Title: Graduate Course on Standard Model & Quantum Field Theory - 4A

Date: Nov 08, 2006 11:00 AM

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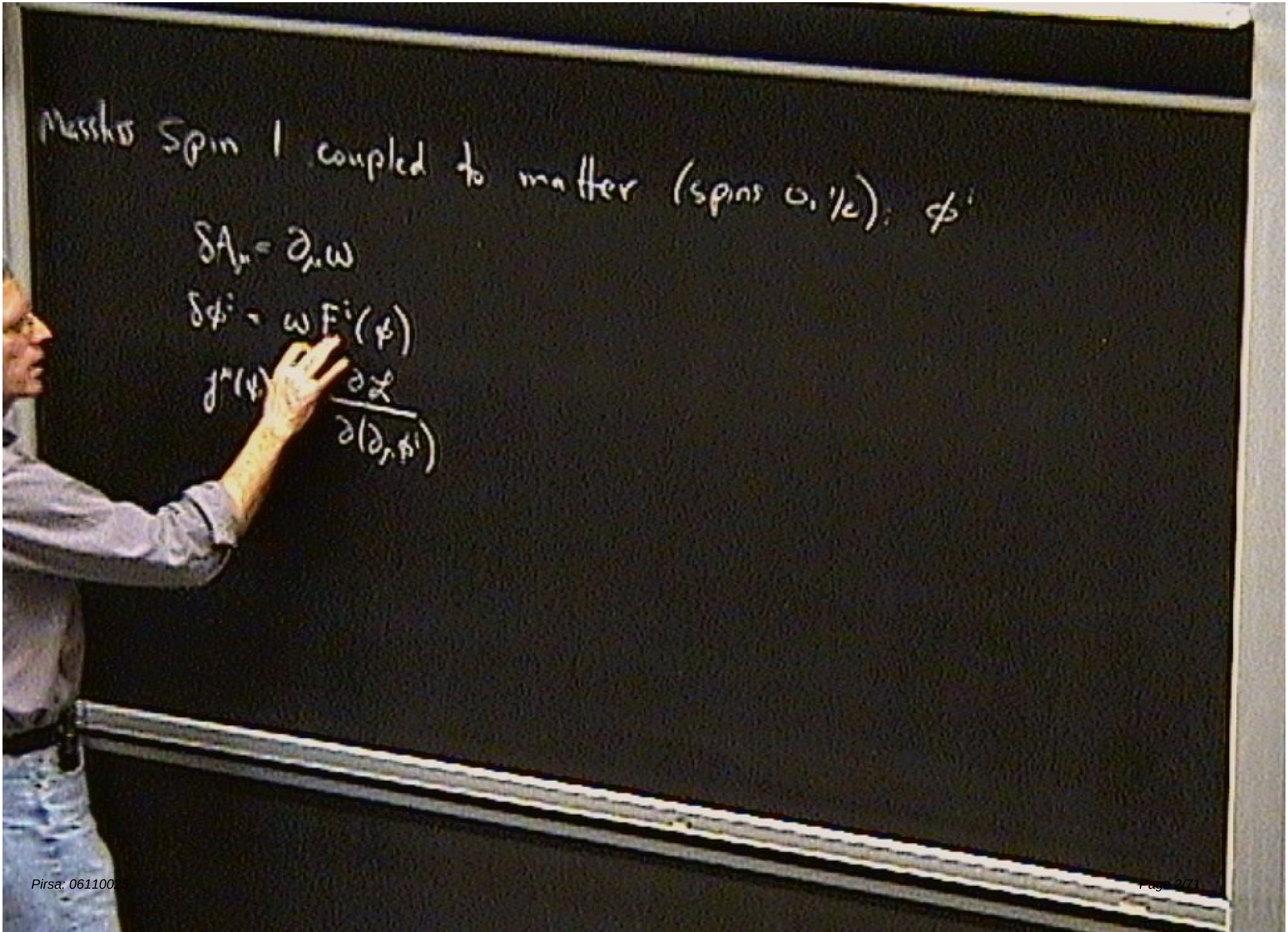
Abstract: Graduate Course on Standard Model & Quantum Field Theory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$M \to O^T M O$

