Title: Graduate Course on Standard Model & Quantum Field Theory - 5B

Date: Nov 15, 2006 03:30 PM

URL: http://pirsa.org/06110028

Abstract: Graduate Course on Standard Model & Quantum Field Theory

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

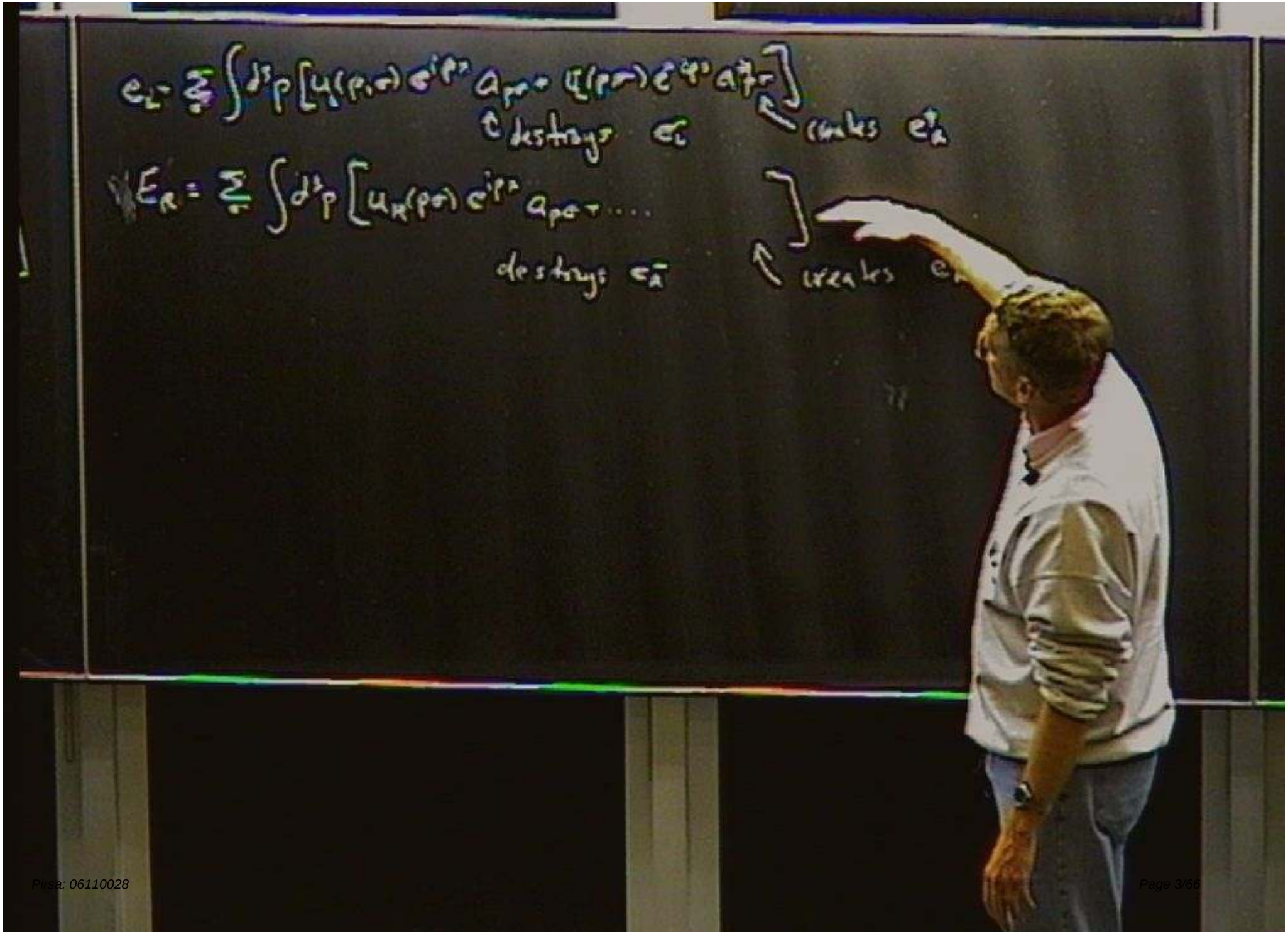
$\nu_i, e_i, E_i$ : $i = 1,2,3$ $(e, \mu, \tau)$

LH $e \xrightarrow{l_i =} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \in (1,2)_{-1/2}$ : $\gamma_L \delta l_i = \gamma_L \left[ \frac{i}{2} \omega_i \tau \cdot l_i = \frac{i}{2} \omega_i l \right]$

RH $e \longrightarrow E_i$ $\in (1,1)_{+1}$ $\gamma_L \delta E_i = \gamma_L [i \omega_i E_i]$

$u_i$ $i = 1,2,3$ $(u, c, t)$

$d_i$ : $(d, s, b)$


destroys
creates
destroys
creates

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

$\nu_i, e_i, E_i$ : $i = 1,2,3$ $(e, \mu, \tau)$

LH $e \xrightarrow{l_i =} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \in (1,2)_{-1/2}$ : $\gamma_L \delta l_i = \gamma_L \left[ \frac{i}{2} \omega_i \tau_i \cdot l_i - \frac{i}{2} \omega_1 l_i \right]$

RH $e \to E_i$ $\in (1,1)_{+1}$ $\gamma_L \delta E_i = \gamma_L [i \omega_1 E_i]$

$u_i$ $i = 1,2,3$ $(u, c, t)$

$d_i$ " $(d, s, b)$

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

$\nu_i, e_i, E_i$ : $i = 1,2,3$ $(e, \mu, \tau)$

LH $e \xrightarrow{l_i =} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \in (1,2)_{-1/2}$ : $\gamma_L \delta l_i = \gamma_L \left[ \frac{i}{2} \omega_i \tau \cdot l_i = \frac{i}{2} \omega_i l_i \right]$

RH $e \rightarrow E_i$ $\in (1,1)_{+1}$ $\gamma_L \delta E_i = \gamma_L [i \omega_i E_i]$

Rule: $Q = t_3 + Y$

$$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ipx}\, a_{pr} + v_L(p,r)\, e^{-ipx}\, a^\dagger_{pr} \right]$$

$a_{pr}$: destroys $e^-$ ; $a^\dagger_{pr}$: creates $e^+_R$

$$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ipx}\, b_{pr} + \dots \right]$$

$b_{pr}$: destroys $e^-_R$ ; creates $e^+_L$

$u_i = (u, c, t)$

$d_i = (d, s, b)$

$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ipx}\, a_{pr} + v_L(p,r)\, e^{-ipx}\, a^\dagger_{pr} \right]$

$a_{pr}$: destroys $e^-$ ; $a^\dagger_{pr}$: creates $e^+$

$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ipx}\, b_{pr} + \dots \right]$

$b$: destroys $e^-$ ; $b^\dagger$: creates $e^+$

$u_i = (u, c, t)$ ← field which destroys LH $u, c, t$.

$d_i = (d, s, b)$ ← " " " " $b$ is $\bar{d}$

$$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ipx}\, a_{pr} + v_L(p,r)\, e^{-ipx}\, a^\dagger_{pr} \right]$$

$a$ destroys $e^-$ ← creates $e^+$

$$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ipx}\, b_{pr} + \dots \right]$$

destroys $e^-$ ← creates $e^+$

$u_i = (u, c, t)$ ← field which destroys LH $u, c, t$.

$d_i = (d, s, b)$ ← " " " " $b, s, d$

$U_i$ " " " RH $u, c, t$

$D_i$ " " " RH $d, s, b$

$$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ipx}\, a_{pr} + v_L(p,r)\, e^{-ipx}\, a^\dagger_{pr} \right]$$

$a_{pr}$: destroys $e_L$; $a^\dagger_{pr}$: creates $e^+_R$

$$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ipx}\, b_{pr} + \dots \right]$$

$b_{pr}$: destroys $e_R$; creates $e^+_L$

"flavour"

$u_i = (u, c, t)$ ← field which destroys LH $u, c, t$

$d_i = (d, s, b)$ ← " " " " " b, s, d

$\left.\begin{matrix} u_i \\ d_i \end{matrix}\right\} q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$

$U_i$ " " " RH $u, c, t$

$D_i$ " " " RH $d, s, b$

$$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ipx}\, a_{pr} + \psi_L(p,r)\, e^{-ipx}\, a^\dagger_{pr} \right]$$

$a$ destroys $e^-$ ← makes $e^+$

$$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ipx}\, b_{pr} + \dots \right]$$

destroys $e^-$ ← creates $e^+$

"flavour"

$u_i^1 = (u, c, t)$ ← field which destroys LH $u, c, t$

$d_i^2 = (d, s, b)$ ← " " " " " $b, s, d$ } $q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$

$U_i^2$ " " " RH $u, c, t$

$D_i^2$ " " " RH $d, s, b$

$i = 1, 2, 3$ colour (red, yellow, blue)

$$E_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ip\cdot x}\, a_{pr} + v_L(p,r)\, e^{-ip\cdot x}\, a^\dagger_{pr} \right]$$

($a_{pr}$: destroys $e^-$; $a^\dagger_{pr}$: creates $e^+$)

$$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ip\cdot x}\, b_{pr} + \dots \right]$$

($b_{pr}$: destroys $e^-$; $b^\dagger$: creates $e^+$)

"flavour"

$u_i^z = (u, c, t)$ ← field which destroys LH $u, c, t$

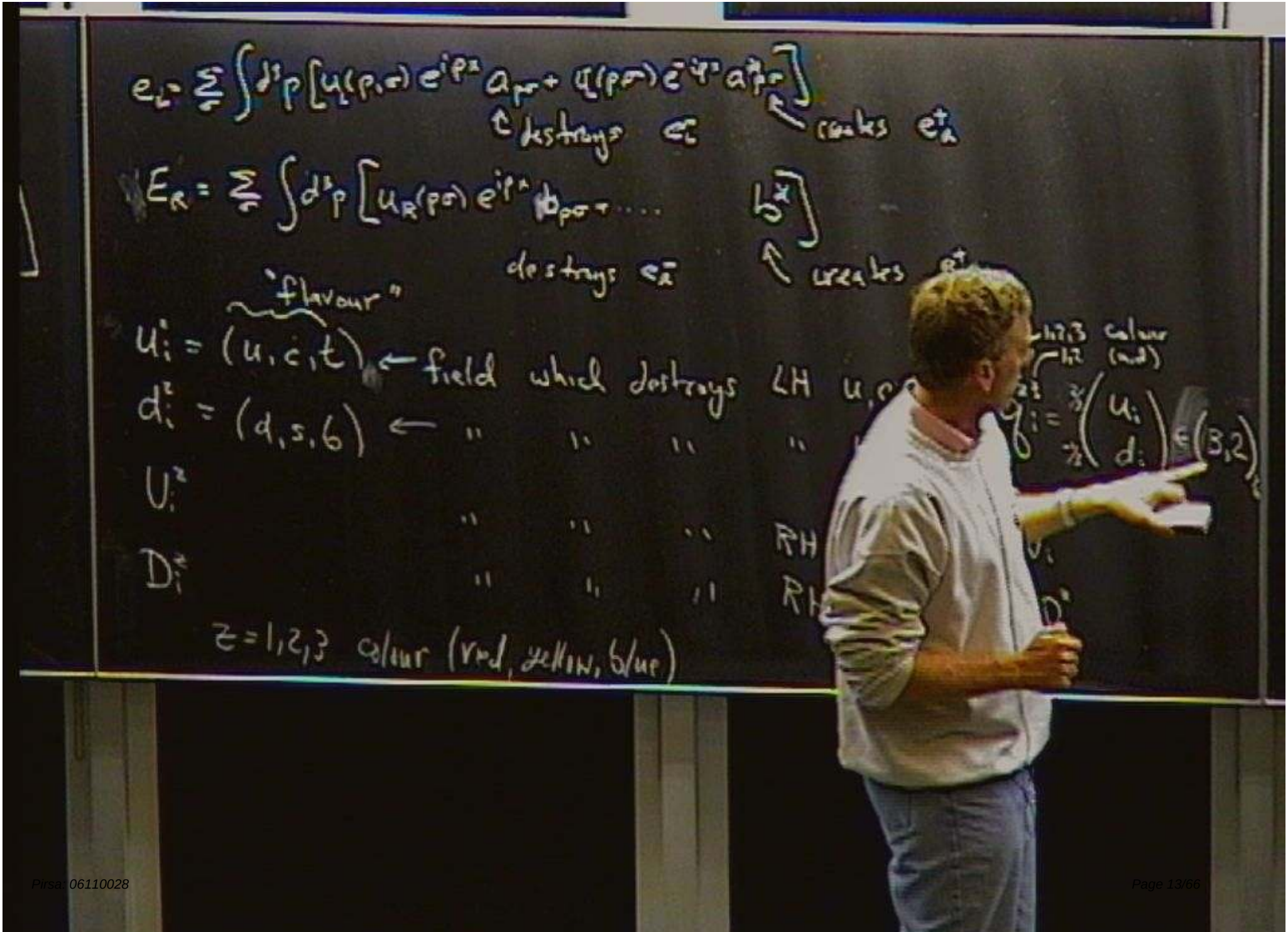
$d_i^z = (d, s, b)$ ← " " " " " $b, s, d$

$q_i^{z\,k} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$ ($k = 1,2,3$ colour (red))

$U_i^z$ " " " RH $u, c, t$ — $U_i^z$

$D_i^z$ " " " RH $d, s, b$ — $D_i^z$

$z = 1,2,3$ colour (red, yellow, blue)


destroys
creates
"flavour"
field which destroys LH
colour
(red)
RH
z = 1,2,3 colour (red, yellow, blue)

$$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ip\cdot x}\, a_{pr} + v_L(p,r)\, e^{-ip\cdot x}\, a^\dagger_{pr} \right]$$

$a$ destroys $e^-$ ; $a^\dagger$ creates $e^+$

$$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ip\cdot x}\, b_{pr} + \dots \quad b^\dagger \right]$$

$b$ destroys $e^-$ ; $b^\dagger$ creates $e^+$

"flavour"

$u_i^1 = (u, c, t)$ ← field which destroys LH $u, c, t$.

$d_i^1 = (d, s, b)$ ← " " " " $b$ is $d$

$U_i^2$ " " " RH $u, c, t$

$D_i^2$ " " " RH $d, s, b$

$q_i^{21} = \begin{pmatrix} u_i \\ d_i \end{pmatrix} \in (3,2)$ — 1,2 flavour, 1,2,3 colour (red)

$U_i^2 \in (3$

$D_i^2$

$i = 1,2,3$ colour (red, yellow, blue)

$$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ip\cdot x}\, a_{pr} + v_L(p,r)\, e^{-ip\cdot x}\, a^\dagger_{pr} \right]$$

$a_{pr}$ destroys $e^-$ ; $a^\dagger_{pr}$ creates $e^+$

$$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ip\cdot x}\, b_{pr} + \dots \right]$$

$b$ destroys $e^-$ ; $b^\dagger$ creates $e^+$

"flavour"

$u_i^1 = (u, c, t)$ ← field which destroys LH $u, c, t$

$d_i^1 = (d, s, b)$ ← " " " " $b$ is $d$

$U_i^2$ " " " RH $u, c, t$

$D_i^2$ " " " RH $d, s, b$

$i = 1,2,3$ colour (red, yellow, blue)

$q_i^{23} = \begin{pmatrix} u_i \\ d_i \end{pmatrix} \in (3,2)_{\frac{1}{6}}$ ($i$: 3 colour (red); 2: 2,3)

$U_i^* \in (\bar{3}, 1)_{-\frac{2}{3}}$

$D_i^* \in (\bar{3}, 1)_{\frac{1}{3}}$

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$$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ip\cdot x}\, a_{pr} + v_L(p,r)\, e^{-ip\cdot x}\, a^\dagger_{pr} \right]$$

$a_{pr}$ destroys $e^-$ ; $a^\dagger_{pr}$ creates $e^+$

$$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ip\cdot x}\, b_{pr} + \dots \right]$$

$b_{pr}$ destroys $e^-$ ; $b^\dagger$ creates $e^+$

"flavour"

