

Title: Inflation after WMAP

Date: Nov 23, 2006 11:00 AM

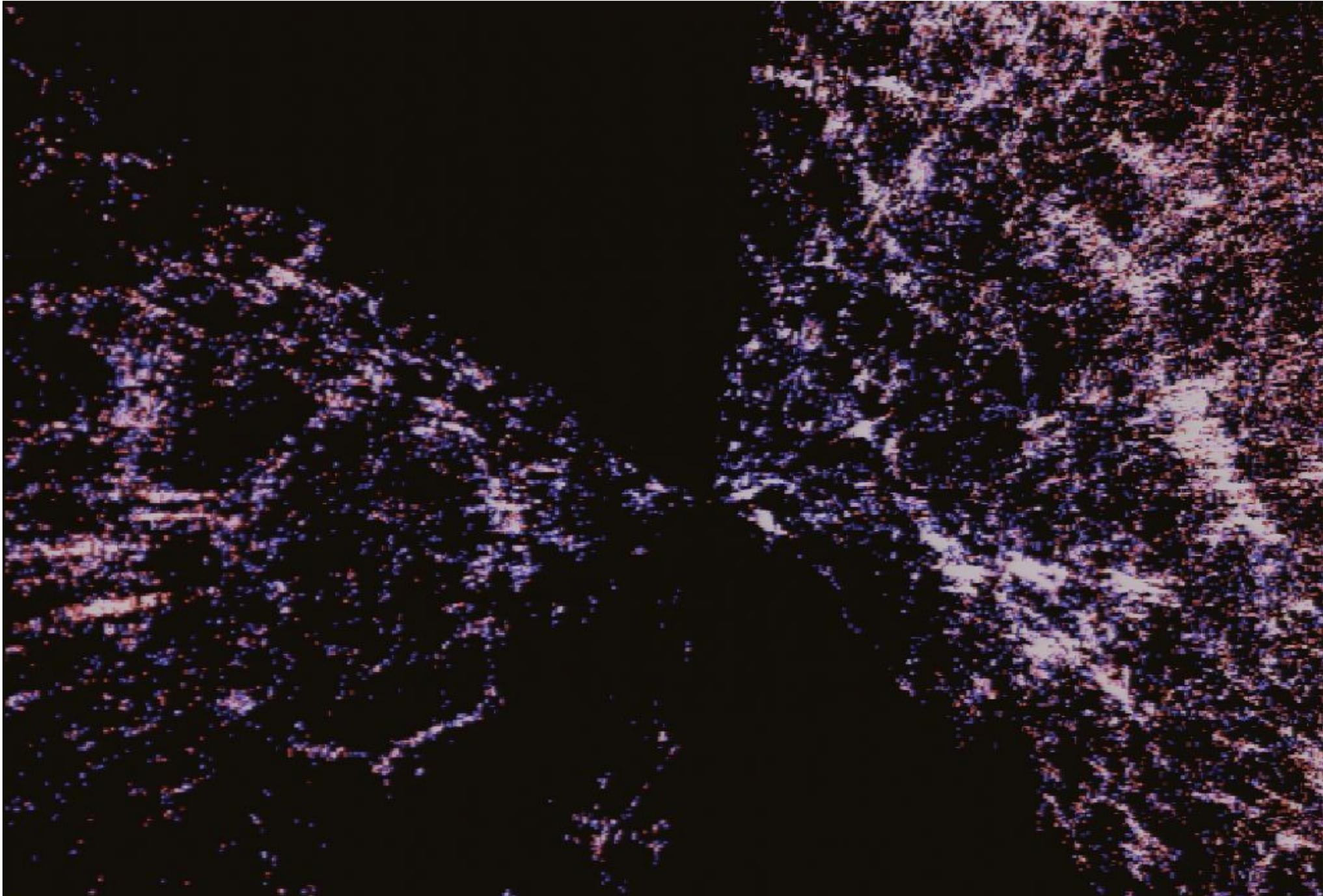
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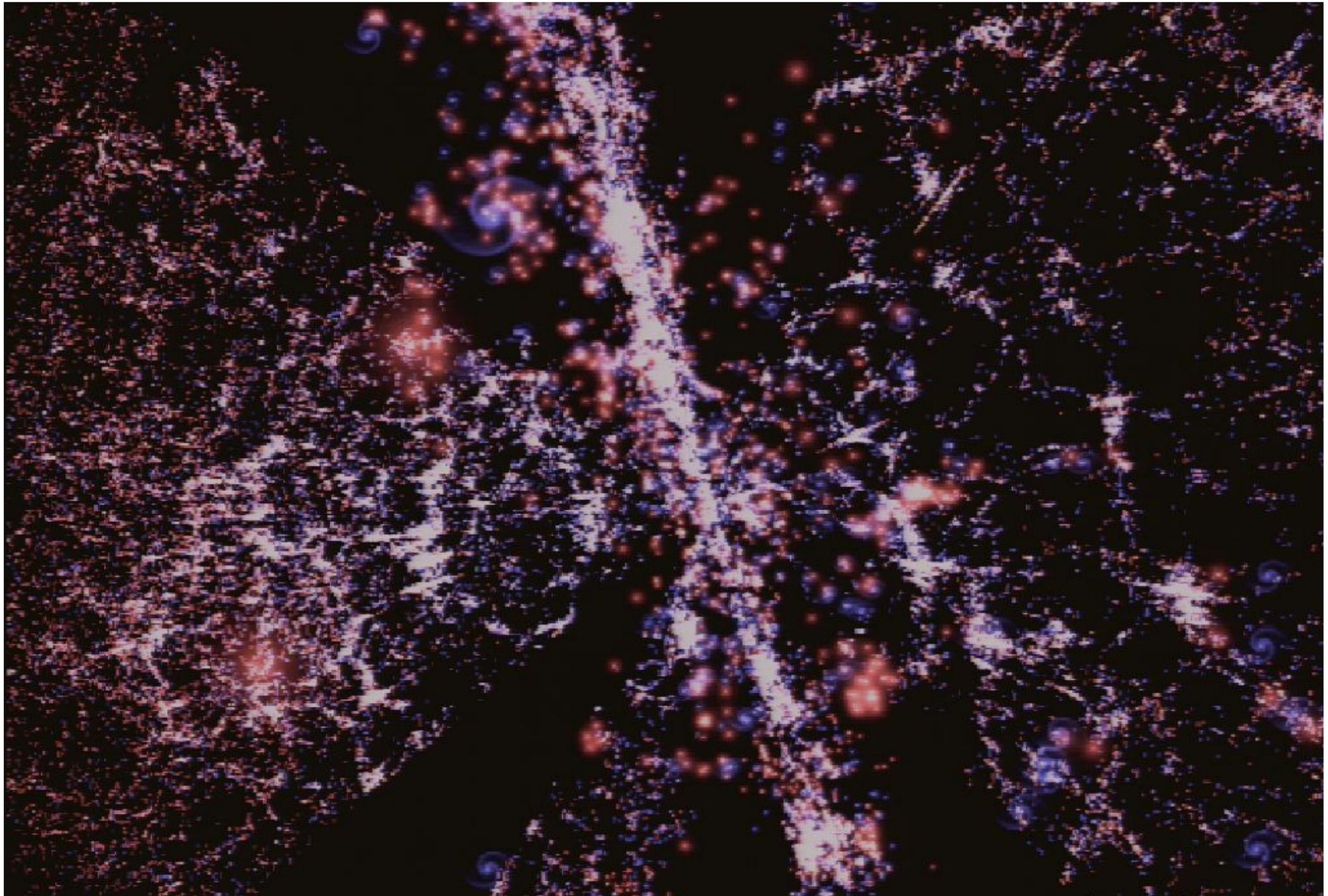
Abstract:

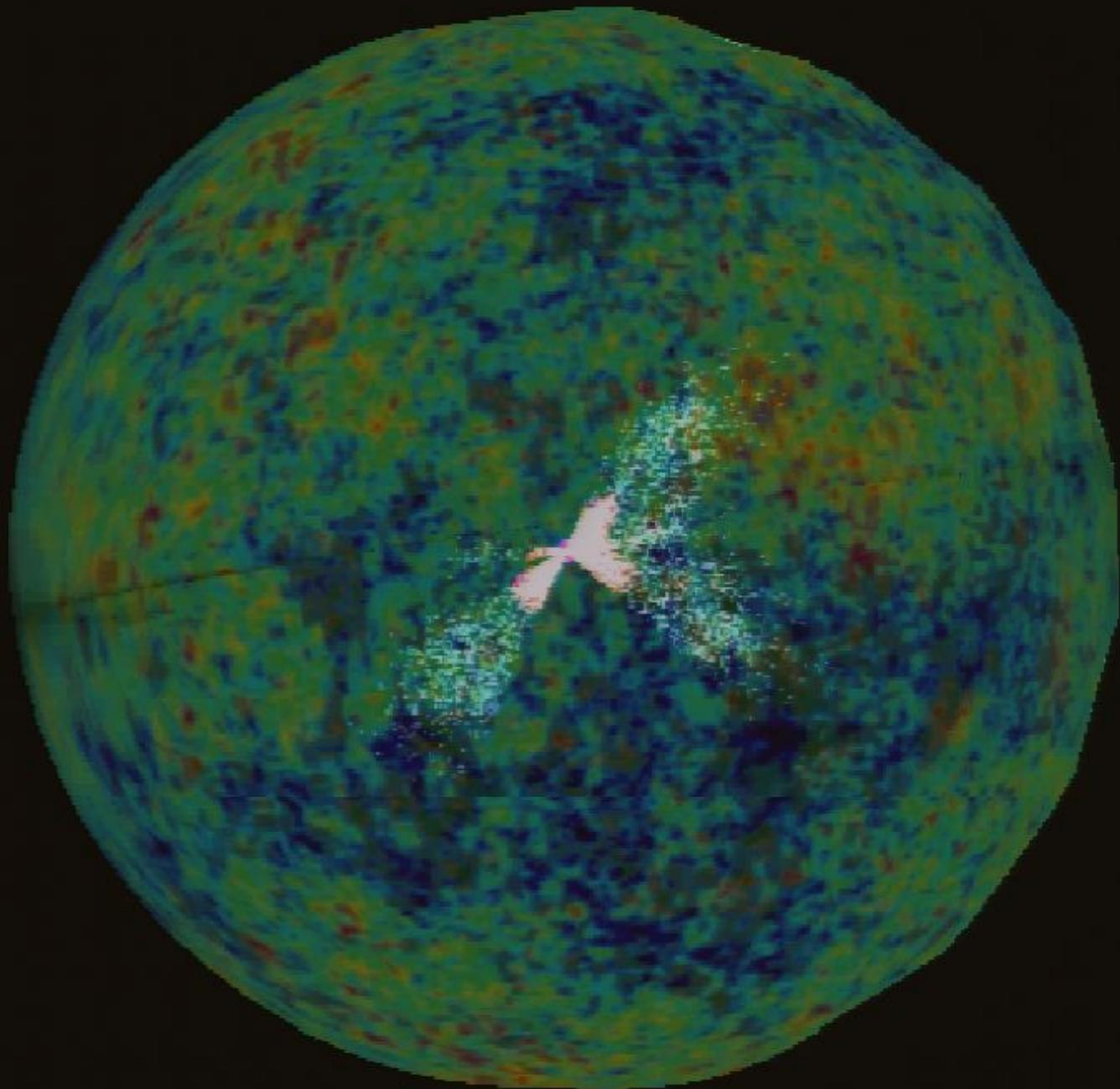
# Inflation after WMAP

V. Mukhanov

Arnold Sommerfeld Center, LMU,  
München



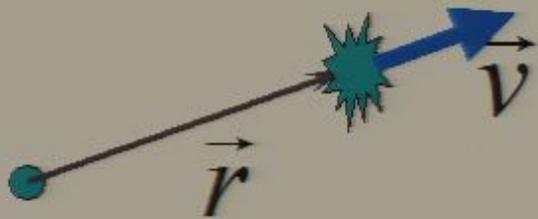




"...our mistake is not that we take our theories too seriously, but that we do not take them seriously enough. It is always hard to realize that these numbers and equations we play with at our desks have something to do with the real world..."

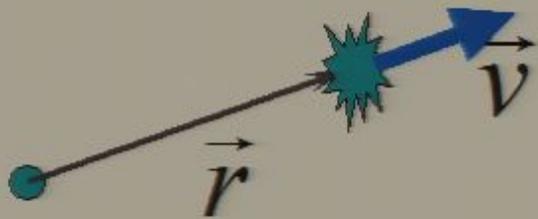
S. Weinberg, "The first three minutes"

● Hubble law

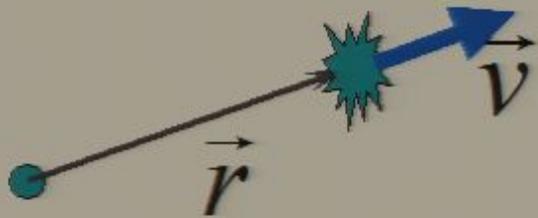


● Hubble law

$$\vec{r} = a(t) \vec{\chi}_{com}$$



● Hubble law



$$\vec{r} = a(t) \vec{\chi}_{com}$$

$$\vec{v} = \dot{a} \vec{\chi}_{com}$$

# ● Matter Distribution

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$t_0 \sim 10^{17}$  sec



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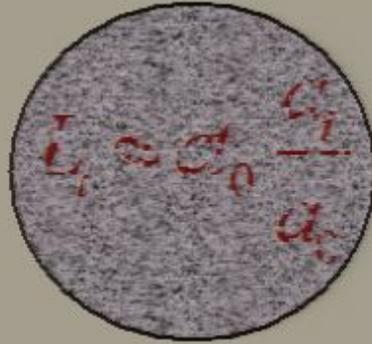
"Initial" moment of time  $t = 10^{-43}$  sec

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"initial" moment of time  $t_i = 10^{-43}$  sec

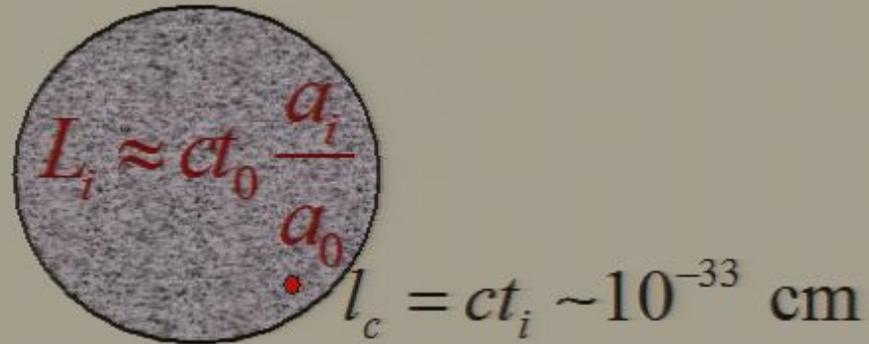
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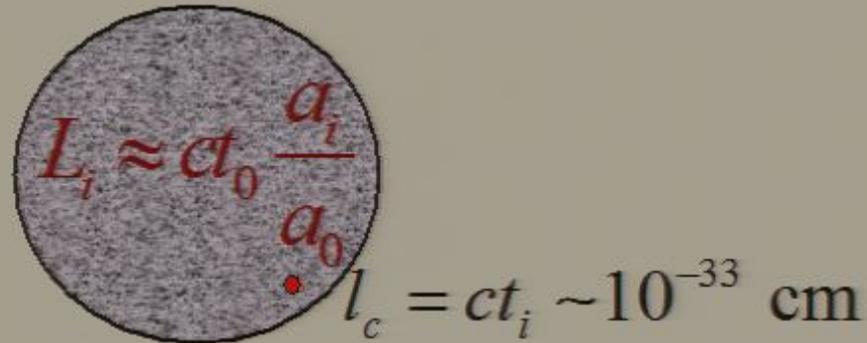
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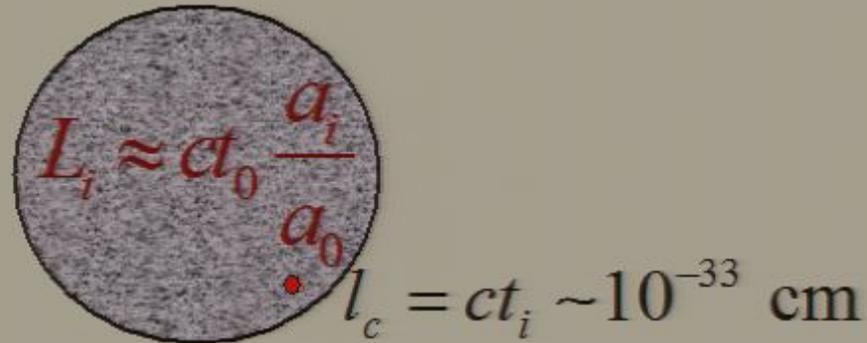
initial rate of expansion

current rate of expansion

$$\frac{a_i}{t_i} \sim a_i'$$

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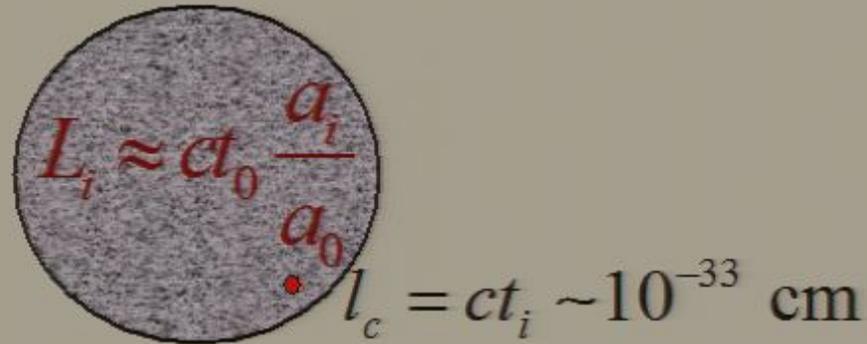
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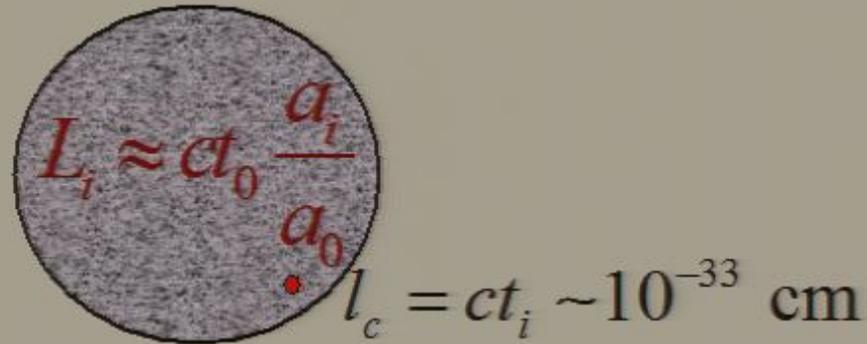
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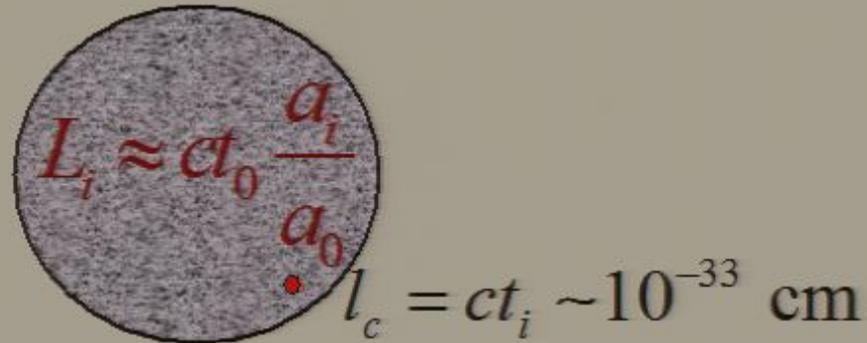
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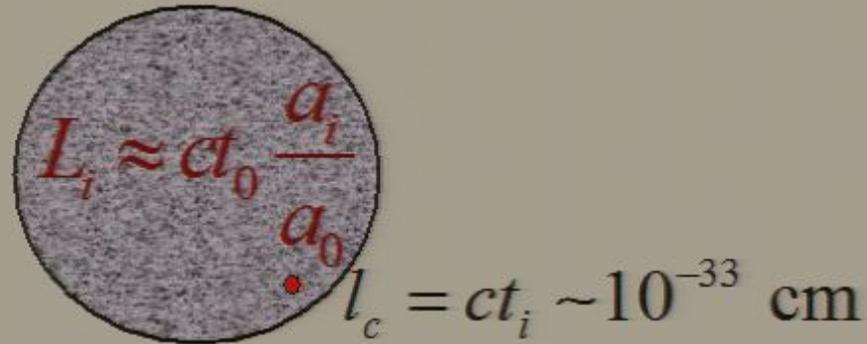
$$\frac{a_i}{t_i} \sim a_i$$

$$a \propto \sqrt{E}$$

$$\begin{aligned} \sqrt{\frac{E_0}{t}} &\sim 10^{17} \\ \frac{PC}{t} &\sim 10^{50} \end{aligned}$$

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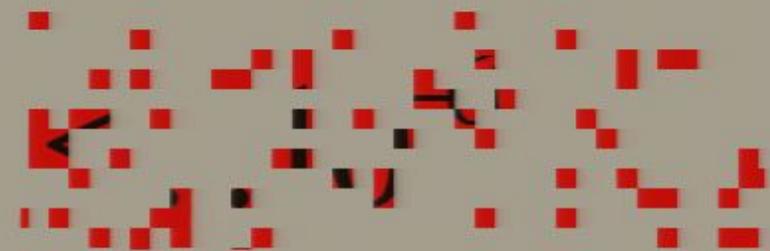
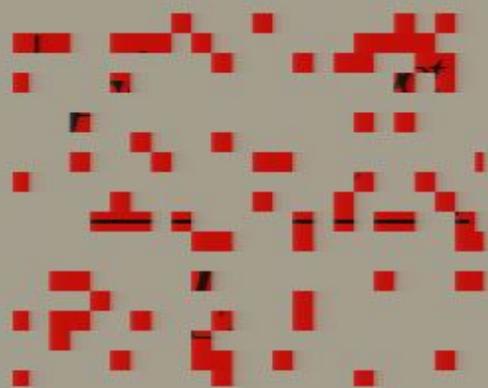
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# ● Initial velocities

$$\mathbf{v}^i = \mathbf{v}^i + \mathbf{v}^i \quad (v^i = 1 \text{ or } 0)$$



## ● Initial velocities

- At "initial moment" ( $t_i \approx 10^{-43}$  sec)

$$\frac{|E_i^{kin} + E_i^{pot}|}{E_i^{kin}} \leq \dots \left( \frac{\dot{a}_0}{\dot{a}_i} \right)^2 \leq \dots 10^{-60}$$

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$$a \propto \sqrt{t}$$

$$\frac{t_0 \sim 10^{17}}{t \sim 10^{30}} \sim 10^{-13}$$

$PC \sim 10^{45} \text{ s}$

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Flatness ( $\equiv$  initial velocities) problem

