

Title: Introduction to Quantum Information and Computation from a Foundational Standpoint

Date: Nov 09, 2006 10:30 AM

URL: <http://pirsa.org/06110033>

Abstract: Classical Information and Quantum Information

'being - thus'

'being - thus'

ABILITY

LITY

'being - thus'

SEPARABILITY

LOCALITY



$\mathcal{A}$
 $\mathcal{A}, \mathcal{A}', \dots$

$\mathcal{B}$
 $\mathcal{B}, \mathcal{B}', \dots$

$$P_{\lambda}^{AB}(a|b) = P_{\lambda}^{AB}(a)$$

$$P_{\lambda}^{AB}(a|c) = P_{\lambda}^{AB}(a)$$

$$P_{\lambda}^{AB}(a) = P_{\lambda}^A(a)$$



$P, P', \dots$

Σ

$B, B', \dots$

$\Rightarrow T$

$$P_{\lambda}^{AB}(a|b) = P_{\lambda}^{AB}(a)$$

$$P_{\lambda}^{AB}(b|c) = P_{\lambda}^{AB}(b)$$

$\Rightarrow C$

$$P_{\lambda}^{AB}(a) = P_{\lambda}^A(a) \quad \text{and similarly for } B, B', \dots$$

$$P_{\lambda}^{AB}(b) = P_{\lambda}^B(b) \quad \dots \dots \dots A', A'' \dots$$

LOC

$$P_{\lambda}^{AB} = P_{\lambda}^A$$
$$P_{\lambda}^{AB} = P_{\lambda}^A$$

and similarly for $B', B'' \dots$
 $\dots \dots \dots A', A'' \dots$

SEP + LOC $\Leftrightarrow$ λ is 'common cause'
of the correlations

LOC

$$P_{\lambda}^{AB}(a,b) = P_{\lambda}^A(a) P_{\lambda}^B(b) \quad \text{and similarly for } B', B'' \dots$$

$$P_{\lambda}^{AB}(a,b) = P_{\lambda}^{A'}(a) P_{\lambda}^{B''}(b) \dots$$

SEP + LOC $\Leftrightarrow$ $\exists$ 'common cause'
of the probabilities

Correlations:

$$P(a,b) \neq P(a) P(b)$$

$$P_{\lambda}(a,b) \neq P_{\lambda}(a) P_{\lambda}(b)$$

$$P(a) = \int P_{\lambda}(a) P(\lambda) d\lambda$$

LOC

$$P_{\lambda}^{AB} = P_{\lambda}^A \quad \text{and similarly } B', B'' \dots$$

$$P_{\lambda}^{AB} = P_{\lambda}^{A'} \quad \dots \dots \dots A', A'' \dots$$

SEP + LOC $\Leftrightarrow$ λ is 'common cause'
of the correlation

Correlations: $P(A, B) \neq P(A) P(B)$

$$P_{\lambda}(A, B) \neq P_{\lambda}(A) P_{\lambda}(B)$$

$$P(A) = \int P_{\lambda}(A) P(\lambda) d\lambda$$

LOC

$$P_{\lambda}^{AB} = P_{\lambda}^A \quad \text{and similarly for } B, B'' \dots$$

$$P_{\lambda}^{AB} = P_{\lambda}^{A''}$$

and similarly for $B, B'' \dots$
 $\dots \dots \dots A', A'' \dots$

A, A'

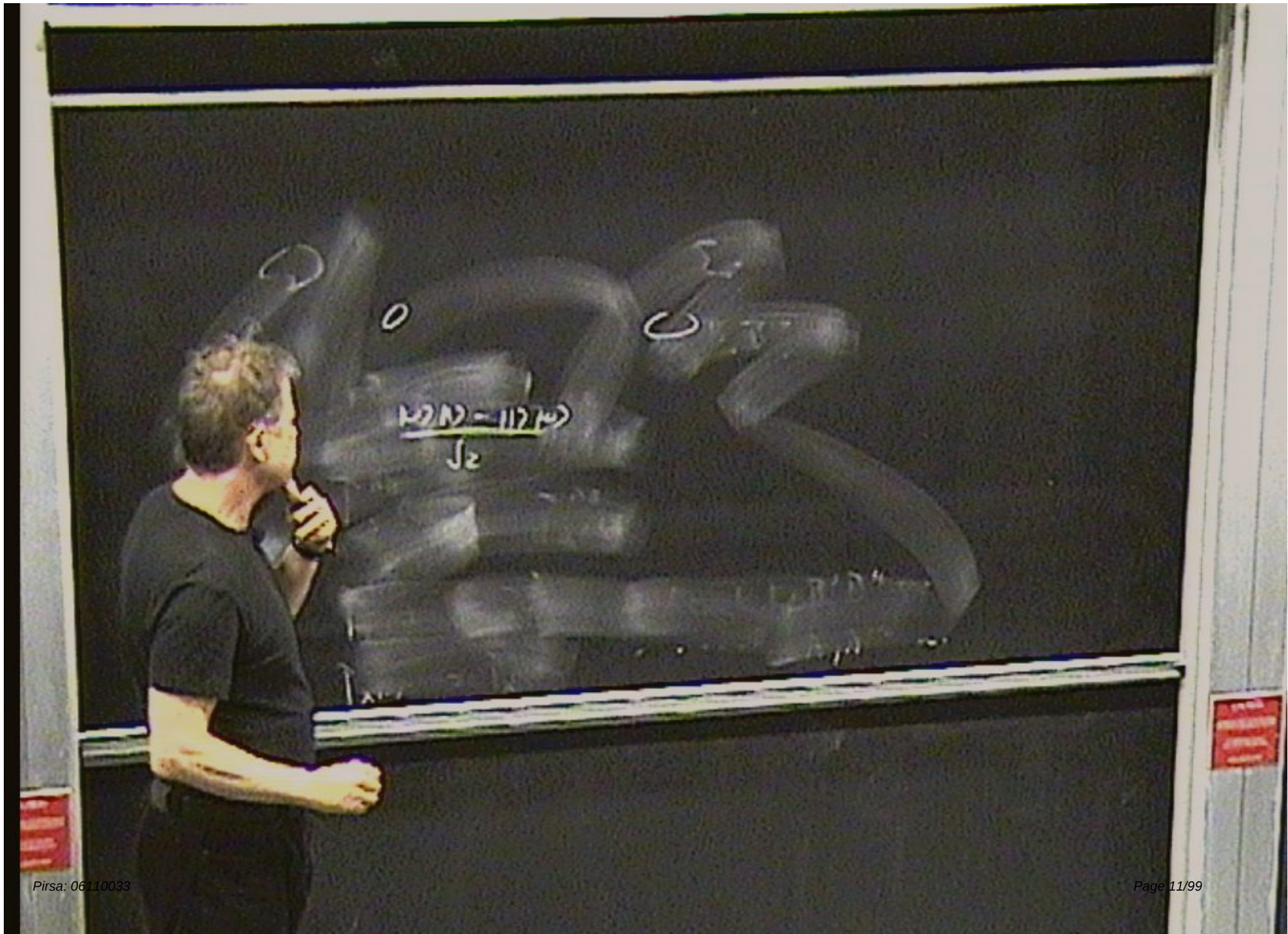
$\circ$ AB SEP + LOC $\Leftrightarrow \lambda$ is 'common cause' of the correlation

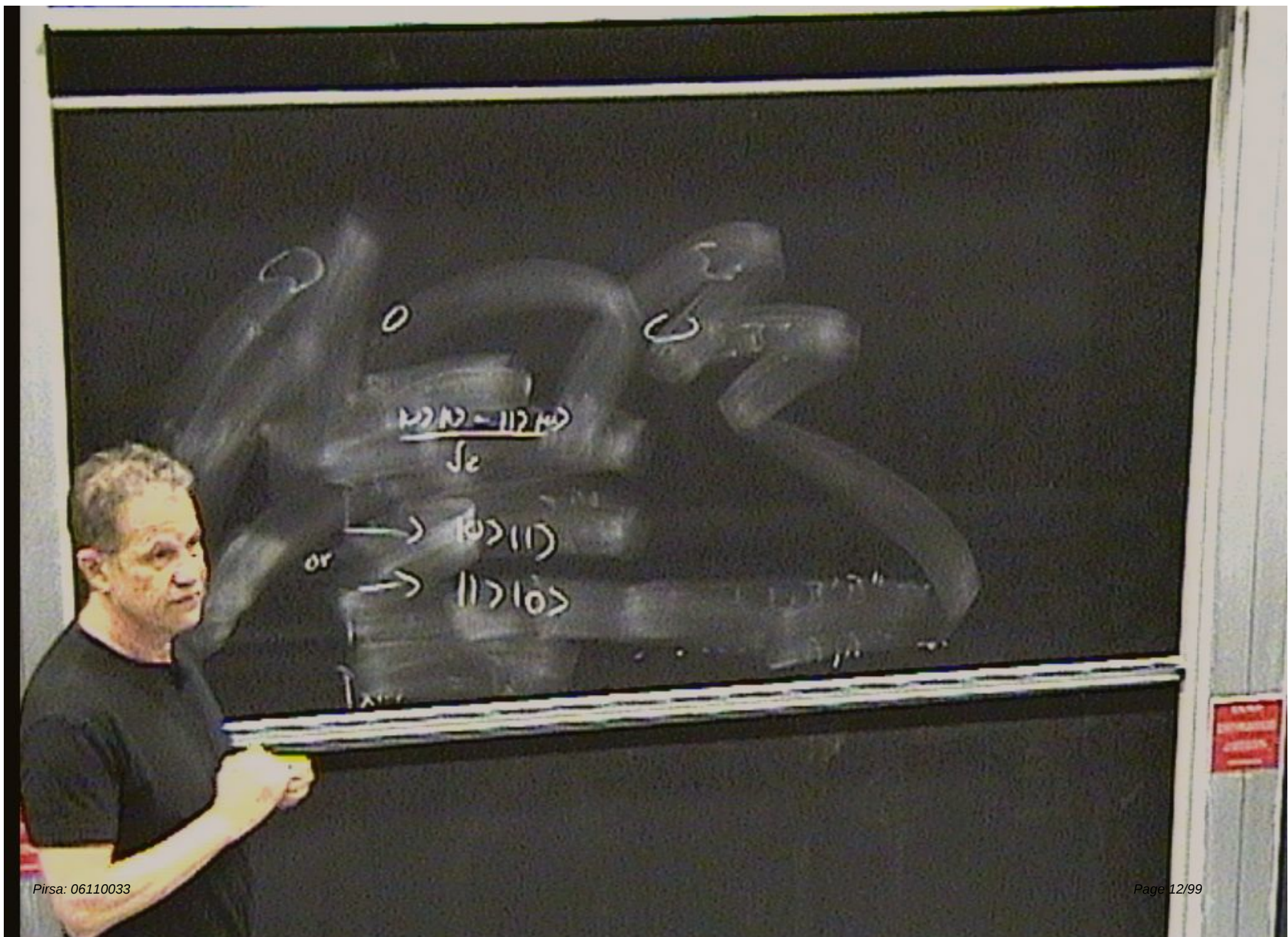
Correlations:

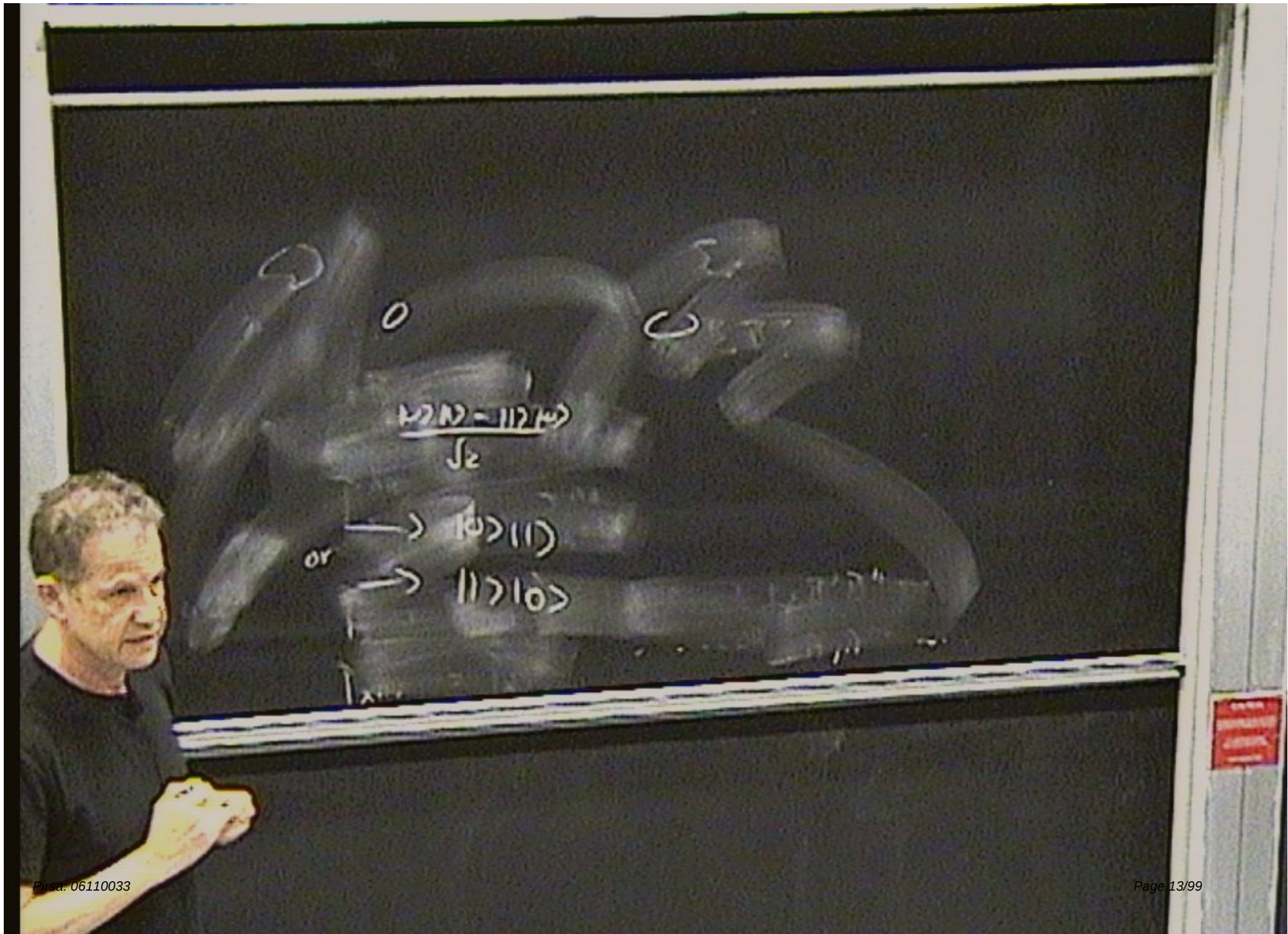
$$P(A \& B) \neq P(A) P(B)$$

$$P_{\lambda}(A \& B) \neq P_{\lambda}(A) P_{\lambda}(B)$$

$$P(A) = \int P_{\lambda}(A) P(\lambda) d\lambda$$







Bell

$$K_t = \alpha [\alpha' (1-\rho) + (1-\alpha') (1-\beta')] \\ + (1-\alpha) [\alpha' \beta' + (1-\alpha') \beta]$$

$$0 \leq \alpha', \beta, \beta' \leq 1$$

$$0 \leq \alpha + \beta + \alpha' \beta' - \alpha' \beta - \alpha \beta' - \alpha' \beta' \leq 1$$

$$p(\alpha, \beta) + p(\alpha', \beta')$$

$$q(\alpha, \beta) + q(\alpha', \beta')$$

$$q(\alpha, \beta) + q(\alpha', \beta')$$

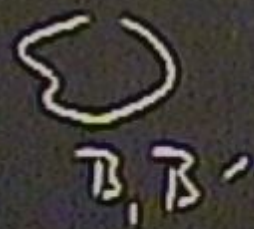
$$K_k = \alpha [\alpha'(1-\rho) + (1-\alpha')(-\beta')] \\ + (1-\alpha) [\alpha'\beta' + (1-\alpha')\beta] \\ \alpha, \alpha', \rho, \beta, \beta' \leq 1 \quad 0 \leq k \leq 1$$

$$K = \alpha [\alpha'(\rho - \rho) + (1 - \alpha')(1 - \beta')] \\ + (1 - \alpha) [\alpha' \beta' + (1 - \alpha') \beta] \\ 0 \leq \alpha, \alpha', \beta, \beta' \leq 1 \quad 0 \leq k \leq 1$$

$$0 \leq K = \alpha + \beta + \alpha'\beta' - \alpha\beta - \alpha'\beta - \alpha\beta' \leq 1$$



$$0 \leq K = \alpha + \beta + \alpha'\beta' - \alpha\beta - \alpha'\beta - \alpha\beta' \leq 1$$



$$0 \leq K = \alpha + \beta + \alpha'\beta' - \alpha\beta - \alpha'\beta - \alpha\beta' \leq 1$$


 A, B
 a, b


 B, B'

$\Rightarrow$ correlations
 $P(A \wedge B) \neq P(A)P(B)$
 etc

Suppose the correlations are the result of
 a common cause λ

i.e. $P_{\lambda}(A \wedge B) = P_{\lambda}(A)P_{\lambda}(B)$

$$0 \leq K = \alpha + \beta + \alpha'\beta' - \alpha\beta - \alpha'\beta - \alpha\beta' \leq 1$$



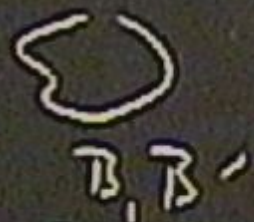
$\Rightarrow$ correlations
 $P(A \& B) \neq P(A)P(B)$
 etc

Suppose the correlations are the result of
 a common cause λ

Define: $P_1(a_+) = \alpha$, etc

i.e. $P_2(a \& b) = P_1(a)P_2(b)$

$$0 \leq K = \alpha + \beta + \alpha'\beta' - \alpha\beta - \alpha'\beta - \alpha\beta' \leq 1$$



$\Rightarrow$ correlations
 $P(A, B) \neq P(A)P(B)$
 etc

Suppose the correlations are the result of
 a common cause λ

Define: $P_\lambda(a_+) = \alpha$, etc

i.e. $P_\lambda(a, b) = P_\lambda(a)P_\lambda(b)$

$$0 \leq p(a) + p(b) + p(c) - p(a \cup b \cup c) = p(a \cup b \cup c) - p(a \cup b \cup c) = 0 \leq 1$$

$$0 \leq p(a_1) + p(a_2) + p(a_3) - p(a_1 \cup a_2) - p(a_1 \cup a_3) - p(a_2 \cup a_3) + p(a_1 \cup a_2 \cup a_3) \leq 1$$

$$0 \leq P(A_1) + P(A_2) + \dots + P(A_n) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - \dots - P(A_{n-1} \cap A_n) \leq 1$$

