

Title: Graduate Course on Standard Model & Quantum Field Theory - 6B

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Abstract: Graduate Course on Standard Model & Quantum Field Theory

Boson Masses:

$$\mathcal{L}_{\text{higgs}} = - D_\mu \phi^\dagger D^\mu \phi - \lambda (\phi^\dagger \phi)^2$$

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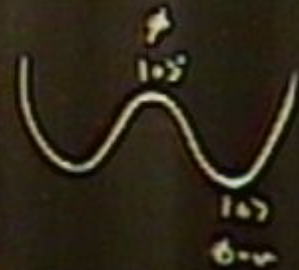
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$$\phi = \frac{1}{\sqrt{2}} [\dots]$$



$$H = H_1 + H_2 + \dots$$

$$\phi = \frac{1}{\sqrt{2}} [\dots]$$

$$b_p |0\rangle = 0$$

$$a_p |0\rangle = 0$$

$$a_i |0\rangle = c |0\rangle$$

$$|0\rangle = \sum_i c_i |0\rangle$$

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$$e^{i\epsilon_1}$$

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$$e^{i\epsilon t}$$

$$H = H_0 + H_{\text{int}}$$

$$\psi = \phi + \dots = \frac{1}{\sqrt{2}} [\dots]$$

$$b_p |0\rangle = 0$$

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$$|0\rangle = \sum_n \frac{c^n}{n!} |n\rangle$$

Boson Masses:

$$\mathcal{L}_{\text{Higgs}} = -D_\mu \phi^\dagger D^\mu \phi - \lambda (\phi^\dagger \phi - v^2/2)^2$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$P^\mu$$

$$J^{\mu\nu}$$

$$P^0$$

$$\phi = \sum \frac{1}{\sqrt{p}} [a_p \dots]$$



$$e^{iEt}$$

$$H = H_0 + H_{\text{int}}$$

$$\psi = \phi + \eta = \sum \frac{1}{\sqrt{p}} [b_p \dots]$$

$$b_p |0\rangle = 0$$

$$a_p |0\rangle = 0$$

$$a_p = b_p + c$$

$$a_p |0\rangle = c |0\rangle$$

$$|0\rangle = \sum e^{i\theta_p} |p_0\rangle$$

Boson Masses:

$$\mathcal{L}_{\text{higgs}} = -D_\mu \phi^\dagger D^\mu \phi - \lambda (\phi^\dagger \phi - v^2/2)^2$$

$$\phi = \begin{pmatrix} 0 \\ (v+H) \end{pmatrix}$$

$$\xi^\mu = \delta^\mu_0$$

$$P^\mu$$

$$J^{\mu\nu}$$

$$P^0 = H$$

$$+ \frac{1}{2} \int d^3x [\dot{\phi}^2]$$



$$e^{i\xi t}$$

$$H = H_0 + H_1$$

$$\psi = \phi + \dots = \int d^3x [\psi]$$

$$b_i |0\rangle = 0$$

$$a_i |0\rangle = 0$$

$$a_i = b_i + c$$

$$a_i |0\rangle = c |0\rangle$$

$$|0\rangle = \sum_i \frac{e^{i\xi t}}{e^{i\xi t}} \psi_i$$

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$$P^0 = H$$



$$\phi = \frac{1}{\sqrt{2}} [a_1 + \dots]$$



$$e^{i\epsilon t}$$

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$$\psi = \phi + \dots = \frac{1}{\sqrt{2}} [a_1 + \dots]$$

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Boson Masses:

$$\mathcal{L}_{\text{higgs}} = -D_\mu \phi^\dagger D^\mu \phi - \lambda (\phi^\dagger \phi - v^2/2)^2$$

$$\phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{pmatrix}$$

$$= -\frac{1}{2} \partial_\mu H \partial^\mu H - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$

$$- \frac{1}{8} g_i^2 (v+H)^2 (W_\mu^i W^{\mu i} + W_\mu^3 W^{\mu 3}) - \frac{1}{8} (v+H)^2 (g_1^2 + g_2^2) Z_\mu Z^\mu$$

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$$\phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{pmatrix}$$

$$\begin{aligned} &= -\frac{1}{2} \partial_\mu H \partial^\mu H - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \\ &\quad - \frac{1}{8} g_i^2 (v+H)^2 (W_\mu^i W^\mu_i + W_\mu^3 W^\mu_3) - \frac{1}{8} (g_1^2 + g_2^2) (v+H)^2 Z_\mu Z^\mu \end{aligned}$$

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$$\begin{aligned} &= -\frac{1}{2} \partial_\mu H \partial^\mu H - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \\ &\quad - \frac{1}{8} g^2 v^2 (W_\mu^+ W_\mu^- + W_\mu^3 W_\mu^3) - \frac{1}{8} v^2 (g_1^2 + g_2^2) Z_\mu Z^\mu \end{aligned}$$

$$M_A = 0. \quad A_y = B_y \cos \theta u + W_y \sin \theta u$$

When is $\delta \phi \Big|_{\phi = (0/a)} = 0$?

$$M_z = 0, \quad A_y = B_y \cos \theta_0 + W_y \sin \theta_0$$

When is $\delta\phi|_{\phi=(0/\pi)} = 0$?

$$\delta\phi|_{\phi=(0/\pi)} = \left[\frac{i}{2} \omega_1 \phi + \frac{i}{2} \omega_2 \tau_a \phi \right]_{\phi=(0/\pi)}$$

$$= \frac{i}{2} \begin{pmatrix} \omega_1 + \omega_2 & \omega_2 - \omega_1 \\ \omega_2 - \omega_1 & \omega_1 - \omega_2 \end{pmatrix} \begin{pmatrix} 0 \\ \pi/2 \end{pmatrix}$$

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$$= \frac{i}{2} \begin{pmatrix} \omega_2 - i\omega_1 \\ \omega_1 - \omega_2 \end{pmatrix} = 0 \quad \text{if } \omega_1 = \omega_2 = \omega$$

$$H_0 \approx A_p = B_p \cos \theta + \omega_p \sin \theta$$

When is $\delta\phi|_{\phi=(0/\pi)} = 0$?

