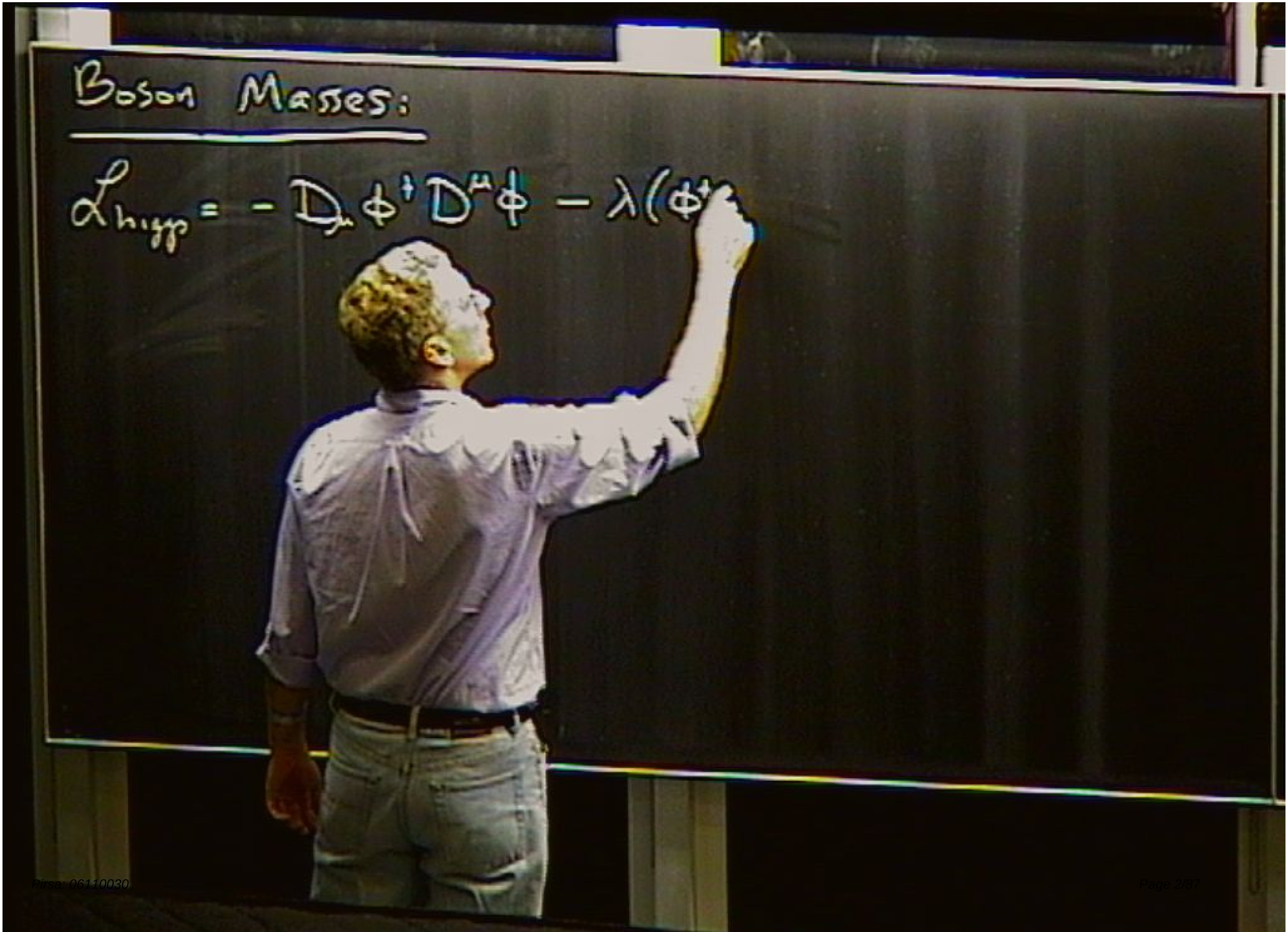
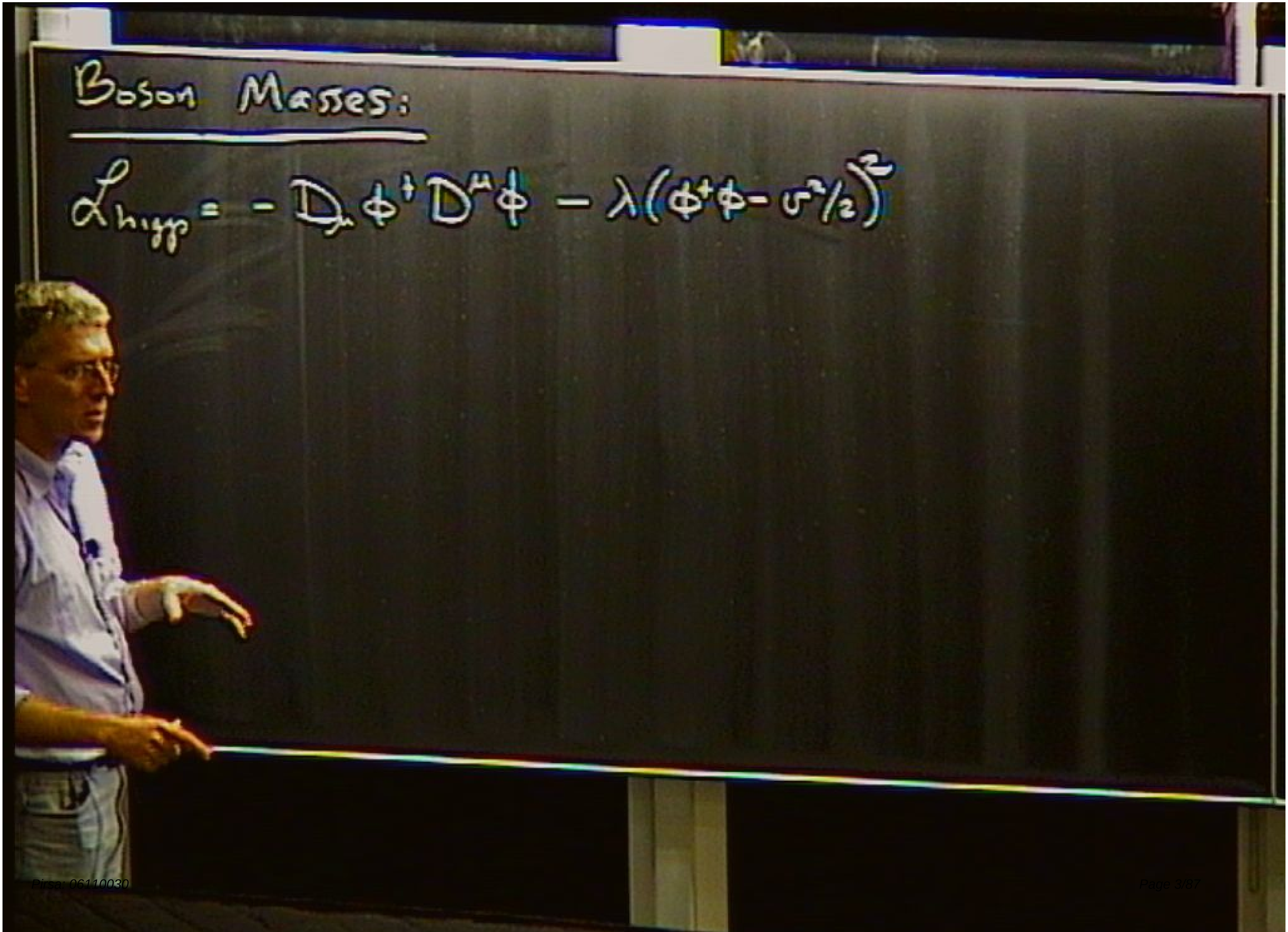
Title: Graduate Course on Standard Model & Quantum Field Theory - 6B

