

Title: Graduate Course on Standard Model & Quantum Field Theory - 4A

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Abstract: Graduate Course on Standard Model & Quantum Field Theory

Massless Spin 1 coupled to matter (spins $0, \frac{1}{2}$): ϕ^i

$$\delta A_{\mu} = \partial_{\mu} \omega$$

$$\delta \phi^i = \omega F^i(\phi)$$

$$J^{\mu}(\phi) = \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \phi^i)}$$



Massless Spin 1 coupled to matter (spins 0, 1/2), ϕ :

$$\delta A_\mu = \partial_\mu \omega$$

$$\delta \phi^i = \omega F^i(\phi)$$

$$j^\mu(\phi) \approx \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^i)} F^i$$

$(\partial_\mu j^\mu = 0)$ follows from $\delta \mathcal{L} = 0$ when ω is const.

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$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_m + j^\mu(\phi) A_\mu$$

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($\partial_\mu j^\mu = 0$ follows from $\delta \mathcal{L}_m = 0$ when ω is constant.)

$$J_\mu = -\partial_\mu \phi^i \delta^i \psi - m \bar{\psi} \gamma^\mu \psi$$

$$\psi \rightarrow e^{i\omega t} \psi$$

$$\delta \psi = i\omega \psi$$

$$j^\mu \propto i\bar{\psi} \gamma^\mu \psi$$

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($\partial_\mu j^\mu = 0$ follows from $\delta \mathcal{L}_m = 0$ when ω is const.)

$$j^\mu = -\bar{\psi} (\gamma_1 m) \psi$$

$$V_m = -\partial_\mu \phi^i \partial^\mu \phi^i - m^2 \phi^i \phi^i$$

$$\psi \rightarrow e^{i\omega t} \psi$$

$$\delta \psi = i\omega \psi$$

$$j^\mu \propto i \bar{\psi} \gamma_\mu \psi$$

Massless Spin 1 coupled to matter (spins ω_1/c) ϕ^i

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$$\delta \phi^i = \omega F^i(\phi)$$

$$j^\mu(\phi) \approx \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^i)} F^i$$

$$\mathcal{L} = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_m + j^\mu(\phi) A_\mu$$

$$j^\mu = -\bar{\psi} (\gamma_1 m) \psi$$

$$A_\mu = -\bar{\psi} \gamma^\mu \gamma^5 \psi - m \bar{\psi} \gamma^\mu \psi$$

$$\delta \psi = i\omega \psi$$

$$\begin{aligned} j^\mu &\propto i\bar{\psi} \gamma^\mu \psi \\ j^\mu &= (\bar{\psi} \gamma^\mu \gamma^5 \psi - \bar{\psi} \gamma^\mu \psi) \end{aligned}$$

Massless Spin 1 coupled to matter (spins $0, \frac{1}{2}$), ϕ^i

$$\delta A_\mu = \partial_\mu \omega$$

$$\delta \phi^i = \omega F^i(\phi)$$

$$j^{\mu}(\phi) \sim \frac{\partial \omega}{\partial (\partial_\mu \phi^i)} F^i$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_m + j^\mu(\phi) A_\mu$$

$$\psi, \delta\psi = \bar{\psi}(\gamma_5 \gamma_0) \psi$$

$$J_\mu = -\partial_\mu \phi^i \partial^\mu \phi^i - m^2 \phi^i \phi^i$$

$$\delta \psi = i \omega \psi$$

$$\delta \phi^i = i \omega \phi^i$$

$$J^\mu = \bar{\psi} \gamma^i \gamma^0 \gamma^1 \psi$$

$$J^\mu = (\gamma^0 \partial^1 \phi^i - \gamma^1 \partial^0 \phi^i)$$

$$\delta j^\mu = -2 \omega \phi^i A_\mu \omega$$

Massless Spin 1 coupled to matter (spins $0, \frac{1}{2}$): ϕ^i

$$\delta A_\mu = \partial_\mu \omega$$

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$$j^\mu(\phi) \propto \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^i)} F^i$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_m + j^\mu(\phi) A_\mu \left(1 + g_2 \phi A_\nu A^\nu \right) \quad \delta j^\mu = -2g_2 \phi A_\nu \omega$$

$$j^\mu = -\bar{\psi} (\gamma_1 m) \psi$$

$$\psi \rightarrow e^{i\omega t} \psi \quad \delta \psi = i\omega \psi$$

$$\delta j^\mu = -\partial_\lambda \delta^\mu_\lambda \delta^\nu \psi - m^2 \phi^\lambda \delta^\mu_\lambda$$

$$\delta \psi = i\omega \psi$$

$$\begin{aligned} \delta^\mu_\lambda & \propto i\bar{\psi} \gamma_\mu \psi \\ \delta^\nu & : (\gamma^\mu \partial^\nu \phi - \phi \partial^\nu \gamma^\mu) \end{aligned}$$

