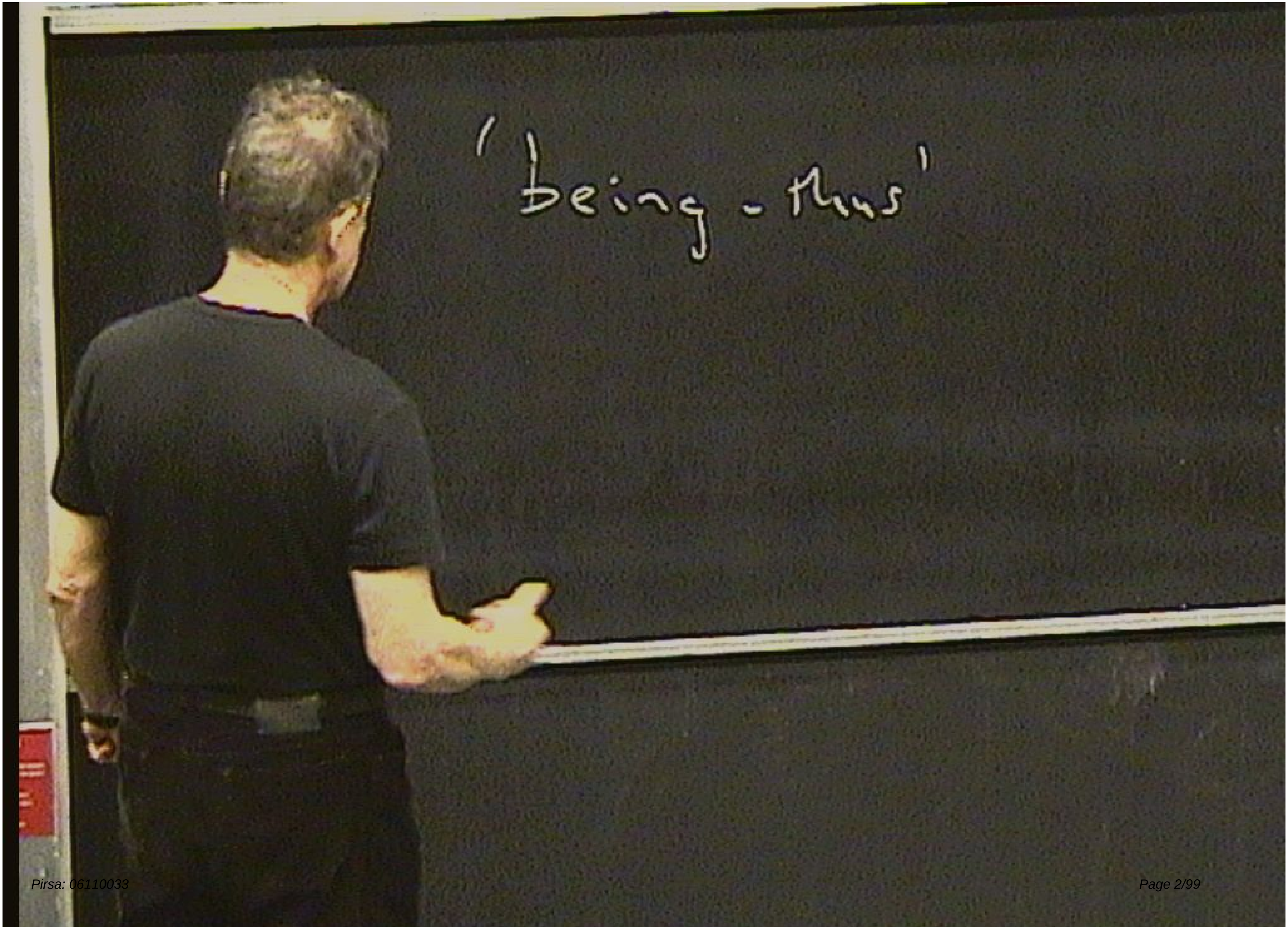
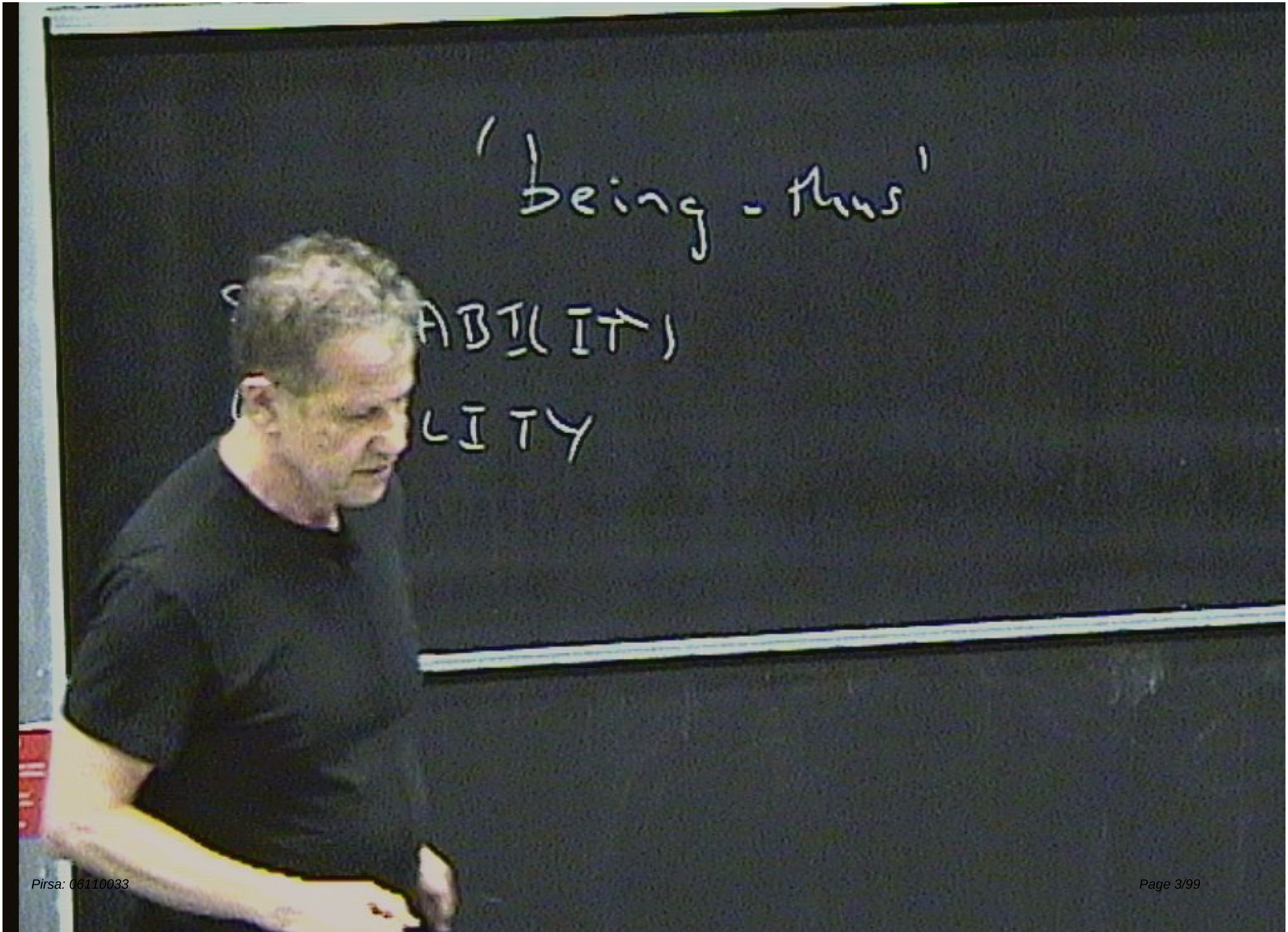
Title: Introduction to Quantum Information and Computation from a Foundational Standpoint

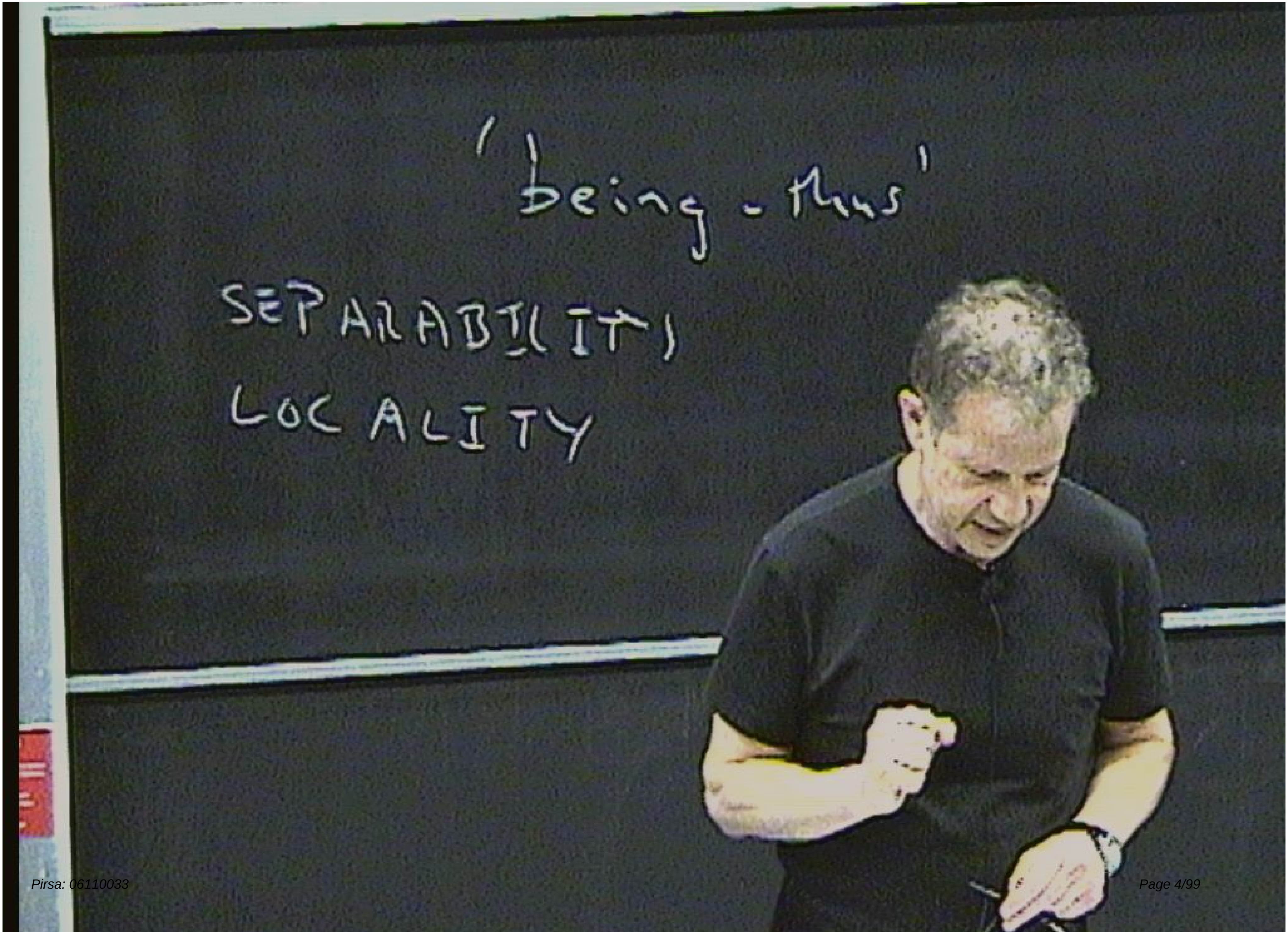
Date: Nov 09, 2006 10:30 AM

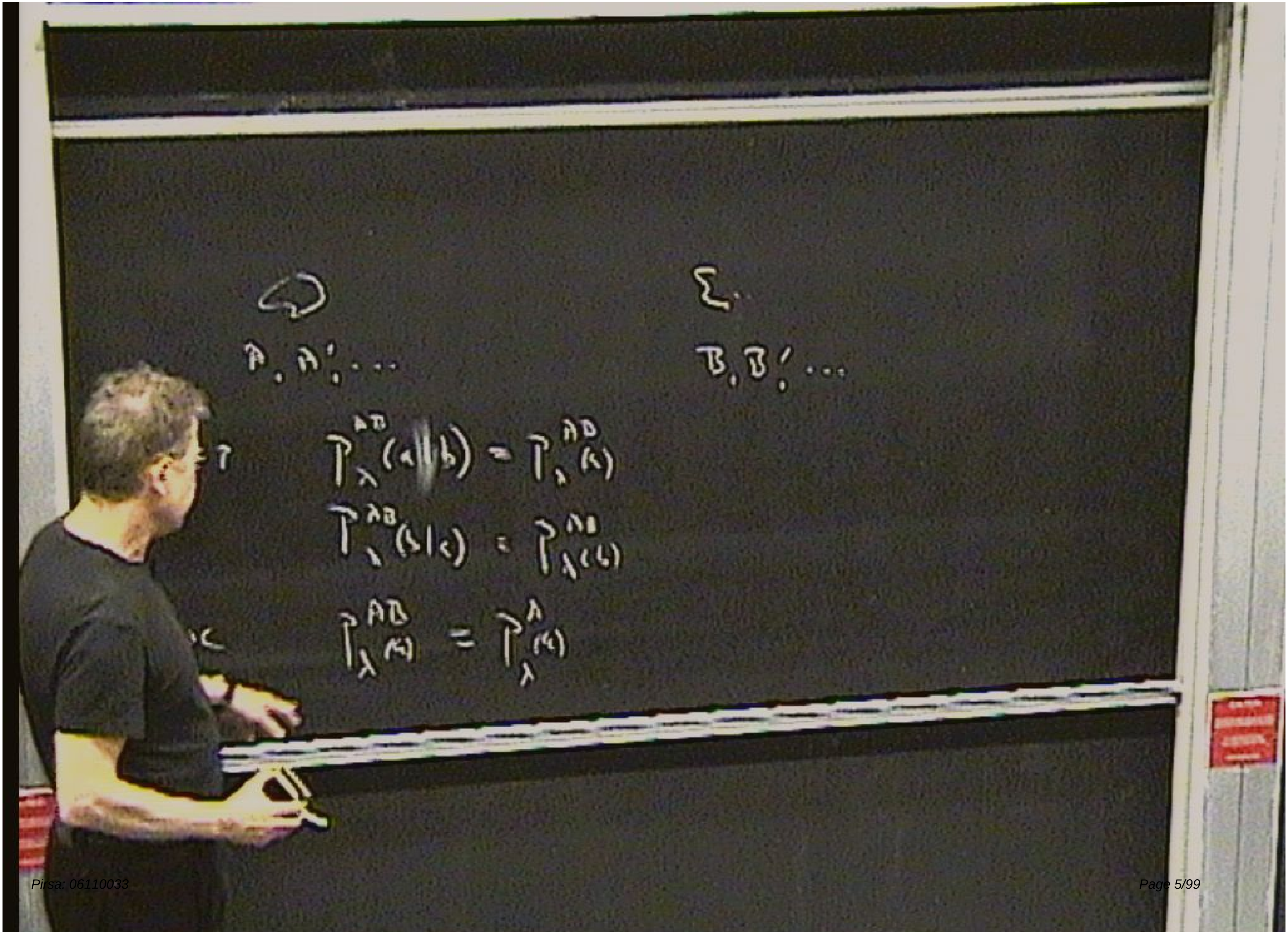
URL: http://pirsa.org/06110033

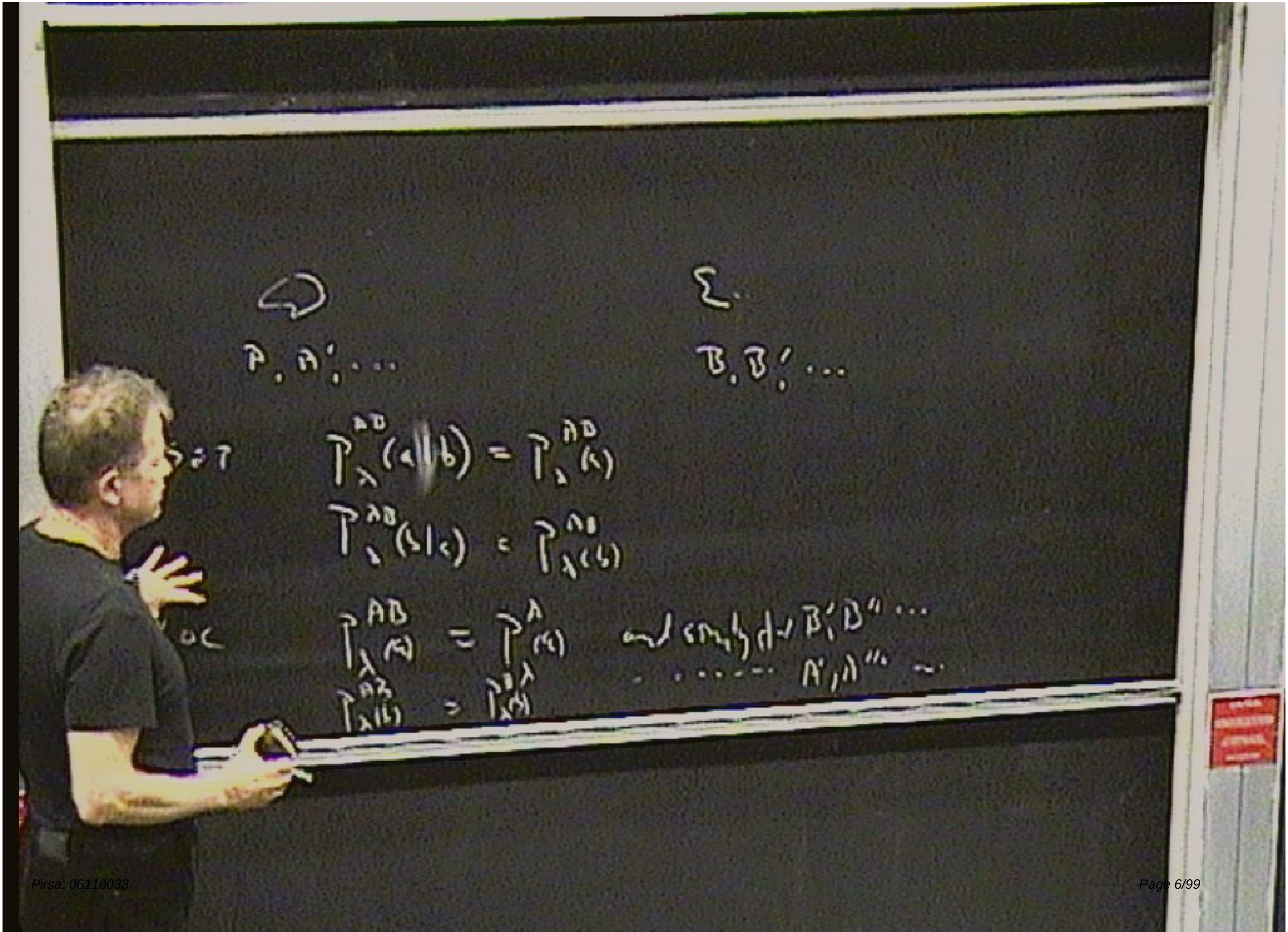
Abstract: Classical Information and Quantum Information

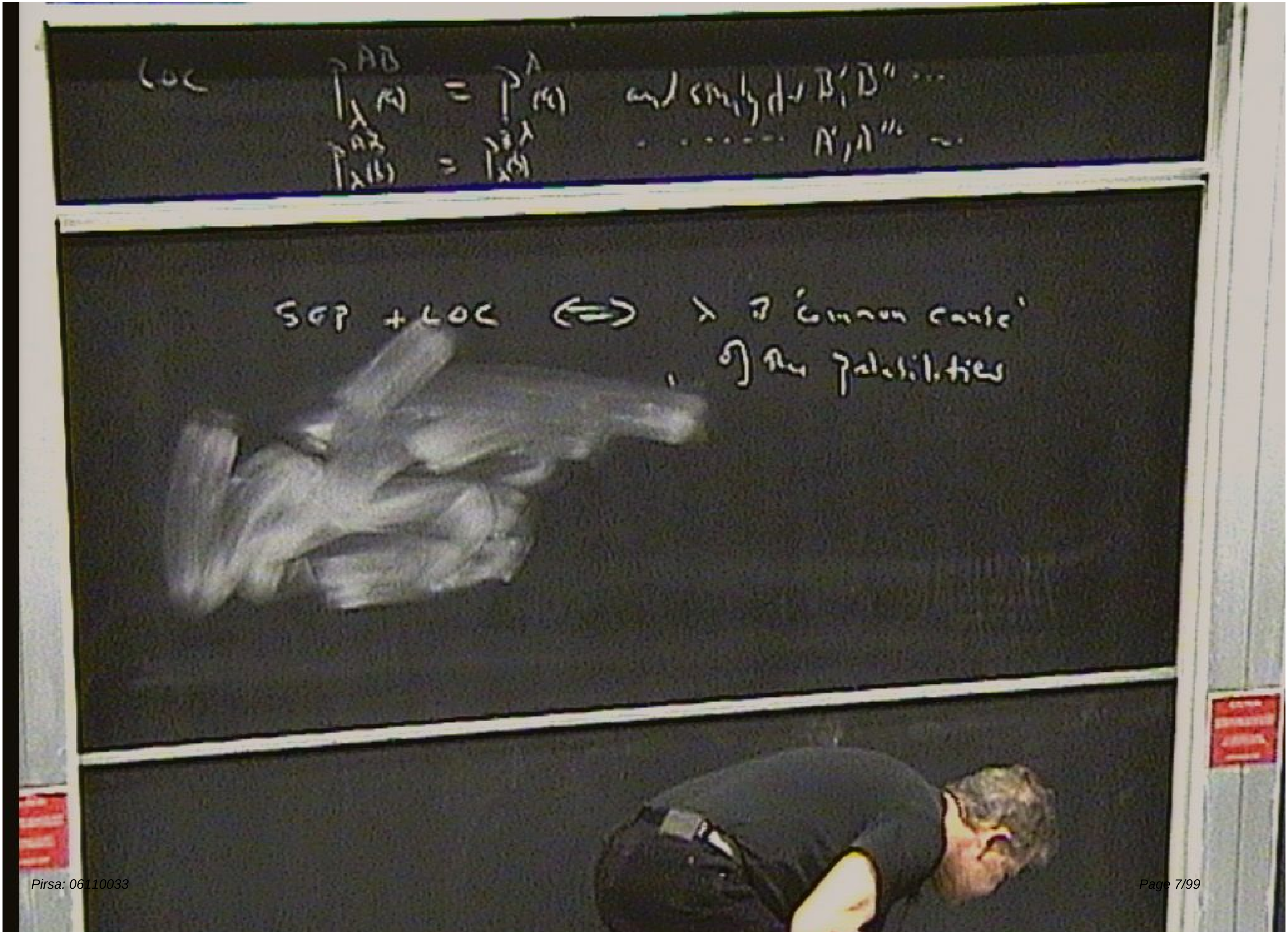

'being - thus'

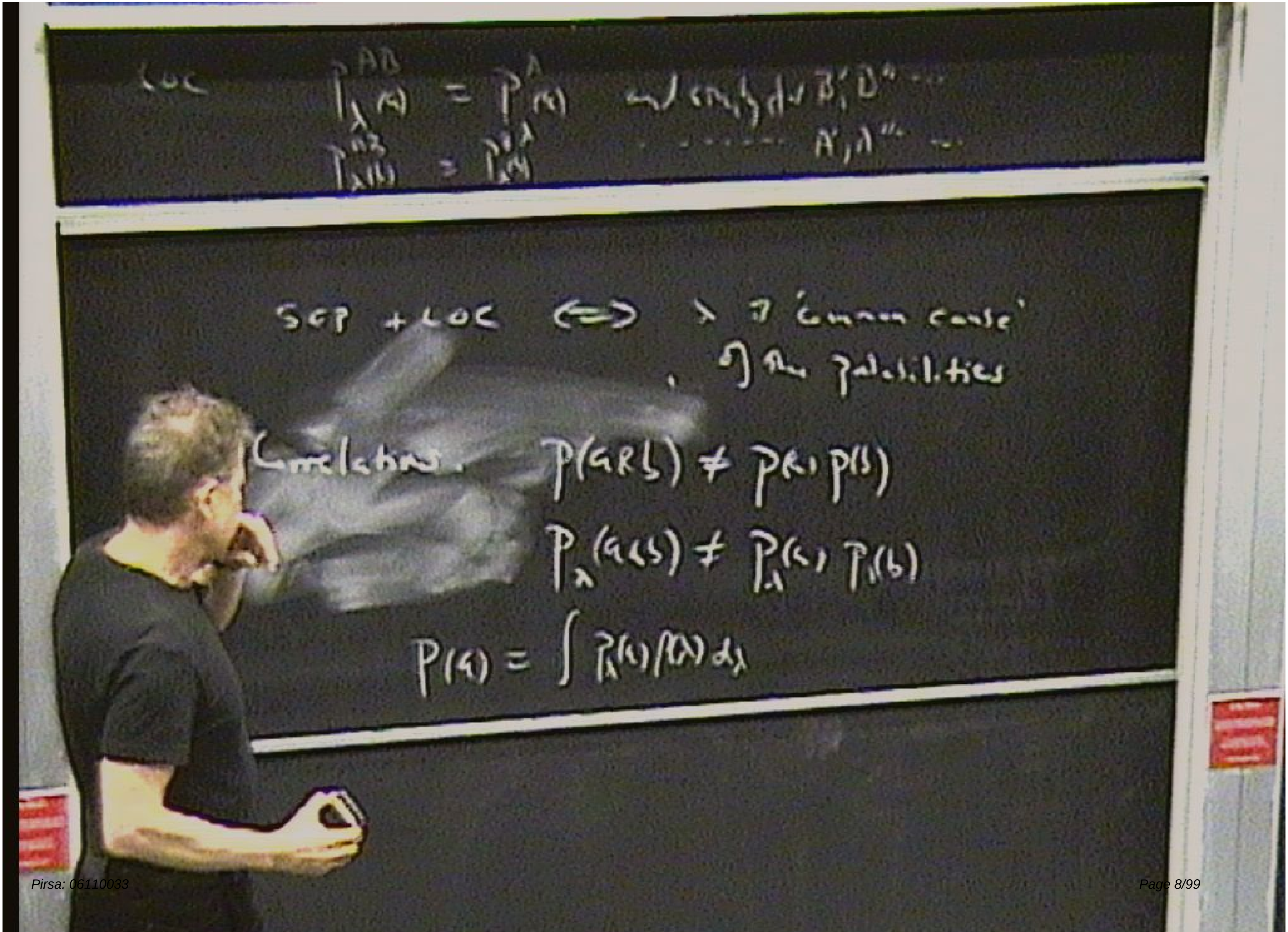

'being - thus'

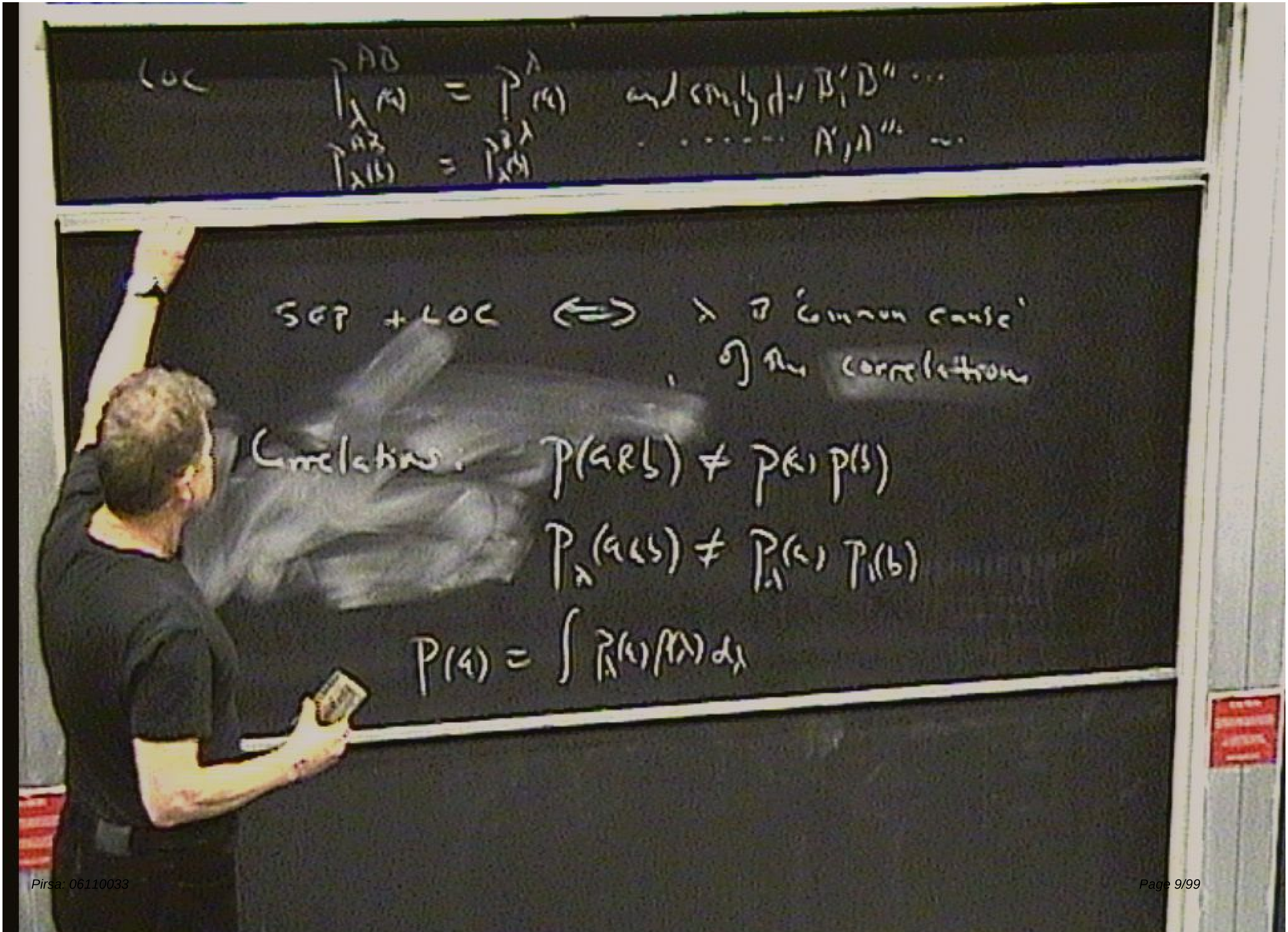

'being - thus'
SEPARABILITY
LOCALITY

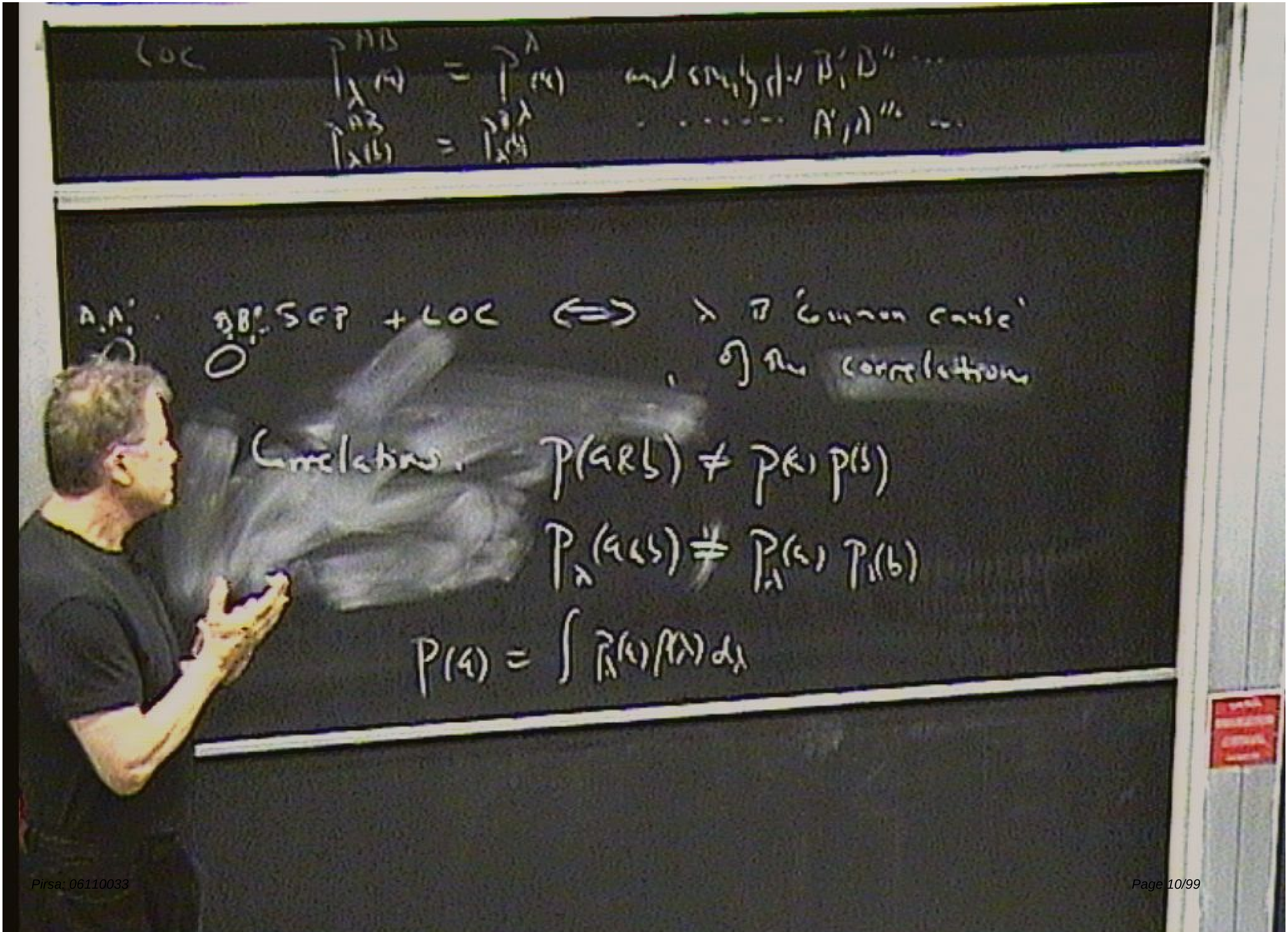

$A, A', \ldots$
$B, B', \ldots$
$P^{AB}_{\lambda}(a|b) = P^{AB}_{\lambda}(a)$
$P^{AB}_{\lambda}(b|a) = P^{AB}_{\lambda}(b)$
$P^{AB}_{\lambda}(a) = P^{A}_{\lambda}(a)$

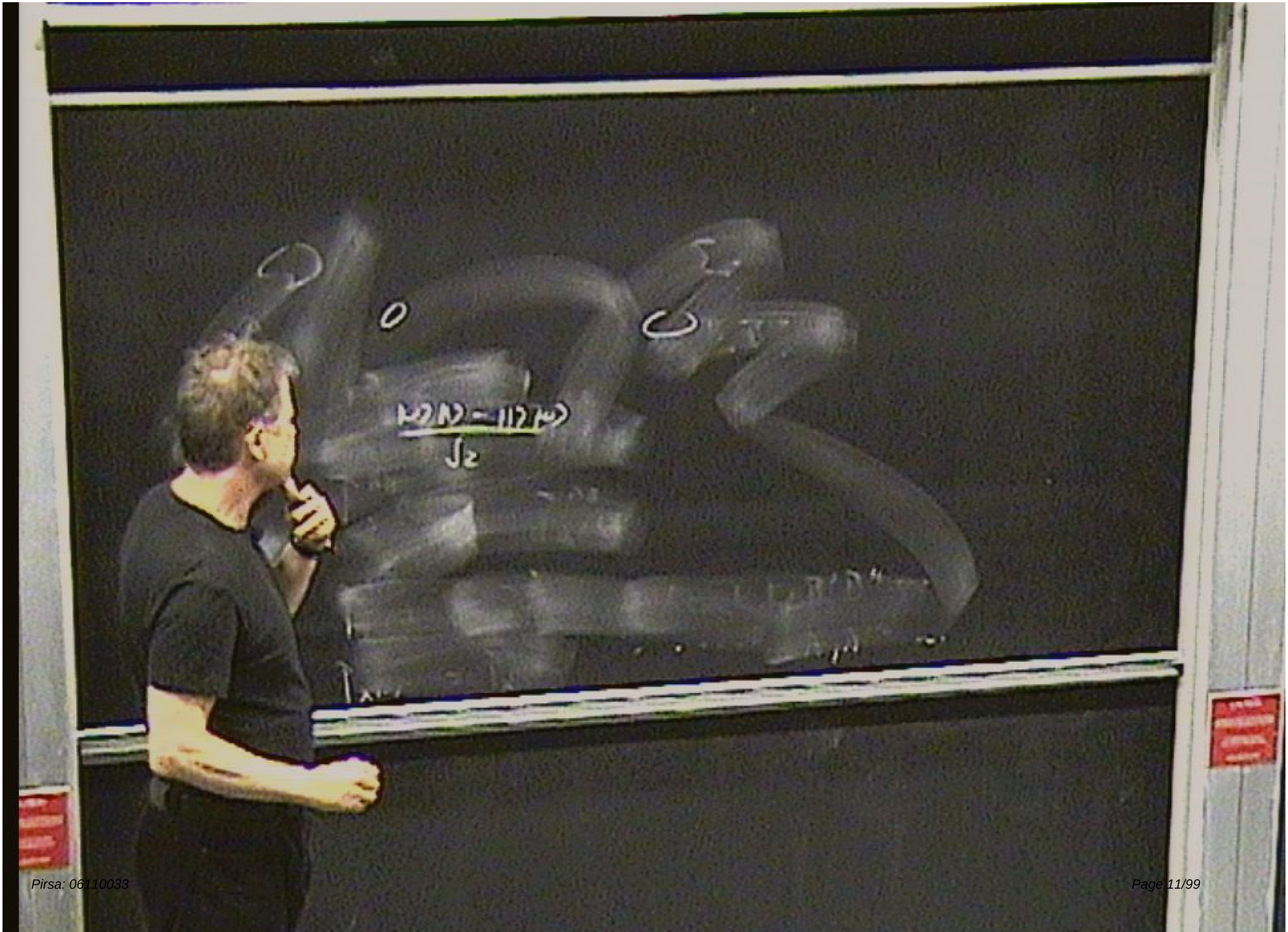

A, A', ...
B, B', ...