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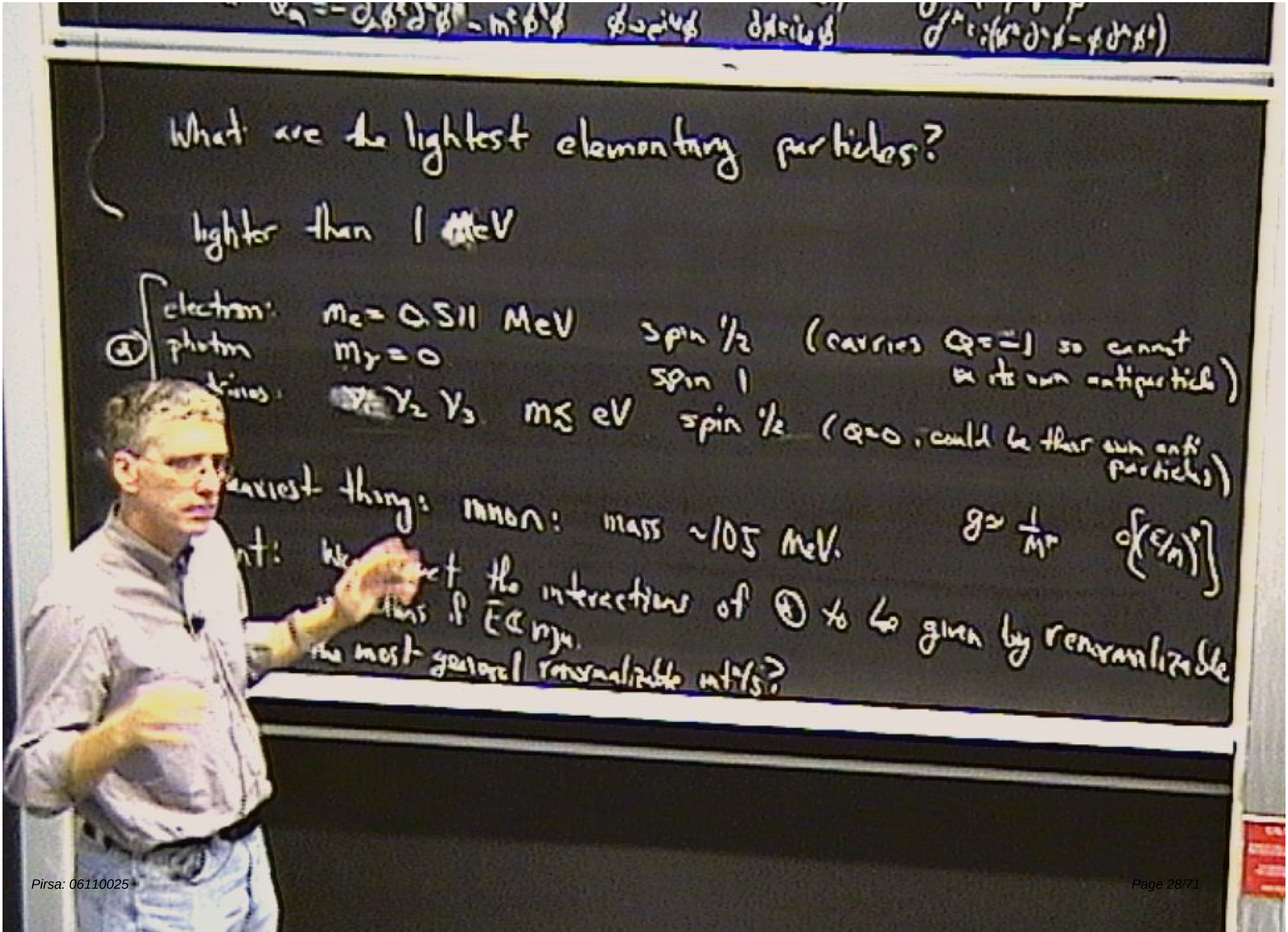
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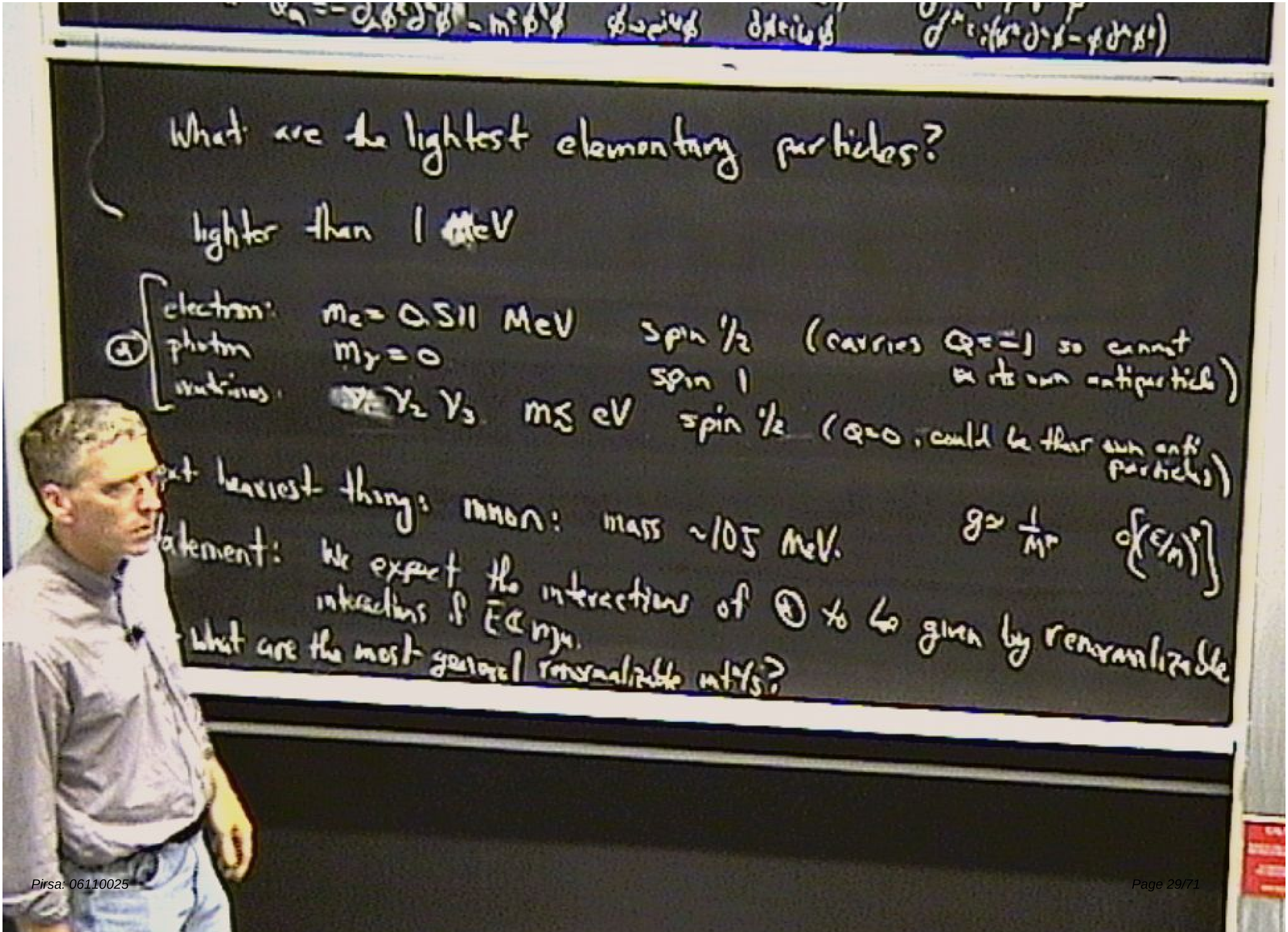
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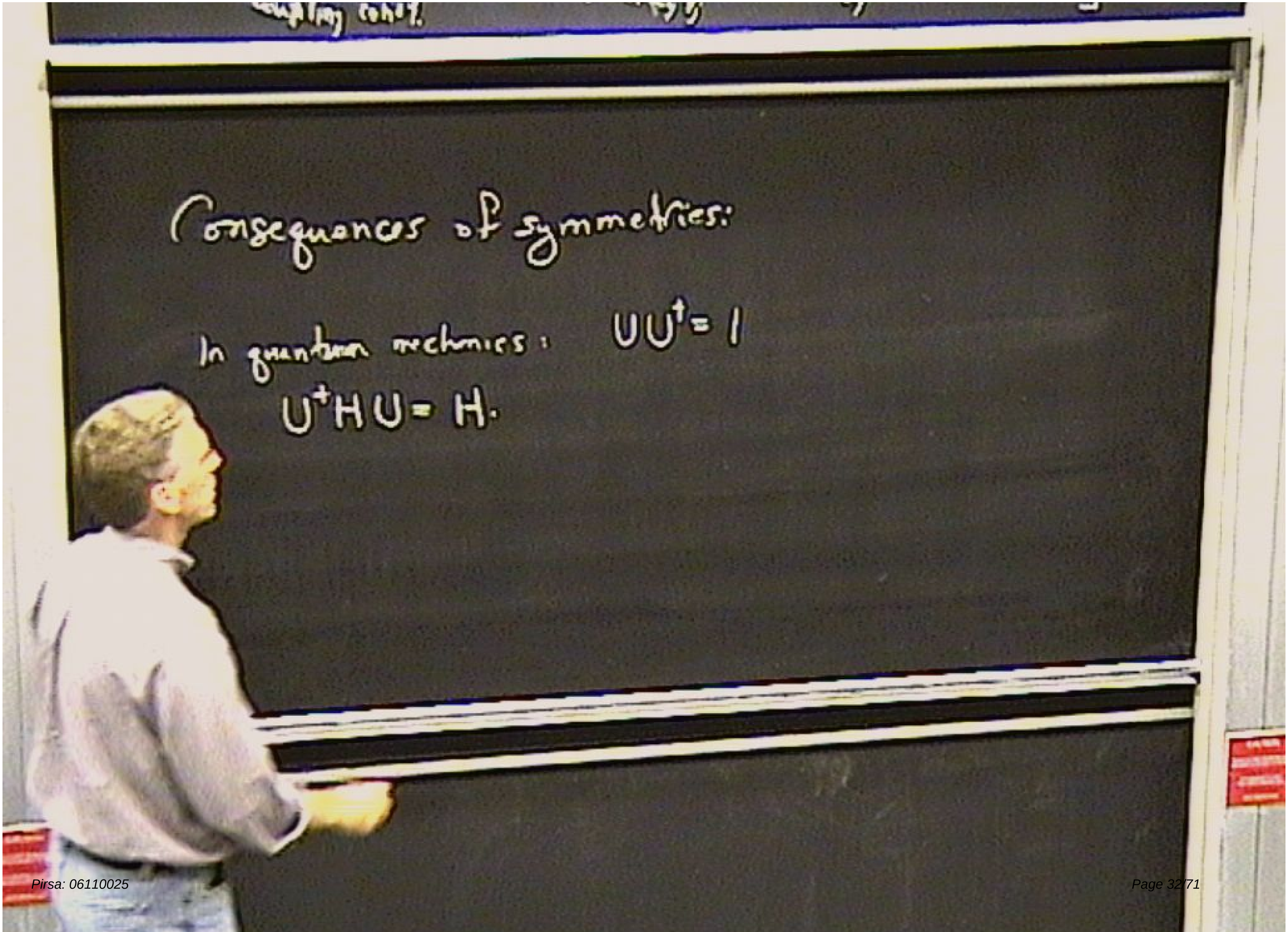
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(**) $\rightarrow U e^{-iHt} = e^{-iHt} U$ (*) preserved for all t.