Massless Spin 1 coupled to matter (spins 0, 1/2), ϕ :

$$\delta A_\mu = \partial_\mu \omega$$

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$$j^\mu(\phi) \leftarrow \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^i)} F^i$$

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$$j^\mu = -\bar{\psi} (\gamma_{\mu+m}) \psi$$

$$A_\mu = -\partial_\mu \phi^i \gamma^i \psi^\dagger - m^2 \phi^i \psi$$

$$\psi \rightarrow e^{i\omega t} \psi$$

$$\delta \psi = i\omega \psi$$

$$j^\mu \propto i\bar{\psi} \gamma^m \psi$$

$$j^\mu = (\gamma^1 \partial^2 \phi - \phi \partial^2 \gamma^1)$$

Massless Spin 1 coupled to matter (spins 0, 1/2), ϕ :

$$\delta A_\mu = \partial_\mu \omega - \omega A_\mu$$

$$\delta \phi^i = \omega F^i(\phi)$$

$$j^\mu(\phi) \leftarrow \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^i)} F^i$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_m + j^\mu(\phi) A_\mu \left(1 + \phi^2 \bar{\psi} \gamma_5 \psi \right) \quad \delta j^\mu = -2 \epsilon_{\mu\nu\rho} \partial^\nu \omega$$

$$\Psi \cdot \mathcal{L}_m = -\bar{\psi} (\partial_\mu m) \psi$$

$$\Psi \rightarrow e^{i\omega t} \Psi \quad \delta \Psi = i\omega \Psi$$

$$\delta \mathcal{L}_m = -\partial_\mu \phi^i \partial^\mu \phi^i - m^2 \phi^i \phi^i$$

$$\begin{aligned} & \delta j^\mu \propto i \bar{\psi} \gamma^\mu \psi \\ & \delta j^\mu = (\bar{\psi}^\mu \partial^\nu \phi^i - \partial^\mu \bar{\psi}^\nu \phi^i) \end{aligned}$$

$$\alpha_s = -\frac{g_s^2 g_F}{m_F^2} \sin^2 \theta_W \left(\frac{g_F^2}{m_F^2} \sin^2 \theta_W - \frac{g_s^2}{m_s^2} \right)$$

What are the lightest elementary particles?

$$\omega_0 = -\partial_x \phi^* \partial_x \phi - m^2 \phi^* \phi \quad \text{gives} \quad \partial_t^2 \phi + (m^2 - \epsilon) \phi = 0$$

What are the lightest elementary particles?
lighter than 1 MeV

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electron: $m_e = 0.511 \text{ MeV}$ spin $\frac{1}{2}$
photon: $m_\gamma = 0$
neutrinos: $\nu_1 \nu_2 \nu_3$ $m \lesssim \text{eV}$ spin 1

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ν_1, ν_2, ν_3

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photon: $m_\gamma = 0$

neutrinos:

ν_1, ν_2, ν_3 $m \lesssim \text{eV}$ spin $\frac{1}{2}$ ($Q=0$, could be their own anti-particles)

$\omega_1 = -\partial \phi^1 \partial \phi^1 - m^2 \phi^1$ describes density $\partial^{\mu}(\phi^1 \partial_{\mu} \phi^1 - \frac{1}{2} m^2 \phi^1)$

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Next heaviest thing: muon: mass $\sim 105 \text{ MeV}$.

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Statement: We expect the interactions of ① to be given by renormalizable

A_μ (photon)
 ψ (Dirac) - electron
 ν_i (Majorana) neutrinos

A_μ (photon)

ψ (Dirac) · electron

ν_i (Majorana) neutrinos

$$\mathcal{L}_{\text{electromagnetism}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial_\mu^2 =$$

A_μ (photon)

ψ (Dirac) : electron

ν_i (Majorana) neutrinos

$$S_{\text{photon}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$S_m = -\bar{\psi}(\not{D} + m_e)\psi - \bar{\nu}_i(\not{D} + m_{\nu_i})\nu_i$$



A_μ (photon)

ψ (Dirac) : electron

ν_i (Majorana) neutrinos

$$\mathcal{L}_{\text{photon}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_m = -\bar{\psi}(\not{D} + m_e)\psi - \bar{\nu}_i(\not{D} + m_{\nu_i})\nu_i \quad \text{symmetry} \quad \psi \rightarrow e^{i\omega t}\psi \\ \delta\psi = i\omega\psi$$

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$$\mathcal{L}_{\text{photon}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

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\mathcal{L}_{int}

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$$\mathcal{L}_{\text{photon}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_m = -\bar{\psi}(\not{D} + m_e)\psi - \bar{\nu}_i(\not{D} + m_{\nu_i})\nu_i \quad \text{symmetry} \quad \begin{cases} \psi \rightarrow e^{i\omega t}\psi \\ \bar{\psi} = i\omega \psi \\ \not{D}_\mu = \partial_\mu - ieA_\mu \end{cases}$$

$$\mathcal{L}_{int}$$

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$$\mathcal{L}_{int} = -ie A_\mu \bar{\psi} \not{D}^\mu \psi$$

coupling const.