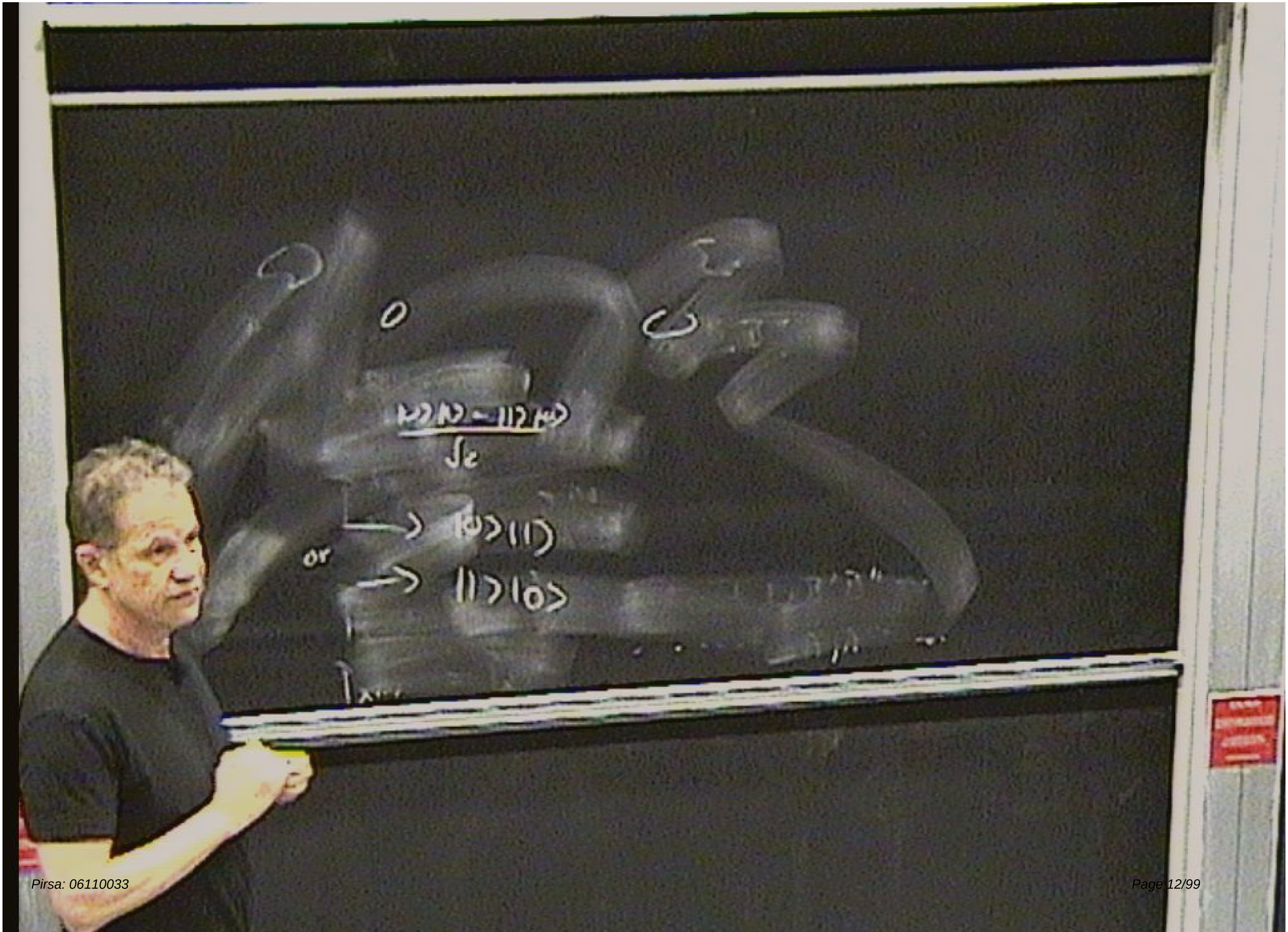

LOC
$p^{AB}_{\lambda}(A) = p^{A\lambda}(A)$ and similarly for B', B'' ...
$p^{AB}_{\lambda}(B) = p^{B\lambda}_{\lambda}(B)$ ...... A', A'' ...
SEP + LOC ⟺ λ 'common cause' of the probabilities

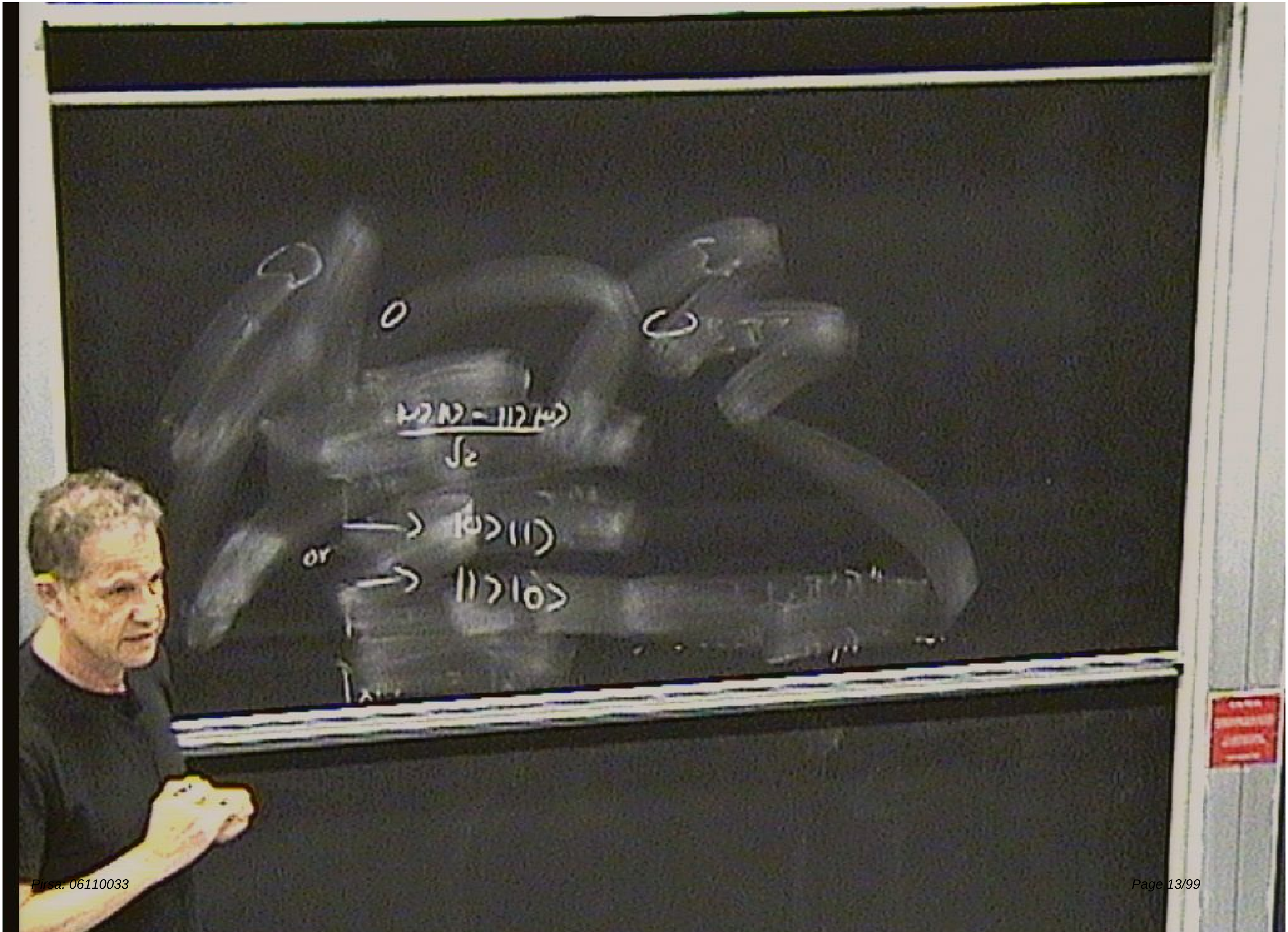

LOC $P^{AB}_\lambda(a) = P^{A}_\lambda(a)$
$P^{AB}_\lambda(b) = P^{B}_\lambda(b)$
SCP + LOC ⟺ ∃ 'common cause'
of the probabilities
Correlations: $P(a\&b) \neq P(a)\,P(b)$
$P_\lambda(a\&b) \neq P_\lambda(a)\,P_\lambda(b)$
$P(a) = \int P_\lambda(a)\,P(\lambda)\,d\lambda$

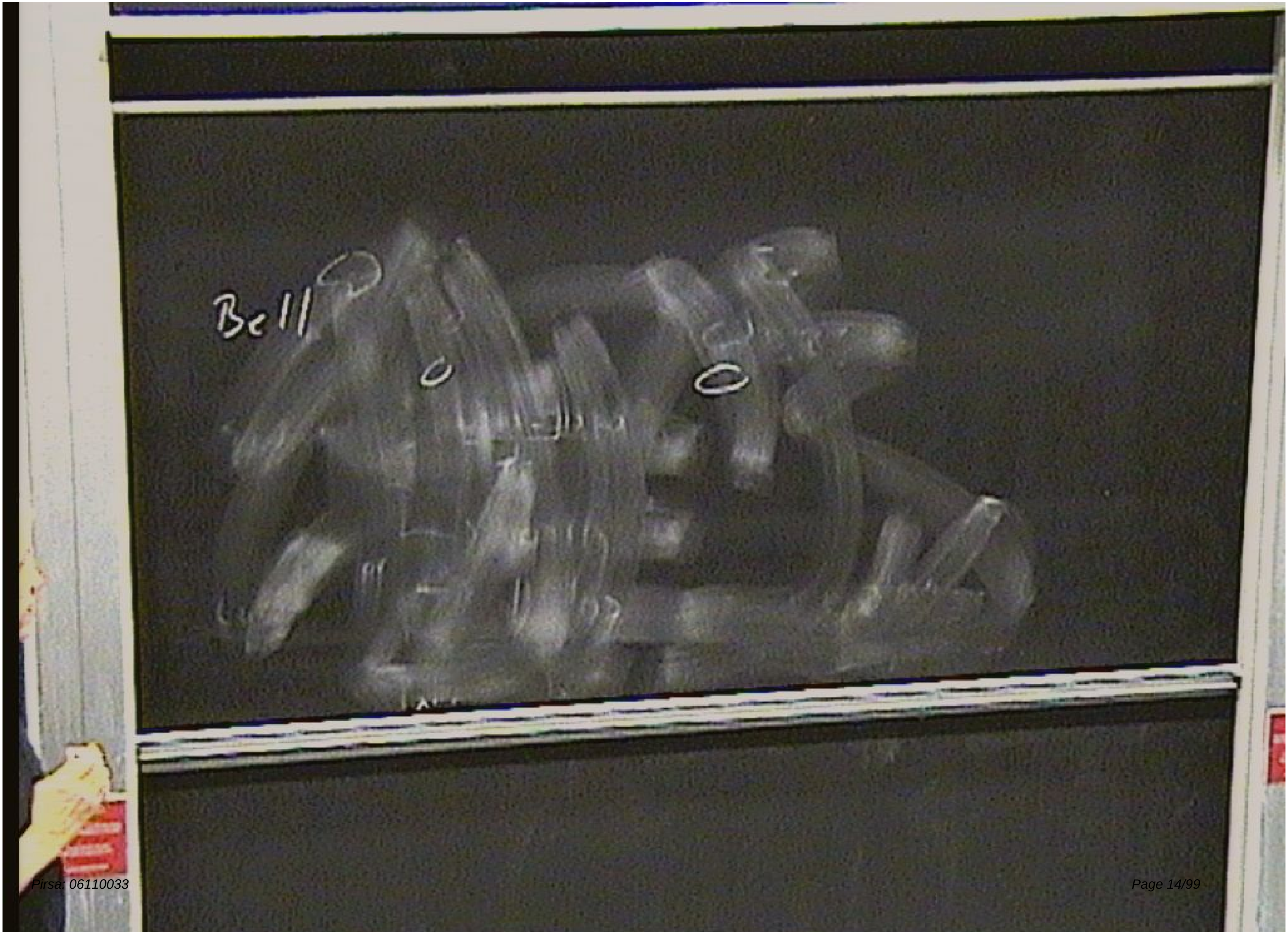

LOC $P^{AB}_{\lambda}(A) = P^{\lambda}(A)$ and similarly for B', B'' ...
$P^{AB}_{\lambda}(B) = P^{B\lambda}_{\lambda}(B)$ .... A', A'' ...
SEP + LOC $\Longleftrightarrow$ λ is 'common cause' of the correlations
Correlations: $P(a\&b) \neq P(a)\,P(b)$
$P_\lambda(a\&b) \neq P_\lambda(a)\,P_\lambda(b)$
$P(a) = \int P_\lambda(a)\,\rho(\lambda)\,d\lambda$

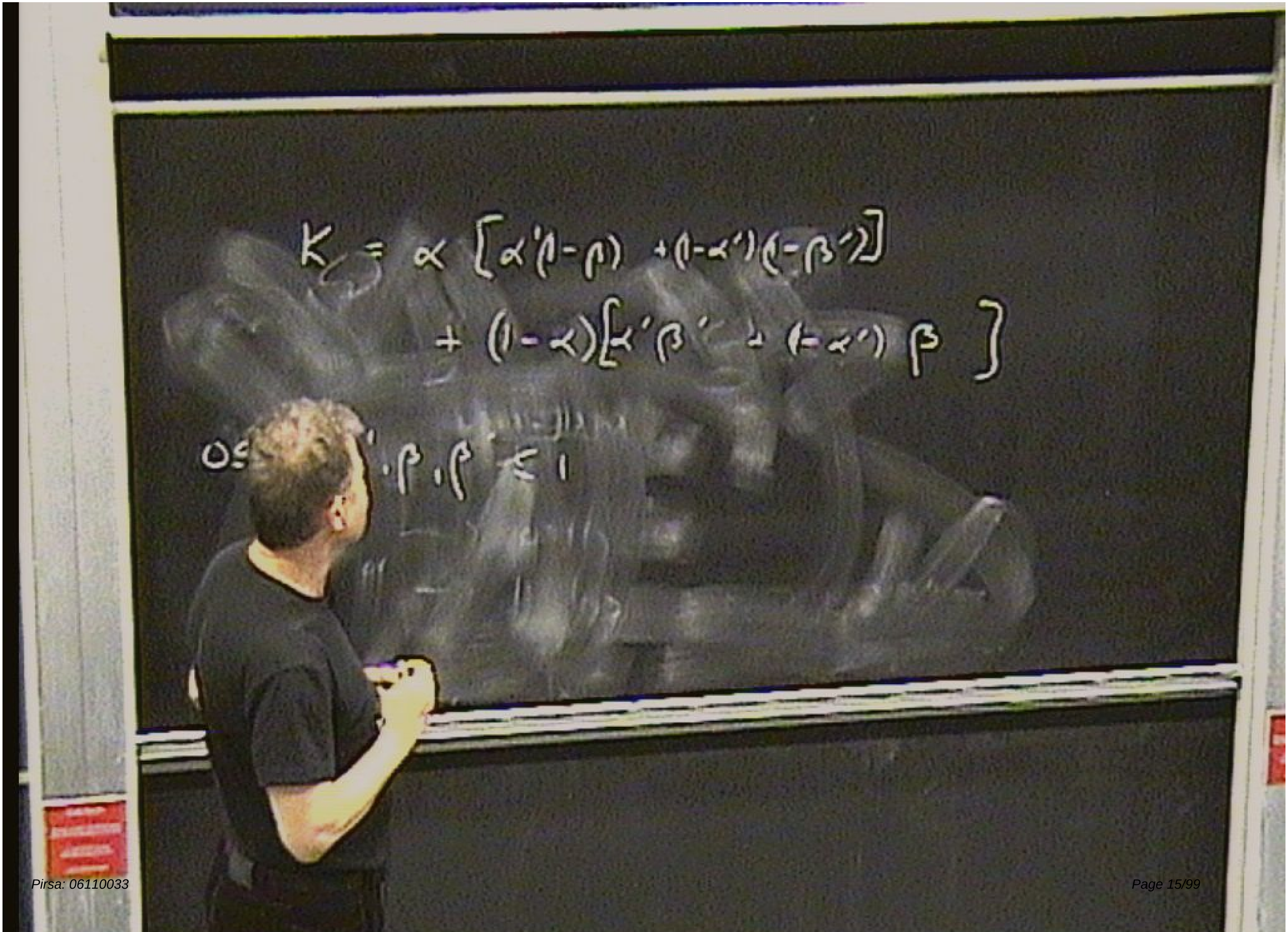

LOC
SEP + LOC
'common cause'
of the correlations
Correlations.

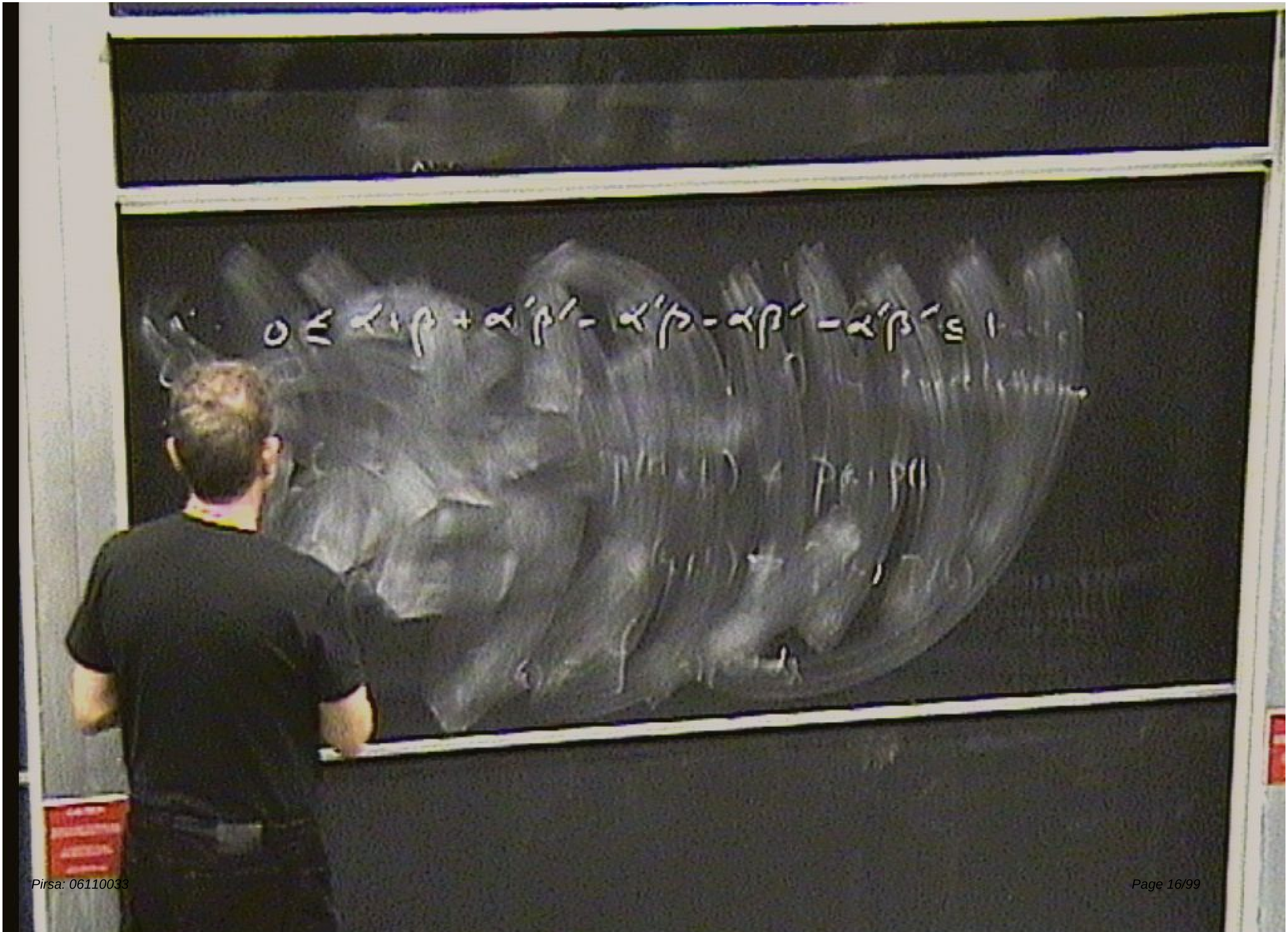


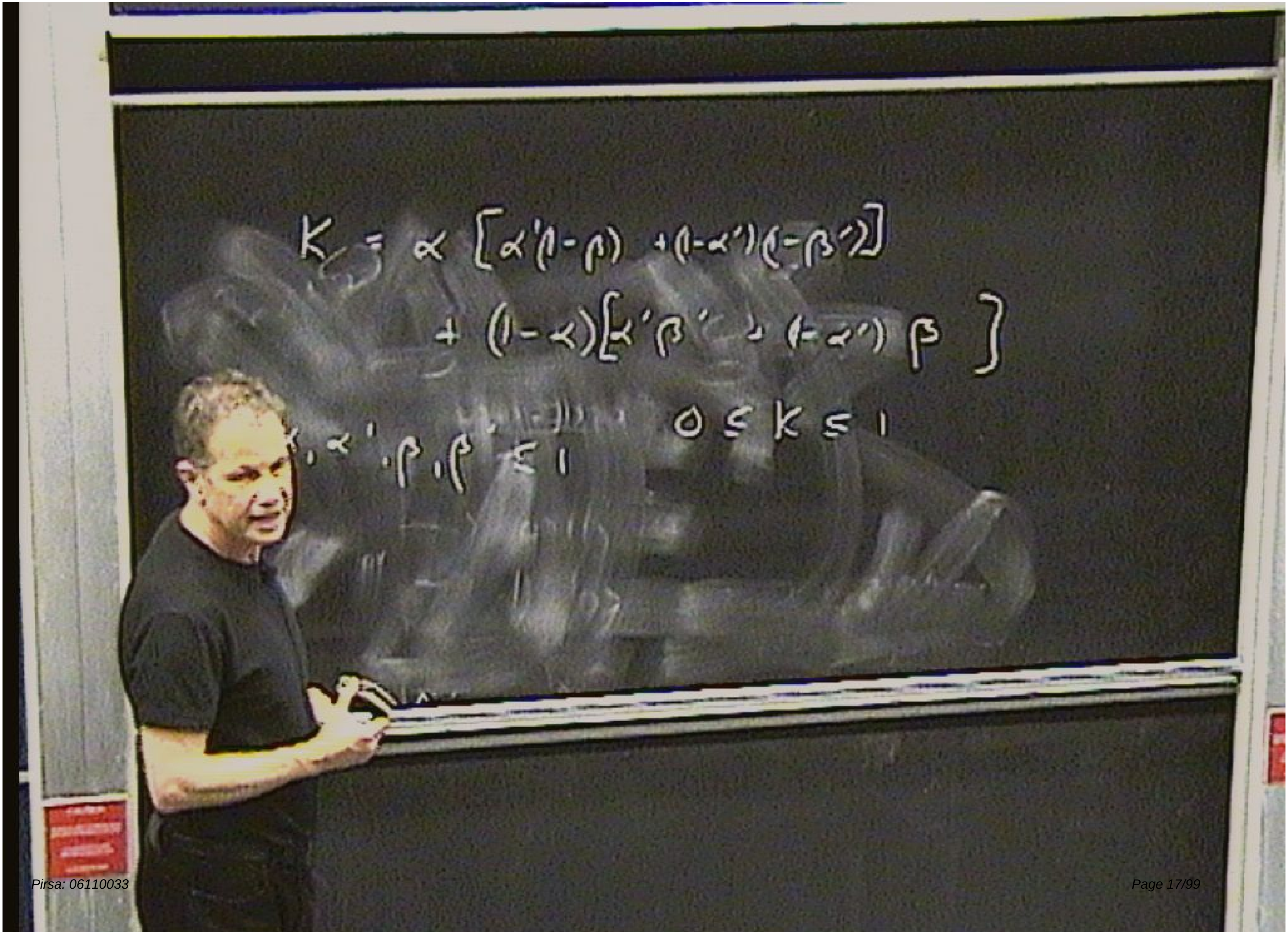

|0>|1>
or
|1>|0>


or
|0>|1>
|1>|0>


Bell


K = α [α'(1-β) + (1-α')(1-β')]
+ (1-α)[α'β' + (1-α') β]
0 ≤ ·β, β' ≤ 1

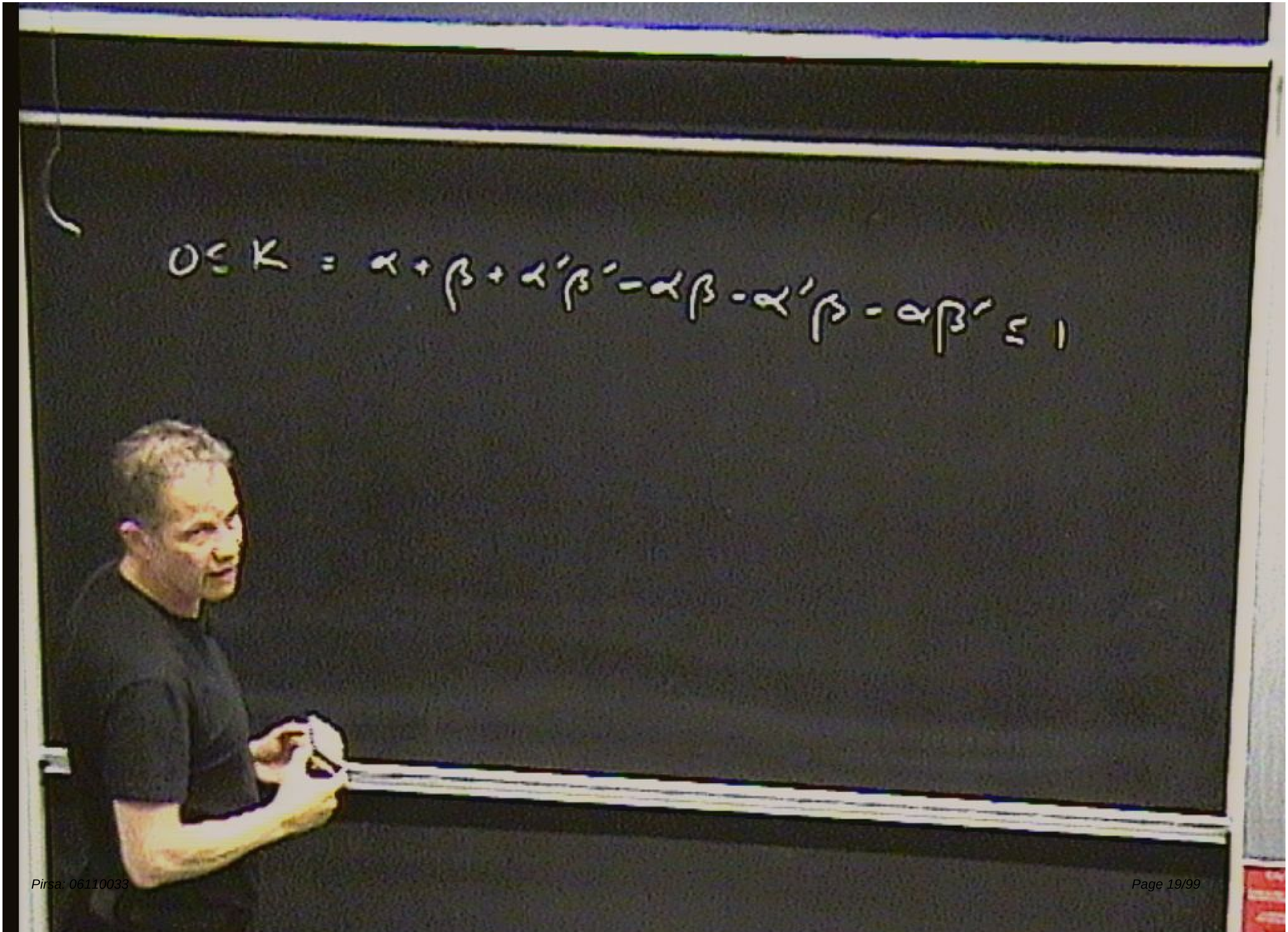

0 ≤ α+β+α'β' − α'β − αβ' − α'β' ≤ 1

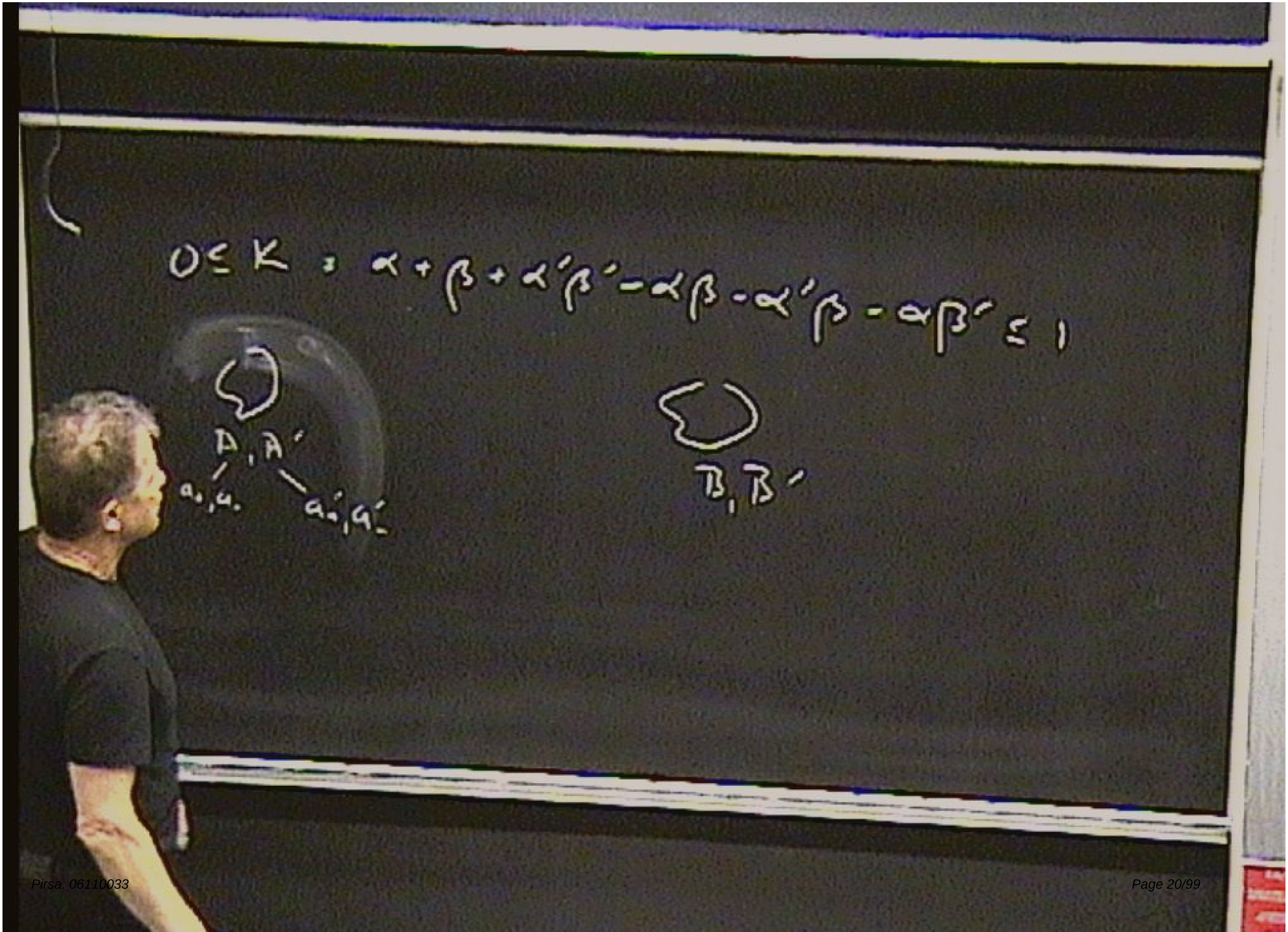

K = α [α'(1-β) + (1-α')(1-β')]
+ (1-α)[α'β' + (1-α') β]
0 ≤ K ≤ 1
α, α', β, β' ≤ 1

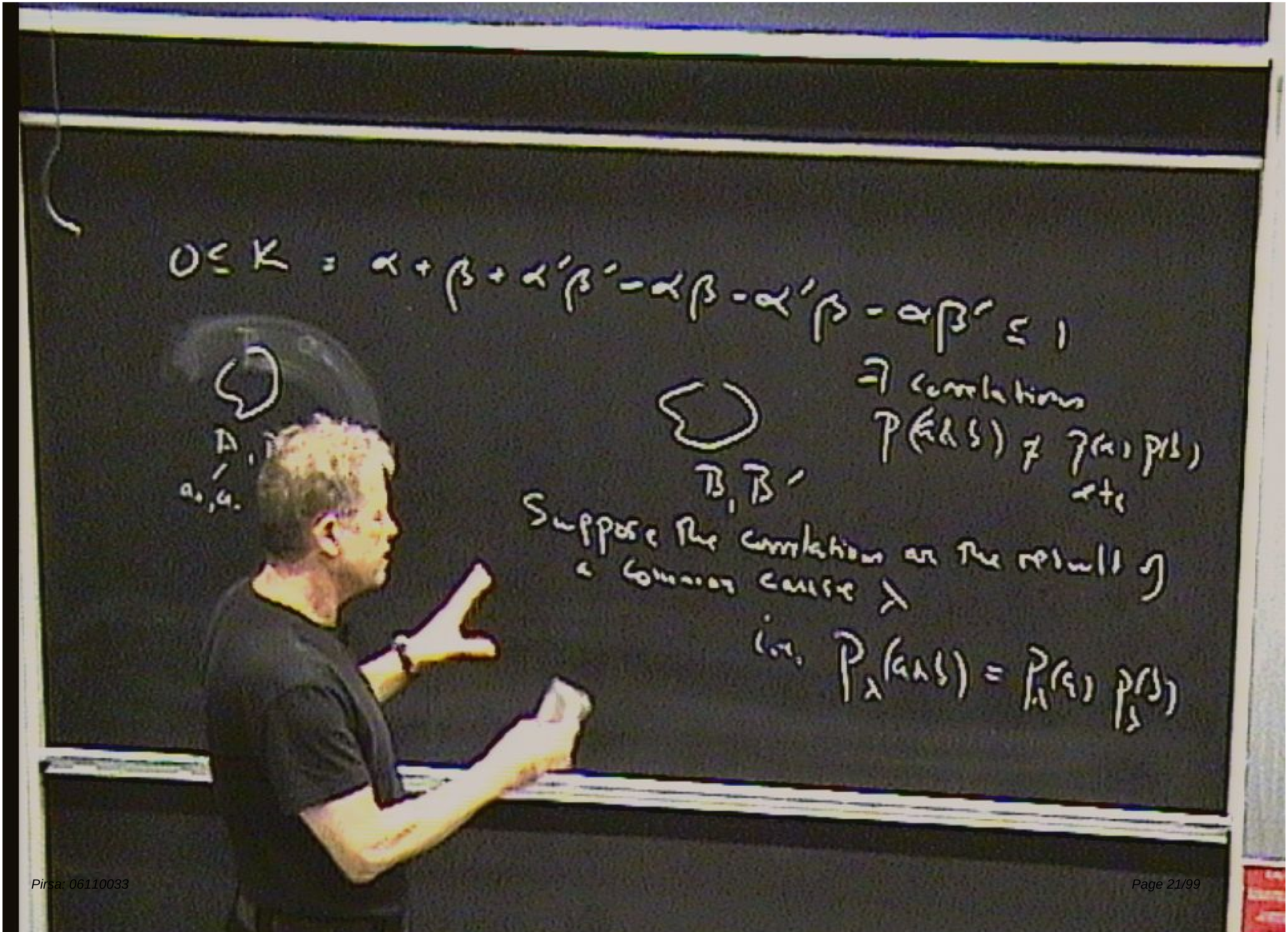
$$K = \alpha\left[\alpha'(1-\beta) + (1-\alpha')(1-\beta')\right]$$
$$+ (1-\alpha)\left[\alpha'\beta' + (1-\alpha')\beta\right]$$

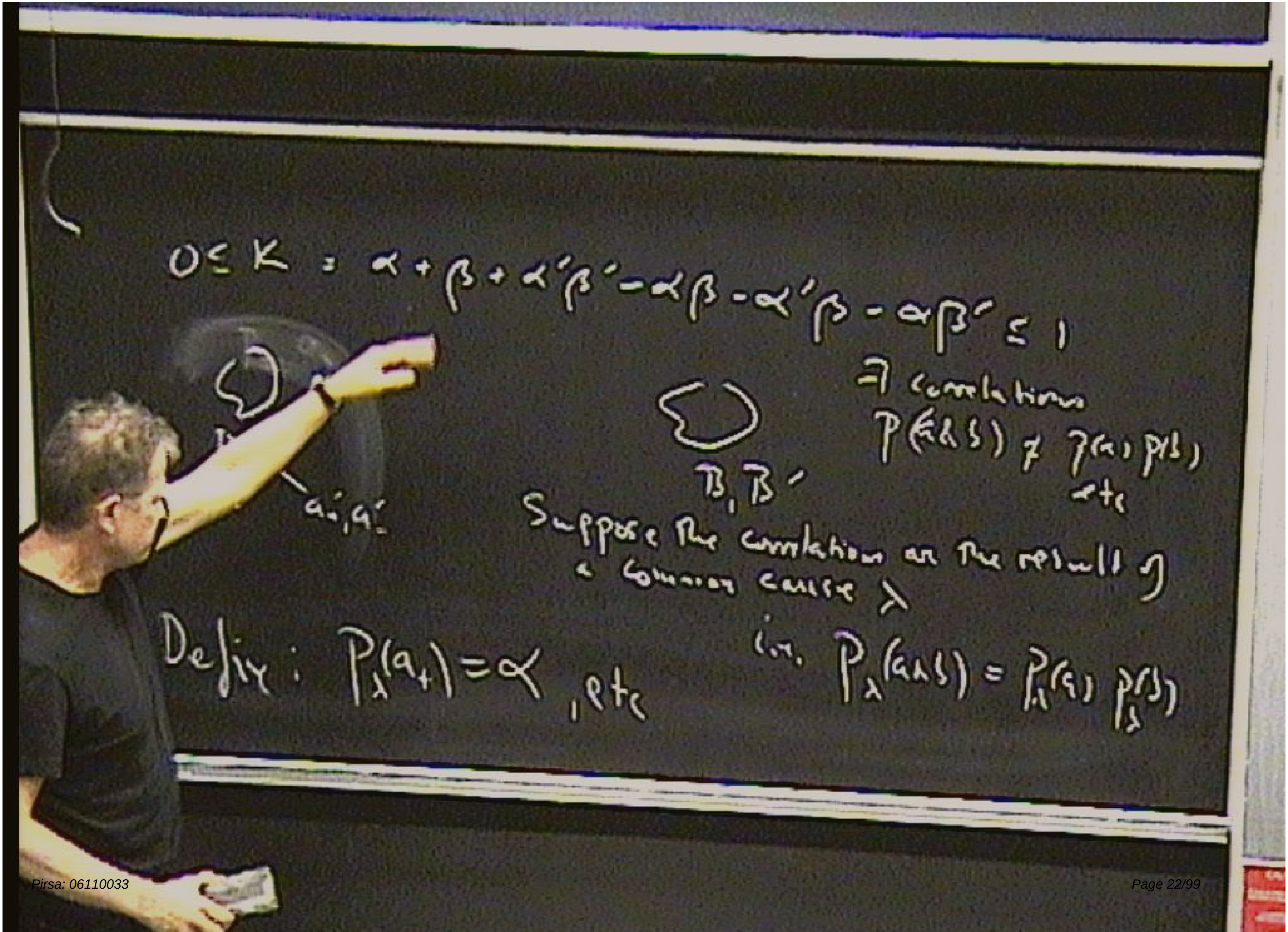
$$0 \le K \le 1$$

$$0 \le \alpha, \alpha', \beta, \beta' \le 1$$


$0 \leq K = \alpha + \beta + \alpha'\beta' - \alpha\beta - \alpha'\beta - \alpha\beta' \leq 1$


0 ≤ K = α + β + α'β' − αβ − α'β − αβ' ≤ 1
A, A'
a0, a1
a0', a1'
B, B'


0 ≤ K = α + β + α'β' − αβ − α'β − αβ' ≤ 1
∃ correlations
p(a∧b) ≠ p(a) p(b), etc
A, A'
B, B'
Suppose the correlations are the result of a common cause λ
i.e. p_λ(a∧b) = p_λ(a) p_λ(b)


$0 \le K = \alpha + \beta + \alpha'\beta' - \alpha\beta - \alpha'\beta - \alpha\beta' \le 1$
$\exists$ correlations
$P(a \wedge b) \neq p(a)\,p(b)$
etc
B, B'
a_+', a_-'
Suppose the correlations are the result of a common cause λ
i.e. $P_\lambda(a \wedge b) = P_\lambda(a)\,p_\lambda(b)$
Define: $P_\lambda(a_+) = \alpha$, etc

$$0 \le K = \alpha + \beta + \alpha'\beta' - \alpha\beta - \alpha'\beta - \alpha\beta' \le 1$$