Date: Nov 22, 2006 03:30 AM

URL: http://pirsa.org/06110030

Abstract: Graduate Course on Standard Model & Quantum Field Theory


Boson Masses:
$\mathcal{L}_{higgs} = -D_\mu \phi^\dagger D^\mu \phi - \lambda((\phi^\dagger$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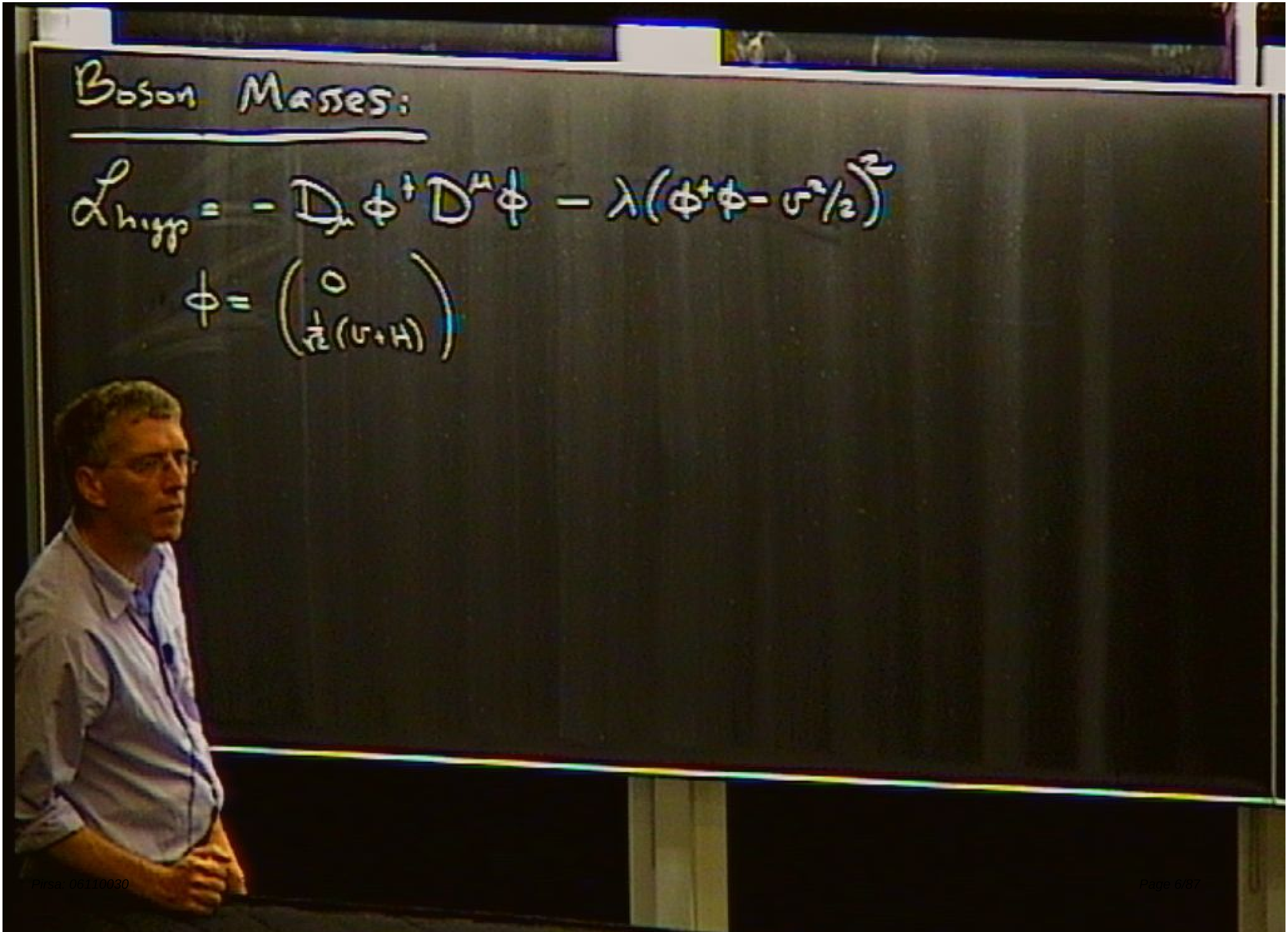
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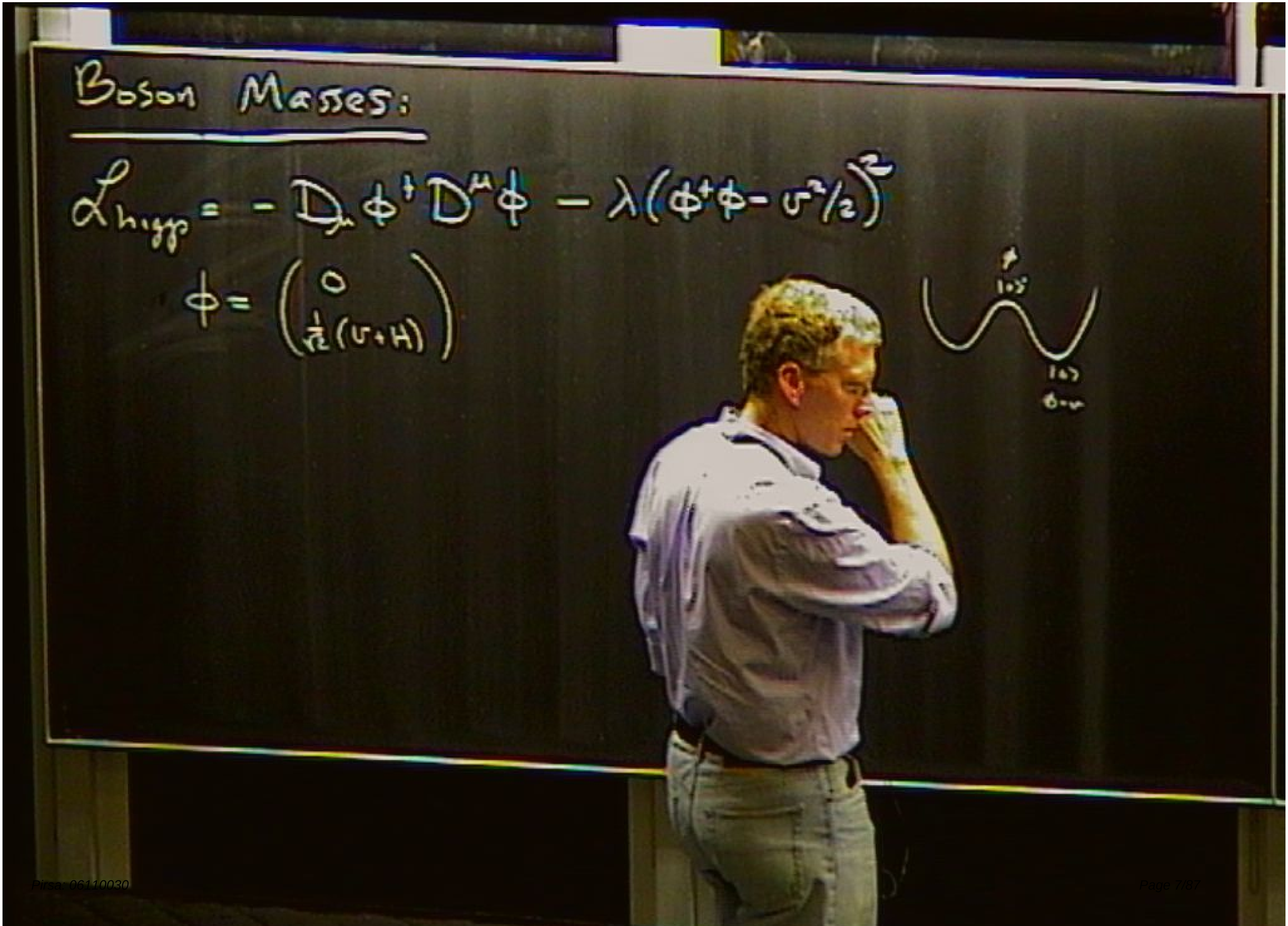
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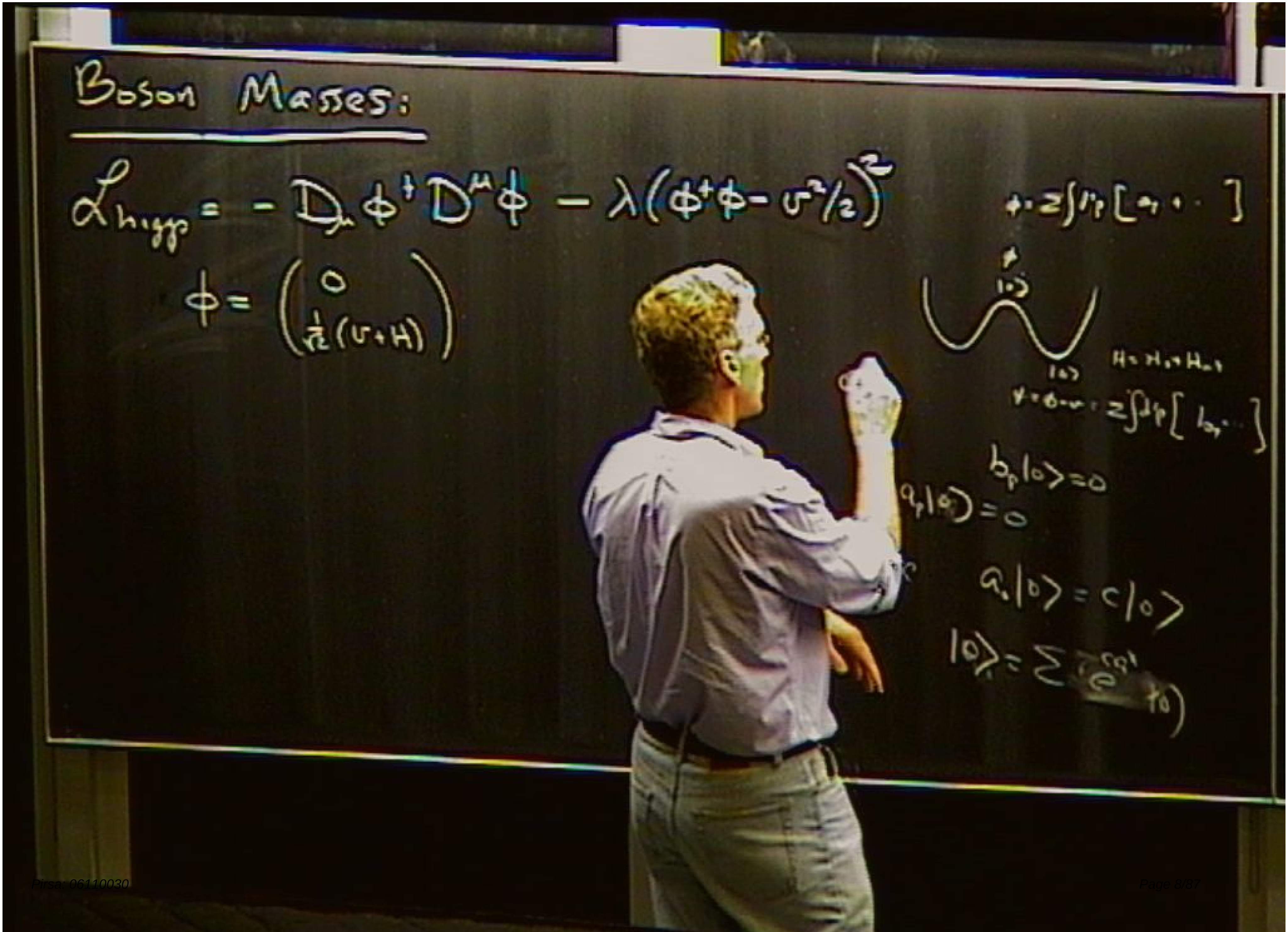
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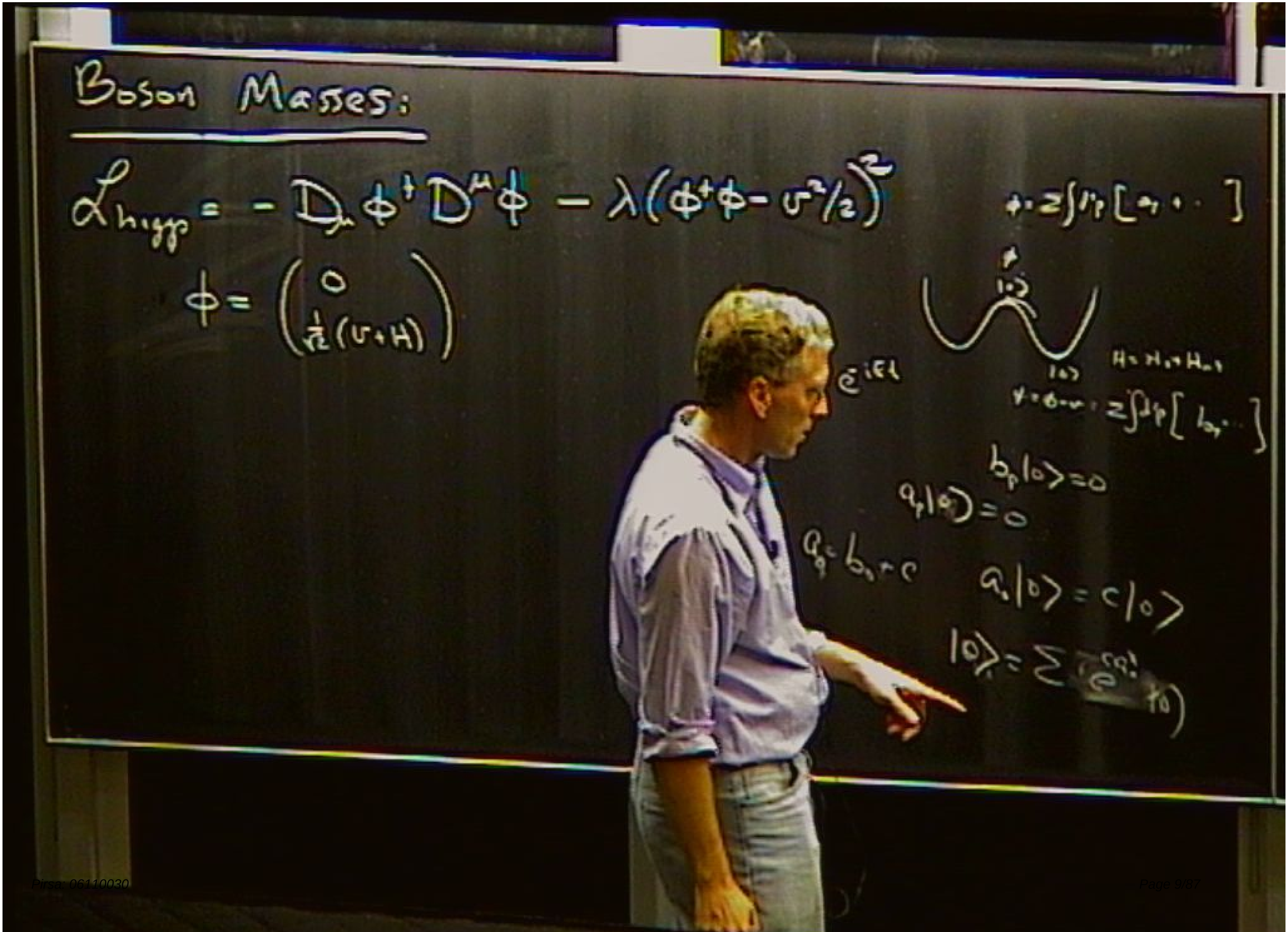
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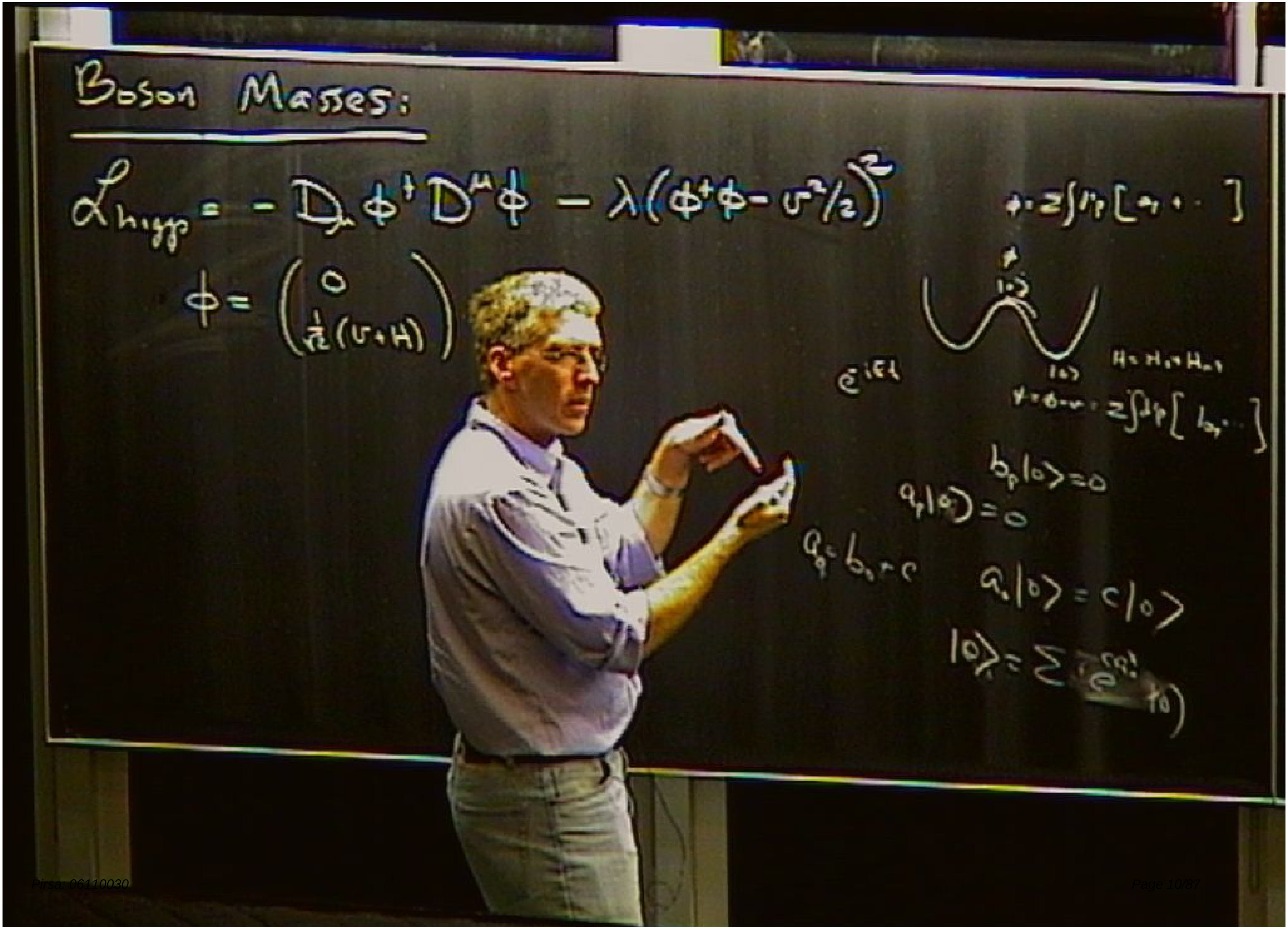
$\phi = \sum\int d^3p\,[a_p + \cdots]$
e^{iEt}
$H = H_0 + H_{int}$
$\psi = \phi - v = \sum\int d^3p\,[b_p \cdots]$
$b_p|0\rangle = 0$
$a_p|0\rangle = 0$
$a_0 = b_0 + c$
$a_0|0\rangle = c|0\rangle$
$|0\rangle = \sum e^{ca^\dagger}|0\rangle$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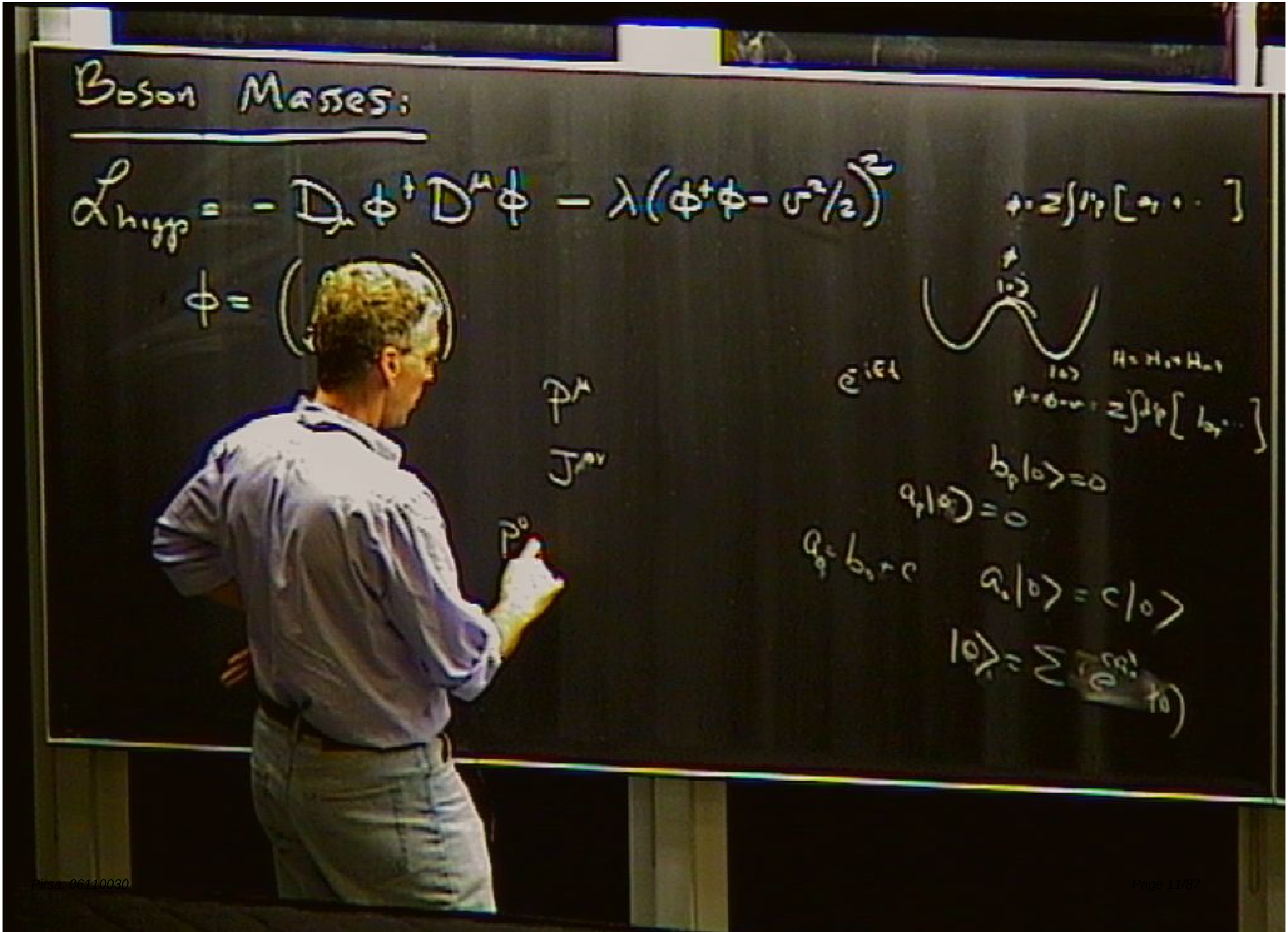
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P^μ
$J^{\mu\nu}$
P^0
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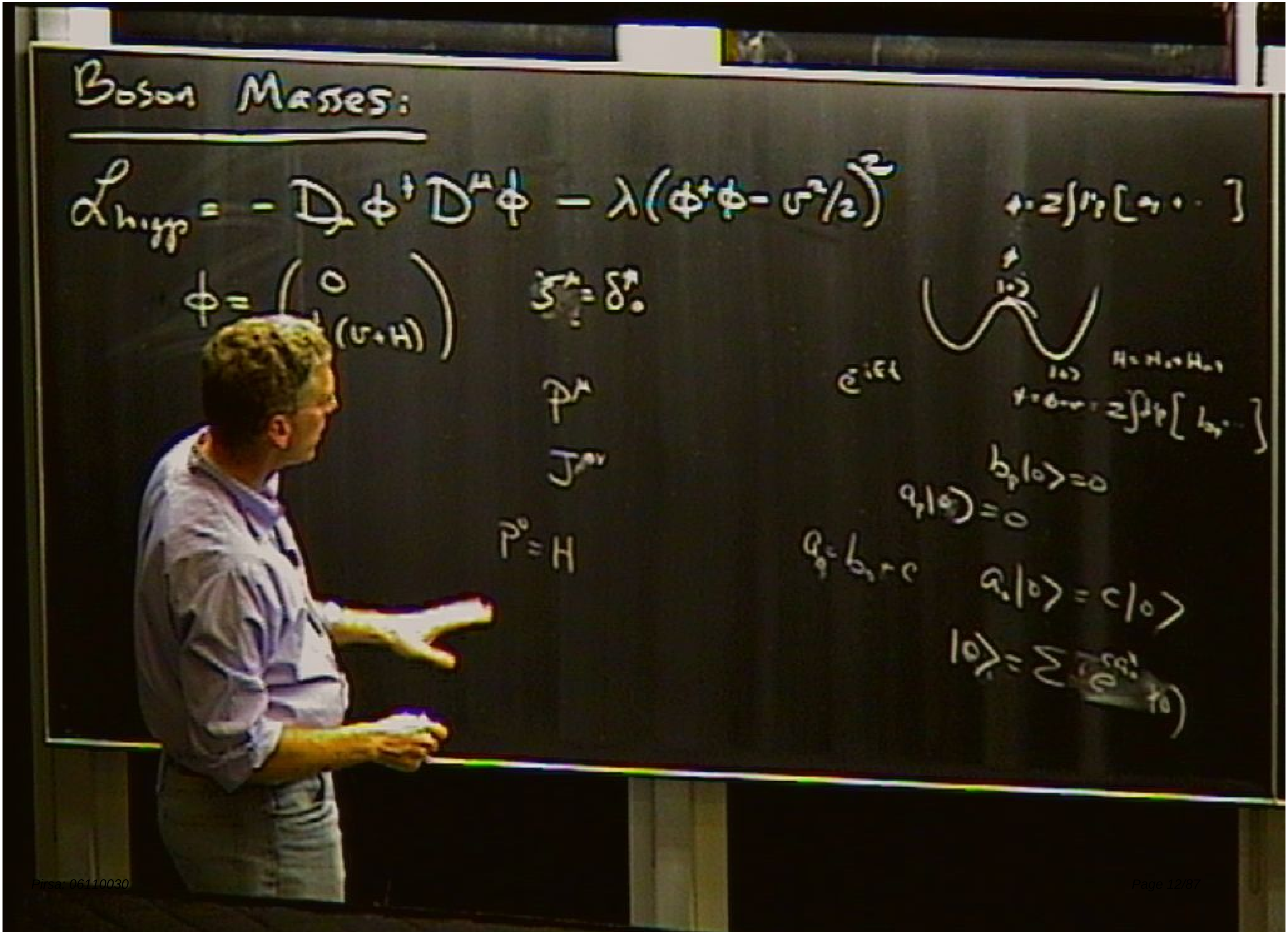
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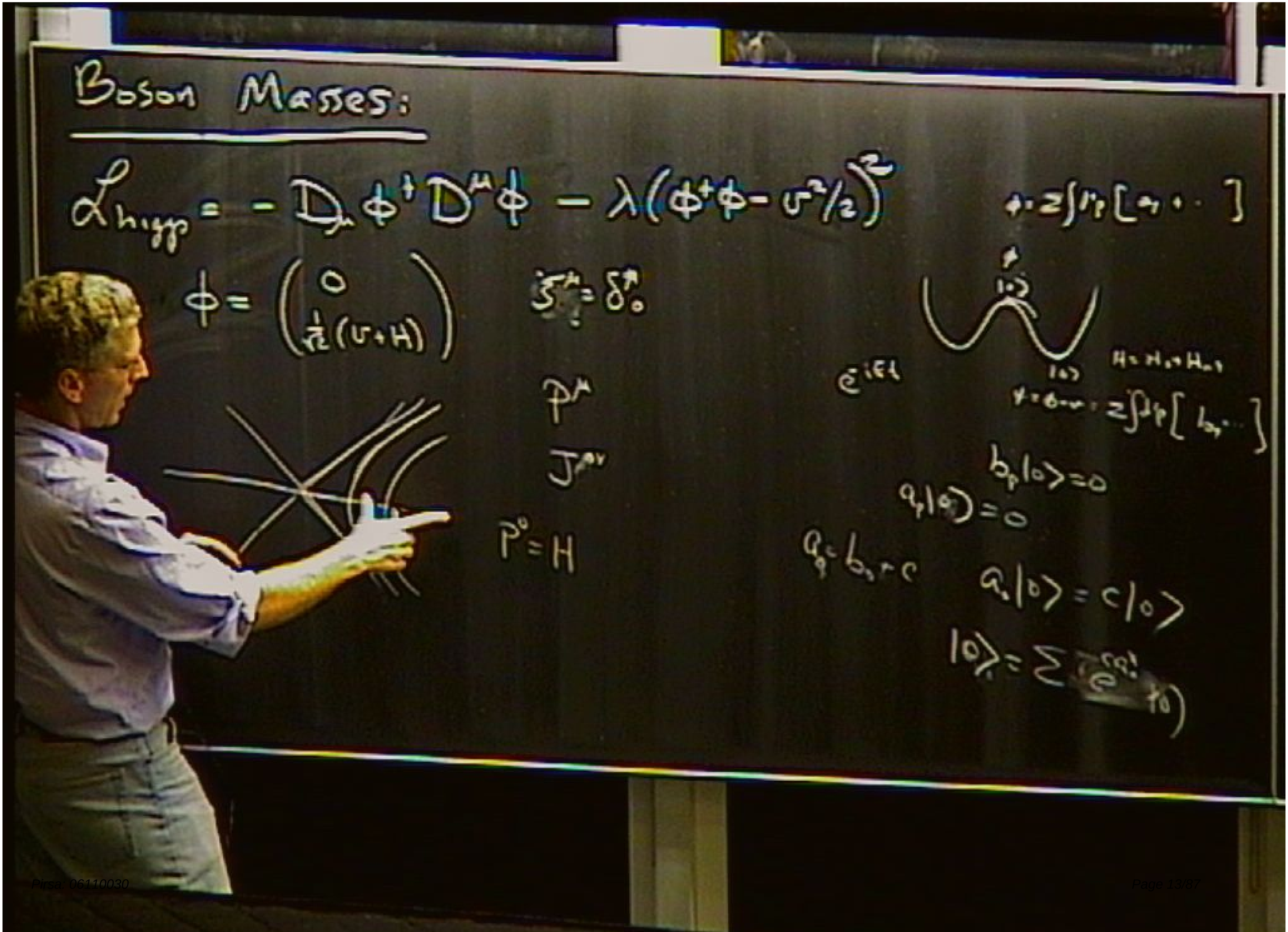
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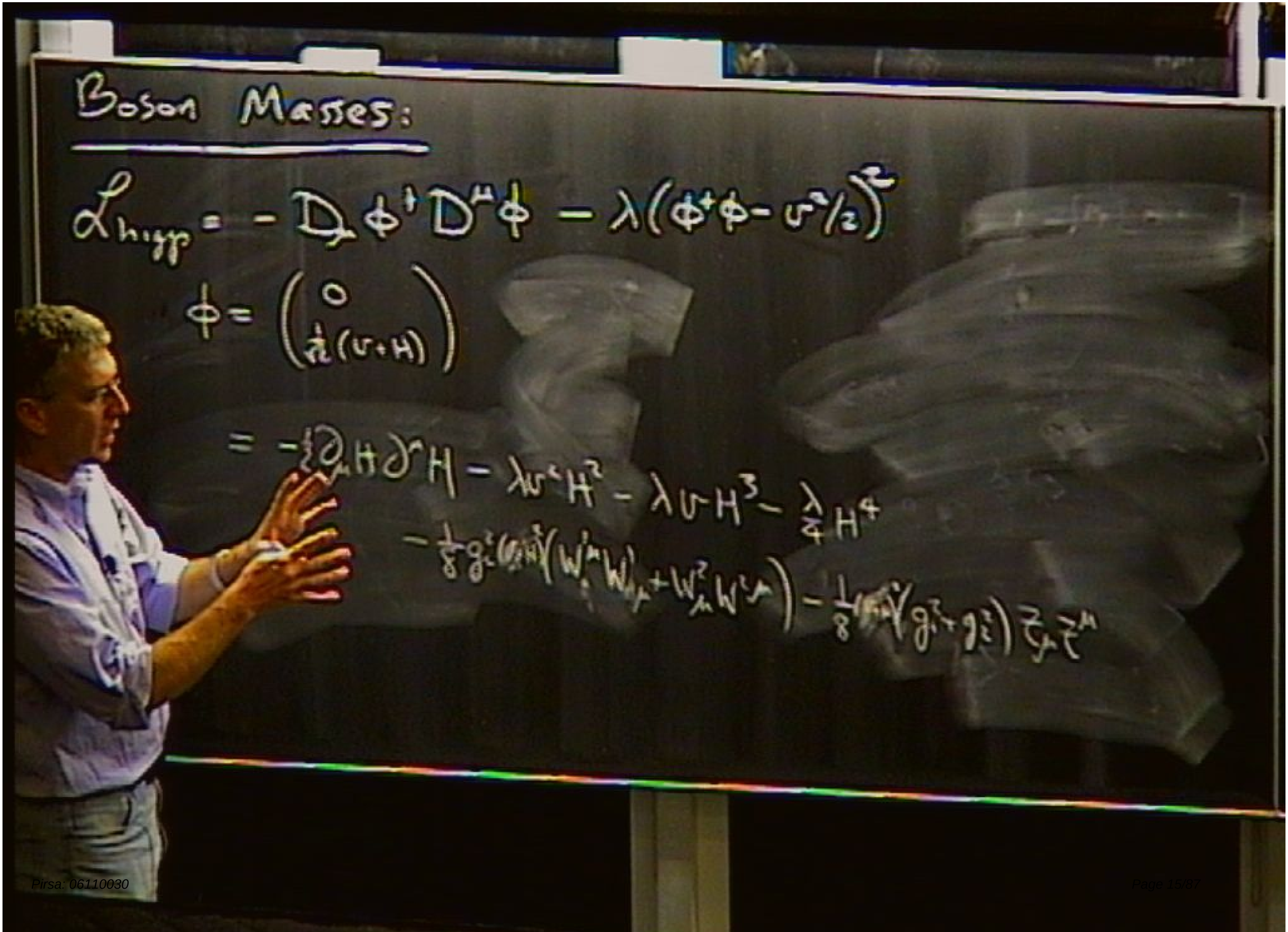
$$= -\tfrac{1}{2}\partial_\mu H \partial^\mu H - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$
$$- \frac{1}{8} g_2^2 (v+H)^2 \left(W_1^\mu W_{1\mu} + W_\mu^2 W^{2\mu}\right) - \frac{1}{8}(v+H)^2\left(g_1^2 + g_2^2\right) Z_\mu Z^\mu$$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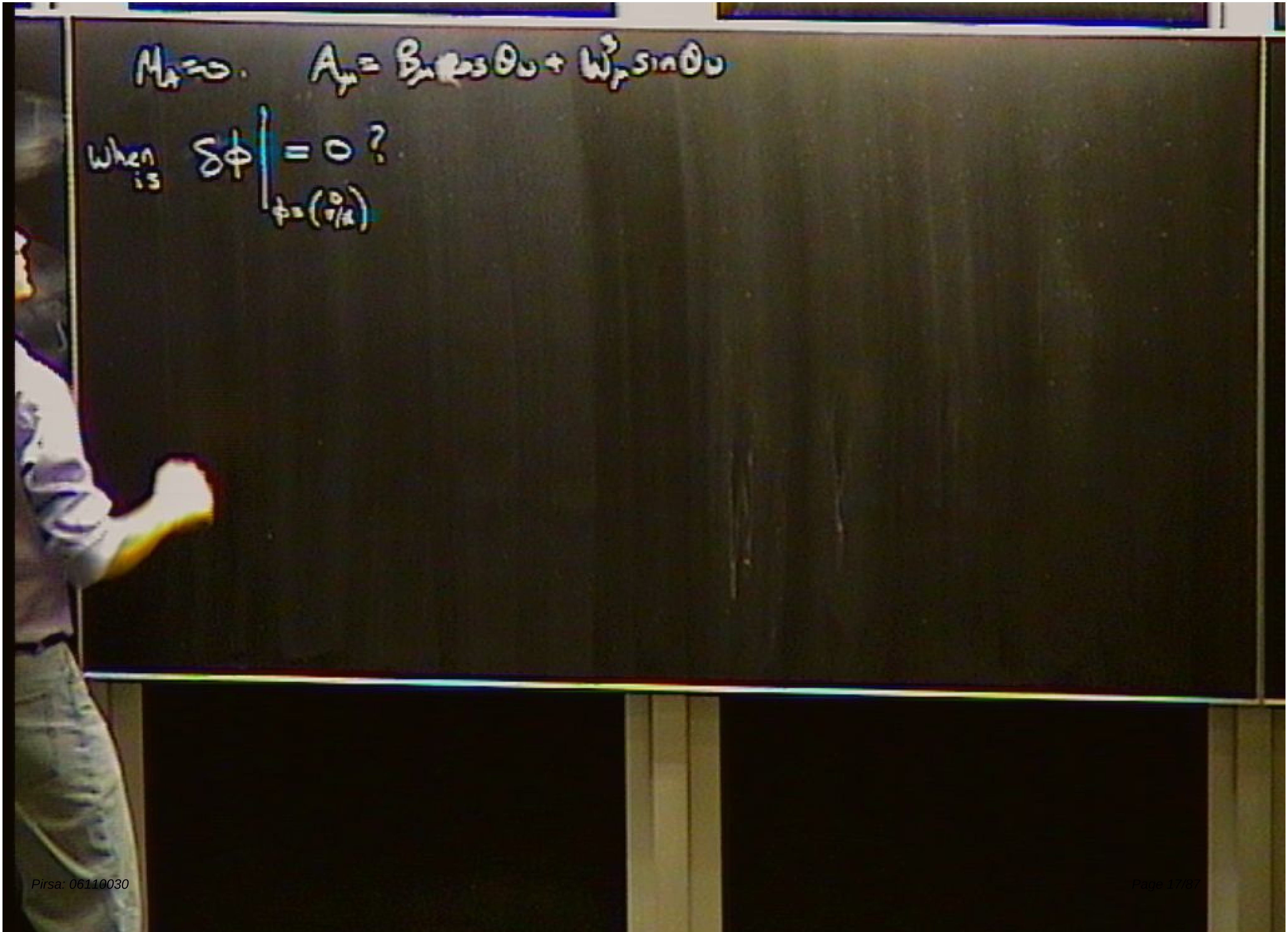
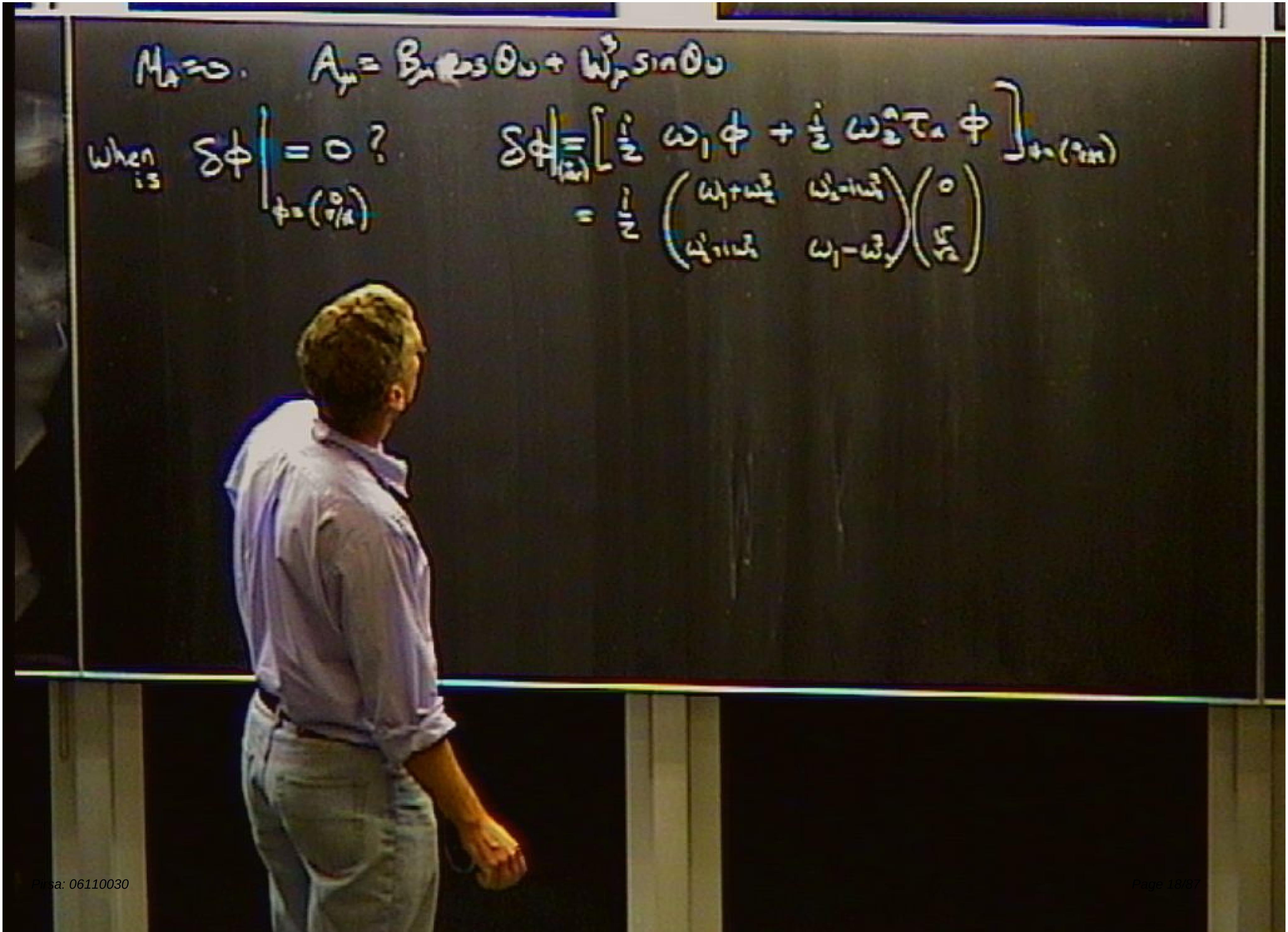