A_μ (photon)

ψ (Dirac) : electron

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$$\gamma_i \rightarrow 0; \bar{\chi} \chi_j$$

$$M \rightarrow O^T M O$$

$$\mathcal{L}_{\text{photon}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}_m = -\overline{\psi} (\not{D} + m_e) \psi - \frac{1}{2} \bar{\nu}_i (\not{D} + m_{\nu_i}) \nu_i \quad \text{symmetry} \quad \begin{cases} \psi \rightarrow e^{i\omega t} \psi \\ \bar{\psi} = i\psi^\dagger \end{cases}$$

$$\mathcal{L}_{int} = -ie A_\mu \overline{\psi} \not{D}^\mu \psi \quad \begin{cases} \delta \nu_i = M_i \nu_i \\ \delta \psi = i \epsilon \psi \end{cases} \quad \not{D}_\mu = \partial_\mu + ie A_\mu$$

coupling const.

What are the lightest elementary particles?

lighter than 1 MeV

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photon: $m_\gamma = 0$ spin 1
neutrinos: ν_1, ν_2, ν_3 mass $\lesssim \text{eV}$ spin $\frac{1}{2}$ ($Q=0$, could be their own antiparticles)]

Next heaviest thing: muon: mass $\sim 105 \text{ MeV}$. $g \approx \frac{1}{M_P} \text{ of } [e/m]$

Statement: We expect the interactions of ① to be given by renormalizable interactions of $\bar{\psi} \psi$.

Q: What are the most general renormalizable intys?

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point: We expect the interactions of Θ to be given by renormalizable terms if $\bar{e} e n \bar{n}$.

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 $\gamma + \gamma \rightarrow \gamma_1 \gamma_2 \gamma_3$ mass ~ 0 eV spin $\frac{1}{2}$ ($Q=0$, could be their own anti-particles)]

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Consequences of symmetries:

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$$U^t H U = H.$$



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$$|\psi'\rangle = U|\psi\rangle \quad \langle\psi'|\chi'\rangle = \langle\psi|\chi\rangle \quad U(\langle\chi|\psi\rangle) = \langle\chi|U|\psi\rangle \text{ (antiunitary)}$$

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$$U e^{-iHt} = e^{-iHt} \Psi$$
$$U(iH) = iH(U)$$

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In quantum mechanics: $\star \underline{UU^\dagger = I}$

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④ $|\psi'\rangle = U|\psi\rangle$ $\langle\psi'|x'\rangle = \langle\psi|x\rangle$ $U(\xi|\psi\rangle) = \langle\xi|U|\psi\rangle$ (antiunitary)

$\hookrightarrow U e^{-iHt} = e^{-iHt} \Psi$ (↑ preserved for all t.)
 $U(iH) = iH(U)$

Discrete symmetry: suppose there is a symmetry for which
 $U^2: I \rightarrow U^\dagger U: I \rightarrow U$ is a homomorphism

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$$U^2 = I \rightarrow U^\dagger U = I \rightarrow U = \text{hermitian}$$

$$U^\dagger H U = H \rightarrow U H = H U$$

so U is hermitian + commutes with H + so is an observable.

$$U|\psi\rangle = \lambda|\psi\rangle$$

$$H|\psi\rangle = E|\psi\rangle$$

$$UH|\psi\rangle = Ue^{i\omega t}|\psi\rangle = e^{i\omega t}U|\psi\rangle$$
$$\rightarrow e^{i\omega t}|\psi\rangle = \lambda|\psi\rangle$$

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$$\rightarrow e^{i\theta H}|\psi\rangle = \lambda|\psi\rangle$$

$\rightarrow \lambda$ is constant

Discrete symmetry: suppose there is a symmetry for which

$$U^2 = I \rightarrow U^\dagger U = I \rightarrow U = \text{hermitian}$$

$$\lambda' = 1 \quad \lambda = \pm 1$$

$$U^\dagger H U = H \rightarrow U H = H U$$

so U is hermitian + commutes with H + τ_0 is an observable.

$$U|\psi\rangle = \lambda|\psi\rangle$$

$$H|\psi\rangle = E|\psi\rangle$$

$$UH(t) = Ue^{iHt}I = e^{iHt}U|0\rangle \\ \rightarrow e^{iHt}|0\rangle = \lambda|0\rangle$$

$\rightarrow \lambda$ is constant

Consequences of symmetries:

In quantum mechanics: $\cancel{U} \cancel{U^\dagger} = I$

$\cancel{U^\dagger H U} = H$ defines what is meant by a symmetry.