$u_i = (u, c, t)$ ← field which destroys LH $u, c, t$

$d_i = (d, s, b)$ ← " " " " " $b$ RH $d$

$U_i$ " " " RH $u, c, t$

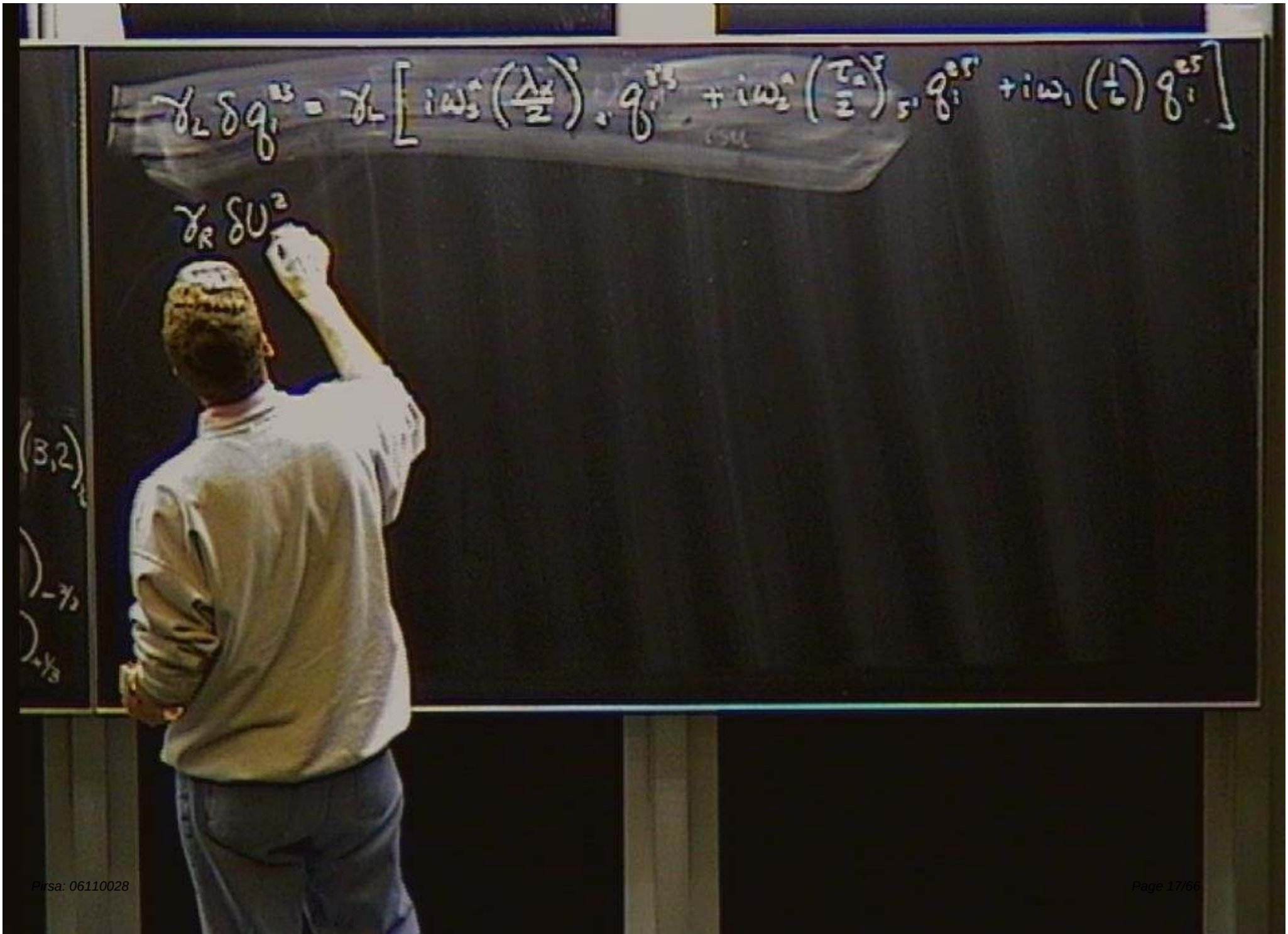
$D_i$ " " " RH $d, s, b$

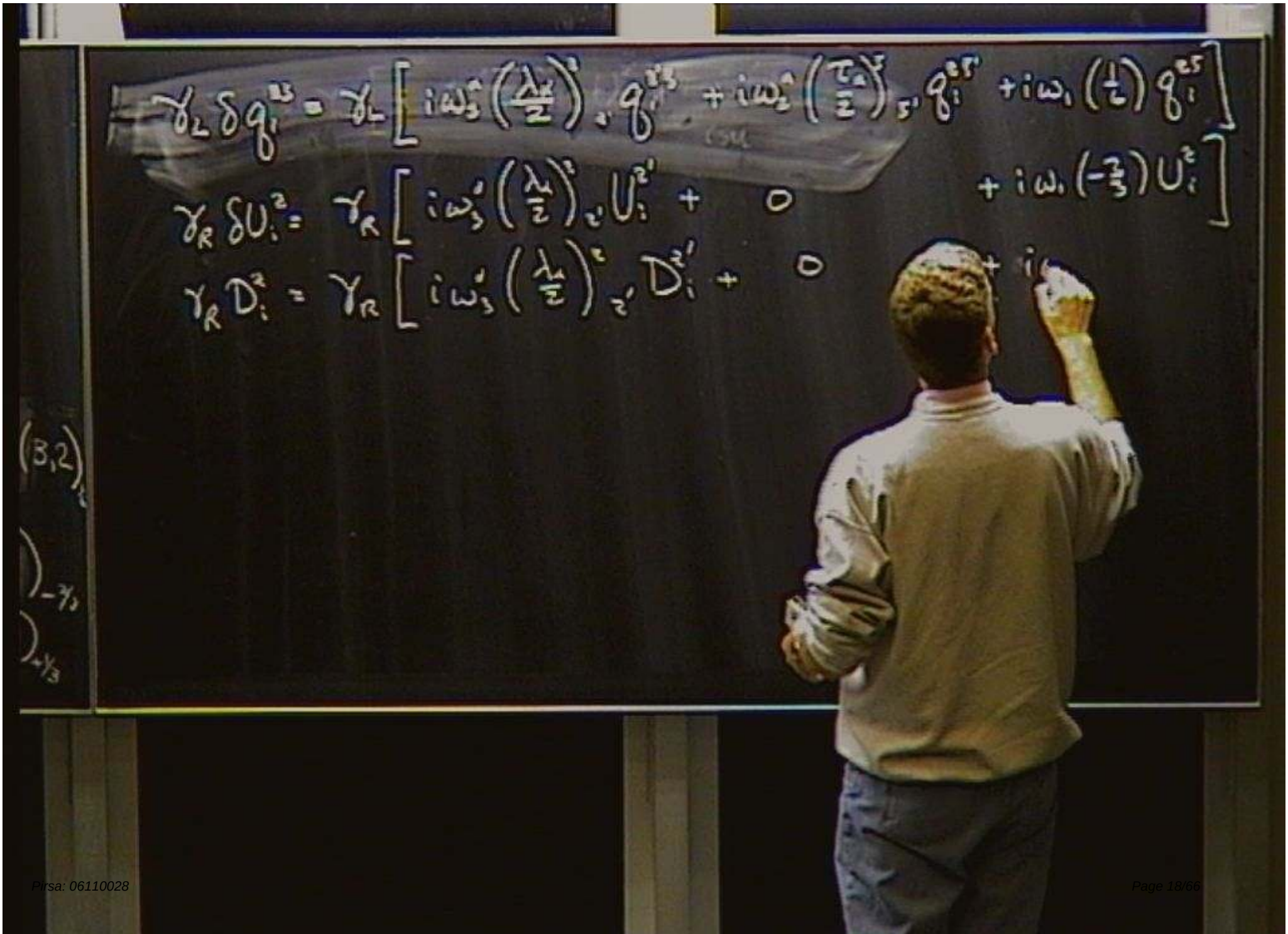
$i = 1, 2, 3$ colour (red, yellow, blue)

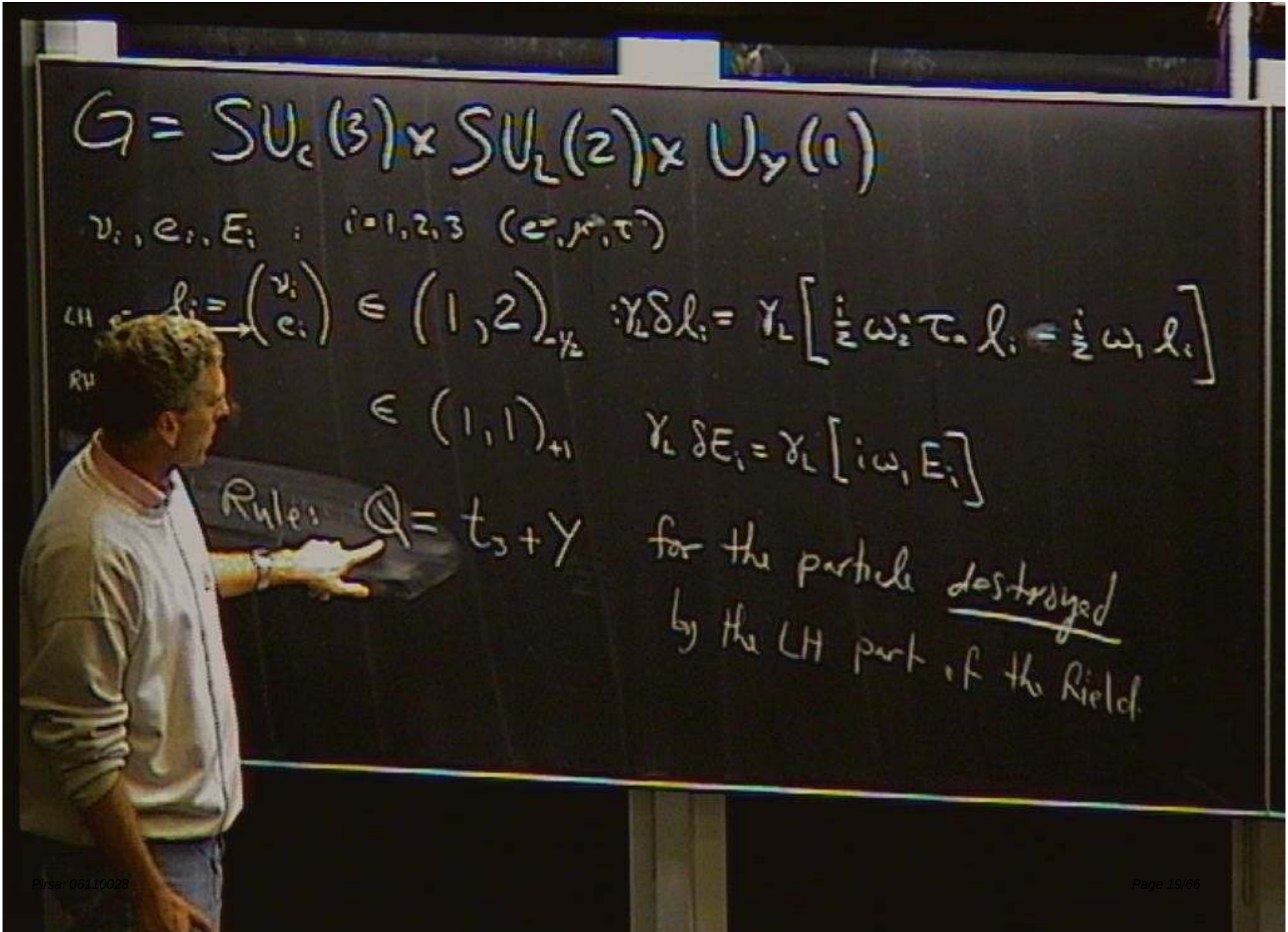
$q_i^{\alpha\beta} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}^{\alpha\beta} \in (3, 2)_{\frac{1}{6}}$ ($\alpha$: 1,2,3 colour (red); $\beta$: 1,2)

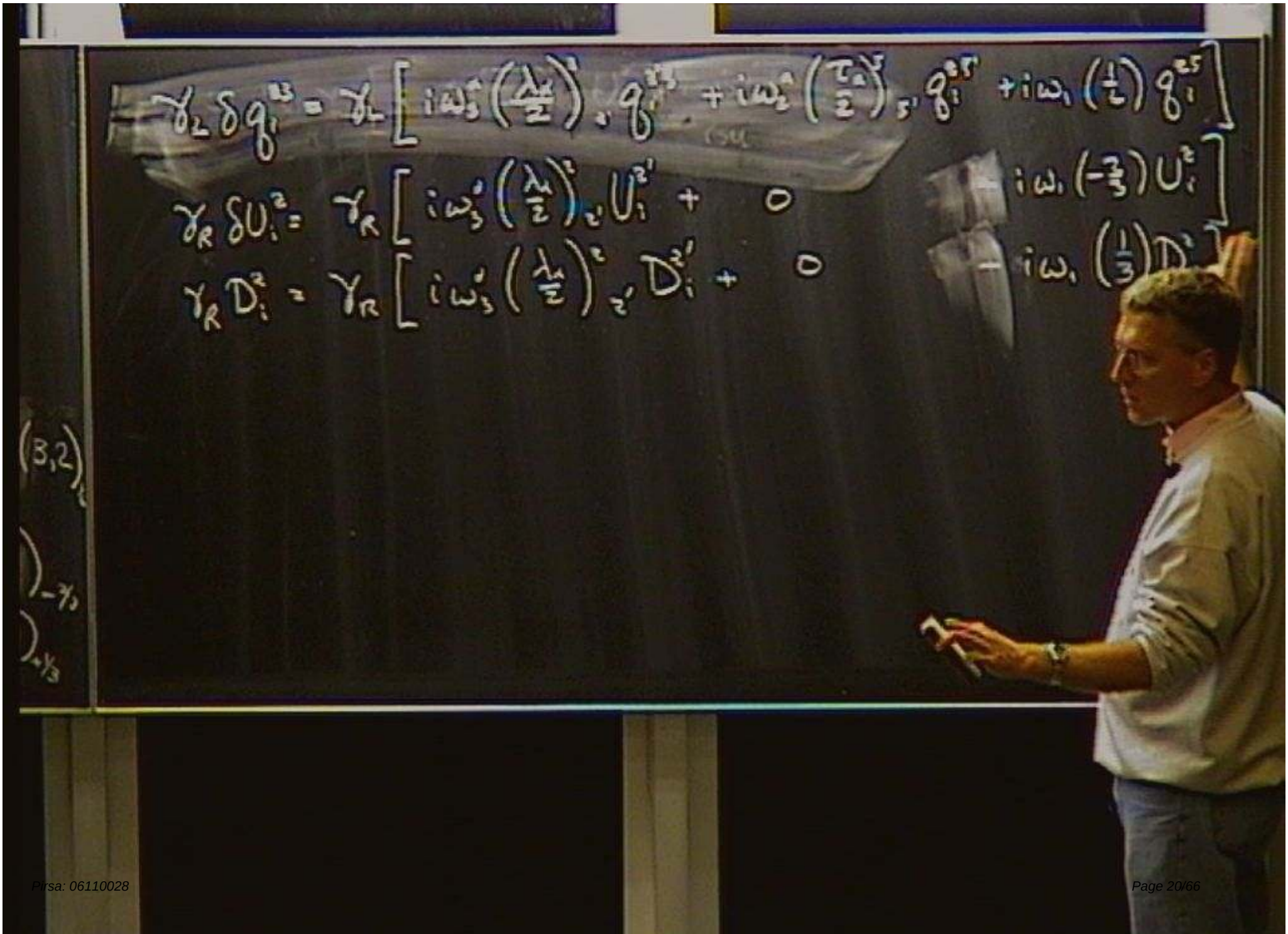
$U_i^\dagger \in (\bar{3}, 1)_{-\frac{2}{3}}$

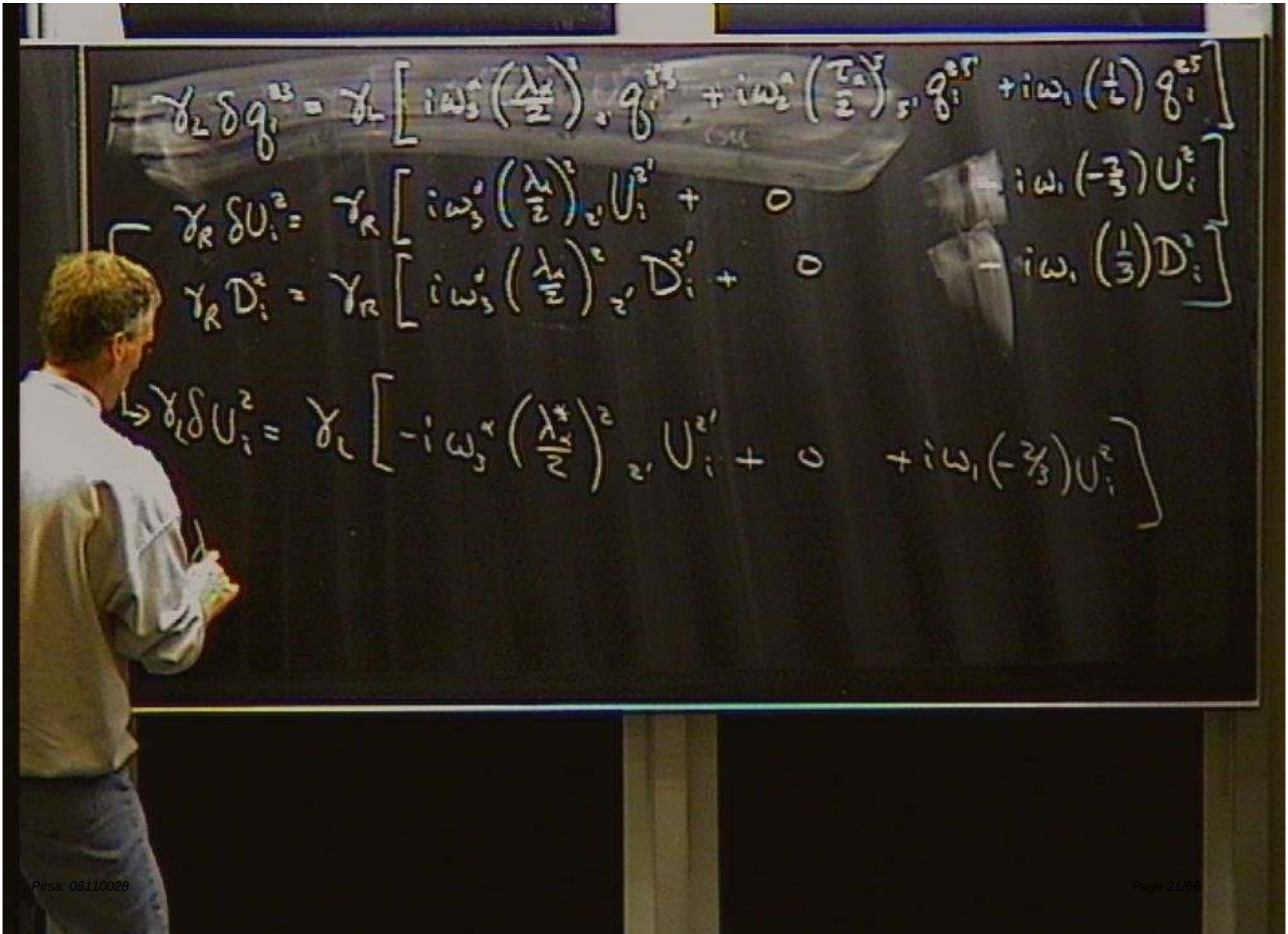
$D_i^\dagger \in (\bar{3}, 1)_{\frac{1}{3}}$


$-\gamma_L \delta q_i^{ij} = \gamma_L \left[ i\omega_3 \left(\frac{\lambda_3}{2}\right)^{ij} q_i^{rs} + i\omega_2 \left(\frac{\tau_2}{2}\right)^{rr'} q_i^{sr'} + i\omega_1 \left(\frac{1}{2}\right) q_i^{sr} \right]$
$\gamma_R \delta U^2$
(3,2)


$-\gamma_L \delta q_i^{23} = \gamma_L \left[ i\omega_3' \left(\frac{\lambda_3}{2}\right)_{3'} q_i^{23} + i\omega_2' \left(\frac{\tau_2}{2}\right)_{5'} q_i^{25} + i\omega_1 \left(\frac{1}{6}\right) q_i^{25} \right]$
$\gamma_R \delta U_i^2 = \gamma_R \left[ i\omega_3' \left(\frac{\lambda_3}{2}\right)_{3'} U_i^{2'} + 0 + i\omega_1 \left(-\frac{2}{3}\right) U_i^2 \right]$
$\gamma_R D_i^2 = \gamma_R \left[ i\omega_3' \left(\frac{\lambda_3}{2}\right)_{3'} D_i^{2'} + 0 + i($
(3,2)


$G = SU_c(3) \times SU_L(2) \times U_Y(1)$
$\nu_i, e_i, E_i$ : $i = 1,2,3$ $(e, \mu, \tau)$
LH $l_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \in (1,2)_{-1/2}$ $: \gamma_L \delta l_i = \gamma_L \left[ \frac{i}{2} \omega_i \tau_i l_i = \frac{i}{2} \omega_i l_i \right]$
RH $\in (1,1)_{+1}$ $\gamma_L \delta E_i = \gamma_L [i \omega_i E_i]$
Rule: $Q = t_3 + Y$ for the particle destroyed by the LH part of the field


$-\gamma_L \delta q_i^{2s} = \gamma_L \left[ i\omega_3' \left(\frac{\lambda_3}{2}\right)_{s'} q_i^{2s'} + i\omega_2' \left(\frac{\tau_a}{2}\right)_{s'} q_i^{2s'} + i\omega_1 \left(\frac{1}{2}\right) q_i^{2s} \right]$
$\gamma_R \delta U_i^{2s} = \gamma_R \left[ i\omega_3' \left(\frac{\lambda_3}{2}\right)_{s'} U_i^{2s'} + 0 - i\omega_1 \left(-\frac{2}{3}\right) U_i^{2s} \right]$
$\gamma_R D_i^{2s} = \gamma_R \left[ i\omega_3' \left(\frac{\lambda_3}{2}\right)_{s'} D_i^{2s'} + 0 - i\omega_1 \left(\frac{1}{3}\right) D_i^{2s} \right]$
(3,2)


$-\gamma_L \delta q_i^{2s} = \gamma_L \left[ i\omega_3' \left(\frac{\lambda_3'}{2}\right)_{s'} q_i^{2s'} + i\omega_2' \left(\frac{\tau_a'}{2}\right)_{s'} q_i^{2s'} + i\omega_1 \left(\frac{1}{2}\right) q_i^{2s} \right]$
$\gamma_R \delta U_i^{2s} = \gamma_R \left[ i\omega_3' \left(\frac{\lambda_3'}{2}\right)_{s'} U_i^{2s'} + 0 - i\omega_1 \left(-\frac{2}{3}\right) U_i^{2s} \right]$
$\gamma_R D_i^{2s} = \gamma_R \left[ i\omega_3' \left(\frac{\lambda_3'}{2}\right)_{s'} D_i^{2s'} + 0 - i\omega_1 \left(\frac{1}{3}\right) D_i^{2s} \right]$
$\hookrightarrow \gamma_L \delta U_i^{2s} = \gamma_L \left[ -i\omega_3' \left(\frac{\lambda_3'^T}{2}\right)_{s'} U_i^{2s'} + 0 + i\omega_1 \left(-\frac{2}{3}\right) U_i^{2s} \right]$

$$-\gamma_L \delta q_i^{a\alpha} = \gamma_L \left[ i\omega_3 \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} q_i^{\beta 3} + i\omega_2 \left(\frac{\tau_A \gamma^r}{2}\right)_{s's} q_i^{as'r} + i\omega_1 \left(\frac{1}{2}\right) q_i^{as} \right]$$

$$\gamma_R \delta U_i^a = \gamma_R \left[ i\omega_3 \left(\frac{\lambda^a}{2}\right)_{z'z} U_i^{z'} + 0 - i\omega_1 \left(-\frac{2}{3}\right) U_i^z \right]$$

$$\gamma_R D_i^a = \gamma_R \left[ i\omega_3 \left(\frac{\lambda^a}{2}\right)_{z'z} D_i^{z'} + 0 - i\omega_1 \left(\frac{1}{3}\right) D_i^z \right]$$

$$\rightarrow \gamma_L \delta U_i^z = \gamma_L \left[ -i\omega_3 \left(\frac{\lambda^{a*}}{2}\right)_{z'z} U_i^{z'} + 0 + i\omega_1 \left(-\frac{2}{3}\right) U_i^z \right]$$