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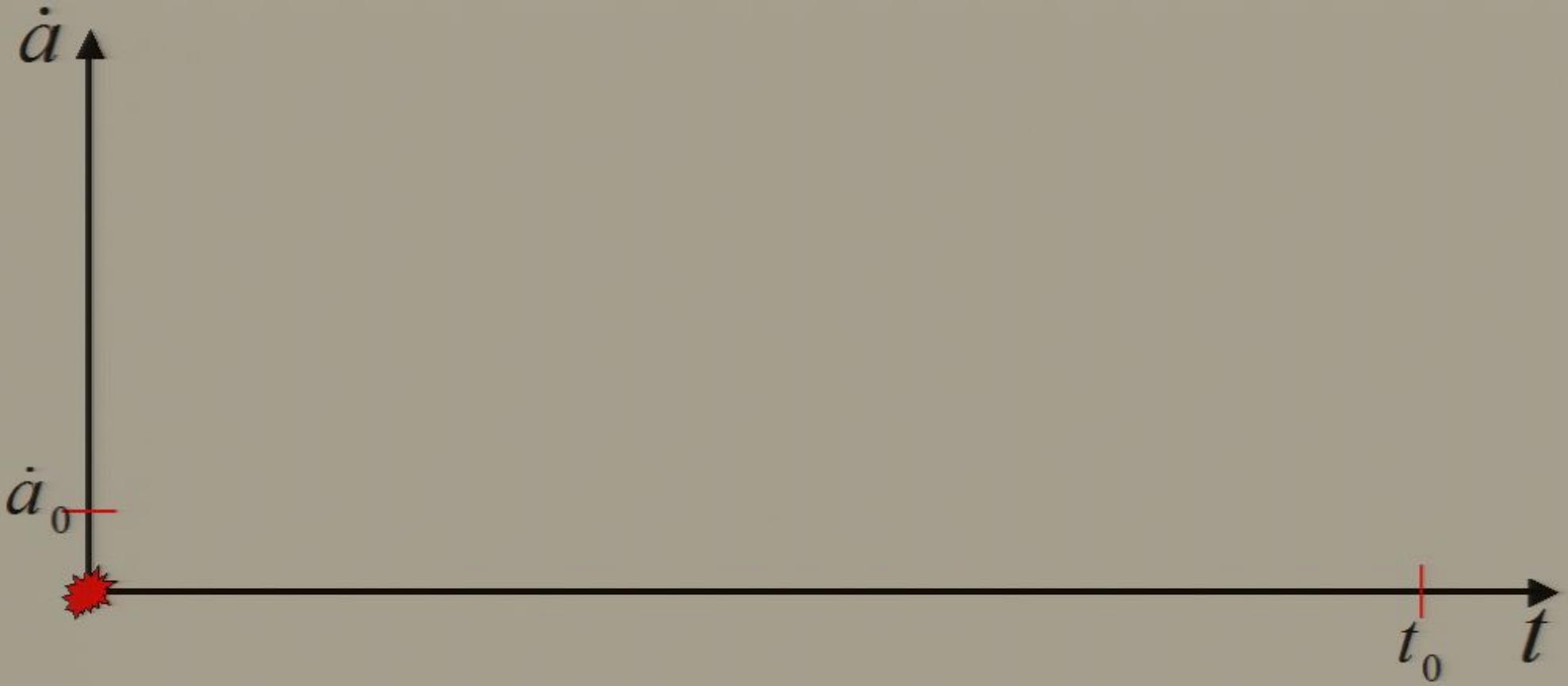
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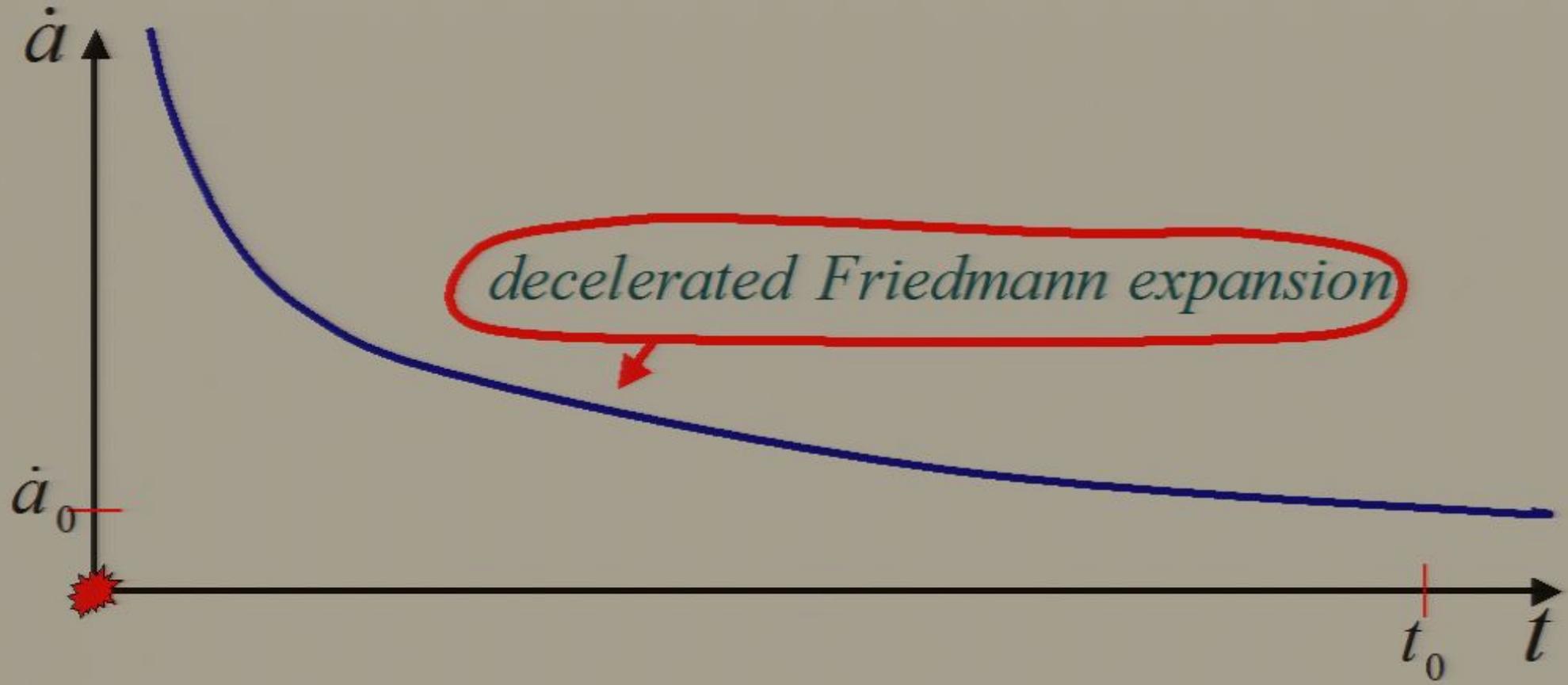
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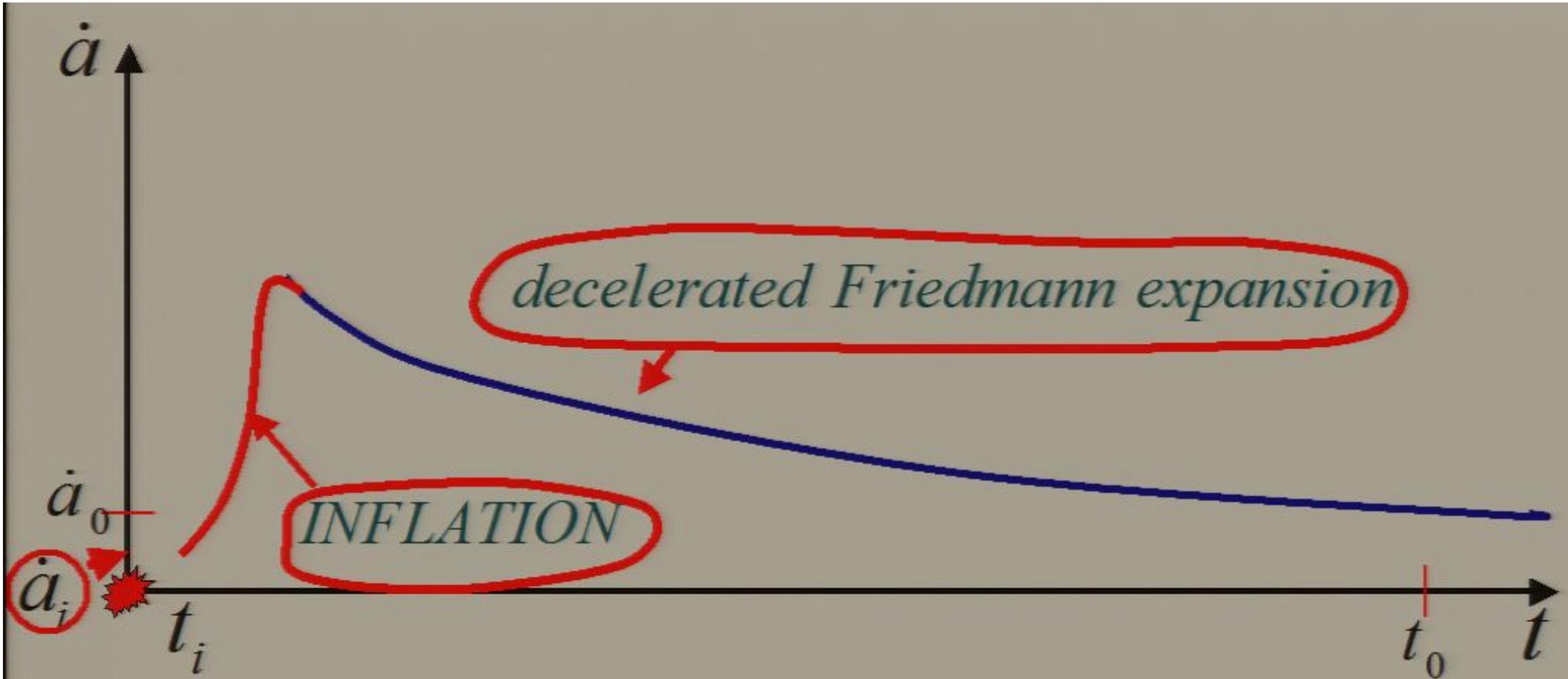


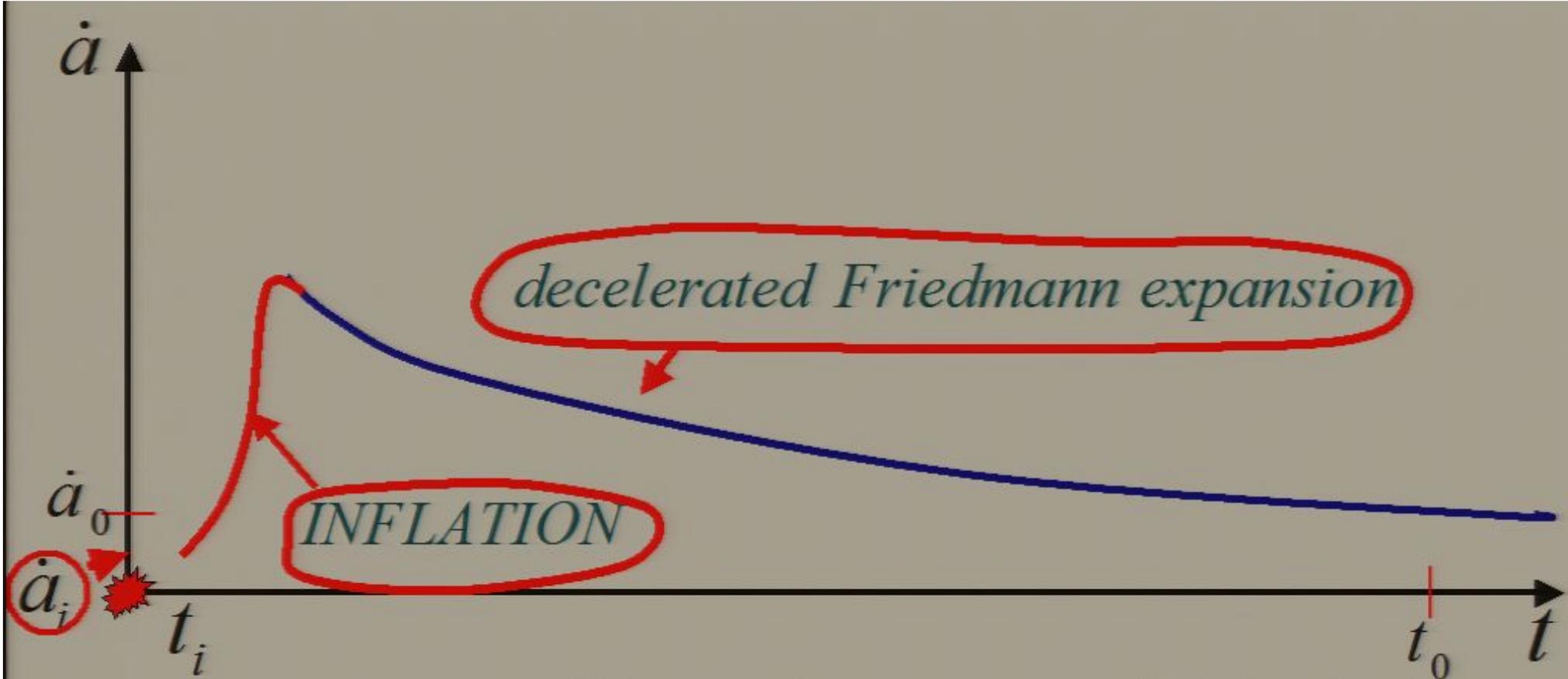
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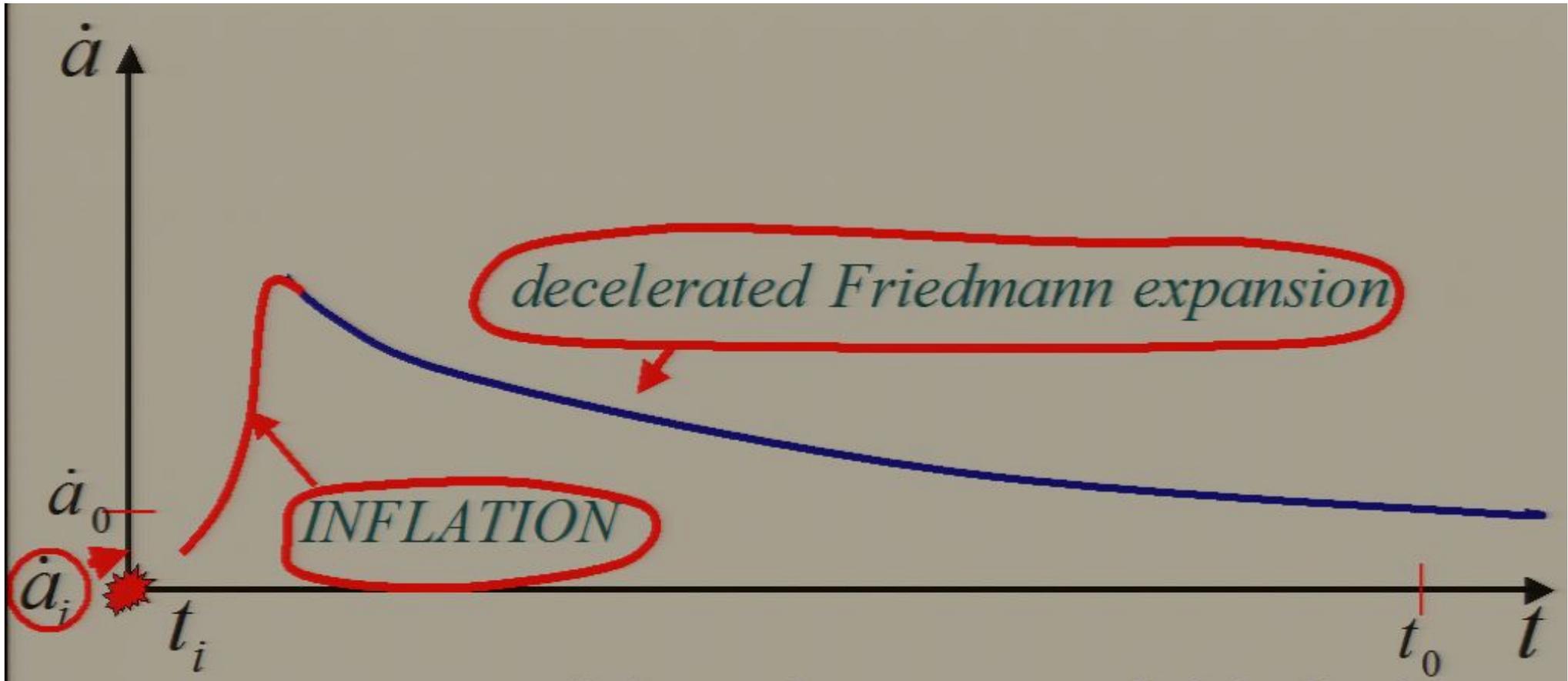




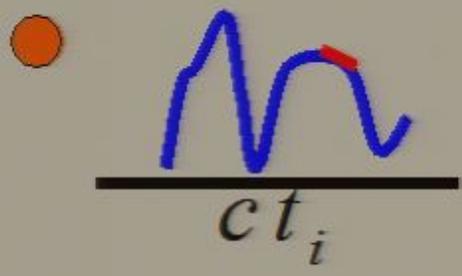


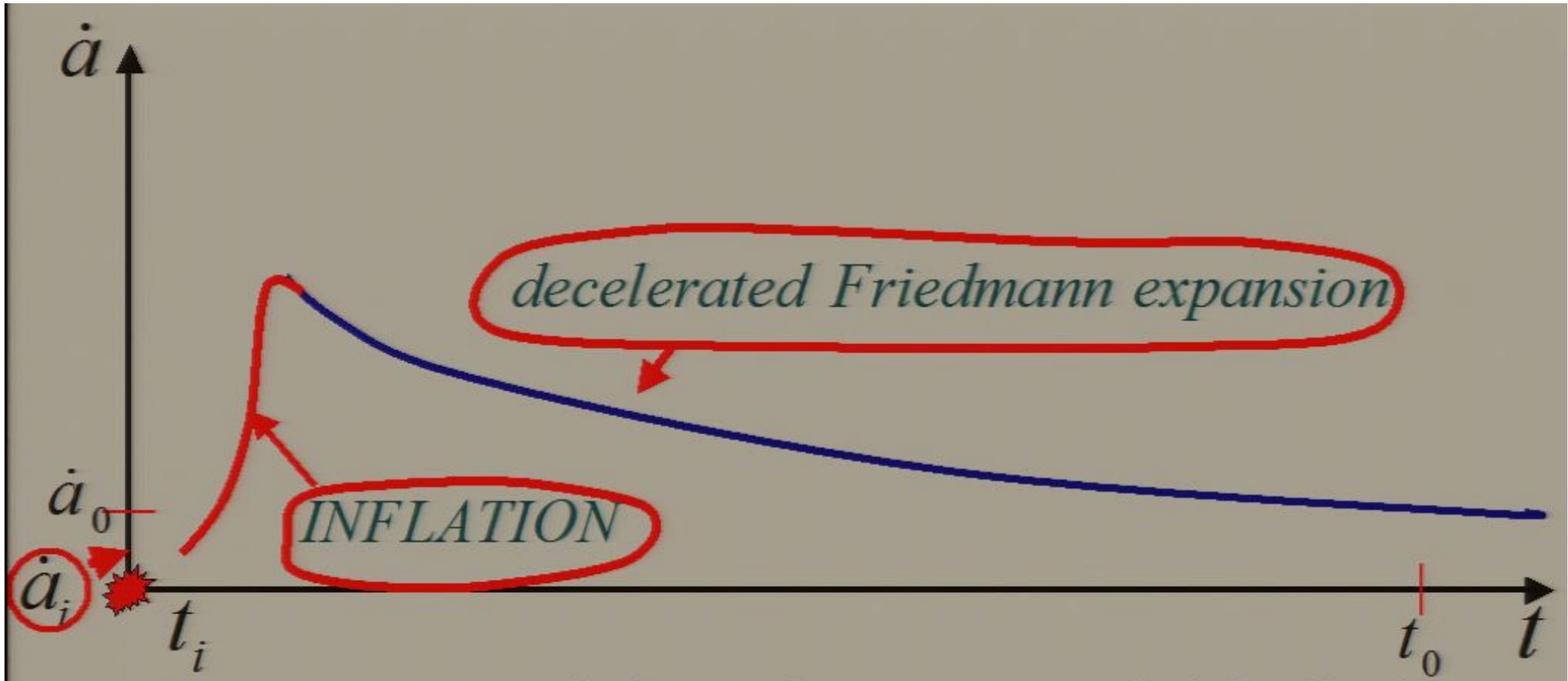


Necessary conditions for successful inflation:



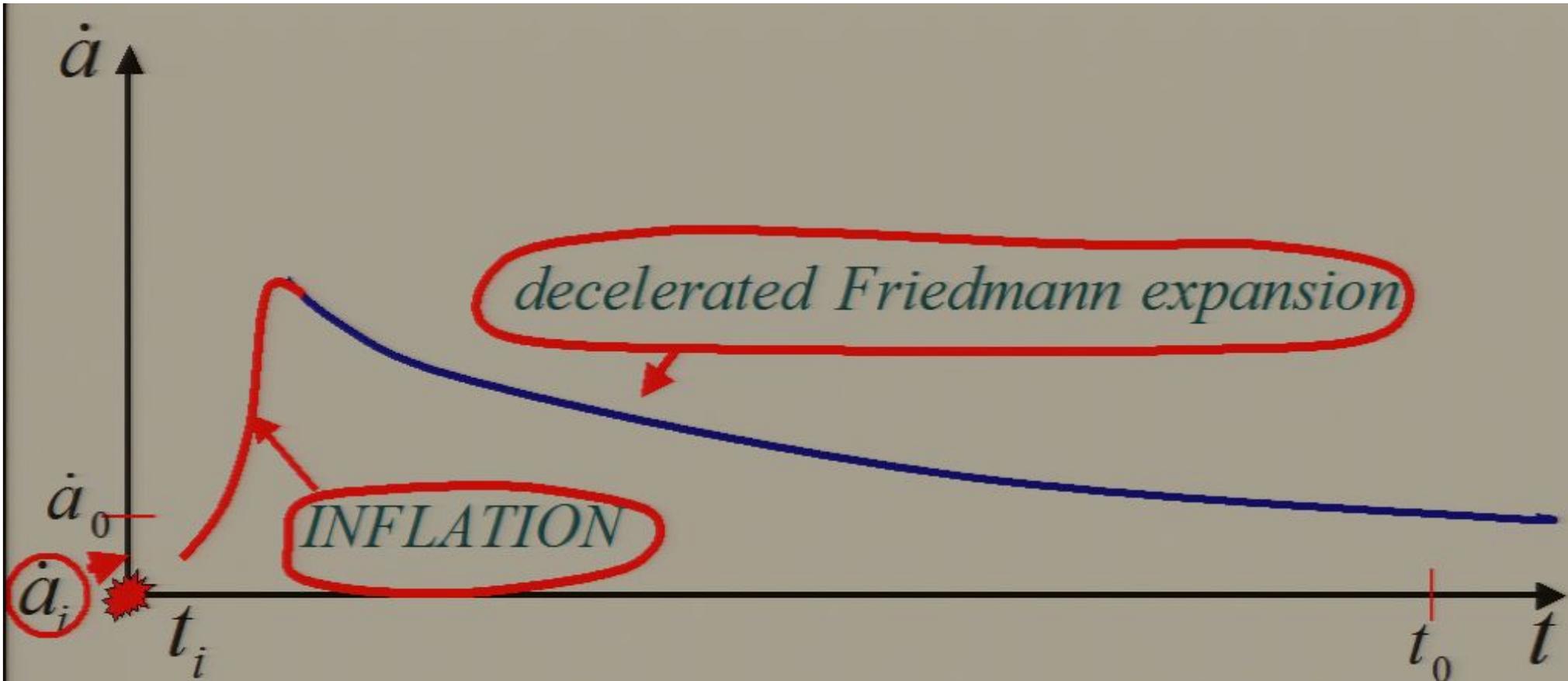
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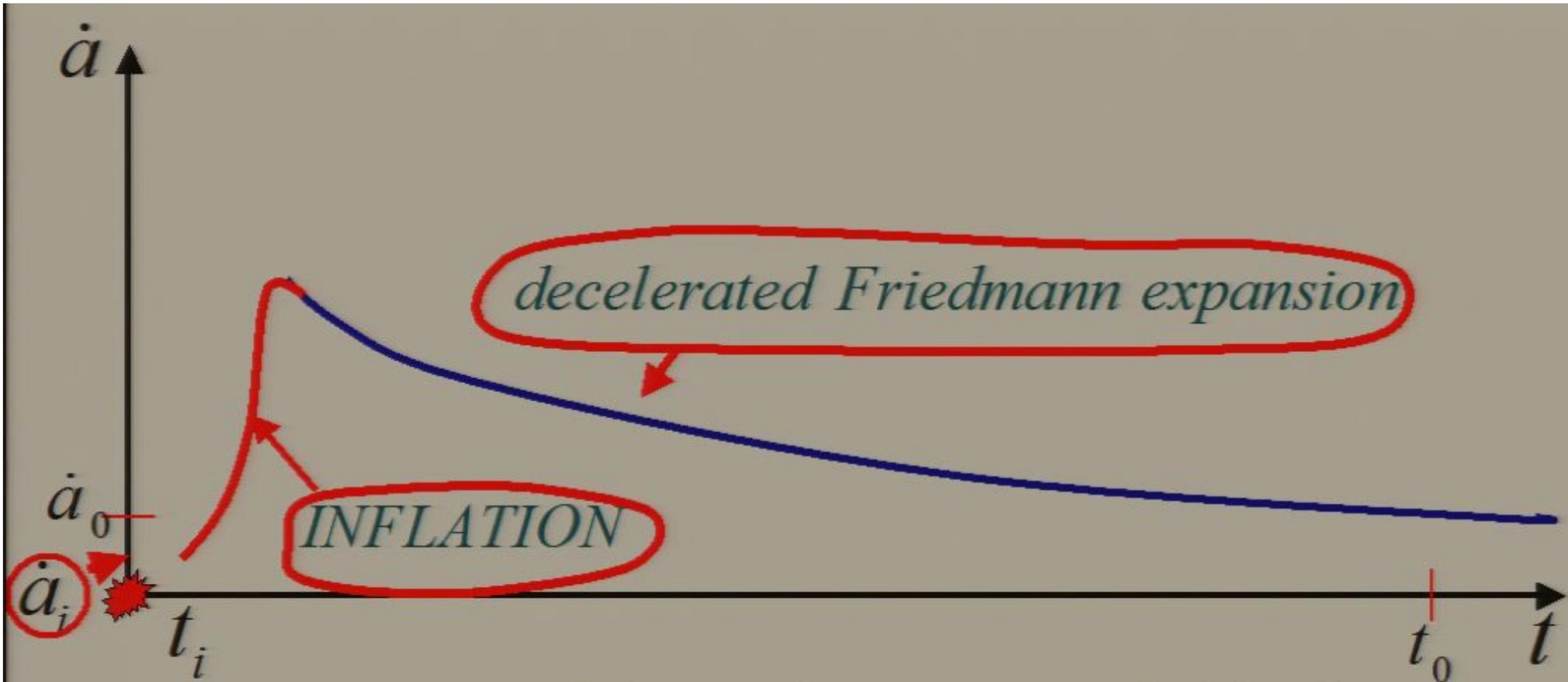
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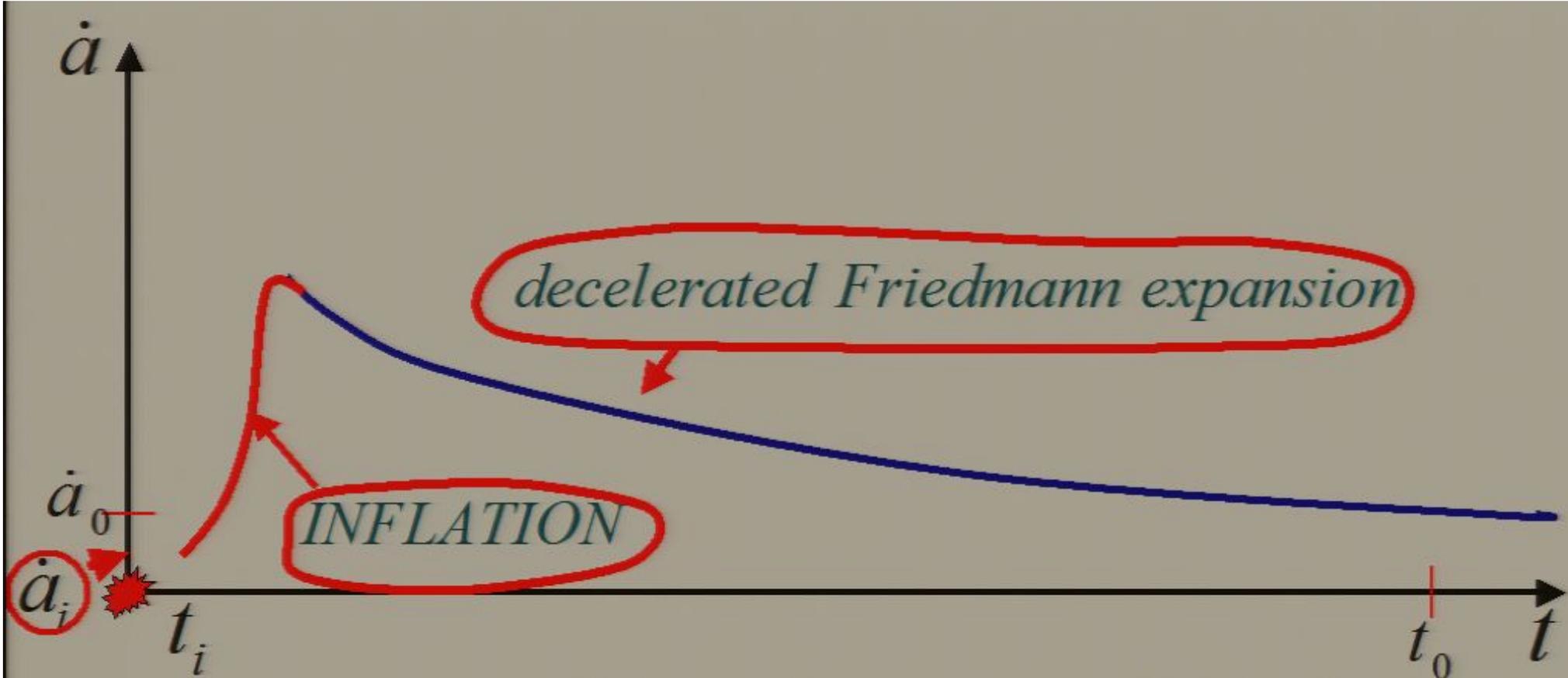
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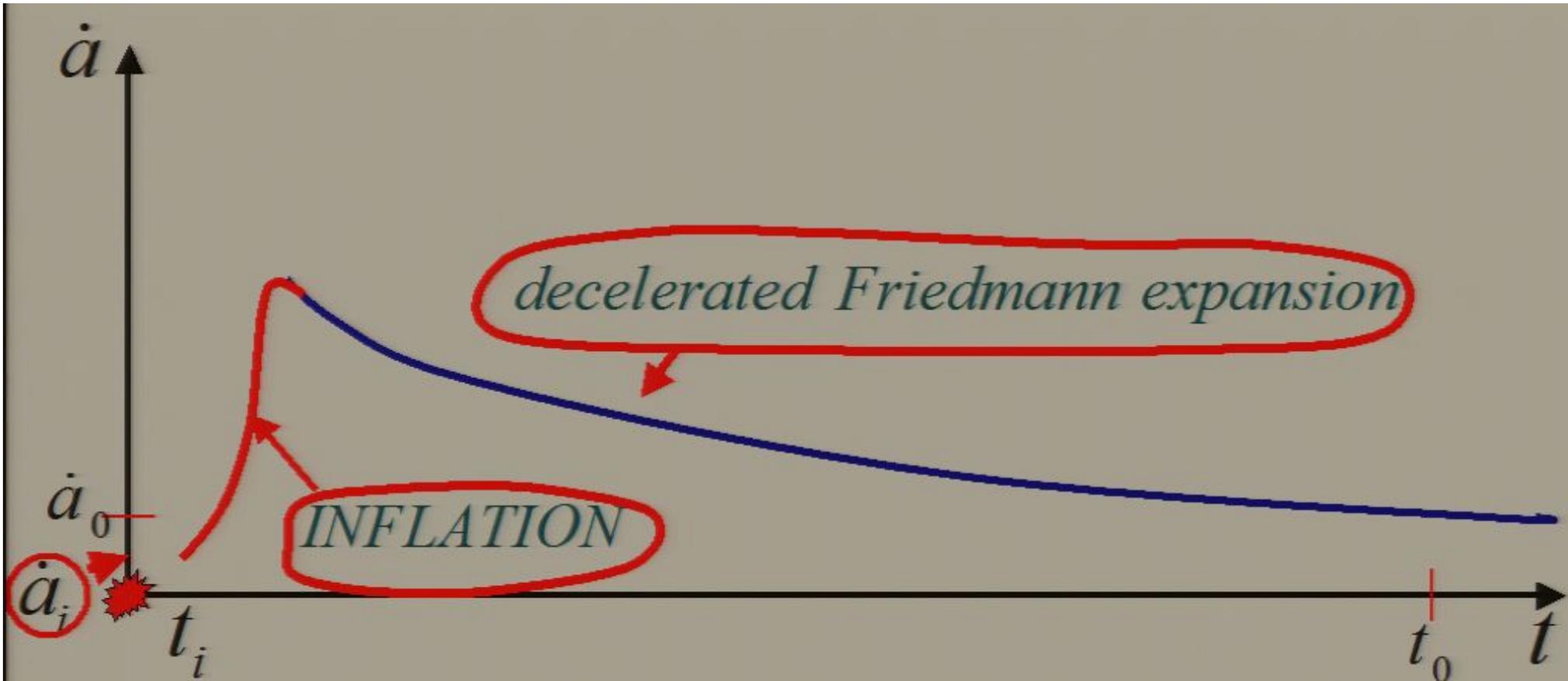
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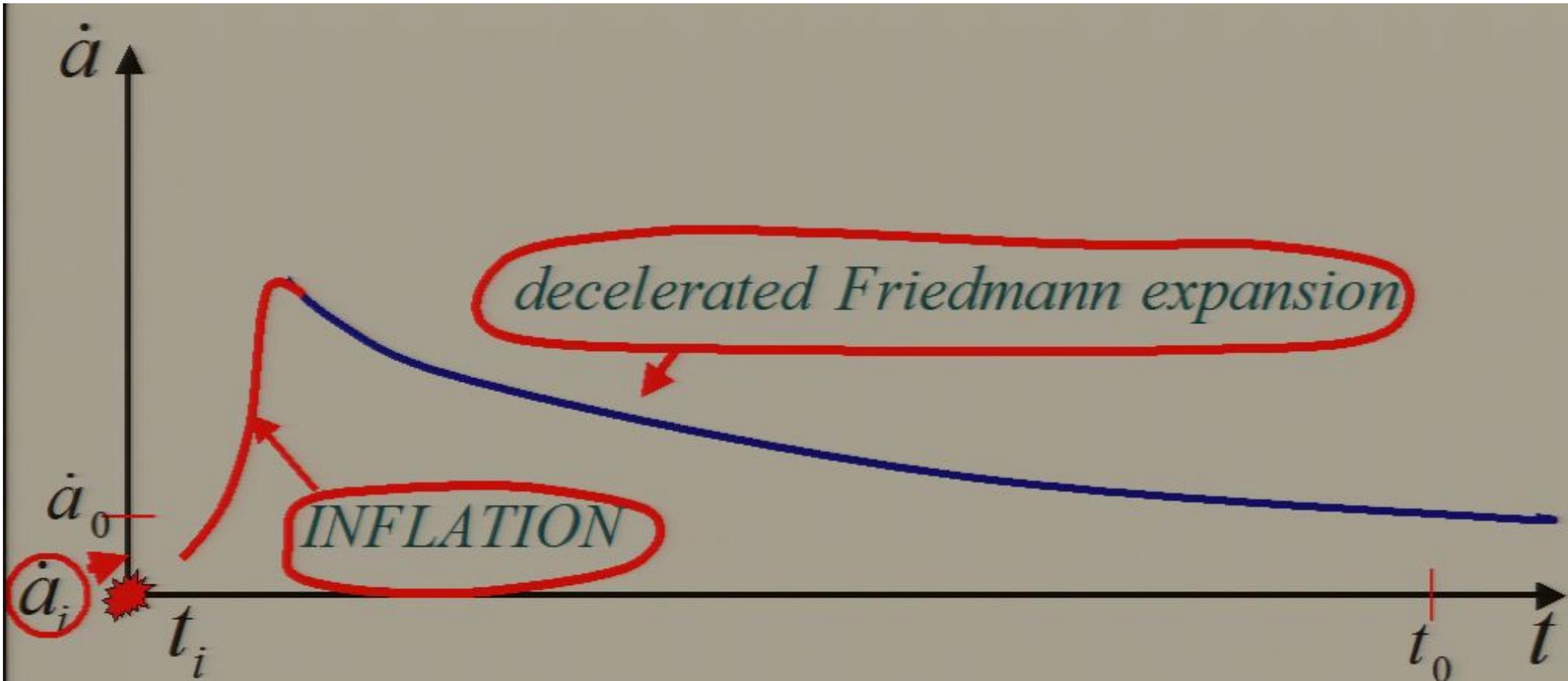
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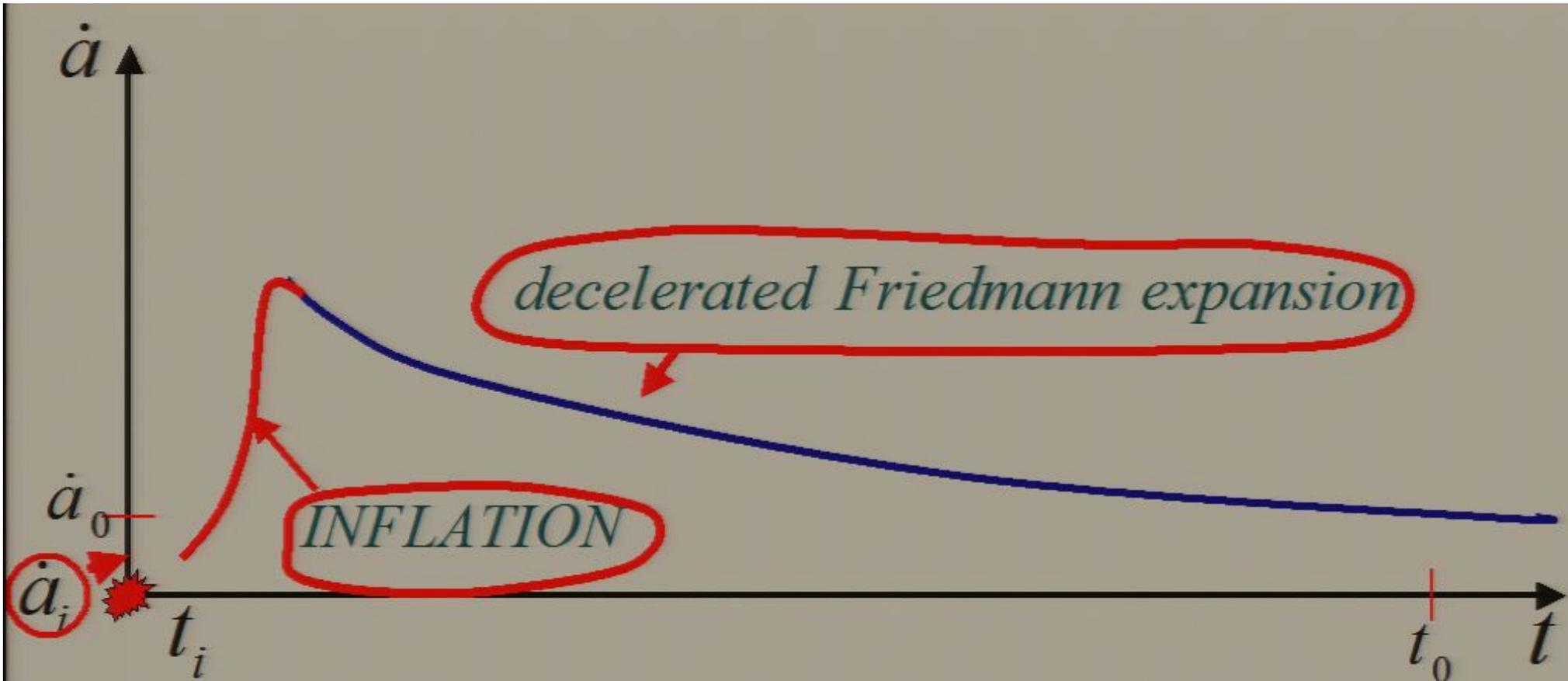
Necessary conditions for successful inflation:

●  $\dot{a}_i \ll \dot{a}_0 \rightarrow \Omega_0 \equiv \frac{|E_0^{pot}|}{E_0^{kin}} = 1 + O(1) \left( \frac{\dot{a}_i}{\dot{a}_0} \right)^2$



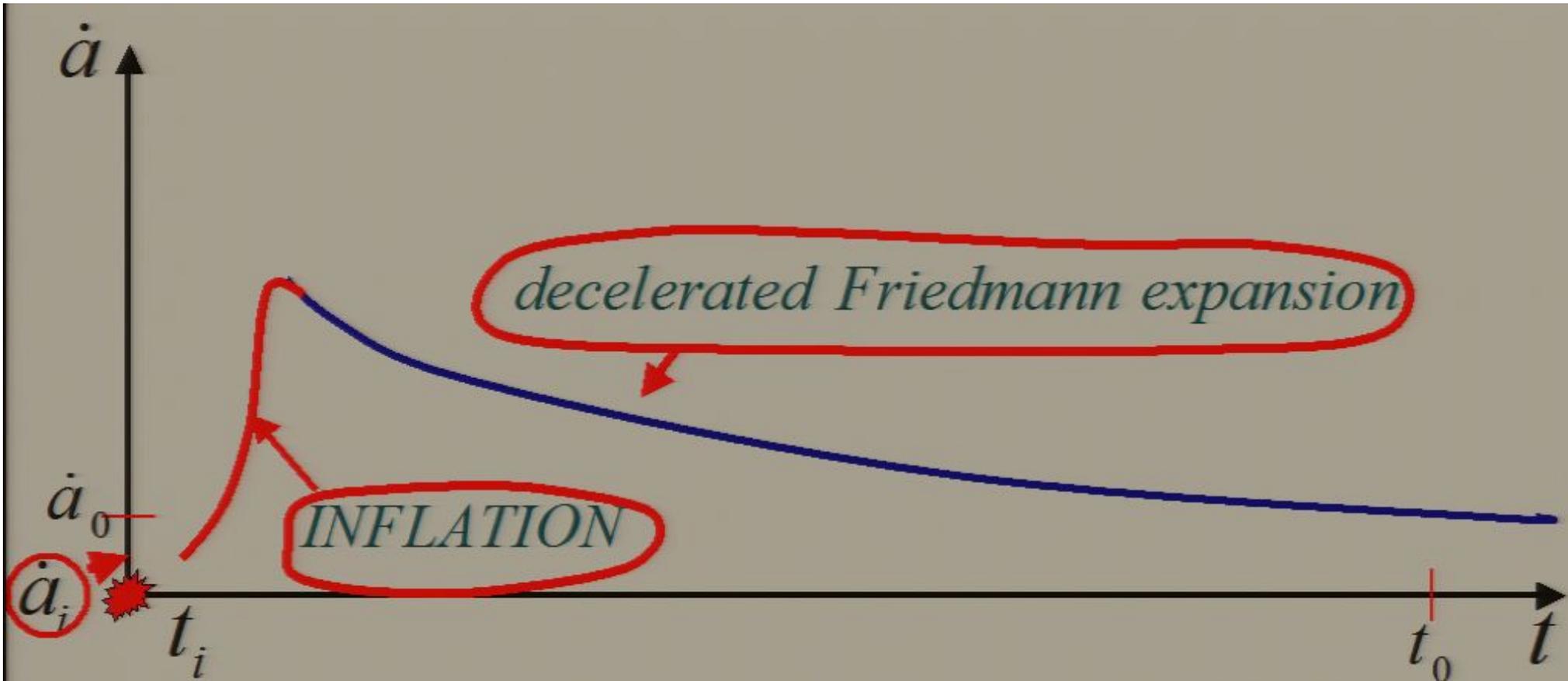
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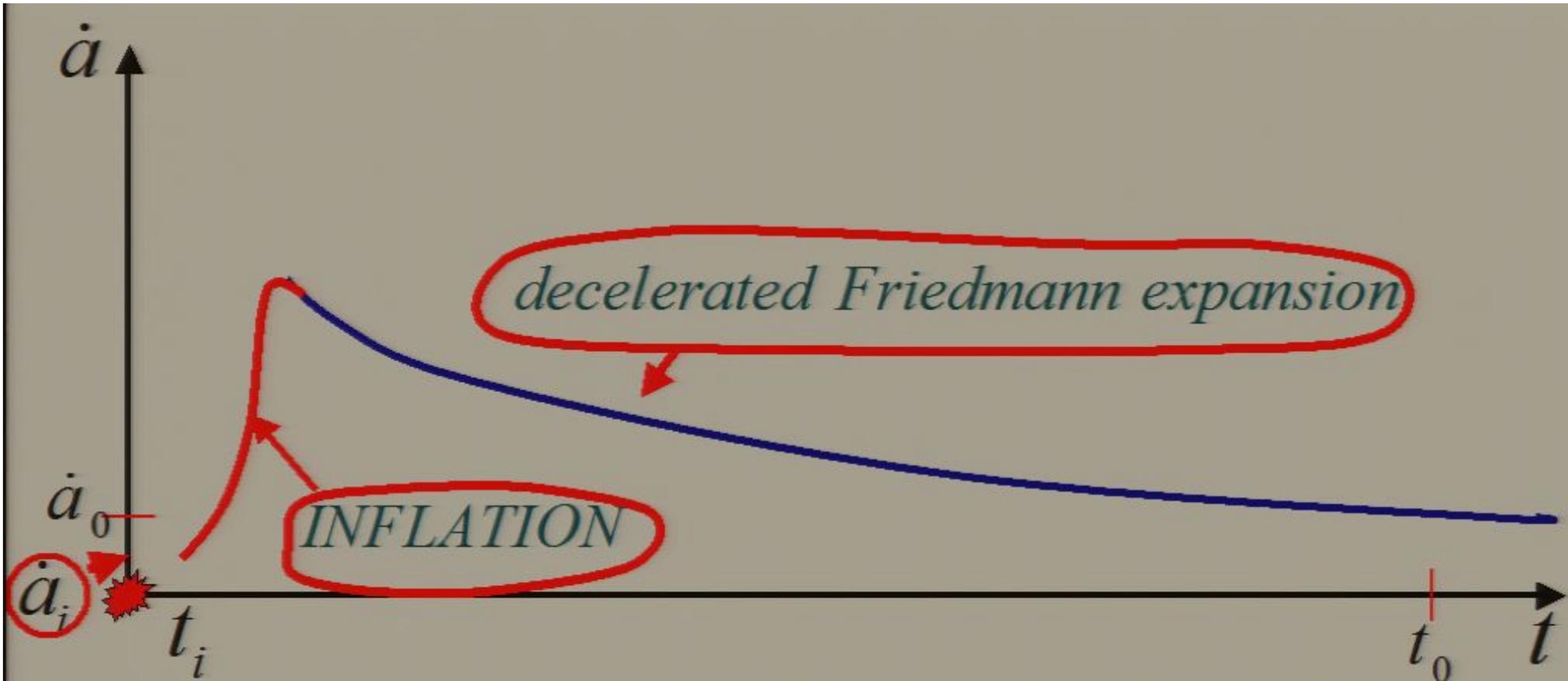
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$\bullet \dot{a}_i \ll \dot{a}_0 \longrightarrow \Omega_0 \equiv \frac{|E_0^{pot}|}{E_0^{kin}} = 1$

Prediction of inflation!



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- How gravity can become "repulsive"?

