Title: Graduate Course on Standard Model & Quantum Field Theory - 5B

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URL: http://pirsa.org/06110028

Abstract: Graduate Course on Standard Model & Quantum Field Theory

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

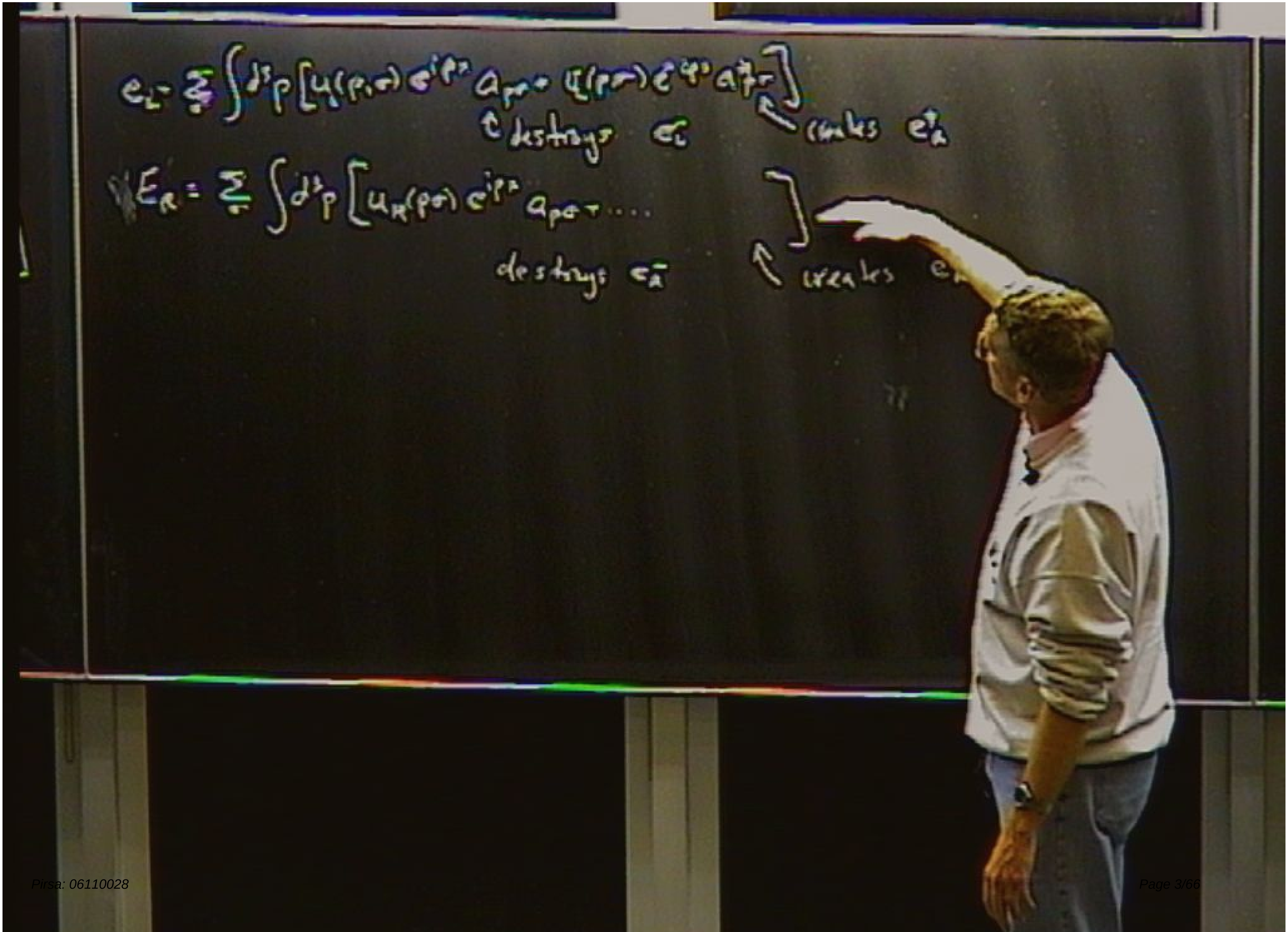
ν_i , e_i , E_i : $i = 1,2,3$ (e^-, μ^-, τ^-)

LH $e \longrightarrow l_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \in (1,2)_{-1/2}$ $\quad \gamma_L \delta l_i = \gamma_L \left[\frac{i}{2} \omega_a \tau_a l_i - \frac{i}{2} \omega_1 l \right]$

RH $e \longrightarrow E_i \in (1,1)_{+1}$ $\quad \gamma_L \delta E_i = \gamma_L \left[i \omega_1 E_i \right]$

u_i $\quad i = 1,2,3 \quad (u, c, t)$

d_i $\quad$ " $\quad (d, s, b)$


$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma) e^{ipx} a_{p\sigma} + v_L(p,\sigma) e^{-ipx} a^\dagger_{p\sigma} \right]$
destroys e_L
creates e^+_R
$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma) e^{ipx} a_{p\sigma} \ldots \right]$
destroys e_R^-
creates

$$G = SU_c(3) \times SU_L(2) \times U_Y(1)$$

ν_i, e_i, E_i : $i = 1,2,3$ (e^-, μ^-, τ^-)

LH $e \xrightarrow{\;l_i =\;} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \in (1,2)_{-1/2}$: $\gamma_L \delta l_i = \gamma_L \left[\frac{i}{2} \omega_a \tau_a l_i - \frac{i}{2} \omega_1 l_i \right]$

RH $e \to E_i$ $\in (1,1)_{+1}$ $\quad \gamma_L \delta E_i = \gamma_L \left[i \omega_1 E_i \right]$

u_i $i = 1,2,3$ (u,c,t)

d_i " (d,s,b)

$$G = SU_c(3)\times SU_L(2)\times U_Y(1)$$

ν_i, e_i, E_i : $i=1,2,3$ (e^-,μ^-,τ^-)

LH $e \xrightarrow{l_i =} \begin{pmatrix}\nu_i\\ e_i\end{pmatrix} \in (1,2)_{-1/2}$: $\gamma_L\delta l_i = \gamma_L\left[\frac{i}{2}\omega_a\tau_a l_i - \frac{i}{2}\omega_1 l_i\right]$

RH $e \to E_i$ $\in (1,1)_{+1}$ $\quad \gamma_L\,\delta E_i = \gamma_L\left[i\omega_1 E_i\right]$

Rule: $Q = t_3 + Y$

$$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma)\, e^{ipx}\, a_{p\sigma} + v_L(p,\sigma)\, e^{-ipx}\, a^\dagger_{p\sigma} \right]$$

destroys e_L^- creates e_R^+

$$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma)\, e^{ipx}\, b_{p\sigma} + \dots \quad b^\dagger \right]$$

destroys e_R^- creates e_L^+

$u_i = (u, c, t)$

$d_i = (d, s, b)$

$$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma)\, e^{ipx}\, a_{p\sigma} + v_L(p,\sigma)\, e^{-ipx}\, a^\dagger_{p\sigma} \right]$$

destroys e_L^- creates e_R^+

$$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma)\, e^{ipx}\, b_{p\sigma} + \dots \quad b^\dagger \right]$$

destroys e_R^- creates e_L^+

$u_i = (u, c, t)$ ← field which destroys LH u, c, t.

$d_i = (d, s, b)$ ← " " " " b, s, d

$$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma)\, e^{ipx}\, a_{p\sigma} + v_L(p\sigma)\, e^{-ipx}\, a^\dagger_{p\sigma} \right]$$

destroys e_L^- creates e_R^+

$$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma)\, e^{ipx}\, b_{p\sigma} + \dots \quad b^\dagger \right]$$

destroys e_R^- creates e_L^+

$u_i = (u, c, t) \leftarrow$ field which destroys LH u, c, t.

$d_i = (d, s, b) \leftarrow$ " " " " b, s, d

U_i " " " RH u, c, t

D_i " " " RH d, s, b

$$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma)\, e^{ip\cdot x}\, a_{p\sigma} + v_L(p,\sigma)\, e^{-ip\cdot x}\, a^\dagger_{p\sigma} \right]$$

↑ destroys e_L^- ↑ creates e_R^+

$$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma)\, e^{ip\cdot x}\, b_{p\sigma} + \dots \quad b^\dagger \right]$$

destroys e_R^- ↑ creates e_L^+

$$\underbrace{u_i}_{\text{"flavour"}}$$

$u_i = (u,c,t)$ ← field which destroys LH u,c,t.

$d_i = (d,s,b)$ ← " " " " b,s,d

$$q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$$

U_i " " " RH u,c,t

D_i " " " RH d,s,b.

$$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma)\, e^{ip\cdot x}\, a_{p\sigma} + v_L(p\sigma)\, e^{-ip\cdot x}\, a^\dagger_{p\sigma} \right]$$

destroys e_L^- creates e_R^+

$$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma)\, e^{ip\cdot x}\, b_{p\sigma} + \dots \quad b^\dagger \right]$$

destroys e_R^- creates e_L^+

$$u_i^z = (\underbrace{u,c,t}_{\text{"flavour"}}) \leftarrow \text{field which destroys LH } u,c,t.$$

$$d_i^z = (d,s,b) \leftarrow \text{" " " " } b,s,d \qquad q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$$

U_i^z " " " RH u,c,t

D_i^z " " " RH d,s,b

$z = 1,2,3$ colour (red, yellow, blue)

$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma) e^{ipx} a_{p\sigma} + v_L(p\sigma) e^{-ipx} a^\dagger_{p\sigma} \right]$

destroys e_L creates e^+_R

$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma) e^{ipx} b_{p\sigma} + \dots \quad b^\dagger \right]$

destroys e_R creates e^+_L

"flavour"

$u_i^z = (u,c,t) \leftarrow$ field which destroys LH u,c,t.

$d_i^z = (d,s,b) \leftarrow$ " " " " b,s,d

$q_i^{z\,a} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$ 1,2,3 colour (red); 1,2

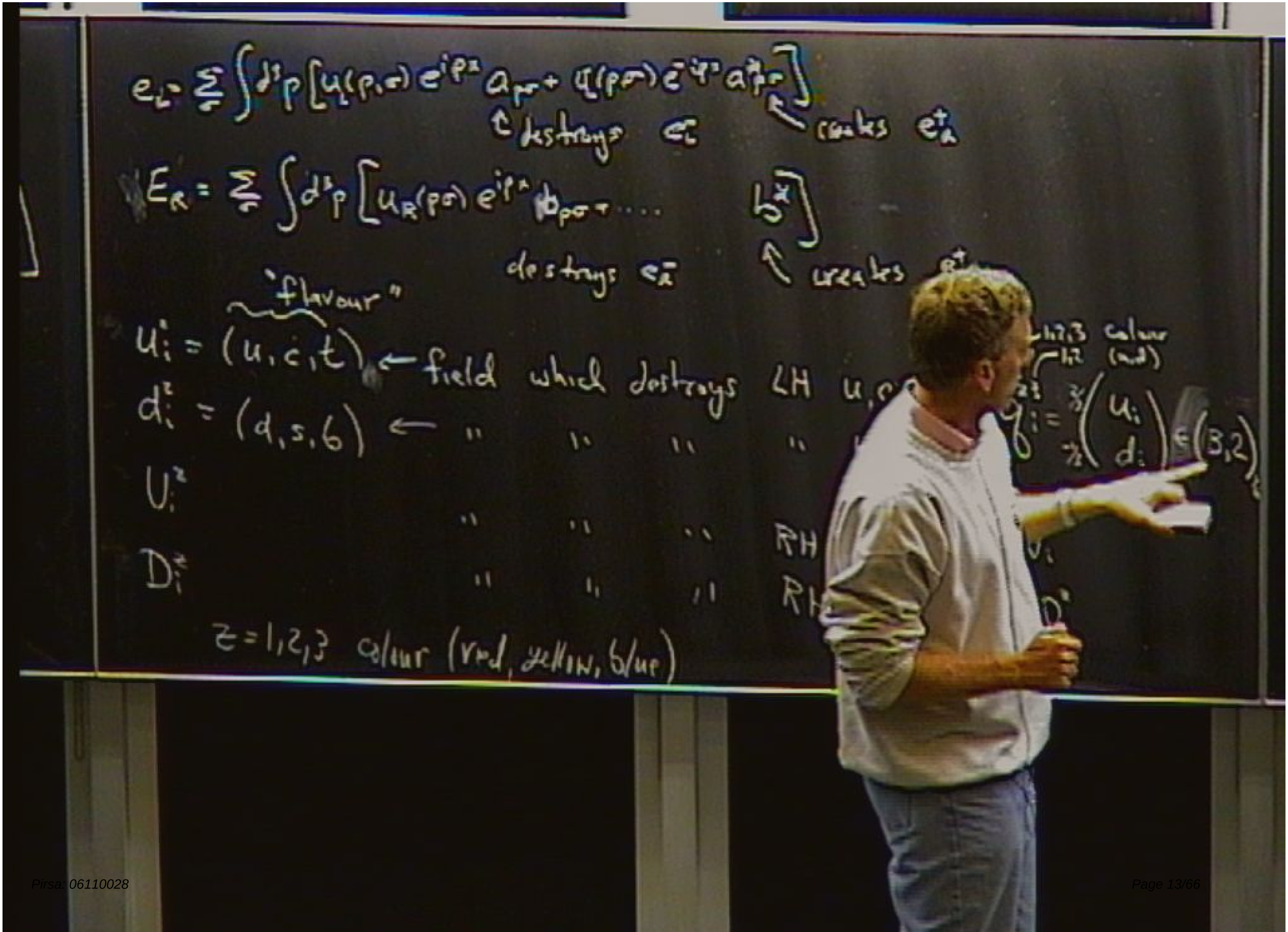
U_i^z " " " RH u,c,t U_i^z

D_i^z " " " RH d,s,b D_i^z

$z = 1,2,3$ colour (red, yellow, blue)

$\gamma_L \delta U_i^{\,j} = \gamma_L \Big[\, i\, \omega_j^{\,i} (\Sigma_a)_j\, U_i^{\,j}$

(su


$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma) e^{ipx} a_{p\sigma} + v_L(p\sigma) e^{-ipx} a^\dagger_{p\sigma} \right]$
destroys e_L^-
creates e_R^+
$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma) e^{ipx} b_{p\sigma} + \dots \; b^\dagger \right]$
destroys e_R^-
creates
"flavour"
$u_i^z = (u, c, t) \leftarrow$ field which destroys LH u, c
$d_i^z = (d, s, b) \leftarrow$ " " " "
U_i^z
" " " RH
D_i^z
" " " RH
z = 1, 2, 3 colour (red, yellow, blue)
colour
$q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix} \in (3,2)$

$$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma) e^{ip\cdot x} a_{p\sigma} + v_L(p,\sigma) e^{-ip\cdot x} a^\dagger_{p\sigma} \right]$$

destroys e_L^- ... creates e_L^+

$$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma) e^{ip\cdot x} b_{p\sigma} + \dots \quad b^\dagger \right]$$

destroys e_R^- ... creates e_R^+

"flavour"

$u_i^z = (u, c, t) \leftarrow$ field which destroys LH u, c, t.

$d_i^z = (d, s, b) \leftarrow$ " " " " b, s, d

$q_i^z = \begin{pmatrix} u_i \\ d_i \end{pmatrix} \in (3,2)_{\frac{1}{6}}$ — z: 1,2,3 colour (red); 1,2

U_i^z " " " RH u, c, t — $U_i^z \in (\bar{3}$

D_i^z " " " RH d, s, b — D_i^z

$z = 1,2,3$ colour (red, yellow, blue)

$$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma) e^{ipx} a_{p\sigma} + v_L(p\sigma) e^{-ipx} a^\dagger_{p\sigma} \right]$$

destroys e_L^- creates e^+_R

$$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma) e^{ipx} b_{p\sigma} + \dots \; b^\dagger \right]$$

destroys e_R^- creates e^+_L

$$\underbrace{\text{"flavour"}}$$

$u_i^z = (u,c,t) \leftarrow$ field which destroys LH u,c,t

$d_i^z = (d,s,b) \leftarrow$ " " " " b,s,d

U_i^z " " " RH u,c,t

D_i^z " " " RH d,s,b

$z = 1,2,3$ colour (red, yellow, blue)

1,2,3 colour

1,2 (wk)

$$q_i^{z\dot z} = \begin{array}{c} 2/3 \\ -1/3 \end{array}\begin{pmatrix} u_i \\ d_i \end{pmatrix} \in (3,2)_{1/6}$$

$$U_i^z \in (\bar 3, 1)_{-2/3}$$

$$D_i^z \in (\bar 3, 1)_{+1/3}$$

$$e_L = \sum_\sigma \int d^3p \left[u_L(p,\sigma) e^{ipx} a_{p\sigma} + v_L(p,\sigma) e^{-ipx} a^{\dagger}_{p\sigma} \right]$$

destroys e_L^- creates e^+_R

$$E_R = \sum_\sigma \int d^3p \left[u_R(p\sigma) e^{ipx} b_{p\sigma} + \ldots \quad b^{\dagger} \right]$$

destroys e_R^- creates e^+_L

"flavour"

$u^z_i = (u, c, t) \leftarrow$ field which destroys LH $u, c, t.$

$d^z_i = (d, s, b) \leftarrow$ " " " " b, s, d

$$q^z_i = \begin{array}{c} \scriptstyle 1/2 \\ \scriptstyle -1/2 \end{array}\begin{pmatrix} u_i \\ d_i \end{pmatrix} \in (3, 2)_{1/6}$$

