Title: Graduate Course on Standard Model & Quantum Field Theory - 4B

Date: Nov 08, 2006 03:30 PM

URL: http://pirsa.org/06110026

Abstract: Graduate Course on Standard Model & Quantum Field Theory

Fields: ϕ, A_μ

Symmetry: $\delta\phi = i\omega\phi \qquad \delta A_\mu = \frac{1}{e}\partial_\mu \omega$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \partial_\mu\phi^*\partial^\mu\phi - \left(a + b\phi^*\phi + \lambda^2(\phi^*\phi)^2\right)$$

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Perturbation theory: $\mathcal{L} = \mathcal{L}_0 + \boxed{\mathcal{L}_{int}}$

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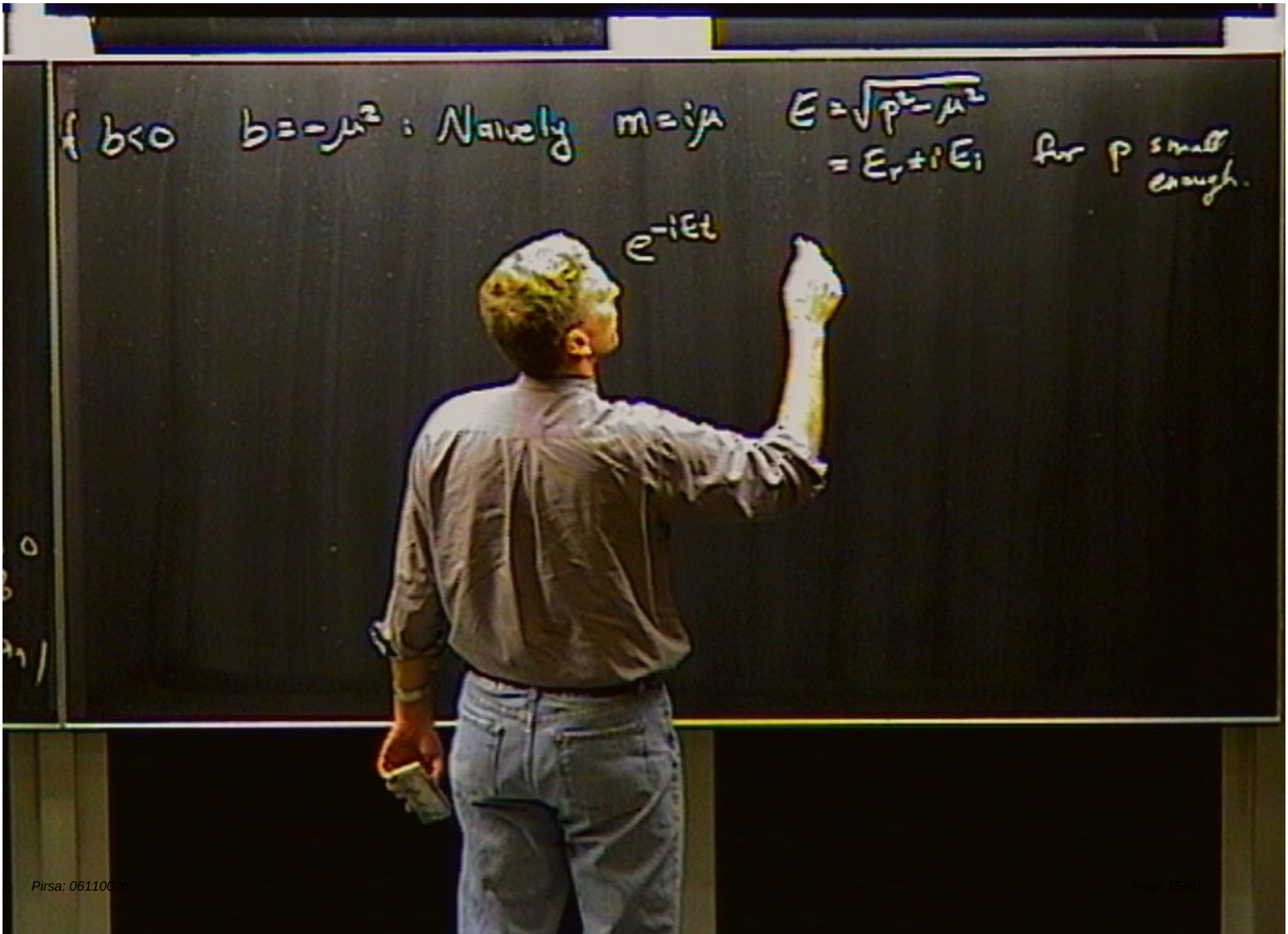
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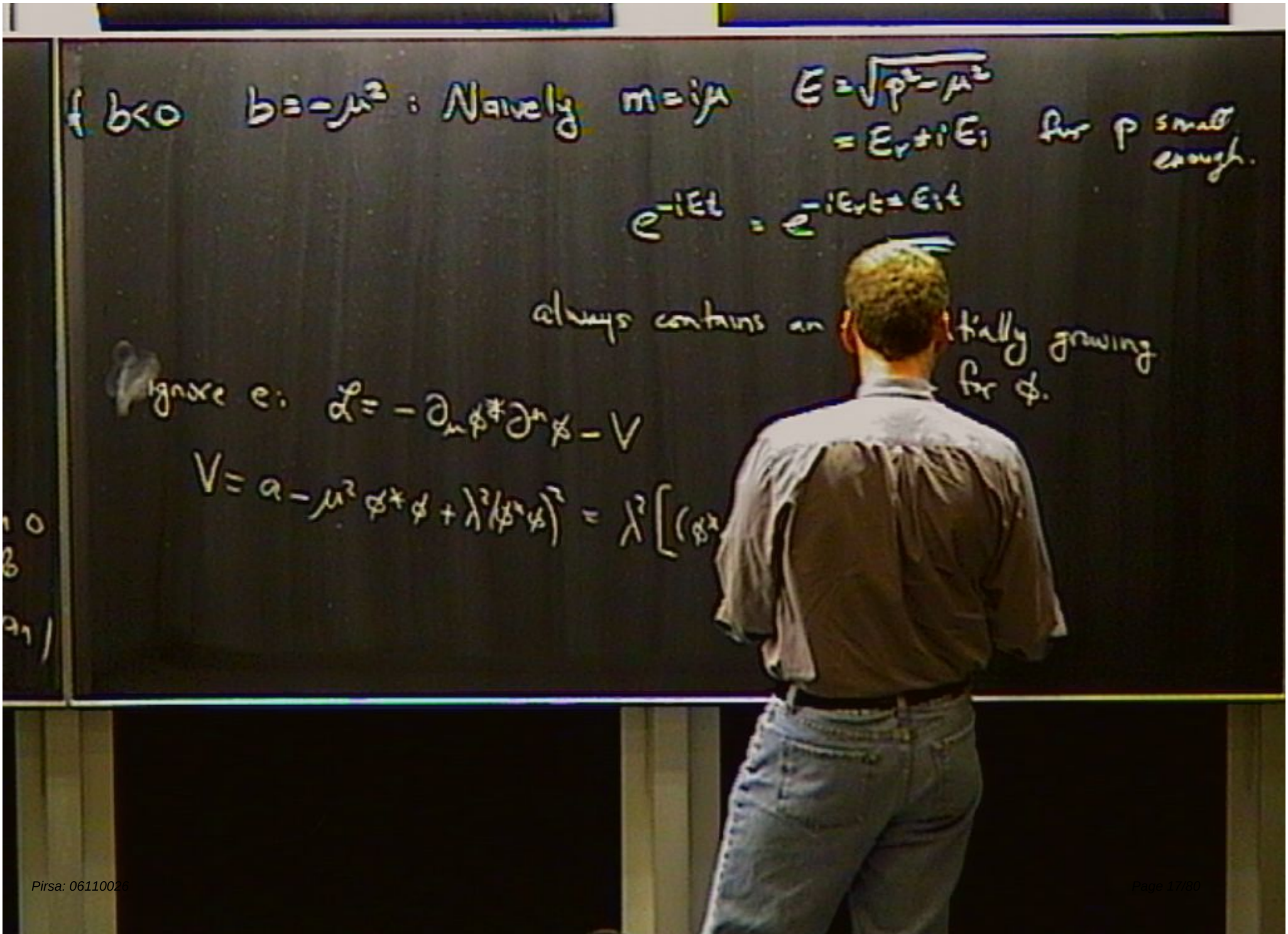
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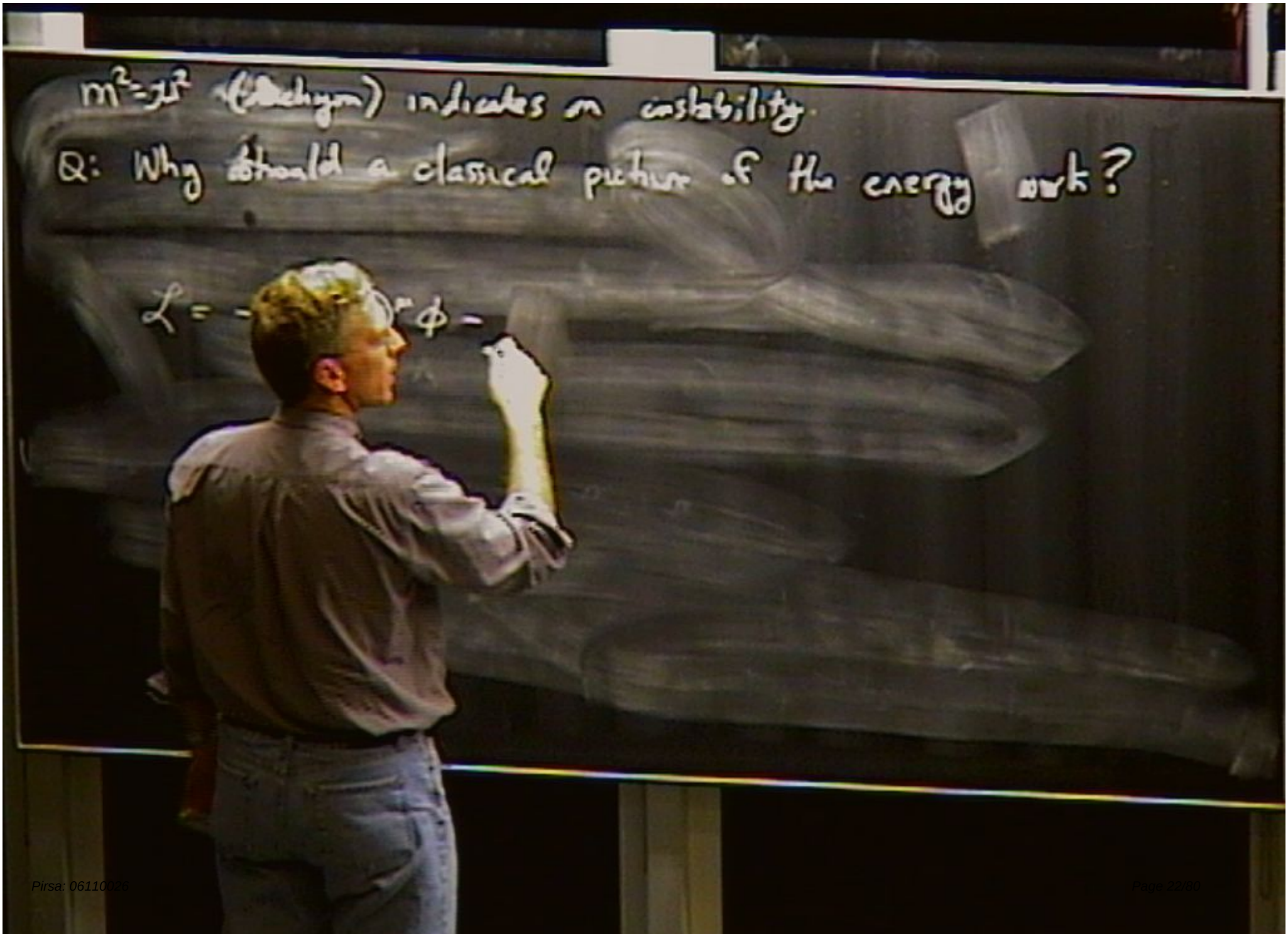
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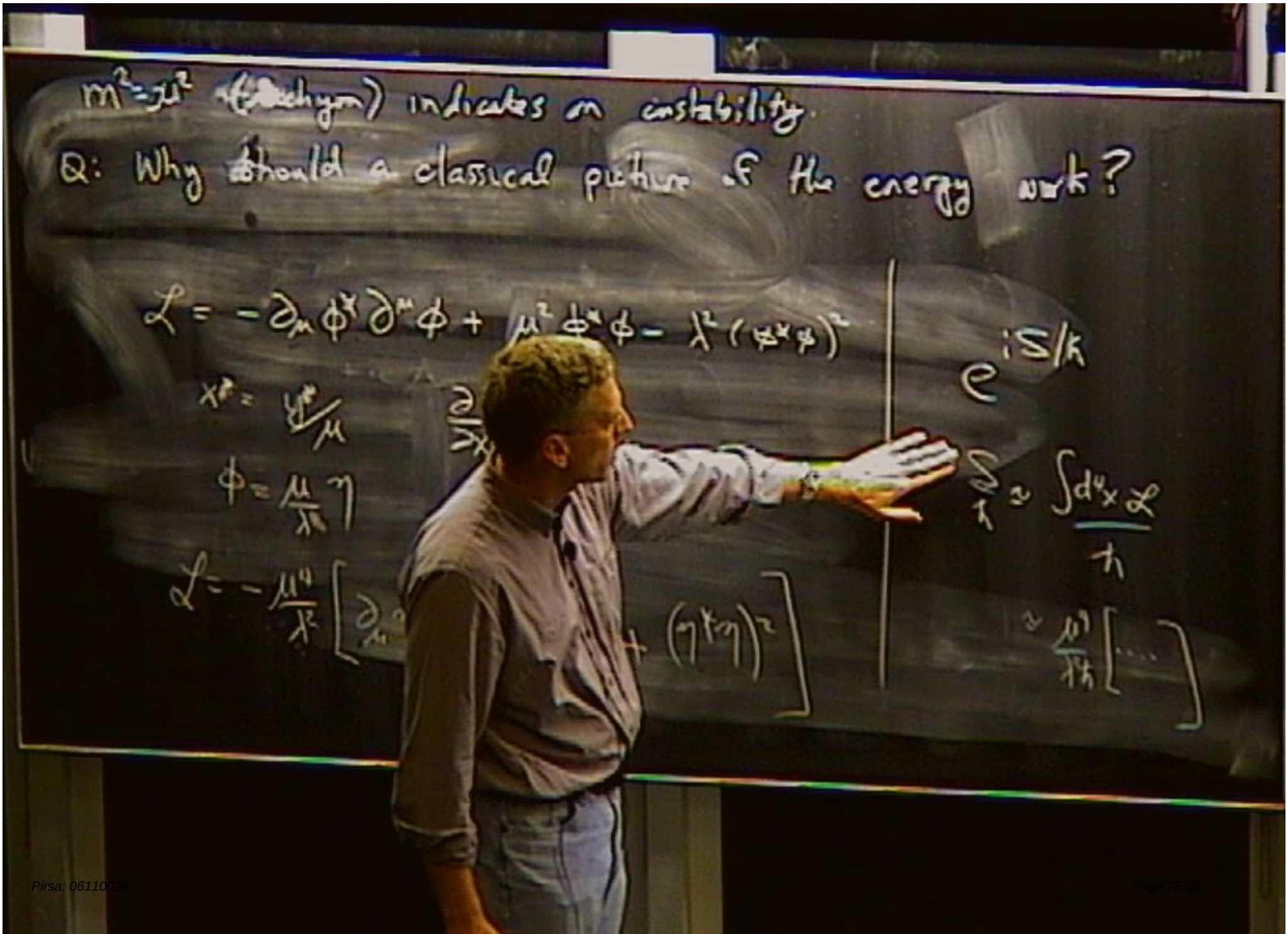
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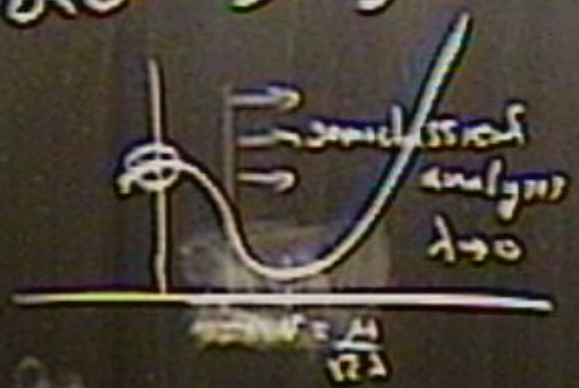