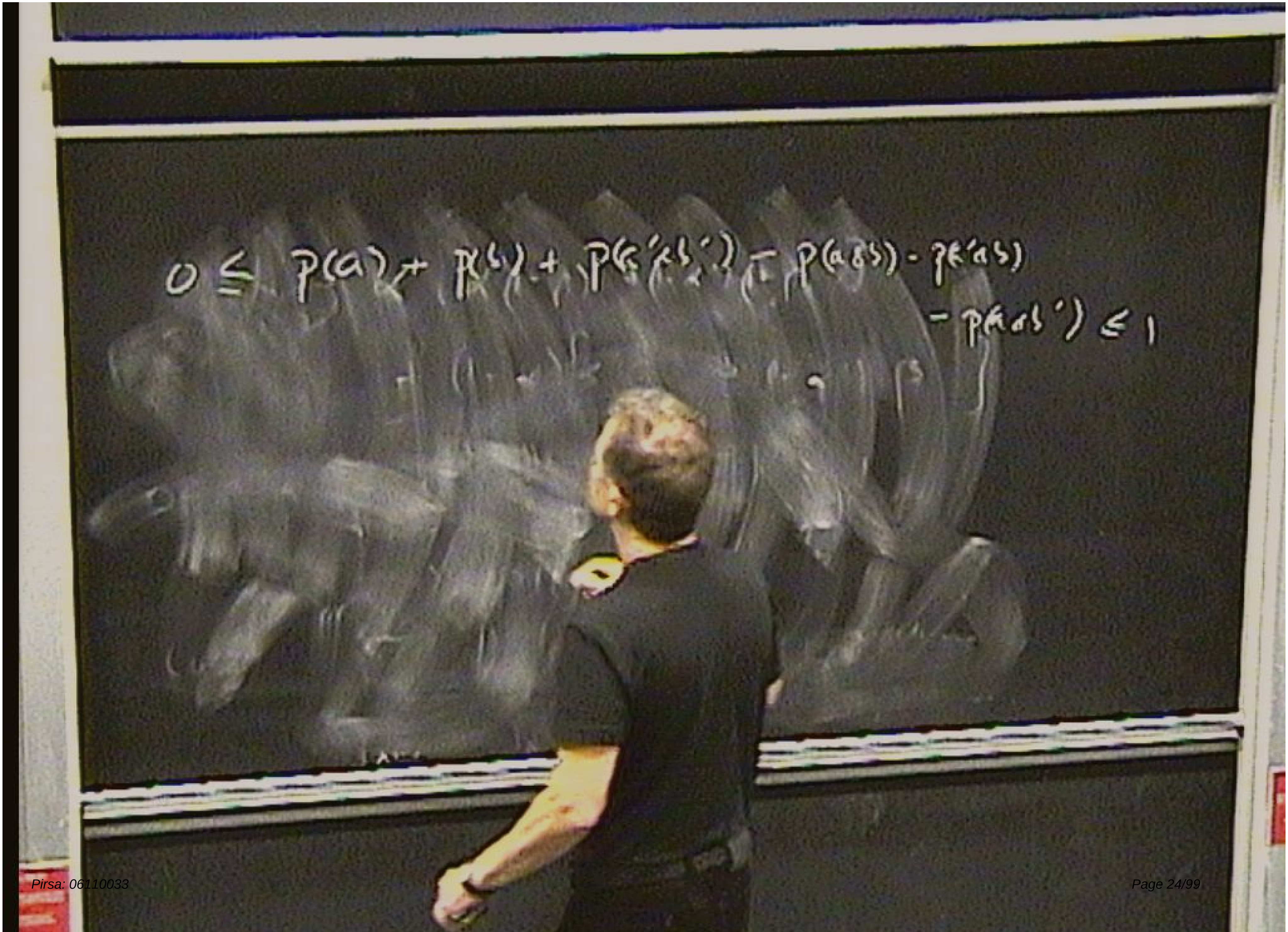
$\exists$ correlations

$P(a \wedge b) \neq P(a)\,P(b)$, etc

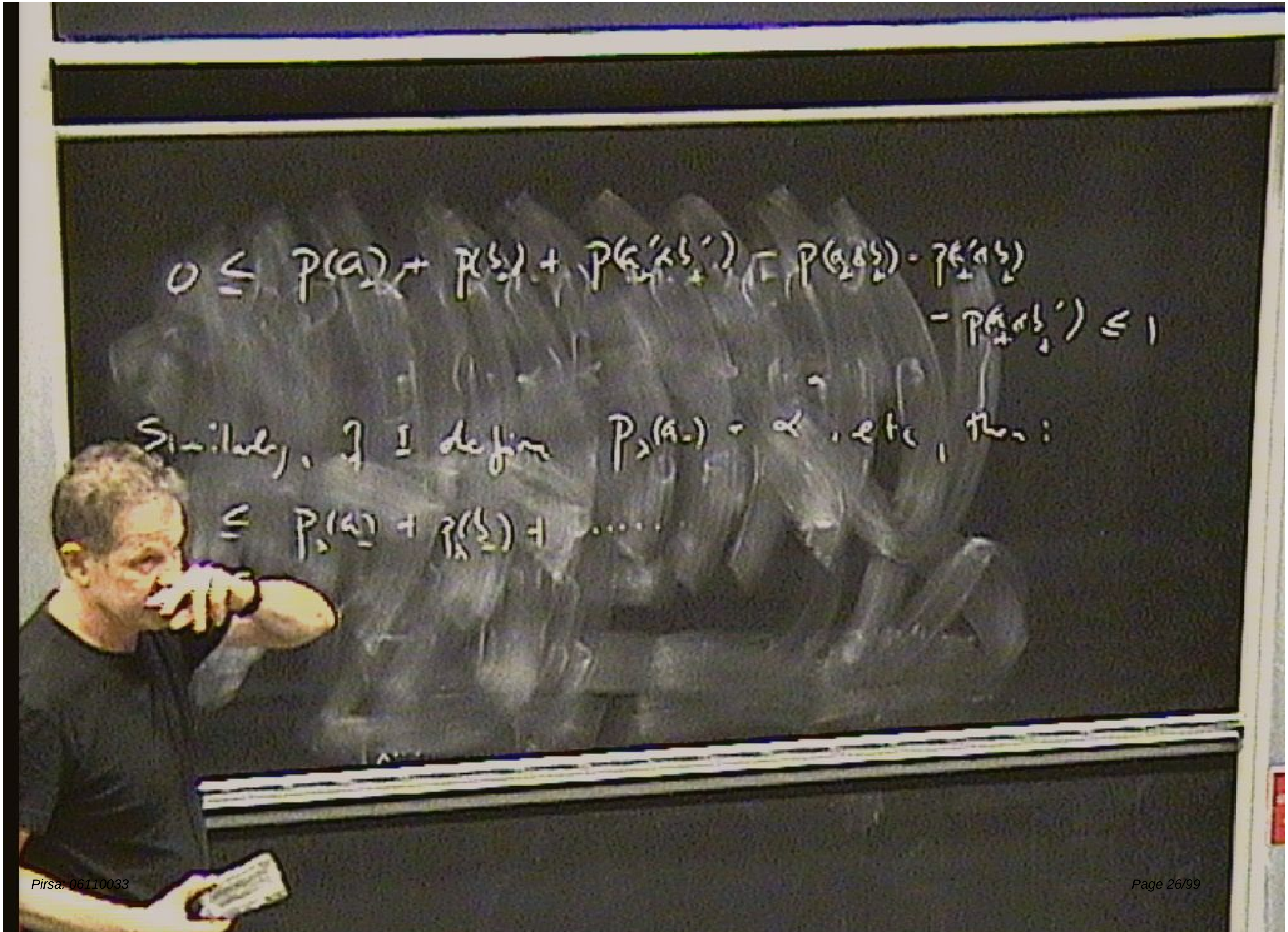
Suppose the correlations are the result of a common cause λ

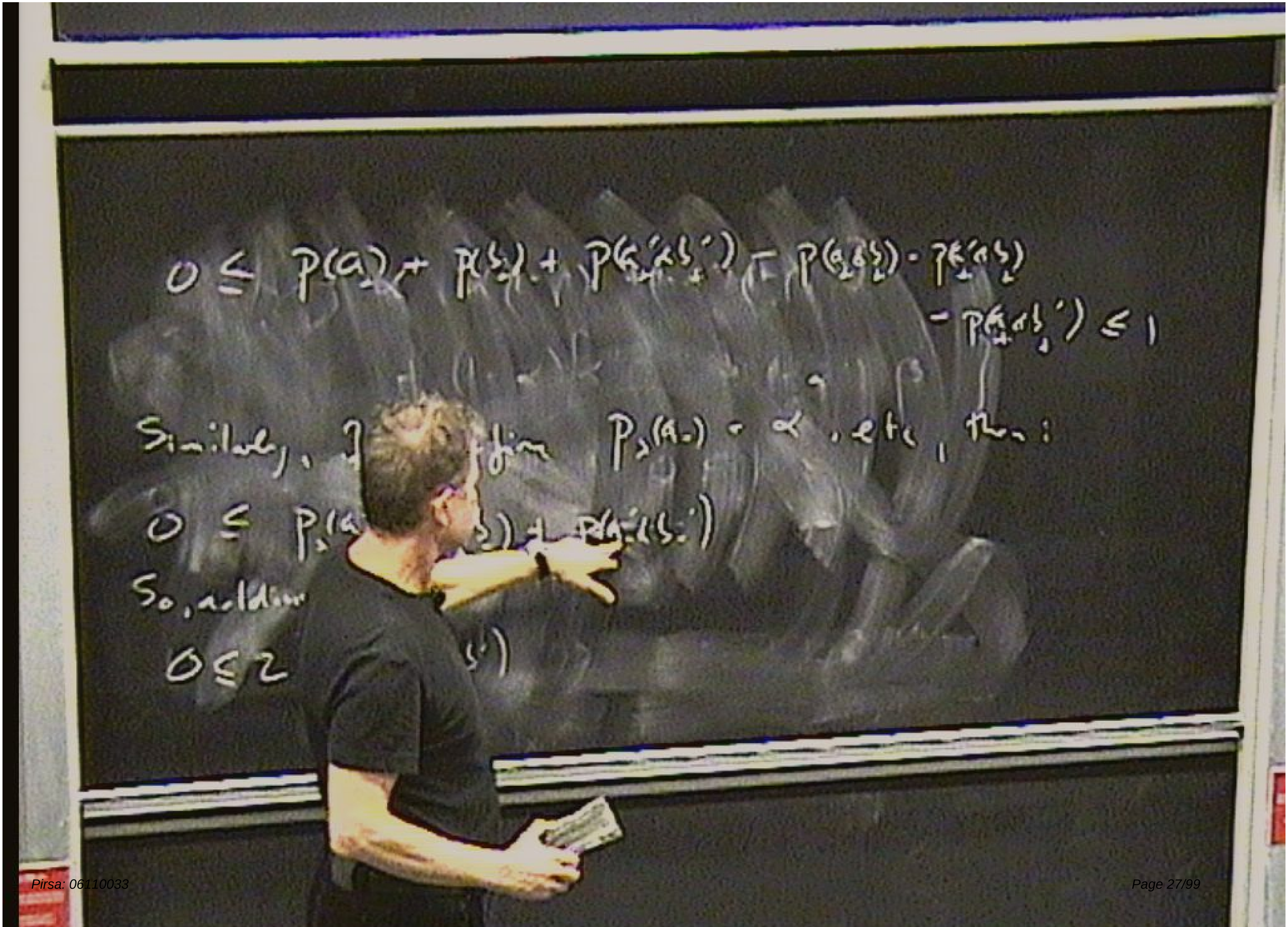
i.e. $P_\lambda(a \wedge b) = P_\lambda(a)\,P_\lambda(b)$

Define: $P_\lambda(a_+) = \alpha$, etc


$0 \leq p(a) + p(b) + p(a' \& b') - p(a \& b) - p(a' \& b)$
$- p(a \& b') \leq 1$

$$0 \leq p(a_2) + p(b_2) + p(a_2' \wedge b_2') - p(a_2 \wedge b_2) - p(a_2' \wedge b_2) - p(a_2 \wedge b_2') \leq 1$$


0 ≤ P(a_1) + P(b_1) + P(a_1' ∧ b_1') − P(a_1 ∧ b_1) − P(a_1 ∧ b_1)
− P(a_1' ∧ b_1') ≤ 1
Similarly, if I define P_λ(a_1) = α, etc, then:
≤ P_λ(a_1) + P_λ(b_1) + ...


0 ≤ P(a) + P(b) + P(a' ∧ b') − P(a ∧ b) − P(a' ∧ b)
− P(a ∧ b') ≤ 1
Similarly, if ... P₂(a) = α, etc., then:
0 ≤ P₂(a ... b) + P(a' ∧ b')
So, adding
0 ≤ 2

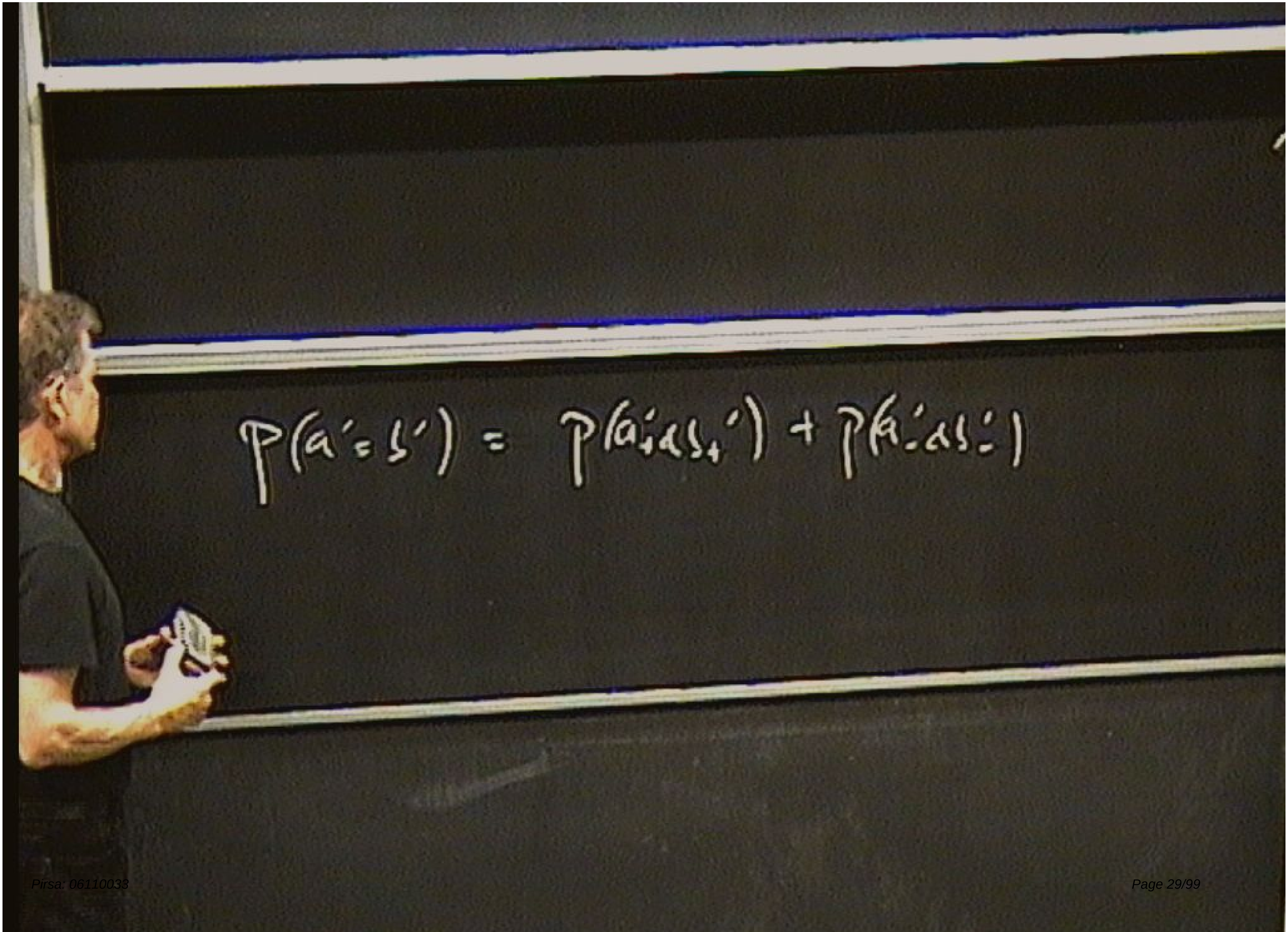
$$0 \leq p(a_2) + p(b_2) + p(a_2' \wedge b_2') - p(a_2 \wedge b_2) - p(a_2' \wedge b_2) - p(a_2 \wedge b_2') \leq 1$$

Similarly, if I define $p_2(a_-) = \alpha$, etc., then:

$$0 \leq p_2(a_-) + p_2(b_-) + p(a_-' \wedge b_-')$$

So, adding:

$$0 \leq 2 + p(a' \wedge b')$$


$p(a'=b') = p(a'_+ \wedge b'_+) + p(a'_- \wedge b'_-)$

$$0 \leq P(a_2) + P(b_2) + P(a_1' \wedge b_1') - P(a_2 \wedge b_2) - P(a_1' \wedge b_2) - P(a_1 \wedge b_1') \leq 1$$

Similarly, if I define $P_3(a_2) = \alpha$, etc, then:

$$0 \leq P_3(a_2) + P(b_2) + P(a_2' \wedge b_2')$$

So, adding:

$$0 \leq 2 + P(a' \wedge b') + P(a \wedge b) - P(a' \wedge b) - P(a \wedge b') \leq 2$$

$$0 \le P(a_-) + P(b_-) + P(a'_+ \wedge b'_+) - P(a_- \wedge b_-) - P(a'_+ \wedge b_-) - P(a_+ \wedge b'_+) \le 1$$

Similarly, if I define $P_2(a_-) = \alpha$, etc., then:

$$0 \le P_2(a_-) + P_2(b_-) + P_2(a'_- \wedge b'_-)$$

So, adding:

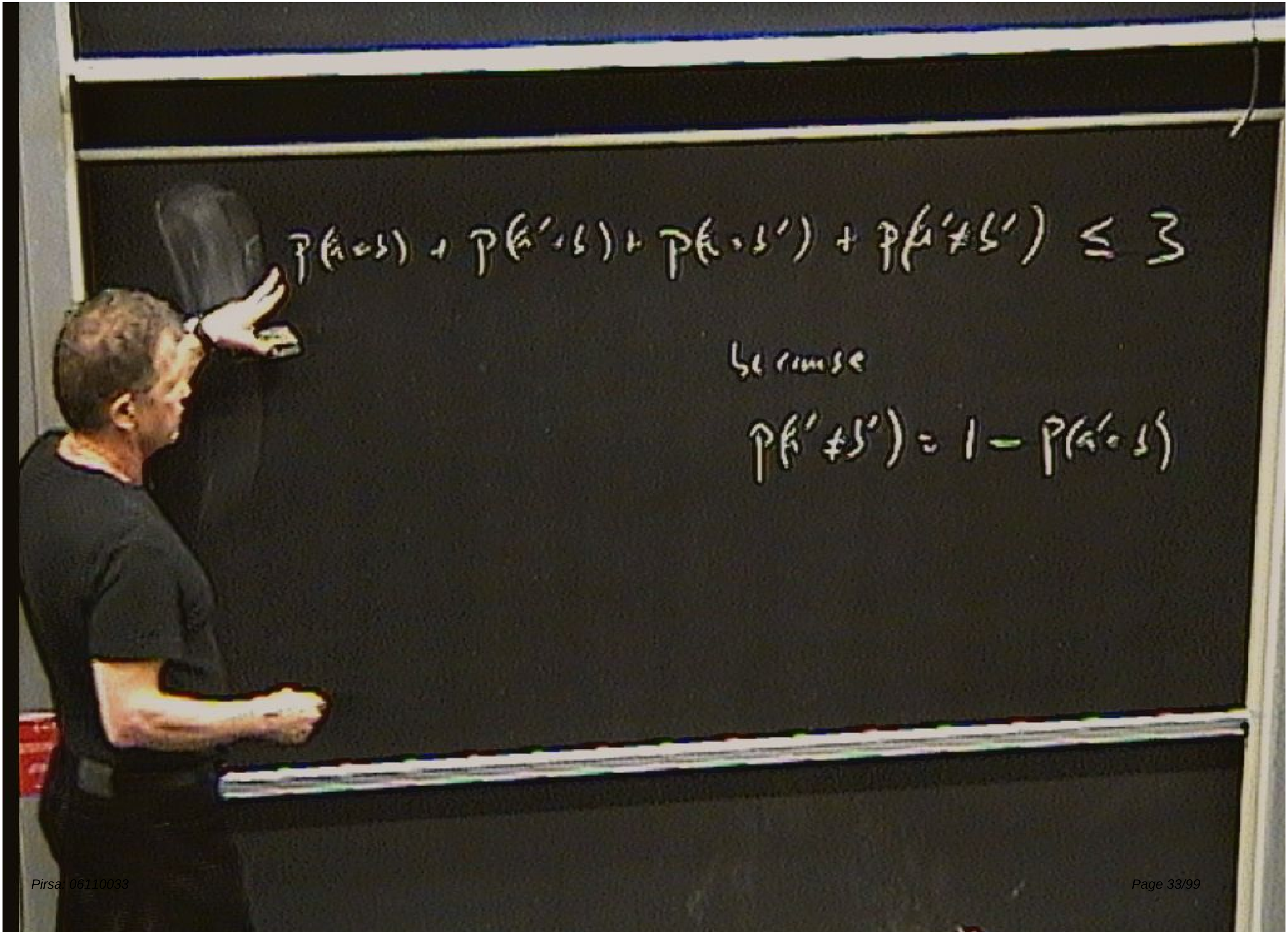
$$0 \le 2 + P(a' = b') + P(a = b) - P(a' = b) - P(a = b') \le 2$$

$$0 \le P(a = b) + P(a' = b) + P(a = b') - P(a' = b') \le 2$$

$$0 \leq p(a=b) + p(a'=b) + p(a=b') + p(a' \neq b') \leq 3$$

because

$$p(a' \neq b') = 1 - p(a'=b)$$


P(a=b) + P(a'=b) + P(a=b') + P(a'≠b') ≤ 3
because
P(a'≠b') = 1 − P(a'=b')

$$p(a=b) + p(a'=b) + p(a=b') + p(a'\neq b') \leq 3$$

because

$$p(a'\neq b') = 1 - p(a'=b')$$

Bell's inequality in a (revised) Clauser-Horne-Shimony-Holt form

$$p(a=b) + p(a'=b) + p(a=b') + p(a'\neq b') \leq 3$$

because

$$p(a'\neq b') = 1 - p(a'=b')$$

Bell's inequality in a (rubBed) Clauser-Horne-Shimony-Holt form

$$E(AB) + E(A'B') + (AB') - E(A'B') \leq 2$$

$$p(a=b) + p(a'=b) + p(a=b') + p(a'\neq b') \leq 3$$

because

$$p(a'\neq b') = 1 - p(a'=b')$$

Bell's inequality in a (revised) Clauser-Horne-Shimony-Holt form

CHSH inequality: $E(AB) + E(A'B') + (AB') - E(A'B') \leq 2$

exp. value

$$\boxed{P(A=B) + P(A'=B) + P(A=B') + P(A' \neq B') \leq 3}$$

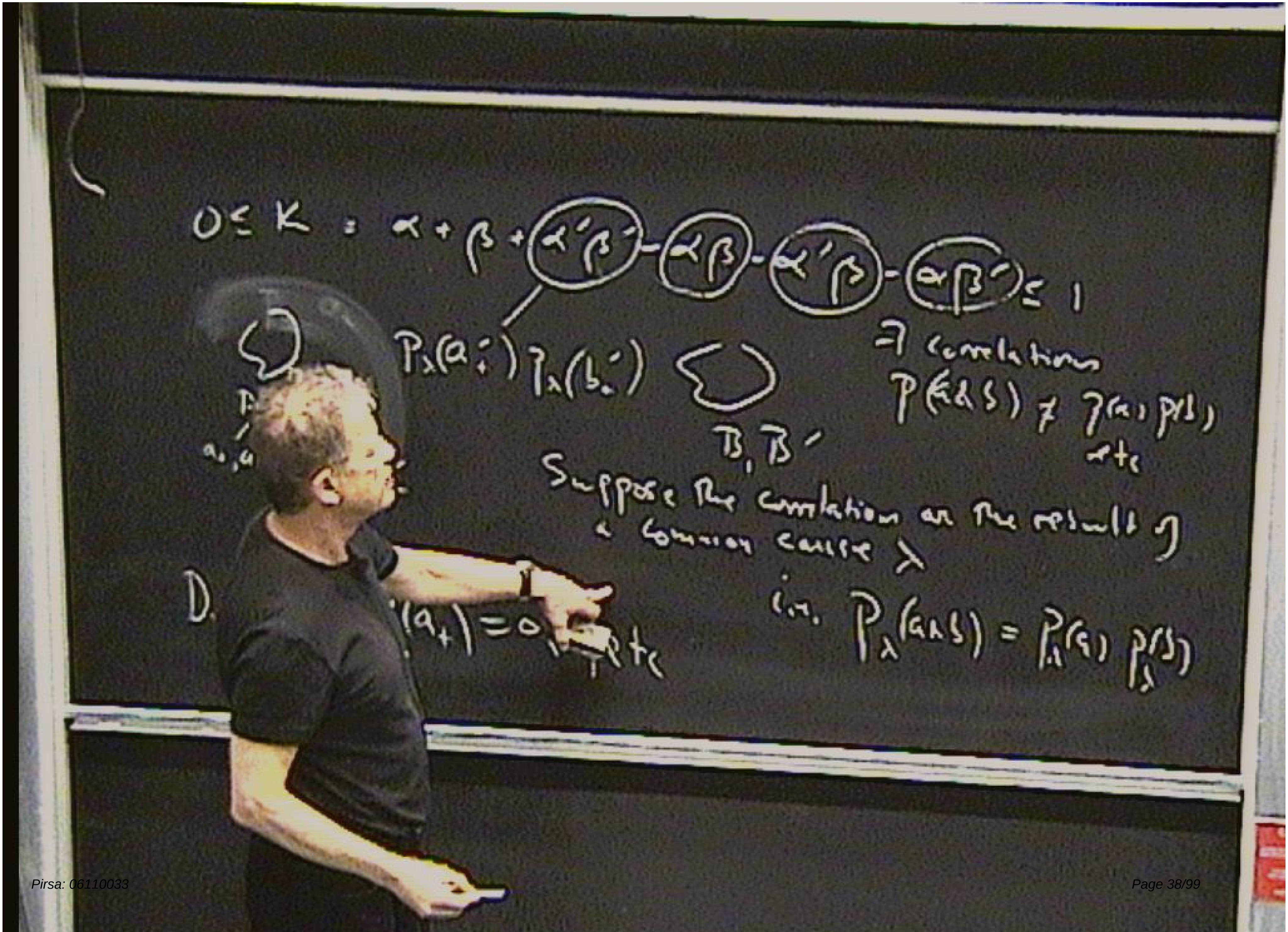
because

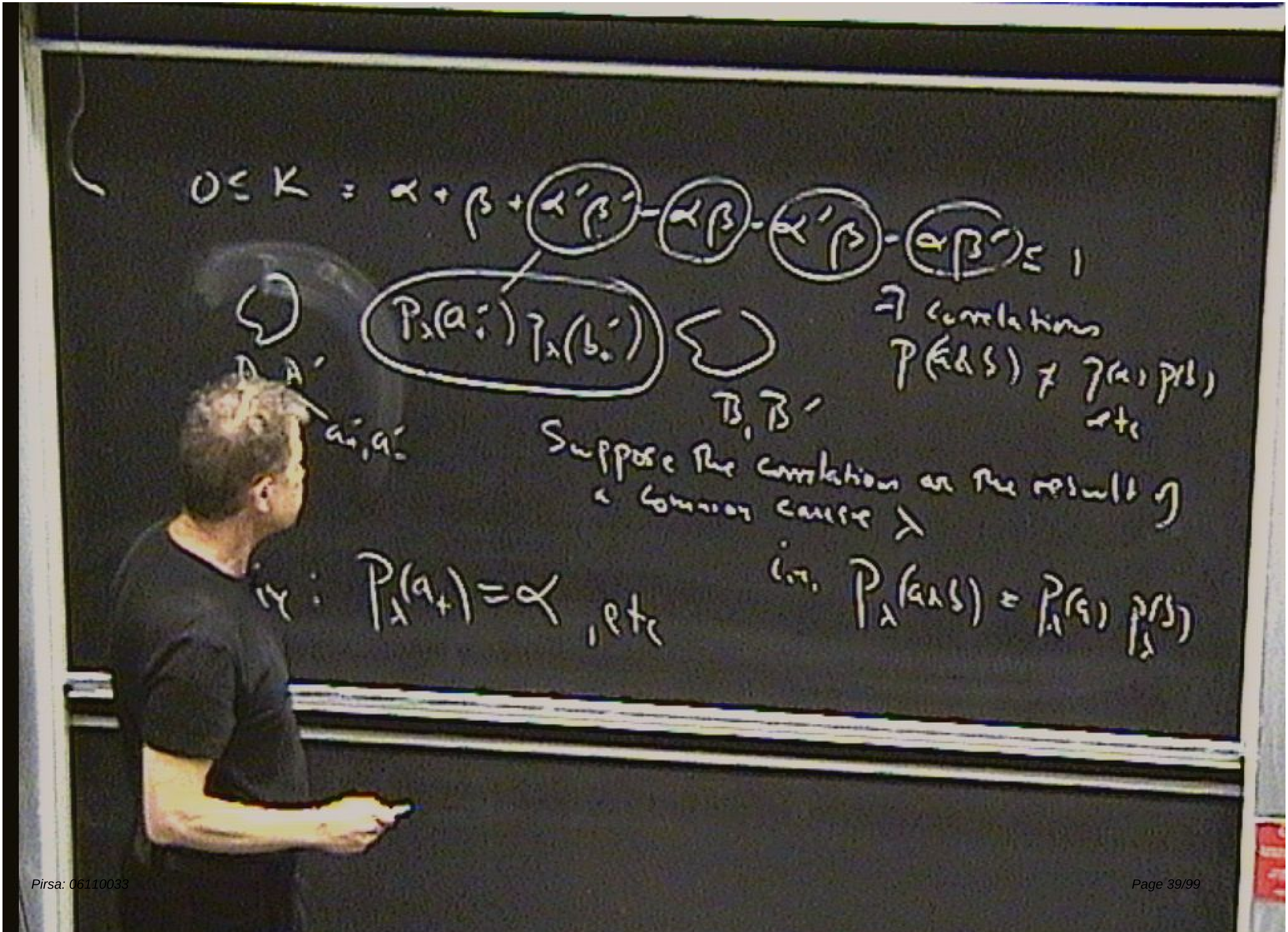
$$P(A' \neq B') = 1 - P(A' = B')$$

Bell's inequality in a (revised) Clauser-Horne-Shimony-Holt form

CHSH inequality: $E(AB) + E(A'B') + (AB') - E(A'B') \leq 2$

exp. value


0 ≤ K = α + β + α′β′ − αβ − α′β − αβ′ ≤ 1
Pλ(a+) Pλ(b+)
B, B′
∃ correlations
P(a & b) ≠ P(a) P(b)
etc
Suppose the correlations are the result of a common cause λ
i.e. Pλ(a&b) = Pλ(a) Pλ(b)
D.
(a+) = 0 , etc


0 ≤ K = α + β + α'β' − αβ − α'β − αβ' ≤ 1
P_λ(a_+') P_λ(b_+')
A, A'
a_+, a_-
B, B'
∃ correlations
P(a∧b) ≠ P(a) P(b), etc
Suppose the correlations are the result of a common cause λ
eg: P_λ(a_+) = α, etc
i.e. P_λ(a∧b) = P_λ(a) P_λ(b)

$$0 \le p(a_2) + p(b_2) + p(a_1' \wedge b_1') - p(a_2 b_2) - p(a_1 b_2) - p(a_1 b_1') \le 1$$

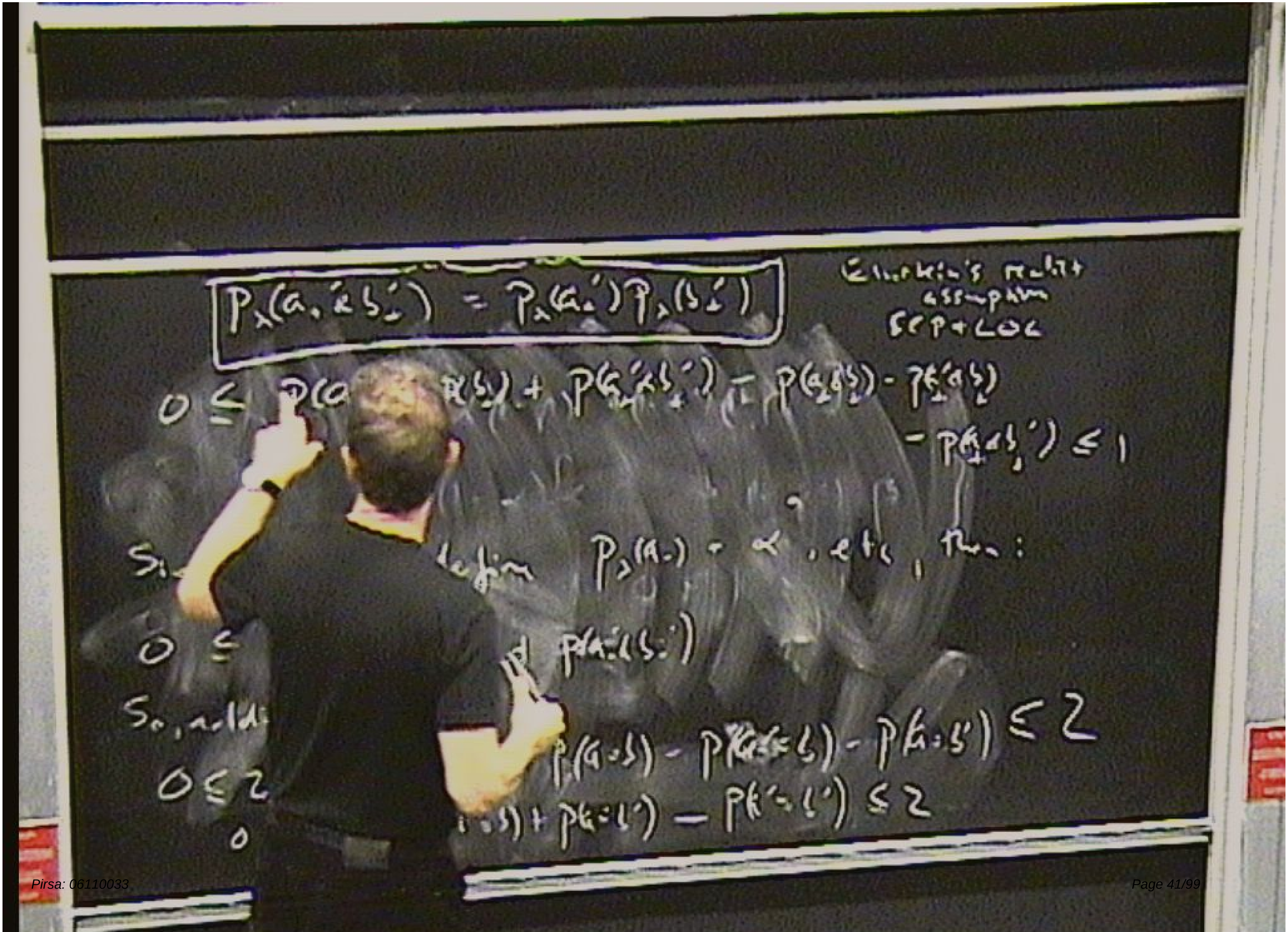
Similarly, if I define $p_\lambda(a_\cdot) = \alpha$, etc, then:

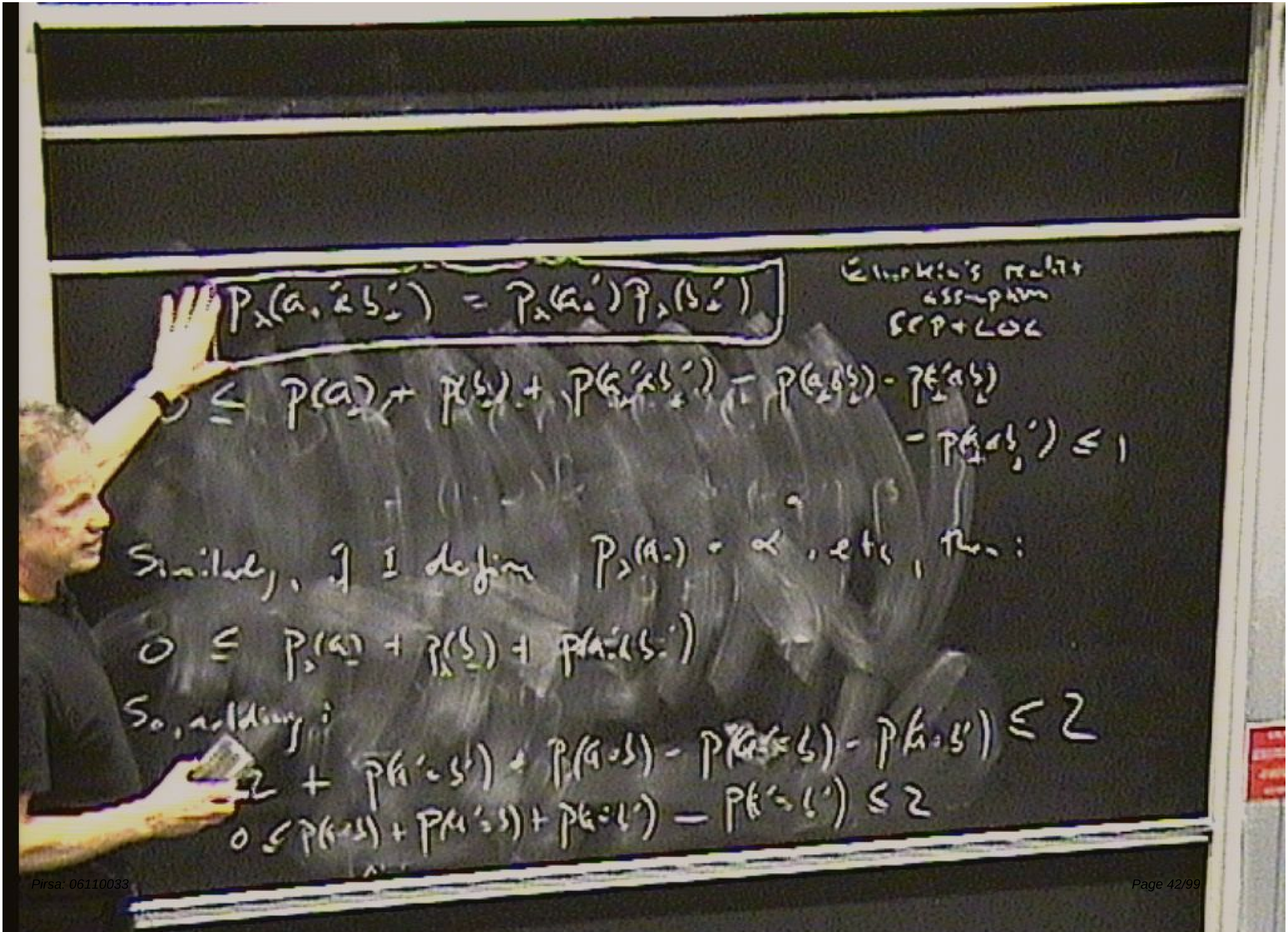
$$0 \le p_\lambda(a_2) + p_\lambda(b_2) + p_\lambda(a_1' \wedge b_1')$$

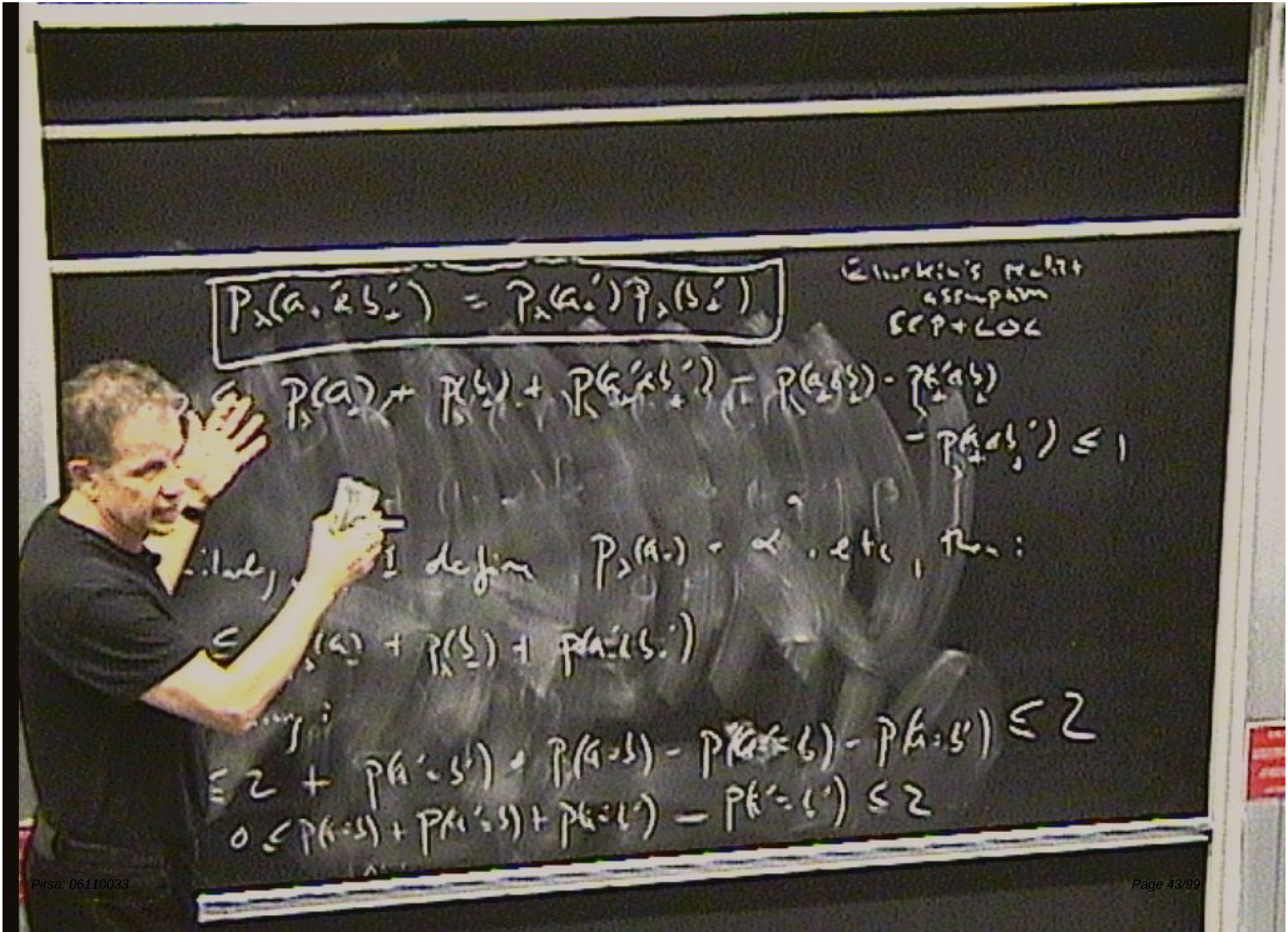
So, adding:

$$0 \le 2 + p(a' = b') + p(a = b) - p(a' = b) - p(a = b') \le 2$$

$$0 \le p(a = b) + p(a' = b) + p(a = b') - p(a' = b') \le 2$$


SEP + LOC
≤ 1
etc
≤ 2
≤ 2


assumption
Similarly, if I define
etc, then:
So, adding:


P_λ(a_+ ∧ b_+') = P_λ(a_+')P_λ(b_+')
Clauser's realist
assumption
SEP + LOC
define P_λ(a_+) = α , etc , then :
≤ 2

$$\boxed{P_\lambda(a_\pm, a' b_\pm') = P_\lambda(a_\pm') P_\lambda(b_\pm')}$$

Clauser's reality assumption SEP + LOC

$$0 \le P(a_+) + P(b_+) + P(a'_+ b'_+) - P(a_+ b_+) - P(a'_+ b_+) - P(a_+ b'_+) \le 1$$

Similarly, if I define $P_\lambda(a_-) = \alpha$, etc, then:

$$0 \le P_\lambda(a_-) + P(b_-) + P(a'_- b'_-)$$

So, adding:

$$0 \le 2 + P(a' = b') + P(a = b) - P(a' = b) - P(a = b') \le 2$$

$$0 \le P(a = b) + P(a' = b) + P(a = b') - P(a' = b') \le 2$$

$$\boxed{P_\lambda(a_+ \,\&\, b_+') = P_\lambda(a_+')P_\lambda(b_+')}$$

Einstein's realist assumption
SEP + LOC

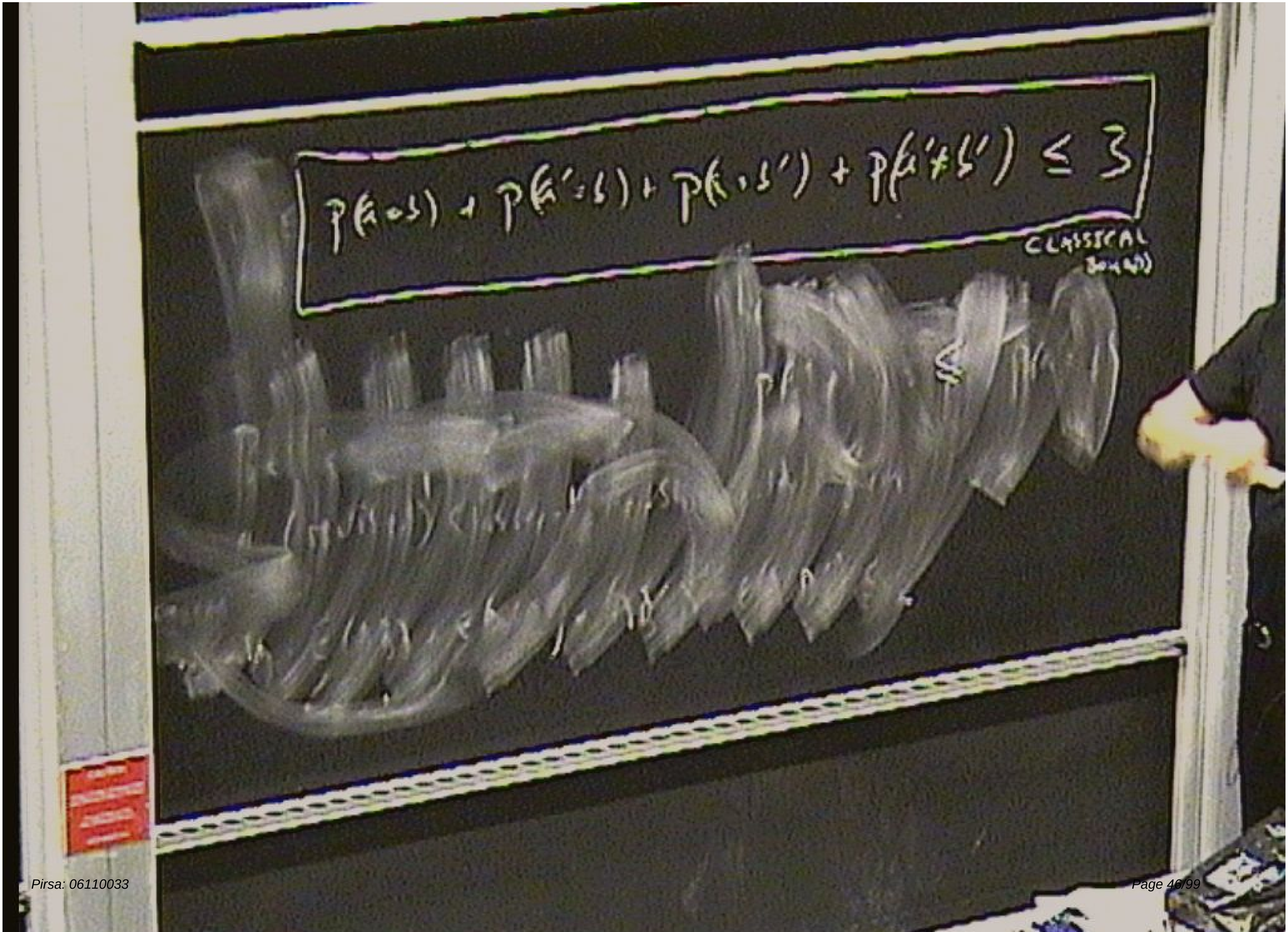
$$0 \le P(a_+) + P(b_+) + P(a_+' \,\&\, b_+') - P(a_+ \,\&\, b_+) - P(a_+' \,\&\, b_+)$$
$$- P(a_+ \,\&\, b_+') \le 1$$

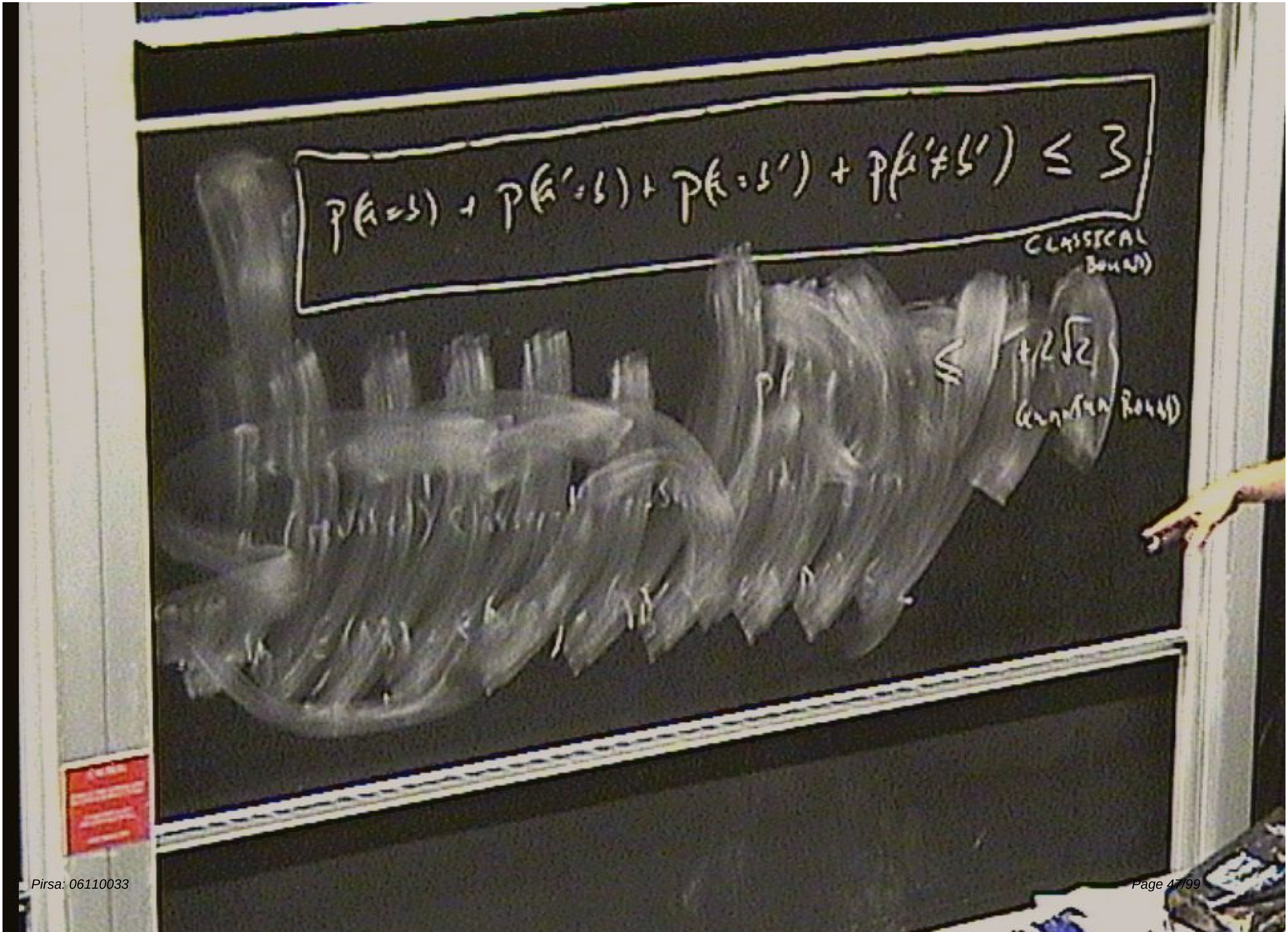
Similarly, if I define $P_\lambda(a_+) = \alpha$, etc., then:

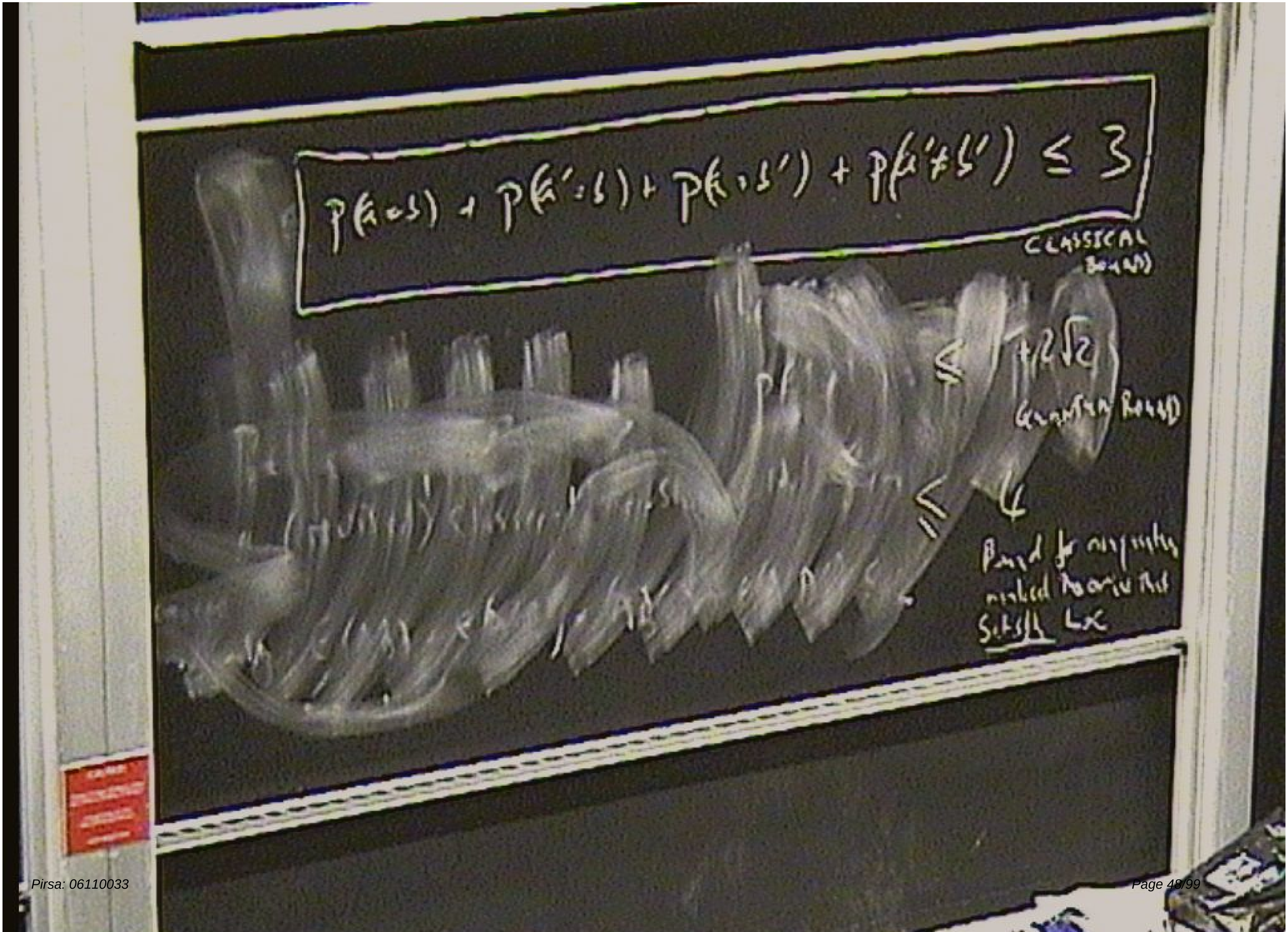
$$0 \le P_\lambda(a_-) + P_\lambda(b_-) + P(a_-' \,\&\, b_-')$$

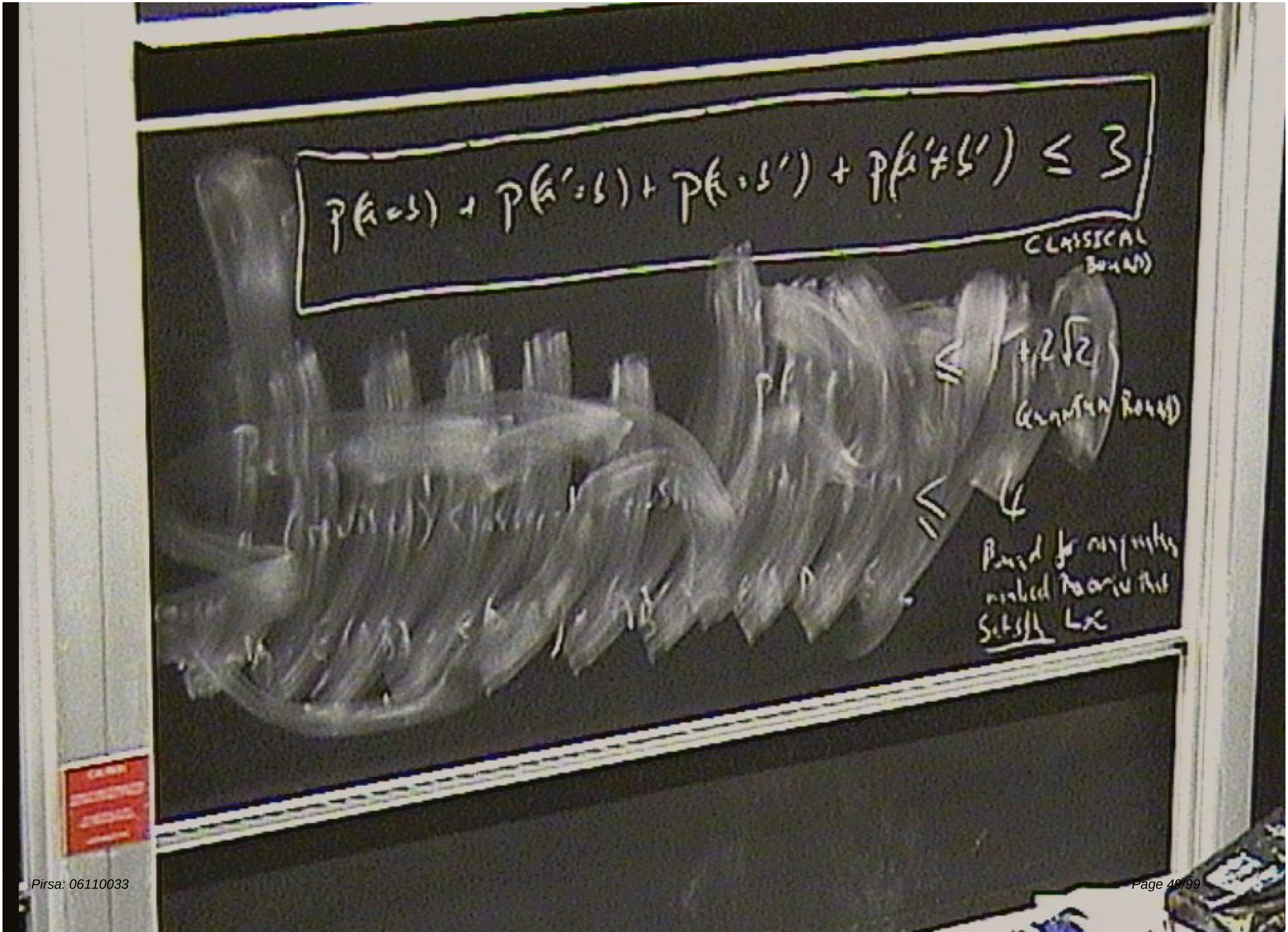
So, adding:

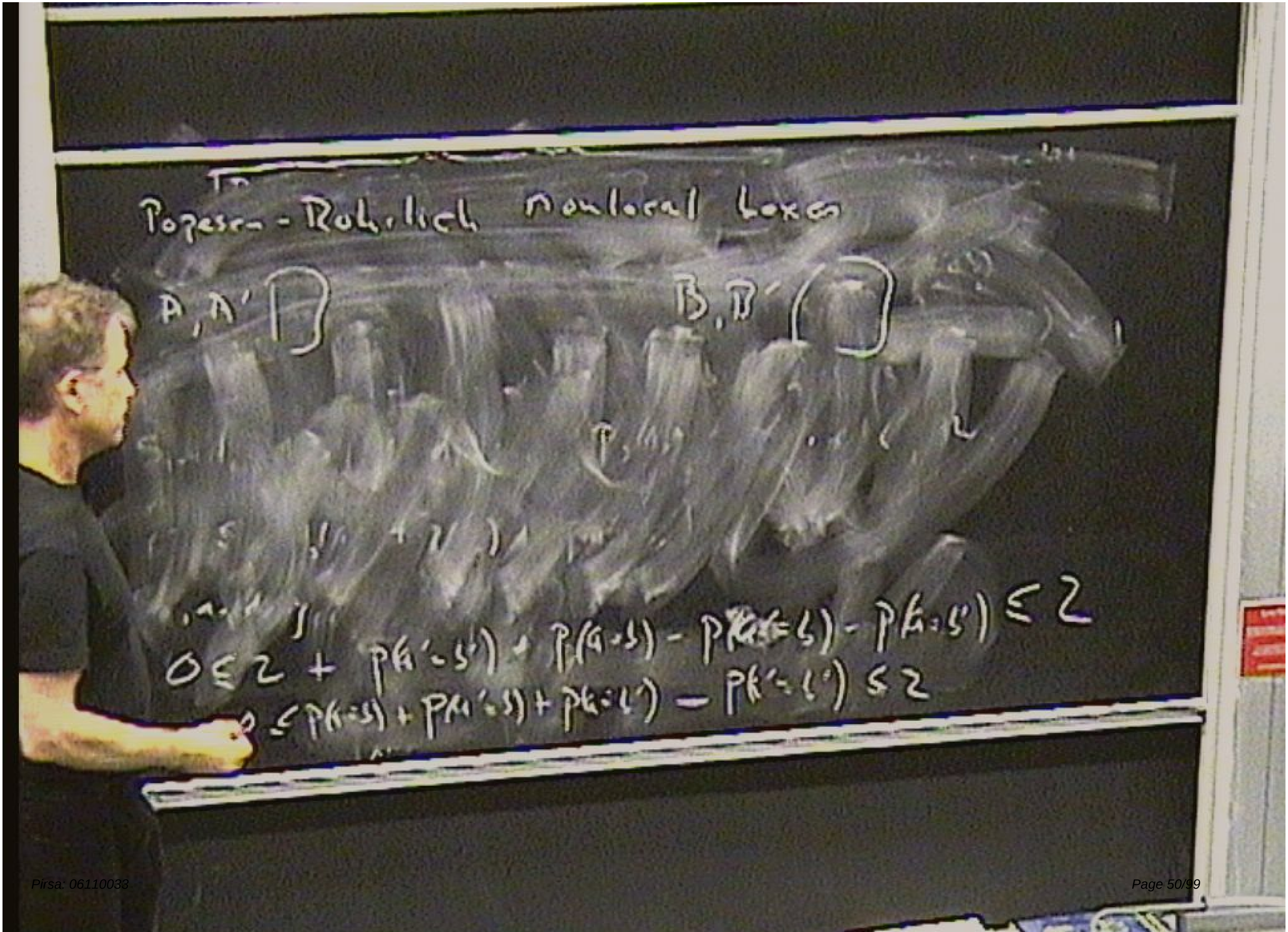
$$0 \le 2 + P(a' = b') + P(a = b) - P(a' = b) - P(a = b') \le 2$$
$$0 \le P(a = b) + P(a' = b) + P(a = b') - P(a' = b') \le 2$$


$p(a=b) + p(a'=b) + p(a=b') + p(a' \neq b') \leq 3$
CLASSICAL BOUND


p(a=b) + p(a'=b) + p(a=b') + p(a'≠b') ≤ 3
CLASSICAL BOUND
≤ 2√2
QUANTUM BOUND


CLASSICAL
BOUND
QUANTUM BOUND


p(a=b) + p(a'=b) + p(a=b') + p(a'≠b') ≤ 3
CLASSICAL BOUND
QUANTUM BOUND


Popescu - Rohrlich nonlocal boxes
A, A'
B, B'

Popescu-Rohrlich nonlocal boxes

A, A' $\qquad\qquad$ B, B'

$$p(a=b)=1$$

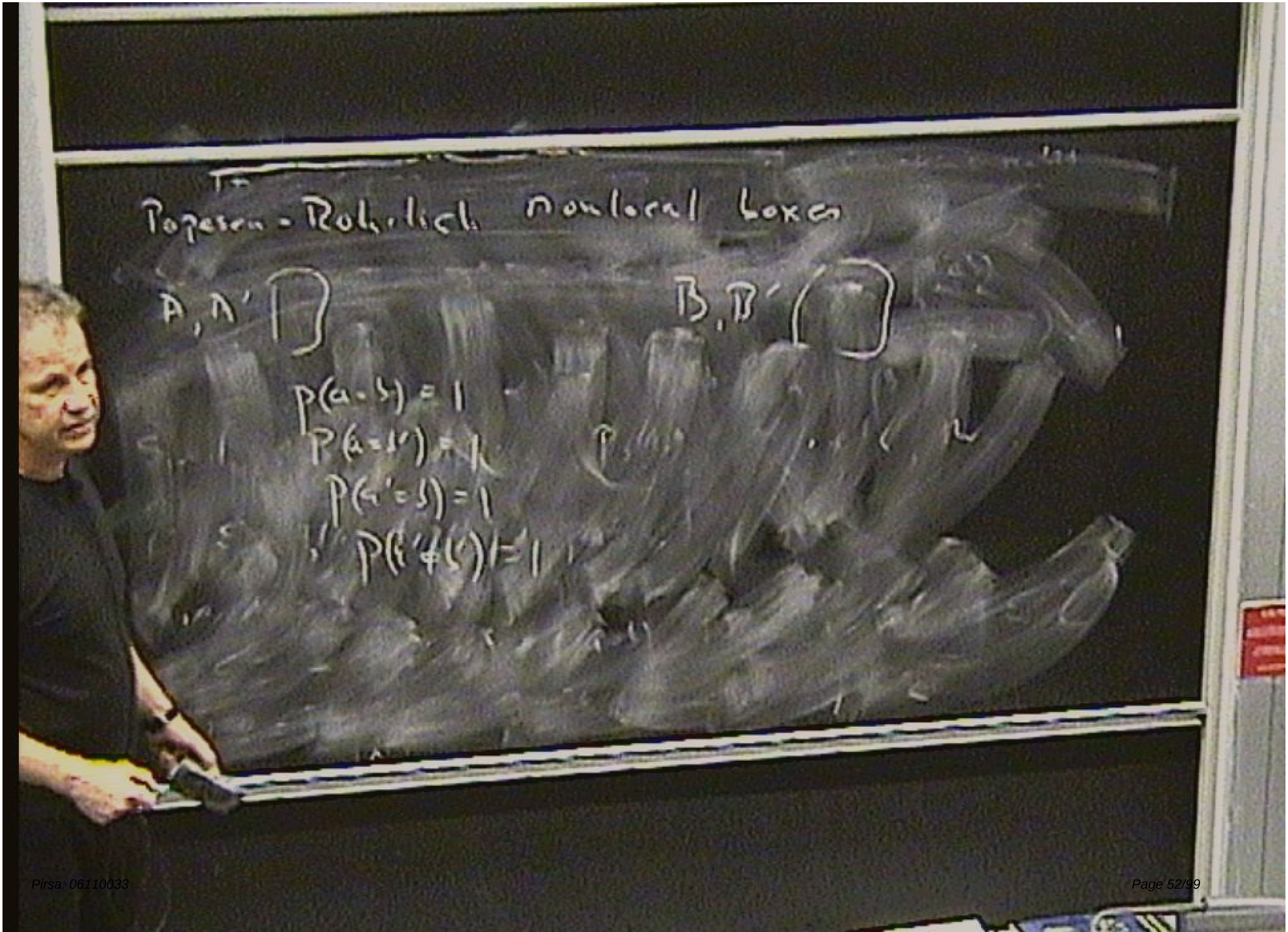
$$p(a=b')=1$$

$$p(a'=b)=1$$

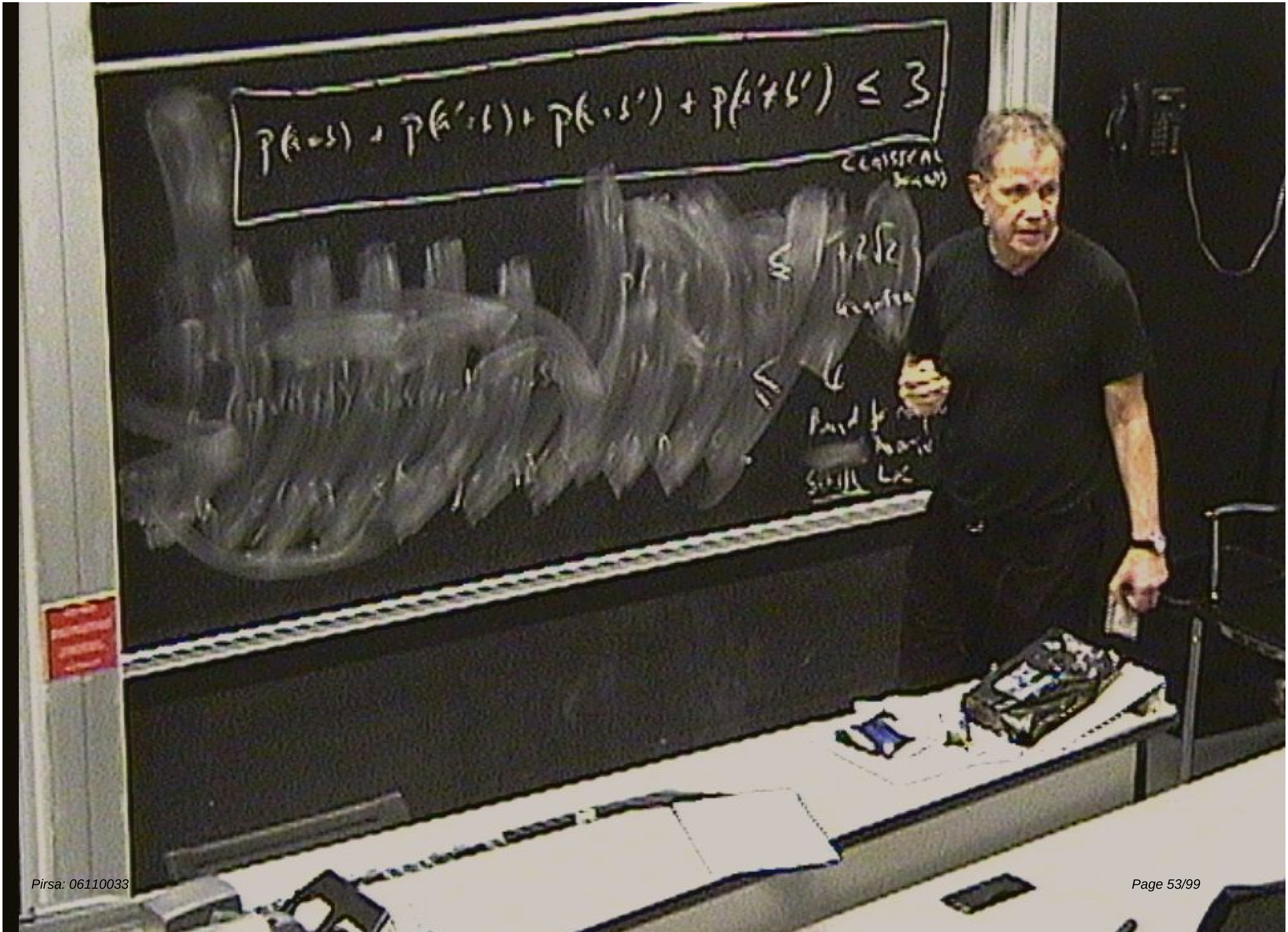
$$p(a'\neq b')=1$$

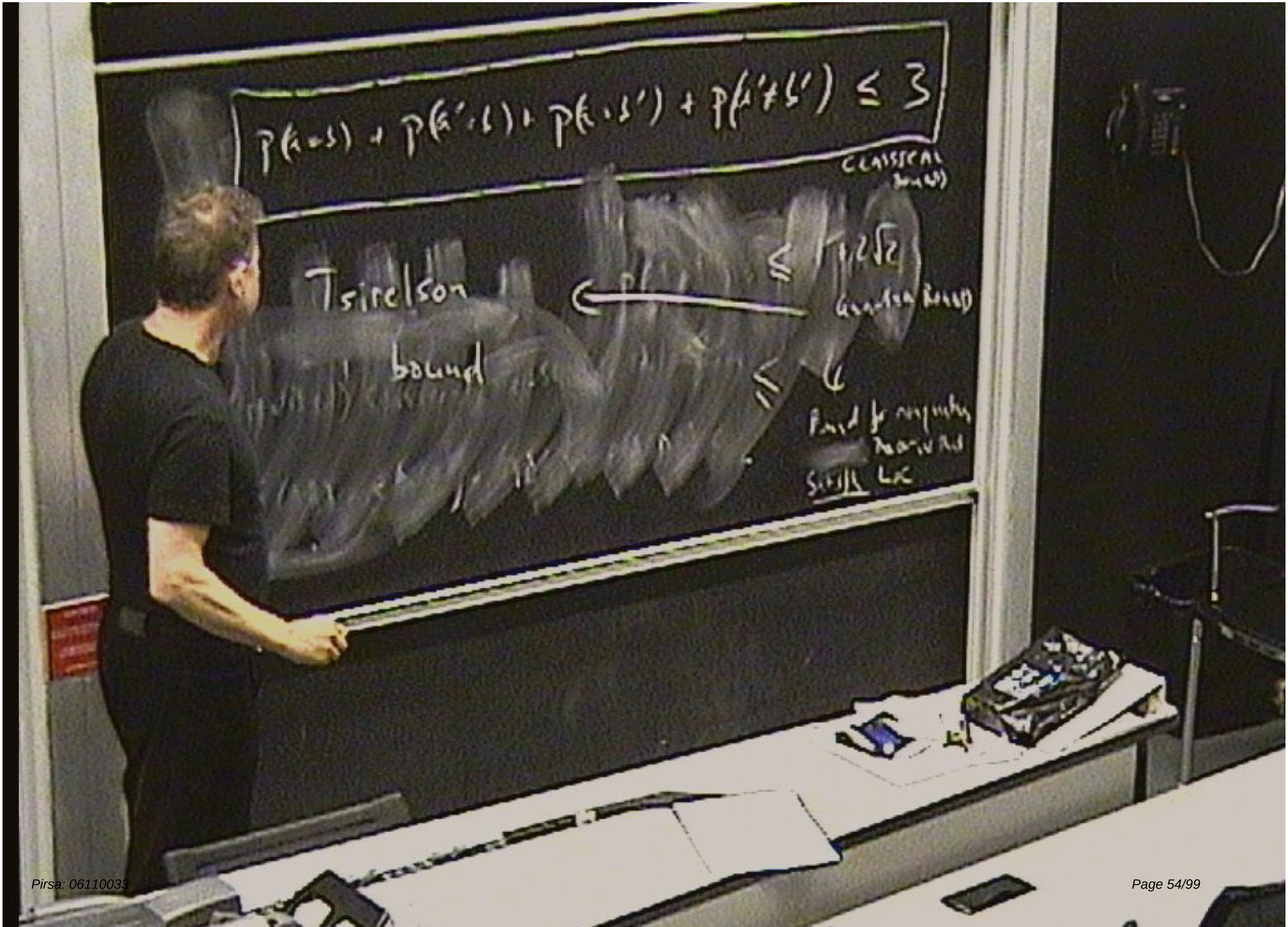
$$0 \leq 2 + p(a'=b') + p(a=b) - p(a'=b) - p(a=b') \leq 2$$

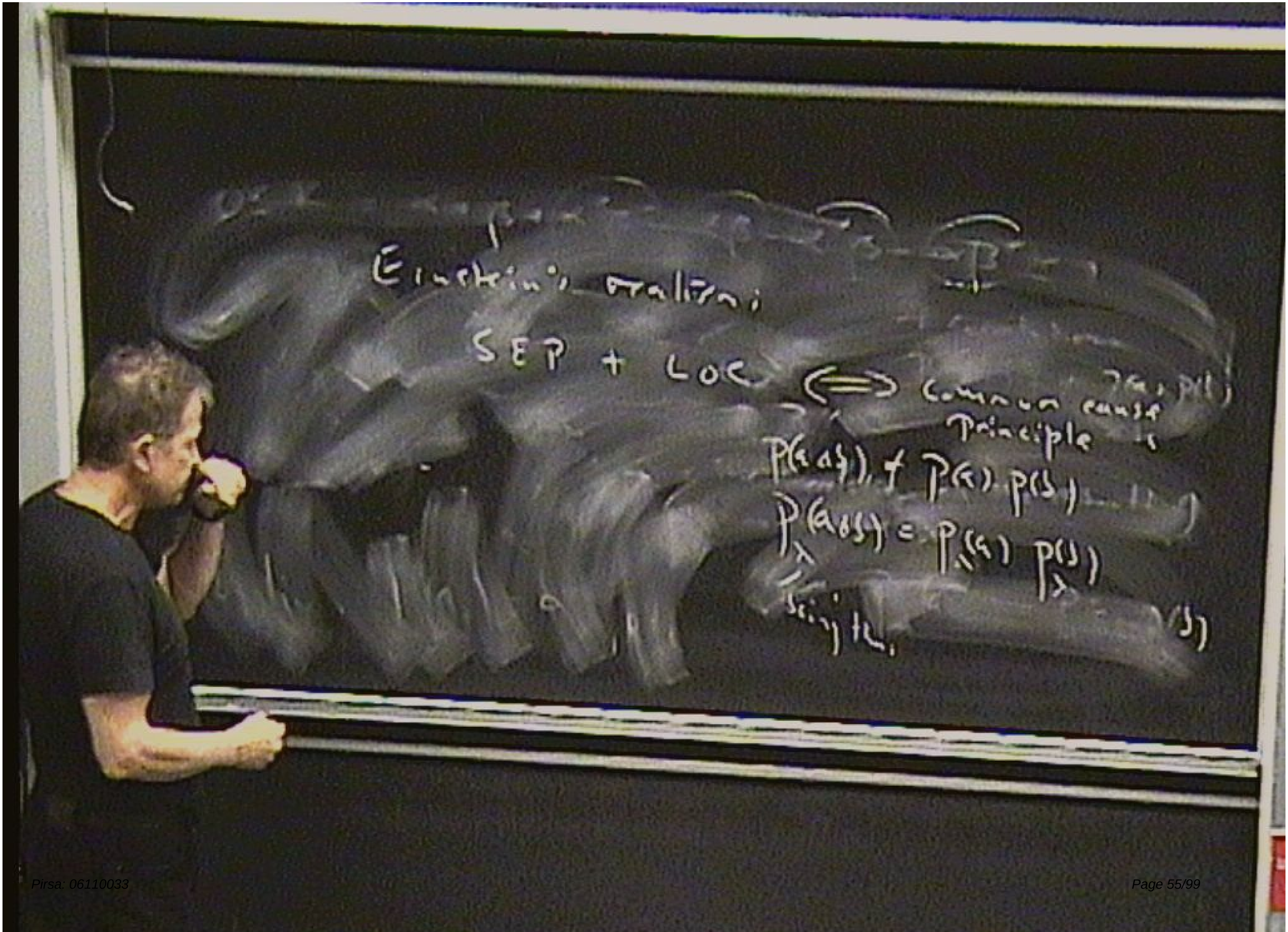
$$0 \leq p(a=b) + p(a'=b) + p(a=b') - p(a'=b') \leq 2$$

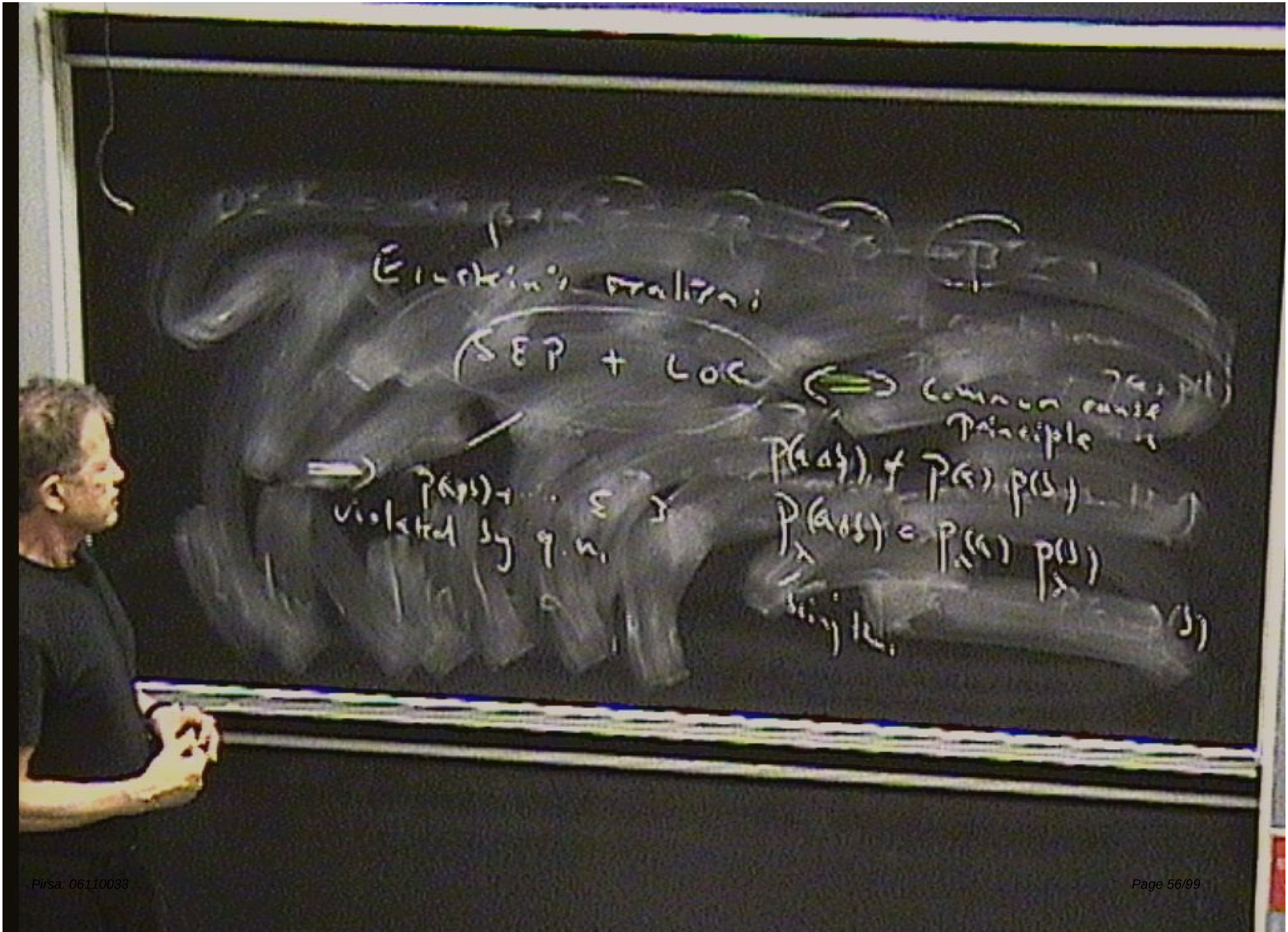


Popescu-Rohrlich nonlocal boxes
A, A'
B, B'
P(a=b)=1
P(a=b')=1
P(a'=b)=1
P(a'≠b')=1


p(a=b) + p(a'=b) + p(a=b') + p(a'≠b') ≤ 3
CLASSICAL LAWS


≤ 3
CLASSICAL
Tsirelson
bound
≤ 4


Einstein's realism
SEP + LOC
⟺ common cause principle
p(a,b) ≠ p(a) p(b)
p_λ(a,b) = p_λ(a) p_λ(b)


Einstein's realism
SEP + LOC
common cause
Principle
violated by q.m.