$z = 1,2,3$ colour (red), $1,2$

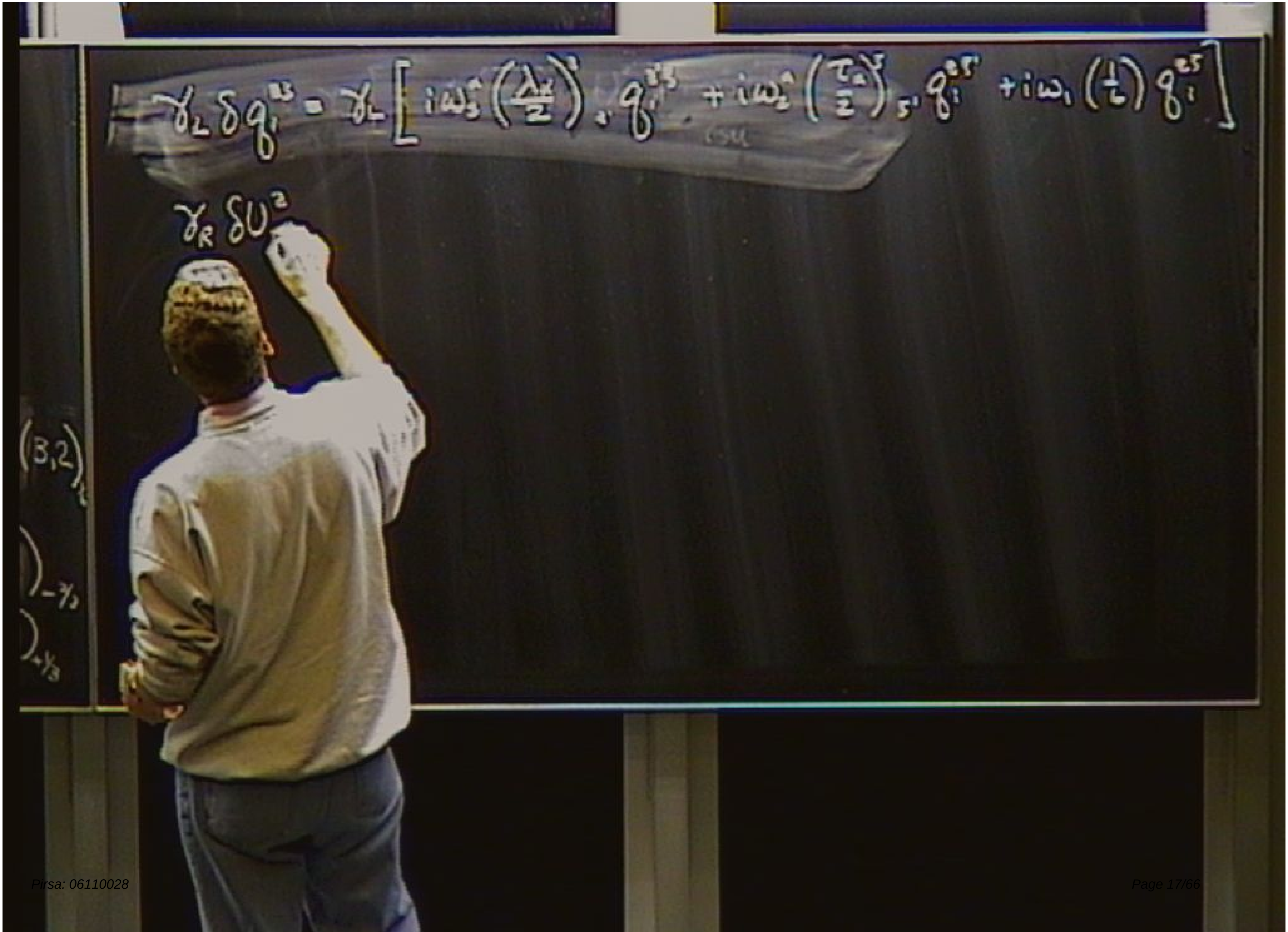
U^z_i " " " RH u, c, t

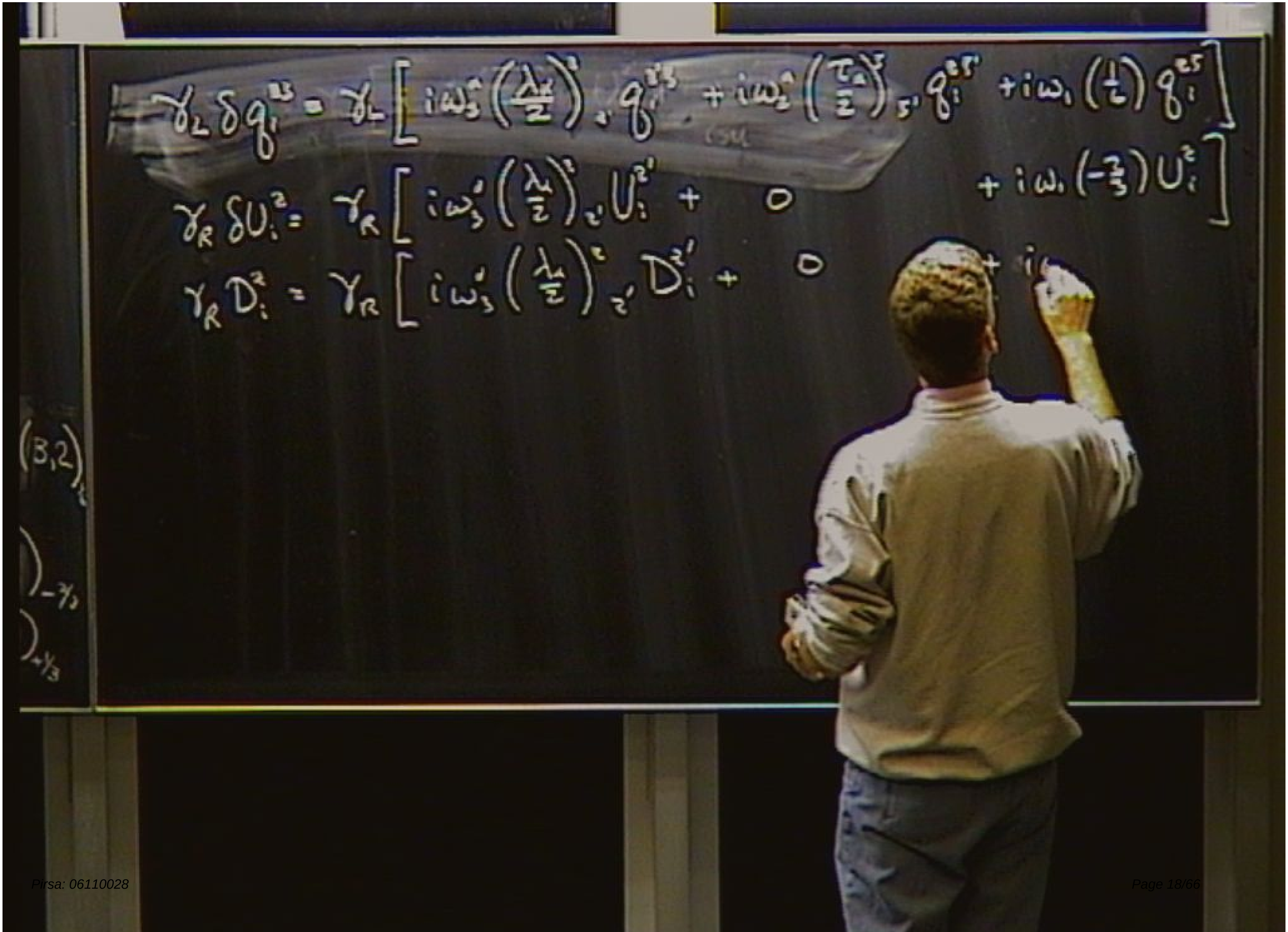
D^z_i " " " RH d, s, b

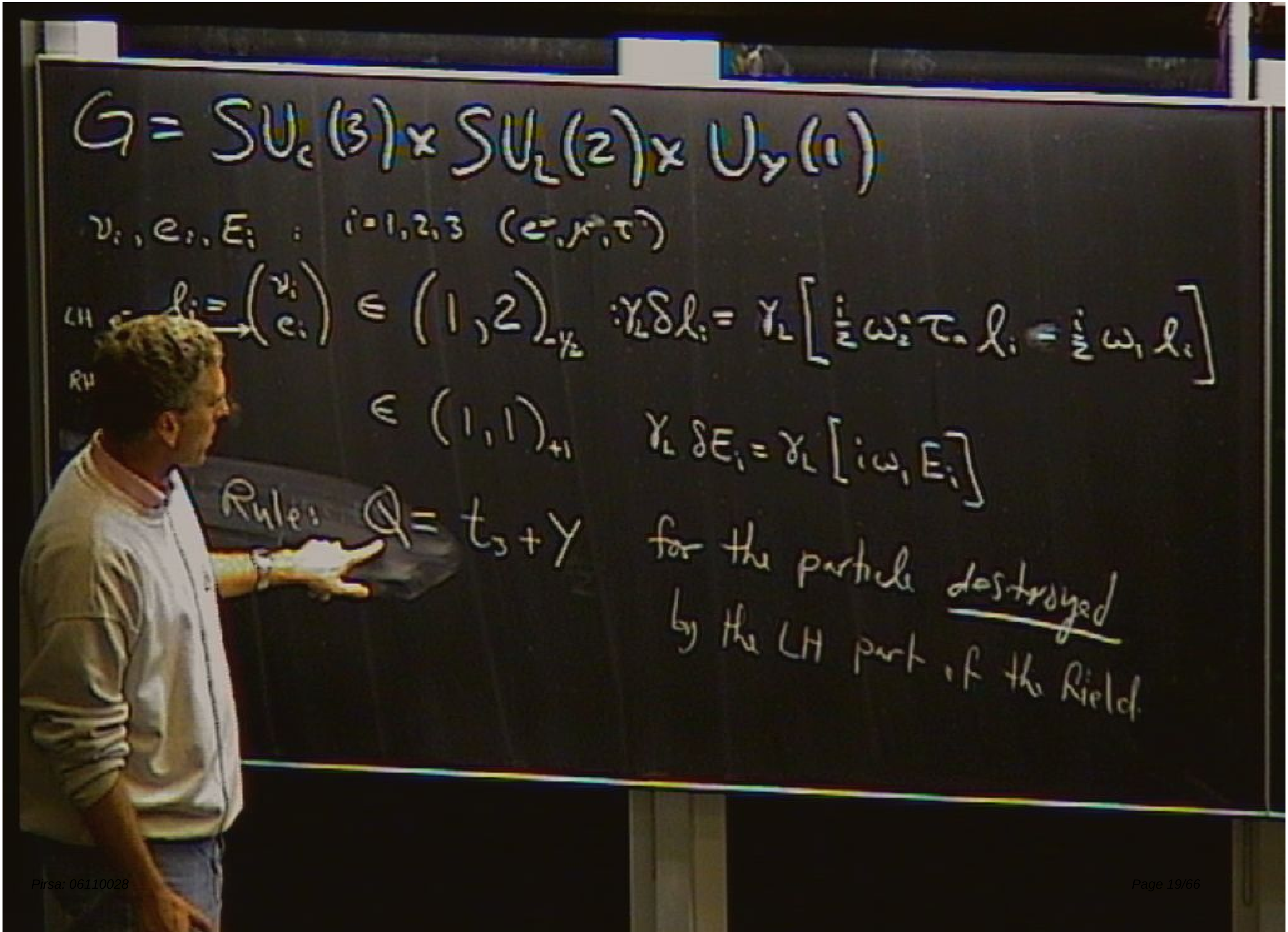
$U^z_i \in (\bar{3}, 1)_{-2/3}$

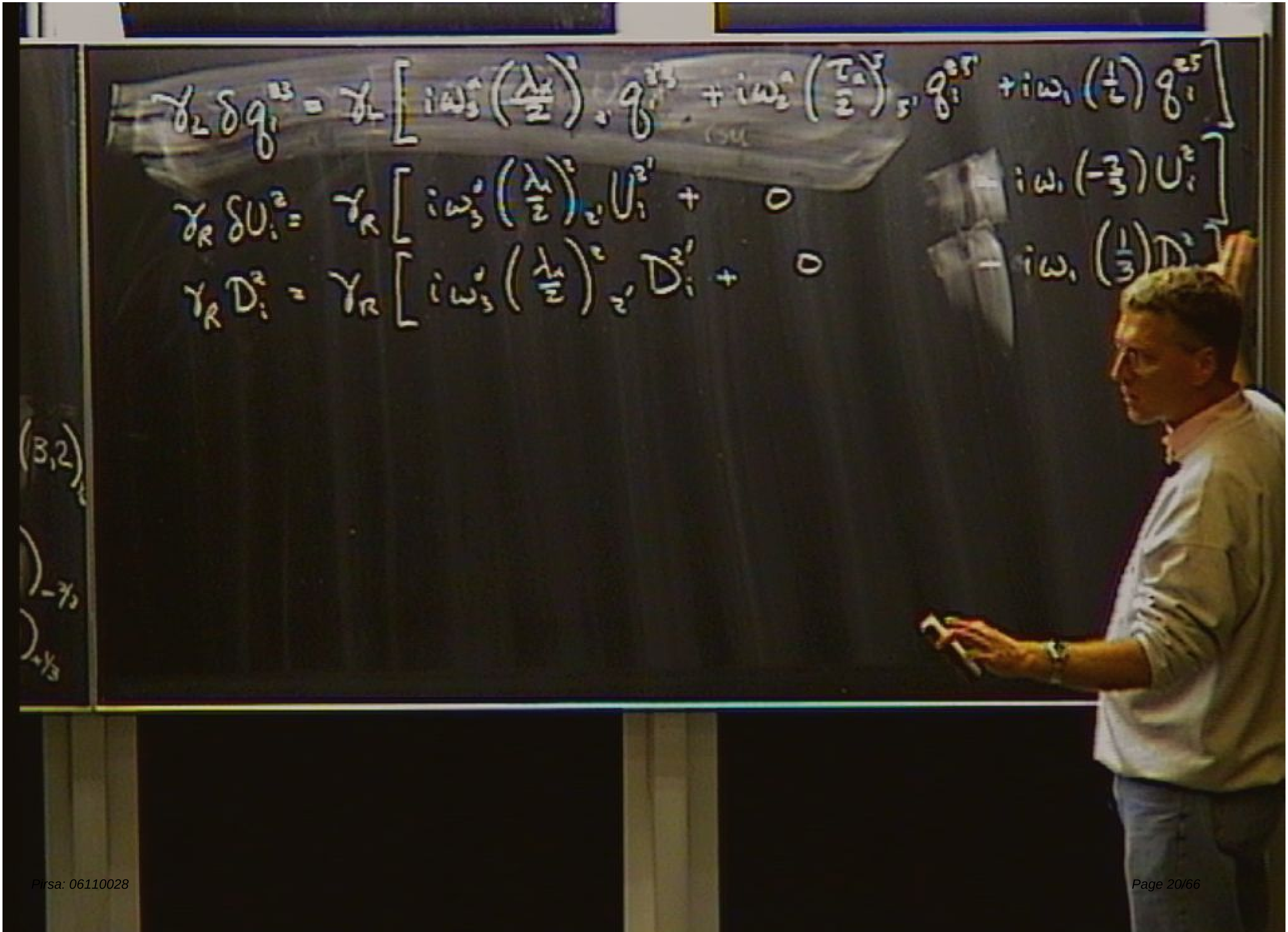
$D^z_i \in (\bar{3}, 1)_{+1/3}$

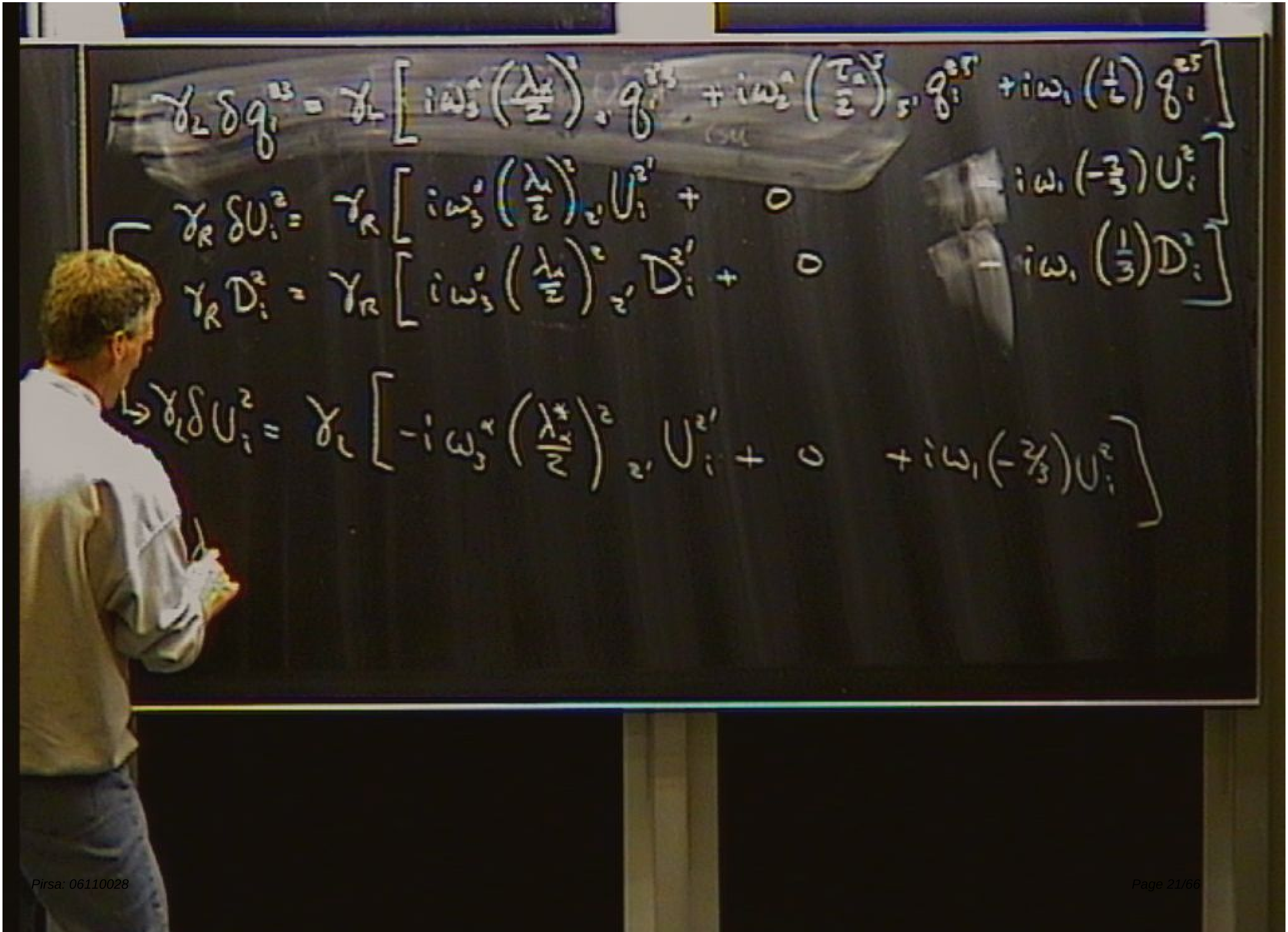
$z = 1,2,3$ colour (red, yellow, blue)




γ_L δq_i^{a3} = γ_L [iω_3^a (λ_a/2)_{33'} q_i^{3'5} + iω_2^a (τ_a/2)_{55'} q_i^{35'} + iω_1 (1/6) q_i^{35}]
γ_R δU_i^2 = γ_R [iω_3^a (λ_a/2)_{22'} U_i^{2'} + 0 + iω_1 (-2/3) U_i^2]
γ_R D_i^2 = γ_R [iω_3^a (λ_a/2)_{22'} D_i^{2'} + 0 + i
(3,2)
)_{-1/3}
)_{+1/3}


G = SU_c(3) x SU_L(2) x U_Y(1)
ν_i , e_i , E_i : i = 1,2,3 (e⁻, μ⁻, τ⁻)
LH
RH
Rule: Q = t_3 + Y for the particle destroyed
by the LH part of the field.


$\gamma_L \delta q_i^{a3} = \gamma_L \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{aa'} q_i^{a'3} + i\omega_2^a \left(\frac{\tau_a}{2}\right)_{33'} q_i^{a3'} + i\omega_1 \left(\frac{1}{6}\right) q_i^{a3} \right]$
$\gamma_R \delta U_i^a = \gamma_R \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{aa'} U_i^{a'} + 0 + i\omega_1 \left(-\frac{2}{3}\right) U_i^a \right]$
$\gamma_R D_i^a = \gamma_R \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{aa'} D_i^{a'} + 0 + i\omega_1 \left(\frac{1}{3}\right) D_i^a \right]$
$(3,2)_{1/6}$
$)_{-2/3}$
$)_{+1/3}$



$\gamma_L \delta q_i^{a3} = \gamma_L \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{3,3'} q_i^{a3'} + i\omega_2^a \left(\frac{\tau_a}{2}\right)_{5,5'} q_i^{a5'} + i\omega_1 \left(\frac{1}{6}\right) q_i^{a5} \right]$
$\gamma_R \delta U_i^2 = \gamma_R \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{2,2'} U_i^{2'} + 0 \quad i\omega_1 \left(-\frac{2}{3}\right) U_i^2 \right]$
$\gamma_R D_i^2 = \gamma_R \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{2,2'} D_i^{2'} + 0 \quad i\omega_1 \left(\frac{1}{3}\right) D_i^2 \right]$
$\gamma_L \delta U_i^2 = \gamma_L \left[-i\omega_3^{a*} \left(\frac{\lambda_a^*}{2}\right)_{2,2'} U_i^{2'} + 0 + i\omega_1 \left(-2/3\right) U_i^2 \right]$

$$\gamma_L \delta q_i^{\alpha 3} = \gamma_L \left[i\omega_3^{\hat{a}} \left(\frac{\lambda_{\hat{a}}}{2}\right)_{\alpha\beta} q_i^{\beta 3} + i\omega_2^{\hat{a}} \left(\frac{\tau_{\hat{a}}}{2}\right)_{s s'} q_i^{\alpha s'} + i\omega_1 \left(\frac{1}{6}\right) q_i^{\alpha s} \right]$$

$$\gamma_R \delta U_i^{\alpha} = \gamma_R \left[i\omega_3^{\hat{a}} \left(\frac{\lambda_{\hat{a}}}{2}\right)_{\alpha\alpha'} U_i^{\alpha'} + 0 + i\omega_1 \left(-\tfrac{2}{3}\right) U_i^{\alpha} \right]$$

$$\gamma_R D_i^{\alpha} = \gamma_R \left[i\omega_3^{\hat{a}} \left(\frac{\lambda_{\hat{a}}}{2}\right)_{\alpha\alpha'} D_i^{\alpha'} + 0 + i\omega_1 \left(\tfrac{1}{3}\right) D_i^{\alpha} \right]$$

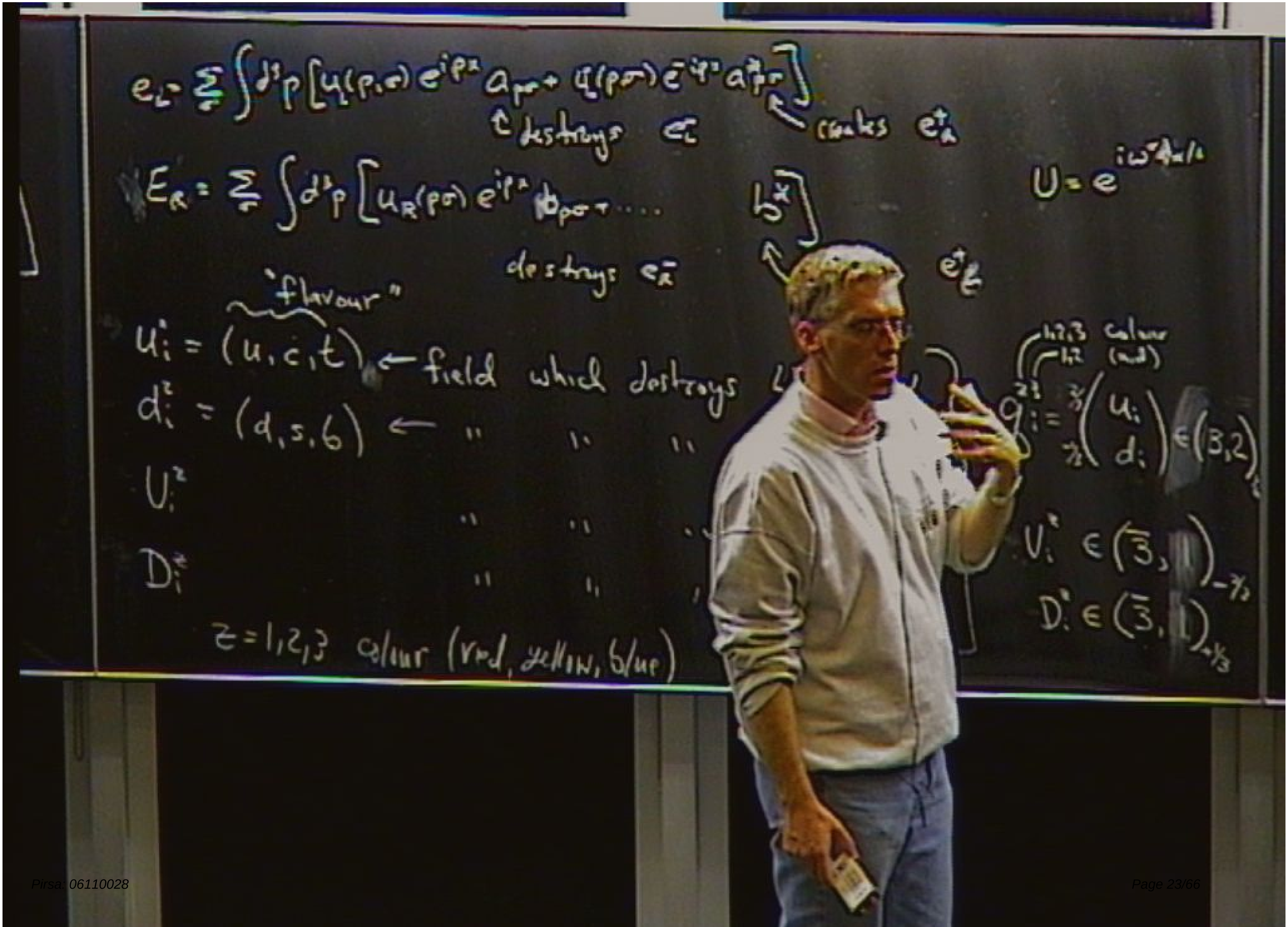
$$\gamma_L \delta U_i^{\alpha} = \gamma_L \left[-i\omega_3^{\hat{a}} \left(\frac{\lambda_{\hat{a}}^*}{2}\right)_{\alpha\alpha'} U_i^{\alpha'} + 0 + i\omega_1 \left(-\tfrac{2}{3}\right) U_i^{\alpha} \right]$$

