Title: Modified Gravity as an Alternative to Dark Energy

Date: Nov 10, 2006 11:30 AM

URL: http://pirsa.org/06110053

Abstract:

9/30/05
X'os

Why Alternate Gravity?

$\boxed{\text{Fact}}$ Cosmic motion ➡ $g_{\mu\nu}$ ➡ $(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)_{rec}$

$\boxed{\text{Problem}}$ $(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)_{rec} \neq 8\pi G (T_{\mu\nu})_{obs}$

$\boxed{\text{Brand X}}$ $(T_{\mu\nu})_{dark} \equiv \frac{1}{8\pi G} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)_{rec} - (T_{\mu\nu})_{obs}$

(a) Always works!

(b) But epicyclic

* $\Omega_B \sim 5\%$
* $\Omega_{DM} \sim 25\%$ $\Big\}$ $\frac{5}{6}$ unknown is too much!
* $\Omega_{DE} \sim 70\%$ ➡ $\frac{19}{20}$ unknown way too much!

$\boxed{\text{Another Fix}}$ $G_{\mu\nu} + \Delta G_{\mu\nu}[g] = 8\pi G (T_{\mu\nu})_{obs}$

$\boxed{\text{Context}}$ Lagrangian Field Theory

➡ $\mathcal{L} = \frac{1}{16\pi G} (R + \Delta R[g]) \sqrt{-g}$

* Try local, invariant ΔR's

➡ depends upon $R^{\rho}{}_{\sigma\mu\nu}$, $R^{\rho}{}_{\sigma\mu\nu;\alpha}$, $R^{\rho}{}_{\sigma\mu\nu;\alpha\beta}$, ...

* Examples

$$\Delta R = \frac{1}{\mu^2} R^{\alpha\beta} R_{\alpha\beta}$$

$$\Delta R = \frac{1}{\mu^4} R \Box R$$

$$\Delta R = \mu^2 \sin\left(\frac{1}{\mu^4} R^{\alpha\beta\rho\sigma} R_{\alpha\beta\rho\sigma}\right)$$

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Tailor-Made Quintessence

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$$

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x} \Rightarrow H(t) \equiv \dot{a}/a$$

① $3H^2 = 8\pi G\left[\frac{1}{2}\dot{\varphi}^2 + V\right]$

② $-2\dot{H} - 3H^2 = 8\pi G\left[\frac{1}{2}\dot{\varphi}^2 - V\right]$

③ $\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$

Usual: $V(\varphi) \Rightarrow a(t)$

Reverse: $a(t) \Rightarrow V(\varphi)$

* $a(t) \Rightarrow H(t)$ and $\dot{H}(t)$ are <u>known</u>
* ① + ② $\Rightarrow -2\dot{H}(t) = 8\pi G\,\dot{\varphi}^2(t)$
* Weak Energy Condition $\Rightarrow \dot{H}(t) < 0$

$$\therefore \varphi(t) = \varphi_0 \pm \int_0^t dt' \sqrt{\frac{-2\dot{H}(t')}{8\pi G}} \Leftrightarrow t(\varphi)$$

* ① − ② $\Rightarrow 2\dot{H} + 6H^2 = 16\pi G V$

$$\therefore V(\varphi) = \frac{1}{8\pi G}\left[\dot{H} + 3H^2\right]\Big|_{t=t(\varphi)}$$

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Why Alternate Gravity?

p.1

$\boxed{\text{Fact}}$ Cosmic motion $\Rightarrow g_{\mu\nu} \Rightarrow (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)_{rec}$

$\boxed{\text{Problem}}$ $(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)_{rec} \neq 8\pi G (T_{\mu\nu})_{obs}$

$\boxed{\text{Brand X}}$ $(T_{\mu\nu})_{dark} \equiv \frac{1}{8\pi G}(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)_{rec} - (T_{\mu\nu})_{obs}$

(a) Always works!

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$\left.\begin{array}{l} * \Omega_B \sim 5\% \\ * \Omega_{DM} \sim 25\% \end{array}\right\}$ $\frac{5}{6}$ unknown is too much!

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$\boxed{\text{Another Fix}}$ $G_{\mu\nu} + \Delta G_{\mu\nu}[g] = 8\pi G (T_{\mu\nu})_{obs}$

$\boxed{\text{Context}}$ Lagrangian Field Theory

$\Rightarrow \mathcal{L} = \frac{1}{16\pi G}(R + \Delta R[g])\sqrt{-g}$

* Try local, invariant ΔR's

$\Rightarrow$ depends upon $R^{\rho}{}_{\sigma\mu\nu}$, $R^{\rho}{}_{\sigma\mu\nu;\alpha}$, $R^{\rho}{}_{\sigma\mu\nu;\alpha\beta}$ ...

* Examples

$\Delta R = \frac{1}{\mu^2} R^{\alpha\beta} R_{\alpha\beta}$

$\Delta R = \frac{1}{\mu^4} R \Box R$

$\Delta R = \mu^2 \sin\left(\frac{1}{\mu^4} R^{\alpha\beta\rho\tau} R_{\alpha\beta\rho\tau}\right)$

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Important No-Go Theorem

P.2

All except $\Delta R = \mu^2 f(\frac{1}{\mu^2} R)$ unstable w. $\Gamma = \infty$

① Why higher $\frac{\partial}{\partial t}$'s almost always unstable;

② Why $\Delta R = \mu^2 f(\frac{1}{\mu^2} R)$ can be stable.

