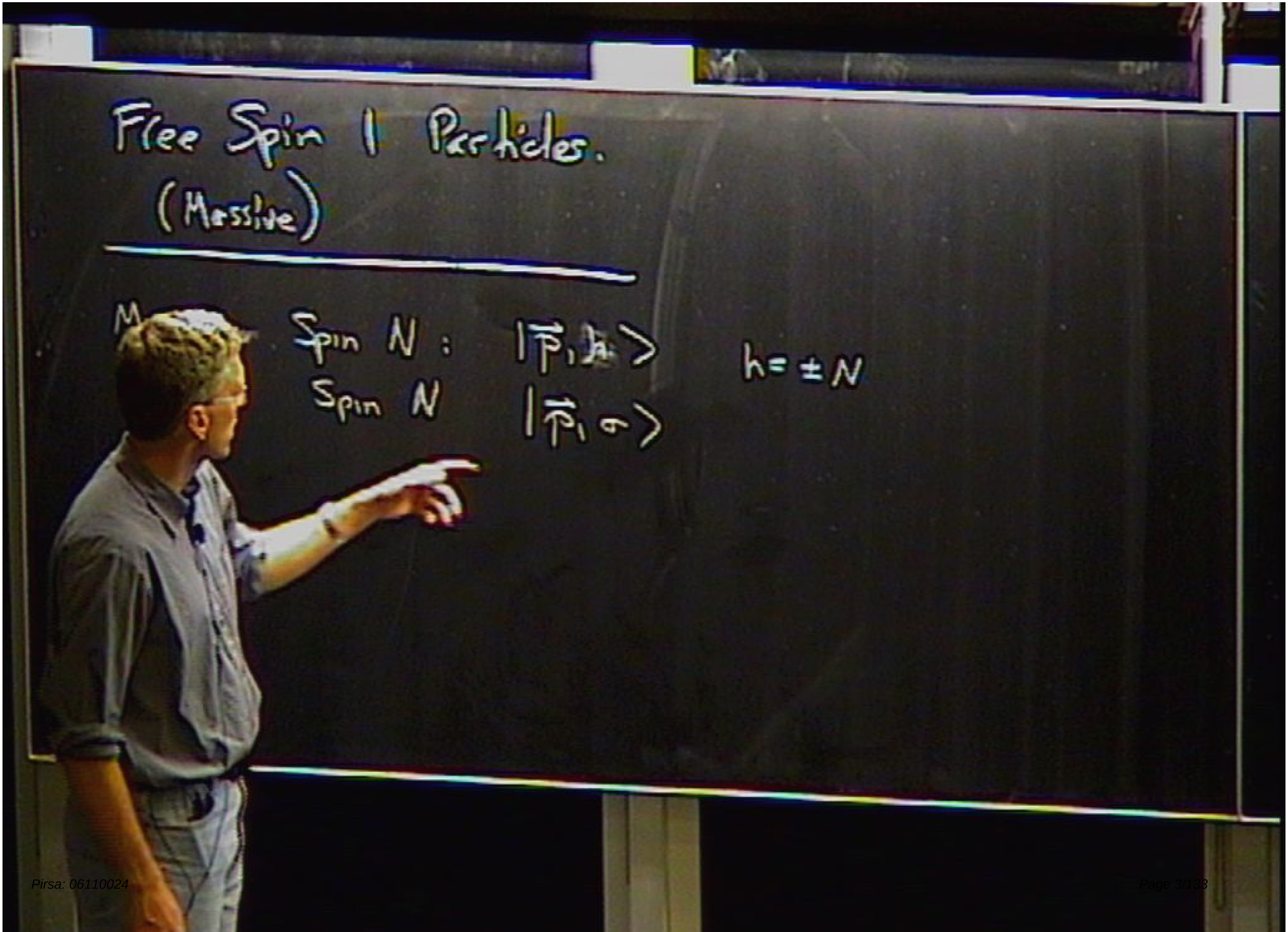
Title: Class 3 B

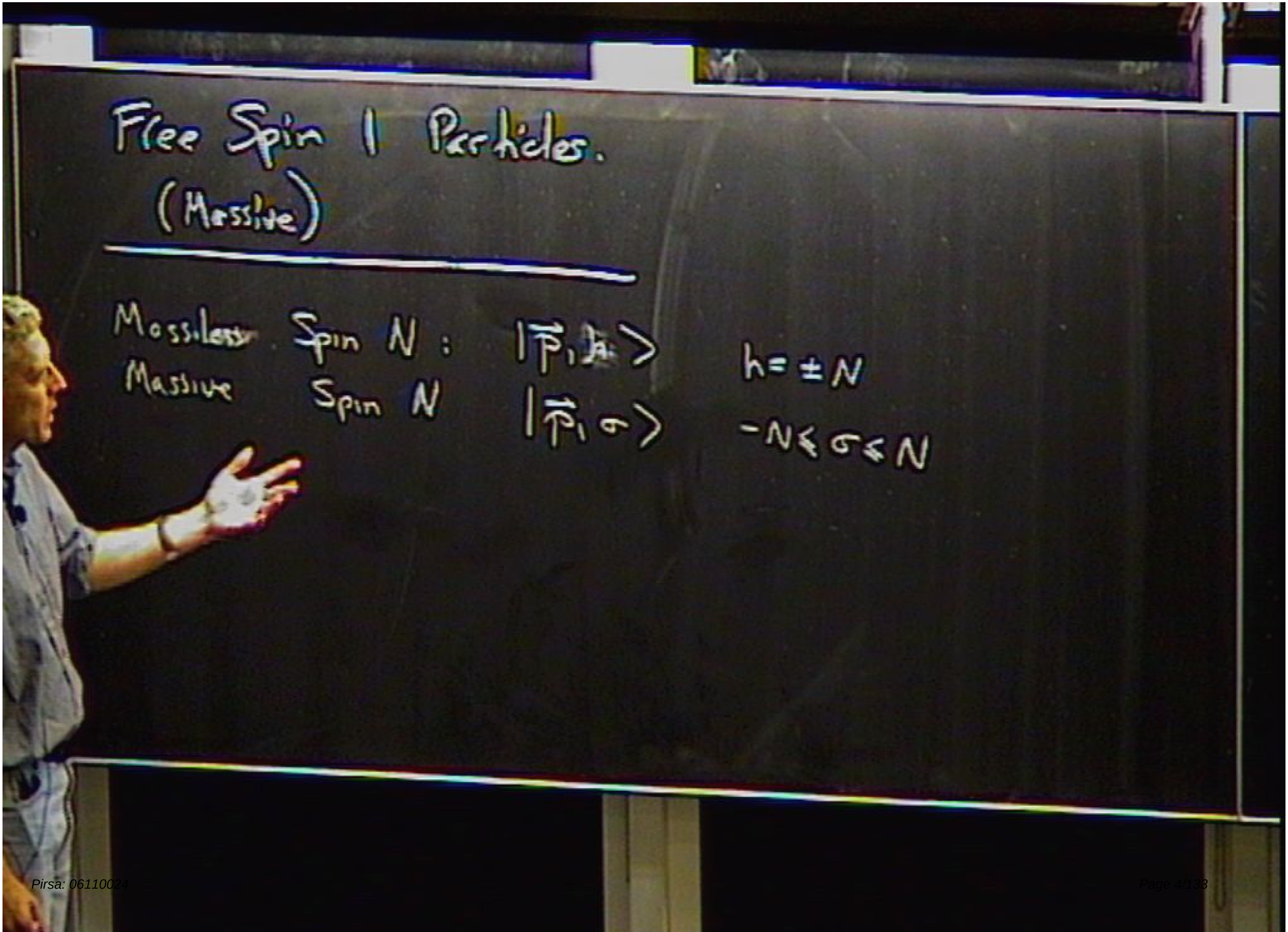
Date: Nov 01, 2006 03:30 AM

URL: http://pirsa.org/06110024

Abstract: Graduate Course on Standard Model & Quantum Field Theory

Free Spin 1 Particles.
(Massive)


Free Spin 1 Particles.
(Massive)
Spin N : $|\vec{p}, h\rangle$
$h = \pm N$
Spin N $|\vec{p}, \sigma\rangle$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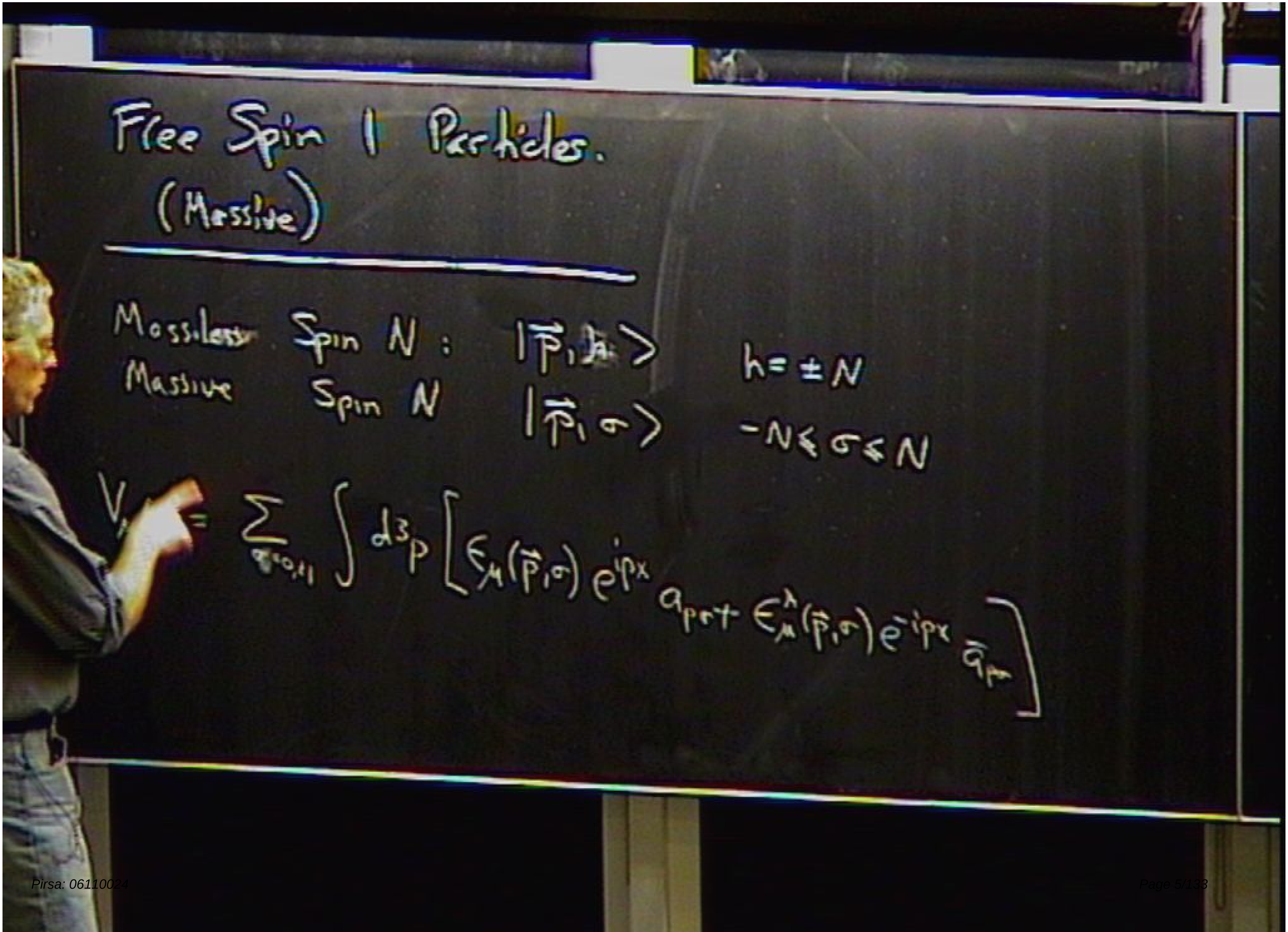
Massive Spin N $|\vec{p}, \sigma\rangle$ $-N \leq \sigma \leq N$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$V_\mu = \sum_{\sigma=0,\pm1} \int d^3p \left[\epsilon_\mu(\vec{p},\sigma) e^{ipx} a_{p\sigma} + \epsilon^*_\mu(\vec{p},\sigma) e^{-ipx} \bar{a}_{p\sigma} \right]$

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$$p^\mu \epsilon_\mu(\vec{p},\sigma) = 0$$

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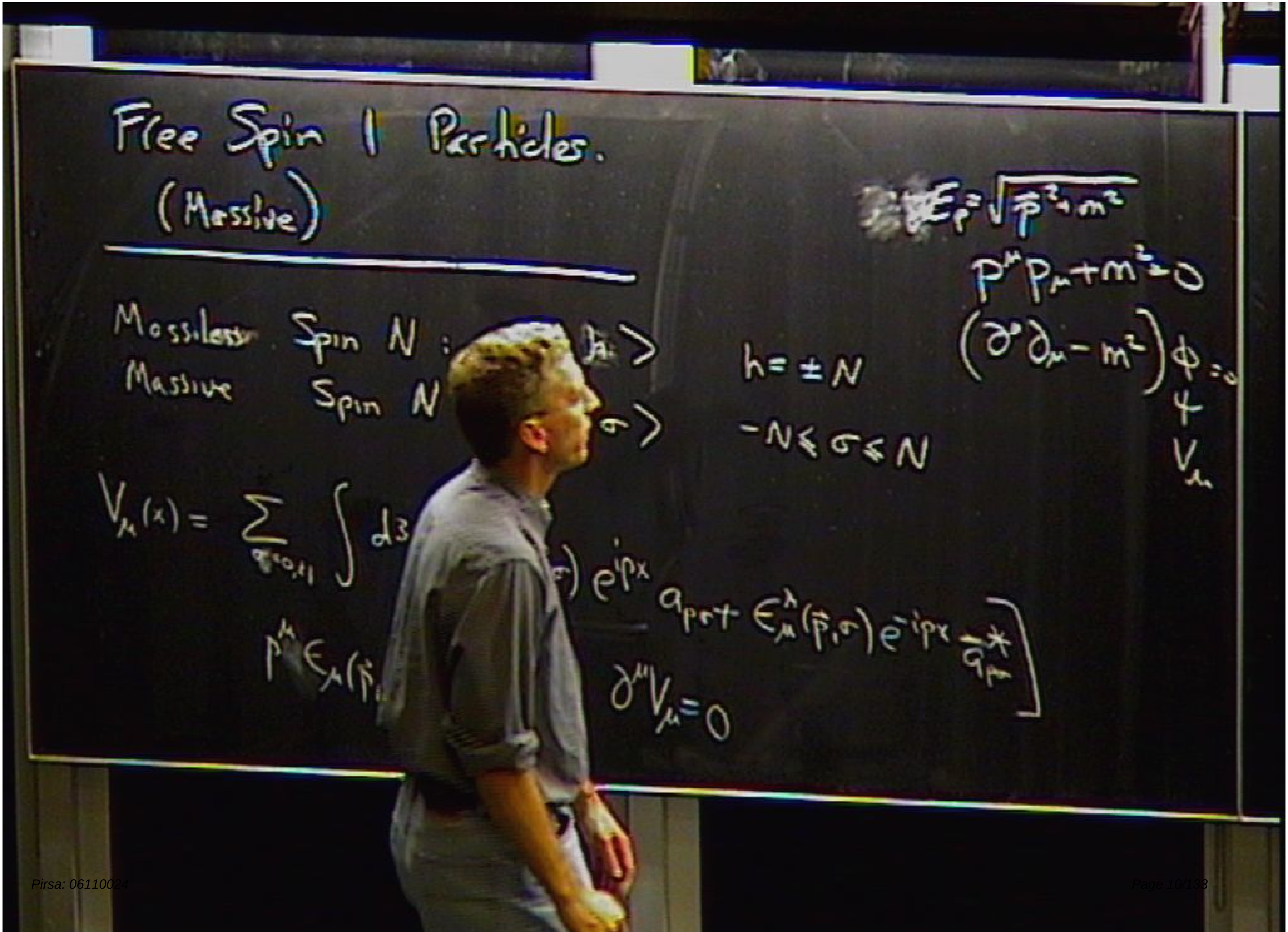
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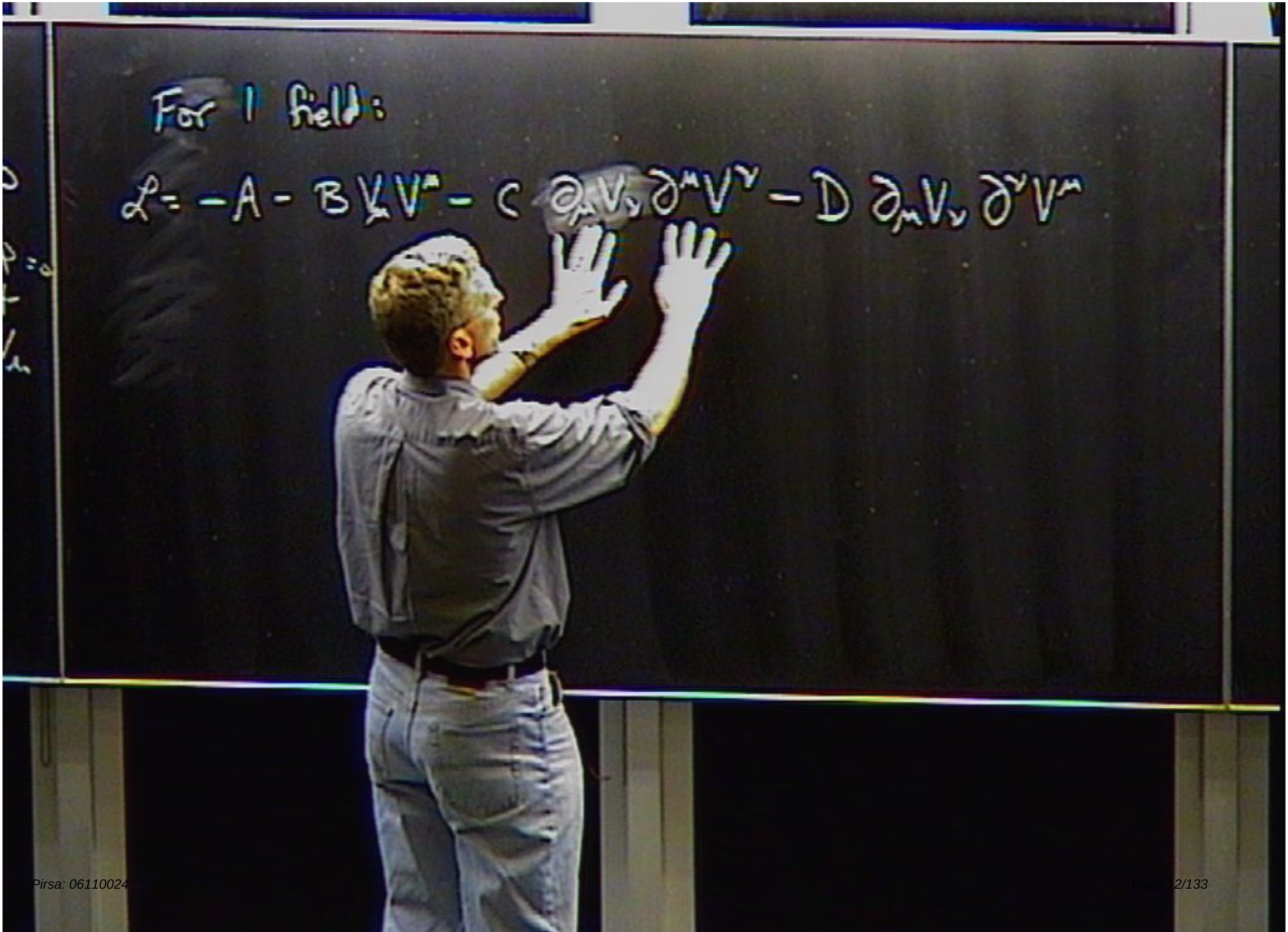
$E_p = \sqrt{\vec{p}^2 + m^2}$
$p^\mu p_\mu + m^2 = 0$
$(\partial^\mu \partial_\mu - m^2)\phi = 0$
ψ
V_μ
Massless Spin N :
$h = \pm N$
Massive Spin N
$-N \leq \sigma \leq N$
$V_\mu(x) = \sum_{\sigma = 0, \pm 1} \int d3$
e^{ipx}
$a_{p\sigma} + \epsilon^*_\mu(\vec{p}, \sigma) e^{-ipx} a^*_{p\sigma}$
$p^\mu \epsilon_\mu(\vec{p}$
$\partial^\mu V_\mu = 0$

For 1 field:

$$\mathcal{L} = -A - B\, V_\mu V^\mu$$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Require $C+D=0$ in order to ensure $\partial^\mu V_\mu = 0$ for all solutions

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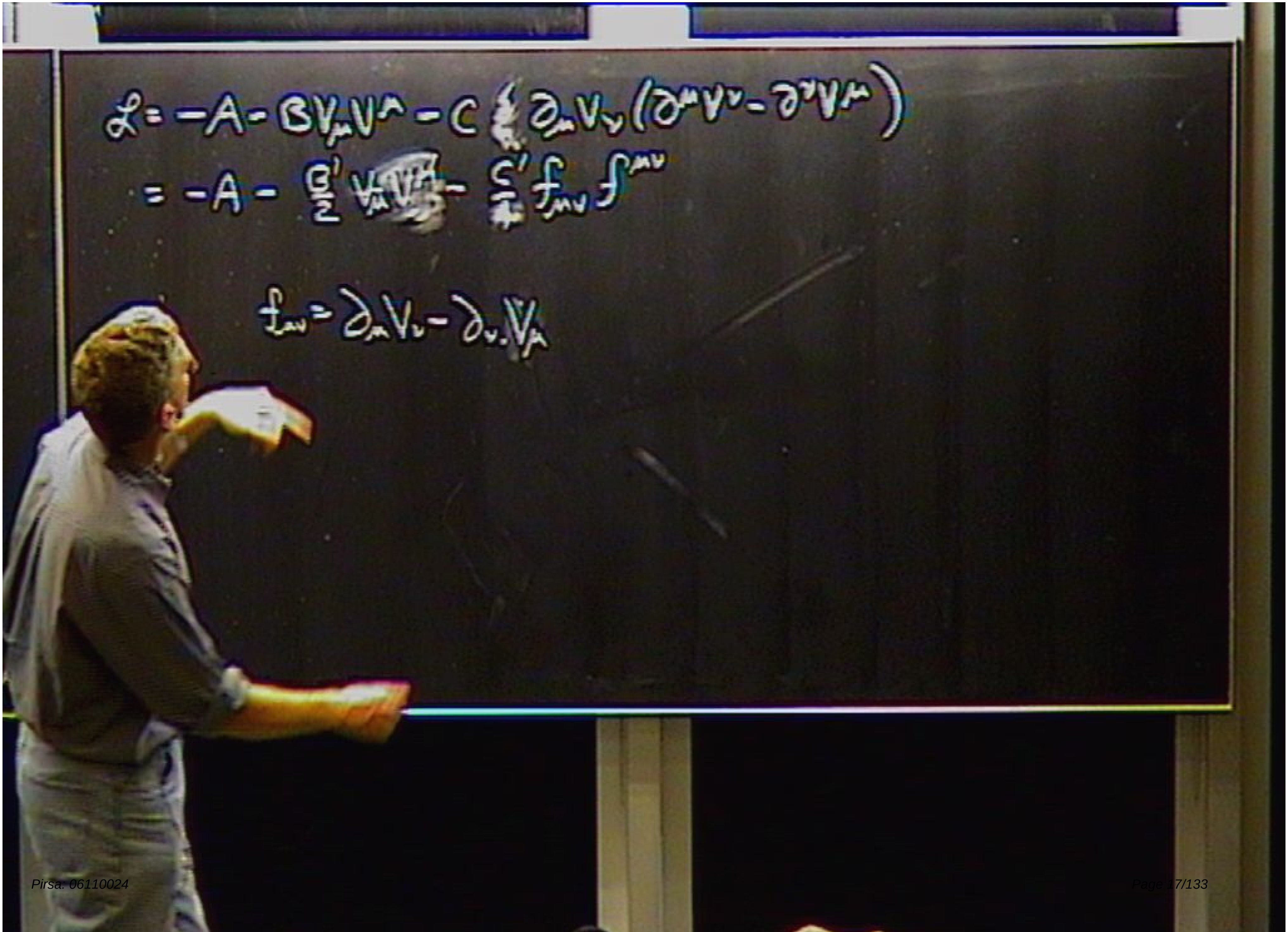
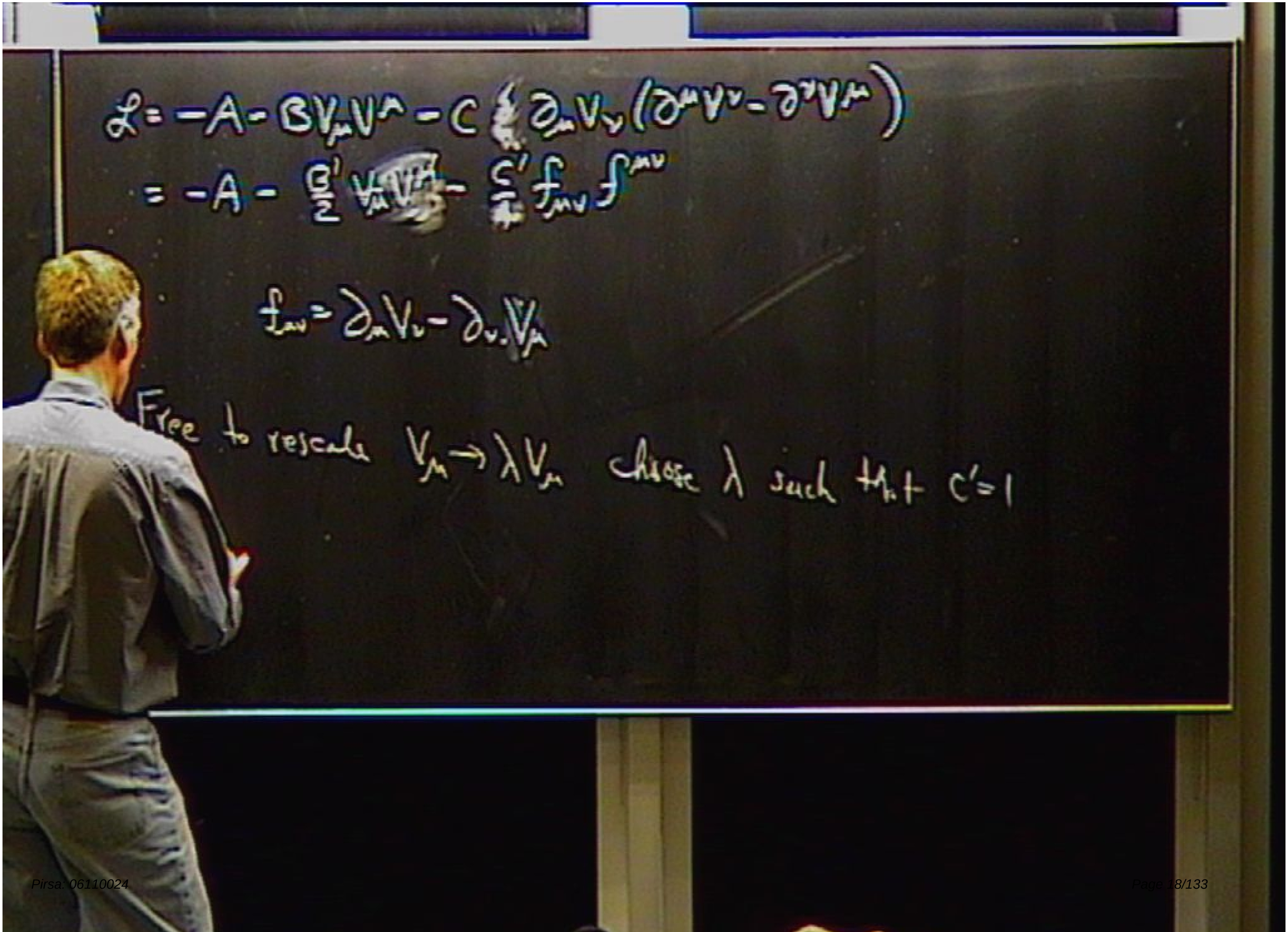

$\mathcal{L} = -A - B V_\mu V^\mu - C\, \partial_\mu V_\nu (\partial^\mu V^\nu - \partial^\nu V^\mu)$
$= -A - \frac{B'}{2} V_\mu V^\mu - \frac{C'}{4} f_{\mu\nu} f^{\mu\nu}$
$f_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$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Free to rescale $V_\mu \to \lambda V_\mu$ choose λ such that $C' = 1$

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Free to rescale $V_\mu \to \lambda V_\mu$ choose λ such that $C' = 1$

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$$\to (\Box - \mu^2) V_\mu = 0$$

Free Spin 1 Particles.
(Massive)