$M_+ \approx 0$. $A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$
When is $\delta\phi \Big|_{\phi=\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}} = 0$?


$M_A = 0$. $A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$
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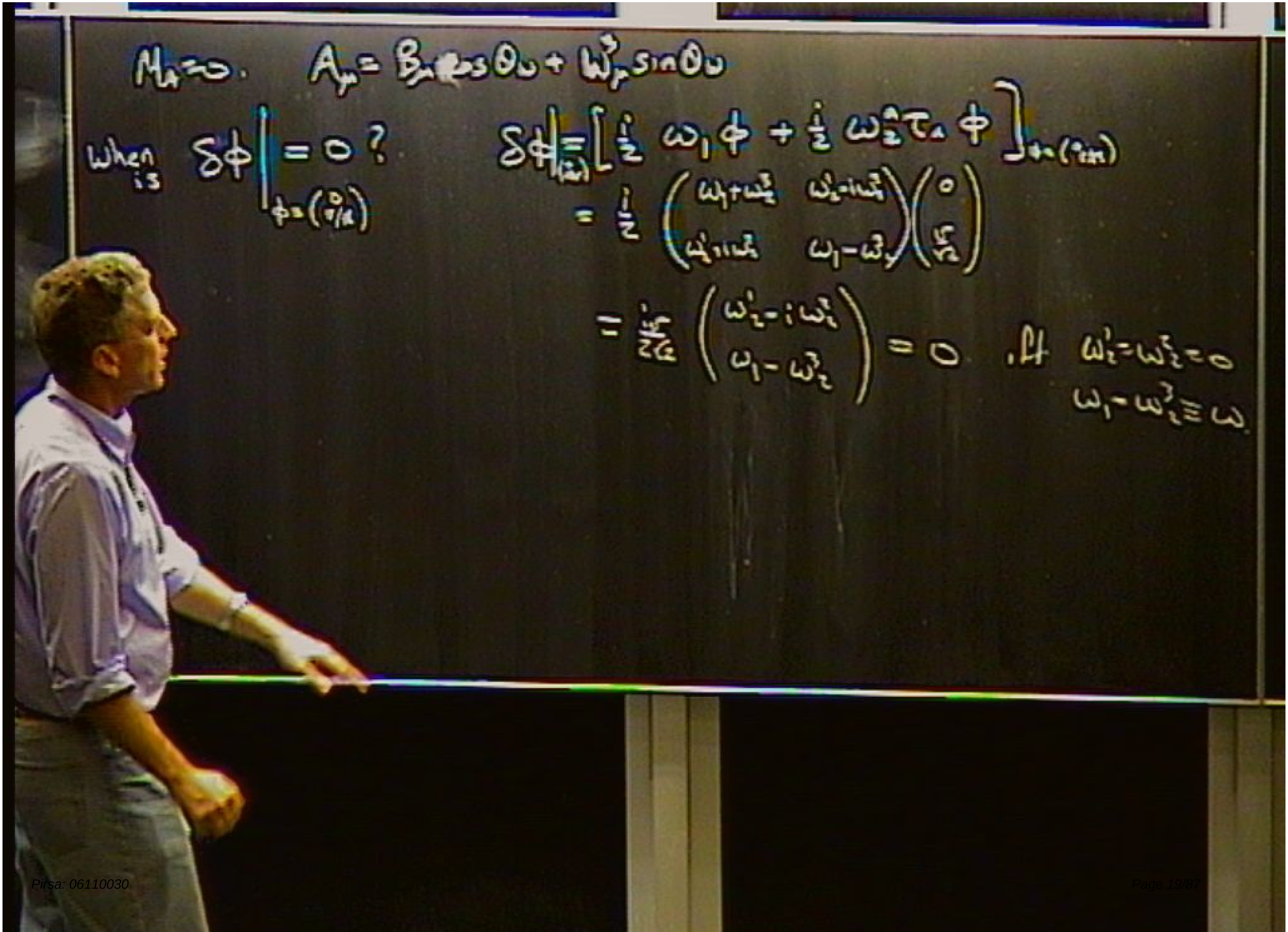
$\delta\phi = \left[\frac{i}{2} \omega_1 \phi + \frac{i}{2} \omega_a \tau_a \phi \right]$
$= \frac{i}{2} \begin{pmatrix} \omega_1 + \omega_3 & \omega_1 - i\omega_2 \\ \omega_1 + i\omega_2 & \omega_1 - \omega_3 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$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When is $\delta\phi\big|_{\phi=\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}} = 0$?
$\delta\phi\big|_{\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}} = \left[\frac{i}{2}\omega_1\phi + \frac{i}{2}\omega_2^a\tau_a\phi\right]_{\phi=\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}}$
$= \frac{i}{2}\begin{pmatrix}\omega_1+\omega_2^3 & \omega_2^1-i\omega_2^2\\ \omega_2^1+i\omega_2^2 & \omega_1-\omega_2^3\end{pmatrix}\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix}$
$= \frac{iv}{2\sqrt{2}}\begin{pmatrix}\omega_2^1-i\omega_2^2\\ \omega_1-\omega_2^3\end{pmatrix} = 0$ iff $\omega_2^1=\omega_2^2=0$
$\omega_1-\omega_2^3 \equiv \omega$

$M_H = \infty$. $\quad A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$

When is $\delta\phi\Big|_{\phi=\binom{0}{v/\sqrt{2}}} = 0$?

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Thus symmetry of the vacuum is $U_{em}(1)$ or $U_Q(1)$

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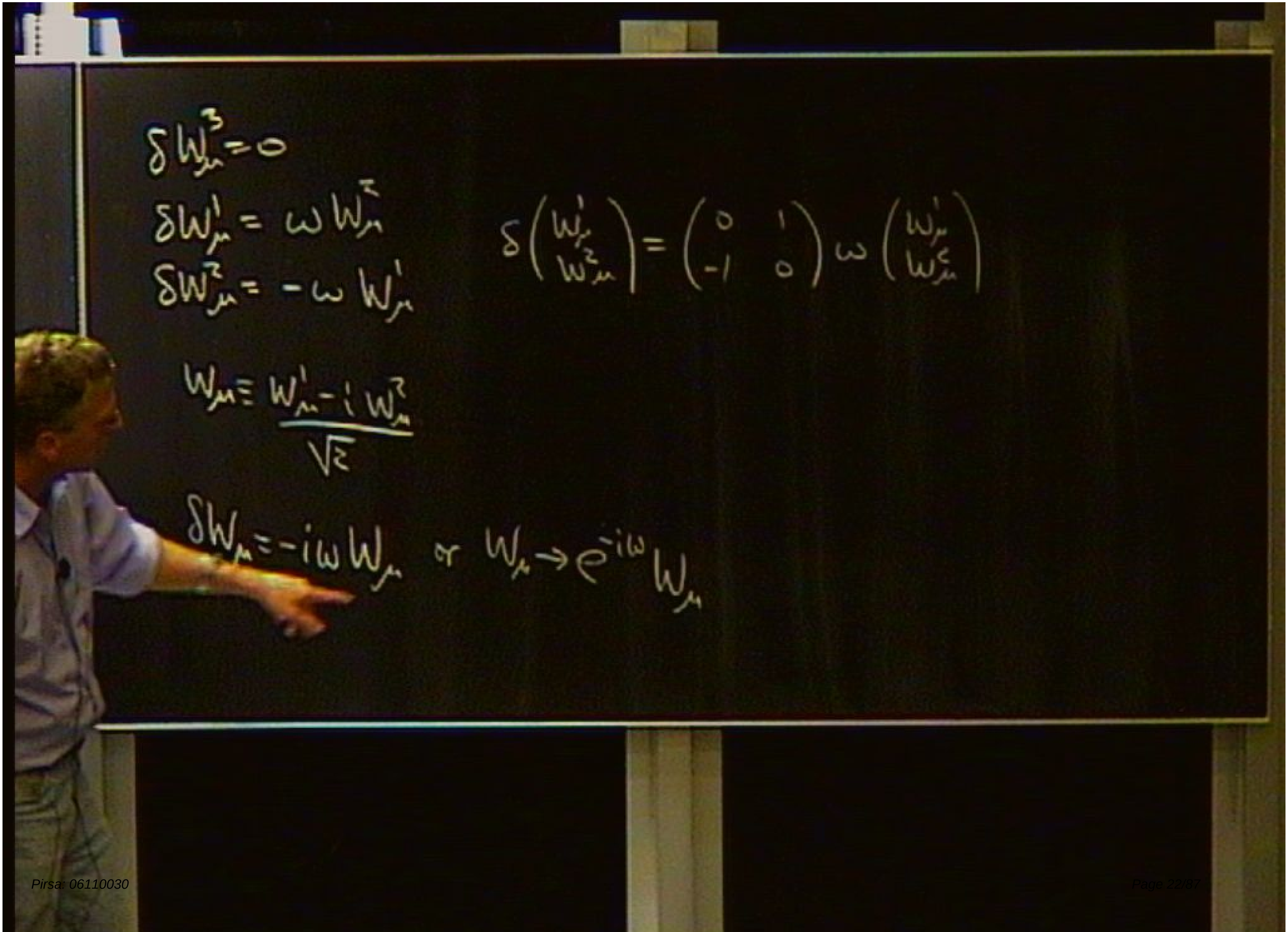
$$= -\epsilon^{a3c}\,\omega W^c_\mu$$



$\delta W^3_\mu = 0$
$\delta W^1_\mu = \omega W^2_\mu$
$\delta W^2_\mu = -\omega W^1_\mu$
$\delta \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \omega \begin{pmatrix} W^1_\mu \\ W^2_\mu \end{pmatrix}$
$W_\mu \equiv \frac{W^1_\mu - i W^2_\mu}{\sqrt{2}}$
$\delta W_\mu = -i\omega W_\mu$ or $W_\mu \to e^{-i\omega} W_\mu$

$$\delta W_\mu^3 = 0$$

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$= 0$

$= \omega$

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$$D_\mu\phi = \partial_\mu\phi - \frac{i}{2}g_2 A^a_\mu\tau_a\phi$$

$$= \frac{iv}{2\sqrt{2}}\begin{pmatrix}\omega_2^1-i\omega_2^2\\ \omega_1-\omega_2^3\end{pmatrix} = 0 \quad \text{iff} \quad \omega_2^1=\omega_2^2=0,\ \ \omega_1=\omega_2^3\equiv\omega$$

Thus symmetry of the vacuum is $U_{em}(1)$ or $U_Q(1)$

$$\delta W^a_\mu = \cancel{\partial_\mu\omega^a} - \epsilon^{abc}\,\omega_2^b W^c_\mu \quad \text{if } \partial_\mu\omega_2 = 0$$

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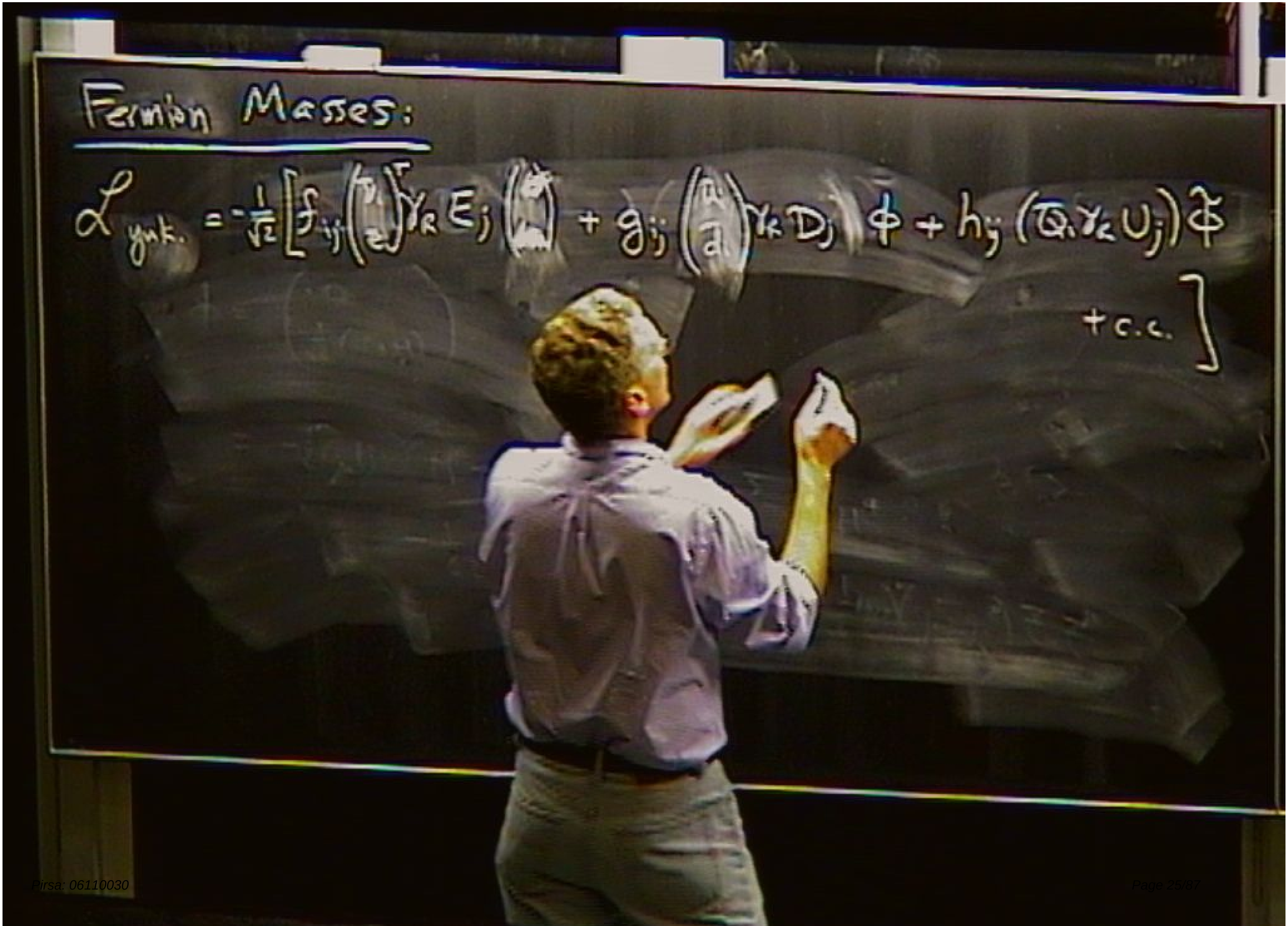
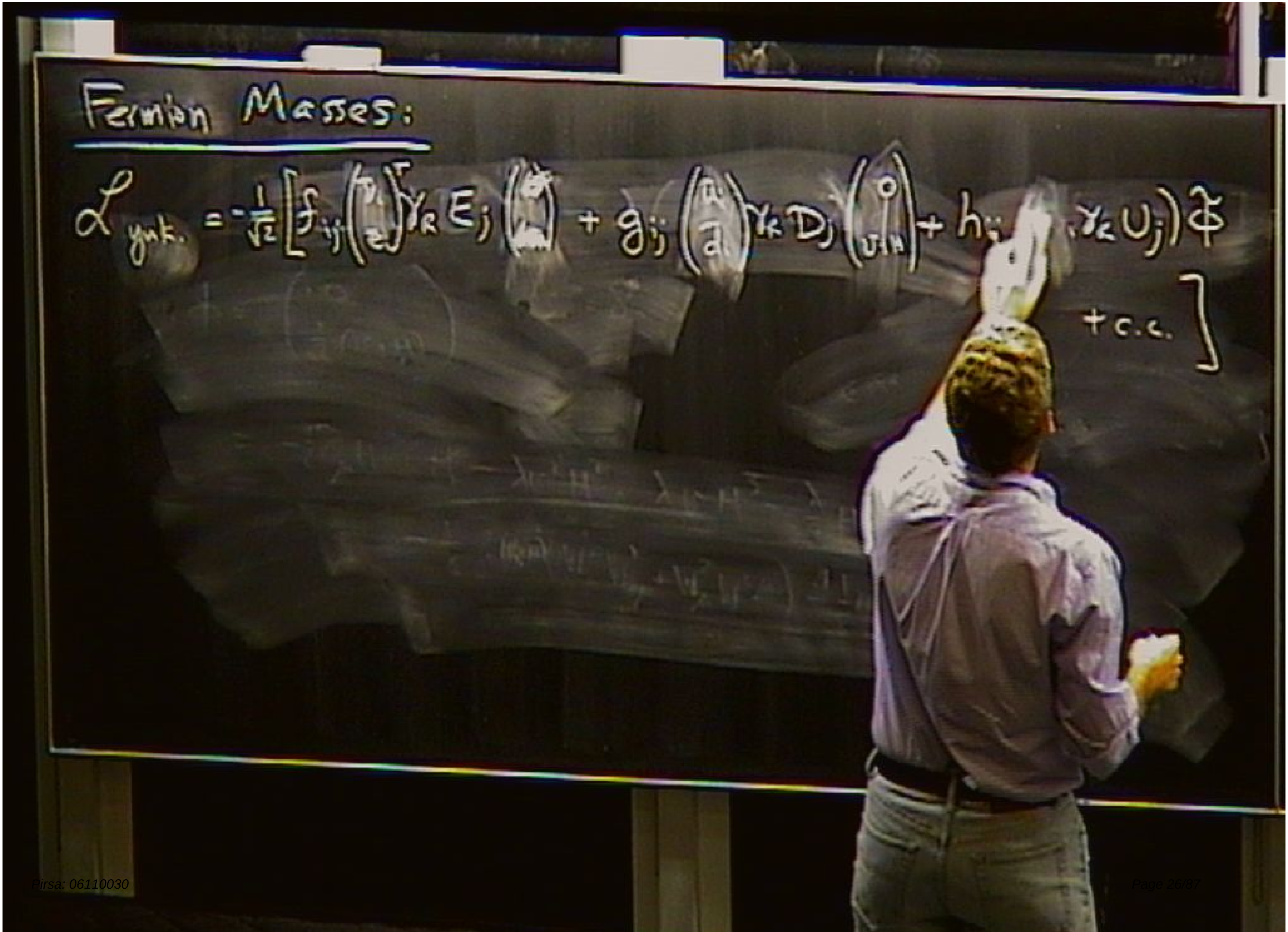

