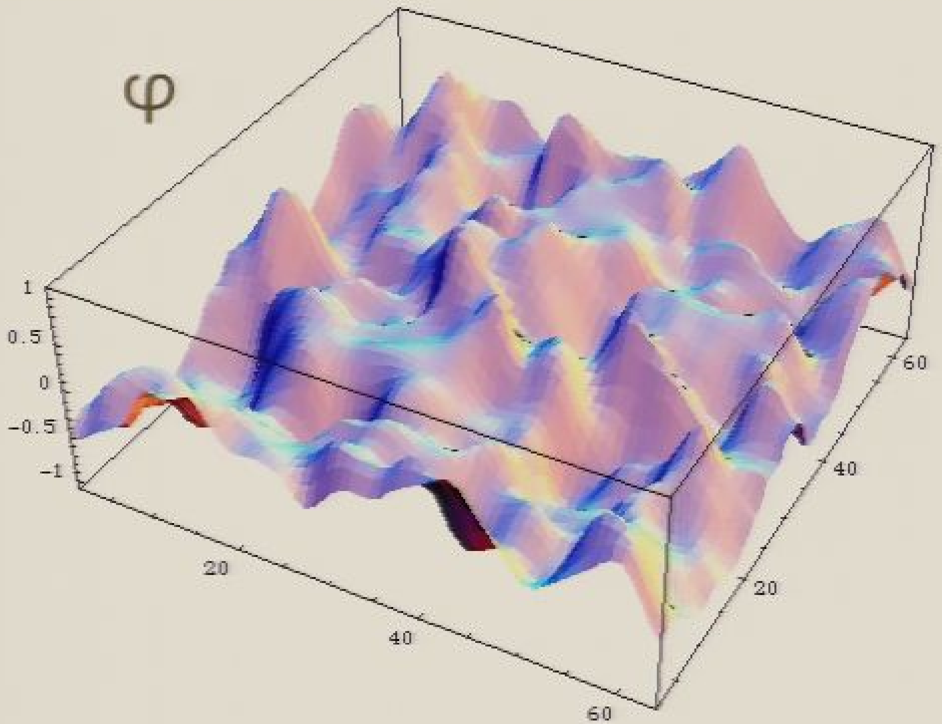
Simulations

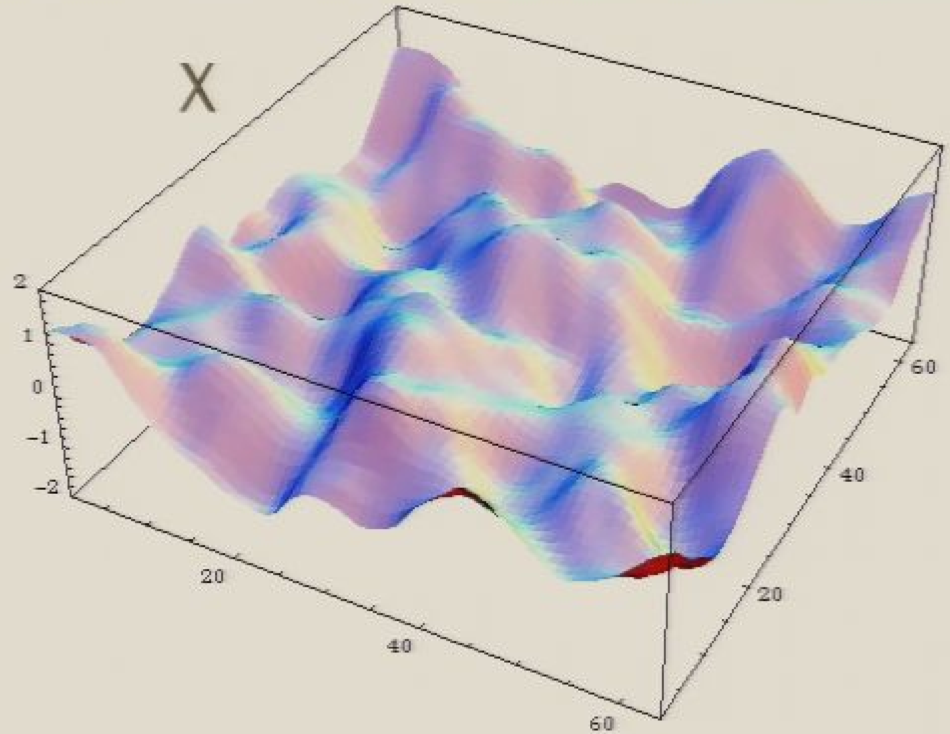


Simulations

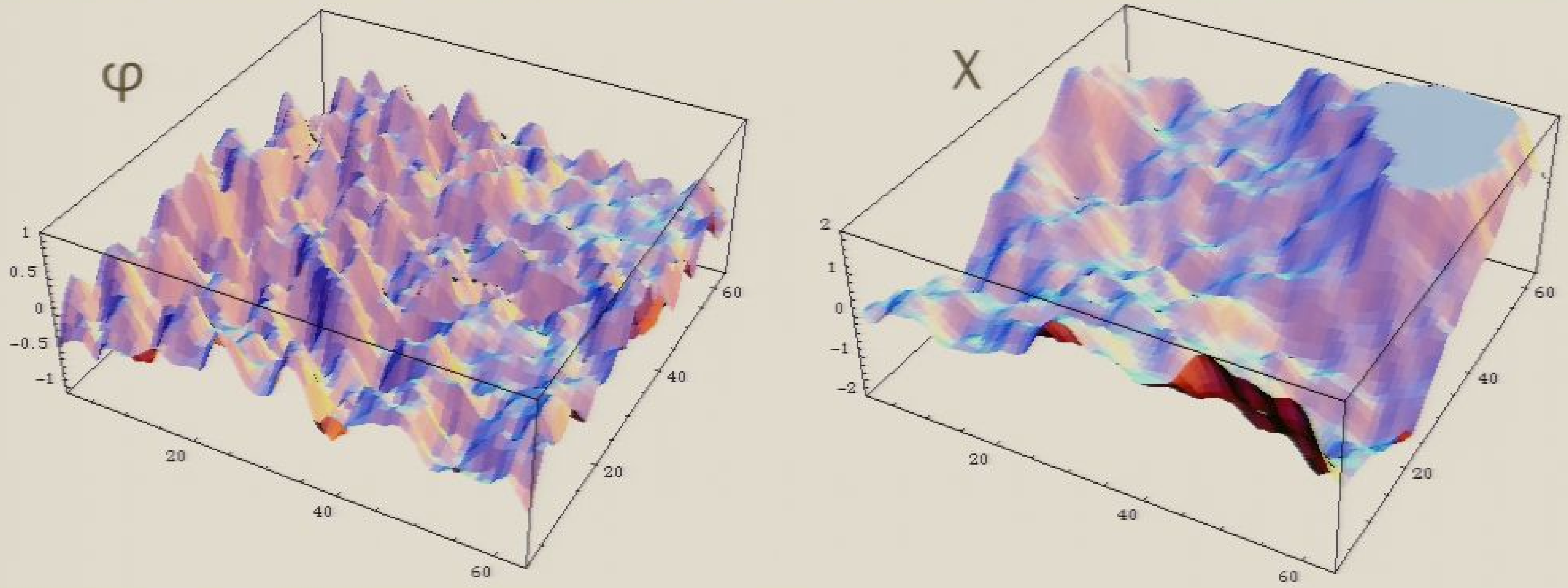


Simulations





Simulations



Production of Gravitational Waves

- First cut a problem: used flat space-space formula
  - Integrated over "boxes"

$$\frac{dE}{d\Omega} = 2G\Lambda_{ij,lm}\omega^2 T^{ij*}(\vec{\mathbf{k}},\omega)T^{lm}(\vec{\mathbf{k}},\omega)d\omega$$

