Title: Renormalizable Non-Metric Quantum Gravity?

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Abstract: We argue that four-dimensional quantum gravity may be essentially renormalizable provided one relaxes the assumption of metricity of the
theory. We work with Plebanski formulation of general relativity in which the metric (tetrad), the connection as well as the curvature are all independent variables and the usual relations among these quantities are only on-shell. One of the Euler-Lagrange equations of this theory guarantees its metricity. We show that quantum corrections generate a counterterm that destroys this metricity property, and that there are no other counterterms, at least at the one-loop level. There is a new coupling constant that controls the non-metric character of the theory. Its beta-function can be computed and is negative, which shows that the non-metricity becomes important in the infra red. The
new IR-relevant term in the action is akin to a curvature dependent
cosmological ``constant\'\' and may provide a mechanism for naturally small ``dark energy\'\'.

Non-Metric Quantum Gravity

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Motivations

Spin foam, loop quantum gravity community uses certain "polynomial" formulations of general relativity as the starting point for (non-perturbative) quantization.

Desirable to see how non-renormalizability of (perturbative) quantum gravity manifests itself in these formulations.

This talk

Will work with Plebanski formulation of GR:

- Jerzy Plebanski, "On the separation of Einsteinian substructures", *J. Math. Phys.* **Vol. 18** 2511-2520 (1977).

Dimensional analysis suggests this theory is incomplete. Other terms (counterterms) must be added. After this is done one gets a new theory that:

- Has different from Einstein GR renormalizability properties.
- Modifies Einstein's GR into a non-metric theory.

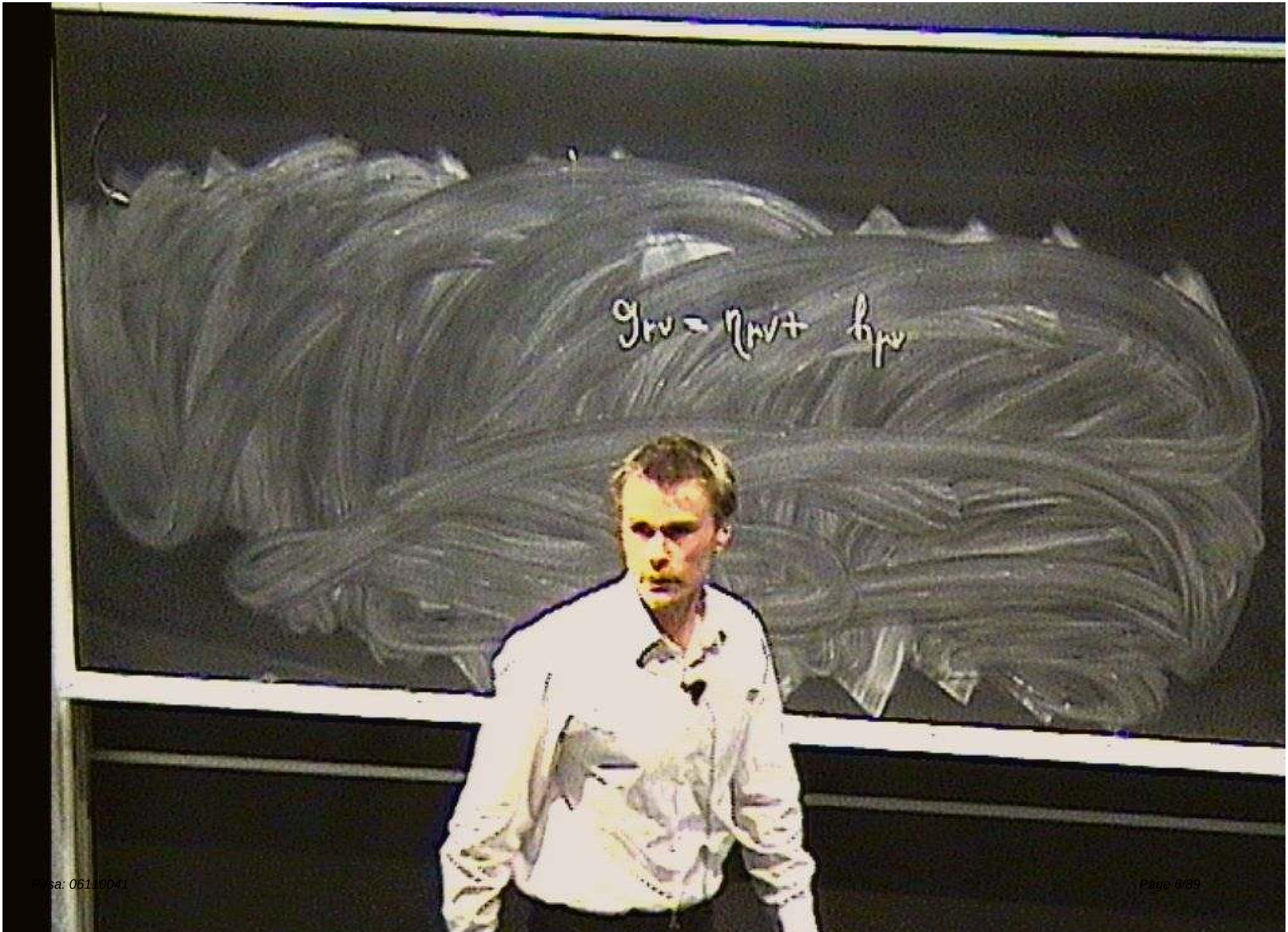
Part I: Polynomial formulations of gravity

Einstein formulated GR as the theory of the metric of spacetime

$$S_{EH}[g] = \frac{1}{16\pi G}\int_M d^4x\sqrt{-\det(g)}(R-2\Lambda). \quad (1)$$

The Newton constant G is absorbed into the fluctuating part of the metric field, giving it the mass dimension 1. Coupling constant - $\sqrt{G}$, mass dimension $[\sqrt{G}] = -1$: non-renormalizable! Hence, do not know its UV completion.

S-matrix is finite at one-loop (one-loop divergences become zero on-shell), but is divergent at two loops.


$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

Appendix A

Our conventions are as follows. We use the space time signature $(-+++)$, and we define the Riemann tensor in terms of the Christoffel symbols as

$$R^{\alpha}{}_{\beta\gamma\delta} = \partial_{\gamma}\Gamma^{\alpha}_{\beta\delta} - \partial_{\delta}\Gamma^{\alpha}_{\beta\gamma} + \Gamma^{\epsilon}_{\beta\delta}\Gamma^{\alpha}_{\epsilon\gamma} - \Gamma^{\epsilon}_{\beta\gamma}\Gamma^{\alpha}_{\epsilon\delta}. \tag{A.1}$$

Then, the Ricci tensor is

$$R_{\alpha\beta} = \delta^{\delta}_{\gamma} R^{\gamma}{}_{\alpha\beta\delta}, \tag{A.2}$$

and we write the Einstein action as

$$L = -2\sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}. \tag{A.3}$$

For completeness, we give below the expansion of the gravity lagrangian up to quartic order in quantum fields, including the gauge fixing term in eq. (2.6). The quantum field is denoted by $h_{\mu\nu}$, and it is implicitly assumed that all other quantities are constructed out of the background metric, $g_{\mu\nu}$. Indices are raised and lowered using the background metric. The terms quadratic in the quantum field are:

$$L_2 = \sqrt{-g}\Big\{-\frac{1}{2}h^{\alpha\beta}{}_{;\gamma}h_{\alpha\beta}{}^{;\gamma} + \frac{1}{4}h^{\alpha}{}_{\alpha;\gamma}h_{\beta}{}^{\beta;\gamma} + h_{\alpha\beta}h_{\gamma\delta}R^{\alpha\gamma\beta\delta} - h_{\alpha\beta}h^{\beta}{}_{\gamma}R^{\delta\alpha\gamma}{}_{\delta}$$
$$+ h^{\alpha}{}_{\alpha}h_{\beta\gamma}R^{\beta\gamma} - \frac{1}{2}h_{\alpha\beta}h^{\alpha\beta}R + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta}R\Big\}. \tag{A.4}$$

The terms cubic in the quantum field are:

$$L_3 = \sqrt{-g}\Big\{-\frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}{}_{;\alpha}h_{\gamma\delta;\beta} + 2h^{\alpha\beta}h^{\gamma\delta}{}_{;\alpha}h_{\beta\gamma;\delta} - h^{\alpha\beta}h^{\gamma}{}_{\gamma;\alpha}h^{\delta}{}_{\beta;\delta} - \frac{1}{2}h^{\alpha}{}_{\alpha}h^{\beta\gamma;\delta}h_{\beta\delta;\gamma}$$
$$+ \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta\gamma;\delta}h_{\beta\gamma;\delta} - h^{\alpha\beta}h^{\gamma}{}_{\gamma;\delta}h^{\delta}{}_{\alpha;\beta} + \frac{1}{2}h^{\alpha\beta}h^{\gamma}{}_{\gamma;\alpha}h^{\delta}{}_{\delta;\beta} - h^{\alpha\beta}h_{\alpha\beta;\gamma}h^{\gamma\delta}{}_{;\delta}$$
$$+ \frac{1}{2}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta;\gamma}h^{\gamma\delta}{}_{;\delta} + h^{\alpha\beta}h_{\alpha\beta;\gamma}h_{\delta}{}^{\delta;\gamma} + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta;\gamma}h_{\delta}{}^{\delta;\gamma} - h^{\alpha\beta}h^{\gamma}{}_{\alpha;\delta}h_{\beta\gamma}{}^{;\delta}$$
$$+ h^{\alpha\beta}h^{\gamma}{}_{\alpha;\beta}h^{\delta}{}_{\beta;\gamma} + R_{\alpha\beta}(2h^{\alpha\gamma}h_{\gamma\delta}h^{\delta\beta} - h^{\gamma}{}_{\gamma}h^{\alpha\delta}h^{\beta}{}_{\delta} - \frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta}$$
$$+ \frac{1}{4}h^{\alpha\beta}h^{\gamma}{}_{\gamma}h^{\delta}{}_{\delta}) + R(-\frac{1}{3}h^{\alpha\beta}h_{\beta\gamma}h^{\gamma}{}_{\alpha} + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta\gamma}h_{\beta\gamma} - \frac{1}{24}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta}h^{\gamma}{}_{\gamma})\Big\}$$
$$\tag{A.5}$$

Finally, the terms quartic in the quantum field are:

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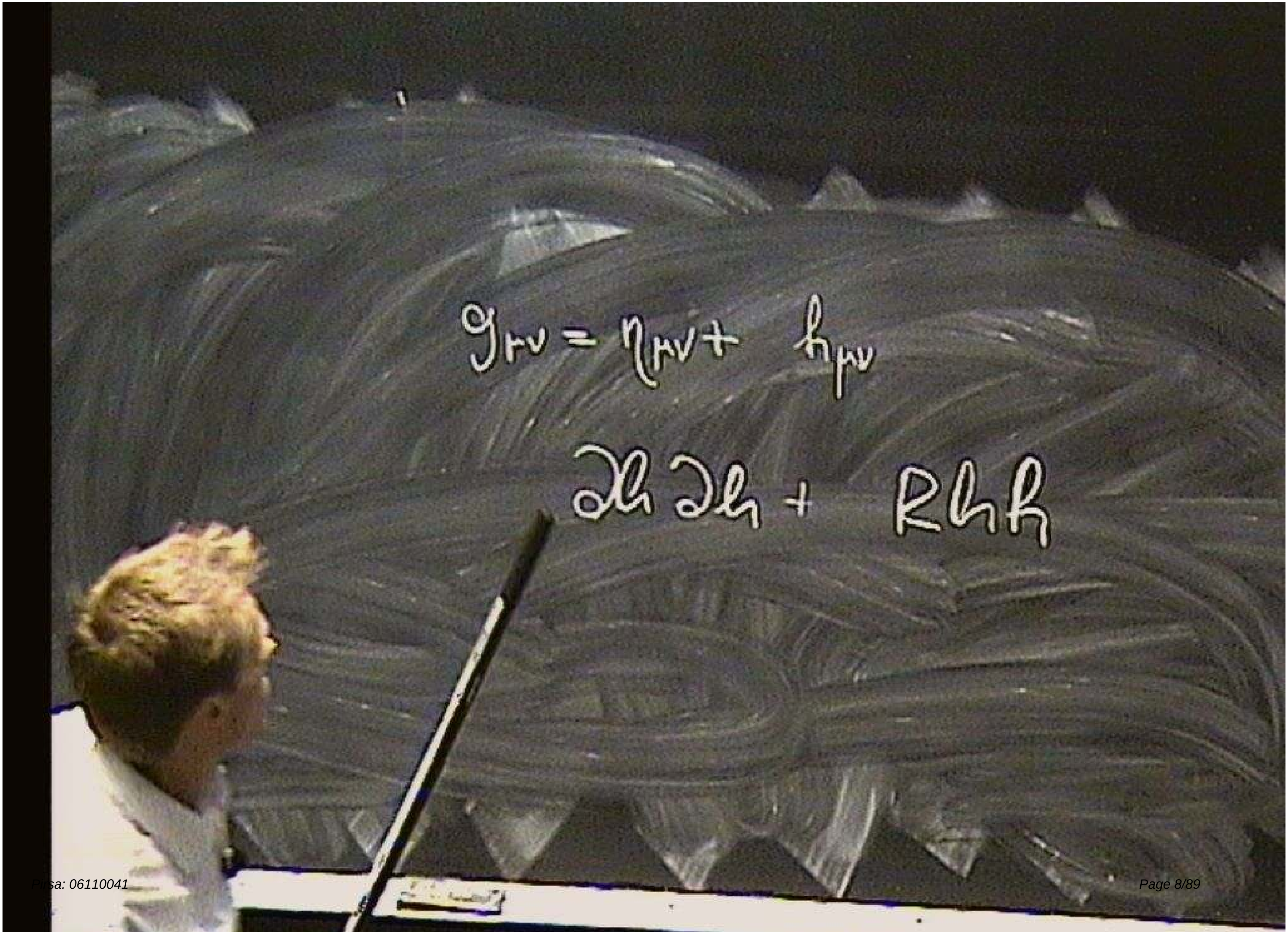
These terms are sufficient for the background field calculation. For the calcu[lation] in normal field theory one needs, in addition, quintic terms.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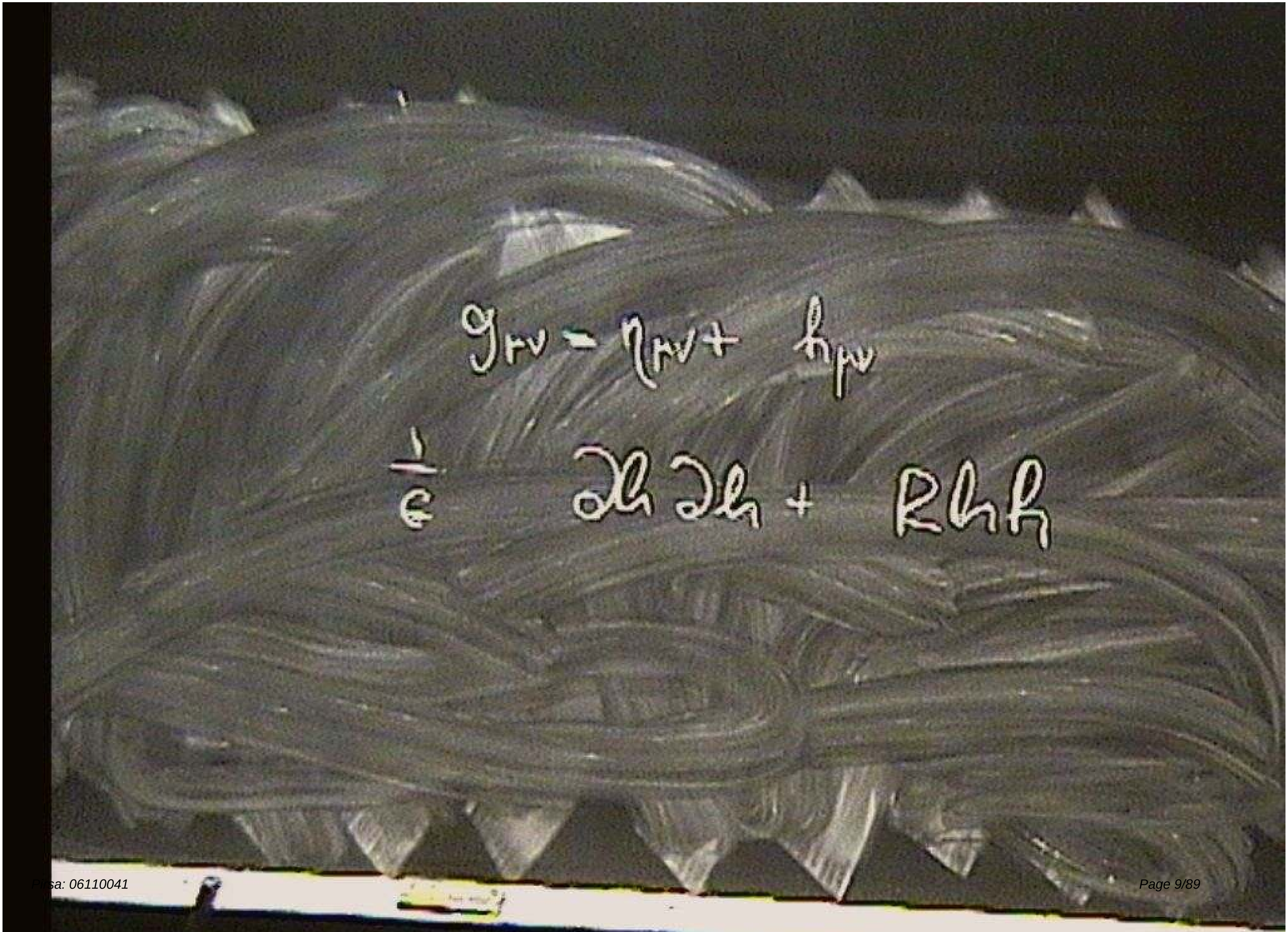
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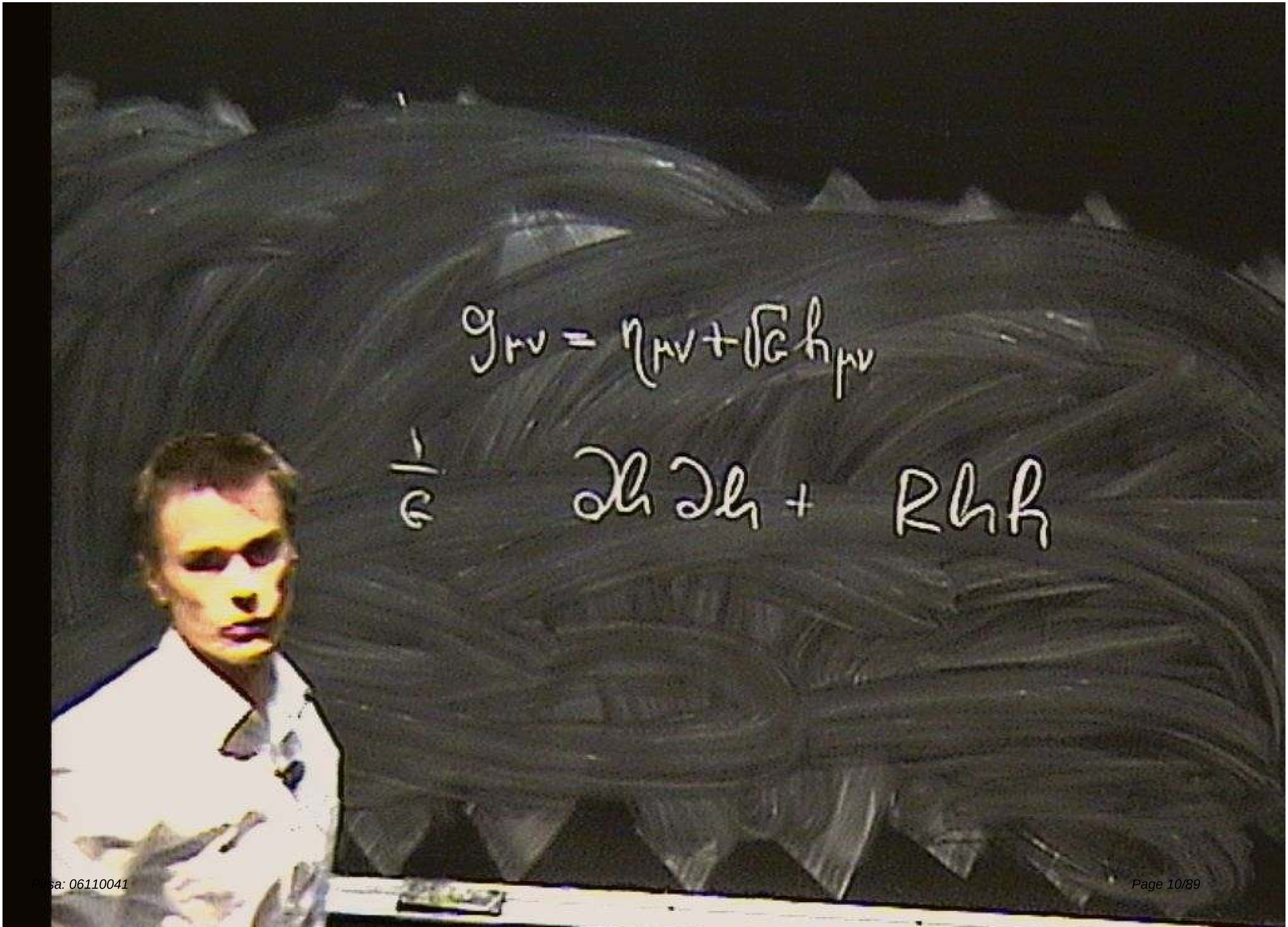
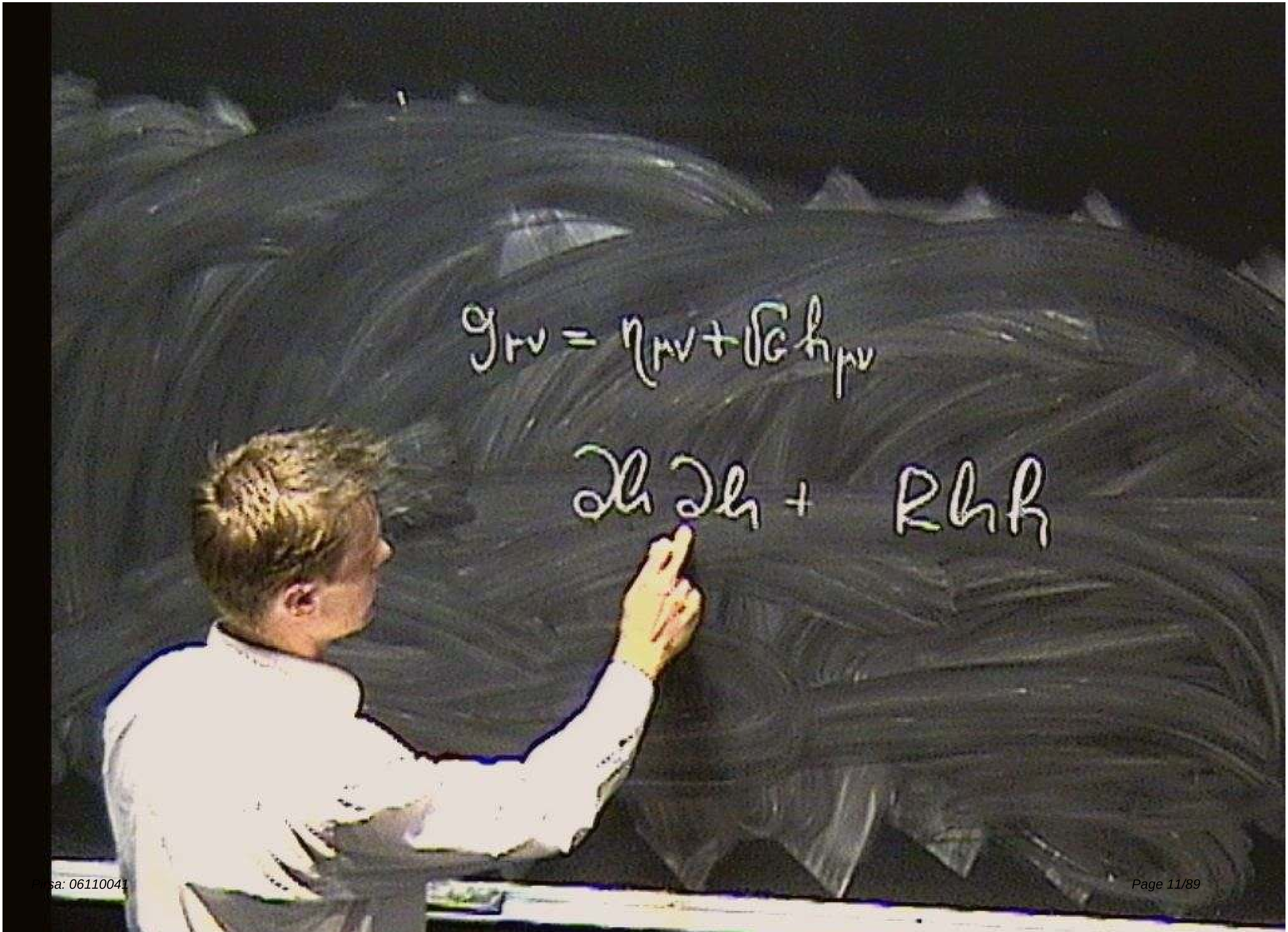