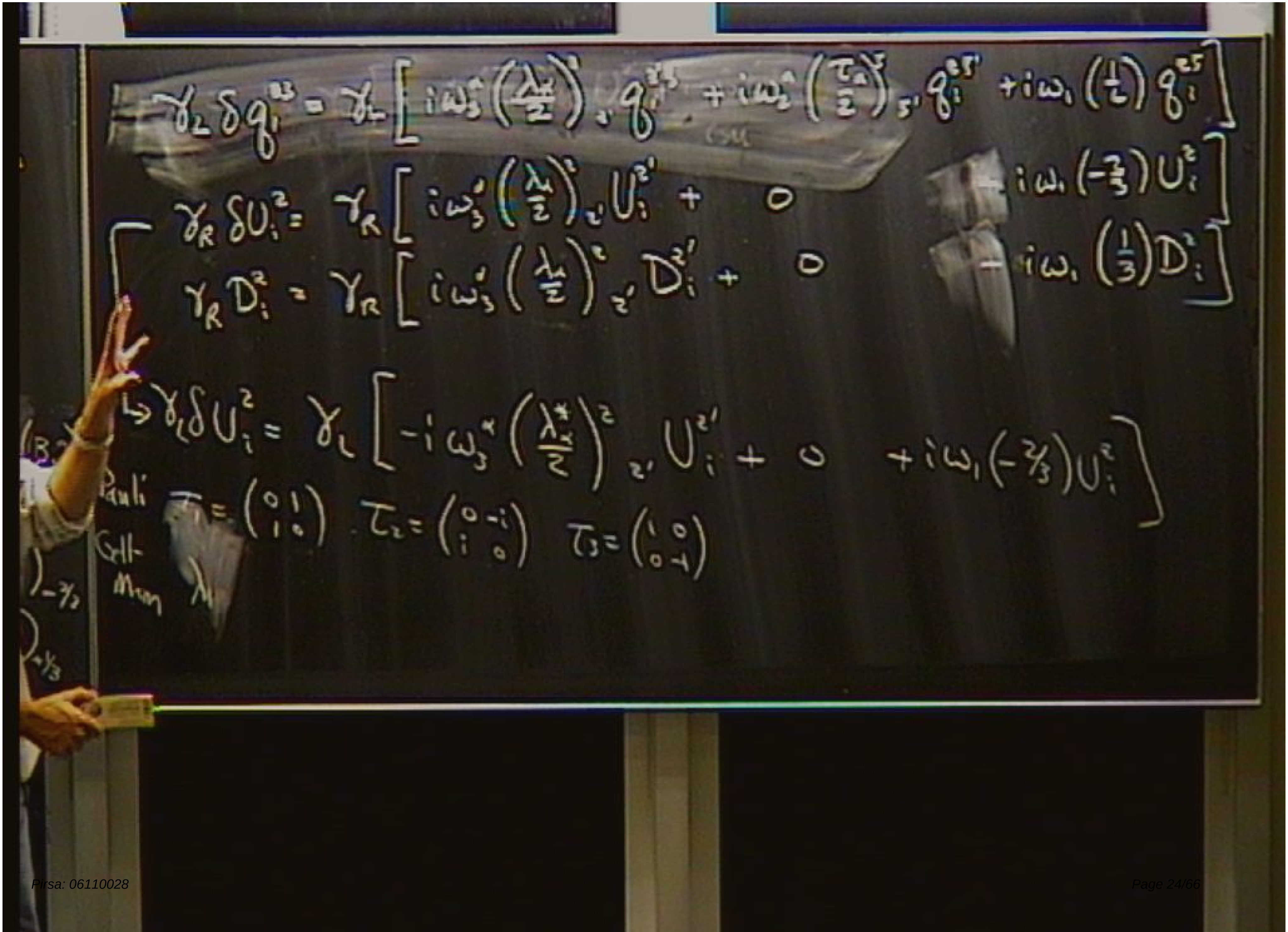
$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

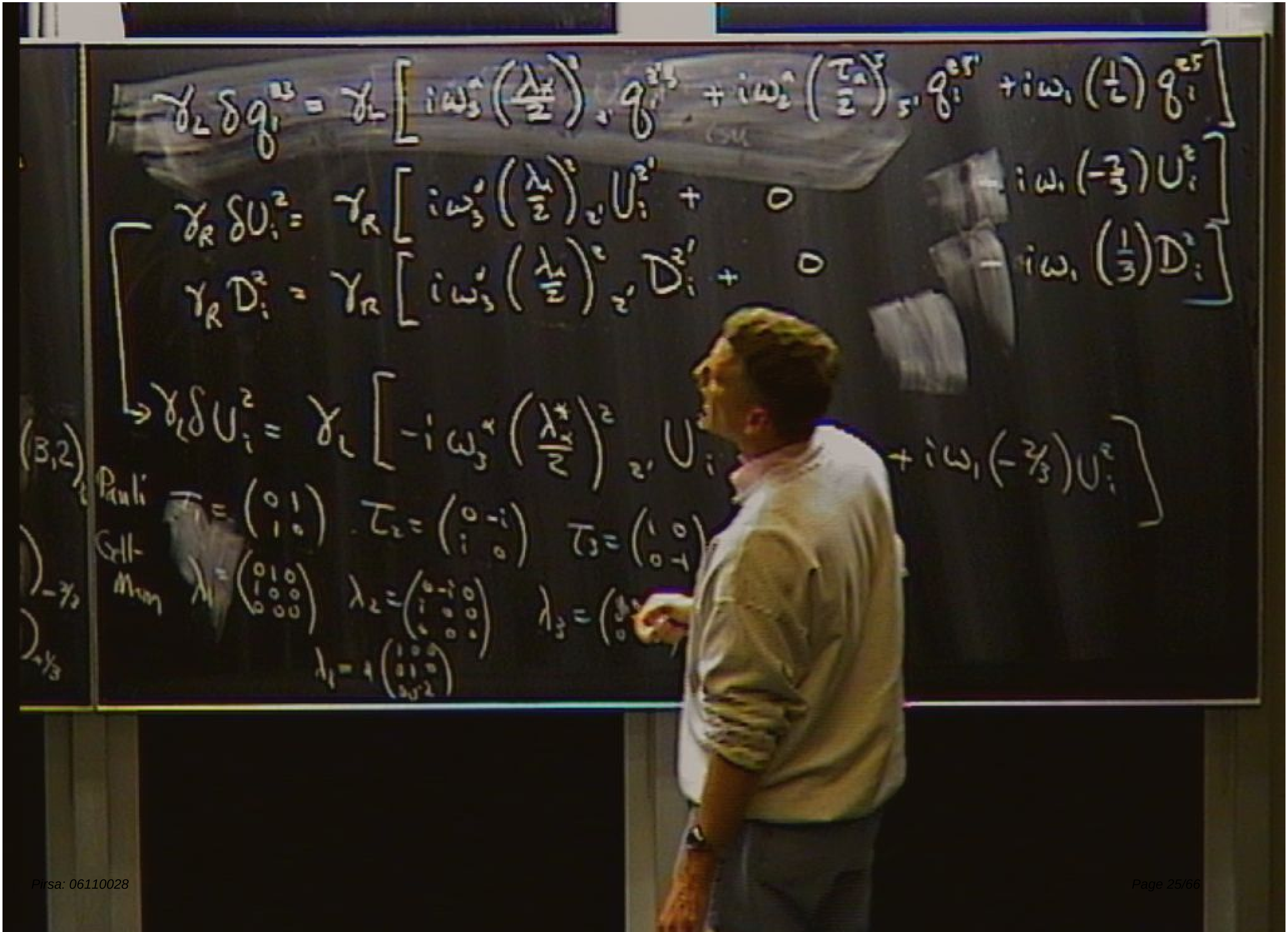
$(3,2)_{1/6}$

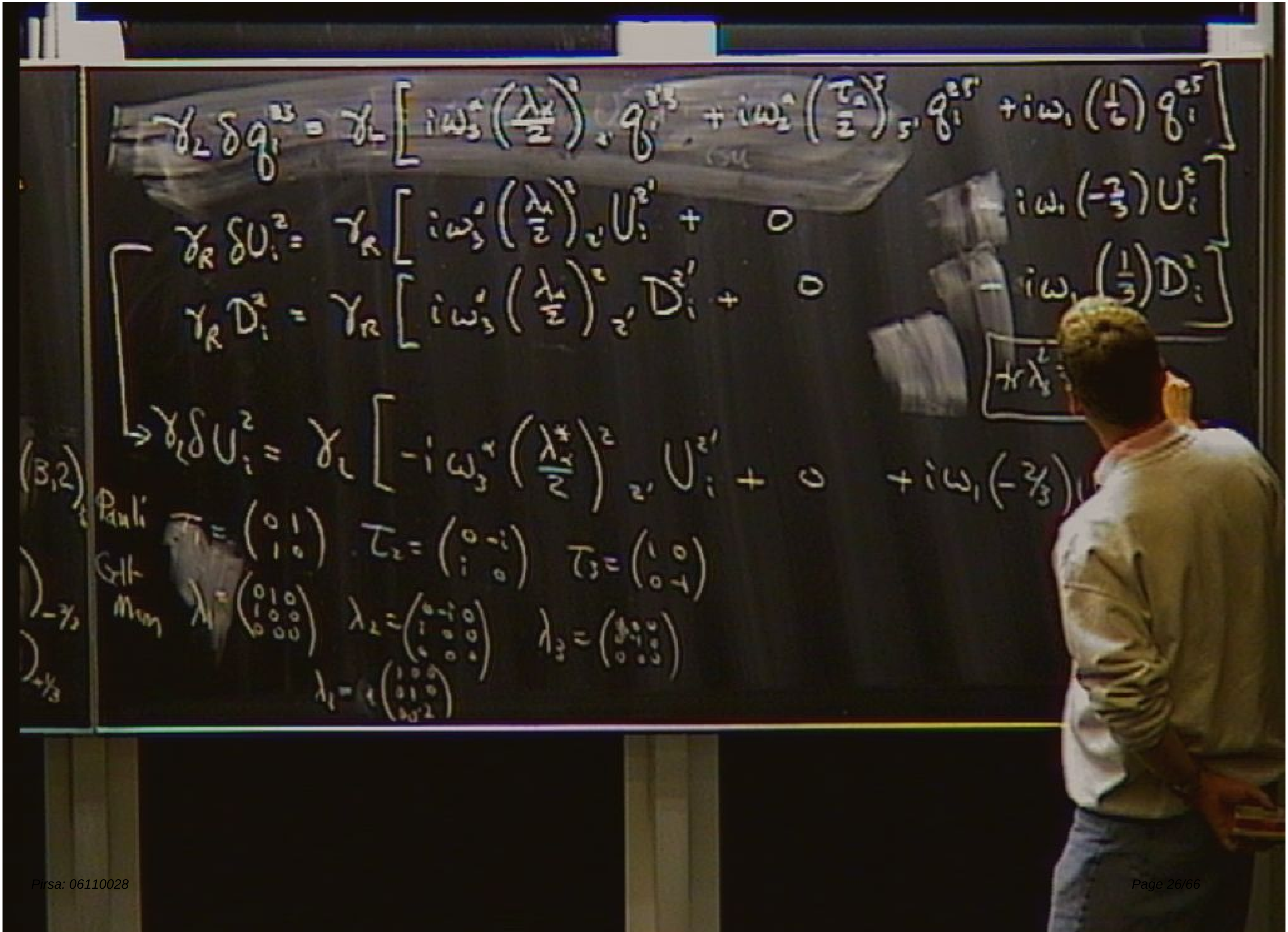
$)_{-2/3}$

$)_{+1/3}$

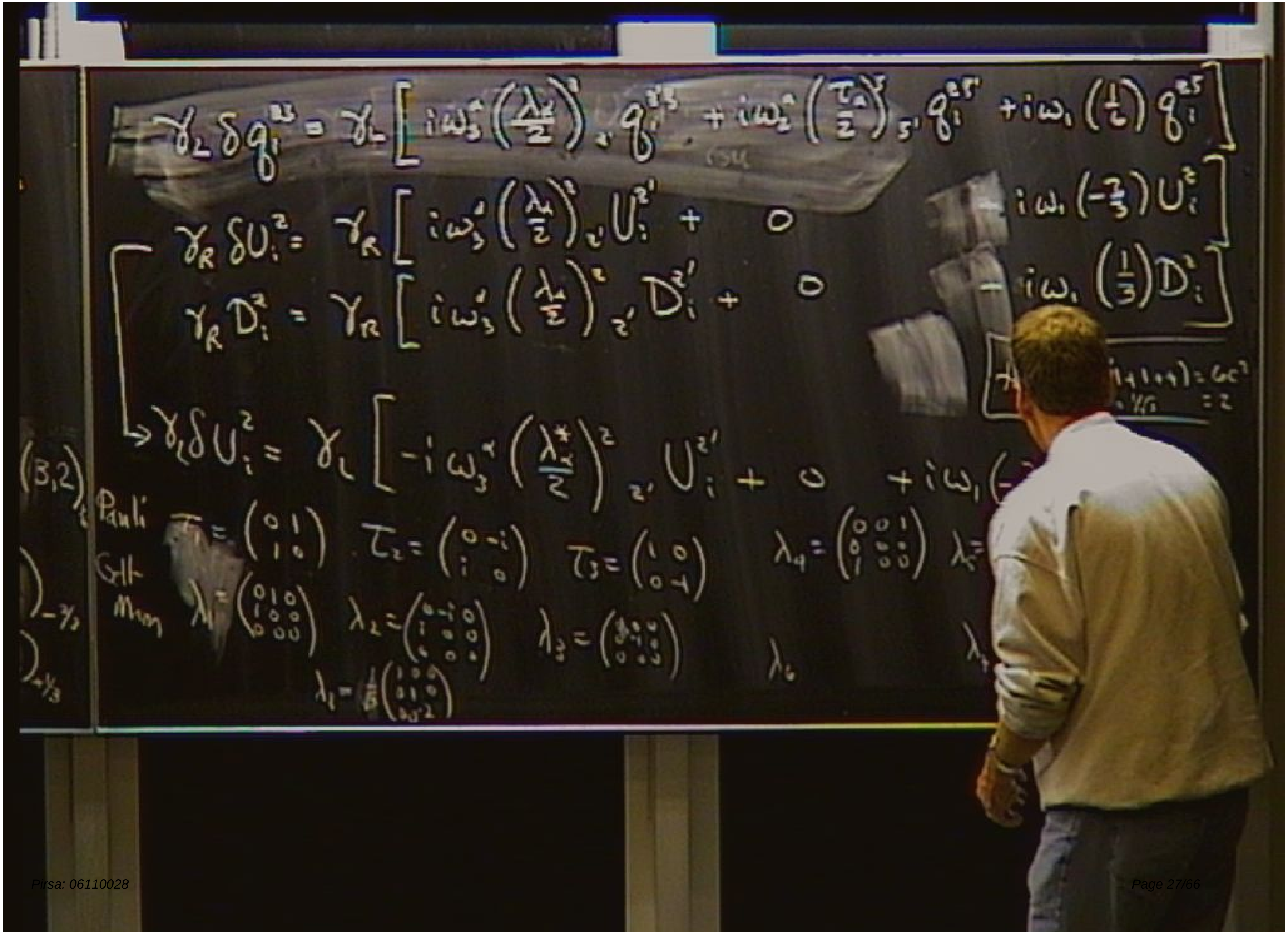

destroys
creates
destroys
"flavour"
field which destroys
colour (red, yellow, blue)
colour (red)

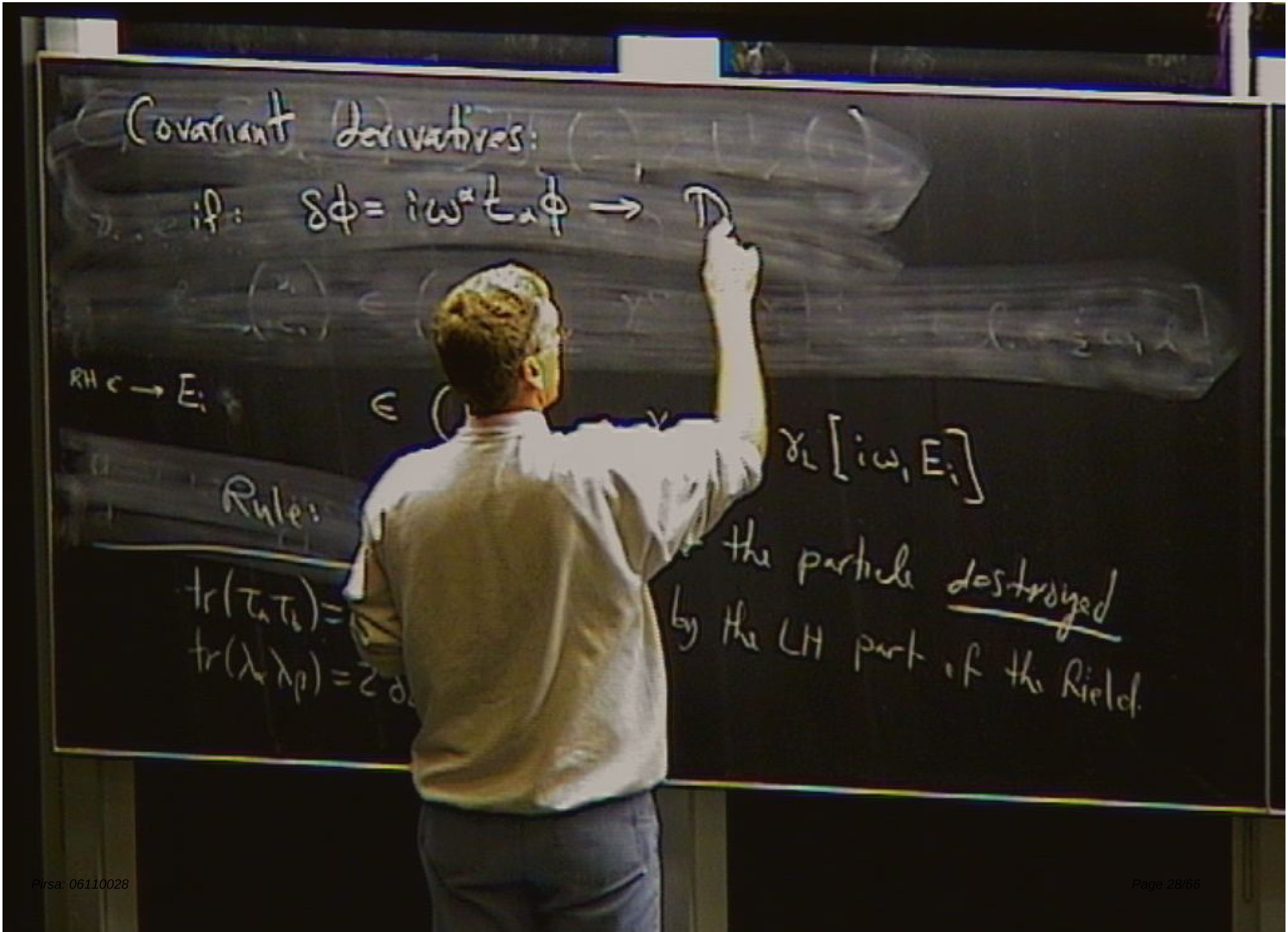

$\gamma_L \delta q_i^{a3} = \gamma_L \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{33'} q_i^{3'5} + i\omega_2^a \left(\frac{\tau_a}{2}\right)_{55'} q_i^{35'} + i\omega_1 \left(\frac{1}{6}\right) q_i^{35} \right]$
$\gamma_R \delta U_i^2 = \gamma_R \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{22'} U_i^{2'} + 0 \quad i\omega_1 \left(-\frac{2}{3}\right) U_i^2 \right]$
$\gamma_R D_i^2 = \gamma_R \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{22'} D_i^{2'} + 0 \quad i\omega_1 \left(\frac{1}{3}\right) D_i^2 \right]$
$\hookrightarrow \gamma_L \delta U_i^2 = \gamma_L \left[-i\omega_3^a \left(\frac{\lambda_a^*}{2}\right)_{22'} U_i^{2'} + 0 + i\omega_1 \left(-2/3\right) U_i^2 \right]$
Pauli $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Gell-Mann λ_a
$)_{-2/3}$
$)_{+1/3}$


$\gamma_L \delta q_i^{\alpha 3} = \gamma_L [i\omega_3^a (\frac{\lambda_a}{2})_{\alpha'} q_i^{\alpha' 3} + i\omega_2^a (\frac{\tau_a}{2})_{s'} q_i^{\alpha s'} + i\omega_1 (\frac{1}{6}) q_i^{\alpha s}]$
$\gamma_R \delta U_i^\alpha = \gamma_R [i\omega_3^a (\frac{\lambda_a}{2})_{\alpha'} U_i^{\alpha'} + 0 + i\omega_1 (-\frac{2}{3}) U_i^\alpha]$
$\gamma_R D_i^\alpha = \gamma_R [i\omega_3^a (\frac{\lambda_a}{2})_{\alpha'} D_i^{\alpha'} + 0 + i\omega_1 (\frac{1}{3}) D_i^\alpha]$
$\gamma_L \delta U_i^\alpha = \gamma_L [-i\omega_3^a (\frac{\lambda_a^*}{2})_{\alpha'} U_i + i\omega_1 (-\frac{2}{3}) U_i^\alpha]$
$(\bar{3},2)$
Pauli $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Gell-Mann $\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_3 =$
$\lambda_8 =$



$\gamma_L \delta q_i^{a3} = \gamma_L \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{33'} q_i^{3'5} + i\omega_2^a \left(\frac{\tau_a}{2}\right)_{55'} q_i^{35'} + i\omega_1 \left(\frac{1}{6}\right) q_i^{35} \right]$
$\gamma_R \delta U_i^2 = \gamma_R \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{22'} U_i^{2'} + 0 \quad i\omega_1 \left(-\frac{2}{3}\right) U_i^2 \right]$
$\gamma_R D_i^2 = \gamma_R \left[i\omega_3^a \left(\frac{\lambda_a}{2}\right)_{22'} D_i^{2'} + 0 \quad - i\omega_1 \left(\frac{1}{3}\right) D_i^2 \right]$
$\gamma_L \delta U_i^2 = \gamma_L \left[-i\omega_3^a \left(\frac{\lambda_a^*}{2}\right)_{22'} U_i^{2'} + 0 \quad + i\omega_1 \left(-\frac{2}{3}\right) \right.$
$(\bar{3},2)$
Pauli $\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Gell-Mann $\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$
$)_{-2/3}$
$)_{+1/3}$
$\mathrm{tr}\,\lambda_8^2$


Pauli
Gell
Mann


Covariant derivatives:
if: δφ = iω^a t_a φ → D
RH e → E_i
γ_L [iω, E_i]
Rule:
the particle destroyed
by the LH part of the field.
tr(τ_a τ_b) =
tr(λ_α λ_β) = 2 δ

Covariant derivatives:

if: $\delta\phi = i\omega^a t_a \phi \longrightarrow D_\mu \phi = \partial_\mu \phi - i A^a_\mu t_a \phi$

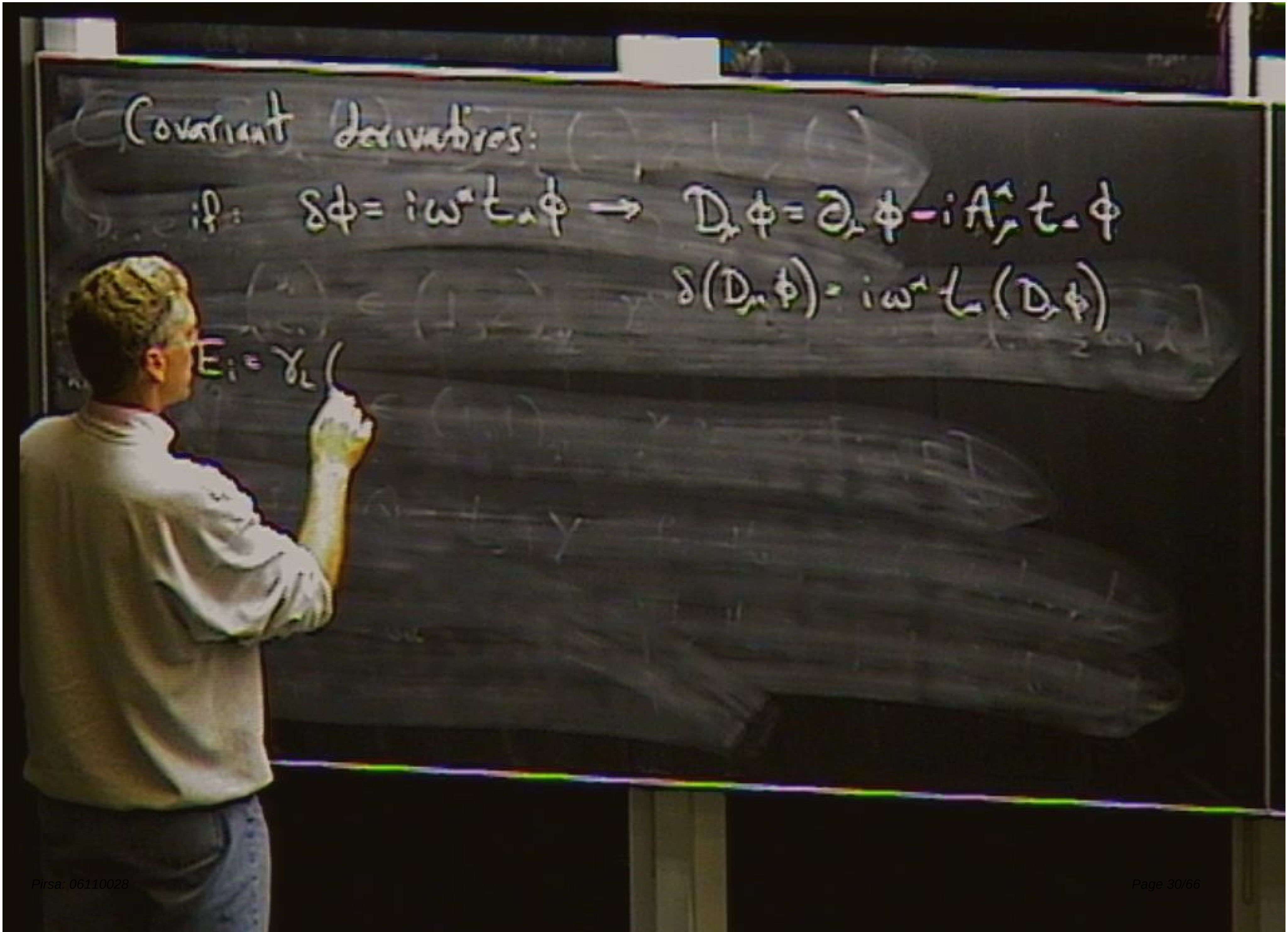
$$\delta(D_\mu \phi) = i\omega^a t_a (D_\mu \phi)$$

RH $e \longrightarrow E_i$ $\in (1,1)_{+1}$ $\quad \gamma_L \, \delta E_i = \gamma_L [i\omega, E_i]$

: $Q = t_3 + Y$ for the particle <u>destroyed</u> by the LH part of the field.