④ $|\psi'\rangle = U|\psi\rangle$ $\langle\psi'|x'\rangle = \langle\psi|x\rangle$ $U(\xi|\psi\rangle) = \cancel{\xi} U|\psi\rangle$ (antiunitary)

$\rightarrow U e^{-iHt} = e^{-iHt} \Psi$ $U(iH) = iH(U)$

(†) preserved for all ξ .

i) $U = e^{i\omega Q}$ for some Q



2) If $U = e^{i\omega Q}$ for some Q + real ω .
then $UU^\dagger = I \Rightarrow Q = Q^\dagger$

$UH = HU \Rightarrow$ ~~for all~~ ω implies $QH = HQ$.

$$\rightarrow Q|\psi\rangle = \omega|\psi\rangle \quad H|\psi\rangle = E|\psi\rangle$$

then $Q|U(t)\rangle = |U(t)|\rangle \dots$

is conserved.

In field theory there are 2 important new features:

i) Conservation is local: $\delta f = 0$ $\Rightarrow j^\mu$ satisfying $\partial_\mu j^\mu = 0$.



$$Q = \int_{\Sigma} d^3x j^0$$

$$\frac{dQ}{dt} = \int_{\Sigma} d^3x \frac{\partial j^0}{\partial t} = - \int_{\Sigma} d^3x \vec{j} \cdot \vec{\nabla} t$$

$$\vec{j} \cdot \vec{\nabla} t = 0$$



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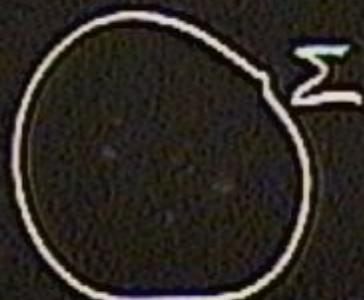
$$Q = \int_{\Sigma} d^3x j^0$$

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$$\frac{\partial j^0}{\partial t} - \nabla \cdot \vec{j} = 0$$

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$$\delta\phi = i\omega\phi$$

$$\phi \rightarrow e^{i\omega t}$$

$$Cl_{in}: U = e^{i\omega t} \quad U^+ \neq U^- e^{-i\omega t}$$

$$Q = \int_{\Sigma} d^3x j^0$$

$$\frac{dQ}{dt} = \int_{\Sigma} d^3x \frac{\partial j^0}{\partial t} - \int_{\Sigma} d^3x \nabla \cdot \vec{j}$$

$$= - \int_{\Sigma} d^3x \vec{n} \cdot \vec{j}$$

$$\vec{j}^0 \cdot \nabla \vec{j} = 0$$

2) If $U = e^{i\omega Q}$ for some Q \rightarrow real ω .

then $UU^\dagger = I \Rightarrow Q = Q^\dagger$

$UH = HV \forall$ real ω implies $QH = HQ$

$\rightarrow Q|\psi\rangle = \delta|\psi\rangle$ $H|\psi\rangle = \epsilon|\psi\rangle$

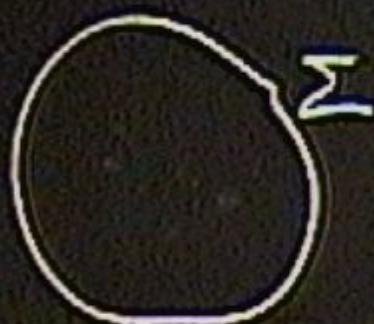
thus $Q|\psi(1)\rangle = |\psi(1)\rangle$...

1 c.ntral.



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$$\delta\phi = i\omega\phi$$

$$q \rightarrow \rho i\omega q$$

$$\text{Ch.} \quad U = \rho i\omega q$$

$$U^* / (U - \rho i\omega q)$$

$$\phi_1 = \frac{\phi_0 + i\omega q}{\sqrt{2}}$$

$$Q = \int_{\Sigma} d^3x j^0$$

$$\vec{n}_L \cdot \nabla \vec{j} = 0$$

$$\frac{dQ}{dt} = \int_S d^3x \frac{\partial j^0}{\partial t} - \int_{\Sigma} d^3x \nabla \vec{j}$$

$$= - \int_{\Sigma} d^3x \vec{n} \cdot \vec{j}$$



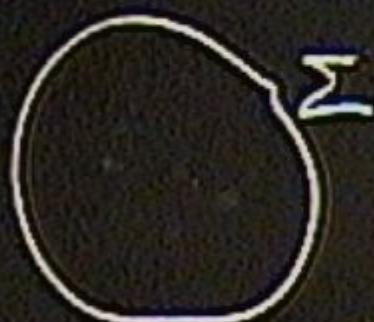
always s.