Fermion Masses:
$\mathcal{L}_{yuk.} = -\frac{1}{\sqrt{2}}\left[f_{ij} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}^T \gamma_R E_j \quad + g_{ij} \begin{pmatrix} u_i \\ d_i \end{pmatrix} \gamma_R D_j \; \phi + h_{ij} \left(\bar{Q}_i \gamma_R U_j\right) \tilde{\phi} \right.$
$\left. + c.c. \right]$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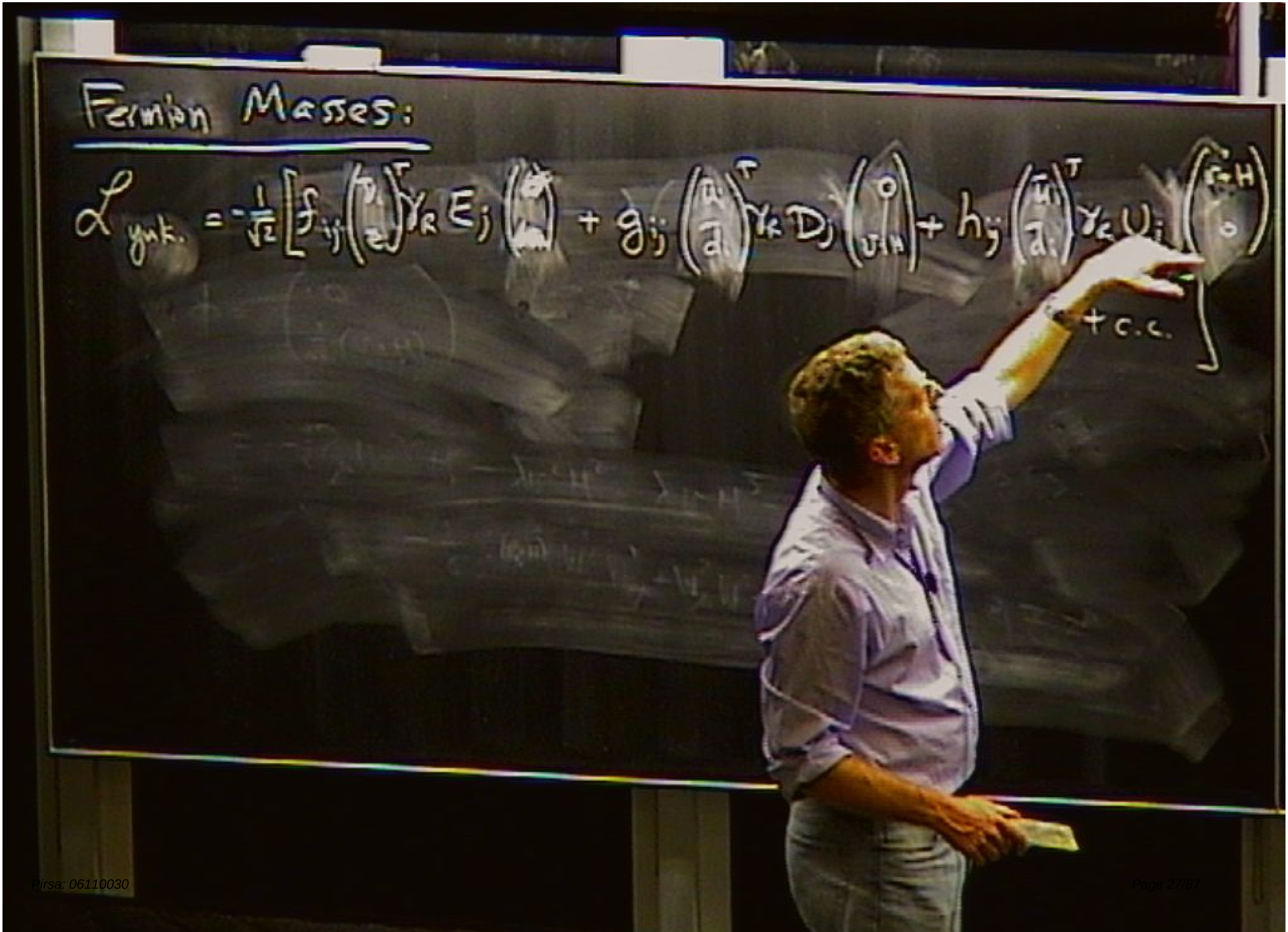
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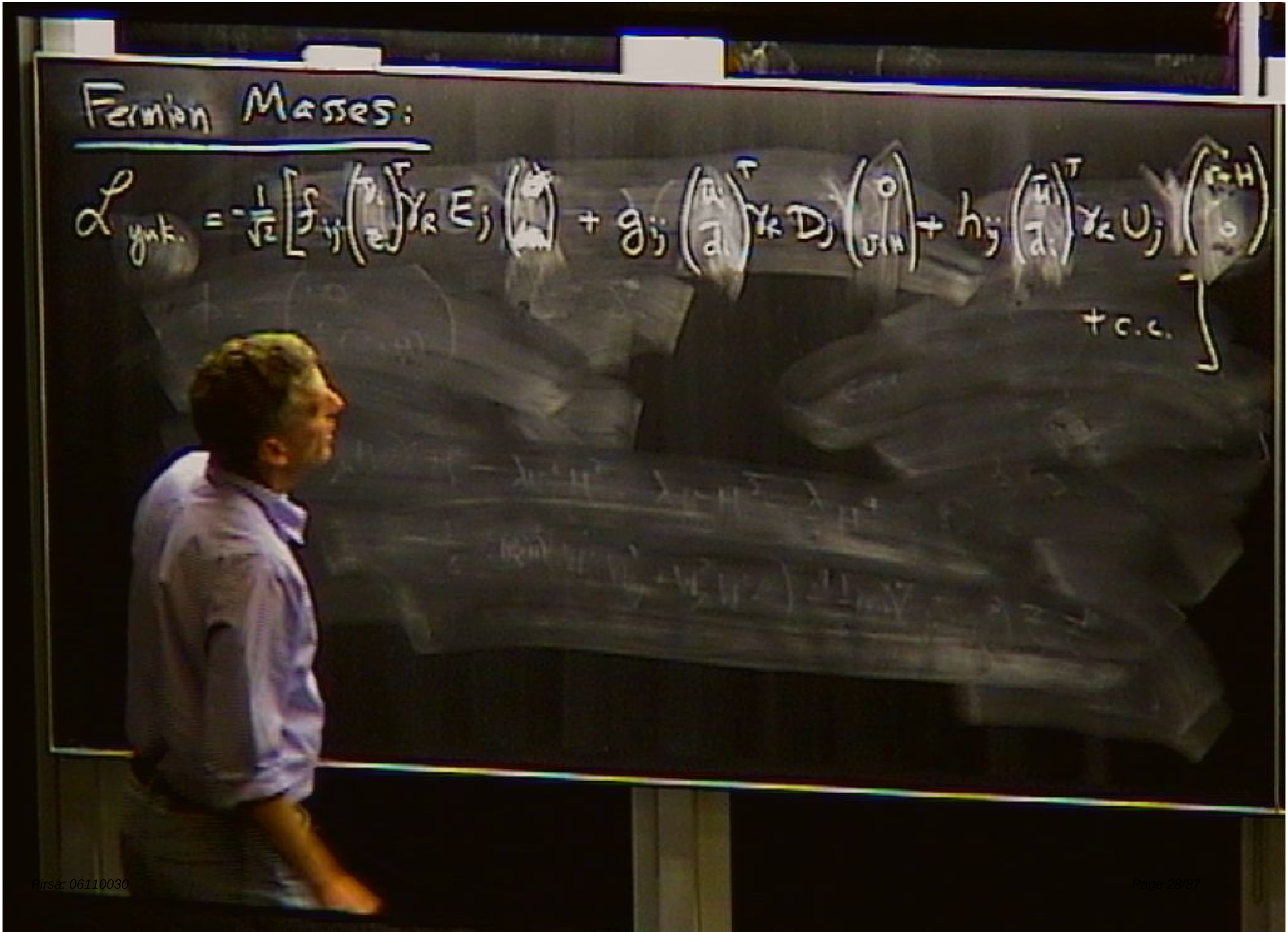
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$$\mathcal{L}_{yuk.} = -\frac{1}{\sqrt{2}}\Big[f_{ij} \begin{pmatrix}\bar{\nu}_i \\ \bar{e}_i\end{pmatrix}^T \gamma_R E_j \begin{pmatrix} \text{[illegible]} \end{pmatrix} + g_{ij} \begin{pmatrix}\bar{u}_i \\ \bar{d}_i\end{pmatrix}^T \gamma_R D_j \begin{pmatrix}0 \\ v+H\end{pmatrix} + h_{ij} \begin{pmatrix}\bar{u}_i \\ \bar{d}_i\end{pmatrix}^T \gamma_R U_j \begin{pmatrix}v+H \\ 0\end{pmatrix} + c.c. \Big]$$

$$= -\Big[\frac{1}{\sqrt{2}} f_{ij} (\bar{e}_i \gamma_R E_j) + g_{ij} (\bar{d}_i \gamma_R D_j) + h_{ij} (\bar{u}_i \gamma_R U_j) \Big] (v+H) + c.c.$$

$Q = T_3 + Y$

$M_A = 0.$ $\quad A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$

When is $\delta\phi\Big|_{\phi=\binom{0}{v/\sqrt2}} = 0$?

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$$D_\mu\phi = \partial_\mu\phi - \frac{i}{2}g_2 A^a_\mu\tau_a\phi$$

$$= \frac{iv}{2\sqrt2}\begin{pmatrix}\omega_2^1-i\omega_2^2\\ \omega_1-\omega_2^3\end{pmatrix} = 0 \quad \text{iff} \quad \omega_2^1=\omega_2^2=0$$

$$\omega_1=\omega_2^3\equiv\omega$$

This symmetry of the vacuum is $U_{em}(1)$ or $U_Q(1)$

$$\delta W^a_\mu = \cancel{\partial_\mu\omega^a} - \epsilon^{abc}\omega_2^b W^c_\mu \quad \text{if } \partial_\mu\omega_2=0$$

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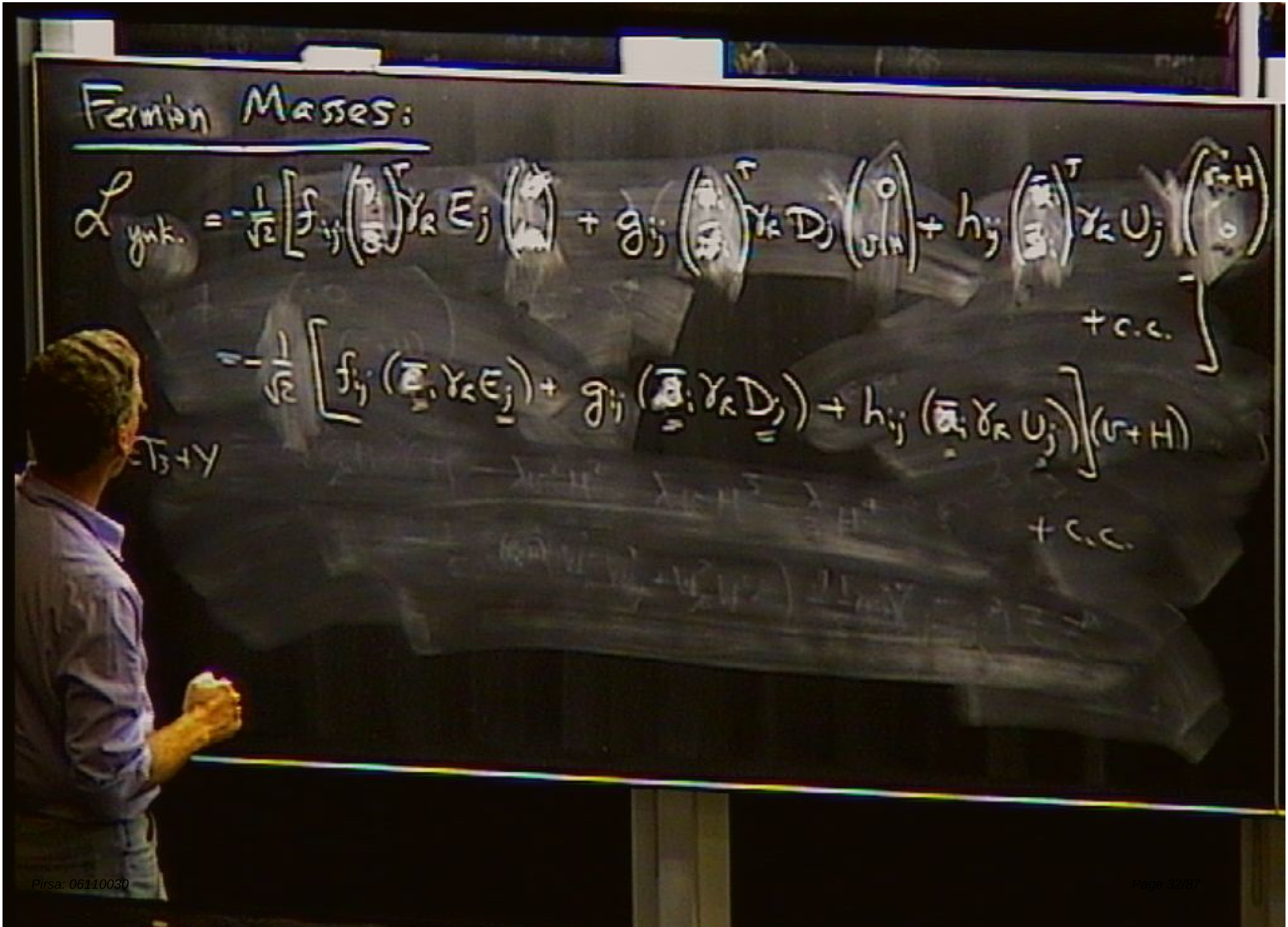
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$+ c.c. \Big]$
$= -\frac{1}{\sqrt{2}} \Big[f_{ij} (\bar{e}_i \gamma_R E_j) + g_{ij} (\bar{d}_i \gamma_R D_j) + h_{ij} (\bar{u}_i \gamma_R U_j) \Big] (v+H)$
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$$= -\frac{1}{\sqrt{2}}\Big[f_{ij}(\bar{e}_i \gamma_R E_j) + g_{ij}(\bar{d}_i \gamma_R D_j) + h_{ij}(\bar{u}_i \gamma_R U_j)\Big](v+H) + c.c.$$

$Q = T_3 + Y$

Conservation of Q makes it convenient to use Dirac spinors for charged leptons + quarks.

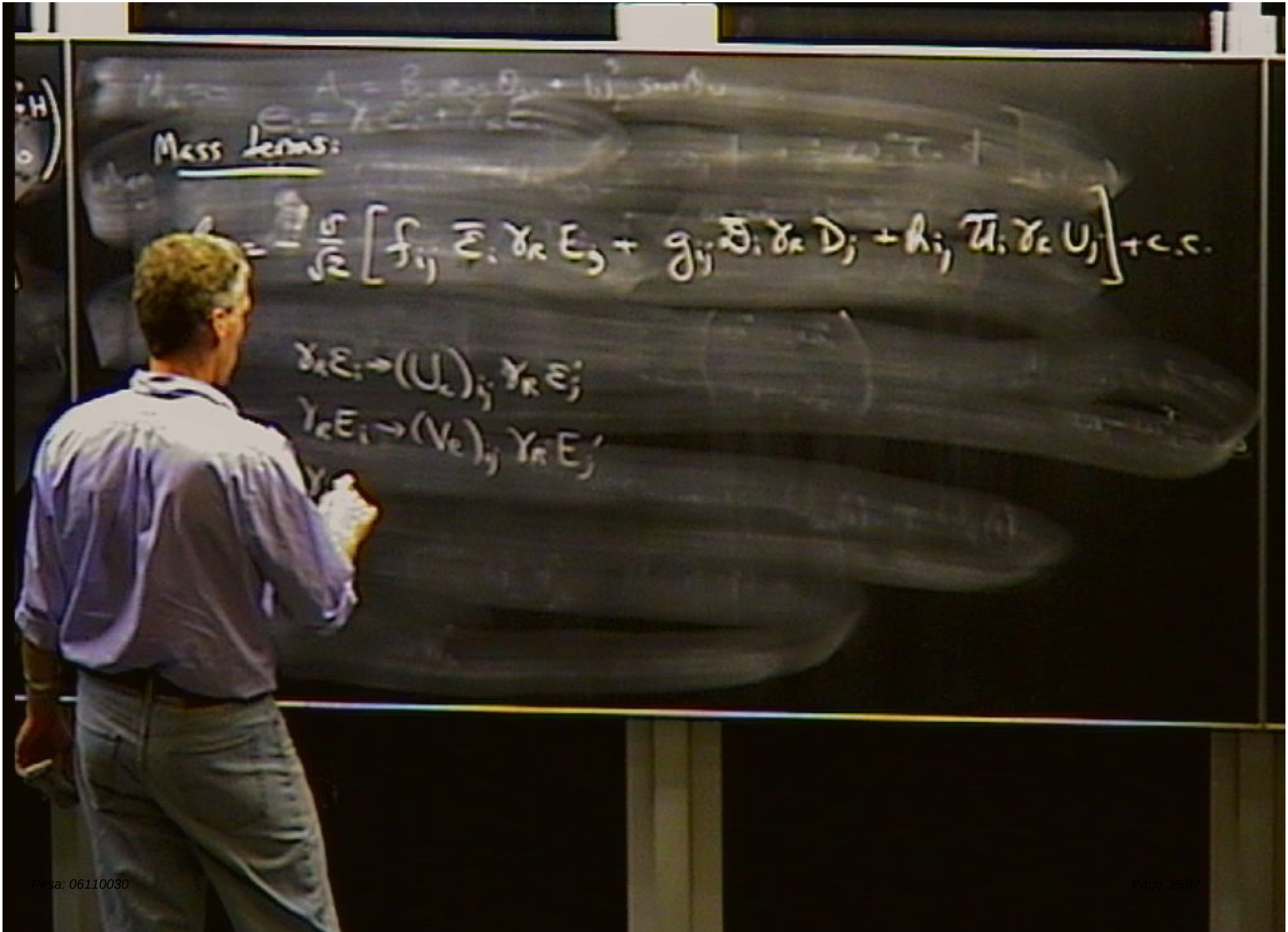
$$e_i = \gamma_L \mathcal{E}_i + \gamma_R E_i$$

$$u_i = \gamma_L \mathcal{U}_i + \gamma_R U_i$$

$$d_i = \gamma_L \mathcal{D}_i + \gamma_R D_i$$

Mass terms:

$$\mathcal{L}_{fm} = -\frac{v}{\sqrt{2}}\left[f_{ij}\,\overline{E}_i \gamma_R E_j + g_{ij}\,\overline{D}_i \gamma_R D_j + h_{ij}\,\overline{U}_i \gamma_R U_j \right] + c.c.$$


Mass terms:
$= -\frac{v}{\sqrt{2}}\left[f_{ij}\,\bar{E}_i\,\gamma_R\,E_j + g_{ij}\,\bar{D}_i\,\gamma_R\,D_j + h_{ij}\,\bar{U}_i\,\gamma_R\,U_j\right] + c.c.$
$\gamma_R E_i \to (U_L)_{ij}\,\gamma_R E_j$
$\gamma_R E_i \to (V_R)_{ij}\,\gamma_R E_j$

Mass terms:

$$\mathcal{L}_{fm} = -\frac{v}{\sqrt{2}}\left[f_{ij}\,\bar{\mathcal{E}}_i\,\gamma_R\,E_j + g_{ij}\,\bar{\mathcal{D}}_i\,\gamma_R\,D_j + h_{ij}\,\bar{\mathcal{U}}_i\,\gamma_R\,U_j \right] + c.c.$$

$$\gamma_R\,\mathcal{E}_i \to (U_e)_{ij}\,\gamma_R\,\mathcal{E}_j' \qquad f_{ij} \to (U_e^T f\, V_e)_{ij}$$

$$\gamma_R\,E_i \to (V_e)_{ij}\,\gamma_R\,E_j' \qquad g_{ij} \to (U_d^T g\, V_d)_{ij}$$

$$\gamma_R\,\mathcal{U}_i \to (U_u)_{ij}\,\gamma_R\,\mathcal{U}_j' \qquad h_{ij} \to (U_u^T h\, V_u)_{ij}$$

$\vdots$

etc

Mass terms:

$$\mathcal{L}_{fm} = -\frac{v}{\sqrt{2}}\left[f_{ij}\,\bar{\mathcal{E}}_i\,\gamma_R\,E_j + g_{ij}\,\bar{\mathcal{D}}_i\,\gamma_R\,D_j + h_{ij}\,\bar{\mathcal{U}}_i\,\gamma_R\,U_j\right] + c.c.$$

$$\gamma_R\,\mathcal{E}_i \to (U_e)_{ij}\,\gamma_R\,\mathcal{E}'_j \qquad f_{ij} \to (U_e^T f\, V_e)_{ij}$$

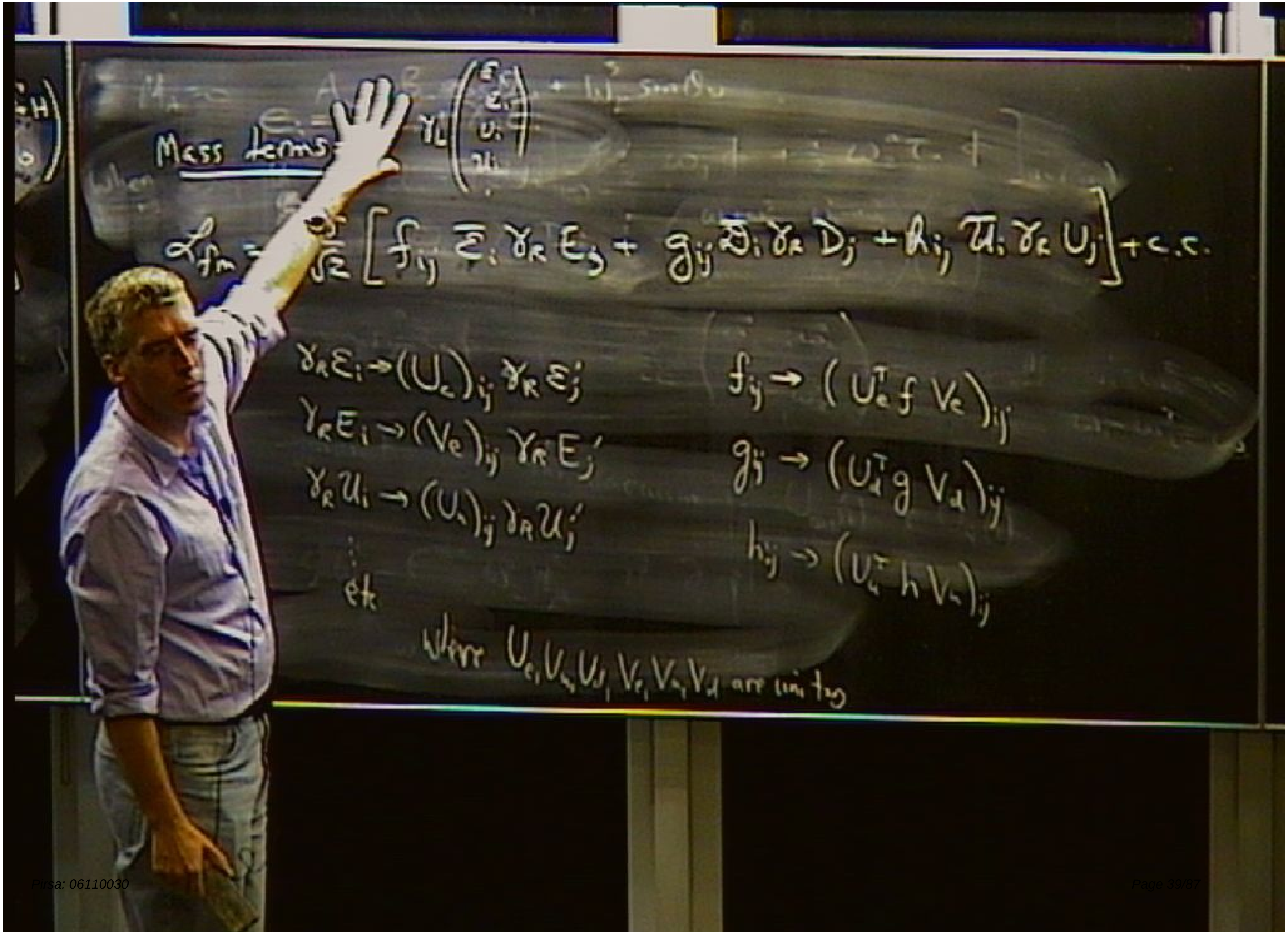
$$\gamma_R\,E_i \to (V_e)_{ij}\,\gamma_R\,E'_j \qquad g_{ij} \to (U_d^T g\, V_d)_{ij}$$

$$\gamma_R\,\mathcal{U}_i \to (U_u)_{ij}\,\gamma_R\,\mathcal{U}'_j \qquad h_{ij} \to (U_u^T h\, V_u)_{ij}$$