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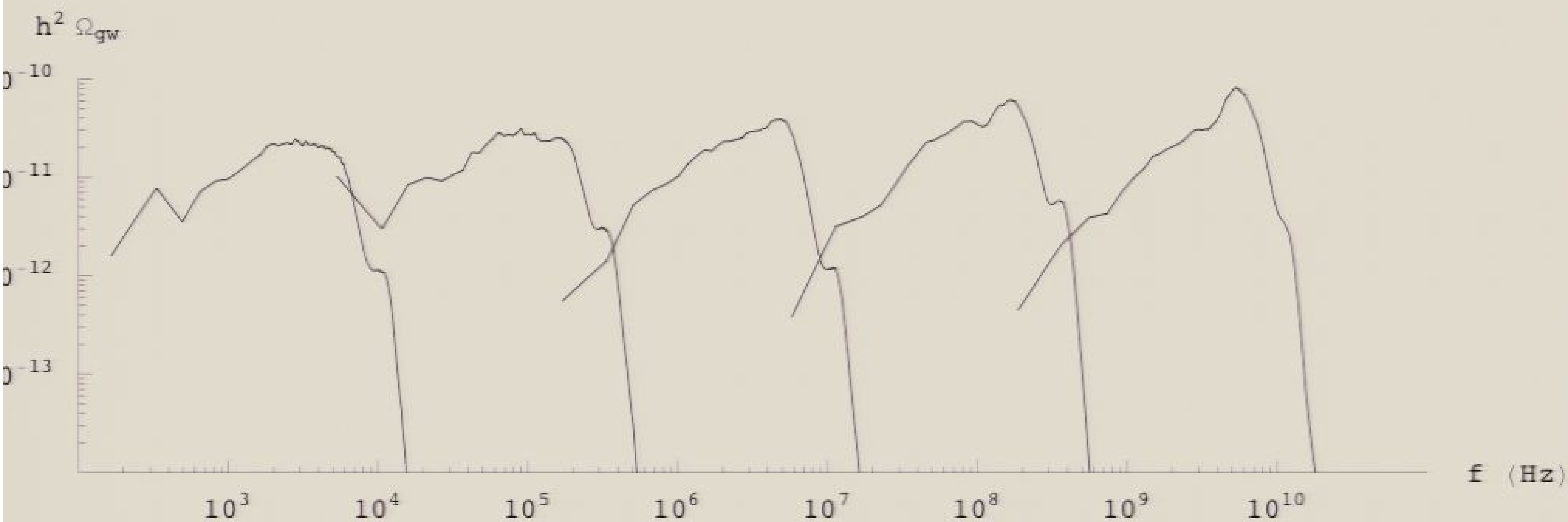
- Current version:
  - Compute field evolution; compute T_{ij}
  - Impose transverse-traceless gauge
  - T_{ij} sources h_{ij}
  - Solve for Fourier components of h_{ij}
  - Spectral method (for better stability)
  - Track tensor signal continuously.