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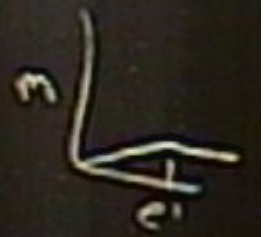

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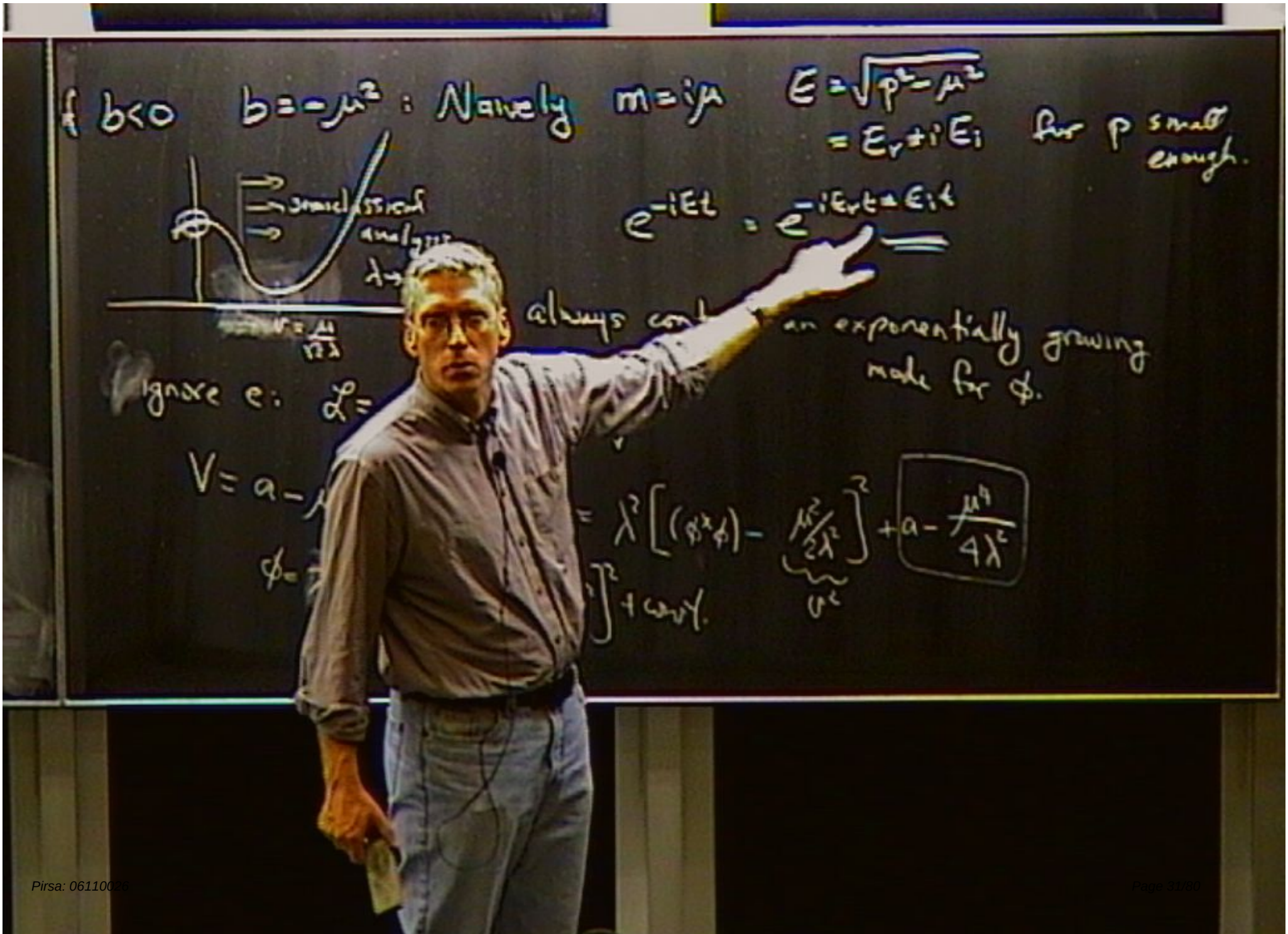
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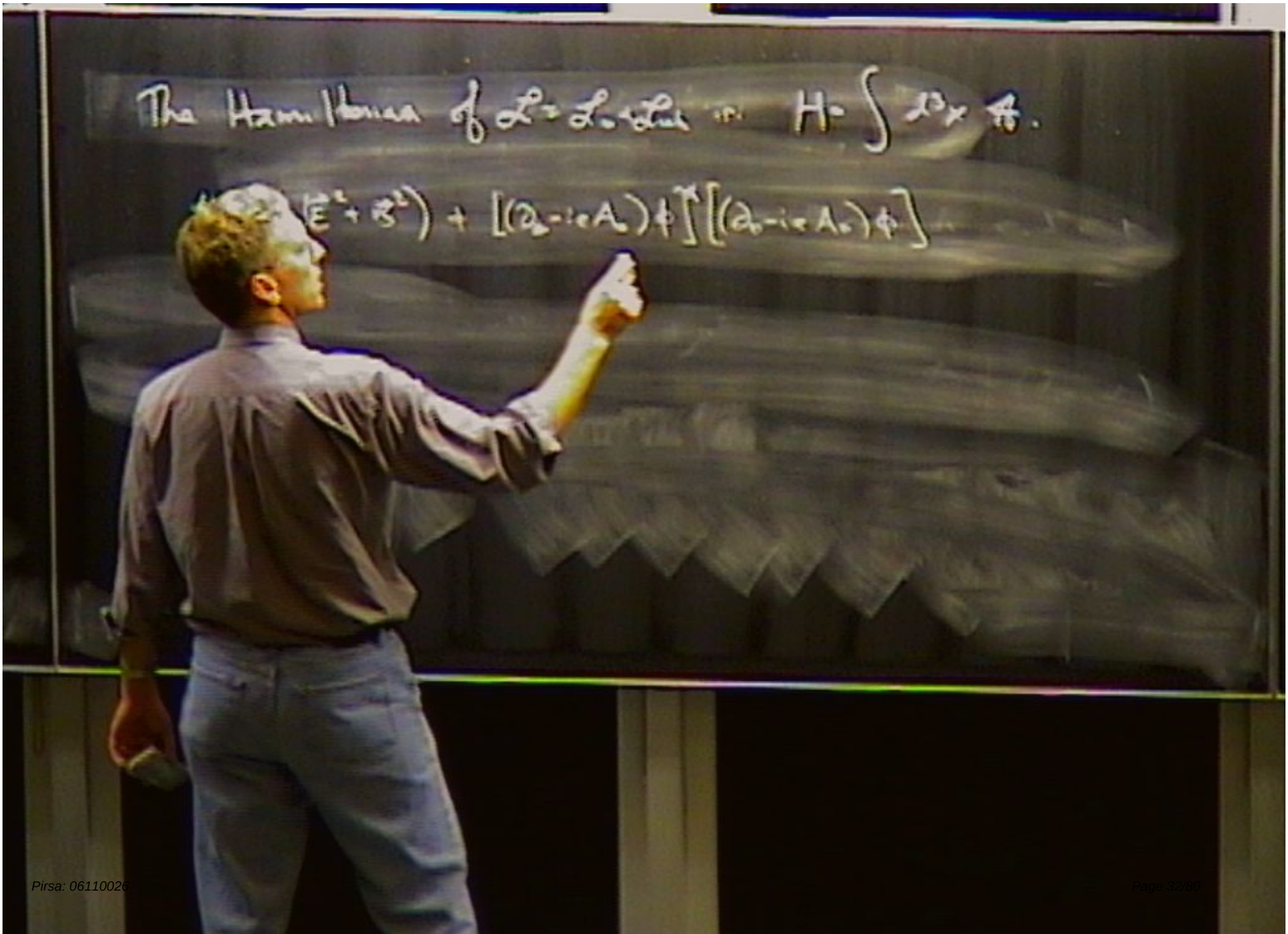
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$(\vec{E}^2 + \vec{B}^2) + [(\partial_0 - ieA_0)\phi]^*[(\partial_0 - ieA_0)\phi]$