$E_p = \sqrt{\vec{p}^2 + m^2}$

$p^\mu p_\mu + m^2 = 0$

$(\partial^\mu \partial_\mu - m^2)\phi = 0$

ψ

V_μ

Massless Spin N : $|\vec{p}, h\rangle$ $h = \pm N$

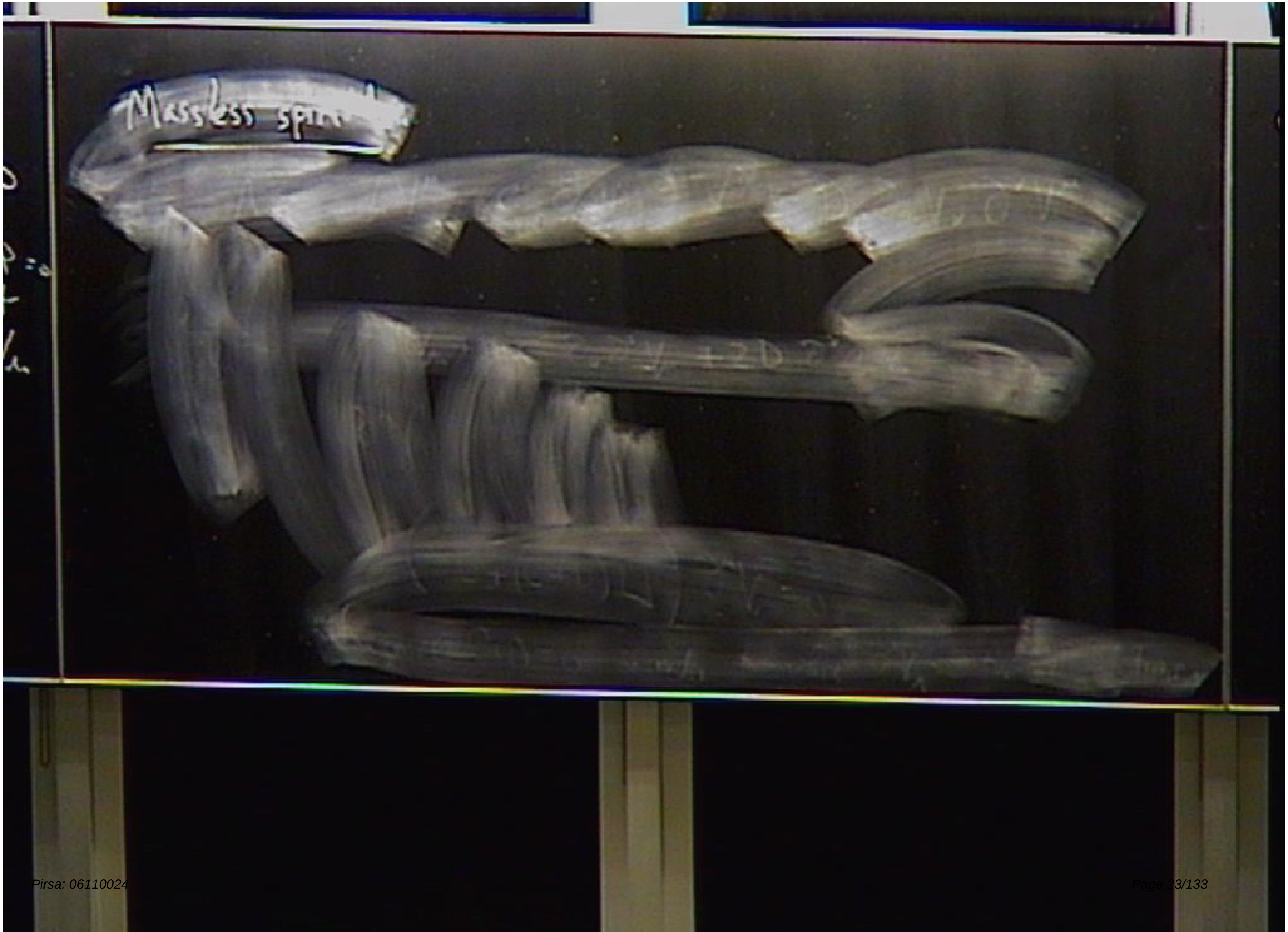
Massive Spin N $|\vec{p}, \sigma\rangle$ $-N \leq \sigma \leq N$

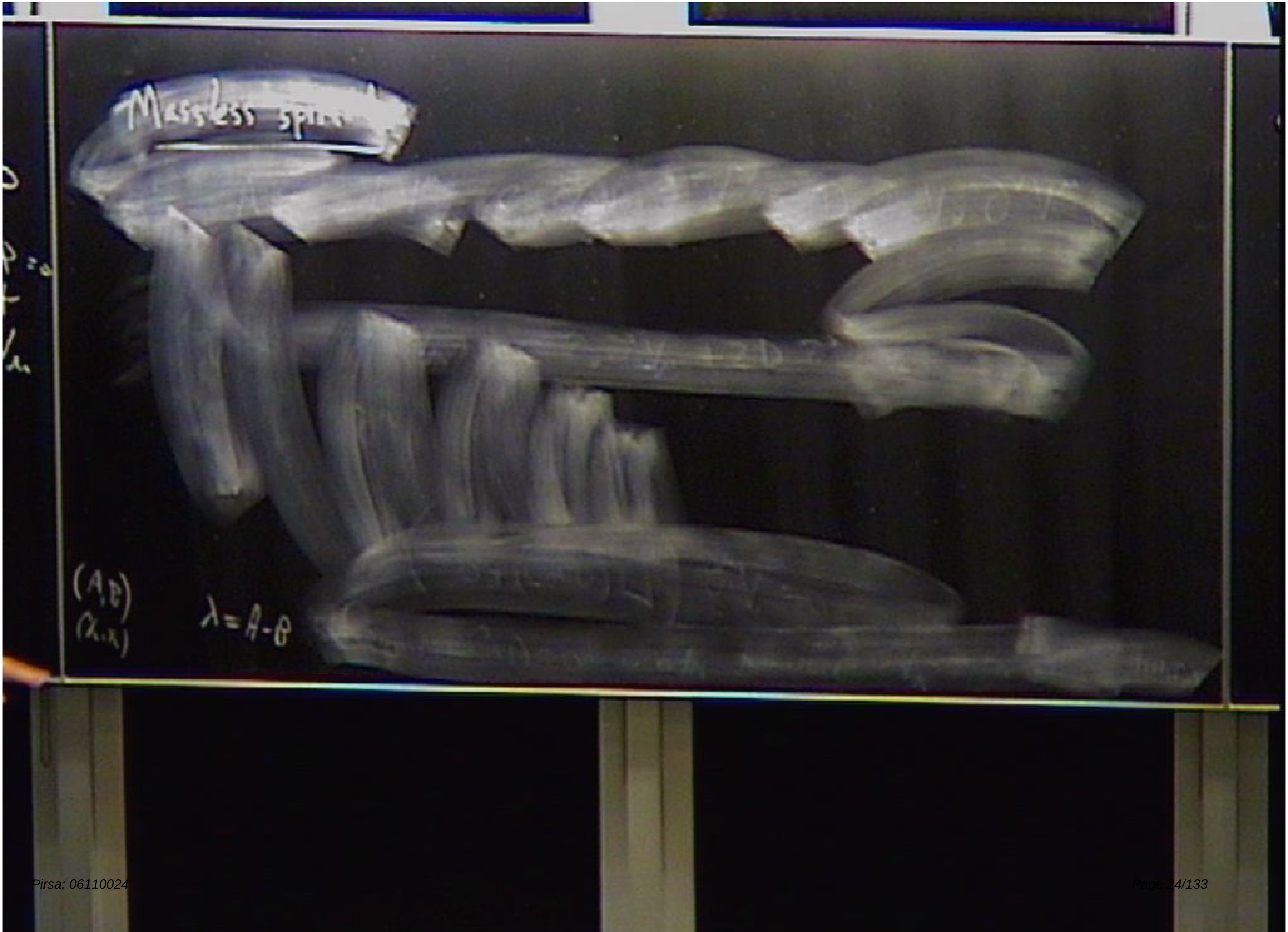
$$V_\mu(x) = \sum_{\sigma=0,\pm1} \int d^3p \left[\epsilon_\mu(\vec{p},\sigma)\, e^{ipx}\, a_{p\sigma} + \epsilon_\mu^*(\vec{p},\sigma)\, e^{-ipx}\, a^*_{p\sigma} \right]$$

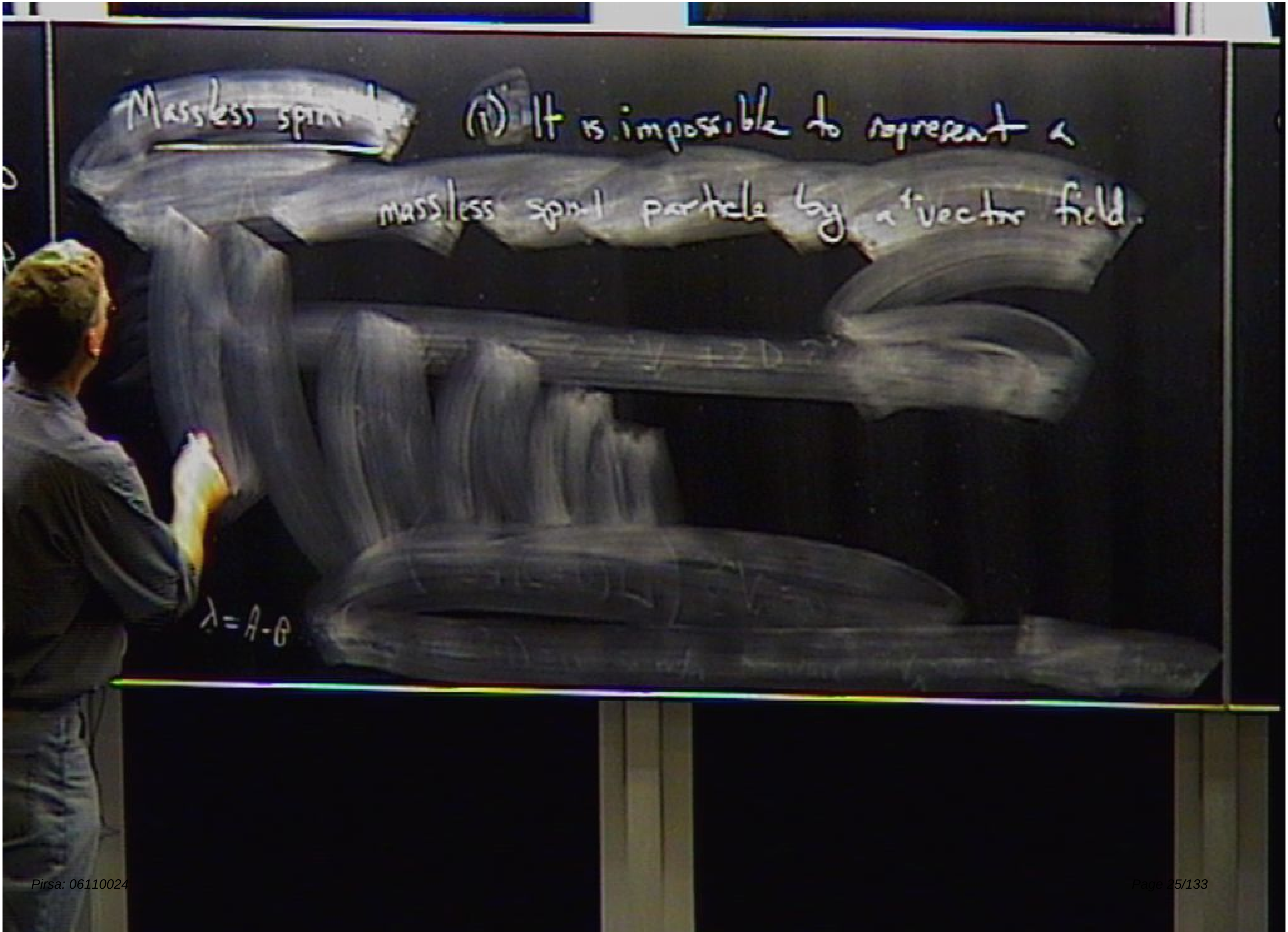
$(i\not{p} + m)u = 0$

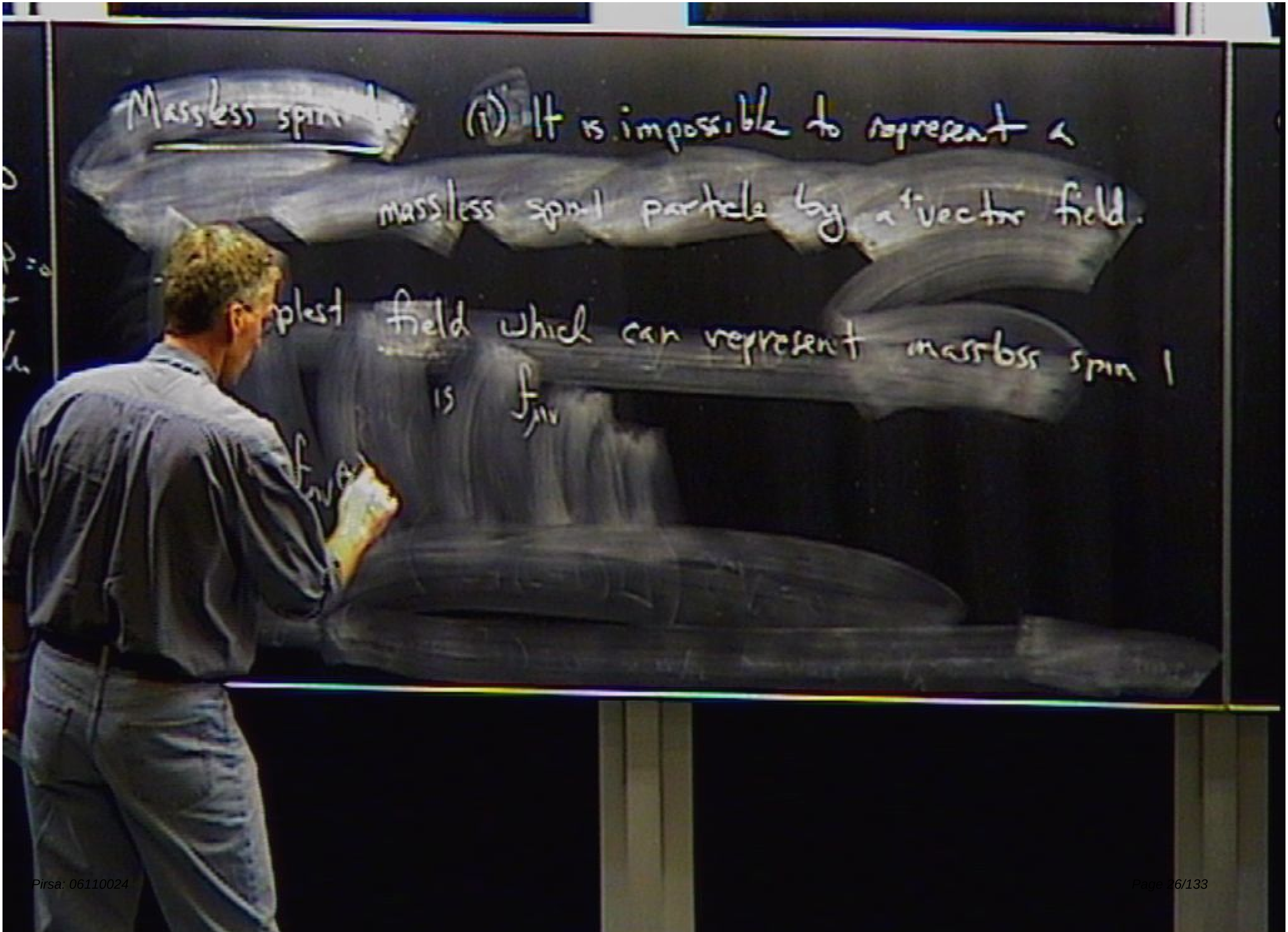
$\bar{p}u = 0$

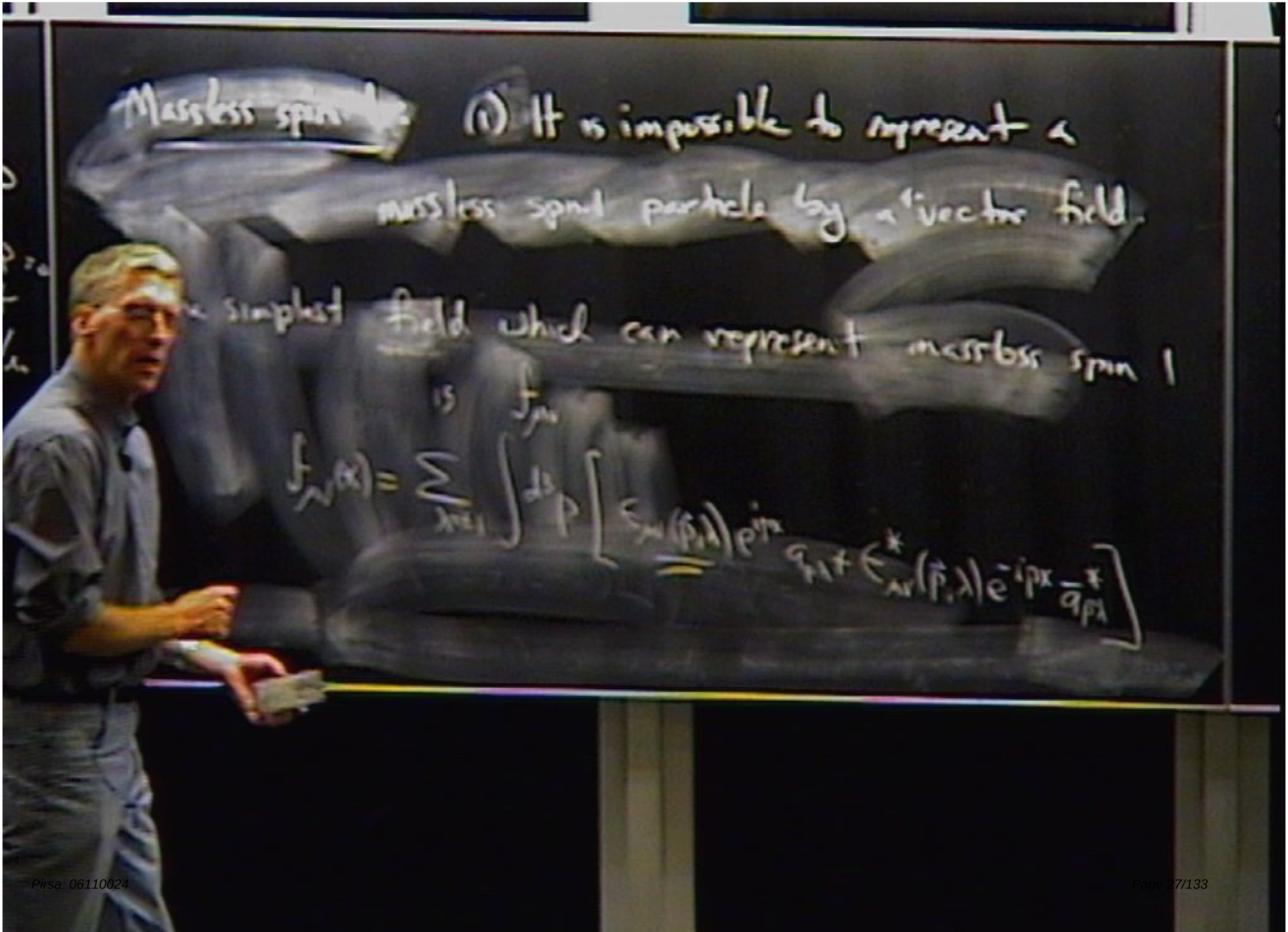
$p^\mu \epsilon_\mu(\vec{p},\sigma) = 0$ — $\partial^\mu V_\mu = 0$


Massless spin


Massless
(A,B)
λ = A - B


Massless spin
(1) It is impossible to represent a massless spin-1 particle by a 4-vector field.
λ = A - θ


Massless spin 1
(i) It is impossible to represent a
massless spin 1 particle by a 4 vector field.
plest field which can represent massless spin 1
is $f_{\mu\nu}$


Massless spin 1:
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$F_{\mu\nu}(x) = \sum_{\lambda=\pm 1} \int d^3p \left[\epsilon_{\mu\nu}(\vec{p},\lambda) e^{ipx} a_{p\lambda} + \epsilon^*_{\mu\nu}(\vec{p},\lambda) e^{-ipx} a^*_{p\lambda} \right]$

Massless spin 1

(ii) It is impossible to represent a massless spin-1 particle by a vector field.

The simplest field which can represent massless spin 1 is $f_{\mu\nu}$

$$f_{\mu\nu}(x) = \sum_{\lambda=\pm 1} \int d^3p \left[\epsilon_{\mu\nu}(\vec{p},\lambda) e^{ipx} a_{p\lambda} + \epsilon^*_{\mu\nu}(\vec{p},\lambda) e^{-ipx} a^*_{p\lambda} \right]$$

$$\epsilon_{\mu\nu} = p_\mu \xi_\nu - p_\nu \xi_\mu \quad \text{where}$$

Massless spin 1

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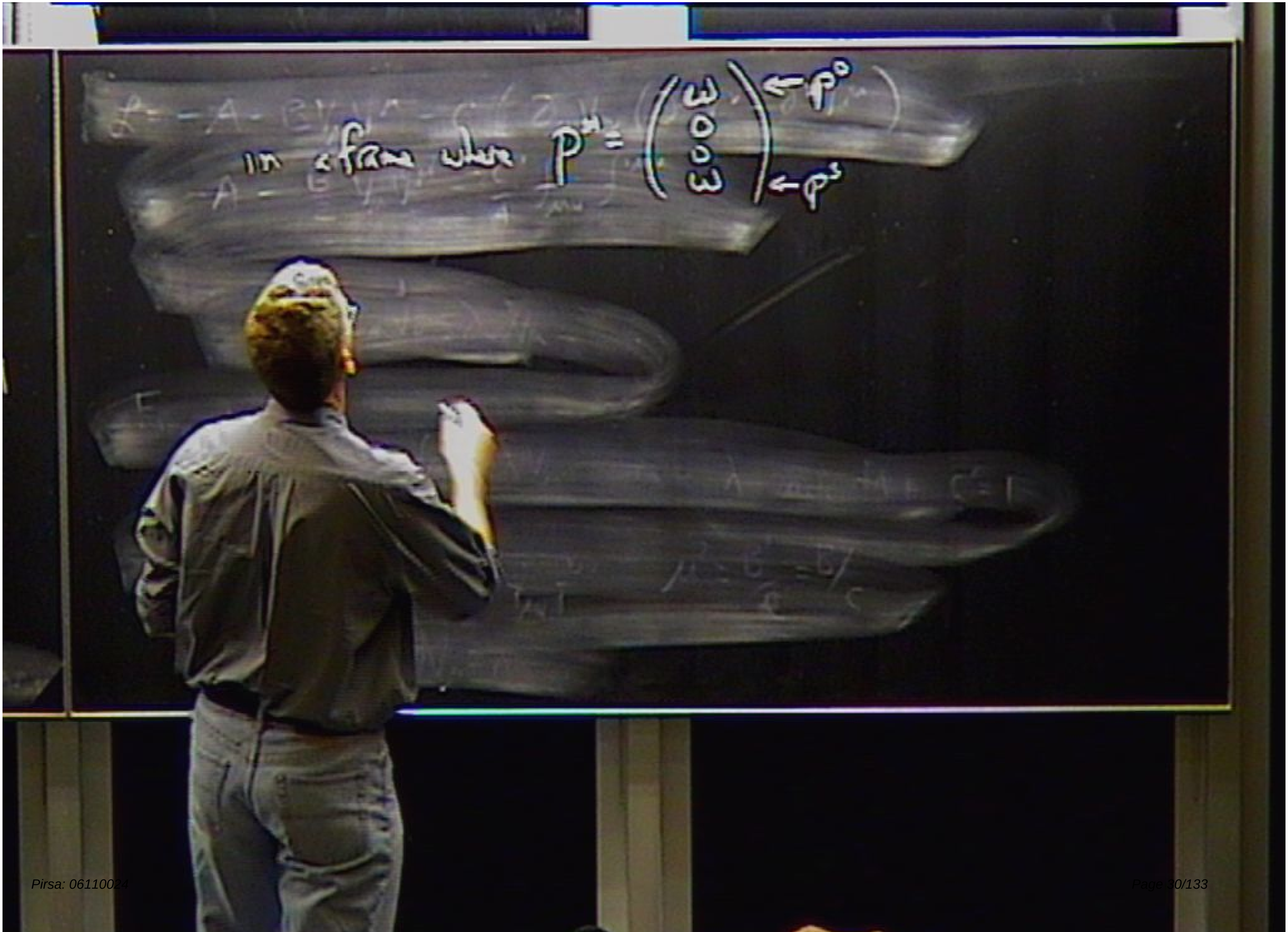
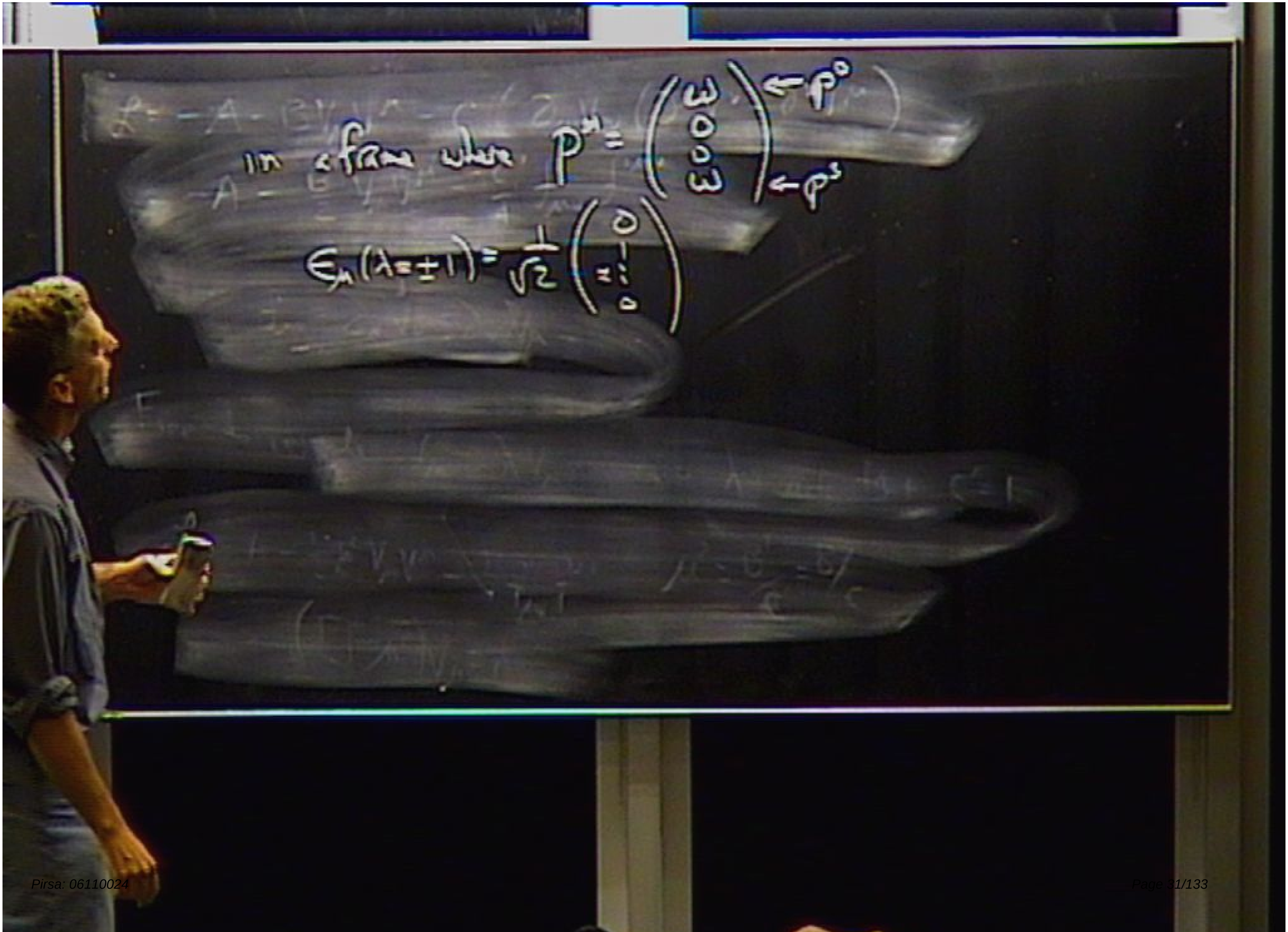
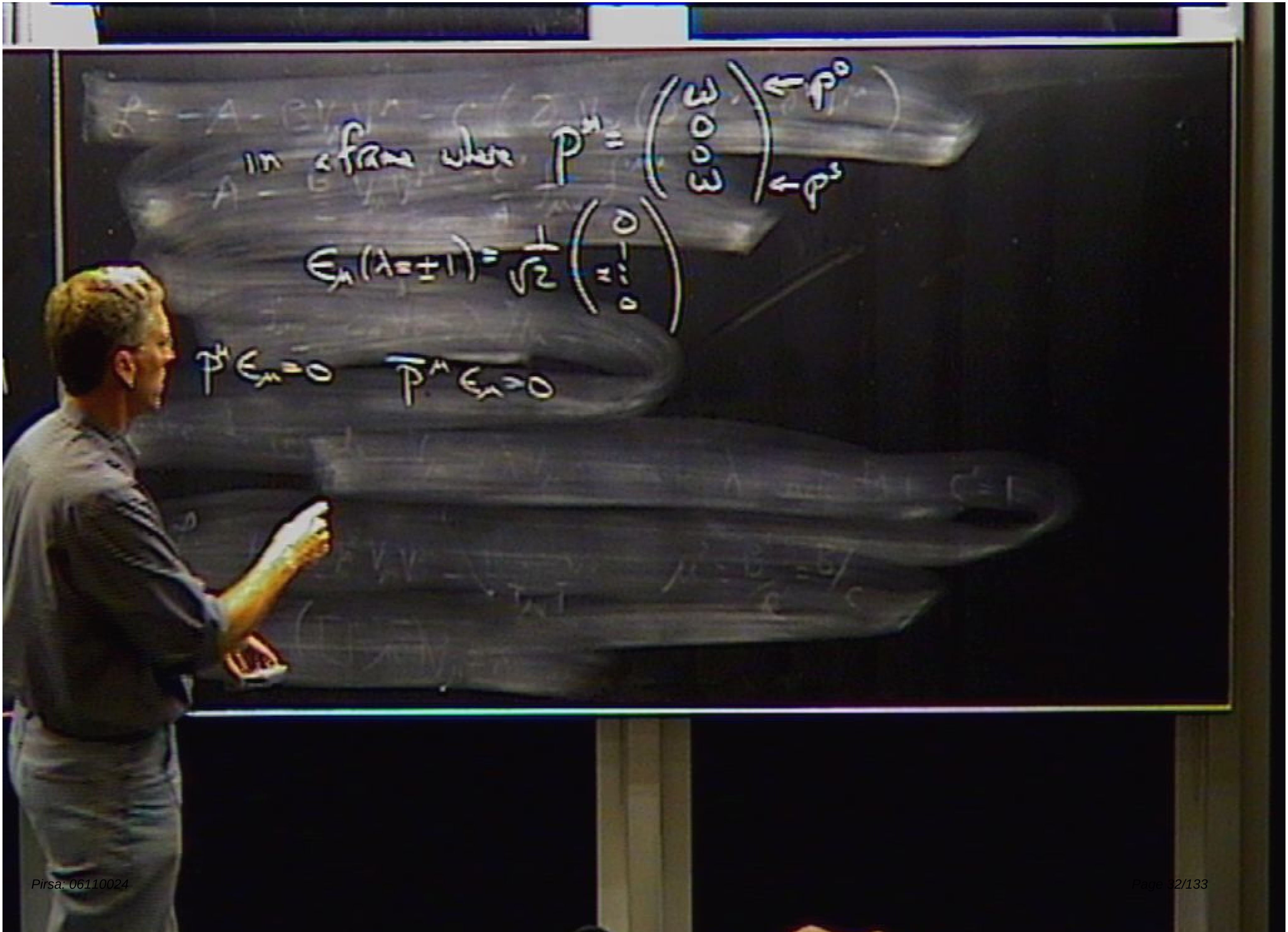
The simplest field which can represent massless spin 1 is $f_{\mu\nu}$

$$f_{\mu\nu}(x) = \sum_{\lambda=\pm1} \int d^3p \left[\epsilon_{\mu\nu}(\vec{p},\lambda)\, e^{ipx}\, a_{p\lambda} + \epsilon^*_{\mu\nu}(\vec{p},\lambda)\, e^{-ipx}\, a^*_{p\lambda} \right]$$