Einstein's realism

LOC = "no signalling" principle

SEP + LOC $\Longleftrightarrow$ common cause principle

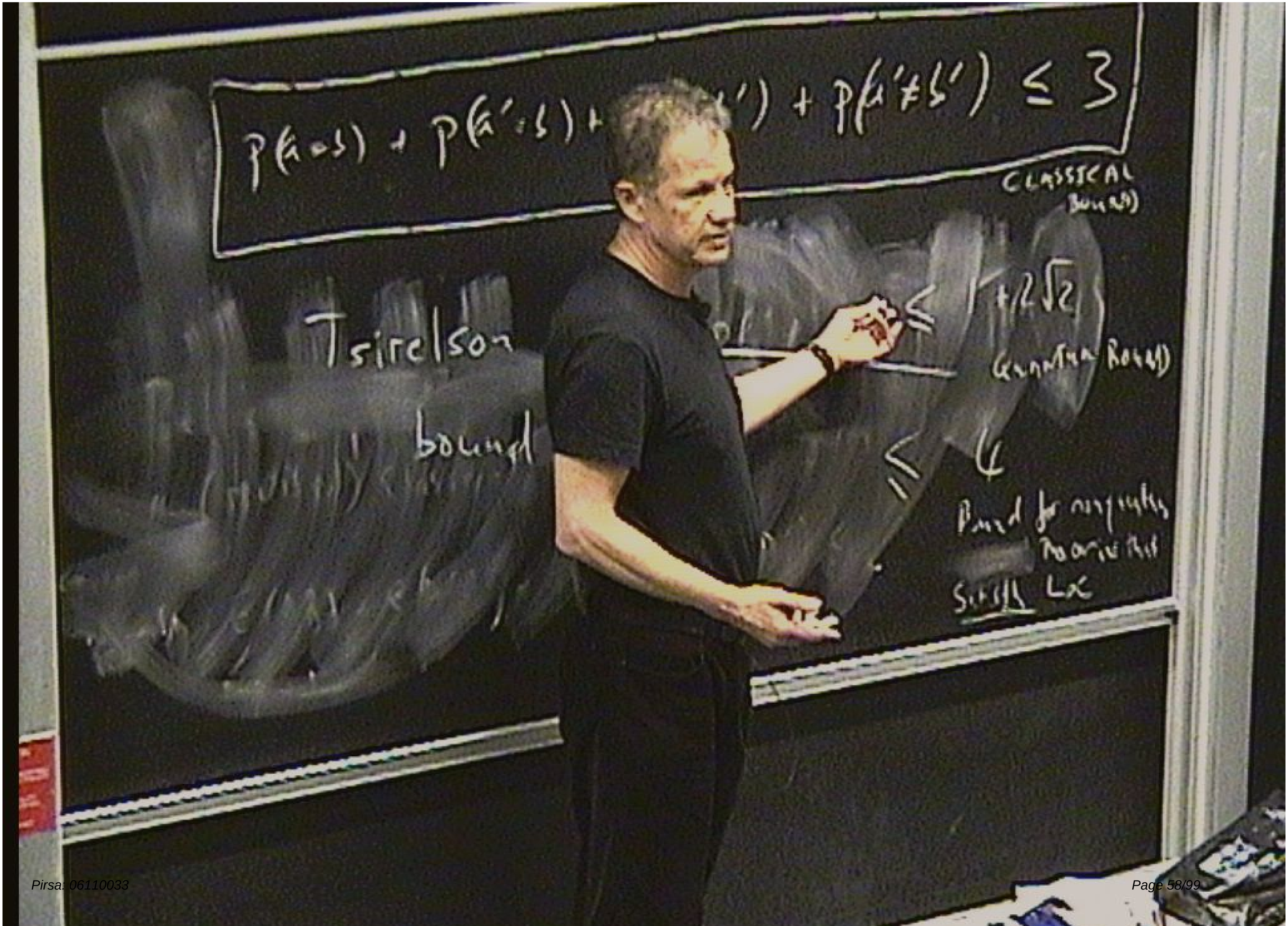
$\Longrightarrow$ $p(a,b)$ ... ε λ

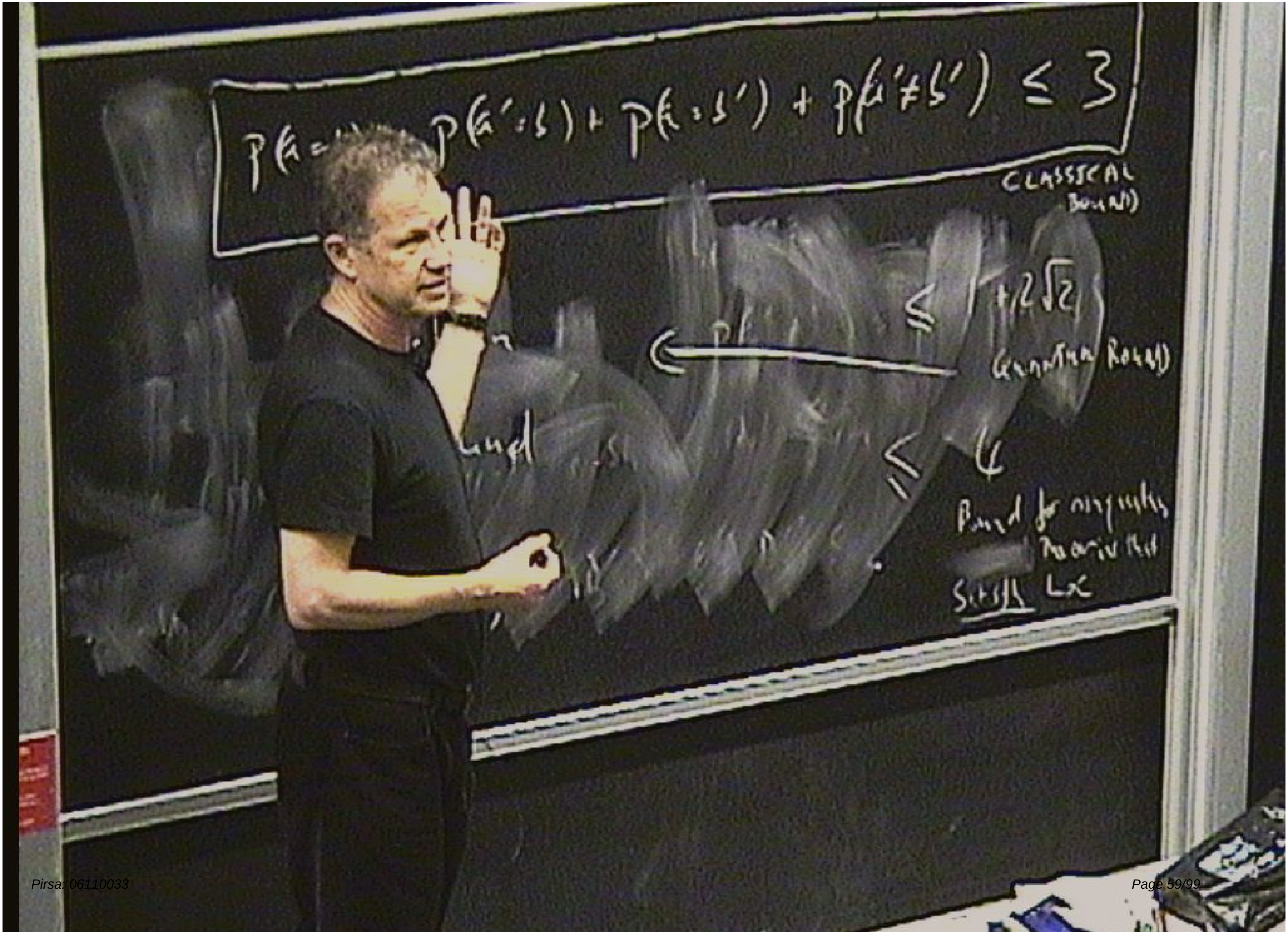
violated by q.m.

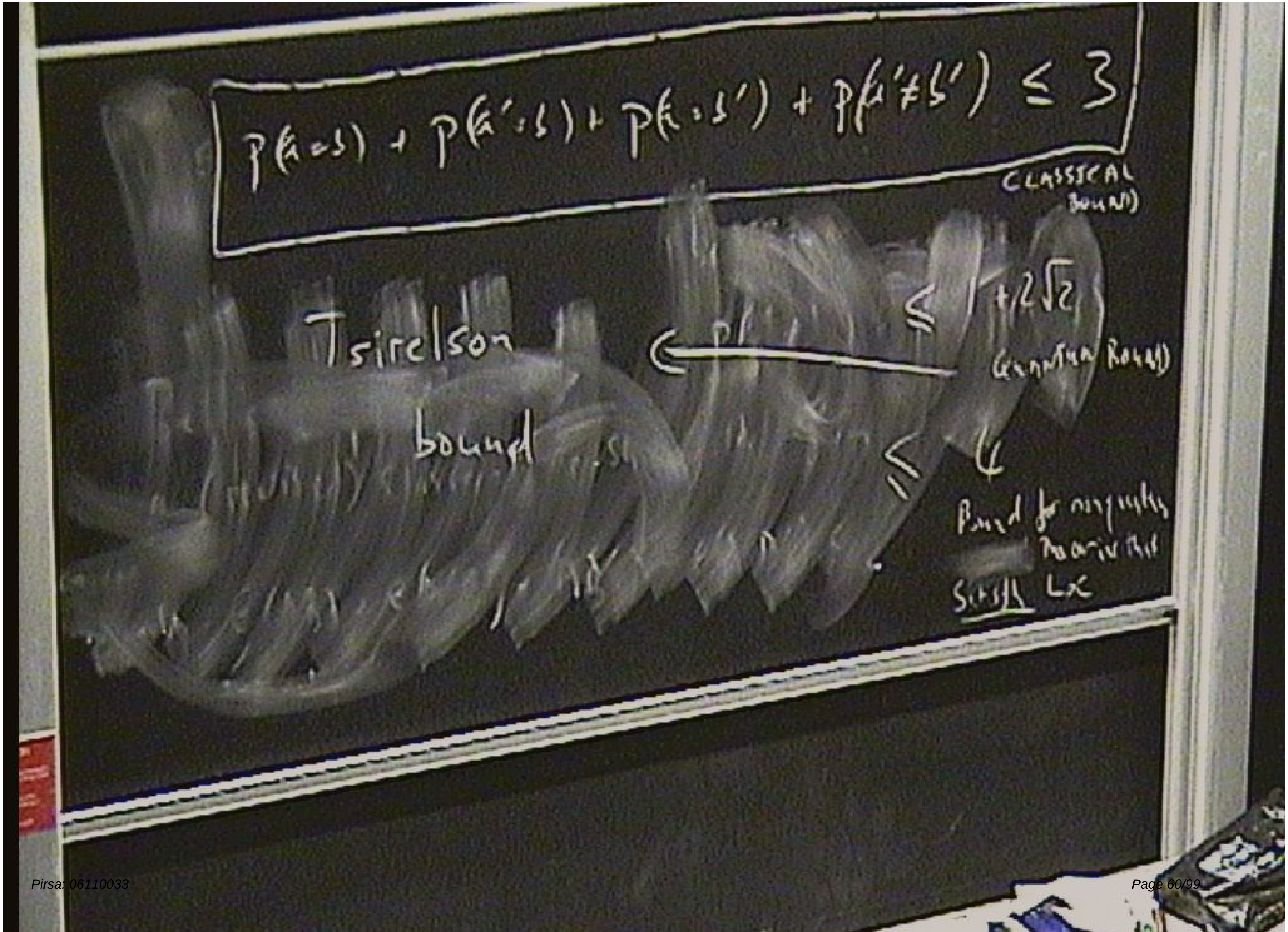
$p(a \wedge b) \neq p(a)\,p(b)$

$p_\lambda(a \wedge b) = p_\lambda(a)\,p_\lambda(b)$

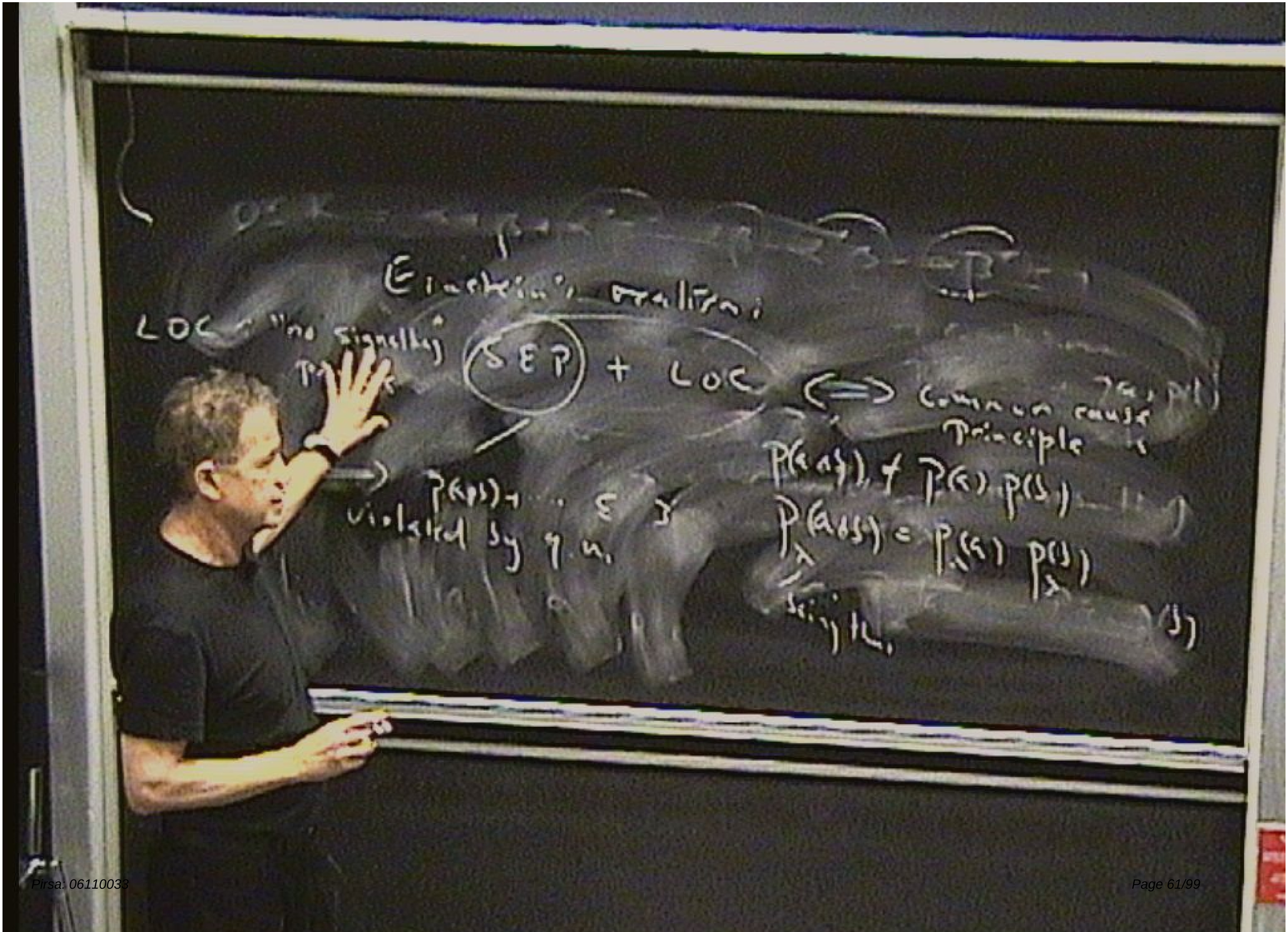
being the ... (b)

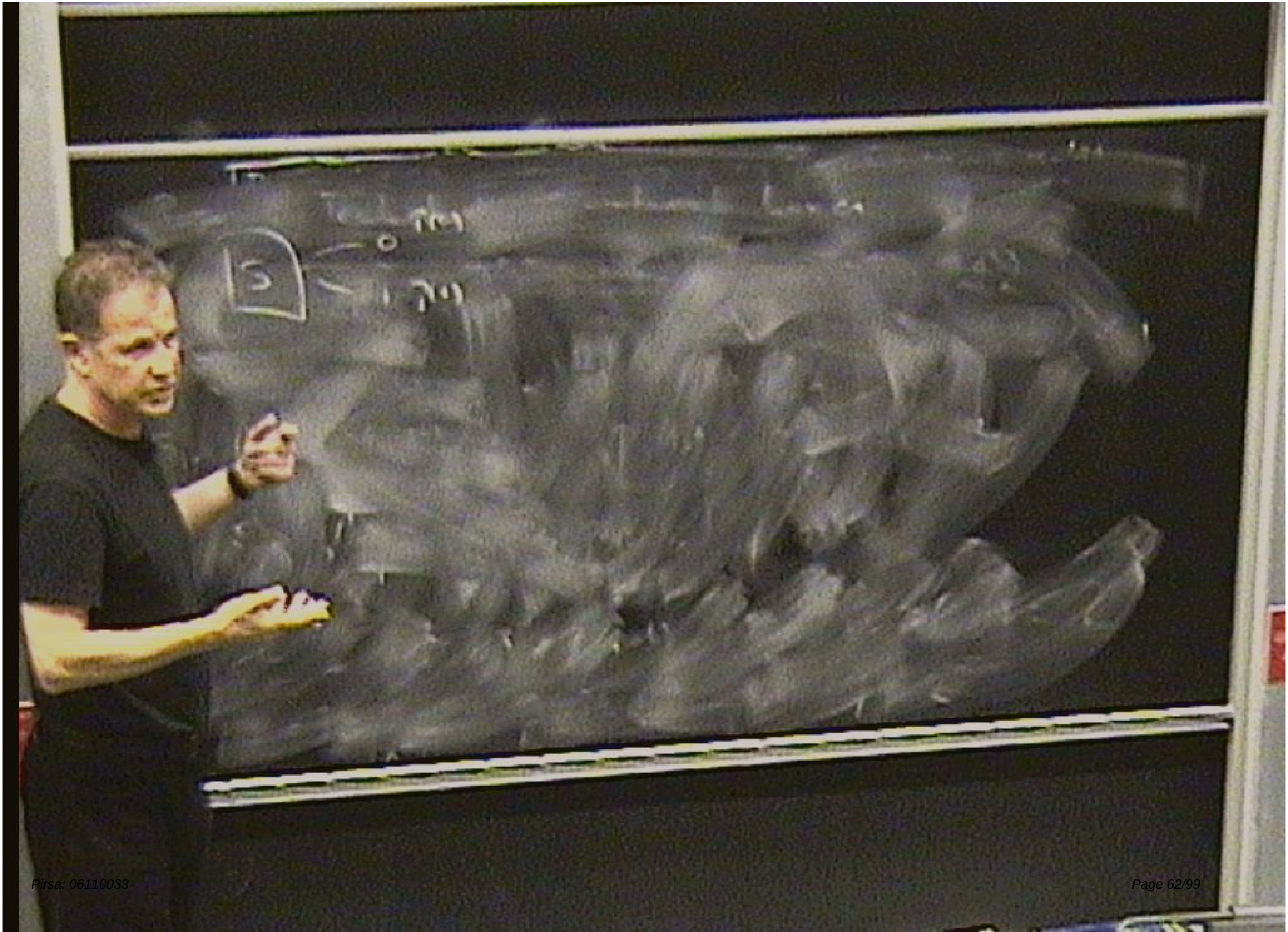

CLASSICAL
Tsirelson
bound

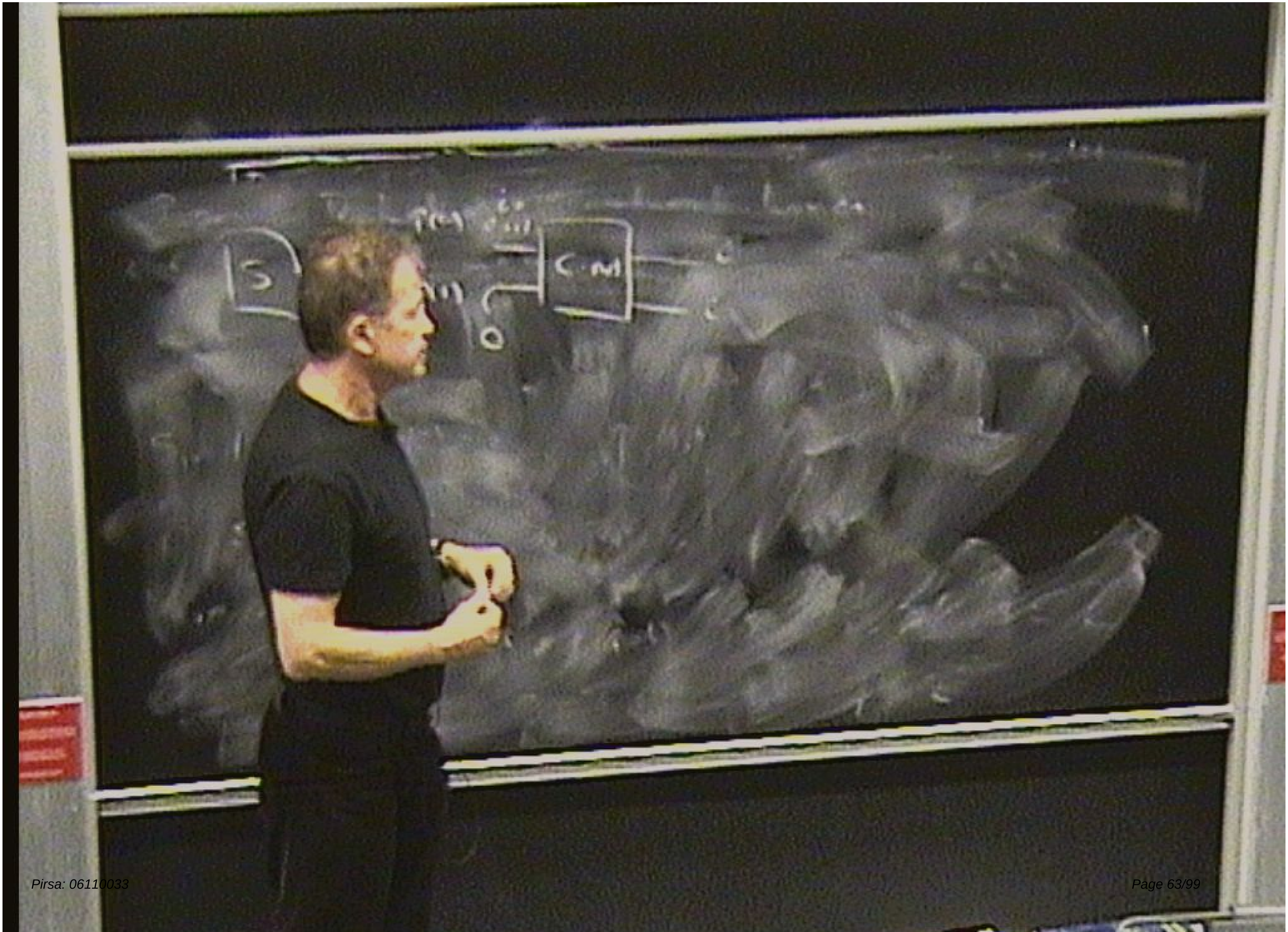

CLASSICAL BOUND
QUANTUM BOUND

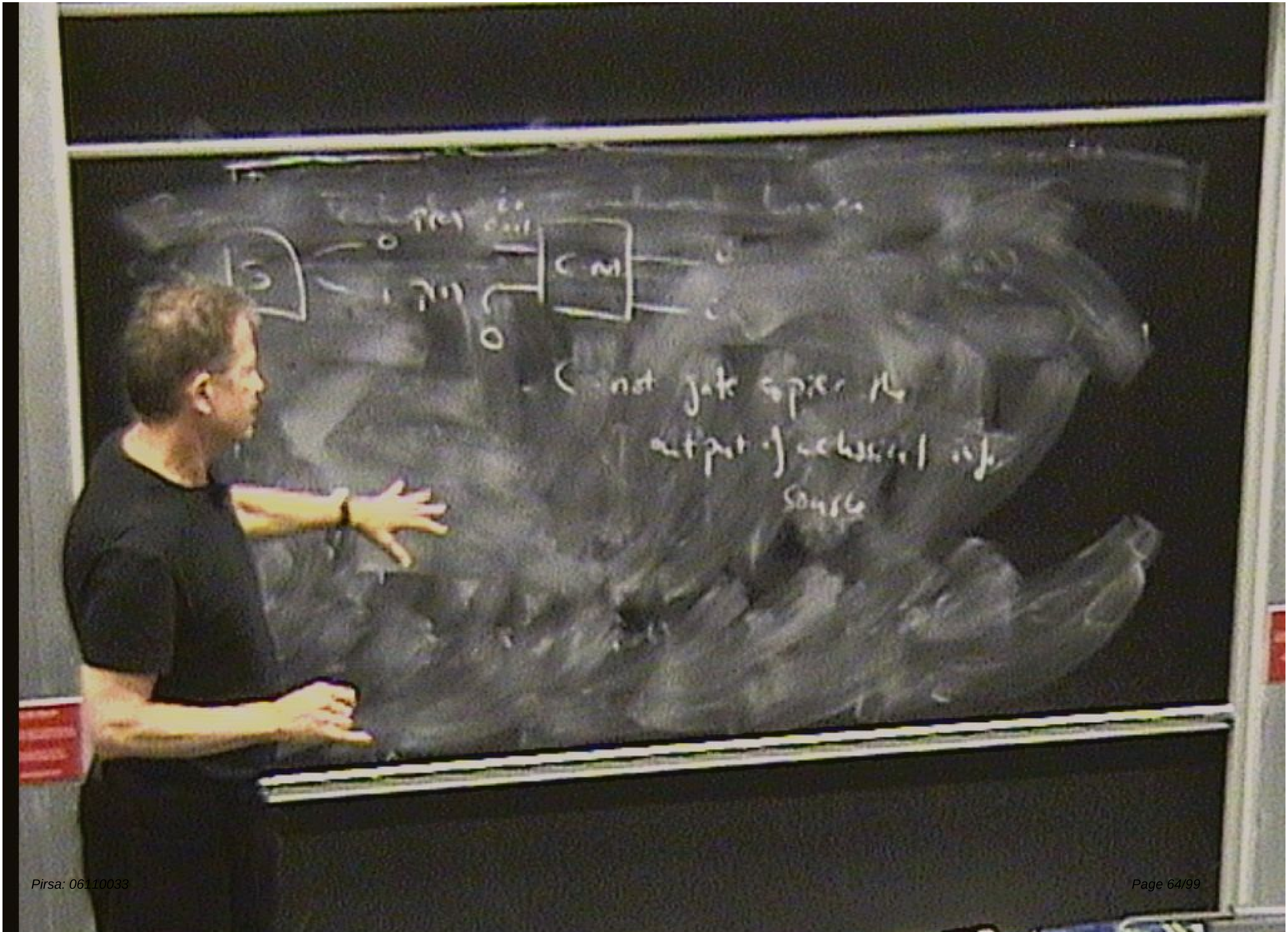


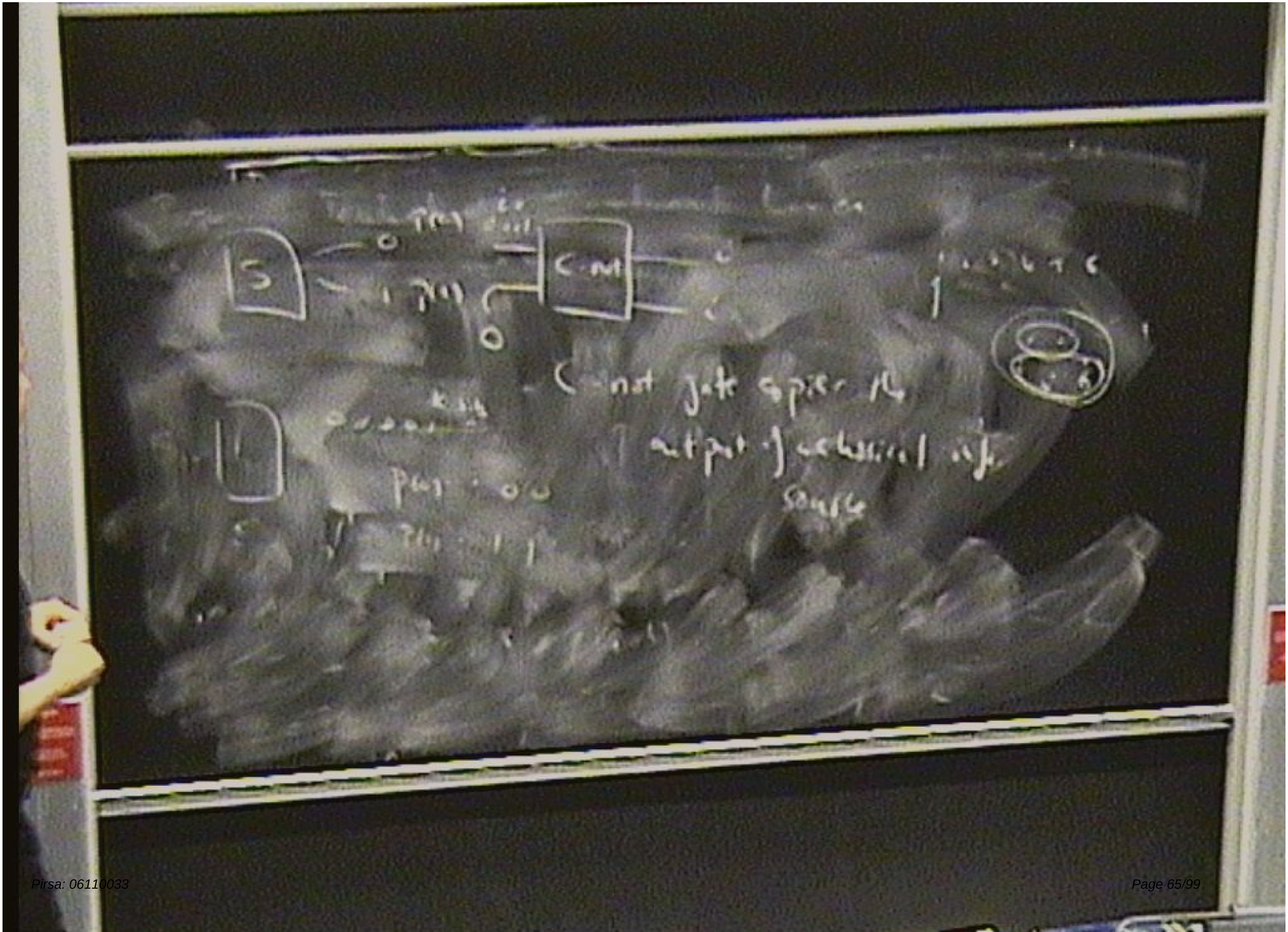
p(a=b) + p(a'=b) + p(a=b') + p(a'≠b') ≤ 3
CLASSICAL BOUND
Tsirelson
bound
QUANTUM BOUND

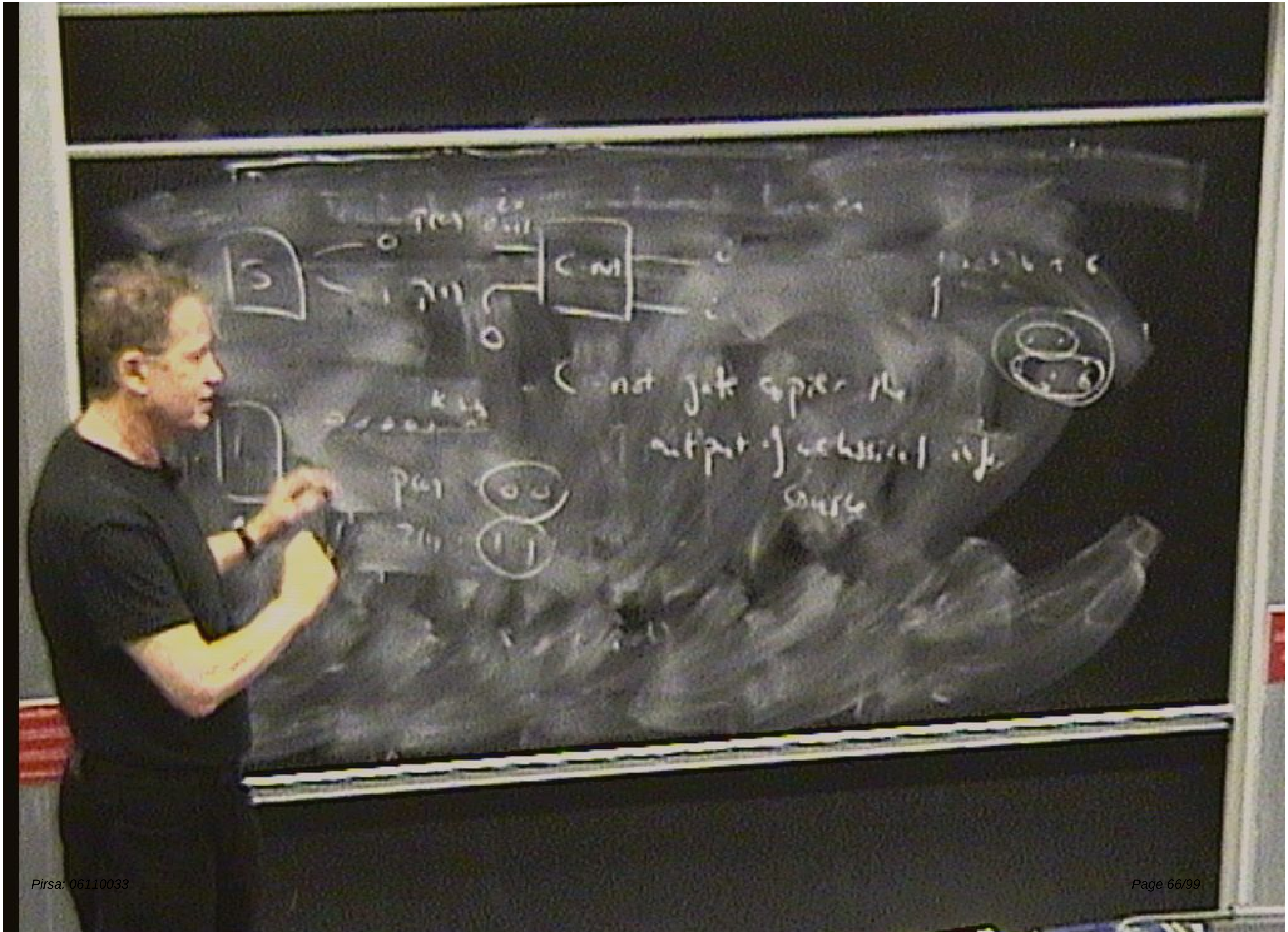

Einstein's realism
LOC = no signalling
SEP + LOC
Common cause
Principle
violated by q.m.