$\boxed{\text{Context for ①}}$ $q(t)$ and $L = L(q, \dot{q}, \ddot{q}, \dots, q^{(N)})$

$\boxed{\text{Recall } L = L(q, \dot{q})}$ $\Rightarrow \frac{\partial L}{\partial q} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}} = 0$

$\therefore \ddot{q} = f(q, \dot{q}) \Rightarrow q = q(t, q_0, \dot{q}_0)$

$\boxed{\text{Canonical}}$

* $Q \equiv q$, $P \equiv \frac{\partial L}{\partial \dot{q}} \Rightarrow \dot{q}(Q, P)$ ("non-degeneracy")
* $H = P\,\dot{q}(Q, P) - L(Q, \dot{q}(Q, P))$
* $\dot{Q} = \frac{\partial H}{\partial P} = \dot{q} + P\frac{\partial \dot{q}}{\partial P} - \frac{\partial L}{\partial \dot{q}}\frac{\partial \dot{q}}{\partial P} = \dot{q}$ ✓
* $\dot{P} = -\frac{\partial H}{\partial Q} = -P\frac{\partial \dot{q}}{\partial Q} + \frac{\partial L}{\partial q} + \frac{\partial L}{\partial \dot{q}}\frac{\partial \dot{q}}{\partial P} = \frac{\partial L}{\partial q}$ ✓
* $t \rightarrow t + \Delta t \Rightarrow H$

$\therefore H$ is "the" energy (up to canonical transformations)

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Ostrogradskiĭ (1850)

P.3

$\boxed{L = L(q, \dot{q}, \ddot{q})}$ $\Rightarrow \frac{\partial L}{\partial q} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}} + \frac{\partial^2}{\partial t^2}\frac{\partial L}{\partial \ddot{q}} = 0$

$\ddddot{q} = f(q, \dot{q}, \ddot{q}, \dddot{q}) \Rightarrow q = q(t, q_0, \dot{q}_0, \ddot{q}_0, \dddot{q}_0)$

$\boxed{\text{Canonical}}$

* $Q_1 \equiv q \qquad P_1 \equiv \frac{\partial L}{\partial \dot{q}} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \ddot{q}}$
* $Q_2 \equiv \dot{q} \qquad P_2 \equiv \frac{\partial L}{\partial \ddot{q}} \quad \Rightarrow \ddot{q}(Q_1, Q_2, P_2)$ (non-deg.)
* $H = \sum_i P_i \dot{Q}_i - L$

$$= P_1 Q_2 + P_2 \ddot{q}(Q_1, Q_2, P_2) - L\Big(Q_1, Q_2, \ddot{q}(Q_1, Q_2, P_2)\Big)$$

* $\dot{Q}_i = \frac{\partial H}{\partial P_i}$
  - $\to i=1 \quad \dot{Q}_1 = Q_2$ ✓
  - $\to i=2 \quad \dot{Q}_2 = \ddot{q}(Q_1, Q_2, P_2)$ ✓
* $\dot{P}_i = -\frac{\partial H}{\partial Q_i}$
  - $\to i=2 \quad \dot{P}_2 = \frac{\partial L}{\partial \dot{q}} - P_1$ ✓
  - $\to i=1 \quad \dot{P}_1 = \frac{\partial L}{\partial q}$ ✓
* $t \to t + \Delta t \Rightarrow H$

$\therefore$ H is "the" energy (up to canonical transformation)

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So What?

P.4

① $H = P_1 Q_2 + P_2 \ddot{q}(Q_1, Q_2, P_2) - L(Q_1, Q_2, \ddot{q}(Q_1, Q_2, P_2))$

is linear in P_1 ⇒ unstable with $\Gamma = \infty$

② True for any non-degenerate $L(q, \dot{q}, \ddot{q})$

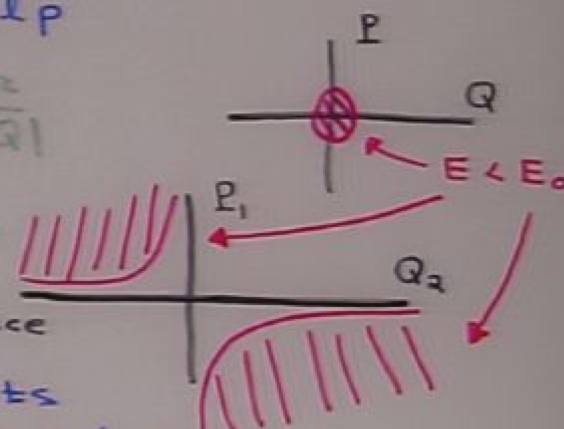
③ $L(q, \dot{q}, \ddot{q}, \ldots, q^{(N)})$ ⇒ H linear in $P_1, \ldots, P_{N-1}$

④ QM doesn't help

* cf $H = \frac{1}{2m} P^2 - \frac{e^2}{|Q|}$

* but $H = P_1 Q_2 + \ldots$

prob. $\sim \frac{1}{2}$ phase space



⑤ Global constraints don't help

* Eg $H = 0$ for $\mathcal{L} = \alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu}$

* Problem NOT $\dot{H} < 0$

rather $\frac{dN_+}{dt} \sim \frac{dN_-}{dt} \gg 0$

* Vast entropy of highly excited states

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No Perturbative Escape

4.5

$$\mathcal{L} = \underbrace{-\tfrac{1}{2}\partial_\mu\varphi\,\partial_\nu\varphi\, g^{\mu\nu}\sqrt{-g}}_{\text{positive K.E.}} - \underbrace{f(\varphi,\Box\varphi)}_{\text{nonlinear in }\Box\varphi}\sqrt{-g}$$

$\boxed{\text{Legendre Transform}}$

* $\Phi \equiv \dfrac{\partial f}{\partial \Box\varphi} \quad \Leftrightarrow \quad \Box\varphi(\varphi,\Phi)$
* $U(\varphi,\Phi) \equiv f\big(\varphi,\Box\varphi(\varphi,\Phi)\big) - \Phi\,\Box\varphi(\varphi,\Phi)$

$\rightarrow \dfrac{\partial U}{\partial \Phi} = -\Box\varphi(\varphi,\Phi)$ $\quad$ [also $\dfrac{\partial U}{\partial\varphi} = \dfrac{\partial f}{\partial \varphi}$]

* $\bar{\mathcal{L}} = -\frac{1}{2}\partial_\mu\varphi\,\partial_\nu\varphi\, g^{\mu\nu}\sqrt{-g} - \Phi\,\Box\varphi\sqrt{-g} - U(\varphi,\Phi)\sqrt{-g}$

$\boxed{\Phi\text{ eqn}}$ $\quad \Box\varphi + \dfrac{\partial U}{\partial \Phi} = 0 \quad \rightarrow \quad \Phi = \dfrac{\partial f}{\partial\Box\varphi}$