Similarly, if I define $P_2(A_i) = \alpha_i$, etc., then:

$$\leq P_2(A_1) + P_2(A_2) + \dots$$

$$0 \leq P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) \leq 1$$

Similarly, if $P_2(A_i) = \alpha$, etc, then:

$$0 \leq P_2(A_1) + P_2(A_2) + P_2(A_3)$$

So, adding

$$0 \leq 2$$

$$0 \leq p(a) + p(b) + p(a'b') - p(a'b) - p(a'b') - p(a'b') \leq 1$$

Similarly, if I define $p_2(a) = \dots$ etc, then:

$$0 \leq p_2(a) + p_2(b) + p_2(a'b')$$

So, adding:

$$0 \leq 2 + p(a'b')$$

A person is standing on the left side of the frame, facing right towards a chalkboard. They are holding a piece of chalk in their right hand and appear to be in the process of writing or have just finished writing the equation on the board. The chalkboard is dark and has several horizontal white lines. The equation is written in white chalk in the center of the board.
$$P(a'=b') = P(a'=a|b_+) + P(a'=a|b_-)$$

$$0 \leq P(a) + P(b) + P(a'b') - P(a'b) - P(a'b') - P(a'b') \leq 1$$

Similarly, if I define $P_2(a) = \alpha$, etc, then:

$$0 \leq P_2(a) + P_2(b) + P_2(a'b') - P_2(a'b) - P_2(a'b') - P_2(a'b') \leq 1$$

So, adding:

$$0 \leq 2 + P(a'b) + P(a'b) - P(a'b) - P(a'b) \leq 2$$

$$0 \leq P(a) + P(b) + P(a', b') - P(a, b) - P(a', b) - P(a, b') \leq 1$$

Similarly, if I define $P_2(a) = \alpha$, etc, then:

$$0 \leq P_2(a) + P_2(b) + P_2(a', b') - P_2(a, b) - P_2(a', b) - P_2(a, b') \leq 1$$

So, adding:

$$0 \leq 2 + P(a', b) + P(a, b) - P(a, b) - P(a', b) \leq 2$$

$$0 \leq P(a, b) + P(a', b) + P(a, b') - P(a', b') \leq 2$$

$$0 \leq P(K=L) + P(K'=L) + P(K=L') + P(K' \neq L') \leq 3$$

because

$$P(K' \neq L') = 1 - P(K' = L')$$

$$P(k=s) + P(k'=s) + P(k=s') + P(k' \neq s') \leq 3$$

by reuse

$$P(k' \neq s') = 1 - P(k'=s')$$

$$P(a=b) + P(a'=b) + P(a=b') + P(a' \neq b') \leq 3$$

because

$$P(a' \neq b') = 1 - P(a' = b')$$

Bell's inequality in
a (revised) Clauser-Horne-Shimony-Polt form

$$P(A=B) + P(A'=B) + P(A=B') + P(A' \neq B') \leq 3$$

because

$$P(A' \neq B') = 1 - P(A'=B')$$

Bell's inequality is

a (revised) Clauser-Horne-Shimony-Ahlborn form

$$E(AB) + E(A'B) + E(AB') - E(A'B') \leq 2$$

$$P(A=B) + P(A' \neq B) + P(A=B') + P(A' \neq B') \leq 3$$

because

$$P(A' \neq B') = 1 - P(A' = B')$$

Bell's inequality is

a (modified) Clauser-Horne-Shimony-Holt form

CHSH

$$\text{inequality: } E(AB) + E(A'B) + E(AB') - E(A'B') \leq 2$$

exactly

$$P(A=B) + P(A'=B) + P(A=B') + P(A' \neq B') \leq 3$$

because

$$P(A' \neq B') = 1 - P(A'=B')$$

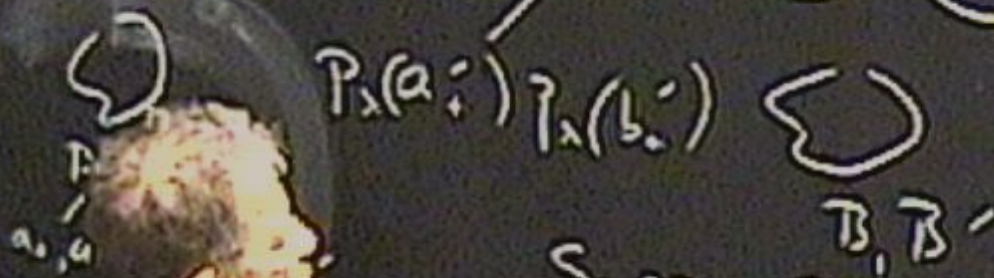
Bell's inequality is

a (revised) Clauser-Horne-Shimony-Holt form

CHSH

$$\text{inequality: } \underbrace{E(AB) + E(A'B') + (AB') - E(A'D')}_{\text{exp. value}} \leq 2$$

$$0 \leq K = \alpha + \beta + \underbrace{\alpha'\beta}_{\text{Cov}(A, B)} - \underbrace{\alpha\beta}_{\text{Cov}(A, B')} - \underbrace{\alpha'\beta'}_{\text{Cov}(A, B')} + \underbrace{\alpha\beta'}_{\text{Cov}(A, B)} \leq 1$$



$\Rightarrow$ correlation
 $P(A \& B) \neq P(A)P(B)$
 etc

Suppose the correlation is the result of
 a common cause λ

i.e. $P_\lambda(A \& B) = P_\lambda(A)P_\lambda(B)$

D. $P(A_+|B) = 0$, etc

$$0 \leq K = \alpha + \beta + \underbrace{\alpha'\beta'}_{\substack{P_A(a_+)P_A(b_+) \\ \text{D, A'}}} - \underbrace{\alpha\beta}_{\substack{P_A(a_-)P_A(b_-) \\ \text{D, A'}}} - \underbrace{\alpha'\beta}_{\substack{P_A(a_+)P_A(b_-) \\ \text{D, A'}}} - \underbrace{\alpha\beta'}_{\substack{P_A(a_-)P_A(b_+) \\ \text{D, A'}}} \leq 1$$

$$\underbrace{P_A(a_+)P_A(b_+)}_{\substack{\text{D, A'} \\ a_+, a_-}} \underbrace{P_A(b_+)}_{\substack{\text{B, B'} \\ b_+, b_-}}$$

$\Rightarrow$ correlations
 $P(A \& B) \neq P(A)P(B)$
 etc

Suppose the correlation is the result of
 a common cause λ

iv: $P_A(a_+) = \alpha$, etc

im: $P_A(a \& b) = P_A(a)P_A(b)$

$$0 \leq P(a) + P(b) + P(a', b') - P(a, b) - P(a', b') \leq 1$$

Similarly, if I define $P_2(a) = \alpha$, etc, then:

$$0 \leq P_2(a) + P_2(b) + P_2(a', b') - P_2(a, b) - P_2(a', b') \leq 1$$

So, adding:

$$0 \leq 2 + P(a', b') + P(a, b) - P(a, b) - P(a', b') \leq 2$$

$$0 \leq P(a, b) + P(a', b) + P(a, b') - P(a', b') \leq 2$$

$$P_2(a_1, b_1') = P_2(a_1') P_2(b_1')$$

Chernoff's result
assumption
 $SEP + LOC$

$$0 \leq P(a_1 = b_1) + P(a_1' = b_1') - P(a_1 = b_1') - P(a_1' = b_1) \leq 1$$

So, define $P_2(a_i) = \alpha_i$, etc, then:

$$0 \leq P(a_i = b_i) + P(a_i' = b_i') - P(a_i = b_i') - P(a_i' = b_i) \leq 2$$

So, add:

$$0 \leq 2$$

$$P(a_i = b_i) + P(a_i' = b_i') - P(a_i = b_i') - P(a_i' = b_i) \leq 2$$

$$P_2(a, b') = P_2(a)P_2(b')$$

Chernoff's result
assumption
 $SP + L \leq C$

$$0 \leq P(a) + P(b) + P(a', b') - P(a, b) - P(a', b) - P(a, b') \leq 1$$

Similarly, if I define $P_2(a) = \alpha$, etc, then:

$$0 \leq P(a) + P(b) + P(a', b')$$

So, adding:

$$2 + P(a', b') + P(a, b) - P(a', b) - P(a, b') \leq 2$$

$$0 \leq P(a, b) + P(a', b) + P(a', b') - P(a', b')$$

$$P_2(a, b) = P_2(a)P_2(b)$$

Chernoff's result +
assumption
 $SP + LOC$

$$P_2(a) + P_2(b) + P_2(a, b) = P_2(a, b) - P_2(a, b) - P_2(a, b) \leq 1$$

Similarly, I define $P_2(a) = \alpha$, etc, then:

$$\leq P_2(a) + P_2(b) + P_2(a, b)$$

$$\leq 2 + P_2(a, b) + P_2(a, b) - P_2(a, b) - P_2(a, b) \leq 2$$

$$0 \leq P_2(a) + P_2(b) + P_2(a, b) - P_2(a, b) \leq 2$$

$$P_2(a, b') = P_2(a)P_2(b')$$

Simplex's result
assumption
 $SP + LC$

$$0 \leq P(a) + P(b) + P(a', b') - P(a, b) - P(a', b) - P(a, b') \leq 1$$

Similarly, if I define $P_2(a) = \alpha$, etc, then:

$$0 \leq P_2(a) + P_2(b) + P_2(a', b')$$

So, adding:

$$0 \leq 2 + P(a', b') + P(a, b) - P(a, b) - P(a', b) \leq 2$$

$$0 \leq P(a, b) + P(a', b) + P(a', b') - P(a', b') \leq 2$$

$$\boxed{P_2(a, b') = P_2(a)P_2(b')}$$

Church's result +
assumption
CP + LCL

$$0 \leq P(a) + P(b) + P(a', b') = P(a, b) - P(a, b) - P(a', b') \leq 1$$

Similarly, if I define $P_2(a) = \alpha$, etc, then:

$$0 \leq P_2(a) + P_2(b) + P_2(a', b')$$

So, adding:

$$0 \leq 2 + P(a', b') + P(a, b) - P(a', b) - P(a, b') \leq 2$$

$$0 \leq P(a, b) + P(a', b) + P(a', b') - P(a', b') \leq 2$$

$$P(k=s) + P(k'=s) + P(k=s') + P(k' \neq s') \leq 3$$

CLASSICAL
30/4/99

$$P(k=s) + P(k'=s) + P(k=s') + P(k' \neq s') \leq 3$$

CLASSICAL
BOUND

$$\leq 1 + 2\sqrt{2}$$

QUANTUM BOUND

$$P(k=s) + P(k'=s) + P(k=s') + P(k' \neq s') \leq 3$$

CLASSICAL
BOARD

$$\leq 1 + 2\sqrt{2}$$

Quantum Board

↓
Based for asymptotic
ranked to arise the
S. K. S. L. C.

$$P(k=s) + P(k'=s) + P(k=s') + P(k' \neq s') \leq 3$$

CLASSICAL
BOUND

$$\leq 1 + 2\sqrt{2}$$

Quantum Bound

↓
Bound for nonlocal
realistic theories
Schrödinger LHC

T_n
Popescu - Rohrlich nonlocal boxes

A, A'

B, B'

$$0 \leq 2 + P(A'=1) + P(A=1) - P(A=1) - P(A=1) \leq 2$$

$$0 \leq P(A=1) + P(A'=1) + P(A=1) - P(A=1) \leq 2$$

Popescu - Rohrlach nonlocal boxes

A, A'

B, B'

$$P(A=1) = 1$$

$$P(A=1') = 1$$

$$P(A'=1) = 1$$

$$P(A' \neq 1') = 1$$

$$0 \leq 2 + P(A=1') + P(A=1) - P(A=1) - P(A=1') \leq 2$$

$$0 \leq P(A=1) + P(A'=1) + P(A=1') - P(A'=1') \leq 2$$

Popescu- Rohrlich nonlocal boxes

A, A'

B, B'

$$P(A=B) = 1$$

$$P(A=B') = 0$$

$$P(A'=B) = 0$$

$$P(A'=B') = 1$$

$$p(k=1) + p(k'=1) + p(k=1') + p(k'=1') \leq 3$$

Central
Theorem

≤ 1.252
Garg

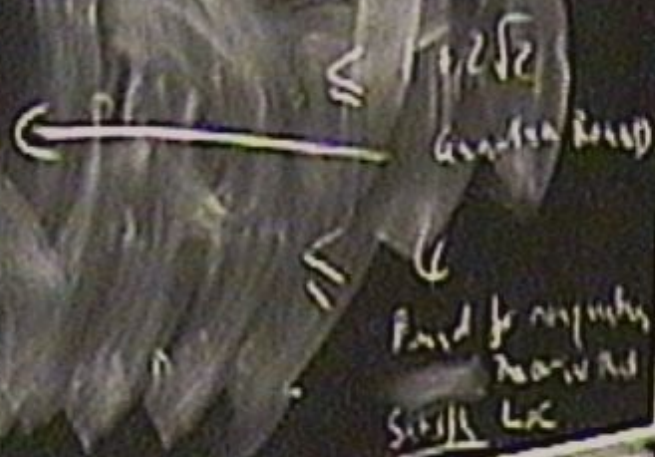
Proof by
hard
Small Log

$$P(a=b) + P(a' \neq b) + P(a \neq b') + P(a' \neq b') \leq 3$$

classical bound

Tsirelson

bound



Einstein's realization:

SEP + LOC $\Leftrightarrow$



Common cause
Principle

$$P(a,b) \neq P(a)P(b)$$

$$P(a,b) = P(a)P(b)$$

being true

Einstein's Realism:

(SEP + LOC)



Common Cause Principle



$P(a,b) \neq \dots$
violated by q.m.

$P(a,b) \neq P(a)P(b)$

$P(a,b) = P(a)P(b)$

violated

Einstein's Realism

LOC - "no signaling" principle

SEP + LOC

$\Leftrightarrow$

Common cause Principle

$\Rightarrow$

$P(a,b) \neq P(a)P(b)$
violated by q.m.

$P(a,b) \neq P(a)P(b)$

$P(a,b) = P(a)P(b)$

being true

$$p(a=b) + p(a' \neq b) + p(a' \neq b') + p(a \neq b') \leq 3$$

CLASSICAL
Bound

Tsirelson

bound

$$\leq 1 + 2\sqrt{2}$$

Quantum Bound

$$\leq 4$$

Based for inequalities
to derive this
SLOCC LDC

$$p(k=l) + p(k=l') + p(k \neq l') \leq 3$$

CLASSICAL
BOUND

$$p(k=l) \leq 1 + 2\sqrt{2}$$

Quantum Bound

↓
Based for inequalities
Tsirelson
S. J. L. C.