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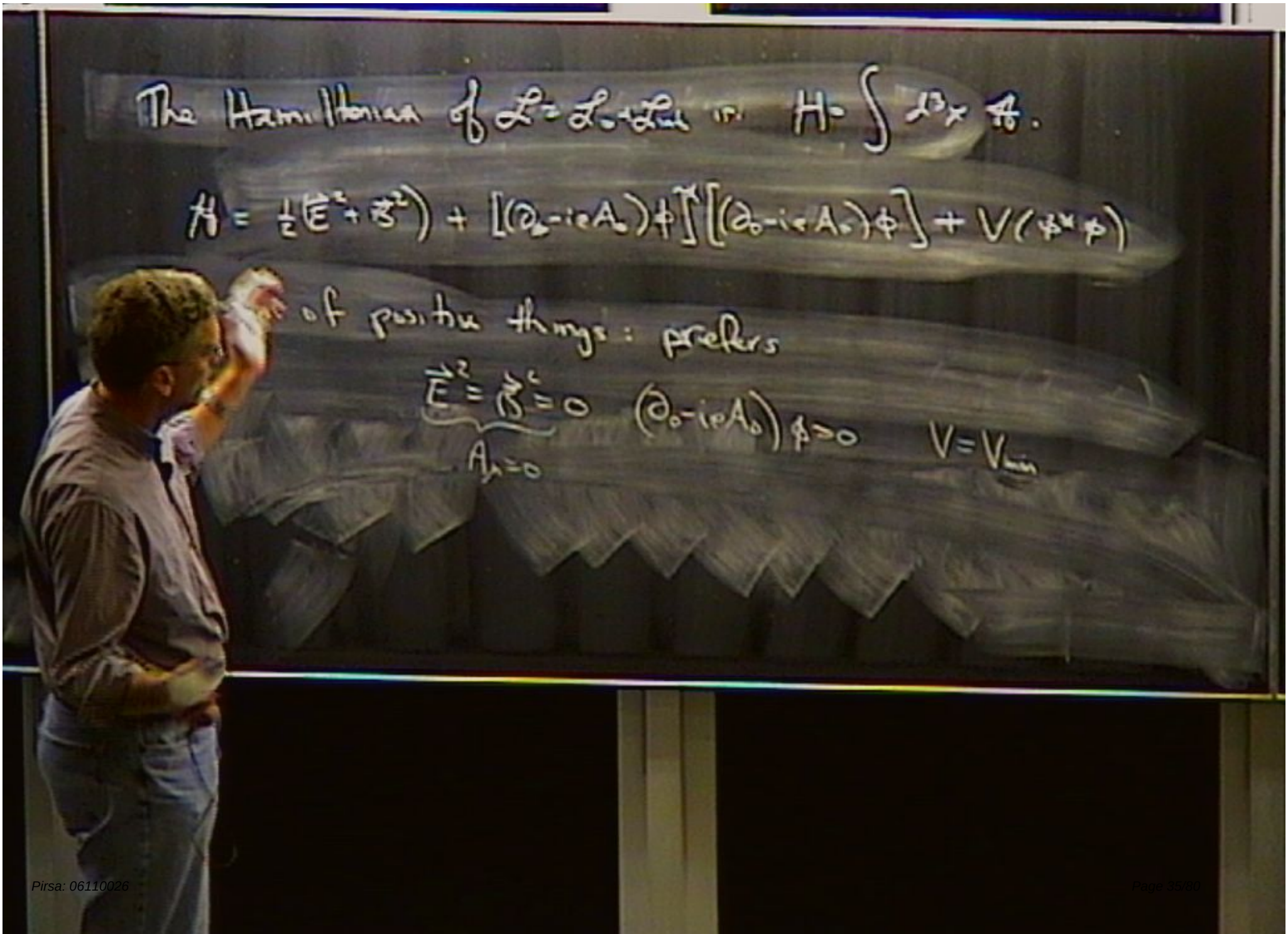
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choose $\phi = \phi_0$ where ϕ_0 minimizes V. $\phi_0 = \frac{\mu}{\sqrt{\lambda}}$

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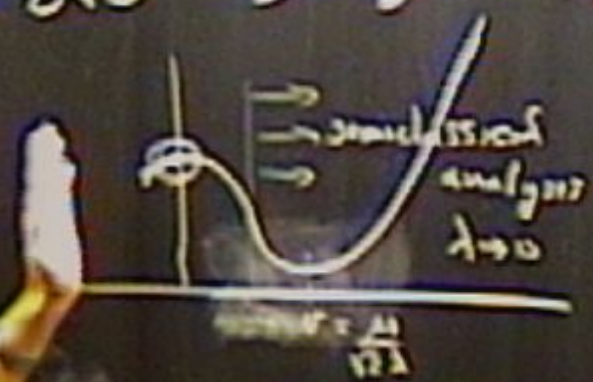