$\boxed{\varphi\text{ eqn}}$ $\quad \Box\varphi - \Box\Phi - \dfrac{\partial U}{\partial\varphi} = 0 \quad \rightarrow$ old φ eqn

* $-\Phi\,\Box\varphi\sqrt{-g} = -\partial_\mu\big(\Phi\,\partial_\nu\varphi\, g^{\mu\nu}\sqrt{-g}\big) + \partial_\mu\Phi\,\partial_\nu\varphi\, g^{\mu\nu}\sqrt{-g}$

$\therefore \bar{\mathcal{L}} \rightarrow -\frac{1}{2}\partial_\mu(\varphi-\Phi)\,\partial_\nu(\varphi-\Phi)\, g^{\mu\nu}\sqrt{-g} + \frac{1}{2}\partial_\mu\Phi\,\partial_\nu\Phi\, g^{\mu\nu}\sqrt{-g} - U(\varphi,\Phi)\sqrt{-g}$

* $\Psi \equiv \varphi - \Phi$

$$\bar{\bar{\mathcal{L}}} = \underbrace{-\tfrac{1}{2}\partial_\mu\Psi\,\partial_\nu\Psi\, g^{\mu\nu}\sqrt{-g}} + \underbrace{\tfrac{1}{2}\partial_\mu\Phi\,\partial_\nu\Phi\, g^{\mu\nu}\sqrt{-g}} - U(\Psi+\Phi,\Phi)\sqrt{-g}$$

$$\mathcal{L} = \underbrace{-\tfrac{1}{2}\partial_\mu\varphi\,\partial_\nu\varphi\, g^{\mu\nu}}_{\text{positive K.E.}}\sqrt{-g} - \underbrace{f(\varphi, \Box\varphi)}_{\text{nonlinear in } \Box\varphi}\sqrt{-g}$$

$\boxed{\text{Legendre Transform}}$

* $\Phi \equiv \dfrac{\partial f}{\partial \Box\varphi} \Longleftrightarrow \Box\varphi(\varphi, \Phi)$

* $U(\varphi, \Phi) \equiv f\big(\varphi, \Box\varphi(\varphi,\Phi)\big) - \Phi\,\Box\varphi(\varphi,\Phi)$

  $\rightarrow \dfrac{\partial U}{\partial \Phi} = -\Box\varphi(\varphi,\Phi)$ $\left[\text{also } \dfrac{\partial U}{\partial\varphi} = \dfrac{\partial f}{\partial\varphi}\right]$

* $\bar{\mathcal{L}} = -\tfrac{1}{2}\partial_\mu\varphi\,\partial_\nu\varphi\, g^{\mu\nu}\sqrt{-g} - \Phi\,\Box\varphi\sqrt{-g} - U(\varphi,\Phi)\sqrt{-g}$

$\boxed{\Phi\text{ eqn}}$ $\Box\varphi + \dfrac{\partial U}{\partial\Phi} = 0$ $\rightarrow \Phi = \dfrac{\partial f}{\partial\Box\varphi}$

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* $-\Phi\,\Box\varphi\sqrt{-g} = -\partial_\mu\left(\Phi\,\partial_\nu\varphi\, g^{\mu\nu}\sqrt{-g}\right) + \partial_\mu\Phi\,\partial_\nu\varphi\, g^{\mu\nu}\sqrt{-g}$

$\therefore \bar{\mathcal{L}} \longrightarrow -\tfrac{1}{2}\partial_\mu(\varphi-\Phi)\,\partial_\nu(\varphi-\Phi)\, g^{\mu\nu}\sqrt{-g} + \tfrac{1}{2}\partial_\mu\Phi\,\partial_\nu\Phi\, g^{\mu\nu}\sqrt{-g} - U(\varphi,\Phi)\sqrt{-g}$

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$$\bar{\bar{\mathcal{L}}} = \underbrace{-\tfrac{1}{2}\partial_\mu\Psi\,\partial_\nu\Psi\, g^{\mu\nu}}_{\text{positive KE}}\sqrt{-g} + \underbrace{\tfrac{1}{2}\partial_\mu\Phi\,\partial_\nu\Phi\, g^{\mu\nu}\sqrt{-g}}_{\text{negative KE}} - U(\Psi+\Phi, \Phi)\sqrt{-g}$$

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Instability NOT Ghosts

P.5

① $L(q,\dot q) \Rightarrow q(t) = a e^{-iEt} + a^{\dagger} e^{iEt}$

② $L(q,\dot q,\ddot q) \Rightarrow q(t) = \sum_{i=1}^{2} \left[a_i e^{-iE_i t} + a_i^{\dagger} e^{iE_i t} \right]$

* $E_1 > 0$ but $E_2 < 0$
* $[a_i, a_j^{\dagger}] = \delta_{ij}$
* $a_i |\Omega\rangle = 0 \Rightarrow \int dQ_1 dQ_2 \, \|\Omega(Q_1,Q_2)\|^2 < \infty$

③ Particle Physicists interchange a_2 and $a_2^{\dagger}$

* $a_1 |\bar\Omega\rangle = 0 = a_2^{\dagger} |\bar\Omega\rangle$
* $a_1^{\dagger} |\bar\Omega\rangle$ has $+E_1 > 0$
* $a_2 |\bar\Omega\rangle$ has $-E_2 > 0$