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coupling const. $\quad \bar\psi_i = i \bar\psi_i \gamma_{ij} \psi_j \quad \ldots \quad \bar\psi \gamma \partial_\mu \psi$

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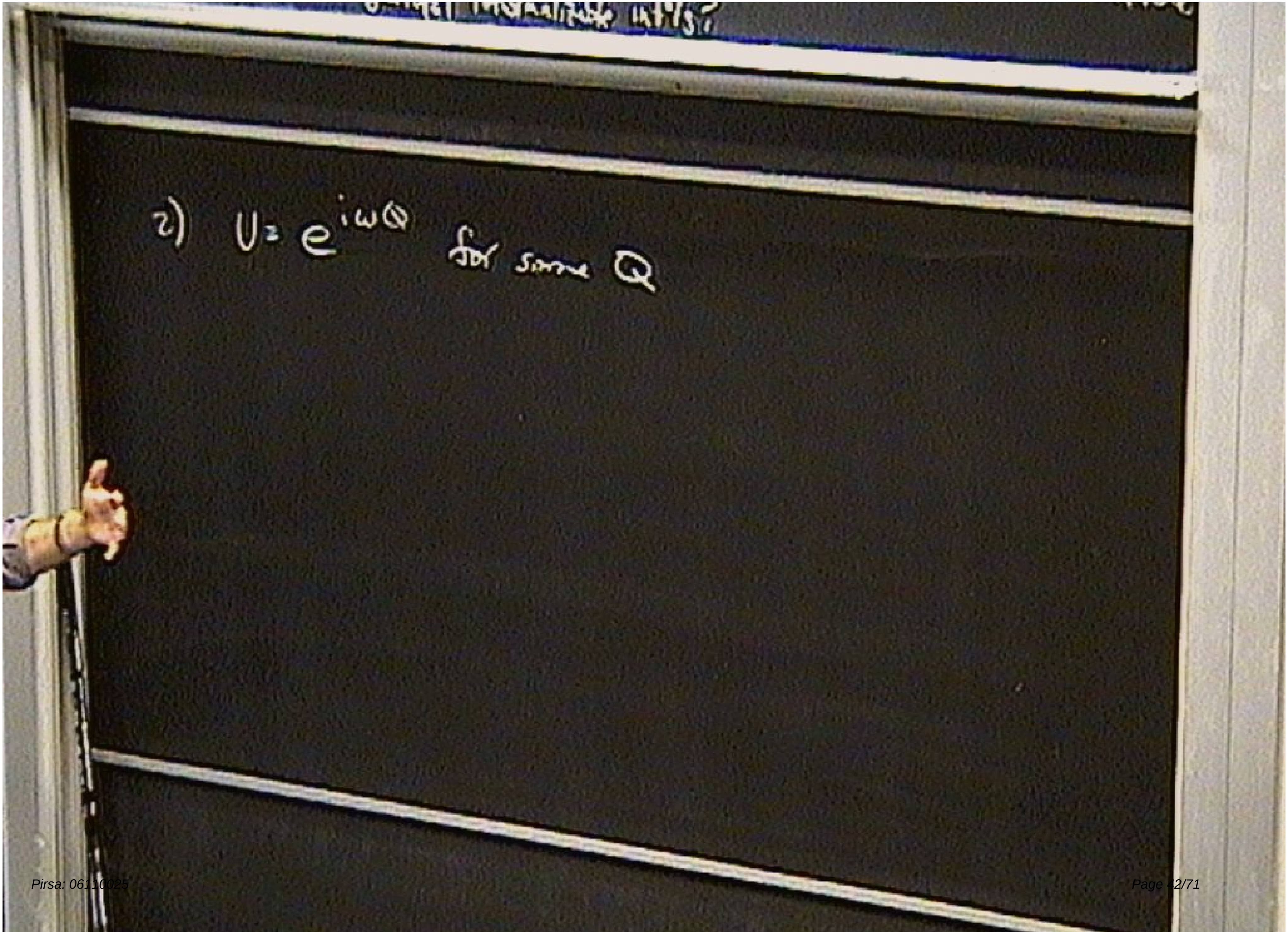
2) $U = e^{i\omega Q}$ for some Q

2) if $U = e^{i\omega Q}$ for some Q + real ω.

then $UU^\dagger = 1 \Leftrightarrow Q = Q^\dagger$

$UH = HU$ for all ω implies $QH = HQ$.

$\rightarrow$ $Q|\psi\rangle = q|\psi\rangle$ $\quad H|\psi\rangle = E|\psi\rangle$

then $Q|\psi(t)\rangle = q|\psi(t)\rangle \ldots$

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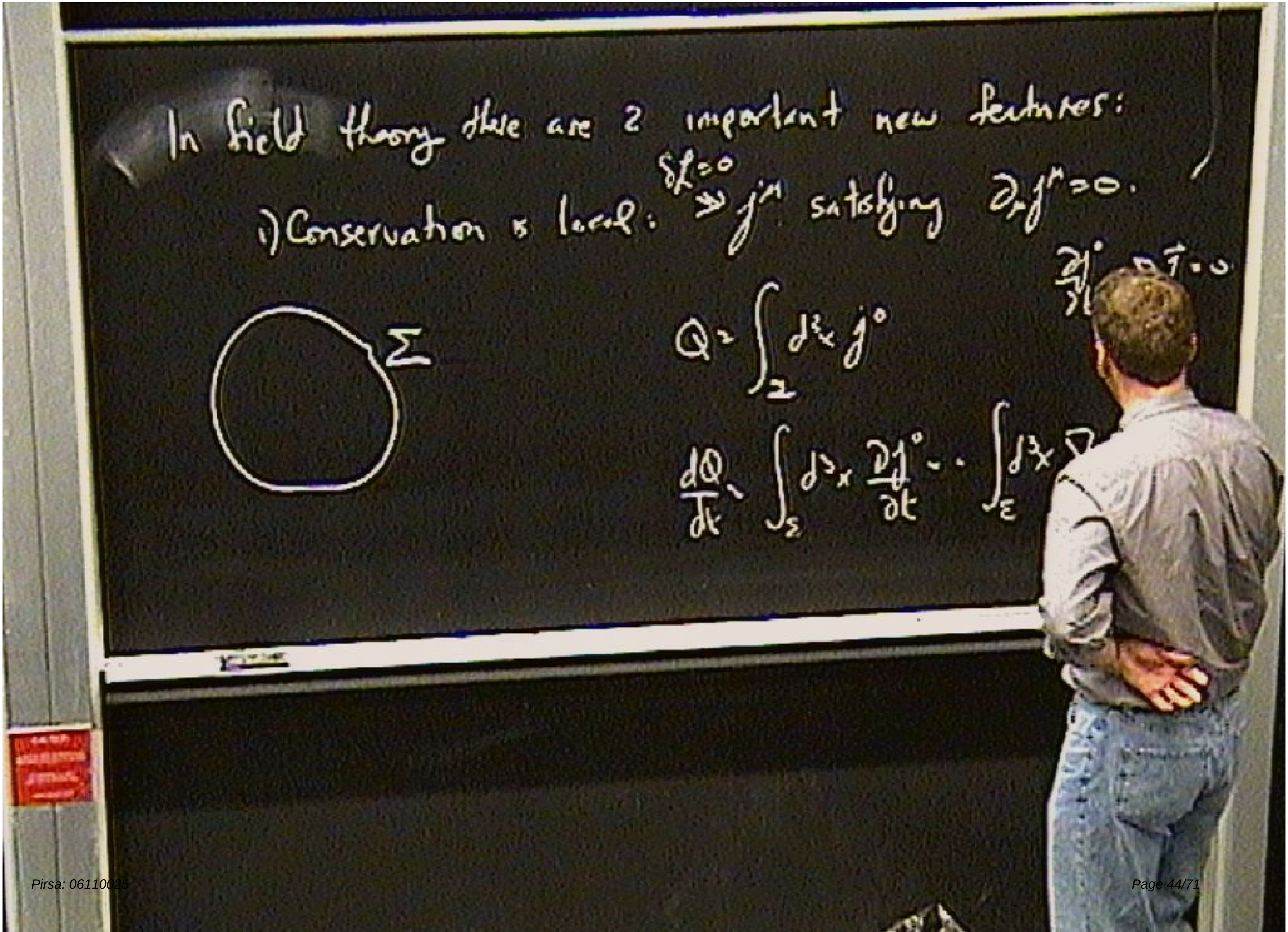
In field theory there are 2 important new features:
i) Conservation is local: δL=0 ⇒ j^μ satisfying ∂_μ j^μ = 0.
∂j^0/∂t + ∇·j = 0
Σ
Q = ∫_Σ d³x j^0
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$$Q = \int_\Sigma d^3x\, j^0$$

$$\frac{dQ}{dt} = \int_\Sigma d^3x\, \frac{\partial j^0}{\partial t} = -\int_\Sigma d^3x\, \nabla\cdot\vec{j}$$

$$= -\int_{\partial\Sigma} d^2x\, \vec{n}\cdot\vec{j}$$

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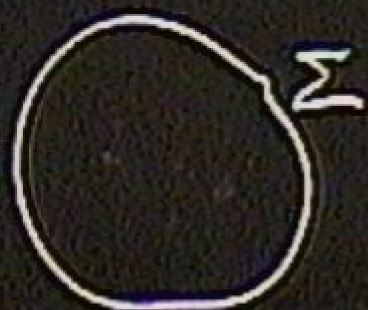

$$Q = \int_\Sigma d^3x\, j^0$$

$$\frac{dQ}{dt} = \int_\Sigma d^3x\, \frac{\partial j^0}{\partial t} = -\int_\Sigma d^3x\, \nabla\cdot\vec{j} = -\int_{\partial\Sigma} d^2x\, \vec{n}\cdot\vec{j}$$