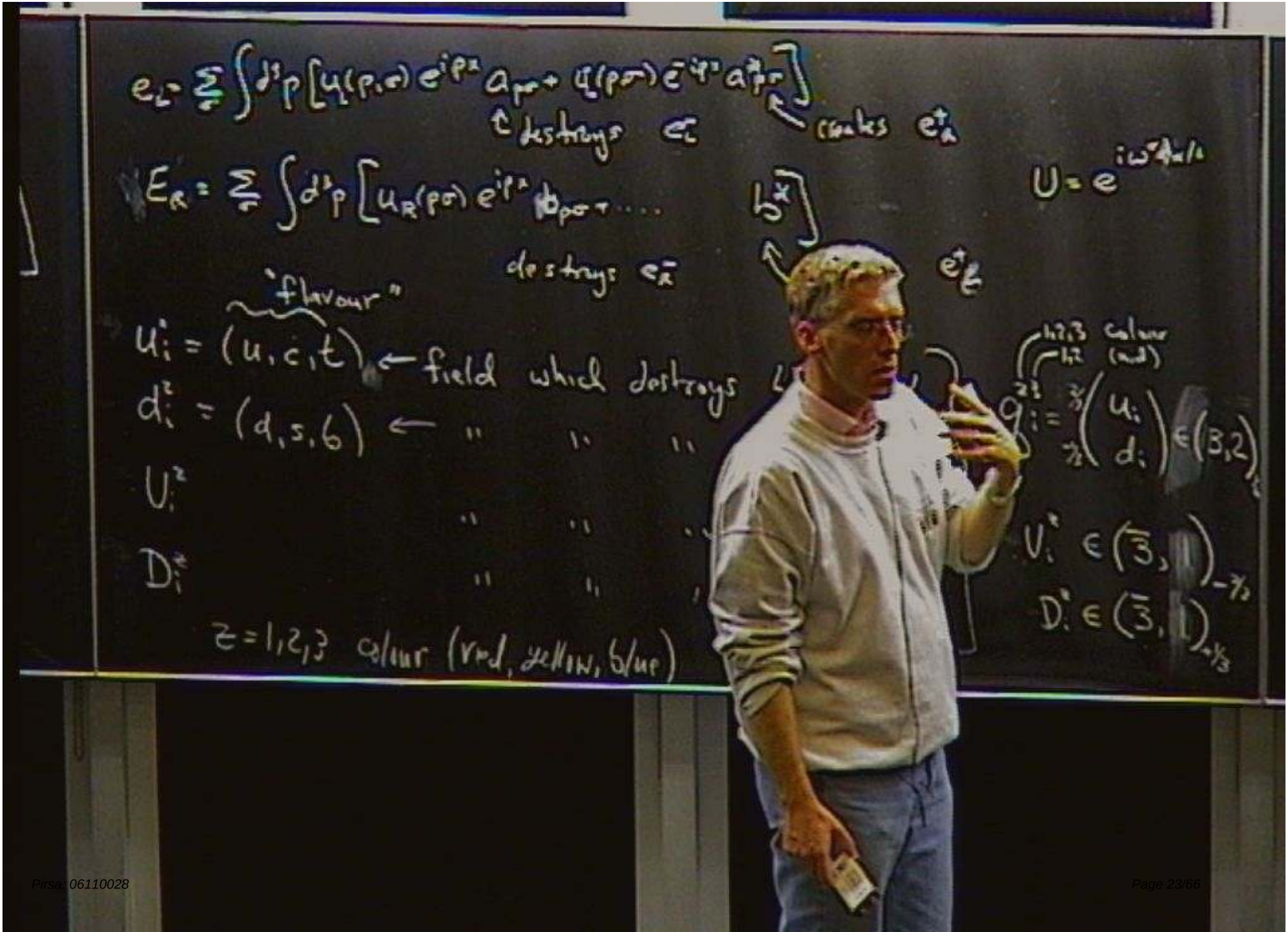
$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\lambda_a$

$(3,2)_{\frac{1}{3}}$

$)_{-\frac{1}{3}}$

$)_{-\frac{1}{3}}$


$e_L = \sum_r \int d^3p \left[ u_L(p,r)\, e^{ipx}\, a_{pr} + v_L(p,r)\, e^{-ipx}\, a^\dagger_{pr} \right]$
destroys $e_L$
creates $e^\dagger_R$
$E_R = \sum_r \int d^3p \left[ u_R(p,r)\, e^{ipx}\, b_{pr} + \ldots \right]$
destroys $e_R$
$U = e^{i\omega^a \lambda_a / 2}$
$e^\dagger_L$
"flavour"
$u_i = (u, c, t)$ ← field which destroys
$d_i = (d, s, b)$ ← " " "
$U_i$ " " "
$D_i$ " " "
$i = 1,2,3$ colour (red, yellow, blue)
$q^{ai}_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix} \in (3,2)_{1/6}$
$U^*_i \in (\bar{3}, 1)_{-2/3}$
$D^*_i \in (\bar{3}, 1)_{+1/3}$

$$-\gamma_L \delta q_i^{a\alpha} = \gamma_L \left[ i\omega_3^{\alpha} \left(\frac{\lambda^{\alpha}}{2}\right)_{a'} q_i^{a'\alpha} + i\omega_2^{\alpha} \left(\frac{\tau_A}{2}\right)_{s'} q_i^{s'\alpha} + i\omega_1 \left(\frac{1}{2}\right) q_i^{a\alpha} \right]$$

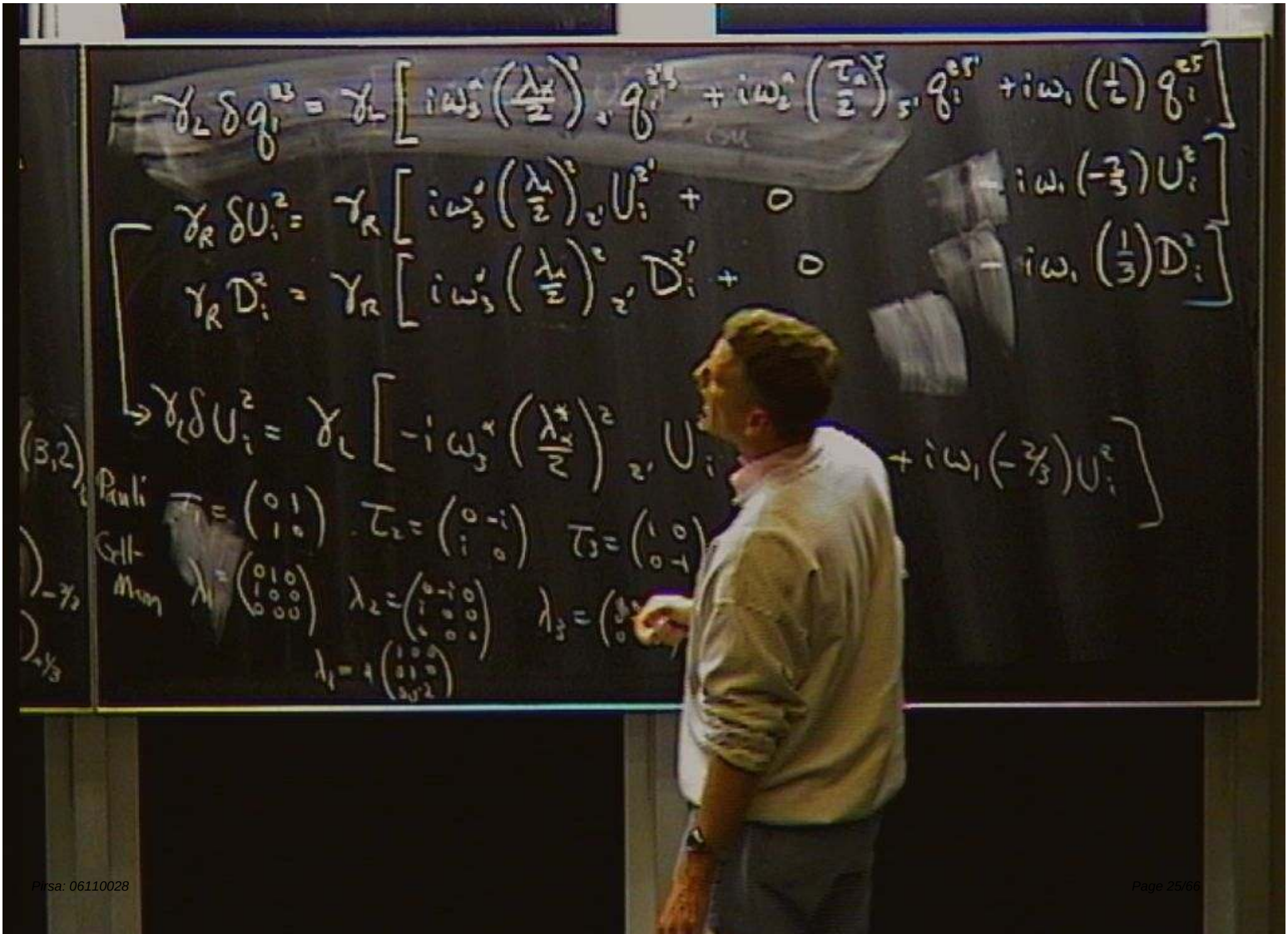
$$\left[ \gamma_R \delta U_i^{a} = \gamma_R \left[ i\omega_3^{\alpha} \left(\frac{\lambda^{\alpha}}{2}\right)_{a'} U_i^{a'} + 0 - i\omega_1 \left(-\frac{2}{3}\right) U_i^{a} \right] \right.$$

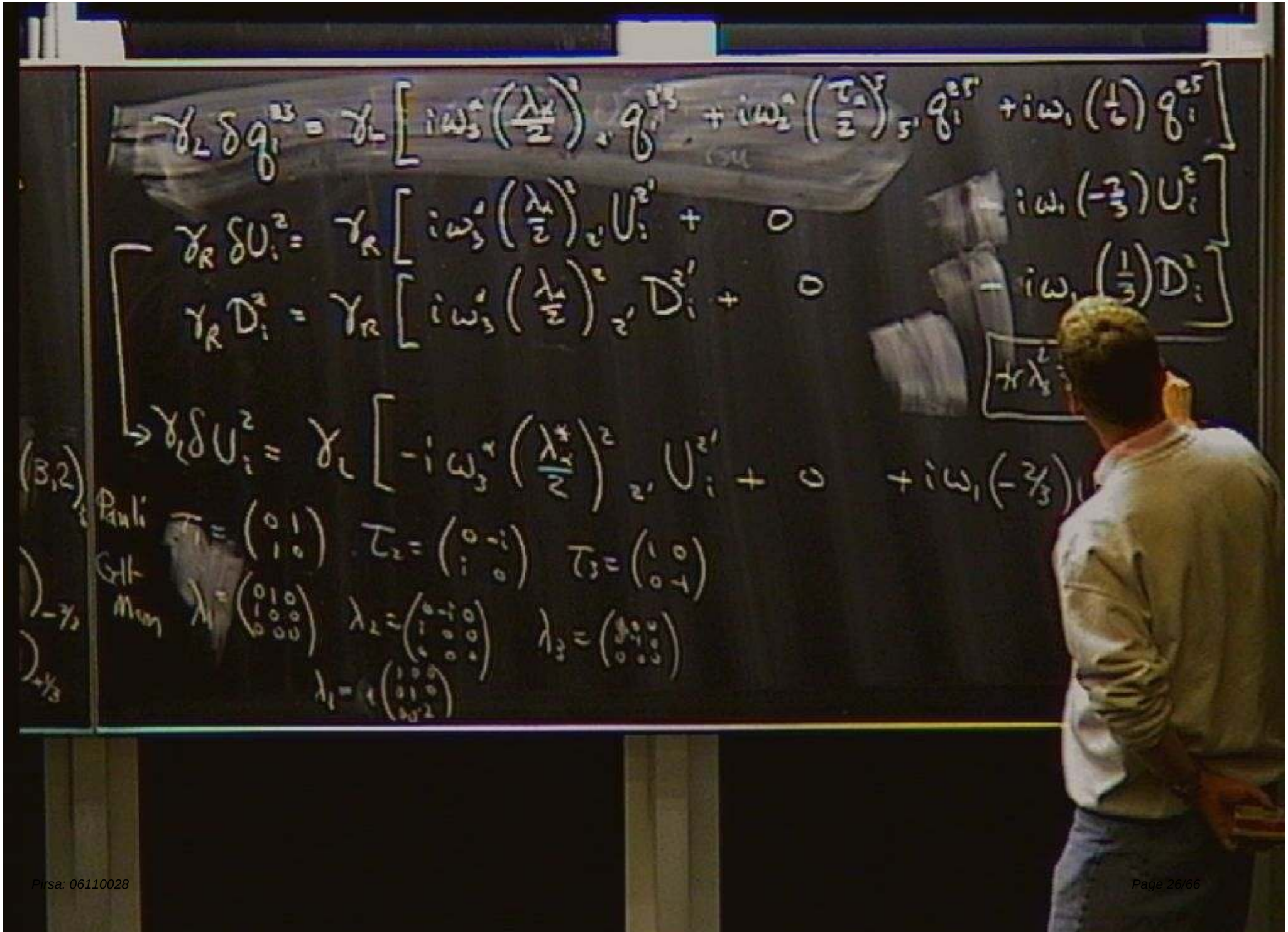
$$\left. \gamma_R D_i^{a} = \gamma_R \left[ i\omega_3^{\alpha} \left(\frac{\lambda^{\alpha}}{2}\right)_{a'} D_i^{a'} + 0 - i\omega_1 \left(\frac{1}{3}\right) D_i^{a} \right] \right]$$

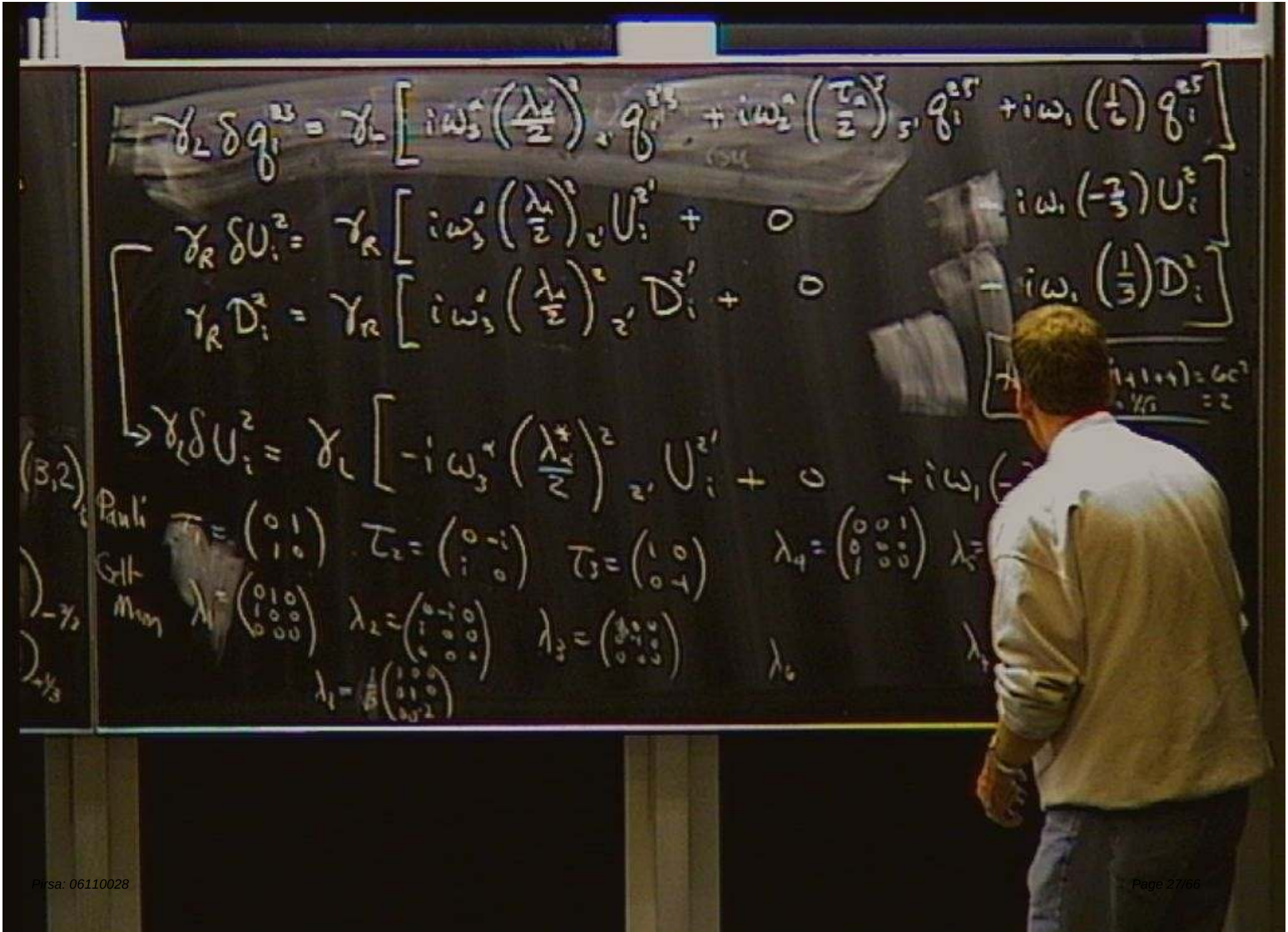
$$\hookrightarrow \gamma_L \delta U_i^{a} = \gamma_L \left[ -i\omega_3^{\alpha} \left(\frac{\lambda^{\alpha}}{2}\right)_{a'} U_i^{a'} + 0 + i\omega_1 \left(-\frac{2}{3}\right) U_i^{a} \right]$$

Pauli Gell-Mann $\lambda_1$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$


Pauli
Gell-Mann


$-\gamma_L \delta q_i^{a\alpha} = \gamma_L \left[ i\omega_3^a \left(\frac{\lambda^a}{2}\right)_{ii'} q_{i'}^{a\alpha} + i\omega_2^a \left(\frac{\tau_2}{2}\right)_{ss'} q_i^{as'} + i\omega_1 \left(\frac{1}{6}\right) q_i^{as} \right]$
$\gamma_R \delta U_i^a = \gamma_R \left[ i\omega_3^a \left(\frac{\lambda^a}{2}\right)_{ii'} U_{i'}^a + 0 - i\omega_1 \left(-\frac{2}{3}\right) U_i^a \right]$
$\gamma_R D_i^a = \gamma_R \left[ i\omega_3^a \left(\frac{\lambda^a}{2}\right)_{ii'} D_{i'}^a + 0 - i\omega_1 \left(\frac{1}{3}\right) D_i^a \right]$
$\gamma_L \delta U_i^a = \gamma_L \left[ -i\omega_3^a \left(\frac{\lambda^{a*}}{2}\right)_{ii'} U_{i'}^a + 0 + i\omega_1 \left(-\frac{2}{3}\right) \right]$
(3,2)
Pauli
Gell-Mann
$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$