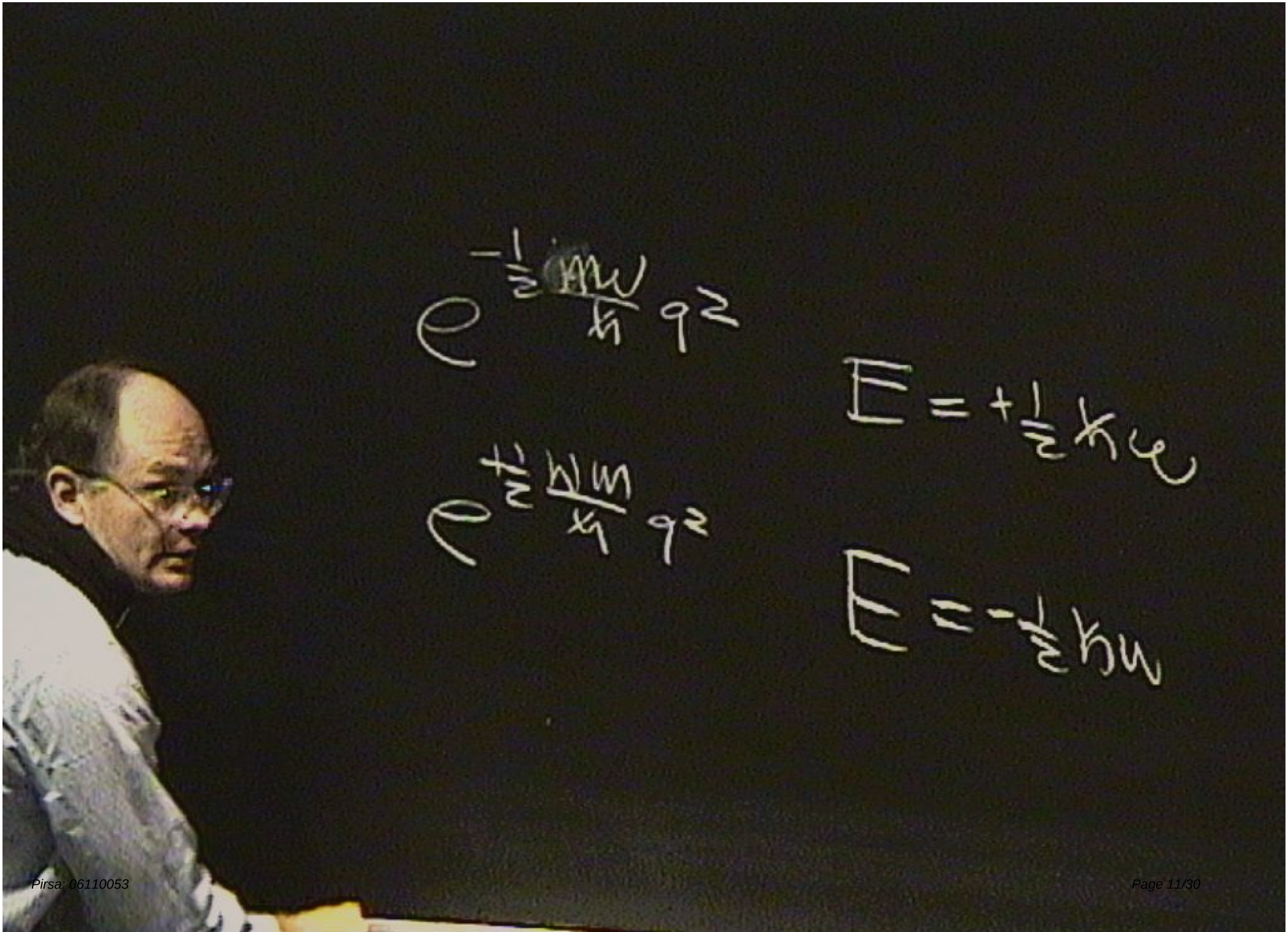
$\Big\}$ no stability problem

* $\langle\bar\Omega| a_2^{\dagger} \cdot a_2 |\bar\Omega\rangle = -\langle\bar\Omega|\bar\Omega\rangle \Rightarrow$ "ghosts"

"but maybe they decouple..."

④ WRONG

* $\int dQ_1 dQ_2 \, \|\bar\Omega(Q_1,Q_2)\|^2 = \infty$
* NB $\exists$ Ψ's $\ni$ $H\Psi = E\Psi$ $\forall E \in \mathbb{C}$
* Normalizability puts the "quantum" in Quantum Mechanics


$e^{-\frac{1}{2}\frac{m\omega}{\hbar}q^2}$
$E = +\frac{1}{2}\hbar\omega$
$e^{+\frac{1}{2}\frac{\omega m}{\hbar}q^2}$
$E = -\frac{1}{2}\hbar\omega$

$$e^{-\frac{1}{2}\frac{m\omega}{\hbar}q^2}$$

$$E = +\frac{1}{2}\hbar\omega$$

$$e^{+\frac{1}{2}\frac{\omega m}{\hbar}q^2}$$

$$E = -\frac{1}{2}\hbar\omega$$

$$H\psi = E\psi$$

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Instability NOT Ghosts

P.5

① $L(q,\dot q) \Rightarrow q(t) = a e^{-iEt} + a^{\dagger} e^{iEt}$

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* $a_1 |\bar\Omega\rangle = 0 = a_2^{\dagger} |\bar\Omega\rangle$
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"but maybe they decouple..."

* $\int dQ_1 dQ_2 \, \| \bar\Omega(Q_1, Q_2) \|^2 = \infty$
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Xiℓos Why is $\Delta R = \mu^2 f(\frac{1}{\mu^2} R)$ OK? P.6

① $(\frac{\partial}{\partial t})^2$ only hits spin zero in R

② Does give new spin zero excit. with opposite E of old one

③ But spin zero in $\mathcal{L} = \frac{1}{16\pi G} R\sqrt{-g}$ has $E < 0$

* cf $U = -\frac{G m_1 m_2}{\|\vec{x}_1 - \vec{x}_2\|}$

* No problem because locally constrained

(NB no new local constraints with HD's)

④ New H.D. spin zero can have $E > 0$

⑤ NB more $\frac{\partial}{\partial t}$'s does mean $E < 0$

⑥ Even $(\frac{\partial}{\partial t})^2$ in $R^{\mu\nu} R_{\mu\nu}$ causes new spin two