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$$= \frac{i}{2} \begin{pmatrix} \omega_2 - \omega_1 \\ \omega_1 - \omega_2 \end{pmatrix} = 0 \quad \text{if } \omega_1 = \omega_2 = \omega$$

This symmetry of the vacuum is $U_{em}(1)$ or $U_1(1)$

$$M_A = 0. \quad A_\mu = B_\mu \cos \theta_0 + W_\mu \sin \theta_0$$

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$$\delta\phi|_{\phi=(0/a)} = \left[\frac{i}{2} \omega_1 \phi + \frac{i}{2} \omega_2^* \tau_a \phi \right]_{\phi=(0/a)}$$

$$= \frac{i}{2} \begin{pmatrix} \omega_1 + \omega_2^* & \omega_2 - i\omega_1^* \\ \omega_1^* + \omega_2 & \omega_1 - \omega_2^* \end{pmatrix} \begin{pmatrix} 0 \\ \frac{a}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{i}{2} \begin{pmatrix} \omega_2 - i\omega_1^* \\ \omega_1 - \omega_2^* \end{pmatrix} = 0 \quad \text{if } \omega_1^* = \omega_2^* = 0$$

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This symmetry of the vacuum is $U_{em}(1)$ or $U_1(1)$

$$\delta W_\mu^a = \partial_\mu \omega^a - \epsilon^{abc} \omega_b^c W_\mu^c \quad \text{if } \partial_\mu \omega_b^c = 0$$

$$= -\epsilon^{abc} \omega_b^c W_\mu^c$$

$$\delta W_{\text{mr}}^3 = 0$$

$$\delta W_{\text{mr}}^1 = \omega W_{\text{mr}}^2$$

$$\delta W_{\text{mr}}^2 = -\omega W_{\text{mr}}^1$$

$$\delta \begin{pmatrix} W_{\text{mr}}^1 \\ W_{\text{mr}}^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \omega \begin{pmatrix} W_{\text{mr}}^1 \\ W_{\text{mr}}^2 \end{pmatrix}$$

$$W_{\text{mr}} = \frac{W_{\text{mr}}^1 - i W_{\text{mr}}^2}{\sqrt{2}}$$

$$\delta W_{\text{mr}} = -i\omega W_{\text{mr}} \quad \text{or} \quad W_{\text{mr}} \rightarrow e^{-i\omega} W_{\text{mr}}$$

$$\delta W_{\mu}^3 = 0$$

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$$W_{\mu} = \frac{W_{\mu}^1 - i W_{\mu}^2}{\sqrt{2}}$$

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When is $\delta\phi|_{\phi=(0/a)} = 0$?
$$\delta\phi_{\text{vac}} = \left[\frac{i}{2} \omega_1 \phi + \frac{i}{2} \omega_2 \tau_3 \phi \right]_{\phi=(0/a)}$$

$$= \frac{i}{2} \begin{pmatrix} \omega_1 + \omega_2^3 & \omega_2 - i\omega_1^3 \\ \omega_2 + i\omega_1^3 & \omega_1 - \omega_2^3 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$D_\mu \phi = \partial_\mu \phi - \frac{i}{2} g A_\mu^a \tau_a \phi = \frac{i}{2} \begin{pmatrix} \omega_2 - i\omega_1^3 \\ \omega_1 - \omega_2^3 \end{pmatrix} = 0. \quad \text{If } \omega_2 = \omega_2^3 = 0, \quad \omega_1 = \omega_1^3 = \omega$$

This symmetry of the vacuum is $U_{em}(1)$ or $U_1(1)$

$$\delta W_\mu^a = \partial_\mu \omega^a - \epsilon^{abc} \omega_b^c W_\mu^c \quad \text{if } \partial_\mu \omega_i^3 = 0$$

$$= -\epsilon^{abc} \omega W_\mu^c$$

Fermion Masses:

$$\mathcal{L}_{\text{Yuk.}} = -\frac{1}{\sqrt{2}} \left[f_{ij} \begin{pmatrix} \bar{\psi}_i \\ \bar{\psi}_j \end{pmatrix} \gamma_k E_j \begin{pmatrix} \psi \\ \psi \end{pmatrix} + g_{ij} \begin{pmatrix} \bar{\psi}_i \\ \bar{\psi}_j \end{pmatrix} \gamma_k D_j \phi + h_{ij} (\bar{\psi}_i \gamma_k U_j) \tilde{\phi} + \text{c.c.} \right]$$

Fermion Masses:

$$\mathcal{L}_{\text{Yuk.}} = -\frac{1}{\sqrt{2}} \left[f_{ij} \begin{pmatrix} \bar{\nu}_i \\ e_i \end{pmatrix}^T \gamma_R E_j \begin{pmatrix} \phi \\ \chi \end{pmatrix} + g_{ij} \begin{pmatrix} \bar{u}_i \\ d_i \end{pmatrix} \gamma_R D_j \begin{pmatrix} \phi \\ \chi \end{pmatrix} + h_{ij} (\bar{\nu}_i \gamma_R U_j) \tilde{\phi} + \text{c.c.} \right]$$

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$$\mathcal{L}_{\text{Yuk.}} = -\frac{1}{\sqrt{2}} \left[f_{ij} \begin{pmatrix} \bar{\psi}_i \\ \bar{e}_i \end{pmatrix}^\top \gamma_\mu E_j \begin{pmatrix} \psi \\ \chi \end{pmatrix} + g_{ij} \begin{pmatrix} \bar{u}_i \\ \bar{d}_i \end{pmatrix}^\top \gamma_\mu D_j \begin{pmatrix} u \\ d \end{pmatrix} + h_{ij} \begin{pmatrix} \bar{u}_i \\ \bar{d}_i \end{pmatrix}^\top \gamma_\mu U_j \begin{pmatrix} \psi + H \\ 0 \end{pmatrix} \right] + \text{c.c.}$$

$$= -\left[\frac{1}{\sqrt{2}} f_{ij} (\bar{e}_i \gamma_\mu E_j) + g_{ij} (\bar{d}_i \gamma_\mu D_j) + h_{ij} (\bar{u}_i \gamma_\mu U_j) \right] (\psi + H) + \text{c.c.}$$

$$Q = T_3 + Y$$

$$M_3 = 0. \quad A_\mu = B_\mu \cos \theta_0 + W_\mu \sin \theta_0$$

When is $\delta\phi|_{\phi=\langle\phi\rangle} = 0$? $\delta\phi|_{\phi=\langle\phi\rangle} = \left[\frac{i}{2} \omega_1 \phi + \frac{i}{2} \omega_2 \tau_3 \phi \right]_{\phi=\langle\phi\rangle}$