$$\phi_2$$

$$\delta\phi_1 = \phi_2$$

In field theory there are 2 important new features:

i) Conservation is local: $\delta f = 0 \Rightarrow j^\mu$ satisfying $\partial_\mu j^\mu = 0$.



$$\text{Ch. m.: } U = \rho i \omega \varphi \quad \delta \varphi = i \omega \delta \varphi \quad \varphi \rightarrow \varphi + i \omega \delta \varphi$$

$$Q = \int_{\Sigma} d^3x j^0$$

$$\vec{j}_L \cdot \nabla \vec{j} = 0$$

$$\frac{dQ}{dt} = \int_{\Sigma} d^3x \frac{\partial j^0}{\partial t} = - \int_{\Sigma} d^3x \nabla \vec{j}$$

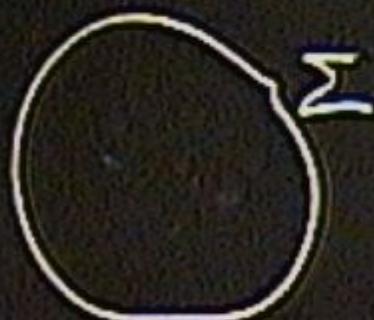
$$t = \frac{\varphi_1 + \varphi_2}{2\pi}$$

$$= - \int_{\Sigma} d^3x \vec{n} \cdot \vec{j}$$



In field theory there are 2 important new features:

i) Conservation is local: $\delta f = 0 \Rightarrow j^\mu$ satisfying $\partial_\mu j^\mu = 0$.



$$\text{Ch. n.: } U = \rho i \omega \varphi \quad U^* \neq U - \rho^* i \omega \varphi$$

$$\delta \varphi_1 = -\dot{\varphi}_2 \quad \delta \varphi_2 = \dot{\varphi}_1$$

$$\delta \varphi = i \omega \varphi$$

$$d \rightarrow \rho i \omega \varphi$$

$$Q = \int_{\Sigma} d^3x j^0$$

$$\vec{n} \cdot \nabla \vec{j} = 0$$

$$\frac{dQ}{dt} = \int_S d^2x \frac{\partial j^0}{\partial t} - \int_{\Sigma} d^3x \nabla \vec{j}$$

$$= - \int_{\partial\Sigma} d^2x \vec{n} \cdot \vec{j}$$

always.

$$\varphi_1 \quad \varphi_2 \quad \delta \varphi_1 = \varphi_c$$

$$\text{Charg.} \quad U = \rho \vec{n} \cdot \vec{q} \quad U^\dagger \neq U - \text{p. i.d.} \quad \frac{\partial \phi}{\partial k} = \frac{q_1 + i q_2}{\sqrt{2}} \quad \frac{\partial^2 \phi}{\partial k^2} = - \int_{\Sigma} d^2x \vec{n} \cdot \vec{j}$$

$$\delta \phi_1 = - q_2 \quad \delta \phi_2 = q_1$$

- 2) Particle states which are related by symmetries
~~symmetries~~ have the same energies, but not
 always.

$$\phi_1 \quad \phi_2 \quad \delta \phi_1 = \phi_2$$

$$\text{Ch.} \quad U = \rho^{ik} q_i \quad U^\dagger U = \rho^{ii} \delta_{ij} \quad \frac{\partial \phi}{\partial k} = \int_S d^3x \frac{\partial \phi}{\partial t} = \int_S \epsilon^{ijk} \partial_j \phi = - \int_S d^3x \vec{n} \cdot \vec{f}$$

$$\delta q_1 = -q_2 \quad \delta q_2 = q_1$$

- e) Particle states which are related by symmetries
 often have the same energies, but not
 always.

$$q_{j2} \quad \delta \phi_1 = \phi_2 \\ \delta \alpha_{j1} = \alpha_{j2}$$

$$\phi_i = \int d^3p \left[\psi(p_i) e^{ip_x x_i} + c.c. \right]$$

$$\text{Charg. } U = \rho^{\text{ind}} \quad U^\dagger \neq U - \rho^{\text{ind}} \quad \phi = \frac{\psi_1 + i\psi_2}{\sqrt{2}} = - \int_{\Sigma} d^3x \vec{n} \cdot \vec{f}$$

$$\delta\phi_1 = -\phi_2 \quad \delta\phi_2 = \phi_1$$

e) Particle states which are related by symmetries
sometimes have the same energies, but not
 always.