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$$\ddot{a} = -\frac{4\pi G}{3}\epsilon a$$

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$$\ddot{a} = -\frac{4\pi G}{3} (\varepsilon + 3p) a$$

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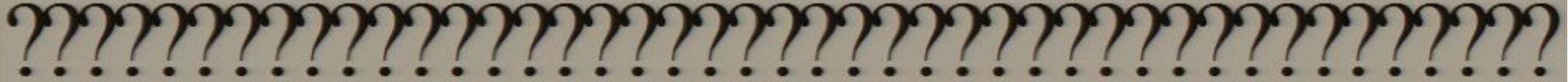
$$\underset{\text{acceleration}}{\ddot{a}} = - \frac{4\pi G}{3} \left( \underset{\substack{\text{energy} \\ \text{density}}}{\varepsilon} + 3 \underset{\text{pressure}}{p} \right) a$$

Only if  $\varepsilon + 3p < 0 \Rightarrow \ddot{a} > 0 \equiv$  "antigravity"

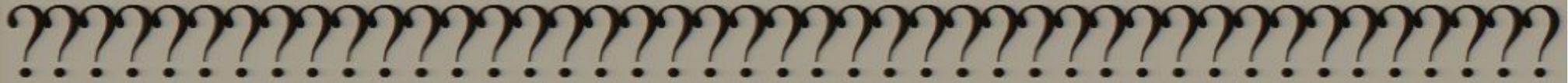


# Scenarios

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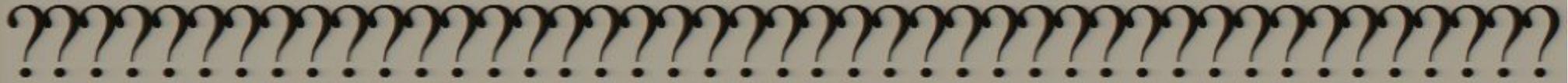


# Scenarios



Energy density  $\varepsilon$ , pressure  $p$

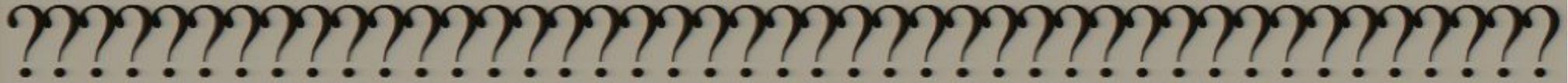
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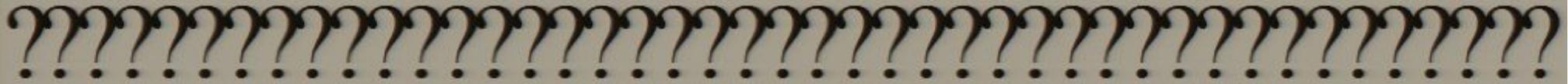


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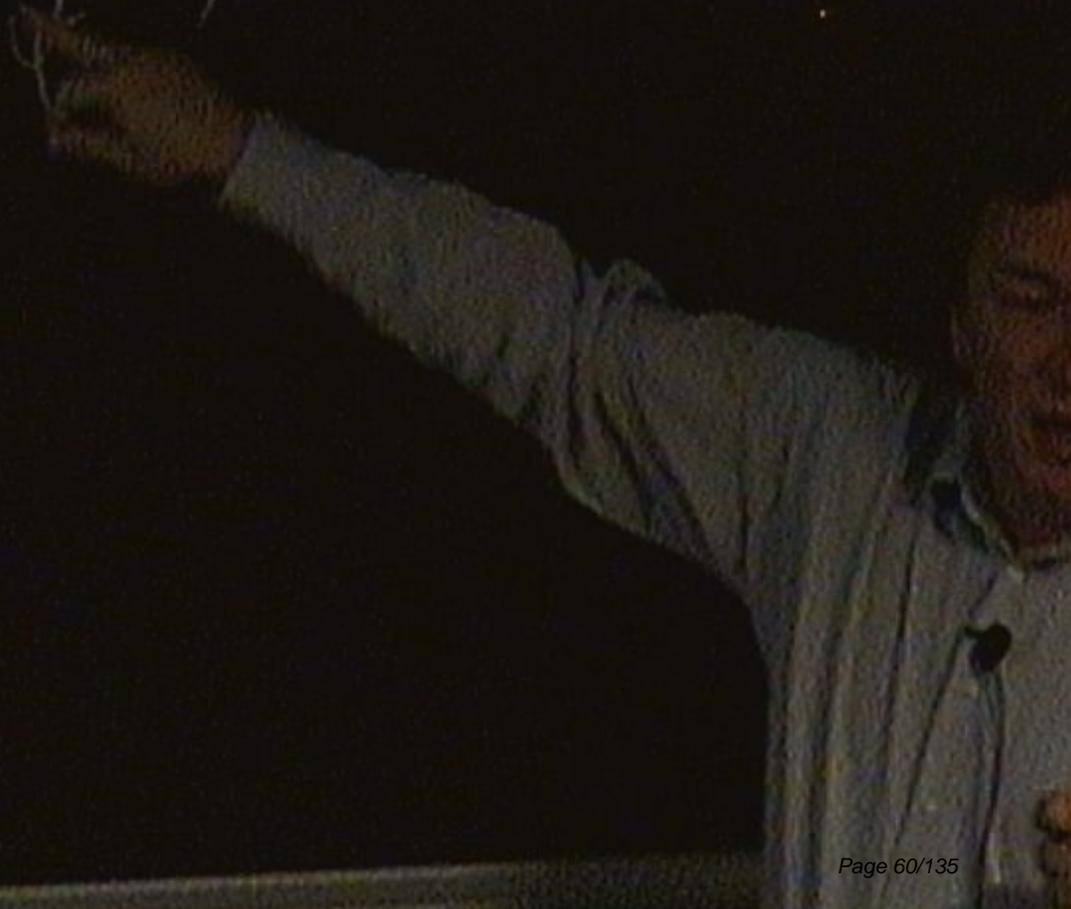
$p + \varepsilon \ll \varepsilon$  for inflation



$$p \approx -\varepsilon$$

Which concrete scenario was realized ???

$$\int P(x, \varphi) \sqrt{-g} d^4x$$



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**Example:** Quantum metric fluctuations in Minkowskii space

$$h_{\lambda} \approx \frac{l_{Pl}}{\lambda} \approx \frac{10^{-33} \text{ cm}}{\lambda}$$

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Can quantum fluctuations be amplified up to

"needed" value  $10^{-5}$  in expanding Universe???



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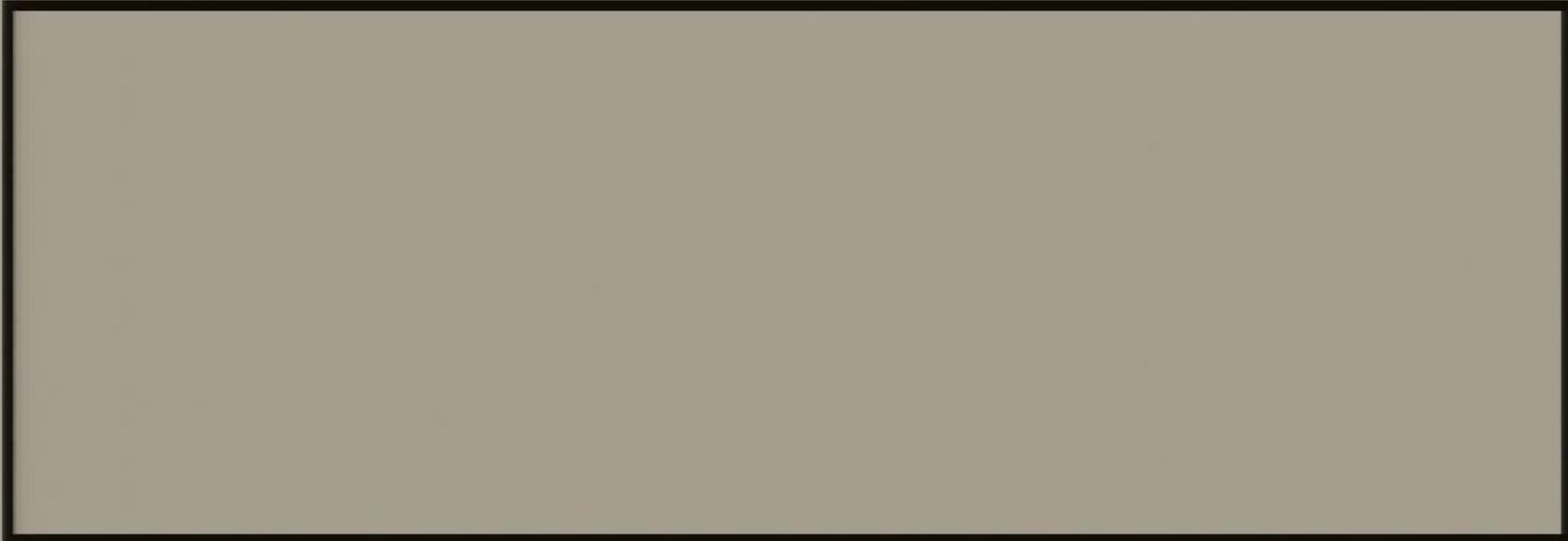
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For given  $k_{com}$ ,  $\lambda_{ph}(\text{cm}) \propto a/k_{com} \propto a(t)$  and the change of the amplitude with time depends on how big is  $\lambda_{phys}$

compared to the curvature scale (size of Einstein lift)  $H^{-1} = a/\dot{a}$

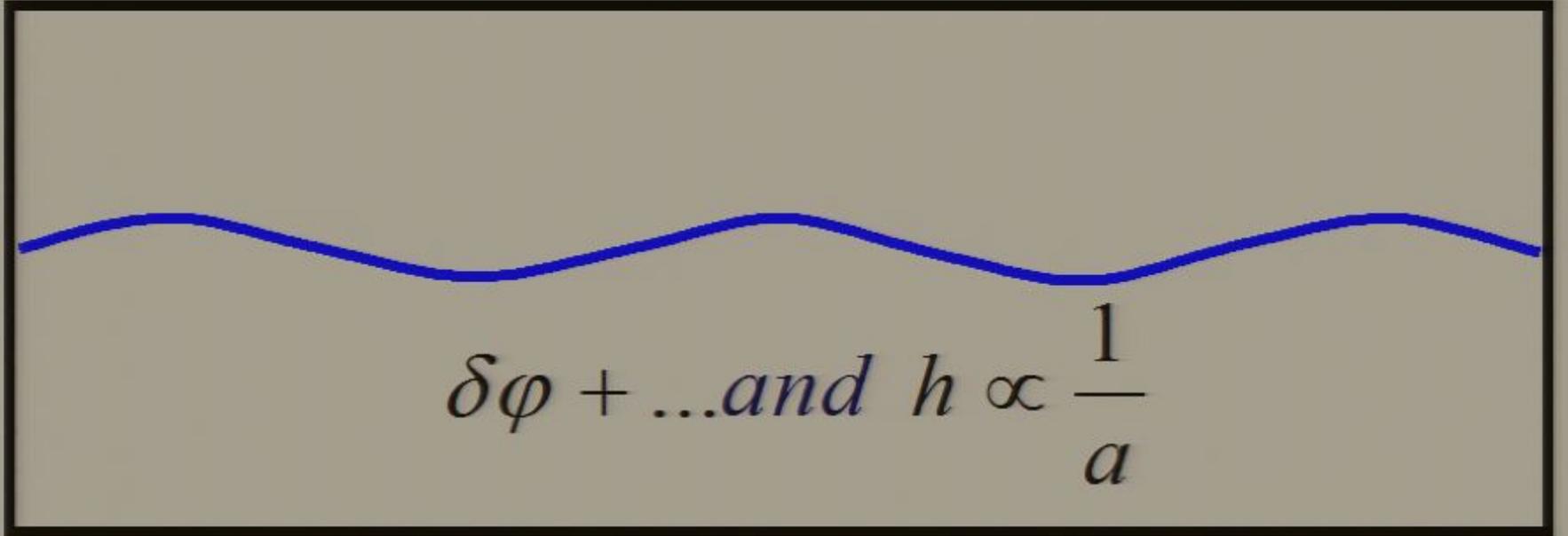




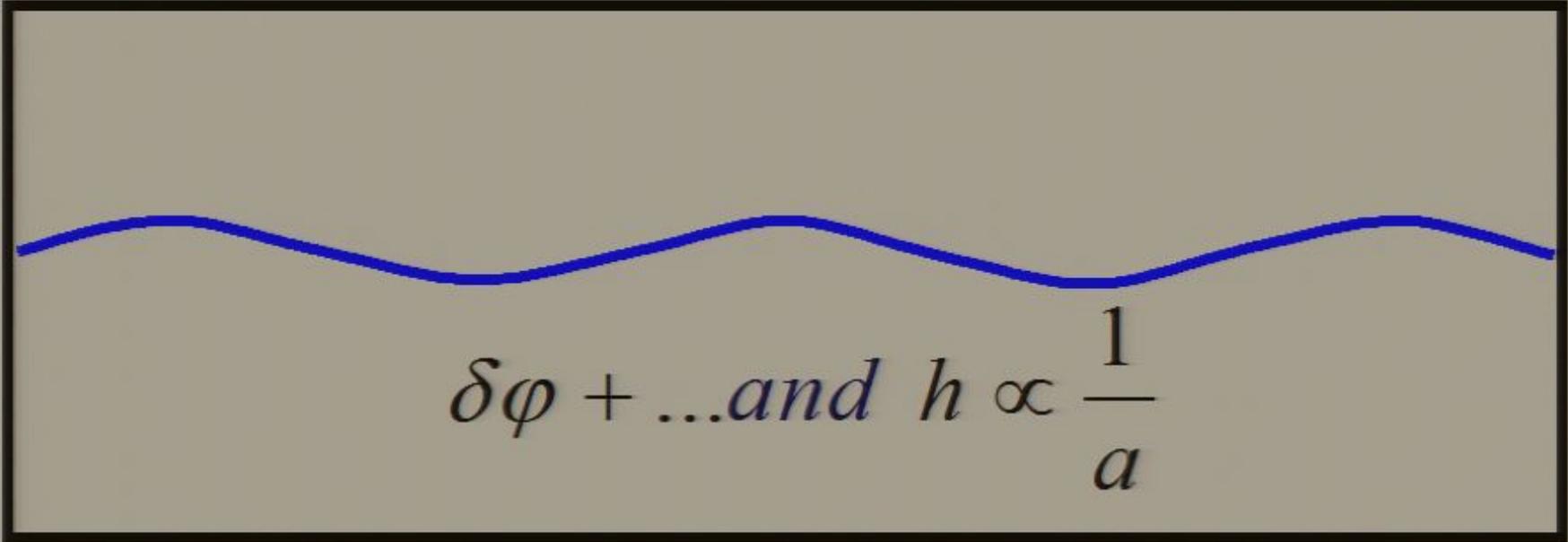
$$H^{-1}$$



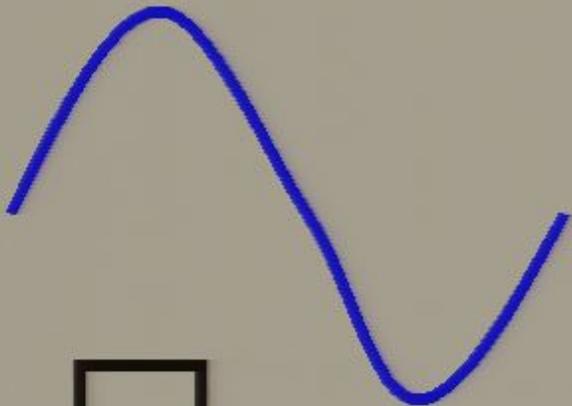
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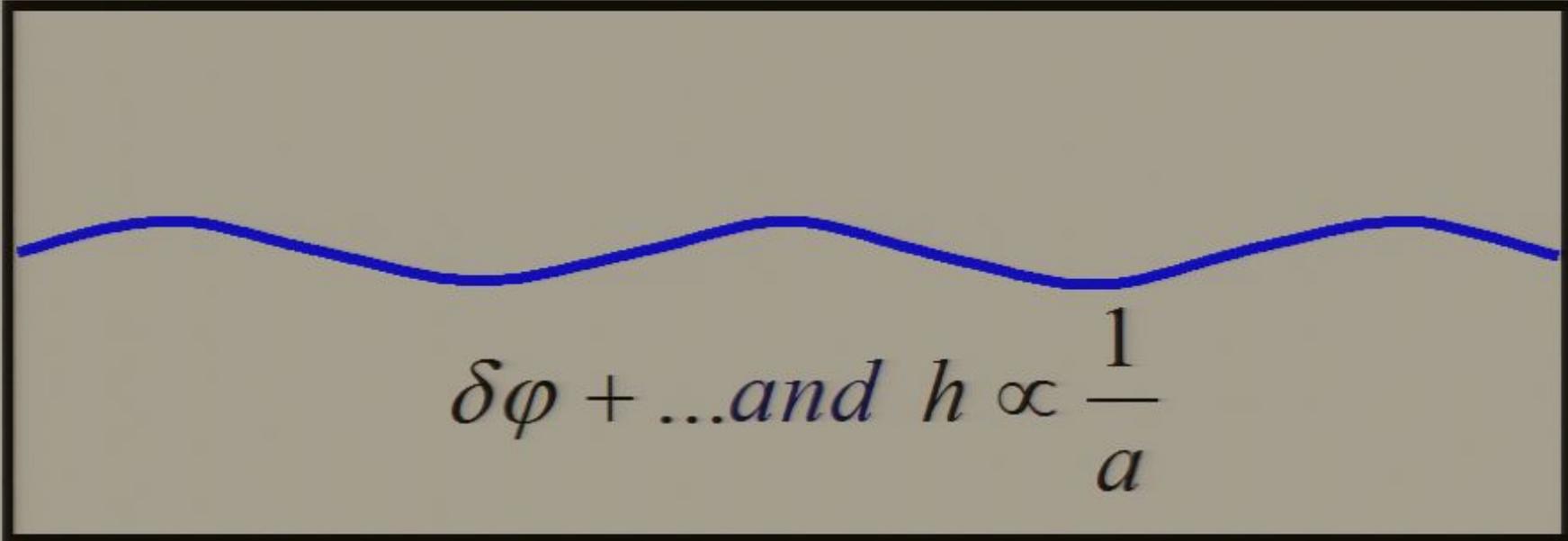
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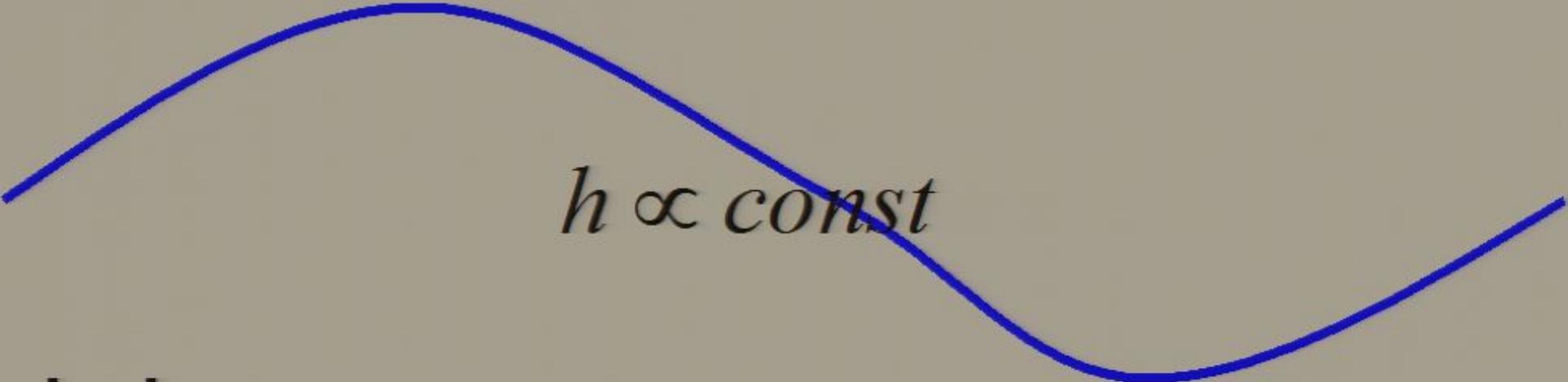
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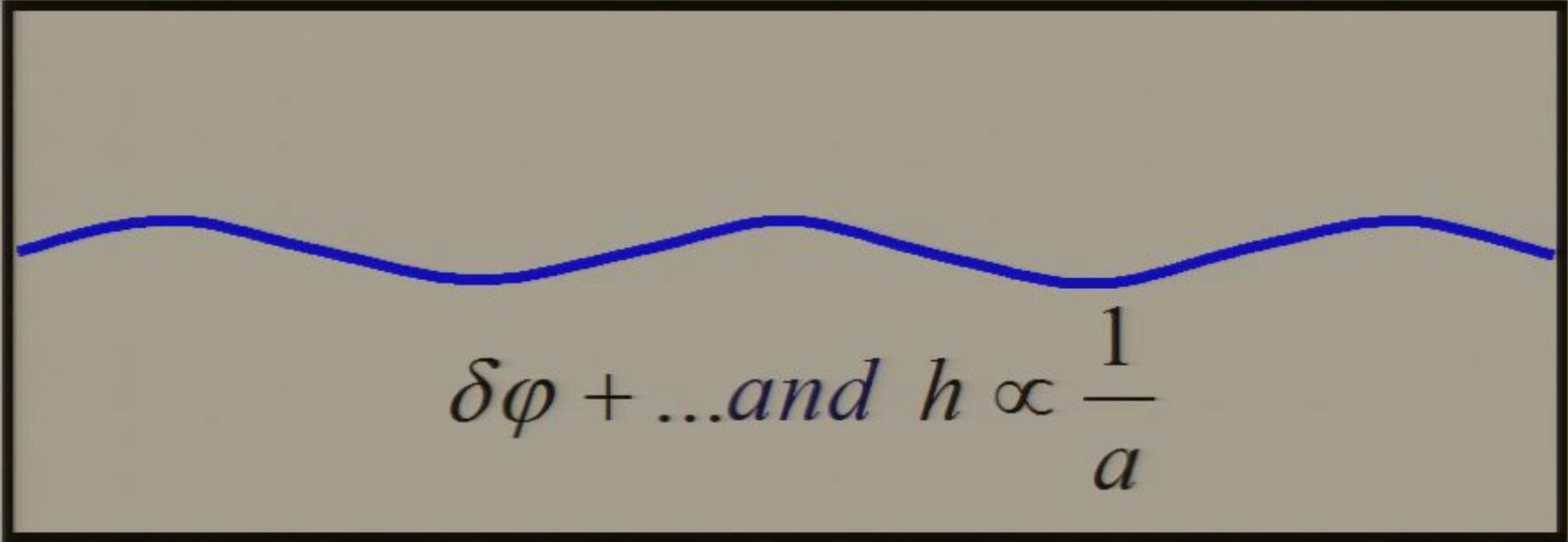


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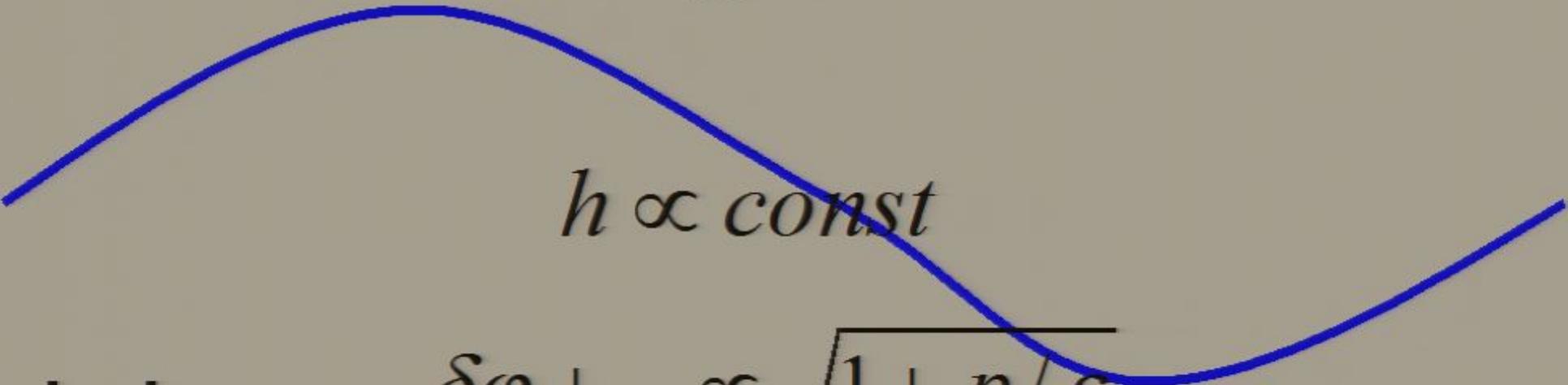