$$= \frac{i}{2} \begin{pmatrix} \omega_1 + \omega_2 & \omega_2 - i\omega_1 \\ \omega_1 - i\omega_2 & \omega_1 + \omega_2 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$D_\mu \phi = \partial_\mu \phi - \frac{i}{2} g A_\mu \tau_3 \phi = \frac{i}{2} \begin{pmatrix} \omega_1 - i\omega_2 \\ \omega_1 - \omega_2 \end{pmatrix} = 0. \quad \text{If } \omega_1 = \omega_2 = 0$$

$$\omega_1 - \omega_2 = \omega$$

This symmetry of the vacuum is $U_{em}(1)$ or $U_1(1)$

$$\delta W_\mu^a = \partial_\mu \omega^a - \epsilon^{abc} \omega_b^c W_\mu^c \quad \text{if } \partial_\mu \omega_2 = 0$$

$$= -\epsilon^{abc} \omega W_\mu^c$$

$$\delta W_{\mu}^3 = 0$$

$$\delta W_{\mu}^1 = \omega W_{\mu}^2$$

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$$\delta \begin{pmatrix} W_{\mu}^1 \\ W_{\mu}^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \omega \begin{pmatrix} W_{\mu}^1 \\ W_{\mu}^2 \end{pmatrix}$$

$$W_{\mu} = \frac{W_{\mu}^1 - i W_{\mu}^2}{\sqrt{2}}$$

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$$\mathcal{L}_{\text{Yuk.}} = -\frac{1}{\sqrt{2}} \left[f_{ij} \begin{pmatrix} \bar{\psi}_i \\ \bar{\psi}_i \end{pmatrix}^T \gamma_L E_j \begin{pmatrix} \psi_j \\ \psi_j \end{pmatrix} + g_{ij} \begin{pmatrix} \bar{\psi}_i \\ \bar{\psi}_i \end{pmatrix}^T \gamma_L D_j \begin{pmatrix} 0 \\ \psi_j \end{pmatrix} + h_{ij} \begin{pmatrix} \bar{\psi}_i \\ \bar{\psi}_i \end{pmatrix}^T \gamma_L U_j \begin{pmatrix} \psi_j + H \\ 0 \end{pmatrix} + \text{c.c.} \right]$$

$$= -\frac{1}{\sqrt{2}} \left[f_{ij} (\bar{\psi}_i \gamma_L E_j) + g_{ij} (\bar{\psi}_i \gamma_L D_j) + h_{ij} (\bar{\psi}_i \gamma_L U_j) \right] (\psi_j + H) + \text{c.c.}$$

Fermion Masses:

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$$= -\frac{1}{\sqrt{2}} \left[f_{ij} (\bar{\nu}_i \gamma_R E_j) + g_{ij} (\bar{u}_i \gamma_R D_j) + h_{ij} (\bar{\nu}_i \gamma_R U_j) \right] (\nu+H) + \text{c.c.}$$

$$Q = T_3 + Y$$

Conservation of Q makes it convenient
to use Dirac spinors for charged leptons, + quarks.

$$e_i = \gamma_L e_{Li} + \gamma_R e_{Ri}$$

$$u_i = \gamma_L u_{Li} + \gamma_R u_{Ri}$$

$$d_i = \gamma_L d_{Li} + \gamma_R d_{Ri}$$

Mass terms:

$$\mathcal{L}_{fm} = -\frac{g}{\sqrt{2}} \left[f_{ij} \bar{\psi}_i \gamma_\mu E_j + g_{ij} \bar{\psi}_i \gamma_\mu D_j + h_{ij} \bar{\psi}_i \gamma_\mu U_j \right] + c.c.$$

Mass terms:

$$= -\frac{1}{\sqrt{2}} \left[f_{ij} \bar{E}_i \gamma_\mu E_j + g_{ij} \bar{D}_i \gamma_\mu D_j + h_{ij} \bar{U}_i \gamma_\mu U_j \right] + \text{c.c.}$$

$$\gamma_\mu E_i \rightarrow (U_c)_i \gamma_\mu E_j$$

$$\gamma_\mu E_i \rightarrow (V_e)_i \gamma_\mu E_j$$

Mass terms:

$$\mathcal{L}_{fm} = -\frac{1}{2} \left[f_{ij} \bar{E}_i \gamma_\mu E_j + g_{ij} \bar{D}_i \gamma_\mu D_j + h_{ij} \bar{U}_i \gamma_\mu U_j \right] + c.c.$$

$$\gamma_\mu E_i \rightarrow (U_e)_{ij} \gamma_\mu E_j$$

$$\gamma_\mu E_i \rightarrow (V_e)_{ij} \gamma_\mu E_j$$

$$\gamma_\mu U_i \rightarrow (U_u)_{ij} \gamma_\mu U_j$$

et

$$f_{ij} \rightarrow (U_e^T f V_e)_{ij}$$

$$g_{ij} \rightarrow (U_d^T g V_d)_{ij}$$

$$h_{ij} \rightarrow (U_u^T h V_u)_{ij}$$

Mass terms:

$$\mathcal{L}_{fm} = -\frac{1}{2} \left[f_{ij} \bar{E}_i \gamma_R E_j + g_{ij} \bar{D}_i \gamma_R D_j + h_{ij} \bar{U}_i \gamma_R U_j \right] + c.c.$$

$$\gamma_R E_i \rightarrow (U_e)_{ij} \gamma_R E_j$$

$$\gamma_R E_i \rightarrow (V_e)_{ij} \gamma_R E_j$$

$$\gamma_R U_i \rightarrow (U_u)_{ij} \gamma_R U_j$$

etc

$$f_{ij} \rightarrow (U_e^T f V_e)_{ij}$$

$$g_{ij} \rightarrow (U_d^T g V_d)_{ij}$$

$$h_{ij} \rightarrow (U_u^T h V_u)_{ij}$$

where U_e, V_e, U_d, V_d are unitary

$$\text{Mass terms} = \bar{\psi}_L \begin{pmatrix} E \\ D \\ U \end{pmatrix} + \text{h.c.} \sin \theta_w$$