$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$
$\frac{1}{G} \quad \partial h \partial h + \quad R h h$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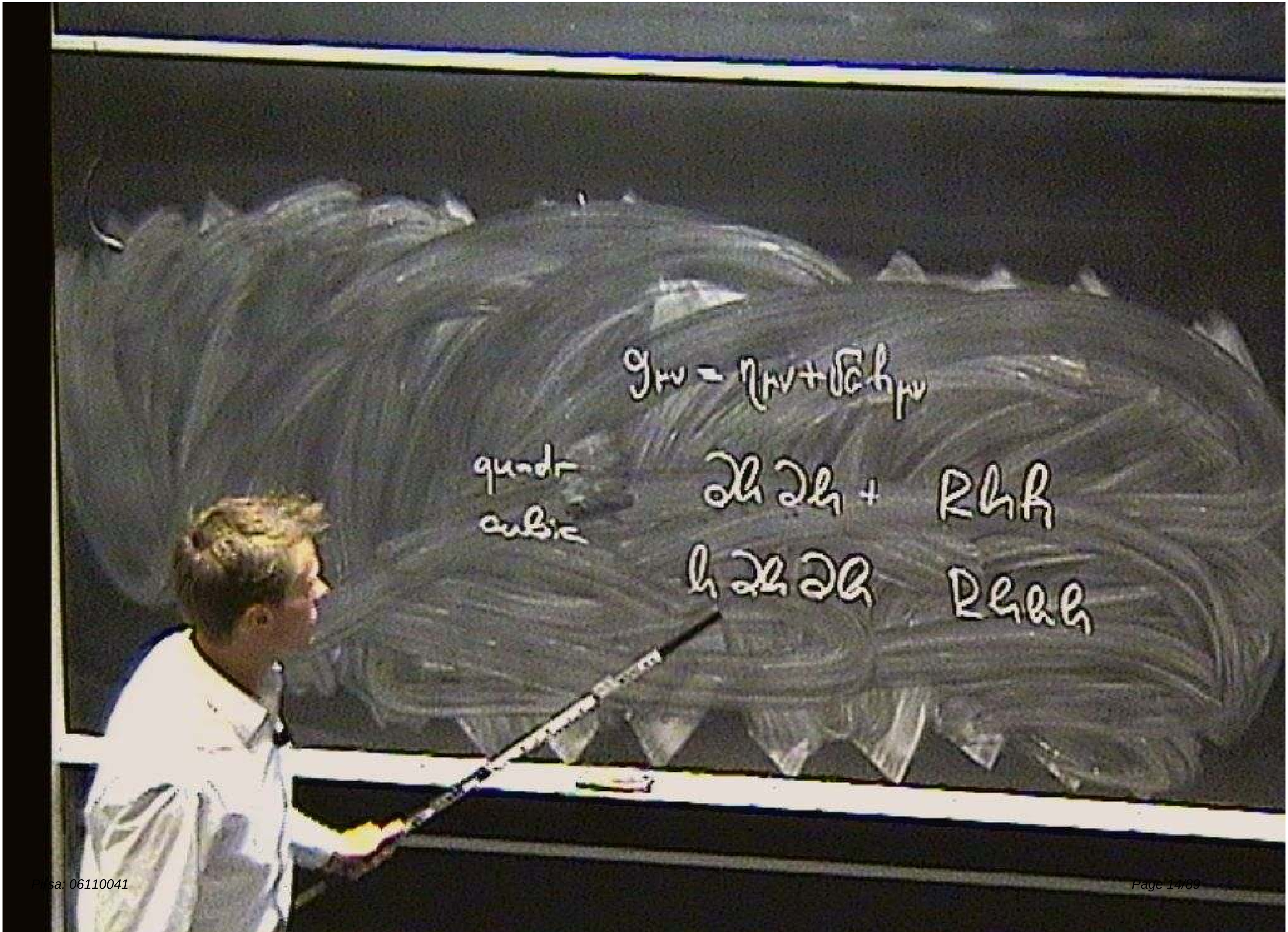
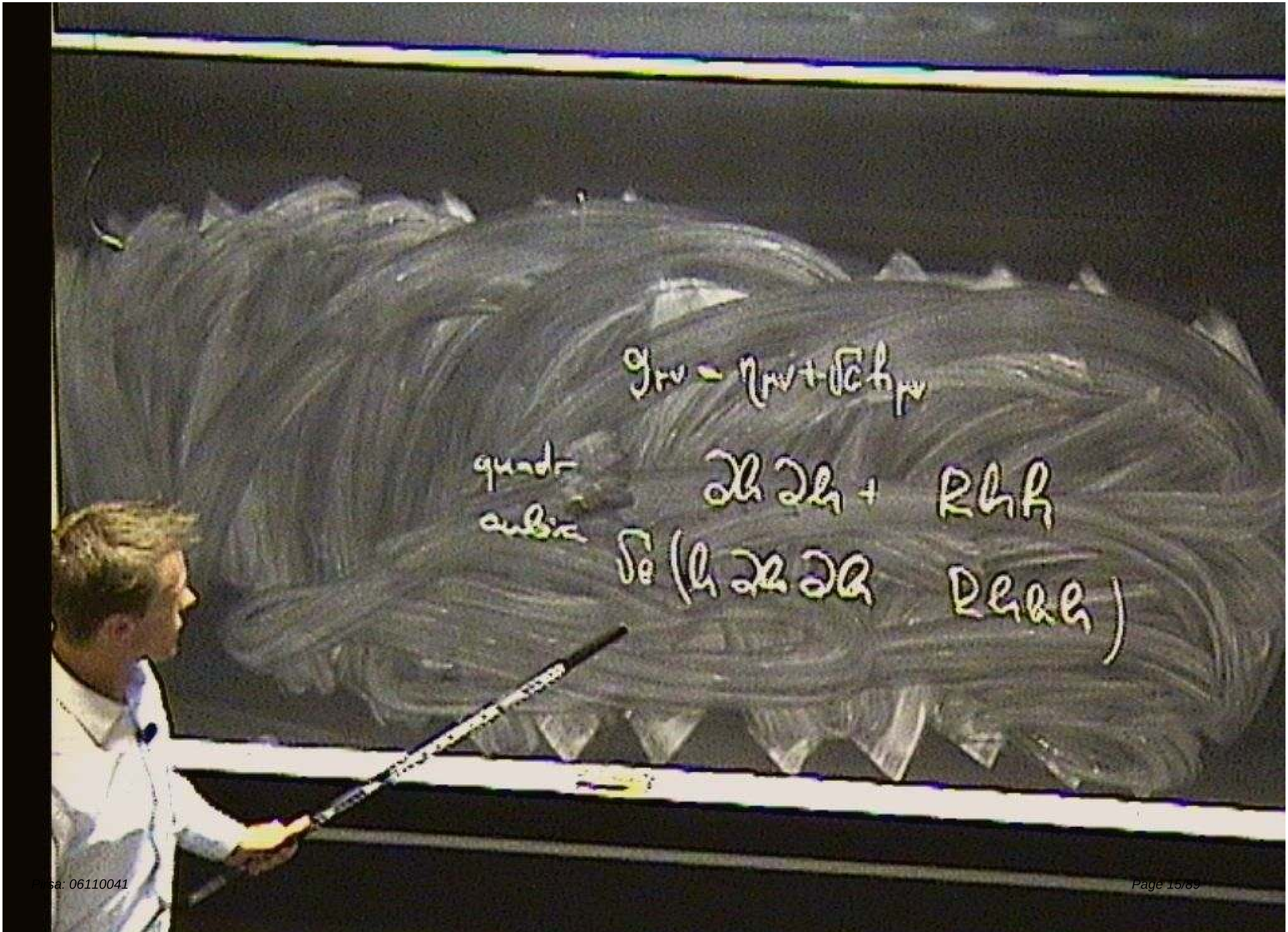
$$- \frac{1}{16}h^{\alpha\beta}h_{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta} - \frac{1}{6}h^{\alpha}{}_{\alpha}h^{\beta\gamma}h_{\gamma\delta}h^{\delta}{}_{\beta})\Big\}.$$

These terms are sufficient for the background field calculation. For the calcu[lation] in normal field theory one needs, in addition, quintic terms.


gμν = ημν + √G hμν
quadr ∂h∂h + Rhh
cubic h∂h∂h Rhhh


$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$
quadr. $\partial h \partial h + R h h$
cubic $\sqrt{G}\,(h \partial h \partial h \quad R h h h)$

Appendix A

Our conventions are as follows. We use the space time signature $(-+++)$, and we define the Riemann tensor in terms of the Christoffel symbols as

$$R^{\alpha}{}_{\beta\gamma\delta} = \partial_\gamma \Gamma^{\alpha}_{\beta\delta} - \partial_\delta \Gamma^{\alpha}_{\beta\gamma} + \Gamma^{\epsilon}_{\beta\delta}\Gamma^{\alpha}_{\epsilon\gamma} - \Gamma^{\epsilon}_{\beta\gamma}\Gamma^{\alpha}_{\epsilon\delta}. \qquad (A.1)$$

Then, the Ricci tensor is

$$R_{\alpha\beta} = \delta^{\delta}_{\gamma} R^{\gamma}{}_{\alpha\beta\delta}, \qquad (A.2)$$

and we write the Einstein action as

$$L = -2\sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}. \qquad (A.3)$$

For completeness, we give below the expansion of the gravity lagrangian up to quartic order in quantum fields, including the gauge fixing term in eq. (2.6). The quantum field is denoted by $h_{\mu\nu}$, and it is implicitly assumed that all other quantities are constructed out of the background metric, $g_{\mu\nu}$. Indices are raised and lowered using the background metric. The terms quadratic in the quantum field are:

$$L_2 = \sqrt{-g}\Big\{-\frac{1}{2}h^{\alpha\beta}{}_{;\gamma}h_{\alpha\beta}{}^{;\gamma} + \frac{1}{4}h^{\alpha}{}_{\alpha;\gamma}h_{\beta}{}^{\beta;\gamma} + h_{\alpha\beta}h_{\gamma\delta}R^{\alpha\gamma\beta\delta} - h_{\alpha\beta}h^{\beta}{}_{\gamma}R^{\delta\alpha\gamma}{}_{\delta}$$
$$+ h^{\alpha}{}_{\alpha}h_{\beta\gamma}R^{\beta\gamma} - \frac{1}{2}h_{\alpha\beta}h^{\alpha\beta}R + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta}R\Big\}. \qquad (A.4)$$

The terms cubic in the quantum field are:

$$L_3 = \sqrt{-g}\Big\{-\frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}{}_{;\alpha}h_{\gamma\delta;\beta} + 2h^{\alpha\beta}h^{\gamma\delta}{}_{;\alpha}h_{\beta\gamma;\delta} - h^{\alpha\beta}h^{\gamma}{}_{\gamma;\alpha}h^{\delta}{}_{\beta;\delta} - \frac{1}{2}h^{\alpha}{}_{\alpha}h^{\beta\gamma;\delta}h_{\beta\delta;\gamma}$$
$$+ \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta\gamma;\delta}h_{\beta\gamma;\delta} - h^{\alpha\beta}h^{\gamma}{}_{\gamma;\delta}h^{\delta}{}_{\alpha;\beta} + \frac{1}{2}h^{\alpha\beta}h^{\gamma}{}_{\gamma;\alpha}h^{\delta}{}_{\delta;\beta} - h^{\alpha\beta}h_{\alpha\beta;\gamma}h^{\gamma\delta}{}_{;\delta}$$
$$+ \frac{1}{2}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta;\gamma}h^{\gamma\delta}{}_{;\delta} + h^{\alpha\beta}h_{\alpha\beta;\gamma}h_{\delta}{}^{\delta;\gamma} + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta;\gamma}h_{\delta}{}^{\delta;\gamma} - h^{\alpha\beta}h^{\gamma}{}_{\alpha;\delta}h_{\beta\gamma}{}^{;\delta}$$
$$+ h^{\alpha\beta}h^{\gamma}{}_{\alpha;\delta}h^{\delta}{}_{\beta;\gamma} + R_{\alpha\beta}(2h^{\alpha\gamma}h_{\gamma\delta}h^{\delta\beta} - h^{\gamma}{}_{\gamma}h^{\alpha\delta}h^{\beta}{}_{\delta} - \frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta}$$
$$+ \frac{1}{4}h^{\alpha\beta}h^{\gamma}{}_{\gamma}h^{\delta}{}_{\delta}) + R(-\frac{1}{3}h^{\alpha\beta}h_{\beta\gamma}h^{\gamma}{}_{\alpha} + \frac{1}{4}h^{\alpha}{}_{\alpha}h^{\beta\gamma}h_{\beta\gamma} - \frac{1}{24}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta}h^{\gamma}{}_{\gamma})\Big\}$$
$$(A.5)$$

Finally, the terms quartic in the quantum field are:

$$L_4 = \sqrt{-g}\Big\{(h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta} - 2h^{\alpha\beta}h_{\alpha\beta})(\frac{1}{16}h^{\gamma\delta;\epsilon}h_{\gamma\delta;\epsilon} - \frac{1}{8}h^{\gamma\delta;\epsilon}h_{\gamma\epsilon;\delta} + \frac{1}{8}h^{\gamma}{}_{\gamma;\delta}h^{\delta\epsilon}{}_{;\epsilon}$$