Pauli
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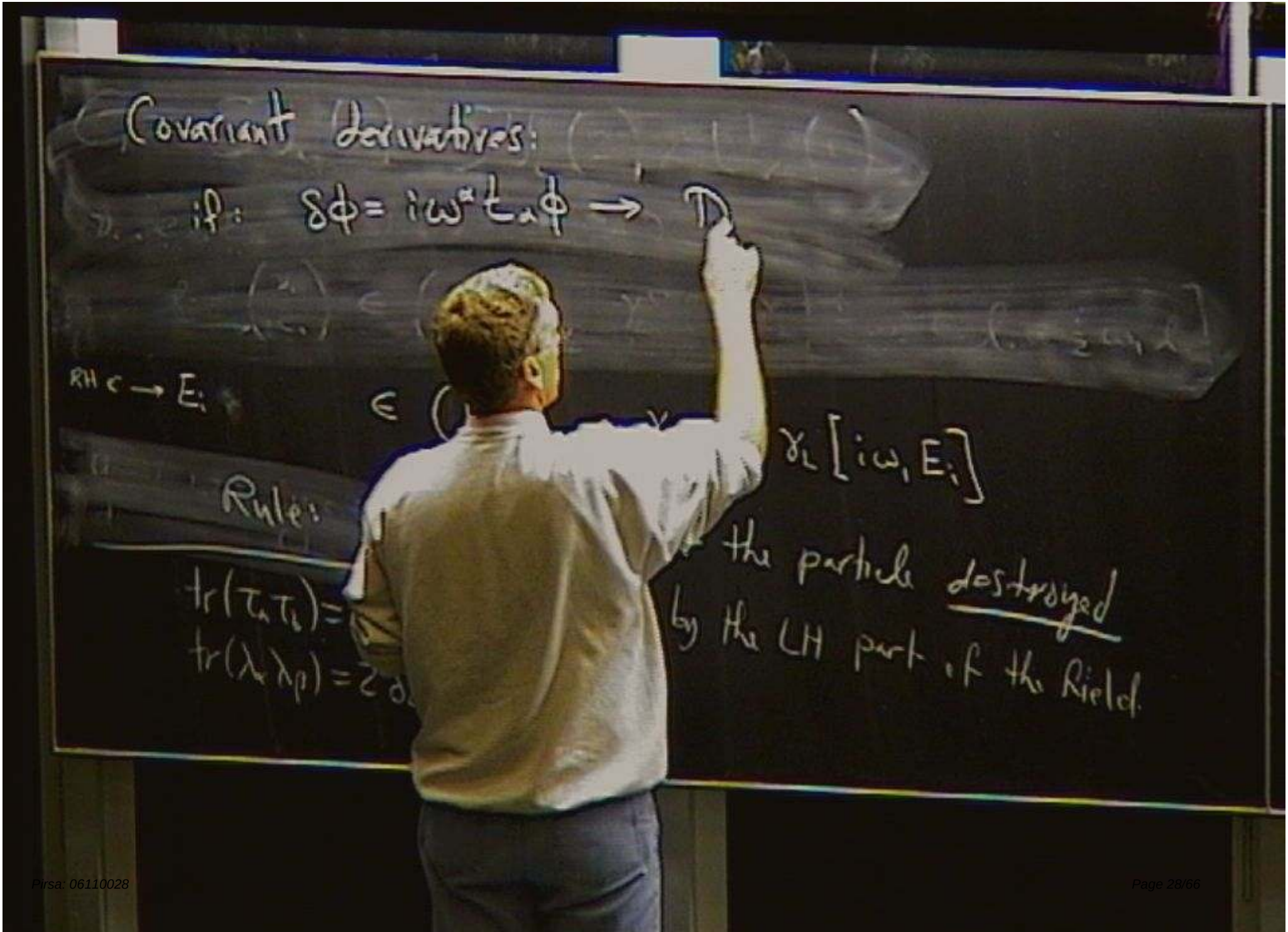

Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow$
RH $c \rightarrow E_i$
$\in$ (
$\gamma_L [i\omega_i E_i]$
Rule:
the particle destroyed
by the LH part of the field.
$tr(\tau_a \tau_b) =$
$tr(\lambda_a \lambda_b) = 2\delta$

Covariant derivatives:

if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - i A^a_\mu t_a \phi$

$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$

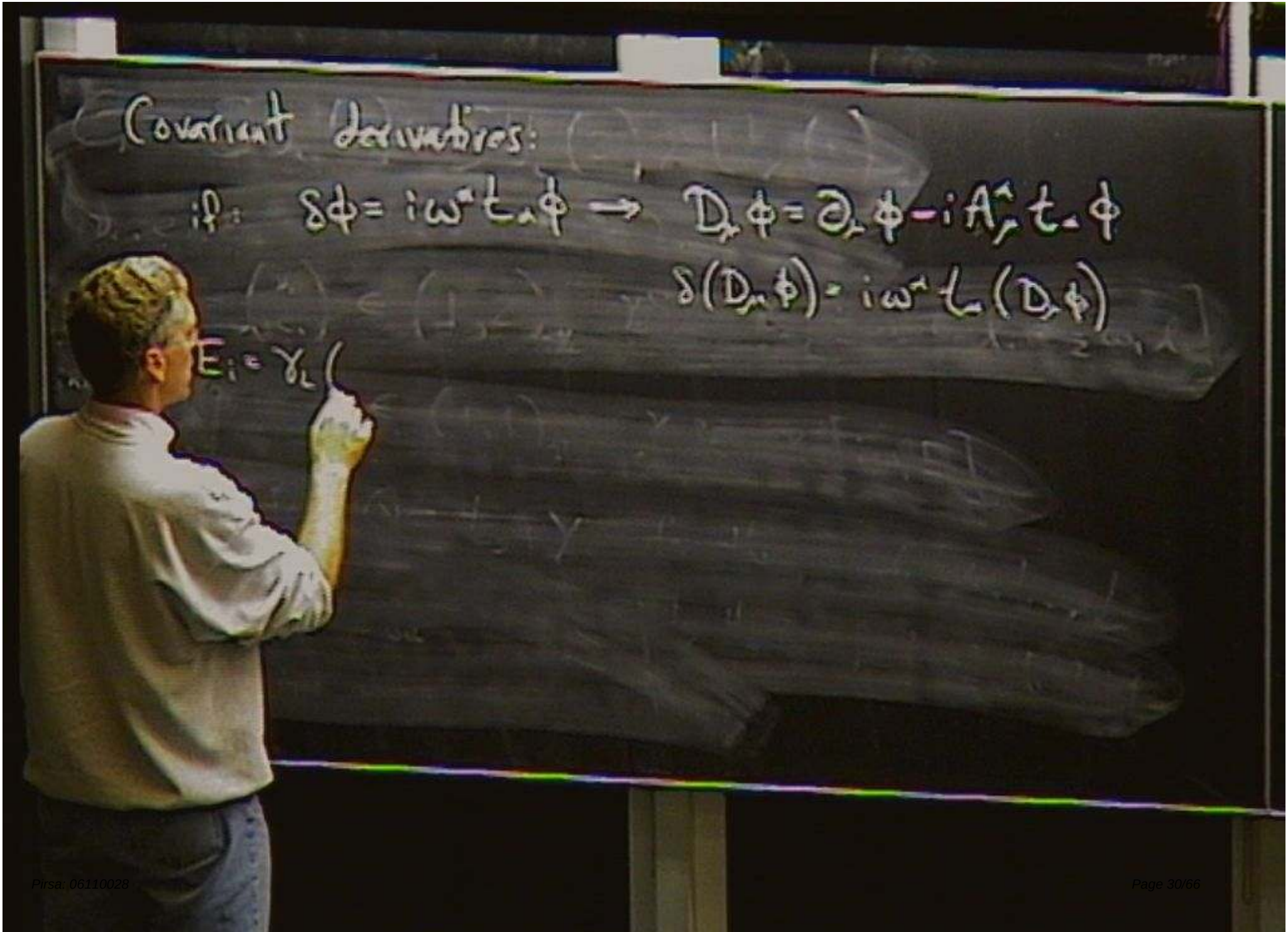
RH $c \rightarrow E_i$ $\in (1,1)_{+1}$ $\gamma_L \delta E_i = \gamma_L [i\omega_i E_i]$

: $Q = t_3 + Y$ for the particle destroyed by the LH part of the field.

$\delta_{ik}$

$\delta_{\alpha\beta}$

Pirsa: 06110028


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - iA^a_\mu t_a\phi$
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$
$E_i = \gamma_L($

Covariant derivatives:

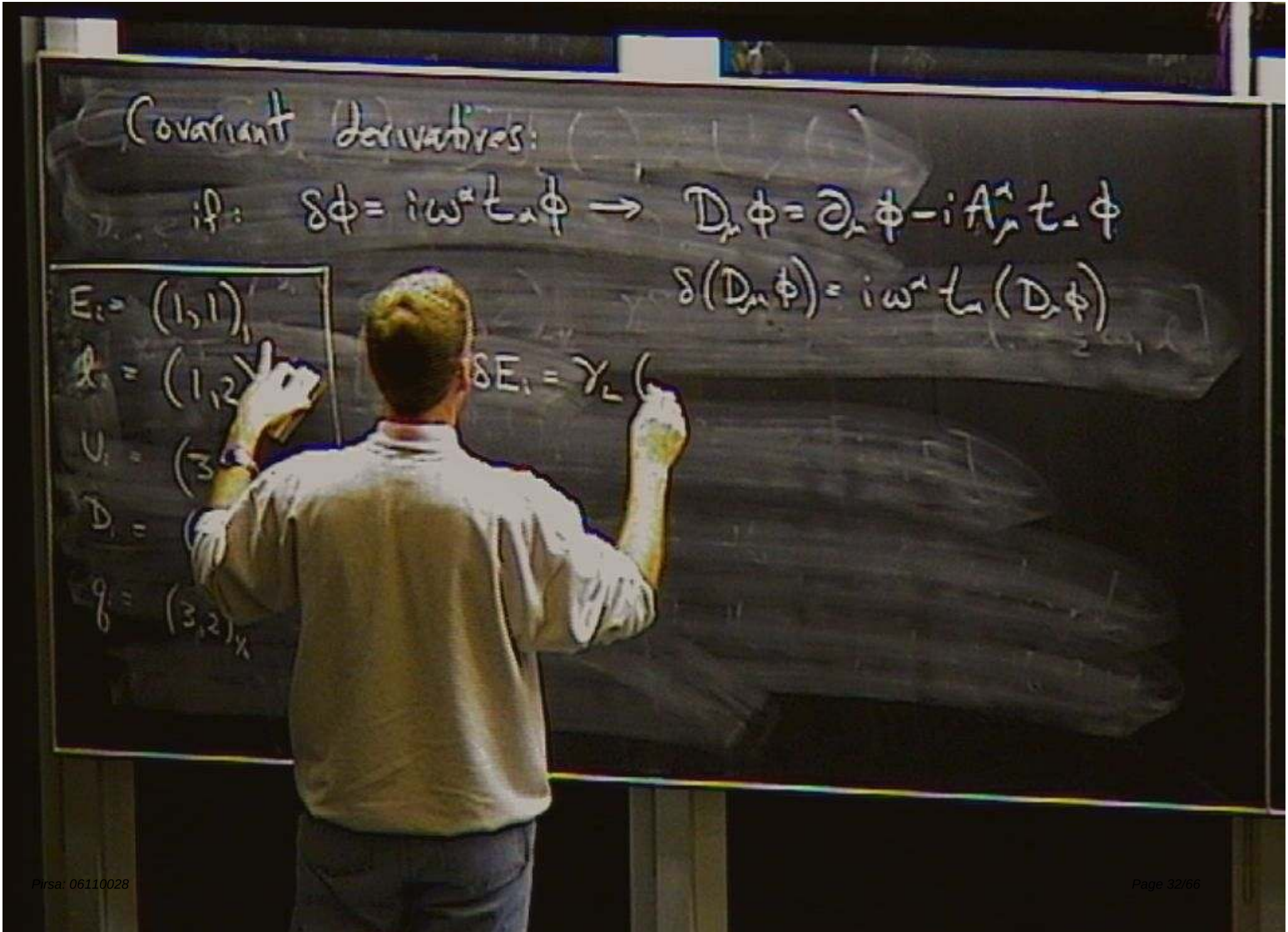
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - i A^a_\mu t_a \phi$

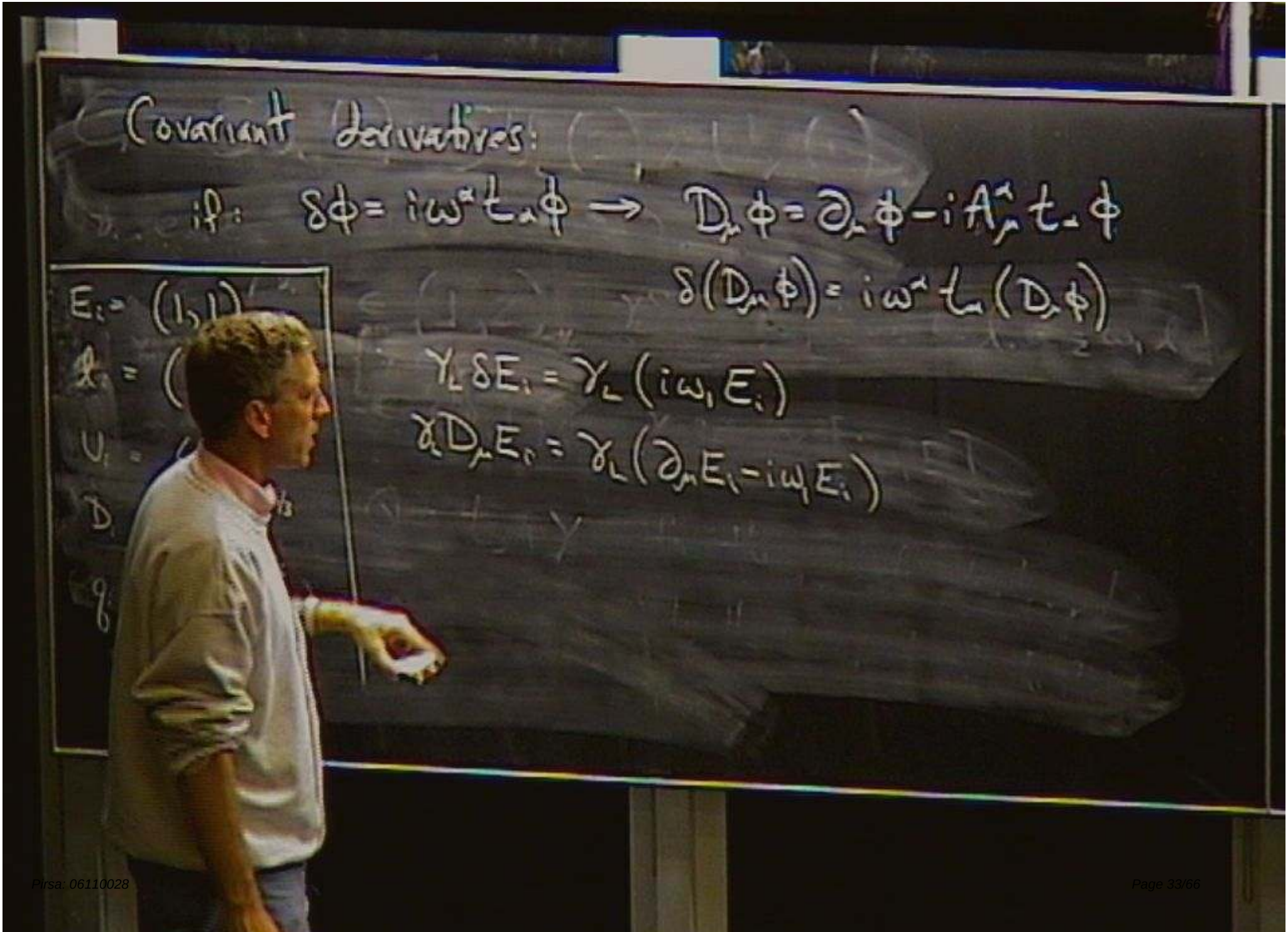
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$

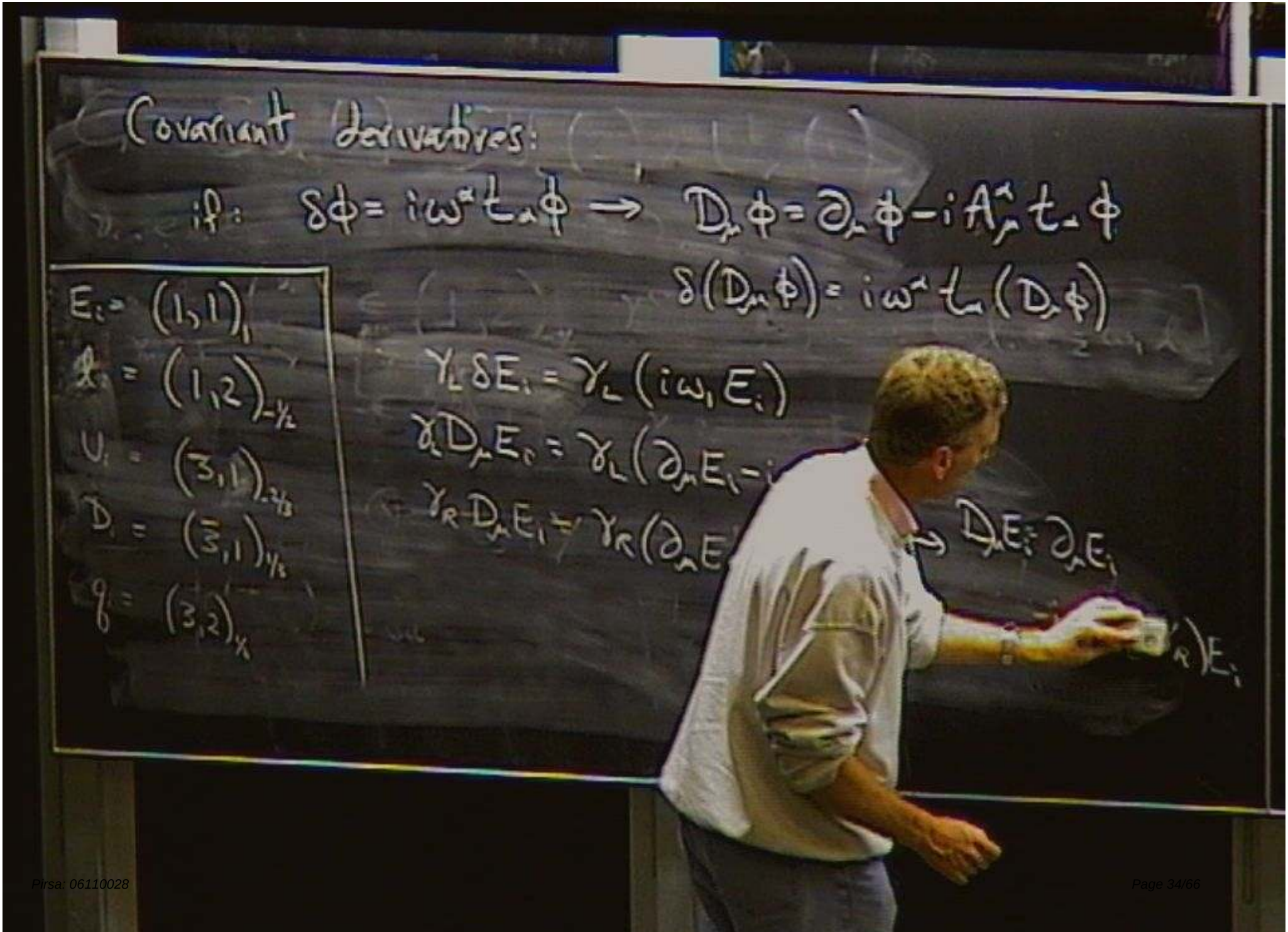
$E_L = (1,2,1)_{-1/2}$

$\ell_L = (1,2)_{-1/2}$

$U$


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - i A^a_\mu t_a \phi$
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$
$E_L = (1,1)_1$
$L_L = (1,2)$
$U_L = (3$
$D_L =$
$Q = (3,2)_{1/6}$
$\delta E_L = \gamma_L$