where $\epsilon_{\mu\nu} = p_\mu \xi_\nu - p_\nu \xi_\mu$


in a frame where $p^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix} \begin{matrix} \leftarrow p^0 \\ \\ \\ \leftarrow p^3 \end{matrix}$


in a frame where $p^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix} \begin{array}{l} \leftarrow p^0 \\ \\ \\ \leftarrow p^3 \end{array}$
$\epsilon_\mu(\lambda = \pm 1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix}$


in a frame where $p^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix}$ ← p^0 ← p^3
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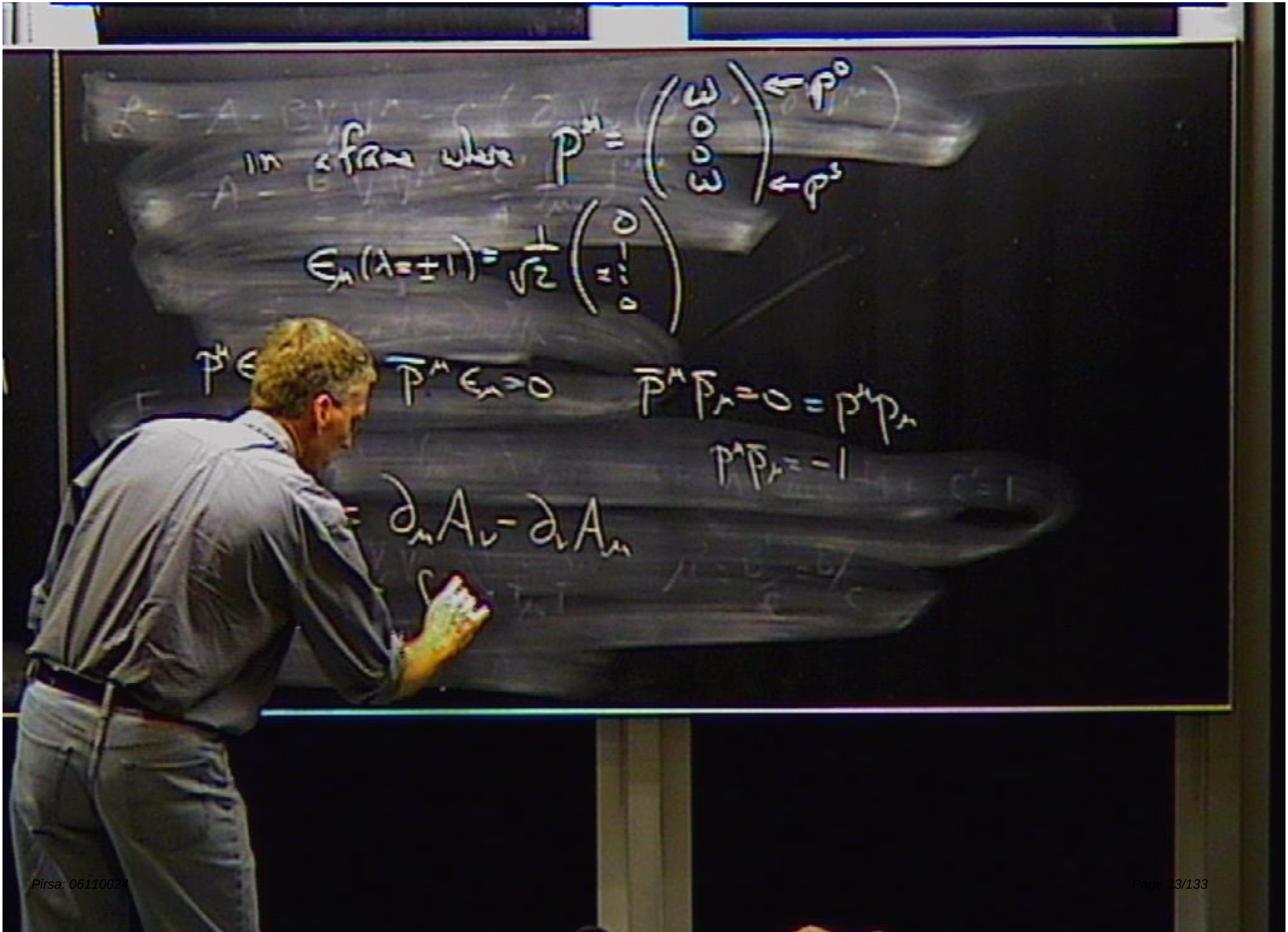
$p^\mu \epsilon_\mu = 0$ $\bar{p}^\mu \epsilon_\mu = 0$


in frame where p^μ = (ω 0 0 ω) ← p^0 ← p^3
ε_μ(λ=±1) = 1/√2 (0 1 ±i 0)
p̄^μ ε_μ = 0
p̄^μ p̄_μ = 0 = p^μ p_μ
p^μ p̄_μ = -1
∂_μ A_ν - ∂_ν A_μ

in a frame where $p^\mu = \begin{array}{cl} \left(\begin{array}{c}\omega\\0\\0\\\omega\end{array}\right. & \begin{array}{l}\leftarrow p^0\\ \\ \\ \leftarrow p^3\end{array}\end{array}$

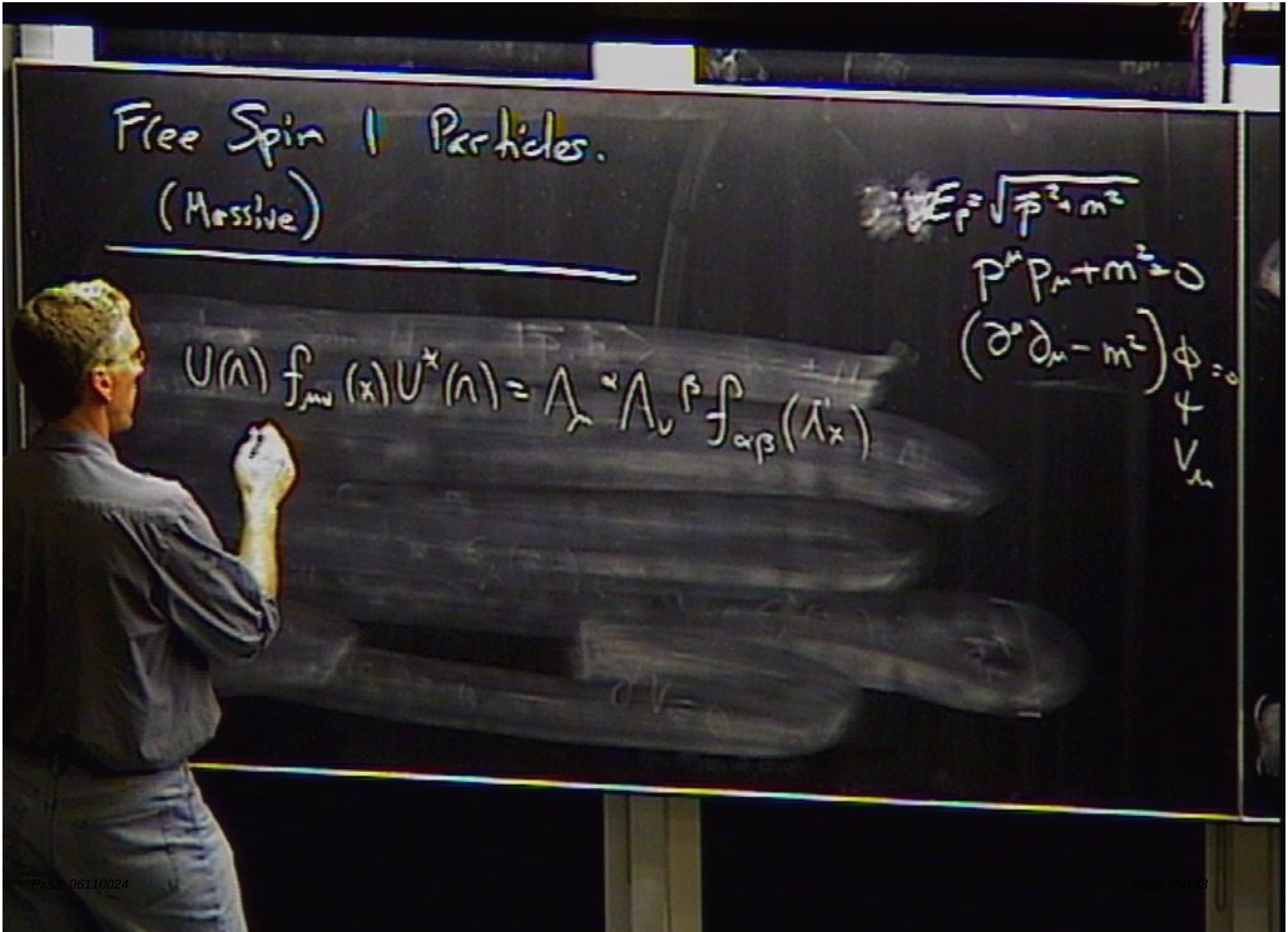
$$\xi_\mu(\lambda=\pm 1) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\\pm i\\0\end{pmatrix}$$

$$p^\mu \xi_\mu = 0 \qquad \bar{p}^\mu \xi_\mu = 0 \qquad \bar{p}^\mu \bar{p}_\mu = 0 = p^\mu p_\mu$$

$$p^\mu \bar{p}_\mu = -1$$

So $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$A_\mu = \sum_{\lambda=\pm 1} \int d^3p \left[\xi_\mu(\vec{p},\lambda)\, e^{ipx}\, a_{p\lambda} + \xi^*_\mu(p,\lambda)\, e^{-ipx}\, a^*_{p\lambda} \right)$$


Free Spin 1 Particles.
(Massive)
$U(\Lambda) f_{\mu\nu}(x) U^{\dagger}(\Lambda) = \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta f_{\alpha\beta}(\Lambda^{-1}x)$
$E_p = \sqrt{\vec{p}^2 + m^2}$
$p^\mu p_\mu + m^2 = 0$
$(\partial^\mu \partial_\mu - m^2)\phi = 0$
ψ
V_μ

Free Spin 1 Particles.
(Massive)

$$U(\Lambda) f_{\mu\nu}(x) U^*(\Lambda) = \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta f_{\alpha\beta}(\Lambda^{-1}x)$$

$$U(\Lambda) A_\mu(x) U^*(\Lambda) = \Lambda_\mu{}^\alpha A_\alpha(\Lambda^{-1}x) + \partial_\mu \Omega$$

for some Ω

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$p^\mu p_\mu + m^2 = 0$

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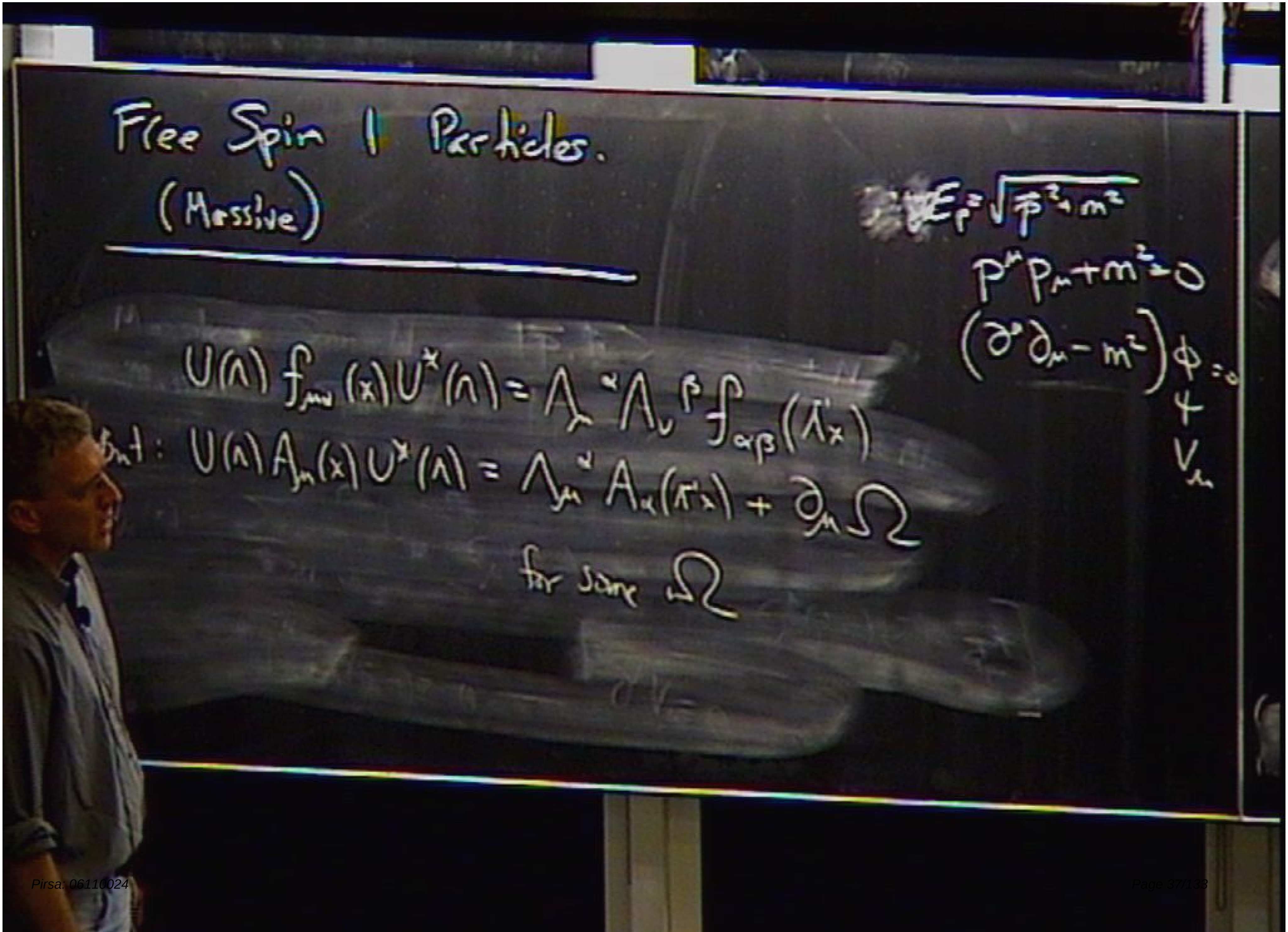
ψ

V_μ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ψ
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$U(\Lambda) f_{\mu\nu}(x) U^*(\Lambda) = \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta f_{\alpha\beta}(\Lambda^{-1}x)$
But: $U(\Lambda) A_\mu(x) U^*(\Lambda) = \Lambda_\mu{}^\alpha A_\alpha(\Lambda^{-1}x) + \partial_\mu \Omega$
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(Massive)

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for some Ω

$$\mathcal{L} = -A - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}$$

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$p^\mu p_\mu + m^2 = 0$

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ψ

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(Massive)

$$U(\Lambda)\, f_{\mu\nu}(x)\, U^*(\Lambda) = \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta f_{\alpha\beta}(\Lambda^{-1}x)$$

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ψ

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$$U(\Lambda)\, f_{\mu\nu}(x)\, U^*(\Lambda) = \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta f_{\alpha\beta}(\Lambda^{-1}x)$$

But: $U(\Lambda)\, A_\mu(x)\, U^*(\Lambda) = \Lambda_\mu{}^\alpha A_\alpha(\Lambda^{-1}x) + \partial_\mu \Omega$

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Free Spin 1 Particles.
(Massive)

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ψ

V_μ

$$U(\Lambda) f_{\mu\nu}(x) U^\dagger(\Lambda) = \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta f_{\alpha\beta}(\Lambda^{-1}x)$$

But: $U(\Lambda) A_\mu(x) U^\dagger(\Lambda) = \Lambda_\mu{}^\alpha A_\alpha(\Lambda^{-1}x) + \partial_\mu \Omega$

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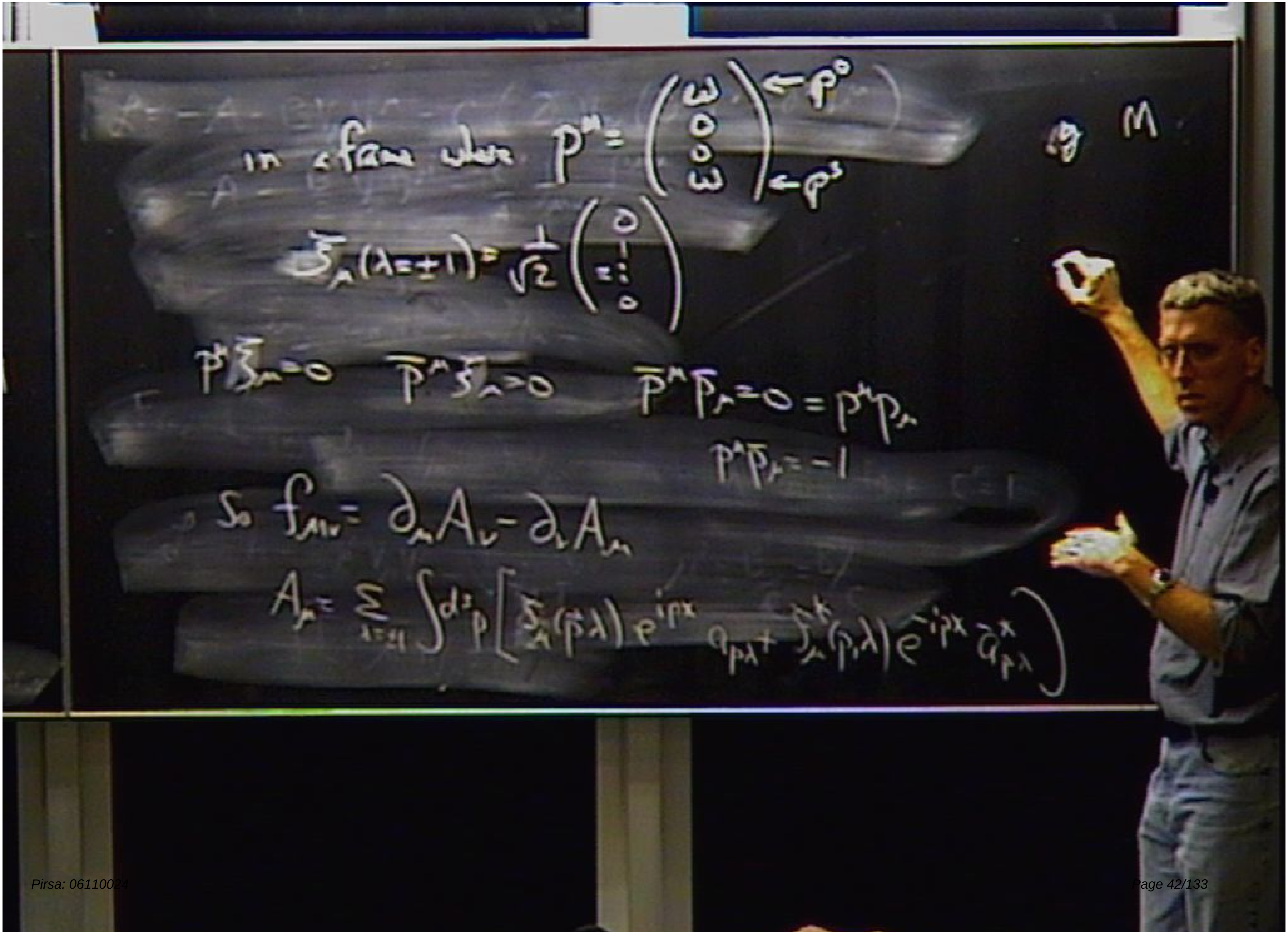
$$\mathcal{L} = -A - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}$$


in a frame where $p^\mu = \begin{pmatrix}\omega\\0\\0\\\omega\end{pmatrix} \begin{matrix}\leftarrow p^0\\ \\ \\ \leftarrow p^3\end{matrix}$
M
$\xi_\mu(\lambda=\pm1) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\\pm i\\0\end{pmatrix}$
$p^\mu \xi_\mu = 0$ $\bar{p}^\mu \xi_\mu = 0$ $\bar{p}^\mu \bar{p}_\mu = 0 = p^\mu p_\mu$
$p^\mu \bar{p}_\mu = -1$
So $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$A_\mu = \sum_{\lambda=\pm1} \int d^3p \left[\xi_\mu(p,\lambda) e^{ipx} a_{p\lambda} + \xi^*_\mu(p,\lambda) e^{-ipx} a^*_{p\lambda} \right)$

in a frame where $p^\mu = \begin{pmatrix}\omega\\0\\0\\\omega\end{pmatrix} \begin{matrix}\leftarrow p^0\\ \\ \\ \leftarrow p^3\end{matrix}$

@ M

$\xi_\mu(\lambda=\pm1) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\\pm i\\0\end{pmatrix}$

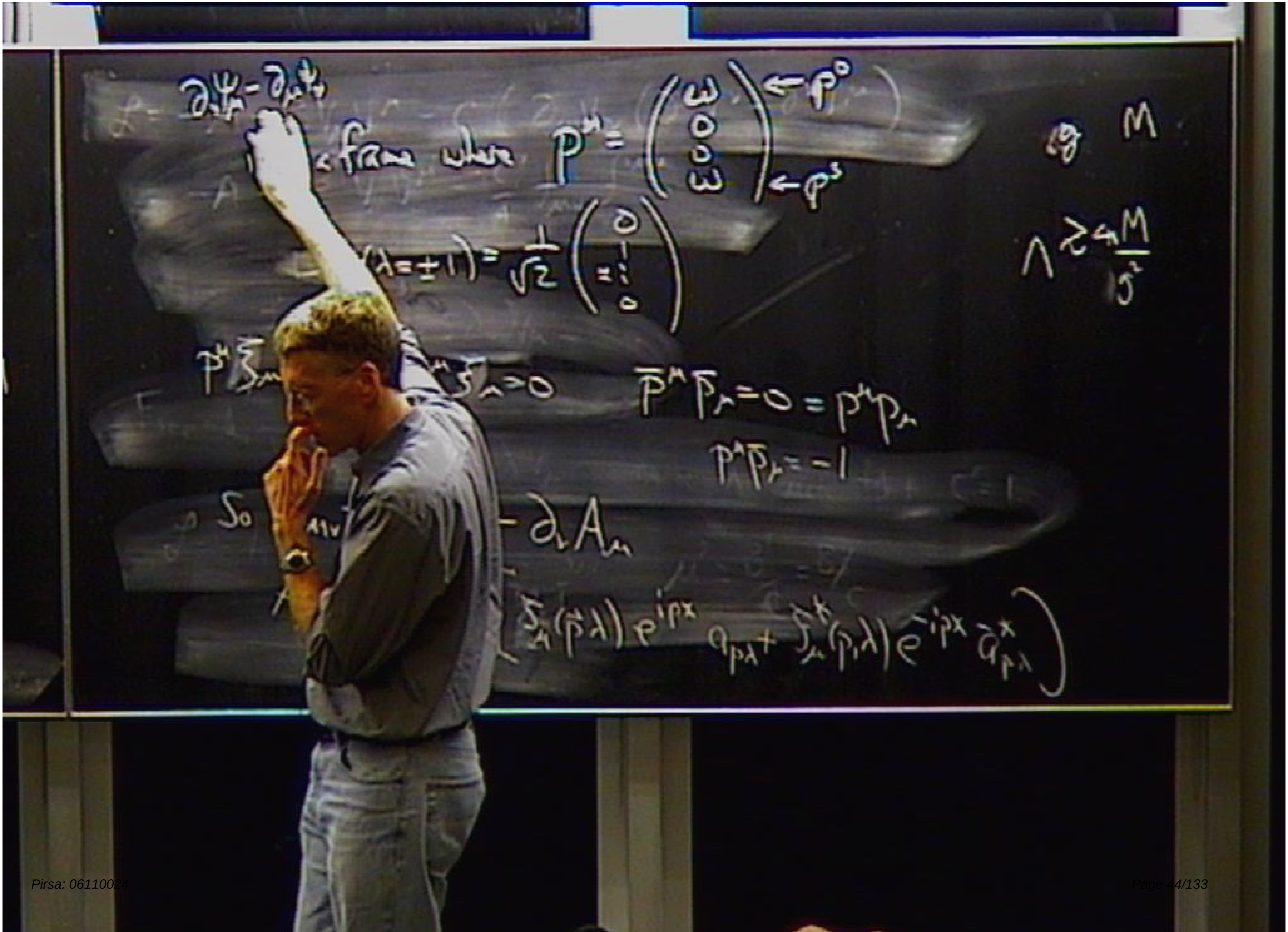
$\Lambda \gtrsim \frac{4M}{g^2}$

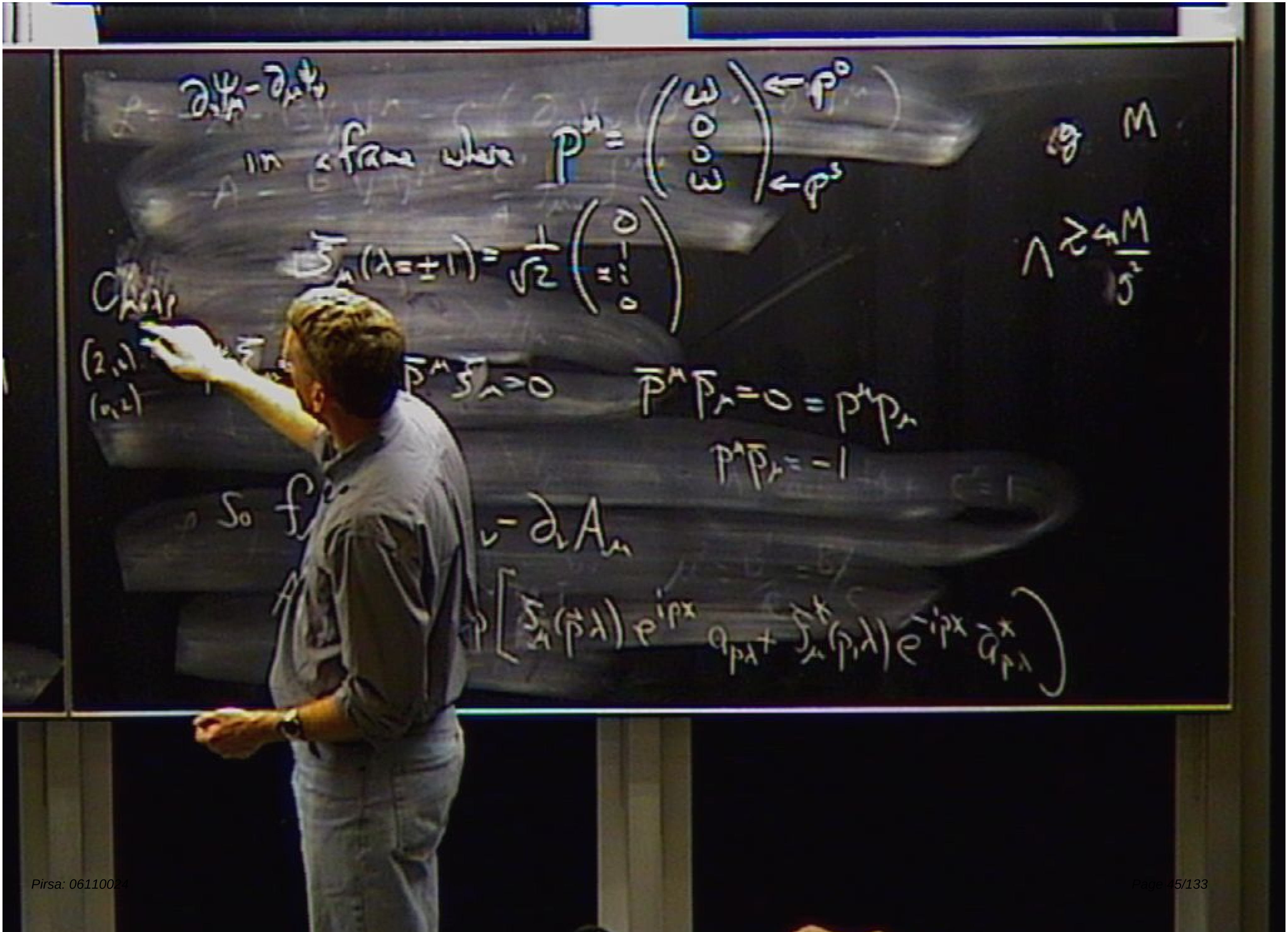
$p^\mu \xi_\mu = 0 \qquad \bar{p}^\mu \xi_\mu = 0 \qquad \bar{p}^\mu \bar{p}_\mu = 0 = p^\mu p_\mu$