$$\phi_1 \quad \phi_2 \quad \delta\phi_1 = \phi_2$$

$$\delta a_{p1} = a_{p2}$$

$$U^\dagger a_{p1} U = a_{p2}$$

$$\phi_i = \int d^3p \left[\psi(p_i) e^{ip_i a_{pi}} + c.c. \right]$$

$$\text{Chm: } U = \rho^{\text{ind}} \quad U^\dagger \neq U - \rho^{\text{ind}} \quad \frac{\partial \phi}{\partial t} = \int_S \vec{J} \cdot \frac{\partial \vec{x}}{\partial t} = - \int_S \vec{J} \cdot \vec{n} \cdot \hat{f}$$

$$\delta \phi_1 = - \vec{k}_2 \quad S \phi_1 = \phi_1$$

2) Particle states which are related by symmetries
 often have the same energies, but not
 always.

$$\phi_1 \quad \phi_2 \quad \delta \phi_1 = \phi_2$$

$$\delta a_{p1} = a_{p2}$$

$$U^\dagger a_{p1} U = a_{p2}$$

$$\phi_i = \int d^3 p \left[\psi(p, r) e^{ip \cdot r} a_{pi} + c.c. \right]$$

Angular mom.

Angular mom
Energy

Angular mom

$$|\vec{p}, \sigma, 1\rangle = a_{p\sigma 1}^* |0\rangle$$

$$|\vec{p}, \sigma, 2\rangle = a_{p\sigma 2}^* |0\rangle$$



Quantum
Mech.

Hydrogen
Atom

Hydrogen

$$|\vec{p}, \sigma, 1\rangle = a_{p\sigma 1}^* |0\rangle$$

$$|\vec{p}, \sigma, 2\rangle = a_{p\sigma 2}^* |0\rangle$$

2) If $U = e^{i\omega Q}$ for some Q + real ω .

then $UU^\dagger = I \Leftrightarrow Q = Q^\dagger$

$UH = HU$ ~~for all~~ ω implies $QH = HQ$.

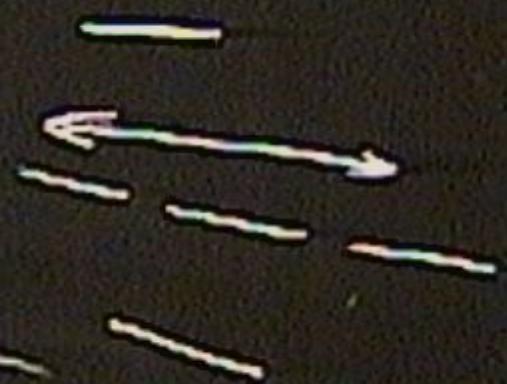
$\Rightarrow Q|\psi\rangle = \varphi|\psi\rangle$ $H|\psi\rangle = E|\psi\rangle$

then $Q|\psi(t)\rangle = \varphi|\psi(t)\rangle$

or c.v. $|\psi\rangle = U|\psi\rangle$...

$$|\psi\rangle = U|\psi\rangle$$

$$|\psi\rangle = HU|\psi\rangle = UHU|\psi\rangle = E|\psi\rangle$$



$$|\vec{p}, \sigma, 1\rangle = a_{p\sigma 1}^* |0\rangle$$

$$|\vec{p}, \sigma, 2\rangle = a_{p\sigma 2}^* |0\rangle$$

$$\begin{aligned} U|\vec{p}, \sigma, 1\rangle &= Ua_{p\sigma 1}^* |0\rangle \\ &= Ua_{p\sigma 1}^* U^* U|0\rangle \\ &= (Ua_{p\sigma 1}^*, U) |0\rangle \end{aligned}$$

$$\text{Charg: } U = \rho^{1/2} q \quad U' \neq U - \rho^{1/2} q \quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \quad = - \int_{\partial\Sigma} d^3x \, n^i \phi$$

$$\delta q_1 = -\phi_2 \quad \delta q_2 = \phi_1$$

- 2) Particle states which are related by symmetries
 often have the same energies, but not
 always.

$$\phi_i = \sum_p \int d^3p [u(p, \tau) e^{ipx} a_{p, i} + c.c.]$$

$$\phi_1 \quad \phi_2 \quad \delta \phi_1 = \phi_2$$

$$+ \begin{pmatrix} q \\ \bar{q} \end{pmatrix}$$

$$\delta a_{p, 1} = a_{p, 2},$$

$$U^\dagger a_{p, 1} U = a_{p, 2}$$

$$\text{Charg.: } U = \rho^i \omega^j \quad U' \neq U - \rho^i \omega^j \quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \quad = - \int_{\partial\Sigma} dX \cdot n \cdot f$$

$$\delta\phi_1 = -\phi_2 \quad \delta\phi_2 = \phi_1$$

2) Particle states which are related by symmetries
sometimes have the same energies, but not
 always.