$$-\frac{1}{16}h^{\gamma}{}_{\gamma;\delta}h_{\epsilon}{}^{\epsilon;\delta}) + h^{\alpha}{}_{\alpha}h^{\beta\gamma}(-\frac{1}{2}h_{\beta\gamma;\delta}h^{\delta\epsilon}{}_{;\epsilon} + \frac{1}{2}h_{\beta\gamma;\delta}h_{\epsilon}{}^{\epsilon;\delta} - \frac{1}{2}h^{\delta}{}_{\delta;\beta}h^{\epsilon}{}_{\epsilon}$$
$$+ \frac{1}{4}h^{\delta}{}_{\delta;\beta}h^{\epsilon}{}_{\epsilon;\gamma} + h^{\delta}{}_{\beta;\epsilon}h^{\epsilon}{}_{\delta;\gamma} - \frac{1}{4}h^{\delta\epsilon}{}_{;\beta}h_{\delta\epsilon;\gamma} - \frac{1}{2}h^{\delta}{}_{\beta;\epsilon}h_{\delta\gamma}{}^{;\epsilon} - \frac{1}{2}h^{\delta}{}_{\delta;\epsilon}h^{\epsilon}$$
$$+ \frac{1}{2}h_{\beta\delta;\epsilon}h_{\gamma}{}^{\epsilon;\delta}) + h^{\alpha}{}_{\beta}h^{\beta\gamma}(h^{\delta}{}_{\delta;\epsilon}h^{\epsilon}{}_{\alpha;\gamma} - h_{\alpha\gamma;\delta}h_{\epsilon}{}^{\epsilon;\delta} + \frac{1}{2}h^{\delta\epsilon}{}_{;\alpha}h_{\delta\epsilon;\gamma}$$
$$- h^{\delta}{}_{\alpha;\epsilon}h^{\epsilon}{}_{\gamma;\delta} - 2h^{\delta}{}_{\alpha;\epsilon}h^{\epsilon}{}_{\delta;\gamma} + h_{\alpha\gamma;\delta}h^{\delta\epsilon}{}_{;\epsilon} + h^{\delta}{}_{\delta;\alpha}h^{\epsilon}{}_{\gamma;\epsilon} - \frac{1}{2}h^{\delta}{}_{\delta;\alpha}h^{\epsilon}{}_{\epsilon;\gamma}$$
$$+ h^{\delta}{}_{\alpha;\epsilon}h_{\gamma\delta}{}^{;\epsilon}) + h^{\alpha\gamma}h^{\beta\delta}(h_{\alpha\gamma;\beta}h^{\epsilon}{}_{\delta;\epsilon} - h_{\alpha\gamma;\epsilon}h^{\epsilon}{}_{\delta;\beta} + \frac{1}{2}h_{\alpha\beta;\epsilon}h_{\gamma\delta}{}^{;\epsilon}$$
$$-\frac{1}{2}h_{\alpha\gamma;\epsilon}h_{\beta\delta}{}^{;\epsilon} + h^{\epsilon}{}_{\alpha;\beta}h_{\gamma\epsilon;\delta} - h^{\epsilon}{}_{\alpha;\beta}h_{\delta\epsilon;\gamma} + h_{\alpha\beta;\delta}h^{\epsilon}{}_{\epsilon;\gamma} - 2h^{\epsilon}{}_{\alpha;\beta}h_{\delta\gamma;\epsilon}$$
$$+ h_{\alpha\gamma;\epsilon}h^{\epsilon}{}_{\beta;\delta}) + R_{\alpha\beta}(-2h^{\alpha\gamma}h_{\gamma\delta}h^{\delta\epsilon}h_{\epsilon}{}^{\beta} + h^{\gamma}{}_{\gamma}h^{\alpha\delta}h_{\delta\epsilon}h^{\epsilon\beta} + \frac{1}{2}h^{\alpha\gamma}h_{\gamma}{}^{\beta}h^{\delta}$$
$$-\frac{1}{4}h^{\alpha\gamma}h_{\gamma}{}^{\beta}h^{\delta}{}_{\delta}h^{\epsilon}{}_{\epsilon} + \frac{1}{3}h^{\alpha\beta}h^{\gamma\delta}h_{\delta\epsilon}h^{\epsilon}{}_{\gamma} - \frac{1}{4}h^{\alpha\beta}h^{\gamma}{}_{\gamma}h^{\delta\epsilon}h_{\delta\epsilon} + \frac{1}{24}h^{\alpha\beta}h^{\gamma}{}_{\gamma}h$$
$$+ R(-\frac{1}{192}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta}h^{\gamma}{}_{\gamma}h^{\delta}{}_{\delta} + \frac{1}{16}h^{\alpha}{}_{\alpha}h^{\beta}{}_{\beta}h^{\gamma\delta}h_{\gamma\delta} + \frac{1}{4}h^{\alpha\beta}h_{\beta\gamma}h^{\gamma\delta}h_{\delta\alpha}$$
$$-\frac{1}{16}h^{\alpha\beta}h_{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta} - \frac{1}{6}h^{\alpha}{}_{\alpha}h^{\beta\gamma}h_{\gamma\delta}h^{\delta}{}_{\beta})\Big\}.$$

These terms are sufficient for the background field calculation. For the calcu
in normal field theory one needs, in addition, quintic terms.

Divergences of Einstein gravity (off-shell)

one-loop

$$\mathcal{L}_{\infty}^{(1)} = \sqrt{-g}\left(c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + c_3\, \epsilon^{\alpha\beta\gamma\delta} \epsilon_{\mu\nu\rho\sigma} R^{\mu\nu}{}_{\alpha\beta} R^{\rho\sigma}{}_{\gamma\delta}\right)$$

two-loop

$$R^{;\mu} R_{;\mu} \qquad R^3 \qquad R_{\alpha\beta;\mu} R^{\alpha\beta;\mu} \qquad R R_{\alpha\beta} R^{\alpha\beta}$$

$$R_{\alpha\gamma} R_{\beta\delta} R^{\alpha\beta\gamma\delta} \qquad R_\alpha{}^\beta R_\beta{}^\gamma R_\gamma{}^\alpha \qquad R R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \qquad R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\epsilon}$$

$$R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\epsilon\xi} R^{\epsilon\xi}{}_{\alpha\beta} \qquad R_{\alpha\beta\gamma\delta} R^\alpha{}_\epsilon{}^\gamma{}_\xi R^{\beta\epsilon\delta\xi}$$

On-shell

One loop: topological term only

Two loops: $\sqrt{-g}\, R^{\alpha\beta}{}_{\gamma\delta} R^{\gamma\delta}{}_{\epsilon\xi} R^{\epsilon\xi}{}_{\alpha\beta}$

First order formalism

Einstein gravity as the second-order formalism: second derivatives in the action. First order formalism available. In its Einstein-Cartan form:

$$S[\theta,\omega] = \frac{1}{8\pi G}\int_M \epsilon^{IJKL}\theta^I \wedge \theta^J \wedge F_\omega^{KL} + \frac{\Lambda}{2}\epsilon^{IJKL}\theta^I \wedge \theta^J \wedge \theta^K \wedge \theta^L. \qquad (2)$$

Here θ^I are the frame field one-forms, $F_\omega = d\omega + (1/2)\omega \wedge \omega$ is the curvature of the spin connection, indices $I, \ldots, K = 0, \ldots, 3$ are the internal ones, and ϵ^{IJKL} is the totally anti-symmetric tensor in the internal space.

Newton's constant

Newton's constant can be absorbed into the fields so that there is no dimensionfull coupling constant left: $\theta/\sqrt{G} = \tilde{\theta}, \Lambda G = \tilde{\Lambda}$. New mass dimensions:

$$[\tilde{\theta}] = 1, \quad [\omega] = 1, \quad [\tilde{\Lambda}] = 0. \tag{3}$$

Note: the presence of G in the usual metric-based perturbation theory (with background $\eta_{\mu\nu}$) is due to the split $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G} h_{\mu\nu}$. No Newton's constant in pure gravity unless there is a background.

Quantization?

Starting point for quantization. Works in 3D, where gravity is shown to be (super) renormalizable in this formulation.

Does not work in higher D: no kinetic term.

Note: "almost-renormalizable" action as there is only a very few terms that can be added to it compatible with mass dimensions and symmetries. In spite of this can't be quantized by the usual perturbative methods.

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Deser, McCarthy, Yang work of 1989

Used the Palatini version of the first order formulation, $S = S[g_{\mu\nu}, \Gamma^{\rho}_{\mu\nu}]$. Kinetic term.

Realized that there is a "mismatch between the symmetries of its quadratic and cubic terms, which makes this ostensibly renormalizable system ill-defined about zero vacuum, and forces the usual expansion of the metric about a background".

Lesson: to start off perturnation theory one is forced to expand around a constant background - this is when dimensionful Newton constant appears, and this is how the theory becomes non-renormalizable.

Plebanski formulation

Why Plebanski: there is a "kinetic term".

Separation of metric, connection A and curvature Ψ from each other.

We will use the original self-dual version (version without the self-dual split is available).

$$S[B, A, \Psi] = \frac{1}{2\pi i G}\int_M B^a F^a_A + \frac{1}{2}(\Lambda\delta^{ab} + \Psi^{ab})B^a B^b. \qquad (4)$$

Here a, b are the $su(2)$ Lie algebra indices, Ψ^{ab} is a field that on-shell becomes the Weyl part of the curvature (it is required to be symmetric traceless), B^a is a Lie algebra valued 2-form field that on-shell becomes expressed through a tetrad.

Euler-Lagrange equations:

$$B^a B^b = \frac{1}{3}\delta^{ab}\delta^{cd}B^c B^d, \quad F_A^a = -(\Lambda\delta^{ab} + \Psi^{ab})B^b, \quad d_A B^a = 0. \tag{5}$$

First implies that B^a is the self-dual part of the two form $B^{IJ} := (1/2)\theta^{[K}\theta^{L]}$ for some tetrad θ^I, second and third identifies Ψ^{ab} as the self-dual part of the Weyl curvature tensor.

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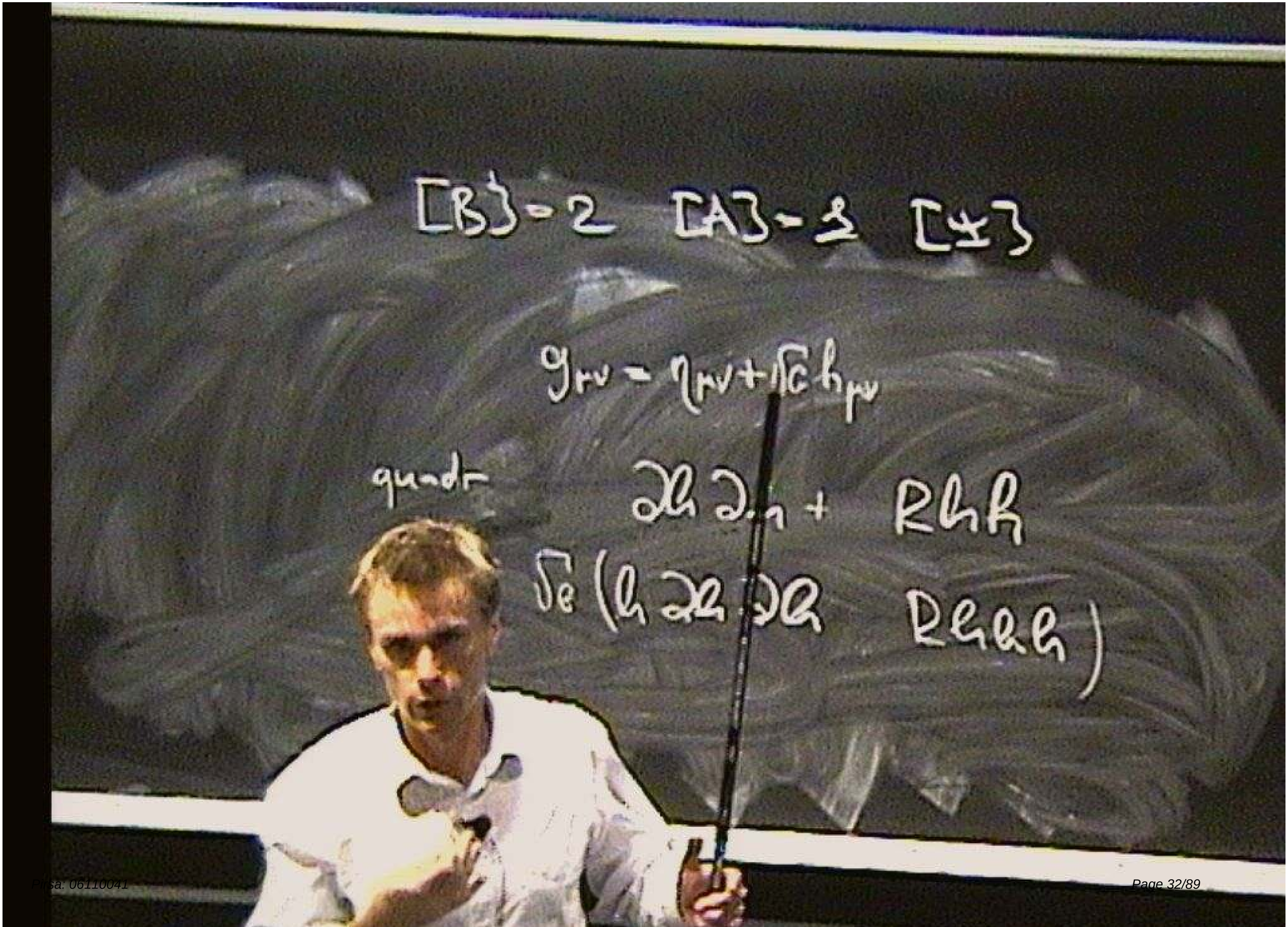
Again, no dimensionfull coupling after field rescalings. There is now a "kinetic term" for the fields, except for Ψ. Let us treat Ψ as an external field (i.e. postpone integration over Ψ).

Compare e.g. QED: can start with fermions in an external electromagnetic field

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu(\partial_\mu - ieA_\mu) + m)\psi. \quad (6)$$

Quantum corrections will generate the kinetic term for A: the usual F^2 Lagrangian. Let us see if anything like this happens for Plebanski theory.


[B] = 2 [A] = 1 [ψ]
g_μν = η_μν + √κ h_μν
quadr
∂h ∂h + Rhh
√κ (h ∂h ∂h Rhhh)

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Another example: Gross-Neveau model

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \sigma\bar{\psi}\psi - \frac{\sigma^2}{2g^2}. \qquad (7)$$

2D theory, mass dimensions $[\psi] = 1/2, [\sigma] = 1$. If no last term - a trivial theory (free left or right movers). The σ^2 term gets generated by quantum corrections. After it is integrated out - interacting $(\bar{\psi}\psi)^2$, asymptotically free theory.

Somewhat similar phenomenon happens in Plebanski theory.

Part II: Possible counterterms

The first step in analyzing the UV behavior is to produce a list of possible divergent terms (of mass dimension four compatible with symmetries).

After G is absorbed into the fields, the mass dimensions are:

$$[A] = 1, \quad [B] = 2, \quad [\Psi] = 0. \tag{8}$$

It is thus obvious that all powers of Ψ will appear.

Thus, in addition to the term $\Psi^{ab}B^aB^b$ need to add the terms of the form

$$\frac{1}{2}(\Psi^{k_1})^{ab}(\mathrm{Tr}(\Psi^2))^{k_2}\ldots(\mathrm{Tr}(\Psi^n))^{k_n}B^aB^b \qquad (9)$$

The theory does seem to be as non-renormalizable as in the usual perturbative quantum gravity. Usual case: dimensionfull Newton's constant; our case - a field Ψ of mass dimension zero.