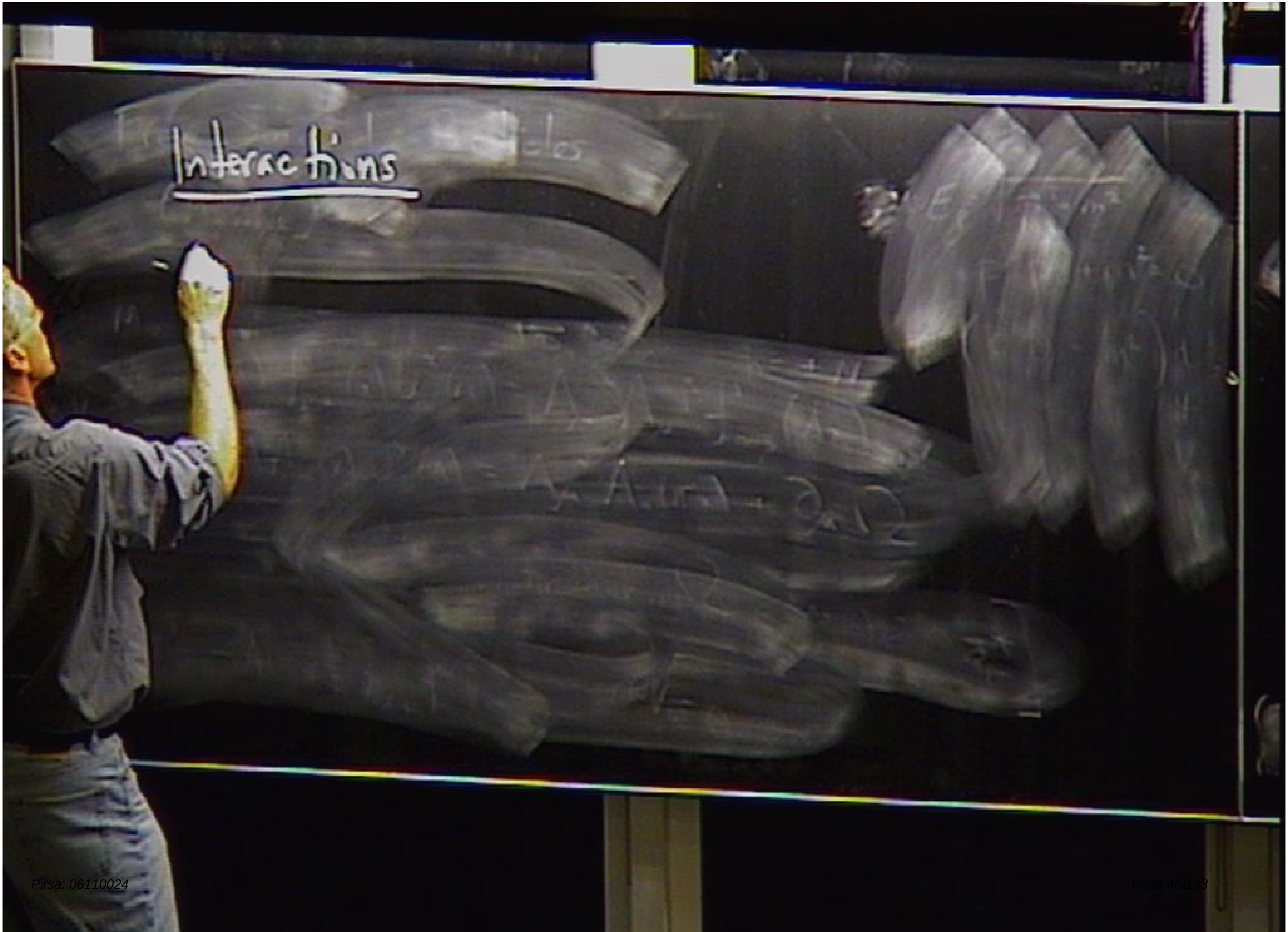
$p^\mu \bar{p}_\mu = -1$

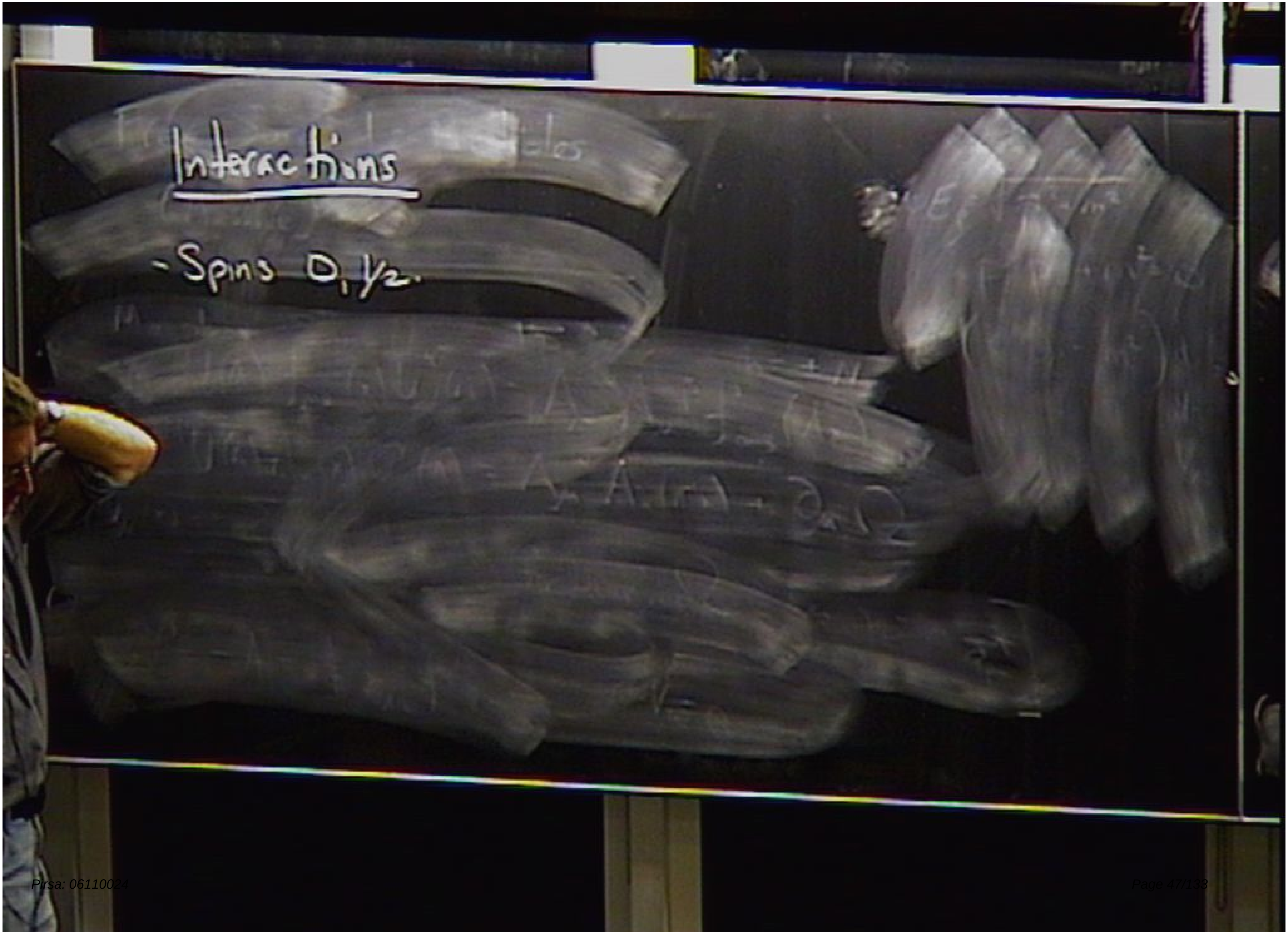
So $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$A_\mu = \sum_{\lambda=\pm1} \int d^3p \left[\xi_\mu(\vec{p},\lambda)\, e^{ipx}\, a_{p\lambda} + \xi^*_\mu(\vec{p},\lambda)\, e^{-ipx}\, a^*_{p\lambda} \right]$


frame where
M
p̄^μ p̄_μ = 0 = p^μ p_μ
p^μ p̄_μ = -1
∂_ν A_μ


in a frame where
So


Interactions


Interactions
- Spins 0, 1/2.

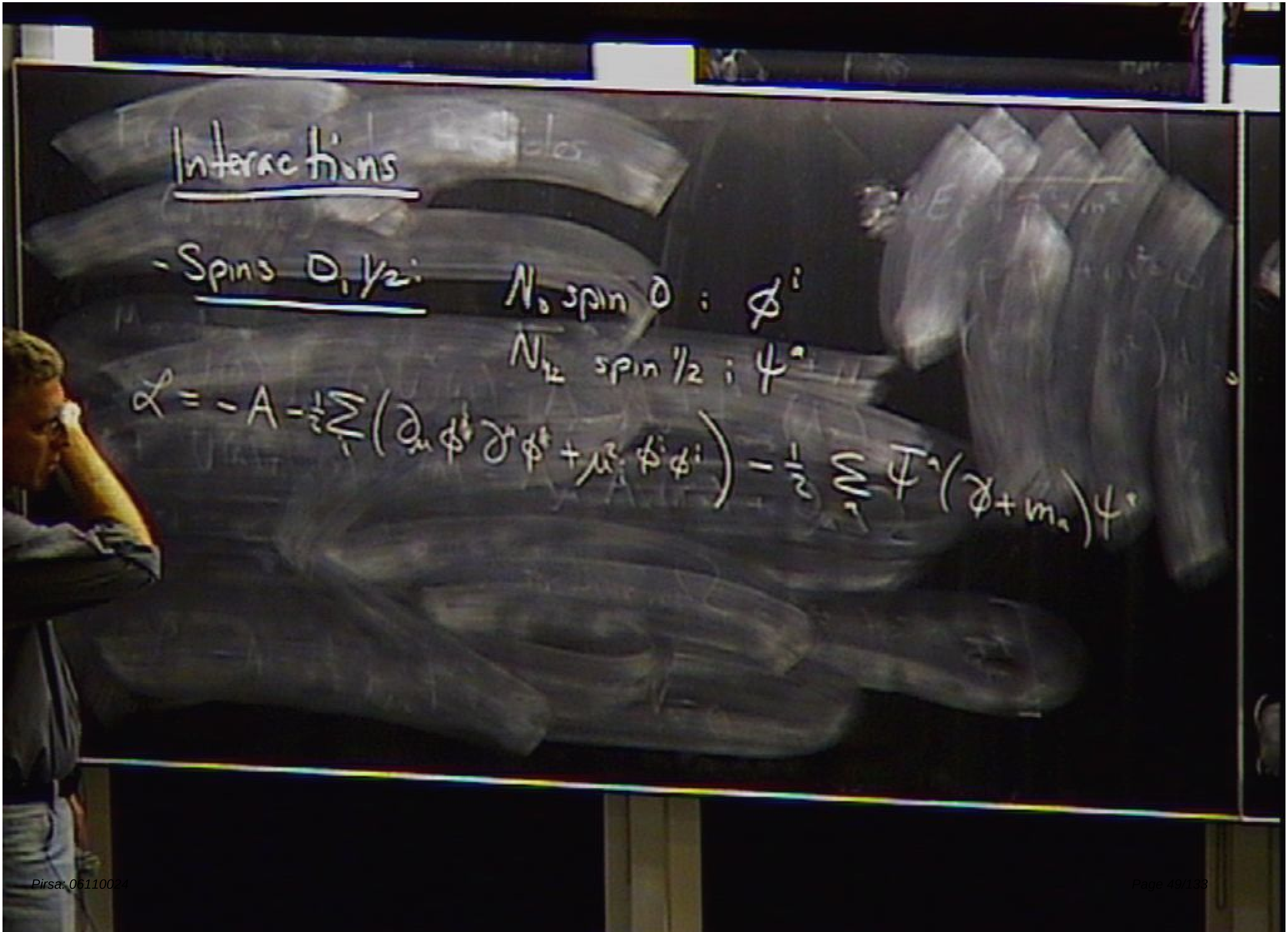
Interactions

- Spins 0, 1/2:

N_0 spin 0 : ϕ^i

$N_{1/2}$ spin 1/2 : ψ^a

$\mathcal{L} =$


Interactions
- Spins 0, 1/2:
N_0 spin 0 : ϕ^i
$N_{1/2}$ spin 1/2 : ψ^a
$\mathcal{L} = -A - \frac{1}{2}\sum_i \left(\partial_\mu \phi^i \partial^\mu \phi^i + \mu_i^2 \phi^i \phi^i\right) - \frac{1}{2}\sum_a \bar{\psi}^a (\not{\partial} + m_a)\psi^a$

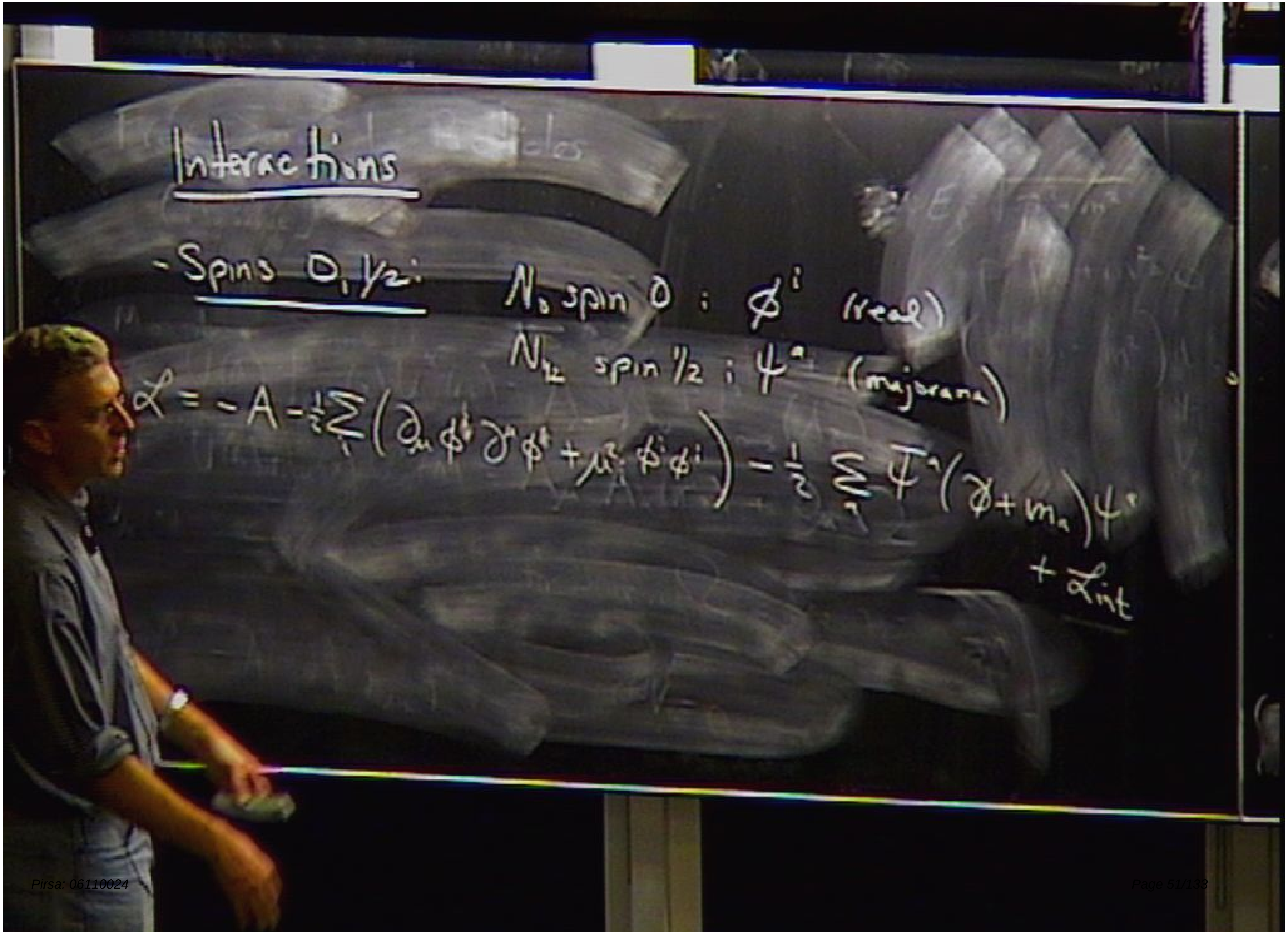
Interactions

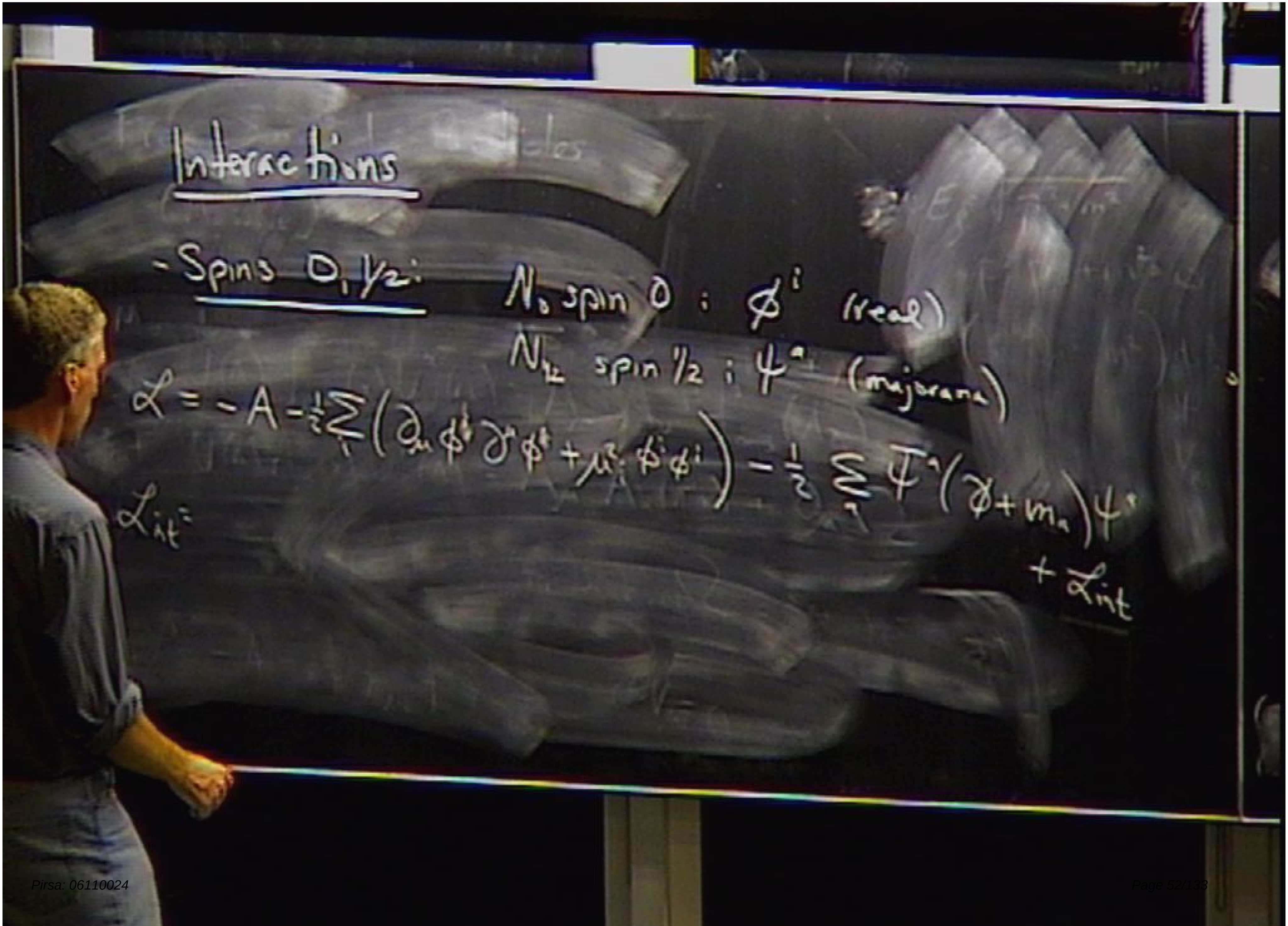
- Spins 0, 1/2:

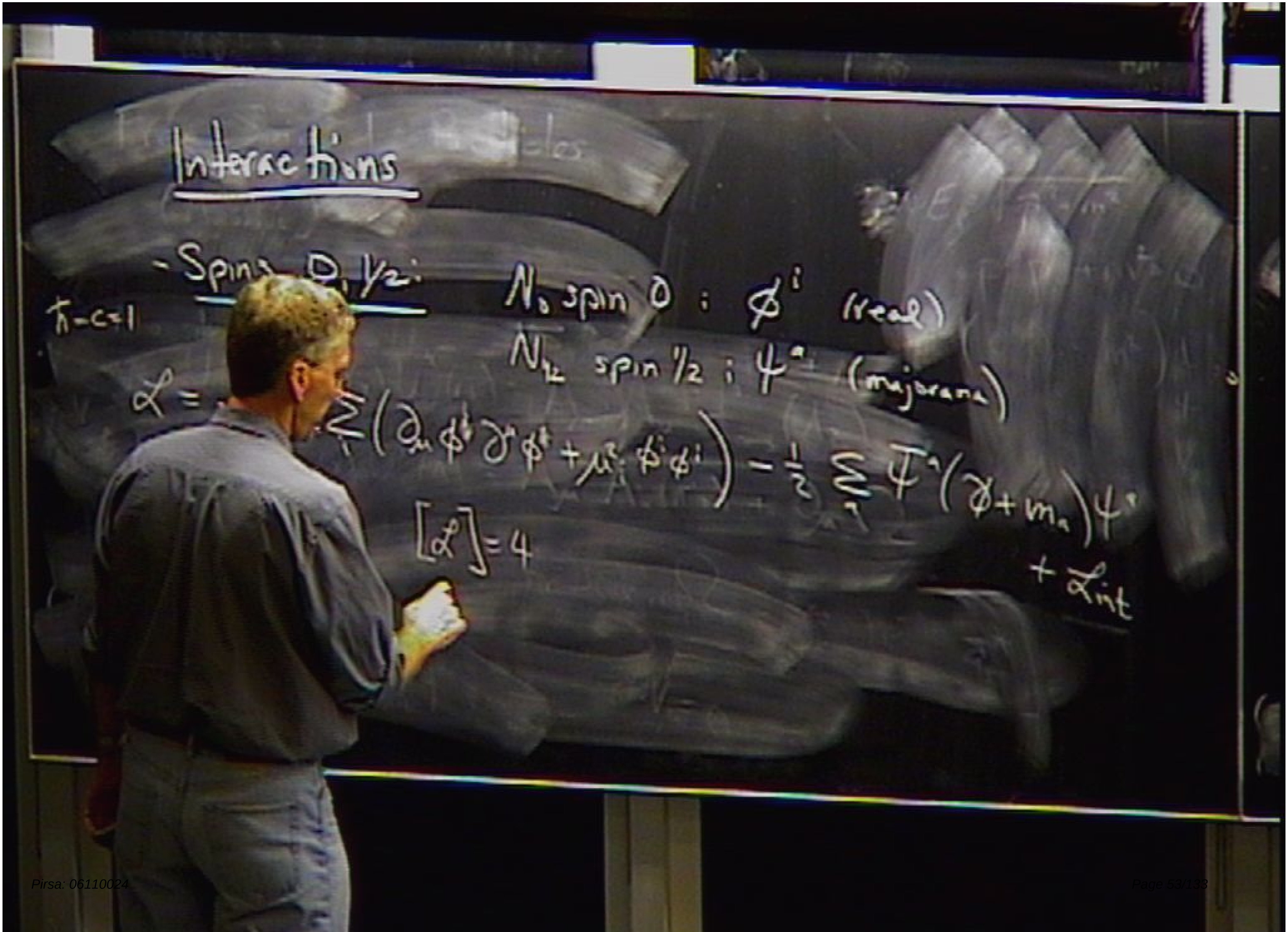
N_0 spin 0 : ϕ^i

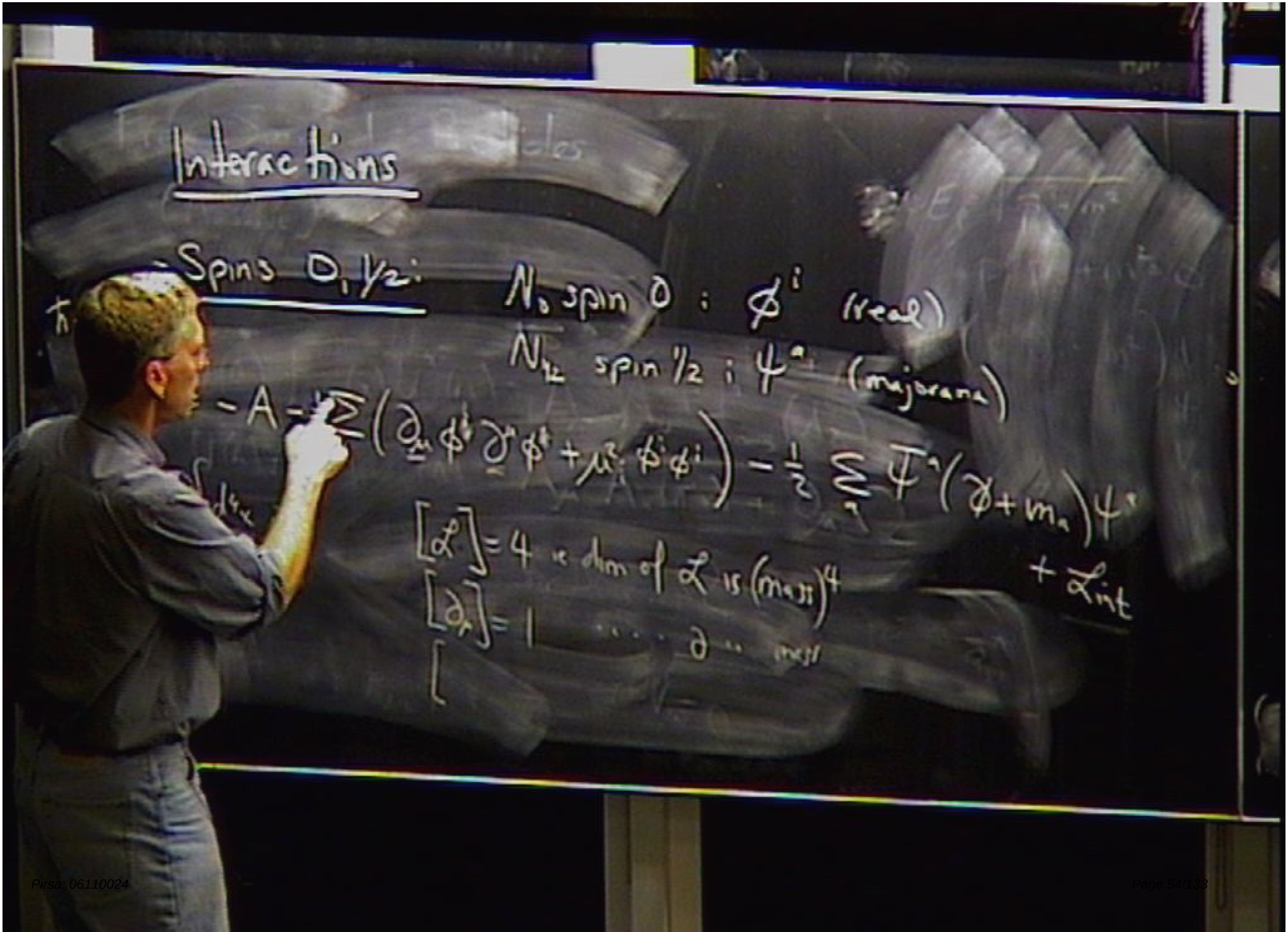
$N_{1/2}$ spin 1/2 : ψ^a

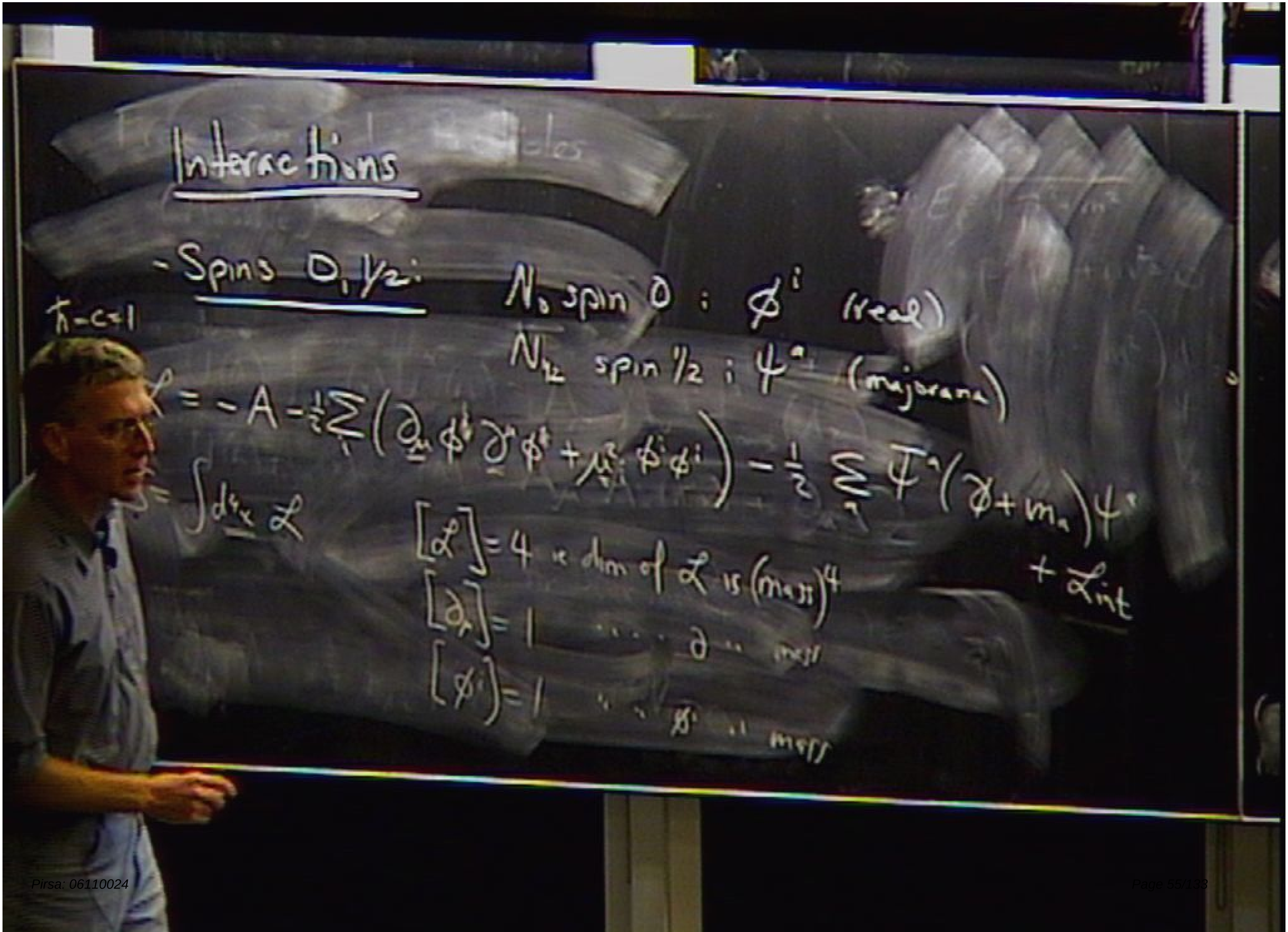
$$\mathcal{L} = -A - \frac{1}{2}\sum_i \left(\partial_\mu \phi^i \partial^\mu \phi^i + \mu_i^2 \phi^i \phi^i \right) - \frac{1}{2} \sum_a \bar{\psi}^a (\not{\partial} + m_a) \psi^a + \mathcal{L}_{int}$$