$H^{-1}$





$$H^{-1}$$



$$\delta\varphi + \dots \propto \sqrt{1 + p/\varepsilon}$$

*Scale*

# *Galactic scales*

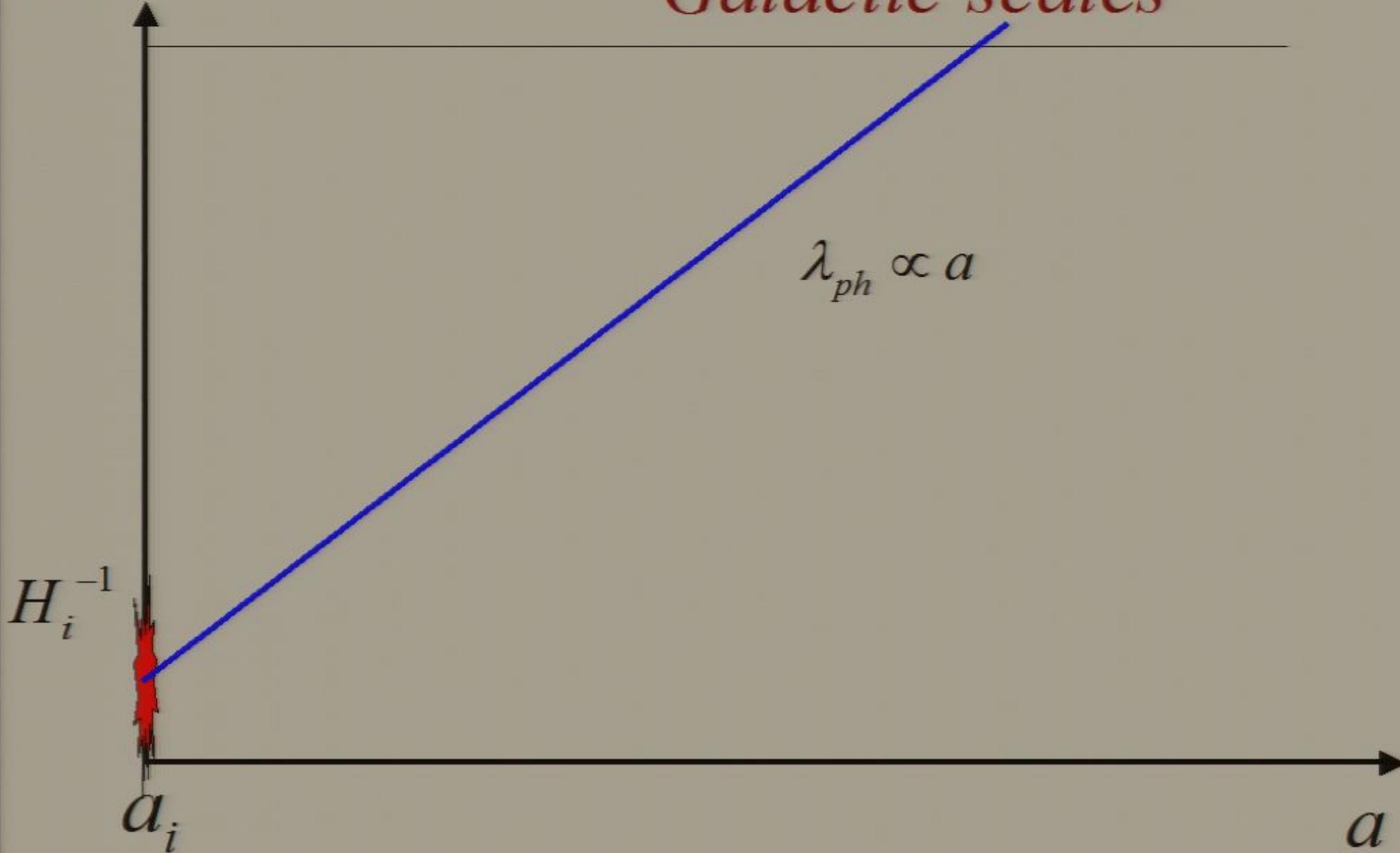
$H_i^{-1}$

$a_i$

$a$

Scale

# Galactic scales



Scale

# Galactic scales

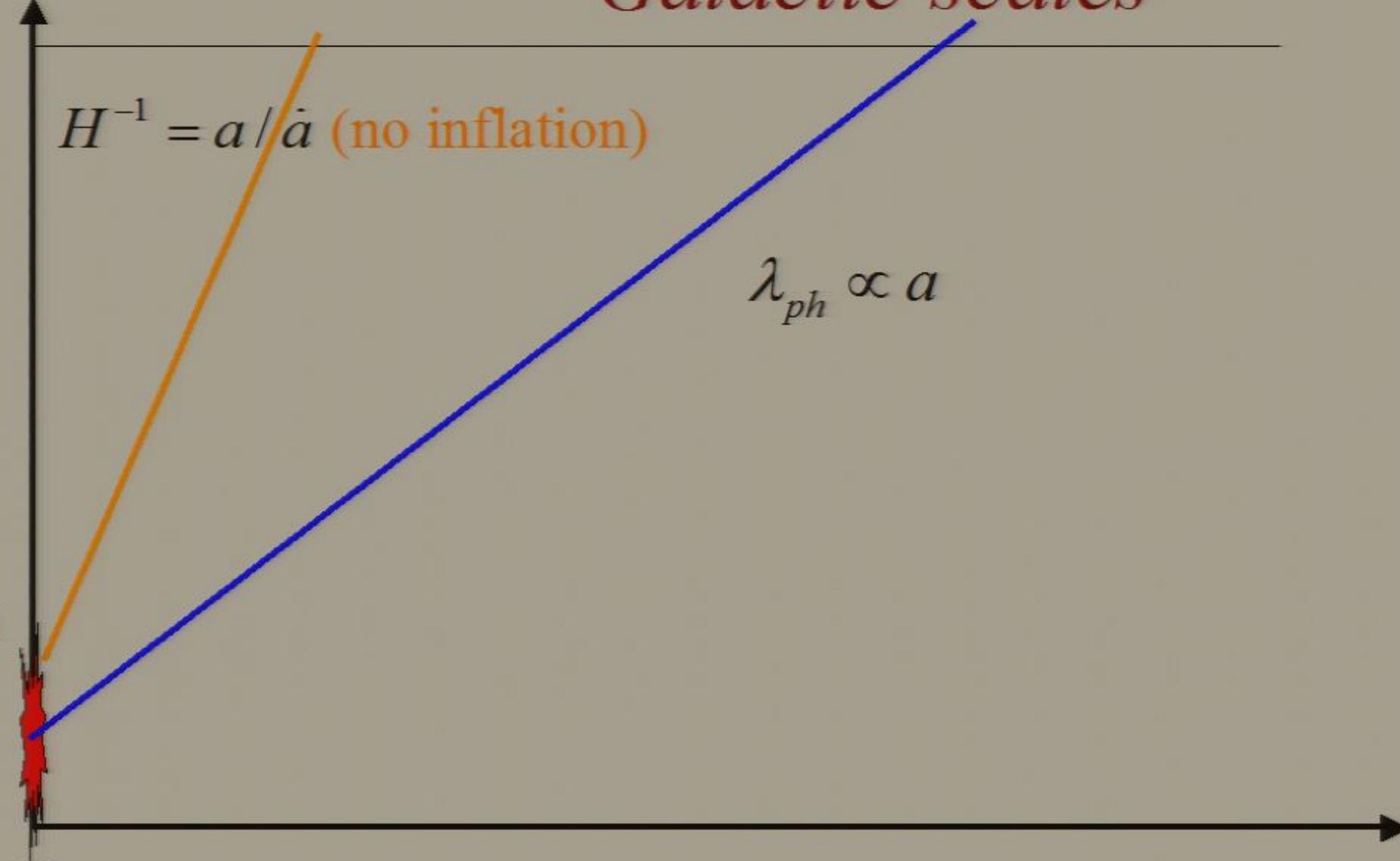
$$H^{-1} = a / \dot{a} \text{ (no inflation)}$$

$$\lambda_{ph} \propto a$$

$H_i^{-1}$

$a_i$

$a$



Scale

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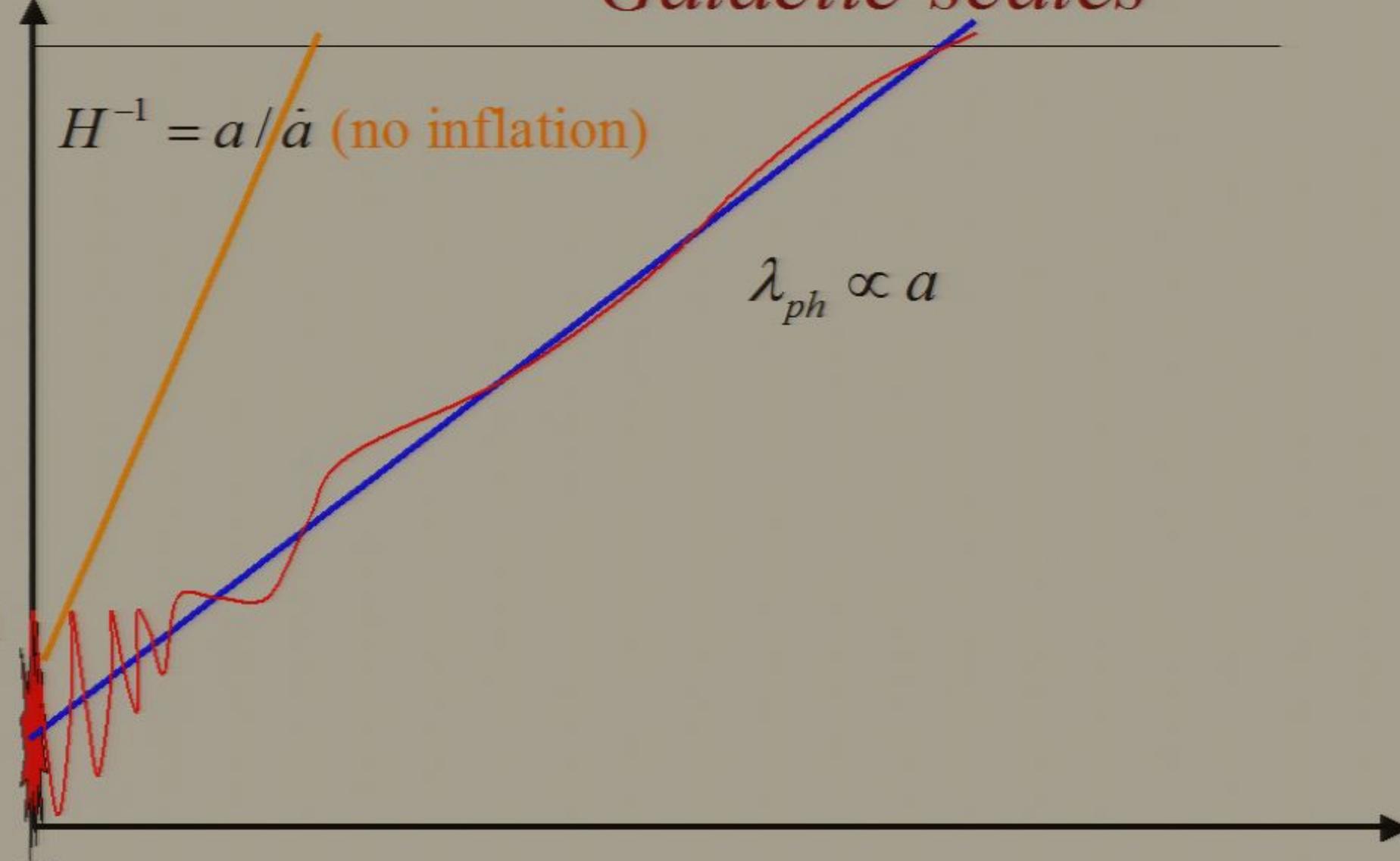
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$$H^{-1} = a / \dot{a} \text{ (no inflation)}$$

No inflation - no chance to get big fluctuations in galactic scales

$$\lambda_{ph} \propto a$$

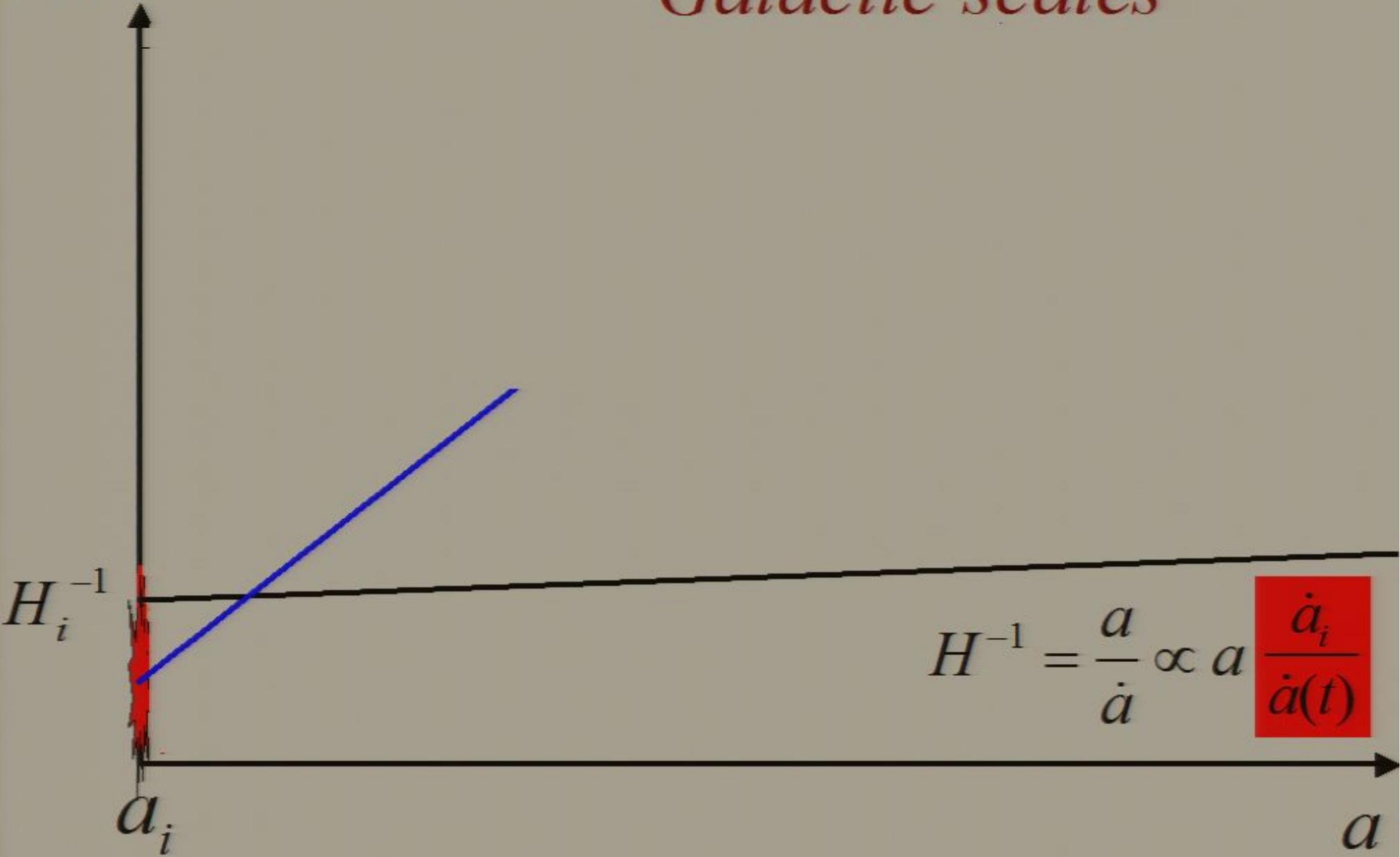
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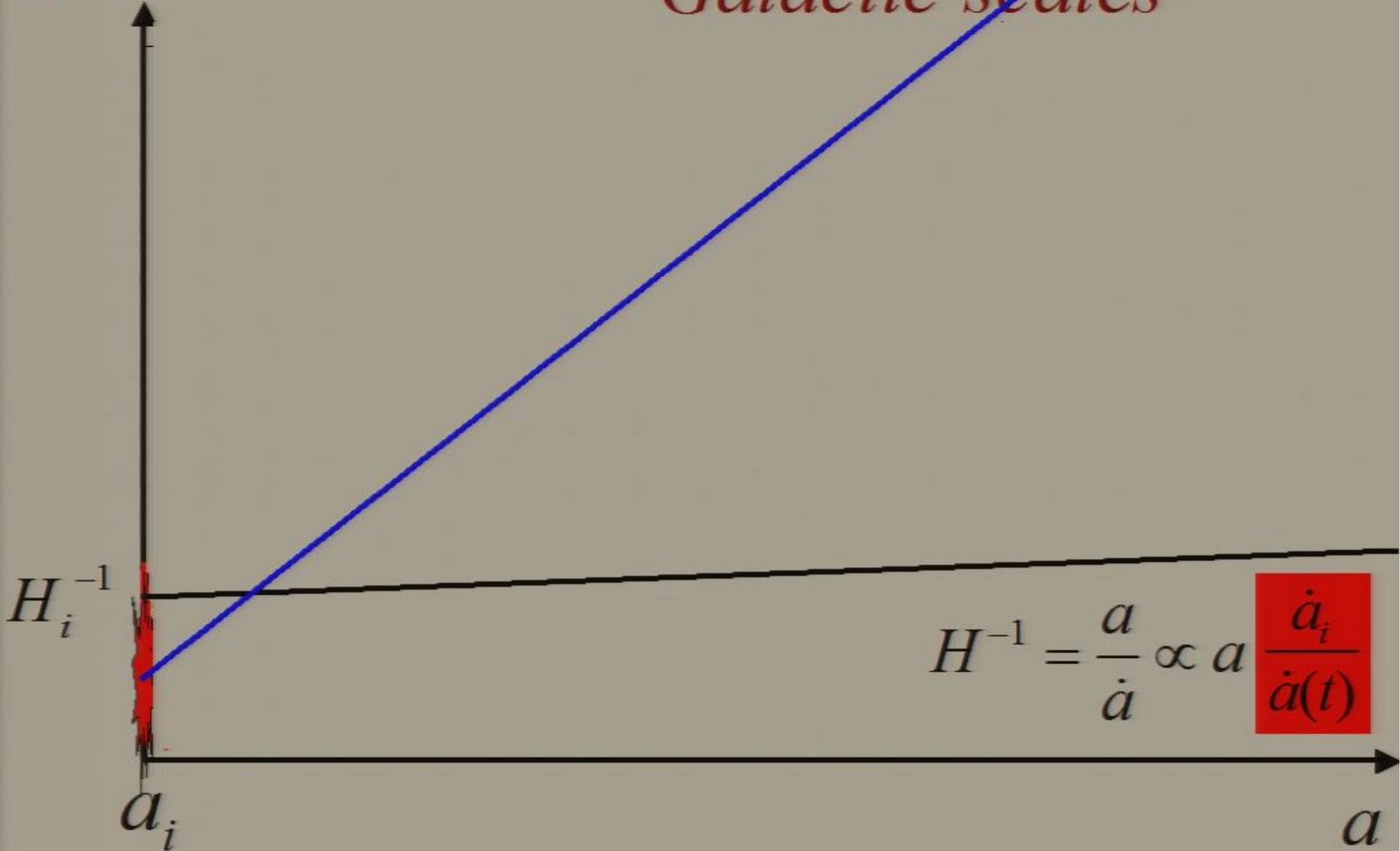
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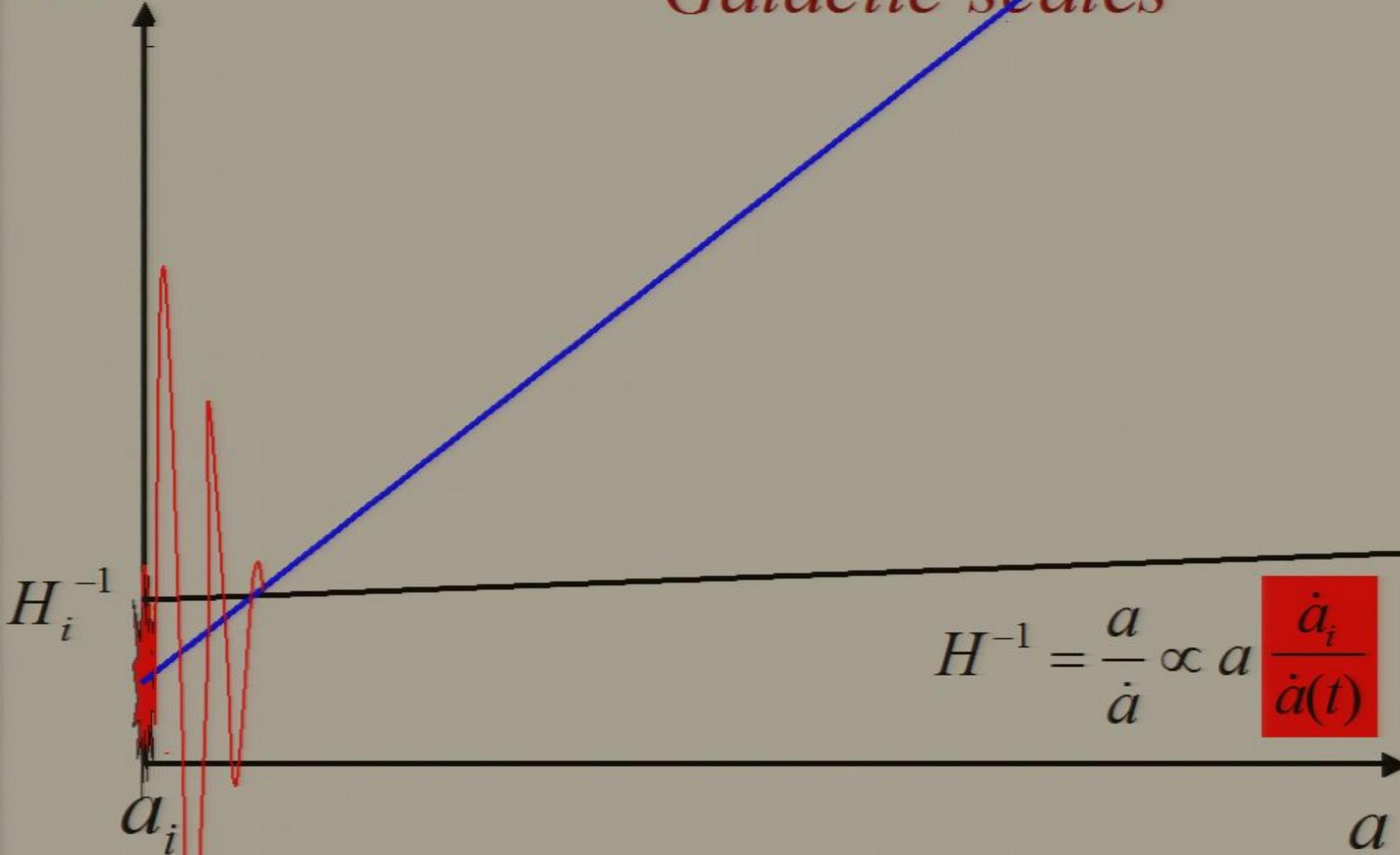
Scale

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Galactic scales

$H_i^{-1}$

$$H^{-1} = \frac{a}{\dot{a}} \propto a \frac{\dot{a}_i}{\dot{a}(t)}$$

$a_i$

$a$

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$$\delta\varphi + (1 + p/\varepsilon)^{1/2} \Phi,$$

$h$



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$$\int P(x, \varphi) \sqrt{g} d^D x$$

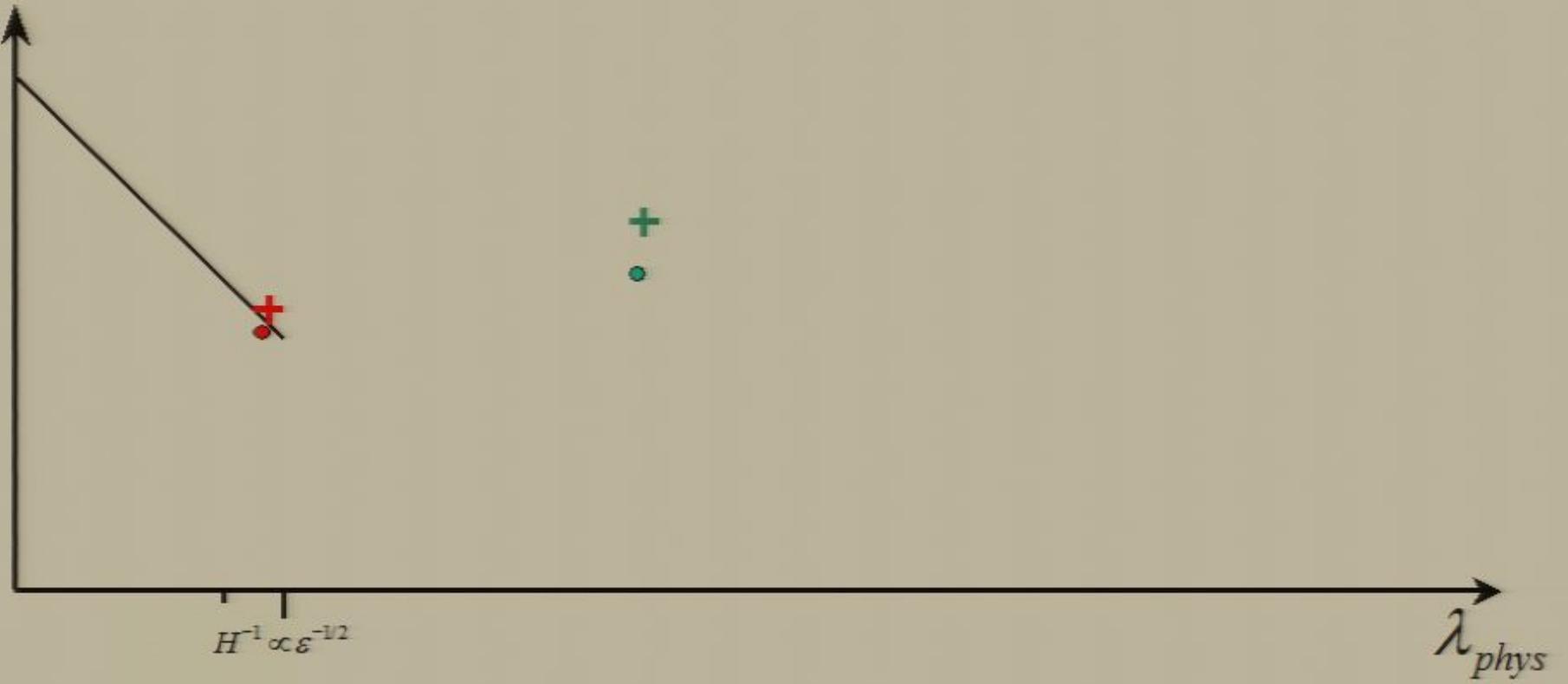
$$\varepsilon = 2 \int P(x, \varphi)$$

$$\| \cdot \| \sim \varepsilon \quad \tilde{\varepsilon} \propto \varepsilon^2 (2 + P)$$