$$P(k=s) + P(k'=s) + P(k=s') + P(k' \neq s') \leq 3$$

CLASSICAL
BOUND

Tsirelson

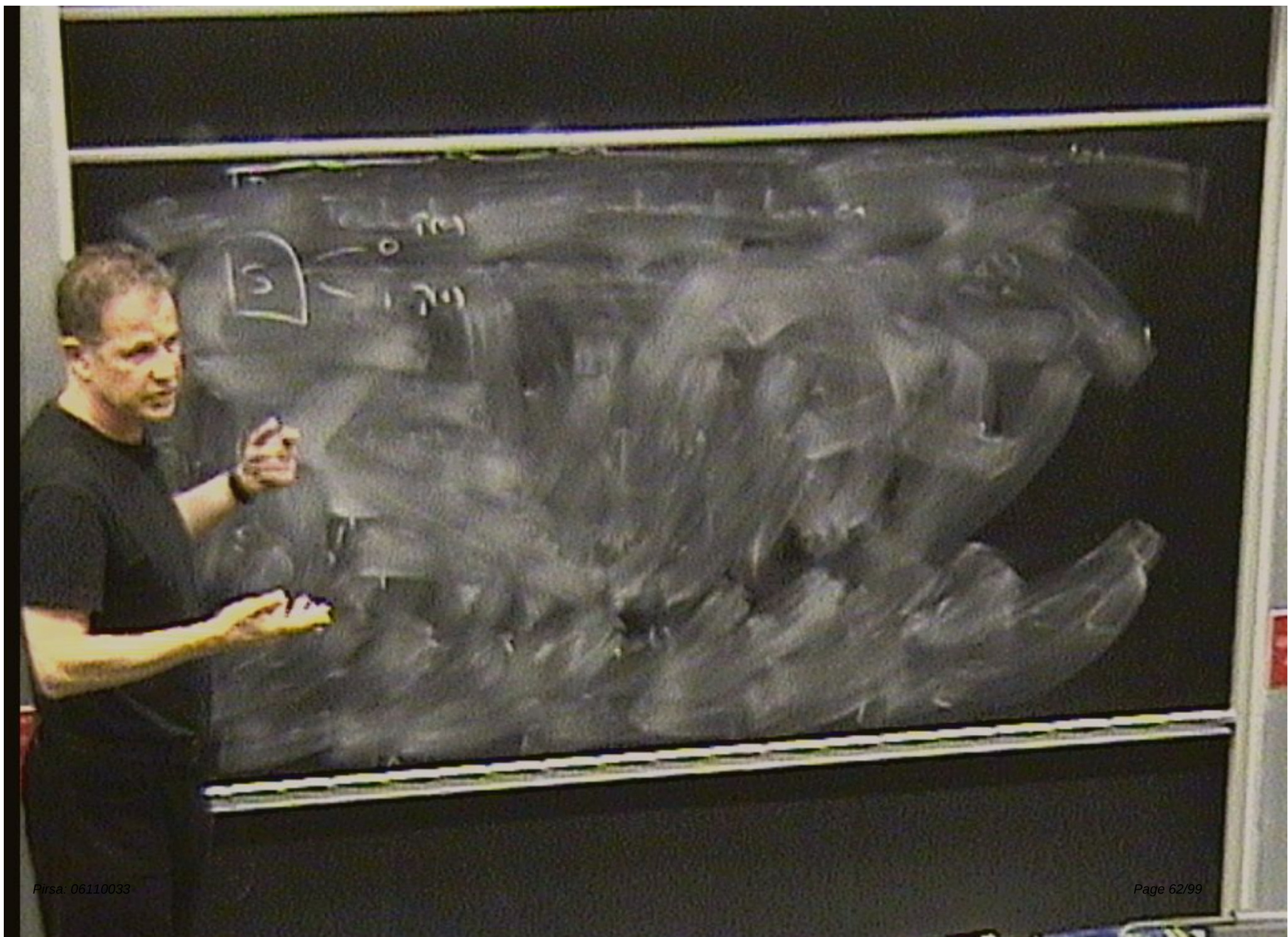
$$\leq 1 + 2\sqrt{2}$$

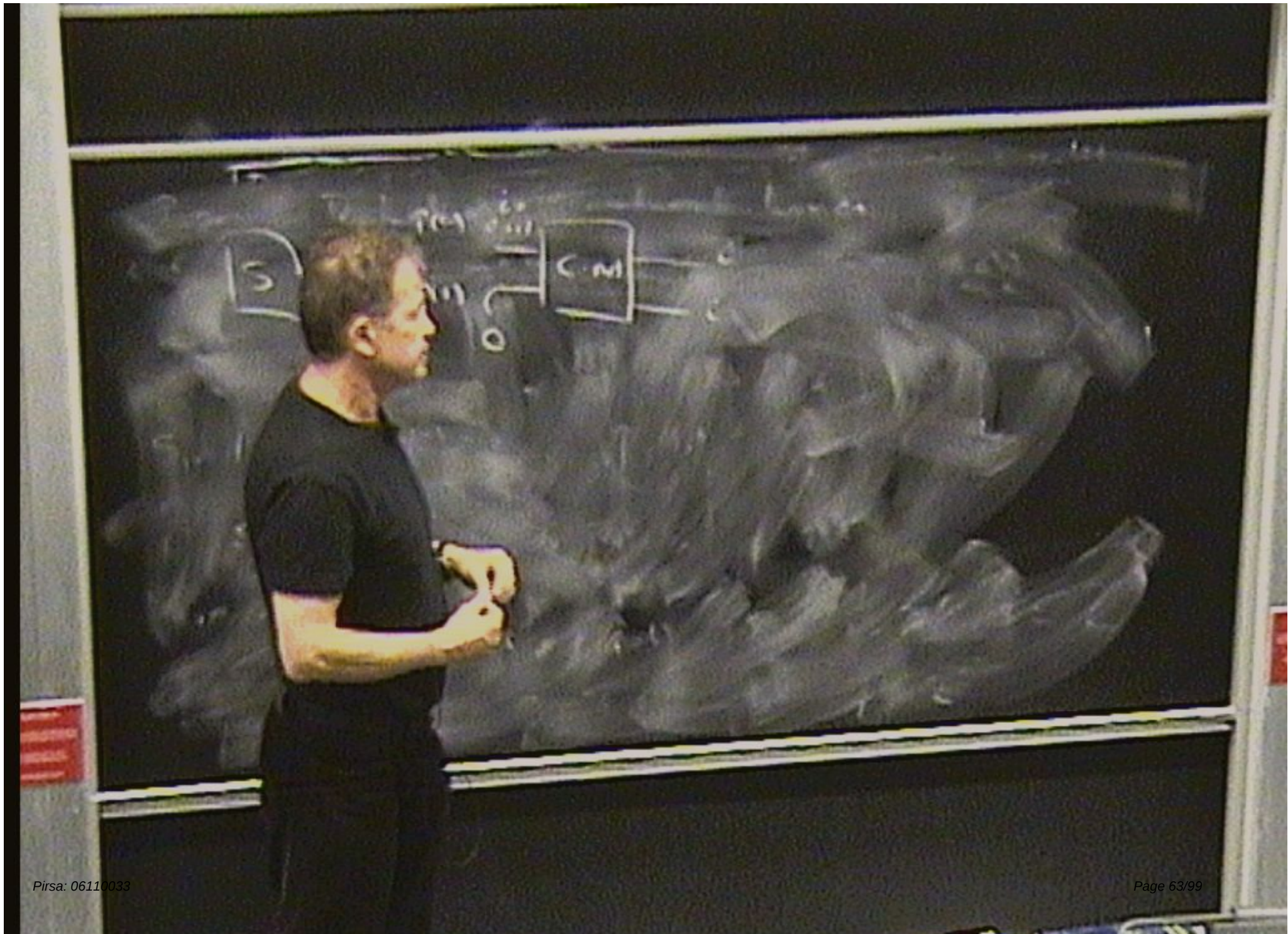
Quantum Bound

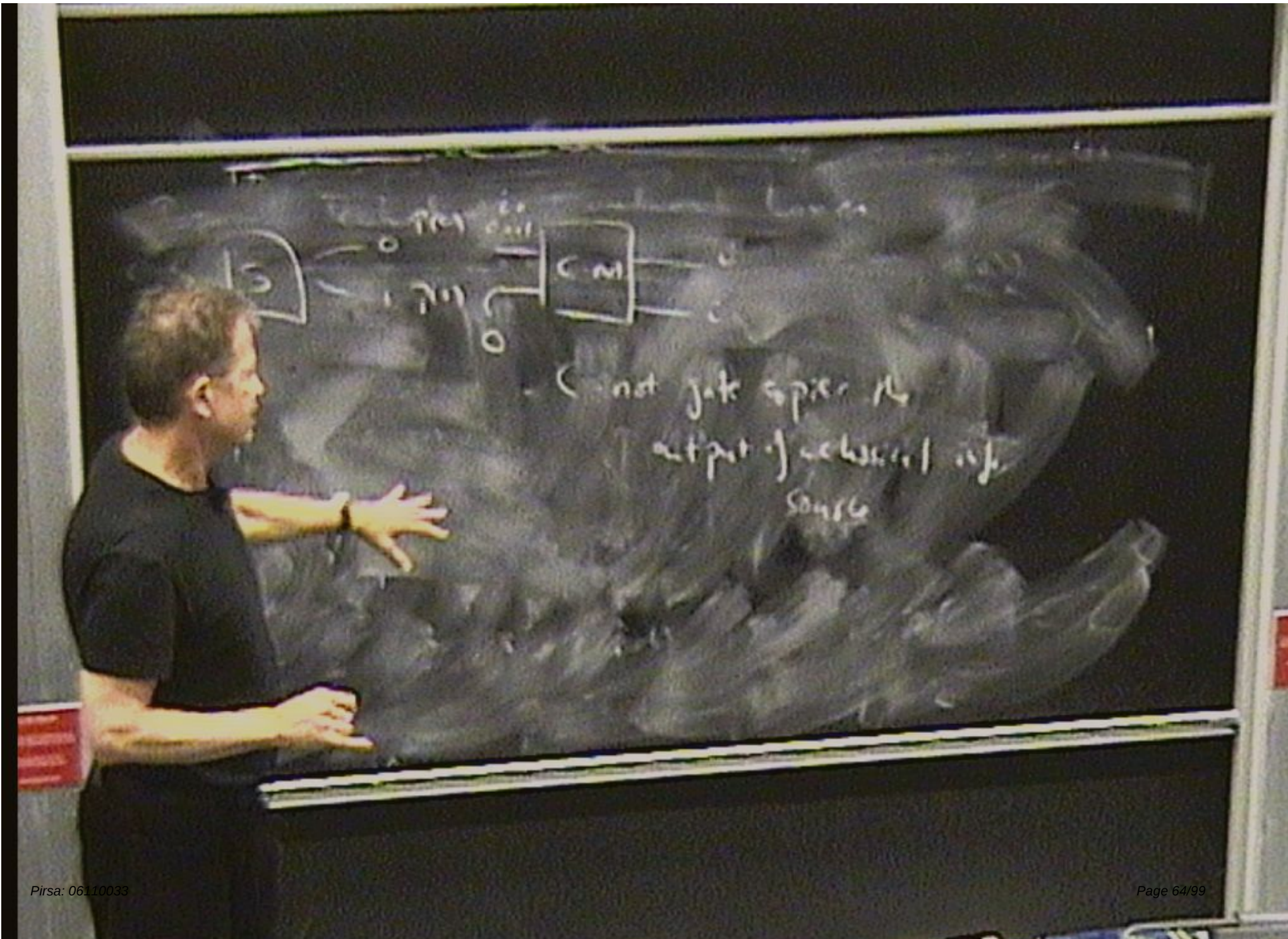
bound

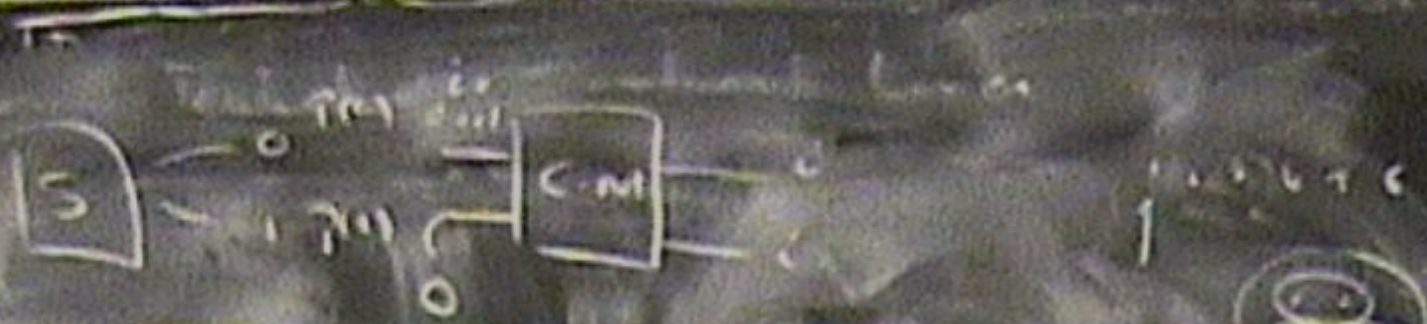
↓
Based for inequalities
Tsirelson
SLOCC Lx

EPR's analysis:
 LOC = "no signaling"
 SEP + LOC $\Leftrightarrow$ Common cause Principle
 $P(a, b) \neq P(a) P(b)$
 $P(a, b) = P(a) P(b)$
 violated by q.m.
 signaling