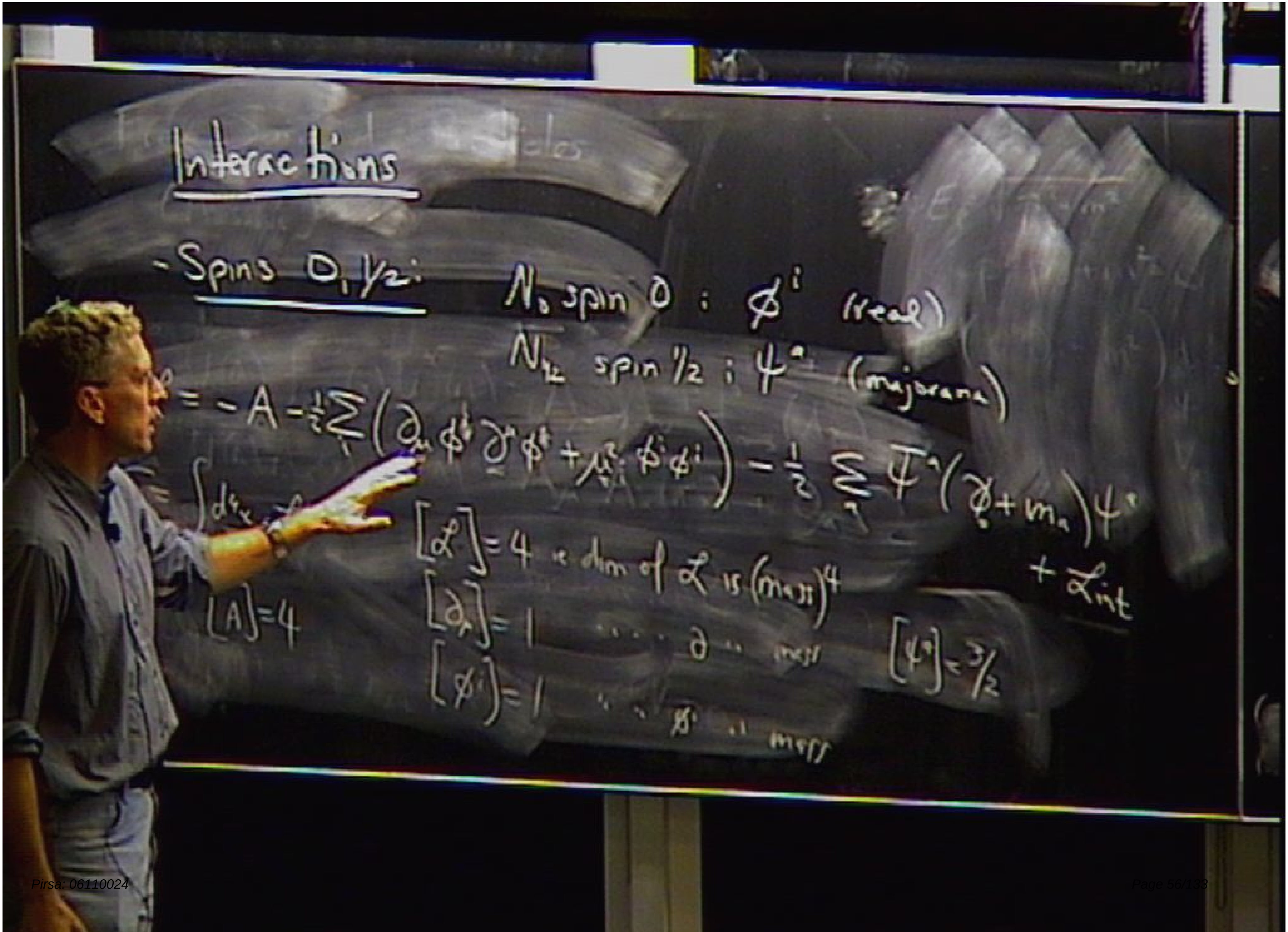

Interactions
- Spins 0, 1/2:
N_0 spin 0 : ϕ^i (real)
$N_{1/2}$ spin 1/2 : ψ^a (majorana)
$\mathcal{L} = -A - \frac{1}{2}\sum_i \left(\partial_\mu \phi^i \partial^\mu \phi^i + \mu_i^2 \phi^i \phi^i\right) - \frac{1}{2}\sum_a \bar{\psi}^a (\partial\!\!\!/ + m_a)\psi^a + \mathcal{L}_{int}$


Interactions
- Spins 0, 1/2:
N_0 spin 0 : ϕ^i (real)
$N_{1/2}$ spin 1/2 : ψ^a (majorana)
$\mathcal{L} = -A - \frac{1}{2}\sum_i \left(\partial_\mu \phi^i \partial^\mu \phi^i + \mu_i^2 \phi^i \phi^i\right) - \frac{1}{2}\sum_a \bar{\psi}^a (\partial\!\!\!/ + m_a)\psi^a + \mathcal{L}_{int}$
$\mathcal{L}_{int} =$


Interactions
- Spin 0, 1/2:
ħ=c=1
N₀ spin 0 : φ^i (real)
N½ spin 1/2 : ψ^a (majorana)
L = Σ_i (∂_μ φ^i ∂^μ φ^i + μ_i² φ^i φ^i) − ½ Σ_a ψ̄^a (∂̸ + m_a) ψ^a
+ L_int
[L] = 4


Interactions
Spins 0, 1/2:
N_0 spin 0 : ϕ^i (real)
$N_{1/2}$ spin 1/2 : ψ^a (majorana)
$+ \mathcal{L}_{int}$
$[\mathcal{L}] = 4$ i.e. dim of $\mathcal{L}$ is $(mass)^4$
$[\partial_\mu] = 1$


Interactions
- Spins 0, 1/2:
ħ = c = 1
N_0 spin 0 : φ^i (real)
N_1/2 spin 1/2 : ψ^a (majorana)
L = - A - 1/2 Σ (∂_μ φ^i ∂^μ φ^i + m_i^2 φ^i φ^i) - 1/2 Σ ψ̄^a (∂̸ + m_a) ψ^a
+ L_int
S = ∫ d^4x L
[L] = 4 i.e. dim of L is (mass)^4
[∂_μ] = 1 ... ∂ ... mass
[φ^i] = 1 ... φ^i ... mass


Interactions
- Spins 0, 1/2:
N0 spin 0 : φ^i (real)
N1/2 spin 1/2 : ψ^a (majorana)
= - A - 1/2 Σ (∂μ φ^i ∂^μ φ^i + m_i^2 φ^i φ^i) - 1/2 Σ ψ̄^a (∂̸ + m_a) ψ^a
+ L_int
∫d^4x
[L] = 4 ie dim of L is (mass)^4
[A] = 4
[∂μ] = 1
[φ^i] = 1
[ψ^a] = 3/2
mass
mass

Interactions

- Spins $0, 1/2$:

$\hbar = c = 1$

N_0 spin 0 : ϕ^i (real)

$N_{1/2}$ spin $1/2$: ψ^a (majorana)

$$\mathcal{L} = -A - \frac{1}{2}\sum_i \left(\partial_\mu \phi^i \partial^\mu \phi^i + m_i^2 \phi^i \phi^i\right) - \frac{1}{2}\sum_a \bar{\psi}^a \left(\not{\partial} + m_a\right)\psi^a + \mathcal{L}_{int}$$

$S = \int d^4x\, \mathcal{L}$

$[\mathcal{L}] = 4$ i.e. dim of $\mathcal{L}$ is $(\text{mass})^4$

$[A] = 4$

$[\partial_\mu] = 1$ ... ∂ .. mass

$[\phi^i] = 1$ ... ϕ^i .. mass

$[\psi^a] = 3/2$

Interactions

- Spins 0, 1/2:

$\hbar = c = 1$

N_0 spin 0 : ϕ^i (real)

$N_{1/2}$ spin 1/2 : ψ^a (majorana)

$$\mathcal{L} = -A - \frac{1}{2}\sum_i \left(\partial_\mu \phi^i \partial^\mu \phi^i + m_i^2 \phi^i \phi^i\right) - \frac{1}{2}\sum_a \bar{\psi}^a \left(\not{\partial} + m_a\right)\psi^a + \mathcal{L}_{int}$$

$$S = \int d^4x \, \mathcal{L}$$

$[\mathcal{L}] = 4$ i.e. dim of $\mathcal{L}$ is $(\text{mass})^4$

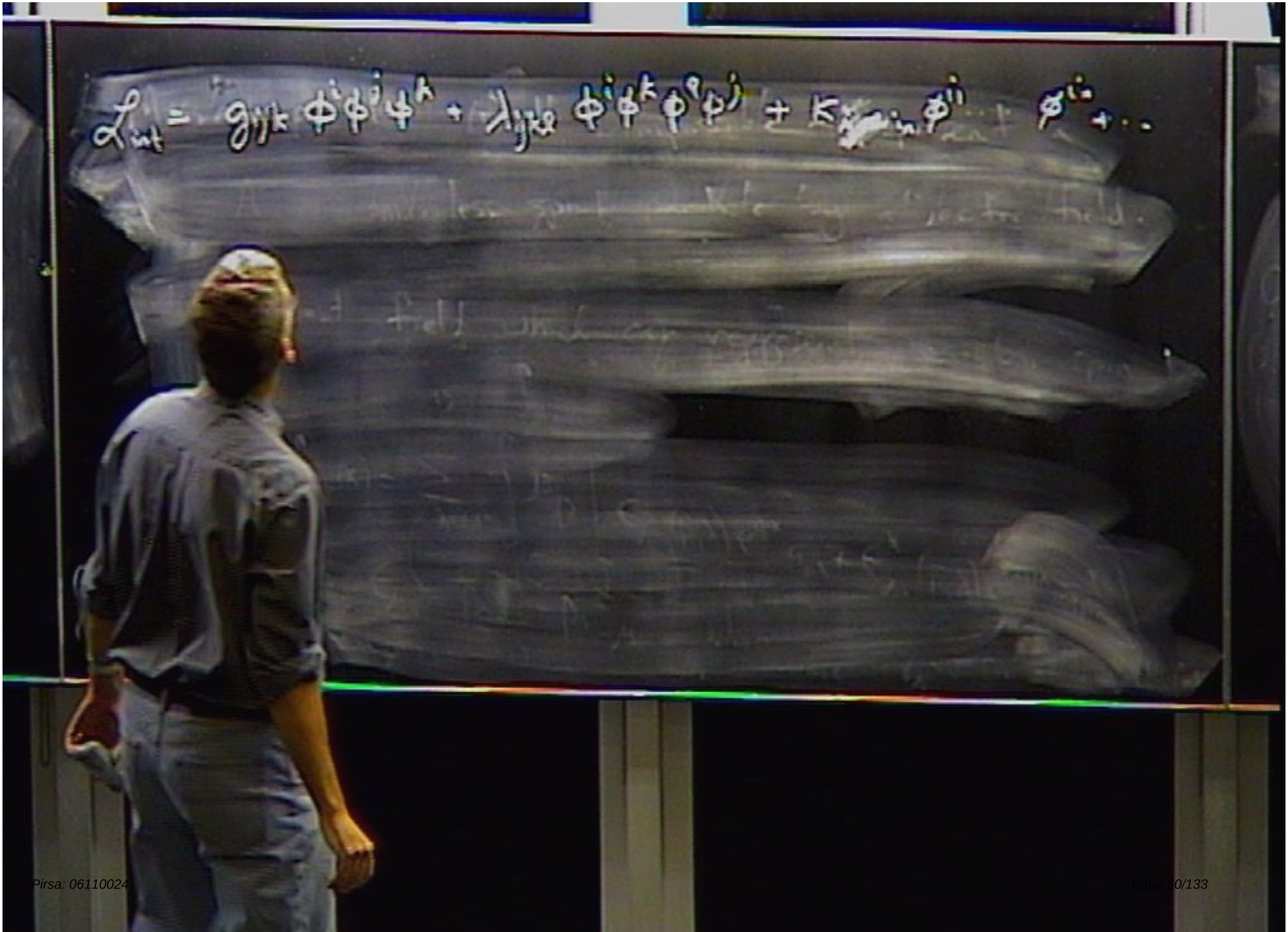
$[A] = 4$

$[\partial_\mu] = 1$ ... ∂ ~ mass

$[\phi^i] = 1$ ... ϕ^i ~ mass

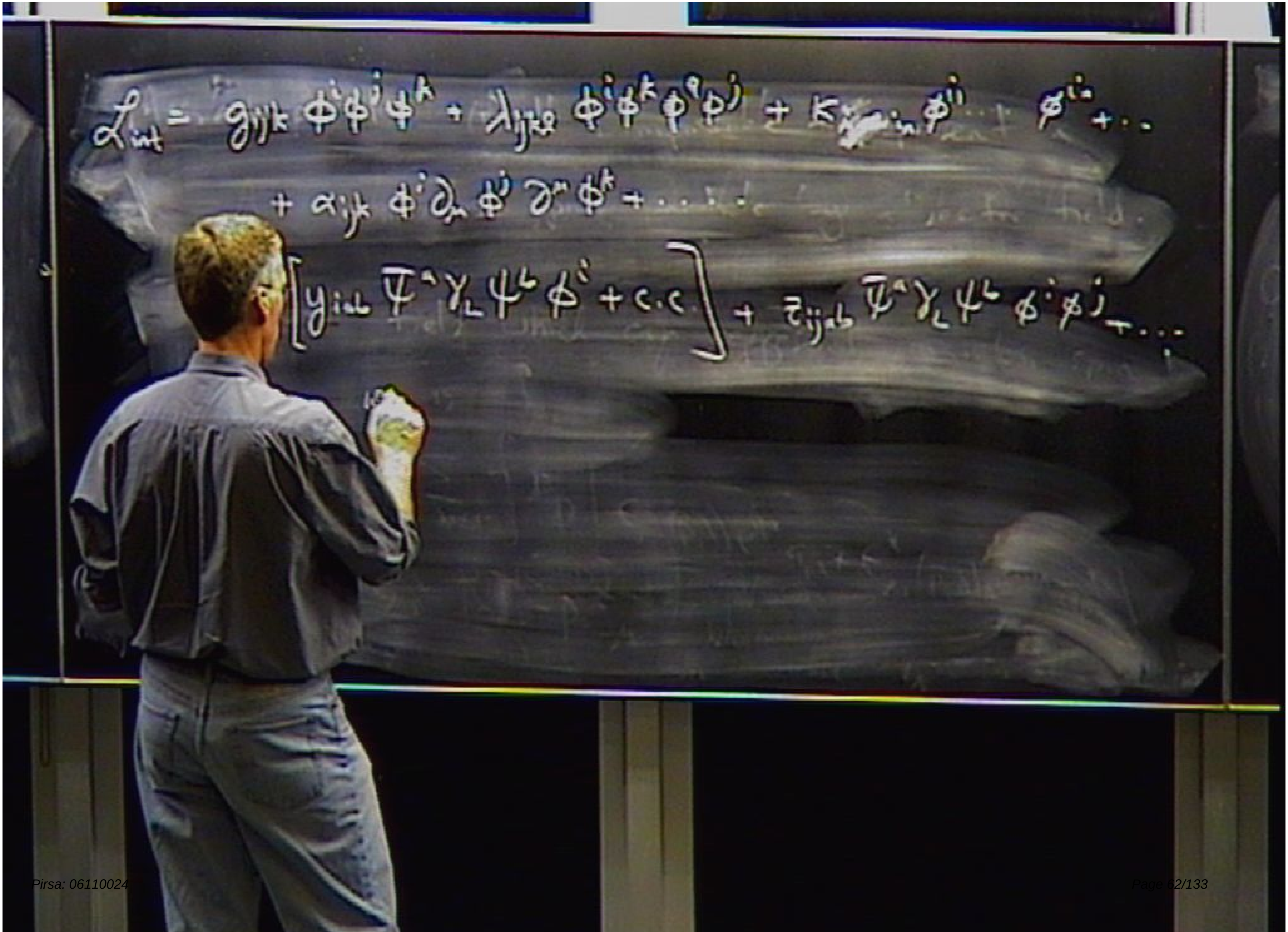
$[\psi^a] = 3/2$

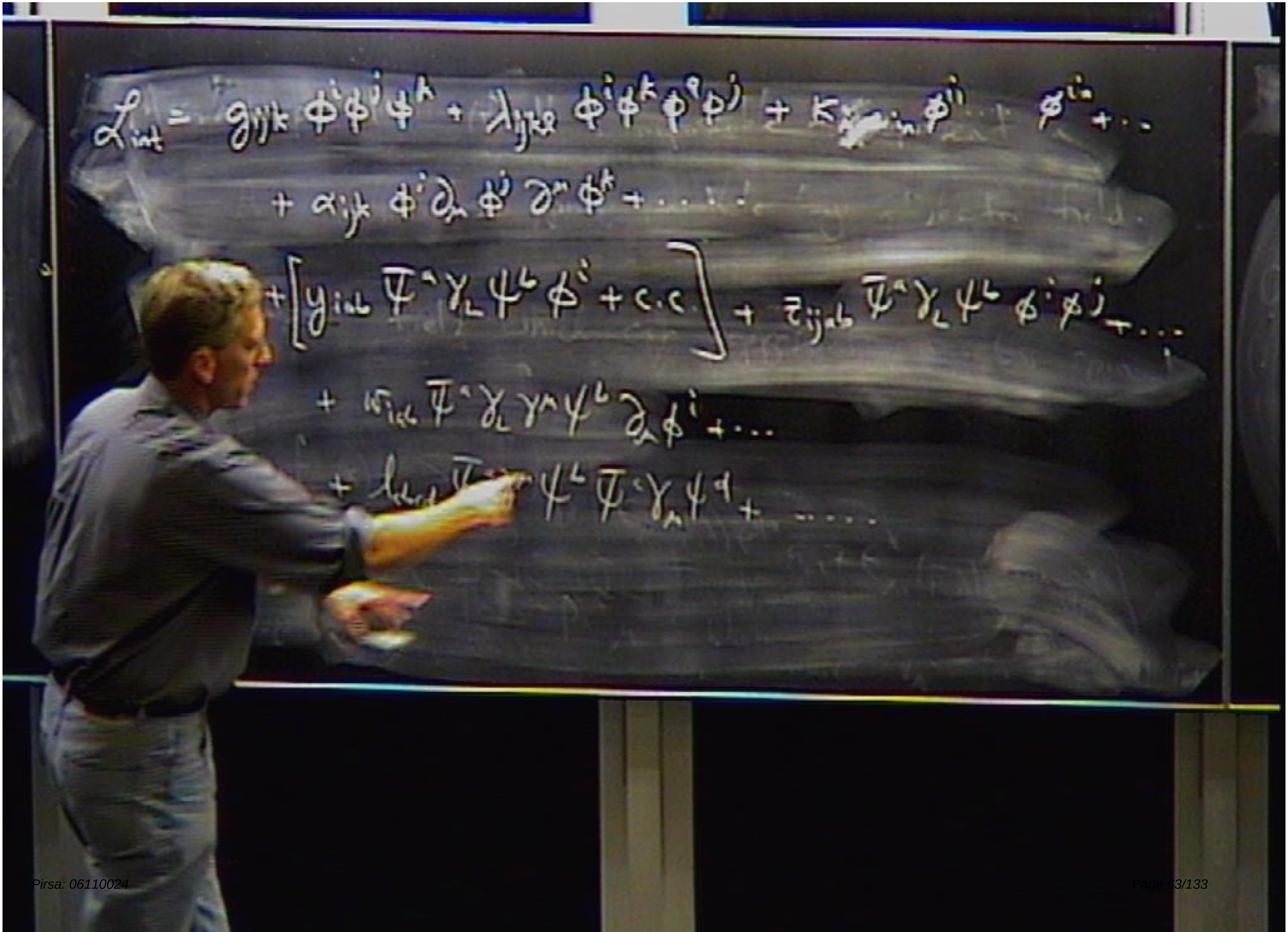
$\mathcal{L}_{int} =$


$\mathcal{L}_{int} = g_{ijk}\phi^i\phi^j\phi^k + \lambda_{ijkl}\phi^i\phi^k\phi^l\phi^j + \ldots$

$$\mathcal{L}_{int} = g_{ijk}\,\phi^i \phi^j \phi^k + \lambda_{ijkl}\,\phi^i \phi^k \phi^l \phi^j + \kappa_{i_1 \ldots i_n}\,\phi^{i_1} \cdots \phi^{i_n} + \ldots$$

$$+ \alpha_{ijk}\,\phi^i \partial_\mu \phi^j \partial^\mu \phi^k + \ldots$$


$\mathcal{L}_{int} = g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^q\phi^j + \ldots$
$+\ \alpha_{ijk}\,\phi^i\partial_\mu\phi^j\partial^\mu\phi^k + \ldots$
$\left[y_{iab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i + c.c\right] + z_{ijab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i\phi^j + \ldots$


$\mathcal{L}_{int} = g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^l\phi^j + \kappa_{i_1 \ldots i_n}\phi^{i_1} \ldots \phi^{i_n} + \ldots$
$+ \alpha_{ijk}\,\phi^i\partial_\mu\phi^j\partial^\mu\phi^k + \ldots$
$+ \left[y_{iab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i + c.c\right] + z_{ijab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i\phi^j + \ldots$
$+ w_{iab}\,\bar{\Psi}^a\gamma_L\gamma^\mu\psi^b\partial_\mu\phi^i + \ldots$
$+ l_{abcd}\,\bar{\Psi}^a \ldots \psi^b\bar{\Psi}^c\gamma_\mu\psi^d + \ldots$

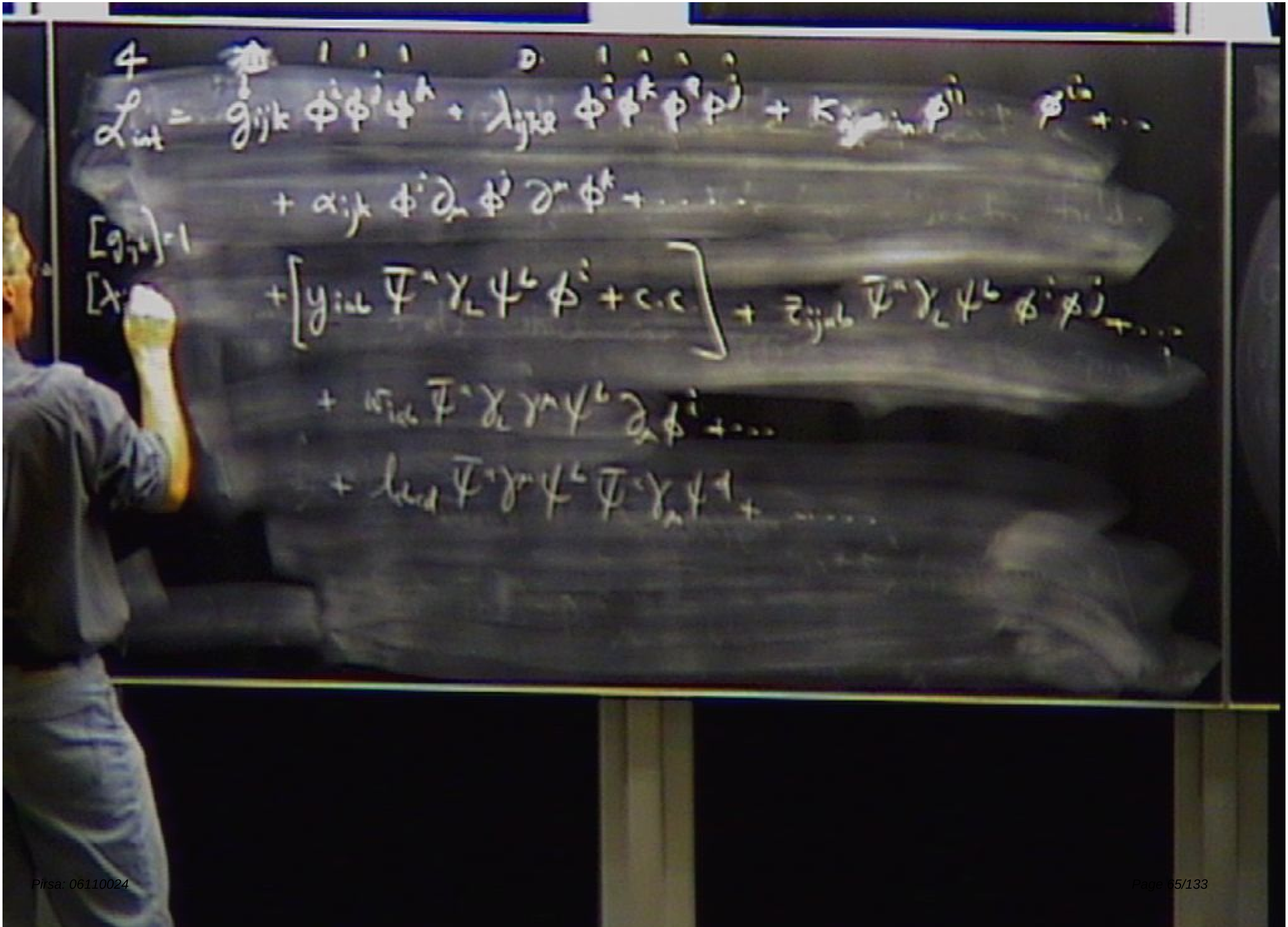
$$\mathcal{L}_{int} = g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^l\phi^j + \kappa_{i_1\ldots i_n}\,\phi^{i_1}\cdots\phi^{i_n} + \ldots$$

$$+\ \alpha_{ijk}\,\phi^i\,\partial_\mu\phi^j\,\partial^\mu\phi^k + \ldots$$

$$+\left[y_{iab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i + c.c\right] + z_{ijab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i\phi^j + \ldots$$

$$+\ w_{iab}\,\bar{\Psi}^a\gamma_L\gamma^\mu\psi^b\,\partial_\mu\phi^i + \ldots$$

$$+\ l_{abcd}\,\bar{\Psi}^a\gamma^\mu\psi^b\,\bar{\Psi}^c\gamma_\mu\psi^d + \ldots\ldots$$


$\mathcal{L}_{int} = g_{ijk}\phi^i\phi^j\phi^k + \lambda_{ijkl}\phi^i\phi^k\phi^l\phi^j + \kappa_{ij\cdots m}\phi^i \cdots \phi^m + \ldots$
$+ \alpha_{ijk}\phi^i\partial_\mu\phi^j\partial^\mu\phi^k + \ldots$
$[g_{ijk}] = 1$
$+ [y_{iab}\bar{\Psi}^a\gamma_L\psi^b\phi^i + c.c] + z_{ijab}\bar{\Psi}^a\gamma_L\psi^b\phi^i\phi^j + \ldots$
$+ w_{iab}\bar{\Psi}^a\gamma_L\gamma^\mu\psi^b\partial_\mu\phi^i + \ldots$
$+ l_{abcd}\bar{\Psi}^a\gamma^\mu\psi^b\bar{\Psi}^c\gamma_\mu\psi^d + \ldots$

$$
\begin{array}{ccccccccc}
4 & & 1\;1\;1 & & 0 & 1\;1\;1\;1 & & 4-n & 1 \cdots 1
\end{array}
$$

$$\mathcal{L}_{int} = g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^l\phi^j + \kappa_{i_1\cdots i_n}\,\phi^{i_1}\cdots\phi^{i_n} + \cdots$$

$$+\ \alpha_{ijk}\,\phi^i\,\partial_\mu\phi^j\,\gamma^\mu\,\phi^k + \cdots$$

$$[g_{ijk}] = 1$$

$$[\lambda_{ijkl}] = 0$$

$$[\kappa_{i_1\cdots i_n}] = 4-n$$

$$+\left[y_{iab}\,\bar\Psi^a\gamma_L\psi^b\phi^i + c.c.\right] + z_{ijab}\,\bar\Psi^a\gamma_L\psi^b\phi^i\phi^j + \cdots$$

$$+\ w_{iab}\,\bar\Psi^a\gamma_L\gamma^\mu\psi^b\,\partial_\mu\phi^i + \cdots$$

$$+\ l_{abcd}\,\bar\Psi^a\gamma^\mu\psi^b\,\bar\Psi^c\gamma_\mu\psi^d + \cdots$$

4 1 1 1 0 1 1 1 1 4-n 1 ... 1

$$\mathcal{L}_{int} = g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^l\phi^j + \kappa_{i_1\cdots i_n}\,\phi^{i_1}\cdots\phi^{i_n} + \dots$$

$$+\ \alpha_{ijk}\,\phi^i\,\partial_\mu\phi^j\,\partial^\mu\phi^k + \dots$$

1 1 1 1 1

$$+\left[y_{iab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i + c.c.\right] + z_{ijab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i\phi^j + \dots$$

3/2 3/2 1 3/2 3/2 1 1

$$+\ w_{iab}\,\bar{\Psi}^a\gamma_L\gamma^\mu\psi^b\,\partial_\mu\phi^i + \dots$$

$$+\ l_{abcd}\,\bar{\Psi}^a\gamma^\mu\psi^b\,\bar{\Psi}^c\gamma_\mu\psi^d + \dots$$

$[g_{ijk}]=1$

$[\lambda_{ijkl}]=0$

$[\kappa_{i_1\cdots i_n}]=4-n$

$[\alpha_{ijk}]=-1$

$[y_{iab}]=0$

[

$$\mathcal{L}_{int} = g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^q\phi^j + K_{i_1\cdots i_n}\,\phi^{i_1}\cdots\phi^{i_n} + \cdots$$

$$+\ \alpha_{ijk}\,\phi^i\partial_\mu\phi^j\partial^\mu\phi^k + \cdots$$

$$+\left[y_{iab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i + c.c.\right] + z_{ijab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i\phi^j + \cdots$$

$$+\ w_{iab}\,\bar{\Psi}^a\gamma_L\gamma^\mu\psi^b\partial_\mu\phi^i + \cdots$$

$$+\ l_{abcd}\,\bar{\Psi}^a\gamma^\mu\psi^b\bar{\Psi}^c\gamma_\mu\psi^d + \cdots$$