In field theory there are 2 important new features:

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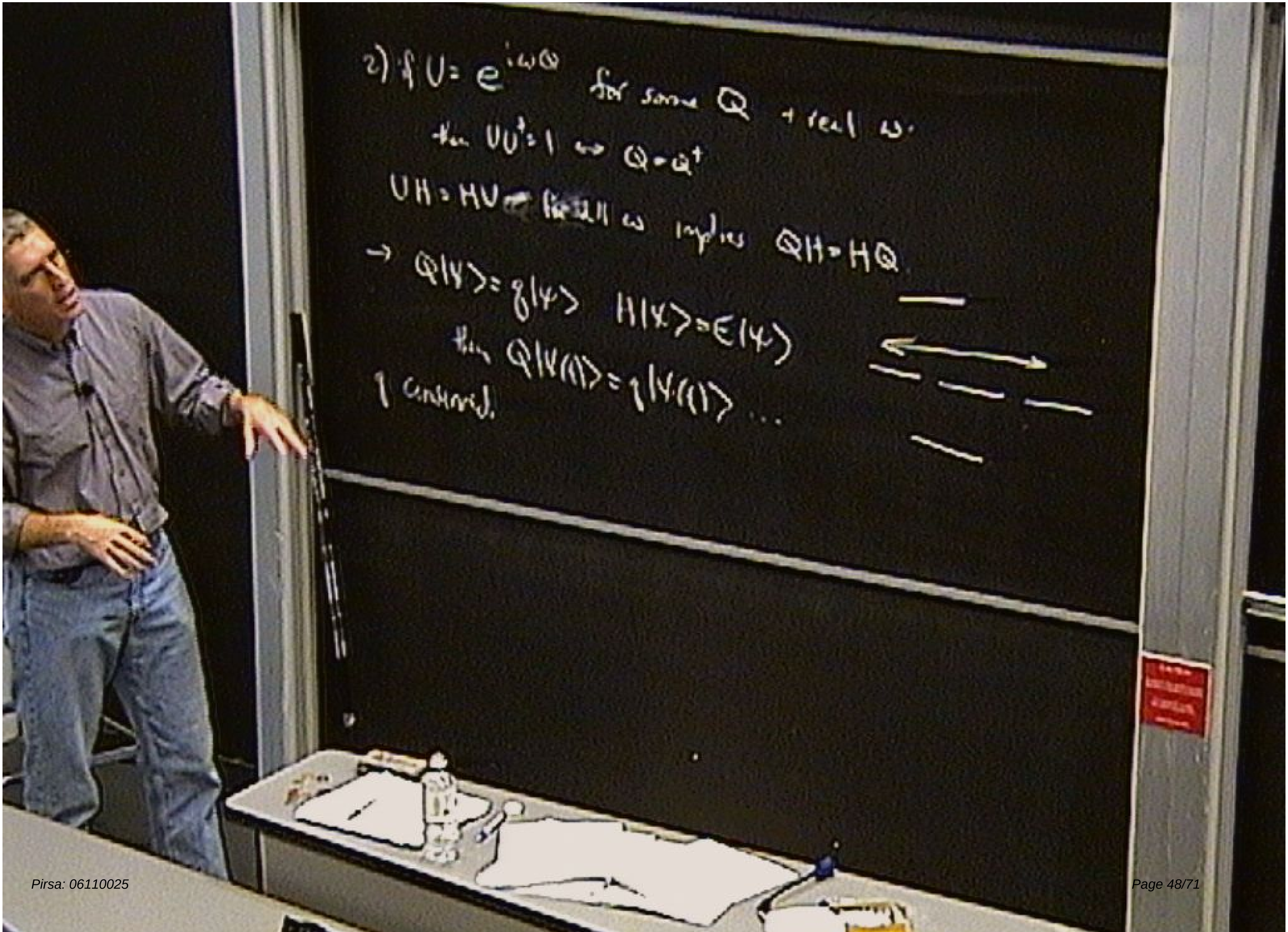
Claim: $U = e^{i\omega Q}$ $\quad U^\dagger \phi U = e^{i\omega}\phi$


2) if U = e^{iωQ} for some Q + real ω
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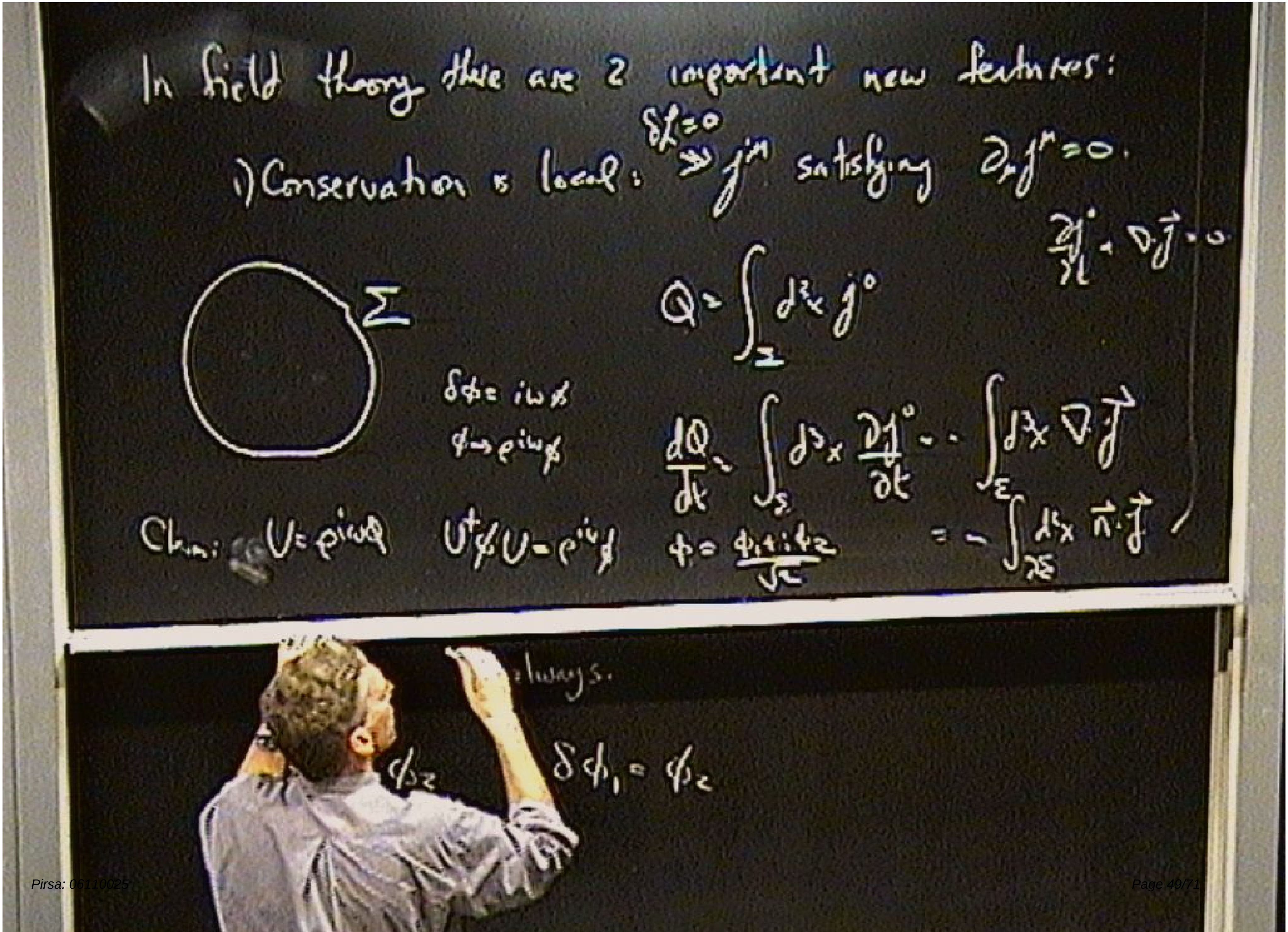
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Chm: $U = e^{i\omega Q}$ $U^\dagger\phi U = e^{i\omega}\phi$ $\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$
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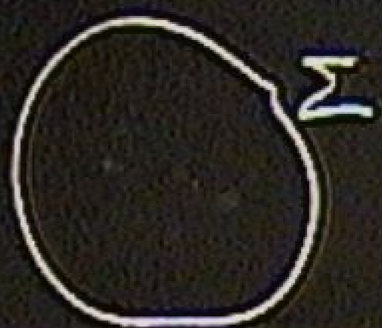

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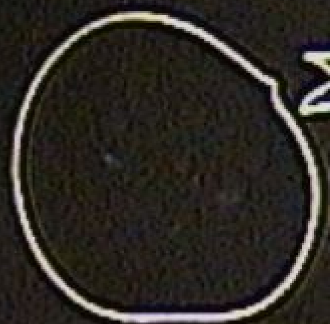

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Claim: $U = e^{i\omega Q}$ $\quad U^\dagger \phi U = e^{i\omega}\phi$ $\quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$

$\delta\phi_1 = -\phi_2 \quad \delta\phi_2 = \phi_1$

always.