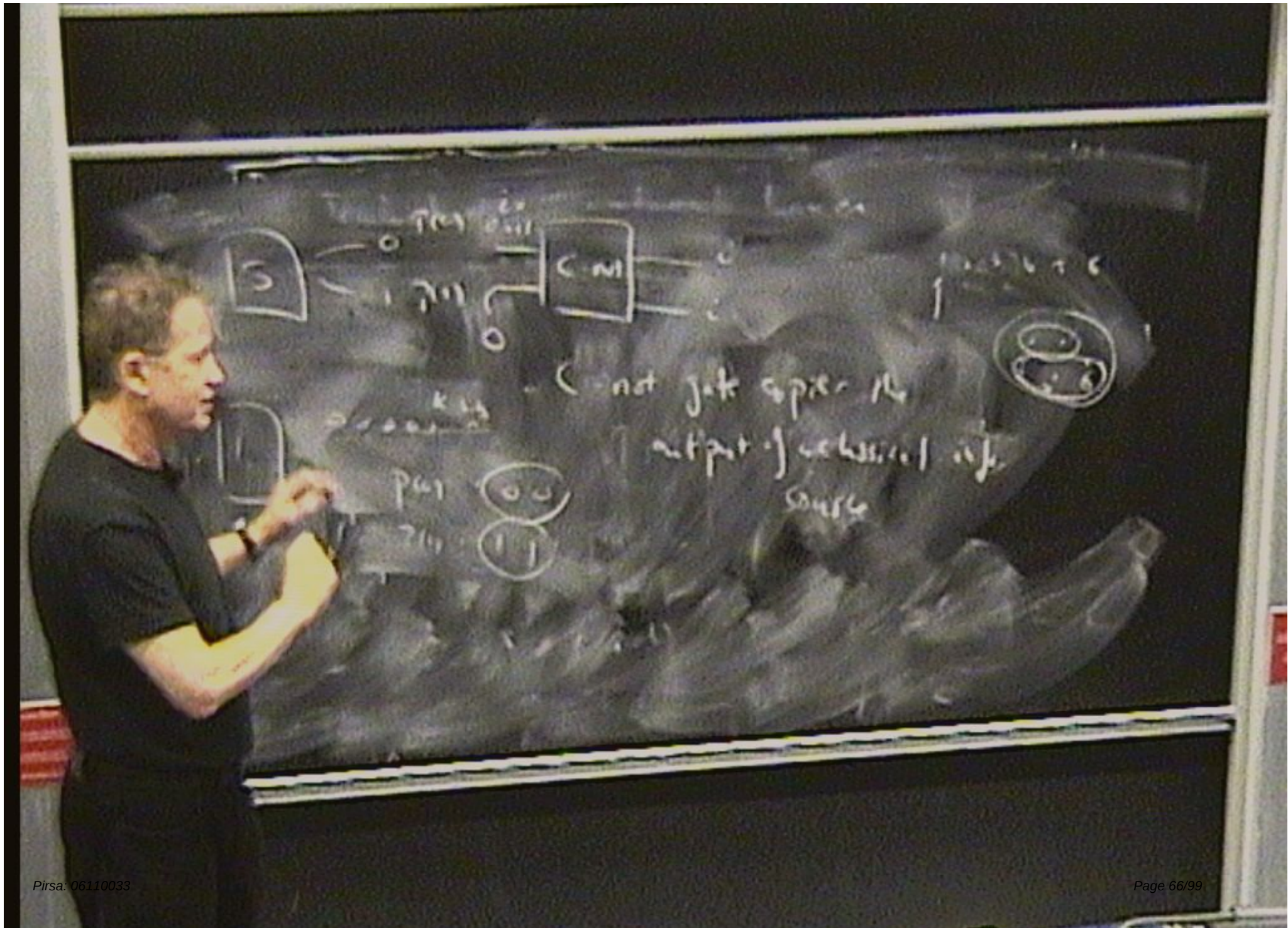


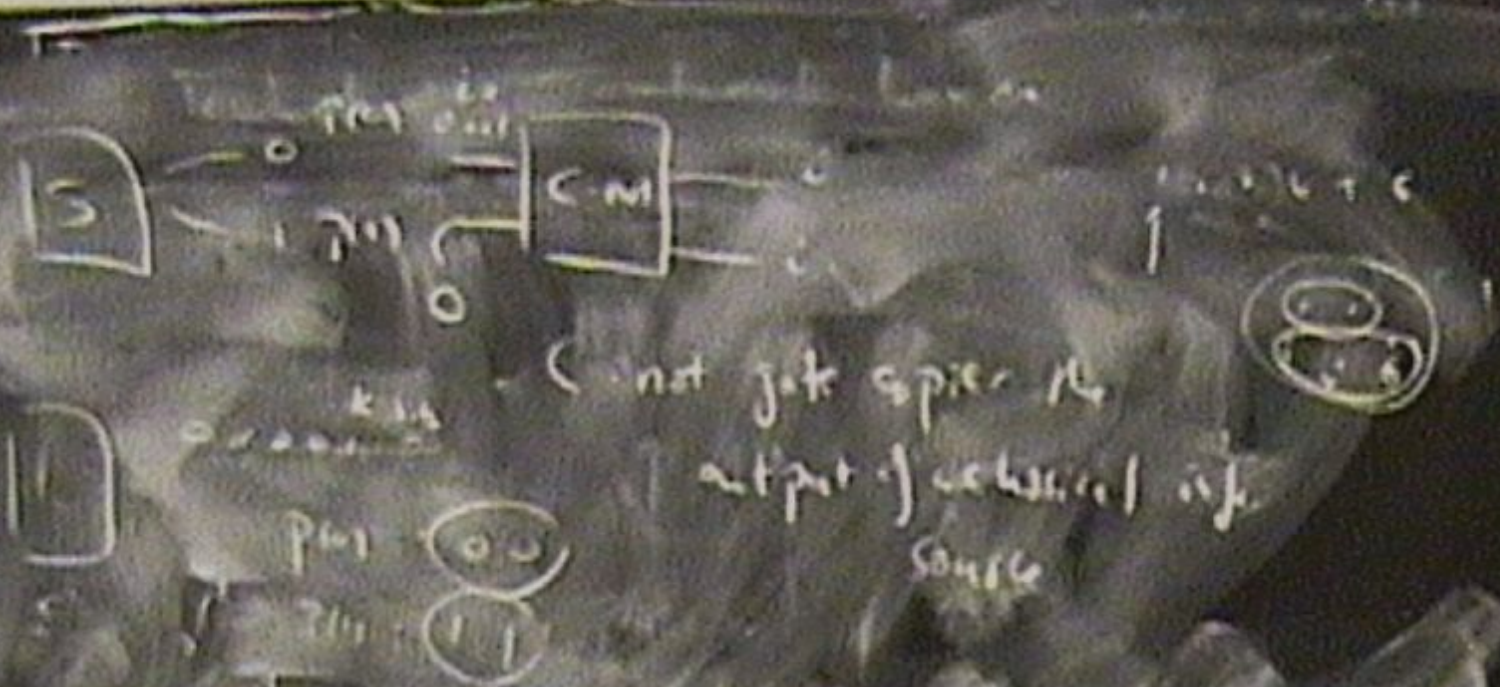


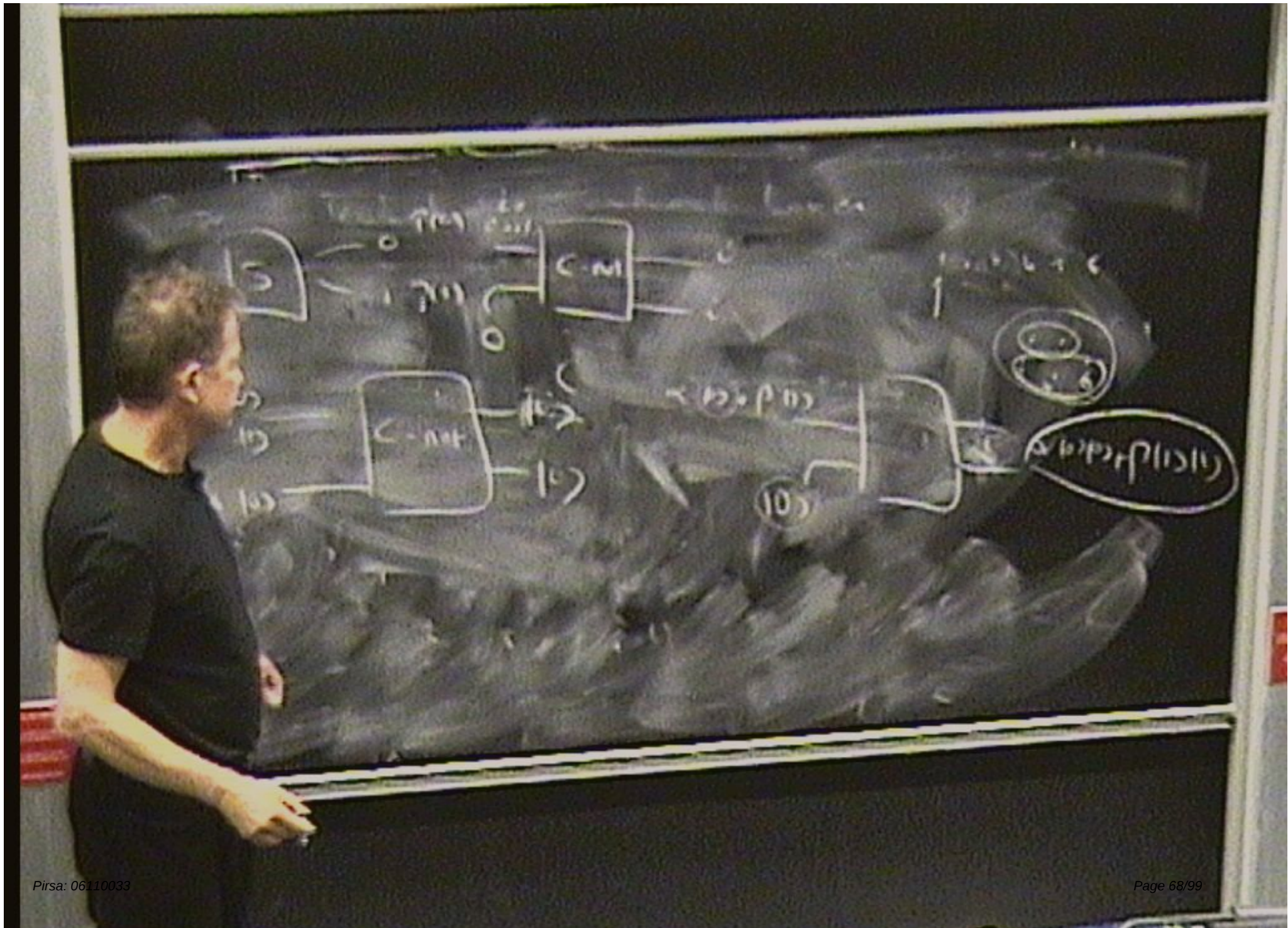
Can not gate spring R₀

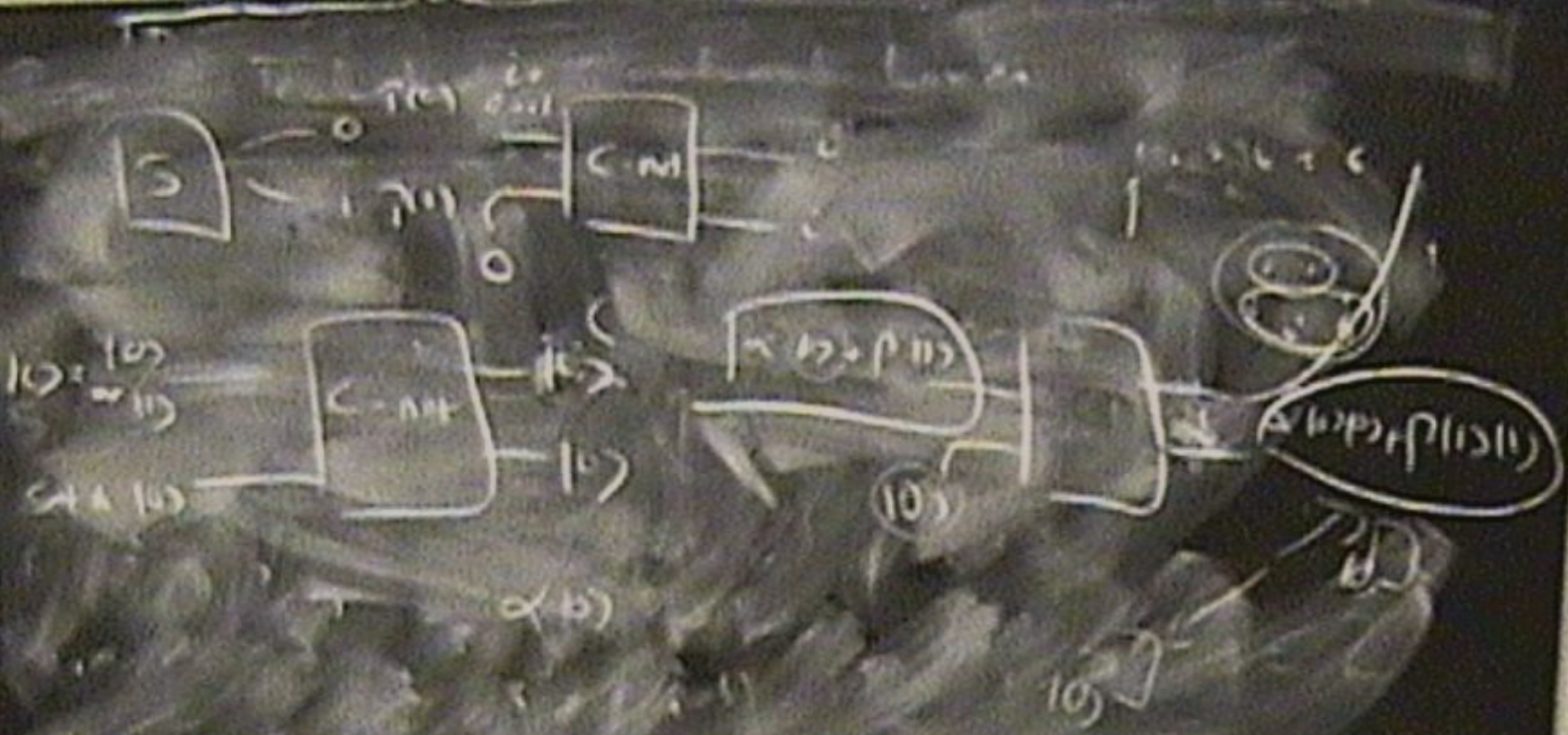
output of mechanical system

Source









Encke's realization

LOC "no signalling" Principle

SEP + (1)