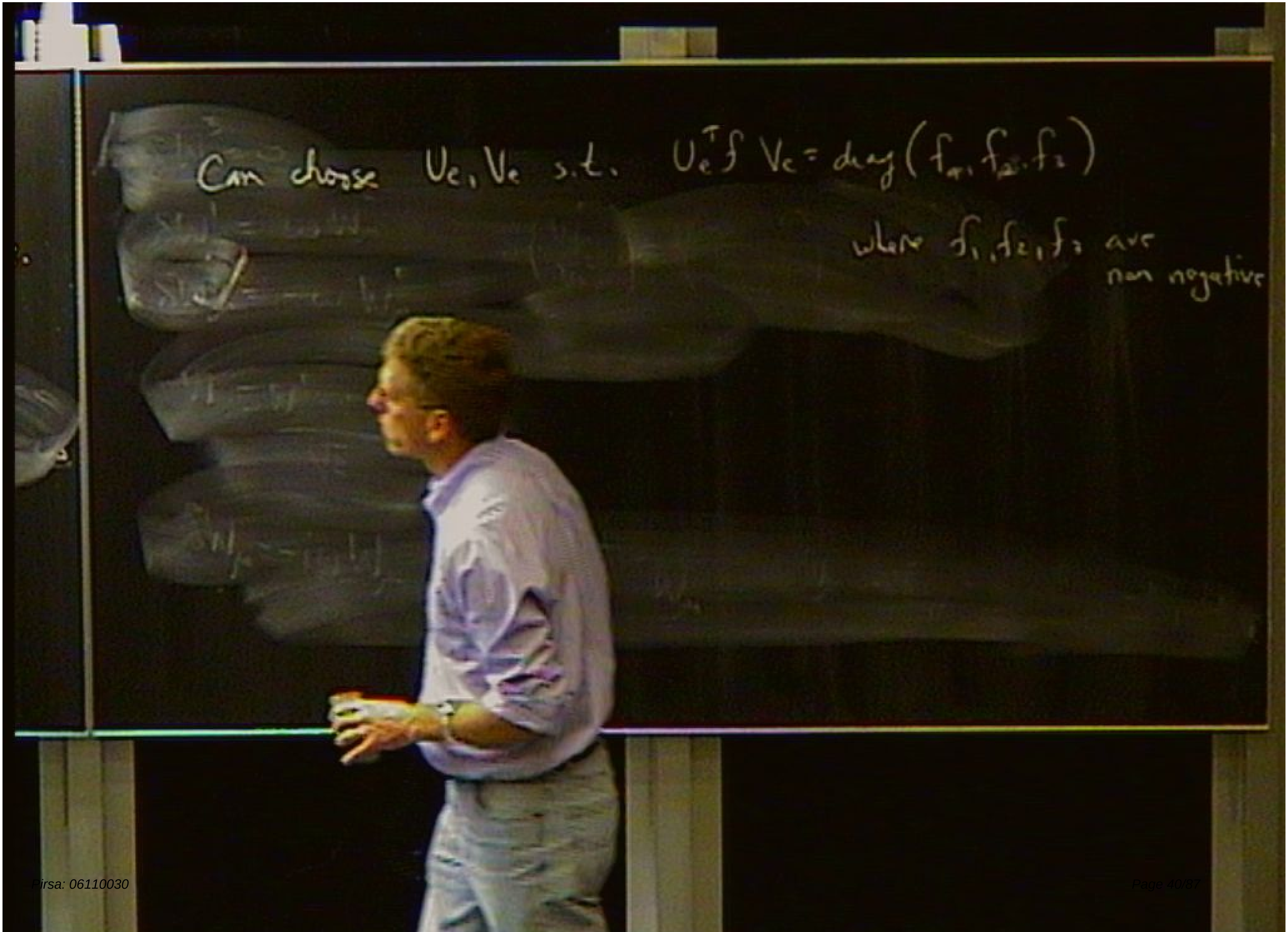
$\vdots$

etc

where $U_e, U_u, U_d, V_e, V_u, V_d$ are unitary



Mass terms
$\mathcal{L}_{fm} = \frac{v}{\sqrt{2}}\left[f_{ij}\, \bar{E}_i \gamma_R E_j + g_{ij}\, \bar{D}_i \gamma_R D_j + h_{ij}\, \bar{U}_i \gamma_R U_j \right] + c.c.$
$\gamma_R \mathcal{E}_i \to (U_e)_{ij}\, \gamma_R \mathcal{E}_j'$
$\gamma_R E_i \to (V_e)_{ij}\, \gamma_R E_j'$
$\gamma_R U_i \to (U_u)_{ij}\, \gamma_R U_j'$
etc
$f_{ij} \to (U_e^T f V_e)_{ij}$
$g_{ij} \to (U_d^T g V_d)_{ij}$
$h_{ij} \to (U_u^T h V_u)_{ij}$
where $U_e, U_u, U_d, V_e, V_u, V_d$ are unitary


Can choose U_e, V_e s.t. $U_e^T f V_e = \mathrm{diag}(f_1, f_2, f_3)$
where f_1, f_2, f_3 are
non negative

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$$\mathcal{L}_{yuk} = -\frac{(v_T + H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{E}_i \gamma_R E_i + g_i \bar{D}_i \gamma_R D_i + h_i \bar{U}_i \gamma_R U_i \right] + c.c.$$

Can choose U_e, V_e s.t. $U_e^T f V_e = \mathrm{diag}(f_1, f_2, f_3)$

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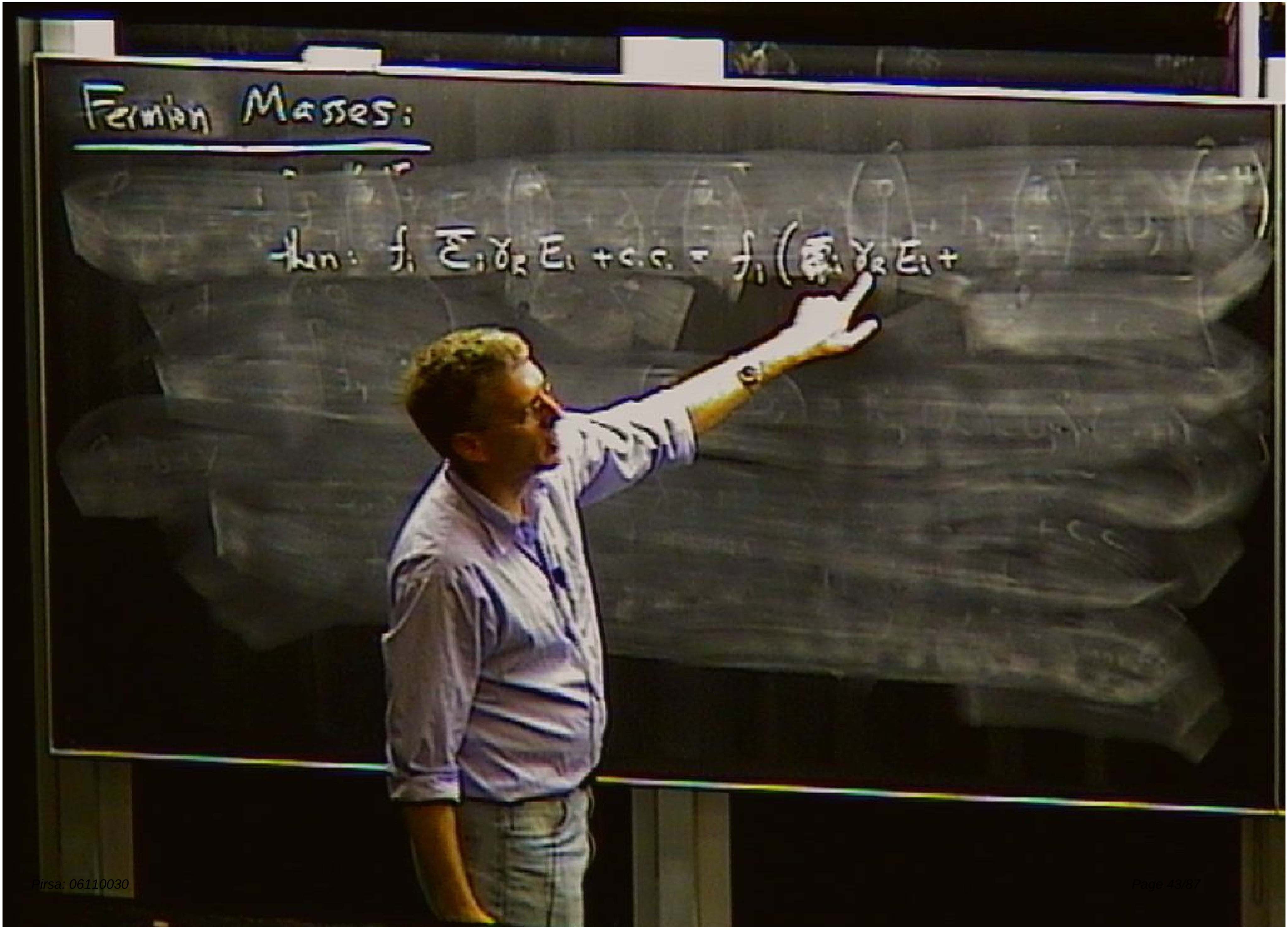
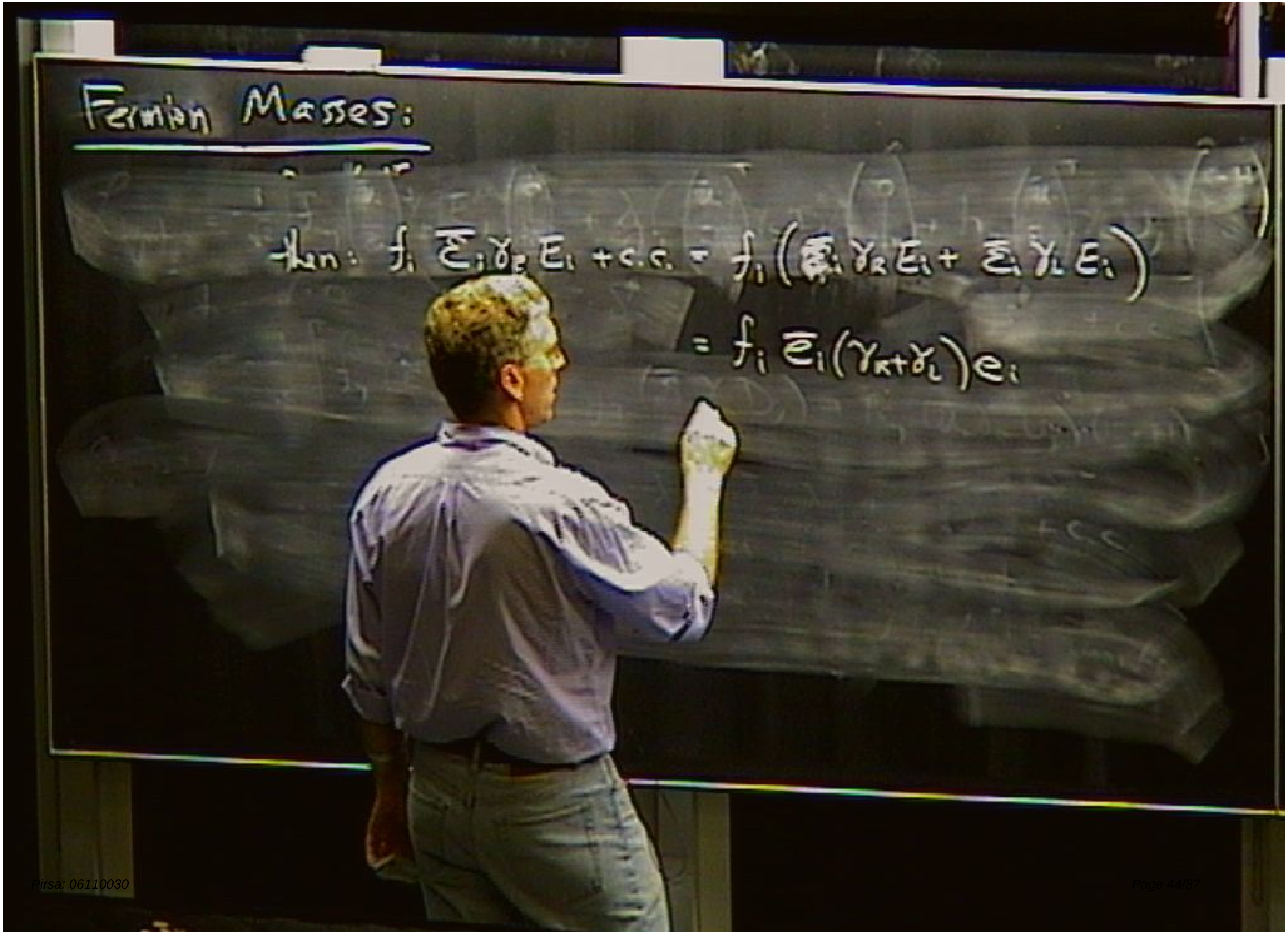
$$\mathcal{L}_{yuk} = -\frac{(v_T + H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{\mathcal{E}}_i \gamma_R E_i + g_i \bar{\mathcal{D}}_i \gamma_R D_i + h_i \bar{\mathcal{U}}_i \gamma_R U_i \right] + c.c.$$

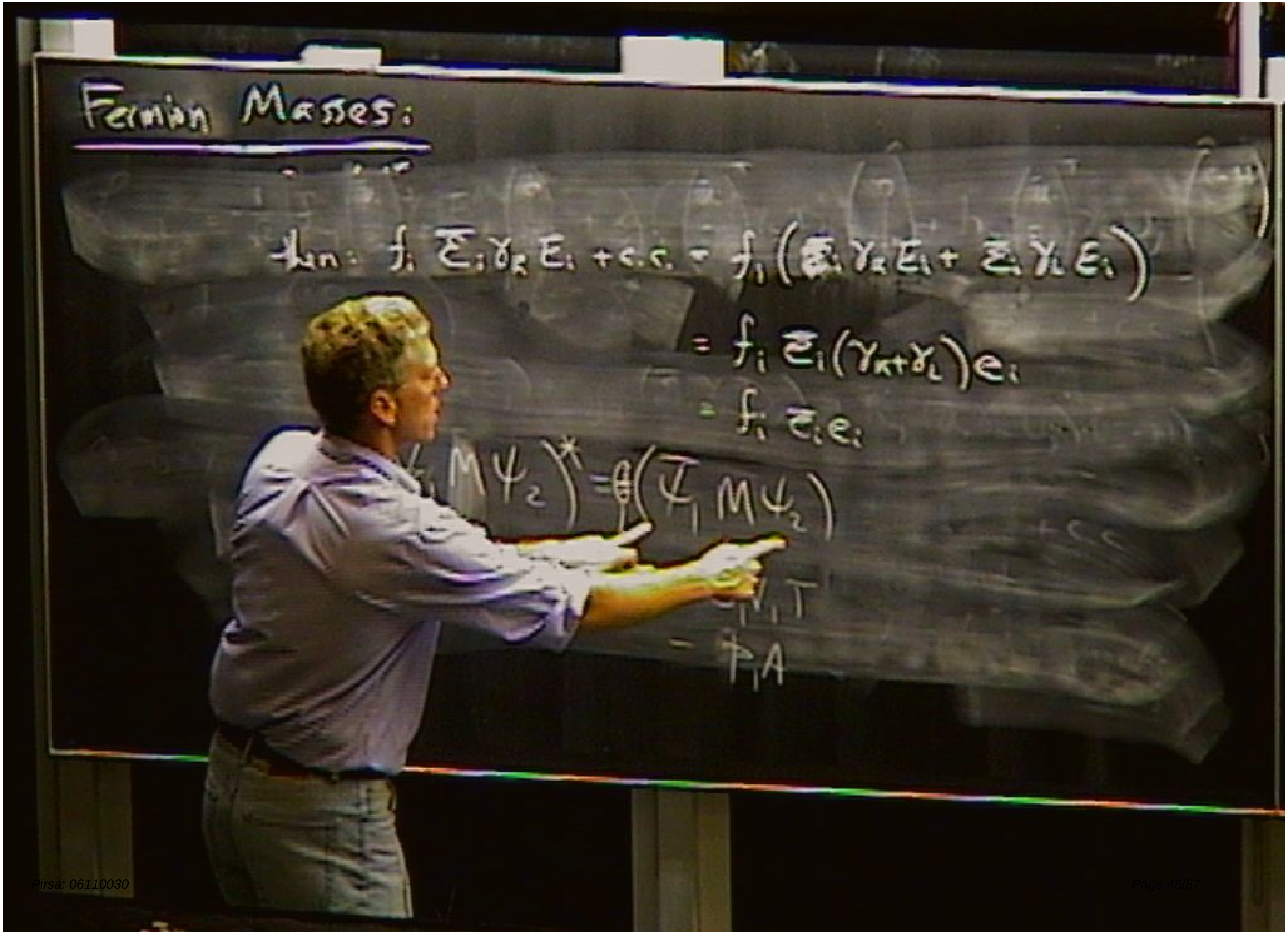
Define Dirac spinors: $e_i = \gamma_L \mathcal{E}_i + \gamma_R E_i$

$u_i = \gamma_L \mathcal{U}_i + \gamma_R U_i$

$d_i = \gamma_L \mathcal{D}_i + \gamma_R D_i$


Fermion Masses:
then: $f_1 \bar{E}_1 \gamma_R E_1 + c.c. = f_1 ($


Fermion Masses:
then: $f_i \bar{E}_i \gamma_R E_i + c.c. = f_i (\bar{E}_i \gamma_R E_i + \bar{E}_i \gamma_L E_i)$
$= f_i \bar{e}_i (\gamma_R + \gamma_L) e_i$


Fermion Masses:
$\mathcal{L}_m$: $f_1 \bar{E}_i \gamma_R E_i$ + c.c. = $f_1(\bar{E}_i \gamma_R E_i + \bar{E}_i \gamma_L E_i)$
= $f_1 \bar{E}_1 (\gamma_R + \gamma_L) e_i$
= $f_1 \bar{e}_i e_i$
$(\bar{\psi}_1 M \psi_2)^* = \theta(\bar{\psi}_1 M \psi_2)$
+c.c.

Fermion Masses:

Term: $f_i \bar{E}_i \gamma_R E_i + \text{c.c.} = f_i \left(\bar{E}_i \gamma_R E_i + \bar{E}_i \gamma_L E_i \right)$

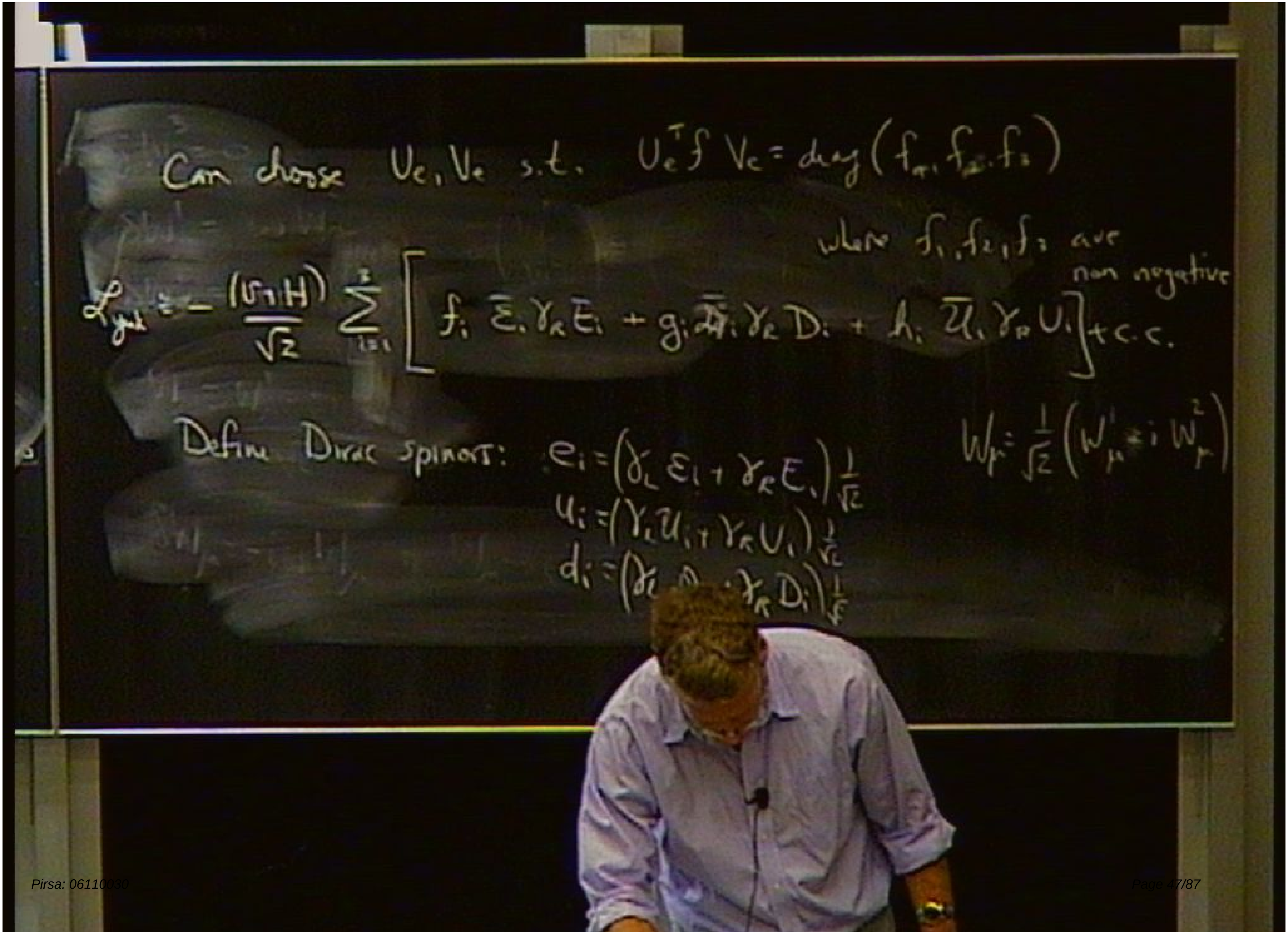
$= f_i \bar{E}_i (\gamma_R + \gamma_L) e_i$