$\phi_1 \quad \phi_2 \qquad \delta\phi_1 = \phi_2$

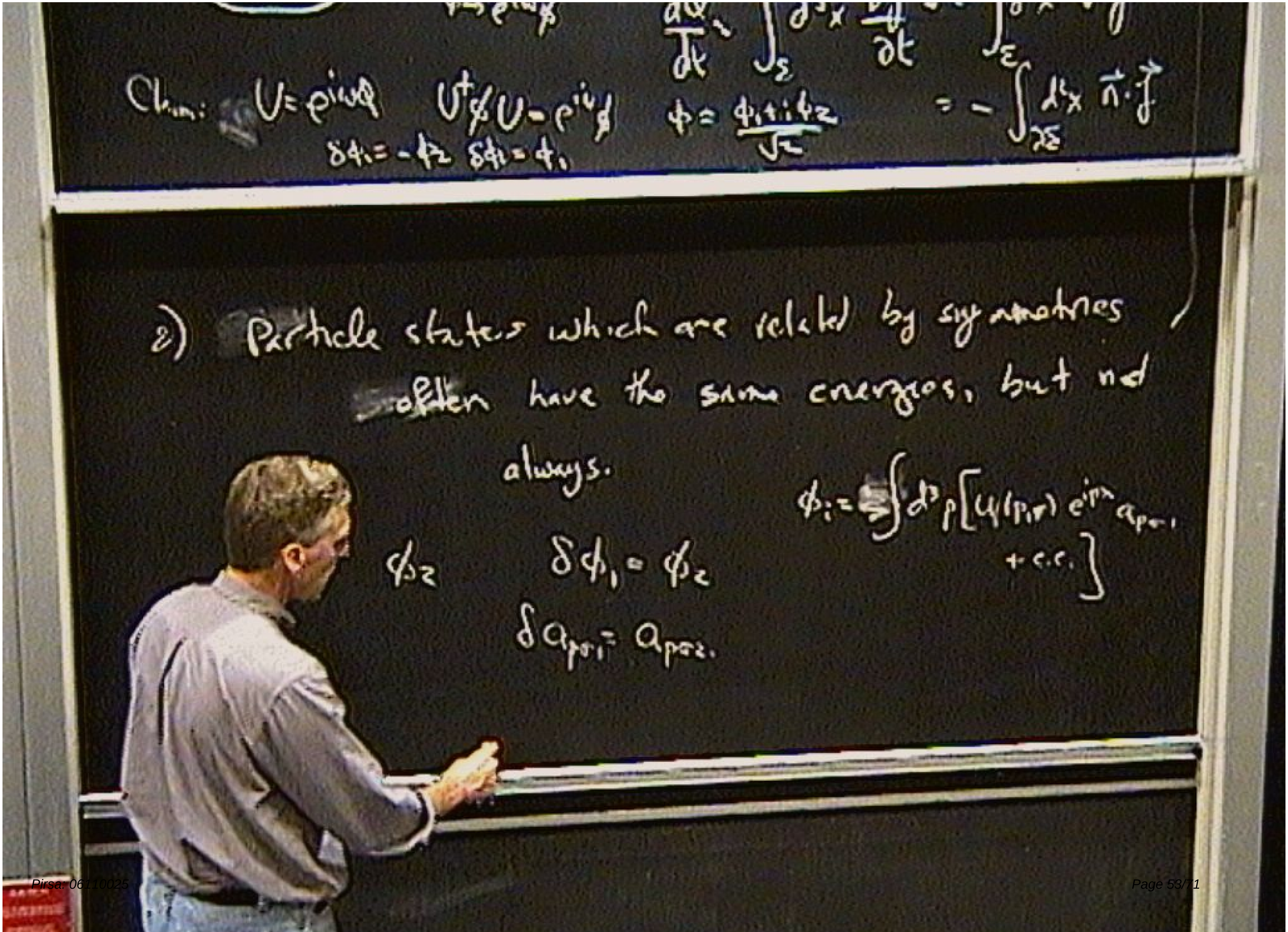
$$\frac{dQ}{dt} = \int_\Sigma d^3x \, \frac{\partial j^0}{\partial t} = \int_\Sigma \ldots$$

Charge: $U = e^{i\alpha Q}$ $\quad U^\dagger \phi U = e^{i\alpha}\phi$ $\quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$ $\quad = -\int_{\partial\Sigma} d^2x \, \vec{n}\cdot\vec{j}$

$\delta\phi_1 = -\phi_2 \quad \delta\phi_2 = \phi_1$

2) Particle states which are related by symmetries often have the same energies, but not always.

$\phi_1 \qquad \phi_2 \qquad \delta\phi_1 = \phi_2$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2) Particle states which are related by symmetries
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$\phi_i = \int d^3p \left[\psi(p,r)\, e^{ipx}\, a_{p,r,i} + \text{c.c.} \right]$
ϕ_2
$\delta\phi_1 = \phi_2$
$\delta a_{p,r,1} = a_{p,r,2}$

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2) Particle states which are related by symmetries often have the same energies, but not always.

$$\phi_i = \int d^3p \left[u_i(p,r)\, e^{ipx} a_{pri} + \text{c.c.} \right]$$

$\phi_1 \quad \phi_2 \qquad \delta\phi_1 = \phi_2$

$$\delta a_{pr1} = a_{pr2}.$$

$$U^\dagger a_{pr1} U = a_{pr2}$$

$\frac{dQ}{dt} = \int_\Sigma d^3x \frac{\partial j}{\partial t} = \int_\Sigma d^3x\, \partial_i j^i$

$= -\int_{\partial\Sigma} d^2x\, \vec{n}\cdot\vec{J}$

Claim: $U = e^{i\epsilon Q}$ $\quad U^\dagger \phi U = e^{i\epsilon}\phi$ $\quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$

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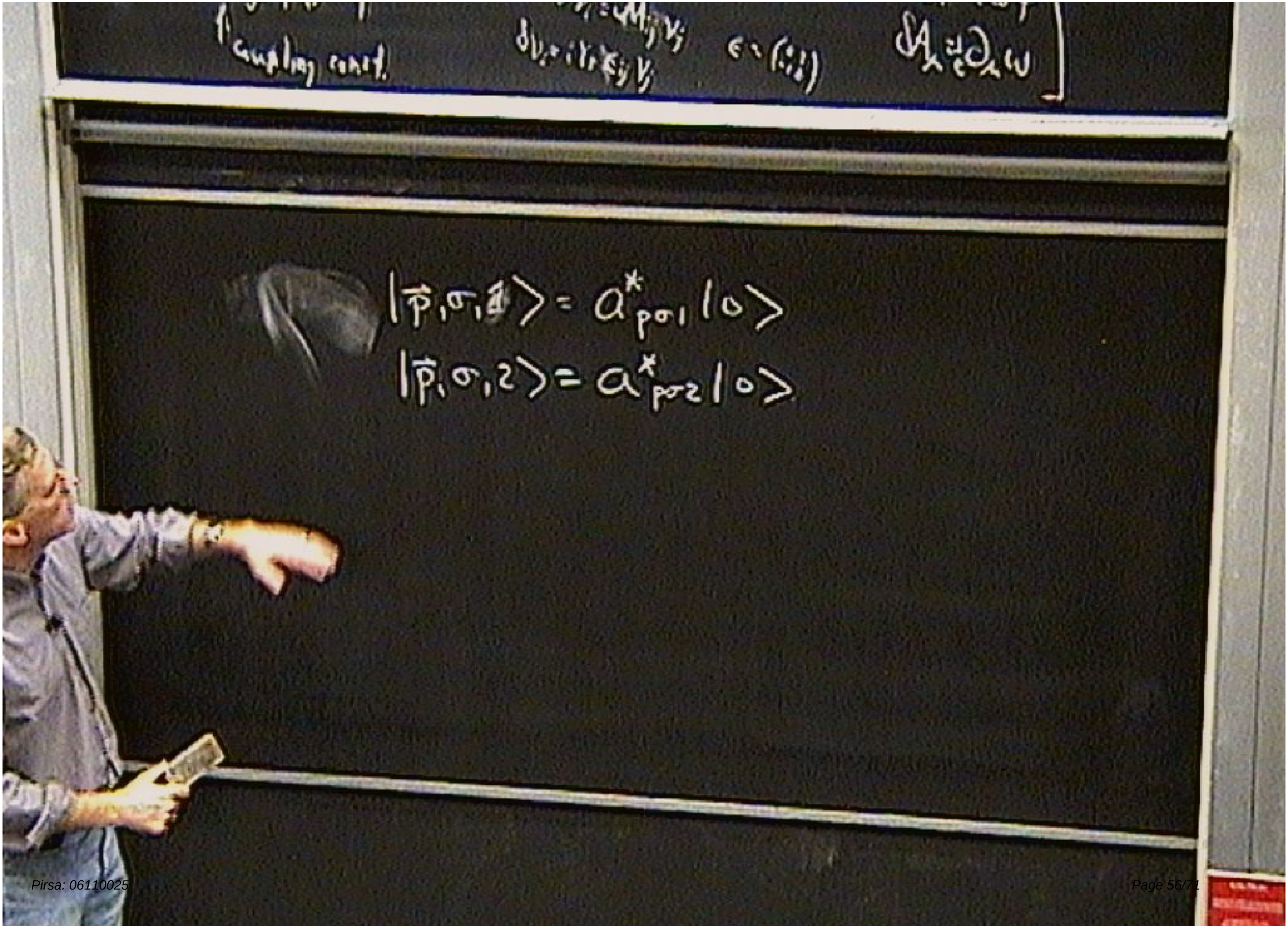
$\phi_1 \qquad \phi_2 \qquad \delta\phi_1 = \phi_2$