→ $\gamma(k_0) + \dots$
violated by γ

$(\alpha + \beta | \gamma)$

$(\alpha + \beta | \gamma)$

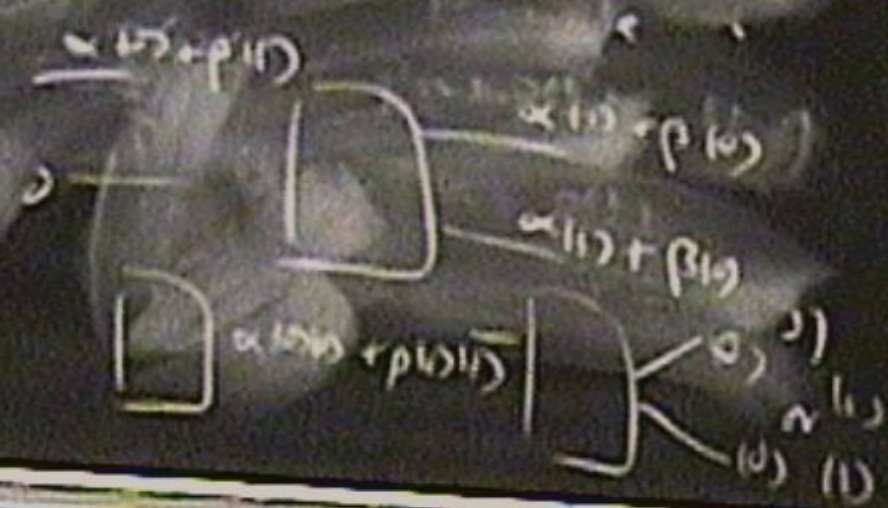
$(\alpha + \beta | \gamma)$

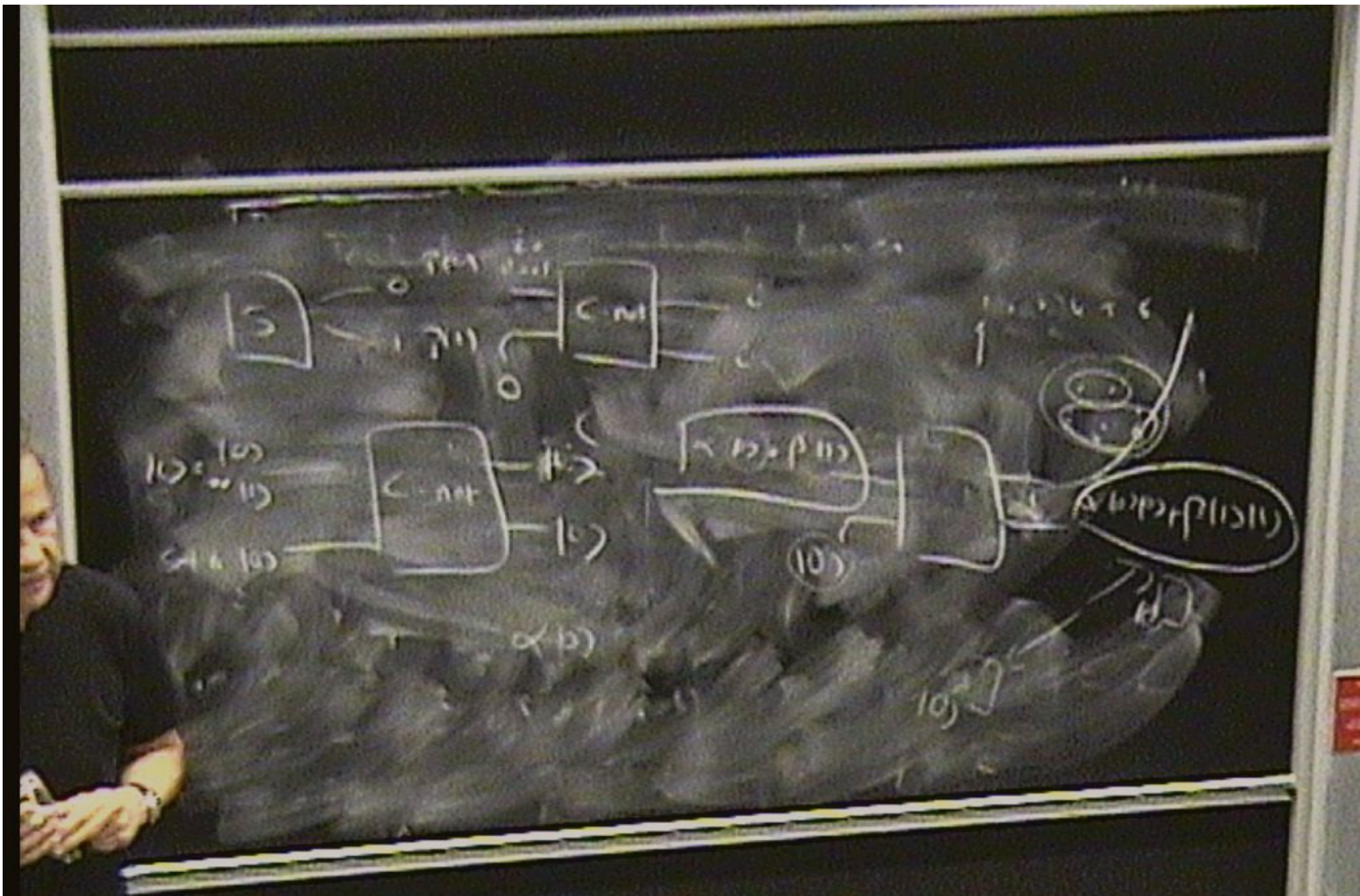
Erstein's realization

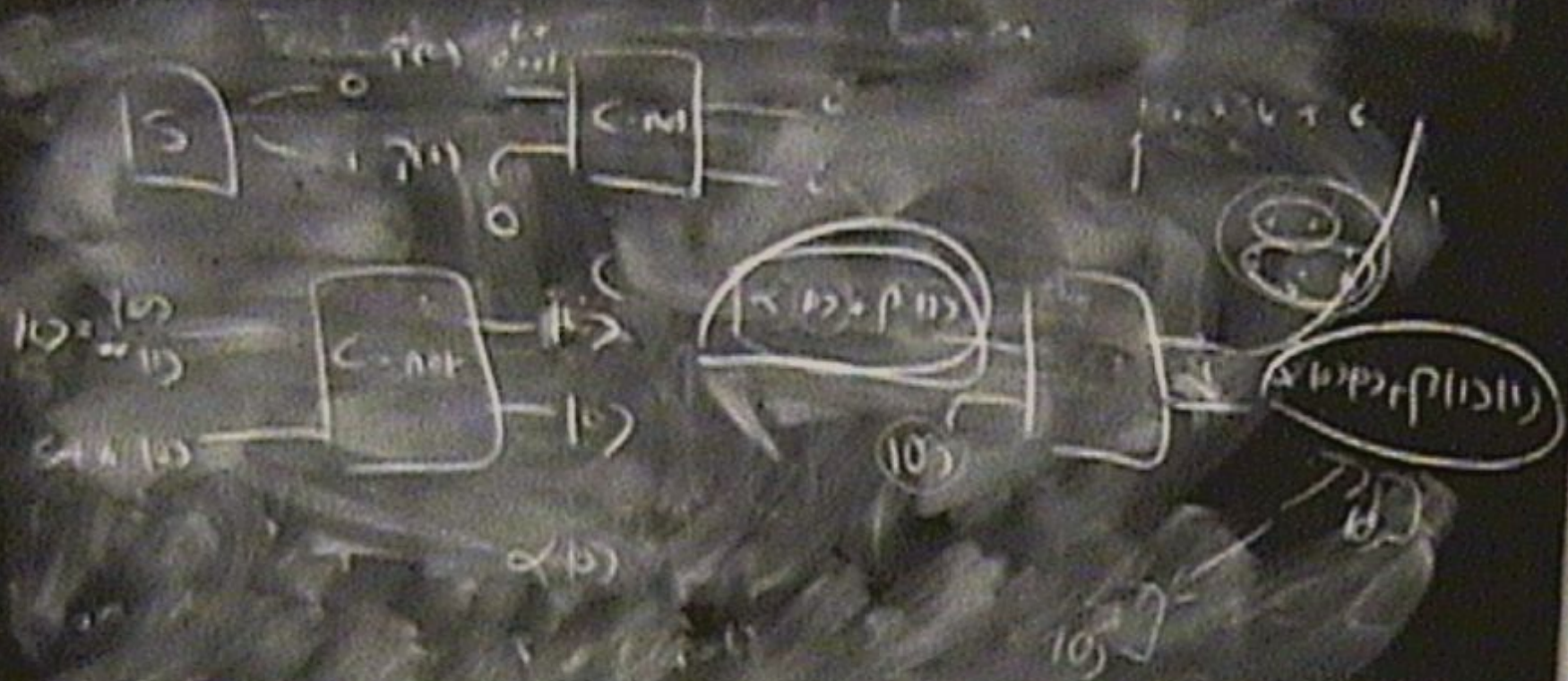
LOC = "no signaling" Principle

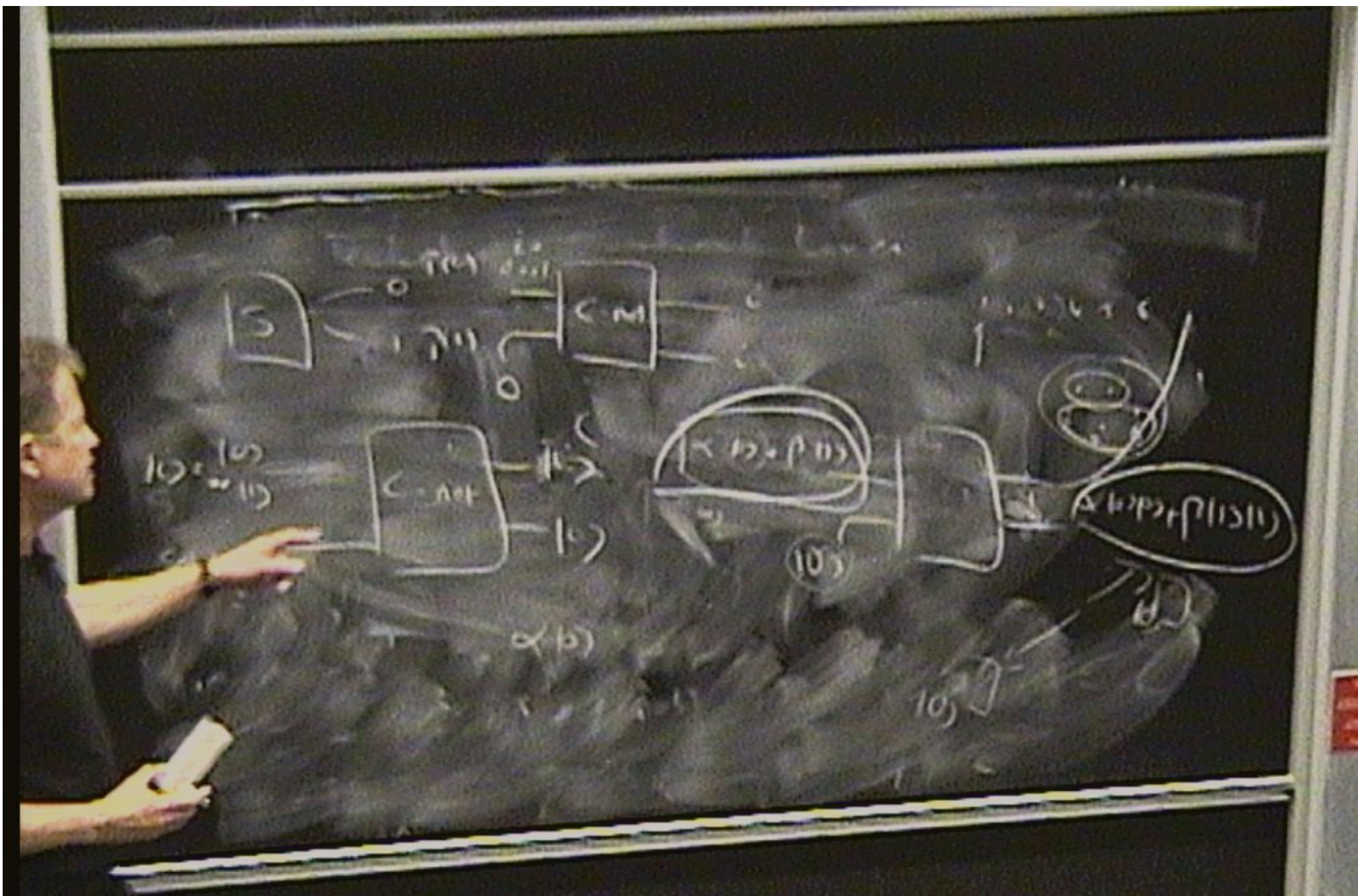
SEP + LOC

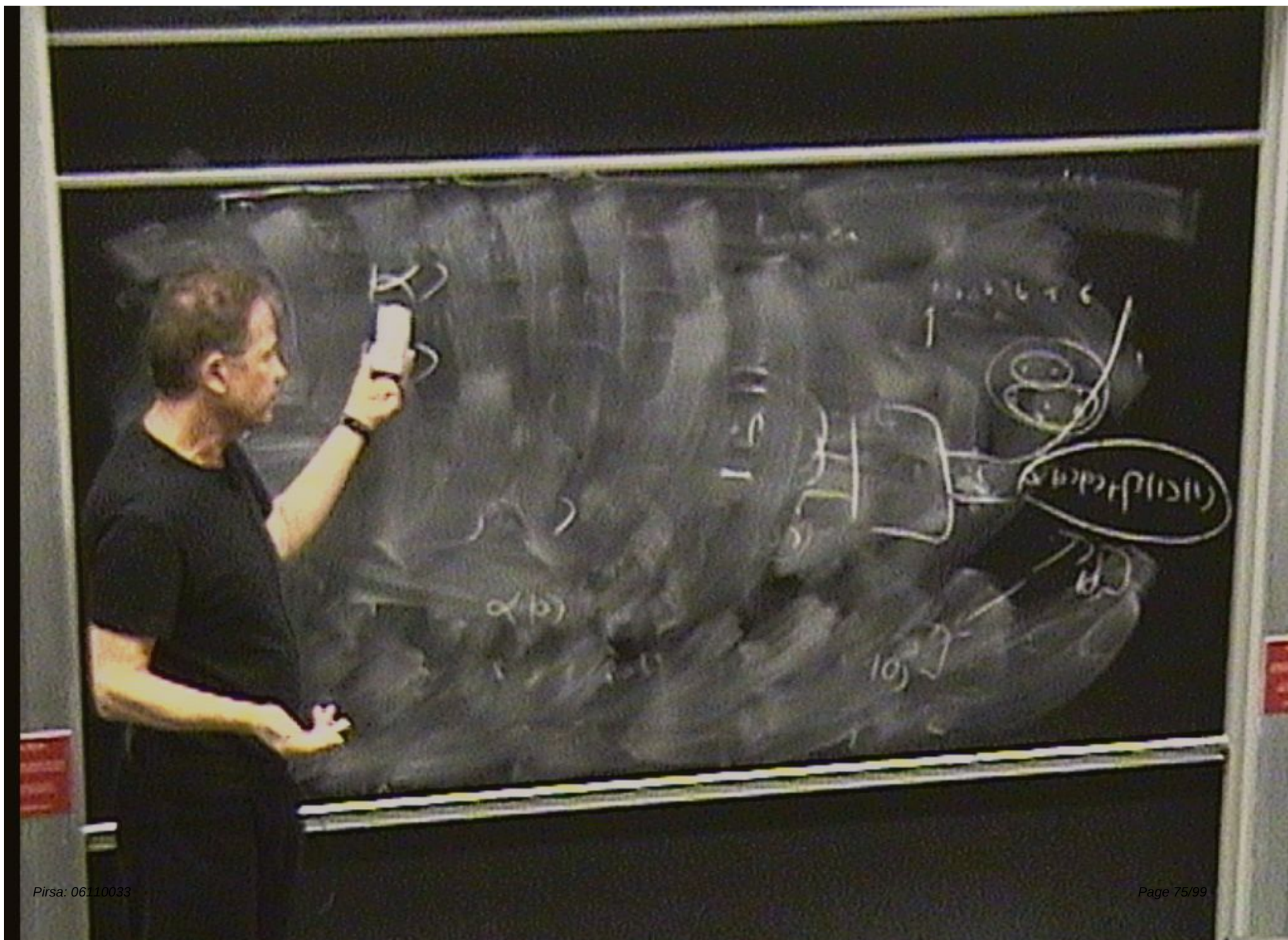
$\Rightarrow$ $\rho(\psi) + \dots \in \mathcal{S}$
 violated by q, u