* old spin two had $E > 0$

* new spin two has $E < 0$

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Non linear Gauss-Bonnet

6.5

* $G \equiv R^{\alpha\beta\rho\sigma}R_{\alpha\beta\rho\sigma} - 4R^{\alpha\beta}R_{\alpha\beta} + R^2$
* $G\sqrt{-g} = \partial_\mu(\quad)$
* but many $\mathcal{L} = \frac{1}{16\pi G}\left(R + f(G)\right)\sqrt{-g}$ papers

cf. Nojiri & Odintsov hep-th/0508049

$\boxed{\text{Synchronous Gauge}}$ $g_{00} = -1$ and $g_{0i} = 0$

* $R_{0i0j} = -\frac{1}{2}\ddot{g}_{ij} + \frac{1}{4}g^{k\ell}\dot{g}_{ik}\dot{g}_{j\ell}$
* $R_{00} = g^{k\ell}R_{0k0\ell}$ and $R_{ij} = -R_{0i0j} + g^{k\ell}R_{ikj\ell}$
* $R = -2g^{k\ell}R_{0k0\ell} + g^{ij}g^{k\ell}R_{ikj\ell}$

$\boxed{\text{3+1 Decomp}}$

* $R^{\alpha\beta\rho\sigma}R_{\alpha\beta\rho\sigma} = 4R^{0i0j}R_{0i0j} + 4R^{0ijk}R_{0ijk} + R^{ijk\ell}R_{ijk\ell}$
* $R^{\alpha\beta}R_{\alpha\beta} = R^{00}R_{00} + 2R^{0i}R_{0i} + R^{ij}R_{ij}$

$\ldots \Rightarrow G = 8\left(g^{ik}g^{j\ell} - \frac{1}{2}g^{ij}g^{k\ell}\right)g^{mn}R_{0i0j}R_{kmln} + \ldots$

* no $\ddot{g}_{ij}$'s for some backgrounds
* only $\sim g^{ij}\ddot{g}_{ij}$ for FRW
* but generally other $\ddot{g}_{ij}$'s $\Rightarrow$ unstable

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$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \Delta R(R)\right)\sqrt{-g}$$

① $\mathcal{G}_{\mu\nu} \equiv \frac{16\pi G}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$

$= [1+\Delta R']R_{\mu\nu} - \frac{1}{2}[R+\Delta R]g_{\mu\nu} + (D_\mu D_\nu - g_{\mu\nu}\Box)\Delta R'$

② $ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x}$

* $H(t) \equiv \dot{a}/a$
* $q(t) \equiv \frac{-a\ddot{a}}{\dot{a}^2} = -1 - \dot{H}/H^2$
* $R_{00} = 3qH^2$
* $R_{ij} = (2-q)H^2 g_{ij}$

$\left.\begin{matrix} R_{00} \\ R_{ij} \end{matrix}\right\}$ $R_{\mu\nu} = \text{const}\; g_{\mu\nu}$ $\Leftrightarrow q = -1 < 0$

③ $\Delta R = -\mu^4/R$ can give acceleration

* $\left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)R g_{\mu\nu} + \left[g_{\mu\nu}\Box - D_\mu D_\nu\right]\frac{\mu^4}{R^2} = 8\pi G T_{\mu\nu}$
* Capozziello, Carloni + Troisi astro-ph/0303041
* Carroll, Duvvuri, Trodden + Turner astro-ph/0306438
* $T_{\mu\nu} = 0$ + $R_{\mu\nu} = \text{const}$

$\Rightarrow \left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)R g_{\mu\nu} = 0$

$\Rightarrow R = \pm\sqrt{3}\mu^2 \Rightarrow R_{\mu\nu} = \pm\frac{\sqrt{3}}{4}\mu^2 g_{\mu\nu}$

④ Tachyonic Instability

* Trace $\Rightarrow -R + \frac{3\mu^4}{R} + \Box\left(\frac{3\mu^4}{R^2}\right) = 0$
* Perturb: $R = +\sqrt{3}\mu^2 + \delta R$

$\Rightarrow -2\delta R - \frac{2}{\sqrt{3}\mu^2}\Box\,\delta R = 0$

* Normal: $(\Box - m^2)\varphi = 0$ Tachyon: $(\Box + m^2)\varphi = 0$

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Equivalent Scalar Representation 7.3

$* \mathcal{L} = \frac{1}{16\pi G}\left(R + f(R)\right)\sqrt{-g}$

$\Rightarrow [1+f'(R)]R_{\mu\nu} - \frac{1}{2}[R+f(R)]g_{\mu\nu} + g_{\mu\nu}(f')^{;\rho}{}_{;\rho} - (f')_{;\mu\nu} = 0$

$\boxed{\text{Legendre Transform}}$

$* \; \phi = 1 + f'(R) \;\Leftrightarrow\; R(\phi)$

$* \; U(\phi) = [\phi - 1]R(\phi) - f(R(\phi)) \rightarrow U'(\phi) = R(\phi)$

$\overline{\mathcal{L}} = \frac{1}{16\pi G}[\phi R - U(\phi)]\sqrt{-g}$

$\boxed{\phi \text{ eqn}}$ $R - U'(\phi) = 0 \rightarrow \phi = 1 + f'(R)$

$\boxed{g_{\mu\nu} \text{ eqn}}$ $\phi R_{\mu\nu} - \frac{1}{2}[\phi R - U(\phi)]g_{\mu\nu} + g_{\mu\nu}\phi^{;\rho}{}_{;\rho} - \phi_{;\mu\nu} = 0$

$\boxed{\text{Conformal Transformation}}$

$\tilde{g}_{\mu\nu} = \phi g_{\mu\nu} \;\Leftrightarrow\; g_{\mu\nu} = \exp\left[-\sqrt{\frac{4\pi G}{3}}\varphi\right]\tilde{g}_{\mu\nu}$

$\varphi = \sqrt{\frac{3}{4\pi G}}\ln(\phi) \;\Leftrightarrow\; \phi = \exp\left[\sqrt{\frac{4\pi G}{3}}\varphi\right]$

$\therefore \overline{\mathcal{L}} = \frac{1}{16\pi G}\tilde{R}\sqrt{-\tilde{g}} - \frac{1}{2}\partial_\mu\varphi\,\partial_\nu\varphi\,\tilde{g}^{\mu\nu}\sqrt{-\tilde{g}} - V(\varphi)\sqrt{-\tilde{g}}$

$V(\varphi) \equiv \frac{1}{16\pi G}U\left(\exp\left[\sqrt{\frac{4\pi G}{3}}\varphi\right]\right)\exp\left[-\sqrt{\frac{4\pi G}{3}}\varphi\right]$