δ_{ab}

$\delta_{\alpha\beta}$


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \longrightarrow D_\mu \phi = \partial_\mu \phi - i A_\mu^a t_a \phi$
$\delta(D_\mu \phi) = i\omega^a t_a (D_\mu \phi)$
$E_i = \gamma_L ($

Covariant derivatives:

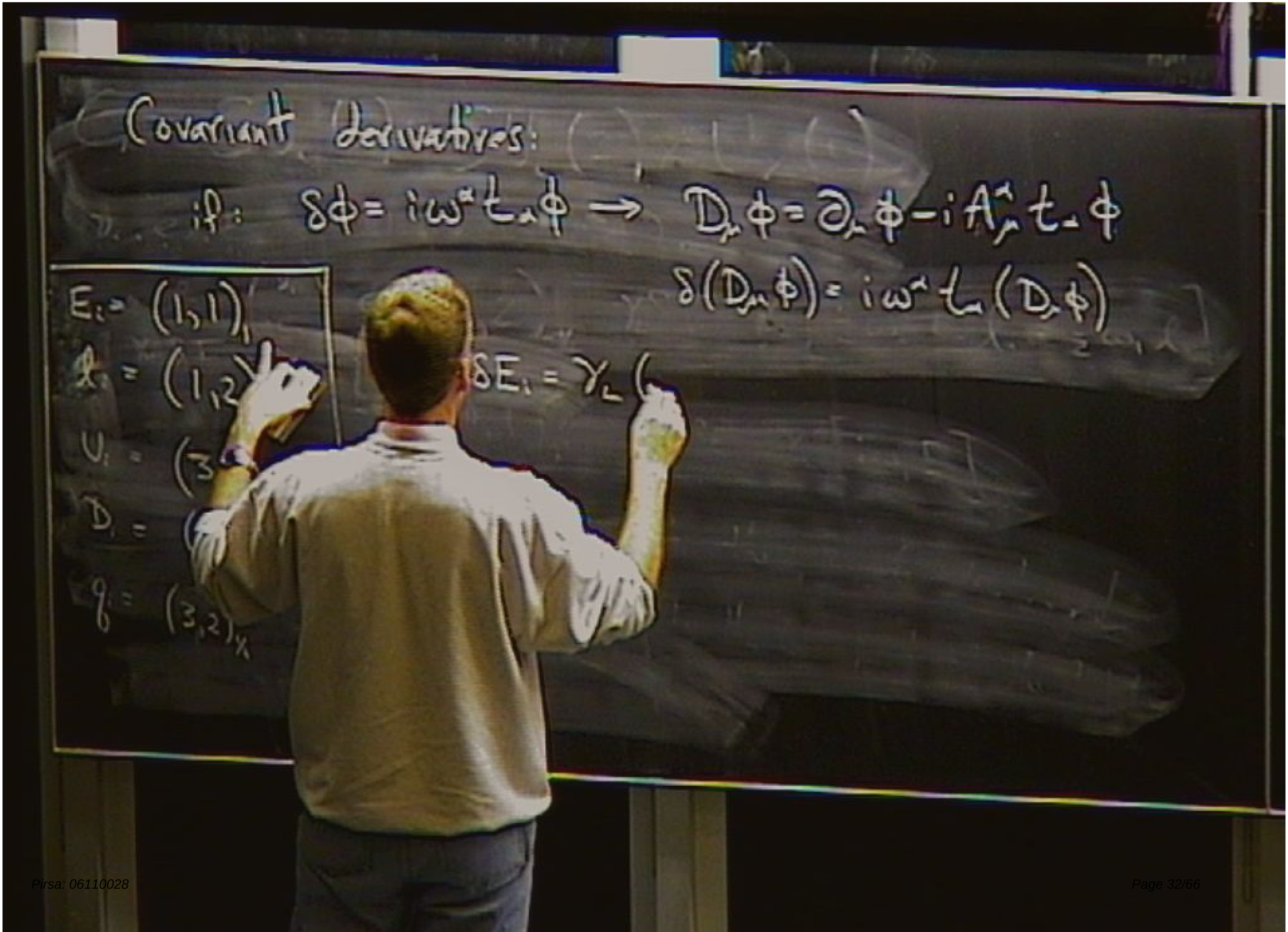
if: $\delta\phi = i\omega^a t_a \phi \longrightarrow D_\mu \phi = \partial_\mu \phi - i A^a_\mu t_a \phi$

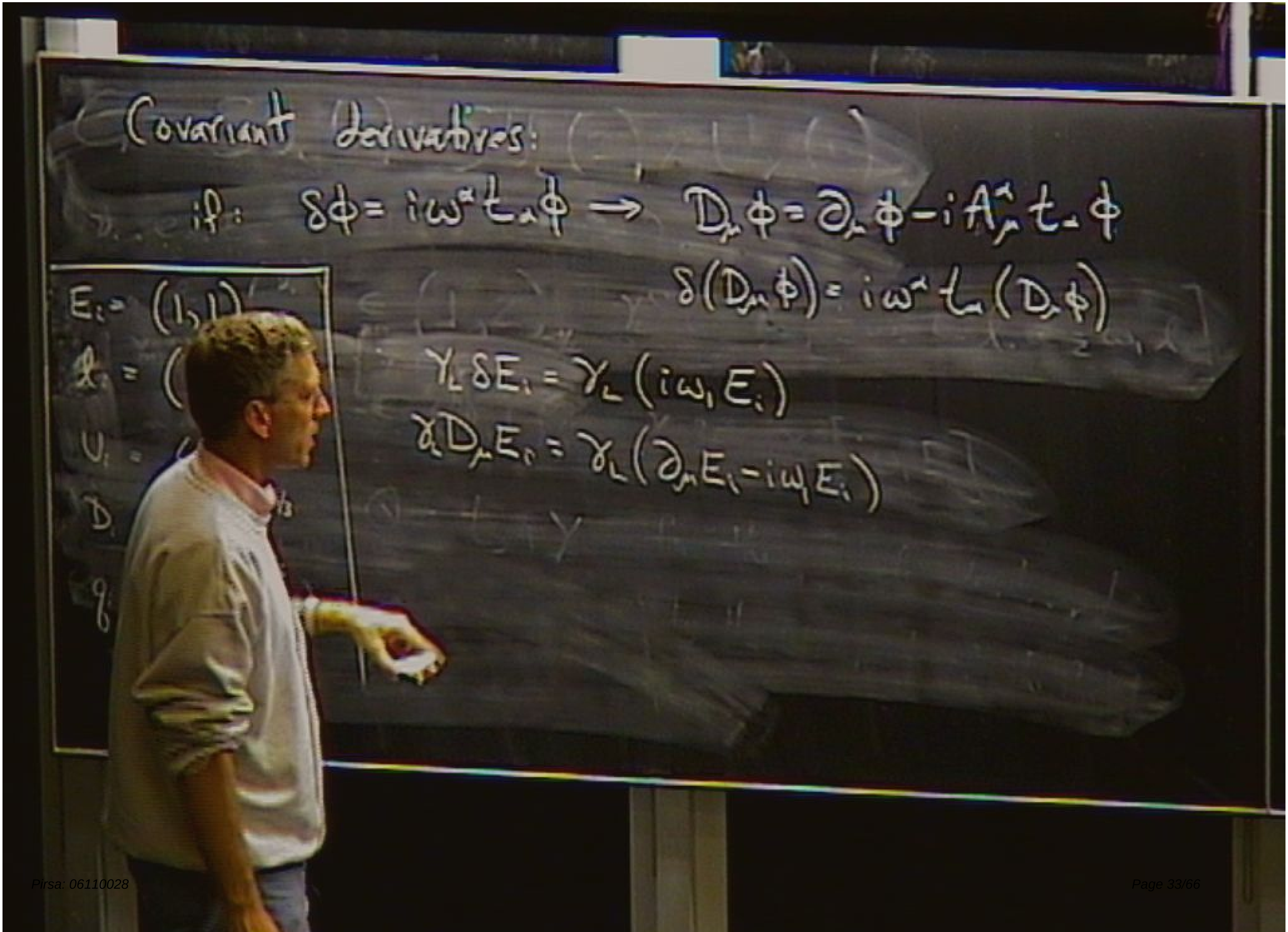
$\delta(D_\mu \phi) = i\omega^a t_a (D_\mu \phi)$

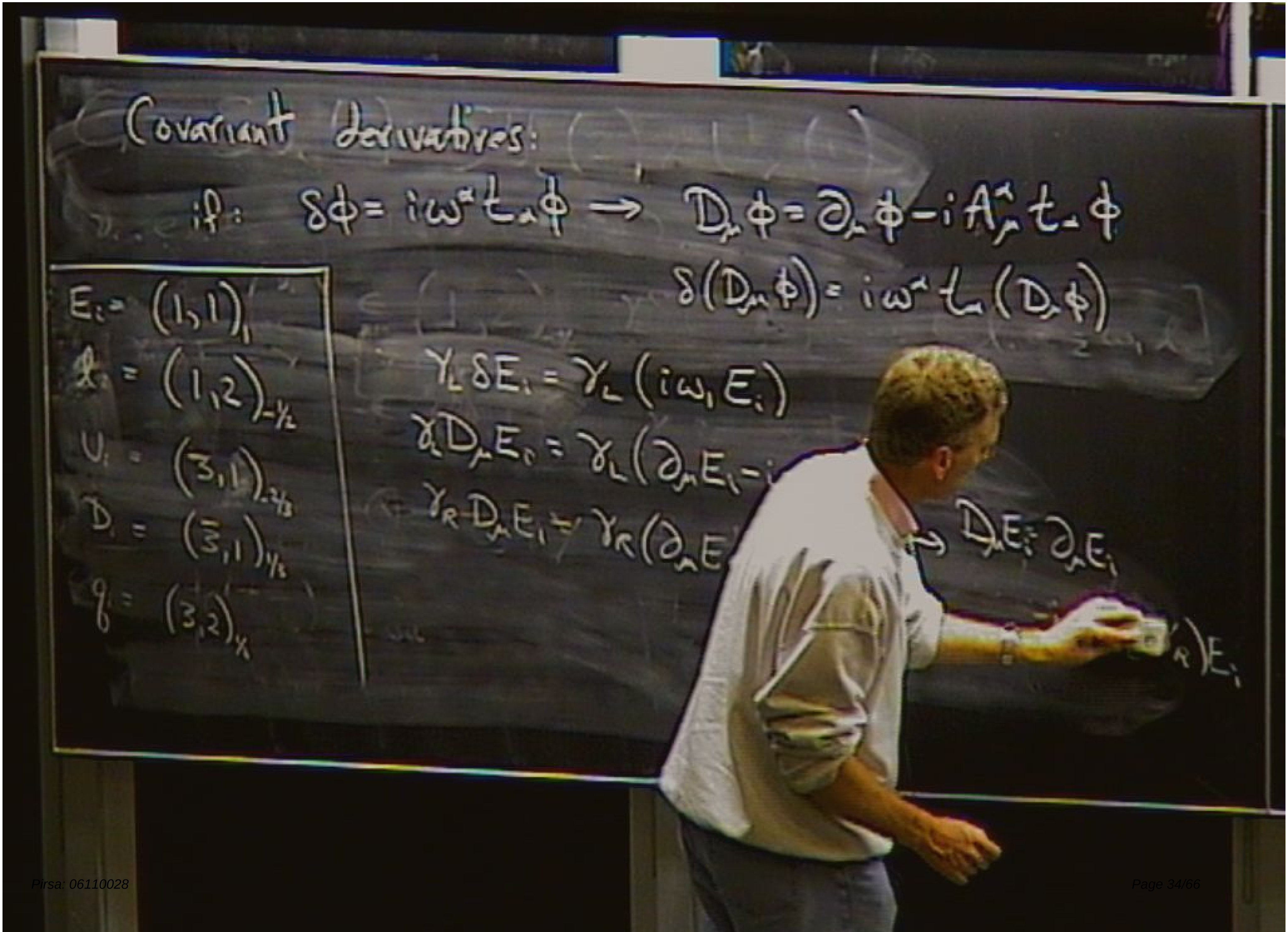
$E_i = (1,1)_1$

$\ell = (1,2)_{-1/2}$

U


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \longrightarrow D_\mu \phi = \partial_\mu \phi - i A_\mu^a t_a \phi$
$\delta(D_\mu \phi) = i\omega^a t_a (D_\mu \phi)$
$E_i = (1,1)_1$
$\ell_i = (1,2)$
$U_i = (3$
$D_i = ($
$q_i = (3,2)_x$
$\delta E_i = \gamma_L ($


Covariant derivatives:
if: $\delta\phi = i\omega^a t_a \phi \longrightarrow D_\mu \phi = \partial_\mu \phi - i A^a_\mu t_a \phi$
$\delta(D_\mu \phi) = i\omega^a t_a (D_\mu \phi)$
$E_i = (1,1)$
$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i)$
$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i - i\omega_1 E_i)$


Covariant derivatives:
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$\delta(D_\mu \phi) = i\omega^a t_a (D_\mu \phi)$
$E_i = (1,1)_1$
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$q_i = (3,2)_{1/6}$
$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i)$
$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i -$
$\gamma_R D_\mu E_i = \gamma_R (\partial_\mu E$
$D_\mu E_i = \partial_\mu E_i$

Covariant derivatives:

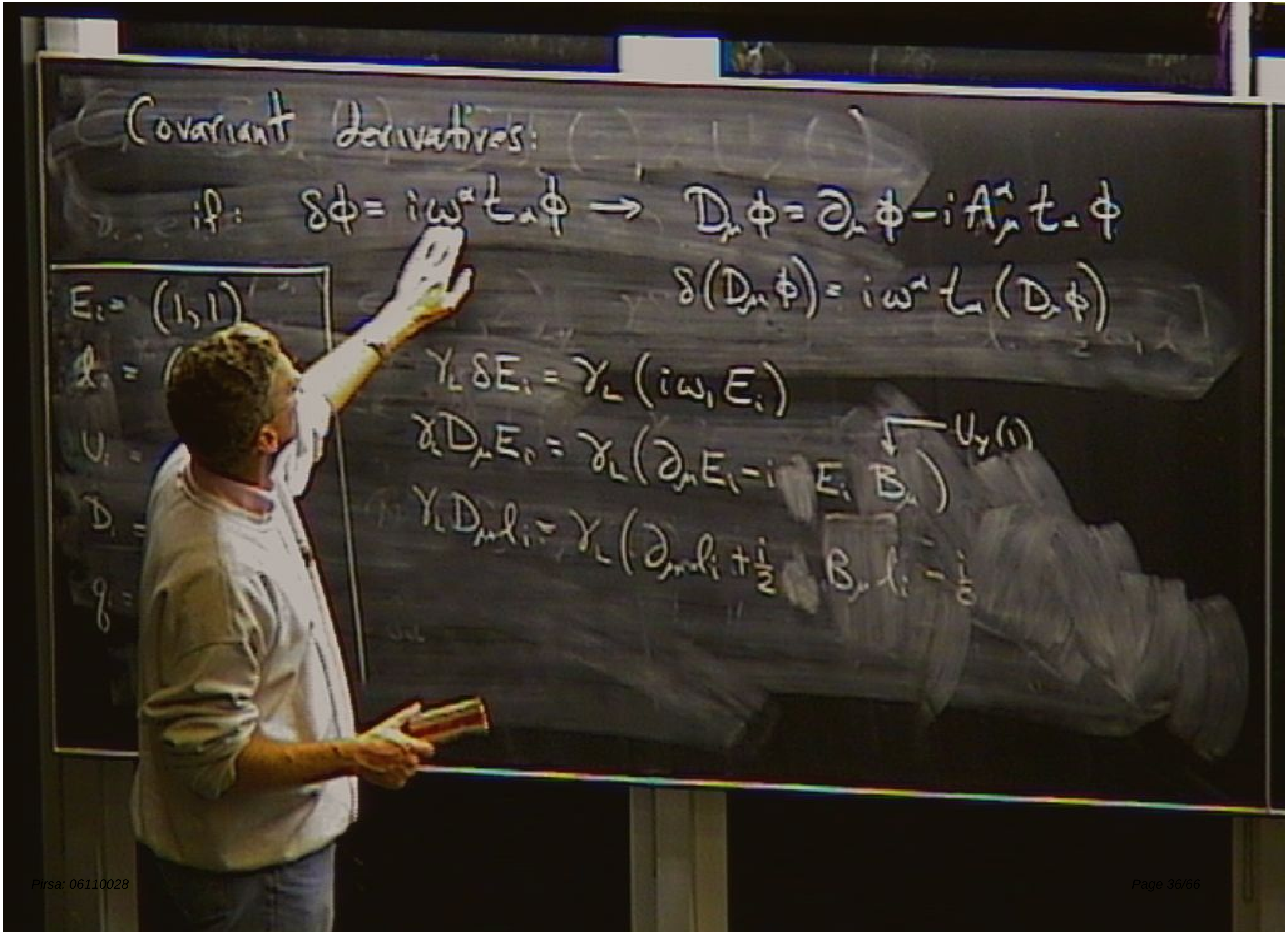
if: $\delta\phi = i\omega^\alpha t_\alpha \phi \rightarrow D_\mu \phi = \partial_\mu \phi - i A_\mu^\alpha t_\alpha \phi$

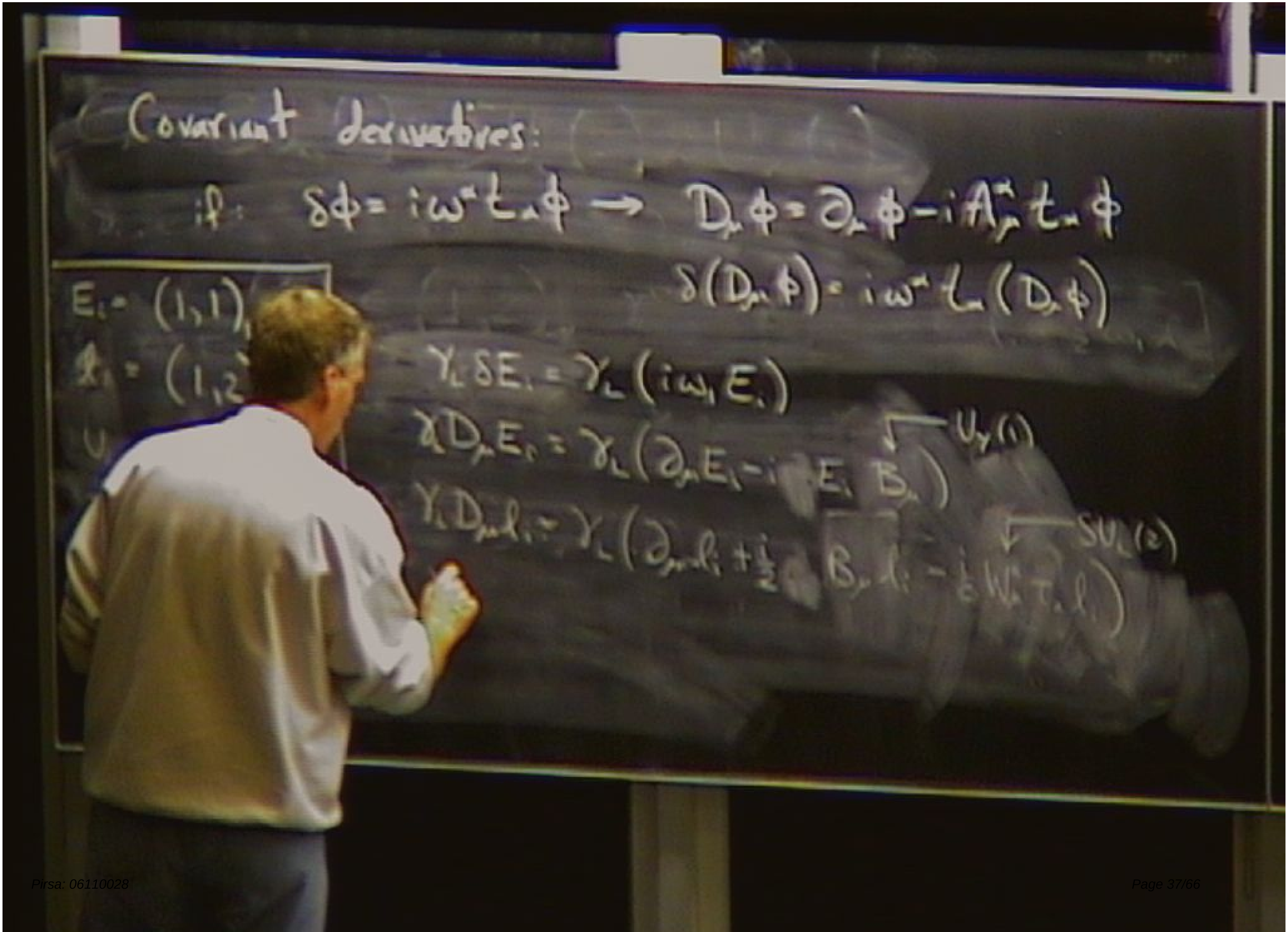
$$\delta(D_\mu \phi) = i\omega^\alpha t_\alpha (D_\mu \phi)$$

$$\boxed{\begin{aligned} E_i &= (1,1)_1 \\ \ell_i &= (1,2)_{-1/2} \\ U_i &= (\bar{3},1)_{-2/3} \\ D_i &= (\bar{3},1)_{1/3} \\ q_i &= (3,2)_{1/6} \end{aligned}}$$

$$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i)$$

$$\left.\begin{aligned} \gamma_L D_\mu E_i &= \gamma_L(\partial_\mu E_i - i\omega_1 E_i) \\ \gamma_R D_\mu E_i &= \gamma_R(\partial_\mu E_i + i\omega_1 E_i) \end{aligned}\right\} \rightarrow D_\mu E_i = \partial_\mu E_i - i\omega_1 \gamma_5 E_i$$