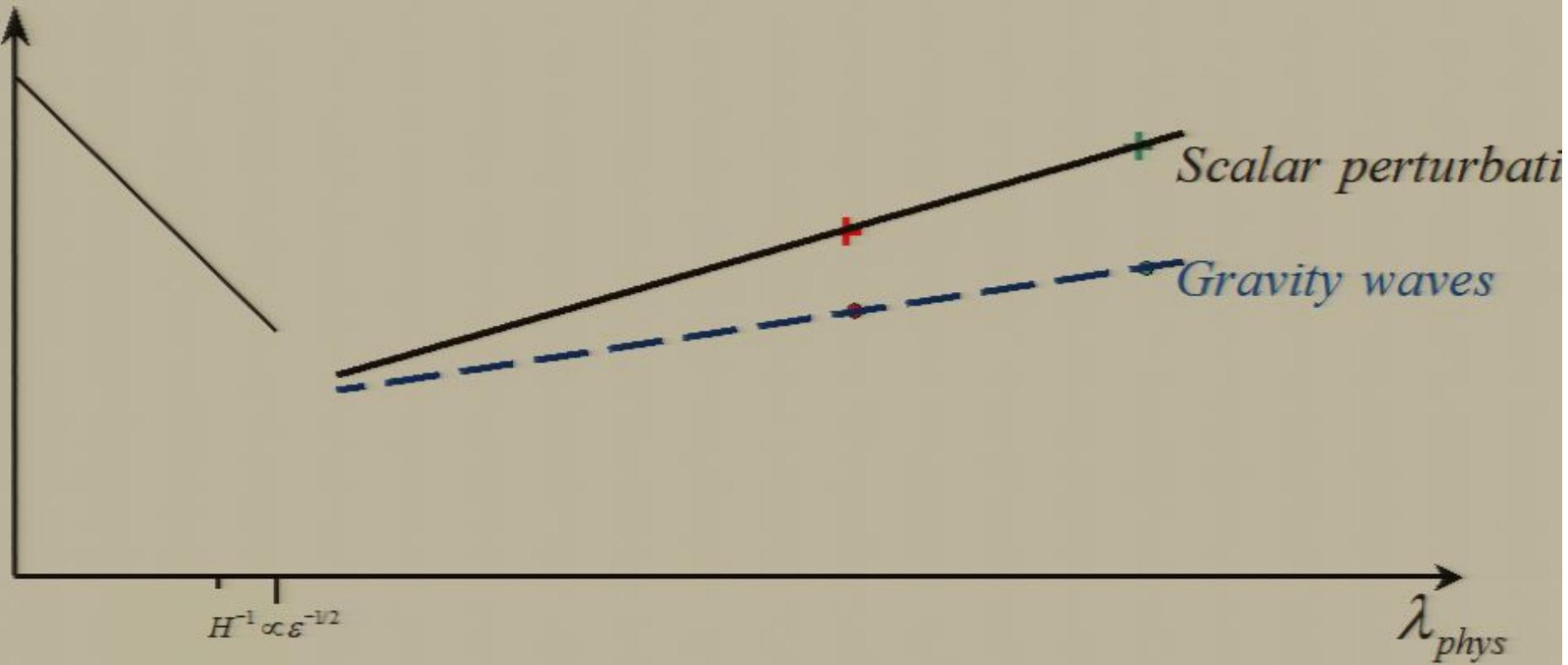
$$\delta\varphi + (1 + p/\varepsilon)^{1/2} \Phi,$$

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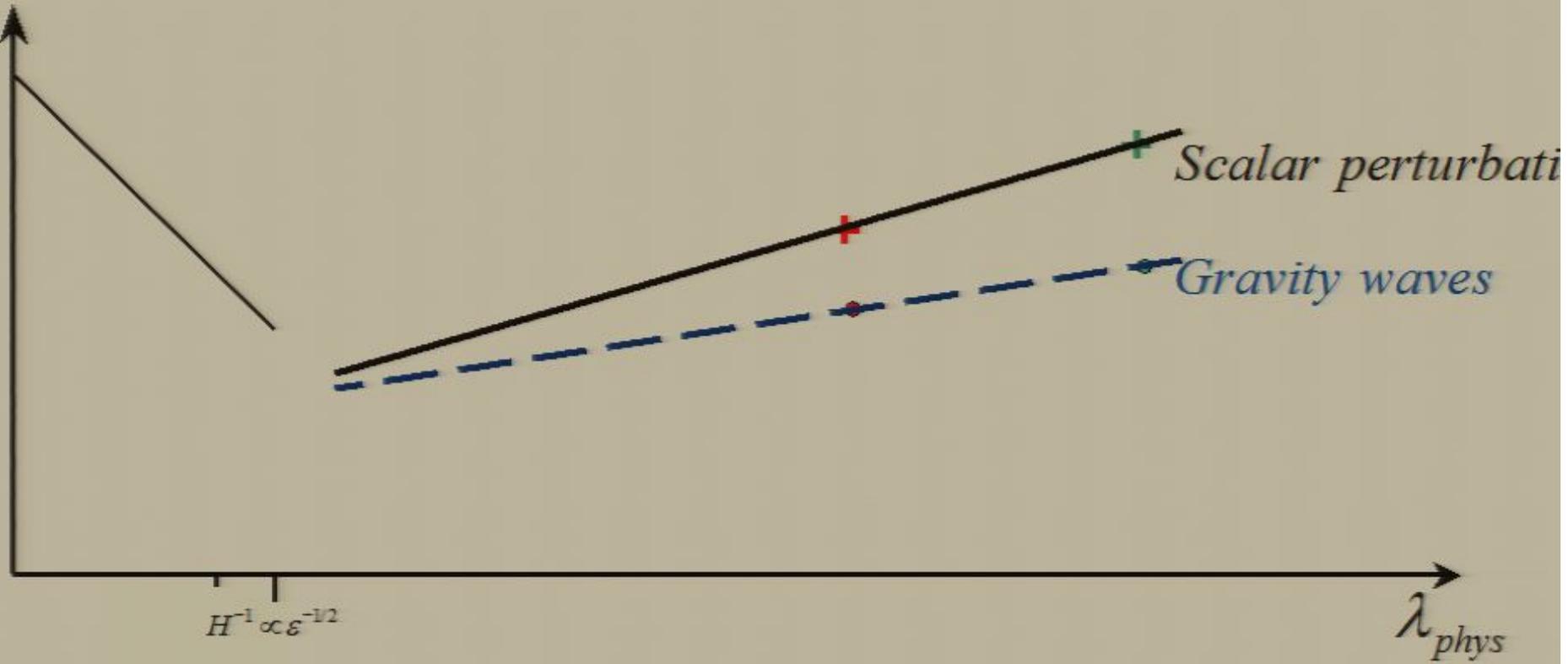
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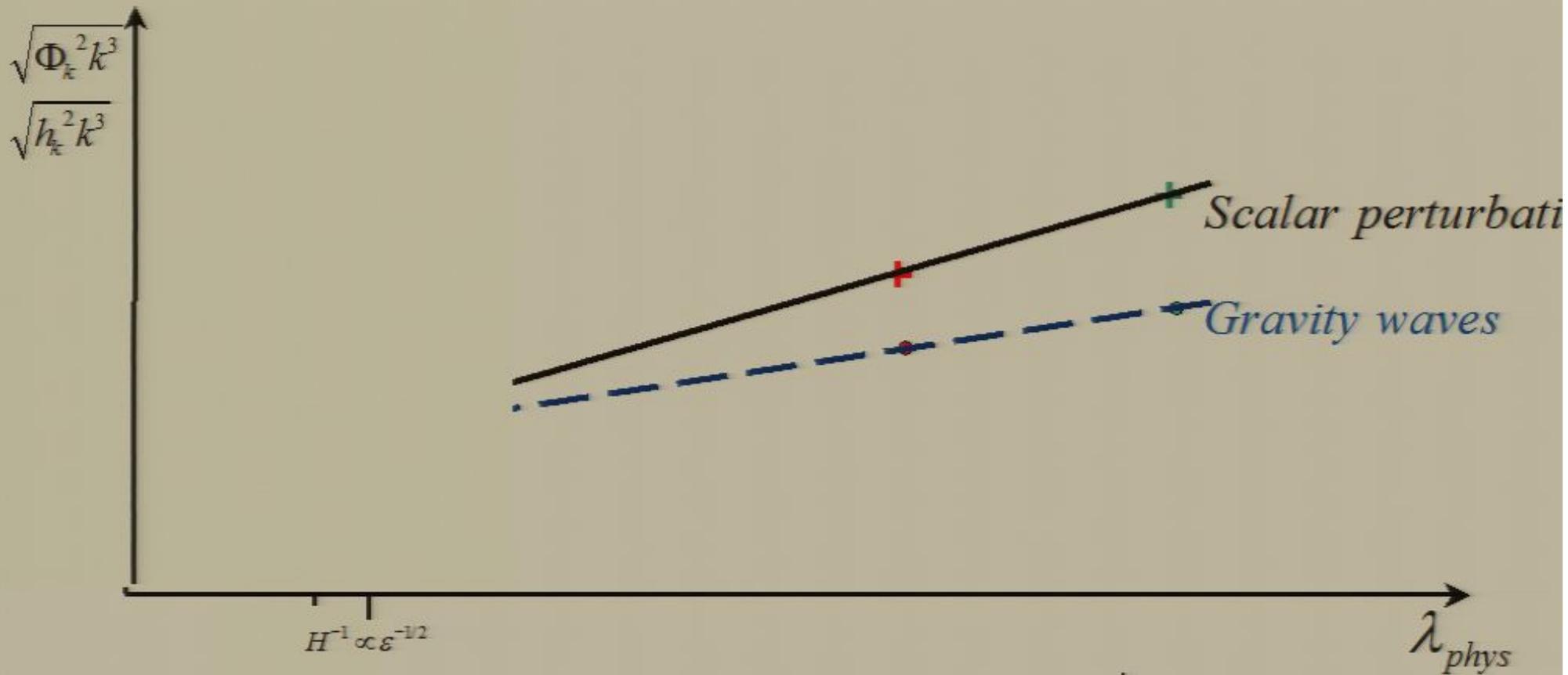
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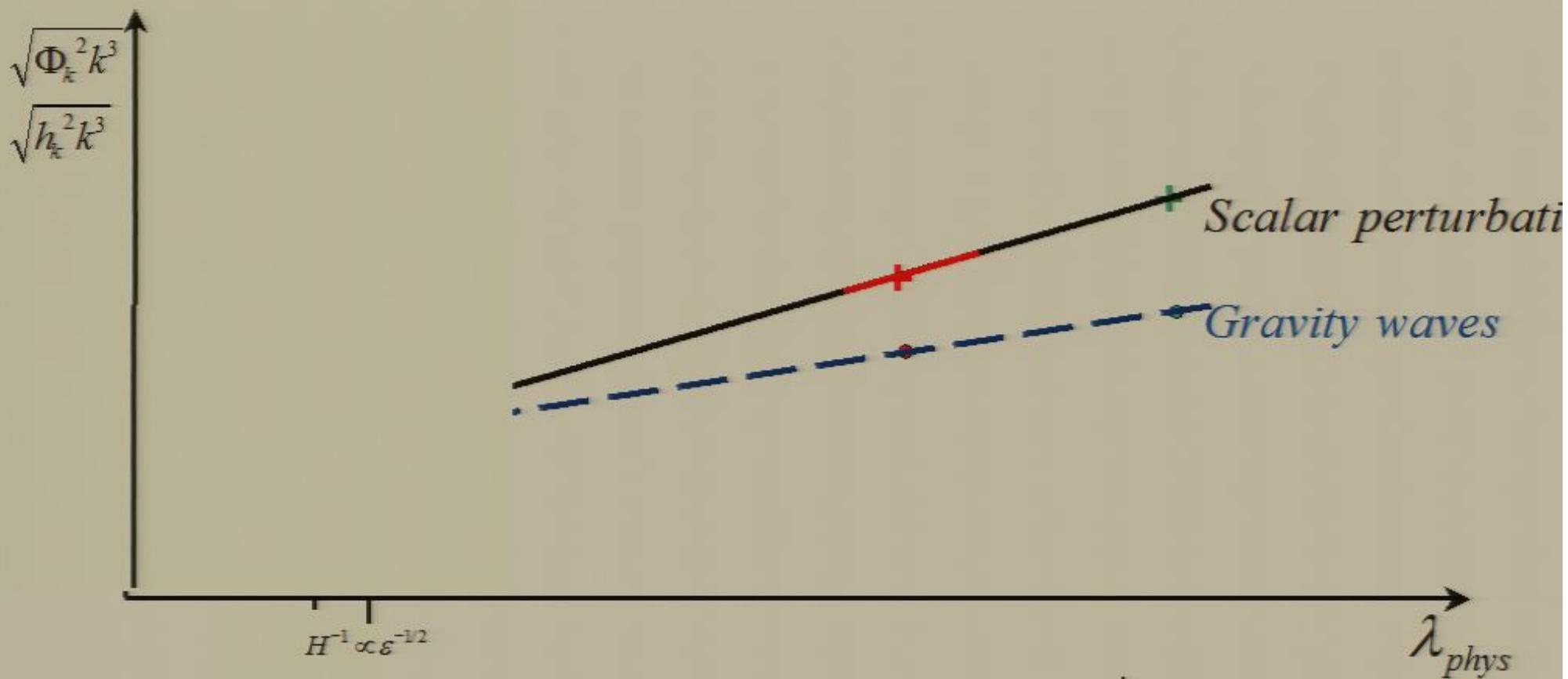
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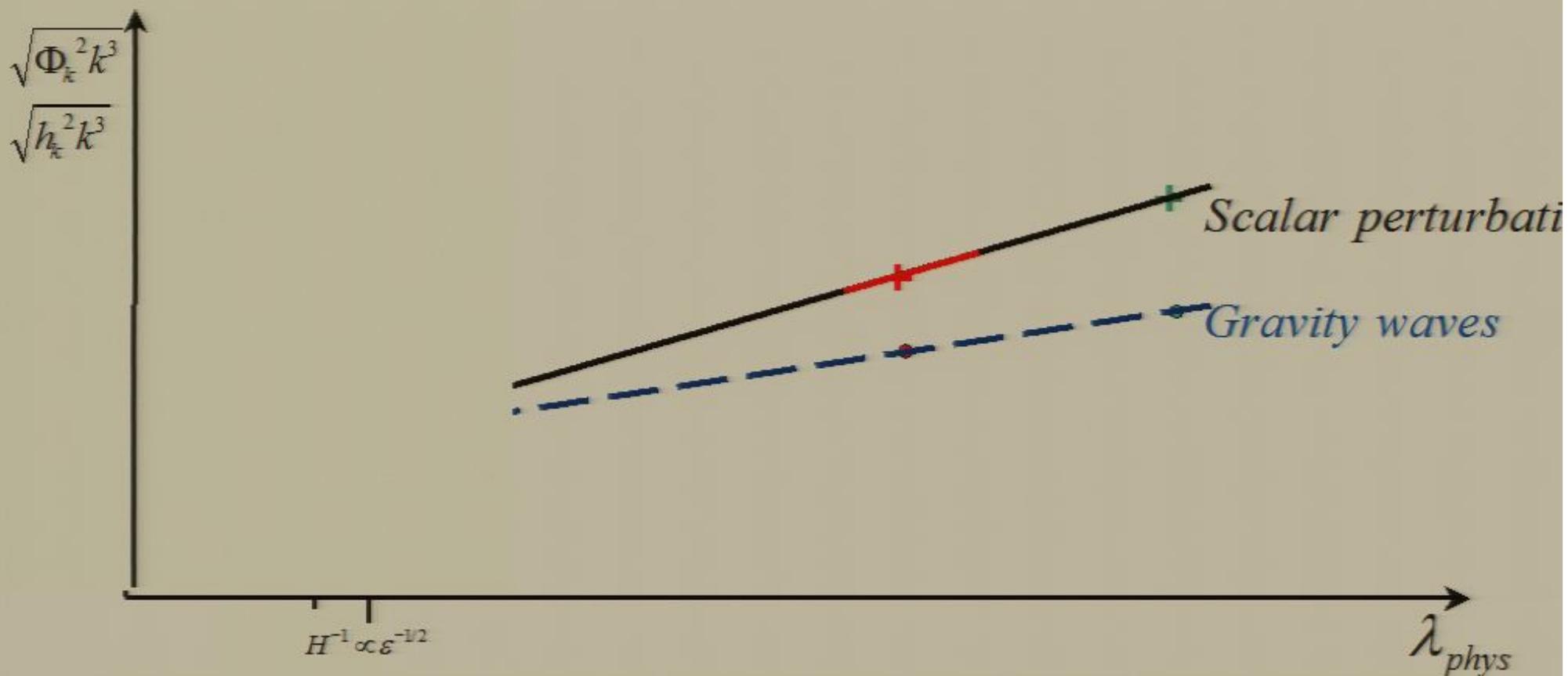




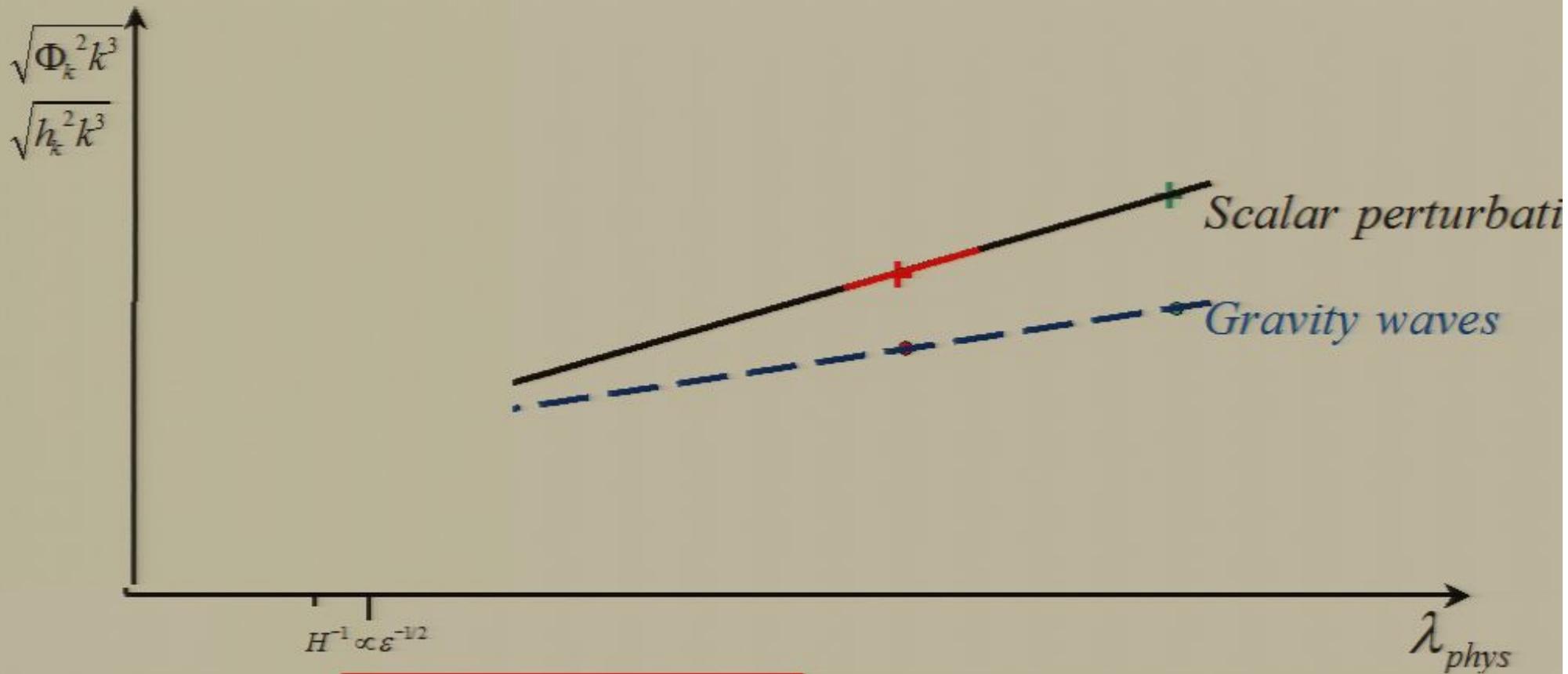
$$\Phi_k^2 k^3 = O(1) \frac{1}{1 + p/\epsilon} \left( \frac{\epsilon}{\epsilon_{Pl}} \right) \Big|_{k \approx Ha}$$



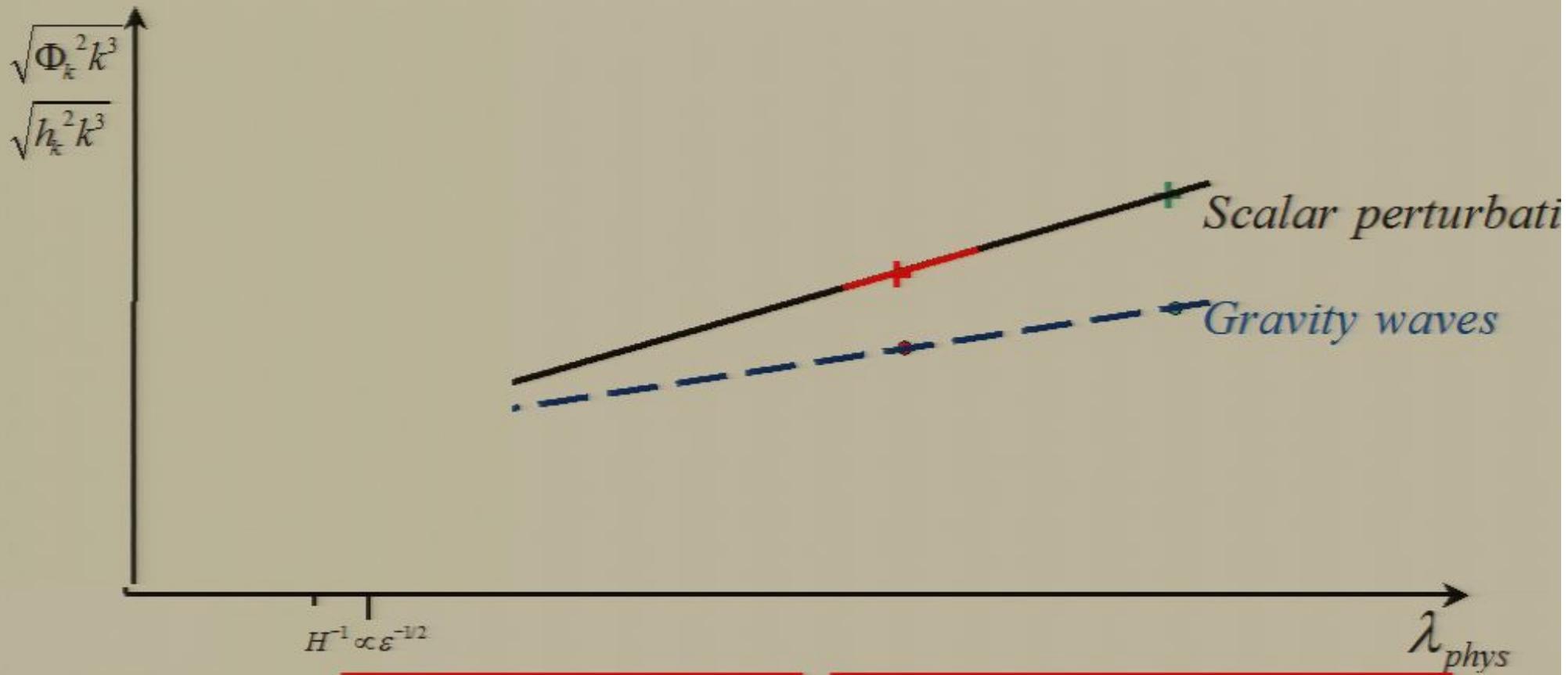
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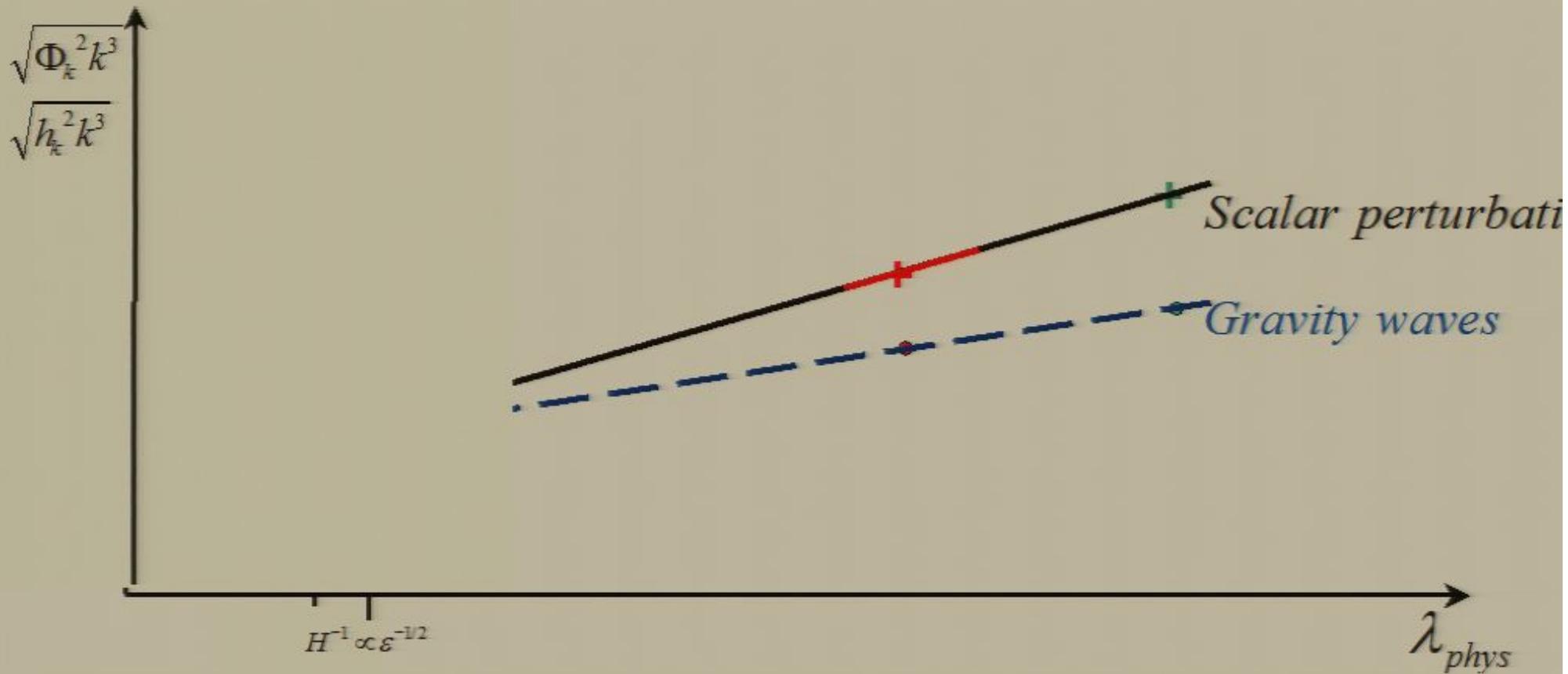
$$n_S - 1 = -3 \left( 1 + \frac{p}{\epsilon} \right)_{k \approx H a} + 3 \frac{d}{d \ln \epsilon} \left( 1 + \frac{p}{\epsilon} \right)_{k \approx H a}$$