$[g_{ijk}]=1$

$[\lambda_{ijkl}]=0$

$[K_{i_1\cdots i_n}]=4-n$

$[\alpha_{ijk}]=-1$

$[y_{iab}]=0$

$[z_{ijab}]=-1$

$[w]=-1$ $\quad [l]=-2$

$$\begin{array}{l} \quad 4 \qquad\qquad 1\ 1\ 1 \qquad\qquad 0 \qquad 1\ 1\ 1\ 1 \qquad\qquad 4-n \qquad 1 \cdots 1 \\ \mathcal{L}_{int} = g_{ijk}\phi^i\phi^j\phi^k + \lambda_{ijkl}\phi^i\phi^k\phi^q\phi^j + K_{i_1\cdots i_n}\phi^{i_1}\cdots\phi^{i_n} + \cdots \end{array}$$

$$+ \alpha_{ijk}\underset{1}{\phi^i}\underset{1}{\partial_\mu}\underset{1}{\phi^j}\underset{1}{\partial^\mu}\underset{1}{\phi^k} + \cdots$$

$$+ \left[y_{iab}\underset{3/2}{\bar{\Psi}^a}\gamma_L\underset{3/2}{\psi^b}\underset{1}{\phi^i} + c.c.\right] + z_{ijab}\underset{3/2}{\bar{\Psi}^a}\gamma_L\underset{3/2}{\psi^b}\underset{1}{\phi^i}\underset{1}{\phi^j} + \cdots$$

$$+ w_{iab}\bar{\Psi}^a\gamma_L\gamma^\mu\psi^b\partial_\mu\phi^i + \cdots$$

$$+ l_{abcd}\bar{\Psi}^a\gamma^\mu\psi^b\bar{\Psi}^c\gamma_\mu\psi^d + \cdots$$

* $[g_{ijk}] = 1$
* $[\lambda_{ijkl}] = 0$

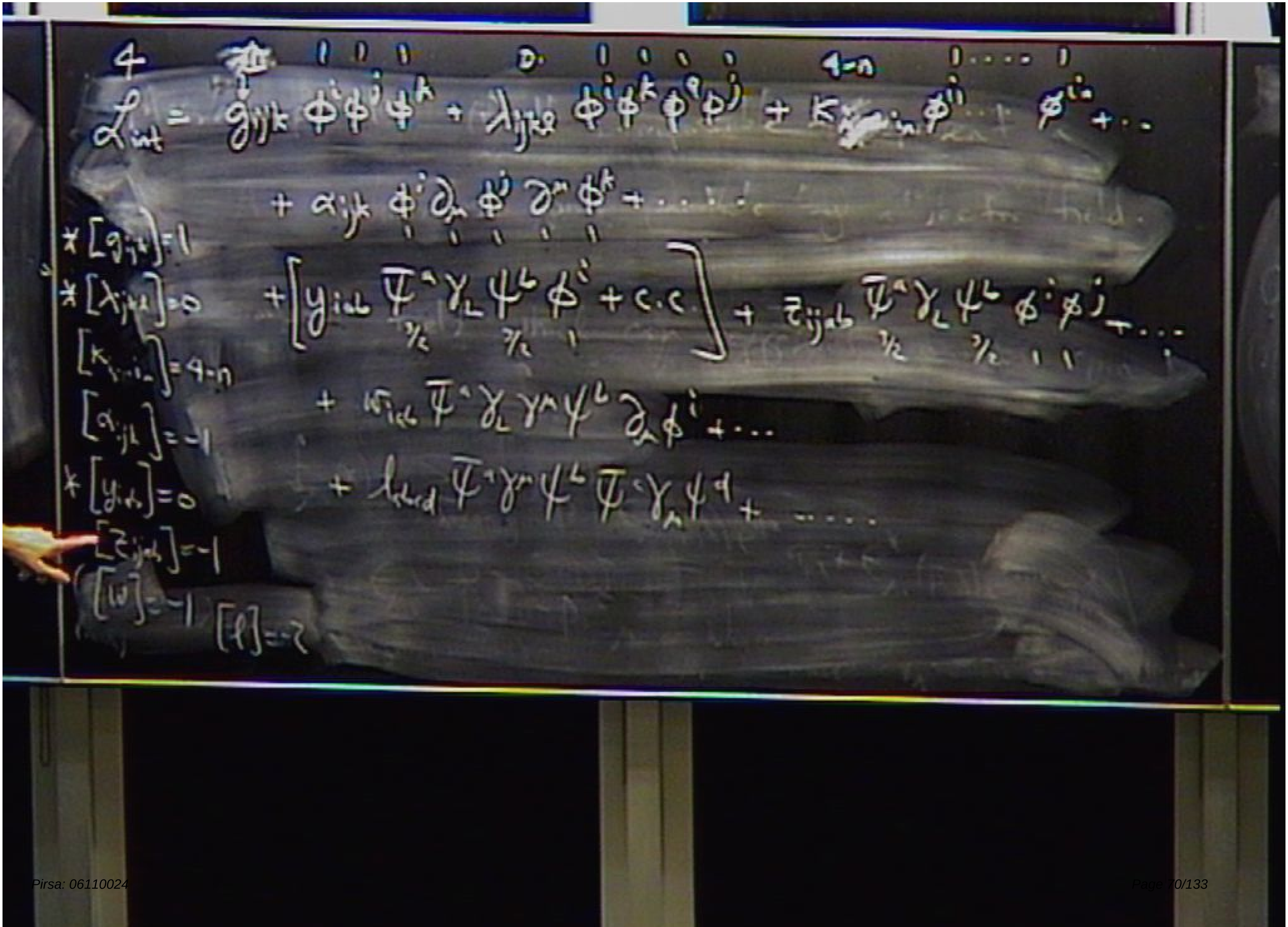
$[K_{i_1\cdots i_n}] = 4-n$

$[\alpha_{ijk}] = -1$

* $[y_{iab}] = 0$

$[z_{ijab}] = -1$

$[w] = -1$ $\quad [l] = -2$



$\mathcal{L}_{int} = g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^l\phi^j + \kappa_{i_1\cdots i_n}\phi^{i_1}\cdots\phi^{i_n} + \cdots$
$+ \alpha_{ijk}\,\phi^i\partial_\mu\phi^j\gamma^\mu\phi^k + \cdots$
$+ \left[y_{iab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i + c.c.\right] + z_{ijab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i\phi^j + \cdots$
$+ w_{iab}\,\bar{\Psi}^a\gamma_L\gamma^\mu\psi^b\partial_\mu\phi^i + \cdots$
$+ l_{abcd}\,\bar{\Psi}^a\gamma^\mu\psi^b\bar{\Psi}^c\gamma_\mu\psi^d + \cdots$
$*\,[g_{ijk}] = 1$
$*\,[\lambda_{ijkl}] = 0$
$[\kappa_{i_1\cdots i_n}] = 4-n$
$[\alpha_{ijk}] = -1$
$*\,[y_{iab}] = 0$
$[z_{ijab}] = -1$
$[w] = -1$
$[l] = -2$

$$4 \quad\quad 1\ 1\ 1 \quad\quad 0 \quad 1\ 1\ 1\ 1 \quad\quad 4-n \quad 1 \cdots 1$$

$$\mathcal{L}_{int} = g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^l\phi^j + \kappa_{i_1\cdots i_n}\,\phi^{i_1}\cdots\phi^{i_n} + \cdots$$

$$+\ \alpha_{ijk}\,\phi^i\partial_\mu\phi^j\partial^\mu\phi^k + \cdots$$

$$\scriptstyle -1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

$$+\left[y_{iab}\,\bar\Psi^a\gamma_L\psi^b\phi^i + c.c.\right] + z_{ijab}\,\bar\Psi^a\gamma_L\psi^b\phi^i\phi^j + \cdots$$

$$\scriptstyle 3/2 \quad 3/2 \quad 1 \quad\quad\quad 3/2 \quad 3/2 \quad 1 \quad 1$$

$$+\ w_{iab}\,\bar\Psi^a\gamma_L\gamma^\mu\psi^b\partial_\mu\phi^i + \cdots$$

$$+\ l_{abcd}\,\bar\Psi^a\gamma^\mu\psi^b\,\bar\Psi^c\gamma_\mu\psi^d + \cdots$$

$*\ [g_{ijk}] = 1$

$*\ [\lambda_{ijkl}] = 0$

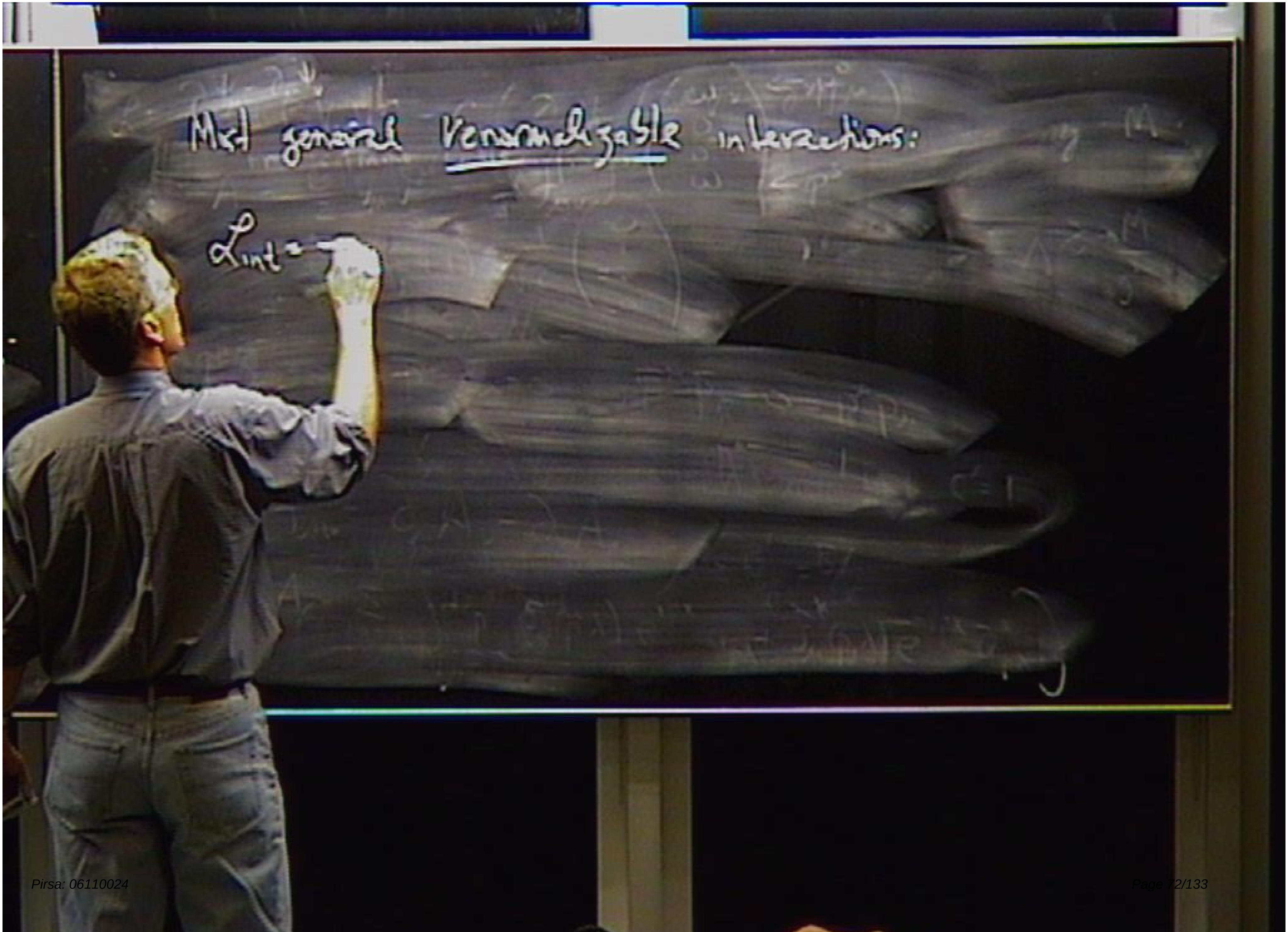
$[\kappa_{i_1\cdots i_n}] = 4-n$

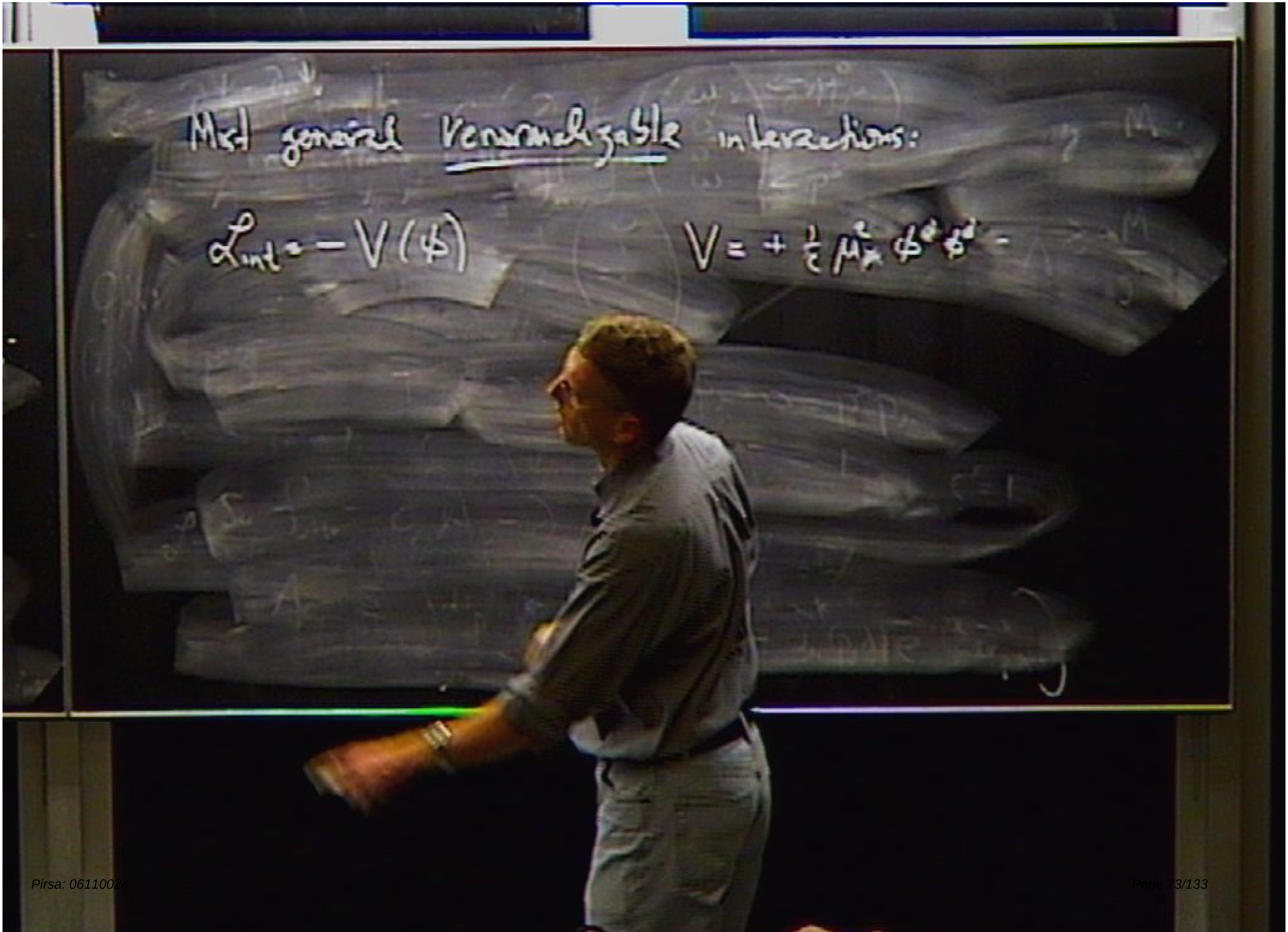
$[\alpha_{ijk}] = -1$

$*\ [y_{iab}] = 0$

$[z_{ijab}] = -1$

$[w] = -1 \quad [l] = -2$


Most general renormalizable interactions:
$\mathcal{L}_{int} =$


Most general renormalizable interactions:
$\mathcal{L}_{int} = -V(\phi)$
$V = + \frac{1}{2}\mu^2_{ab}\phi^a\phi^b -$

$$\mathcal{L}_{int} = -g_{ijk}\phi^i\phi^j\phi^k - \lambda_{ijkl}\phi^i\phi^k\phi^l\phi^j - \kappa_{i_1\ldots i_n}\phi^{i_1}\cdots\phi^{i_n} + \ldots$$

$$+ \alpha_{ijk}\phi^i\partial_\mu\phi^j\gamma^\mu\phi^k + \ldots$$

$$+\left[y_{iab}\bar{\Psi}^a\gamma_L\psi^b\phi^i + c.c.\right] + z_{ijab}\bar{\Psi}^a\gamma_L\psi^b\phi^i\phi^j + \ldots$$

$$+ w_{iab}\bar{\Psi}^a\gamma_L\gamma^\mu\psi^b\partial_\mu\phi^i + \ldots$$

$$+ l_{abcd}\bar{\Psi}^a\gamma^\mu\psi^b\bar{\Psi}^c\gamma_\mu\psi^d + \ldots$$

$*\ [g_{ijk}] = 1$

$*\ [\lambda_{ijkl}] = 0$

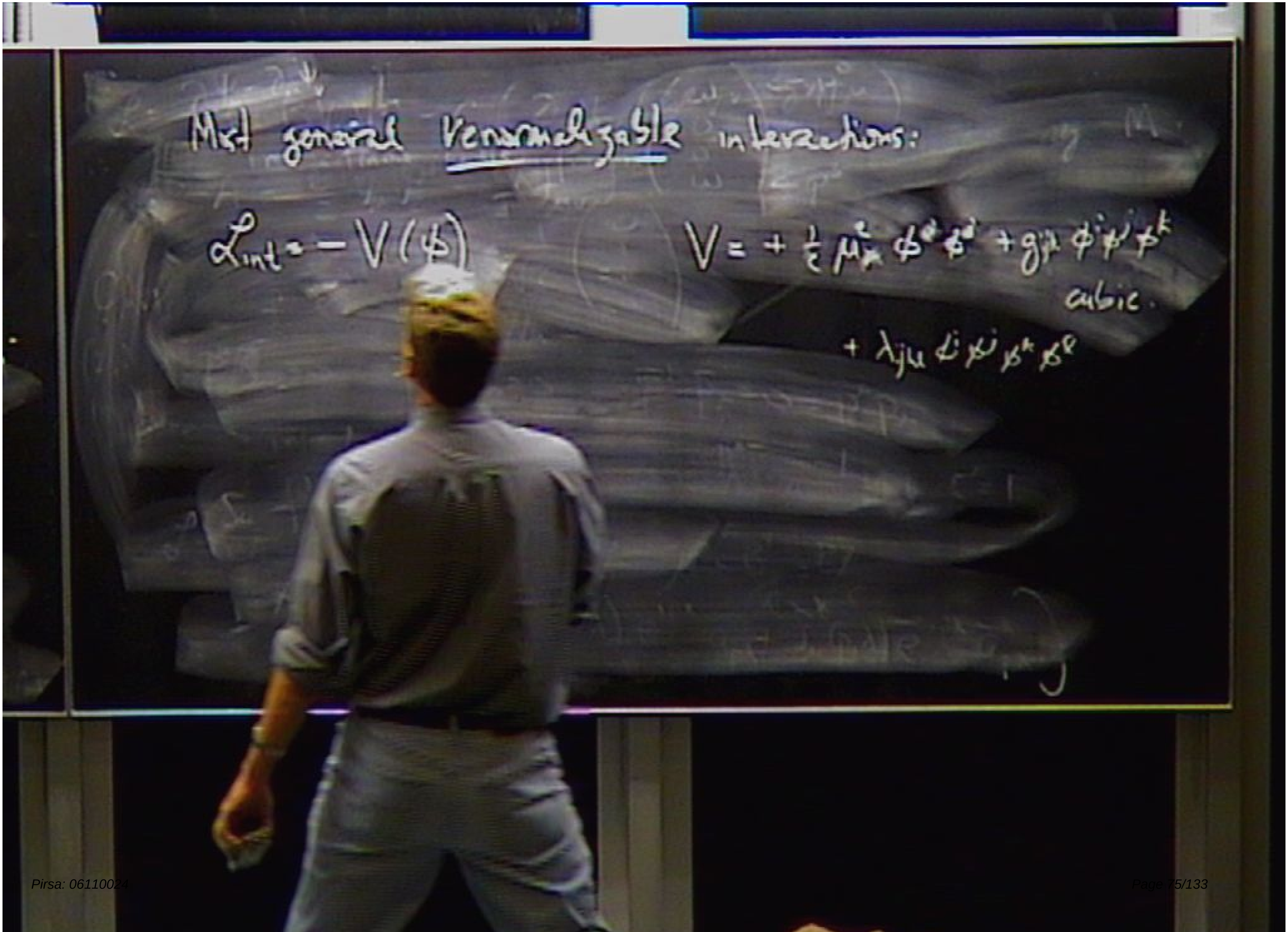
$[\kappa_{i_1\ldots i_n}] = 4-n$

$[\alpha_{ijk}] = -1$

$*\ [y_{iab}] = 0$

$[z_{ijab}] = -1$

$[w] = -1 \quad [l] = -2$


Most general renormalizable interactions:
$\mathcal{L}_{int} = -V(\phi)$
$V = + \frac{1}{2}\mu^2_{ab}\phi^a\phi^b + g_{ijk}\phi^i\phi^j\phi^k$
cubic.
$+ \lambda_{ijkl}\phi^i\phi^j\phi^k\phi^l$

$$\begin{array}{l} \qquad 4 \qquad\quad 1\;\;1\;\;1 \qquad\qquad 0 \quad 1\;\;1\;\;1\;\;1 \qquad\quad 4-n \quad 1 \cdots 1 \\ \mathcal{L}_{int} = -g_{ijk}\,\phi^i\phi^j\phi^k + \lambda_{ijkl}\,\phi^i\phi^k\phi^l\phi^j + \kappa_{i_1\cdots i_n}\,\phi^{i_1}\cdots\phi^{i_n} + \cdots \end{array}$$

$$\begin{array}{l} + \alpha_{ijk}\,\phi^i\,\partial_\mu\phi^j\,\partial^\mu\phi^k + \cdots \\ \qquad\quad\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \end{array}$$

$* \; [g_{ijk}] = 1$

$* \; [\lambda_{ijkl}] = 0$

$[\kappa_{i_1\cdots i_n}] = 4-n$

$[\alpha_{ijk}] = -1$

$* \; [y_{iab}] = 0$

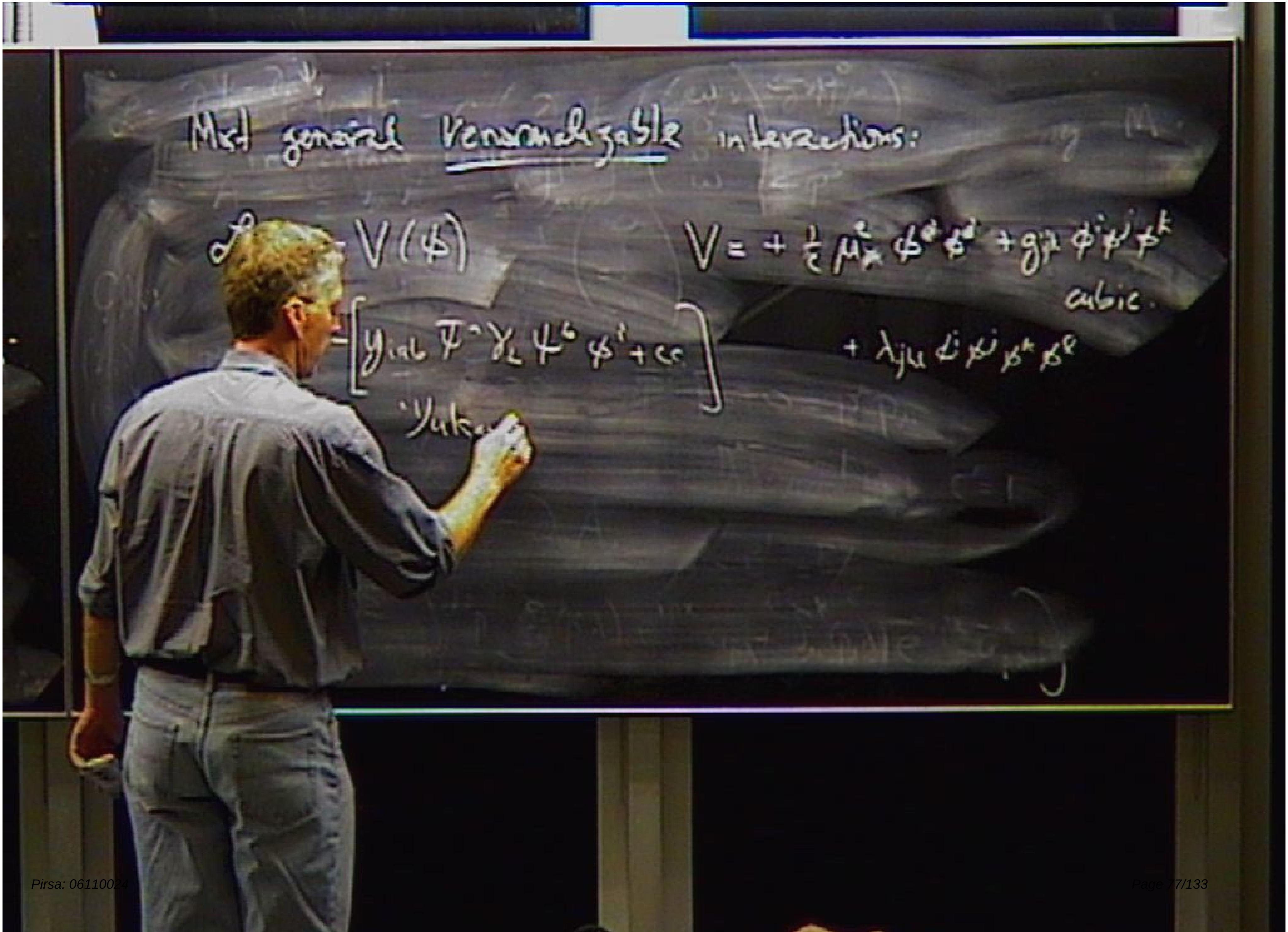
$[z_{ijab}] = -1$

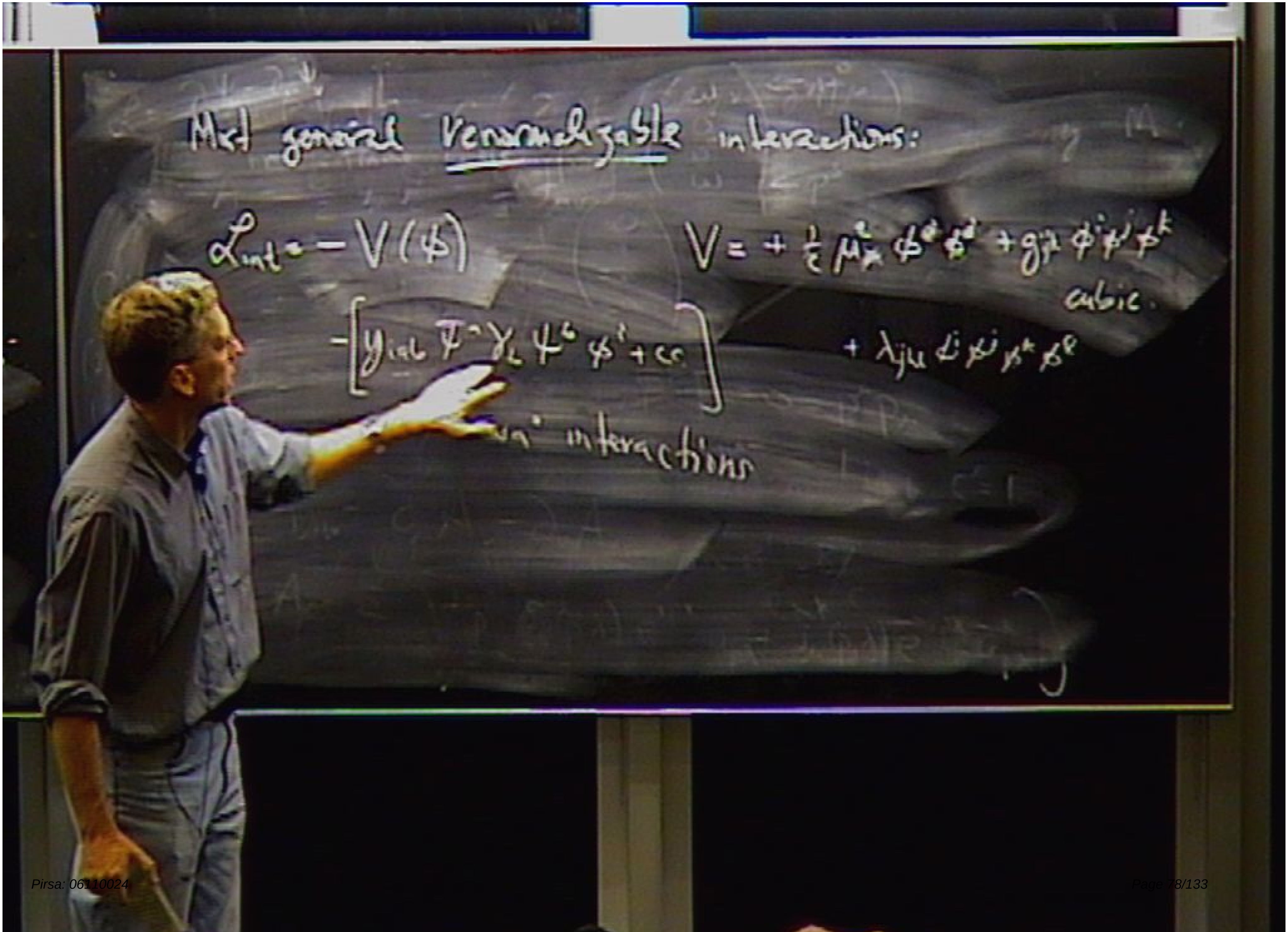
$[w] = -1 \quad [l] = -2$

$$\begin{array}{l} -\left[y_{iab}\,\bar\psi^a\gamma_L\psi^b\phi^i + c.c.\right] + z_{ijab}\,\bar\psi^a\gamma_L\psi^b\phi^i\phi^j + \cdots \\ \qquad\quad\;\; 3/2 \qquad\; 3/2 \;\; 1 \qquad\qquad\qquad\quad 3/2 \qquad 3/2 \;\; 1 \;\; 1 \end{array}$$

$$+ w_{iab}\,\bar\psi^a\gamma_L\gamma^\mu\psi^b\,\partial_\mu\phi^i + \cdots$$

$$+ l_{abcd}\,\bar\psi^a\gamma^\mu\psi^b\,\bar\psi^c\gamma_\mu\psi^d + \cdots$$


Most general renormalizable interactions:
$V = + \frac{1}{2}\mu^2_{ij}\phi^i\phi^j + g_{ijk}\phi^i\phi^j\phi^k$
cubic
$+ \lambda_{ijkl}\phi^i\phi^j\phi^k\phi^l$
$[y_{iab}\bar{\psi}^a\gamma_L\psi^b\phi^i + c.c.]$


Most general renormalizable interactions:
$\mathcal{L}_{int} = -V(\phi)$
$V = +\frac{1}{2}\mu^2_{ab}\phi^a\phi^b + g_{ijk}\phi^i\phi^j\phi^k$
cubic
$-\left[y_{iab}\,\bar{\Psi}^a\gamma_L\psi^b\phi^i + cc\right]$
$+\lambda_{ijkl}\phi^i\phi^j\phi^k\phi^l$
interactions

Most general renormalizable interactions:

$$\mathcal{L}_{int} = -V(\phi) \qquad V = +\tfrac{1}{2}\mu^2_{ij}\,\phi^i\phi^j + g_{ijk}\,\phi^i\phi^j\phi^k$$

cubic

$$-\left[y_{iab}\,\bar\psi^a \gamma_L \psi^b \phi^i + cc\right] \qquad + \lambda_{ijkl}\,\phi^i\phi^j\phi^k\phi^l$$