$\phi_i = \int d^3p \left[\psi(p,r)\, e^{ipx} a_{p\sigma i} + \text{c.c.} \right]$

$\delta a_{p\sigma 1} = a_{p\sigma 2}.$

$U^\dagger a_{p\sigma 1} U = a_{p\sigma 2}$


coupling const.
$|\vec{p},\sigma,1\rangle = a^{*}_{p\sigma 1}|0\rangle$
$|\vec{p},\sigma,2\rangle = a^{*}_{p\sigma 2}|0\rangle$

coupling const.

$\delta v_i = i \gamma_5 \epsilon_{ij} v_j$ $\quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\quad \delta A_\mu = \frac{1}{e}\partial_\mu \omega$

$$|\vec{p}, \sigma, 1\rangle = a^*_{p\sigma 1}|0\rangle$$

$$|\vec{p}, \sigma, 2\rangle = a^*_{p\sigma 2}|0\rangle$$

2) if $U = e^{i\omega Q}$ for some Q + real ω.

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$UH = HU$ for all ω implies $QH = HQ$.

$\Rightarrow$ $Q|\psi\rangle = q|\psi\rangle$ $\quad H|\psi\rangle = E|\psi\rangle$

then $Q|\psi(t)\rangle = q|\psi(t)\rangle$ ...

q conserved.

$|\psi\rangle = U|\phi\rangle$

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$$U|\vec{p}, \sigma, 1\rangle = U a^*_{p\sigma 1}|0\rangle$$

$$= U a^*_{p\sigma 1} U^* U|0\rangle$$

$$= (U^{*} a_{p\sigma 1} U)^*$$

Claim: $U = e^{i\omega Q}$ $\quad U^\dagger \phi U = e^{i\omega}\phi$ $\quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$ $\quad = -\int_{\partial\Sigma} d^2x\, n\cdot j$

$\delta\phi_1 = -\phi_2 \quad \delta\phi_2 = \phi_1$

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$\phi_1 \qquad \phi_2 \qquad \delta\phi_1 = \phi_2$

$$\begin{matrix} + \\ + \end{matrix} \begin{pmatrix} q \\ \vdots \\ q \end{pmatrix} \qquad \delta a_{p\sigma 1} = a_{p\sigma 2}.$$

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$U^{\dagger} a_{p\sigma 1} U^{*} = a_{p\sigma 2}$

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Claim: $U = e^{i\omega Q}$ $U^{*}\phi U = e^{i\omega}\phi$ $\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$ $= -\int_{\partial S} d^2x\, \hat{n}\cdot \vec{j}$

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$U^{*} a_{p\sigma 1} U = a_{p\sigma 2}$