Other terms

Clear that all powers of Ψ will get generated also in front of BF and FF terms. The renormalized action with all these counterterms is:

$$i\mathcal{L} = \frac{1}{2}\bar{X}(\Psi)^{ab} F_A^a F_A^b + \bar{Y}(\Psi)^{ab} B^a F_A^b + \frac{1}{2}\bar{Z}(\Psi)^{ab} B^a B^b, \qquad (10)$$

where $\bar{X}(\Psi), \bar{Y}(\Psi), \bar{Z}(\Psi)$ are all tensors, polynomials in Ψ and its traces. The coefficients of these polynomials are undetermined. Infinite number of them, seemingly no predictive power. Usual non-renormalizability.

Field B redefinition

Can redefine the field $B \to B + H(\Psi)F(A)$ to get rid of the $F^a F^b$ term. Can then "rescale" the field B to map the $B^a F^b$ term into its canonical form. After this B field redefinitions one gets

$$i\mathcal{L} = \tilde{B}^a F_A^a + \frac{1}{2}\tilde{\Psi}(\Psi)^{ab}\tilde{B}^a\tilde{B}^b, \tag{11}$$

where

$$\tilde{\Psi}(\Psi) = (Y(\Psi)^T Z(\Psi)^{-1} Y(\Psi) - X(\Psi))^{-1}. \tag{12}$$

Field Ψ redefinition

The whole effect of the counterterms is to replace the curvature field Ψ^{ab} by a non-trivial, depending on many new parameters (coupling constants) functional $\tilde{\Psi}^{ab}(\Psi)$.

Rewrite:

$$\tilde{\Psi}(\Psi)^{ab} = \Phi^{ab}(\Psi) + \frac{1}{3}\delta^{ab}\phi(\Psi), \qquad (13)$$

where $\Phi^{ab}(\Psi)$ is the traceless part of $\tilde{\Psi}$. The field Φ^{ab} just replaces the original field Ψ^{ab} after the renormalization!

Euler-Lagrange equations:

$$B^a B^b = \frac{1}{3}\delta^{ab}\delta^{cd}B^c B^d, \quad F_A^a = -(\Lambda\delta^{ab} + \Psi^{ab})B^b, \quad d_A B^a = 0. \qquad (5)$$

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Renormalized action

The effect of counterterms is in replacing the bare curvature field Ψ by the renormalized one Φ, and in appearance in the action of a new "trace" term:

$$\frac{1}{i}\int_M B^a F^a_A + \frac{1}{2}\left(\Lambda\delta^{ab} + \Phi^{ab} + \frac{\delta^{ab}}{3}\phi(\Phi)\right) B^a B^b, \qquad (14)$$

with $\phi(\Phi) = (\tilde{g}/2)\mathrm{Tr}(\Phi)^2 + O(\Phi^3)$, where $\tilde{g}$ is one of the new couplings.

Still non-renormalizable in the strict sense of the word (as still an infinite number of undetermined constants). More on this below.

Modifications of gravity

The metricity equations are modified to

$$B^a B^b = \frac{1}{3}\left(\delta^{ab} - \frac{d\phi(\Phi)}{d\Phi_{ab}}\right)\delta^{cd} B^c B^d. \qquad (15)$$

This equation no longer implies that the two-form field B^a is metric. We see that non-metricity is unavoidable whenever there is non-zero "curvature" Φ.

The full (quantum corrected) theory is no longer about metrics! Compare Gross-Neveau model with and without σ^2 term: more DOF.

Scale of the corrections

Not important at our scales: typical dimensionless curvatures in our Solar system

$$L^2\Psi \sim \frac{GM_{solar}}{R_{solar}} \sim 10^{-5}. \qquad (16)$$

Here L is the scale $L \sim R_{solar}$. Higher orders in Ψ are not of any significance.

Corrections become important on intergalactic (Hubble) scales, where the scale approaches the radius of curvature of the Universe.

Thus, should expect that gravity on large scales is not about metric. See also beta-functions below.

Dark energy

The term modifying GR (implying non-metricity) has the form of a curvature dependent "cosmological constant".

A mechanism for a naturally small "dark energy"?

Average curvature squared as dark energy.

Other counterterms

Other counterterms are possible, of high power in Ψ, e.g.

$$\Psi^{aa_1}(d_A\Psi)^{a_1a_2}(d_A\Psi)^{a_2a_3}(d_A\Psi)^{a_3a_4}(d_A\Psi)^{a_4a}, \quad (17)$$

$$f^{abc}\Psi^{ba_1}(d_A\Psi)^{a_1a_2}(d_A\Psi)^{a_2c}F^a, \quad (18)$$

as well as terms similar to the last one with B^a instead of F^a.

Don't have to worry about these for small curvatures, but important for understanding renormalizability properties. More on these below.

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Don't have to worry about these for small curvatures, but important for understanding renormalizability properties. More on these below.

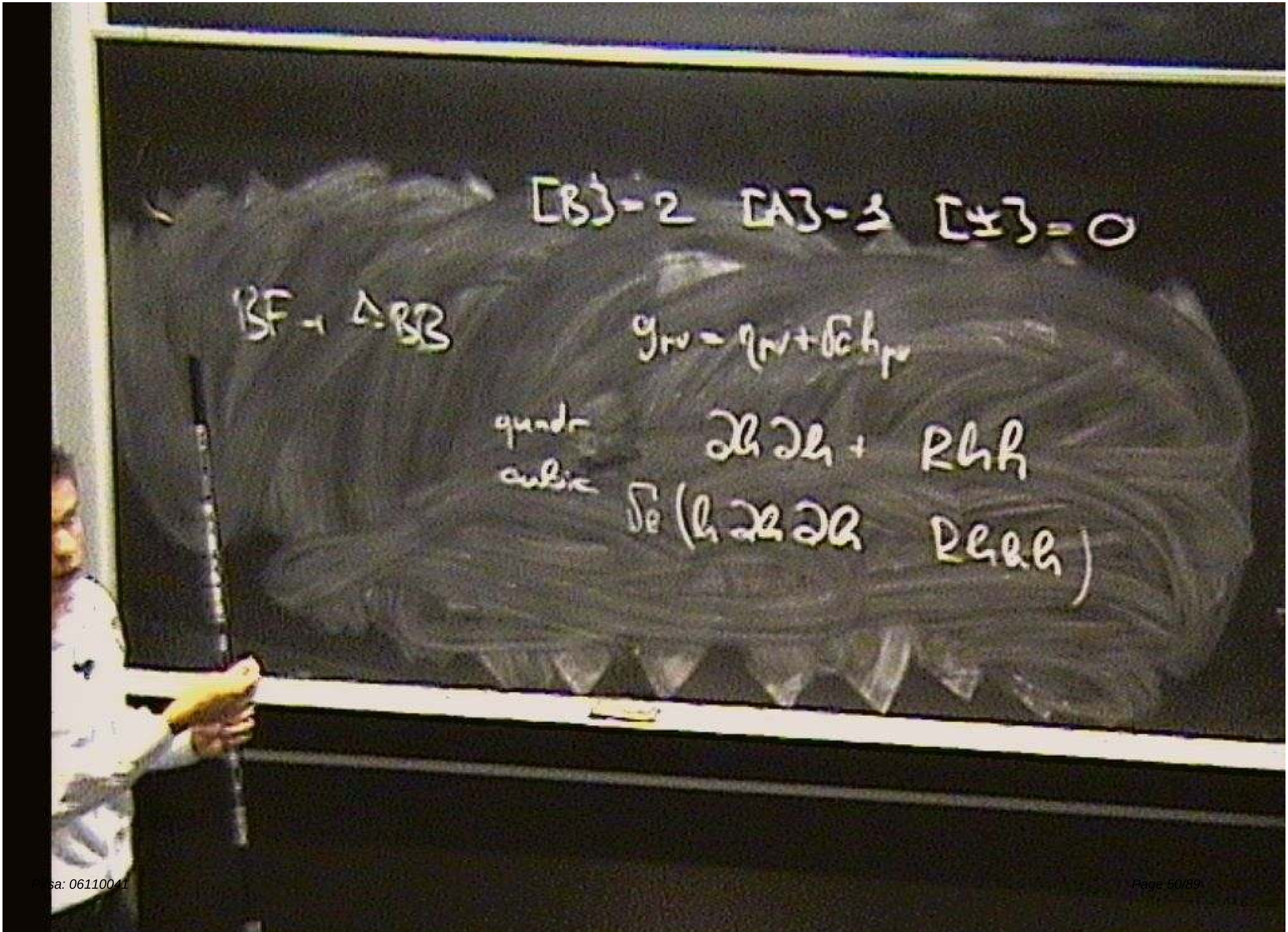
Part III: Quantum Theory

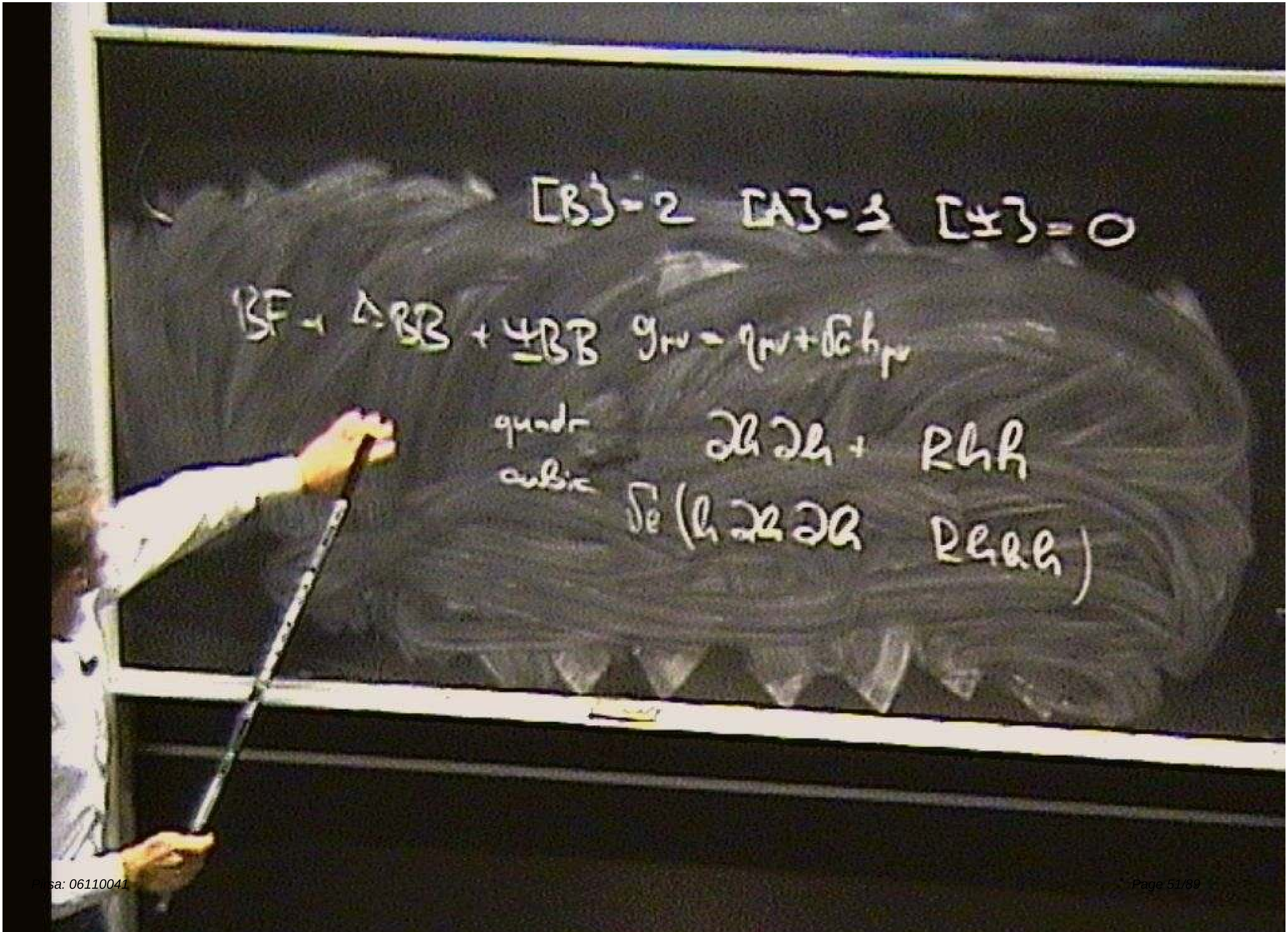
Would like to verify which of the counterterms do appear, and find the regime (UV or IR) where they are relevant - compute the corresponding beta-functions.

Problem: Even with Ψ non-fluctuating, there is still a mismatch between the symmetries of the quadratic and cubic terms.

This is what "killed" the Deser et all attempt at quantization in Palatini formulation.

Choose a constant background - not learn anything new.


[B] = 2 [A] = 1 [ψ] = 0
BF + ΛBB
g_μν = η_μν + √G h_μν
quadr ∂h∂h + Rhh
cubic √G (h∂h∂h Rhhh)


[B] = 2 [A] = 1 [Ψ] = 0
BF + ΛBB + ΨBB g_μν = η_μν + √G h_μν
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Part III: Quantum Theory

Would like to verify which of the counterterms do appear, and find the regime (UV or IR) where they are relevant - compute the corresponding beta-functions.

Problem: Even with Ψ non-fluctuating, there is still a mismatch between the symmetries of the quadratic and cubic terms.