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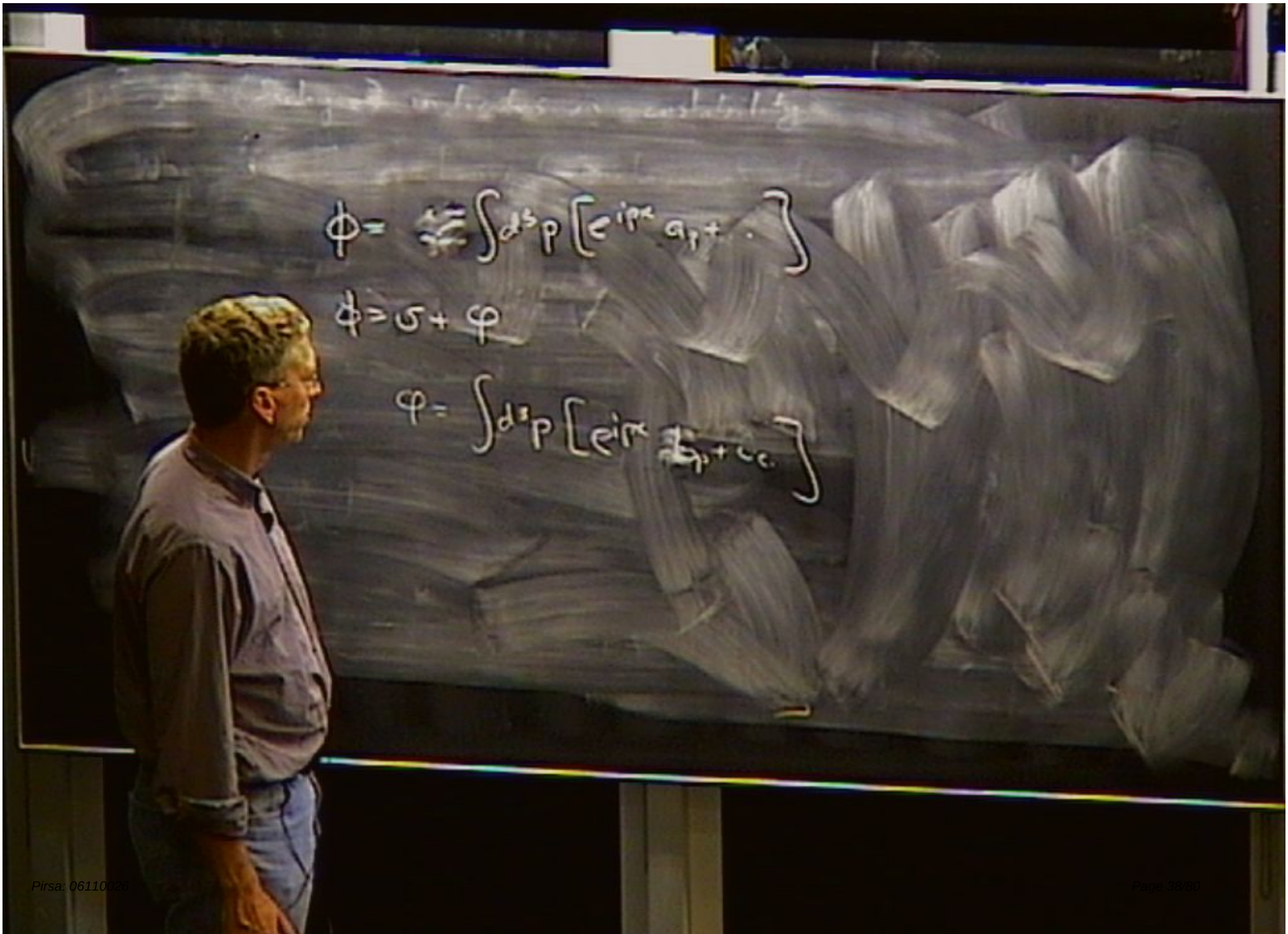
$\phi = \int d^3p \left[e^{ipx} a_p + \ldots \right]$
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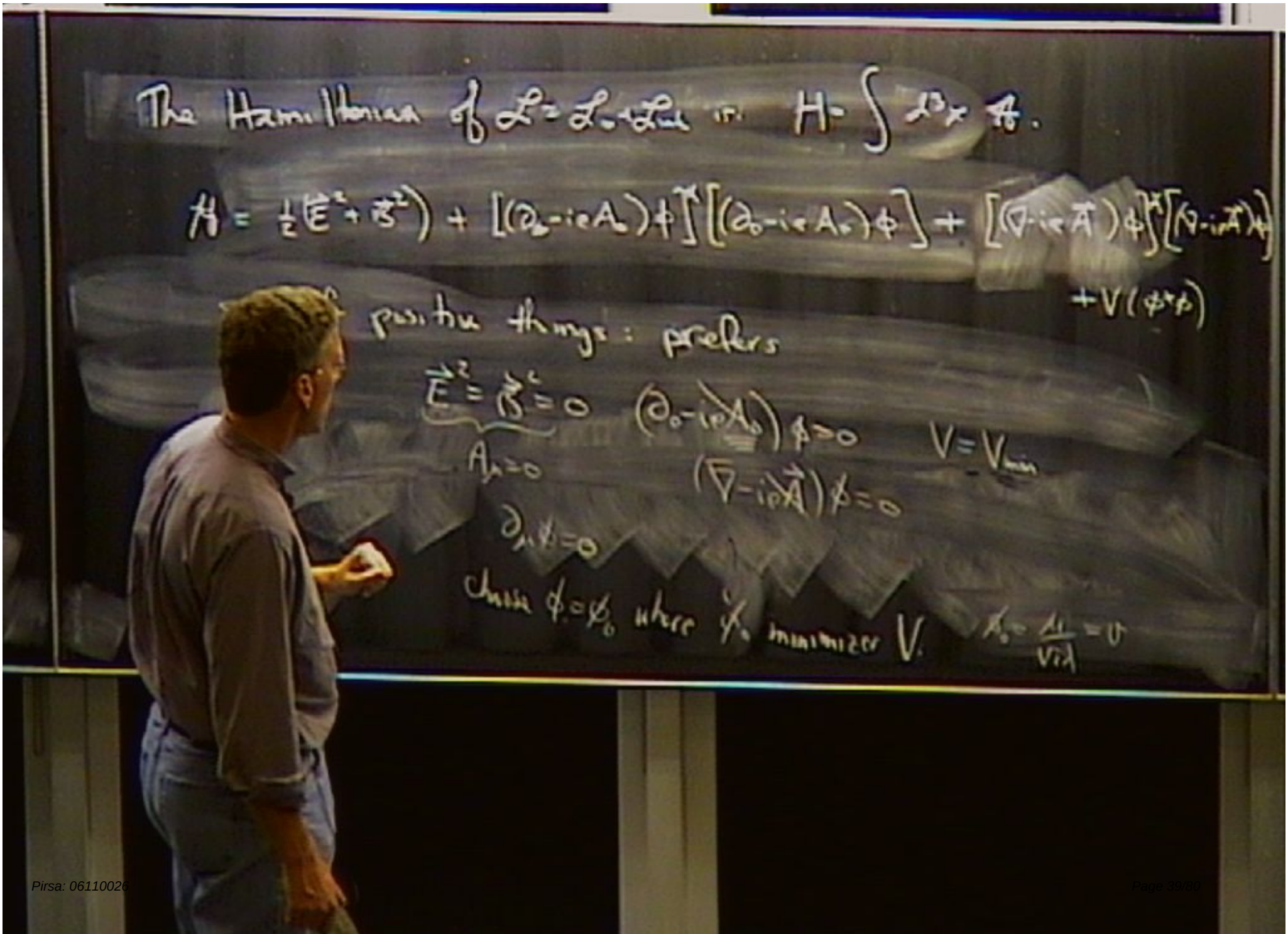
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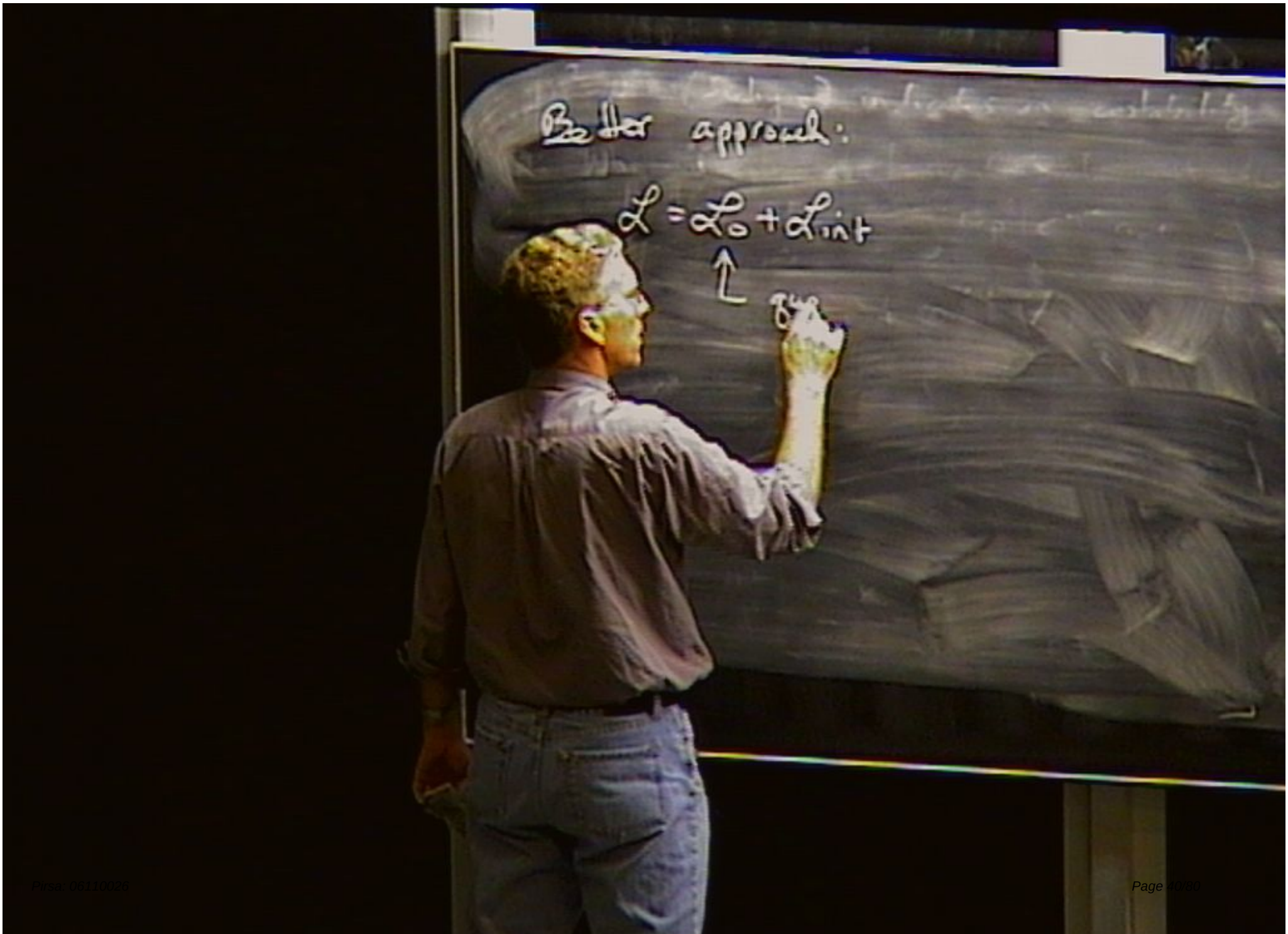
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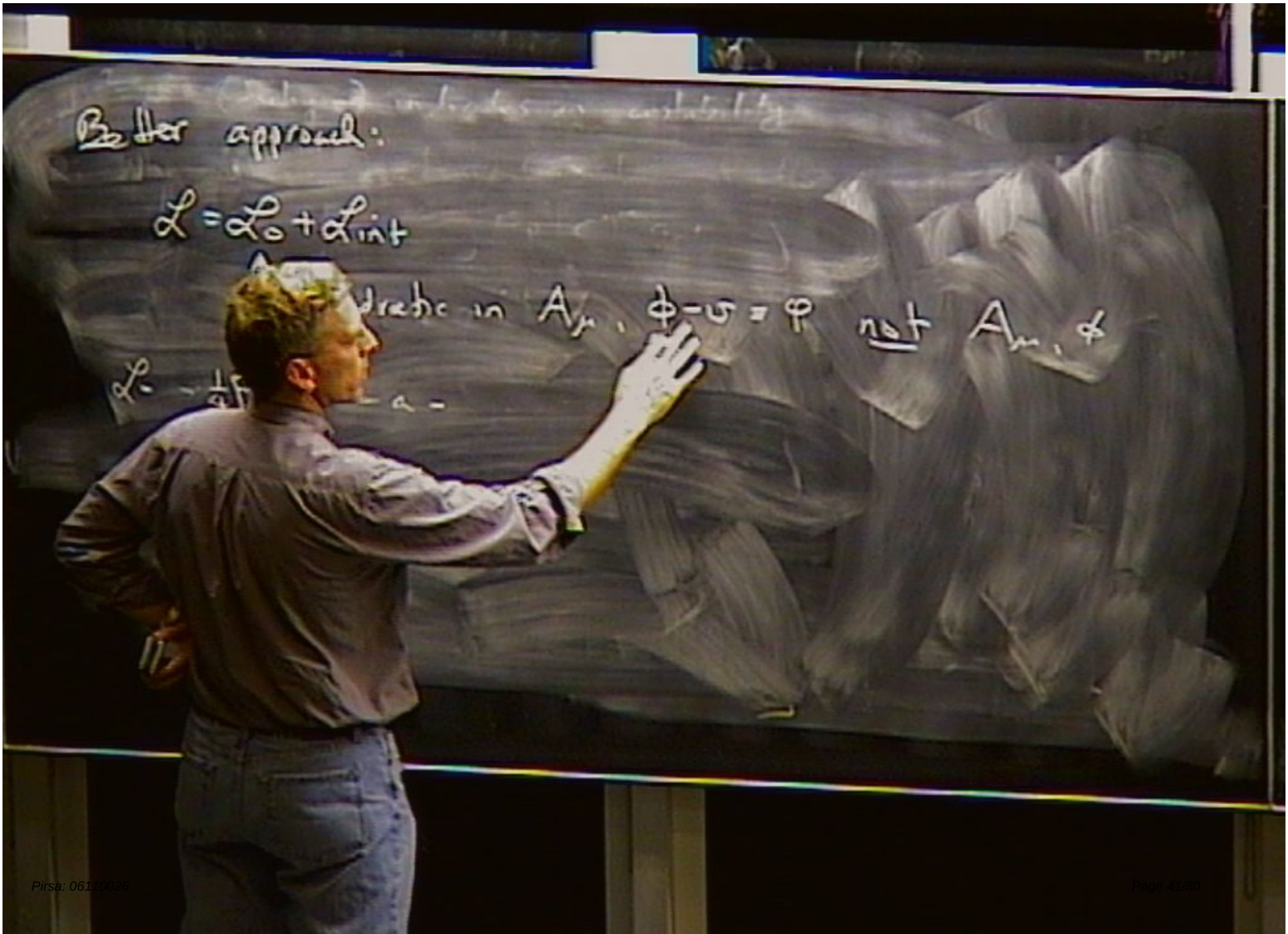
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$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \qquad v = \frac{\Lambda}{\sqrt{2\lambda}}$$