$$\begin{pmatrix} q \\ \vdots \\ q \end{pmatrix}$$

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if $U|0\rangle = |0\rangle$

$$= U a^*_{p\sigma 1} U^* U|0\rangle$$

then $U|p\sigma 1\rangle = |p\sigma 2\rangle$

$$= (U a_{p\sigma 1} U^*)^* U|0\rangle$$

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and so $m_1 = m_2$

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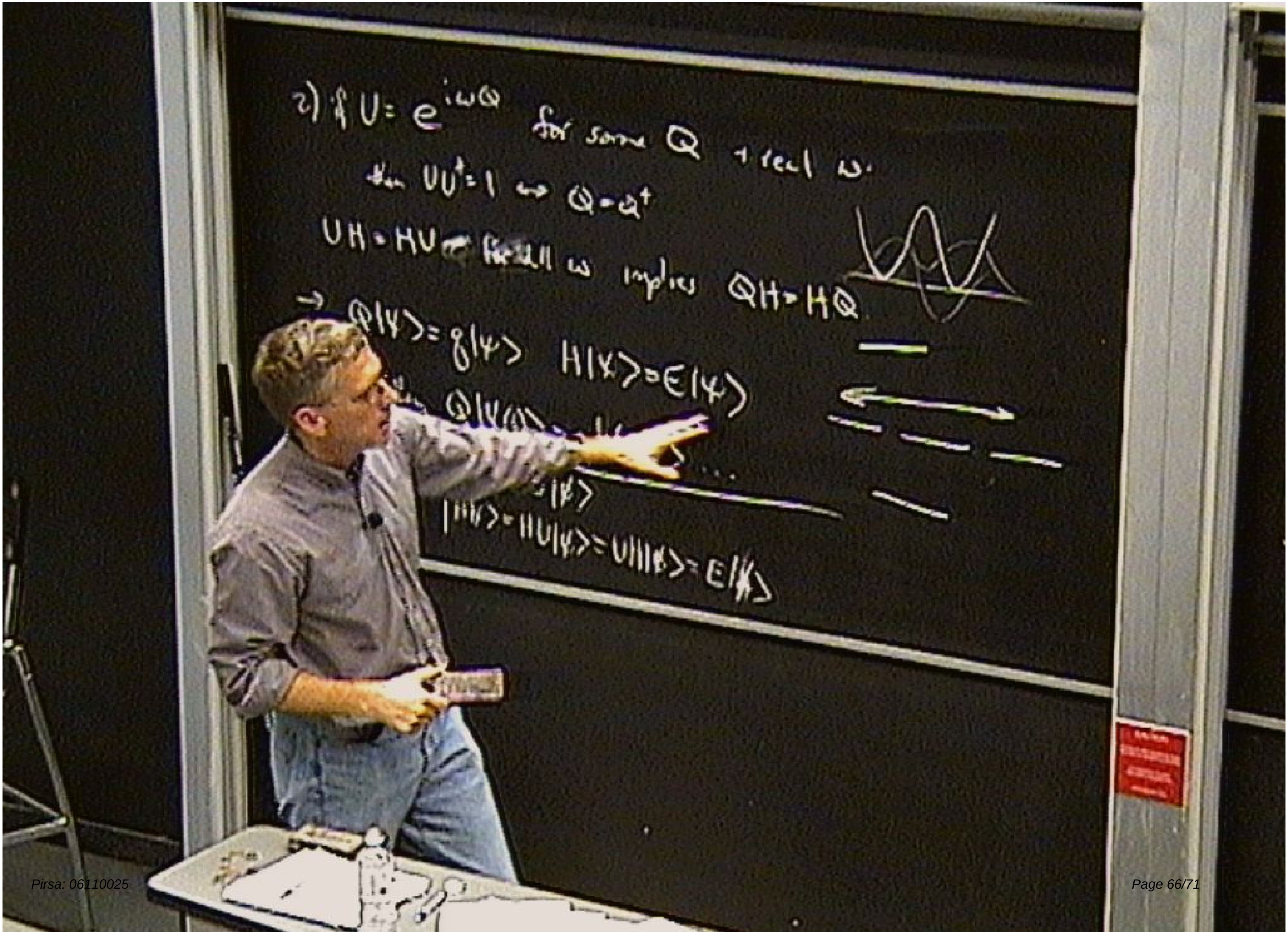
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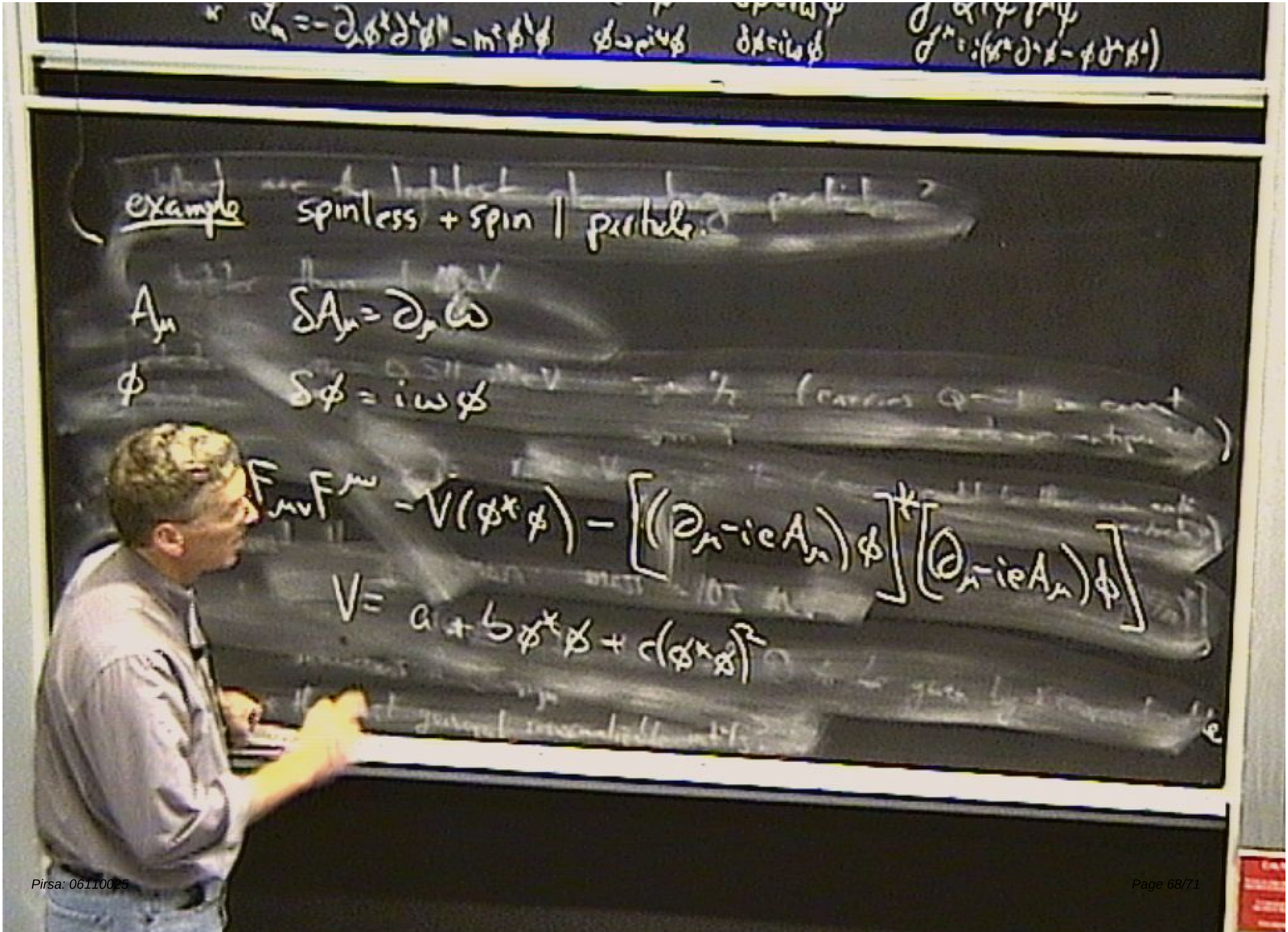
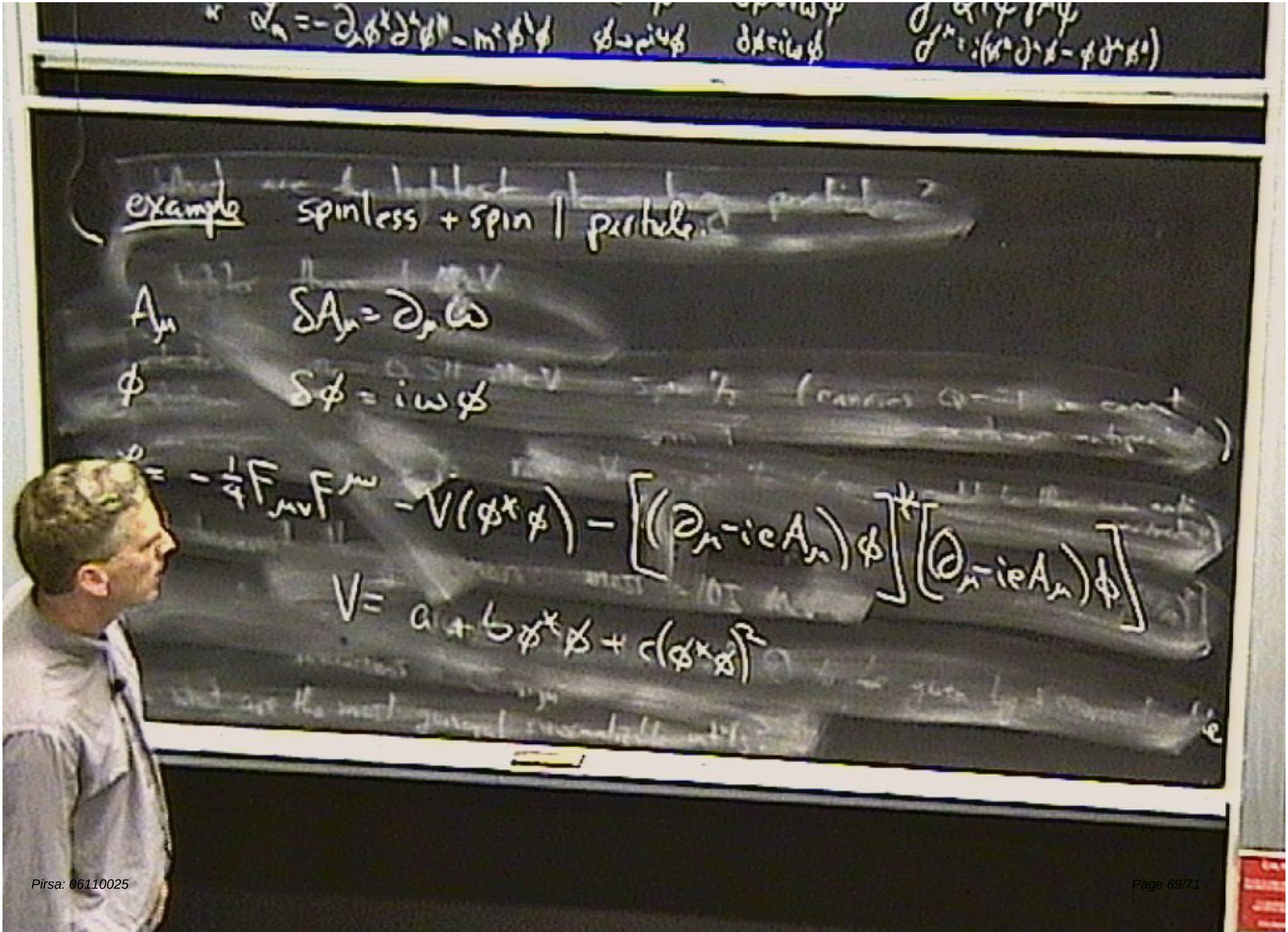
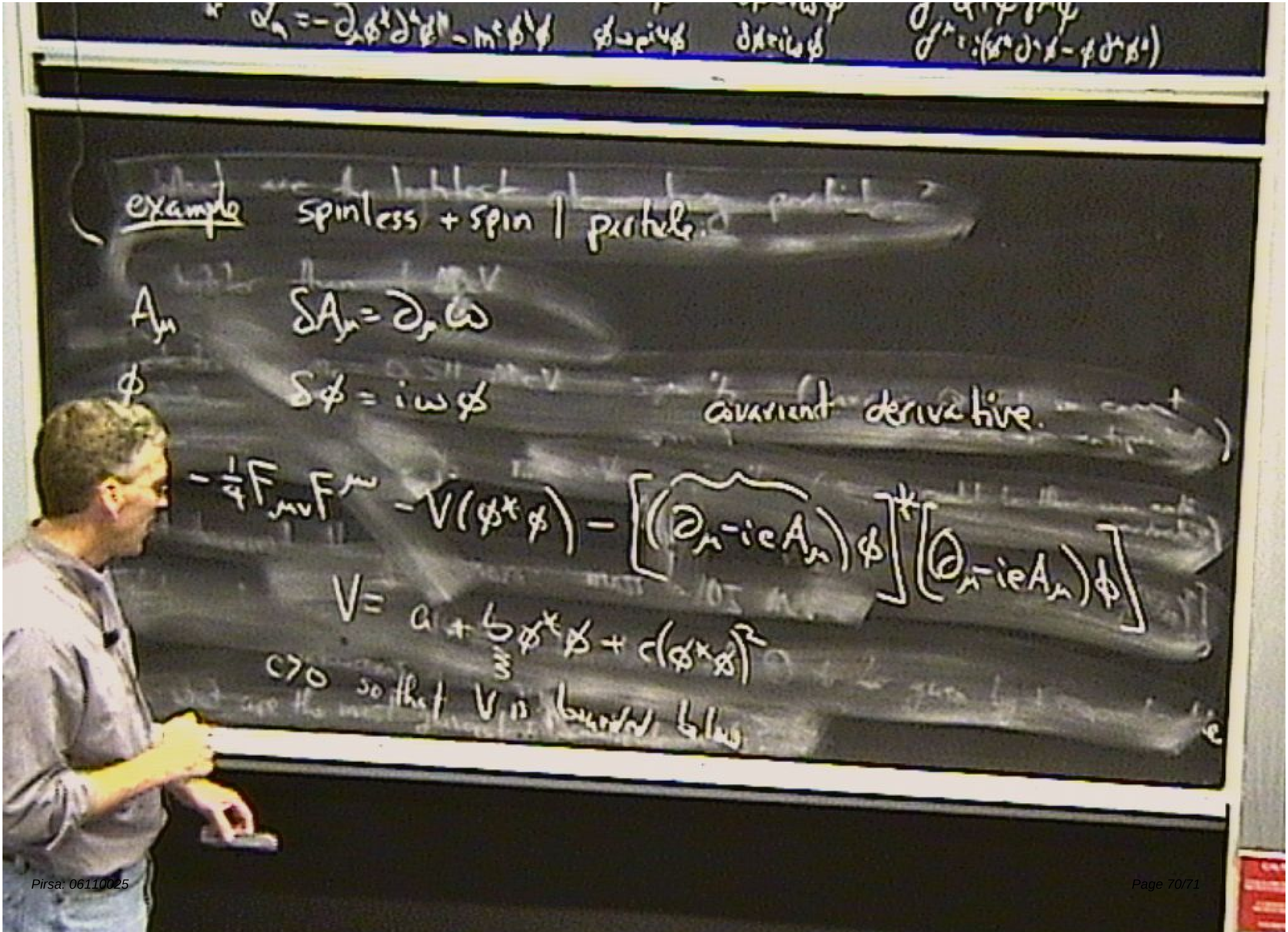