Covariant derivatives:
if: δφ = iω^α t_α φ → D_μ φ = ∂_μ φ − i A^α_μ t_α φ
δ(D_μ φ) = iω^α t_α (D_μ φ)
E_i = (1,1)
Y_L δE_i = Y_L (iω_1 E_i)
U_Y(1)
Y_L D_μ E_i = Y_L (∂_μ E_i − i E_i B_μ)
Y_L D_μ l_i = Y_L (∂_μ l_i + i/2 B_μ l_i − i/2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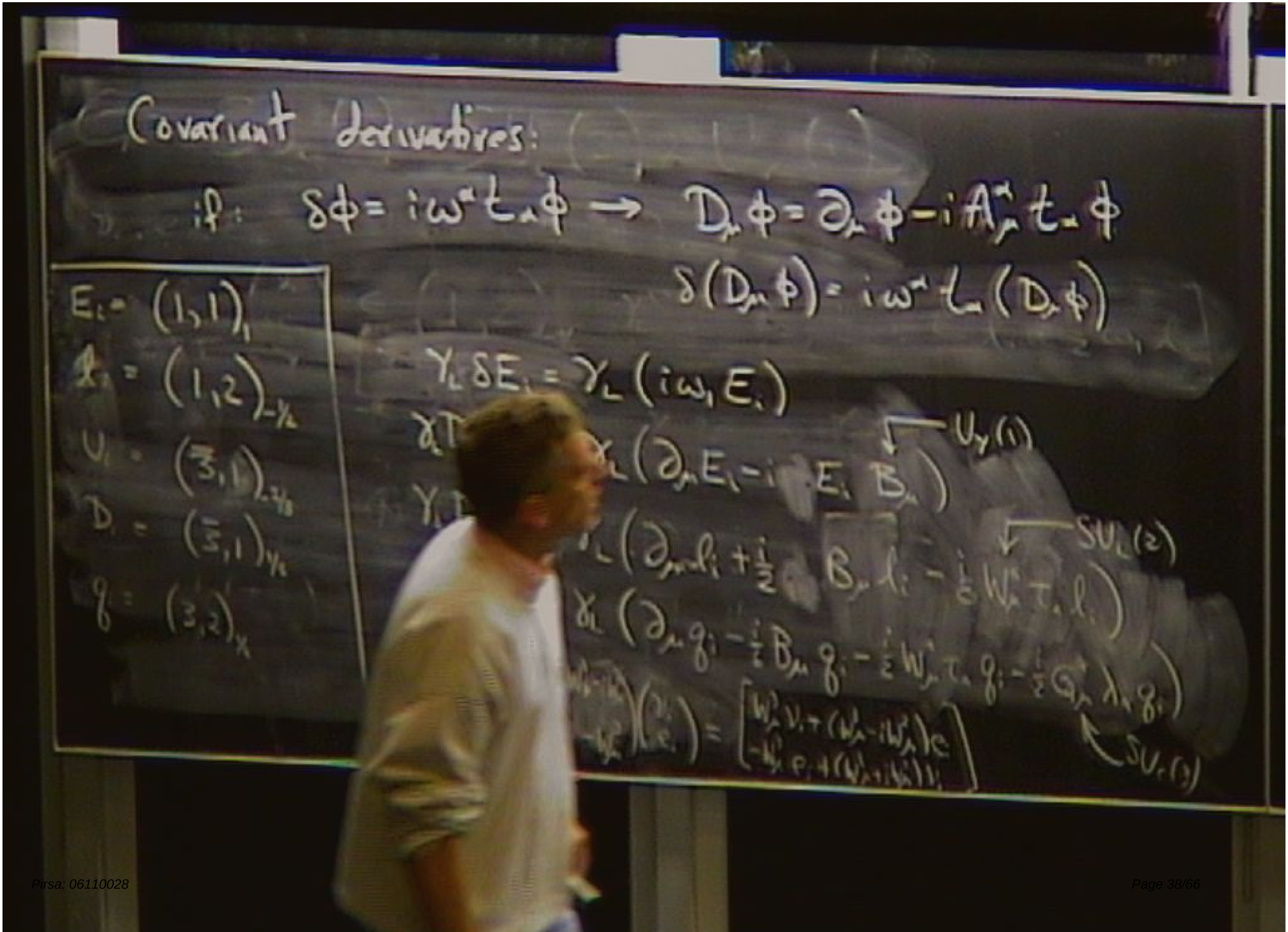
$E_i = (1,1)$
$U_Y(1)$
$SU_L(2)$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$SU_c(3)$

Covariant derivatives:

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$$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i)$$

$U_Y(1)$

$$\gamma_L D_\mu E_i = \gamma_L(\partial_\mu E_i - i E_i B_\mu)$$

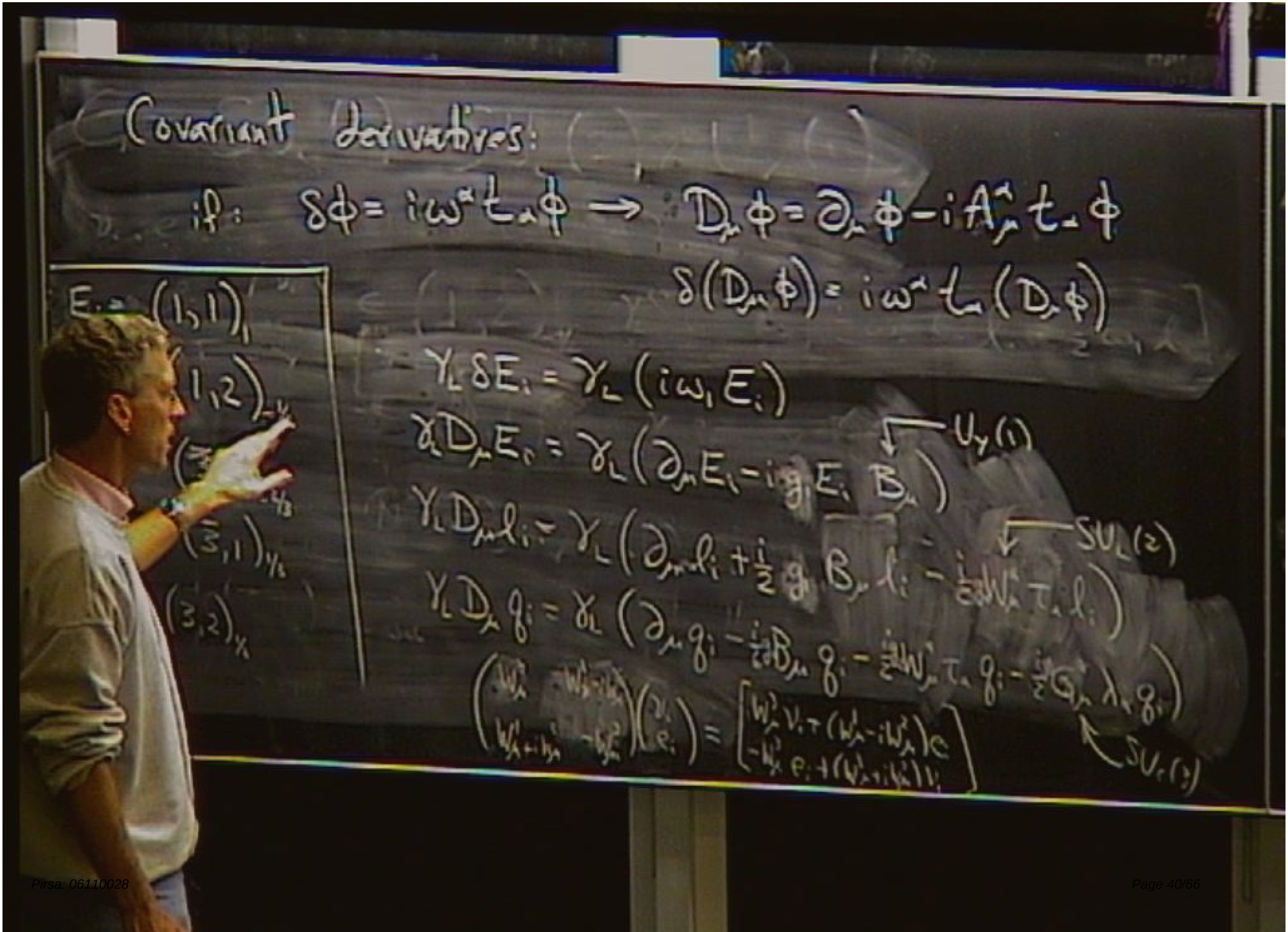
$SU_L(2)$

$$\gamma_L D_\mu l_i = \gamma_L\left(\partial_\mu l_i + \frac{i}{2} B_\mu l_i - \frac{i}{2} W^a_\mu \tau_a l_i\right)$$

$$\gamma_L D_\mu q_i = \gamma_L\left(\partial_\mu q_i - \frac{i}{2} B_\mu q_i - \frac{i}{2} W^a_\mu \tau_a q_i - \frac{i}{2} G^a_\mu \lambda_a q_i\right)$$

$SU_c(3)$

$$\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\begin{pmatrix} \nu_i \\ e_i \end{pmatrix} = \begin{bmatrix} W^3_\mu \nu_i + (W^1_\mu - iW^2_\mu) e_i \\ -W^3_\mu e_i + (W^1_\mu + iW^2_\mu)\nu_i \end{bmatrix}$$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(3,2)_{1/6}
γ_L δE_i = γ_L (iω_1 E_i)
γ_L D_μ E_i = γ_L (∂_μ E_i − i g_1 E_i B_μ)
U_Y(1)
γ_L D_μ l_i = γ_L (∂_μ l_i + i/2 g_1 B_μ l_i − i/2 g W_μ^a τ_a l_i)
SU_L(2)
γ_L D_μ q_i = γ_L (∂_μ q_i − i/6 B_μ q_i − i/2 W_μ^a τ_a q_i − i/2 G_μ^a λ_a q_i)
SU_c(3)

Covariant derivatives:

if: $\delta\phi = i\omega^a t_a \phi \longrightarrow D_\mu \phi = \partial_\mu \phi - i A^a_\mu t_a \phi$

$$\delta(D_\mu \phi) = i\omega^a t_a (D_\mu \phi)$$

$$\boxed{\begin{aligned} E_i &= (1,1)_1 \\ l_i &= (1,2)_{-1/2} \\ U_i &= (\bar{3},1)_{-2/3} \\ D_i &= (\bar{3},1)_{1/3} \\ q_i &= (3,2)_{1/6} \end{aligned}}$$

$$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i)$$

$U_Y(1)$

$$\gamma_L D_\mu E_i = \gamma_L (\partial_\mu E_i - i g_1 E_i B_\mu)$$

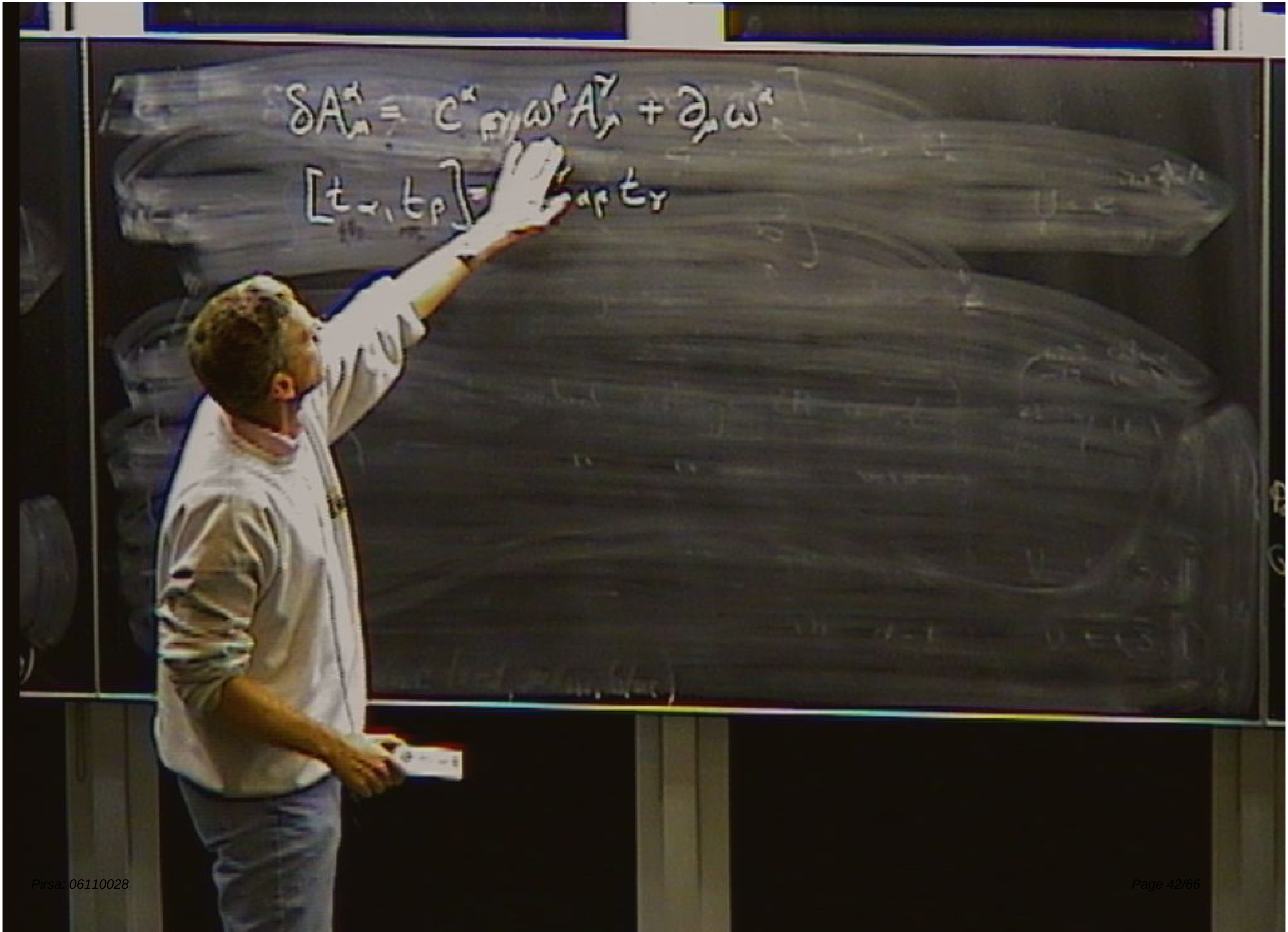
$SU_L(2)$

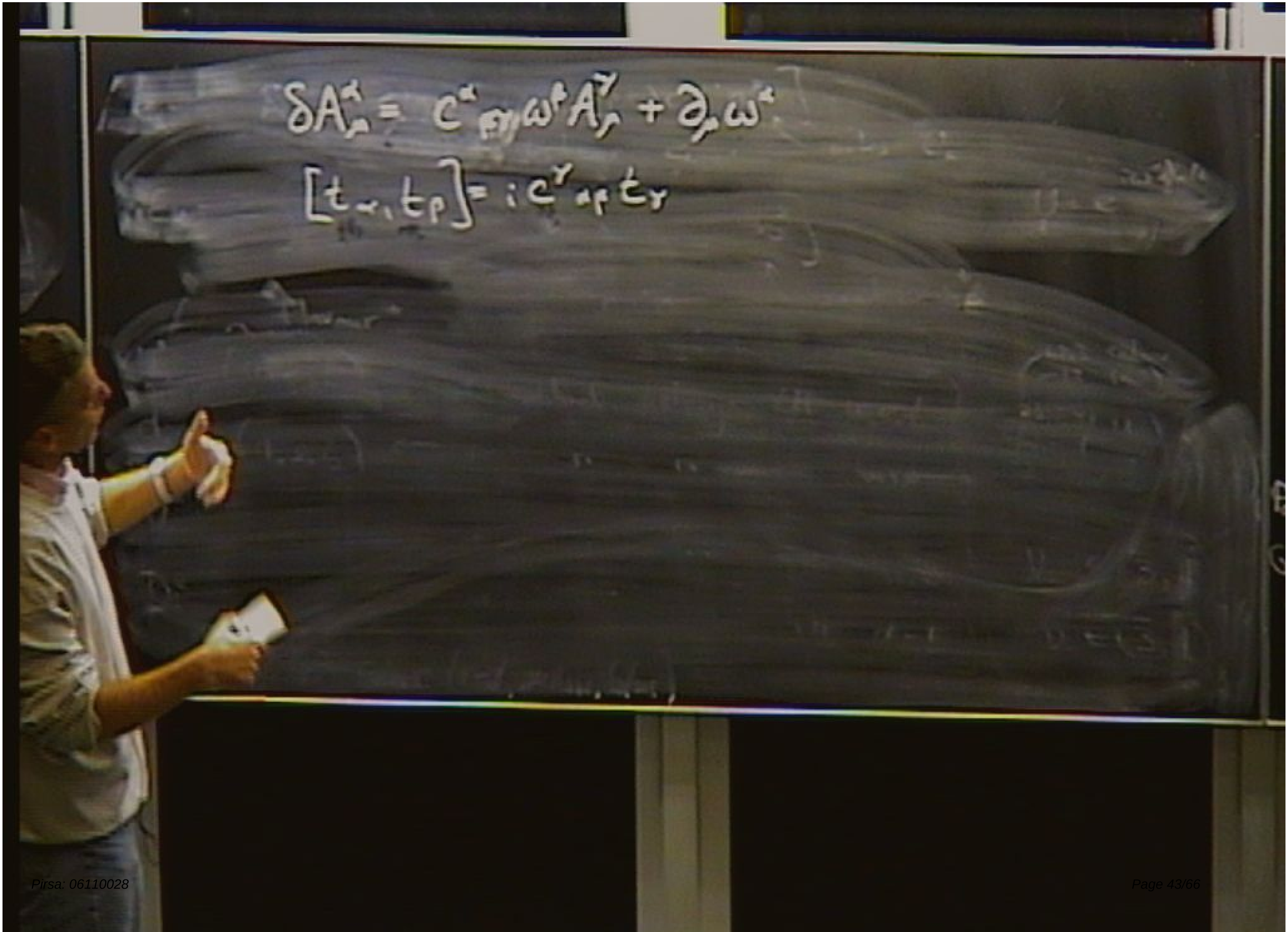
$$\gamma_L D_\mu l_i = \gamma_L \left(\partial_\mu l_i + \tfrac{i}{2} g_1 B_\mu l_i - \tfrac{i}{2} g W^a_\mu \tau_a l_i\right)$$

$$\gamma_L D_\mu q_i = \gamma_L \left(\partial_\mu q_i - \tfrac{i}{6} g_1 B_\mu q_i - \tfrac{i}{2} g W^a_\mu \tau_a q_i - \tfrac{i}{2} g_s G^a_\mu \lambda_a q_i\right)$$

$SU_c(3)$

$$\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix} \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} = \begin{bmatrix} W^3_\mu \nu_i + (W^1_\mu - iW^2_\mu) e_i \\ -W^3_\mu e_i + (W^1_\mu + iW^2_\mu) \nu_i \end{bmatrix}$$


$\delta A^\alpha_\mu = C^\alpha{}_{\beta\gamma}\,\omega^\beta A^\gamma_\mu + \partial_\mu \omega^\alpha$
$[t_\alpha, t_\beta] =$ $_{\alpha\beta}\, t_\gamma$


$\delta A^\alpha_\mu = C^\alpha{}_{\beta\gamma}\,\omega^\beta A^\gamma_\mu + \partial_\mu \omega^\alpha$
$[t_\alpha, t_\beta] = i\,C^\gamma{}_{\alpha\beta}\,t_\gamma$

$$\delta A^{\alpha}_{\mu} = C^{\alpha}{}_{\beta\gamma}\,\omega^{\beta} A^{\gamma}_{\mu} + \partial_{\mu}\omega^{\alpha}$$

$$[t_{\alpha}, t_{\beta}] = i C^{\gamma}{}_{\alpha\beta}\, t_{\gamma} \qquad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = i\epsilon_{abc}\left(\frac{\tau_c}{2}\right)$$

$$\delta B_{\mu} = \partial_{\mu}\omega_{1}$$

$$\delta W^{a}_{\mu} = \partial_{\mu}\omega^{a}_{2} + \epsilon^{a}{}_{bc}\,\omega^{b}_{2} W^{c}_{\mu}$$

$$\delta A^{\alpha}_{\mu} = C^{\alpha}{}_{\beta\gamma}\,\omega^{\beta} A^{\gamma}_{\mu} + \partial_{\mu}\omega^{\alpha}$$

$$[t_{\alpha}, t_{\beta}] = i C^{\gamma}{}_{\alpha\beta}\, t_{\gamma} \qquad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = i\epsilon_{abc}\left(\frac{\tau_c}{2}\right)$$

$$\delta B_{\mu} = \partial_{\mu}\omega_1$$

$$\delta W^{a}_{\mu} = \partial_{\mu}\omega^{a}_{2} + \epsilon^{a}{}_{bc}\,\omega^{b}_{2} W^{c}_{\mu}$$

$$\delta G^{\alpha}_{\mu} = \partial_{\mu}\omega^{\alpha}_{3} + f^{\alpha}{}_{\beta\gamma}\,\omega^{\beta}_{3} G^{\gamma}_{\mu}$$

$$\delta A^{\alpha}_{\mu} = C^{\alpha}{}_{\beta\gamma}\,\omega^{\beta} A^{\gamma}_{\mu} + \partial_{\mu}\omega^{\alpha}$$

$$[t_{\alpha}, t_{\beta}] = i\,C^{\gamma}{}_{\alpha\beta}\, t_{\gamma} \qquad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = i\epsilon_{abc}\left(\frac{\tau_c}{2}\right)$$

$$\delta B_{\mu} = \frac{1}{g_1}\partial_{\mu}\omega_1$$

$$\delta W^{a}_{\mu} = \frac{1}{g_2}\partial_{\mu}\omega^{a}_{2} + \epsilon^{a}{}_{bc}\,\omega^{b}_{2} W^{c}_{\mu}$$

$$\delta G^{\alpha}_{\mu} = \frac{1}{g_3}\partial_{\mu}\omega^{\alpha}_{3} + f^{\alpha}{}_{\beta\gamma}\,\omega^{\beta}_{3} G^{\gamma}_{\mu}$$