$$\phi_1 \quad \phi_2 \quad \delta\phi_1 = \phi_2$$

$$+ \begin{pmatrix} q \\ \bar{q} \end{pmatrix}$$

$$\delta a_{p1} = a_{p2},$$

$$U a_{p1} U^* = a_{p2}$$

$$\phi_i = \int d^3 p \left[\psi(p_i) e^{ipx} a_{pi} + c.c. \right]$$

$$\text{Ch.} \quad U = \rho^{\mu\nu}\partial_\mu\phi \quad U^\nu_U = \rho^{\mu\nu}\partial_\mu\phi \quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \quad = - \int \frac{d^3x}{2\pi} \rho^{\mu\nu} \partial_\mu\phi$$

$$\delta\phi_1 = -i\phi_2 \quad \delta\phi_2 = \phi_1$$

2) Particle states which are related by symmetries
sometimes have the same energies, but not
always.

$$\phi_1 \quad \phi_2 \quad \delta\phi_1 = \phi_2$$

$$+ \begin{pmatrix} q \\ \vdots \\ q \end{pmatrix}$$

$$\delta\phi_2 = \phi_1$$

$$U^* \phi_1 U = \phi_2$$

$$\phi_i = \sum_{p=1}^n \int d^3p \left[\psi(p_i) e^{ipx} a_{p,i} + c.c. \right]$$

$$|\vec{p}, \sigma, 1\rangle = a_{p\sigma 1}^* |0\rangle$$

$$|\vec{p}, \sigma, 2\rangle = a_{p\sigma 2}^* |0\rangle$$

$$\begin{aligned} U|\vec{p}, \sigma, 1\rangle &= Ua_{p\sigma 1}^* |0\rangle \\ &= Ua_{p\sigma 1}^* U^* U|0\rangle \\ &= (Ua_{p\sigma 1}^*, U) |0\rangle \end{aligned}$$

$$|\vec{p}, \sigma, 1\rangle = a_{p\sigma 1}^* |0\rangle$$

$$|\vec{p}, \sigma, 2\rangle = a_{p\sigma 2}^* |0\rangle$$

$$U |\vec{p}, \sigma, 1\rangle = U a_{p\sigma 1}^* |0\rangle$$

$$\text{if } U|0\rangle = |0\rangle$$

$$\begin{aligned} \text{then } U |\vec{p}\sigma 1\rangle &= |\vec{p}\sigma 2\rangle \\ &= U a_{p\sigma 1}^* U^* U |0\rangle \\ &= (U a_{p\sigma 1}^* U)^* U |0\rangle \\ &= a_{p\sigma 2}^* U |0\rangle \end{aligned}$$

$$|\vec{p}, \sigma, 1\rangle = a_{p\sigma 1}^* |0\rangle$$

$$|\vec{p}, \sigma, 2\rangle = a_{p\sigma 2}^* |0\rangle$$

$$U |\vec{p}, \sigma, 1\rangle = U a_{p\sigma 1}^* |0\rangle$$

$$\text{if } U|0\rangle \approx |0\rangle$$

$$= U a_{p\sigma 1}^* U^* |0\rangle$$

$$\text{then } U |\vec{p}\sigma 1\rangle = |\vec{p}\sigma 2\rangle = (U a_{p\sigma 1}^* U)^* |0\rangle$$

$$\text{if } s_1 = m_1, m_1 = m_2$$

$$= a_{p\sigma 2}^* |0\rangle$$

i) If $U = e^{i\omega Q}$ for some Q is real.

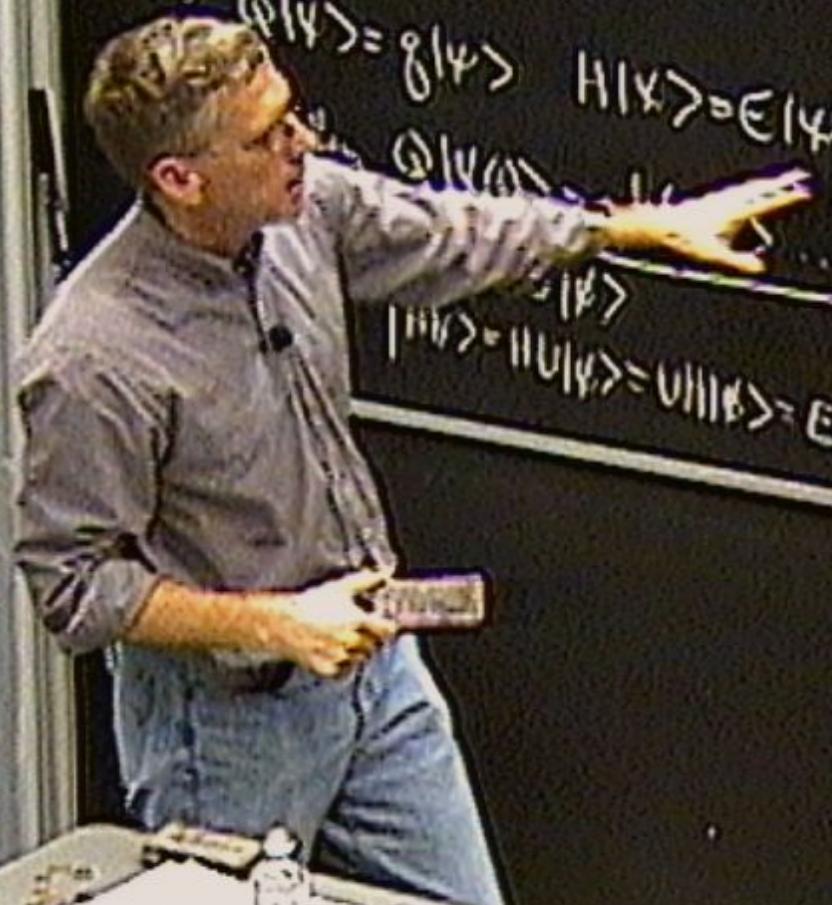
$$\text{then } UU^* = I \Leftrightarrow Q = Q^*$$