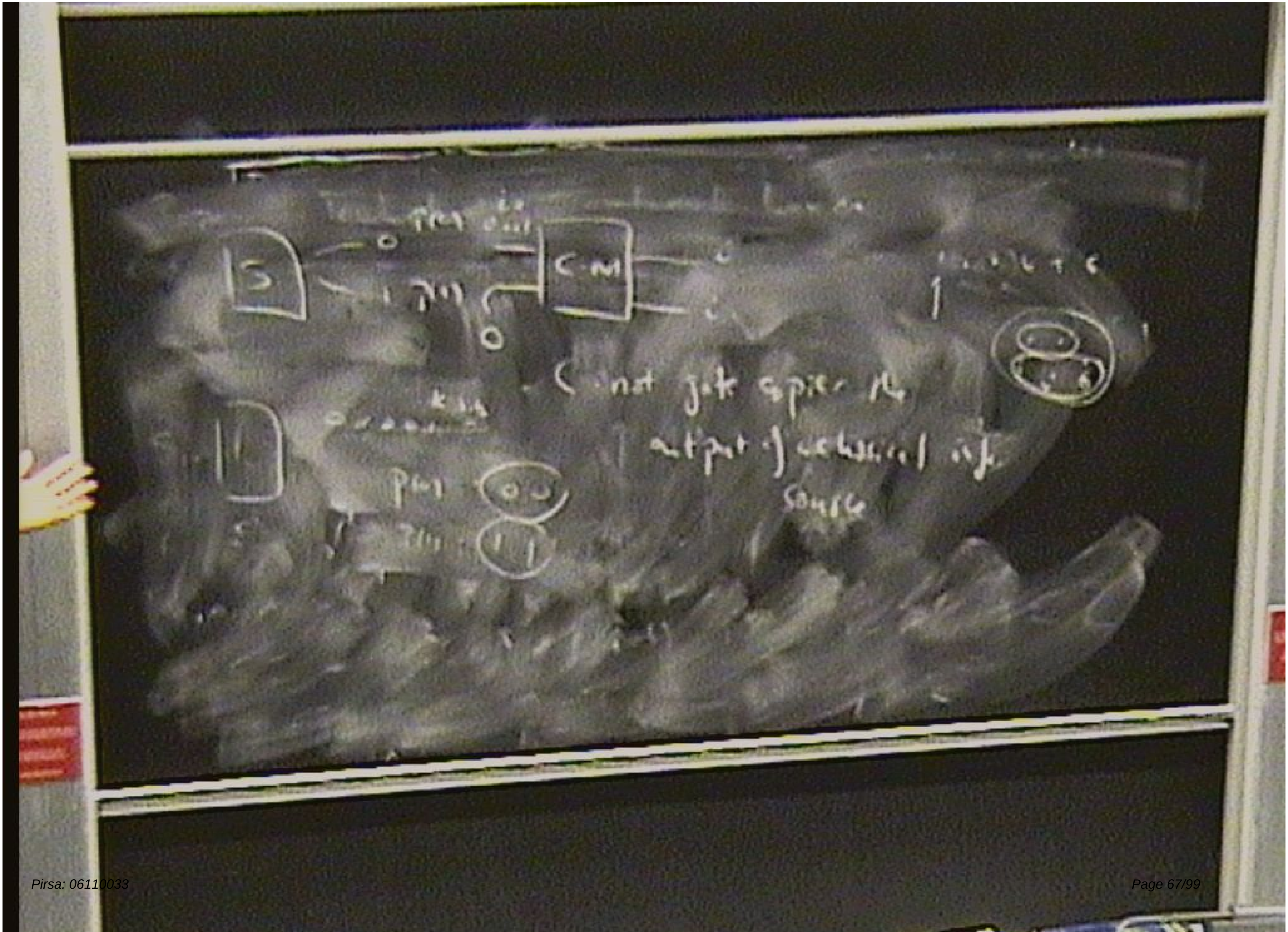


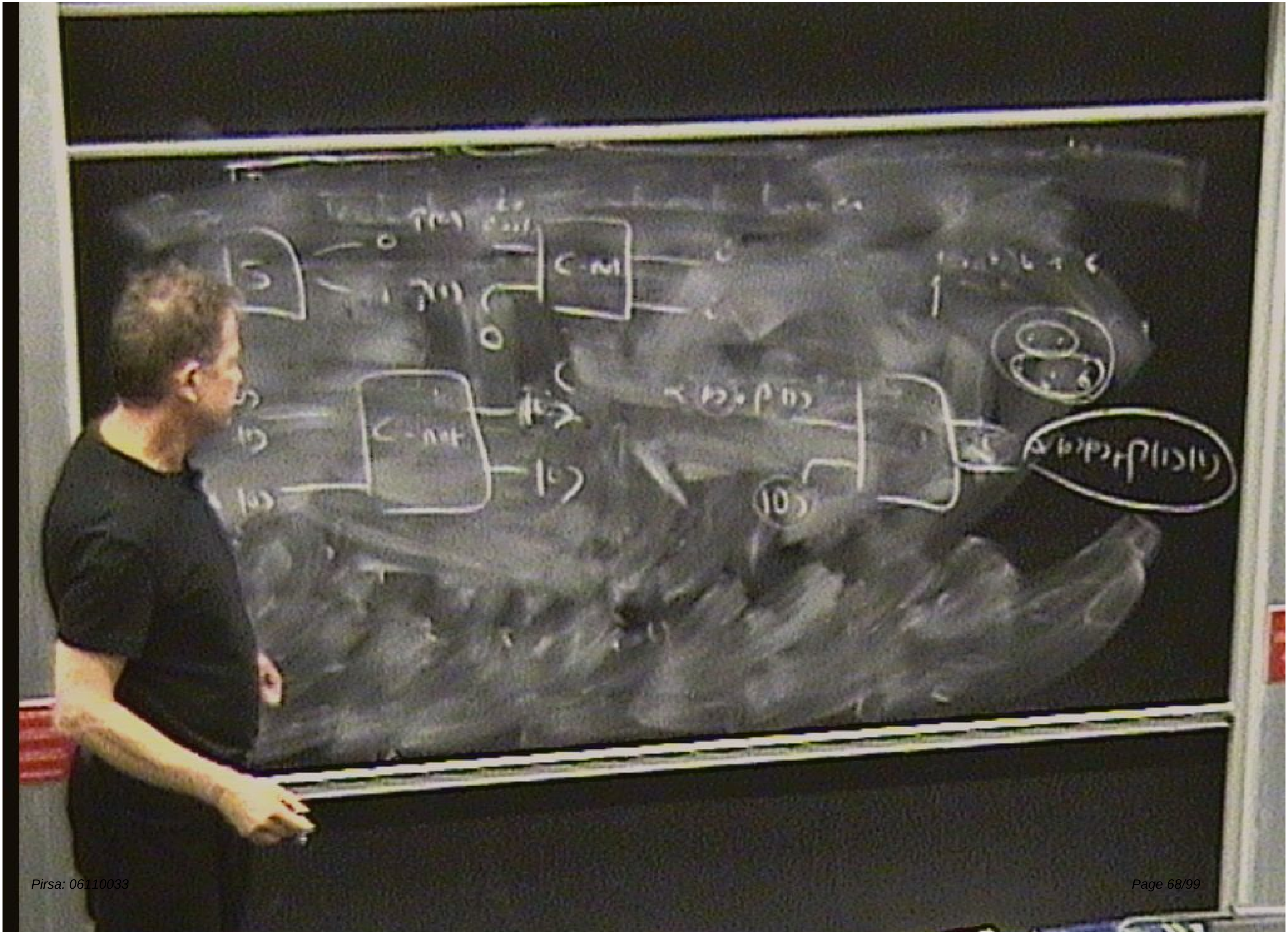


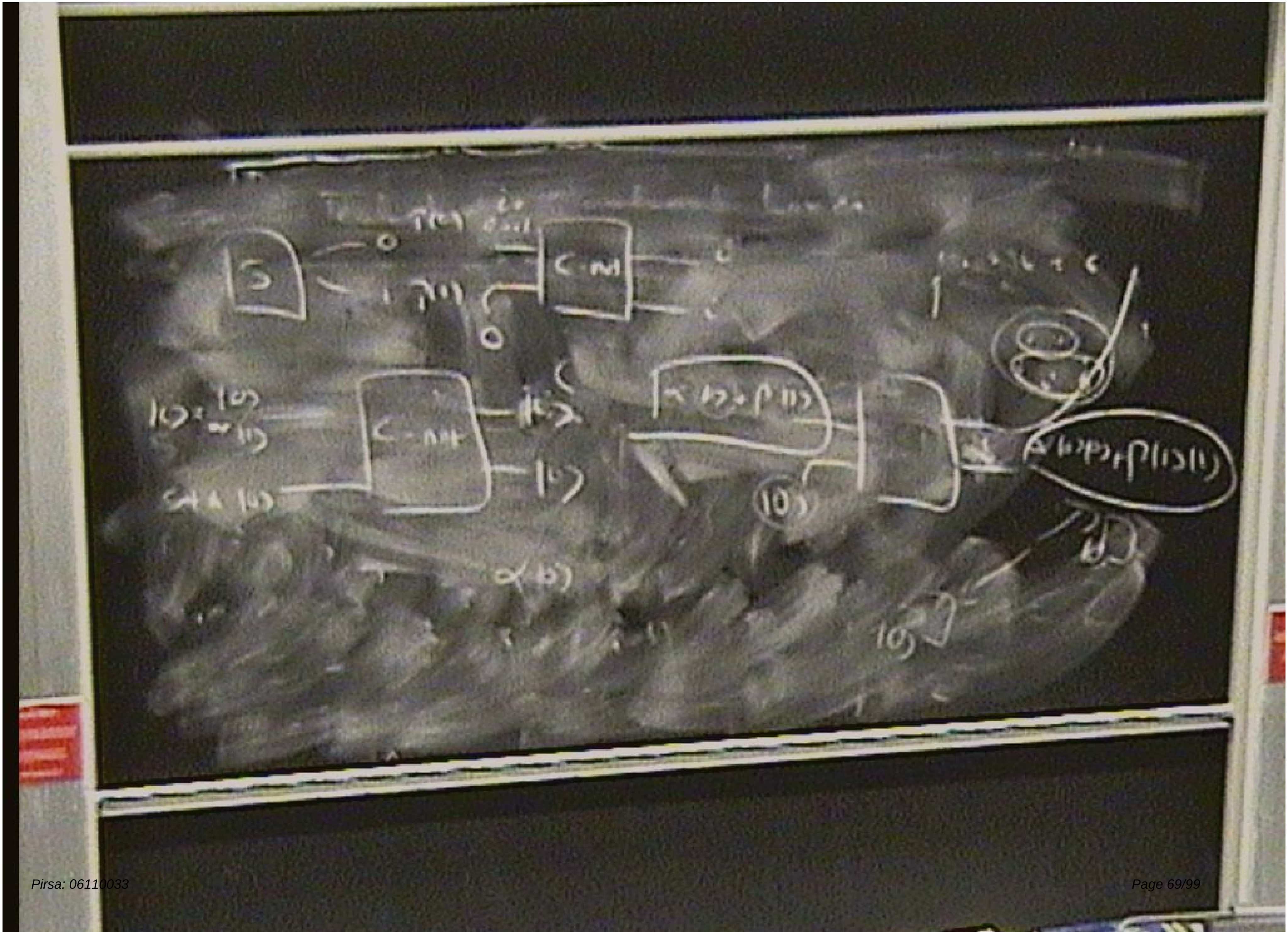


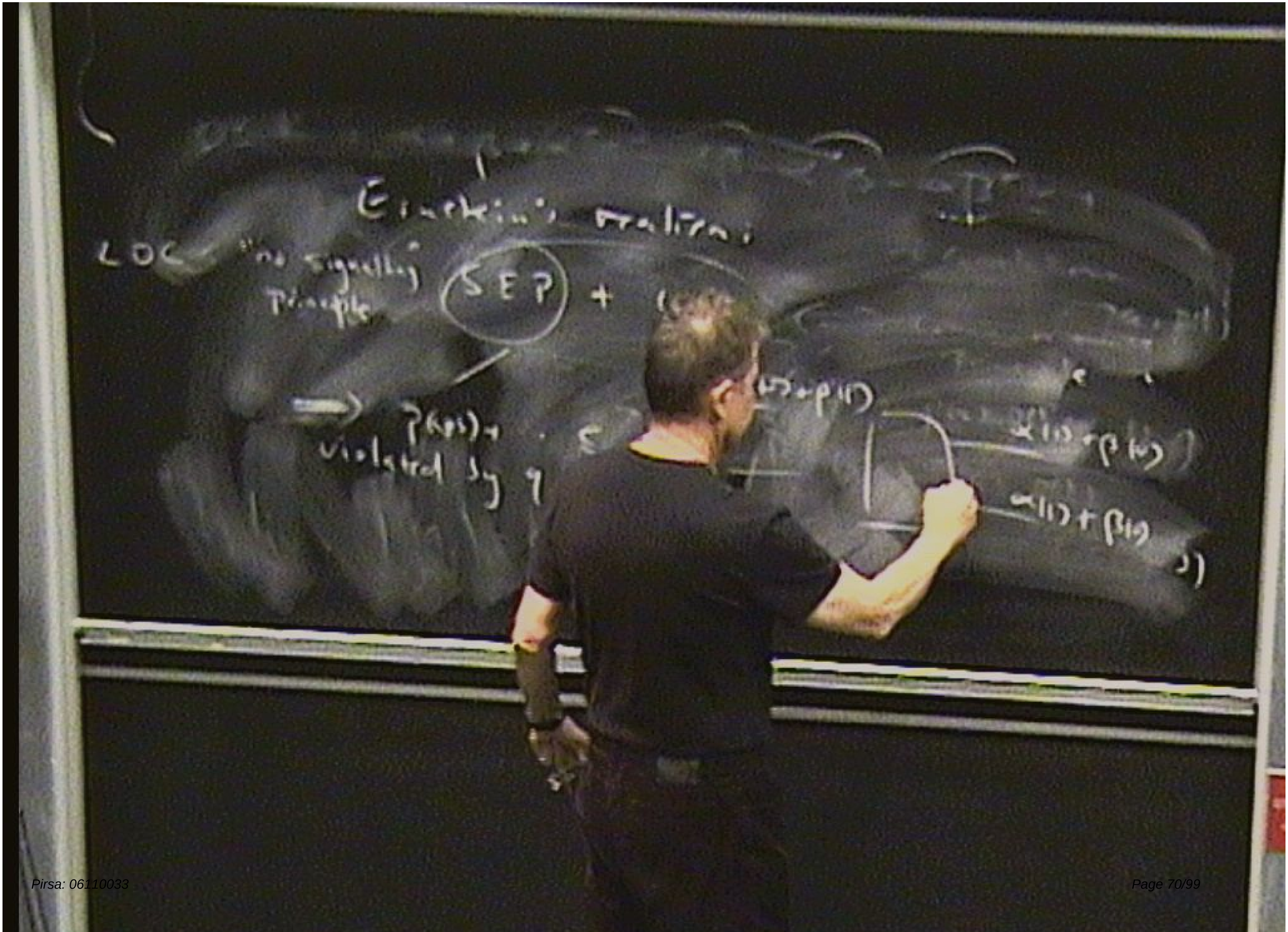


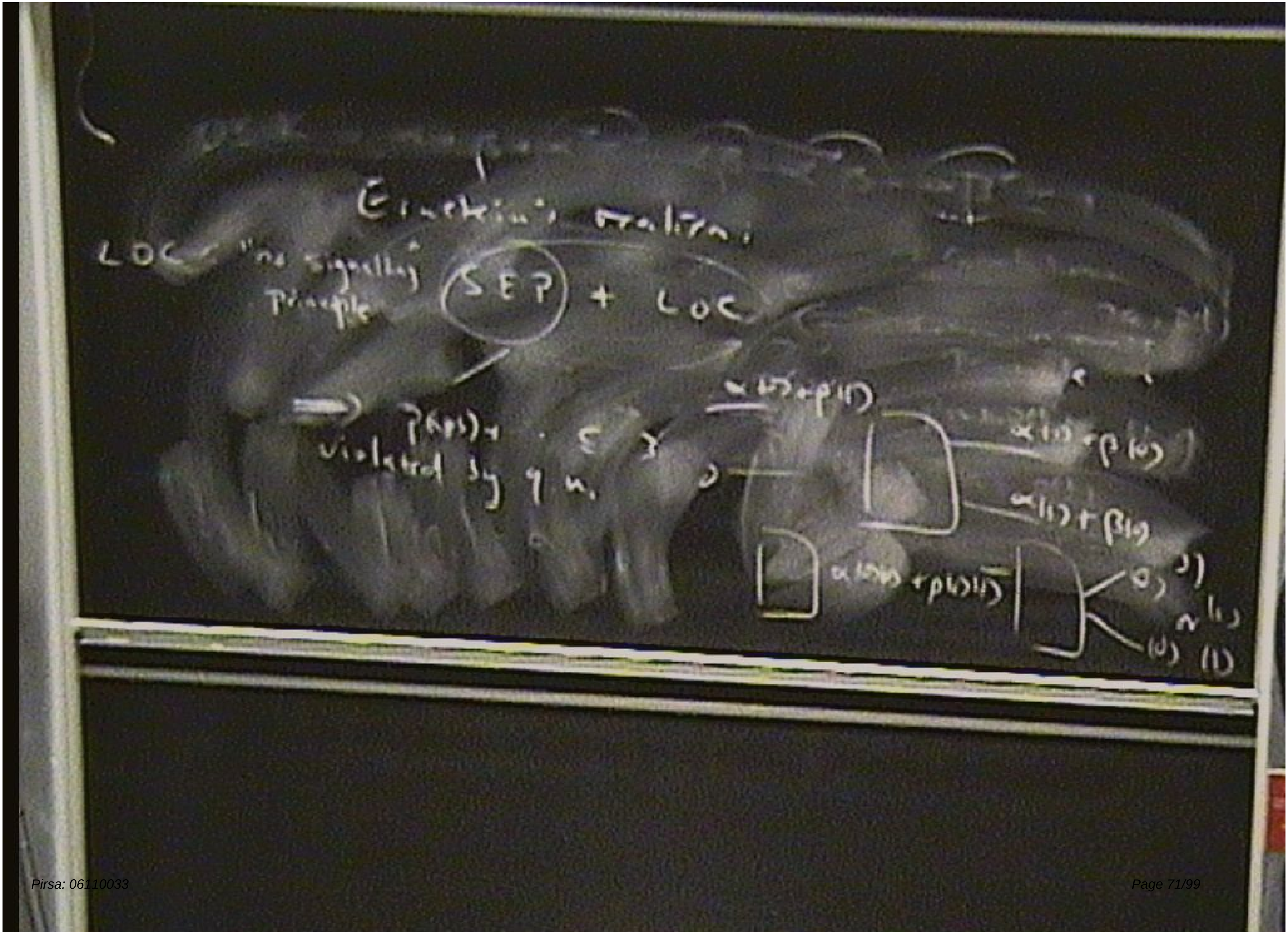


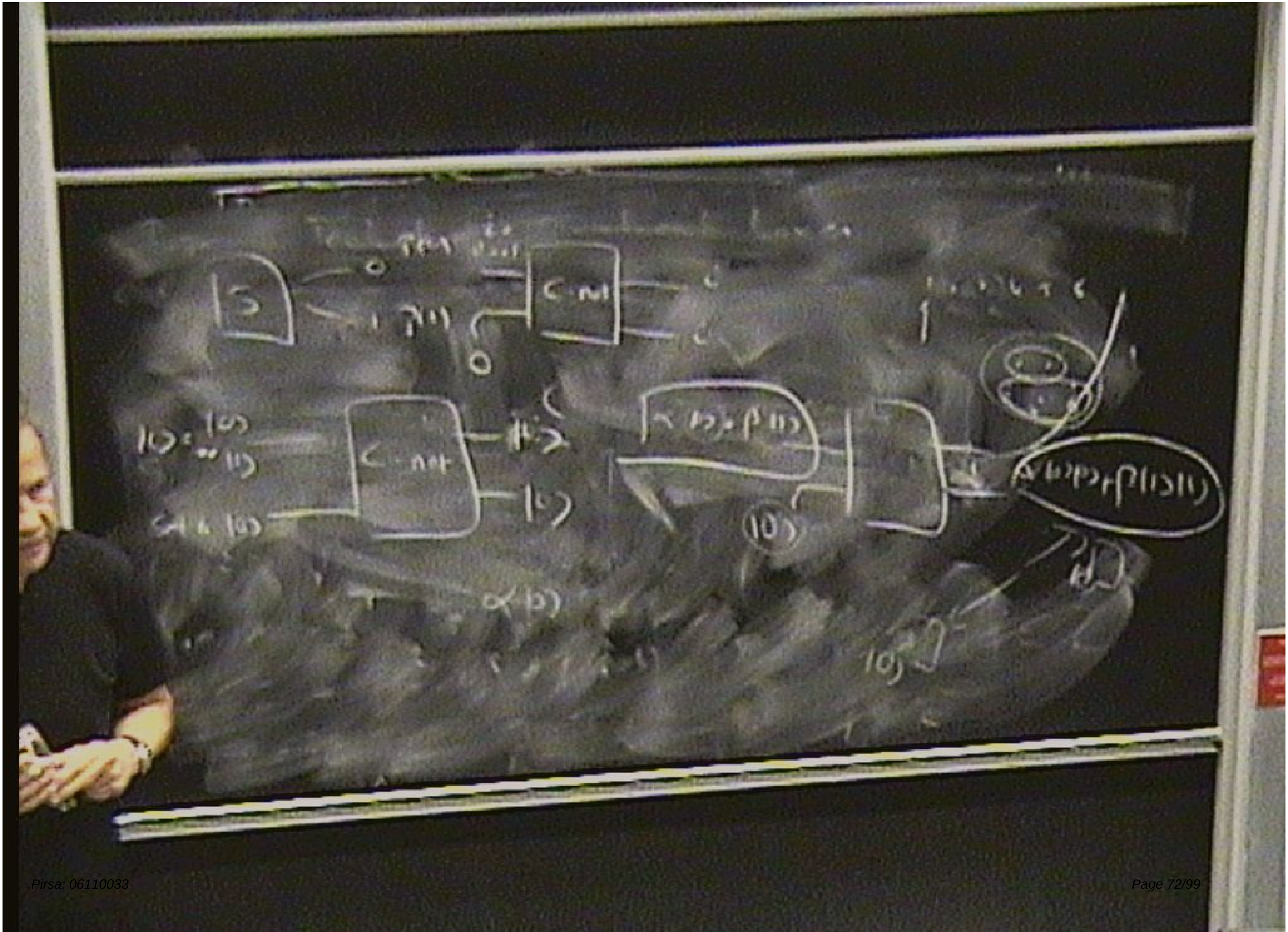


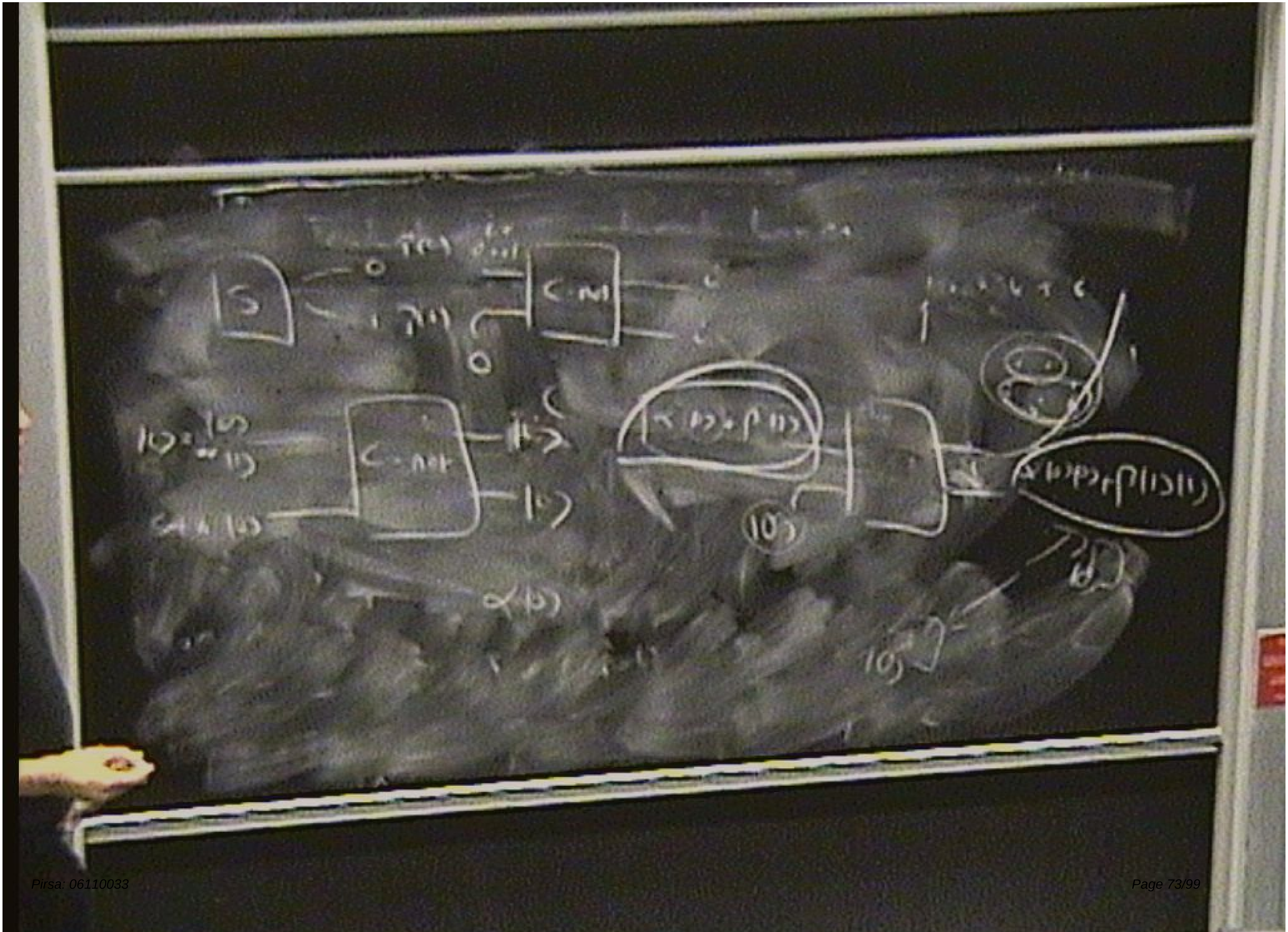


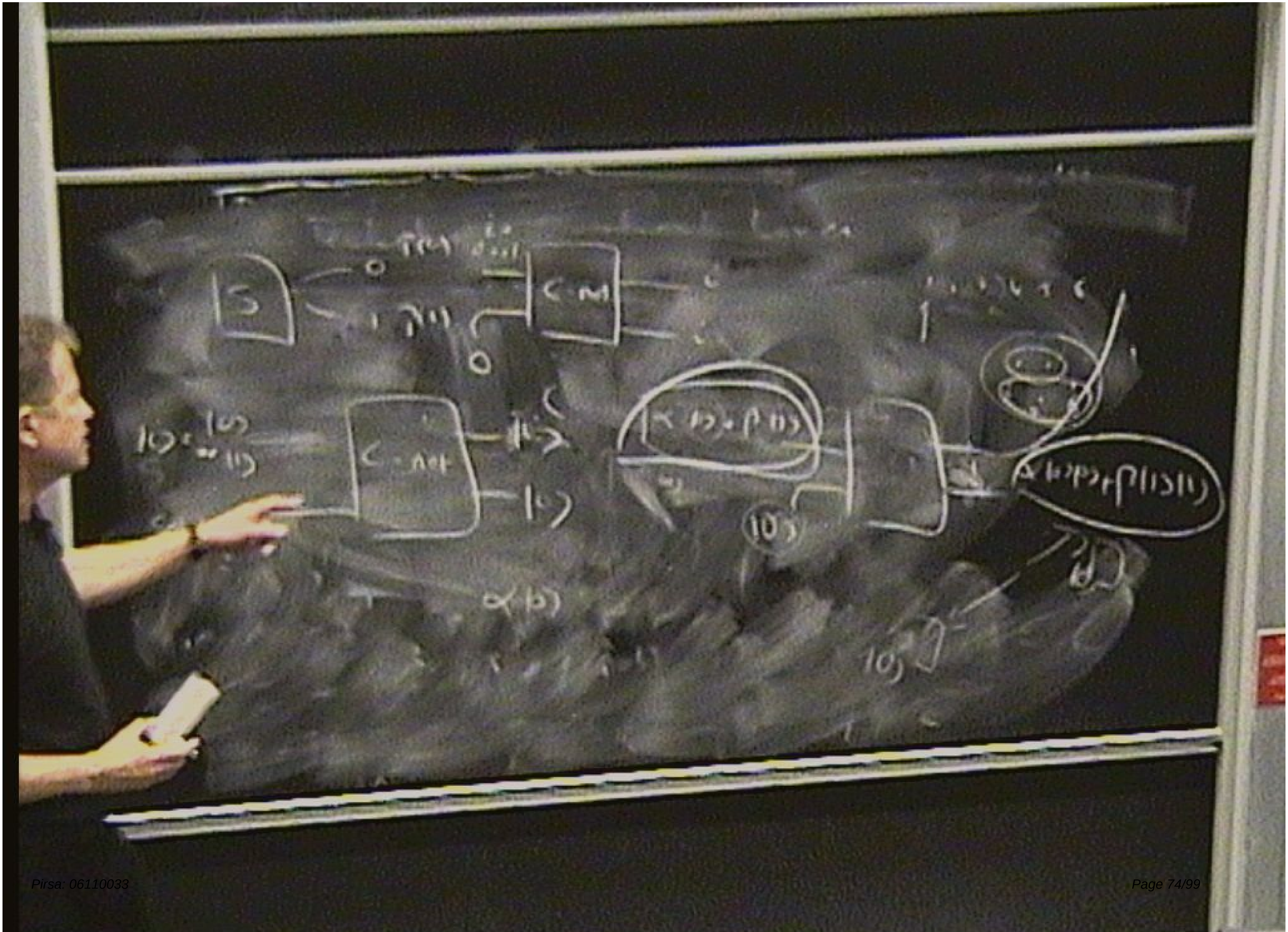


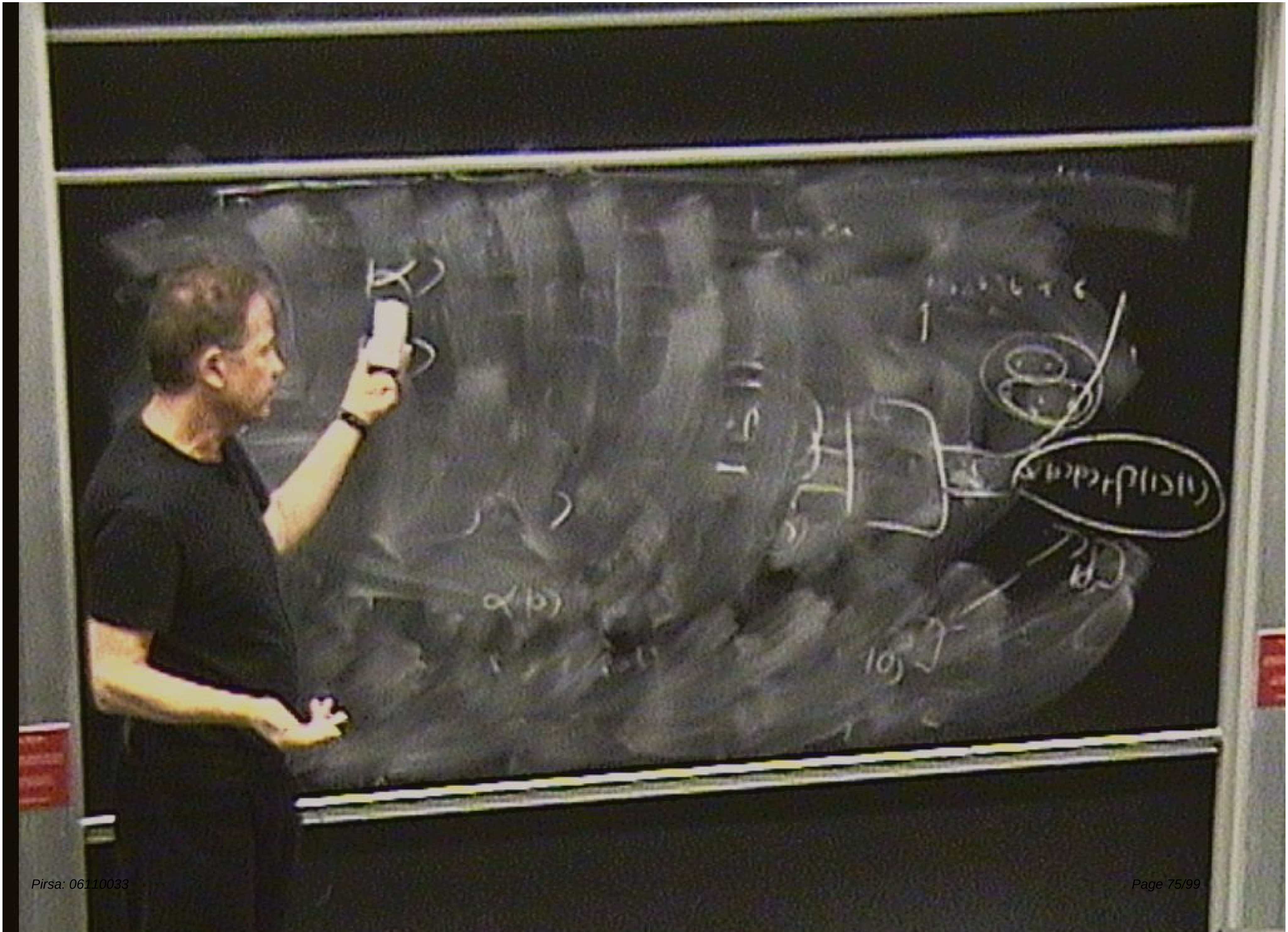

LOC
SEP
violated by

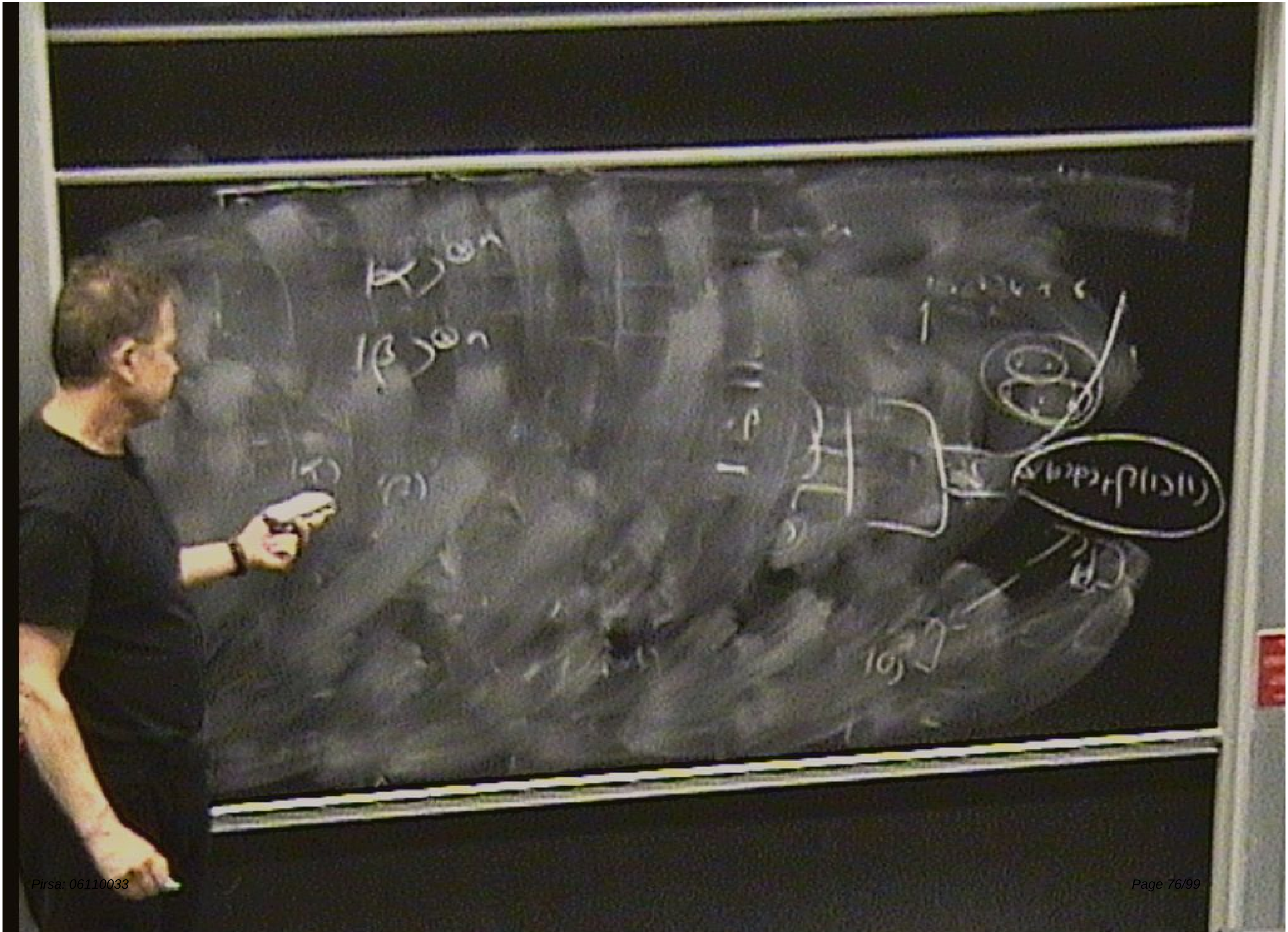

Einstein's realism
LOC = no signalling principle
SEP + LOC
violated by q m.