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$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - a + \frac{\Lambda^4}{4\lambda^2} -$$

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$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu} - a + \frac{\mu^4}{4\lambda^2} - \lambda^2\left[(v+\varphi)^*(v+\varphi) - v^2\right]^2$$

$$- \partial_\mu\varphi^*\partial^\mu\varphi - ieA_\mu\left[(v+\varphi)\partial^\mu\varphi^* - (v+\varphi)^*\partial^\mu\varphi\right] - e^2(v+\varphi)^*(v+\varphi)A_\mu A^\mu$$

Better approach:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \qquad v = \frac{\mu}{\sqrt{2}\lambda}$$

quadratic in A_μ, $\phi - v = \varphi$ not A_μ, ϕ

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - a + \frac{\mu^4}{4\lambda^2} - \lambda^2\left[(v+\varphi)^*(v+\varphi) - v^2\right]^2$$
$$- \partial_\mu\varphi^*\partial^\mu\varphi - ieA_\mu\left[(v+\varphi)\partial^\mu\varphi^* - (v+\varphi)^*\partial^\mu\varphi\right] - e^2(v+\varphi)^*(v+\varphi)A_\mu A^\mu$$

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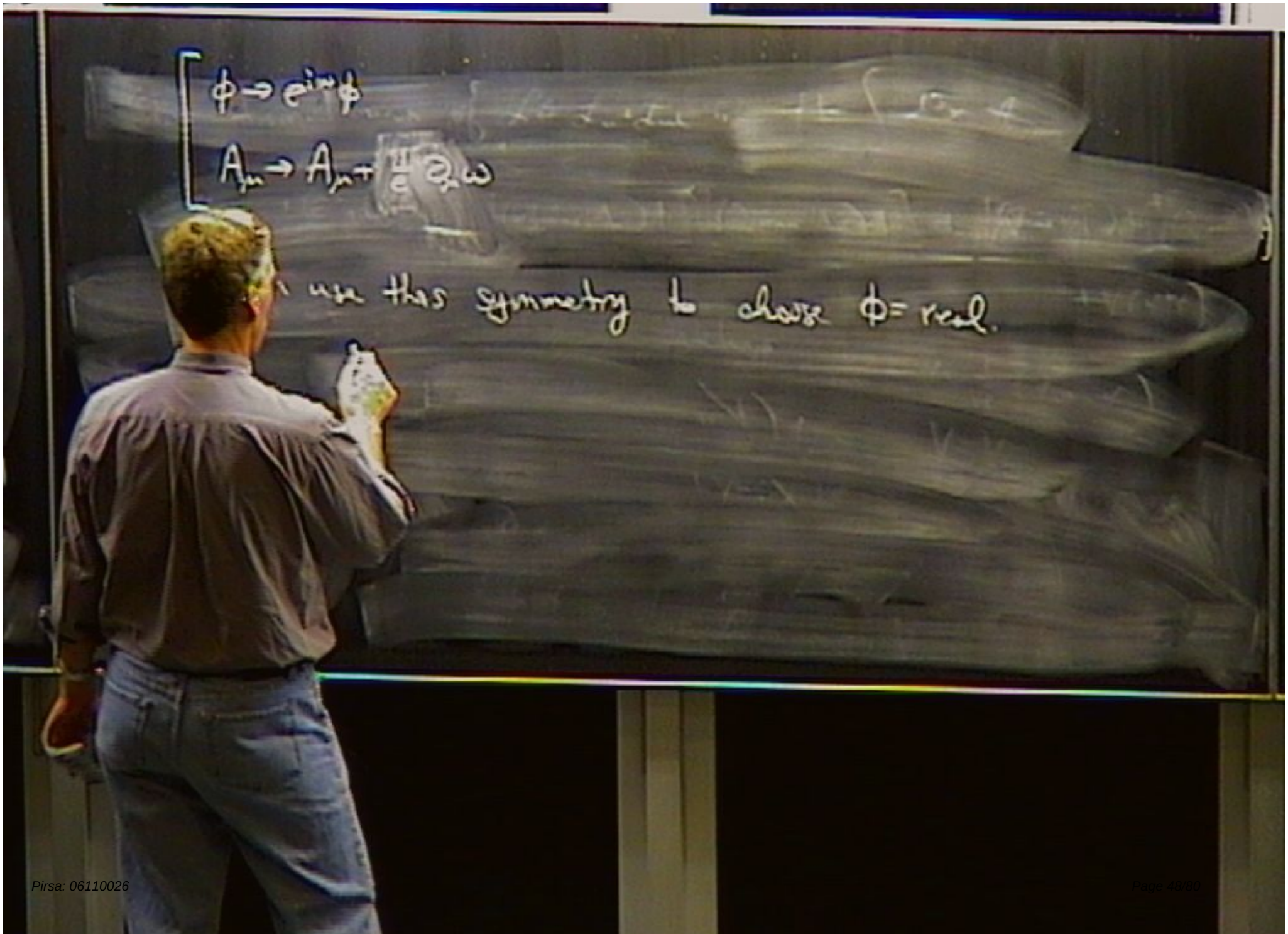
$$\phi \to e^{i\omega}\phi$$

$$A_\mu \to A_\mu + \frac{\hbar}{e}\partial_\mu \omega$$


$\phi \to e^{i\omega}\phi$
$A_\mu \to A_\mu + \frac{\hbar}{e}\partial_\mu \omega$
use this symmetry to choose ϕ = real.

$$\begin{cases} \phi \to e^{i\omega}\phi \\ A_\mu \to A_\mu + \frac{\hbar}{e}\partial_\mu \omega \end{cases}$$

We can use this symmetry to choose $\phi =$ real.

$\Rightarrow \phi = v + \varphi$, φ is also real.

Better approach:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \qquad v = \frac{\mu}{\sqrt{2}\lambda}$$

quadratic in A_μ, $\phi - v = \varphi$ not A_μ, ϕ

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$$\mathcal{L}_0 = -a + \mu^4/4\lambda^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \partial_\mu\varphi^*\partial^\mu\varphi - \lambda^2 v^2(\varphi+\varphi^*)^2 - e^2v^2A_\mu A^\mu - \underline{\underline{iev A_\mu\partial^\mu(\varphi^* - \varphi)}}$$

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We can use this symmetry to choose ϕ = real.

$\Rightarrow \phi = v + \varphi/\sqrt{2}$, φ is also real.

With this choice $\mathcal{L}_0$ simplifies:

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - 2\lambda^2 v^2\varphi^2 - e^2 v^2 A_\mu A^\mu$$

$$\left[\begin{array}{l} \phi \to e^{i\omega}\phi \\ A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \omega \end{array}\right\} \text{gauge symmetry}$$

We can use this symmetry to choose ϕ = real.

$\Rightarrow \phi = v + \varphi/\sqrt{2}$, φ is also real. : "gauge choice"

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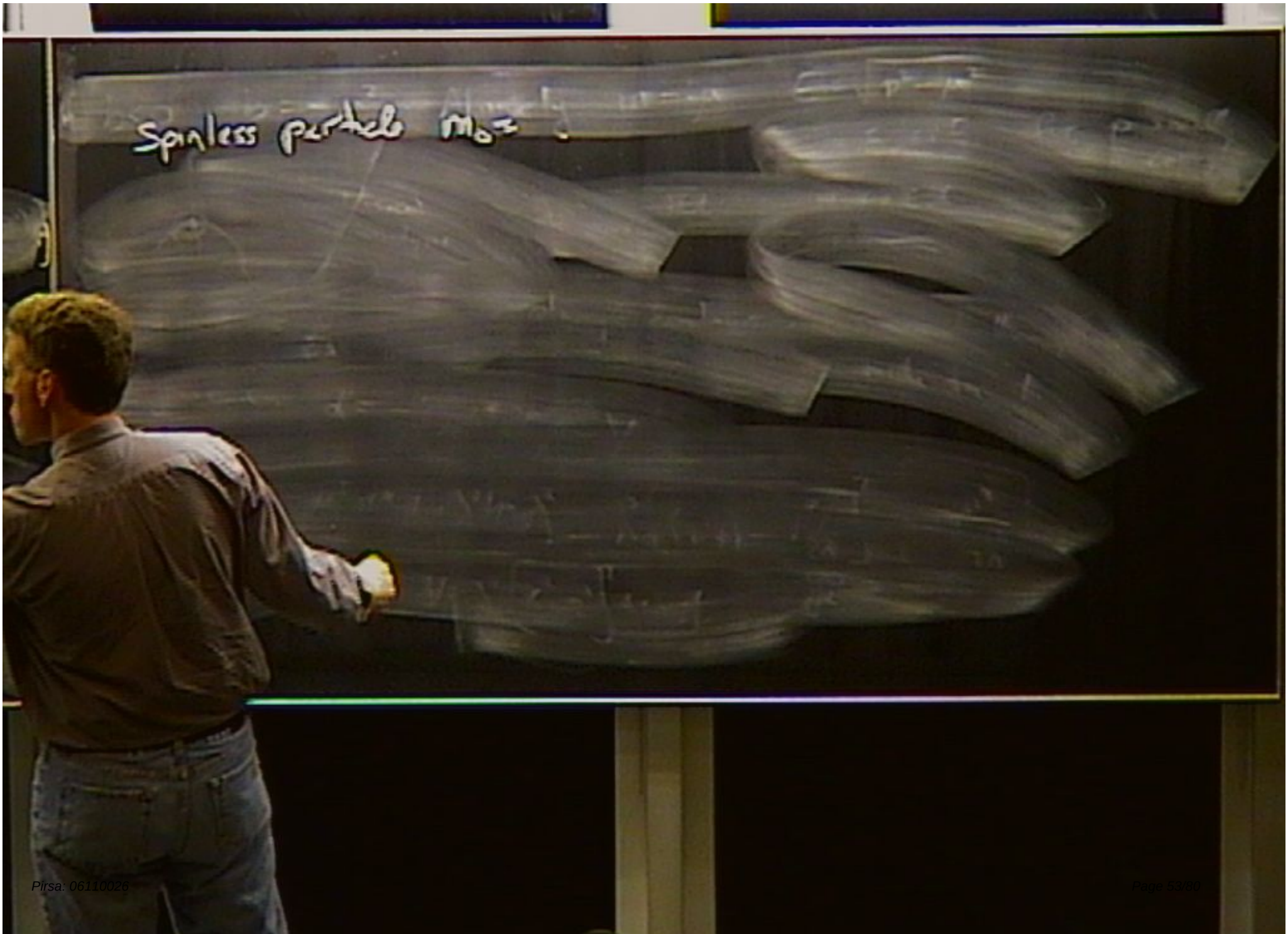
$\longleftarrow$ "Unitary gauge"