$$\mathcal{L}_{fm} = \frac{1}{\sqrt{2}} \left[f_{ij} \bar{\psi}_i \gamma_\mu \psi_j + g_{ij} \bar{\psi}_i \gamma_\mu \psi_j + h_{ij} \bar{\psi}_i \gamma_\mu \psi_j \right] + \text{c.c.}$$

$$\gamma_\mu \psi_i \rightarrow (U_e)_{ij} \gamma_\mu \psi_j$$

$$\gamma_\mu \psi_i \rightarrow (V_e)_{ij} \gamma_\mu \psi_j$$

$$\gamma_\mu \psi_i \rightarrow (U_u)_{ij} \gamma_\mu \psi_j$$

etc

$$f_{ij} \rightarrow (U_e^\dagger f V_e)_{ij}$$

$$g_{ij} \rightarrow (U_u^\dagger g V_u)_{ij}$$

$$h_{ij} \rightarrow (U_d^\dagger h V_d)_{ij}$$

where $U_e, U_u, U_d, V_e, V_u, V_d$ are unitary

Can choose U_e, V_e s.t. $U_e^T S V_e = \text{diag}(f_1, f_2, f_3)$

where f_1, f_2, f_3 are
non negative

Can choose U_e, V_e s.t. $U_e^\dagger f V_e = \text{diag}(f_1, f_2, f_3)$

where f_1, f_2, f_3 are non negative

$$\mathcal{L}_{\text{Yuk}} = -\frac{(U_1 H)}{\sqrt{2}} \sum_{i=1}^3 \left[f_i \bar{E}_i \gamma_R E_i + g_i \bar{D}_i \gamma_R D_i + h_i \bar{U}_i \gamma_R U_i \right] + \text{c.c.}$$

Can choose U_e, V_e s.t. $U_e^\dagger f V_e = \text{diag}(f_1, f_2, f_3)$

where f_1, f_2, f_3 are non negative

$$\mathcal{L}_{\text{Yuk}} = -\frac{(U^\dagger H)}{\sqrt{2}} \sum_{i=1}^3 \left[f_i \bar{E}_i \gamma_R E_i + g_i \bar{D}_i \gamma_R D_i + h_i \bar{U}_i \gamma_R U_i \right] + \text{c.c.}$$

Define Dirac spinors:

$$\begin{aligned} e_i &= \gamma_L E_i + \gamma_R E_i \\ u_i &= \gamma_L U_i + \gamma_R U_i \\ d_i &= \gamma_L D_i + \gamma_R D_i \end{aligned}$$

Fermion Masses:

$$\text{then: } \int d^4x \bar{\psi}_i \gamma_\mu \psi_i + \text{c.c.} = \int d^4x (\bar{\psi}_i \gamma_\mu \psi_i +$$

Fermion Masses:

$$\begin{aligned}\text{then: } f_i \bar{e}_i \gamma_5 e_i + \text{c.c.} &= f_i (\bar{e}_i \gamma_R e_i + \bar{e}_i \gamma_L e_i) \\ &= f_i \bar{e}_i (\gamma_R + \gamma_L) e_i\end{aligned}$$

Fermion Masses:

$$\begin{aligned}
 \text{then: } f_i \bar{\psi}_i \gamma_5 \psi_i + \text{c.c.} &= f_i (\bar{\psi}_i \gamma_5 \psi_i + \bar{\psi}_i \gamma_5 \psi_i) \\
 &= f_i \bar{\psi}_i (\gamma_5 + \gamma_5) \psi_i \\
 &= f_i \bar{\psi}_i \psi_i
 \end{aligned}$$

$$\begin{aligned}
 (f_i M \psi_2)^* &= \theta (\bar{\psi}_1 M \psi_2) \\
 &\quad \begin{matrix} \text{---} V, T \\ \text{---} P, A \end{matrix}
 \end{aligned}$$

Fermion Masses:

$$\begin{aligned}\text{then: } f_i \bar{\psi}_i \gamma_R \psi_i + \text{c.c.} &= f_i (\bar{\psi}_i \gamma_R \psi_i + \bar{\psi}_i \gamma_L \psi_i) \\ &= f_i \bar{\psi}_i (\gamma_R + \gamma_L) \psi_i \\ &= f_i \bar{\psi}_i \psi_i\end{aligned}$$

$$(\bar{\psi}_1 M \psi_2)^* = \theta (\bar{\psi}_1 M \psi_2)$$

↓
+ S, V, T
- P, A

Can choose U_e, V_e s.t. $U_e^\dagger f V_e = \text{diag}(f_1, f_2, f_3)$

where f_1, f_2, f_3 are non negative

$$\mathcal{L}_{\text{Yuk}} = -\frac{(v_1 H)}{\sqrt{2}} \sum_{i=1}^3 \left[f_i \bar{E}_i \gamma_R E_i + g_i \bar{D}_i \gamma_R D_i + h_i \bar{U}_i \gamma_R U_i \right] + \text{c.c.}$$

Define Dirac spinors:

$$e_i = (\gamma_L E_i + \gamma_R E_i) \frac{1}{\sqrt{2}}$$

$$u_i = (\gamma_L U_i + \gamma_R U_i) \frac{1}{\sqrt{2}}$$

$$d_i = (\gamma_L D_i + \gamma_R D_i) \frac{1}{\sqrt{2}}$$

$$W_\mu = \frac{1}{\sqrt{2}} (W_\mu^1 + i W_\mu^2)$$

Can choose U_e, V_e s.t. $U_e^\dagger f V_e = \text{diag}(f_1, f_2, f_3)$

where f_1, f_2, f_3 are non negative

$$\mathcal{L}_{\text{Yuk}} = -\frac{(v_1 H)}{\sqrt{2}} \sum_{i=1}^3 \left[f_i \bar{E}_i \gamma_L E_i + g_i \bar{D}_i \gamma_L D_i + h_i \bar{U}_i \gamma_L U_i \right] + \text{c.c.}$$