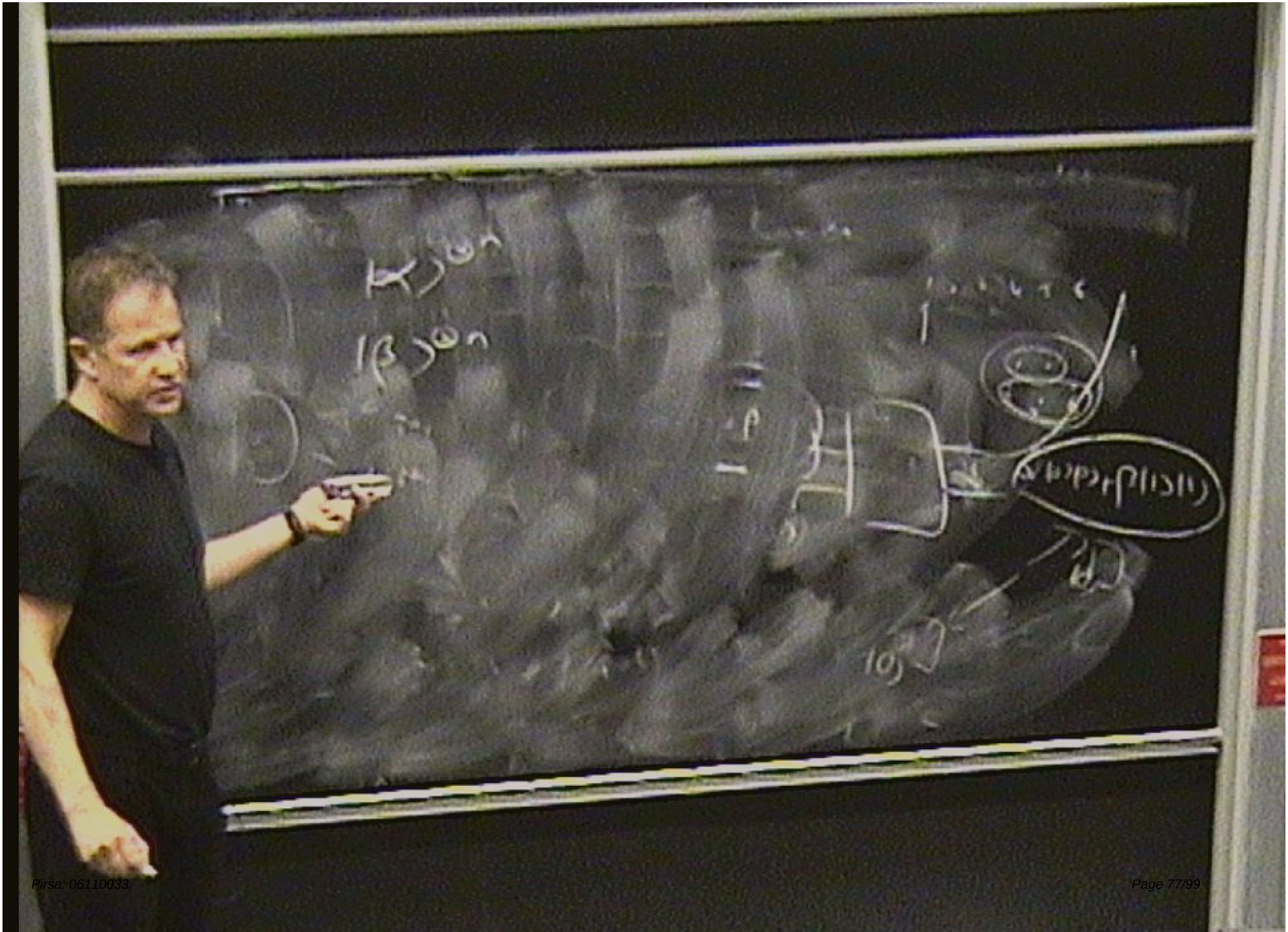


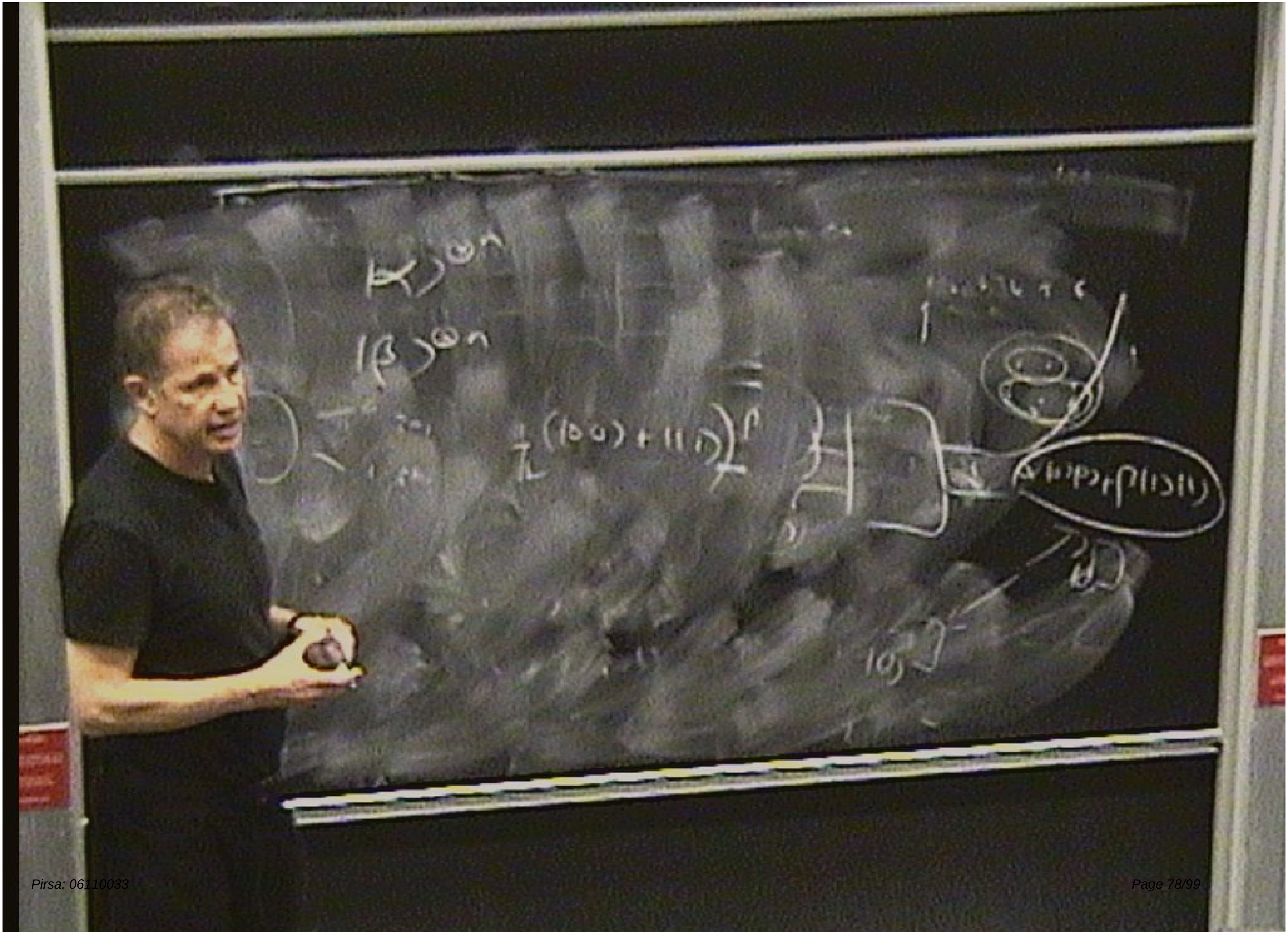


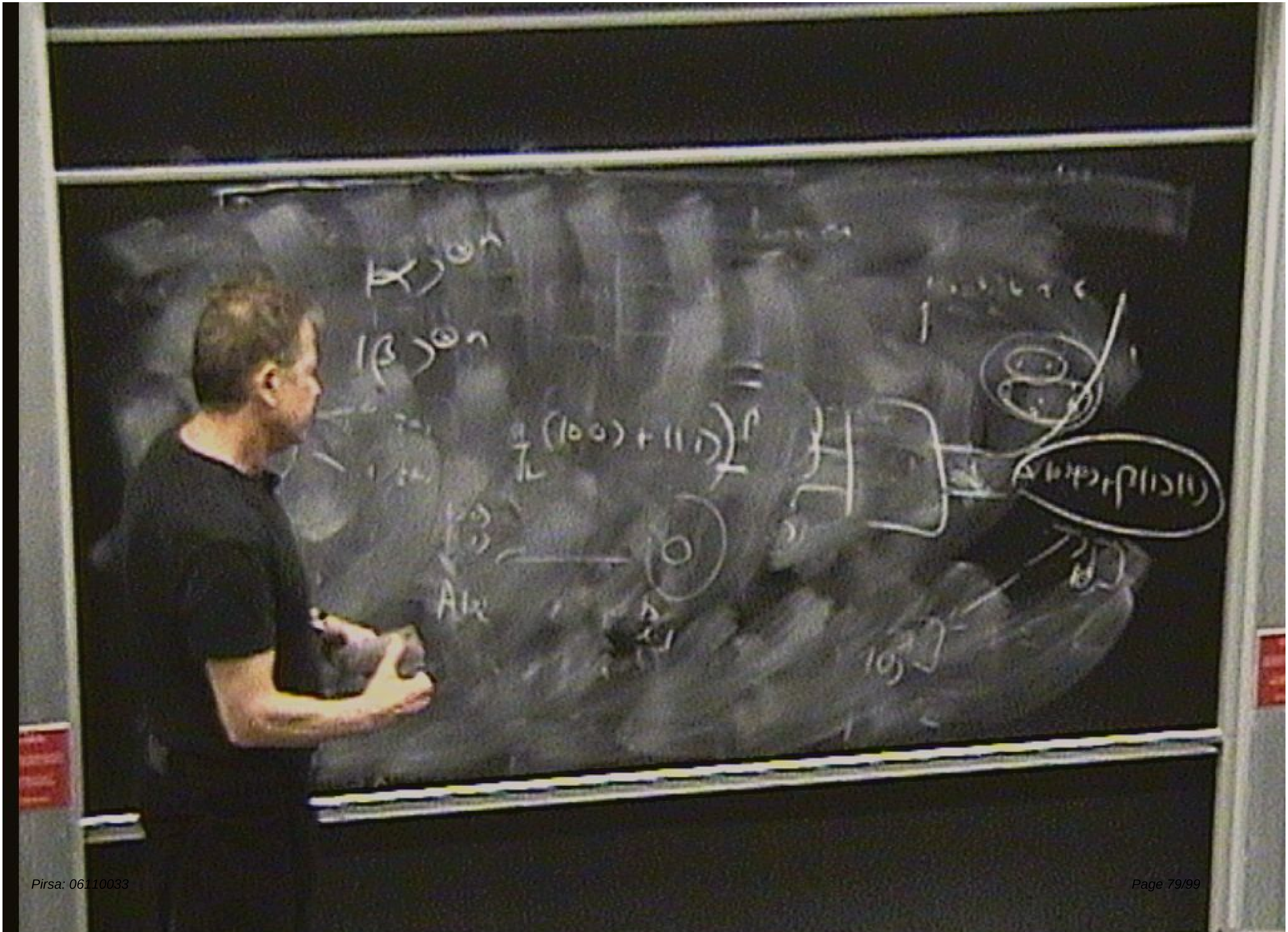


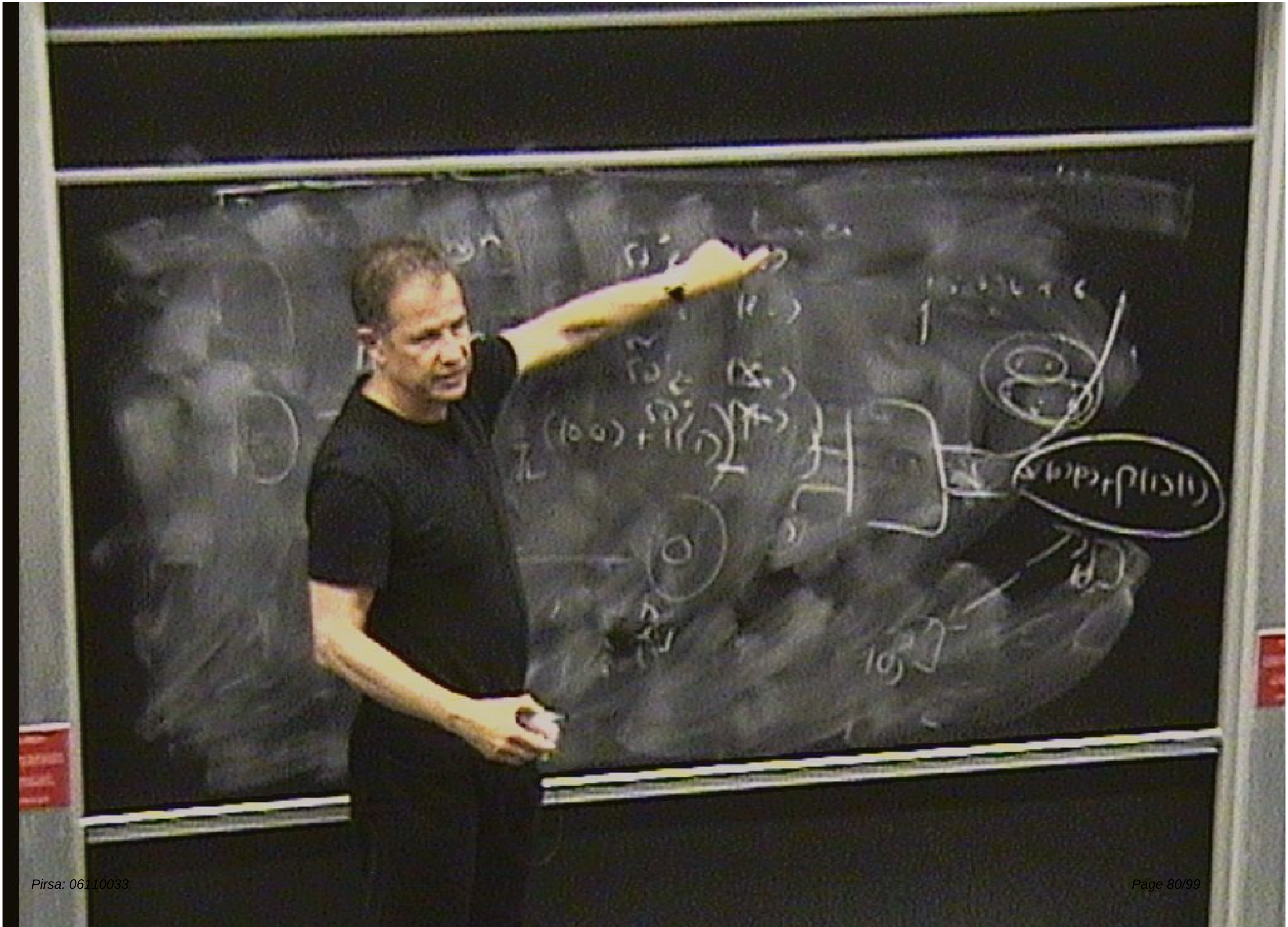


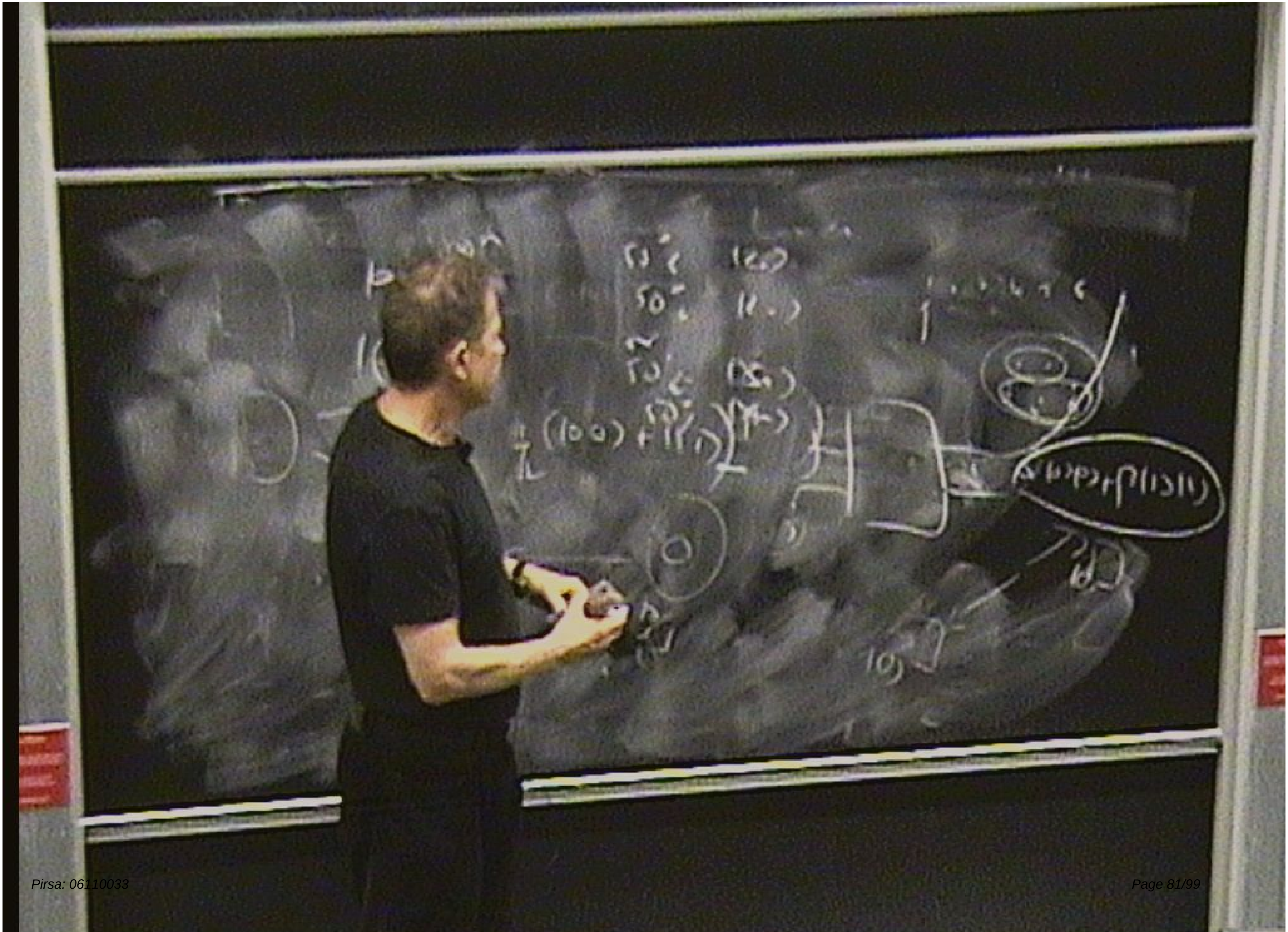


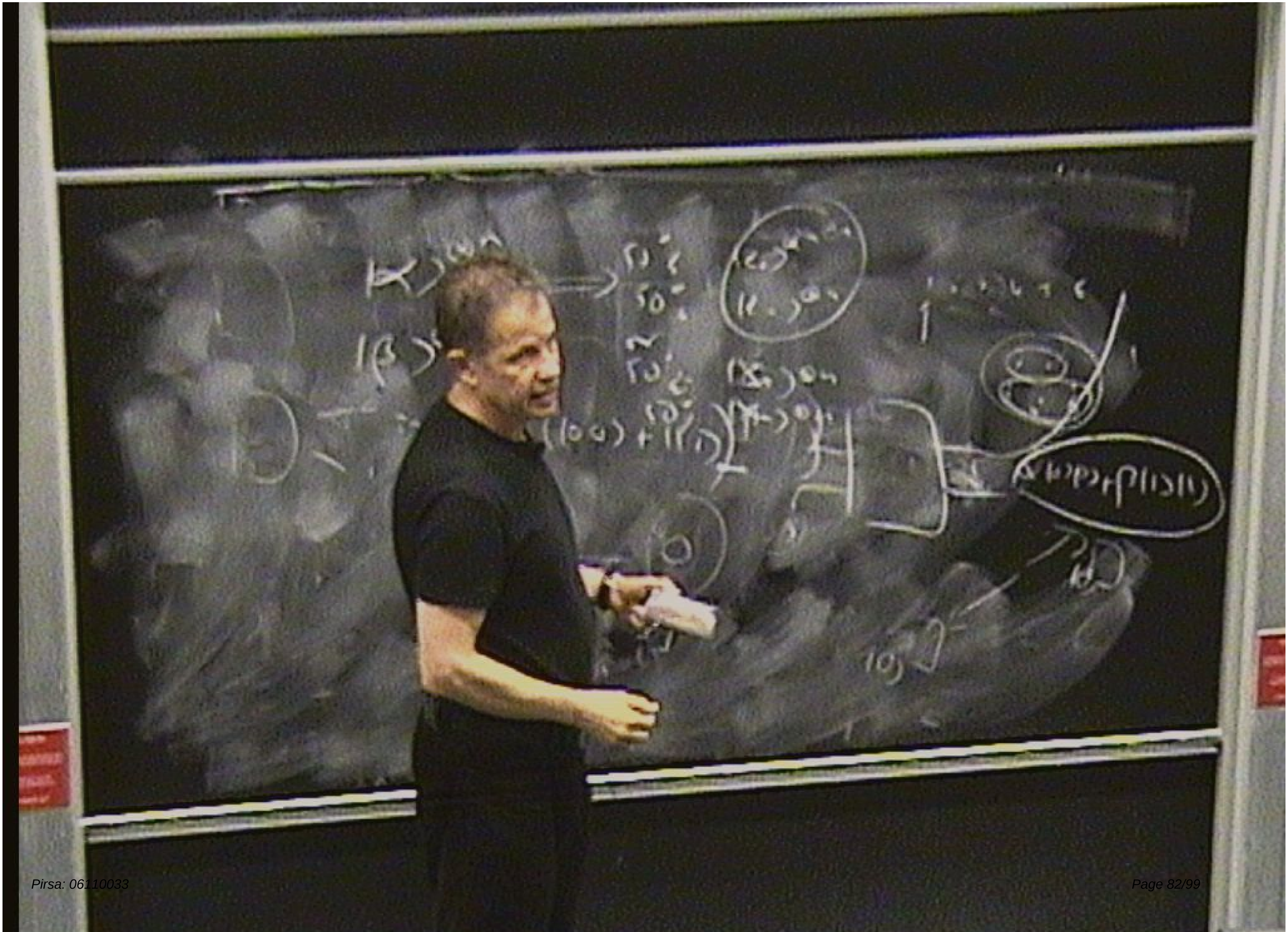


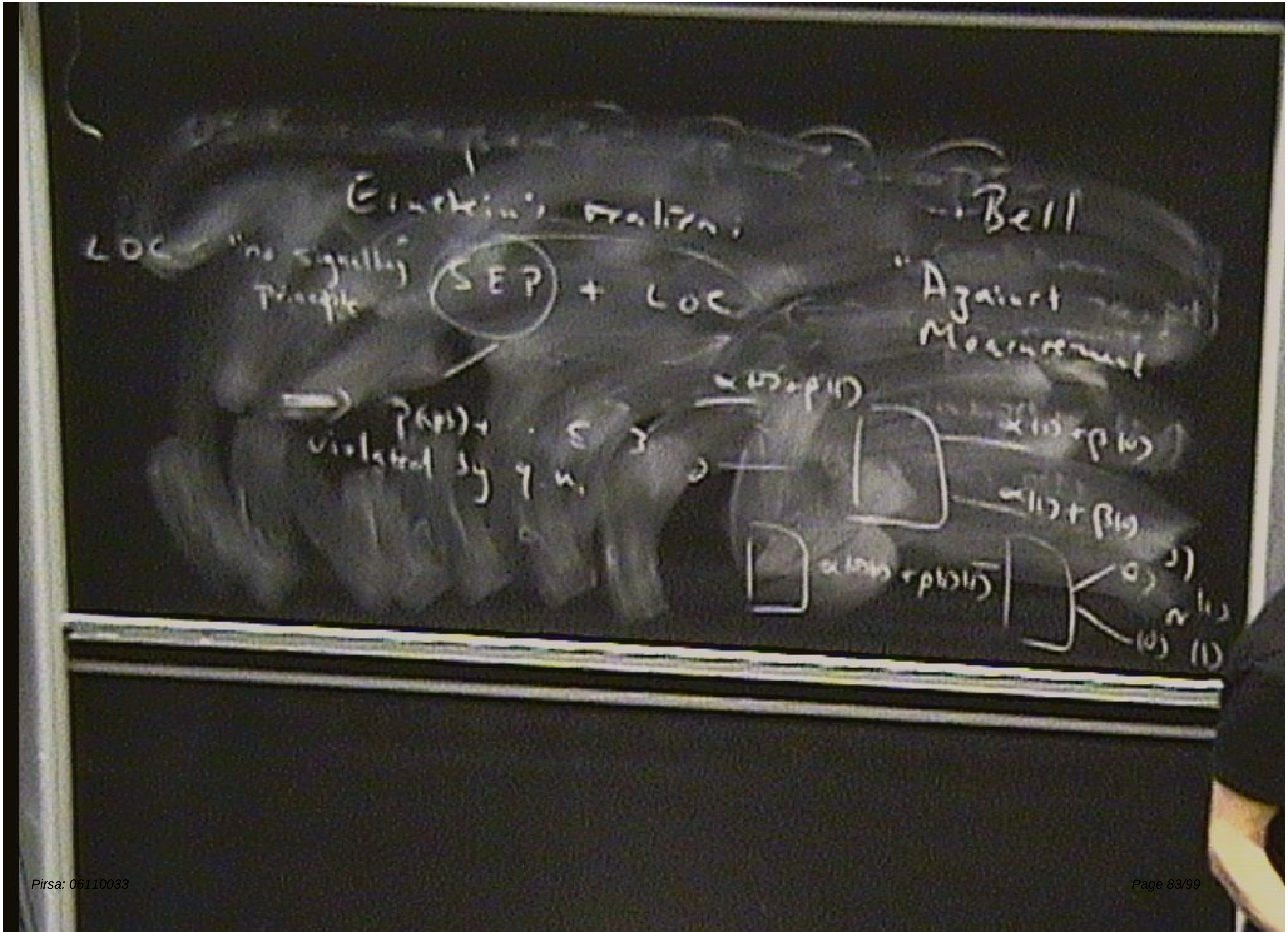

|α⟩⊗n
|β⟩⊗n

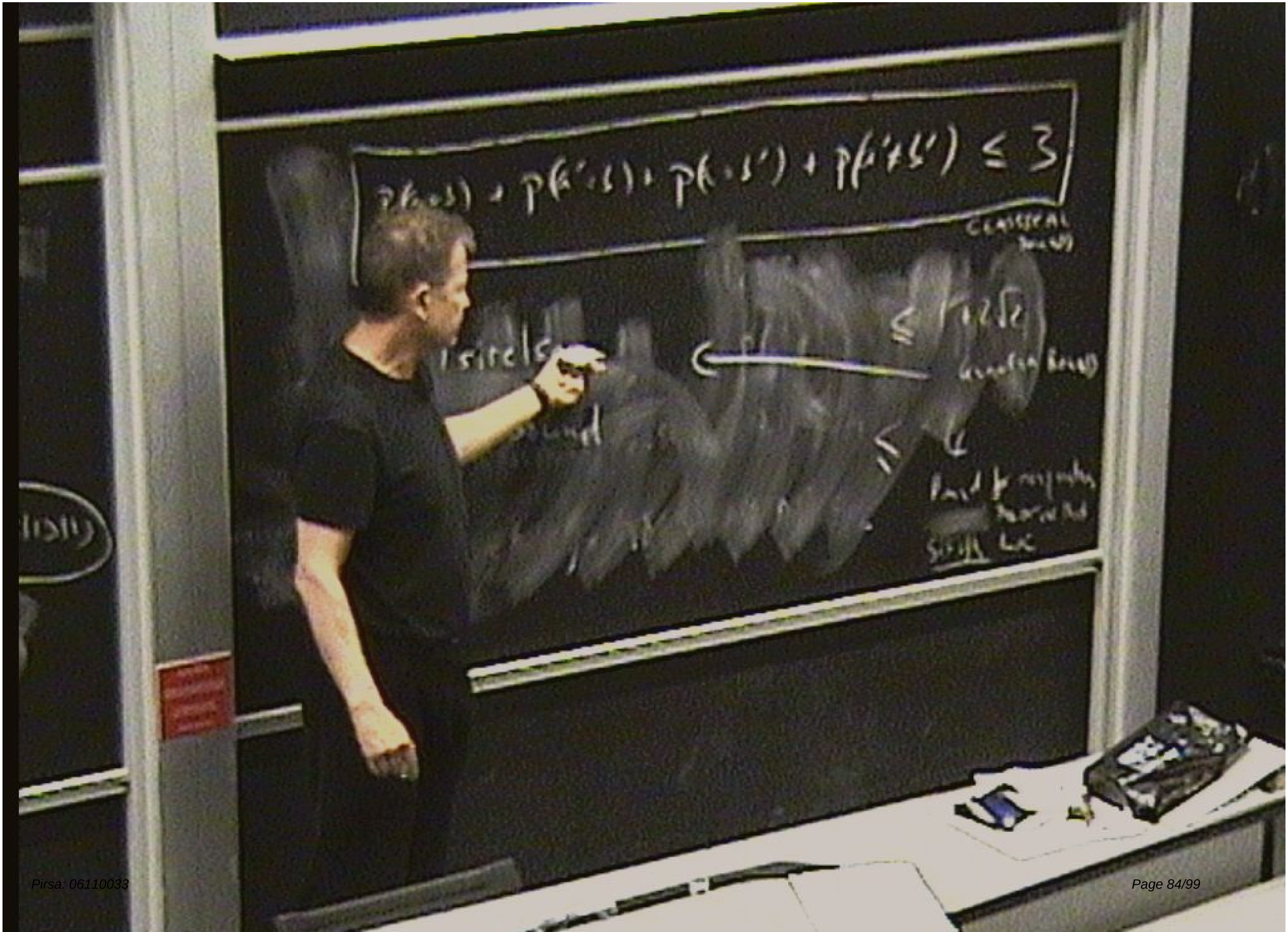


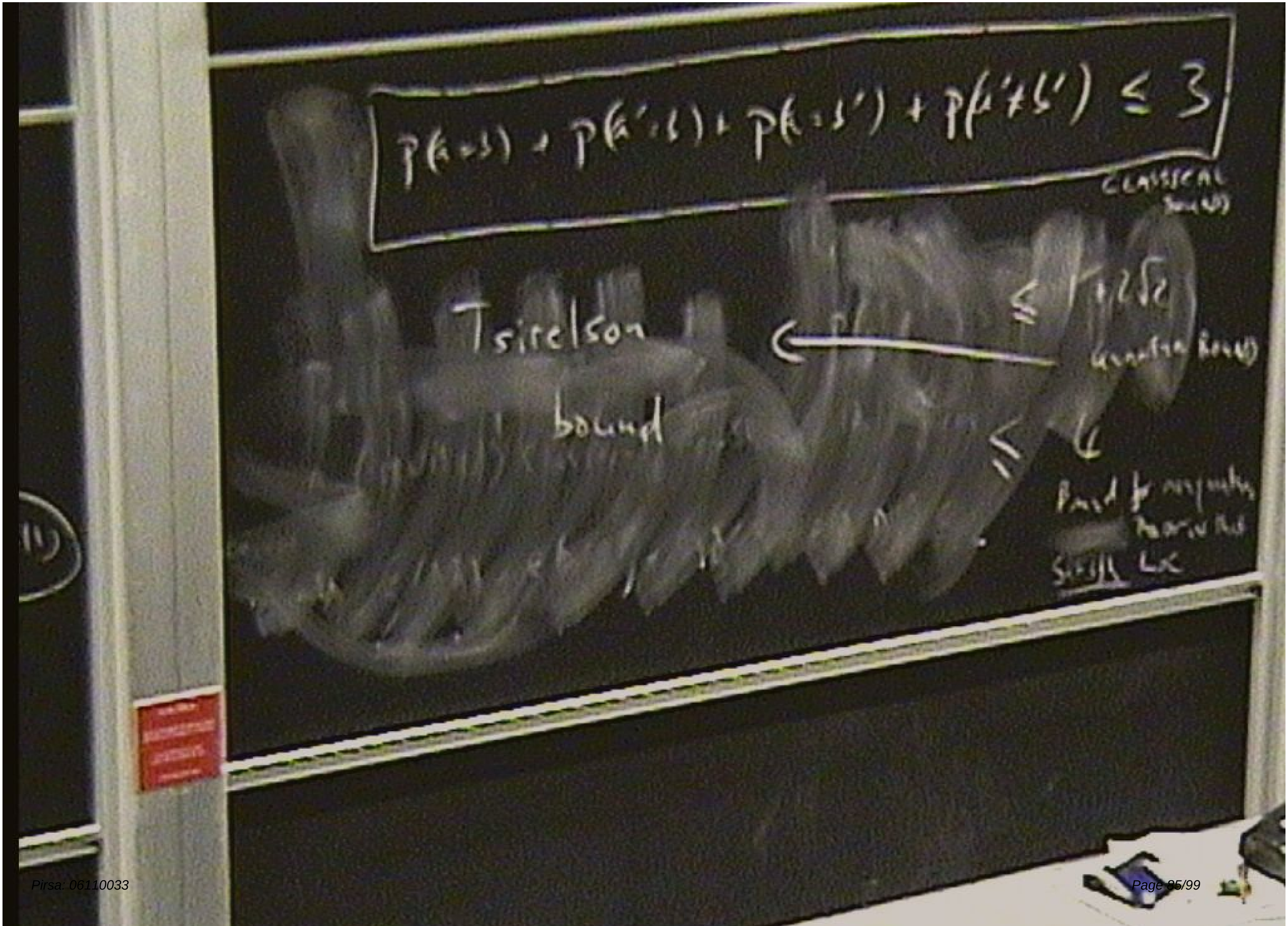


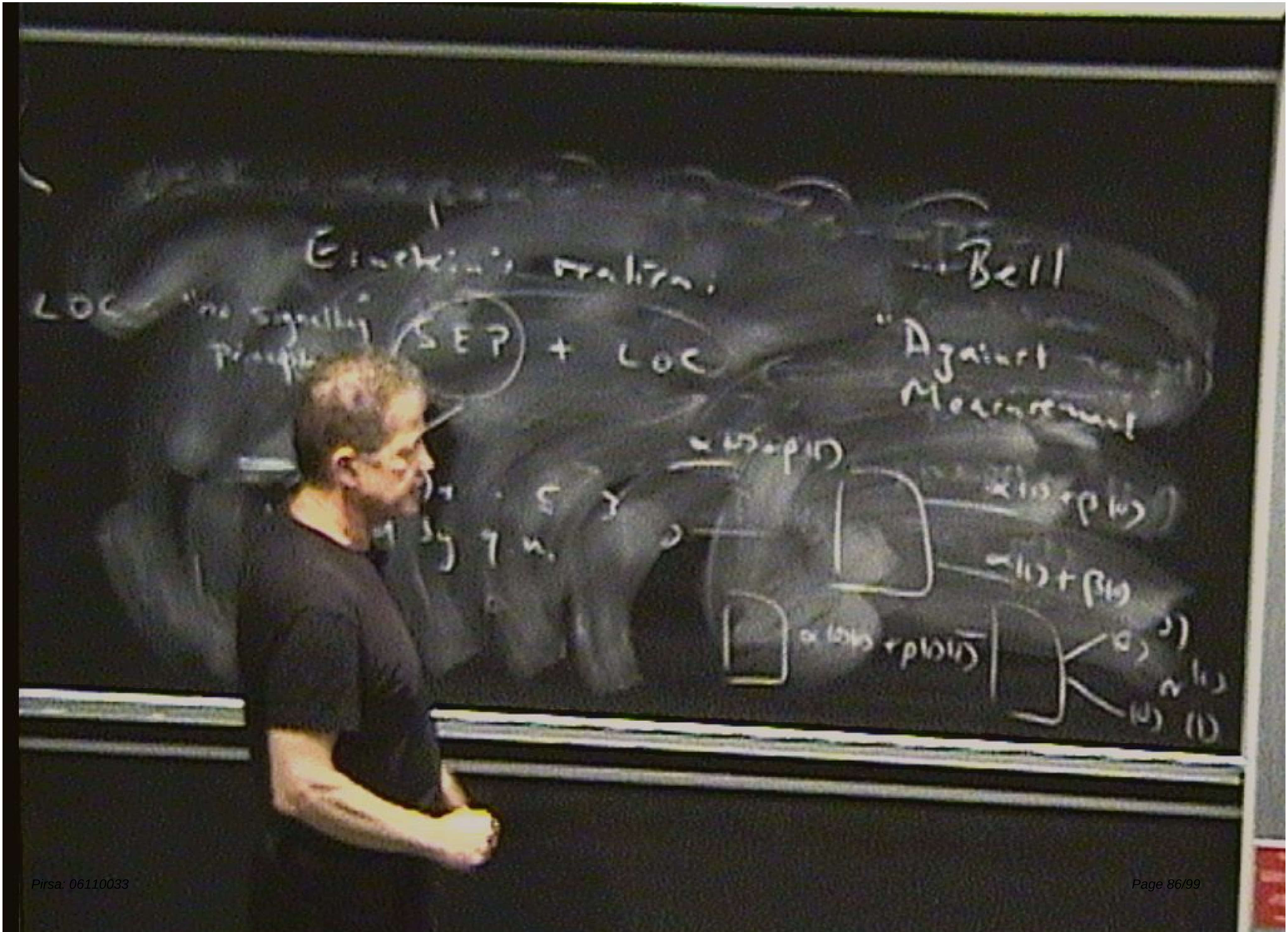


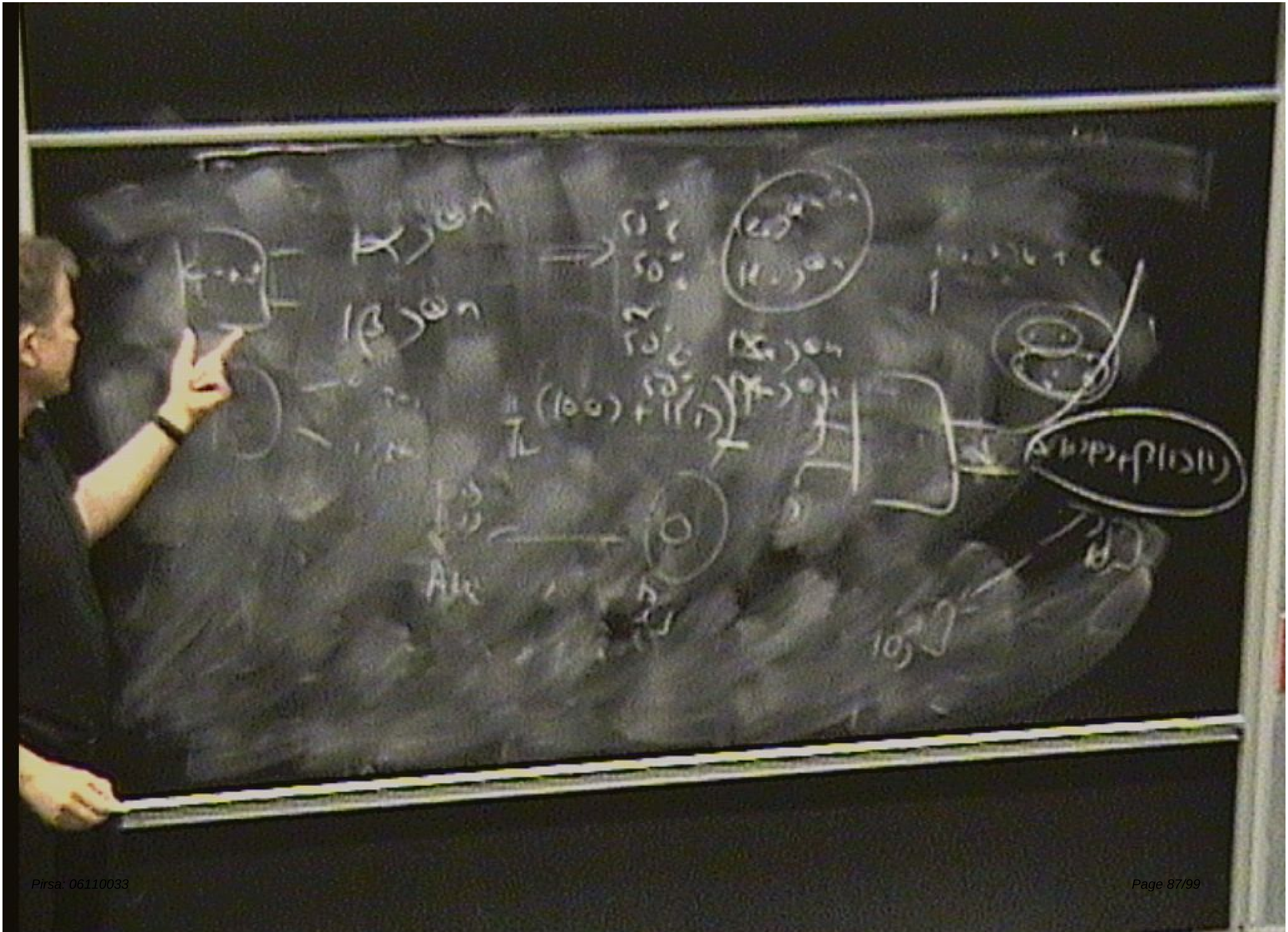


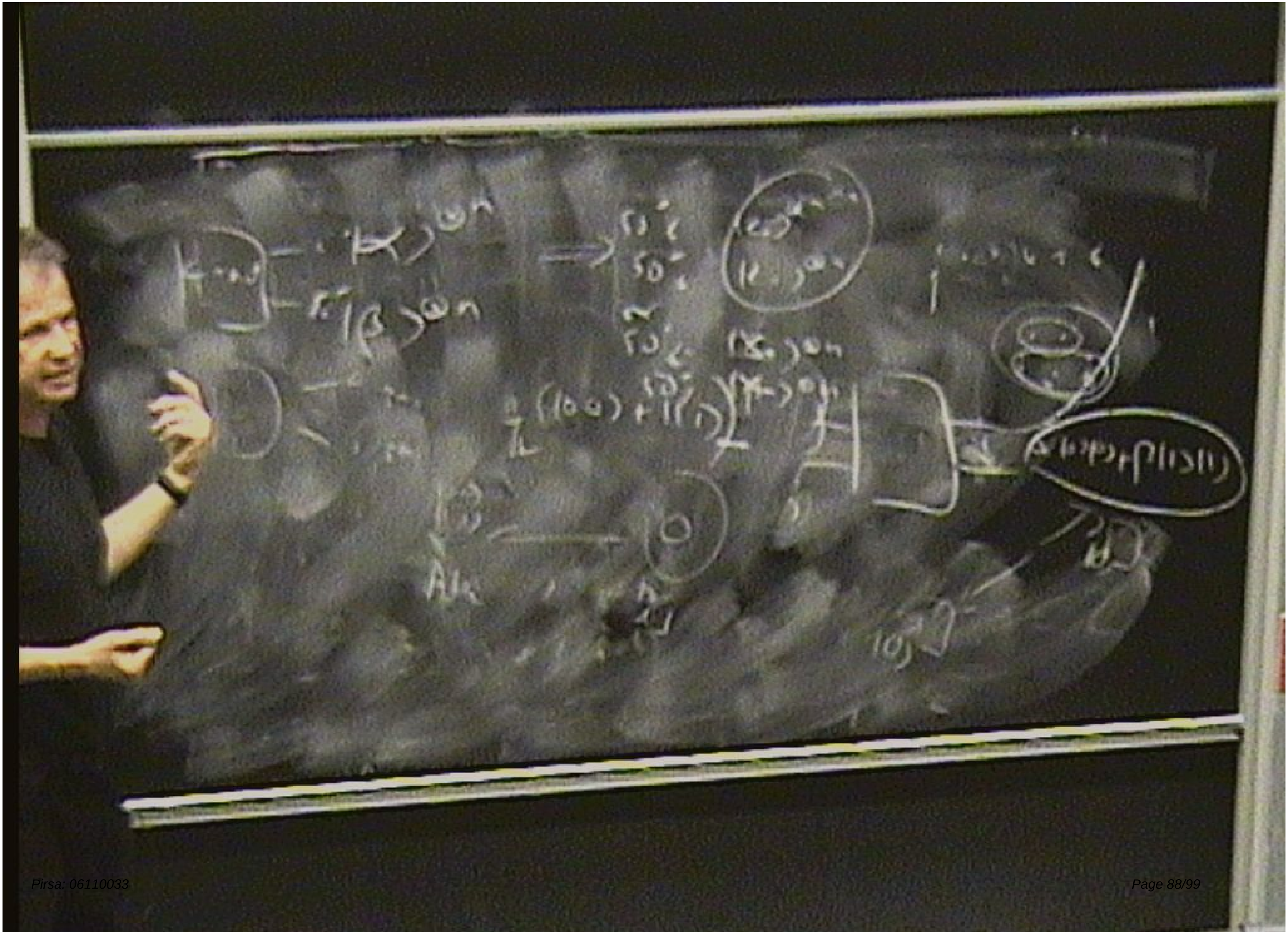

Einstein's realism
Bell
LOC
"no signalling" principle
SEP + LOC
"Against Measurement"
violated by q.m.