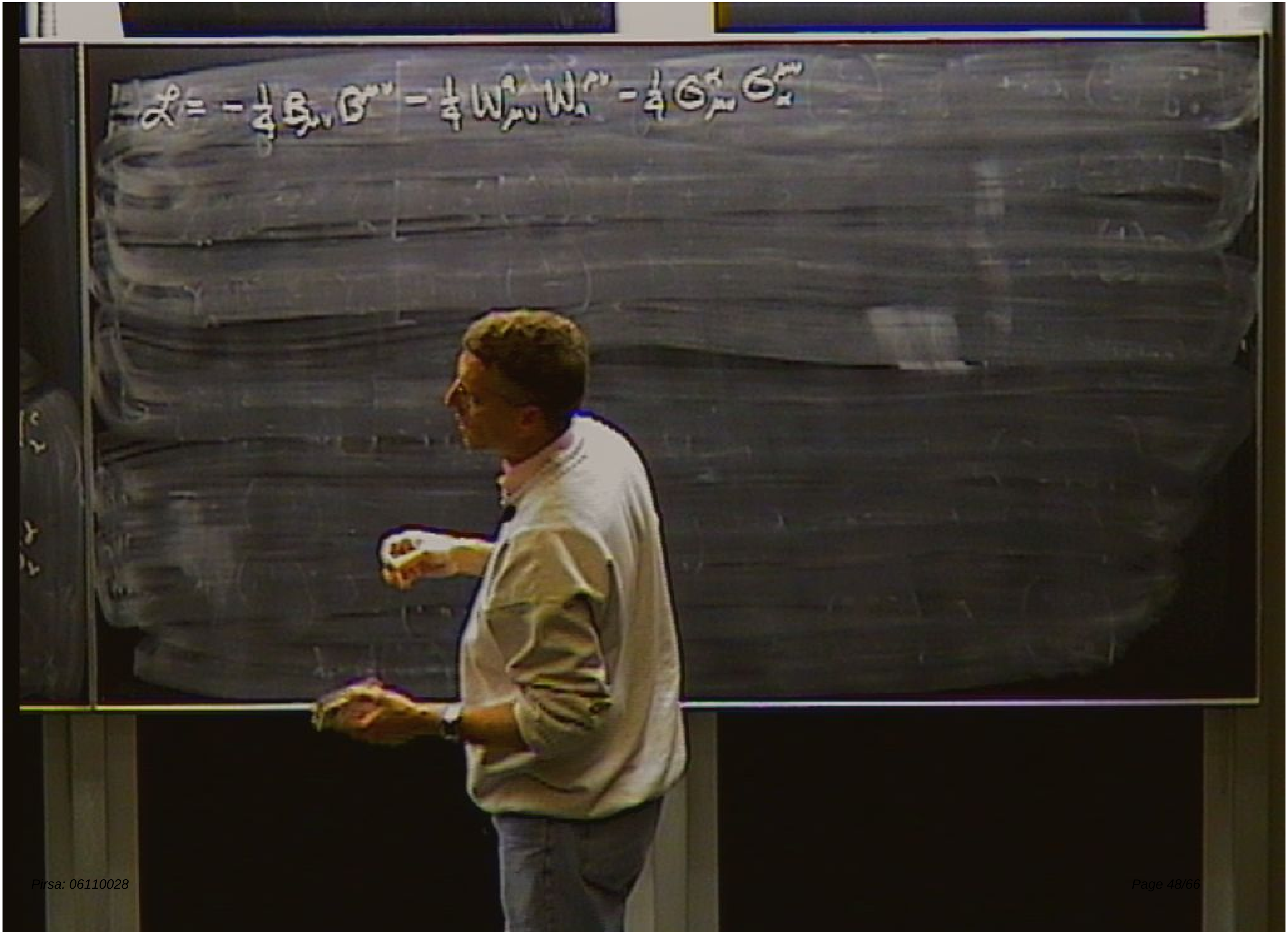
$$\delta A^{\alpha}_{\mu} = C^{\alpha}{}_{\beta\gamma}\,\omega^{\beta} A^{\gamma}_{\mu} + \partial_{\mu}\omega^{\alpha}$$

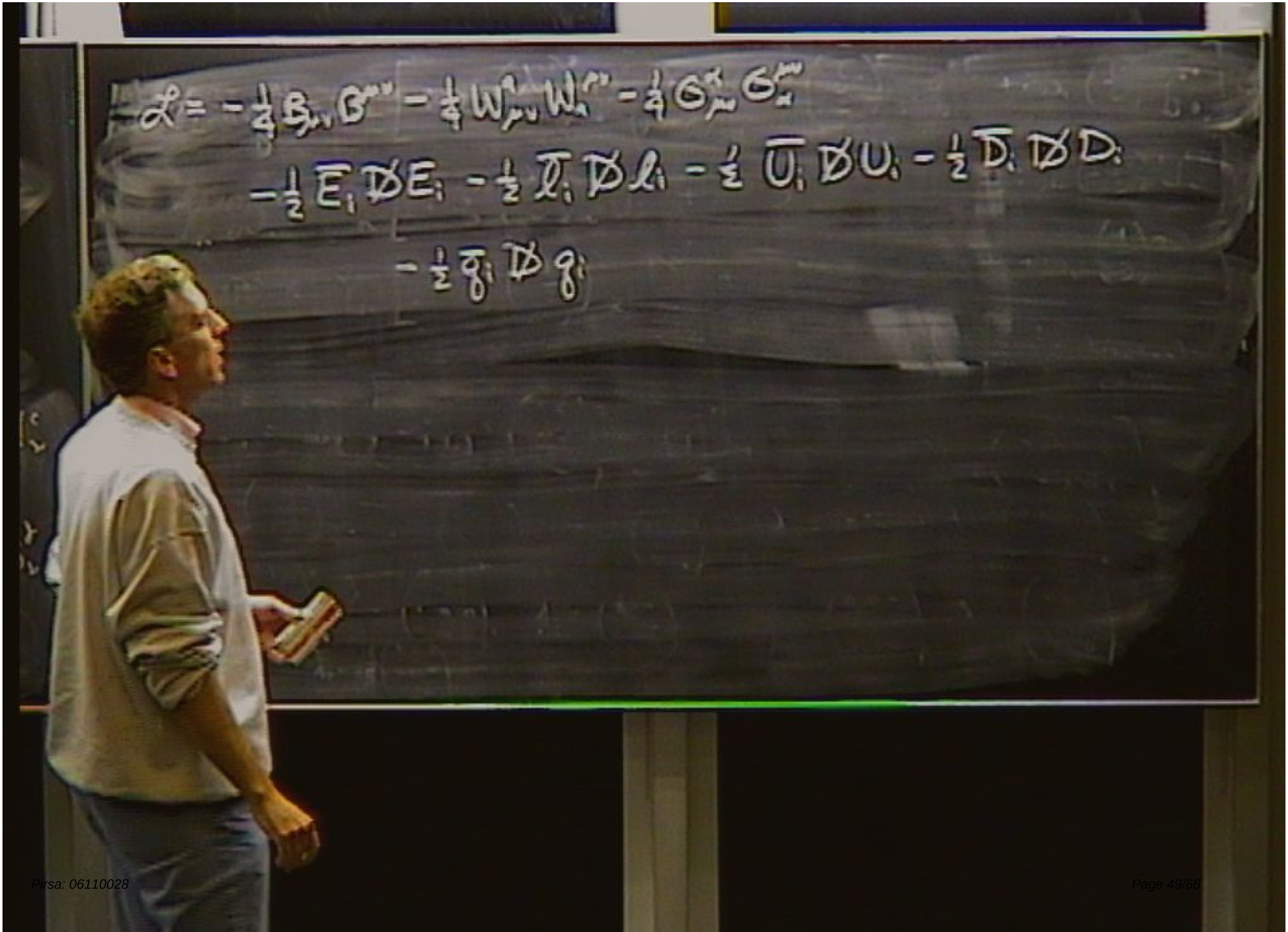
$$[t_{\alpha}, t_{\beta}] = i C^{\gamma}{}_{\alpha\beta}\, t_{\gamma} \qquad \left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = i\epsilon_{abc}\left(\frac{\tau_c}{2}\right)$$

$$\delta B_{\mu} = \frac{1}{g_1}\partial_{\mu}\omega_1 \longrightarrow B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$$

$$\delta W^{a}_{\mu} = \frac{1}{g_2}\partial_{\mu}\omega^{a}_{2} + \epsilon^{a}{}_{bc}\,\omega^{b}_{2} W^{c}_{\mu} \qquad W^{a}_{\mu\nu} = \partial_{\mu}W^{a}_{\nu} - \partial_{\nu}W^{a}_{\mu} + g_2\,\epsilon^{a}{}_{bc} W^{b}_{\mu} W^{c}_{\nu}$$

$$\delta G^{\alpha}_{\mu} = \frac{1}{g_3}\partial_{\mu}\omega^{\alpha}_{3} + f^{\alpha}{}_{\beta\gamma}\,\omega^{\beta}_{3} G^{\gamma}_{\mu}$$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$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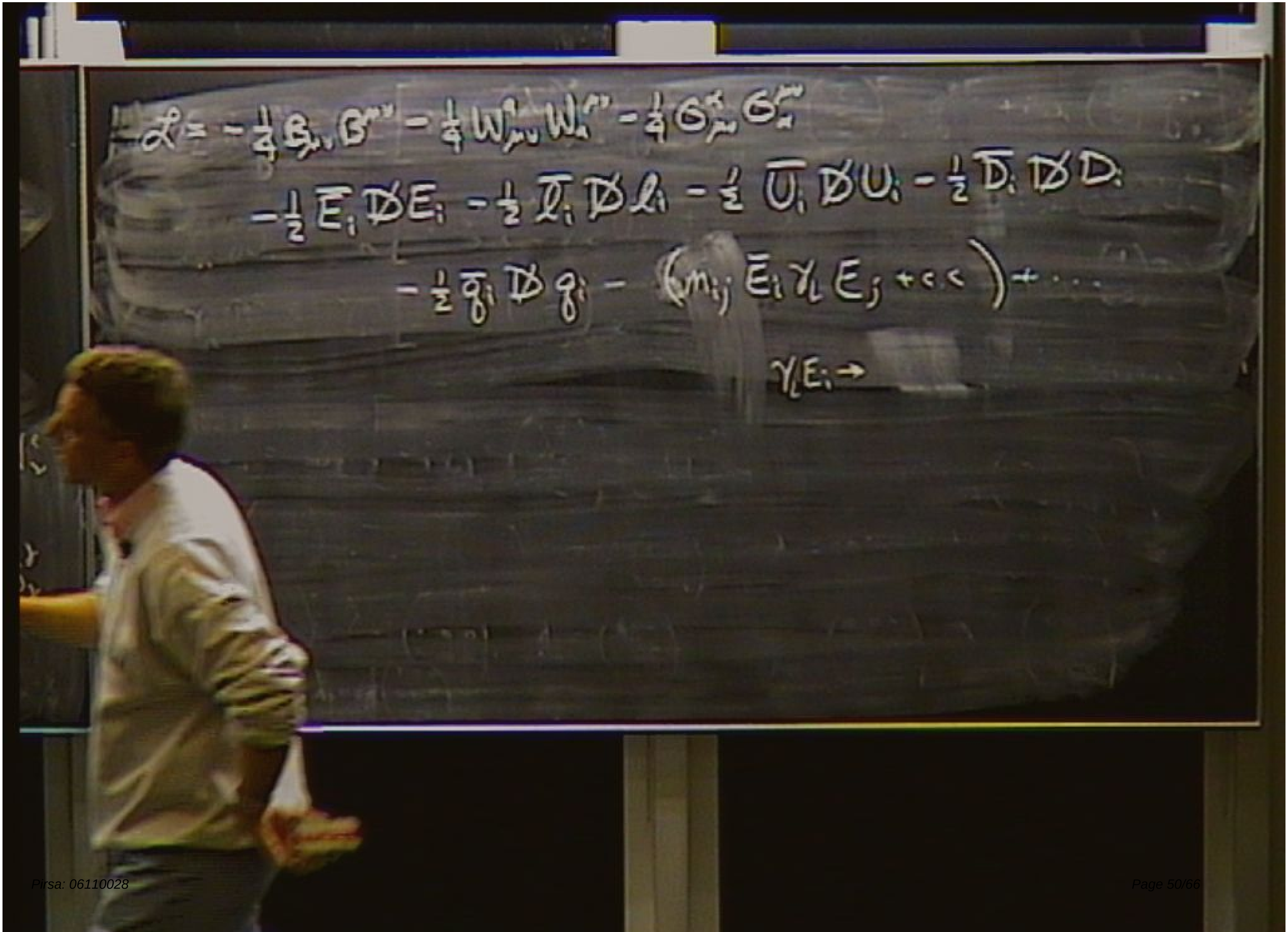
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{\ell}_i \not{D} \ell_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^\alpha_{\mu\nu} G_\alpha^{\mu\nu}$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{\ell}_i \not{D} \ell_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i - (m_{ij} \bar{E}_i \gamma_L E_j + c.c.) + \ldots$
$\gamma_L E_i \rightarrow$

Covariant derivatives:

if: $\delta\phi = i\omega^a t_a \phi \;\longrightarrow\; D_\mu \phi = \partial_\mu \phi - i A_\mu^a t_a \phi$

$$\delta(D_\mu\phi) = i\omega^a t_a (D_\mu \phi)$$

$$\boxed{\begin{aligned} E_i &= (1,1)_1 \\ \ell_i &= (1,2)_{-1/2} \\ U_i &= (\bar{3},1)_{-2/3} \\ D_i &= (\bar{3},1)_{1/3} \\ q_i &= (3,2)_{1/6} \end{aligned}}$$

$$\gamma_L \delta E_i = \gamma_L (i\omega_1 E_i) \rightarrow \gamma_L E_i \rightarrow e^{i\omega_1}\gamma_L E_i$$

$U_Y(1)$

$$\gamma_L D_\mu E_i = \gamma_L\left(\partial_\mu E_i - i g_1 E_i B_\mu\right)$$

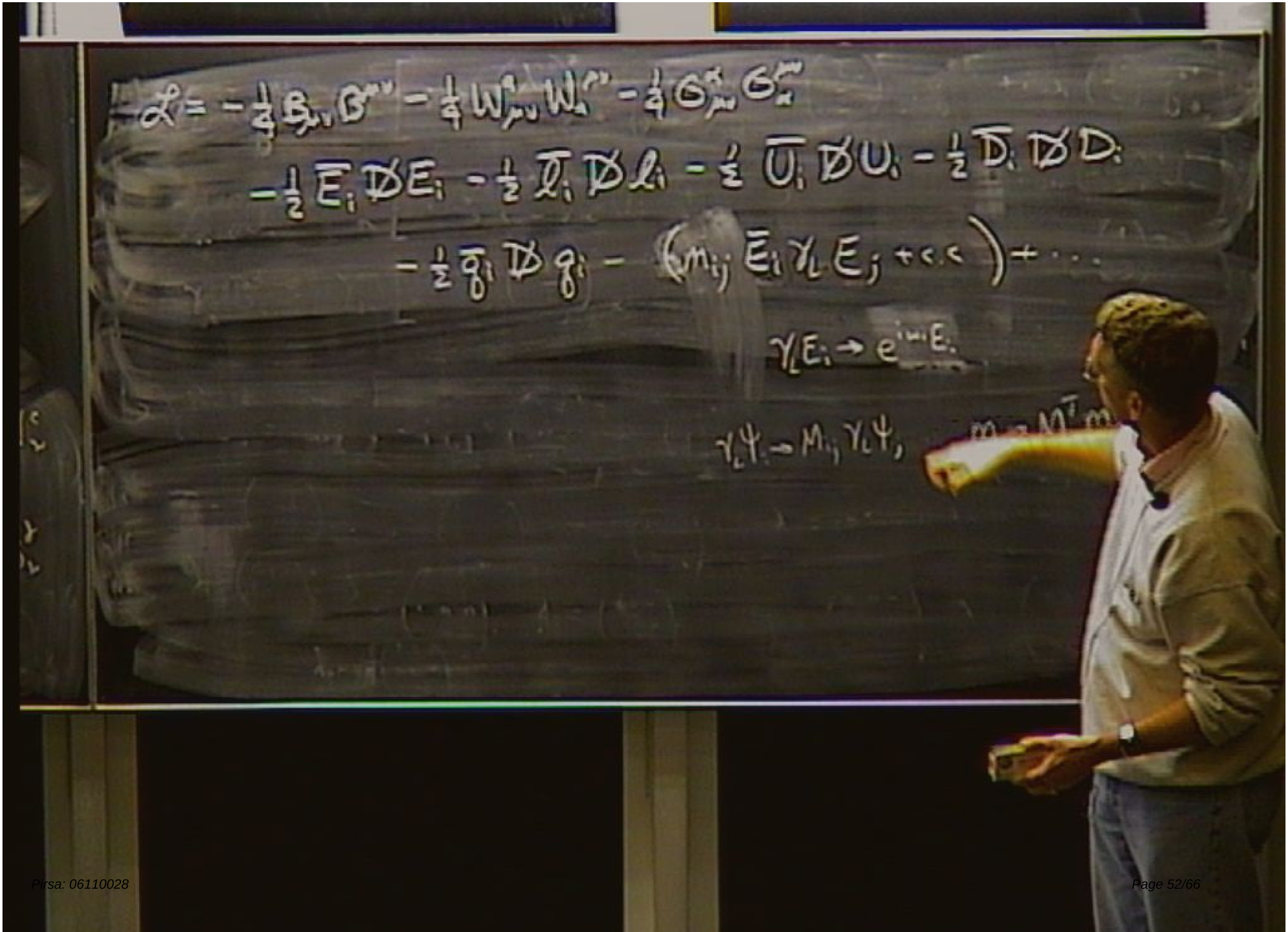
$$\gamma_L D_\mu \ell_i = \gamma_L\left(\partial_\mu \ell_i + \tfrac{i}{2} g_1 B_\mu \ell_i - \tfrac{i}{2} g_2 W_\mu^a \tau_a \ell_i\right)$$

$SU_L(2)$

$$\gamma_L D_\mu q_i = \gamma_L\left(\partial_\mu q_i - \tfrac{i}{6} g_1 B_\mu q_i - \tfrac{i}{2} g_2 W_\mu^a \tau_a q_i - \tfrac{i}{2} g_3 G_\mu^a \lambda_a q_i\right)$$

$SU_c(3)$

$$\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}\begin{pmatrix} \nu_i \\ e_i \end{pmatrix} = \begin{bmatrix} W_\mu^3 \nu_i + (W_\mu^1 - iW_\mu^2) e \\ -W_\mu^3 e_i + (W_\mu^1 + iW_\mu^2)\nu_i \end{bmatrix}$$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{\ell}_i \not{D} \ell_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i - (m_{ij} \bar{E}_i \gamma_L E_j + c.c.) + \dots$
$\gamma_L E_i \to e^{i\omega} E_i$
$\gamma_L \psi_i \to M_{ij} \gamma_L \psi_j$

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^{a}_{\mu\nu} W_{a}^{\mu\nu} - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha}$$

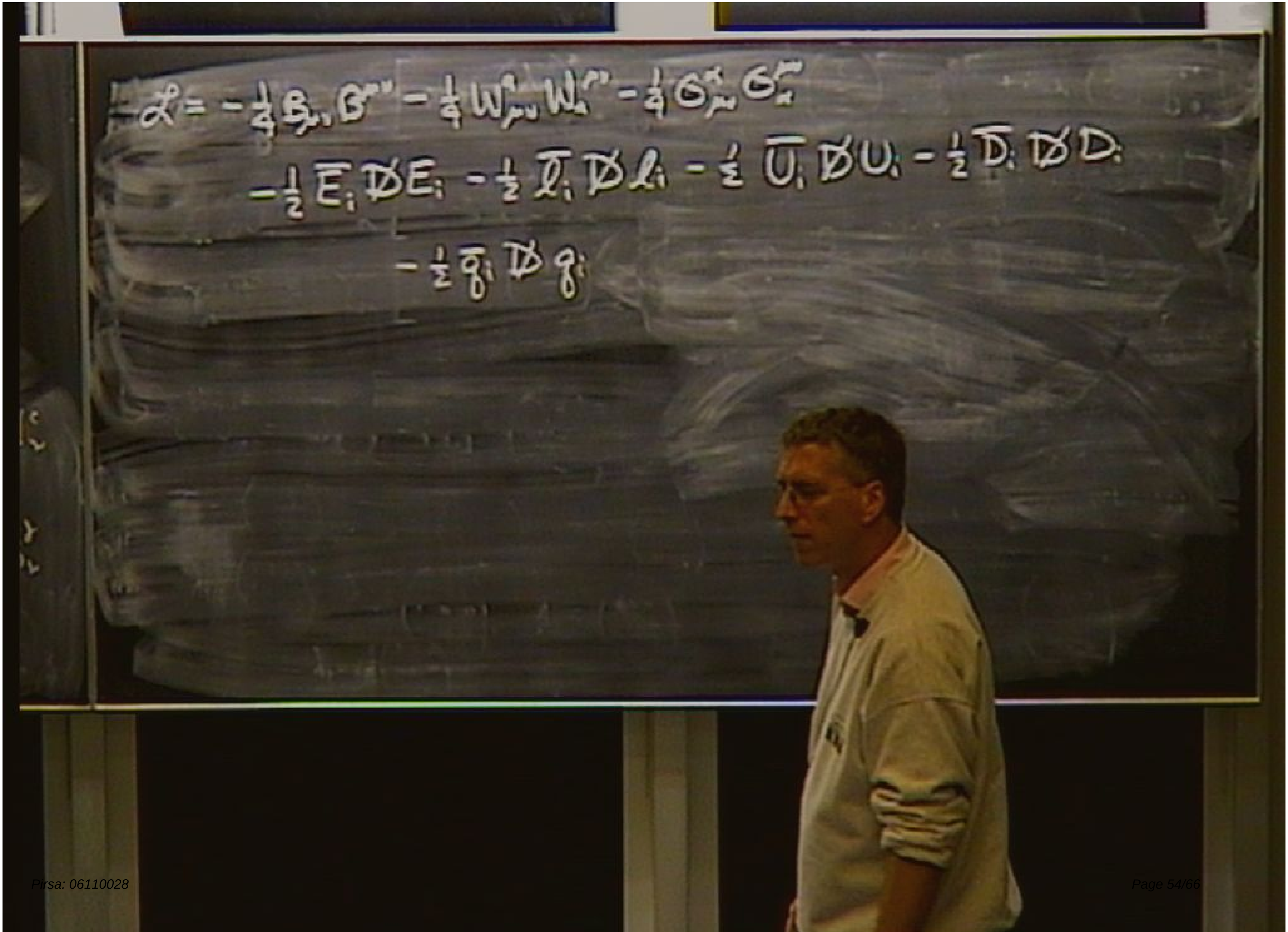
$$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{\ell}_i \not{D} \ell_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$$

$$-\frac{1}{2} \bar{q}_i \not{D} q_i - \cancel{\left(m_{ij} \bar{E}_i \gamma_L E_j + c.c.\right)} + \dots$$