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - i A^a_\mu t_a \phi$
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$
$\gamma_L \delta E_i = \gamma_L (i\omega_i E_i)$
$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i - i\omega_i E_i)$


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - iA^a_\mu t_a\phi$
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$
$E_i = (1,1)_1$
$L_i = (1,2)_{-1/2}$
$U_i = (3,1)_{-2/3}$
$D_i = (\bar{3},1)_{1/3}$
$Q_i = (3,2)_{1/6}$
$\gamma_L \delta E_i = \gamma_L (i\omega_i E_i)$
$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i -$
$\gamma_R D_\mu E_i = \gamma_R (\partial_\mu E$
$\rightarrow D_\mu E_i = \partial_\mu E_i$
$\gamma_R) E_i$

Covariant derivatives:

if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - iA^a_\mu t_a \phi$

$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$

$E_i = (1,1)_1$

$L_i = (1,2)_{-1/2}$

$U_i = (\bar{3},1)_{-2/3}$

$D_i = (\bar{3},1)_{1/3}$

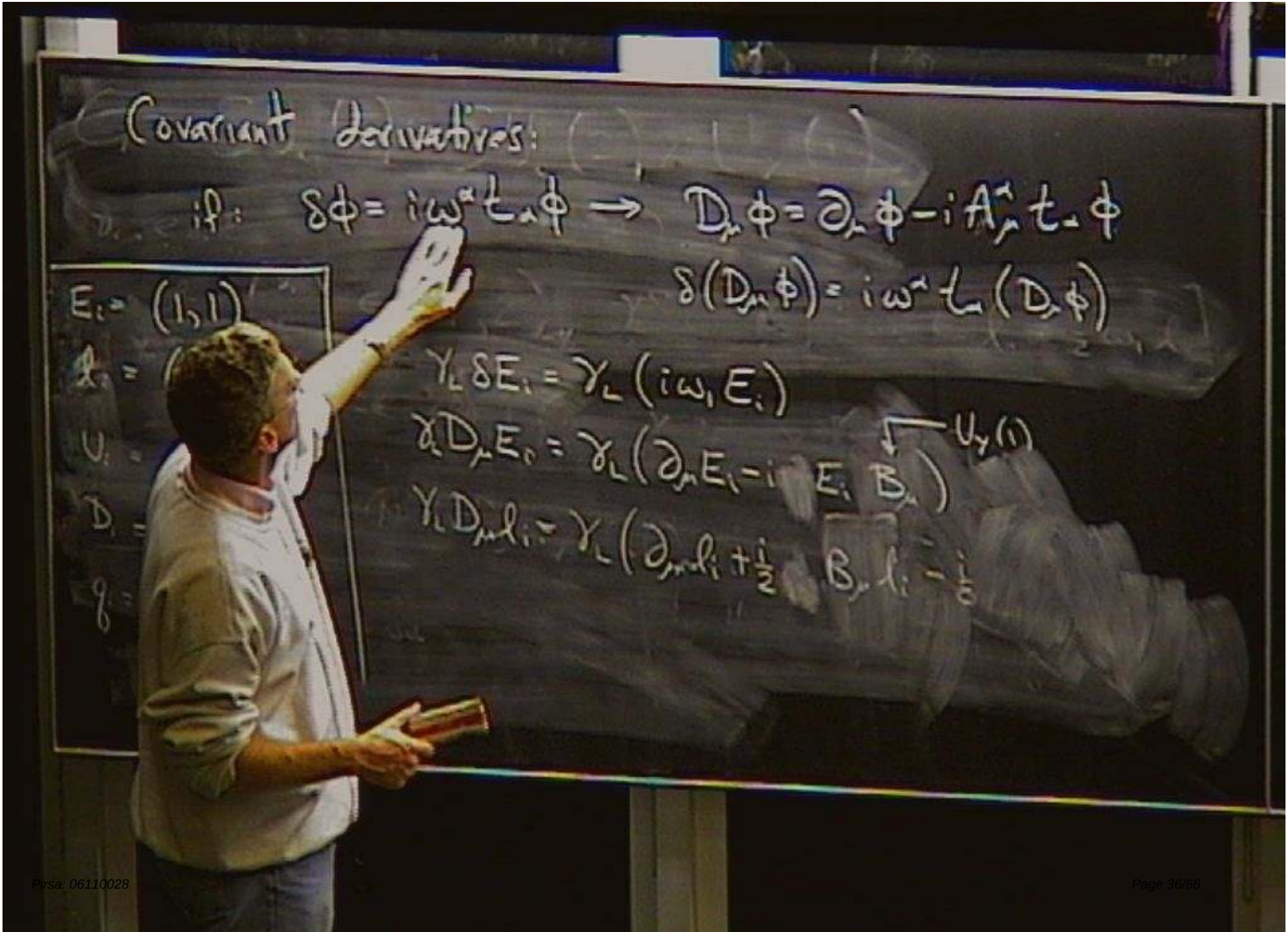
$Q = (3,2)_{1/6}$

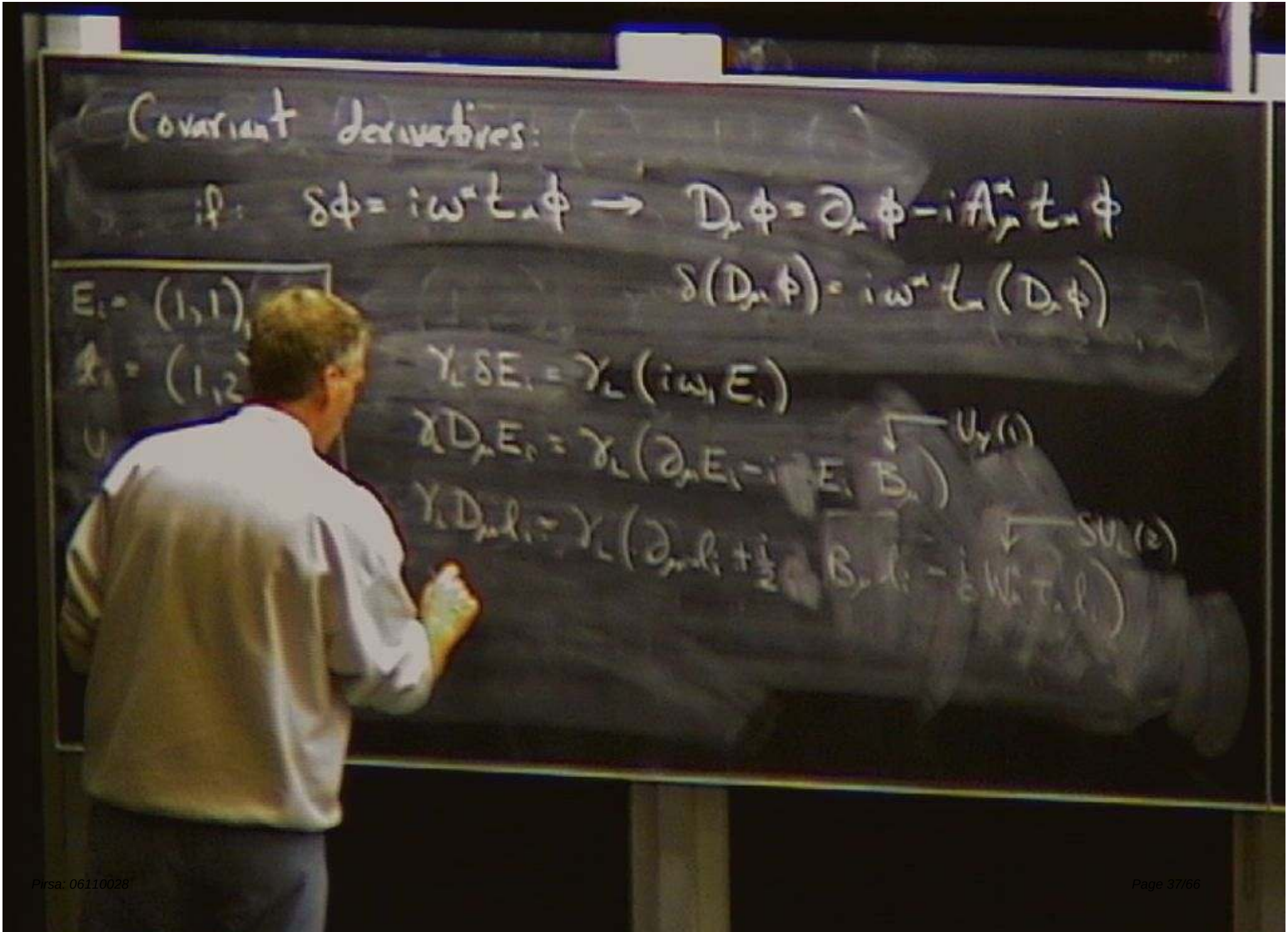
$\gamma_L \delta E_i = \gamma_L (i\omega_i E_i)$

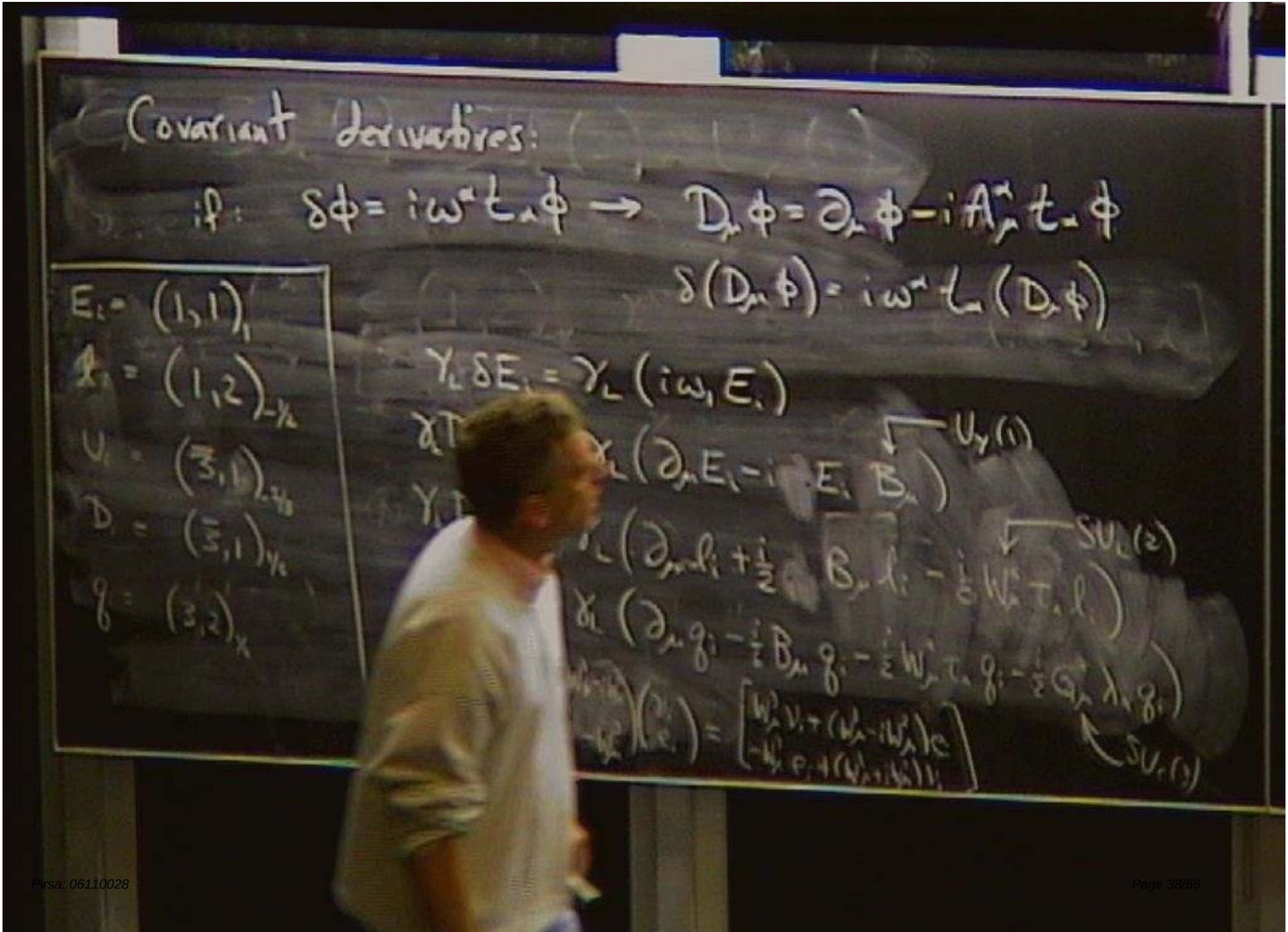
$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i - i\omega_i E_i)$

$\gamma_R D_\mu E_i = \gamma_R (\partial_\mu E_i + i\omega_i E_i)$

$\rightarrow D_\mu E_i = \partial_\mu E_i - i\omega_i \gamma_5 E_i$


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - i A^a_\mu t_a \phi$
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$
$E_i = (1,1)$
$\gamma_L \delta E_i = \gamma_L (i\omega_i E_i)$
$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i - i$ $E_i$ $B_\mu)$
$U_Y(1)$
$\gamma_L D_\mu l_i = \gamma_L (\partial_\mu l_i + \frac{i}{2}$ $B_\mu l_i - \frac{i}{2}$


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - i A^a_\mu t_a \phi$
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$
$E_L = (1,1)_{-1}$
$\ell_L = (1,2)_{-\frac{1}{2}}$
$\gamma_L \delta E_L = \gamma_L (i\omega_1 E_L)$
$\gamma D_\mu E_L = \gamma_L (\partial_\mu E_L - i$ $E_L B_\mu)$
$\gamma_L D_\mu \ell_L = \gamma_L (\partial_\mu \ell_L + \frac{i}{2}$ $B_\mu \ell_L - \frac{i}{2} W^a_\mu \tau_a \ell_L)$
$U_Y(1)$
$SU_L(2)$


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - i A^a_\mu t_a \phi$
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$
$E_L = (1,1)_1$
$l_L = (1,2)_{-1/2}$
$U_R = (3,1)_{-4/3}$
$D_R = (3,1)_{2/3}$
$q_L = (3,2)_{1/3}$
$\gamma_L \delta E_L = \gamma_L (i\omega_1 E_L)$
$U_Y(1)$
$SU_L(2)$
$SU_C(3)$

Covariant derivatives:

if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - iA^a_\mu t_a\phi$

$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$

$E_i = (1,1)_1$

$l_i = (1,2)_{-1/2}$

$U_i = (\bar{3},1)_{-2/3}$

$D_i = (\bar{3},1)_{1/3}$

$q_i = (3,2)_{1/6}$

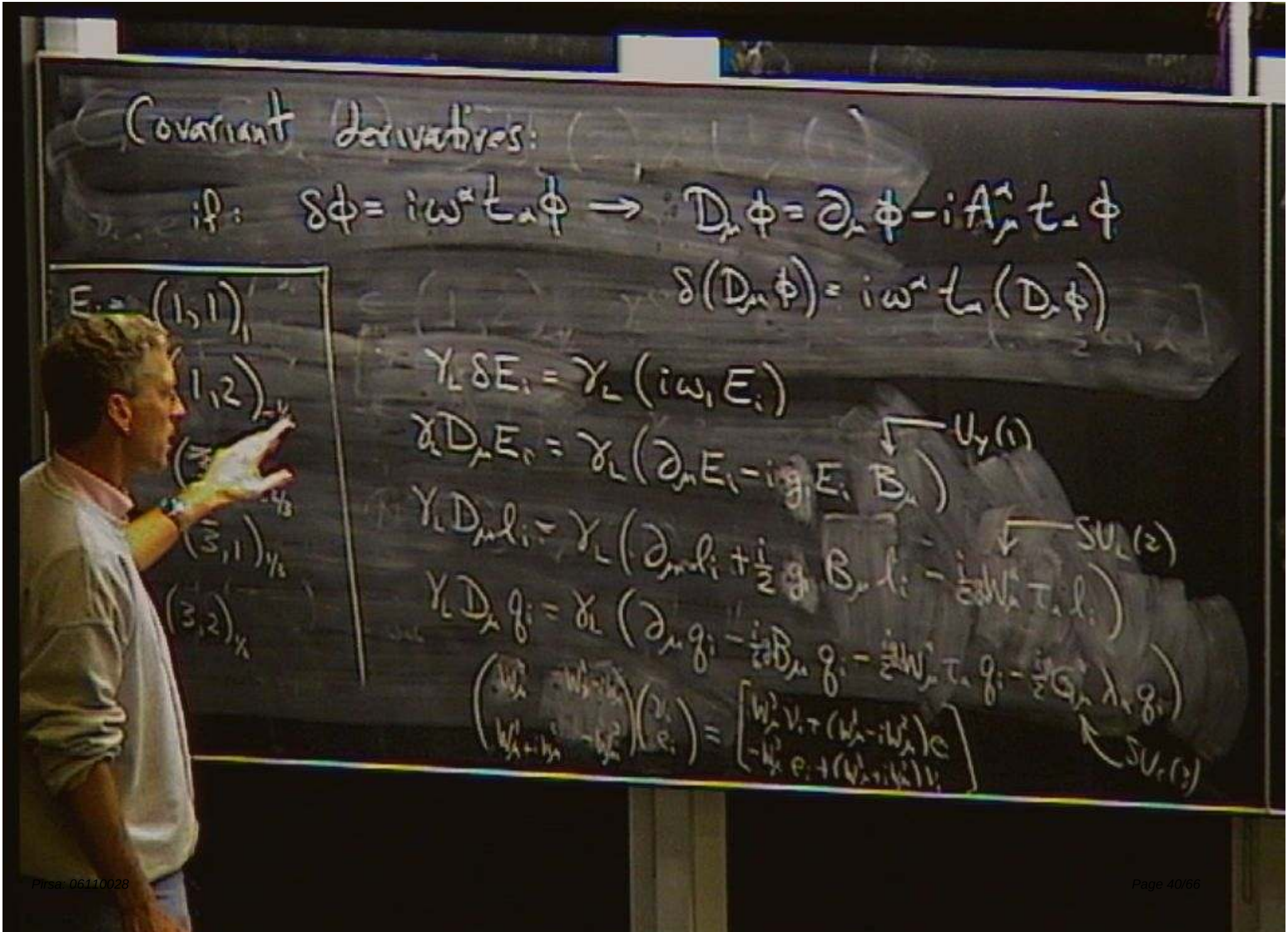
$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i)$

$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i - i\ldots E_i B_\mu)$ — $U_Y(1)$

$\gamma_L D_\mu l_i = \gamma_L (\partial_\mu l_i + \frac{i}{2} B_\mu l_i - \frac{i}{2} W^a_\mu \tau_a l_i)$ — $SU_L(2)$

$\gamma_L D_\mu q_i = \gamma_L (\partial_\mu q_i - \frac{i}{6} B_\mu q_i - \frac{i}{2} W^a_\mu \tau_a q_i - \frac{i}{2} G^a_\mu \lambda_a q_i)$ — $SU_C(3)$

$\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} = \begin{bmatrix} W^3_\mu \nu_e + (W^1_\mu - iW^2_\mu) e \\ -W^3_\mu e + (W^1_\mu + iW^2_\mu) \nu_e \end{bmatrix}$