This is what “killed” the Deser et all attempt at quantization in Palatini formulation.

Choose a constant background - not learn anything new.

A possible solution

Will use a trick due to Stueckelberg, which is to introduce an extra field so that the full action acquires the desired symmetry (that of the kinetic term). The symmetry can then be gauge fixed.

Action with the Stueckelberg field

Convenient to define: $\Lambda^{ab} = \Lambda\delta^{ab} + \Psi^{ab}$, and rescale B^a by Λ^{ab}. The original action then becomes:

$$i\mathcal{L} = (\Lambda^{-1})^{ab}\left(B^a F^b + \frac{1}{2}B^a B^b\right). \tag{19}$$

We introduce a new field η - Lie algebra valued one form. The new action is:

$$i\mathcal{L} = \frac{1}{2}(\Lambda^{-1})^{ab}\left((B + F_{A+\eta})^a (B + F_{A+\eta})^b - F^a_A F^b_A\right). \tag{20}$$

Reduces to the original action when $\eta = 0$.

Symmetries

The (topological) symmetry of the kinetic term becomes that of the full action:

$$\eta \to \eta + \tau, \qquad B \to B - d_{A+\eta}\tau - (1/2)[\tau, \tau]. \tag{21}$$

This symmetry is sufficient to set $\eta = 0$. Another, more interesting gauge is the *self-dual* one given by $B^{-} = 0$.

External fields

Even after the gauge fixing there is no kinetic term for Ψ and A. Could again try to choose a constant background for A. More interesting option is to keep A classical as well.

Interpretation: fixing a background for the Ψ and the connection A, and integrating over the fluctuation η of the connection and over the "geometrical" field B. Quantum gravity in the background of A, Ψ.

Honest quantum theory without need for a constant background. Price: external fields. What are the physical questions that can be asked in this theory?

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Integrating our B

After the gauge-fixing (self-dual), the result of integration over B is:

$$\mathcal{L} = -\frac{1}{2}(\Lambda^{-1})^{ab}(F^{+}_{A+\eta})^{\mu\nu a}(F^{+}_{A+\eta})^{a}_{\mu\nu}, \quad (22)$$

plus a term that only depends on the background, plus a set of gauge-fixing terms for η (essentially the same as in Donaldson theory).

Integrating out η

We have only performed the computation to one loop order. However, the theory is simple enough - not much more complicated then YM. Higher loops are also not hard. Results:

- No divergences containing derivatives of Ψ appear. The counterterm corrected Lagrangian is then

$$\mathcal{L} = -\frac{1}{2}(\tilde{\Lambda}(\Lambda^{-1}))^{ab}(F_A^+)^{\mu\nu a}(F_A^+)^a_{\mu\nu}, \qquad (23)$$

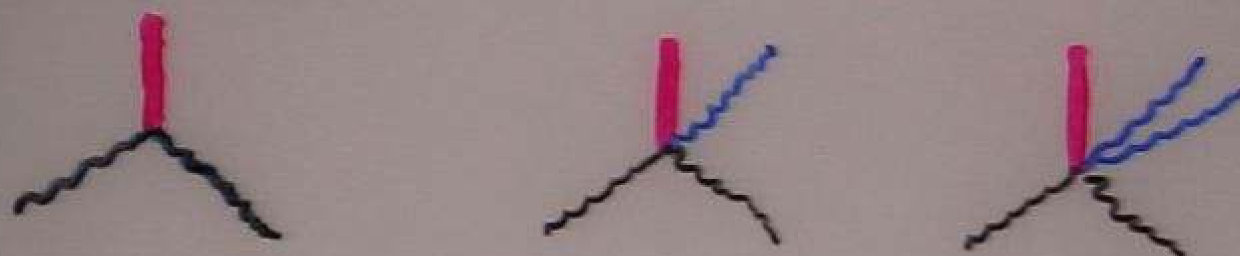
plus the background fields term of the original action.

Types of vertices in the theory

Gauge-field only : same as in YM (in external field)

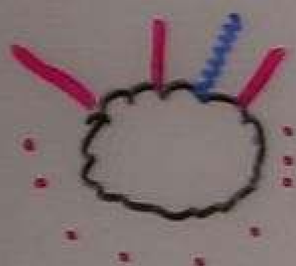
propagator fo
the fluctuat
gauge fiel

Curvature field



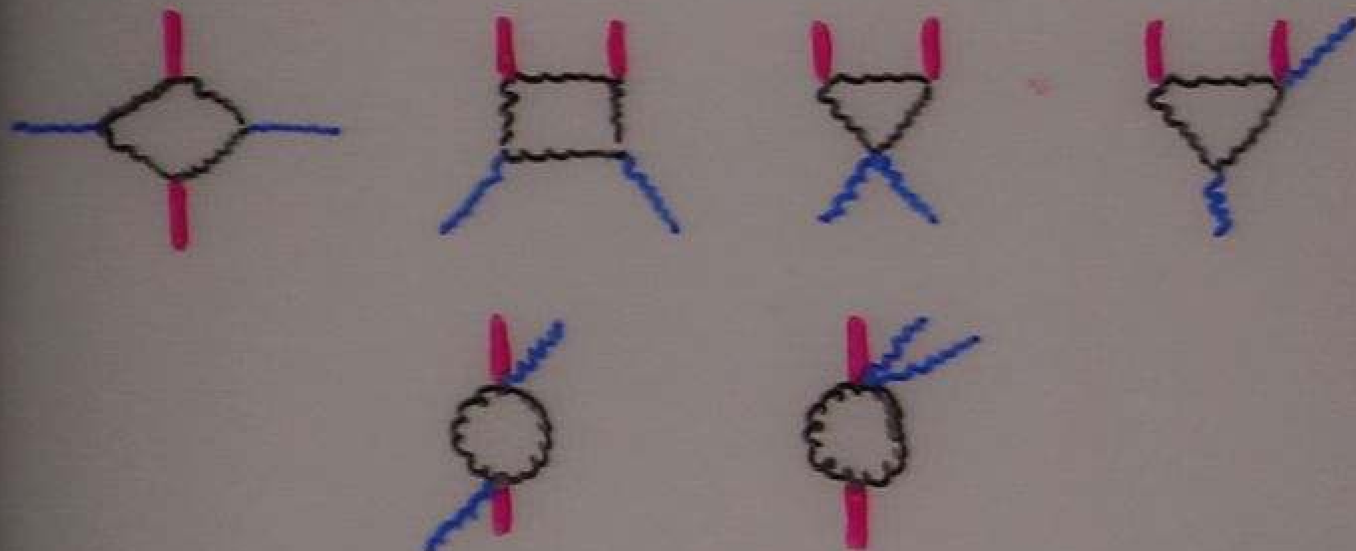
No derivatives of Ψ

Diverges as K^4, cancelled by $(\det \Psi)^*$



vanish after summed over all possible insertions of the gauge field.

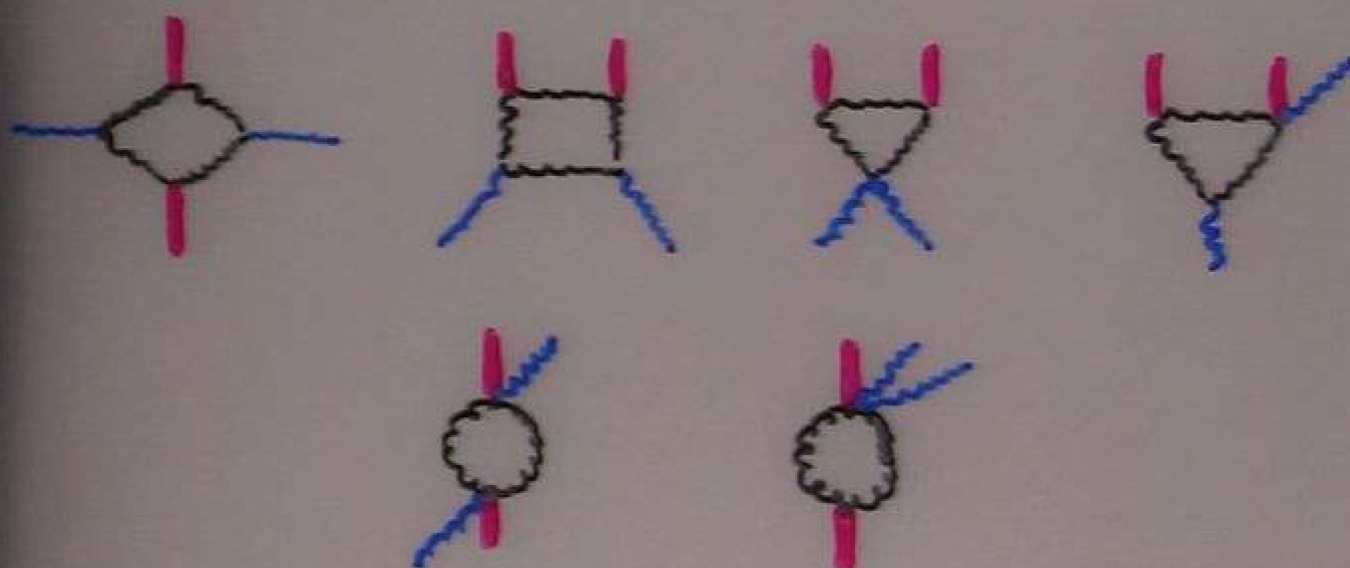
Order 2 in Ψ



Diagrams computed

Order 1 in Ψ

Order 2 in Ψ



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plus the background fields term of the original action.

The beta-functions

$$\frac{d\alpha}{d\log\mu} = -4C_2\alpha^2, \qquad \frac{d\tilde{g}}{d\log\mu} = -16C_2\alpha\tilde{g}. \tag{24}$$

where C_2 is the quadratic Casimir in the fundamental representation ($C_2 = 2$ in our case), and $\alpha := g^2/(4\pi)^2$, where g is the gauge field coupling. The negative sign of the beta-function for $\tilde{g}$ shows that non-metricity becomes important in the IR.

Renormalized action

The effect of counterterms is in replacing the bare curvature field Ψ by the renormalized one Φ, and in appearance in the action of a new "trace" term:

$$\frac{1}{i}\int_M B^a F_A^a + \frac{1}{2}\left(\Lambda\delta^{ab} + \Phi^{ab} + \frac{\delta^{ab}}{3}\phi(\Phi)\right) B^a B^b, \qquad (14)$$

with $\phi(\Phi) = (\tilde{g}/2)\mathrm{Tr}(\Phi)^2 + O(\Phi^3)$, where $\tilde{g}$ is one of the new couplings.

Still non-renormalizable in the strict sense of the word (as still an infinite number of undetermined constants). More on this below.