Define Dirac spinors: $e_i = (\gamma_L E_i + \gamma_R E_i) \frac{1}{\sqrt{2}}$

$$u_i = (\gamma_L U_i + \gamma_R U_i) \frac{1}{\sqrt{2}}$$

$$d_i = (\gamma_L D_i + \gamma_R D_i) \frac{1}{\sqrt{2}}$$

$$W_\mu = \frac{1}{\sqrt{2}} (W_\mu^1 + i W_\mu^2)$$

Fermion Masses:

$$\bar{\epsilon}_i \beta \gamma_\mu = \epsilon_i^\dagger \gamma_\mu \beta - (\gamma_\mu \epsilon)_i^\dagger \beta$$

$$\text{then: } \int \bar{\epsilon}_i \gamma_\mu \epsilon_i + \text{c.c.} = \int (\bar{\epsilon}_i \gamma_\mu \epsilon_i + \bar{\epsilon}_i \gamma_\mu \epsilon_i)$$

$$= \int \bar{\epsilon}_i (\gamma_\mu + \gamma_\mu) \epsilon_i = \int \bar{\epsilon}_i \epsilon_i$$

$$\psi_2^* = \theta(\bar{\psi}_1 M \psi_2)$$

$$\begin{aligned} \gamma_L \epsilon &= \gamma_L \epsilon \\ \gamma_R \epsilon &= \gamma_R \epsilon \end{aligned}$$

$$+ S, V, T$$

$$- P, A$$

Fermion Masses:

$$\bar{e}_i \gamma_\mu e_i = \bar{e}_i \gamma_\mu e_i - (\gamma_\mu e)_i^\dagger (e)_i$$

$$\text{then: } \int \bar{e}_i \gamma_\mu e_i + c.c. = \int (\bar{e}_i \gamma_\mu e_i + \bar{e}_i \gamma_\mu e_i)$$

$$= \int \bar{e}_i (\gamma_\mu + \gamma_\mu) e_i = \int \bar{e}_i e_i$$

$$(\bar{\psi}_1 M \psi_2)^* = \theta (\bar{\psi}_1 M \psi_2)$$

+ S, V, T
- P, A

$\gamma_\mu e = \gamma_\mu e$
 $\gamma_\mu e = \gamma_\mu e$
 $\gamma_\mu e$

Fermion Masses:

$$\bar{\epsilon}_i \beta \gamma_R = \bar{\epsilon}_i^+ \gamma_L \beta = (\gamma_L \epsilon)_i^\dagger \beta$$

$$\text{then: } f_i \bar{\epsilon}_i \gamma_R \epsilon_i + \text{c.c.} = f_i (\bar{\epsilon}_i \gamma_R \epsilon_i + \bar{\epsilon}_i \gamma_L \epsilon_i)$$

$$(\bar{\psi}_1 M \psi_2) = \theta (\bar{\psi}_2 M \psi_1)$$

$$= f_i \bar{\epsilon}_i (\gamma_R + \gamma_L) \epsilon_i$$

$$= f_i \bar{\epsilon}_i \epsilon_i$$

$$+ \frac{[S, P] A}{V, T}$$

$$(\bar{\psi}_1 M \psi_2)^* = \theta (\bar{\psi}_1 M \psi_2)$$

$$+ S, V, T$$

$$- \bar{\psi}, A$$

Fermion Masses:

$$\bar{\epsilon}_i \beta \gamma_\mu \epsilon_i + \bar{\epsilon}_i^\dagger \gamma_\mu \beta - (\gamma_\mu \epsilon)_i^\dagger \beta$$

$$\text{then: } f_1 \bar{\epsilon}_i \gamma_\mu \epsilon_i + \text{c.c.} = f_1 \left(\underbrace{\bar{\epsilon}_i}_{(\gamma_\mu \epsilon)_i^\dagger} \underbrace{\gamma_\mu \epsilon_i}_{\gamma_\mu \epsilon_i} + \bar{\epsilon}_i \gamma_\mu \epsilon_i \right)$$

$$(\bar{\psi}_1 M \psi_2) = \theta (\bar{\psi}_2 M \psi_1)$$

$$= f_1 \bar{\epsilon}_i (\gamma_\mu + \gamma_\mu) \epsilon_i$$

$$= f_1 \bar{\epsilon}_i \epsilon_i$$

$$+ \frac{S_1 P}{V_T} A$$

$$(\bar{\psi}_1 M \psi_2)^* = \theta (\bar{\psi}_1 M \psi_2)$$

$$+ S_1 V_1$$

$$- P_1 A$$

Fermion Masses:

$$\bar{e}_i \beta \gamma_\mu = \bar{e}_i^\dagger \gamma_\mu \beta = -(\gamma_\mu e)_i^\dagger$$

$$\text{then: } f_1 \bar{e}_1 \gamma_\mu e_1 + c.c. = f_1 \left(\bar{e}_1 \gamma_\mu e_1 + \bar{e}_1 \gamma_\mu e_1 \right)$$

$$(\bar{\psi}_1 M \psi_2) = \theta (\bar{\psi}_2 M \psi_1)$$

$$= f_1 \bar{e}_1 (\gamma_\mu + \gamma_\mu) e_1$$

$$= f_1 \bar{e}_1 e_1$$

$$+ \frac{S, P, A}{V, T}$$

$$(\bar{\psi}_1 M \psi_2)^* = \theta (\bar{\psi}_1 M \psi_2)$$

$$+ S, V, T$$

$$- P, A$$

$\mathcal{L} =$

$$\gamma_L E_i \rightarrow (U_e)_{ij} \gamma_L E_j$$

$$\gamma_R E_i \rightarrow (V_e)_{ij} \gamma_R E_j$$

$$\gamma_R U_i \rightarrow (U_u)_{ij} \gamma_R U_j$$

$\vdots$
 e_k

$$f_{ij} \rightarrow (U_e^T f V_e)_{ij}$$

$$g_{ij} \rightarrow (U_d^T g V_d)_{ij}$$

$$h_{ij} \rightarrow (U_u^T h V_u)_{ij}$$

where $U_e, U_u, U_d, V_e, V_u, V_d$ are unitary

$$\mathcal{L}_{\text{quark}} = -\frac{(U+H)}{\sqrt{2}} \sum_{i=1}^3 [f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i]$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} \bar{E}_i \not{\partial} E_i - \frac{1}{2} \bar{E}_i \not{\partial} E_i$$