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - i A^a_\mu t_a \phi$
$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$
$E_i = (1,1)_1$
$(1,2)_{-1/2}$
$(3,1)_{1/3}$
$(3,2)_{1/6}$
$\gamma_L \delta E_i = \gamma_L (i\omega_i E_i)$
$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i - i g_1 E_i B_\mu)$
$U_Y(1)$
$\gamma_L D_\mu l_i = \gamma_L (\partial_\mu l_i + \frac{i}{2} g_1 B_\mu l_i - \frac{i}{2} g_2 W^a_\mu \tau_a l_i)$
$SU_L(2)$
$\gamma_L D_\mu q_i = \gamma_L (\partial_\mu q_i - \frac{i}{6} g_1 B_\mu q_i - \frac{i}{2} g_2 W^a_\mu \tau_a q_i - \frac{i}{2} g_3 G^a_\mu \lambda_a q_i)$
$SU_C(3)$

Covariant derivatives:

if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - iA^a_\mu t_a \phi$

$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$

$E_i = (1,1)_1$

$l_i = (1,2)_{-1/2}$

$U_i = (\bar{3},1)_{-2/3}$

$D_i = (\bar{3},1)_{1/3}$

$q_i = (3,2)_{1/6}$

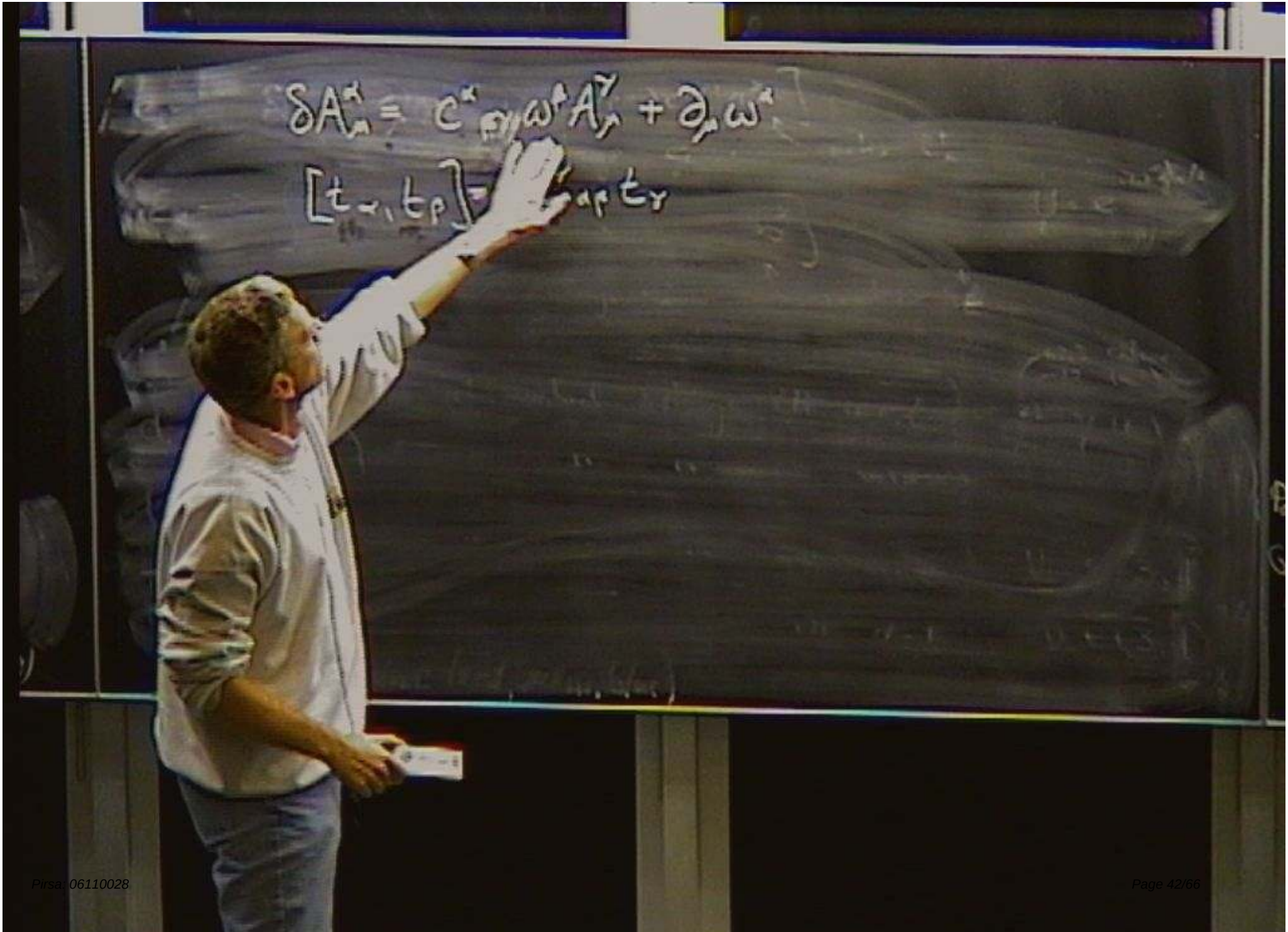
$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i)$

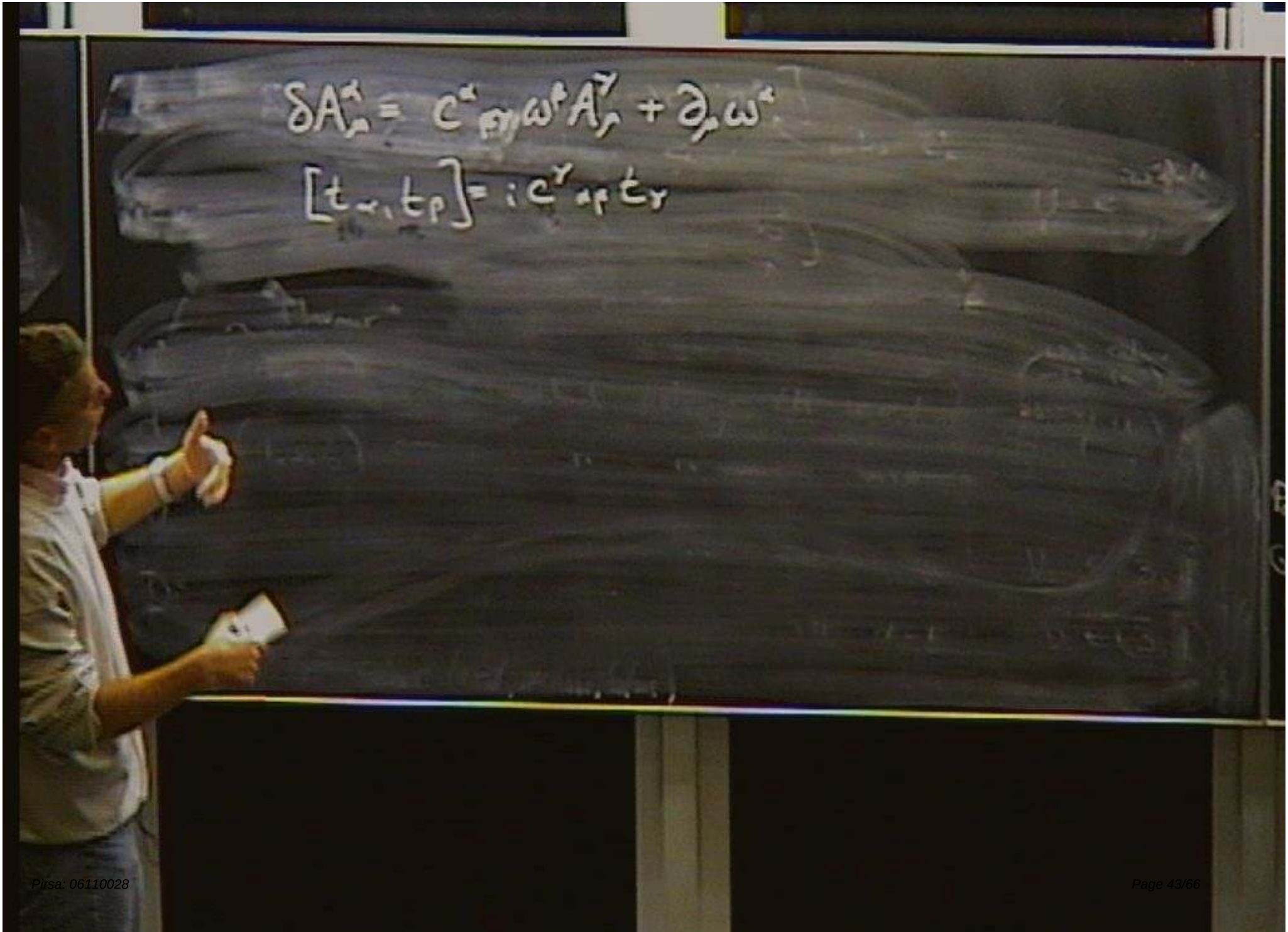
$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i - i g_1 E_i B_\mu)$ ← $U_Y(1)$

$\gamma_L D_\mu l_i = \gamma_L (\partial_\mu l_i + \frac{i}{2} g_1 B_\mu l_i - \frac{i}{2} g_2 W^a_\mu \tau_a l_i)$ ← $SU_L(2)$

$\gamma_L D_\mu q_i = \gamma_L (\partial_\mu q_i - \frac{i}{6} g_1 B_\mu q_i - \frac{i}{2} g_2 W^a_\mu \tau_a q_i - \frac{i}{2} g_3 G^a_\mu \lambda_a q_i)$ ← $SU_c(3)$

$\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} = \begin{bmatrix} W^3_\mu \nu_i + (W^1_\mu - iW^2_\mu) e \\ -W^3_\mu e_i + (W^1_\mu + iW^2_\mu) \nu_i \end{bmatrix}$


$\delta A^{\alpha}_{\mu} = C^{\alpha}{}_{\beta\gamma}\omega^{\beta}A^{\gamma}_{\mu} + \partial_{\mu}\omega^{\alpha}$
$[t_{\alpha}, t_{\beta}] = C_{\alpha\beta}{}^{\gamma} t_{\gamma}$


$\delta A^a_\mu = C^a{}_{bc}\,\omega^b A^c_\mu + \partial_\mu \omega^a$
$[t_\alpha, t_\beta] = i C^\gamma{}_{\alpha\beta}\, t_\gamma$

$$\delta A^a_\mu = C^a_{\beta\gamma} \omega^\beta A^\gamma_\mu + \partial_\mu \omega^a$$

$$[t_\alpha, t_\beta] = i C^\gamma_{\alpha\beta} t_\gamma \qquad \left[\frac{\tau_i}{2}, \frac{\tau_j}{2}\right] = i \epsilon_{ijk} \left(\frac{\tau_k}{2}\right)$$

$$\delta B_\mu = \partial_\mu \omega_1$$

$$\delta W^a_\mu = \partial_\mu \omega^a_L + \epsilon^a{}_{bc} \omega^b_L W^c_\mu$$

$$\delta A^{\alpha}_{\mu} = C^{\alpha}_{\beta\gamma}\omega^{\beta}A^{\gamma}_{\mu} + \partial_{\mu}\omega^{\alpha}$$

$$[t_{\alpha}, t_{\beta}] = i C^{\gamma}_{\alpha\beta} t_{\gamma} \qquad [T^{a}_{\Sigma}, T^{b}_{\Sigma}] = i\epsilon_{abc}(T^{c}_{\Sigma})$$

$$\delta B_{\mu} = \partial_{\mu}\omega_{1}$$

$$\delta W^{a}_{\mu} = \partial_{\mu}\omega^{a}_{L} + \epsilon^{a}{}_{bc}\omega^{b}_{L}W^{c}_{\mu}$$

$$\delta G^{\alpha}_{\mu} = \partial_{\mu}\omega^{\alpha}_{3} + f^{\alpha}{}_{\beta\gamma}\omega^{\beta}_{3}G^{\gamma}_{\mu}$$

$$\delta A^{\alpha}_{\mu} = C^{\alpha}_{\beta\gamma}\omega^{\beta}A^{\gamma}_{\mu} + \partial_{\mu}\omega^{\alpha}$$

$$[t_{\alpha}, t_{\beta}] = i C^{\gamma}_{\alpha\beta} t_{\gamma} \qquad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = i\epsilon_{abc}\left(\frac{\tau_c}{2}\right)$$

$$\delta B_{\mu} = \frac{1}{g_1}\partial_{\mu}\omega_1$$

$$\delta W^{a}_{\mu} = \frac{1}{g_2}\partial_{\mu}\omega^{a}_{2} + \epsilon^{a}_{bc}\omega^{b}_{2}W^{c}_{\mu}$$

$$\delta G^{\alpha}_{\mu} = \frac{1}{g_3}\partial_{\mu}\omega^{\alpha}_{3} + f^{\alpha}_{\beta\gamma}\omega^{\beta}_{3}G^{\gamma}_{\mu}$$

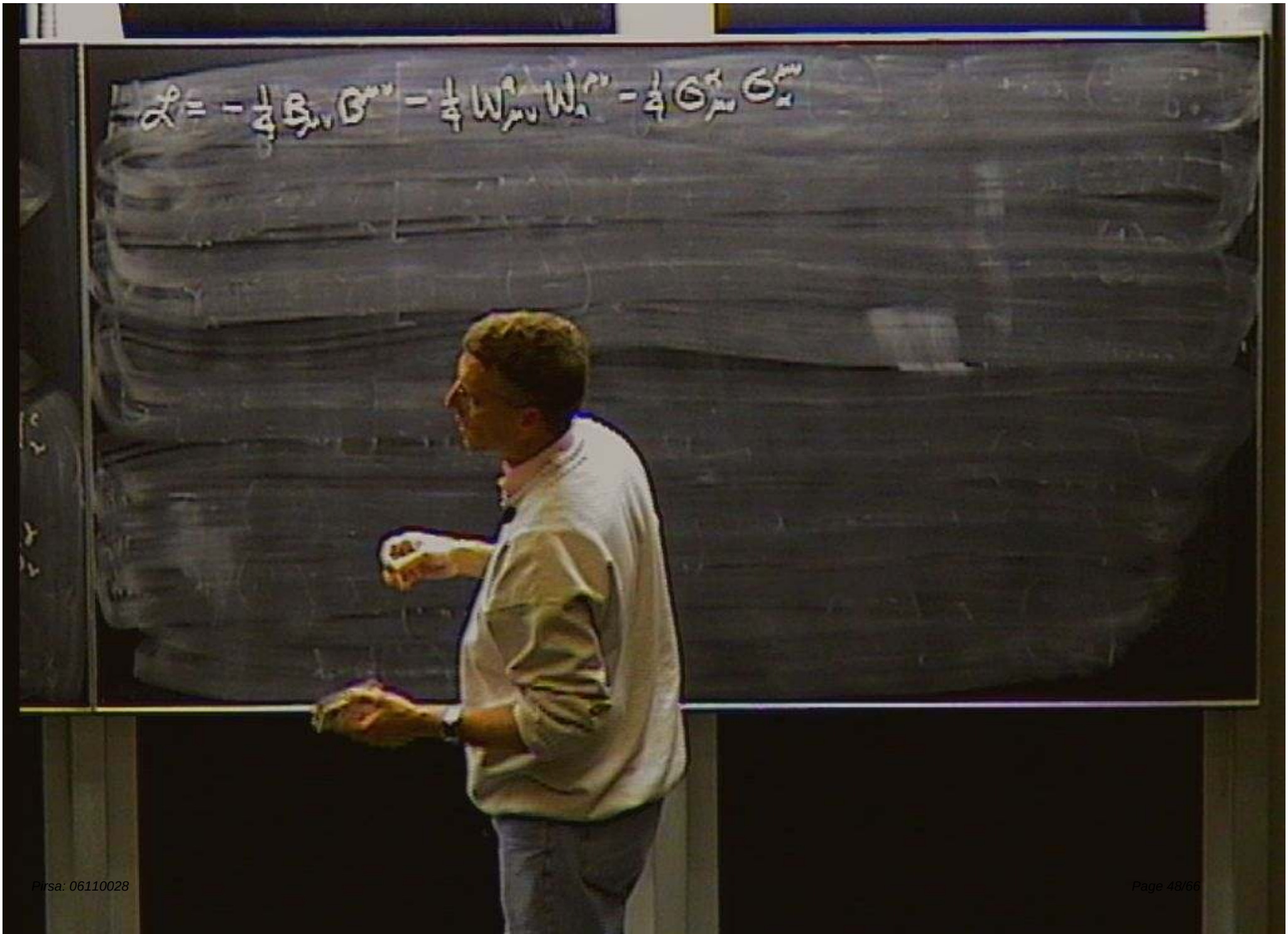
$$\delta A^{\alpha}_{\mu} = C^{\alpha}_{\beta\gamma}\omega^{\beta}A^{\gamma}_{\mu} + \partial_{\mu}\omega^{\alpha}$$

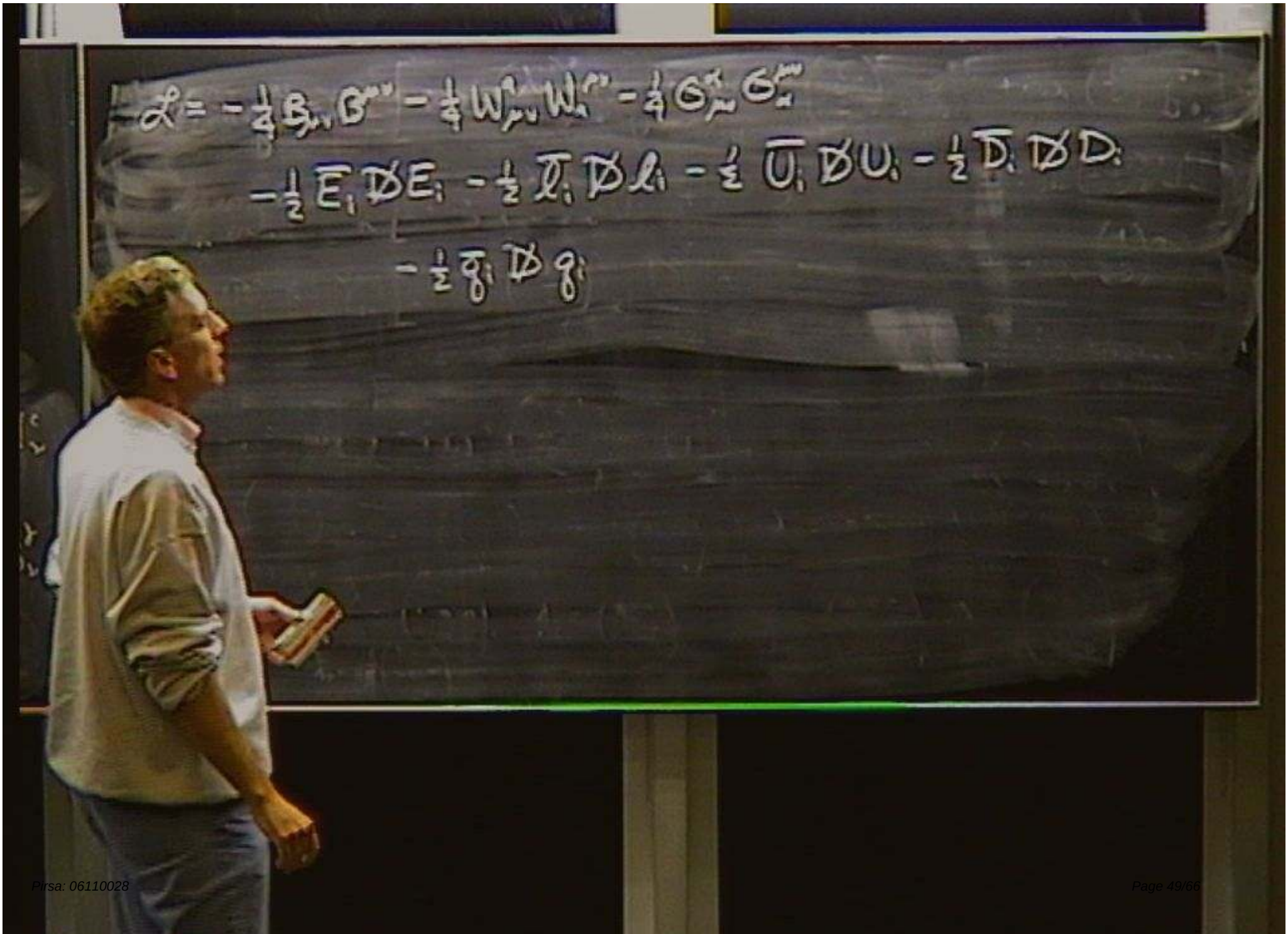
$$[t_{\alpha}, t_{\beta}] = i C^{\gamma}_{\alpha\beta} t_{\gamma} \qquad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = i\epsilon_{abc}\left(\frac{\tau_c}{2}\right)$$

$$\delta B_{\mu} = \frac{1}{g_1}\partial_{\mu}\omega_1 \longrightarrow B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$$

$$\delta W^{a}_{\mu} = \frac{1}{g_2}\partial_{\mu}\omega^{a}_{L} + \epsilon^{a}{}_{bc}\omega^{b}_{L}W^{c}_{\mu} \qquad W^{a}_{\mu\nu} = \partial_{\mu}W^{a}_{\nu} - \partial_{\nu}W^{a}_{\mu} + g_2\epsilon^{a}{}_{bc}W^{b}_{\mu}W^{c}_{\nu}$$

$$\delta G^{a}_{\mu} = \frac{1}{g_3}\partial_{\mu}\omega^{a}_{3} + f^{\alpha}_{\beta\gamma}\omega^{\beta}_{3}G^{\gamma}_{\mu}$$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$