"Yukawa" interactions

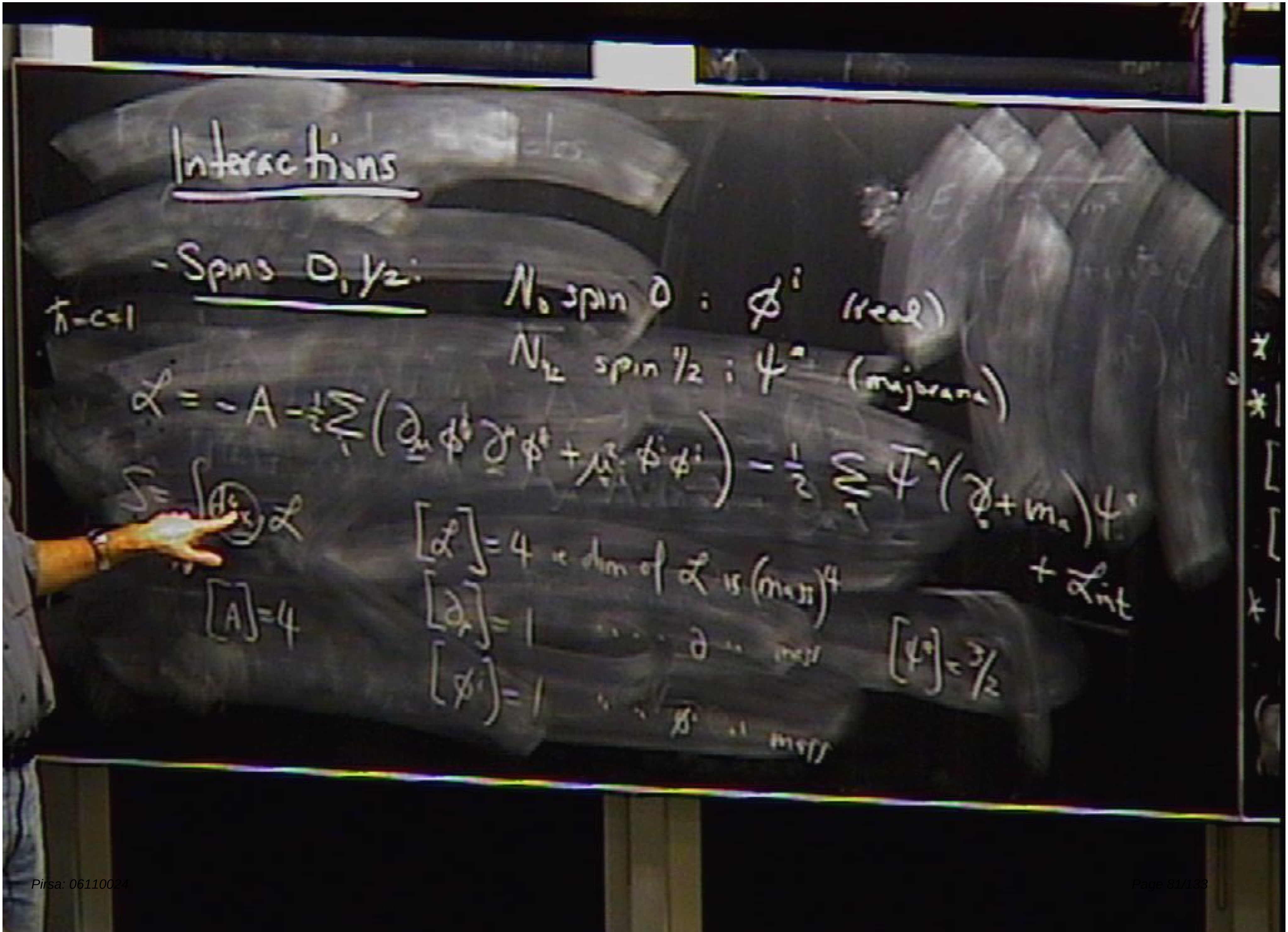
Most general renormalizable interactions:

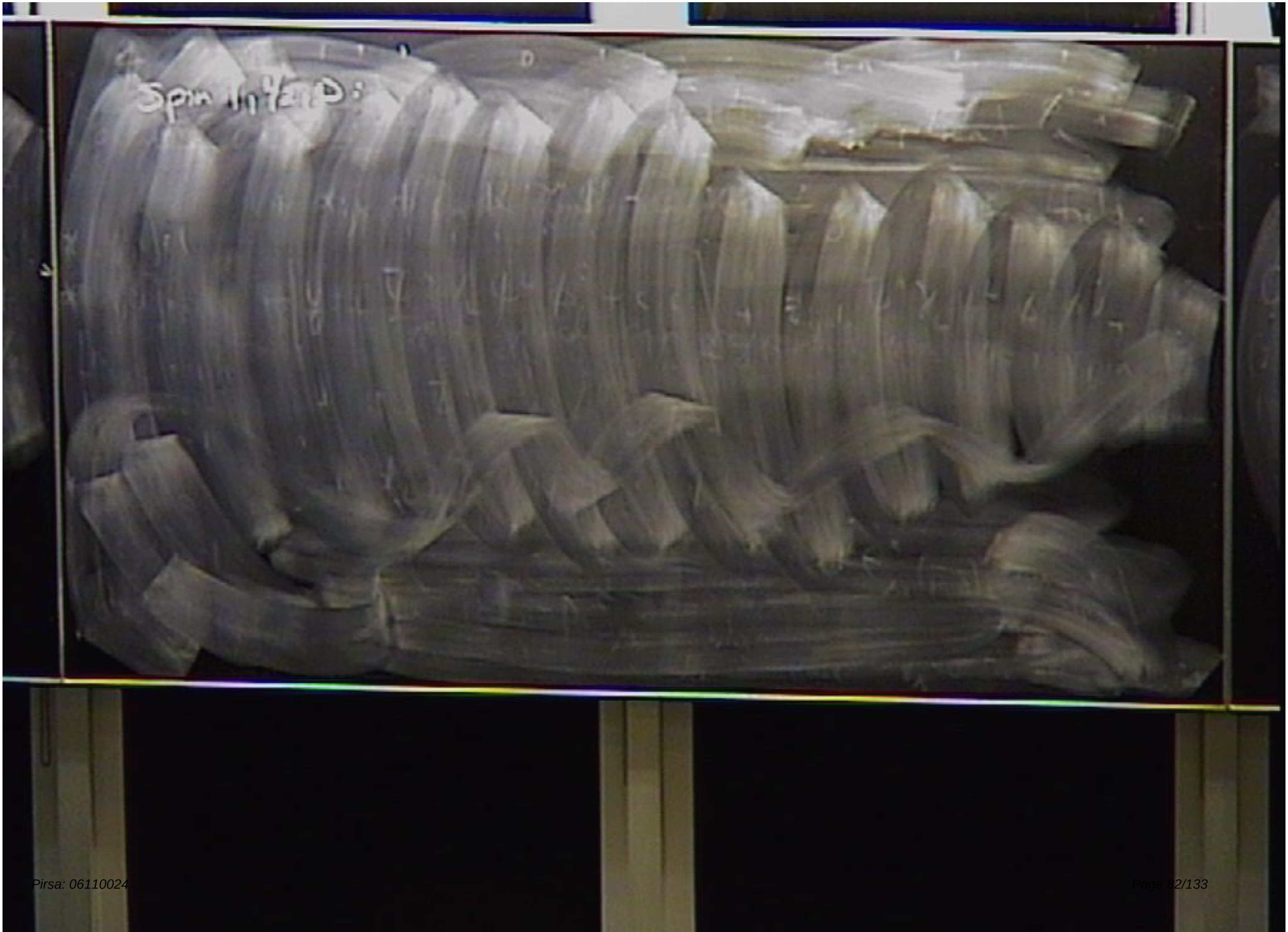
$$\mathcal{L}_{int} = -V(\phi) \qquad V = +\frac{1}{2}\mu^2_{ij}\phi^i\phi^j + g_{ijk}\phi^i\phi^j\phi^k$$

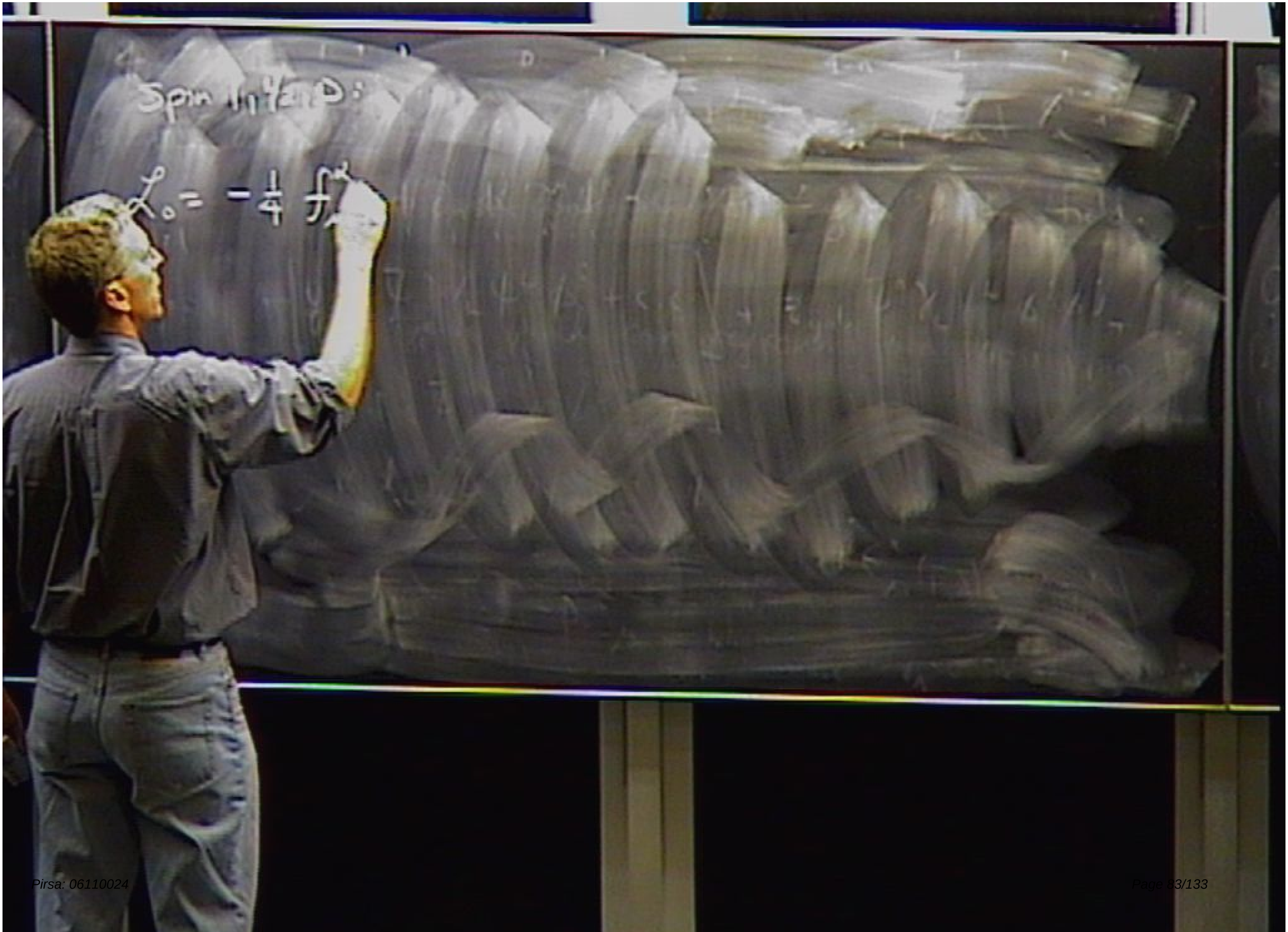
cubic

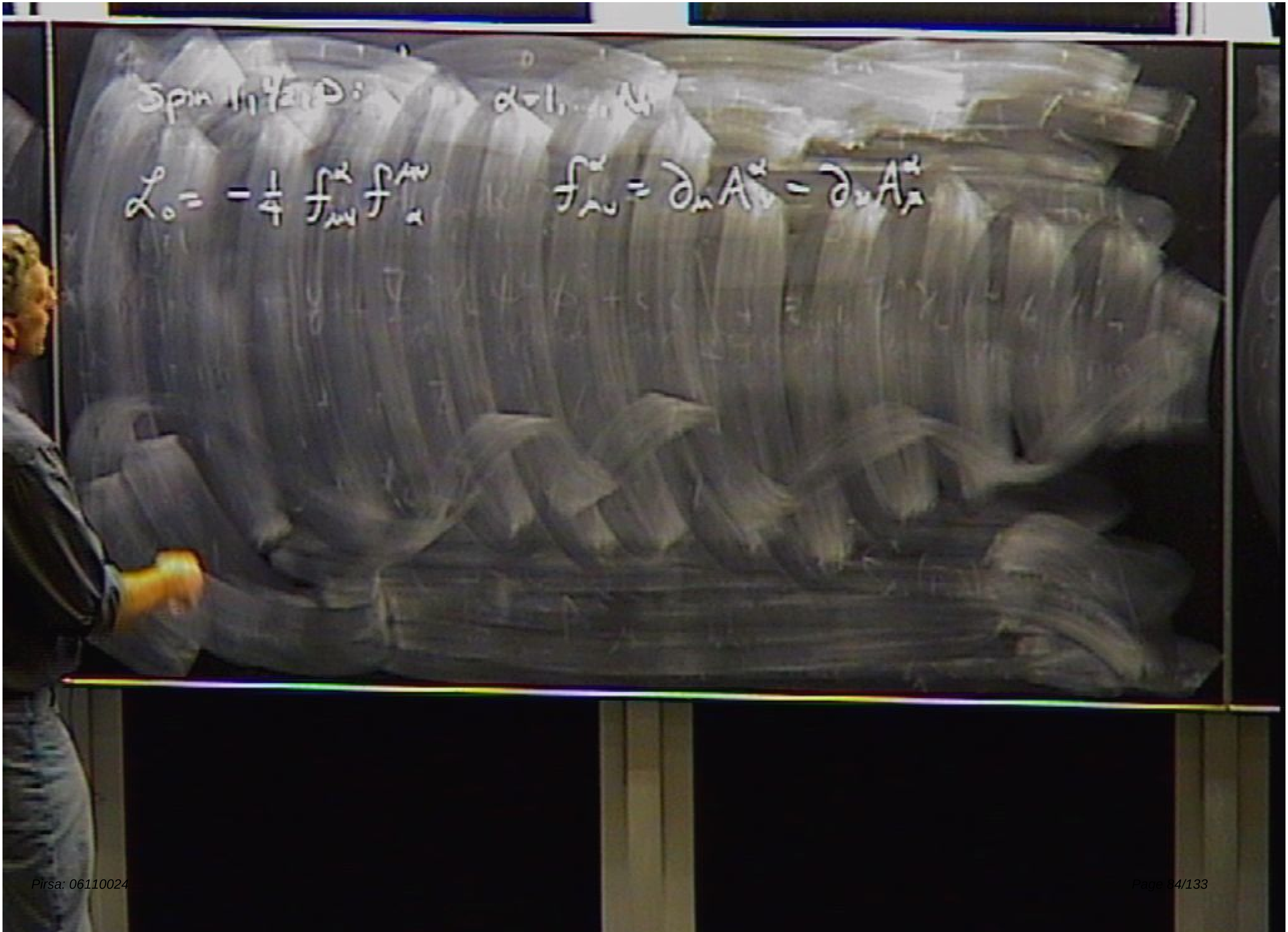
$$-\left[y_{iab}\bar{\psi}^a\gamma_L\psi^b\phi^i + c.c.\right] \qquad + \lambda_{ijkl}\phi^i\phi^j\phi^k\phi^l$$

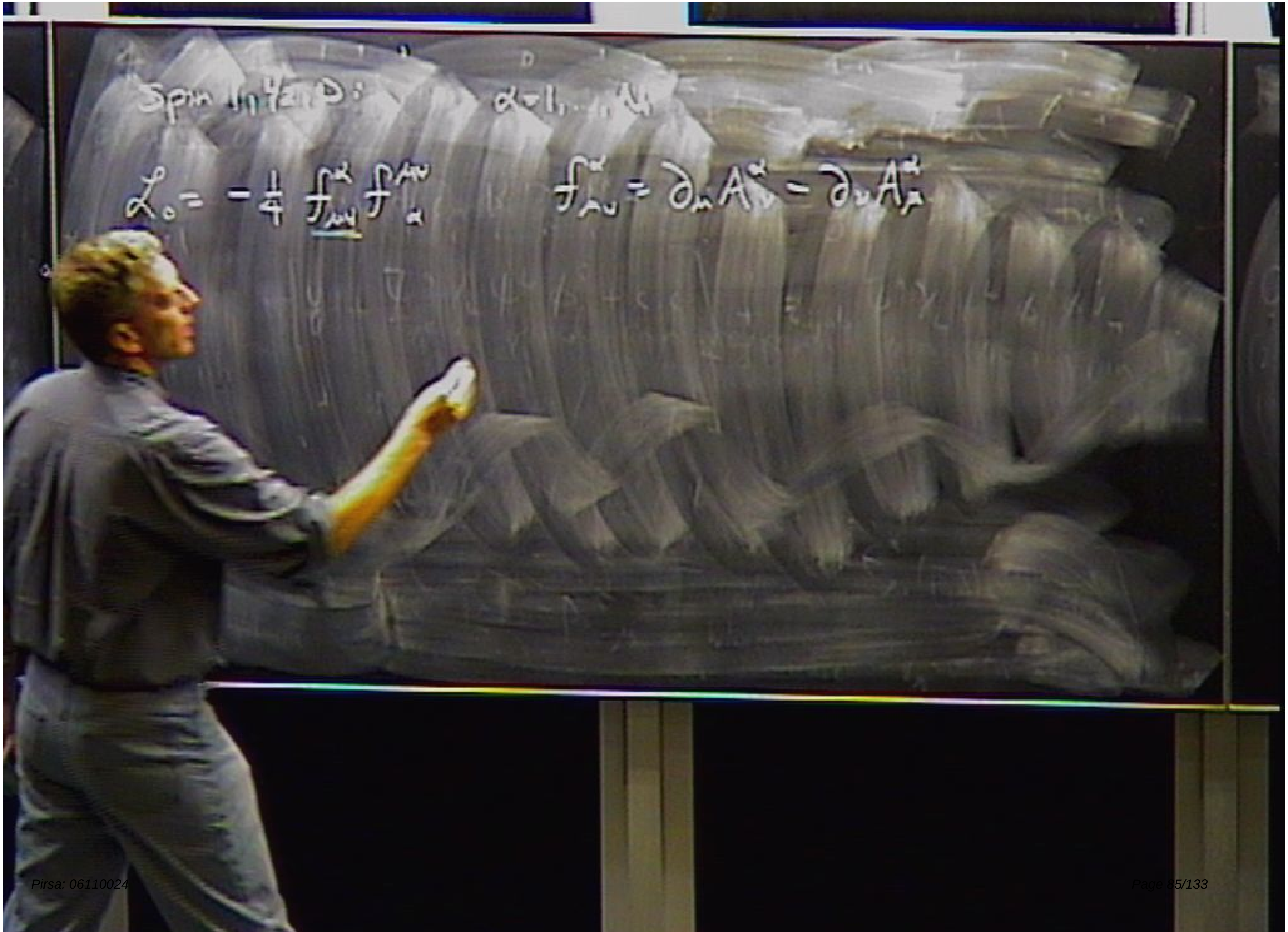
"Yukawa" interactions


Interactions
- Spins 0, 1/2:
$\hbar = c = 1$
N_0 spin 0 : ϕ^i (real)
$N_{1/2}$ spin 1/2 : ψ^a (majorana)
$\mathcal{L} = -A - \frac{1}{2}\sum_i \left(\partial_\mu \phi^i \partial^\mu \phi^i + \mu_i^2 \phi^i \phi^i\right) - \frac{1}{2}\sum_a \bar{\psi}^a \left(\partial\!\!\!/ + m_a\right)\psi^a + \mathcal{L}_{int}$
$S = \int d^4x \, \mathcal{L}$
$[\mathcal{L}] = 4 \Leftarrow$ dim of $\mathcal{L}$ is $(\text{mass})^4$
$[A] = 4$
$[\partial_\mu] = 1$ ... ∂ ~ mass
$[\phi^i] = 1$ ... ϕ^i ~ mass
$[\psi^a] = 3/2$

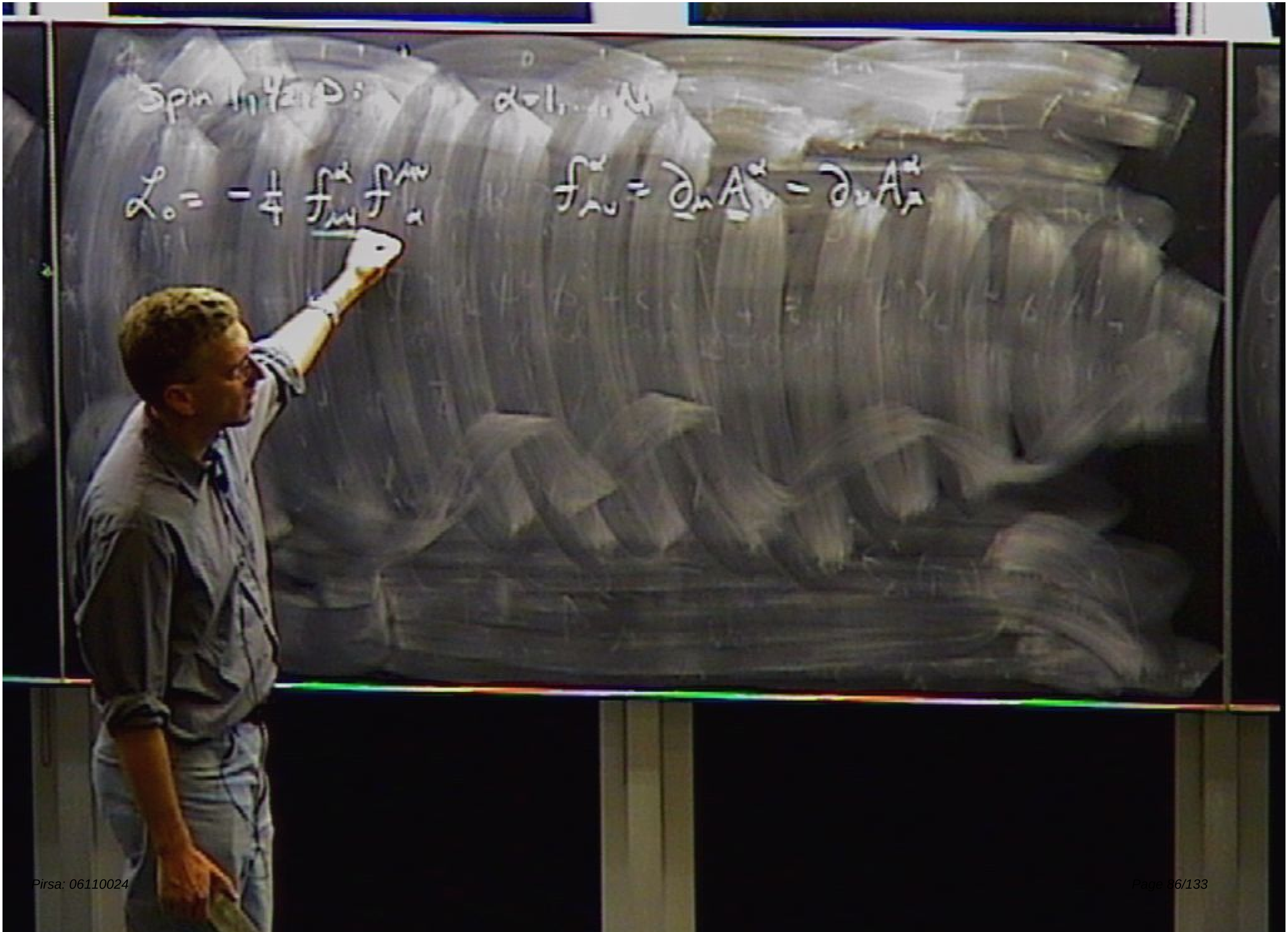

Spin in 4D:

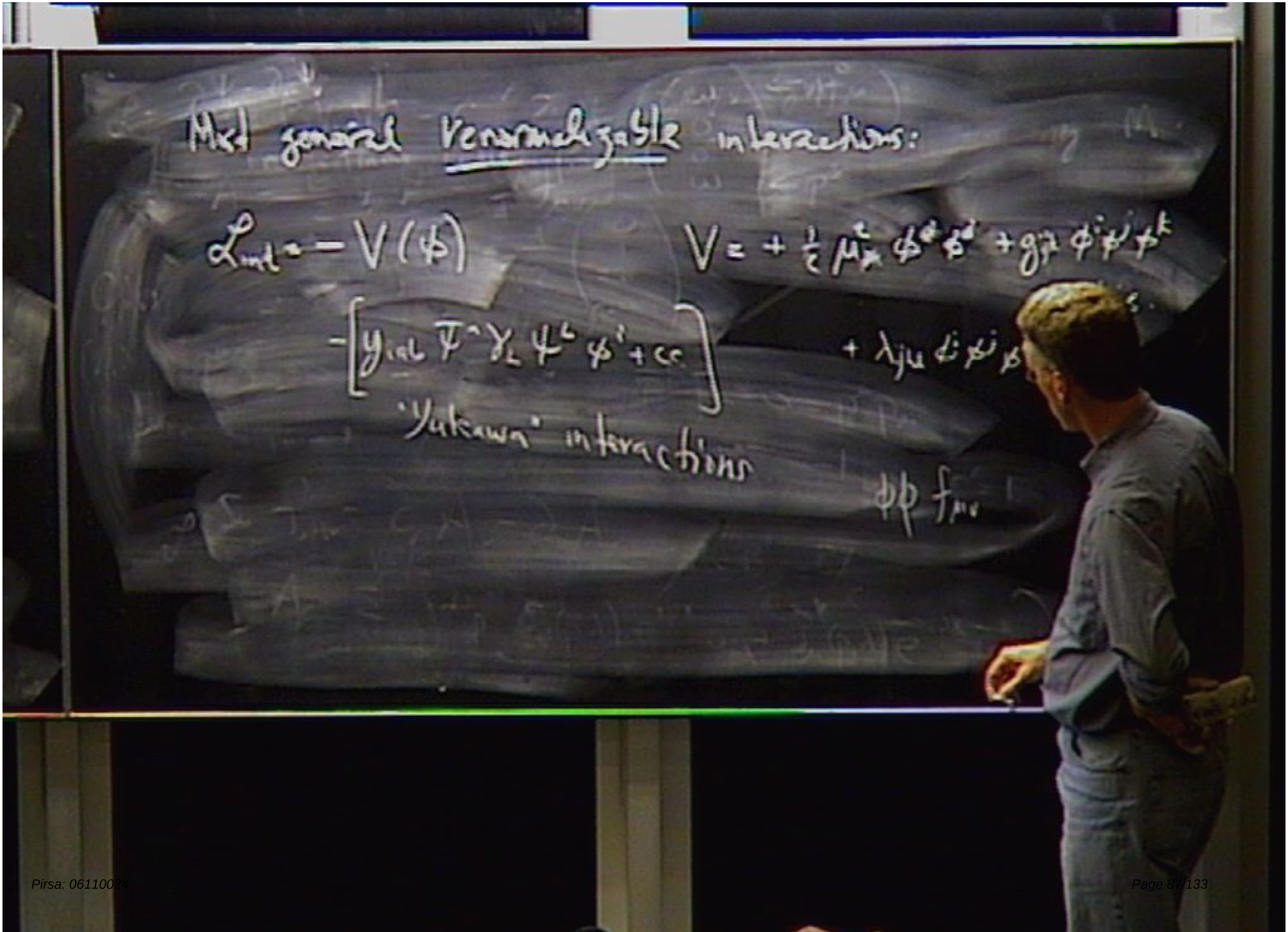

Spin 1 in 4 D:
$\mathcal{L}_0 = -\frac{1}{4} f^2$

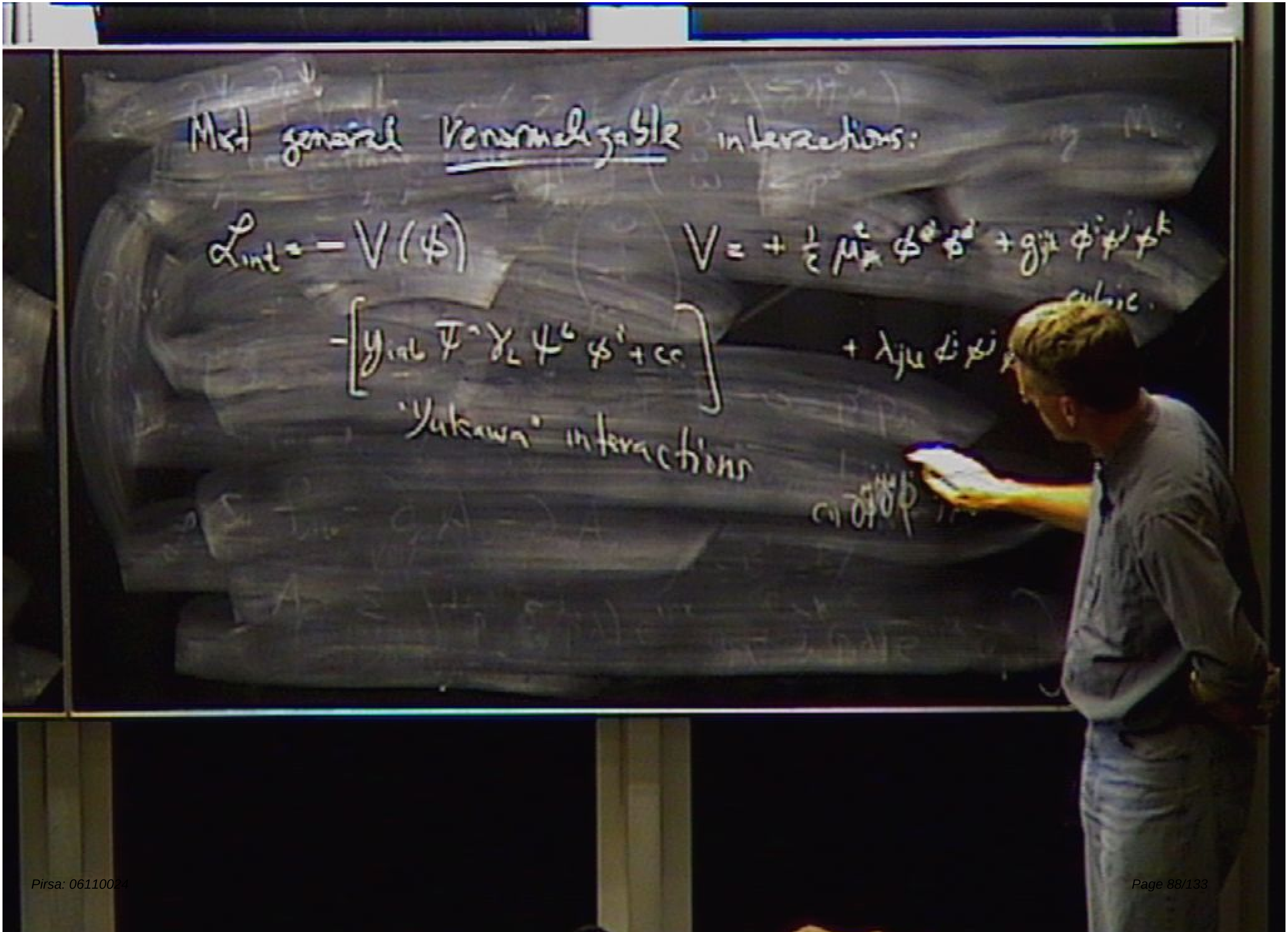

Spin 1, 1/2, 0:
$\alpha = 1, \ldots, N$
$\mathcal{L}_0 = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha}$
$f^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu}$