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Equivalent Scalar Representation 7.3

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$\rightarrow [1+f'(R)]R_{\mu\nu} - \frac{1}{2}[R+f(R)]g_{\mu\nu} + g_{\mu\nu}(f')^{;\rho}{}_{\rho} - (f')_{;\mu\nu} = 0$

$\boxed{\text{Legendre Transform}}$

* $\phi = 1 + f'(R) \leftrightarrow R(\phi)$
* $U(\phi) = [\phi - 1]R(\phi) - f(R(\phi)) \rightarrow U'(\phi) = R(\phi)$

$\overline{\mathcal{L}} \equiv \frac{1}{16\pi G}[\phi R - U(\phi)]\sqrt{-g}$

$\boxed{\phi\text{ eqn}}$ $R - U'(\phi) = 0 \rightarrow \phi = 1 + f'(R)$

$\boxed{g_{\mu\nu}\text{ eqn}}$ $\phi R_{\mu\nu} - \frac{1}{2}[\phi R - U(\phi)]g_{\mu\nu} + g_{\mu\nu}\phi^{;\rho}{}_{\rho} - \phi_{;\mu\nu} = 0$

$\boxed{\text{Conformal Transformation}}$

$\tilde{g}_{\mu\nu} = \phi g_{\mu\nu} \leftrightarrow g_{\mu\nu} = \exp\left[-\sqrt{\frac{4\pi G}{3}}\varphi\right]\tilde{g}_{\mu\nu}$

$\varphi = \sqrt{\frac{3}{4\pi G}}\ln(\phi) \leftrightarrow \phi = \exp\left[\sqrt{\frac{4\pi G}{3}}\varphi\right]$

$\therefore \overline{\mathcal{L}} = \frac{1}{16\pi G}\tilde{R}\sqrt{-\tilde{g}} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi\,\tilde{g}^{\mu\nu}\sqrt{-\tilde{g}} - V(\varphi)\sqrt{-\tilde{g}}$

$V(\varphi) \equiv \frac{1}{16\pi G}U\left(\exp\left[\sqrt{\frac{4\pi G}{3}}\varphi\right]\right)\exp\left[-\sqrt{\frac{4\pi G}{3}}\varphi\right]$

Tailoring $f(R)$ to Support $a(t)$

$\boxed{\text{Conformal Coords}}$

* $ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x} = a^2(-d\eta^2 + d\vec{x}\cdot d\vec{x})$
* $d\tilde{s}^2 = \tilde{a}^2(-d\eta^2 + d\vec{x}\cdot d\vec{x}) = -d\tilde{t}^2 + \tilde{a}^2(\tilde{t})\, d\vec{x}\cdot d\vec{x}$

$$\therefore \frac{\partial}{\partial\eta} = a\frac{\partial}{\partial t} = \tilde{a}\frac{\partial}{\partial\tilde{t}}$$

$$\boxed{a(t) = \tilde{a}(\tilde{t}) \exp\left[-\sqrt{\frac{\pi G}{3}}\, \varphi_0(\tilde{t})\right]}$$

* Recall also $\dot{\varphi}_0(\tilde{t}) = \sqrt{\frac{-2\dot{\tilde{H}}(\tilde{t})}{8\pi G}}$

$$\therefore \frac{a'}{a} = \frac{\tilde{a}'}{\tilde{a}} - \sqrt{\frac{\pi G}{3}}\, \varphi_0' = \underbrace{\frac{\tilde{a}'}{\tilde{a}} - \sqrt{-\frac{1}{12}\tilde{a}'}}_{\text{solve 1st order diffeqn for } \tilde{a}(\tilde{t})}$$

assumed known

* then use previous constr. to get $V(\varphi)$

$\Rightarrow U(\phi) = 16\pi G\, \phi^2\, V\left(\sqrt{\frac{3}{4\pi G}}\, \ln(\phi)\right)$

* then $U'(\phi) = R \Leftrightarrow \phi(R)$

$\Rightarrow f(R) = [\phi(R) - 1]R - U(\phi(R))$

9/30/05

Xcos

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \Delta R(R)\right)\sqrt{-g}$$

① $\mathcal{G}_{\mu\nu} \equiv \frac{16\pi G}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$

$= [1+\Delta R']R_{\mu\nu} - \frac{1}{2}[R+\Delta R]g_{\mu\nu} + (D_\mu D_\nu - g_{\mu\nu}\Box)\Delta R'$

② $ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x}$

* $H(t) \equiv \dot{a}/a$
* $q(t) \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \dot{H}/H^2$
* $R_{00} = 3qH^2$
* $R_{ij} = (2-q)H^2 g_{ij}$

$\left.\begin{matrix} \\ \end{matrix}\right\}$ $R_{\mu\nu} = \text{const}\; g_{\mu\nu}$

$\Leftrightarrow q = -1 < 0$

③ $\Delta R = -\mu^4/R$ can give acceleration

* $\left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)R g_{\mu\nu} + [g_{\mu\nu}\Box - D_\mu D_\nu]\frac{\mu^4}{R^2} = 8\pi G T_{\mu\nu}$
* Capozziello, Carloni + Troisi astro-ph/0303041
* Carroll, Duvvuri, Trodden + Turner astro-ph/0306438
* $T_{\mu\nu} = 0$ + $R_{\mu\nu} = \text{const}$

$\Rightarrow \left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)R g_{\mu\nu} = 0$

$\Rightarrow R = \pm\sqrt{3}\mu^2 \Rightarrow R_{\mu\nu} = \pm\frac{\sqrt{3}}{4}\mu^2 g_{\mu\nu}$

④ Tachyonic Instability

* Trace $\Rightarrow -R + \frac{3\mu^4}{R} + \Box\left(\frac{3\mu^4}{R^2}\right) = 0$
* Perturb: $R = +\sqrt{3}\mu^2 + \delta R$

$\Rightarrow -2\delta R - \frac{3}{\sqrt{3}\mu^2}\Box\delta R = 0$

* Normal: $(\Box - m^2)\varphi = 0$ Tachyon: $(\Box + m^2)\varphi = 0$
* But $\mu \sim 10^{-33}$ eV