$-\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{L}_i \not{D} L_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{Q}_i \not{D} Q_i$

$$-\mathcal{L} = -\tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - \tfrac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \tfrac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$- \tfrac{1}{2} \bar{E}_i \not{D} E_i - \tfrac{1}{2} \bar{L}_i \not{D} L_i - \tfrac{1}{2} \bar{U}_i \not{D} U_i - \tfrac{1}{2} \bar{D}_i \not{D} D_i$$

$$- \tfrac{1}{2} \bar{Q}_i \not{D} Q_i - \left( Y_{ij} \bar{E}_i \gamma_L E_j + \text{c.c.} \right) + \dots$$

$\gamma_L E_i \to$

Covariant derivatives:

if: $\delta\phi = i\omega^a t_a \phi \rightarrow D_\mu\phi = \partial_\mu\phi - iA^a_\mu t_a\phi$

$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu\phi)$

$E_i = (1,1)_1$

$l_i = (1,2)_{-1/2}$

$U_i = (\bar{3},1)_{-2/3}$

$D_i = (\bar{3},1)_{1/3}$

$q_i = (3,2)_{1/6}$

$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i) \rightarrow \gamma_L E_i \rightarrow e^{i\omega_1}\gamma_L E_i$ ($U_Y(1)$)

$\gamma_L D_\mu E_i = \gamma_L(\partial_\mu E_i - i g_1 E_i B_\mu)$

$\gamma_L D_\mu l_i = \gamma_L(\partial_\mu l_i + \frac{i}{2} g_1 B_\mu l_i - i g_2 W^a_\mu \tau_a l_i)$ ($SU_L(2)$)

$\gamma_L D_\mu q_i = \gamma_L(\partial_\mu q_i - \frac{i}{6} g_1 B_\mu q_i - i g_2 W^a_\mu \tau_a q_i - i g_3 G^a_\mu \lambda_a q_i)$ ($SU_C(3)$)

$\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\begin{pmatrix} \nu_i \\ e_i \end{pmatrix} = \begin{bmatrix} W^3_\mu \nu_i + (W^1_\mu - iW^2_\mu)e \\ -W^3_\mu e_i + (W^1_\mu + iW^2_\mu)\nu_i \end{bmatrix}$

$$-\mathcal{L} = -\tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - \tfrac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \tfrac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$-\tfrac{1}{2}\bar{E}_i \not{D} E_i - \tfrac{1}{2}\bar{L}_i \not{D} L_i - \tfrac{1}{2}\bar{U}_i \not{D} U_i - \tfrac{1}{2}\bar{D}_i \not{D} D_i$$

$$-\tfrac{1}{2}\bar{Q}_i \not{D} Q_i - \left( m_{ij}\, \bar{E}_i \gamma_L E_j + \text{c.c.} \right) + \cdots$$

$$\gamma_L E_i \to e^{i\omega_i} E_i$$

$$\gamma_L \psi \to M_{ij} \gamma_L \psi, \qquad m \to M^T m$$

$$-\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

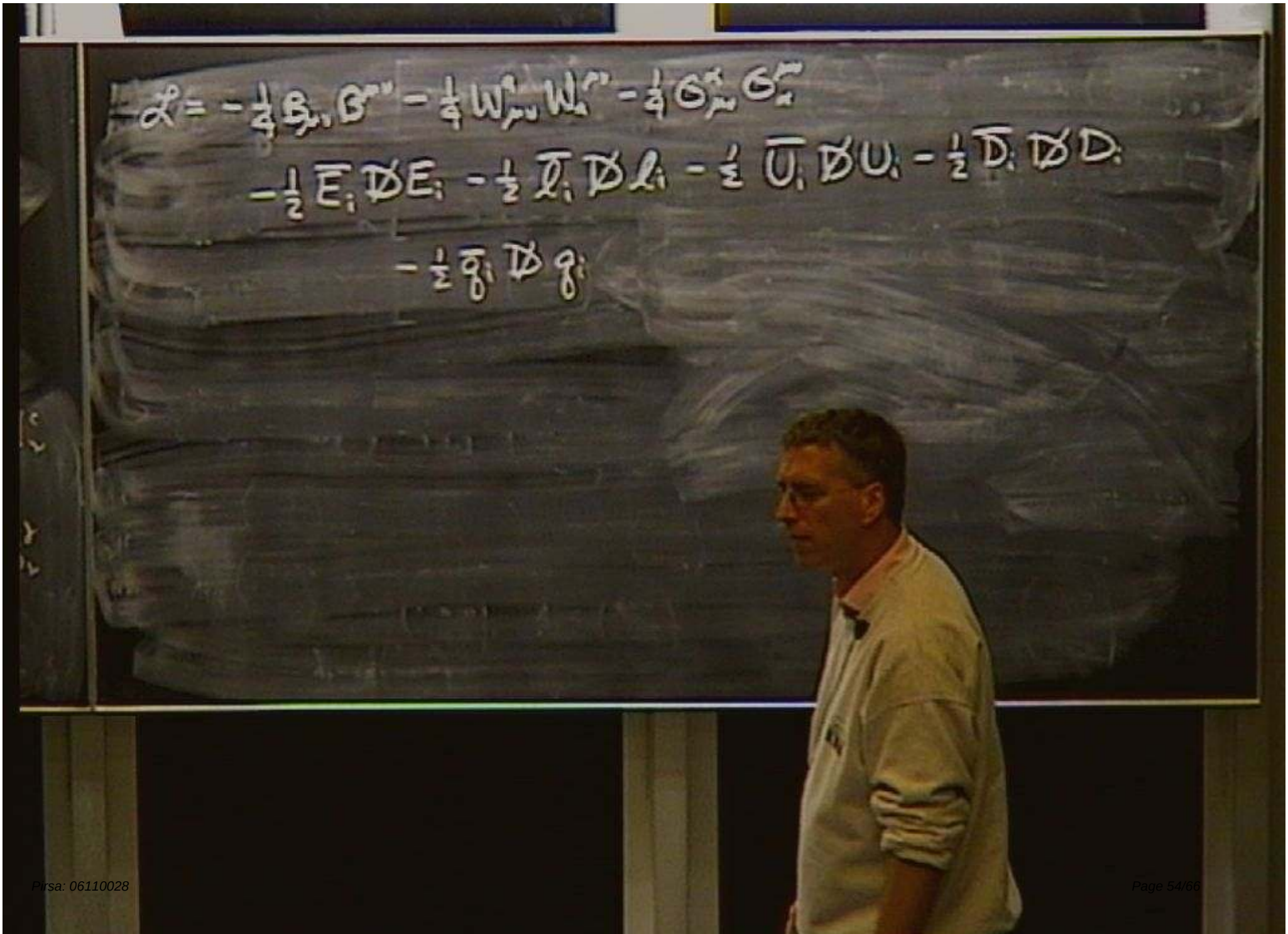
$$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{l}_i \not{D} l_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$$

$$-\frac{1}{2} \bar{q}_i \not{D} q_i - \cancel{\left( m_{ij} E_i \gamma_L E_j + \text{c.c.} \right)} + \dots$$

$\gamma_L E_i \to e^{i\omega_i} E_i$ $\quad \bar{E}_i \gamma_L \to e^{i\omega_i} \bar{E}_i \gamma_L$

$\bar{E}_i \gamma_L E_j \to e^{2i\omega_i} \bar{E}_i \gamma_L E_j$

not invariant under $U_Y(1)$


$-\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{L}_i \not{D} L_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{Q}_i \not{D} Q_i$

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^\alpha_{\mu\nu} G^{\alpha\mu\nu}$$

$$- \frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{L}_i \not{D} L_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$$

$$- \frac{1}{2} \bar{q}_i \not{D} q_i$$

$$= \mathcal{L}_0 + \mathcal{L}_{int}$$

$$\mathcal{L}_1 = \mathcal{L}(g_1, g_2, g_3 = 0)$$

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

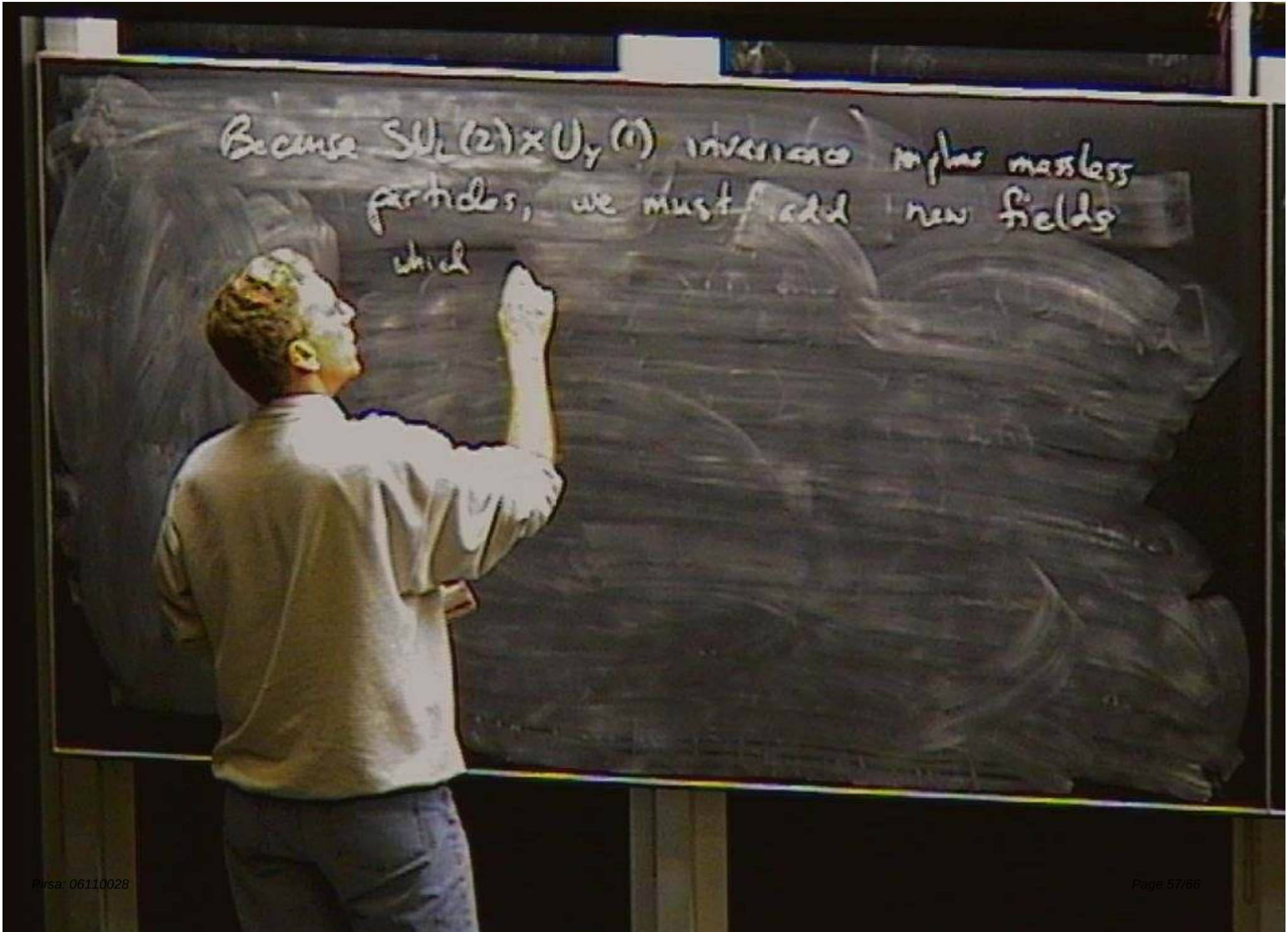
$$- \frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{l}_i \not{D} l_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$$

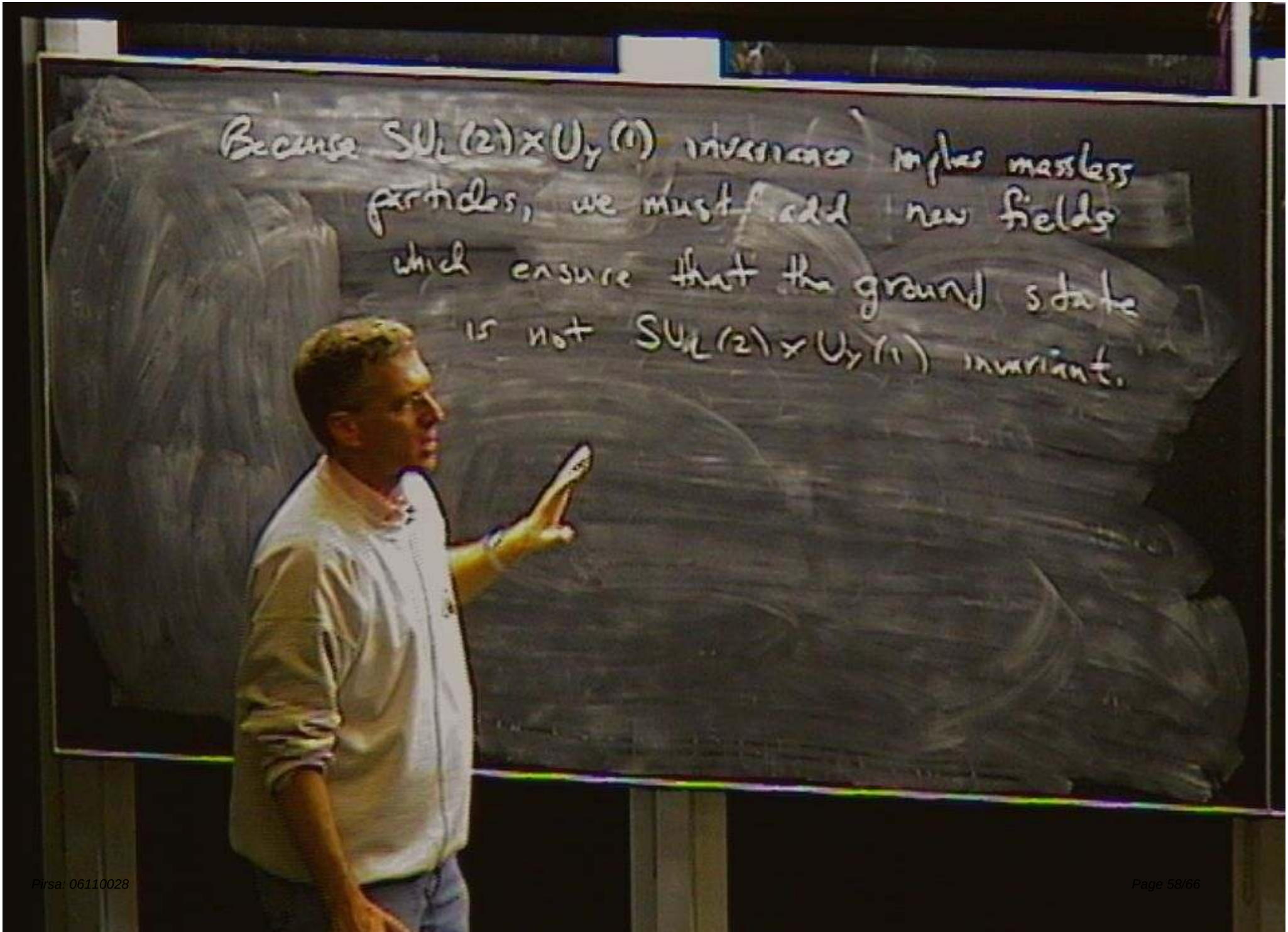
$$- \frac{1}{2} \bar{q}_i \not{D} q_i$$

$$= \mathcal{L}_0 + \mathcal{L}_{int}$$

$\mathcal{L}_0 = \mathcal{L}(g_1, g_2, g_3 = 0)$

= kinetic terms all around with no masses.