$$n_S - 1 = \mathbf{< 0} + 3 \frac{d}{d \ln \epsilon} \left( 1 + \frac{p}{\epsilon} \right)_{k \approx H a}$$



$$n_S - 1 = < 0 \quad - \quad < 0$$



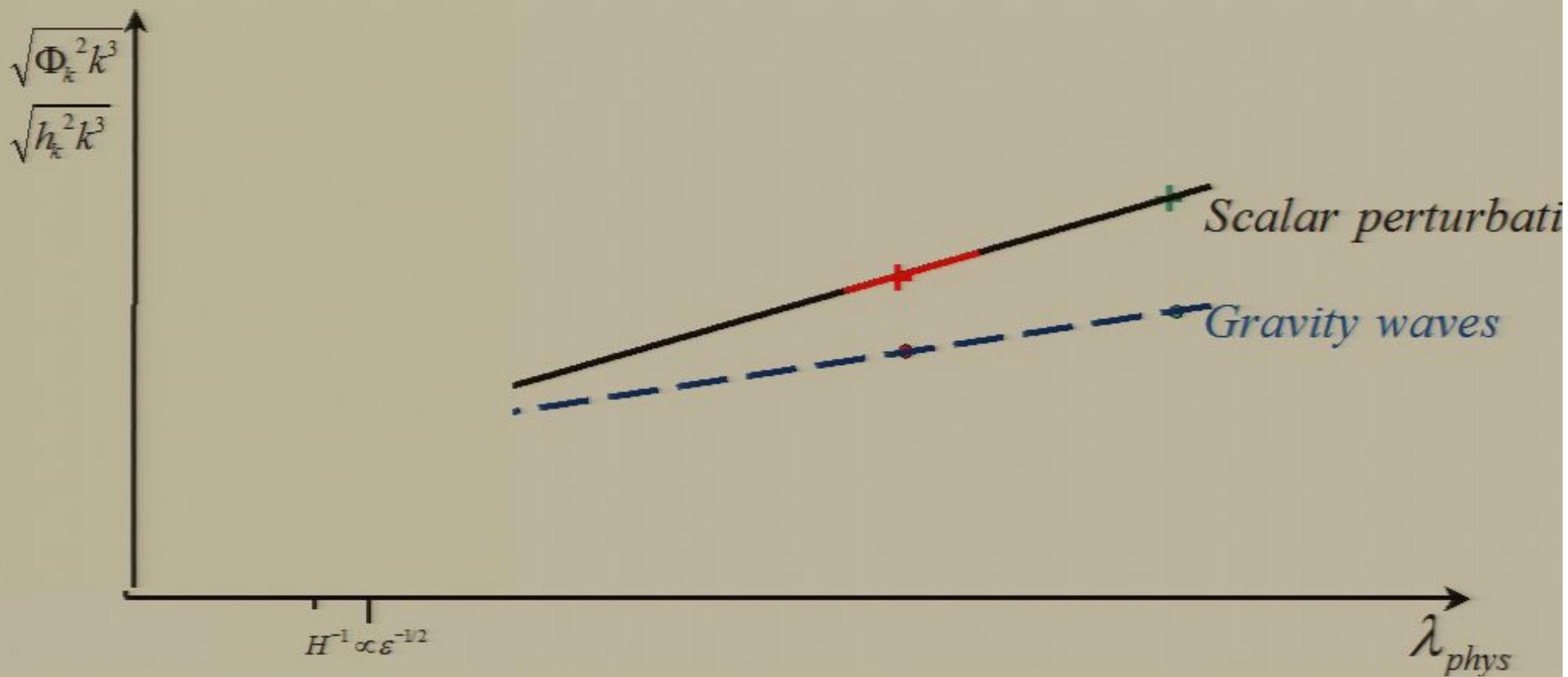
$$0.92 < n_s < 0.97$$

$$H \sim \mathcal{E}$$

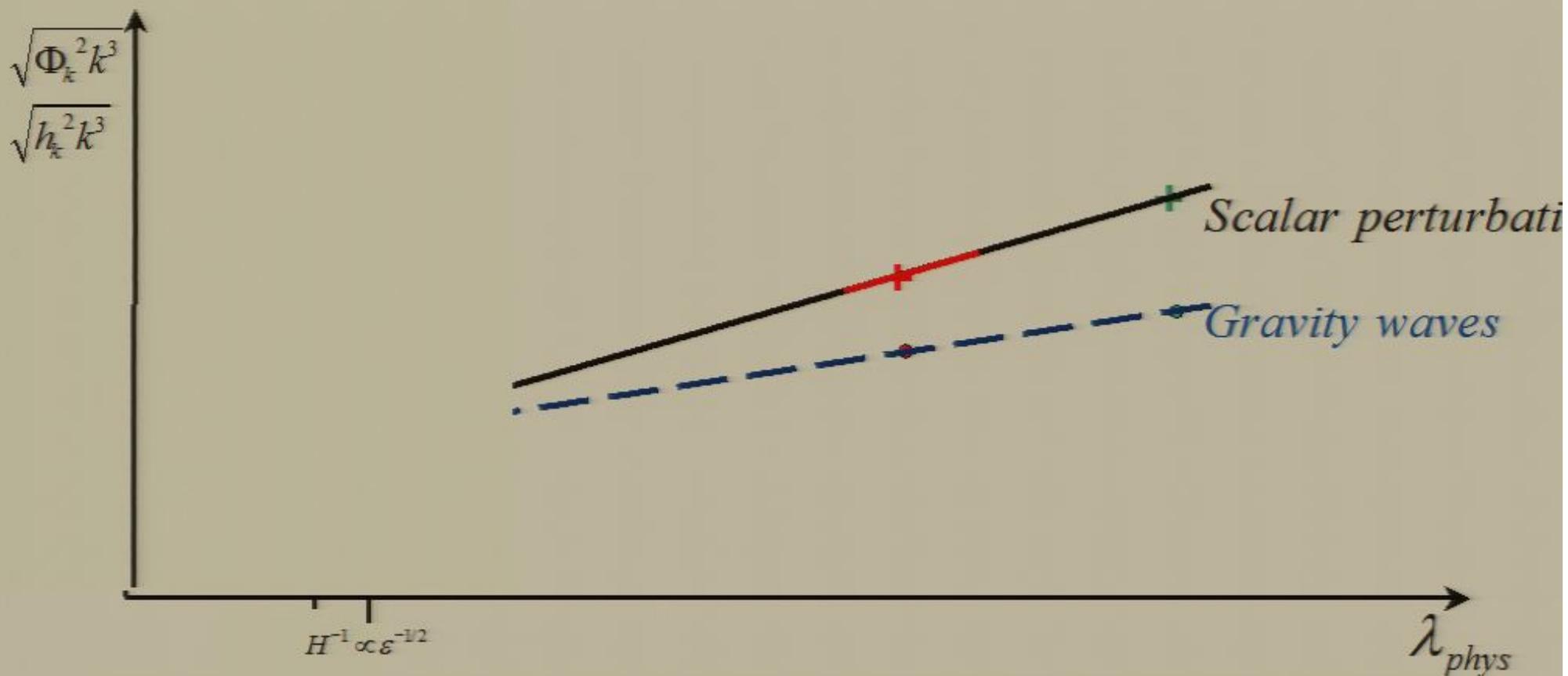
$$\dot{\mathcal{E}} \propto -2(\mathcal{E} - P)$$

$$\int P(X, \varphi) \sqrt{-g} d^4x$$

$$C_S^2 = \frac{P_{,X} - \dot{\varphi}^2}{2X} = 2X P_{,X} - P$$



$$0.92 < n_s < 0.97$$



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$$\frac{T}{S} = O(1) \left( 1 + \frac{p}{\epsilon} \right)^{1/2} \quad k \approx Ha$$

# Summary

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Idea and basic properties of inflation are established:

Inflation is the stage of accelerated expansion of the universe with graceful exit to Friedmann stage

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QUESTION



- Robust predictions:
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  - Slightly **red-tilted** spectrum of scalar perturbations ( $0,92 < n_s < 0,97$ )
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  - Gravity waves



# Comparison with observations: Present

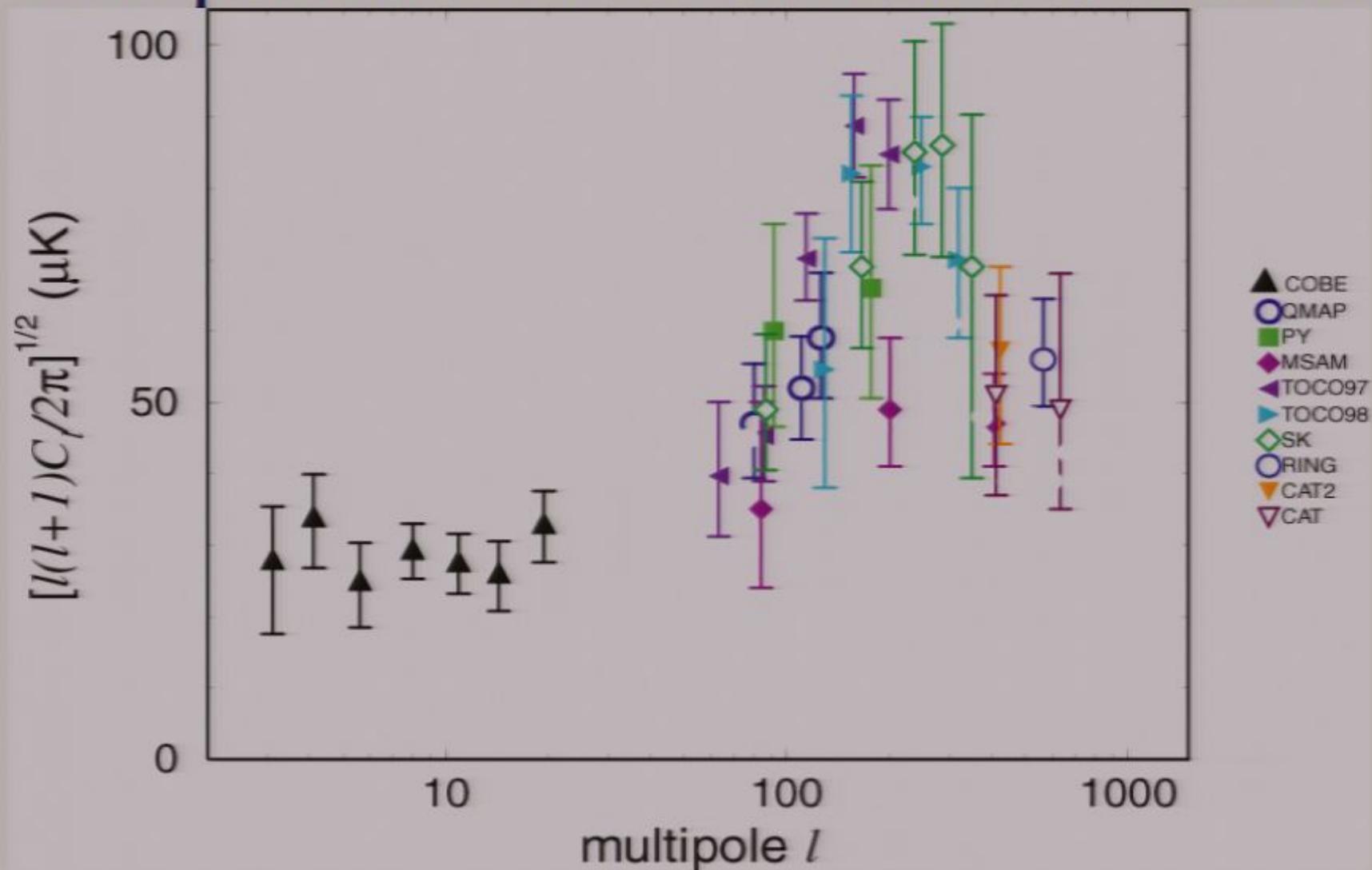
$$\left\langle \frac{\delta T}{T}(\varphi) \frac{\delta T}{T}(\varphi + \theta) \right\rangle_{\varphi} = \frac{1}{4\pi} \sum (2l + 1) C_l P_l(\cos \theta)$$