$\gamma_L E_i \to e^{i\omega} E_i$ $\quad$ $\bar{E}_i \gamma_L \to e^{i\omega} \bar{E}_i \gamma_L$

$\bar{E}_i \gamma_L E_j \to \underline{e^{2i\omega}} \bar{E}_i \gamma_L E_j$

not invariant under $U_Y(1)$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{\ell}_i \not{D} \ell_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i$

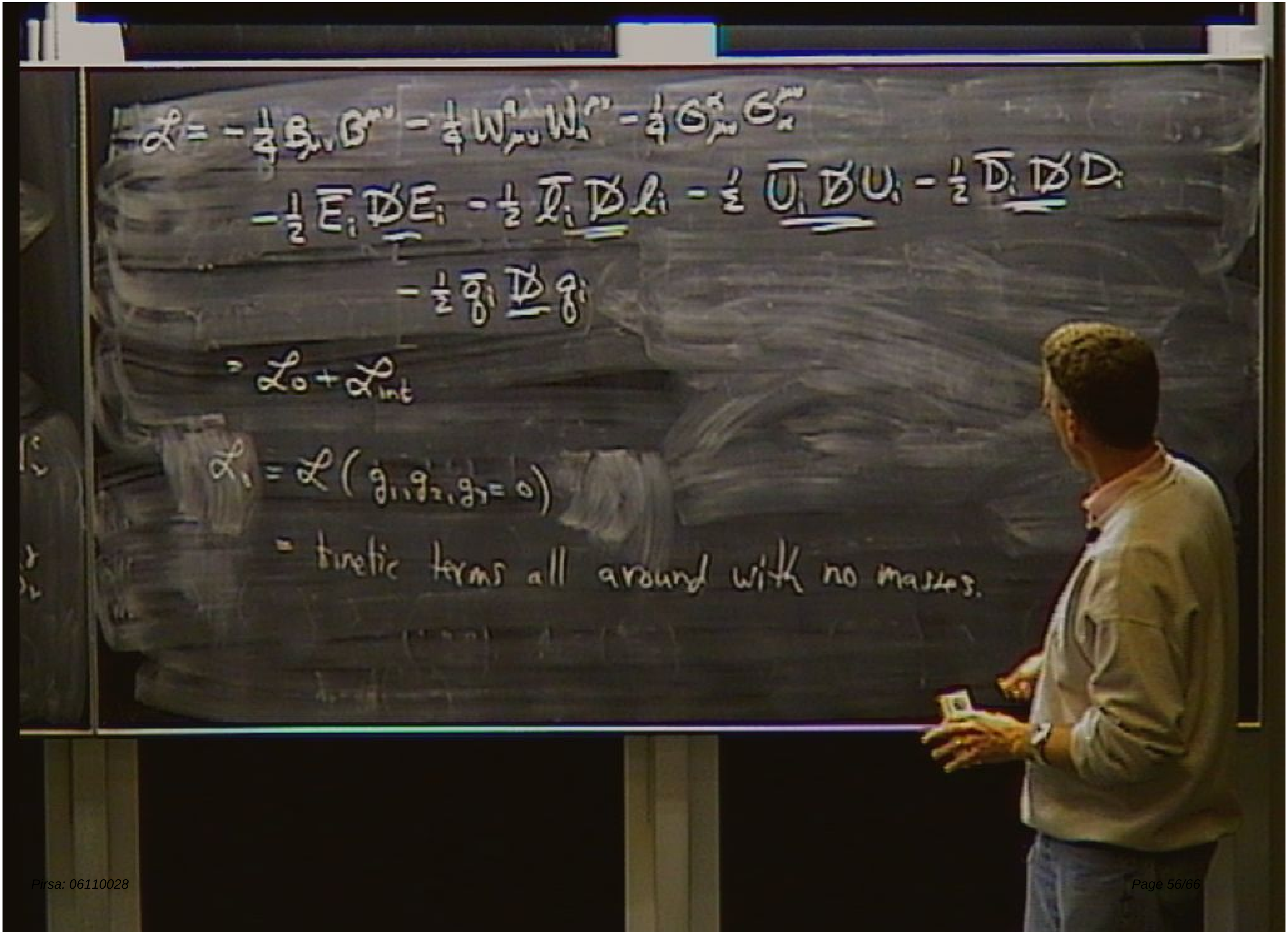
$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^\alpha_{\mu\nu} G^{\mu\nu}_\alpha$$

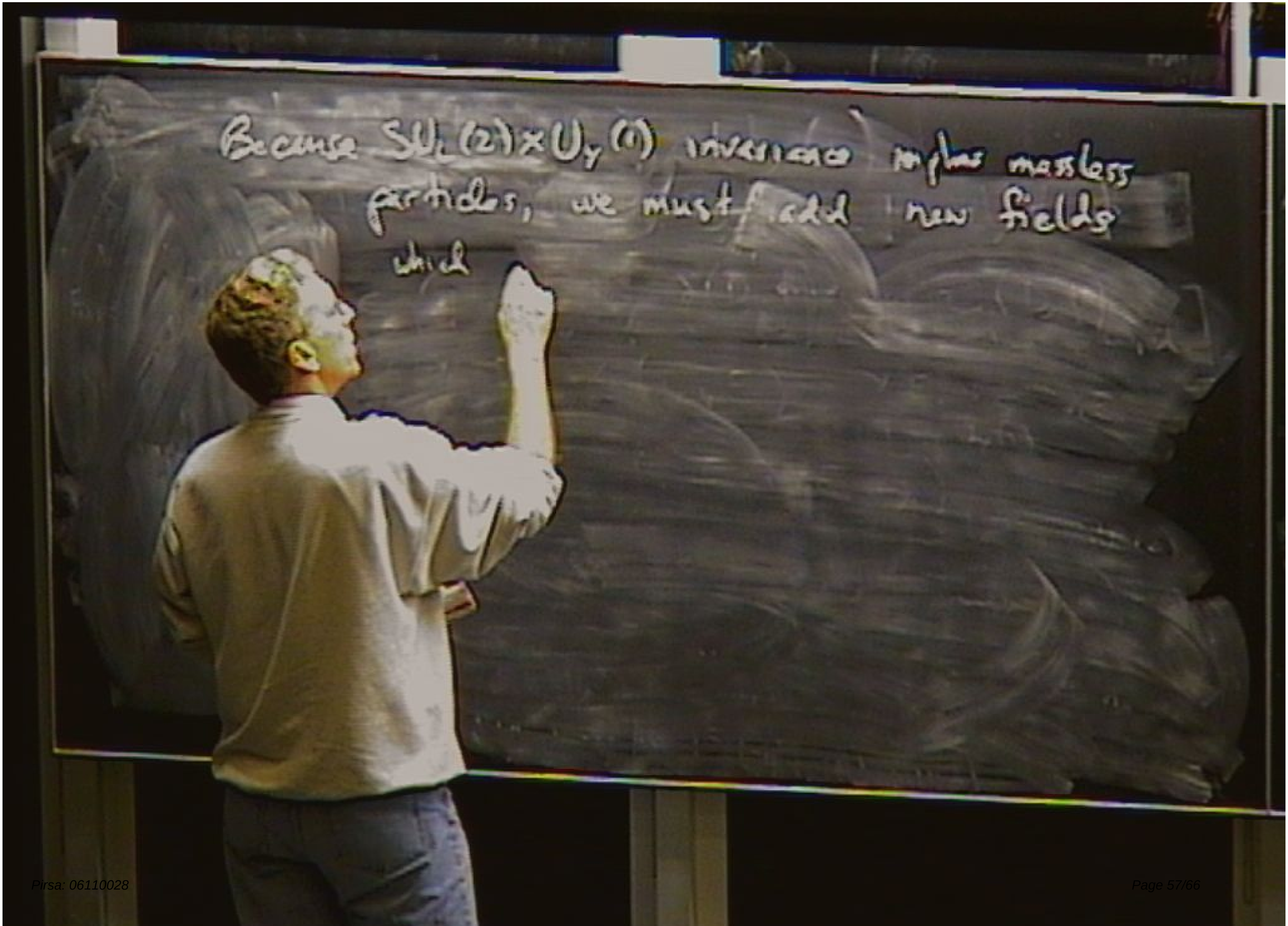
$$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{\ell}_i \not{D} \ell_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$$

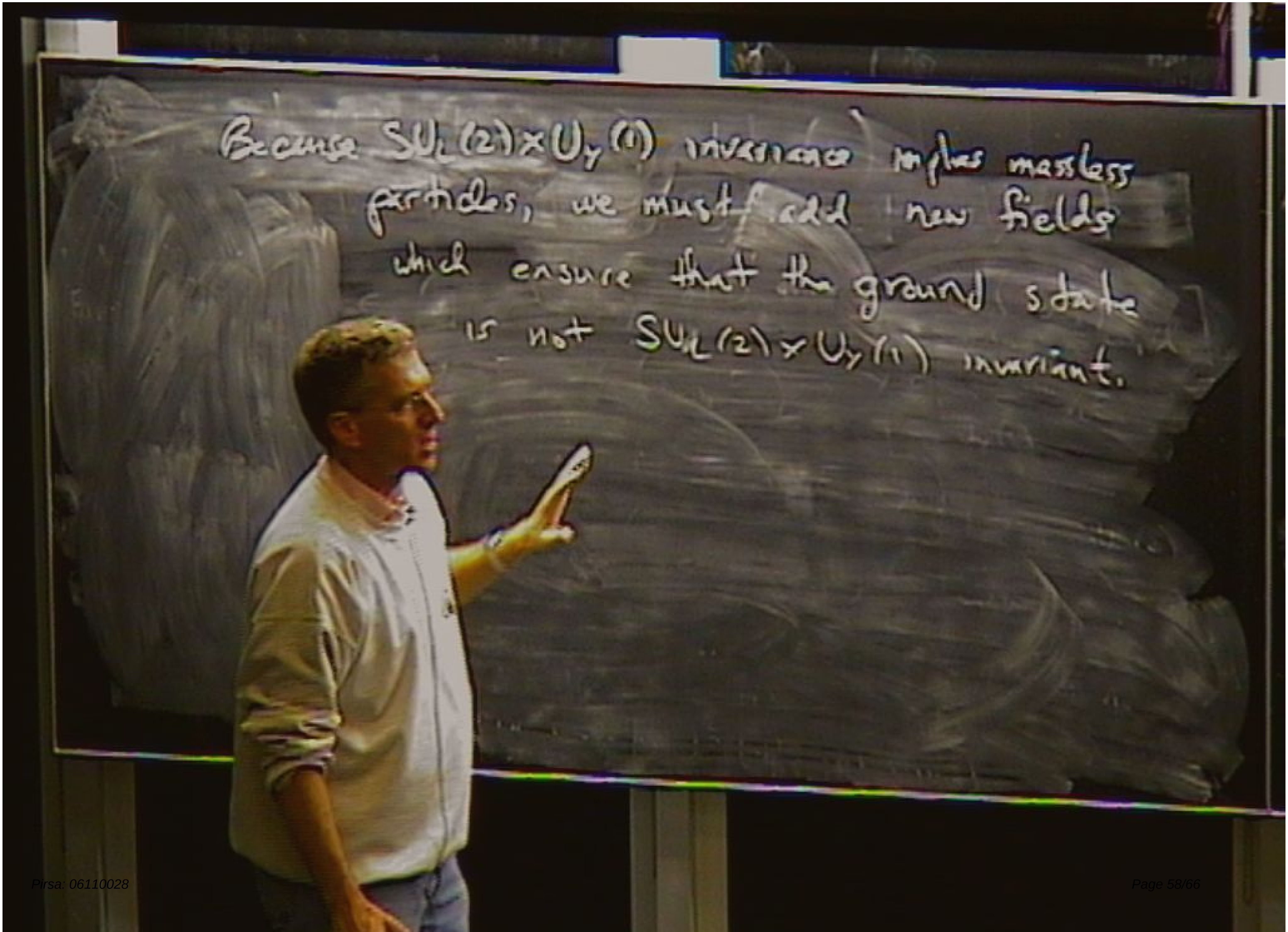
$$-\frac{1}{2} \bar{g}_i \not{D} g_i$$

$$= \mathcal{L}_0 + \mathcal{L}_{int}$$

$$\mathcal{L}_1 = \mathcal{L}(g_1, g_2, g_3 = 0)$$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^\alpha_{\mu\nu} G^{\mu\nu}_\alpha$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{l}_i \not{D} l_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i$
$= \mathcal{L}_0 + \mathcal{L}_{int}$
$\mathcal{L}_1 = \mathcal{L}(g_1, g_2, g_3 = 0)$
= kinetic terms all around with no masses.


Because $SU_L(2) \times U_Y(1)$ invariance implies massless
particles, we must add new fields
which