$$UH = HU \quad \text{for all } \omega \text{ implies } QH = HQ$$

$$\Rightarrow Q|\psi\rangle = \omega|\psi\rangle \quad H|\psi\rangle = E|\psi\rangle$$

$$Q|\psi\rangle = \omega|\psi\rangle$$

$$|U\psi\rangle = |U|\psi\rangle = U|\psi\rangle = E|\psi\rangle$$



$$|\vec{p}, \sigma, 1\rangle = a_{p\sigma 1}^* |0\rangle$$

$$|\vec{p}, \sigma, 2\rangle = a_{p\sigma 2}^* |0\rangle$$

$$U|\vec{p}, \sigma, 1\rangle = Ua_{p\sigma 1}^* |0\rangle$$

$$\frac{U|0\rangle = |0\rangle}{\text{then } U|\vec{p}\sigma 1\rangle = |\vec{p}\sigma 2\rangle} = Ua_{p\sigma 1}^* U^* U|0\rangle$$

= $(Ua_{p\sigma 1}^* U)^* U|0\rangle$

= $a_{p\sigma 2}^* U|0\rangle$

$$J_\mu = -\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

deriving
 $\partial_\mu = (\partial_t, \nabla)$

example spinless + spin 1 particle

$$A_\mu \quad \delta A_\mu = \partial_\mu \omega$$

$$\phi \quad \delta \phi = i \omega \phi$$

$$F_{\mu\nu} F^\mu - V(\phi^* \phi) - [(\partial_\mu - ieA_\mu)\phi]^* [(\partial_\mu - ieA_\mu)\phi]$$

$$V = a + b\phi^* \phi + c(\phi^* \phi)^2$$



$$S_\mu = -\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

dissipative
 $\delta^{\mu\nu} = i(\partial^\mu \phi^* - \phi \partial^\mu \phi)$

example spinless + spin 1 particle

$$A_\mu$$

$$\delta A_\mu = \partial_\mu \omega$$

$$\phi$$

$$\delta \phi = i\omega \phi$$

$$S = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi^* \phi) - [(\partial_\mu - ieA_\mu)\phi]^* [(\partial_\mu - ieA_\mu)\phi]$$

$$V = a + b\phi^* \phi + c(\phi^* \phi)^2$$



$$\alpha_\mu = -\partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi \quad \text{gauge field} \quad \partial^\mu \phi^\dagger (\partial_\mu \phi - \phi \partial_\mu \phi)$$

example spinless + spin 1 particle.

$$A_\mu$$

$$\delta A_\mu = \partial_\mu \omega$$

$$\phi$$

$$\delta \phi = i\omega \phi$$

covariant derivative.

$$-\frac{1}{4} F_{\mu\nu} F^\mu - V(\phi^* \phi) - [(\partial_\mu - ieA_\mu)\phi]^* [(\partial_\mu - ieA_\mu)\phi]$$

$$V = a + b\phi^* \phi + c(\phi^* \phi)^2$$

$c > 0$ so that V is bounded below

$$L = -\partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi + \frac{1}{2} \partial_\mu \phi^\dagger \partial^\mu \phi + \frac{1}{4!} (\phi^\dagger \phi)^2$$

example spinless + spin 1 particle

$$A_\mu \quad \delta A_\mu = \partial_\mu \omega$$

$$\phi \quad \delta \phi = i \omega \phi$$

covariant derivative.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi^\dagger \phi) - [(\partial_\mu - ie A_\mu) \phi]^\dagger [(\partial_\mu - ie A_\mu) \phi]$$

$$V = a + b \phi^\dagger \phi + c (\phi^\dagger \phi)^2$$

$c > 0$ so that V is bounded below