Spinless particle

$$\left[\begin{array}{l} \phi \to e^{i\omega}\phi \\ A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \omega \end{array}\right\} \text{gauge symmetry}$$

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$$-\frac{1}{2}m_\varphi^2\varphi^2 \qquad -\frac{1}{2}m_A^2 A_\mu A^\mu$$

$\leftarrow$ "Unitary gauge"

$v = \hbar/\sqrt{2}\lambda$

Spinless particle $m_0 = 2\lambda v = \sqrt{2}\mu$

spin-1 $m_1 = \sqrt{2}\, e v = \frac{e\mu}{\lambda}$

$b = -\mu^2$ $\qquad$ $v = \mu/\sqrt{2\lambda}$

real spinless particle $m_0 = 2\lambda v = \sqrt{2}\mu$

spin-1 $\qquad$ $m_1 = \sqrt{2}\, e v = \frac{e\mu}{\lambda}$

$b = +\mu^2$ $\qquad$ complex (2 real) spinless particles $m_0 = \mu$ $\quad$ 2

1 massless spin 1. $\qquad$ 2 spin

$b = -\mu^2$ $\qquad v = \mu/\sqrt{2\lambda}$

real spinless particle $m_0 = 2\lambda v = \sqrt{2}\mu$ 1 state.

spin-1 " $m_1 = \sqrt{2}ev = \frac{e\mu}{\lambda}$ 3 states

Higgs mechanism "eat kn"

$b = +\mu^2$ complex (2 real) spinless particles $m_0 = \mu$

1 massless spin 1. 2 spin

$$\left[\begin{array}{l} \phi \to e^{i\omega}\phi \\ A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \omega \end{array}\right\} \text{gauge symmetry}$$

$$\phi = \chi e^{i\theta}$$

We can use this symmetry to choose ϕ = real.

$\Rightarrow \phi = v + \varphi/\sqrt{2}$, φ is also real. : "gauge choice"

With this choice $\mathcal{L}_0$ simplifies:

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - 2\lambda^2 v^2\varphi^2 - e^2 v^2 A_\mu A^\mu$$

$$-\frac{1}{2}m_\varphi^2\varphi^2 \qquad -\frac{1}{2}m_A^2 A_\mu A^\mu$$

$\leftarrow$ "Unitary gauge"

$b = -\mu^2$ $\qquad v = \mu/\sqrt{2\lambda}$

real spinless particle $m_0 = 2\lambda v = \sqrt{2}\mu$ 1 state.

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1 massless spin 1. 2 spin

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Higgs mechanism "eat kn"

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1 massless spin 1. 2 spin

Better approach:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \qquad v = \frac{\mu}{\sqrt{2}\lambda}$$

quadratic in A_μ, $\phi - v = \varphi$ $\quad \overset{m}{\updownarrow}$ not A_μ, ϕ

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - a + \frac{\mu^4}{4\lambda^2} - \lambda^2\left[\overbrace{(v+\varphi)^*(v+\varphi)}^{v^2+v(\varphi+\varphi^*)+\varphi^*\varphi} - \frac{\mu^2}{2\lambda^2}\right]^2$$

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Better approach: $\lambda \sim \frac{m}{v}$

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$ $\quad v = \frac{\Lambda}{\sqrt{2}\lambda}$ $\quad v = 246\ \text{GeV}$

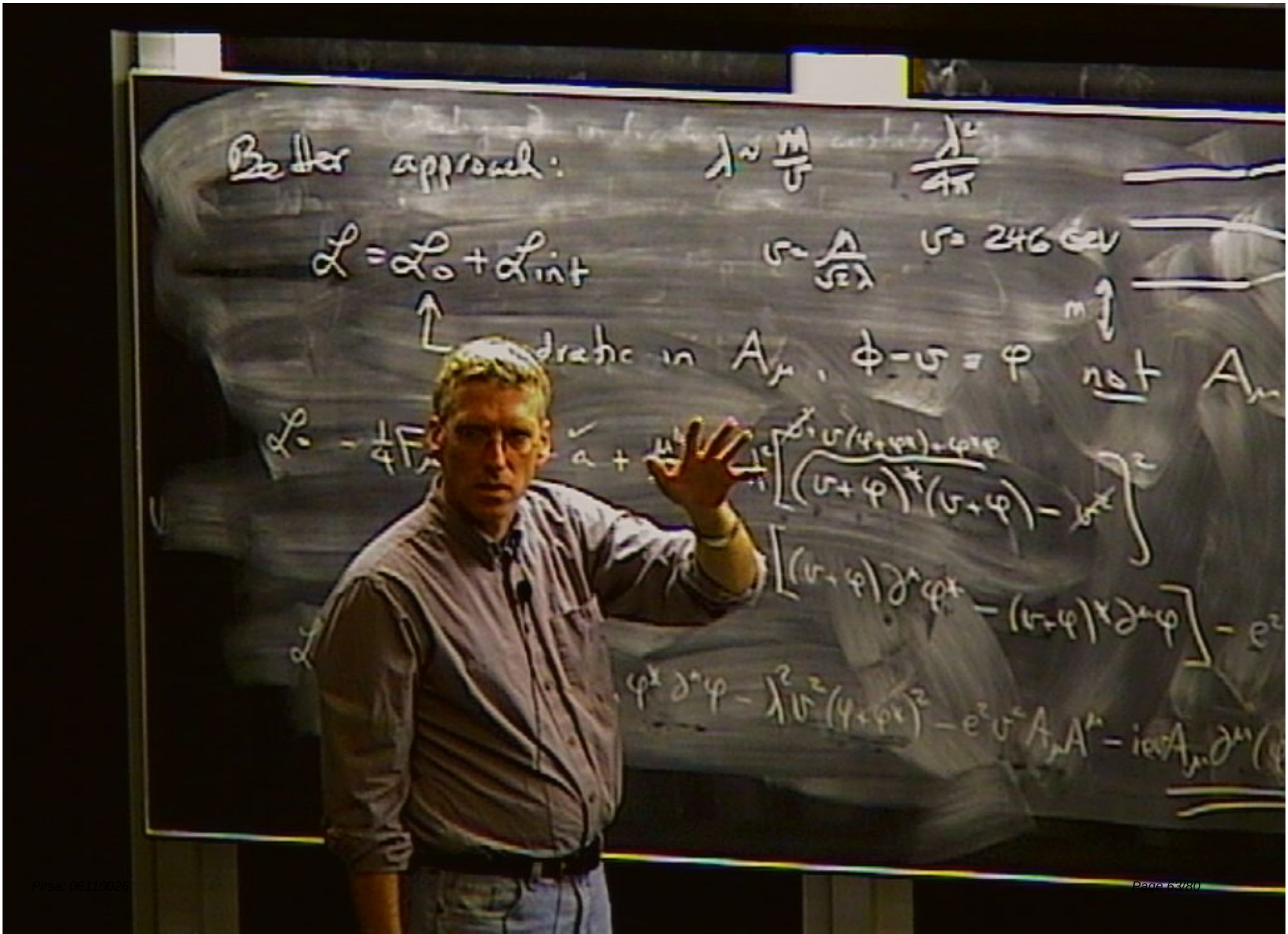
m

quadratic in A_μ, $\phi - v = \varphi$ not A_μ

$$\mathcal{L} = F_{\mu\nu}F^{\mu\nu} - a + \frac{\mu^4}{4\lambda^2} - \lambda^2\left[\overbrace{(v+\varphi)^*(v+\varphi) - \mu^2}^{v(\varphi+\varphi^*)+\varphi^*\varphi}\right]^2$$

$$\partial_\mu\varphi^*\partial^\mu\varphi - ie A_\mu\left[(v+\varphi)\partial^\mu\varphi^* - (v+\varphi)^*\partial^\mu\varphi\right] - e^2$$

$$F_{\mu\nu}F^{\mu\nu} - \partial_\mu\varphi^*\partial^\mu\varphi - \lambda^2 v^2(\varphi+\varphi^*)^2 - e^2 v^2 A_\mu A^\mu - iev A_\mu\partial^\mu(\varphi$$


Better approach:
$\lambda \sim \frac{m}{v}$
$\frac{\lambda^2}{4\pi}$
$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$
$v \sim \frac{\Lambda}{\sqrt{2}\lambda}$
$v = 246$ GeV
m
not A_μ
$\phi - v = \varphi$

Better approach: $\lambda \sim \frac{m}{v}$ $\quad \frac{\lambda^2}{4\pi}$

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$ $\quad v \sim \frac{1}{\sqrt{G_F}}$ $\quad v = 246$ GeV

m

quadratic in A_μ, $\phi - v = \varphi$ not A_μ, φ

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\mu^4}{4\lambda^2} - \lambda^2\left[(v+\varphi)^*(v+\varphi) - v^2\right]^2$$