Because $SU_L(2) \times U_Y(1)$ invariance implies massless
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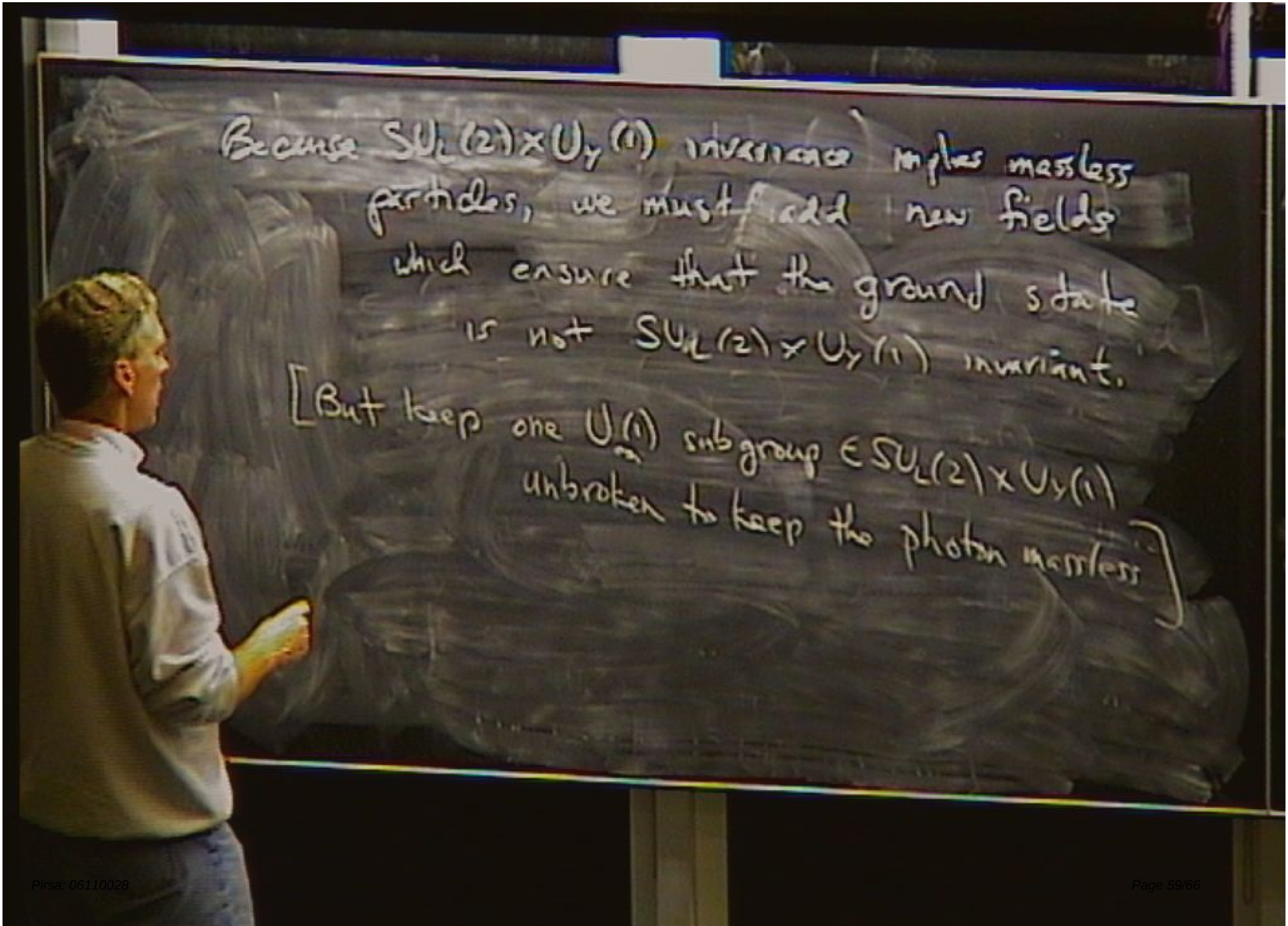
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[But keep one $U_{em}(1)$ subgroup $\in SU_L(2)\times U_Y(1)$
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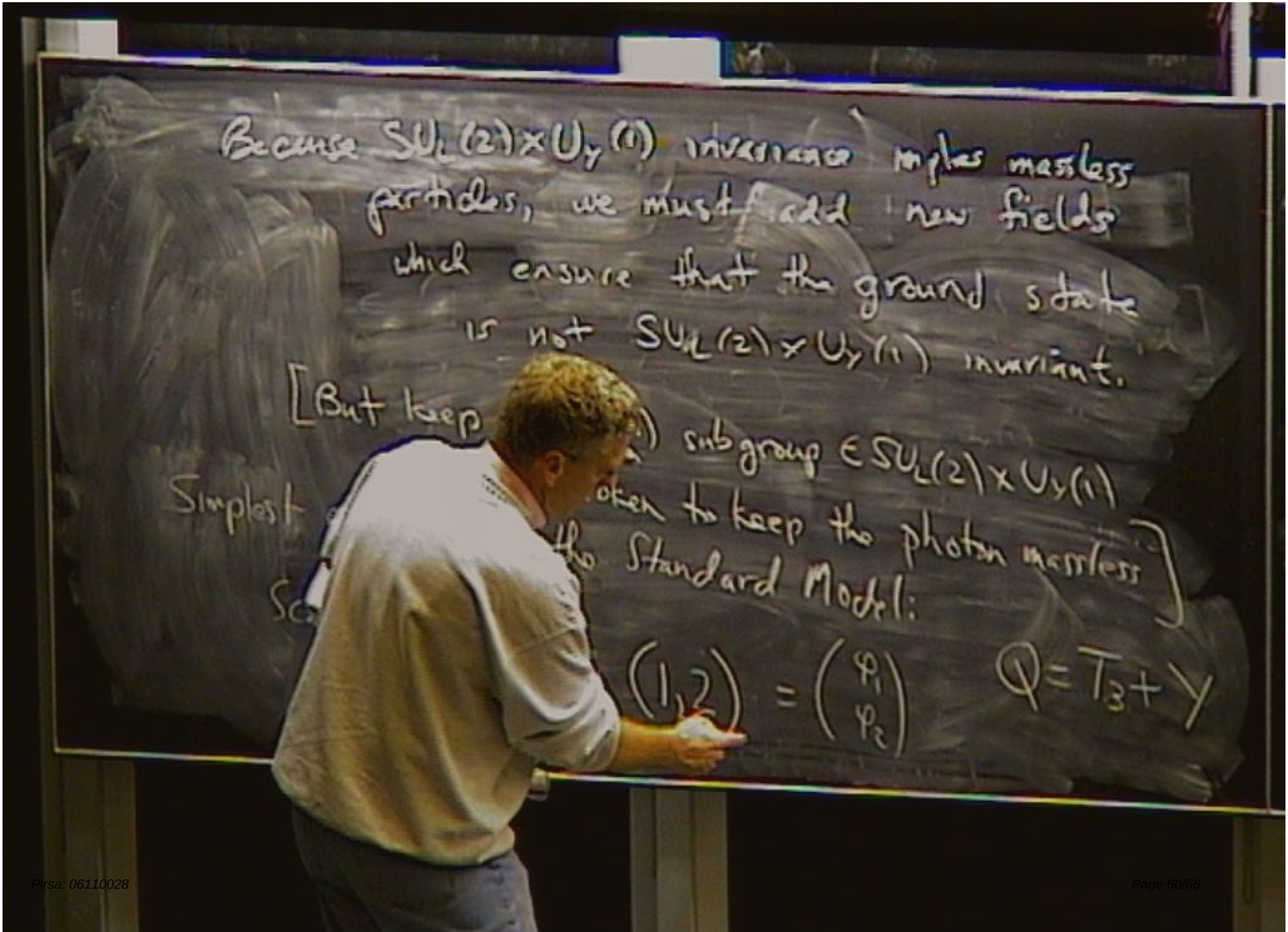
Because SU_L(2)×U_Y(1) invariance implies massless
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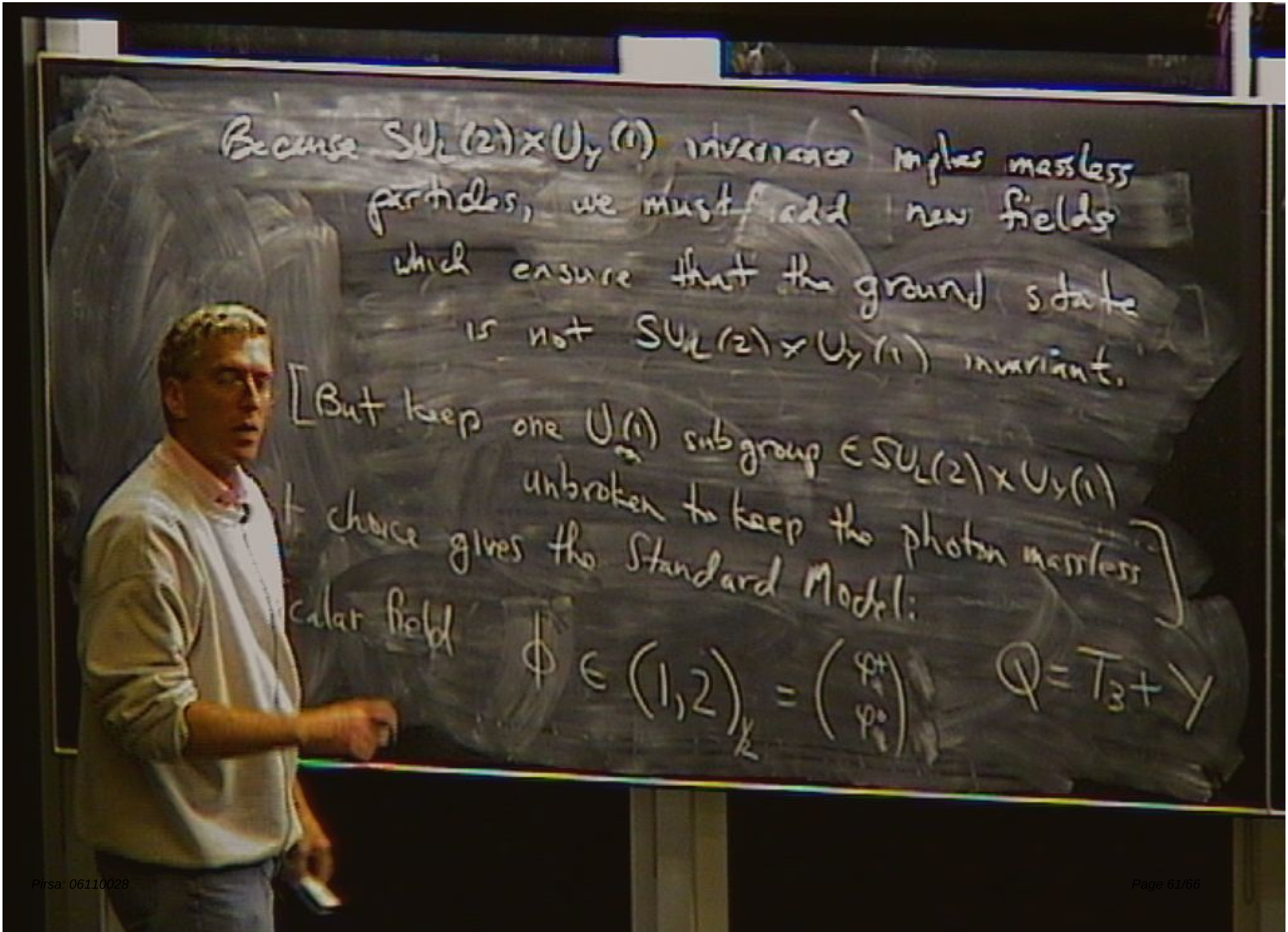
[But keep
subgroup ∈ SU_L(2)×U_Y(1)
Simplest
oken to keep the photon massless]
Standard Model:
(1,2) = (φ1 φ2)
Q = T3 + Y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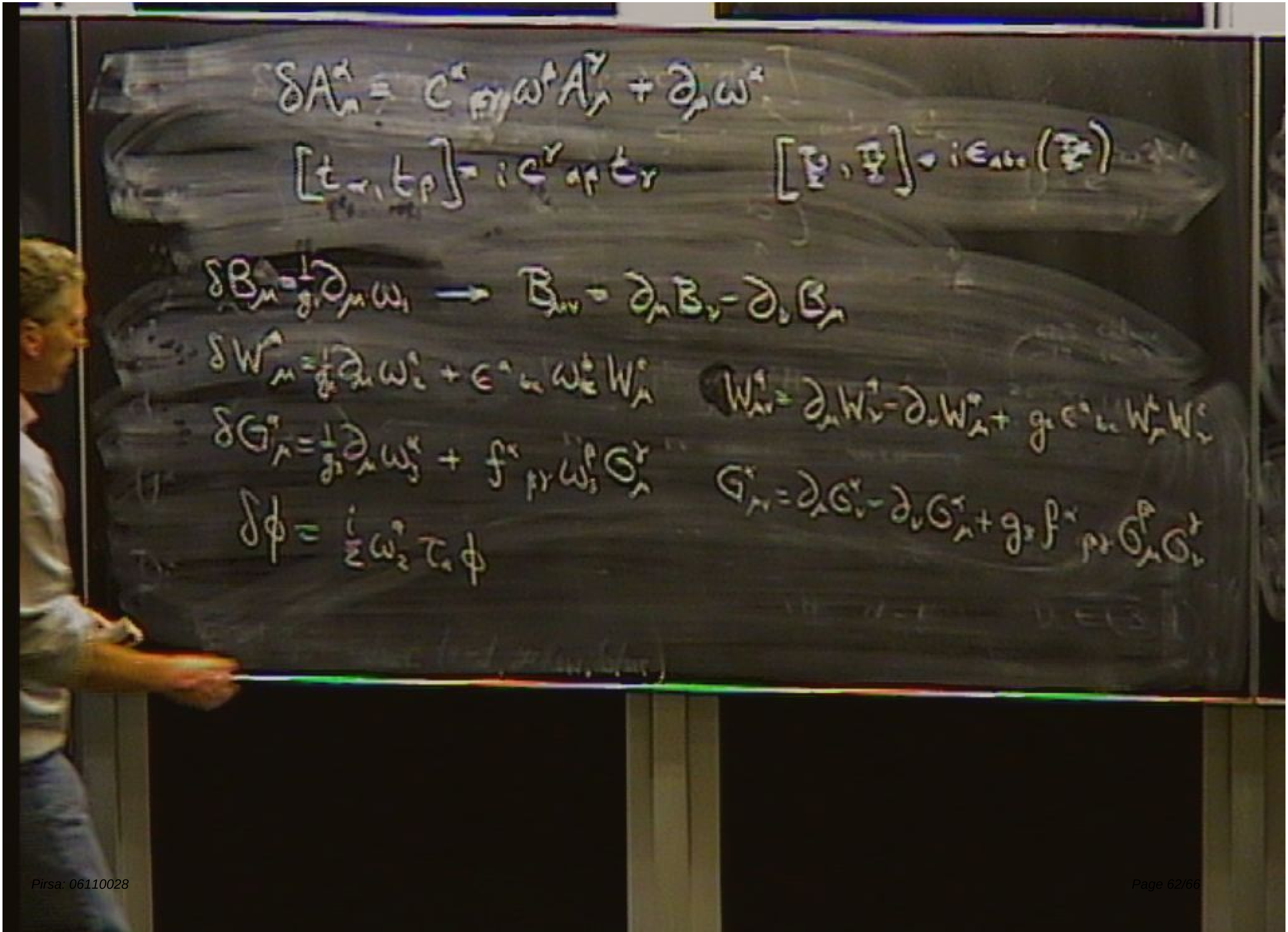
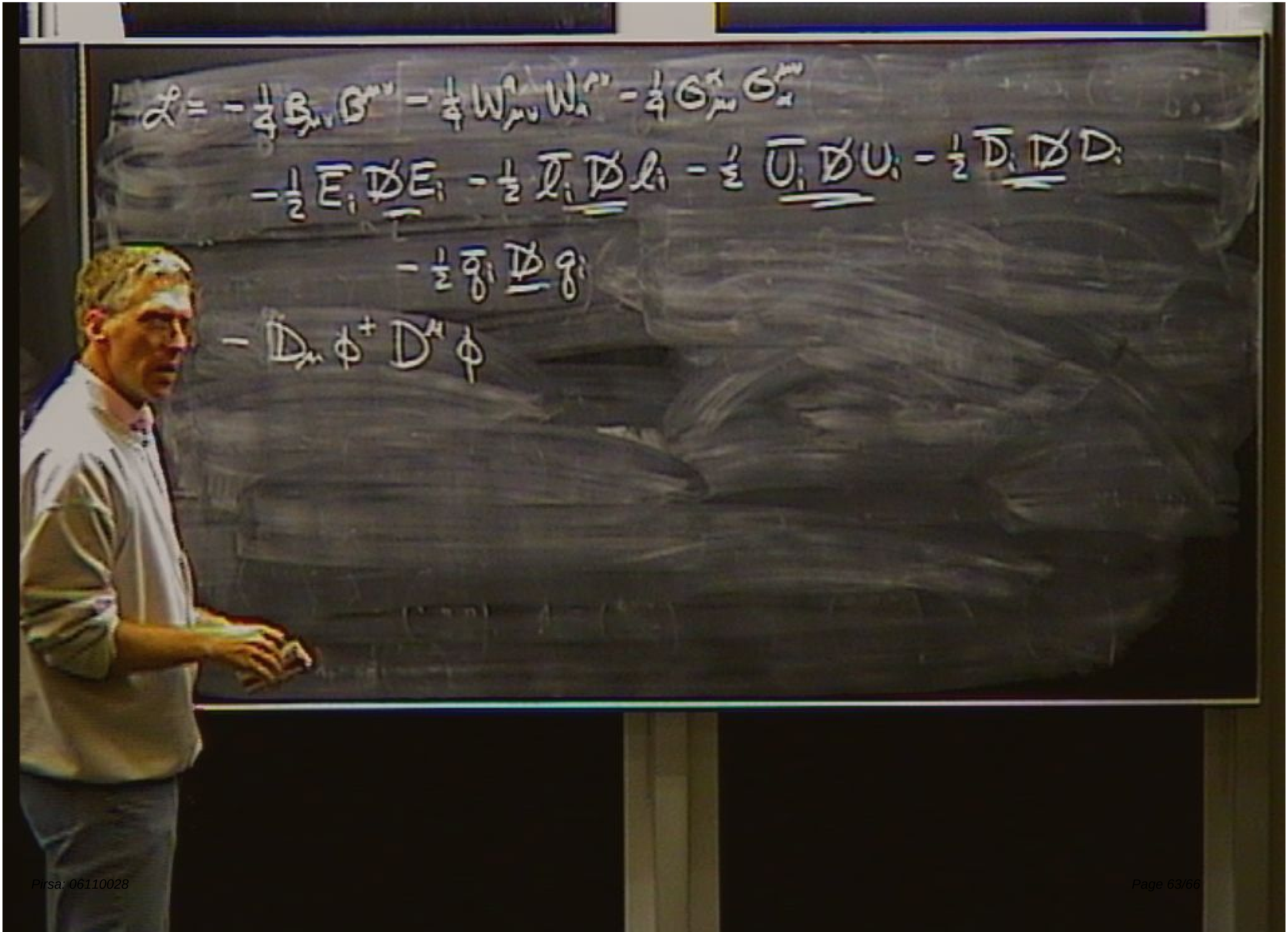
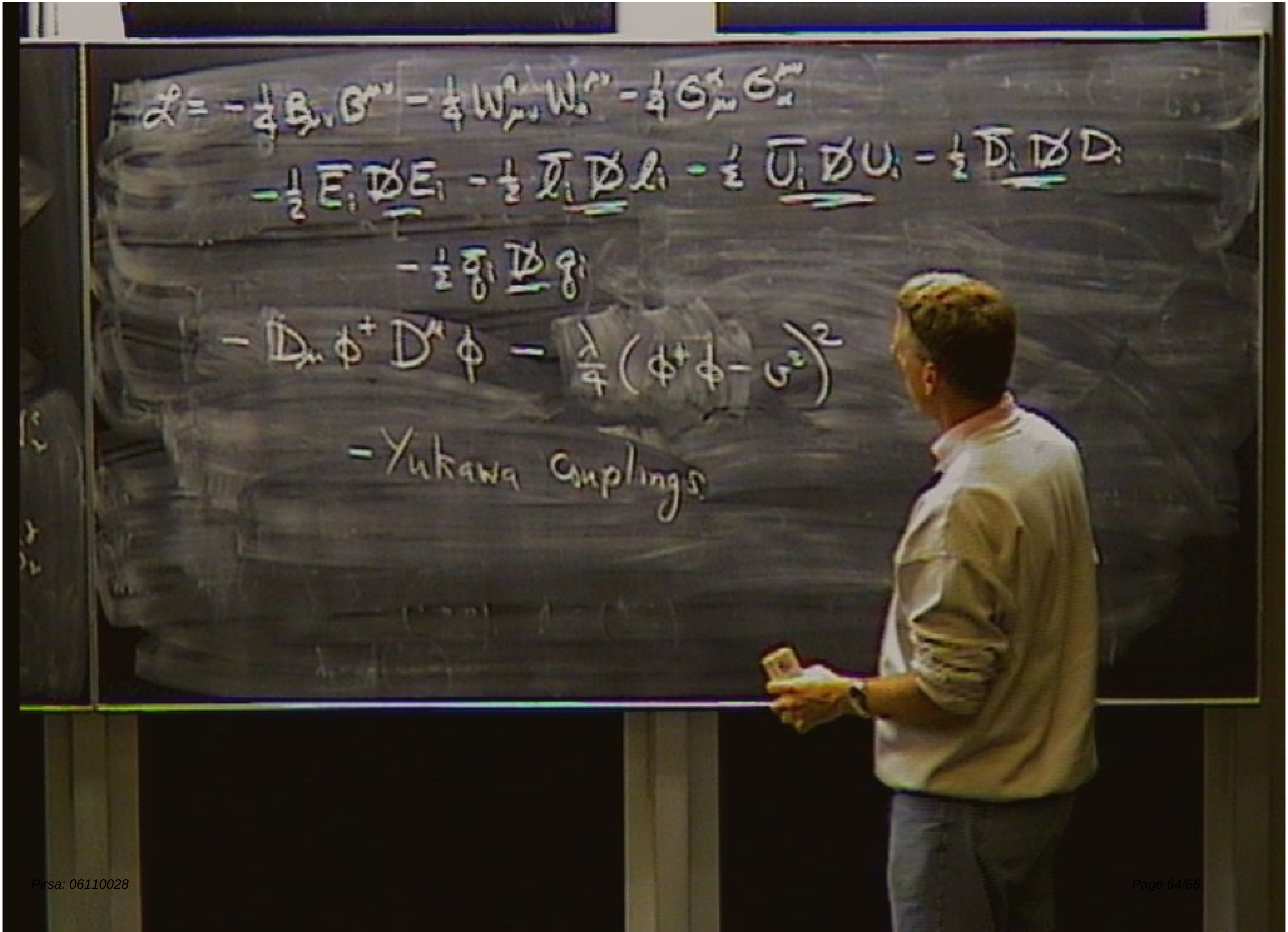
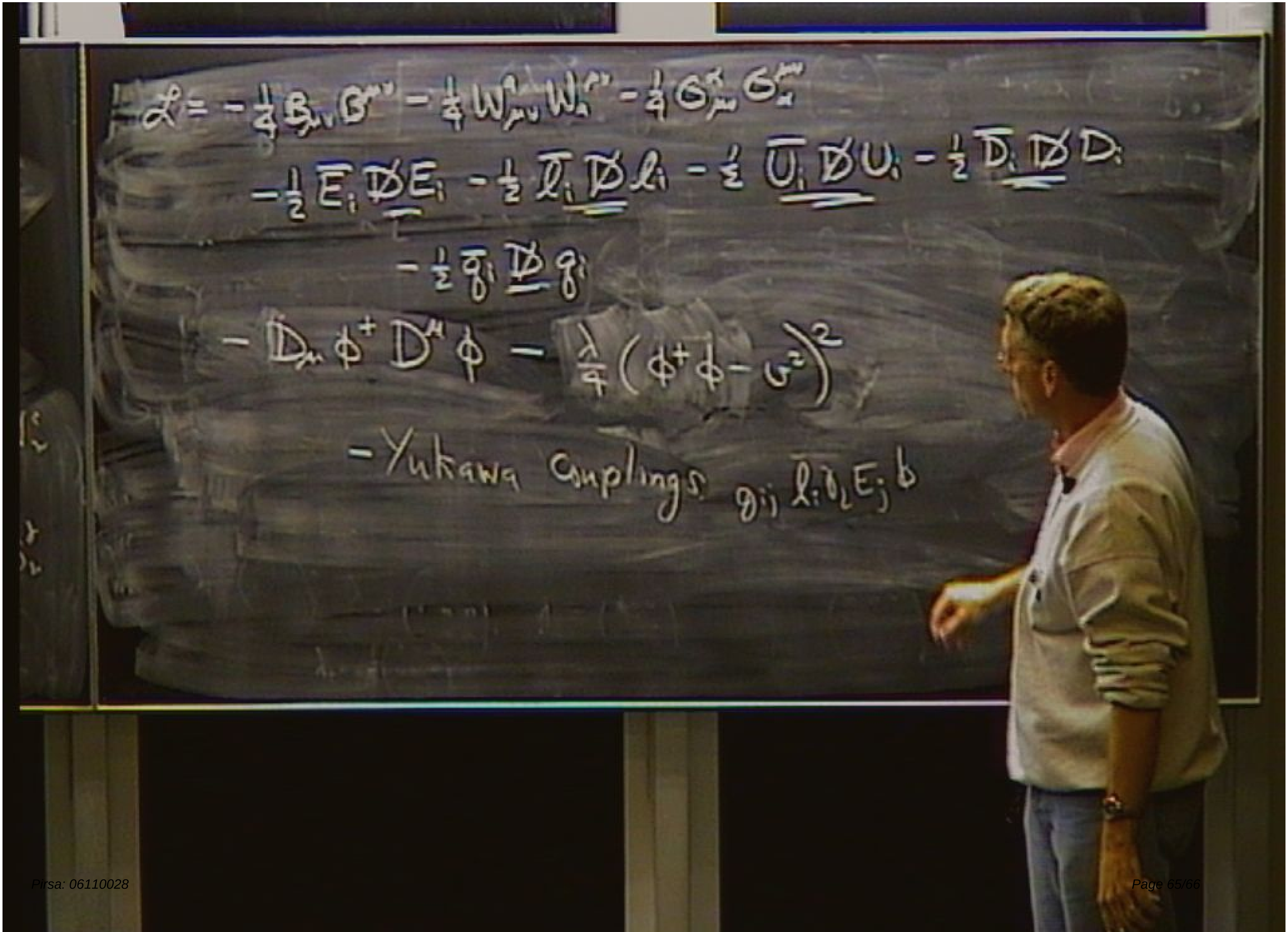
unbroken to keep the photon massless]
choice gives the Standard Model:
calar field $\phi \in (1,2)_{1/2} = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$
$Q = T_3 + Y$

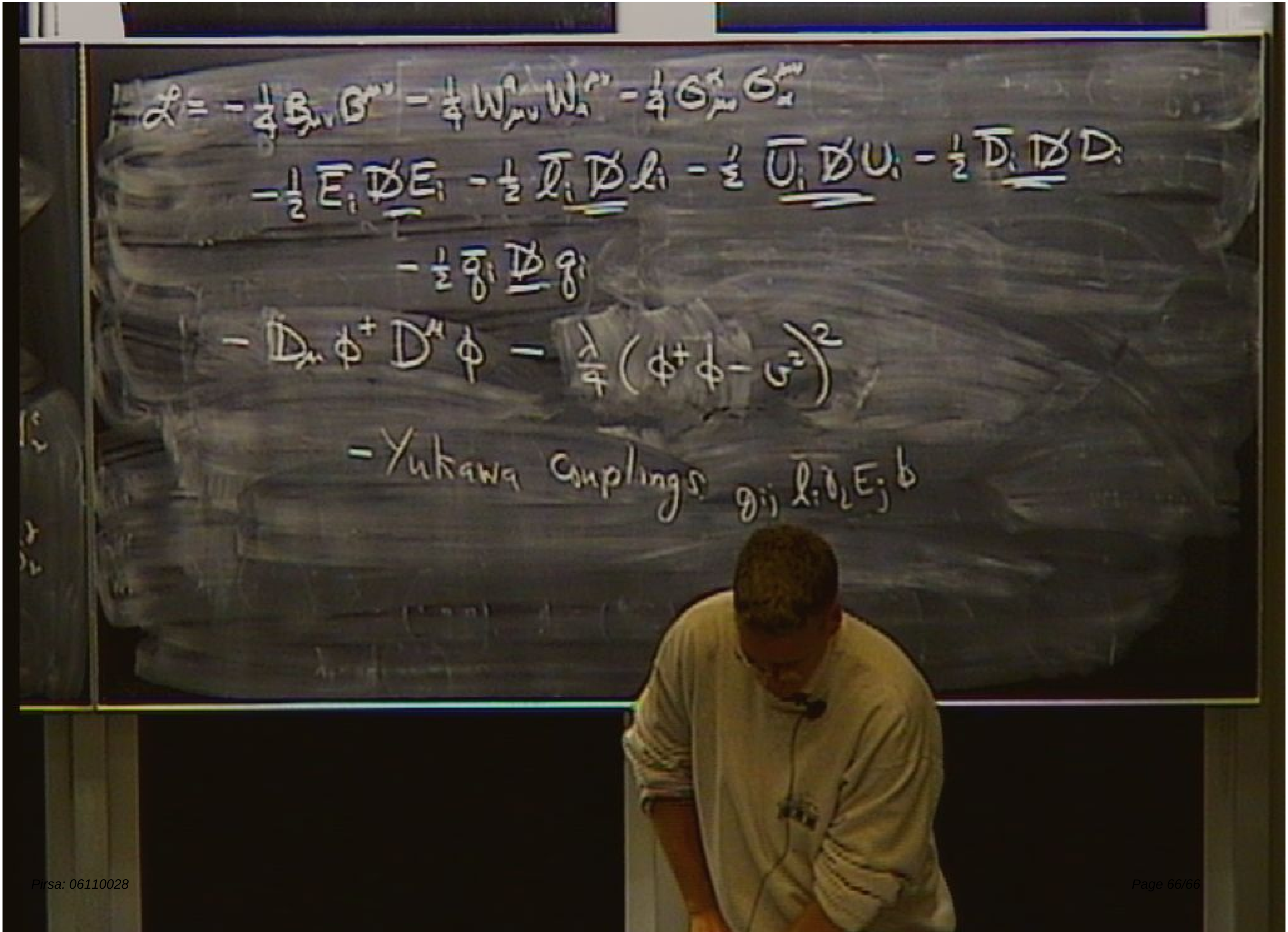


$\delta A^\alpha_\mu = C^\alpha{}_{\beta\gamma}\, \omega^\beta A^\gamma_\mu + \partial_\mu \omega^\alpha$
$[t_\alpha, t_\beta] = i C^\gamma{}_{\alpha\beta}\, t_\gamma$
$[\frac{\tau_a}{2}, \frac{\tau_b}{2}] = i \epsilon_{abc} (\frac{\tau_c}{2})$
$\delta B_\mu = \frac{1}{g_1} \partial_\mu \omega_1 \Longrightarrow B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$
$\delta W^a_\mu = \frac{1}{g_2} \partial_\mu \omega^a_2 + \epsilon^a{}_{bc}\, \omega^b_2 W^c_\mu$
$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^a{}_{bc} W^b_\mu W^c_\nu$
$\delta G^\alpha_\mu = \frac{1}{g_3} \partial_\mu \omega^\alpha_3 + f^\alpha{}_{\beta\gamma}\, \omega^\beta_3 G^\gamma_\mu$
$G^\alpha_{\mu\nu} = \partial_\mu G^\alpha_\nu - \partial_\nu G^\alpha_\mu + g_3 f^\alpha{}_{\beta\gamma} G^\beta_\mu G^\gamma_\nu$
$\delta \phi = \frac{i}{2} \omega^a_2 \tau_a \phi$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^\alpha_{\mu\nu} G_\alpha^{\mu\nu}$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{\ell}_i \not{D} \ell_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i$
$- D_\mu \phi^+ D^\mu \phi$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{\ell}_i \not{D} \ell_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i$
$- D_\mu \phi^+ D^\mu \phi - \frac{\lambda}{4} (\phi^+ \phi - v^2)^2$
$-$Yukawa Couplings


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^\alpha_{\mu\nu} G^{\mu\nu}_\alpha$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{l}_i \not{D} l_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i$
$- D_\mu \phi^+ D^\mu \phi - \frac{\lambda}{4} (\phi^+ \phi - v^2)^2$
$-$Yukawa couplings $g_{ij}\, l_i \phi_L E_j b$


$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$
$-\frac{1}{2} \bar{E}_i \not{D} E_i - \frac{1}{2} \bar{l}_i \not{D} l_i - \frac{1}{2} \bar{U}_i \not{D} U_i - \frac{1}{2} \bar{D}_i \not{D} D_i$
$-\frac{1}{2} \bar{q}_i \not{D} q_i$
$- D_\mu \phi^+ D^\mu \phi - \frac{\lambda}{4} (\phi^+ \phi - v^2)^2$
$-$ Yukawa Couplings: $g_{ij} \, \bar{l}_i \phi_2 E_j \, b$