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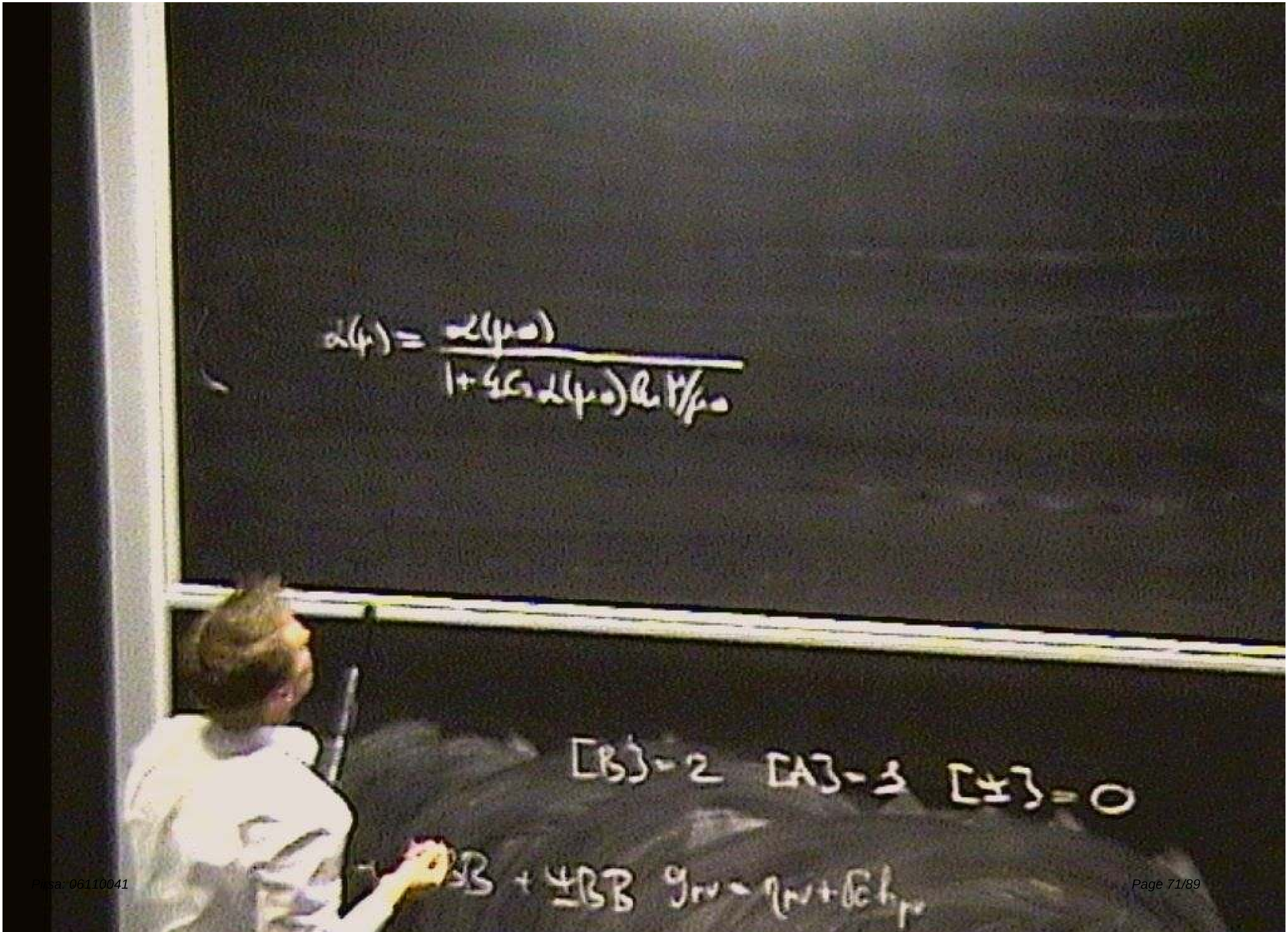
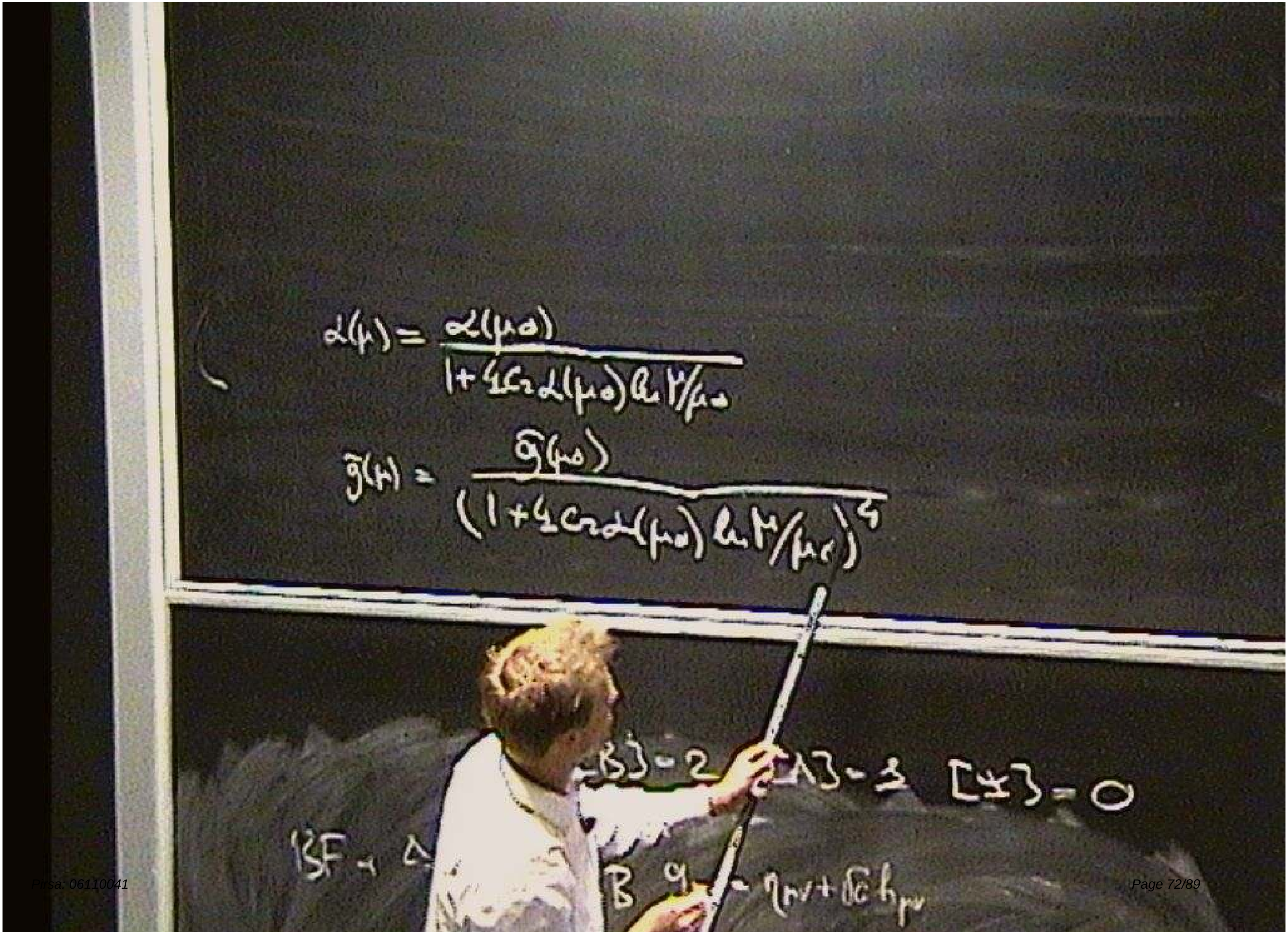

[B] = 2 [A] = 3 [ψ] = 0



α(μ) = α(μo) / (1 + 4C2 α(μo) ln M/μo)
g̃(μ) = g̃(μo) / (1 + 4C2 α(μo) ln M/μo)^5
[B] = 2 [Λ] = 3 [ψ] = 0
g = ημν + √G hμν

$$\alpha(\mu) = \frac{\alpha(\mu_0)}{1 + 4 C_2 \alpha(\mu_0) \ln \mu/\mu_0}$$

$$\tilde{g}(\mu) = \frac{\tilde{g}(\mu_0)}{\left(1 + 4 C_2 \alpha(\mu_0) \ln \mu/\mu_0\right)^{\varsigma}}$$

$[B] = 2 \quad [A] = 1 \quad [\Psi] = 0$

$BF + \Lambda BB + \Psi BB \quad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$

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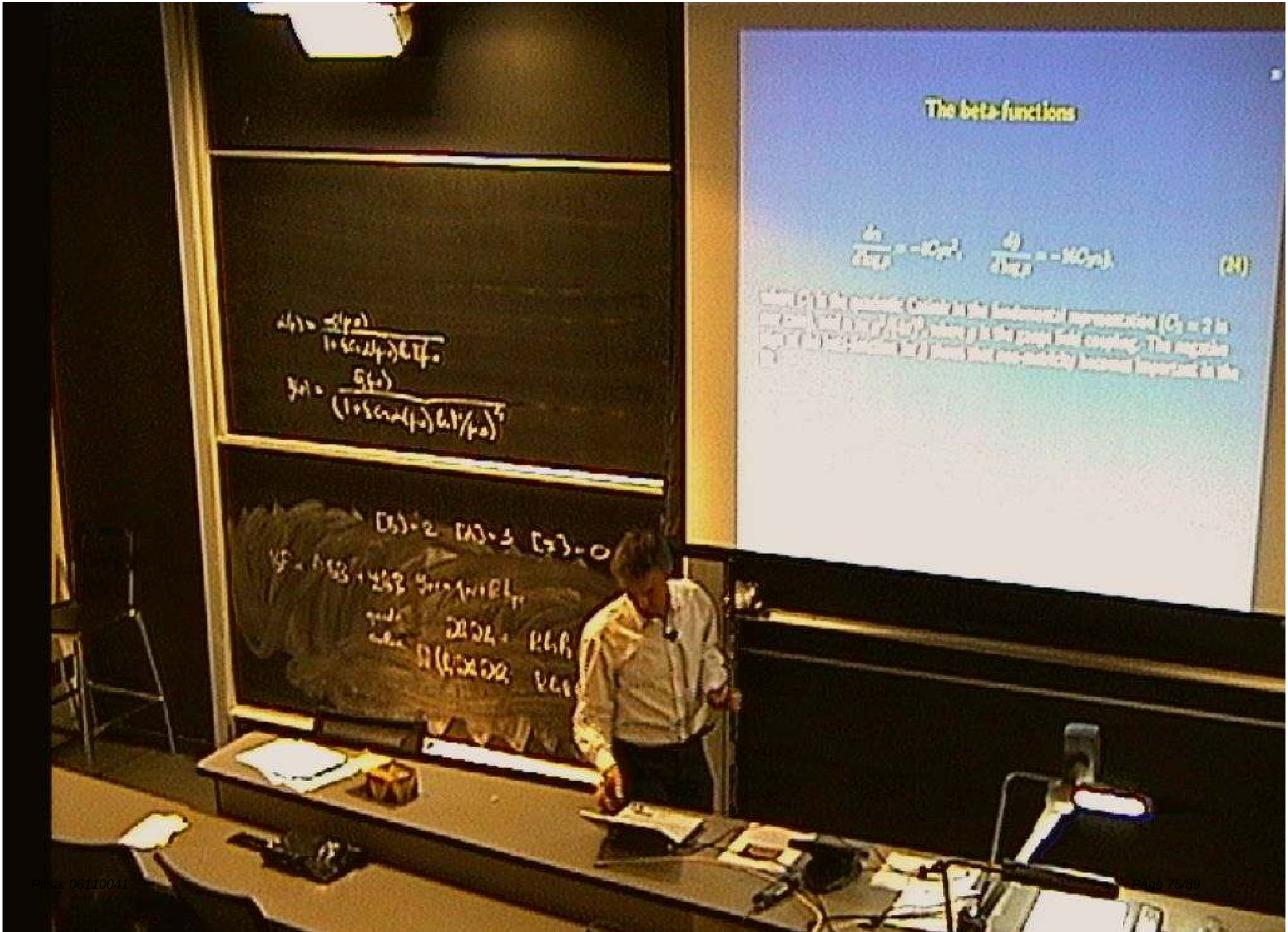

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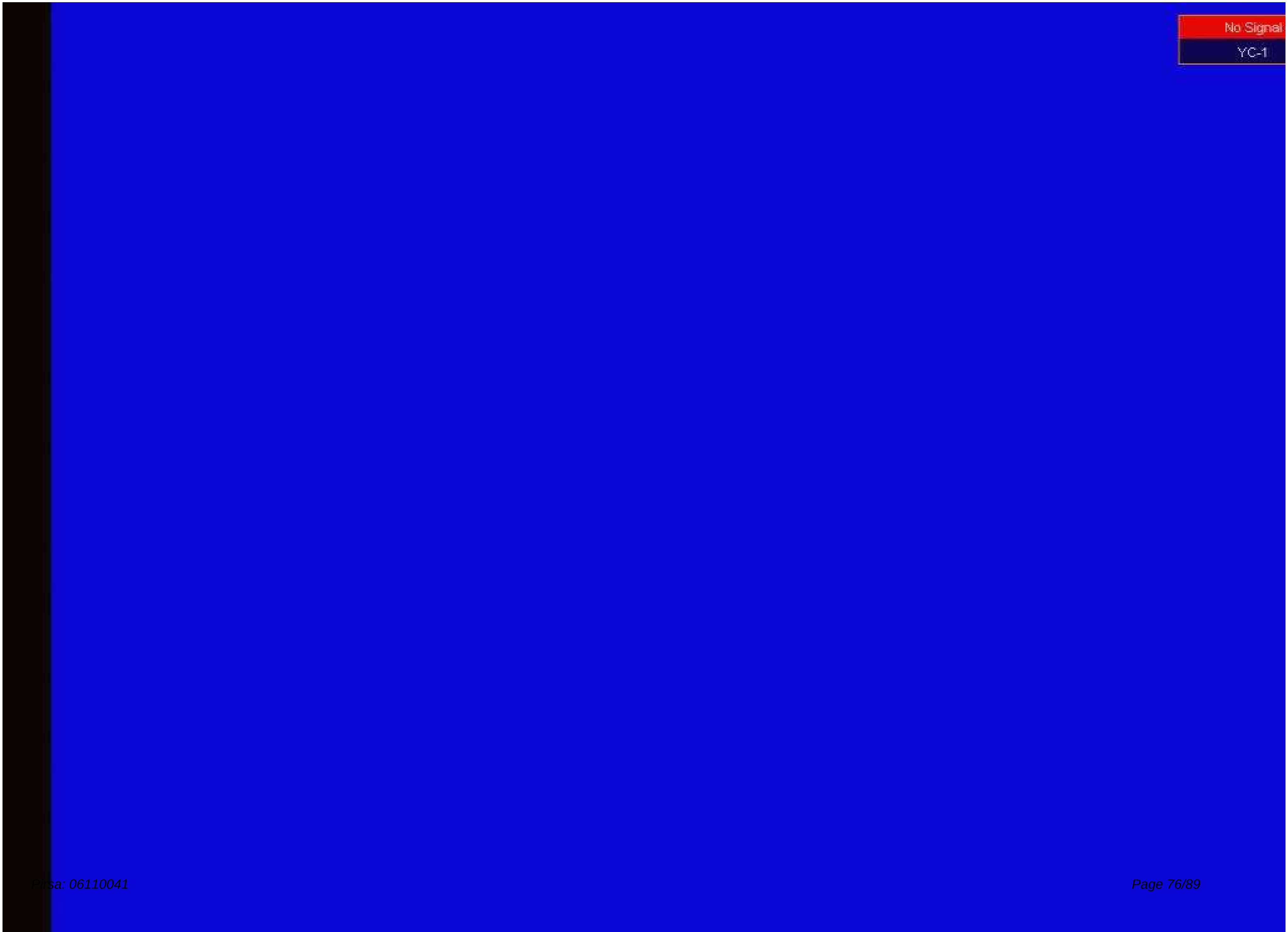

No Signal
YC-1

$$\alpha(\mu) = \frac{\alpha(\mu_0)}{1 + 4C_2\,\alpha(\mu_0)\ln \mu/\mu_0}$$

$$\tilde{g}(\mu) = \frac{\tilde{g}(\mu_0)}{\left(1 + 4C_2\,\alpha(\mu_0)\ln \mu/\mu_0\right)^4}$$