$= f_i \bar{E}_i e_i$

$$(\bar{\Psi}_1 M \Psi_2)^* = \theta (\bar{\Psi}_1 M \Psi_2)$$

$+ \; S, V, T$

$- \; P, A$


Can choose U_e, V_e s.t. $U_e^T f V_e = \mathrm{diag}(f_1, f_2, f_3)$
where f_1, f_2, f_3 are non negative
$\mathcal{L}_{yuk} = -\frac{(v_T + H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{\mathcal{E}}_i \gamma_R E_i + g_i \bar{\mathcal{D}}_i \gamma_R D_i + h_i \bar{\mathcal{U}}_i \gamma_R U_i \right] + c.c.$
Define Dirac spinors:
$e_i = (\gamma_L \mathcal{E}_i + \gamma_R E_i)\frac{1}{\sqrt{2}}$
$u_i = (\gamma_L \mathcal{U}_i + \gamma_R U_i)\frac{1}{\sqrt{2}}$
$d_i = (\gamma_L \mathcal{D}_i + \gamma_R D_i)\frac{1}{\sqrt{2}}$
$W_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$

Can choose U_e, V_e s.t. $U_e^T f V_e = \mathrm{diag}(f_1, f_2, f_3)$

where f_1, f_2, f_3 are non negative

$$\mathcal{L}_{yuk} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{\mathcal{E}}_i \gamma_R E_i + g_i \bar{\mathcal{D}}_i \gamma_R D_i + h_i \bar{\mathcal{U}}_i \gamma_R U_i \right] + c.c.$$

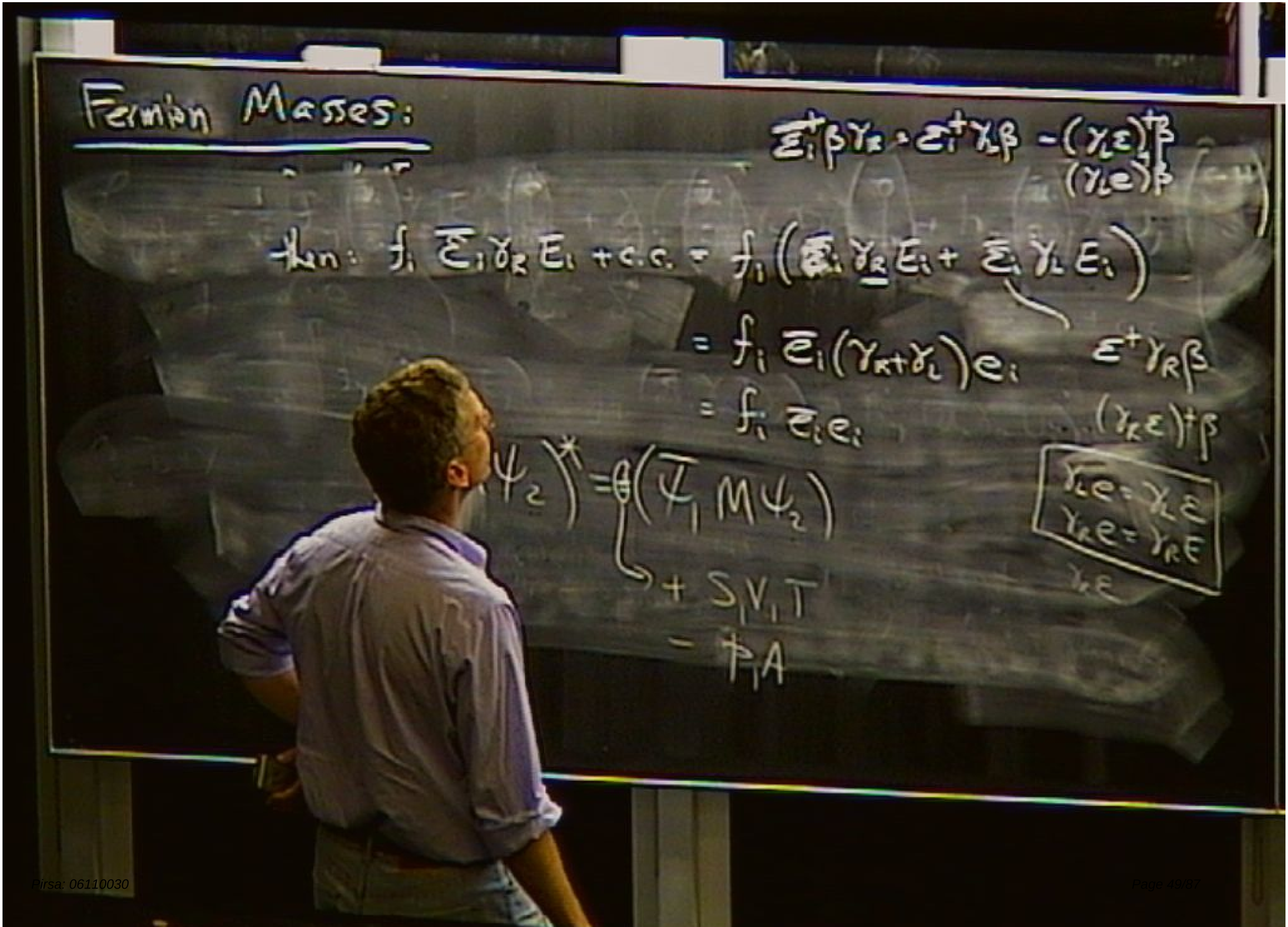
$$W_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right)$$

Define Dirac spinors:

$$e_i = \left(\gamma_L \mathcal{E}_i + \gamma_R E_i\right)\frac{1}{\sqrt{2}}$$

$$u_i = \left(\gamma_L \mathcal{U}_i + \gamma_R U_i\right)\frac{1}{\sqrt{2}}$$

$$d_i = \left(\gamma_L \mathcal{D}_i + \gamma_R D_i\right)\frac{1}{\sqrt{2}}$$


Fermion Masses:
$\mathcal{E}_i^\dagger \beta \gamma_R = \mathcal{E}_i^\dagger \gamma_L \beta = (\gamma_L \mathcal{E})^\dagger \beta$
$(\gamma_L e)^\dagger \beta$
then: $f_i \bar{\mathcal{E}}_i \gamma_R E_i + c.c. = f_i (\bar{\mathcal{E}}_i \gamma_R E_i + \bar{\mathcal{E}}_i \gamma_L E_i)$
$= f_i \bar{e}_i (\gamma_R + \gamma_L) e_i$
$\mathcal{E}^\dagger \gamma_R \beta$
$= f_i \bar{e}_i e_i$
$(\gamma_R \mathcal{E})^\dagger \beta$
$\gamma_L e = \gamma_L \mathcal{E}$
$\gamma_R e = \gamma_R \mathcal{E}$
$\gamma_L \mathcal{E}$
$\psi_2)^* = \theta(\bar{\psi}_1 M \psi_2)$
$+ S, V, T$
$- P, A$

Fermion Masses:

$\bar{\mathcal{E}}_i^\dagger \beta \gamma_L = \mathcal{E}_i^\dagger \gamma_L \beta = (\gamma_L \mathcal{E})^\dagger \beta$

$(\gamma_L e)\beta$

$\mathcal{L}_m: \; f_1 \bar{\mathcal{E}}_1 \gamma_L E_1 + c.c. = f_1 \left(\bar{\mathcal{E}}_1 \gamma_{\underline{L}} E_1 + \bar{\mathcal{E}}_1 \gamma_L E_1 \right)$

$= f_1 \bar{\mathcal{E}}_1 (\gamma_R + \gamma_L) e_1$ $\qquad \mathcal{E}^\dagger \gamma_R \beta$

$= f_1 \bar{\mathcal{E}}_1 e_1$ $\qquad (\gamma_L \mathcal{E})^\dagger \beta$

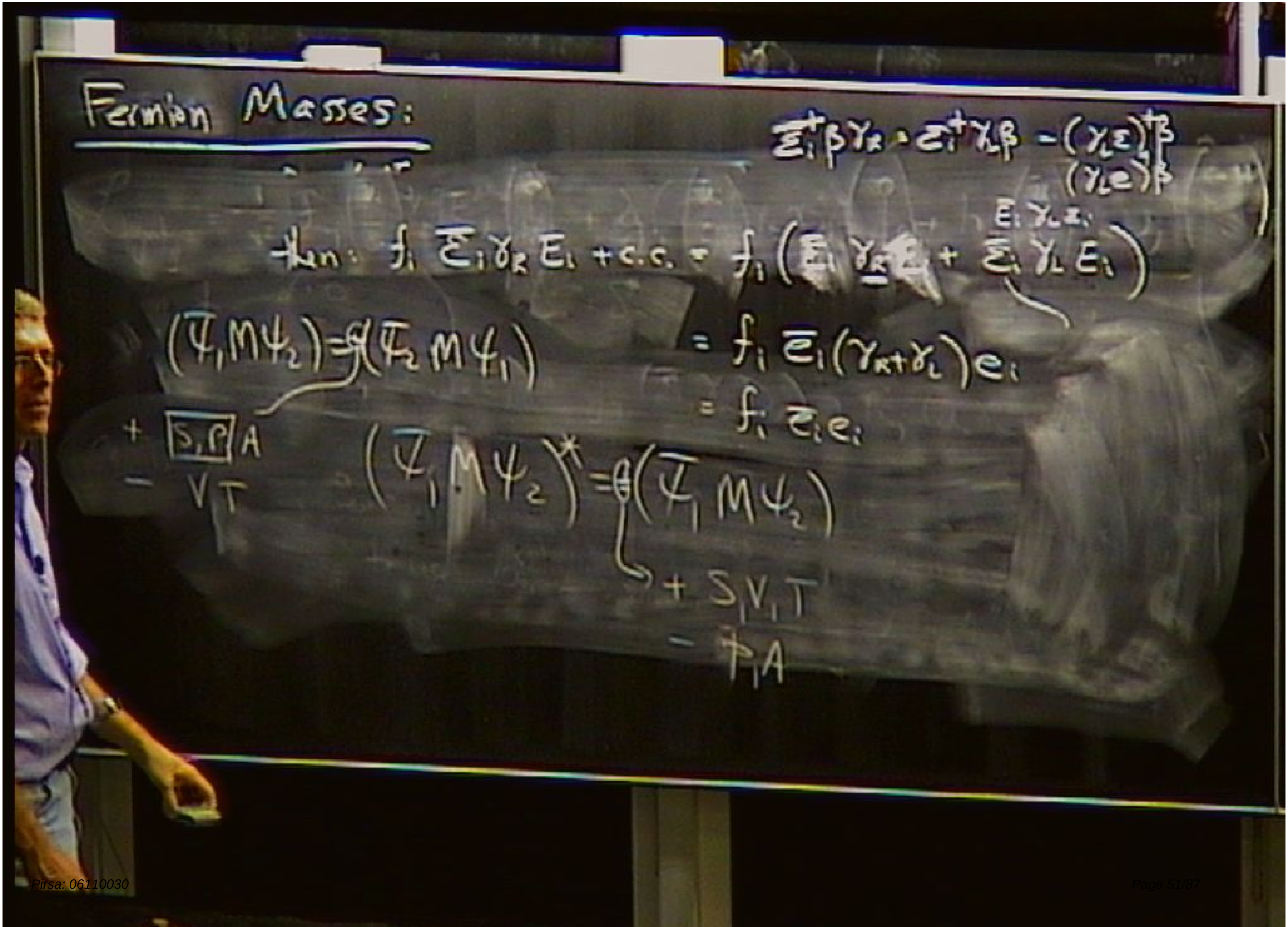
$$\boxed{\begin{array}{l} \gamma_L e = \gamma_L E \\ \gamma_R e = \gamma_R E \end{array}}$$

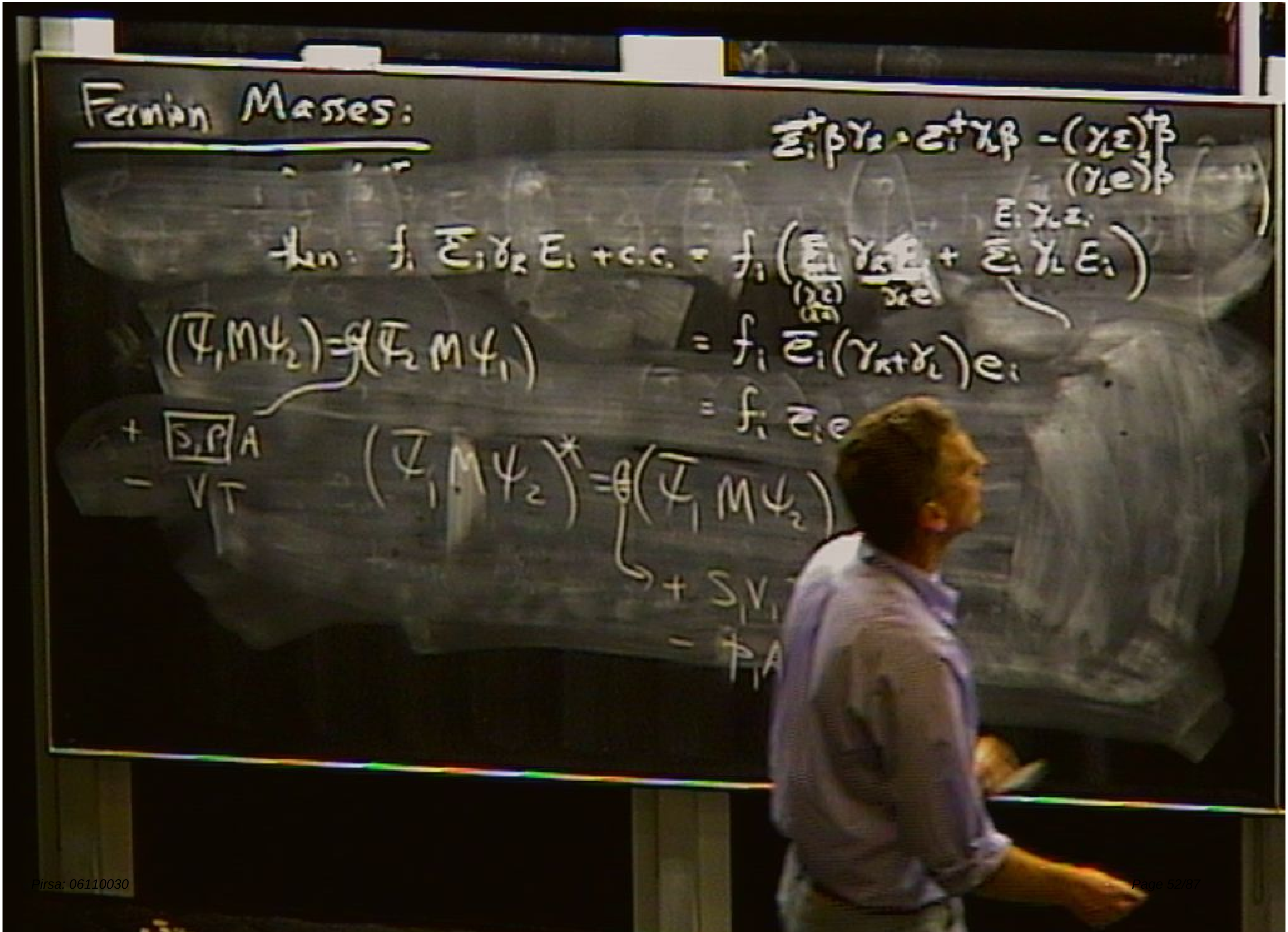
$\gamma_L E$

$(\bar{\Psi}_1 M \Psi_2)^* = \theta (\bar{\Psi}_1 M \Psi_2)$

$+ \; S, V, T$

$- \; P, A$


Fermion Masses:
$\bar{\Xi}_i \beta \gamma_R = \Xi_i^\dagger \gamma_L \beta = (\gamma_L \Xi)^\dagger \beta$
$(\gamma_L e)^\dagger \beta$
$\bar{E}_i \gamma_L E_i$
then: $f_i \bar{\Xi}_i \gamma_R E_i + c.c. = f_i (\bar{E}_i \gamma_R E_i + \bar{\Xi}_i \gamma_L E_i)$
$= f_i \bar{e}_i (\gamma_R + \gamma_L) e_i$
$= f_i \bar{e}_i e_i$
$(\bar{\Psi}_1 M \Psi_2) = \theta (\bar{\Psi}_2 M \Psi_1)$
$+ \boxed{S, P}\, A$
$-\ V, T$
$(\bar{\Psi}_1 M \Psi_2)^* = \theta (\bar{\Psi}_1 M \Psi_2)$
$+\ S, V, T$
$-\ P, A$


Fermion Masses:
$\mathcal{L}_{m}: f_i \bar{E}_i \gamma_R E_i + \text{c.c.} = f_i \left(\bar{E}_i \gamma_R E_i + \bar{E}_i \gamma_L E_i \right)$
$(\bar{\Psi}_1 M \psi_2) = \theta (\bar{\Psi}_2 M \psi_1)$
$= f_i \bar{e}_i (\gamma_R + \gamma_L) e_i$
$= f_i \bar{e}_i e$
$+ \boxed{S, P}\ A$
$-\ V\ T$
$(\bar{\Psi}_1 M \psi_2)^* = \theta (\bar{\Psi}_1 M \psi_2)$

Fermion Masses:

$\bar{\varepsilon}_i \beta \gamma_R = \varepsilon_i^{\dagger} \gamma_L \beta = (\gamma_L \varepsilon)^{\dagger} \beta$

$(\gamma_L e)^{\dagger} \beta$

then: $f_1 \bar{\varepsilon}_1 \gamma_R \varepsilon_1 + \text{c.c.} = f_1 \left(\bar{\varepsilon}_1 \gamma_R e_1 + \bar{\varepsilon}_1 \gamma_L \varepsilon_1 \right)$

$= f_1 \bar{\varepsilon}_1 (\gamma_R + \gamma_L) e_1$

$= f_1 \bar{e}_1 e_1$

$(\bar{\Psi}_1 M \Psi_2) = \theta (\bar{\Psi}_2 M \Psi_1)$

$+ \boxed{S, P}\, A$

$- V, T$

$(\bar{\Psi}_1 M \Psi_2)^* = \theta (\bar{\Psi}_1 M \Psi_2)$

$+ S, V, T$

$- P, A$

$\mathcal{L} = \ldots$

$\gamma_R \mathcal{E}_i \to (U_e)_{ij}\, \gamma_R \mathcal{E}_j'$ $\qquad$ $f_{ij} \to (U_e^T f V_e)_{ij}$

$\gamma_R E_i \to (V_e)_{ij}\, \gamma_R E_j'$ $\qquad$ $g_{ij} \to (U_d^T g V_d)_{ij}$

$\gamma_R U_i \to (U_u)_{ij}\, \gamma_R U_j'$ $\qquad$ $h_{ij} \to (U_u^T h V_u)_{ij}$

$\vdots$

etc

where $U_e, U_u, U_d, V_e, V_u, V_d$ are unitary

$$\mathcal{L}_{yuk} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i \right]$$

$$\mathcal{L}_{kin} = -\frac{1}{2} \bar{E}_i \not\partial E_i - \frac{1}{2} \bar{E}_i \not\partial E_i$$

Can choose U_e, V_e s.t. $U_e^T f V_e = \mathrm{diag}(f_1, f_2, f_3)$

where f_1, f_2, f_3 are non negative

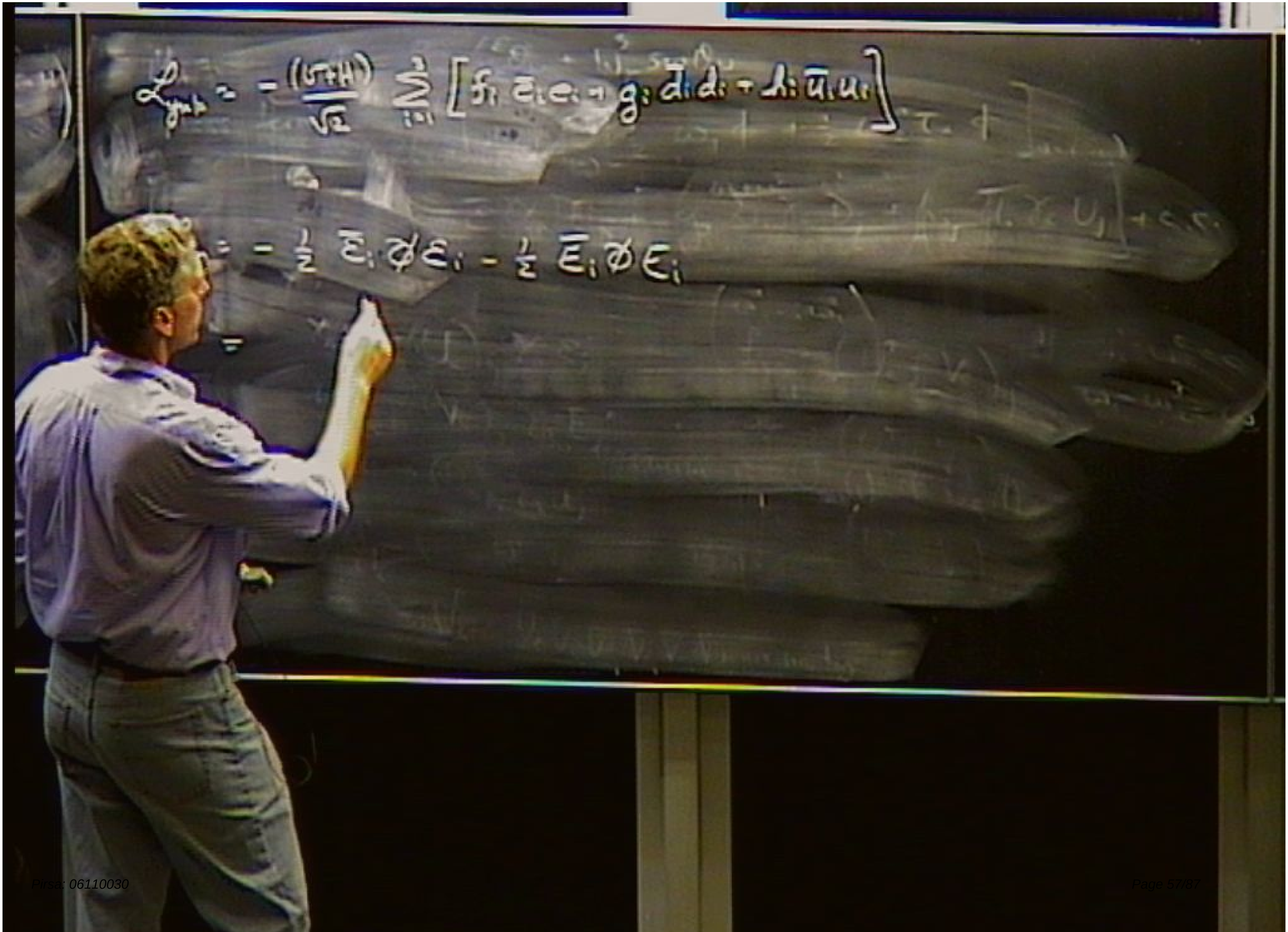
$$\mathcal{L}_{yuk} = -\frac{(v_1 + H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{\mathcal{E}}_i \gamma_R E_i + g_i \bar{\mathcal{D}}_i \gamma_R D_i + h_i \bar{\mathcal{U}}_i \gamma_R U_i \right] + c.c.$$

$$W_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu - i W^2_\mu\right)$$

Define Dirac spinors: $e_i = (\gamma_L \mathcal{E}_i + \gamma_R E_i)$

$u_i = (\gamma_L \mathcal{U}_i + \gamma_R U_i)$

$d_i = (\gamma_L \mathcal{D}_i + \gamma_R D_i)$


$\mathcal{L}_{yuk} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i \right]$
$= -\frac{1}{2} \bar{E}_i \not{\partial} E_i - \frac{1}{2} \bar{E}_i \not{\partial} E_i$
$=$

$$\mathcal{L}_{yuk} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i \right]$$

$$(\gamma_L + \gamma_R) \qquad\qquad (\gamma_L + \gamma_R)$$

$$\mathcal{L}_{kin} = -\frac{1}{2} \bar{E}_i \not\partial E_i - \frac{1}{2} \bar{E}_i \not\partial E_i$$

$$= -\bar{e}_i \not\partial e_i \qquad \text{(check this)}$$

$$\mathcal{L}_{yuk} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i \right]$$

$$\mathcal{L}_{kin} = -\frac{1}{2} \bar{e}_i \overset{(\gamma_L+\gamma_R)}{\not\partial} e_i - \frac{1}{2} \bar{e}_i \overset{(\gamma_L+\gamma_R)}{\not\partial} e_i$$

$$= -\bar{e}_i \not\partial e_i \qquad \text{(check this)}$$

Standard form $\mathcal{L} = -\bar{\Psi}(\not\partial + m)\Psi$

$$m_{e_i} = \frac{1}{\sqrt{2}} f_i v \qquad m_{d_i} = \frac{1}{\sqrt{2}} g_i v \qquad m_{u_i} = \frac{1}{\sqrt{2}} h_i v$$