Equivalent Scalar Representation 7.3

$$\frac{1}{16\pi G}\left(R+f(R)\right)\sqrt{-g}$$

$$[1+f'(R)]R_{\mu\nu}-\tfrac{1}{2}[R+f(R)]g_{\mu\nu}+g_{\mu\nu}(f')^{;\rho}{}_{\rho}-(f')_{;\mu\nu}=0$$

$\boxed{\text{endre Transform}}$

$$\phi = 1+f'(R) \leftrightarrow R(\phi)$$

$$U(\phi) = [\phi-1]R(\phi)-f(R(\phi)) \rightarrow U'(\phi)=R(\phi)$$

$$= \frac{1}{16\pi G}\left[\phi R-U(\phi)\right]\sqrt{-g}$$

$\boxed{\text{eqn}}$ $R-U'(\phi)=0 \rightarrow \phi = 1+f'(R)$

$\boxed{\text{eqn}}$ $\phi R_{\mu\nu}-\tfrac{1}{2}[\phi R-U(\phi)]g_{\mu\nu}+g_{\mu\nu}\phi^{;\rho}{}_{\rho}-\phi_{;\mu\nu}=0$

$\boxed{\text{onformal Transformation}}$

$$\tilde{g}_{\mu\nu}=\phi g_{\mu\nu} \leftrightarrow g_{\mu\nu}=\exp\left[-\sqrt{\tfrac{4\pi G}{3}}\varphi\right]\tilde{g}_{\mu\nu}$$

$$\varphi=\sqrt{\tfrac{3}{4\pi G}}\ln(\phi) \leftrightarrow \phi=\exp\left[\sqrt{\tfrac{4\pi G}{3}}\varphi\right]$$

$$\therefore \mathcal{L}=\frac{1}{16\pi G}\tilde{R}\sqrt{-\tilde{g}}-\tfrac{1}{2}\partial_\mu\varphi\,\partial_\nu\varphi\,\tilde{g}^{\mu\nu}\sqrt{-\tilde{g}}-V(\varphi)\sqrt{-\tilde{g}}$$

$$V(\varphi)\equiv\frac{1}{16\pi G}U\left(\exp\left[\sqrt{\tfrac{4\pi G}{3}}\varphi\right]\right)\exp\left[-\sqrt{\tfrac{4\pi G}{3}}\varphi\right]$$

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Xios

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \Delta R(R)\right)\sqrt{-g}$$

P. 7

① $\mathcal{G}_{\mu\nu} \equiv \frac{16\pi G}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$

$= [1+\Delta R']R_{\mu\nu} - \frac{1}{2}[R+\Delta R]g_{\mu\nu} + (D_\mu D_\nu - g_{\mu\nu}\Box)\Delta R'$

② $ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x}$

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$R_{\mu\nu} = \text{const}\ g_{\mu\nu}$
$\Leftrightarrow q = -1 < 0$

③ $\Delta R = -\mu^4/R$ can give acceleration

* $\left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)R g_{\mu\nu} + \left[g_{\mu\nu}\Box - D_\mu D_\nu\right]\frac{\mu^4}{R^2} = 8\pi G T_{\mu\nu}$
* Capozziello, Carloni + Troisi astro-ph/0303041
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* $T_{\mu\nu} = 0$ + $R_{\mu\nu} = \text{const}$

$\Rightarrow \left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)R g_{\mu\nu} = 0$

$\Rightarrow R = \pm\sqrt{3}\mu^2 \Rightarrow R_{\mu\nu} = \pm\frac{\sqrt{3}}{4}\mu^2 g_{\mu\nu}$

④ Tachyonic Instability

* Trace $\Rightarrow -R + \frac{3\mu^4}{R} + \Box\left(\frac{3\mu^4}{R^2}\right) = 0$
* Perturb: $R = +\sqrt{3}\mu^2 + \delta R$

$\Rightarrow -2\delta R - 3$ [illegible]

9/30/05

Xios

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \Delta R(R)\right)\sqrt{-g}$$

① $\mathcal{G}_{\mu\nu} \equiv \frac{16\pi G}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$

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* $R_{00} = 3qH^2$
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$\left.\begin{array}{l} R_{00} \\ R_{ij} \end{array}\right\}$ $R_{\mu\nu} = \text{const}\ g_{\mu\nu} \Leftrightarrow q = -1 < 0$

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* $T_{\mu\nu} = 0 + R_{\mu\nu} = \text{const}$

$\rightarrow \left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)R g_{\mu\nu} = 0$

$\rightarrow R = \pm\sqrt{3}\mu^2 \rightarrow R_{\mu\nu} = \pm\frac{\sqrt{3}}{4}\mu^2 g_{\mu\nu}$

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χ'ios

Inside Matter

① Dolgov + Kawasaki, astro-ph/0307285

② $T_{\mu\nu} = \rho\, \delta^0_\mu \delta^0_\nu$ with $8\pi G \rho \equiv M^2 >> \mu^2$

Trace $\rightarrow$ $-R + \frac{3\mu^4}{R} + \Box\left(\frac{3\mu^4}{R^2}\right) = -M^2$

③ Static Solution

$$R_0 = \frac{1}{2}\left[M^2 + \sqrt{M^4 + 12\mu^4}\right] \cong M^2$$

④ Perturb : $R = R_0 + \delta R$

Linearize $\rightarrow$ $-\delta R - \frac{3\mu^4}{R_0^2}\delta R - \frac{6\mu^4}{R_0^3}\Box \delta R = 0$

$$\therefore \left(\Box + \frac{M^6}{6\mu^4}\right)\delta R \cong 0$$

$$\frac{1}{\Gamma} \sim 10^{-26} \text{ sec}$$

⑤ Dolgov's humor

9/30/05

Xcos

Inside Matter

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$\frac{1}{\Gamma} \sim 10^{-26}$ sec

⑤ Dolgov's humor

9/30/05
Χίος

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Outside Matter

① Soussa + Woodard, astro-ph/0308114

② $ds^2 = -(1-h_{00})dt^2 + 2a(t)h_{0i}\,dt\,dx^i + a^2(t)[\delta_{ij} + h_{ij}]dx^i dx^j$