Because $SU_L(2) \times U_Y(1)$ invariance implies massless
particles, we must add new fields
which


Because $SU_L(2) \times U_Y(1)$ invariance implies massless
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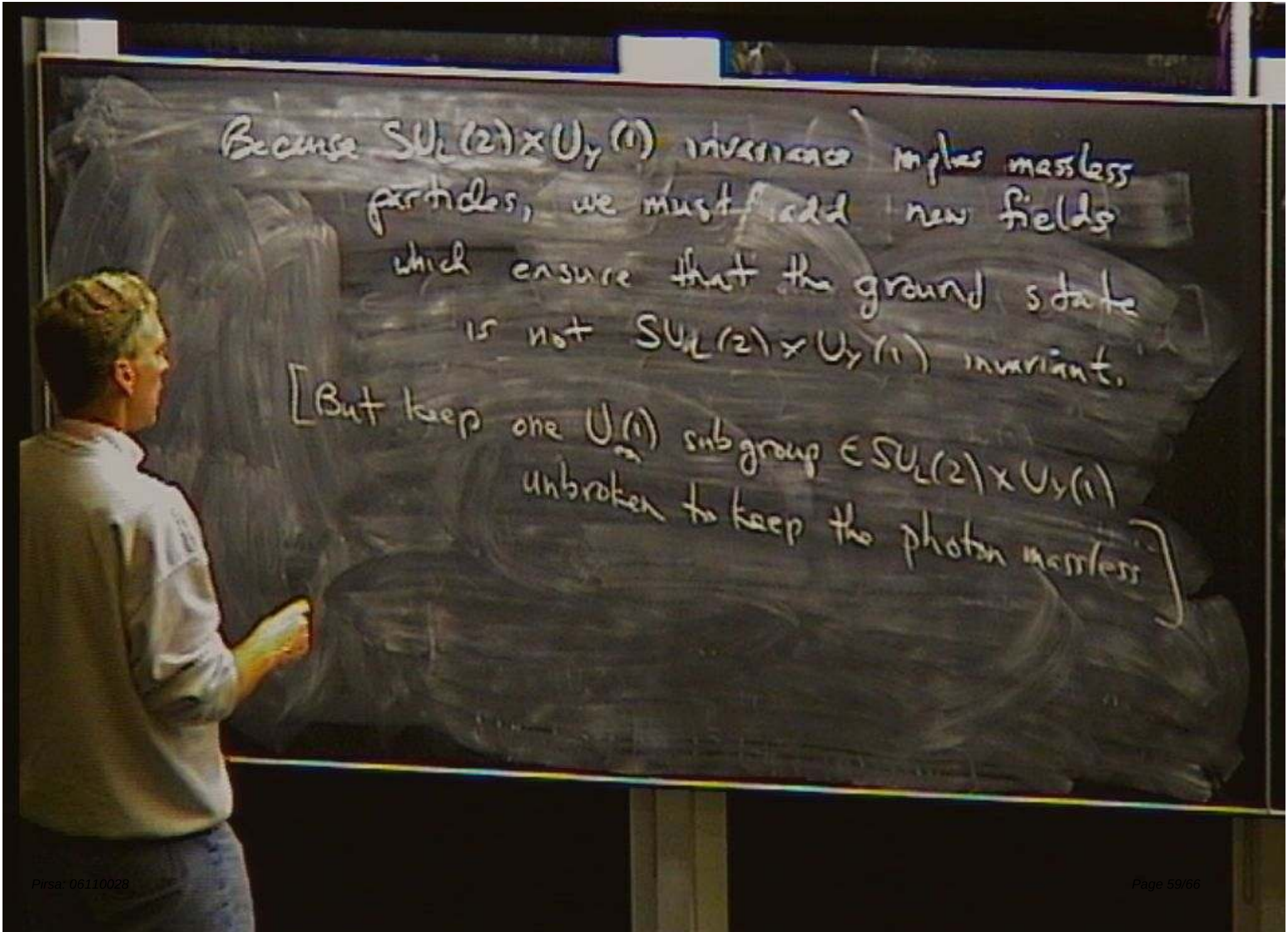
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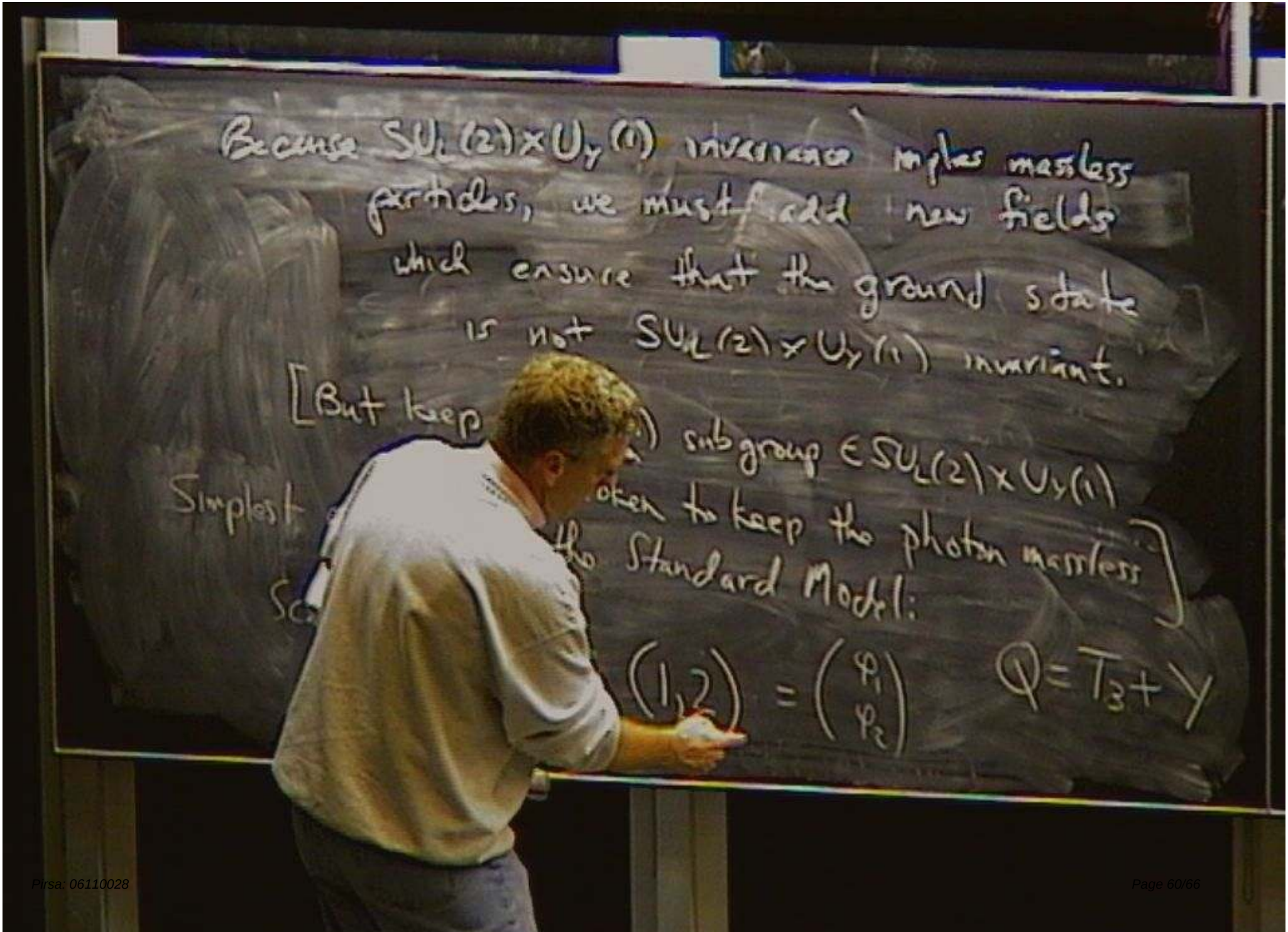
[But keep one $U_{EM}(1)$ subgroup $\in SU_L(2) \times U_Y(1)$
unbroken to keep the photon massless]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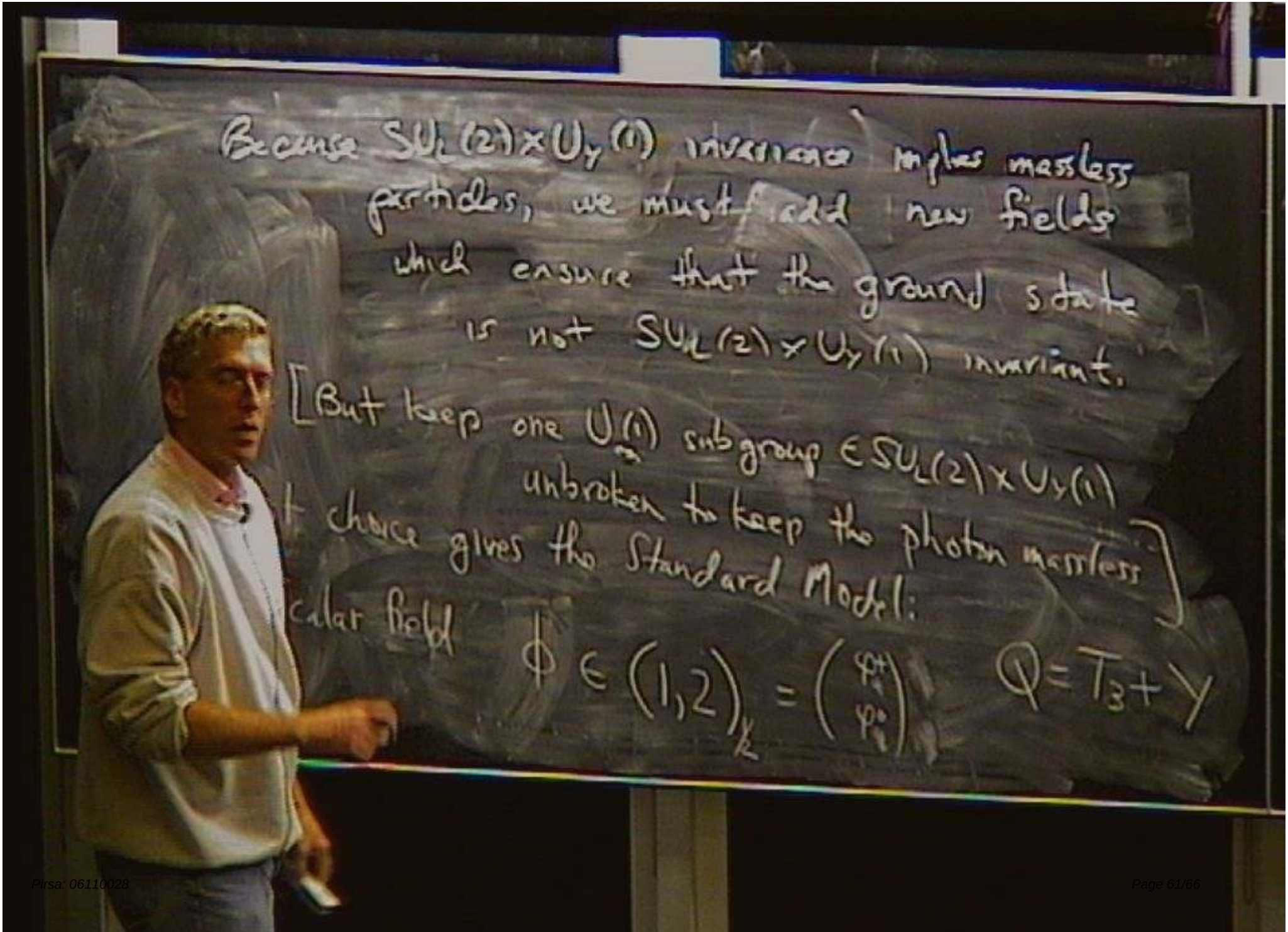
[But keep
subgroup $\in SU_L(2) \times U_Y(1)$
Simplest
oken to keep the photon massless
Standard Model:
$(1,2) = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$
$Q = T_3 + Y$


Because $SU_L(2) \times U_Y(1)$ invariance implies massless particles, we must add new fields which ensure that the ground state is not $SU_L(2) \times U_Y(1)$ invariant.
[But keep one $U_{em}(1)$ subgroup $\in SU_L(2) \times U_Y(1)$ unbroken to keep the photon massless
This choice gives the Standard Model:
Scalar field $\phi \in (1,2)_k = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ $Q = T_3 + Y$

$$\delta A^a_\mu = C^a_{\beta\gamma}\omega^\beta A^\gamma_\mu + \partial_\mu \omega^a$$

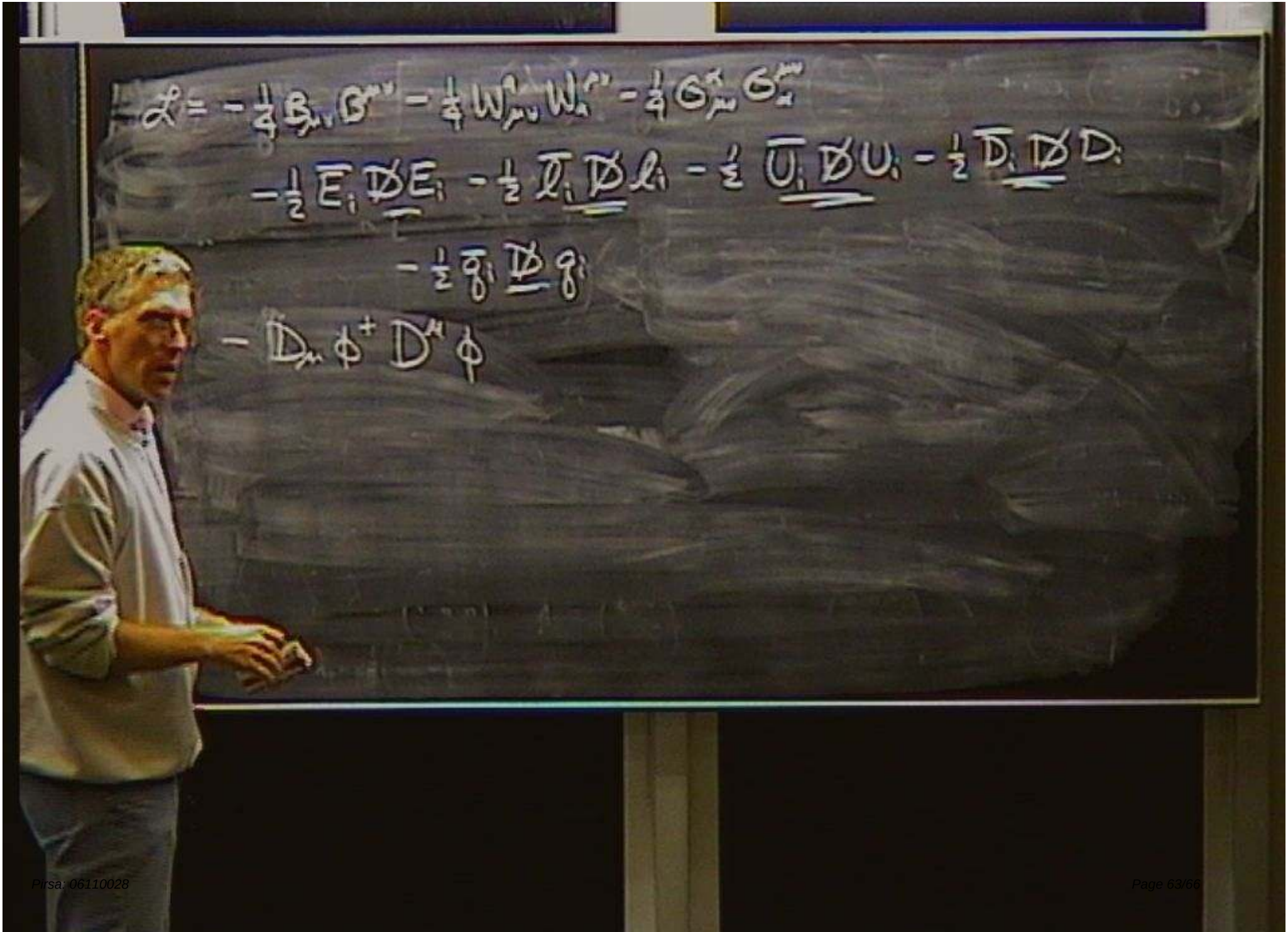
$$[t_\alpha, t_\beta] = i C^\gamma_{\alpha\beta} t_\gamma \qquad [\tau_i, \tau_j] = i\epsilon_{ijk}(\tau_k)$$

$$\delta B_\mu = \frac{1}{g'}\partial_\mu \omega_1 \longrightarrow B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\delta W^a_\mu = \frac{1}{g}\partial_\mu \omega^a_2 + \epsilon^a{}_{bc}\omega^b_2 W^c_\mu \qquad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^a{}_{bc} W^b_\mu W^c_\nu$$

$$\delta G^a_\mu = \frac{1}{g_s}\partial_\mu \omega^a_3 + f^a{}_{\beta\gamma}\omega^\beta_3 G^\gamma_\mu \qquad G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^a{}_{\beta\gamma} G^\beta_\mu G^\gamma_\nu$$

$$\delta\phi = \frac{i}{2}\omega^a_2 \tau_a \phi$$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{L}_i \not{D} L_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{Q}_i \not{D} Q_i$
$- D_\mu \phi^+ D^\mu \phi$

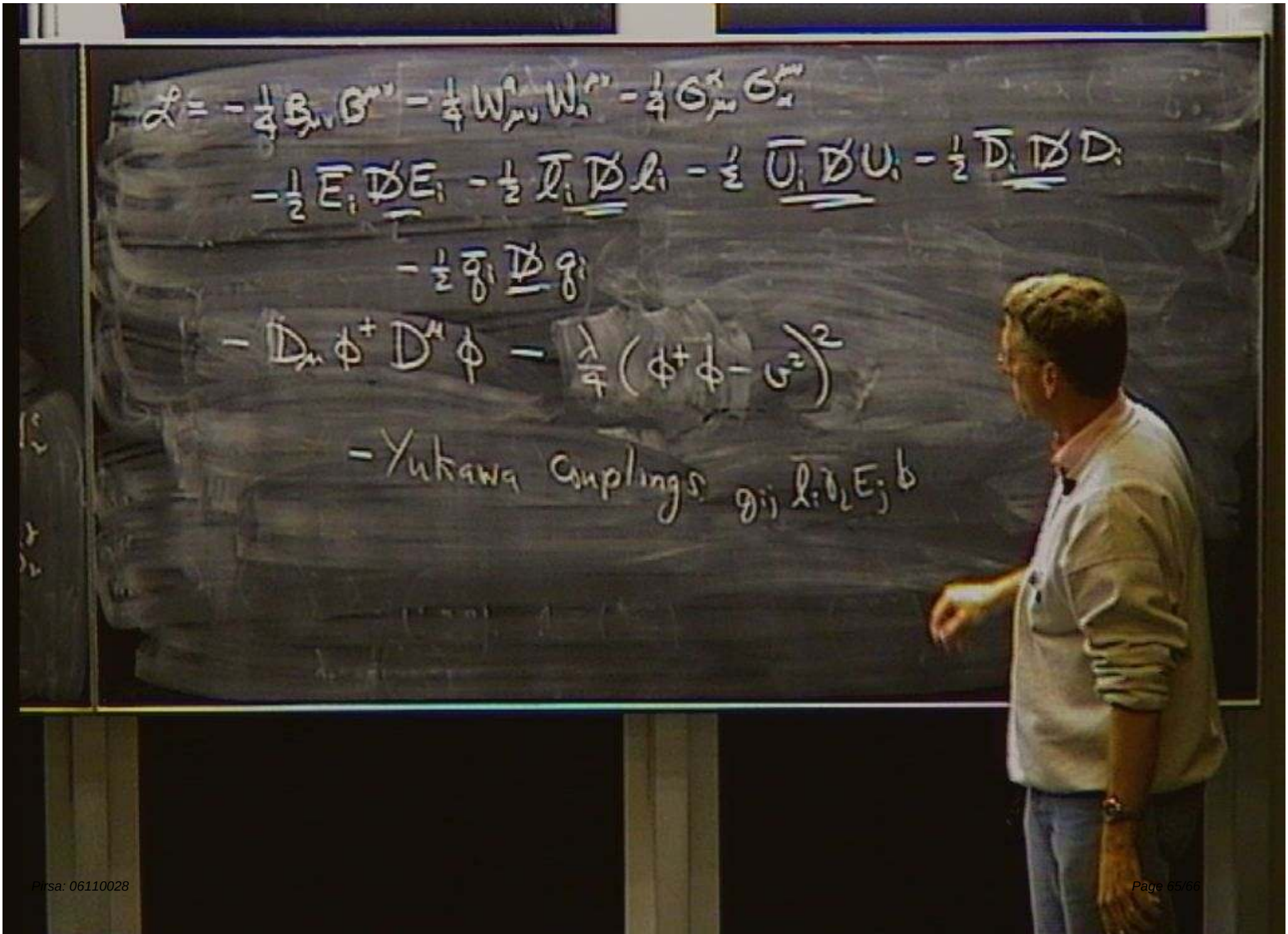
$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$- \frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{l}_i \not{D} l_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$$

$$- \frac{1}{2} \bar{q}_i \not{D} q_i$$

$$- (D_\mu \phi^+)(D^\mu \phi) - \frac{\lambda}{4} \left( \phi^+ \phi - v^2 \right)^2$$

- Yukawa Couplings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$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{l}_i \not{D} l_i - \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i$
$- D_\mu \phi^+ D^\mu \phi - \frac{\lambda}{4} (\phi^+ \phi - v^2)^2$
- Yukawa Couplings: $g_{ij} \bar{l}_i \phi l_j E_j$ b

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$- \frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{l}_i \not{D} l_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$$

$$- \frac{1}{2} \bar{q}_i \not{D} q_i$$

$$- (D_\mu \phi^+)(D^\mu \phi) - \frac{\lambda}{4} \left( \phi^+ \phi - v^2 \right)^2$$

- Yukawa couplings: $g_{ij}\, \bar{l}_i \phi_L E_j$ + h.c.