Can choose U_e, V_e s.t. $U_e^\dagger f V_e = \text{diag}(f_1, f_2, f_3)$

where f_1, f_2, f_3 are non negative

$$\mathcal{L}_{\text{Yuk}} = - \frac{(v_1 H)}{\sqrt{2}} \sum_{i=1}^3 \left[f_i \bar{E}_i \gamma_R E_i + g_i \bar{D}_i \gamma_R D_i + h_i \bar{U}_i \gamma_R U_i \right] + \text{c.c.}$$

Define Dirac spinors: $e_i = (\gamma_L E_i + \gamma_R E_i)$

$$u_i = (\gamma_L U_i + \gamma_R U_i)$$

$$d_i = (\gamma_L D_i + \gamma_R D_i)$$

$$W_\mu = \frac{1}{\sqrt{2}} (W_\mu^1 + i W_\mu^2)$$

$$\mathcal{L}_{\text{gib}} = -\frac{(G+H)}{\sqrt{2}} \sum_{i=1}^3 [f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i]$$

$$= -\frac{1}{2} \bar{E}_i \phi E_i - \frac{1}{2} \bar{E}_i \phi E_i$$

$$\mathcal{L}_{\text{gib}} = -\frac{(G+H)}{\sqrt{2}} \sum_{i=1}^3 [f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i]$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} \bar{E}_i \not{\partial} E_i - \frac{1}{2} \bar{E}_i \not{\partial} E_i$$

$$= -\bar{E}_i \not{\partial} E_i \quad (\text{check this})$$

$$\mathcal{L}_{\text{g.f.}} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^3 [f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i]$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} \bar{e}_i \not{\partial} e_i - \frac{1}{2} \bar{e}_i \not{\partial} e_i$$

$$= -\bar{e}_i \not{\partial} e_i \quad (\text{check this})$$

Standard form $\mathcal{L} = \bar{\psi} (\not{\partial} + m) \psi$

$$m_{e_i} = \frac{1}{\sqrt{2}} f_i v$$

$$m_{d_i} = \frac{1}{\sqrt{2}} g_i v$$

$$m_{u_i} = \frac{1}{\sqrt{2}} h_i v$$

Notice: Yukawa couplings are diagonal in flavour space
when written in mass basis.

$$\mathcal{L}_{\text{gib}} = -\frac{(\bar{v} + H)}{\sqrt{2}} \sum_{i=1}^3 [f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i]$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} \bar{E}_i \not{\partial} E_i - \frac{1}{2} \bar{E}_i \not{\partial} E_i$$

$$= -\bar{E}_i \not{\partial} E_i \quad (\text{check this})$$

Standard form $\mathcal{L} = \bar{\Psi} (\not{\partial} + m) \Psi$

$$m_{e_i}^2 = \frac{1}{v^2} f_i v$$

$$m_{d_i} = \frac{1}{\sqrt{2}} g_i v$$

$$m_{u_i} = \frac{1}{\sqrt{2}} h_i v$$

Notice: Yukawa couplings are diagonal in flavour space
when written in mass basis.

if we write $\mathcal{L}_{\text{YH}} = -y_f H \bar{f} f$

$y_f =$

Notice: Yukawa couplings are diagonal in flavour space
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if we write $\mathcal{L}_{\text{YH}} = -y_f H \bar{f} f$

$$y_f = \frac{m_f}{v}$$

Notice: Yukawa couplings are diagonal in flavour space
when written in mass basis.

if we write $\mathcal{L}_{\text{YH}} = -y_f H \bar{f} f$

$$y_f = \frac{m_f}{v}$$

$$m_t = 180 \text{ GeV}$$

$$v \approx 246 \text{ GeV}$$

Notice: Yukawa couplings are diagonal in flavour space when written in mass basis.

if we write $\mathcal{L}_{Yuk} = -y_f H \bar{f} f$

$$y_f = \frac{m_f}{v}$$

$$m_t = 180 \text{ GeV}$$

$$v \approx 246 \text{ GeV}$$

$$\frac{y^2}{4}$$

$$r = \frac{e^2}{4\pi}$$

Notice: Yukawa couplings are diagonal in flavour space when written in mass basis.

if we write $\mathcal{L}_{\text{Yuk}} = -y_f H \bar{f} f$

$$y_f = \frac{m_f}{v}$$

$$m_t = 180 \text{ GeV}$$

$$v \approx 246 \text{ GeV}$$

$$\frac{y^2}{4\pi}$$

$$\alpha = \frac{e^2}{4\pi}$$

$$\mathcal{L}_{\text{gub}} = -\frac{(U+H)}{\sqrt{2}} \sum_{i=1}^3 [f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i]$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} \bar{E}_i \not{\partial} E_i - \frac{1}{2} \bar{E}_i \not{\partial} E_i$$

$$= -\bar{E}_i \not{\partial} E_i \quad (\text{check this})$$

Standard form $\mathcal{L} = \bar{\psi}(\not{\partial} + m)\psi$

$$m_e = \frac{1}{\sqrt{2}} f_i v$$

$$m_d = \frac{1}{\sqrt{2}} g_i v$$

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if we write $\mathcal{L}_{Yuk} = -y_f H \bar{f} f$

$$y_f = \frac{m_f}{v}$$

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$$v \approx 246 \text{ GeV}$$

$$\frac{y_f^2}{4\pi}$$

$$\alpha = \frac{e^2}{4\pi}$$

Notice: Yukawa couplings are diagonal in flavour space when written in mass basis.

if we write $\mathcal{L}_{Yuk} = -y_f H \bar{f} f$

$$y_f = \frac{m_f}{v}$$

$$\frac{y_f^2}{4\pi}$$

$$\alpha = \frac{e^2}{4\pi}$$

$$m_t = 180 \text{ GeV}$$

$$v \approx 246 \text{ GeV}$$

$$M_W = 80 \text{ GeV}$$

$$M_Z \approx 90 \text{ GeV}$$

$$\mathcal{L}_{\text{g.f.}} = -\frac{(b+H)}{\sqrt{2}} \sum_{i=1}^3 [f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i]$$

$$\mathcal{L}_{\text{m}} = -\frac{1}{2} \bar{e}_i \not{\partial} e_i - \frac{1}{2} \bar{e}_i \not{\partial} e_i$$

$$= -\bar{e}_i \not{\partial} e_i \quad (\text{check this})$$

Standard form $\mathcal{L} = \bar{\psi}(\not{\partial} + m)\psi$

$$m_e = \frac{1}{\sqrt{2}} f_i v \quad m_d = \frac{1}{\sqrt{2}} g_i v \quad m_u = \frac{1}{\sqrt{2}} h_i v$$