$[B] = 2 \qquad [A] = 1$

Interpretation

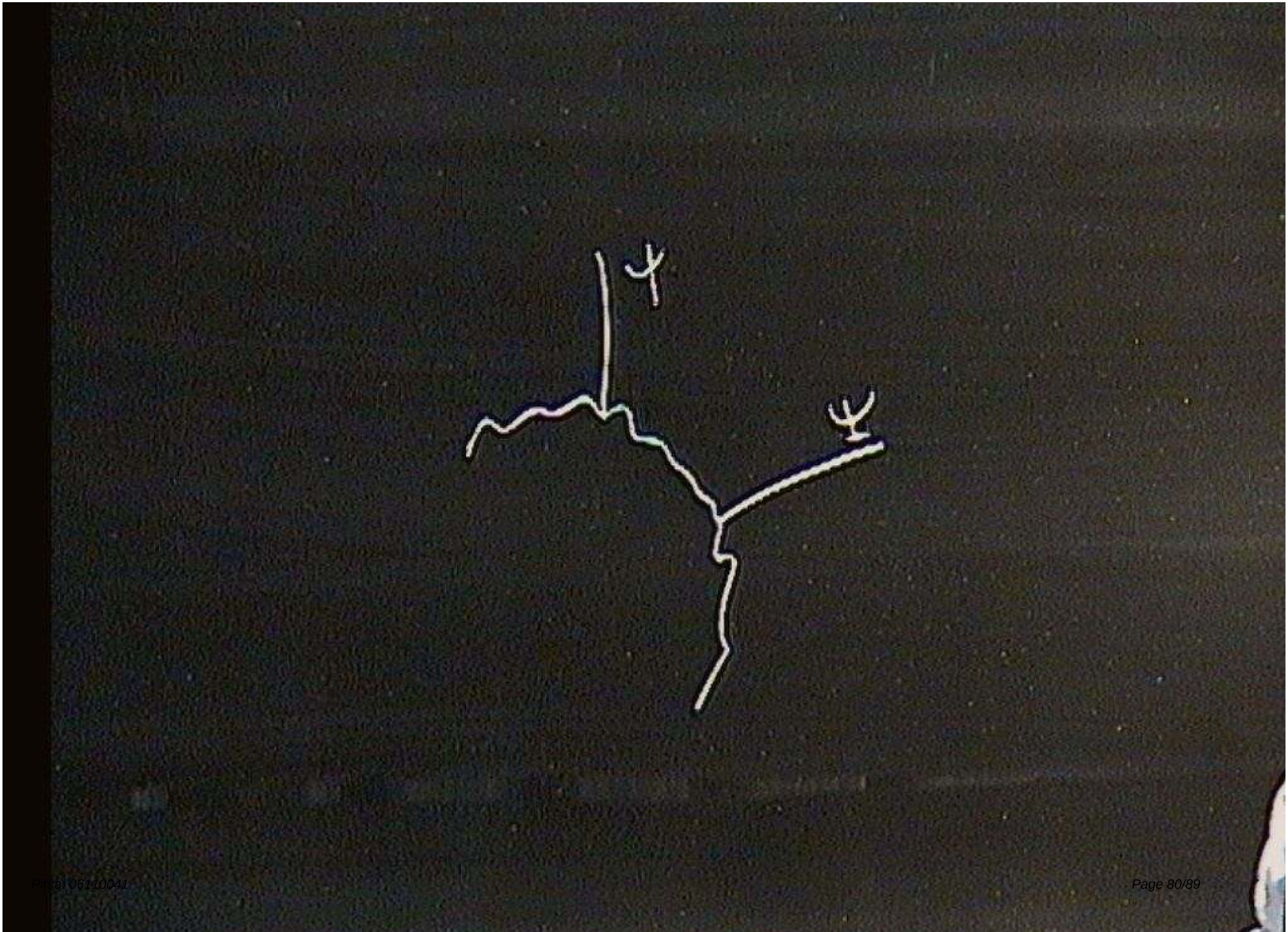
There is just a single scale in the theory, that set by the running of the gauge field coupling. To be checked for higher order in Φ couplings.

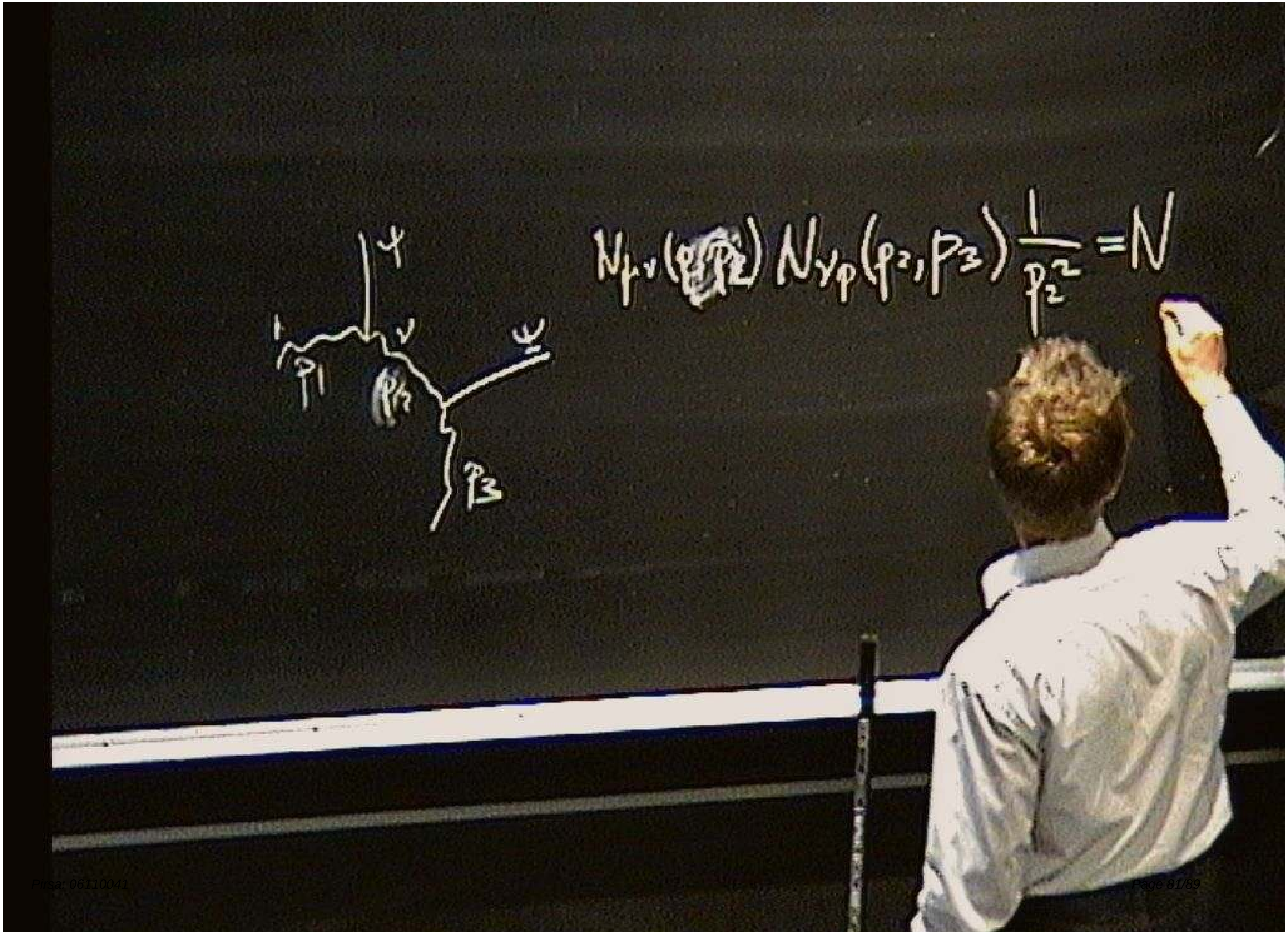
The scale determines where the higher order in Φ terms become important. But these are also the terms that induce modifications to gravity. Thus, this scale must be much larger than the scale of our solar system.

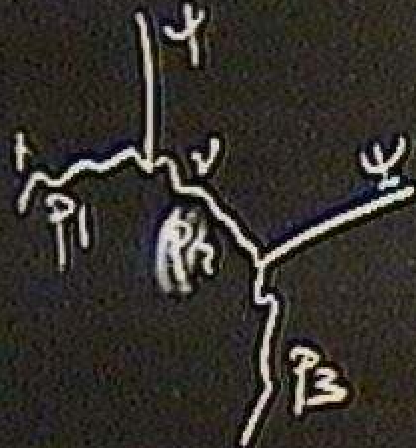
Can this be the cosmological scale?

Summary

- Plebanski gravity is incomplete, and must be supplemented by a curvature dependent "trace" term. This term renders gravity non-metric. Non-metricity becomes important for large curvatures.
- Studied the quantum theory perturbatively, in a background of fixed classical fields Ψ, A.
- The structure of the divergences is very transparent, their effect is in producing the renormalized curvature field together with the trace term.
- The one-loop beta-functions can be computed. For the lowest order coupling $\tilde{g}$ it is negative. Expect to have all of them negative, and a single scale in the theory.
- Speculated that we live above that scale, and the theory we have is UV complete. No Planck scale, no spacetime foam.




p1
p3



$$N_{\mu\nu}(\text{[illegible]})\, N_{\nu\rho}(p_2, p_3)\, \frac{1}{p_2^2} = N_{\mu\rho}(p_1, p_3)$$

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$$N_{\mu\nu} = (pq)\, g^{\mu\nu} - q^{\mu} p^{\nu} + \epsilon^{\cdots\cdots} p q$$

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Open questions

- Interpretation of the quantum theory in the external field. What are gravitons, can one extract their scattering aplitudes from the effective action for A, Ψ? What physical questions can be asked?
- Dependence on the gauge?
- Higher loops?
- Lots of interesting questions about the classical theory (modified gravity) as soon as it is coupled to matter. Matter couplings are known, work in progress.

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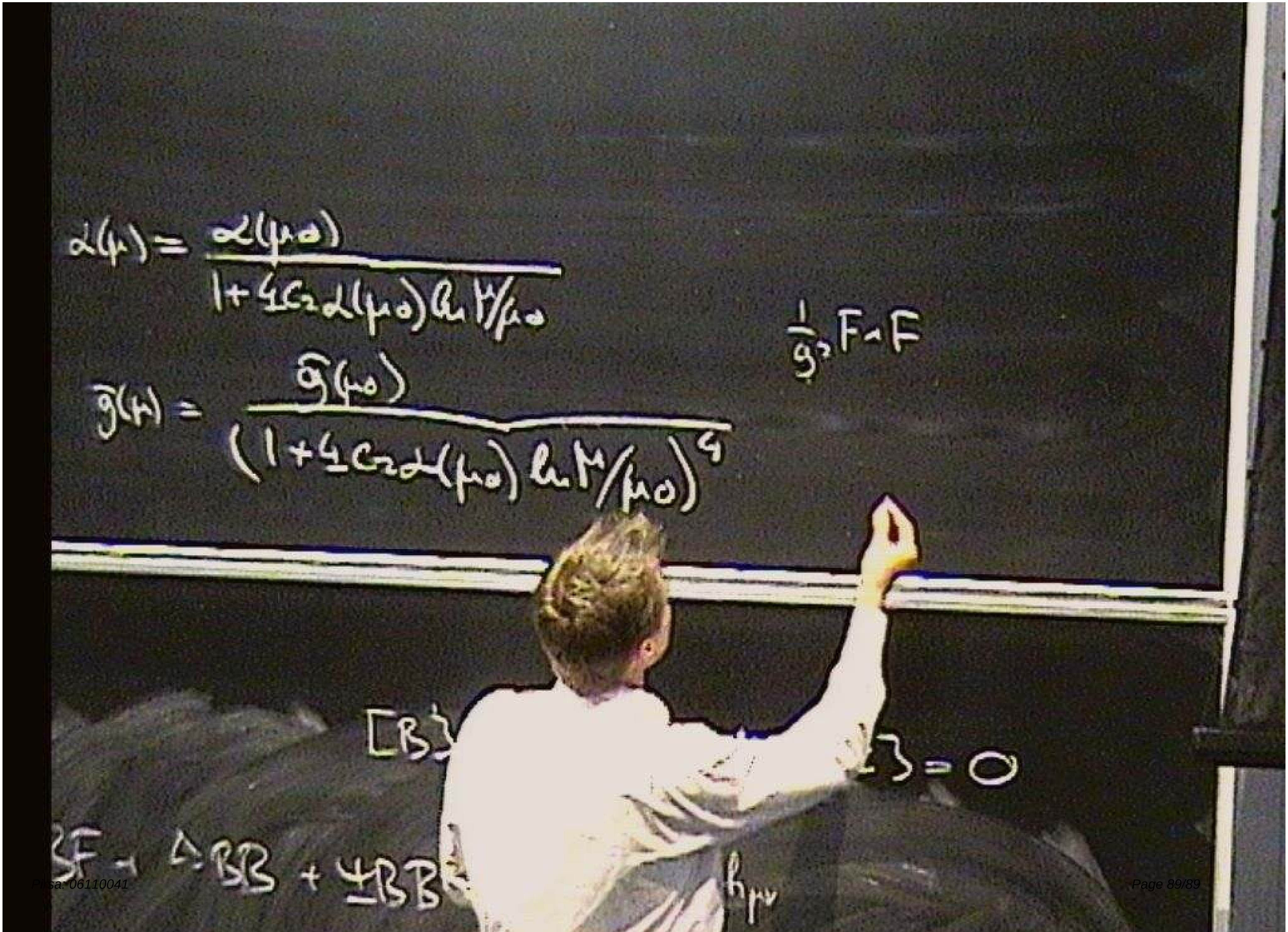
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BB + BB
hμν