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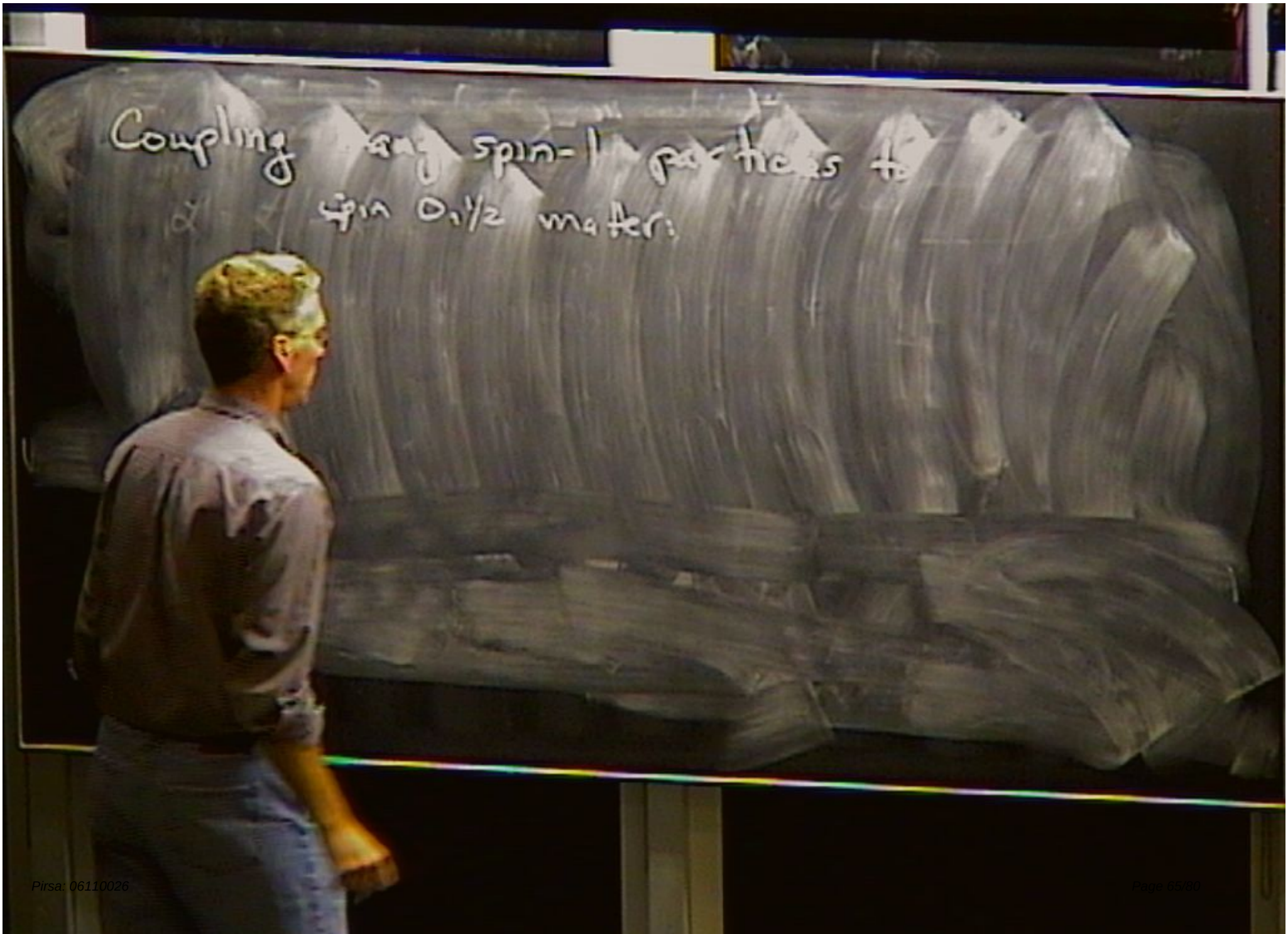
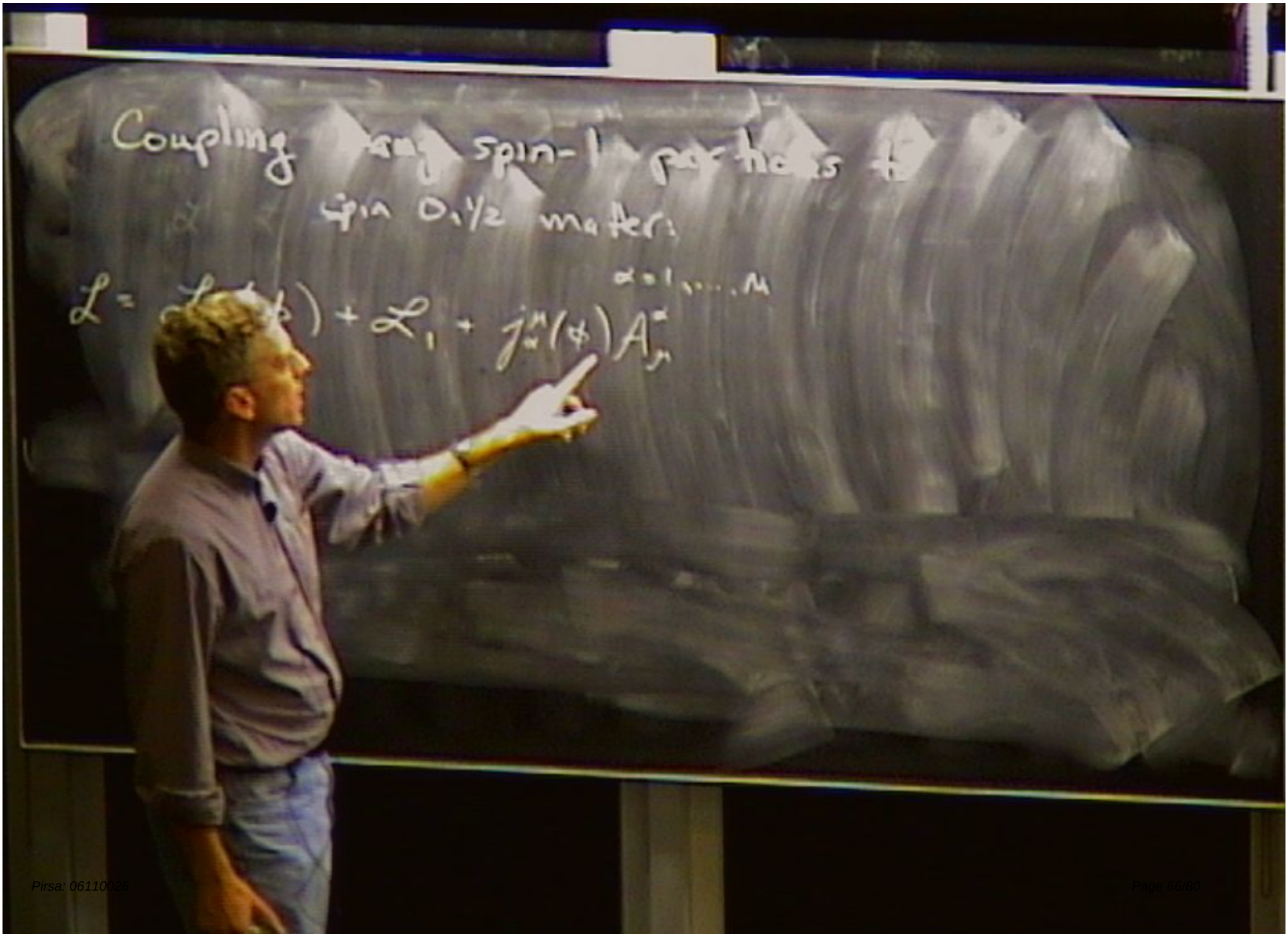
$$\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \partial_\mu\varphi^*\partial^\mu\varphi - \lambda^2 v^2(\varphi+\varphi^*)^2 - e^2v^2A_\mu A^\mu - iev A_\mu\partial^\mu(\varphi^*-\varphi)$$


Coupling
spin-1
spin 0, 1/2 matter:


Coupling
spin-1
spin 0, 1/2 matter
$\alpha = 1, \ldots, M$
$\mathcal{L} = \mathcal{L}(\phi) + \mathcal{L}_1 + j^\mu_\alpha(\phi) A^\alpha_\mu$

Coupling any spin-1 particles to spin 0, 1/2 matter:

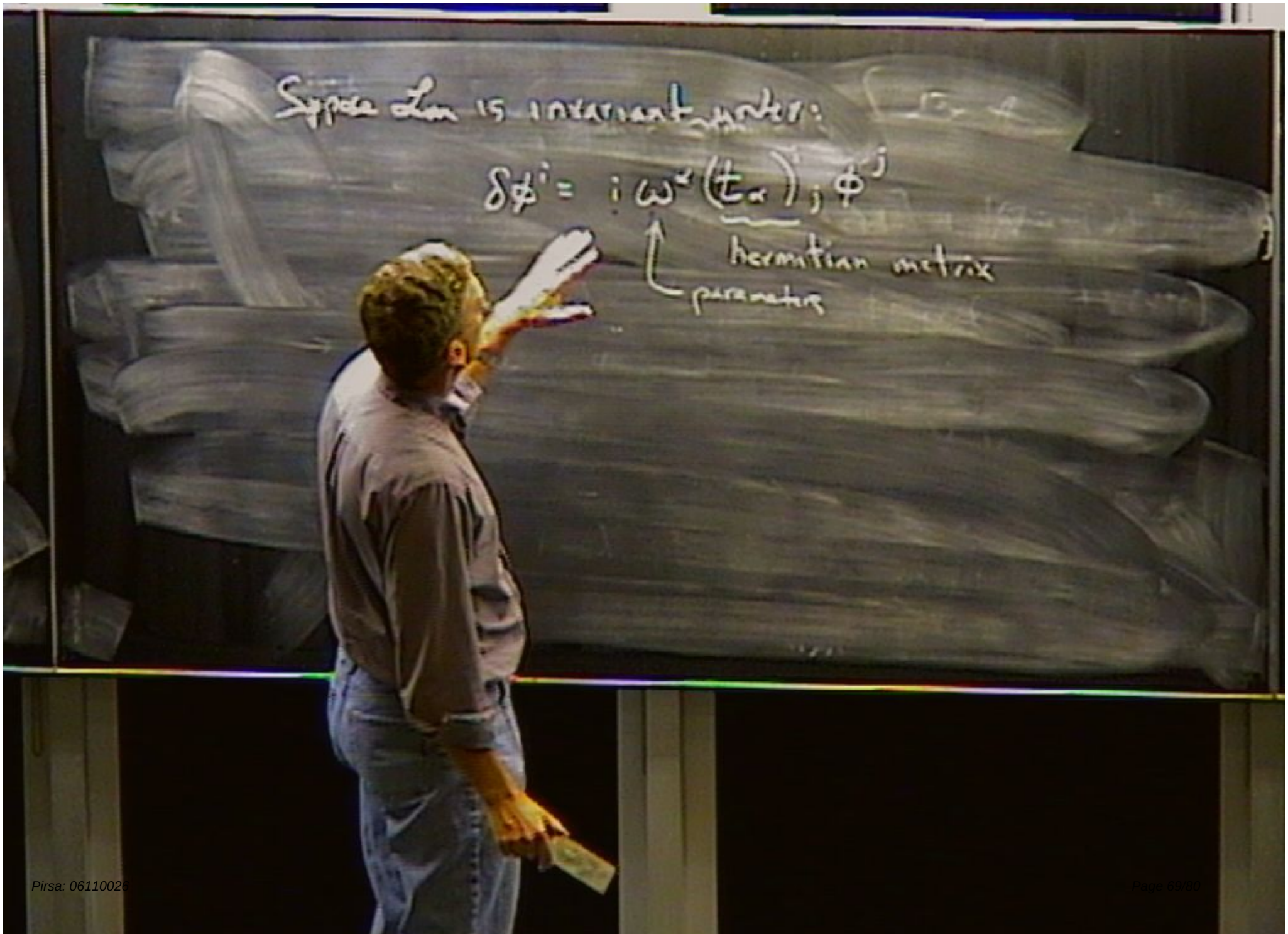
$$\mathcal{L} = \mathcal{L}_m(\phi) + \mathcal{L}_1 + j^\mu_\alpha(\phi) A^\alpha_\mu + \dots$$

$\alpha = 1, \dots, N$

Coupling any spin-1 particles to spin 0, 1/2 matter:

$$\mathcal{L} = \mathcal{L}_m(\phi) + \mathcal{L}_1 + j^\mu_\alpha(\phi) A^\alpha_\mu + \dots$$

$\alpha = 1, \dots, N_1$


Suppose $\mathcal{L}_m$ is invariant under:
$\delta\phi^i = i\omega^\alpha (t_\alpha)^i{}_j \phi^j$
hermitian matrix
parameter

Suppose $\mathcal{L}_m$ is invariant under:

$$\delta\phi^i = i\,\omega^\alpha (t_\alpha)^i{}_j\,\phi^j$$

hermitian matrix

parameters

$$\phi \to g(\omega)\,\phi$$

$$g(\omega_1)\,g(\omega_2) = g(\omega_1,\omega_2)$$

Suppose $\mathcal{L}_m$ is invariant under:

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hermitian matrix

parameters

$$\phi \to g(\omega)\,\phi$$

$$g(\omega_1)\,g(\omega_2) = g(\omega_1, \omega_2)$$

Suppose $\mathcal{L}_{int}$ is invariant under:

$$\delta\phi^i = i\,\omega^\alpha (t_\alpha)^i{}_j\, \phi^j$$

hermitian matrix

parameters

$$\phi \to g(\omega)\,\phi$$

$$g(\omega_1)\,g(\omega_2) = g(\omega_1, \omega_2)$$

Suppose $\mathcal{L}_m$ is invariant under:

$$\delta\phi^i = i\,\omega^\alpha (\underline{t_\alpha})^i{}_j\, \phi^j$$

hermitian matrix

parameters

$$\begin{pmatrix} \phi^1 \\ \vdots \\ \phi^n \end{pmatrix}$$

$$\phi \to g(\omega)\,\phi$$

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Suppose $\mathcal{L}_m$ is invariant under:

$$\delta\phi^i = i\,\omega^\alpha (t_\alpha)^i{}_j\, \phi^j$$

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$$\begin{pmatrix} \phi^1 \\ \vdots \\ \phi^N \end{pmatrix}$$

$$\delta\phi^i = i\,\omega^\alpha (t_\alpha)^i{}_j\, \phi^j$$

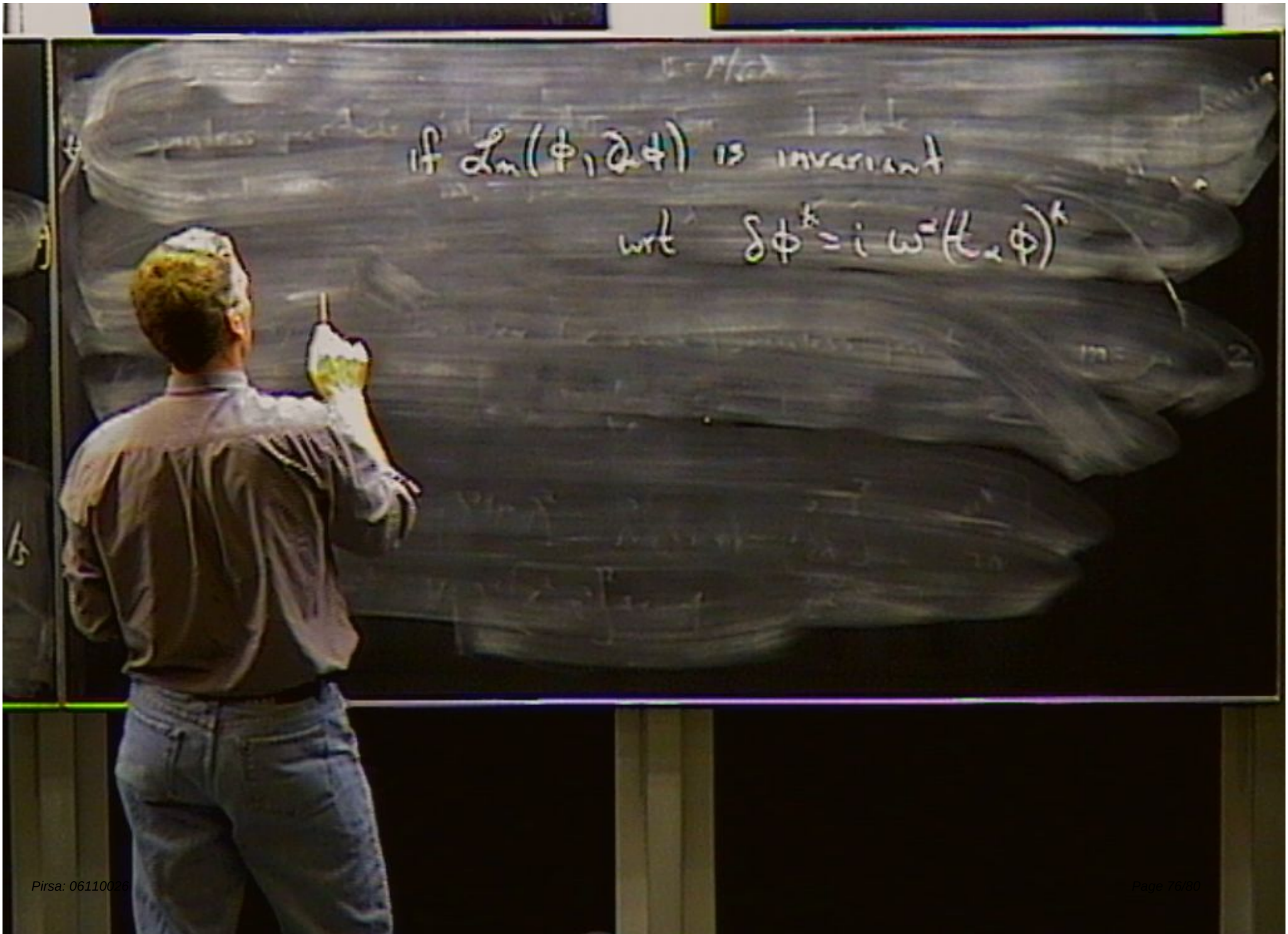
hermitian matrix

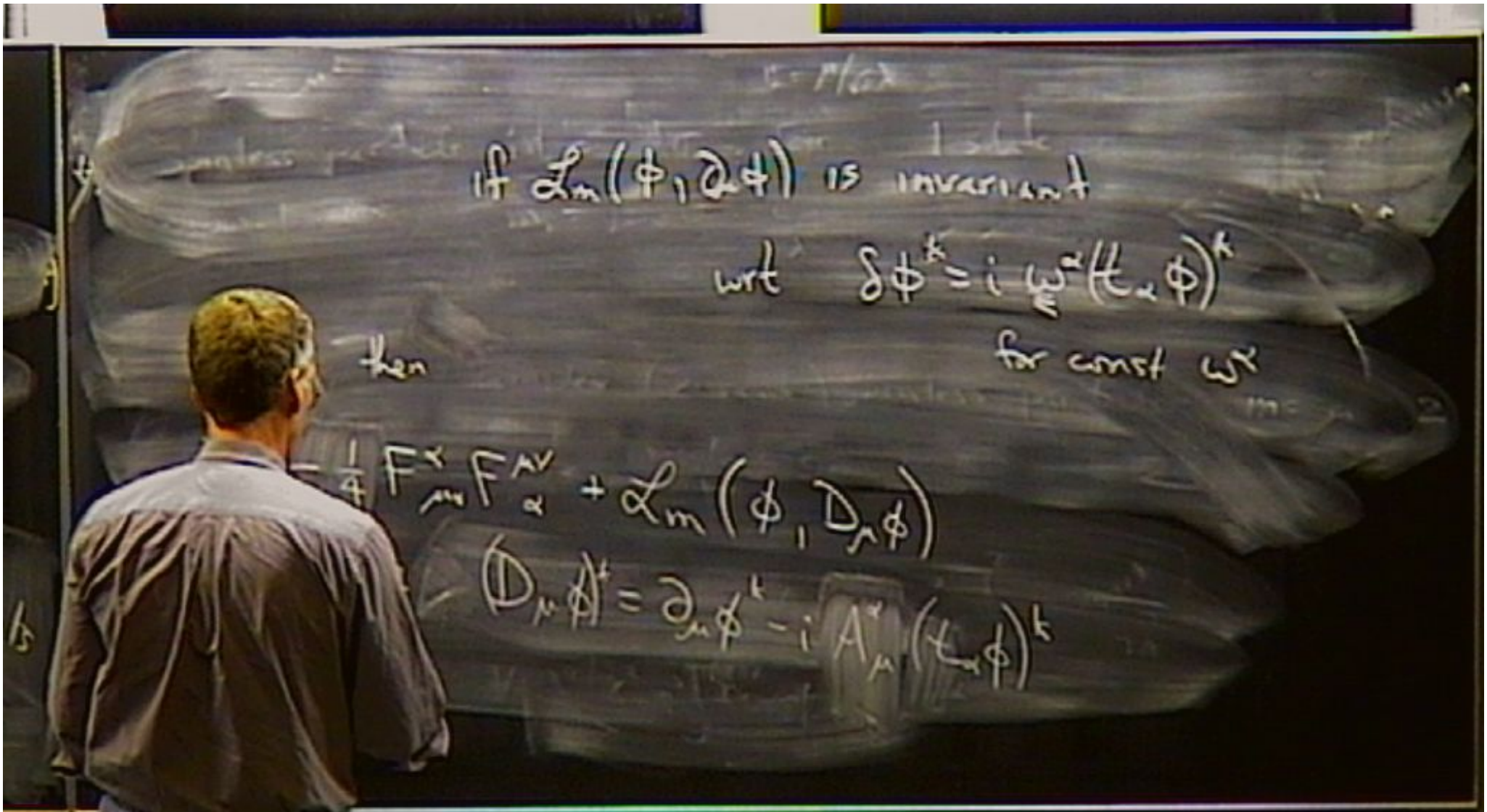
parameters

$$\phi \to g(\omega)\,\phi \qquad g = e^{i\omega^\alpha t_\alpha}$$

$$\boxed{\cancel{g(\omega_1)\,g(\omega_2) = g(\omega_1,\omega_2)}} \longrightarrow [t_\alpha, t_\beta] = C^\gamma{}_{\alpha\beta}\, t_\gamma$$

structure constants


if $\mathcal{L}_m(\phi, \partial\phi)$ is invariant
wrt $\delta\phi^k = i\,\omega^a (t_a \phi)^k$


if $\mathcal{L}_m(\phi, \partial\phi)$ is invariant
wrt $\delta\phi^k = i\,\omega^\alpha (t_\alpha \phi)^k$
for const ω^α
then
$-\frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu}_\alpha + \mathcal{L}_m(\phi, D_\mu\phi)$
$(D_\mu\phi)^k = \partial_\mu \phi^k - i A^\alpha_\mu (t_\alpha \phi)^k$

if $\mathcal{L}_m(\phi, \partial_\mu \phi)$ is invariant

wrt $\delta\phi^k = i\,\omega^\alpha (t_\alpha \phi)^k$

for const ω^α

then

$$\mathcal{L} = -\frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu}_\alpha + \mathcal{L}_m(\phi, D_\mu \phi)$$

where $(D_\mu \phi)^k = \partial_\mu \phi^k - i A^\alpha_\mu (t_\alpha \phi)^k$

$$F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + c^\alpha_{\beta\gamma} A^\beta_\mu A^\gamma_\nu$$

where $(D_\mu \phi)^k = \partial_\mu \phi^k - i A^\alpha_\mu (t_\alpha \phi)^k$

and $F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + c^\alpha_{\beta\gamma} A^\beta_\mu A^\gamma_\nu$

invariant wrt:

$\delta \phi^k = i \omega^\alpha (t_\alpha \phi)^k$

$\delta A^\alpha_\mu = \partial_\mu \omega^\alpha + c^\alpha_{\beta\gamma} \omega^\beta A^\gamma_\mu$

where $(D_\mu \phi)^k = \partial_\mu \phi^k - i A^\alpha_\mu (t_\alpha \phi)^k$

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is invariant wrt:

$$\delta \phi^k = i \omega^\alpha (t_\alpha \phi)^k$$

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and ω^α <u>not</u> const.