* $a(t) = e^{H_0 t}$
* $R_0 = \sqrt{3}\mu^2 = 12H_0^2$
* $R = R_0 + \delta R$
* then $\delta R = \frac{1}{2}(-\partial^2 h + 4H\partial_0 h)$

for $h \equiv -h_{00} + h_{ii}$ in $h_{\mu\nu}{}^{;\nu} - \frac{1}{2}h_{;\mu} + 3h^{\nu}{}_{\mu}(\ln a)_{,\nu}$ [illegible]

③ $\rho = \begin{cases} 0 & \text{outside} \\ \dfrac{m}{\frac{4}{3}\pi R^3} & \text{inside} \end{cases}$ boundary at $a(t)\|\vec{x}\| = R$

④ New coordinate $y \equiv a(t)H\|\vec{x}\|$

* $\left[(1-y^2)\frac{d^2}{dy^2} + \frac{2}{y}(1-2y^2)\frac{d}{dy} + 12\right]\delta R = 0 \qquad y > HR$

$\Rightarrow \delta R = \beta_1 f_0(y) + \beta_2 f_{-1}(y)$

* $f_0(y) = 1 - 2y^2 + \frac{1}{5}y^4 + \ldots \qquad \beta_1 \cong \frac{3mG}{R^3}$
* $f_{-1}(y) = \frac{1}{y}\left[1 - 7y^2 + \frac{14}{3}y^4 + \ldots\right] \qquad \beta_2 \cong -12\,Gm\,H^3$

⑤ Solve for $h \Rightarrow \left[(y^2-1)\frac{d}{dy} + \frac{1}{y}(5y^2-2)\right]h'(y) = \frac{2}{H^2}\delta R$

* $h'(y) = -\frac{2Gm}{H^2R^3}\,y + \mathcal{O}(y^3)$
* cf GR $\Rightarrow h'(y) = -\frac{4GmH}{\ldots} + \mathcal{O}(1)$

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χ'cos

Outside Matter

① Soussa + Woodard, astro-ph/0308114

② $ds^2 = -(1-h_{00})dt^2 + 2a(t)h_{0i}\,dt\,dx^i + a^2(t)[\delta_{ij}+h_{ij}]dx^i dx^j$

* $a(t) = e^{H_0 t}$
* $R_0 = \sqrt{3}\mu^2 = 12 H_0^2$
* $R = R_0 + \delta R$
* then $\delta R = \frac{1}{2}(-\partial^2 h + 4H\partial_0 h)$

for $h \equiv -h_{00} + h_{ii}$ in $h_{\mu\nu}{}^{,\nu} - \frac{1}{2}h_{,\mu} + 3h^{\nu}{}_{\mu}$ [illegible]

③ $$\rho = \begin{cases} 0 & \text{outside} \\ \dfrac{m}{\frac{4}{3}\pi R^3} & \text{inside} \end{cases}$$ boundary at $a(t)\|\vec{x}\| = R$

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* $\left[(1-y^2)\frac{d^2}{dy^2} + \frac{2}{y}(1-2y^2)\frac{d}{dy} + 12\right]\delta R = 0$ $\quad y > HR$

$\Rightarrow \delta R = \beta_1 f_0(y) + \beta_2 f_{-1}(y)$

* $f_0(y) = 1 - 2y^2 + \frac{1}{5}y^4 + \dots$ $\qquad \beta_1 \cong \frac{3mG}{R^3}$
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⑤ Solve for $h \Rightarrow \left[(y^2-1)\frac{d}{dy} + \frac{1}{y}(5y^2-2)\right]h'(y) = \frac{2}{H^2}\delta R$

* $h'(y) = -\frac{2Gm}{H^2R^3}\,y + \mathcal{O}(y^3)$
* cf GR $\Rightarrow h'(y) = -\frac{4GmH}{y^2} + \mathcal{O}(1)$

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9.5

What about Nonlocal Eqns?

$\boxed{\text{Justification}}$ QFT produces them!

* Schwinger-Keldysh Eff. Eqns are causal

* No essential stability problems

$\boxed{\text{More Important}}$ Cosmologically than local

* $m = 0$ loops ➡ factors of $\ln(p^2)$

for $c/H_0 \sim 6{,}000$ Mpc $p^2 \approx 0$!

* Eg 1 loop QG

$$R \ln\left(\frac{\Box}{\mu^2}\right) R \sqrt{-g} \quad \text{vs} \quad R^2 \sqrt{-g}$$

unique and finite

depends on counter-term

AND $|p^4 \ln(p^2)|$ > than $|p^4|$ for $p^2 \to 0$

cf Espriu, Multamaki and Vagenas, gr-qc/0503033

* $\Lambda \neq 0$ ➡ a dimension THREE coupling

∴ expect big IR corrections during inflation

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χ'os

Conclusions

① Worth studying $\mathcal{L} = \frac{1}{16\pi G}\left(R + \Delta R[g]\right)\sqrt{-g}$

② For local ΔR only $f(R)$ <u>might</u> be stable

③ $\Delta R = \frac{-\mu^4}{R}$ can give acceleration

④ But wildly unstable inside stars & planets

⑤ $\Delta R = \frac{-\mu^4}{R} + \frac{\alpha}{2\mu^2} R^2$ can give $q < 0$ + stability inside

⑥ But outside force too strong
and can't get both in and out ok

⑦ My favorite : nonlocal ΔR from QFT

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What about Nonlocal Eqns?

9.5

$\boxed{\text{Justification}}$ QFT produces them!

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* $m = 0$ loops ➡ factors of $\ln(p^2)$

for $c/H_0 \sim 6{,}000$ Mpc $p^2 \approx 0$!

* Eg 1 loop QG

$R \ln\left(\frac{\square}{\mu^2}\right) R \sqrt{-g}$ vs $R^2 \sqrt{-g}$

unique and finite

depends on counter-term

AND $|p^4 \ln(p^2)|$ > than $|p^4|$ for $p^2 \to 0$

cf Espriu, Multamaki and Vagenas, gr-qc/0503033

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