Notice: Yukawa couplings are diagonal in flavour space when written in mass basis.

if we write $\mathcal{L}_{Yuk} = -y_f H \bar{f} f$

$$y_f = \frac{m_f}{v}$$

$$\frac{y^2}{4\pi}$$

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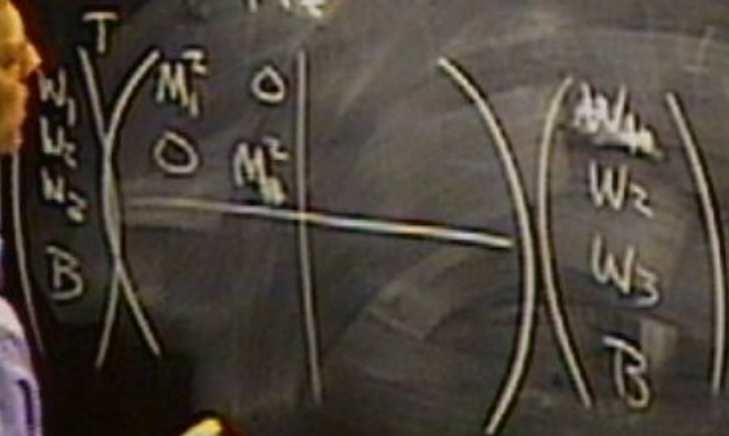
'Custodial' $SU(2)$ approximate symmetry:

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$$\frac{M_W}{M_Z} = \cos \theta_W.$$

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'Custodial' $SU(2)$ approximate symmetry:

$$\frac{M_W}{M_Z} = \cos \theta_W.$$

$$\begin{pmatrix} W_1^T \\ W_2^T \\ W_3^T \\ B \end{pmatrix} \begin{pmatrix} M_1^2 & 0 & 0 \\ 0 & M_2^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}$$

is the most general mass term
consistent with unbroken $U_{em}(1)$ invariance.

'Custodial' $SU(2)$ approximate symmetry:

$$\frac{M_W}{M_Z} = \cos \theta_w$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}^T \begin{pmatrix} M_1^2 & 0 & 0 & 0 \\ 0 & M_2^2 & 0 & 0 \\ 0 & 0 & M_3^2 & m^c \\ 0 & 0 & m^c & M_4^2 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}$$

is the most general mass term
consistent with unbroken $U(1)$ invariance.

$$\det \begin{pmatrix} M_3^2 & m^c \\ m^c & M_4^2 \end{pmatrix} = M_3^2 M_4^2 - m^{c2} = 0$$

the massive eigenvector

$$\begin{pmatrix} \cos \theta_w \\ -\sin \theta_w \end{pmatrix}$$

massless eigenvector

$$\begin{pmatrix} \sin \theta_w \\ \cos \theta_w \end{pmatrix}$$

$$\Rightarrow \tan \theta_L = \frac{m^2}{M_3^2} = \left| \frac{M_0}{M_3} \right|$$

$$M_{\text{tot}}^2 = \text{tr} \begin{pmatrix} M_3^2 & m^2 \\ m^2 & M_0^2 \end{pmatrix} = M_3^2 + M_0^2$$

$$\Rightarrow \tan \theta_W = \frac{m}{M_3} = \left| \frac{M_0}{M_3} \right|$$

$$\begin{aligned} M_{\text{tot}}^2 &= \text{tr} \begin{pmatrix} M_3^2 & m^c \\ m^c & M_0^2 \end{pmatrix} = M_3^2 + M_0^2 \\ &= M_3^2 (1 + \tan^2 \theta_W) \\ &= M_3^2 \sec^2 \theta_W \end{aligned}$$

$$M_3 = M_Z \cos \theta_W$$

$$M_H = M_1 = M_Z \cos \theta_W \Leftrightarrow M_1 = M_3 = M_H$$

'Custodial' $SU(2)$ approximate symmetry:

$$\frac{M_W}{M_Z} = \cos \theta_w$$

$$\begin{pmatrix} W_1^T \\ W_2^T \\ W_3^T \\ B \end{pmatrix} \begin{pmatrix} M_W^2 & 0 & 0 & 0 \\ 0 & M_W^2 & 0 & 0 \\ 0 & 0 & M_Z^2 & m^c \\ 0 & 0 & m^c & M_Z^2 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}$$

is the most general mass term
consistent with unbroken $U_{em}(1)$ invariance.

$$\det \begin{pmatrix} M_Z^2 & m^c \\ m^c & M_Z^2 \end{pmatrix} = M_Z^4 - m^{c2} = 0$$

the massive eigenvector

$$\begin{pmatrix} \cos \theta_w \\ -\sin \theta_w \end{pmatrix}$$

$$\text{massless eigenvector} \begin{pmatrix} \sin \theta_w \\ \cos \theta_w \end{pmatrix}$$

"Custodial" $SU(2)$ approximate symmetry:

$$\frac{M_W}{M_Z} = \cos \theta_w$$

$$W_\mu^a \bar{\psi} T_a \psi$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\begin{pmatrix} M_W^2 & 0 \\ 0 & M_W^2 \end{pmatrix} \quad \begin{pmatrix} M_Z^2 & m^2 \\ m^2 & M_Z^2 \end{pmatrix} \quad \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}$$

is the most general mass term consistent with unbroken $U_{em}(1)$ invariance.

the massive eigenvector $\begin{pmatrix} \cos \theta_w \\ -\sin \theta_w \end{pmatrix}$ massless eigenvector $\begin{pmatrix} \sin \theta_w \\ \cos \theta_w \end{pmatrix}$

$$\det \begin{pmatrix} M_Z^2 & m^2 \\ m^2 & M_Z^2 \end{pmatrix} = M_Z^4 - m^4 = 0$$

'Custodial' $SU(2)$ approximate symmetry:

$$\frac{M_W}{M_Z} = \cos \theta_w$$

$$W_{\mu}^a \bar{\psi} T_a \psi$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\begin{pmatrix} W_1 & W_2 & W_3 \\ W_2 & W_3 & B \end{pmatrix} \begin{pmatrix} M_W^2 & 0 & 0 \\ 0 & M_W^2 & 0 \\ 0 & 0 & M_Z^2 \end{pmatrix} \begin{pmatrix} m^2 & m^c \\ m^c & M_Z^2 \end{pmatrix}$$

is the most general mass term consistent with unbroken $U_{em}(1)$ invariance.