Notice: Yukawa couplings are diagonal in flavour space
when written in mass basis.

$$\mathcal{L}_{yuk} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i \right]$$

$$\mathcal{L}_{kin} = -\frac{1}{2} \bar{\varepsilon}_i \not{\partial} (\gamma_L + \gamma_R) \varepsilon_i - \frac{1}{2} \bar{\varepsilon}_i \not{\partial} (\gamma_L + \gamma_R) \varepsilon_i$$

$$= -\bar{e}_i \not{\partial} e_i \qquad \text{(check this)}$$

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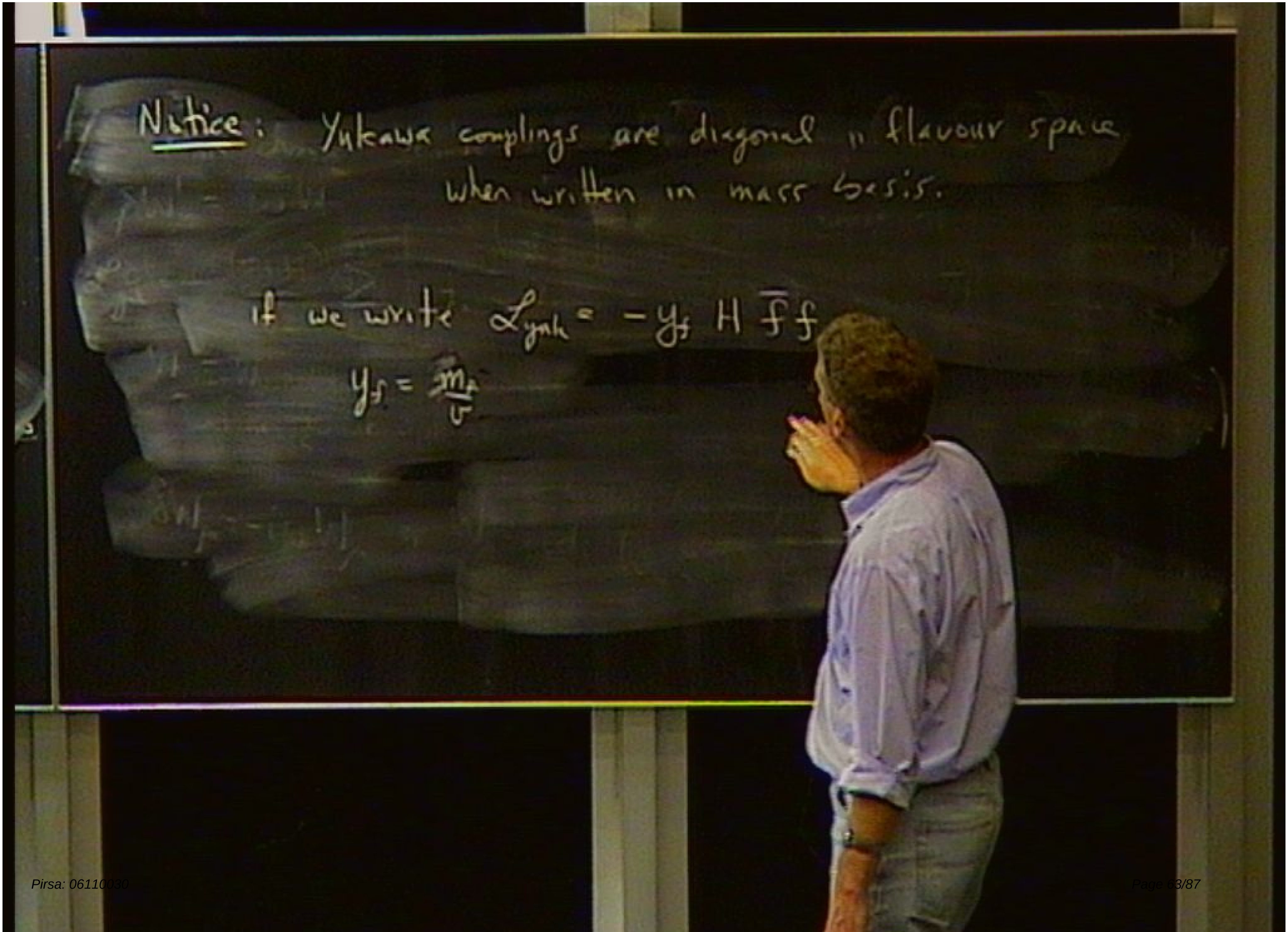
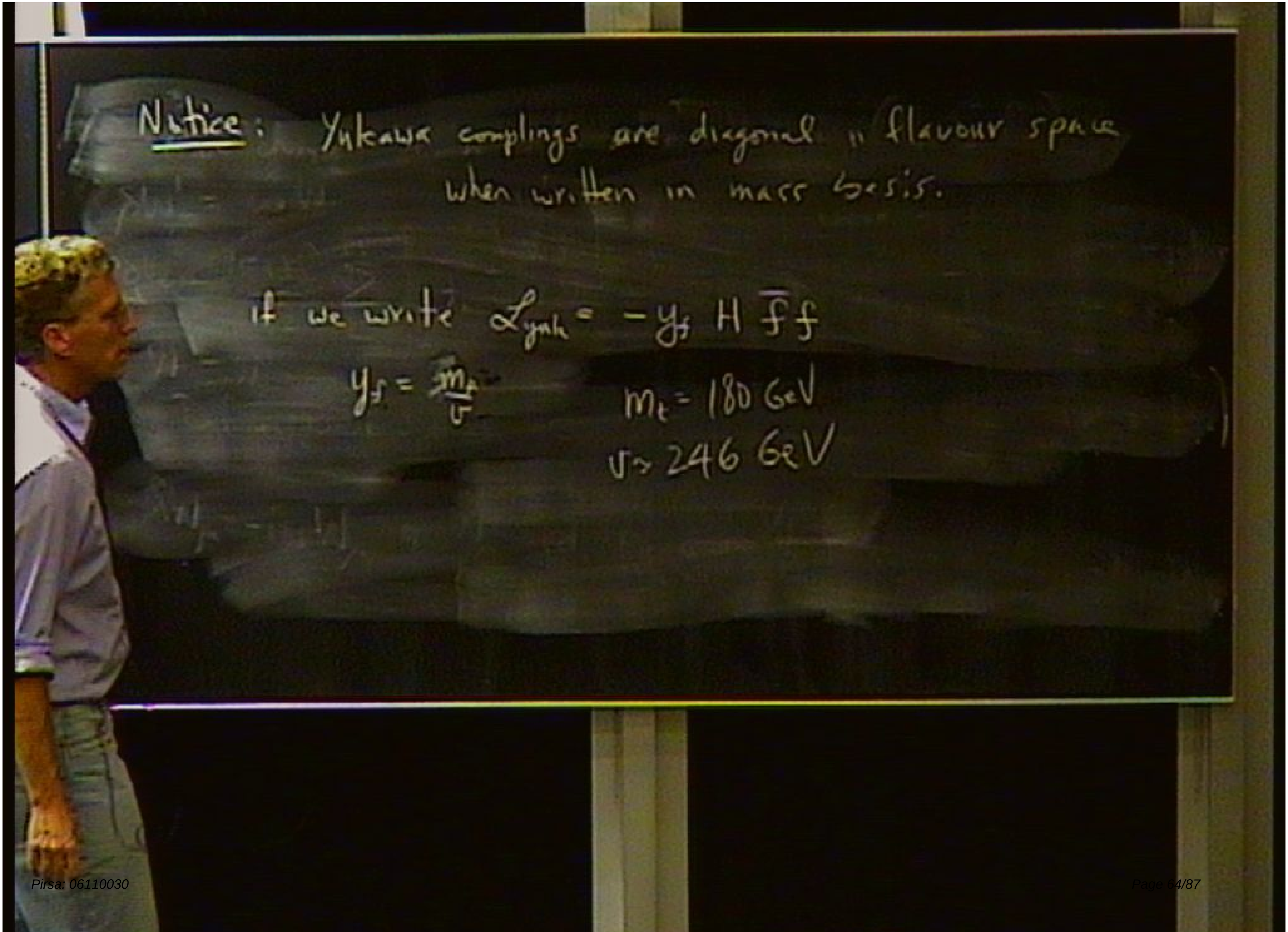
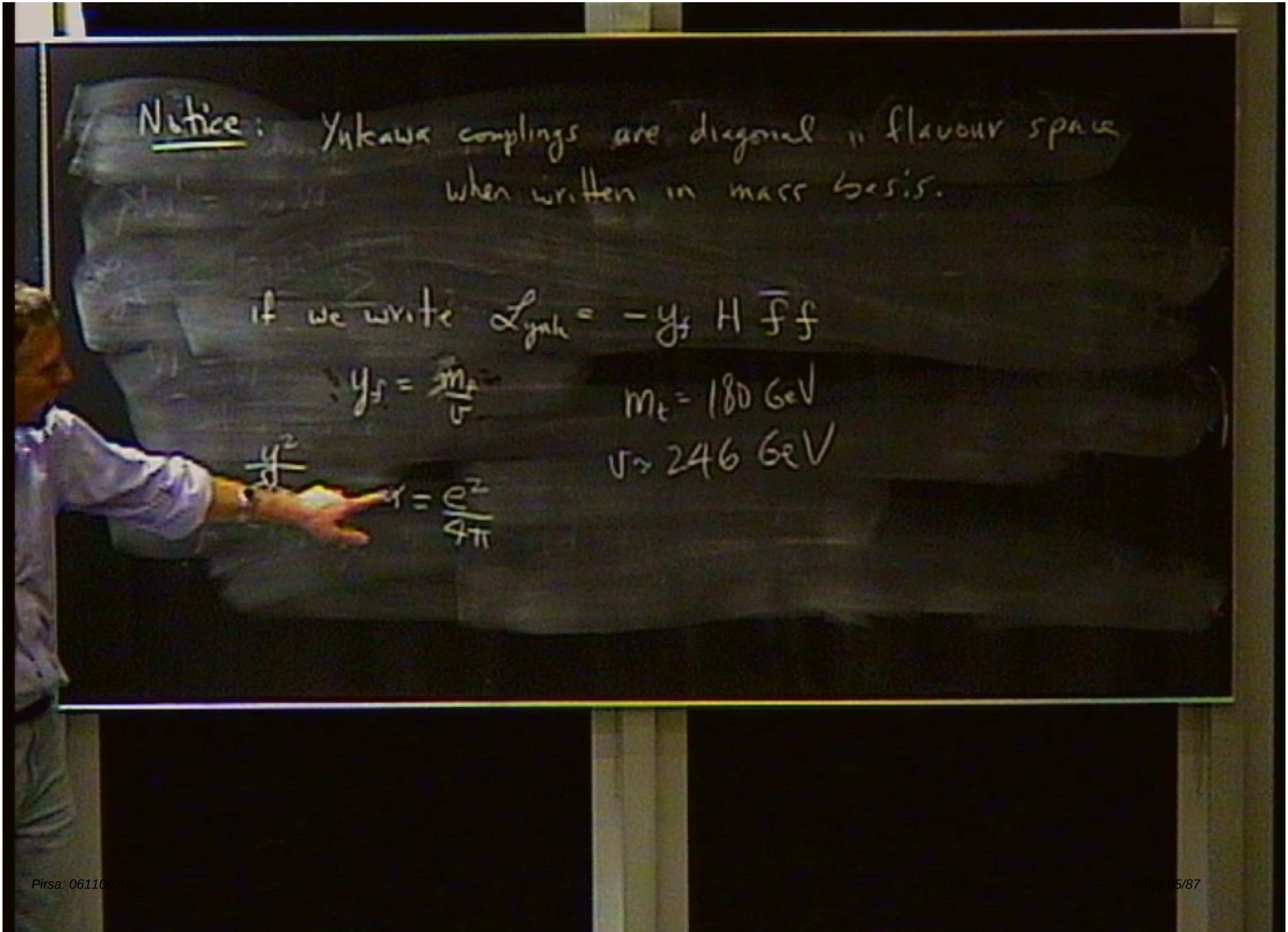
<u>Notice</u>: Yukawa couplings are diagonal in flavour space when written in mass basis.

If we write $\mathcal{L}_{yuk} = -y_f H \bar{f} f$

$y_f =$


Notice: Yukawa couplings are diagonal in flavour space
when written in mass basis.
If we write $\mathcal{L}_{yuk} = -y_f H \bar{f} f$
$y_f = \frac{m_f}{v}$


Notice: Yukawa couplings are diagonal in flavour space
when written in mass basis.
if we write $\mathcal{L}_{yuk} = -y_f H \bar{f} f$
$y_f = \frac{m_f}{v}$
$m_t = 180$ GeV
$v \simeq 246$ GeV


Notice: Yukawa couplings are diagonal in flavour space
when written in mass basis.
If we write $\mathcal{L}_{yuk} = -y_f H \bar{f} f$
$y_f = \frac{m_f}{v}$
$m_t = 180$ GeV
$v \approx 246$ GeV
y^2
$\alpha = \frac{e^2}{4\pi}$

Notice: Yukawa couplings are diagonal in flavour space when written in mass basis.

if we write $\mathcal{L}_{yuk} = -y_f H \bar{f} f$

$y_f = \frac{m_f}{v}$

$m_t = 180$ GeV

$v \simeq 246$ GeV

$\frac{y^2}{4\pi}$

$\alpha = \frac{e^2}{4\pi}$

$$\mathcal{L}_{yuk} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i \right]$$

$$\mathcal{L}_{kin} = -\frac{1}{2} \bar{\mathcal{E}}_i \overset{(\gamma_L+\gamma_R)}{\not\partial} \mathcal{E}_i - \frac{1}{2} \bar{\mathcal{E}}_i \overset{(\gamma_L+\gamma_R)}{\not\partial} \mathcal{E}_i$$

$$= -\bar{e}_i \not\partial e_i$$

(check this)

Standard form $\mathcal{L} = -\bar{\Psi}(\not\partial + m)\Psi$

$$m_{e_i} = \frac{1}{\sqrt{2}} f_i v \qquad m_{d_i} = \frac{1}{\sqrt{2}} g_i v \qquad m_{u_i} = \frac{1}{\sqrt{2}} h_i v$$

Notice: Yukawa couplings are diagonal in flavour space when written in mass basis.

if we write $\mathcal{L}_{yuk} = -y_f \, H \bar{f} f$

$y_f = \frac{m_f}{v}$

$m_t = 180 \text{ GeV}$

$v \simeq 246 \text{ GeV}$

$\frac{y^2}{4\pi}$

$\alpha = \frac{e^2}{4\pi}$

Notice: Yukawa couplings are diagonal in flavour space when written in mass basis.

If we write $\mathcal{L}_{yuk} = -y_f H \bar{f} f$

$M_W = 80$ GeV

$y_f = \frac{m_f}{v}$

$m_t = 180$ GeV

$M_Z \simeq 90$ GeV

$v \simeq 246$ GeV

$\frac{y^2}{4\pi}$

$\alpha = \frac{e^2}{4\pi}$

$$\mathcal{L}_{yuk} = -\frac{(v+H)}{\sqrt{2}} \sum_{i=1}^{3} \left[f_i \bar{e}_i e_i + g_i \bar{d}_i d_i + h_i \bar{u}_i u_i \right]$$

$$\mathcal{L}_{kin} = -\frac{1}{2} \bar{e}_i \not{\partial} (\gamma_L + \gamma_R) e_i - \frac{1}{2} \bar{e}_i \not{\partial} (\gamma_L + \gamma_R) e_i$$

$$= -\bar{e}_i \not{\partial} e_i \quad \text{(check this)}$$

Standard form $\mathcal{L} = -\bar{\Psi}(\not{\partial} + m)\Psi$

$$m_{e_i} = \frac{1}{\sqrt{2}} f_i v \qquad m_{d_i} = \frac{1}{\sqrt{2}} g_i v \qquad m_{u_i} = \frac{1}{\sqrt{2}} h_i v$$

Notice: Yukawa couplings are diagonal in flavour space when written in mass basis.

If we write $\mathcal{L}_{yuk} = -y_f H \bar{f} f$

$M_W = 80$ GeV

$M_Z = 90$ GeV

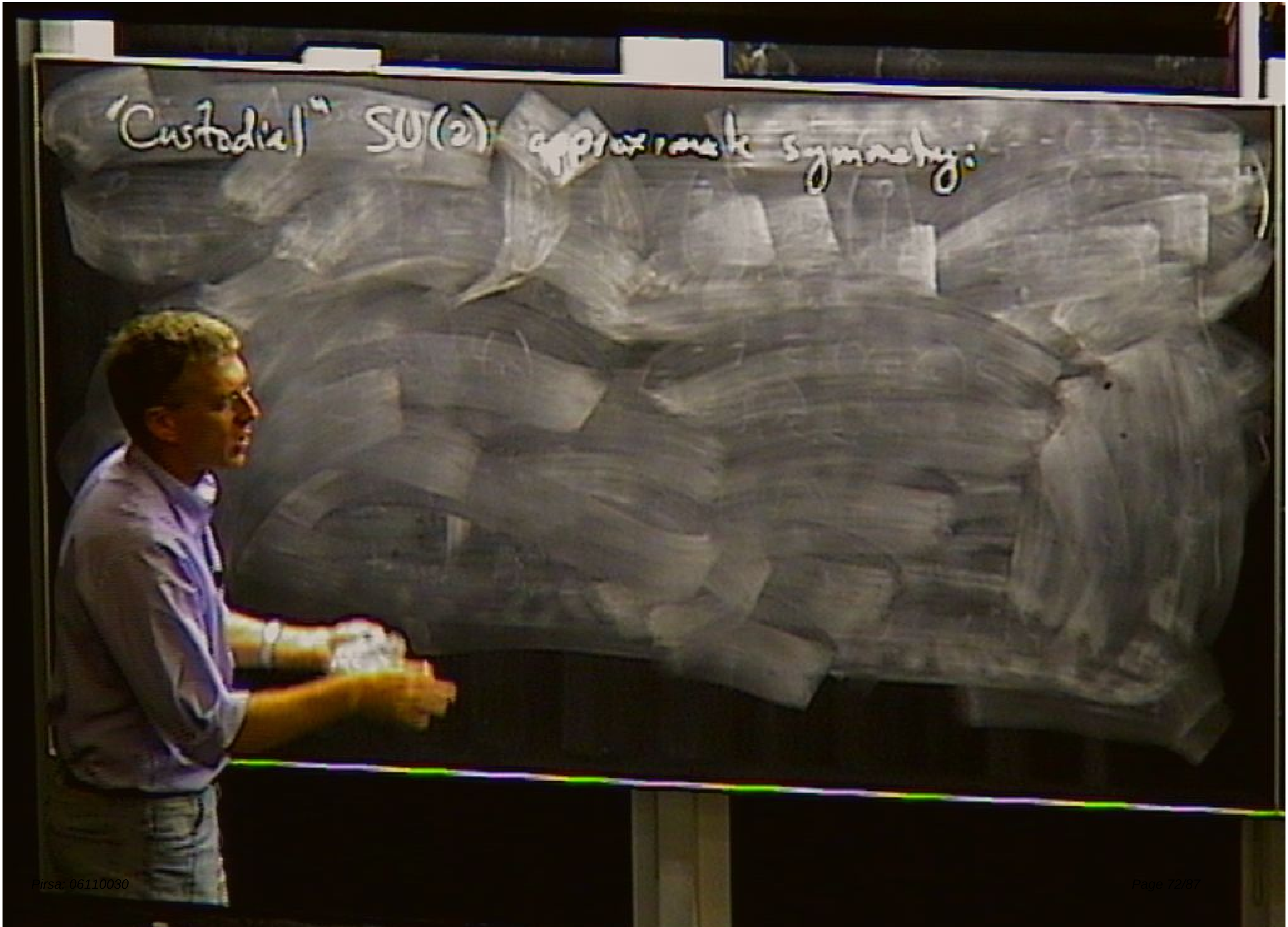
$y_f = \frac{m_f}{v}$

$m_t = 180$ GeV

$v \simeq 246$ GeV

$\frac{y^2}{4\pi}$

$\alpha = \frac{e^2}{4\pi}$


"Custodial" SU(2): approximate symmetry:

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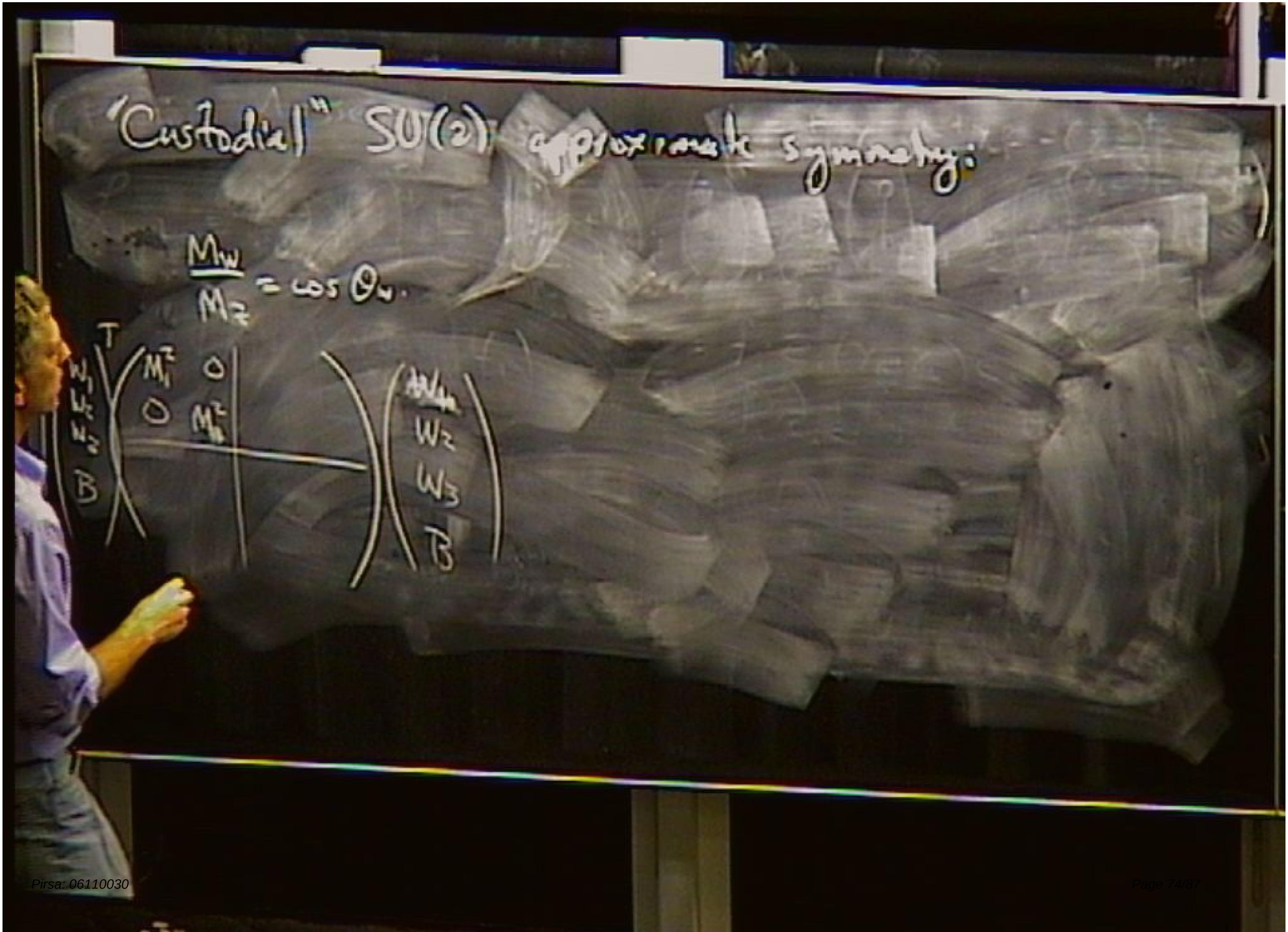
$$\frac{M_W}{M_Z} = \cos\theta_W .$$


"Custodial" SU(2) approximate symmetry:
$\frac{M_W}{M_Z} = \cos\theta_w$
T
W_1
W_2
W_3
B
M_1^2 0
0 M_W^2
W_2
W_3
B

"Custodial" SU(2) approximate symmetry:

$$\frac{M_W}{M_Z} = \cos\theta_W$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}^T \left(\begin{array}{cc|c} M_1^2 & 0 & 0 \\ 0 & M_1^2 & \\ \hline 0 & & \end{array}\right) \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}$$

is the most general mass term consistent with unbroken $U_{em}(1)$ invariance.

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$$\det\begin{pmatrix} M_3^2 & m^2 \\ m^2 & M_0^2 \end{pmatrix} = M_0^2 M_3^2 - m^2 = 0$$

the massive eigenvector $\begin{pmatrix} \cos\theta_W \\ -\sin\theta_W \end{pmatrix}$ massless eigenvector $\begin{pmatrix} \sin\theta_W \\ \cos\theta_W \end{pmatrix}$

$$\Rightarrow \quad \tan\theta_W = \frac{m^2}{M_3^2} = \left|\frac{M_0}{M_3}\right|$$

$$M^2 = \mathrm{tr}\begin{pmatrix} M_3^2 & m^2 \\ m^2 & M_0^2 \end{pmatrix} = M_3^2 + M_0^2$$

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$$M_Z^2 = \mathrm{tr}\begin{pmatrix} M_3^2 & m^2 \\ m^2 & M_0^2 \end{pmatrix} = M_3^2 + M_0^2$$

$$= M_3^2\left(1 + \tan^2\theta_W\right)$$

$$= M_3^2 \sec^2\theta_W.$$

$$\boxed{M_3 = M_Z\cos\theta_W}$$

$$M_W = M_1 = M_Z\cos\theta_W \Longleftrightarrow M_1 = M_3 = M_W$$

"Custodial" SU(2) approximate symmetry:

$$\boxed{\frac{M_W}{M_Z} = \cos\theta_W}$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}^T \begin{pmatrix} M_W^2 & 0 & & \\ 0 & M_W^2 & & \\ & & M_3^2 & m^2 \\ & & m^2 & M_0^2 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}$$

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$$\det\begin{pmatrix} M_3^2 & m^2 \\ m^2 & M_0^2 \end{pmatrix} = M_0^2 M_3^2 - m^4 = 0$$

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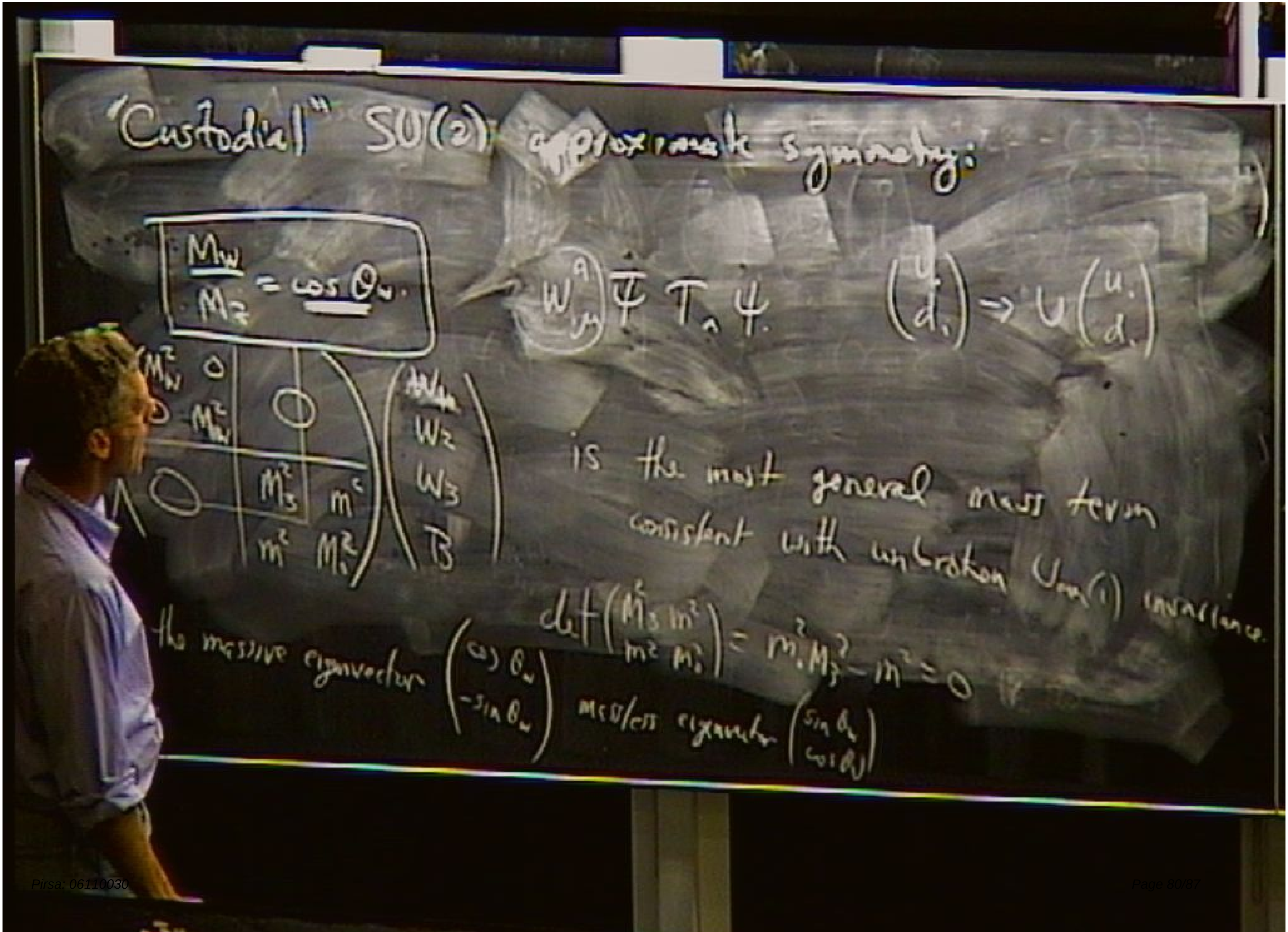
"Custodial" SU(2) approximate symmetry:
$\frac{M_W}{M_Z} = \cos\theta_W$
$W^a_\mu \bar{\Psi} T_a \Psi$
$\begin{pmatrix} u_i \\ d_i \end{pmatrix} \to U \begin{pmatrix} u_i \\ d_i \end{pmatrix}$
is the most general mass term
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"Custodial" SU(2) approximate symmetry:

$$\boxed{\frac{M_W}{M_Z} = \cos\theta_w}$$

$$W^a_\mu \bar{\psi} T_a \psi \qquad \begin{pmatrix} u \\ d \end{pmatrix} \to U \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}^T \begin{pmatrix} M_W^2 & 0 & & \\ 0 & M_W^2 & & \\ & & M_3^2 & m^2 \\ & & m^2 & M_0^2 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ B \end{pmatrix}$$

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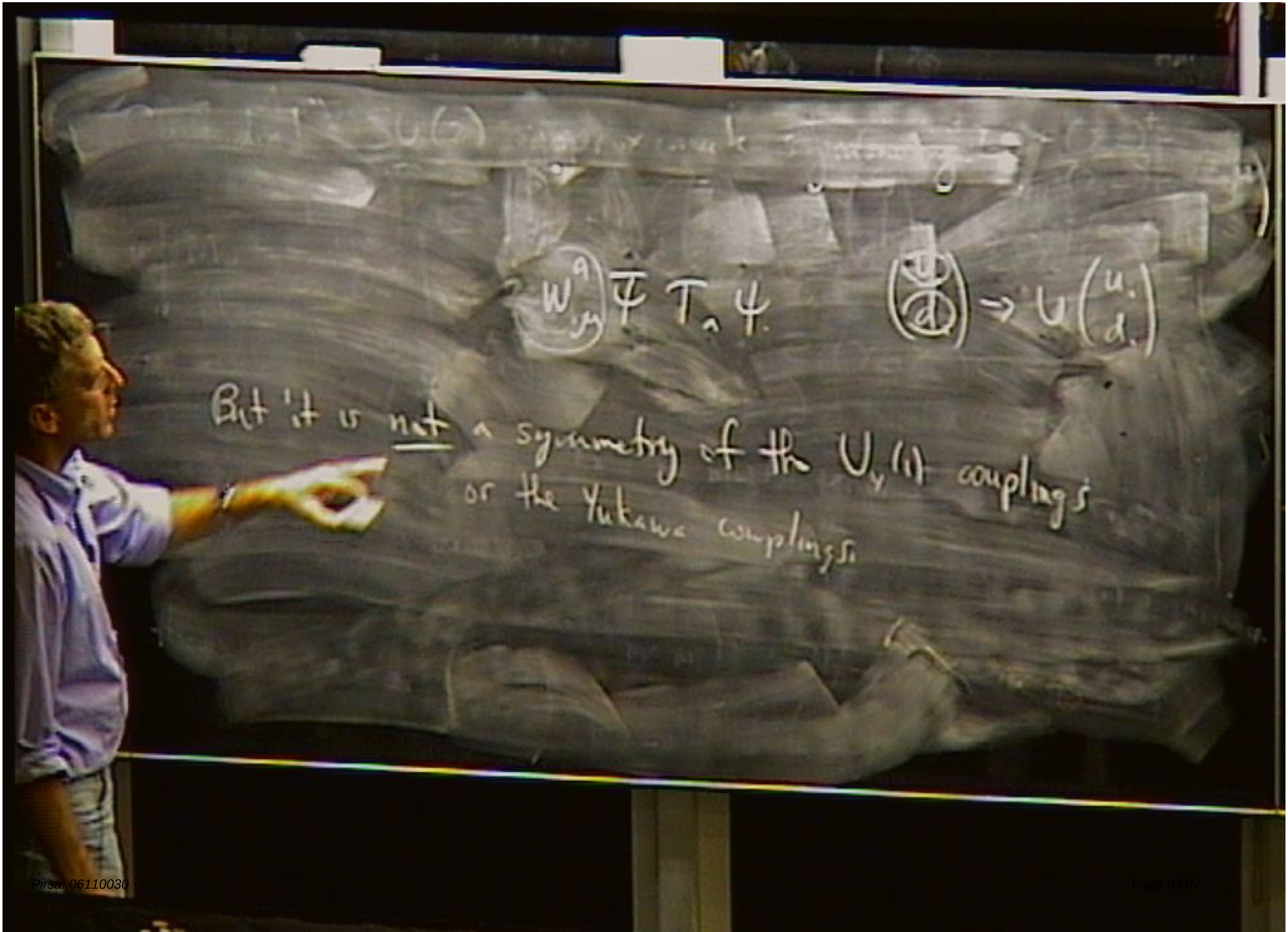
So $M_W = M_Z \cos\theta_W$ would be a consequence of a symmetry of the form

$$\begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix} \to O \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix}$$

$O(3) \sim SU_c(2)$

This can be promoted to a symmetry of the strong interactions, as well as of the $SU_L(2)$ interactions.

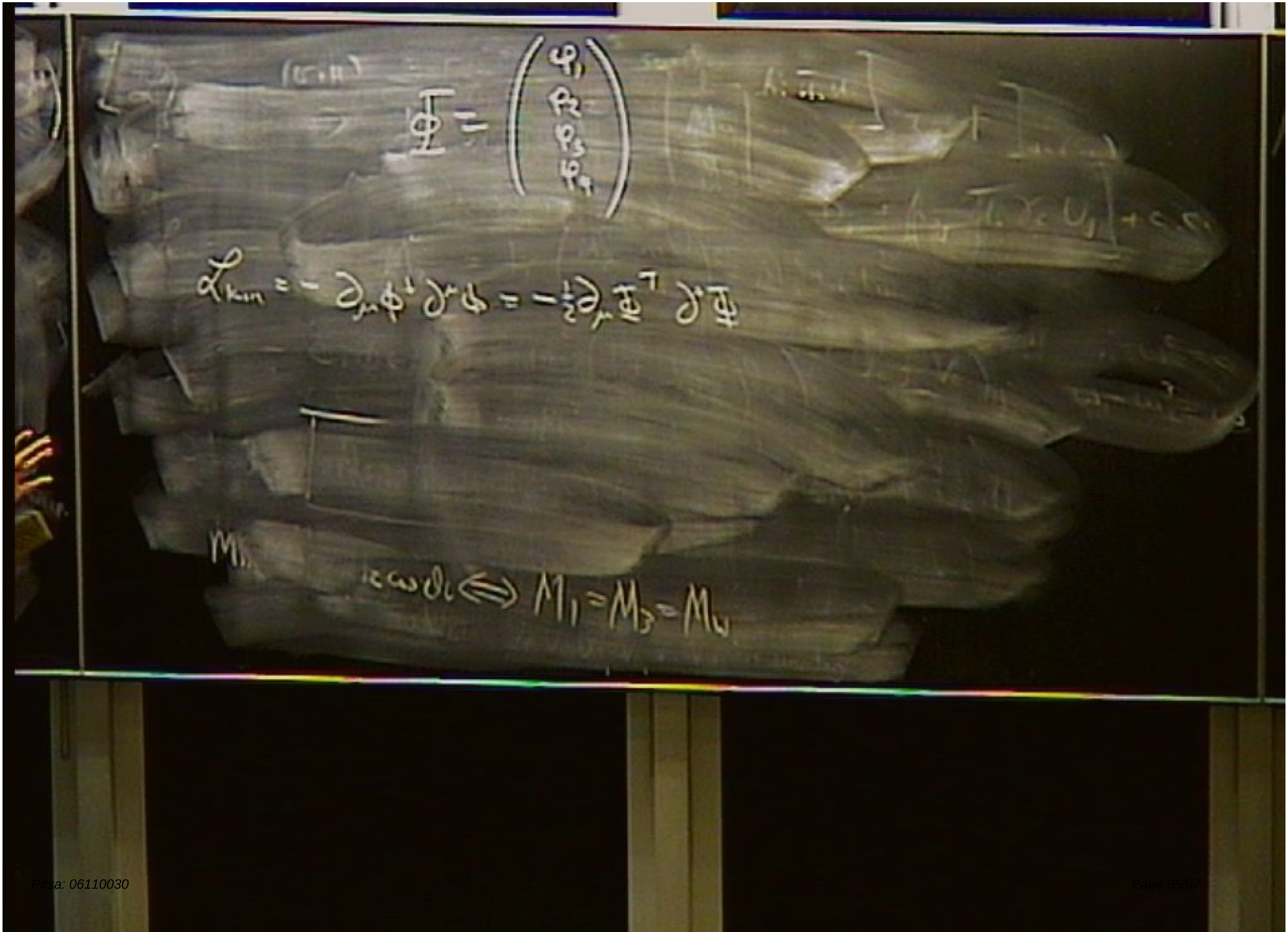

$W^a_{\mu\nu} \bar{\psi} T_a \psi$
$\begin{pmatrix} u \\ d \end{pmatrix} \to U \begin{pmatrix} u \\ d \end{pmatrix}$
But it is not a symmetry of the $U_Y(1)$ couplings
or the Yukawa couplings.

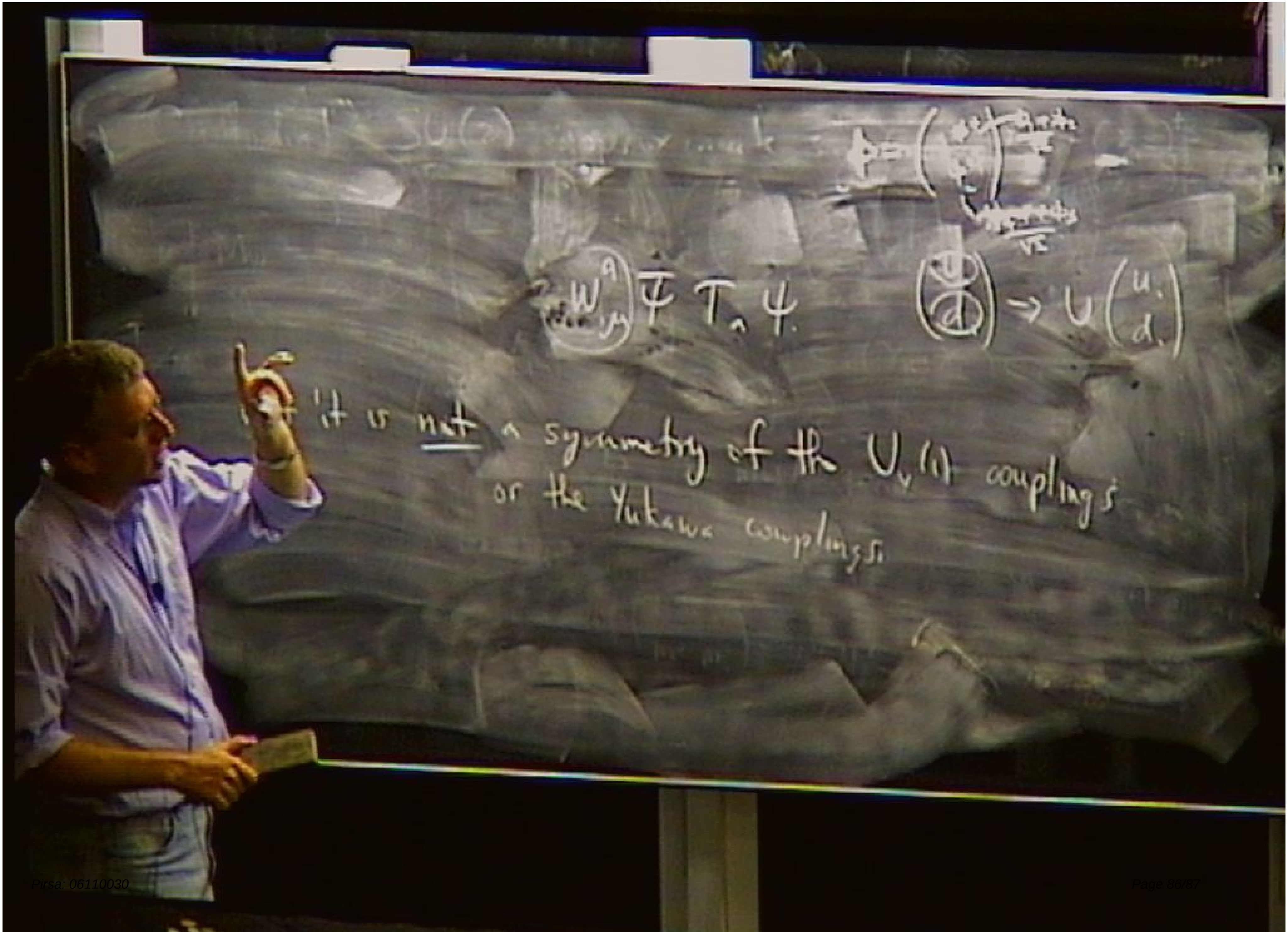
SU(2)

$\Phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$

$W^A_\mu \bar{\Psi} T_A \Psi$ $\qquad$ $\begin{pmatrix}u \\ d\end{pmatrix} \to U \begin{pmatrix}u_i \\ d_i\end{pmatrix}$

But it is not a symmetry of the $U_Y(1)$ couplings or the Yukawa couplings.


$\Phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}$
$\mathcal{L}_{kin} = -\partial_\mu \phi^\dagger \partial^\mu \phi = -\frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi$
$\Longleftrightarrow M_1 = M_3 = M_4$


it is not a symmetry of the U(1) couplings
or the Yukawa couplings.

$$\Phi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix} \qquad \langle \Phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L}_{kin} = -\partial_\mu \phi^\dagger \partial^\mu \phi = -\tfrac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi$$

the scalar potential = $V = \lambda\left(\phi^\dagger\phi - v^2/2\right)^2$

$= \lambda\left[\Phi^T\Phi - v^2/2\right]^2$

V has an "accidental" O(4) symmetry.