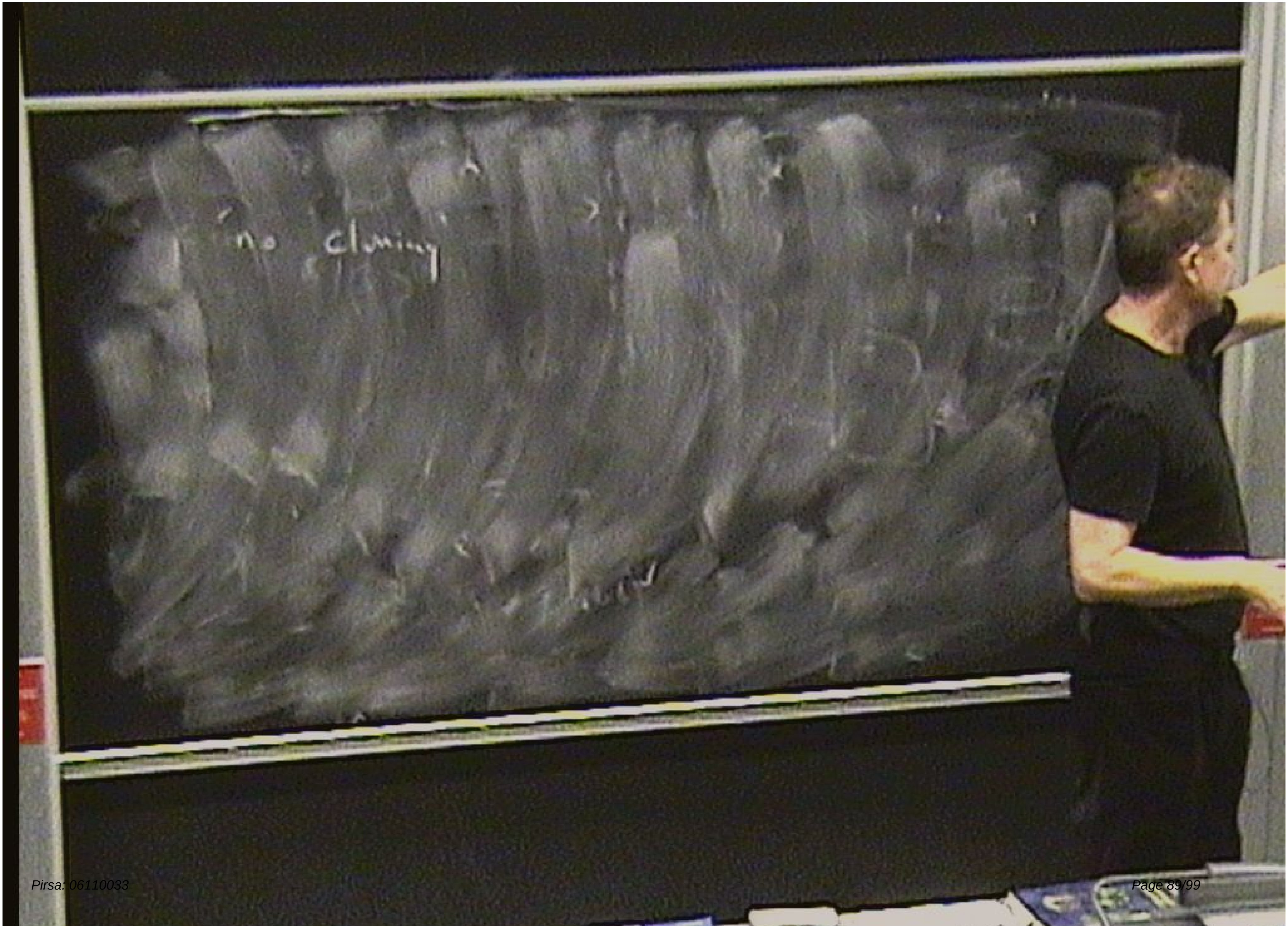

CLASSICAL

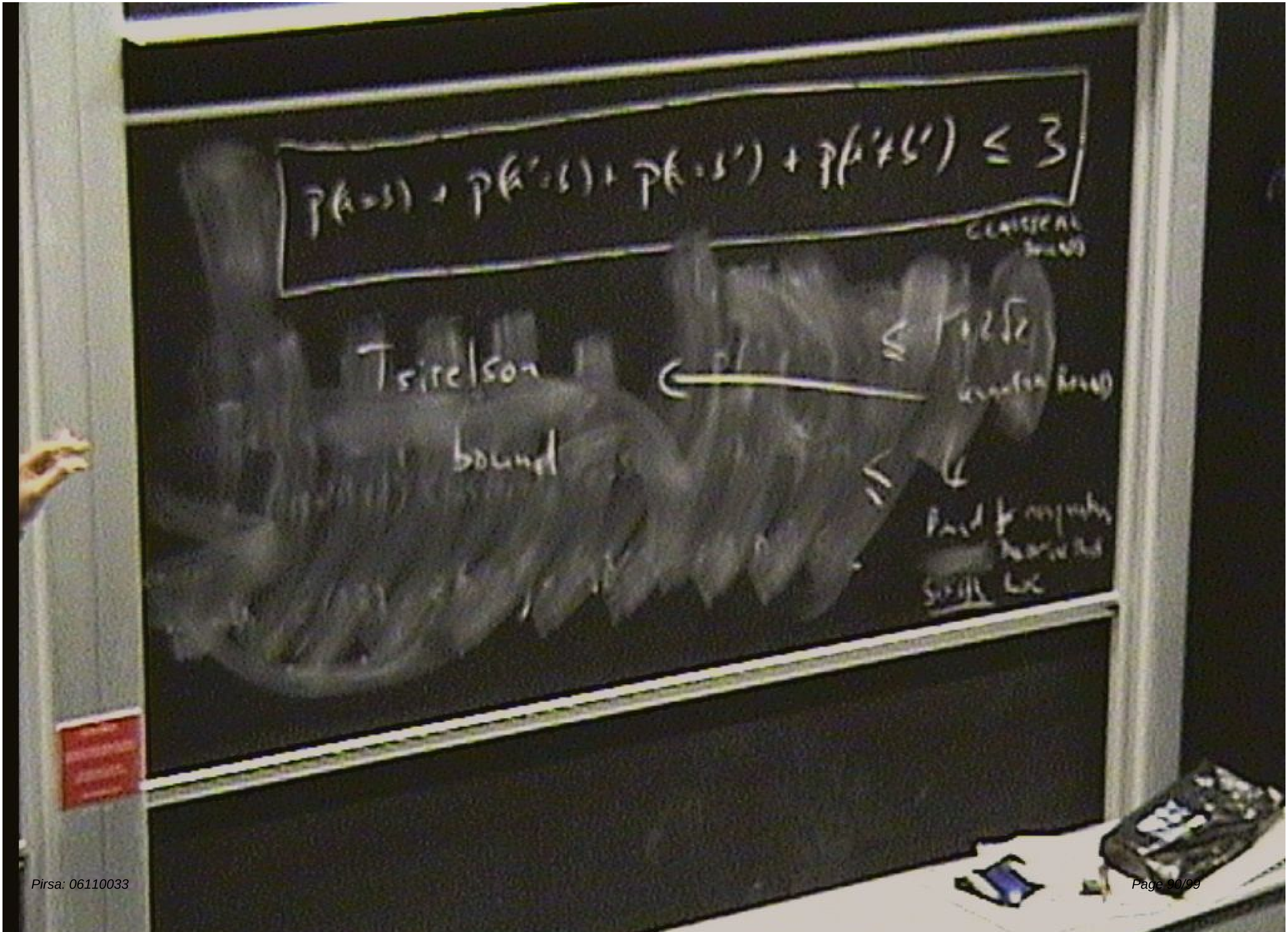

CLASSICAL
Tsirelson
bound

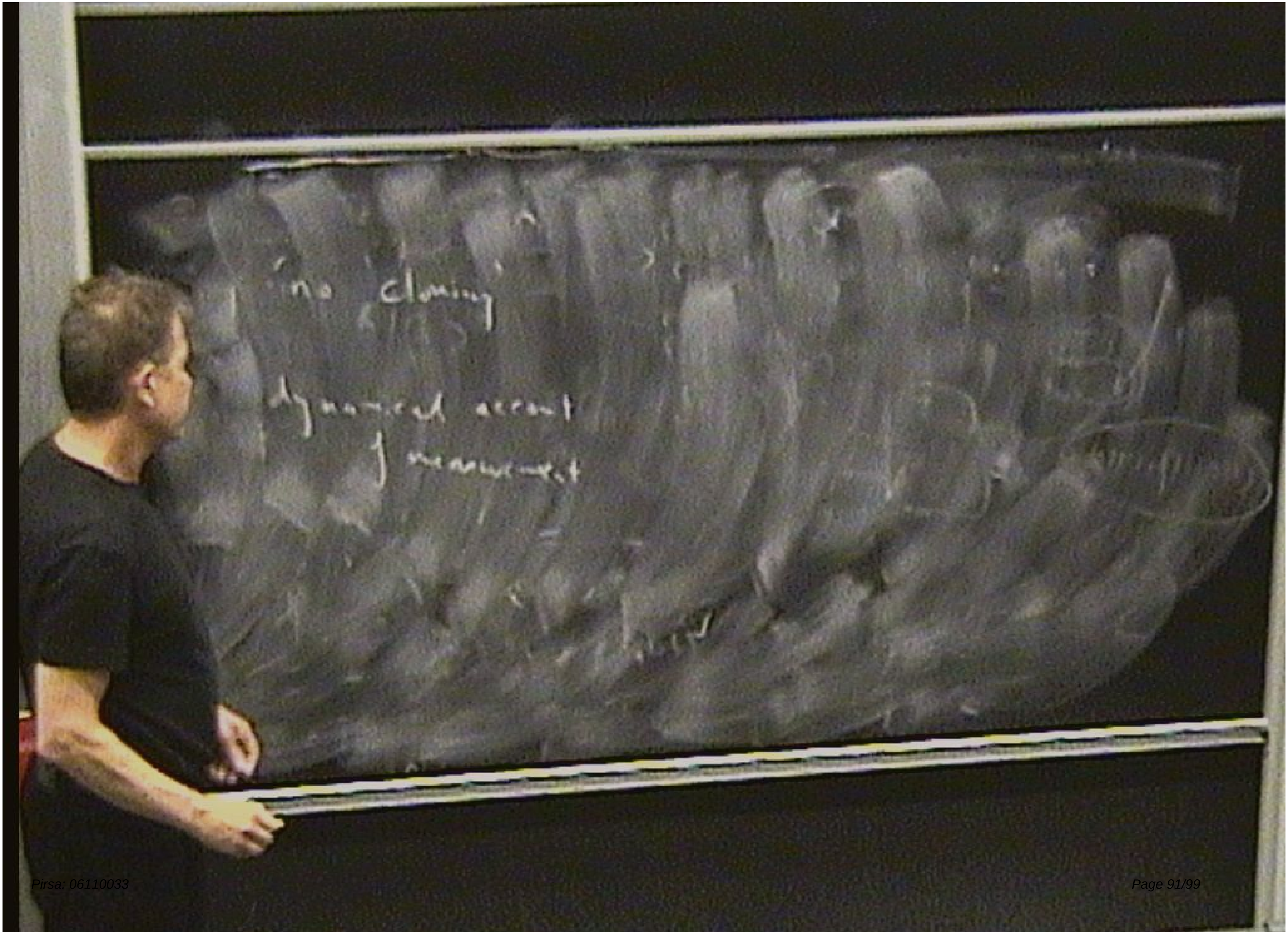

Einstein's realism
Bell
LOC
no signalling principle
SEP + LOC
"Against
Measurement"

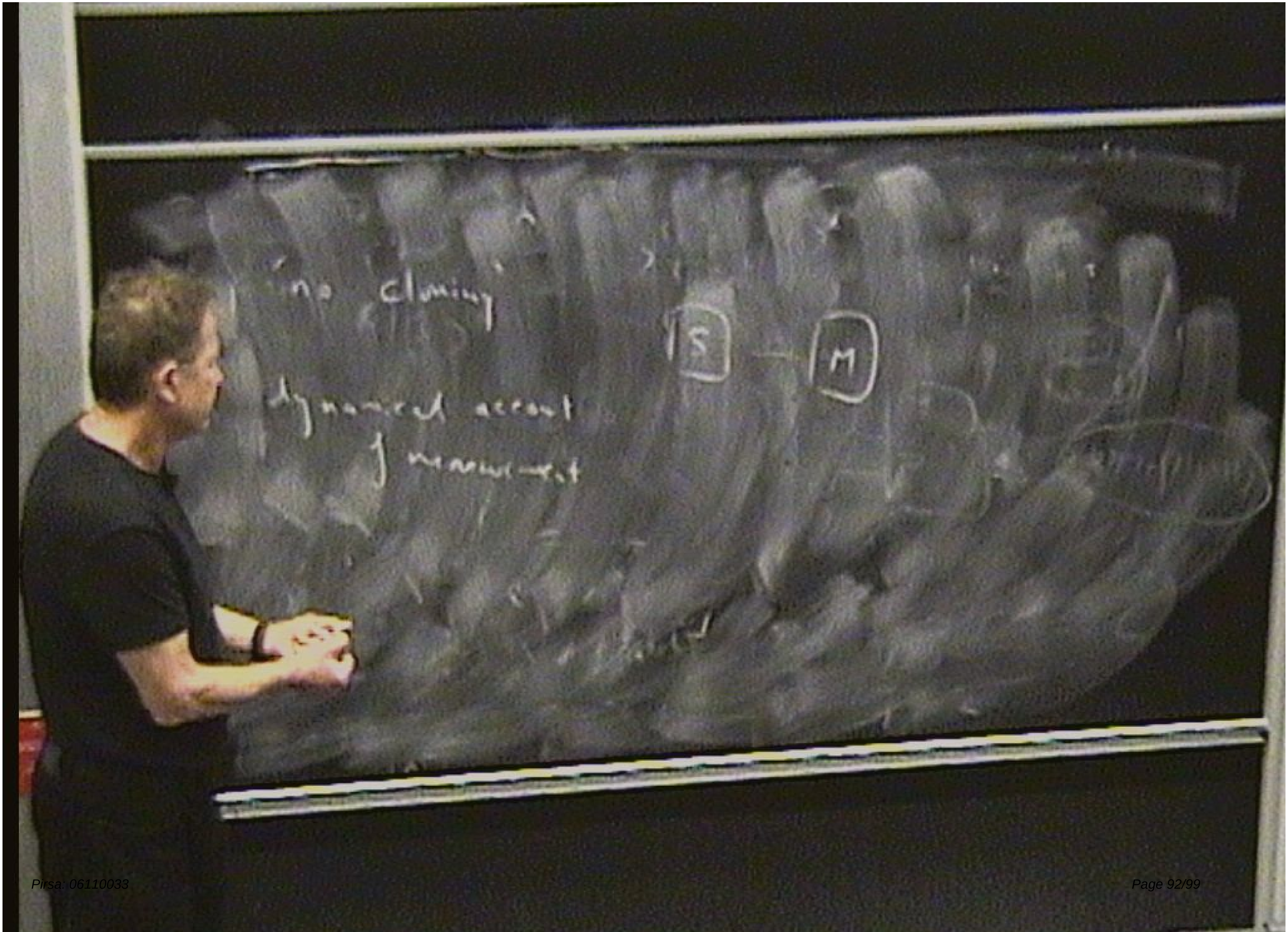


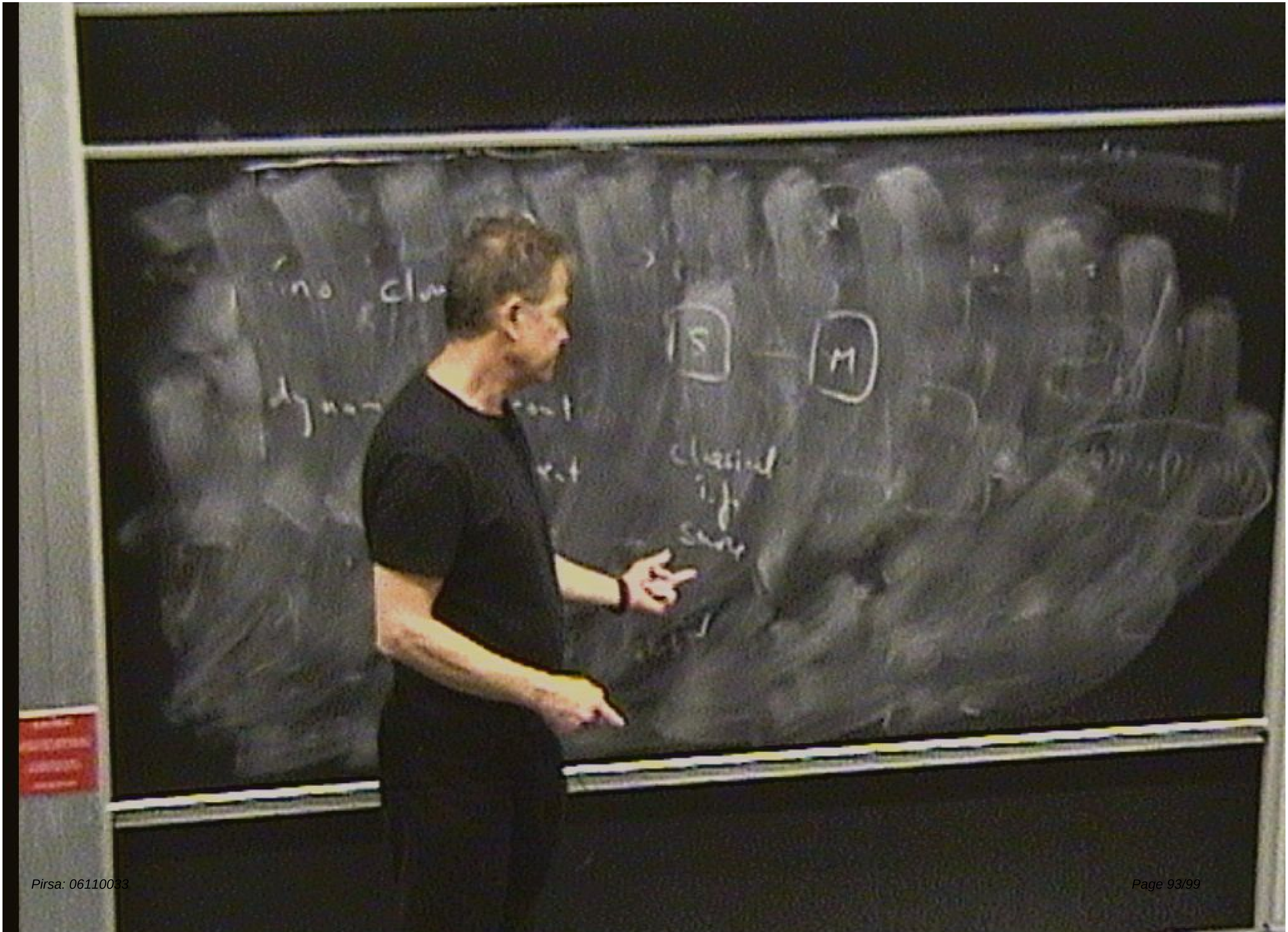


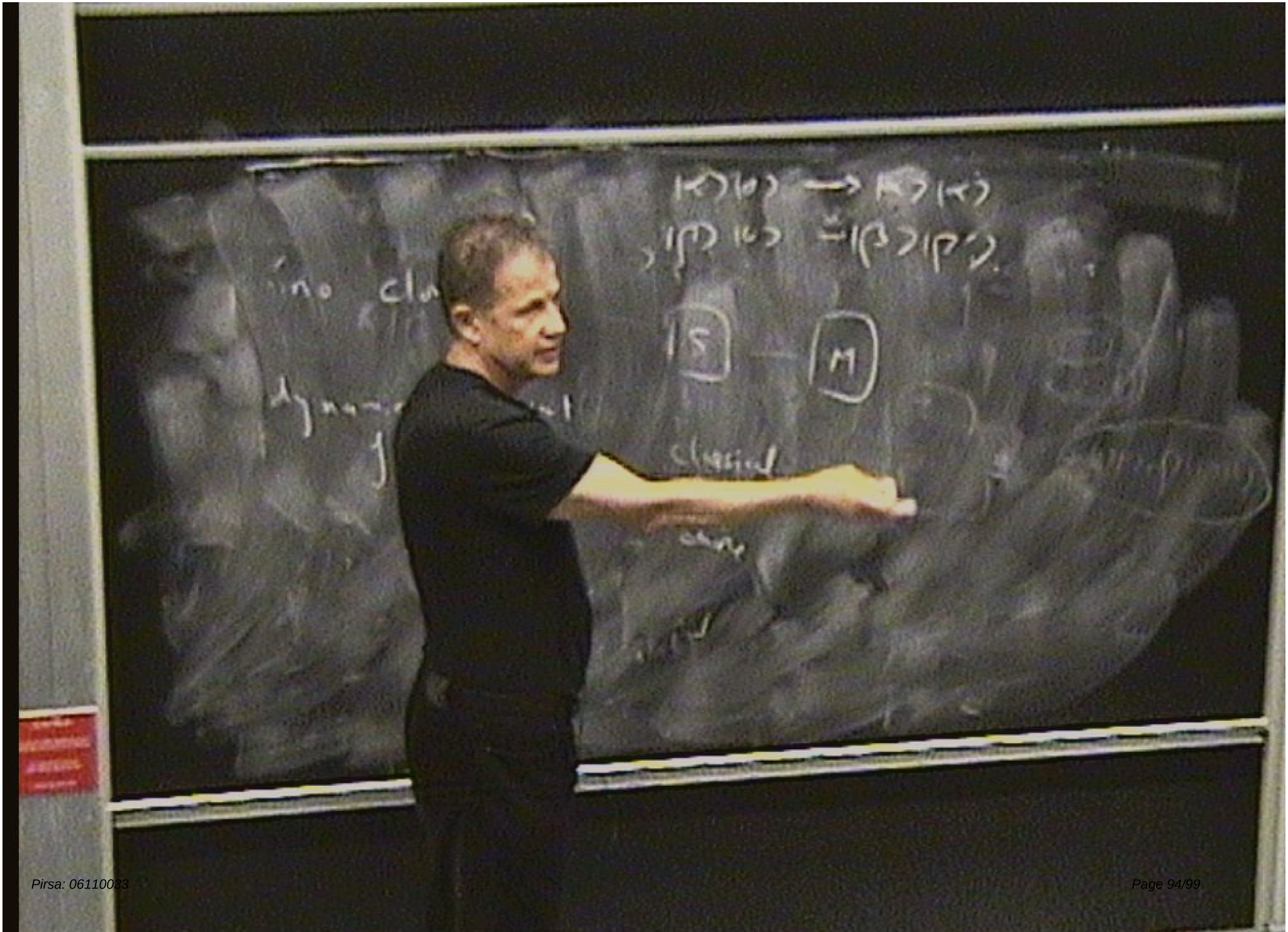

no cloning

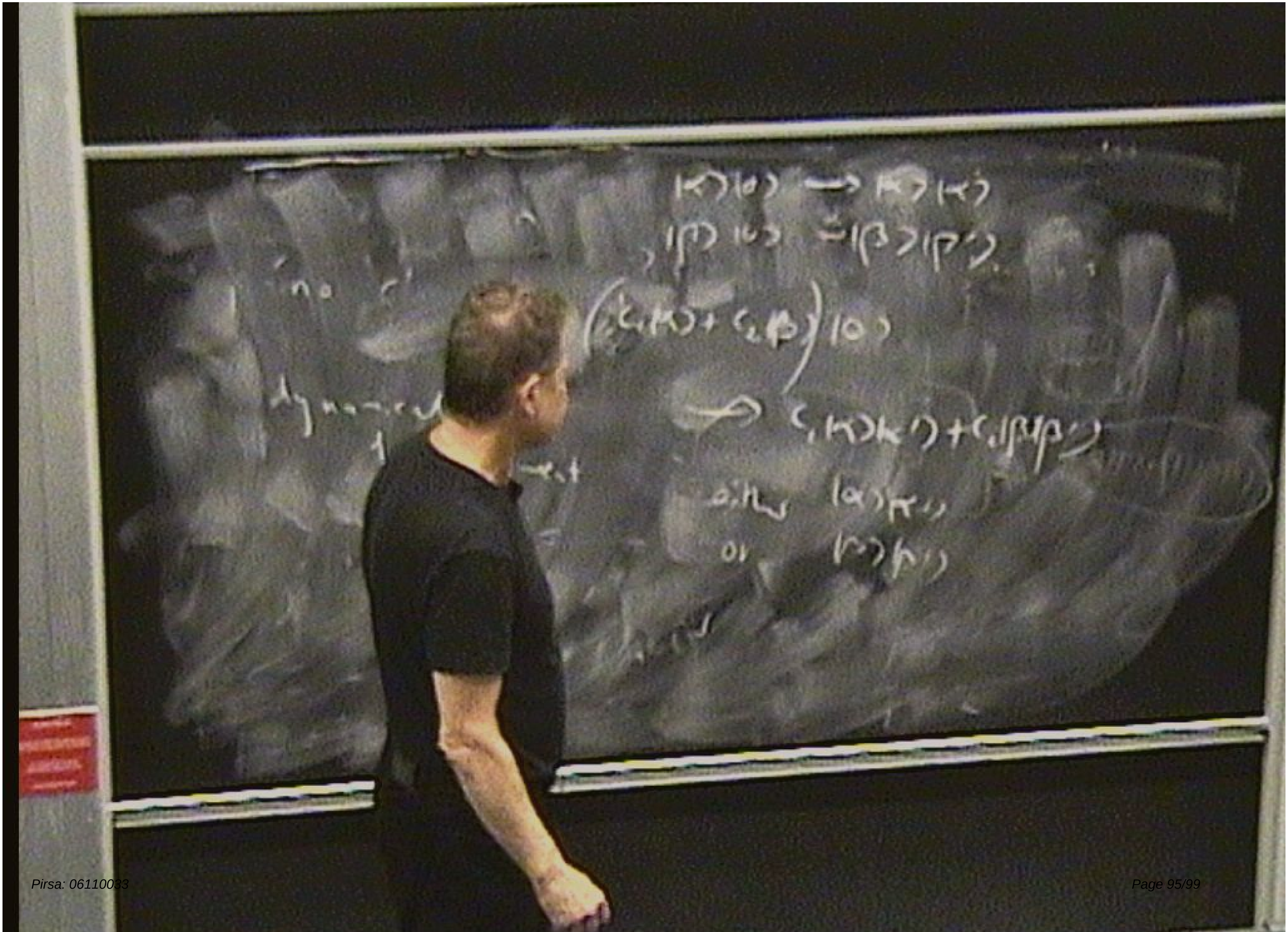

p(a=b) + p(a'=b) + p(a=b') + p(a'≠b') ≤ 3
CLASSICAL bound
Tsirelson bound
quantum bound

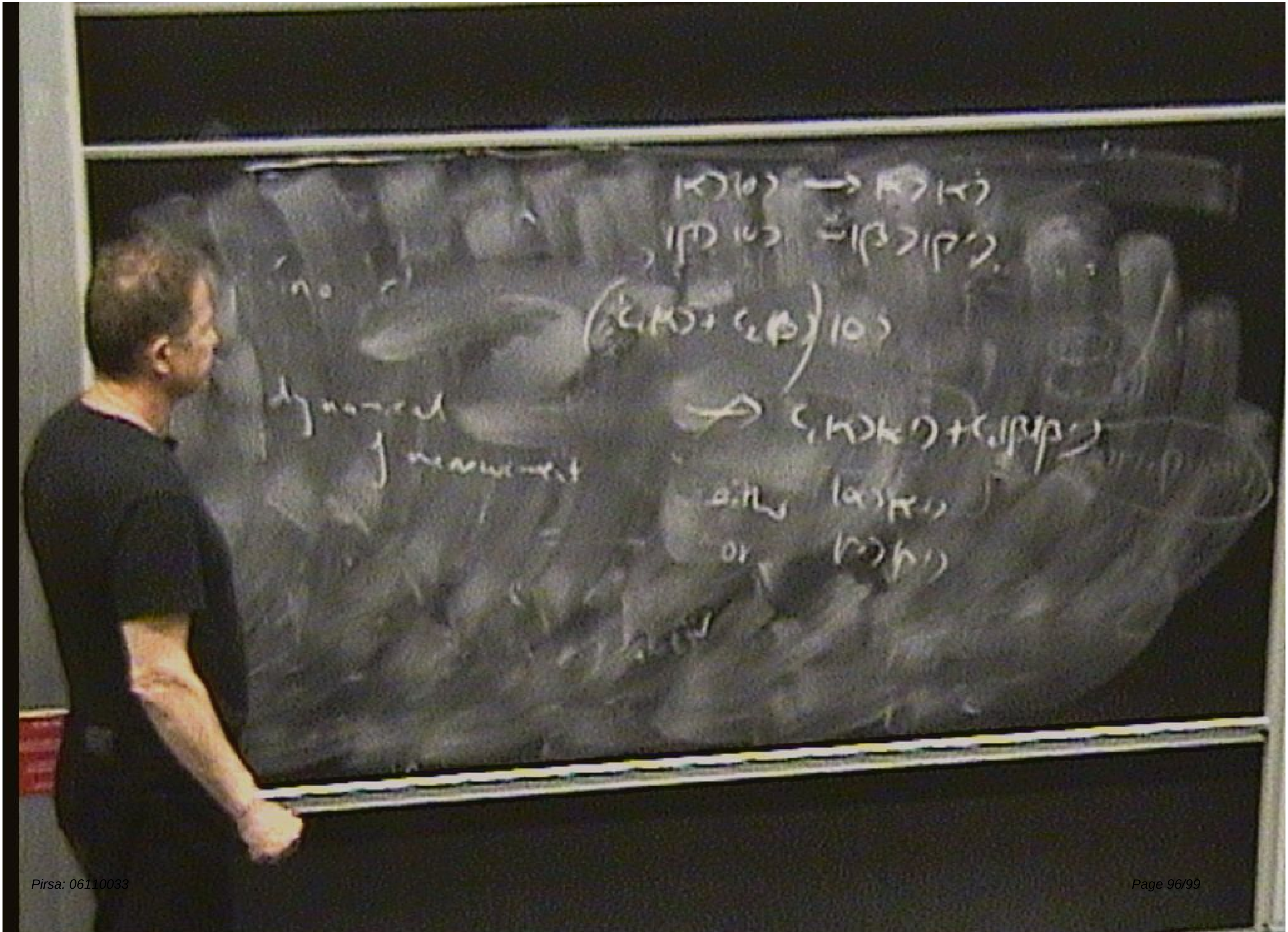

no cloning
dynamical account
of measurement

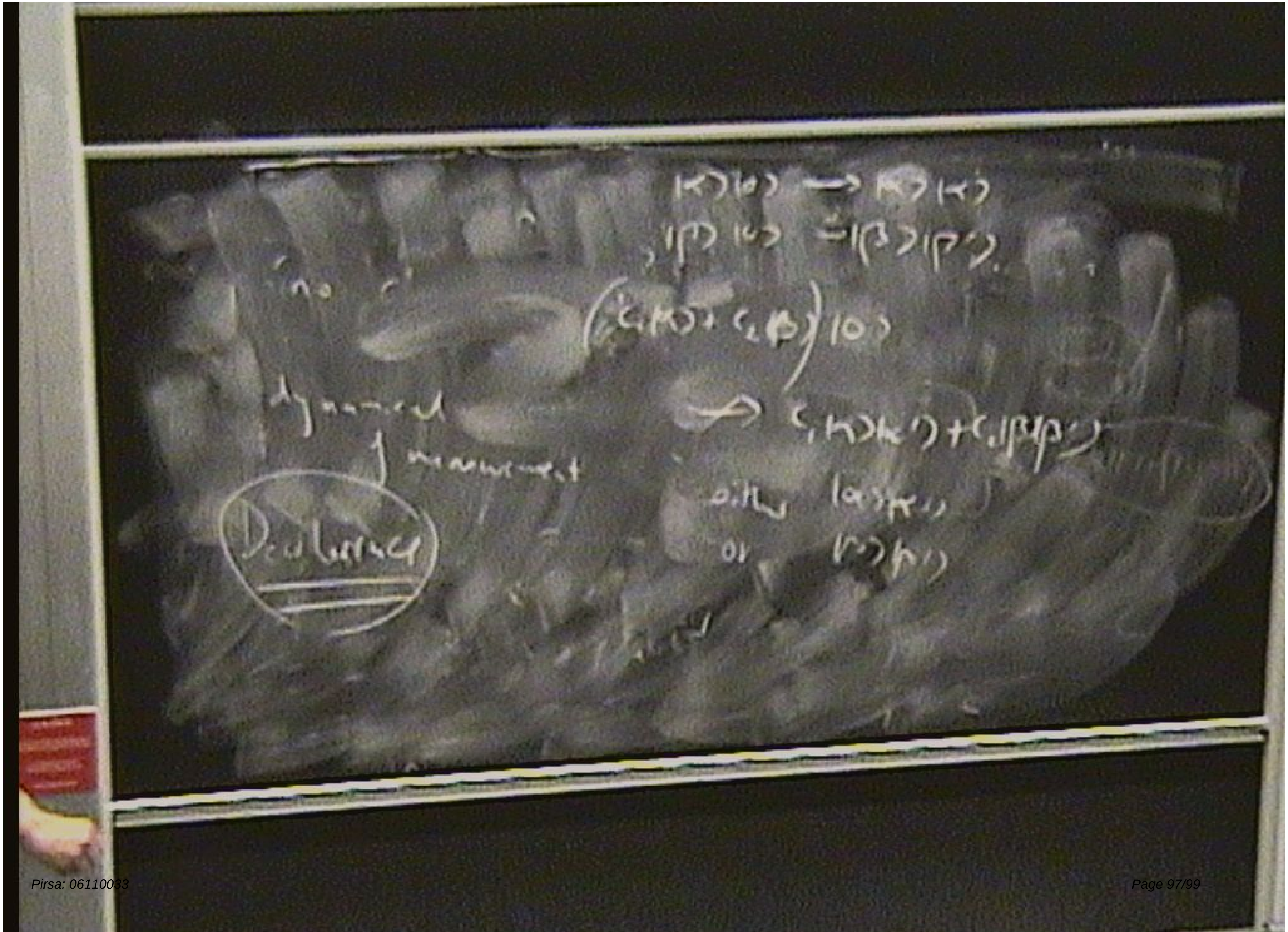

no cloning
dynamical account
of measurement
S
M

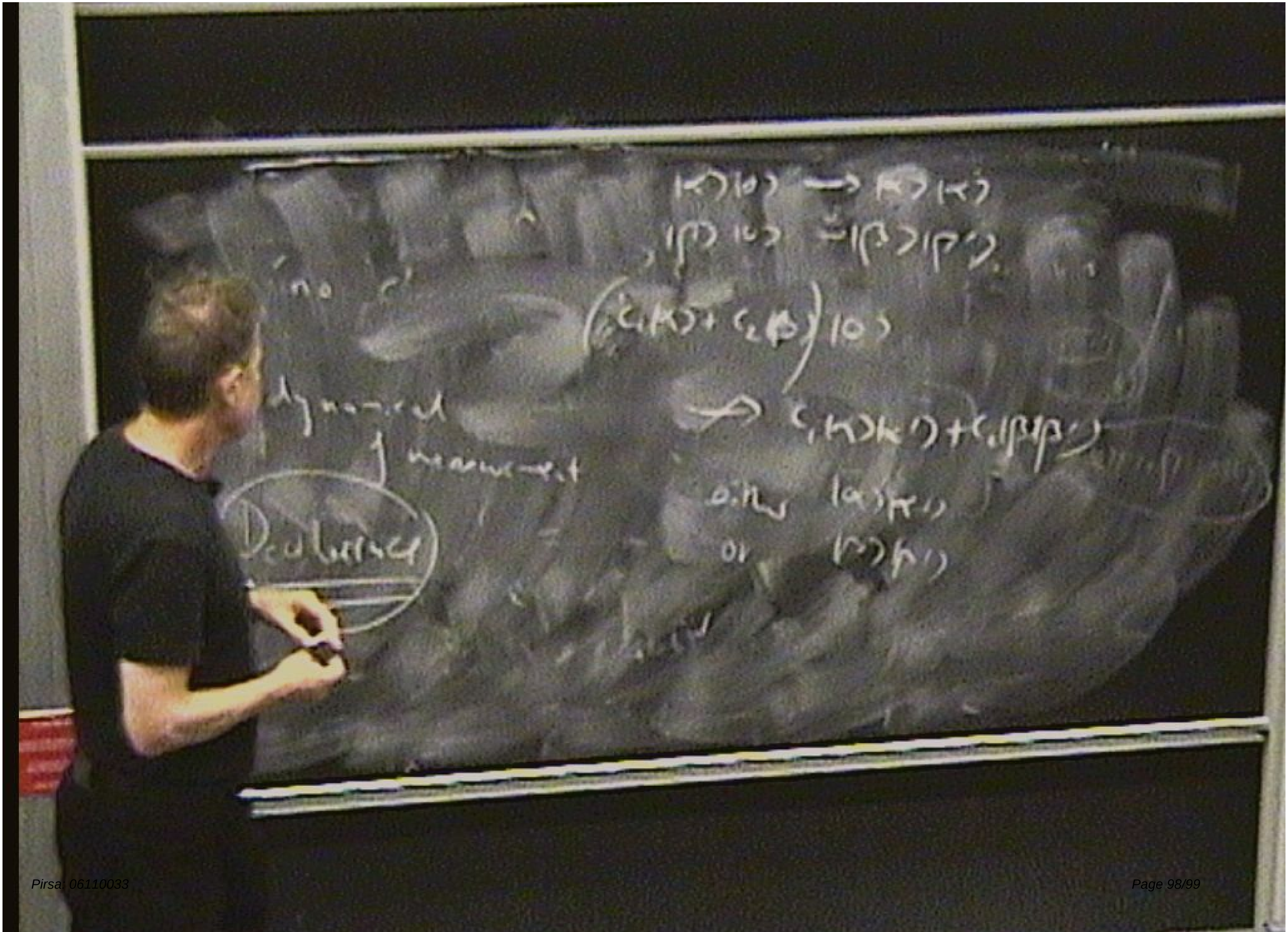

S
M
classical

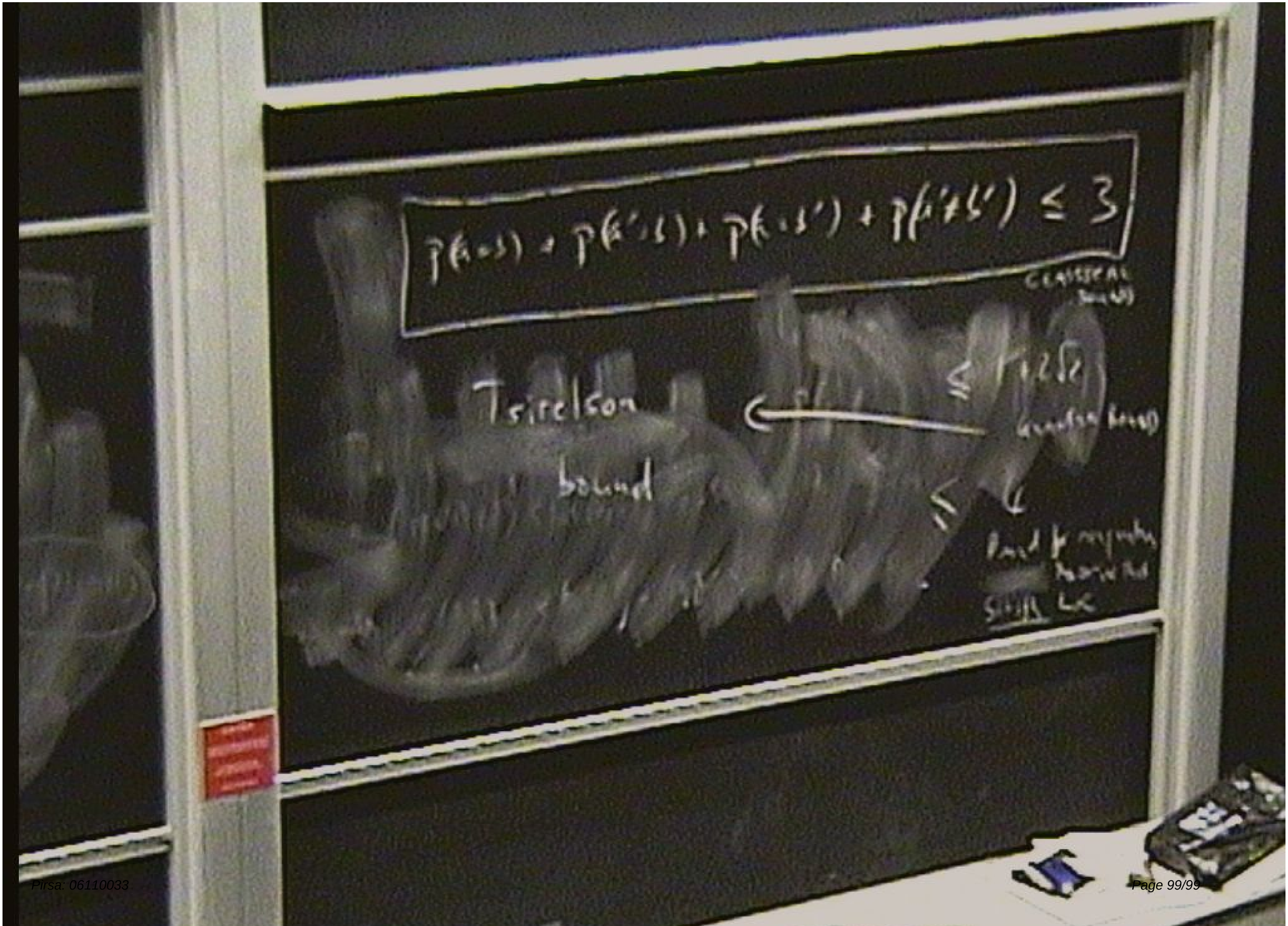

S
M
classical




dynamical
measurement
either
or


$|\alpha\rangle|0\rangle \to |\alpha\rangle|\alpha\rangle$
$|\beta\rangle|0\rangle \to |\beta\rangle|\beta\rangle$
$(c_1|\alpha\rangle + c_2|\beta\rangle)|0\rangle$
$\to c_1|\alpha\rangle|\alpha\rangle + c_2|\beta\rangle|\beta\rangle$
either $|\alpha\rangle|\alpha\rangle$
or $|\beta\rangle|\beta\rangle$
dynamical
measurement
Decoherence


dynamical
measurement
Decoherence


≤ 3
Tsirelson
bound
≤ 1 + 2√2
quantum bound