Power in Gravitational Waves from Preheating

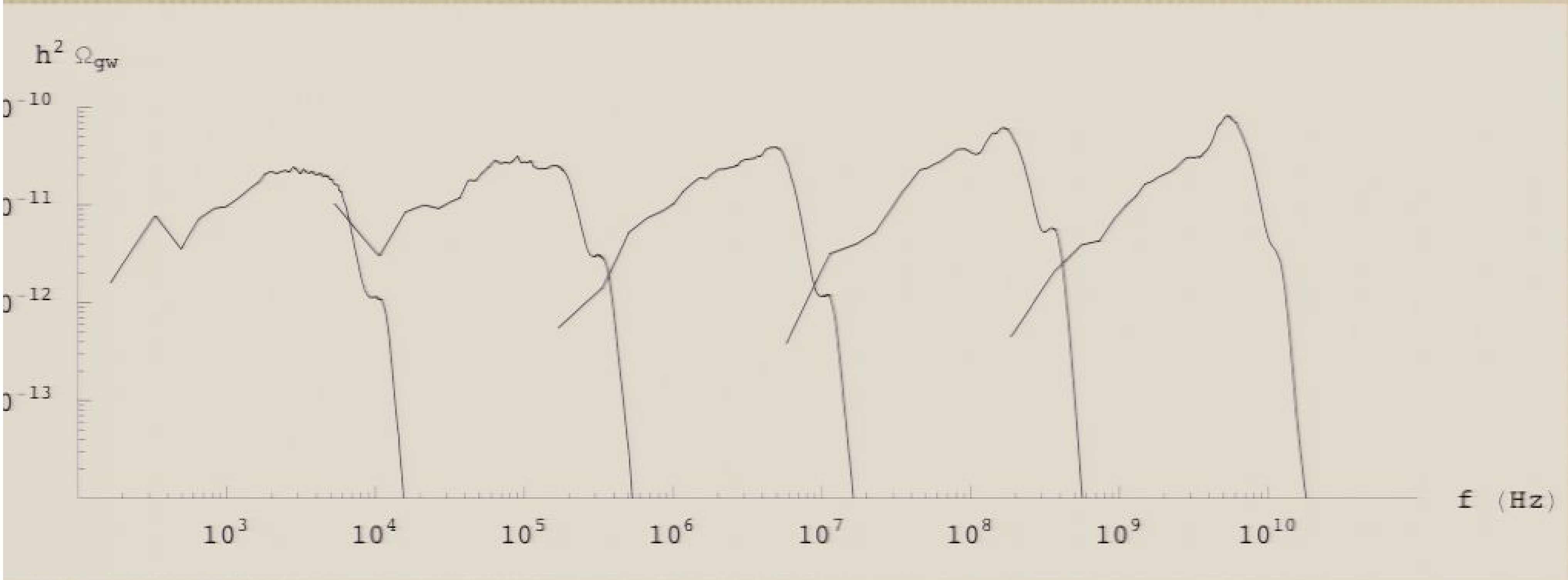


TeV Scale

GUT scale

128^3 Lattices

Power in Gravitational Waves from Preheating



TeV Scale

GUT scale

128^3 Lattices

Peak Location

tion ends

UT scale

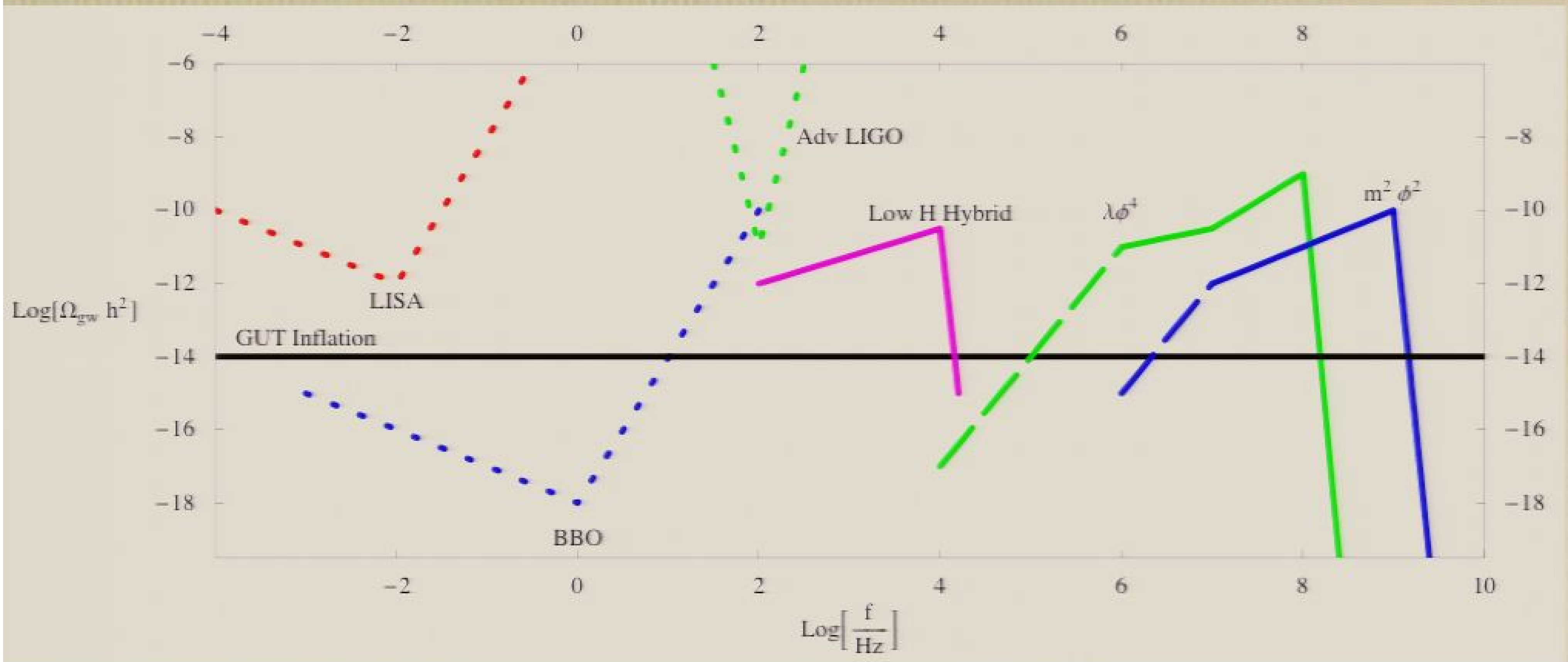
meter

Height of Peak

Summary

Primordial	Preheating
Quantum source	Classical source
Scale Invariant	Peaked near observable range
Low H : Low amplitude	Low H : redder peak
Probes Inflation scale	Probes oscillation scale
Amplitude lower bound	Amplitude probably large
$\Omega_{gw,inf} h^2 < 10^{-14}$	$\Omega_{gw} h^2 \approx 10^{-13} \sim 10^{-10}$

Summary Plot



Conclusions-III

- Still work in progress
  - But opens a new window into inflationary physics