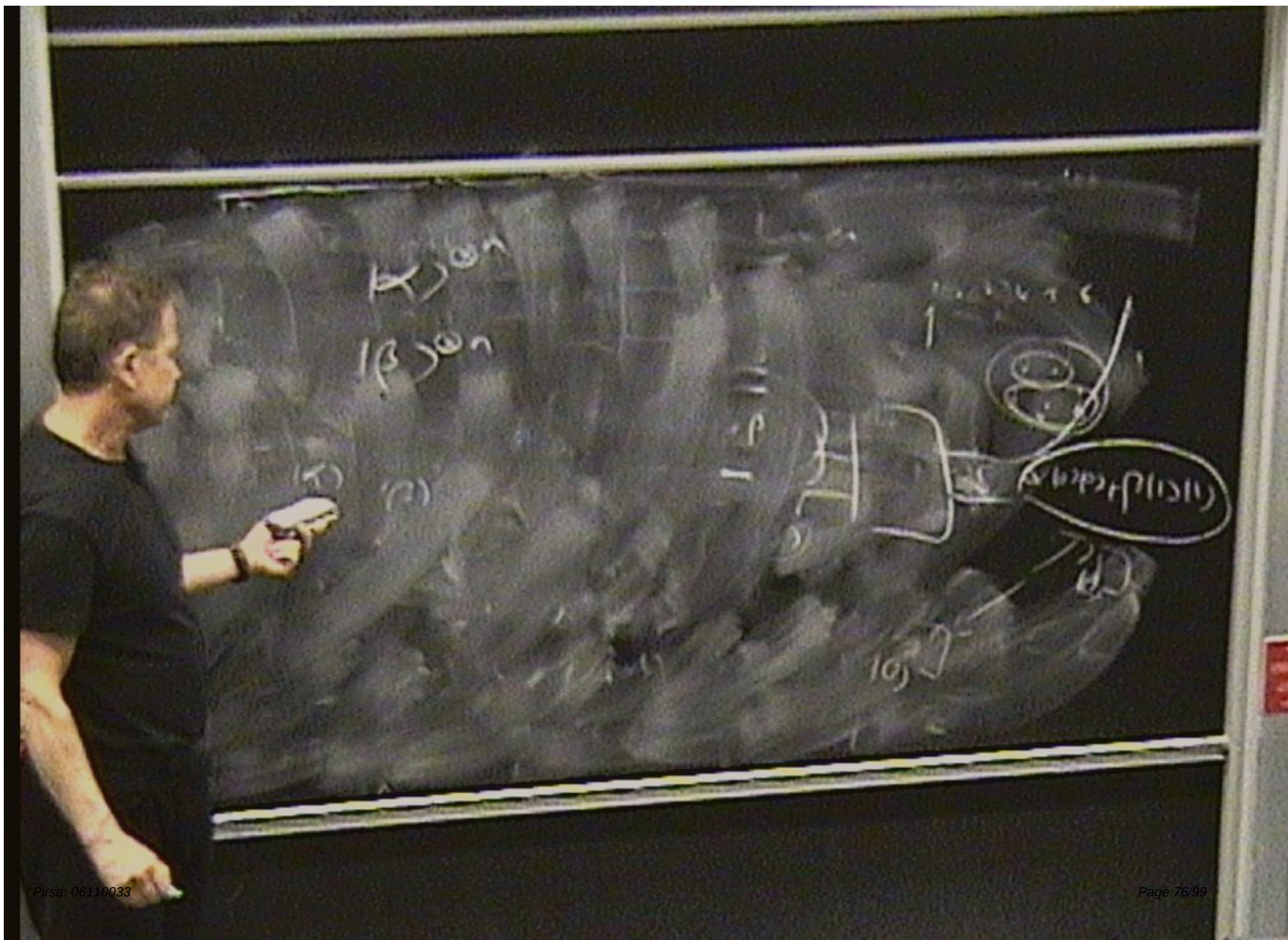


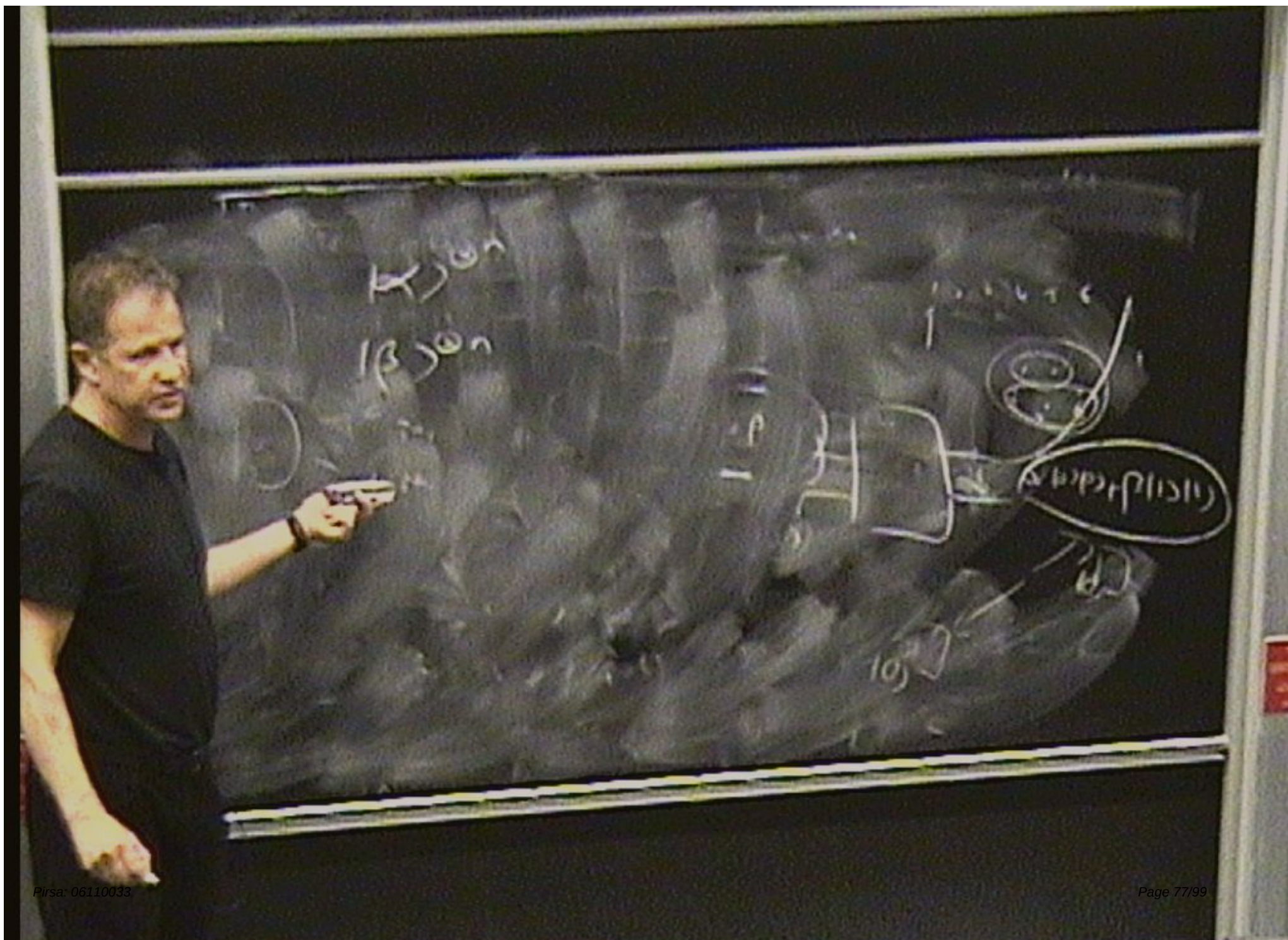


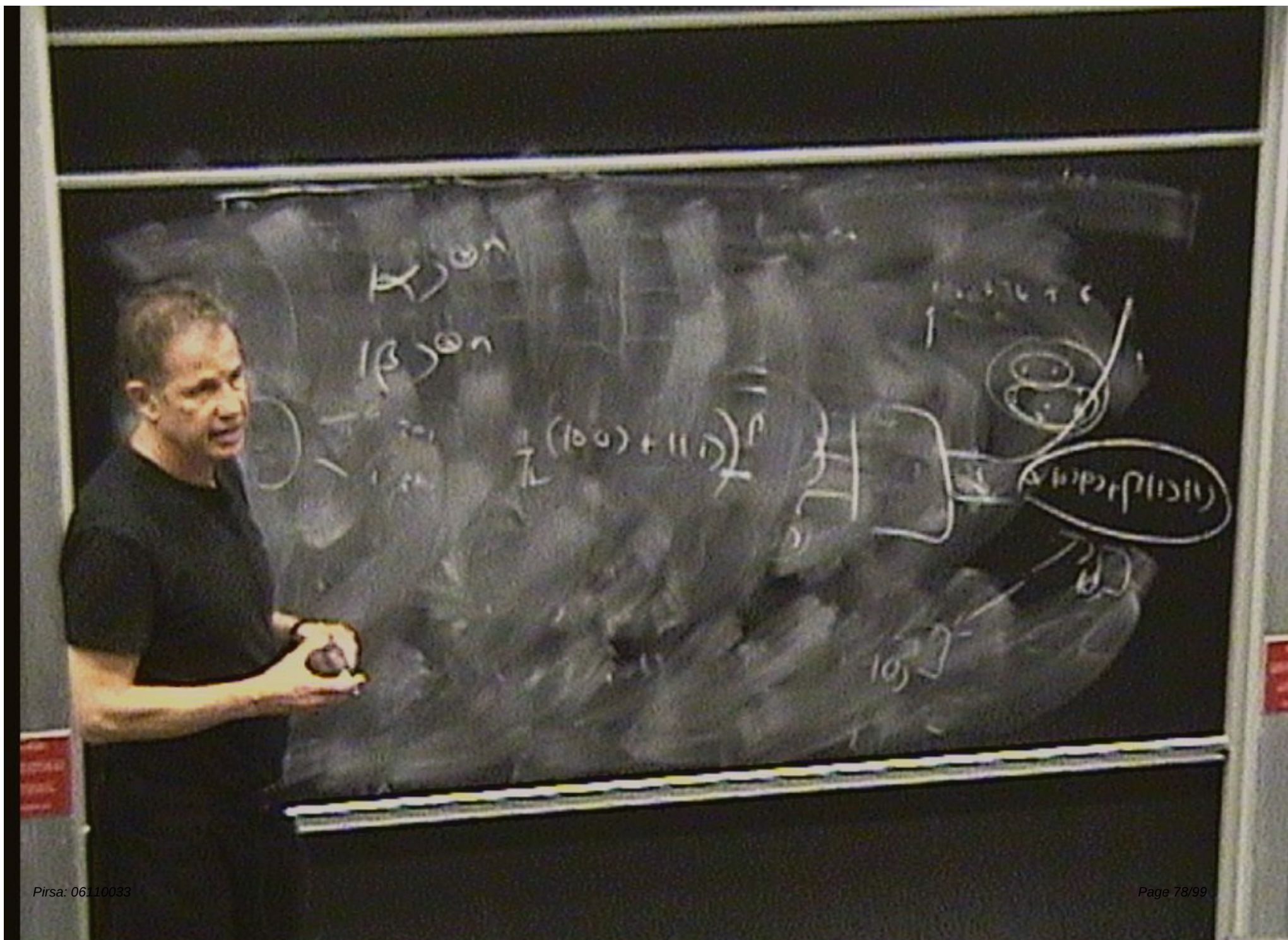


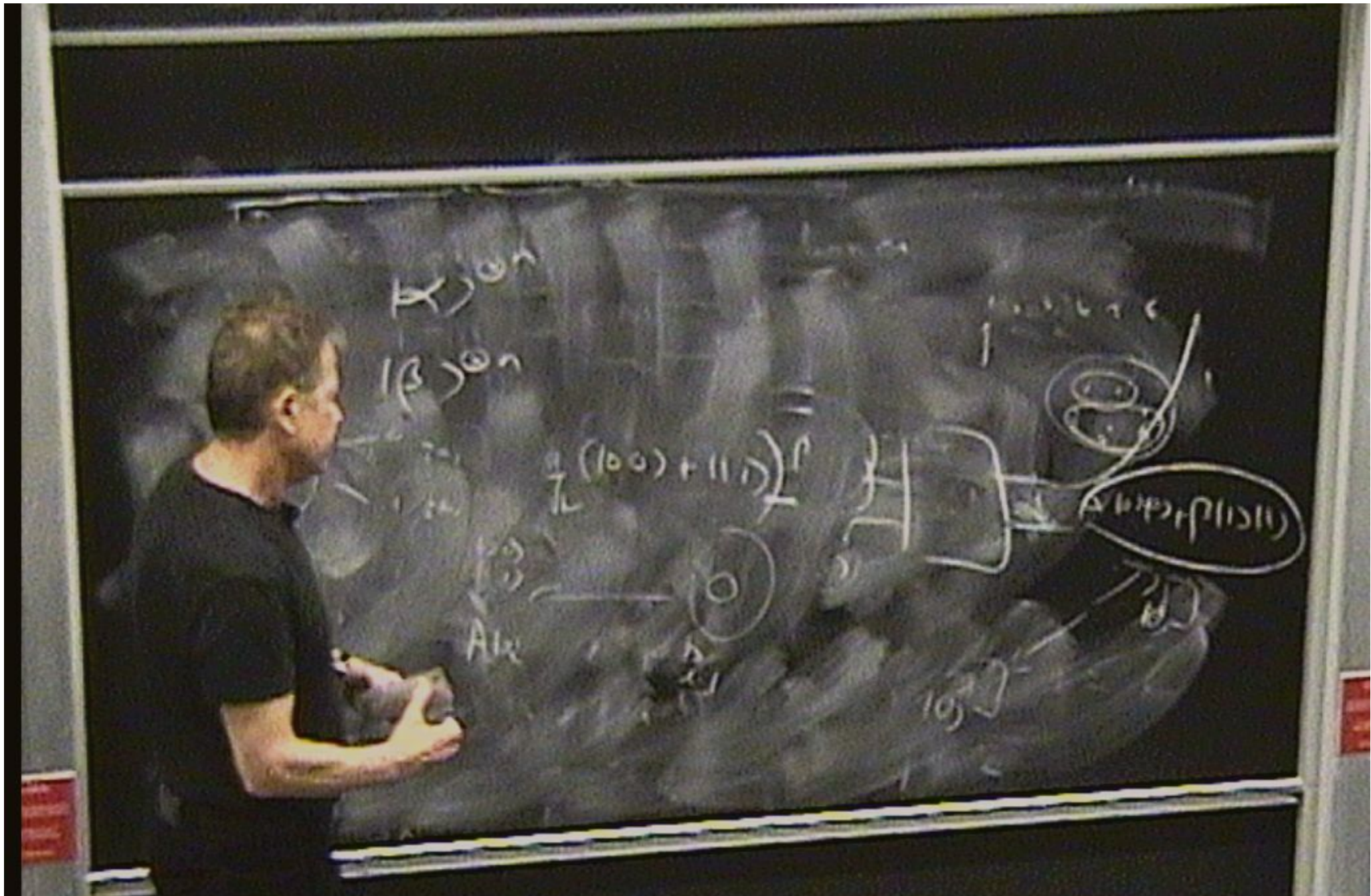


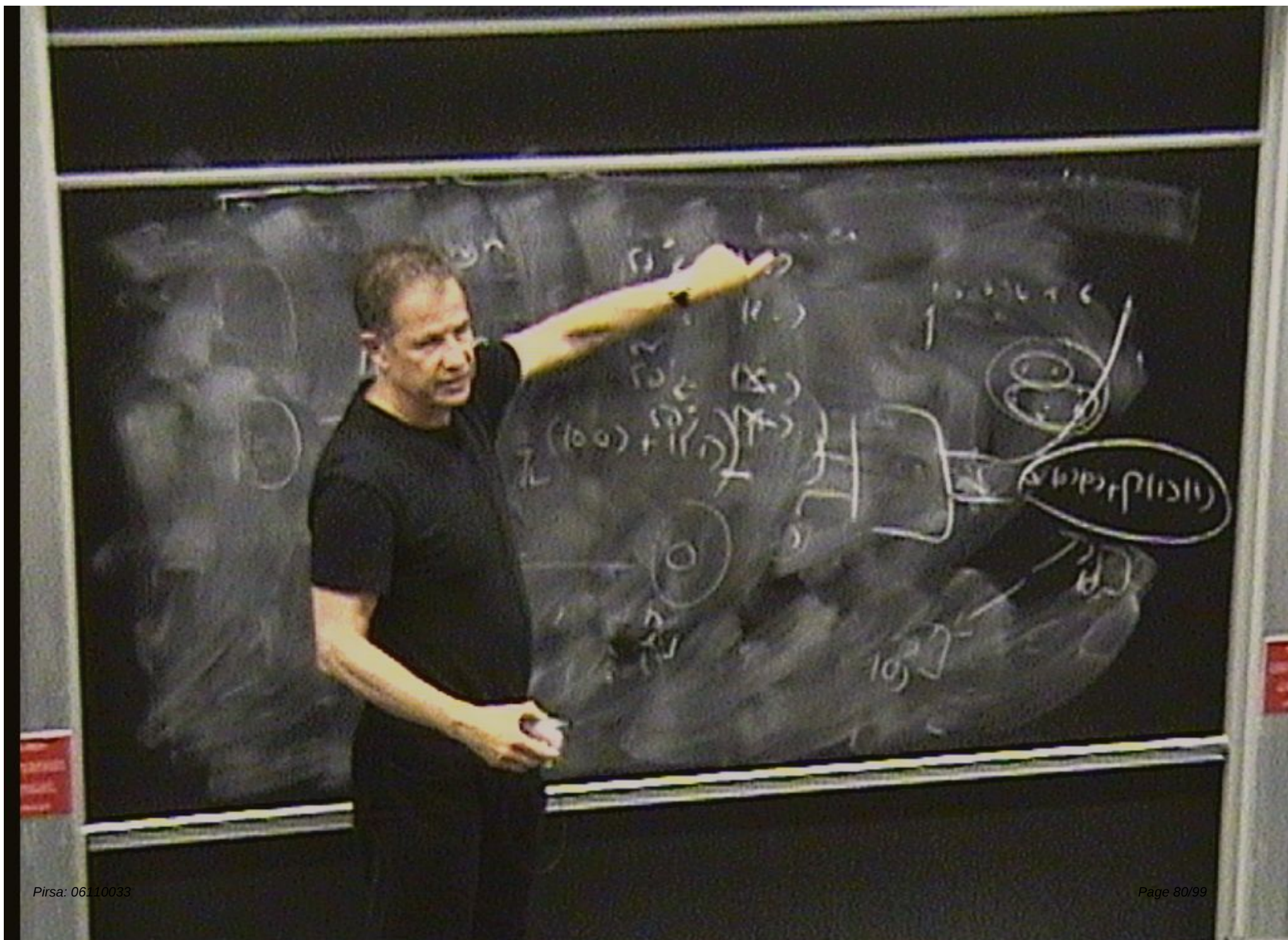


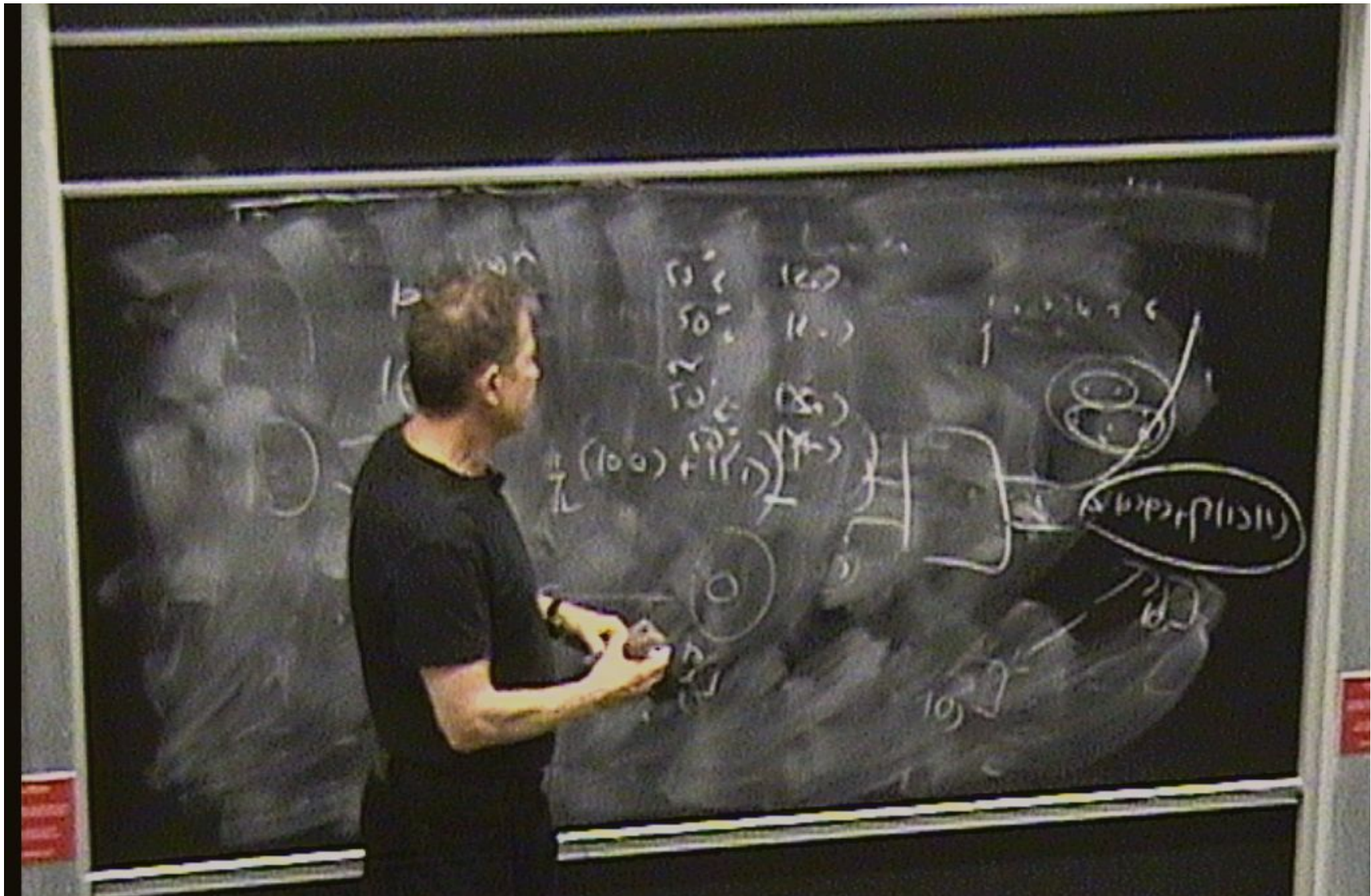


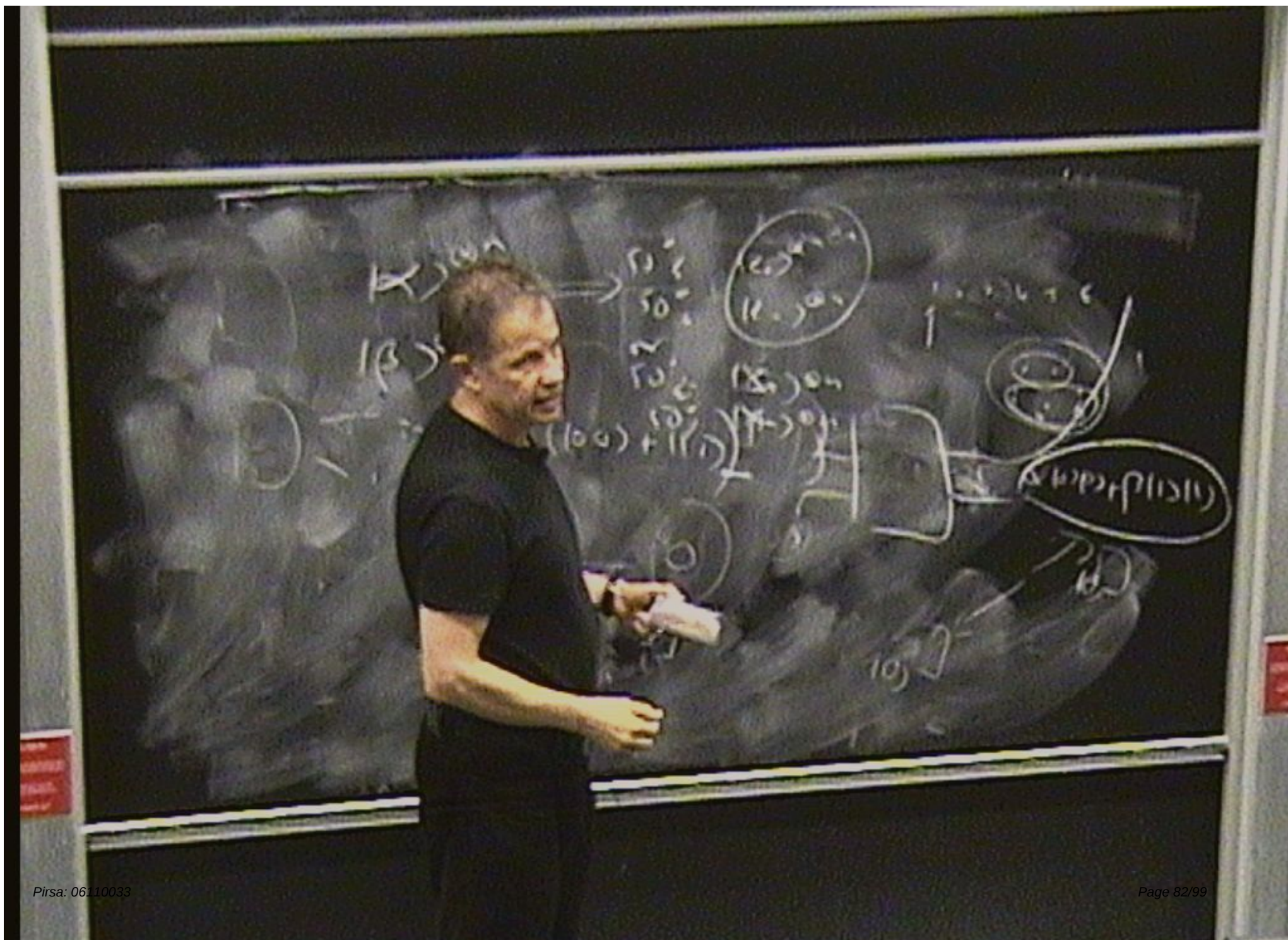












Einstein's realization:

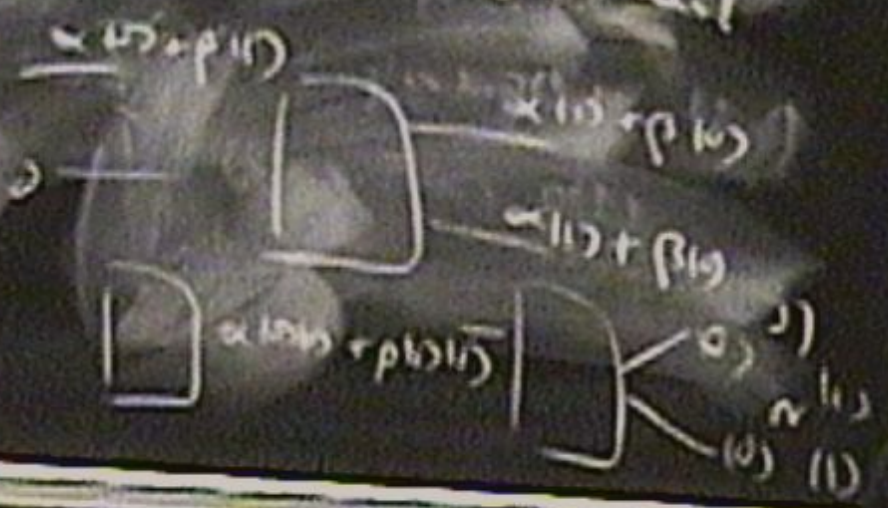
LOC "no signaling" Principle

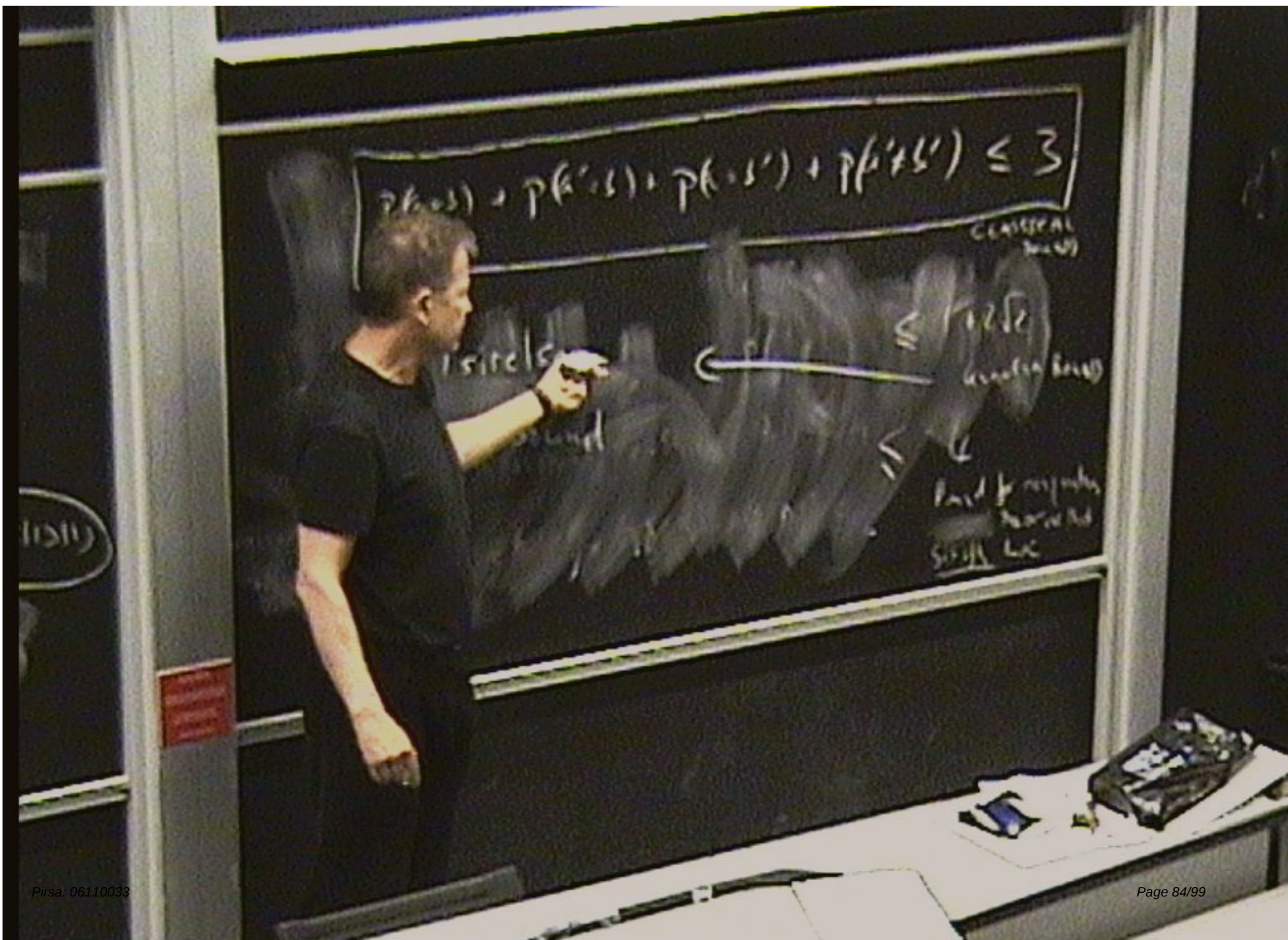
SEP + LOC

Bell

"Against Measurement"

$\Rightarrow P(a,b) \dots \epsilon$
violated by q.m.





$$p(k=s) + p(k'=s) + p(k=s') + p(k' \neq s') \leq 3$$

CLASSICAL
BOUND

Tsirelson

$$\leq \sqrt{2} + 2\sqrt{2}$$

Quantum Bound

bound

Proof for nonnegative
probabilities
Simple LDC

Einstein's realization:

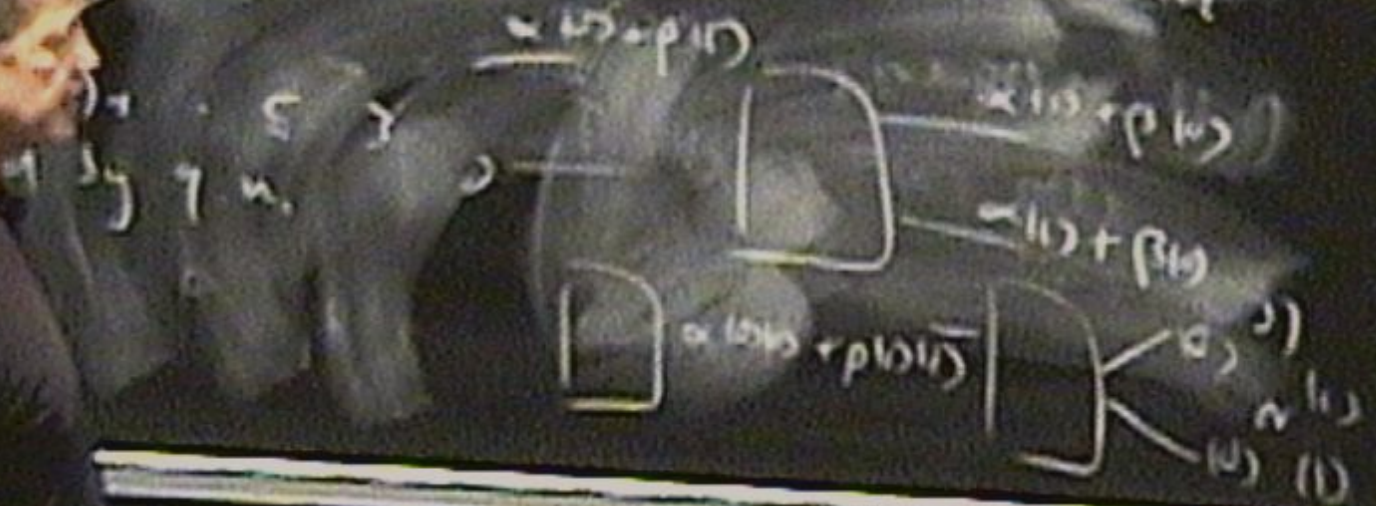
LOC

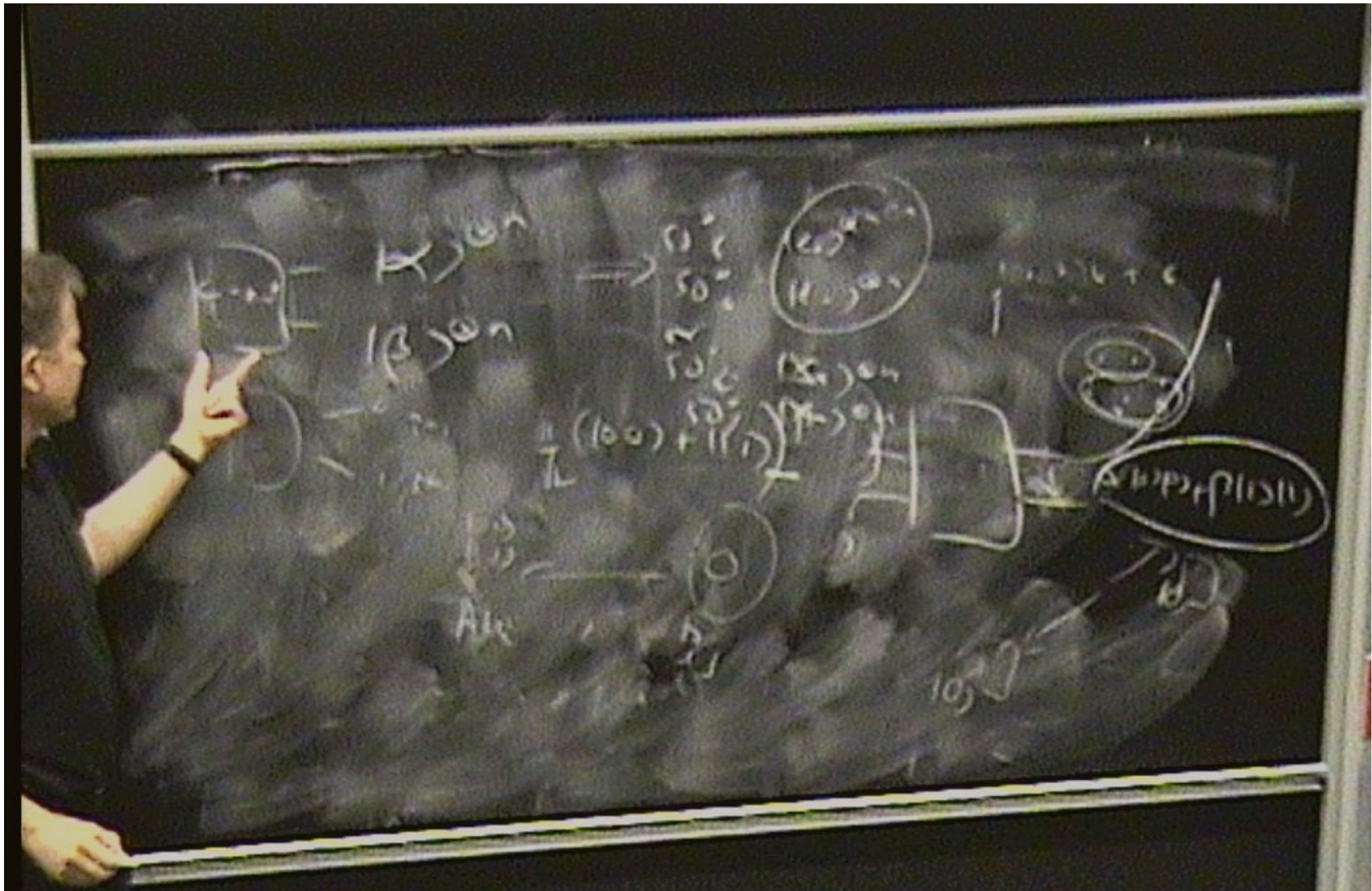
"no signalling"
Principle

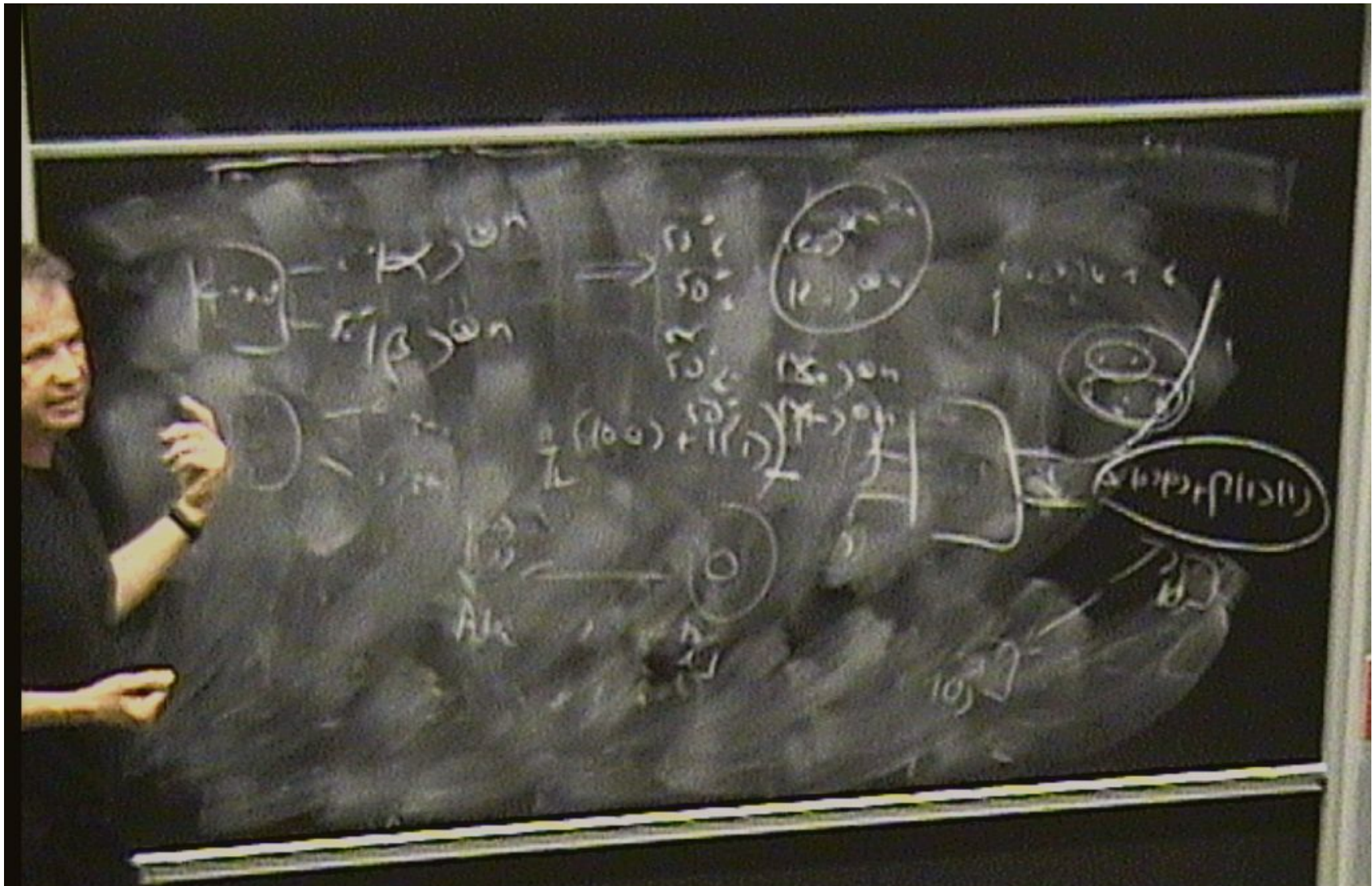
SEP + LOC

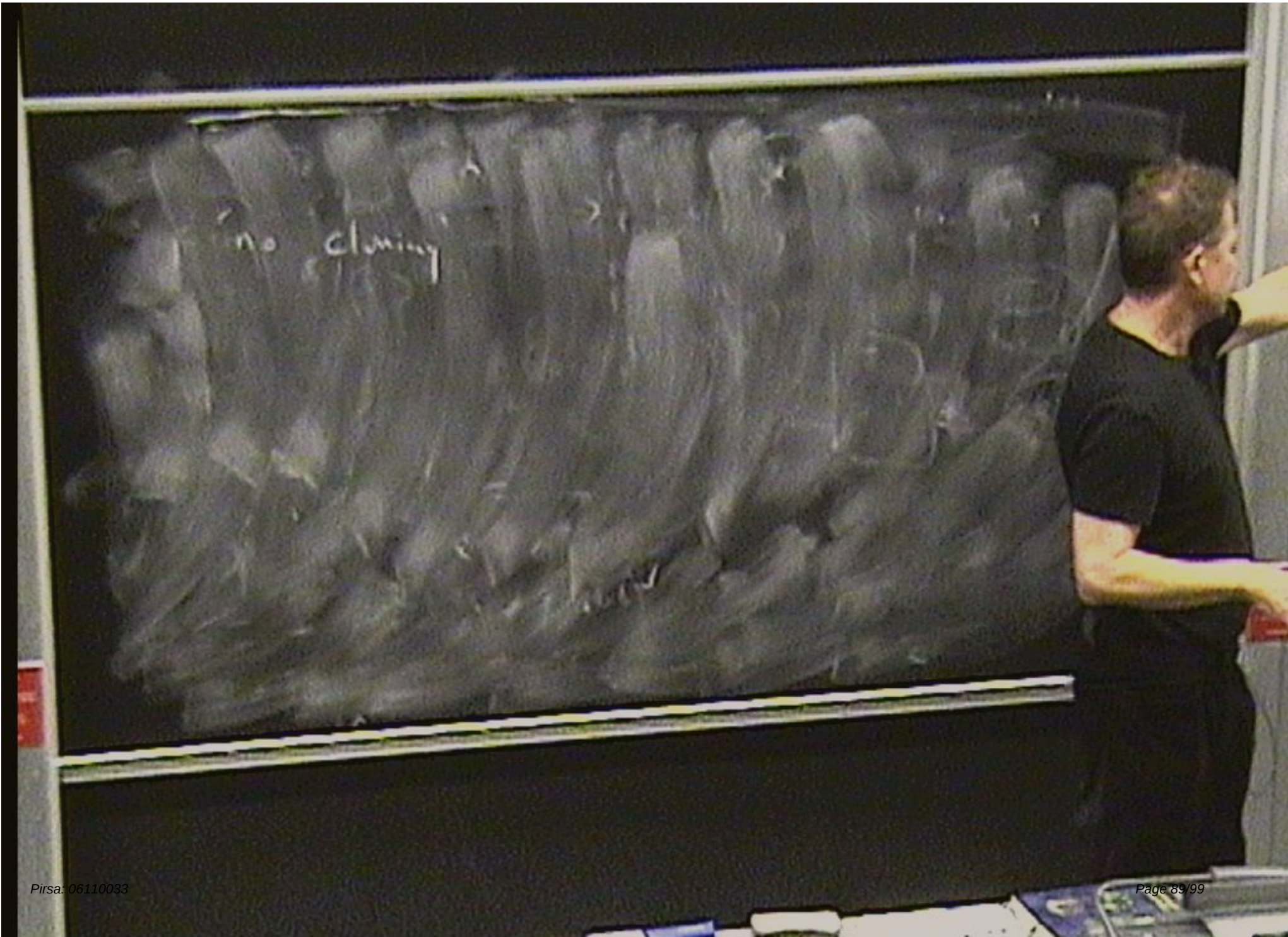
Bell

"Against
Measurement"