example spinless + spin 1 particles.
A_μ $\delta A_\mu = \partial_\mu \omega$
ϕ $\delta\phi = i\omega\phi$
$F_{\mu\nu}F^{\mu\nu} - V(\phi^*\phi) - [(\partial_\mu - ieA_\mu)\phi]^*[(\partial_\mu - ieA_\mu)\phi]$
$V = a + b\phi^*\phi + c(\phi^*\phi)^2$



$\mathcal{L}_m = -\partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi$
$\phi \to e^{i\omega}\phi$
$\delta\phi = i\omega\phi$
$J^\mu = i(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*)$
example spinless + spin 1 particles
A_μ $\delta A_\mu = \partial_\mu\omega$
ϕ $\delta\phi = i\omega\phi$
$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi^*\phi) - [(\partial_\mu - ieA_\mu)\phi]^*[(\partial_\mu - ieA_\mu)\phi]$
$V = a + b\phi^*\phi + c(\phi^*\phi)^2$


example spinless + spin 1 particle.
A_μ $\delta A_\mu = \partial_\mu \omega$
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covariant derivative.
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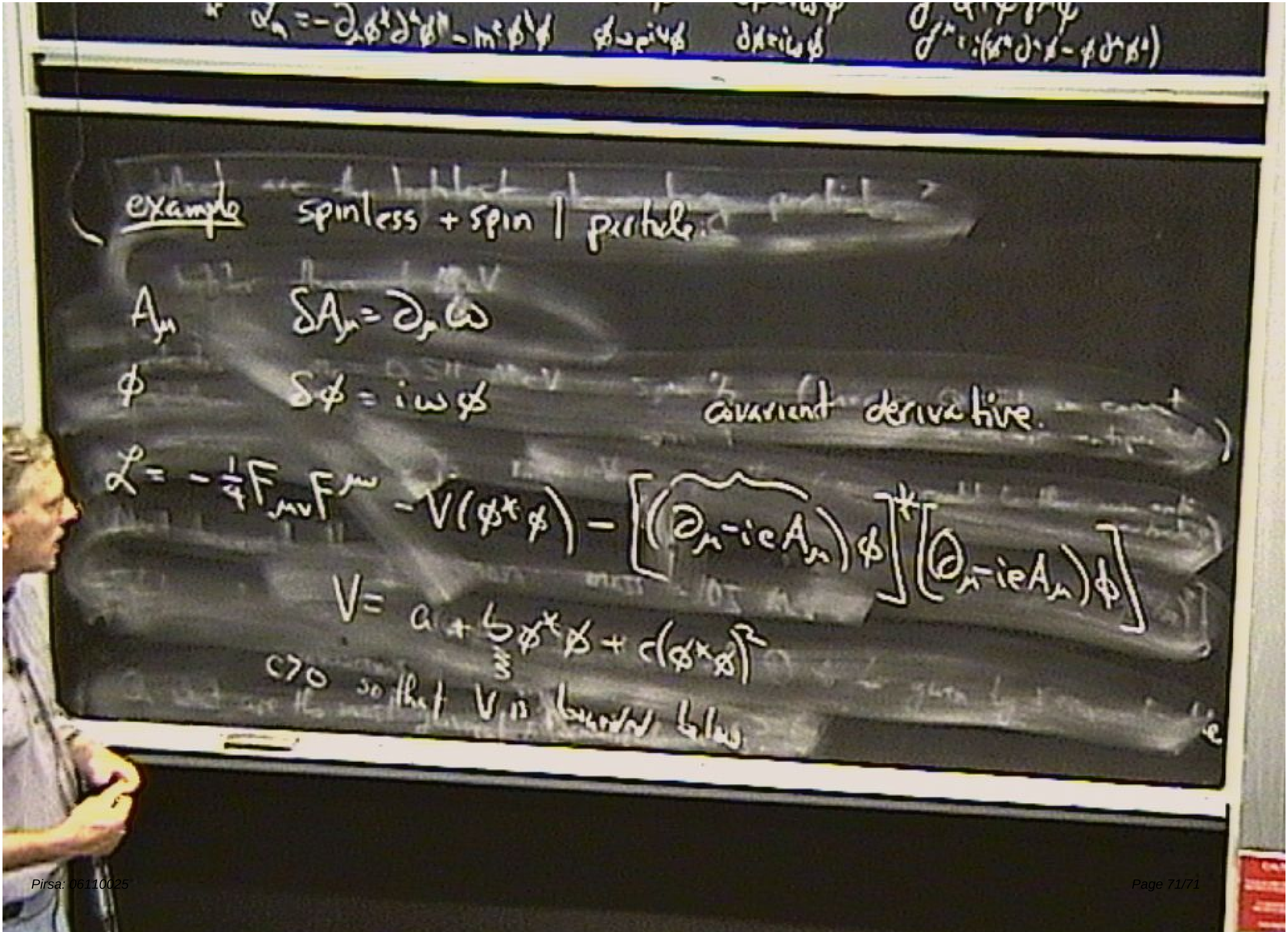
$c > 0$ so that V is bounded below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