the massive eigenvector $\begin{pmatrix} \cos \theta_w \\ -\sin \theta_w \end{pmatrix}$ massless eigenvector $\begin{pmatrix} \sin \theta_w \\ \cos \theta_w \end{pmatrix}$

$$\det \begin{pmatrix} M_Z^2 & m^2 \\ m^2 & M_Z^2 \end{pmatrix} = M_Z^4 - m^4 = 0$$

So $M_W = M_Z \cos \theta_W$ would be a consequence of a symmetry of the form

$$\begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix} \rightarrow \bigcirc \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix}$$

$$O(3) \simeq SU_c(2)$$

This can be promoted to a symmetry of the strong interactions, as well as of the $SU_L(2)$ interactions.

$$W_\mu^a \bar{\psi} T_a \psi$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix}$$

But it is not a symmetry of the $U_V(1)$ couplings
or the Yukawa couplings.

Standard Model $SU(2)$ gauge theory

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$W_\mu^a \bar{\psi} T_a \psi$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow U \begin{pmatrix} u \\ d \end{pmatrix}$$

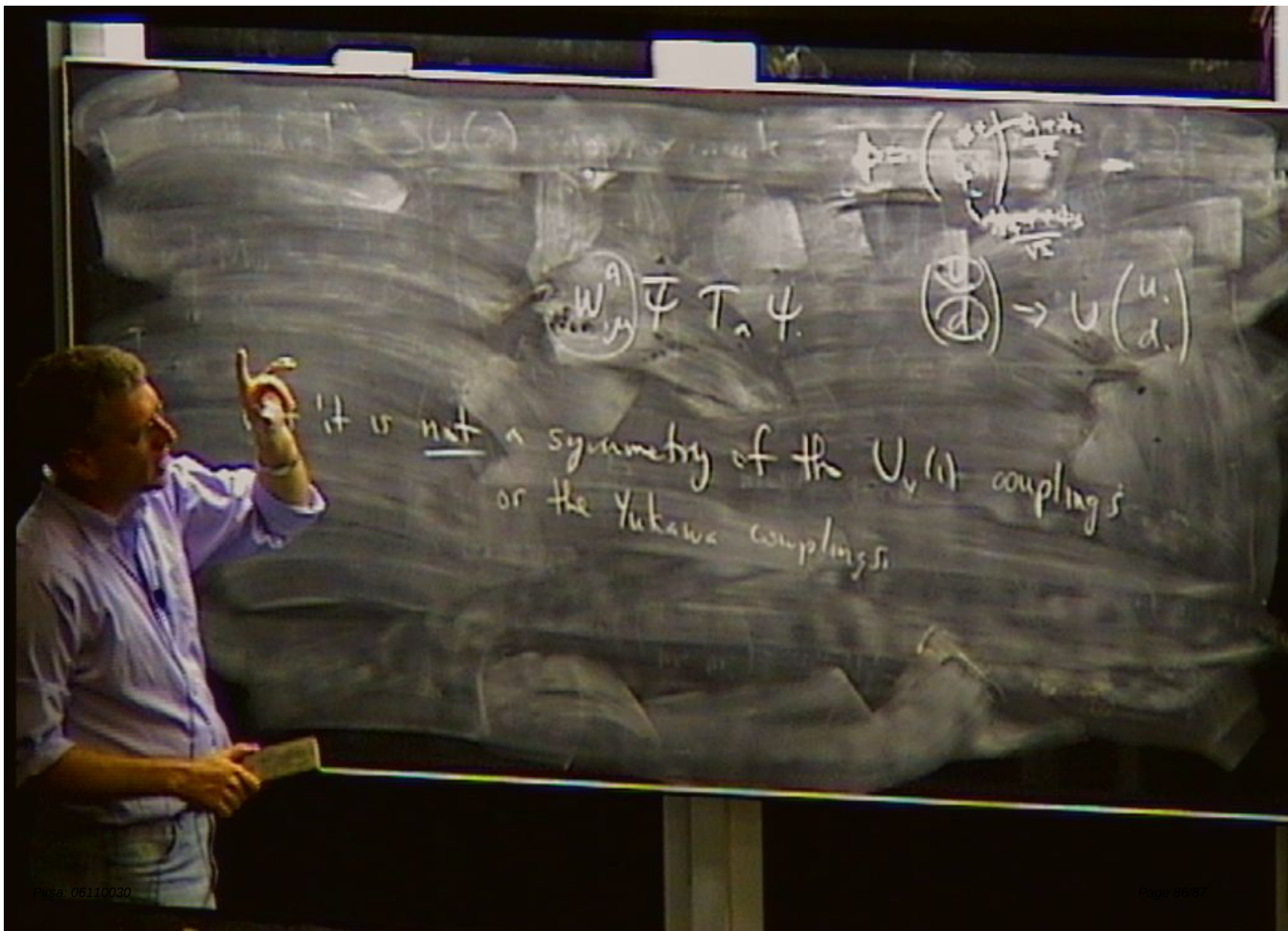
But it is not a symmetry of the $U(1)$ couplings
or the Yukawa couplings

$$\Phi = \begin{pmatrix} \phi \\ \phi^* \end{pmatrix}$$

$$\mathcal{L}_{\text{kin}} = -\partial_\mu \phi^\dagger \partial^\mu \phi = -\frac{1}{2} \partial_\mu \Phi^\dagger \partial^\mu \Phi$$

M_1

$$12 \cos \theta_1 \Leftrightarrow M_1 = M_3 = M_4$$



$$\Phi = \begin{pmatrix} \phi \\ \phi^c \end{pmatrix}$$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 19000 \end{pmatrix}$$

$$\mathcal{L}_{kin} = -\partial_\mu \phi^\dagger \partial^\mu \phi = -\frac{1}{2} \partial_\mu \Phi^\dagger \partial^\mu \Phi$$

the scalar potential = $V = \lambda (\phi^\dagger \phi - v^2/2)^2$

V has an "accidental" = $\lambda [\Phi^\dagger \Phi - v^2/2]^2$

$O(4)$ symmetry