$$P(k=s) + P(k'=s) + P(k=s') + P(k' \neq s') \leq 3$$

classical bound

Tsirelson

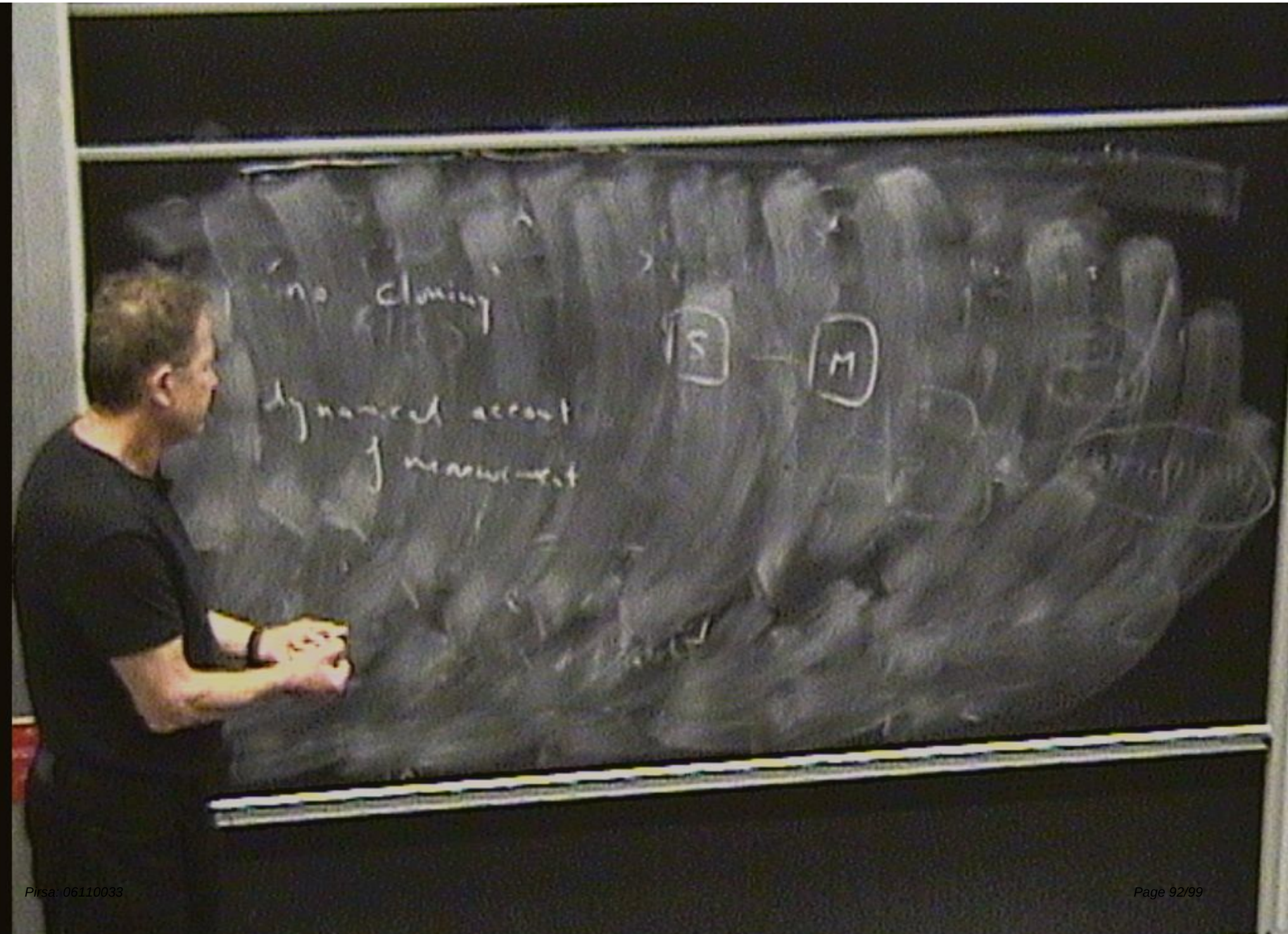
bound

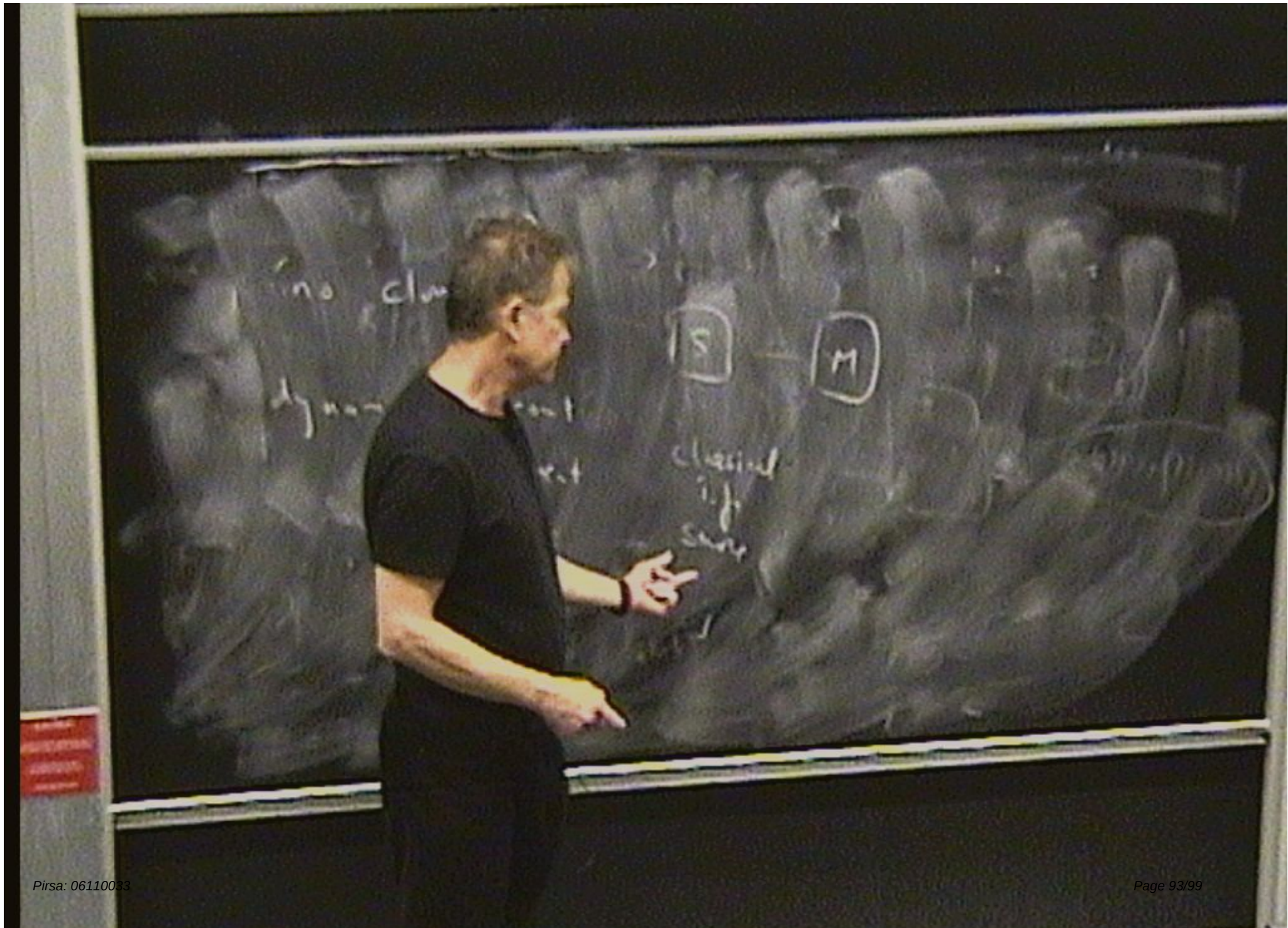
$$\leq \sqrt{2}$$

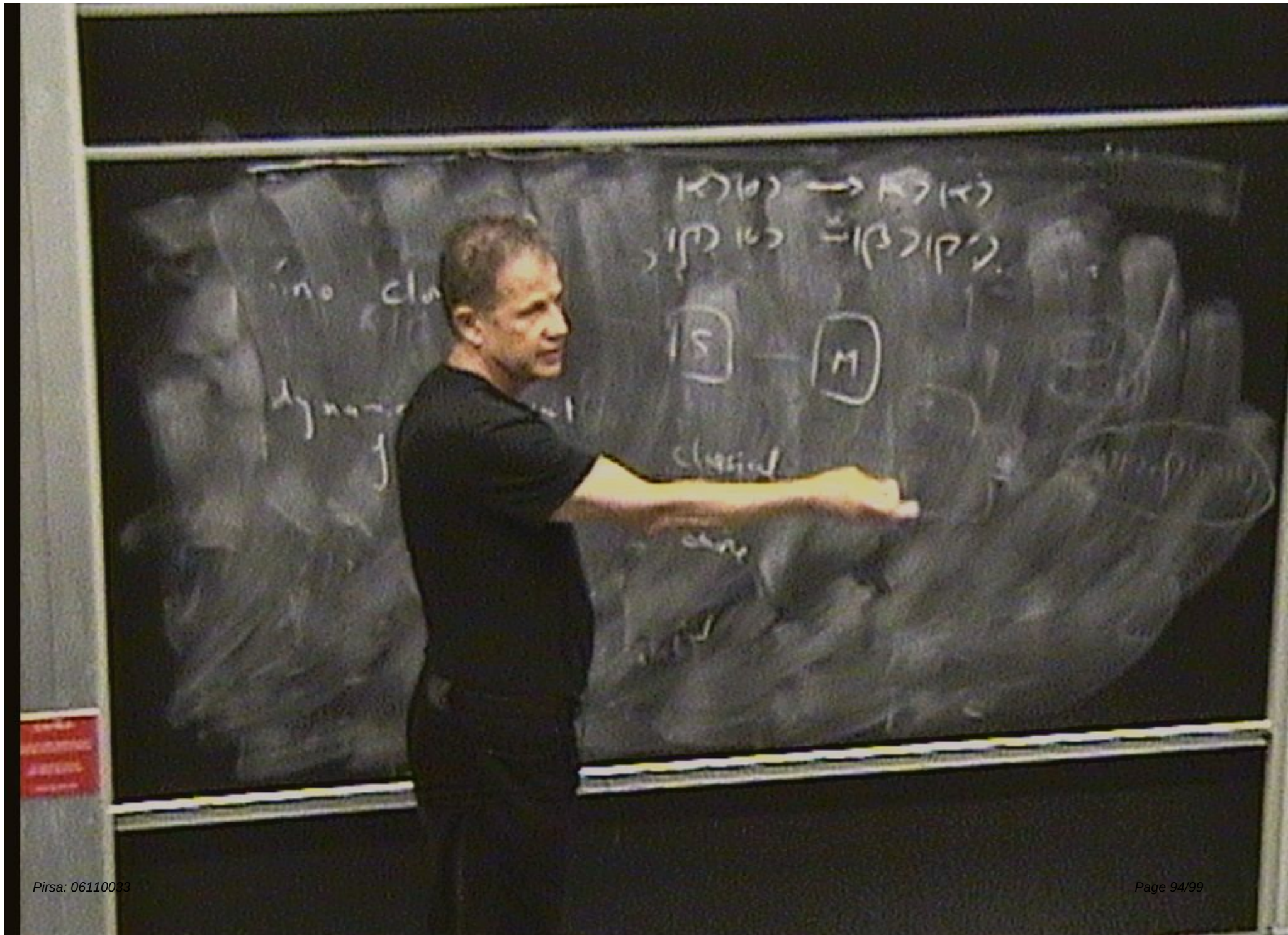
quantum bound

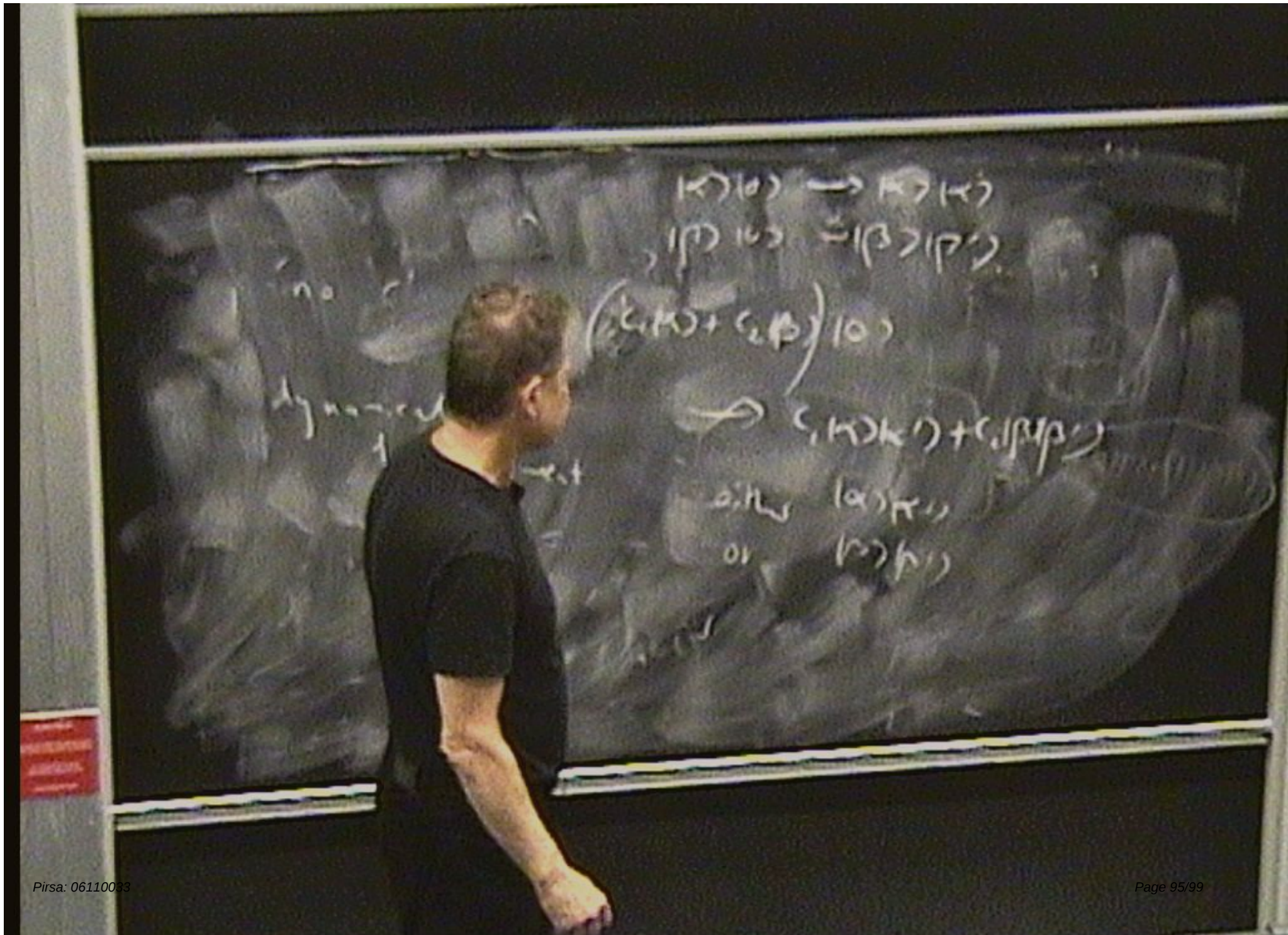
Proof for nonlocal hidden variables
Still LDC

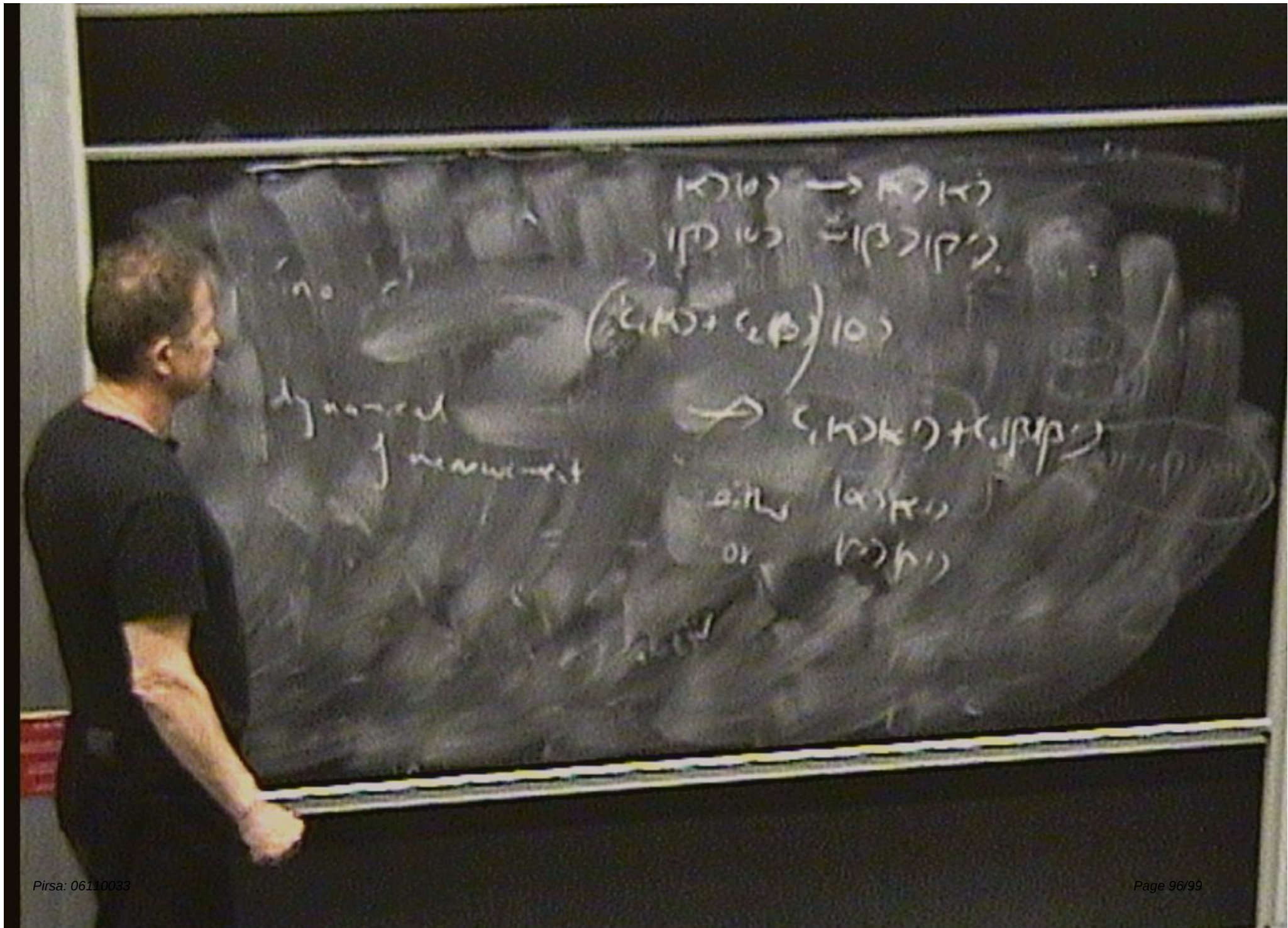
no cloning
dynamical account
of measurement











$$|K\rangle|0\rangle \rightarrow |K\rangle|K\rangle$$
$$|P\rangle|0\rangle \rightarrow |P\rangle|P\rangle$$

$$(|\psi_1\rangle|K\rangle + |\psi_2\rangle|P\rangle)|0\rangle$$

$$\rightarrow |\psi_1\rangle|K\rangle|K\rangle + |\psi_2\rangle|P\rangle|P\rangle$$

either $|K\rangle|K\rangle$
or $|P\rangle|P\rangle$

dynamical
measurement

$$|k\rangle \rightarrow |k'\rangle$$

$$|p\rangle \rightarrow |p'\rangle$$

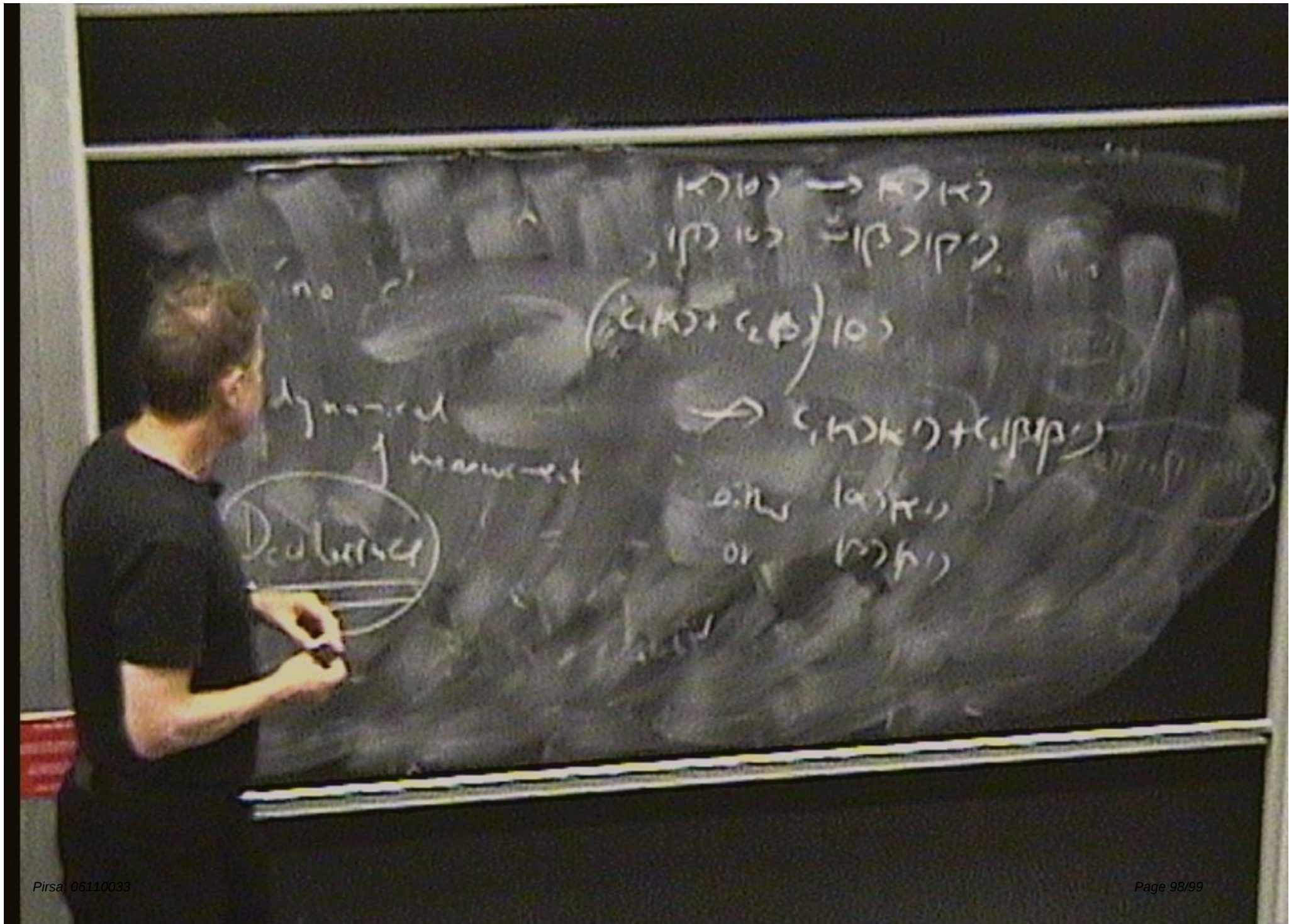
$$(|k\rangle + |p\rangle) |0\rangle$$

dynamic
↑ measurement

$$\rightarrow |k\rangle + |p\rangle$$

either $|k\rangle$
or $|p\rangle$

Declarative



no

dynamical
measurement

Decoherence

$$|A\rangle \rightarrow |A\rangle$$

$$|B\rangle \rightarrow |B\rangle$$

$$(|A\rangle + |B\rangle) |0\rangle$$

$$\rightarrow |A\rangle |1\rangle + |B\rangle |0\rangle$$

$$\text{or } |A\rangle |1\rangle$$

$$\text{or } |B\rangle |0\rangle$$

$$p(k=1) + p(k'=1) + p(k=1') + p(k' \neq 1') \leq 3$$

CENTRAL
BOUND

Tsirelson
bound

$\leq \sqrt{2} \sqrt{2}$
Grover bound

$\leq$
Bound for regular
Tsirelson
Still LC