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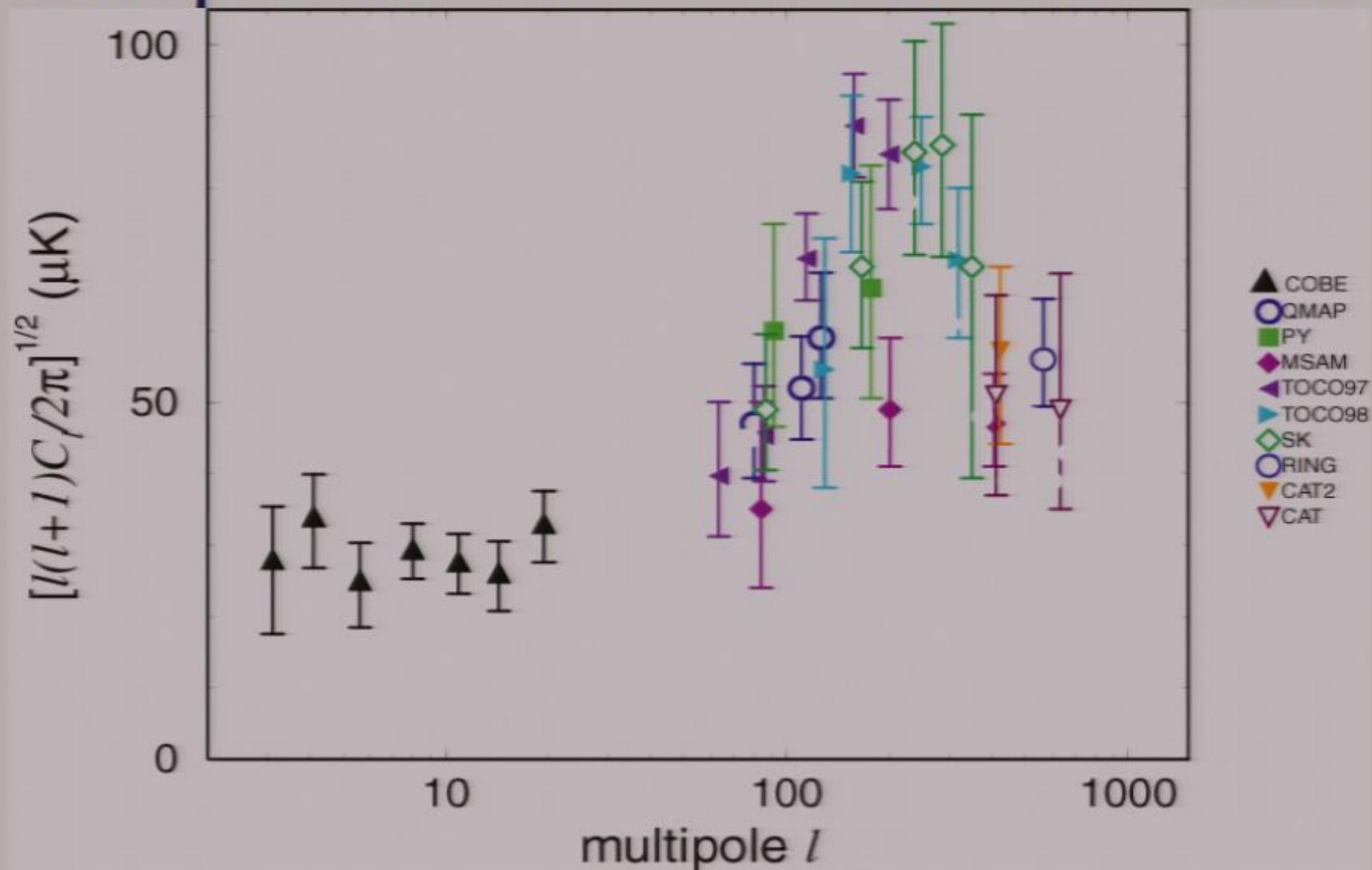
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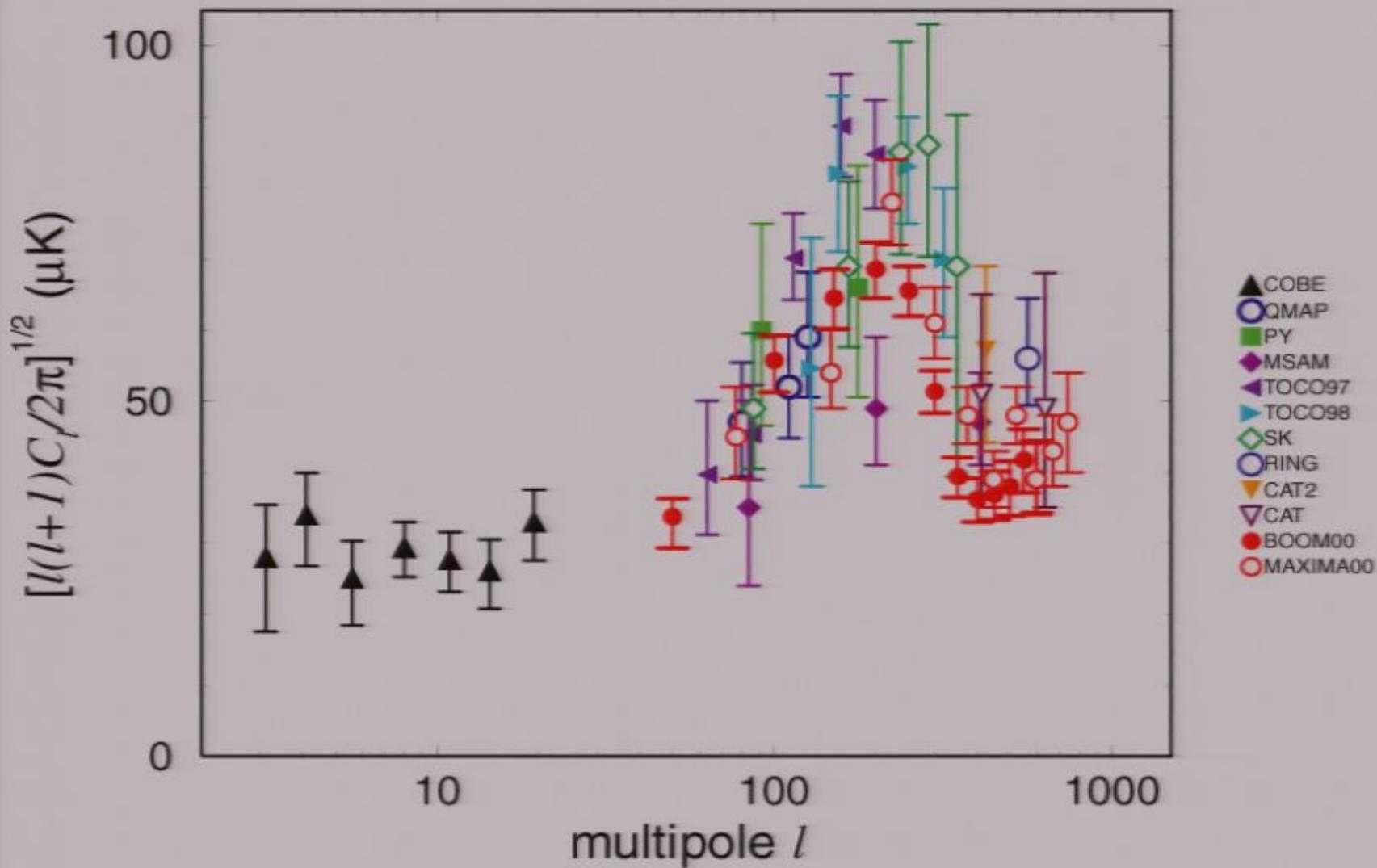
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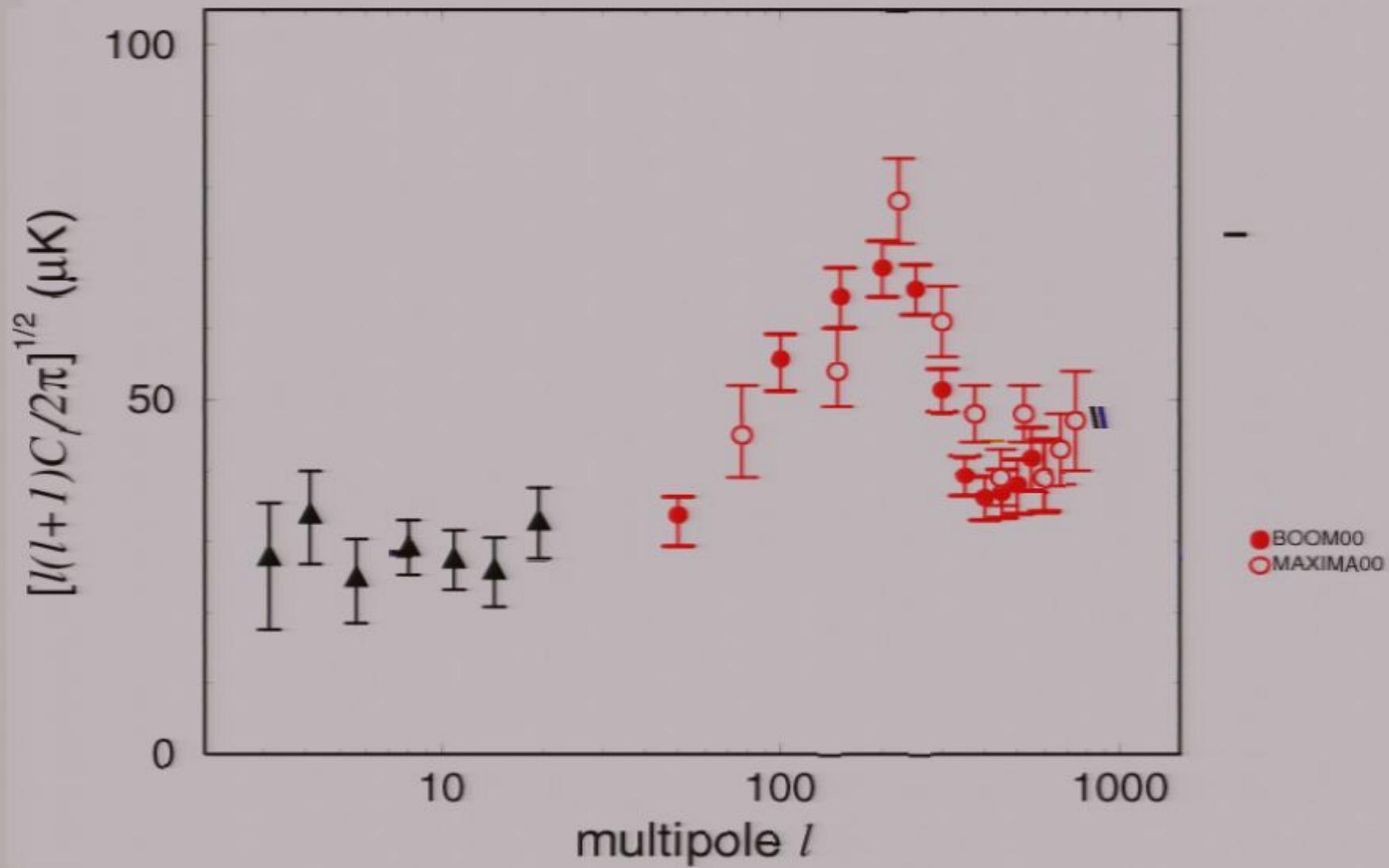
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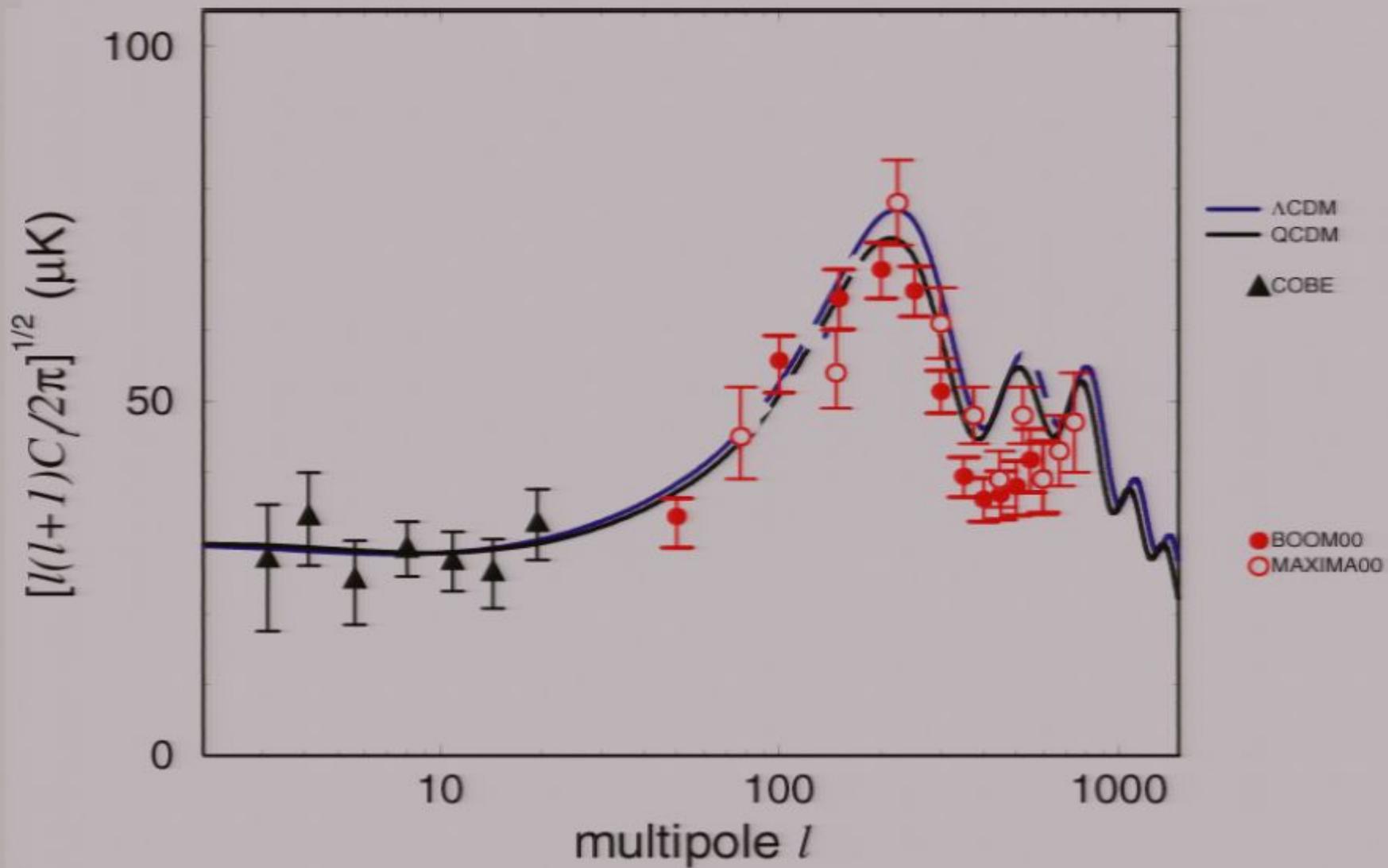
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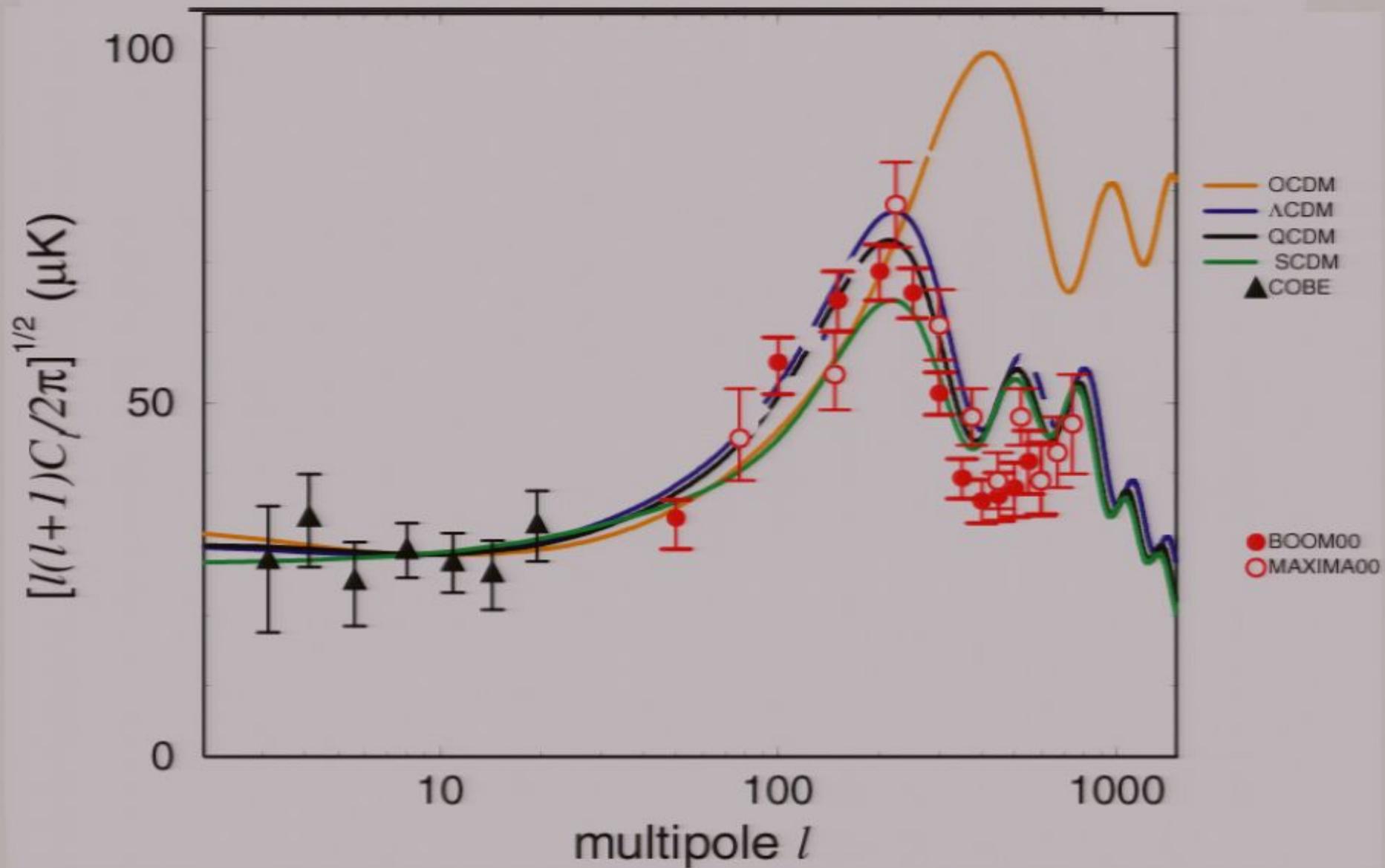
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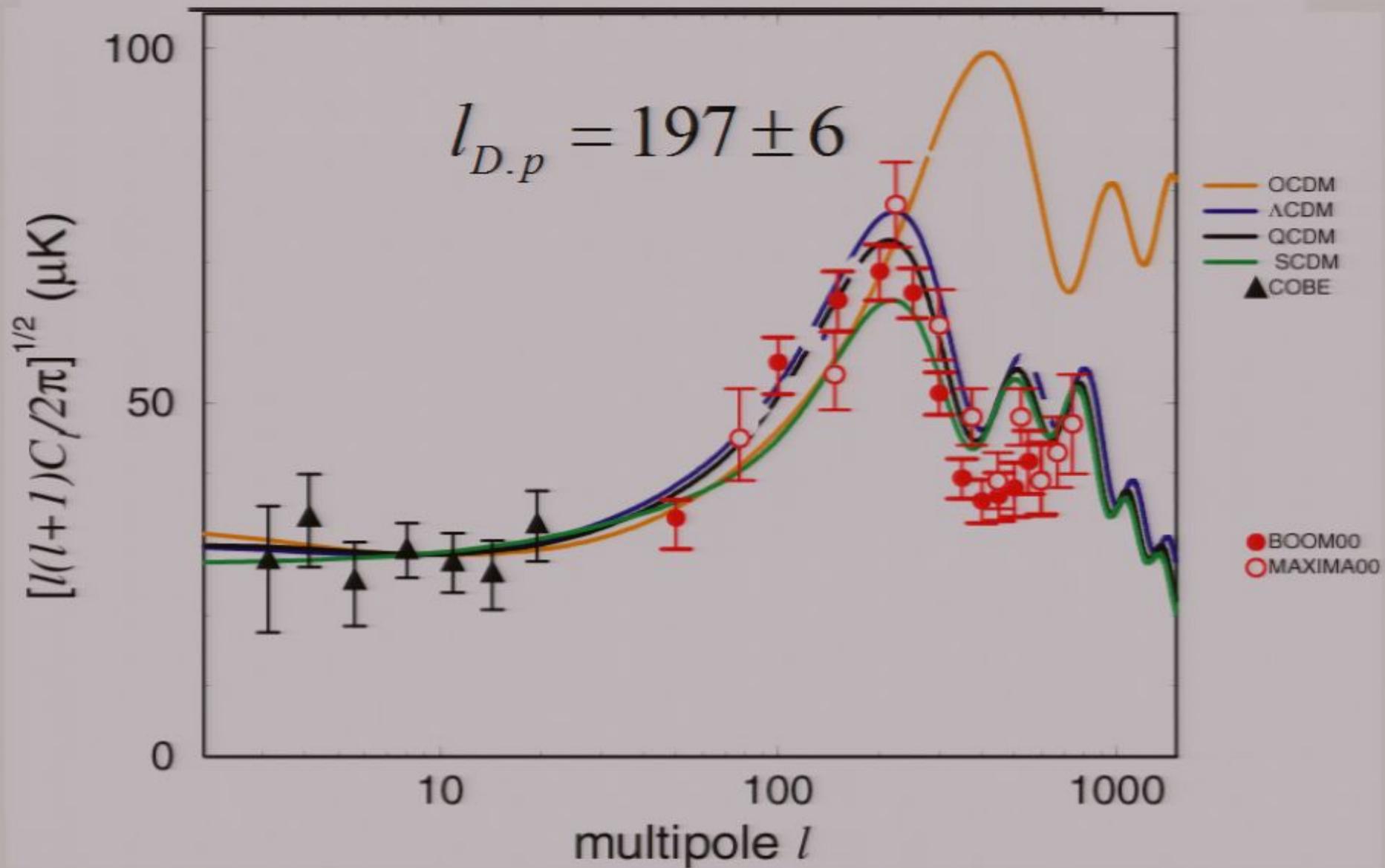


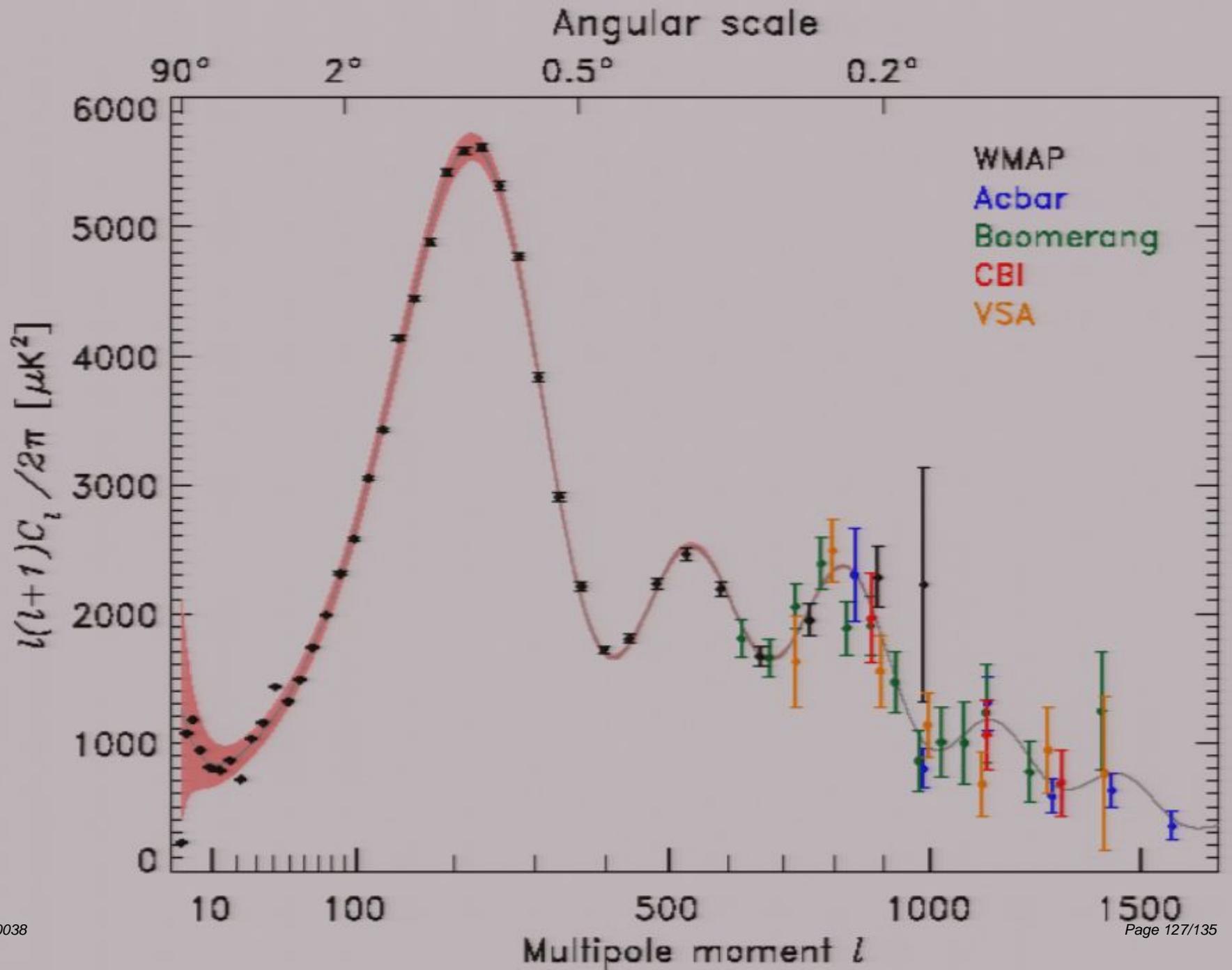


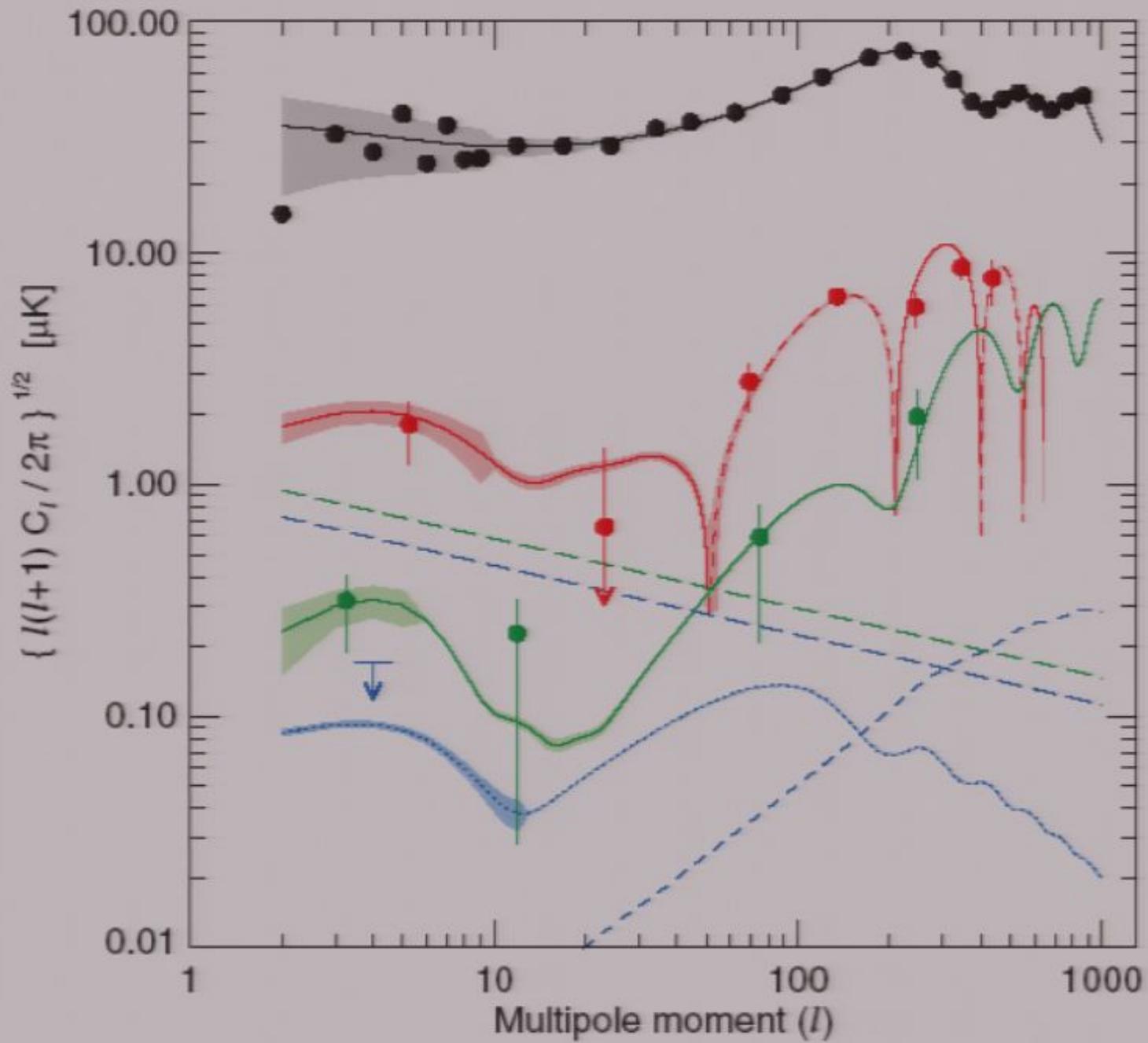


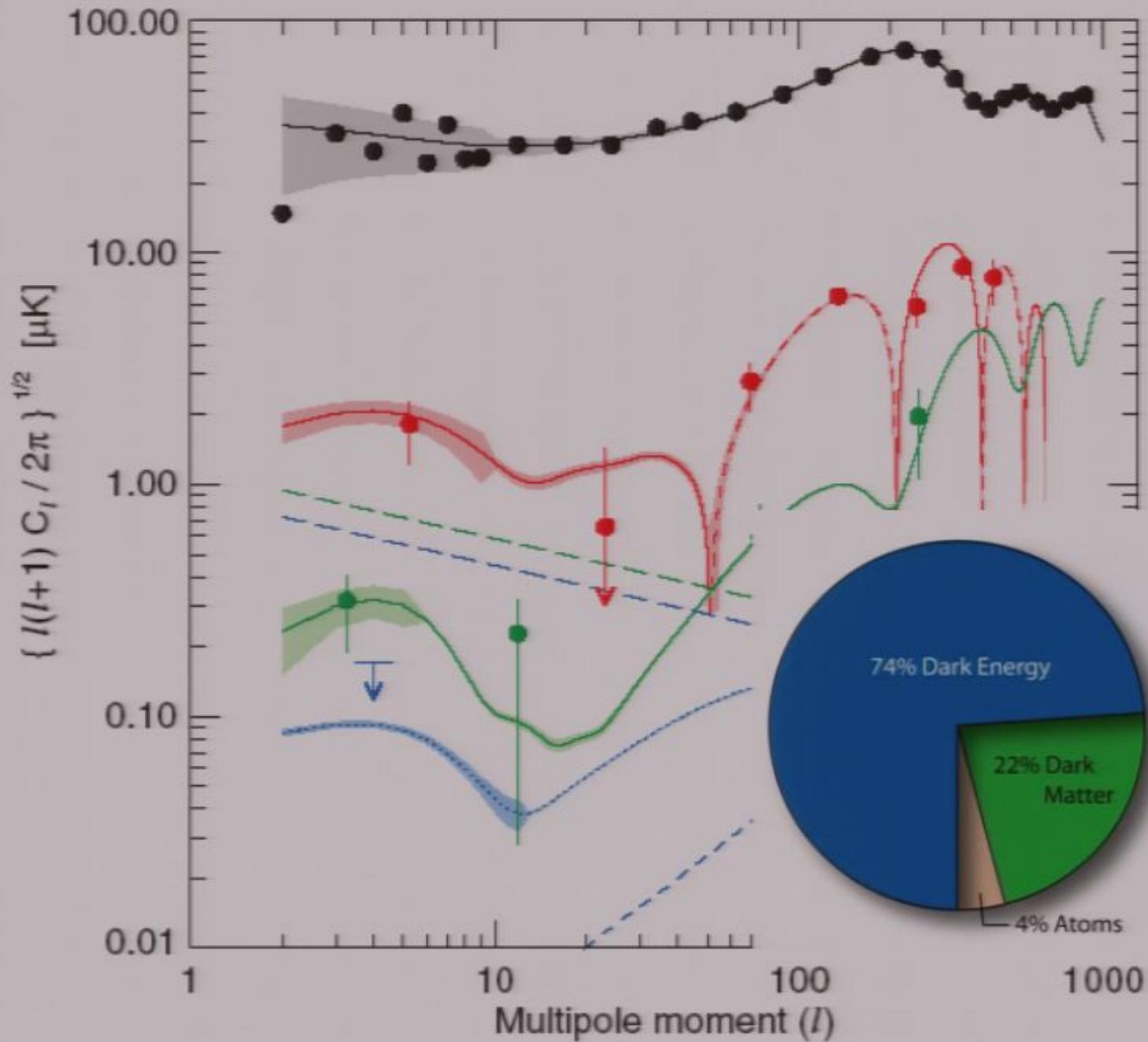












"A finite duration of the de Sitter stage does not by itself rule out the possibility that this stage may exist as an intermediate stage in the evolution of the universe. An interesting question arises here: Might not perturbations of the metric, which would be sufficient for the formation of galaxies and galactic clusters, arise in this stage?.....

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In terms of my own money, I'd bet a lot (many thousands )