Spin 1, 4=D:
α = 1, ..., N
$\mathcal{L}_0 = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha}$
$f^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu}$


Spin 1 in 4 D:
α = 1, ..., N
$\mathcal{L}_0 = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha}$
$f^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_\nu - \partial_\nu A^{\alpha}_\mu$


Most general renormalizable interactions:
L_int = − V(φ)
V = + ½ μ²_ij φ^i φ^j + g_ijk φ^i φ^j φ^k
+ λ_ijkl φ^i φ^j φ
− [y_iab Ψ̄^a γ_L ψ^b φ^i + c.c.]
"Yukawa" interactions
φφ F_μν


Most general renormalizable interactions:
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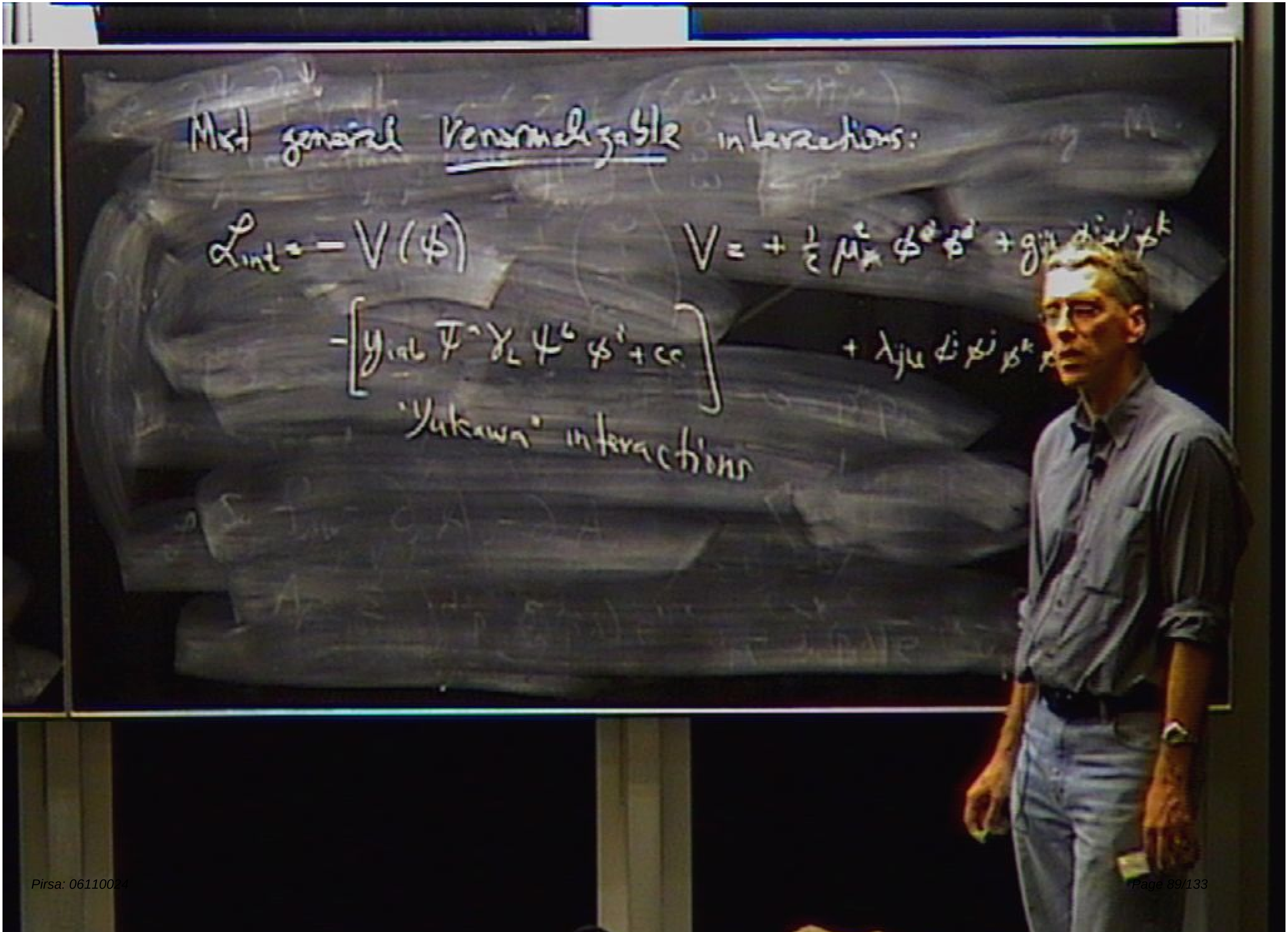
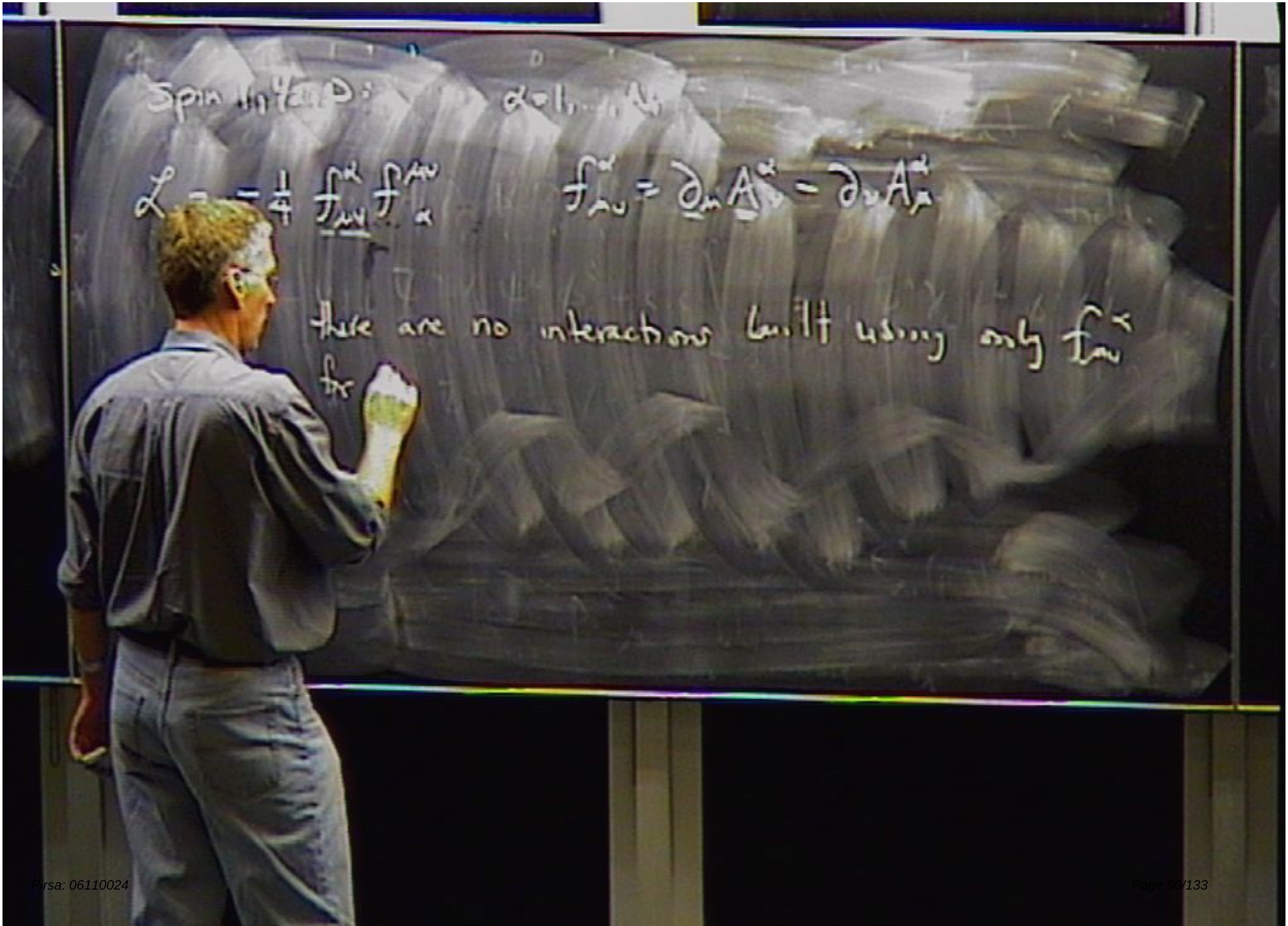
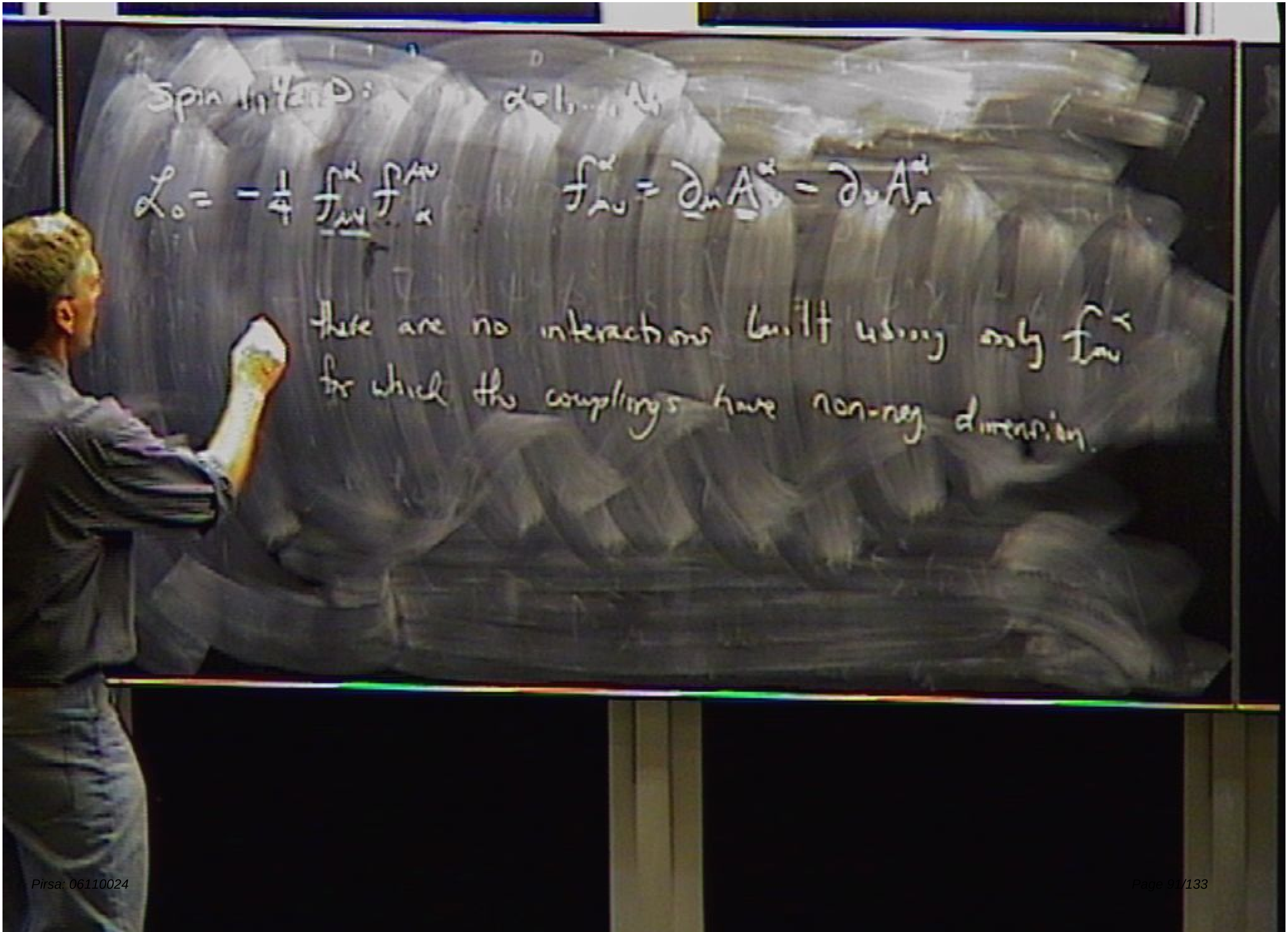
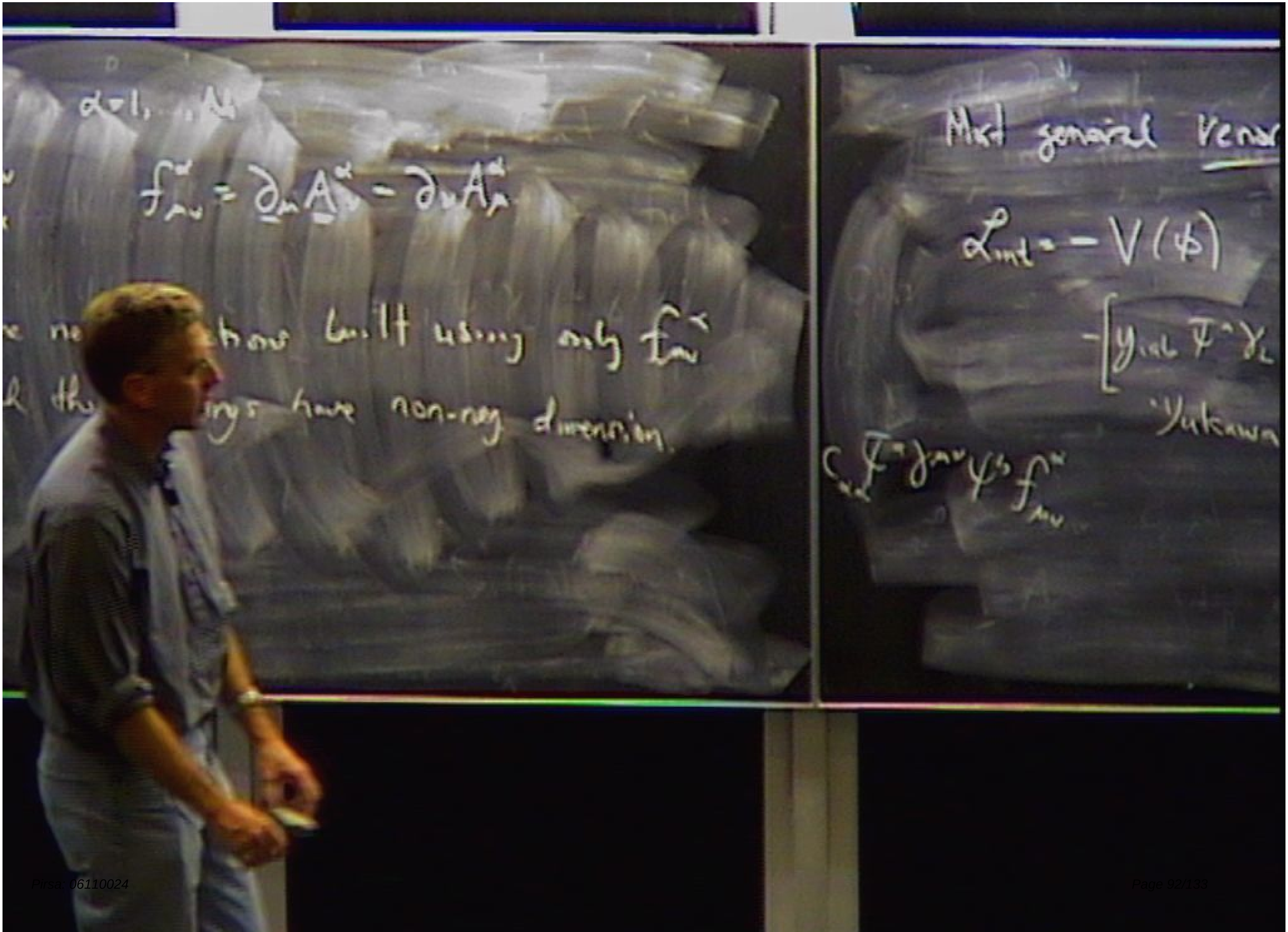
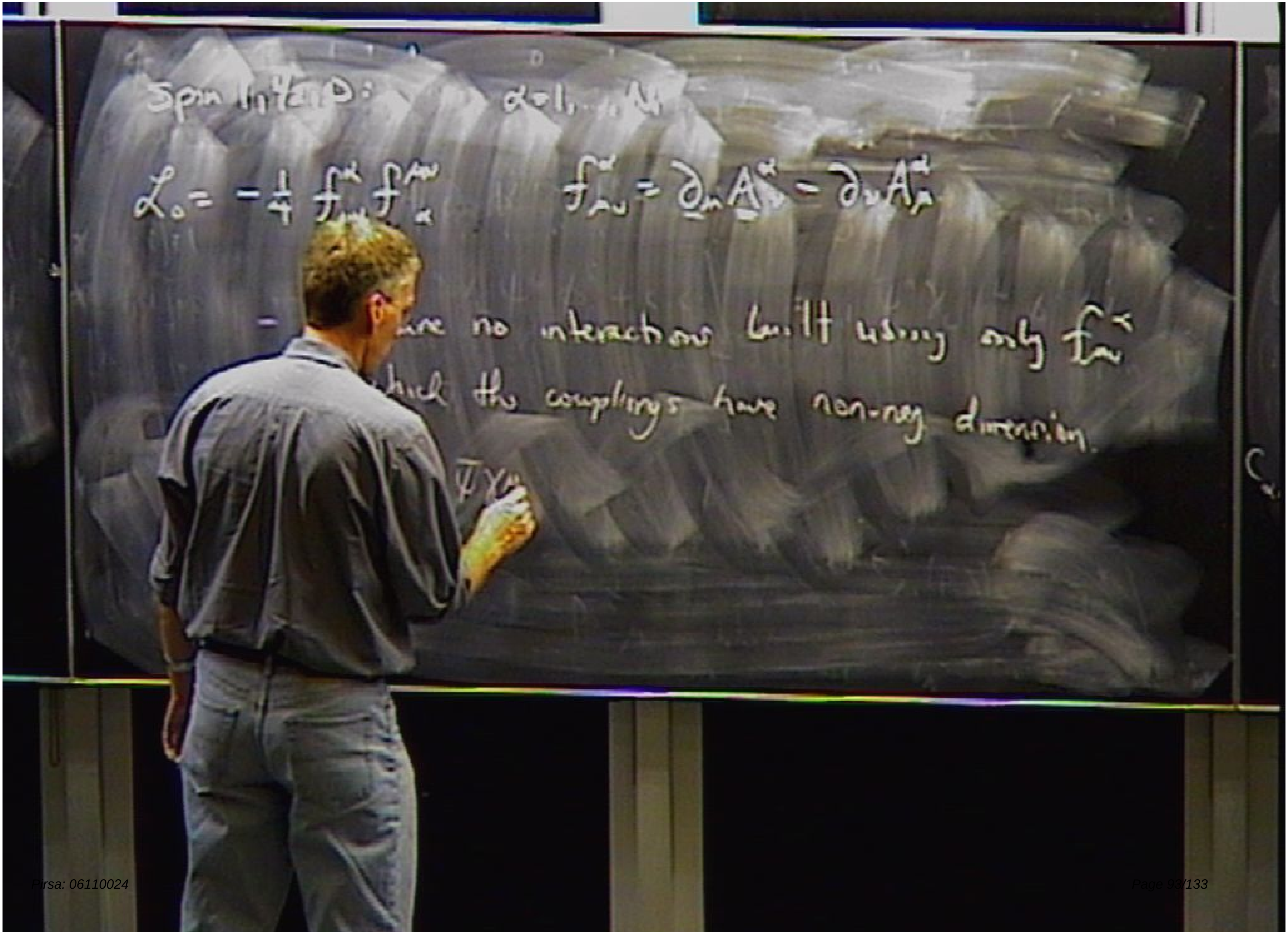
$-\left[y_{iab}\bar{\psi}^a\gamma_L\psi^b\phi^i + c.c.\right]$
"Yukawa" interactions

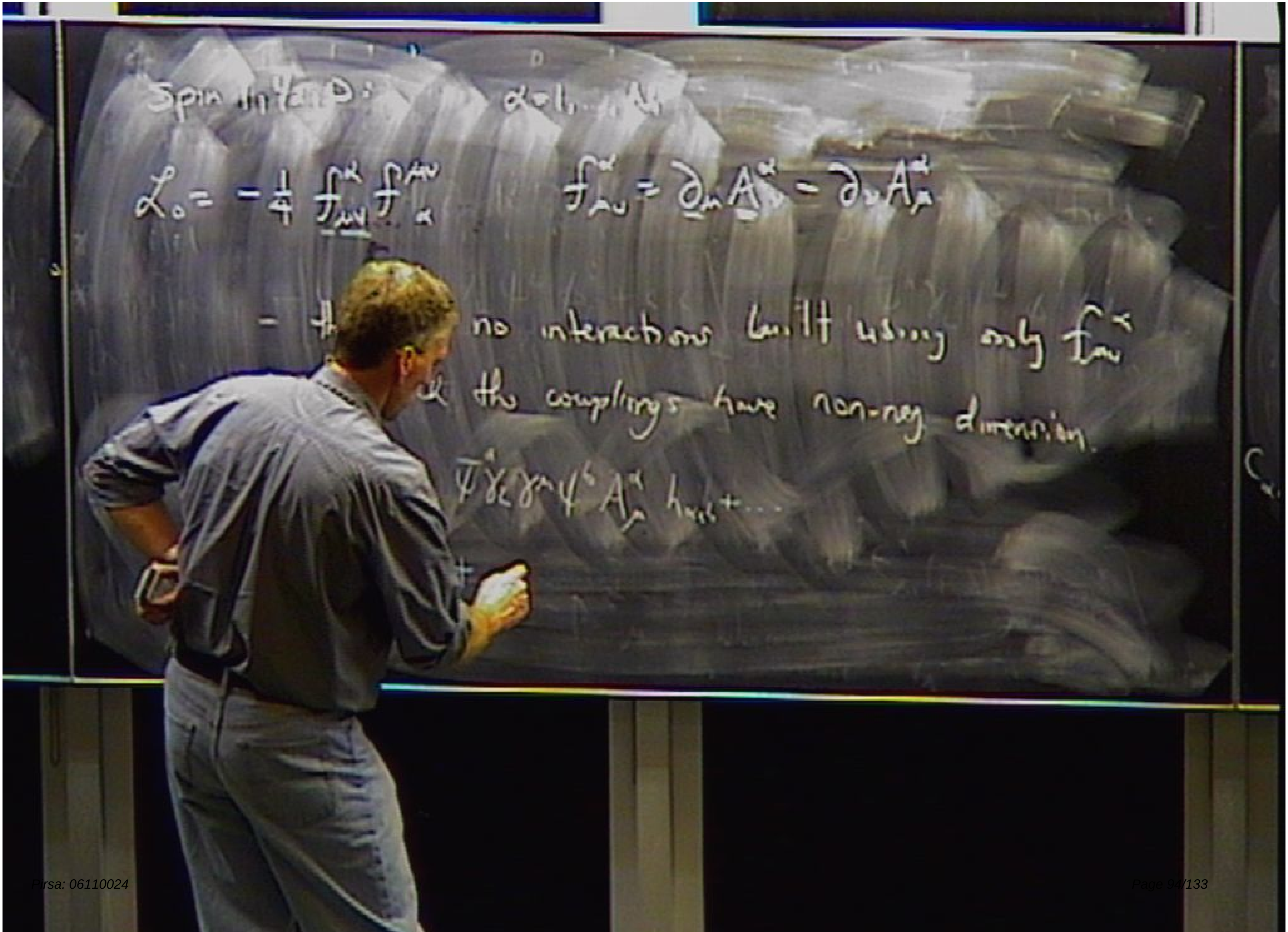

Most general renormalizable interactions:
L_int = − V(φ)
V = + ½ μ² φ φ + g
−[y_iab ψ̄ γ_L ψ φ + c.c.]
+ λ φ φ φ
"Yukawa" interactions


Spin 1 in 4D: α = 1, ..., N
$\mathcal{L} = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha}$
$f^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu}$
there are no interactions built using only $f^{\alpha}_{\mu\nu}$
for


Spin 1 in 4D: $\alpha = 1, \ldots, N$
$\mathcal{L}_0 = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha}$ $f^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu}$
there are no interactions built using only $f^{\alpha}_{\mu\nu}$
for which the couplings have non-neg dimension


f^α_μν = ∂_μ A^α_ν − ∂_ν A^α_μ
built using only f^α_μν
have non-neg dimension
Most general renor
L_int = − V(φ)
Yukawa


Spin 1 in 4D: α=1,...,N
L₀ = −¼ f^α_μν f^μν_α   f^α_μν = ∂_μ A^α_ν − ∂_ν A^α_μ
− ...re no interactions built using only f^α_μν
...hich the couplings have non-neg dimension


Spin in 4D:
α=1,...,N
no interactions built using only f
the couplings have non-neg dimension

Spin 1 in 4D: $\alpha = 1, \dots, N$

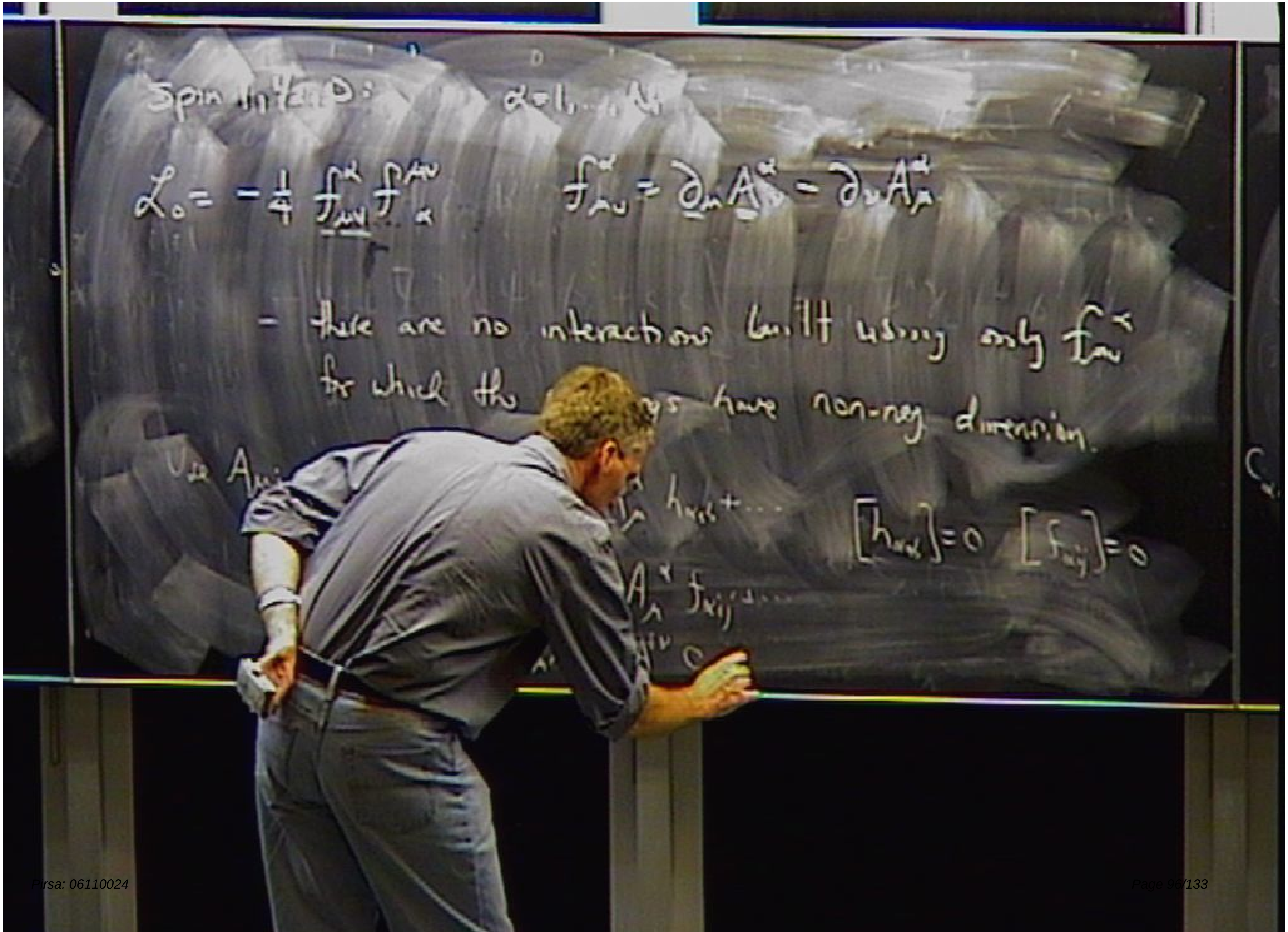
$$\mathcal{L}_0 = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha} \qquad f^{\alpha}_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu$$

- there are no interactions built using only $f^\alpha_{\mu\nu}$ for which the couplings have non-neg dimension.

Use A_μ:

$$\mathcal{L}_{int} = \bar{\psi}^a \gamma_L \gamma^\mu \psi^b A^\alpha_\mu h_{\alpha ab} + \dots + \phi^i \partial^\mu \phi^j A^\alpha_\mu f_{\alpha ij} + \dots$$

$$[h_{\alpha ab}] = 0 \qquad [f_{\alpha ij}] = 0$$


Spin in 4-D: α = 1, ..., N
L₀ = −1/4 f^α_μν f^μν_α
f^α_μν = ∂_μ A^α_ν − ∂_ν A^α_μ
– there are no interactions built using only f^α_μν
for which the
have non-neg dimension
[h_μν] = 0
[f_αij] = 0

Spin 1 in 4D: $\alpha = 1, \ldots, N$

$$\mathcal{L}_0 = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha} \qquad f^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu}$$

- there are no interactions built using only $f^{\alpha}_{\mu\nu}$ for which the couplings have non-neg dimension

Use $A_{\mu i}$
$$\mathcal{L}_{int} = \overline{\psi}^a \gamma_L \gamma^\mu \psi^b A^{\alpha}_{\mu} h_{\alpha ab} + \ldots \qquad [h_{\alpha ab}] = 0 \quad [f_{\alpha ij}] = 0$$
$$+ \phi^i \partial^\mu \phi^j A^{\alpha}_{\mu} f_{\alpha ij} + \ldots$$
$$+ A^{\alpha}_{\mu} A^{\beta}_{\nu} A^{\beta \mu} A^{\delta \nu} c_{\alpha\beta\gamma\delta} + AA\partial A$$

Spin 1 in 4D: $\alpha = 1, \dots, N$

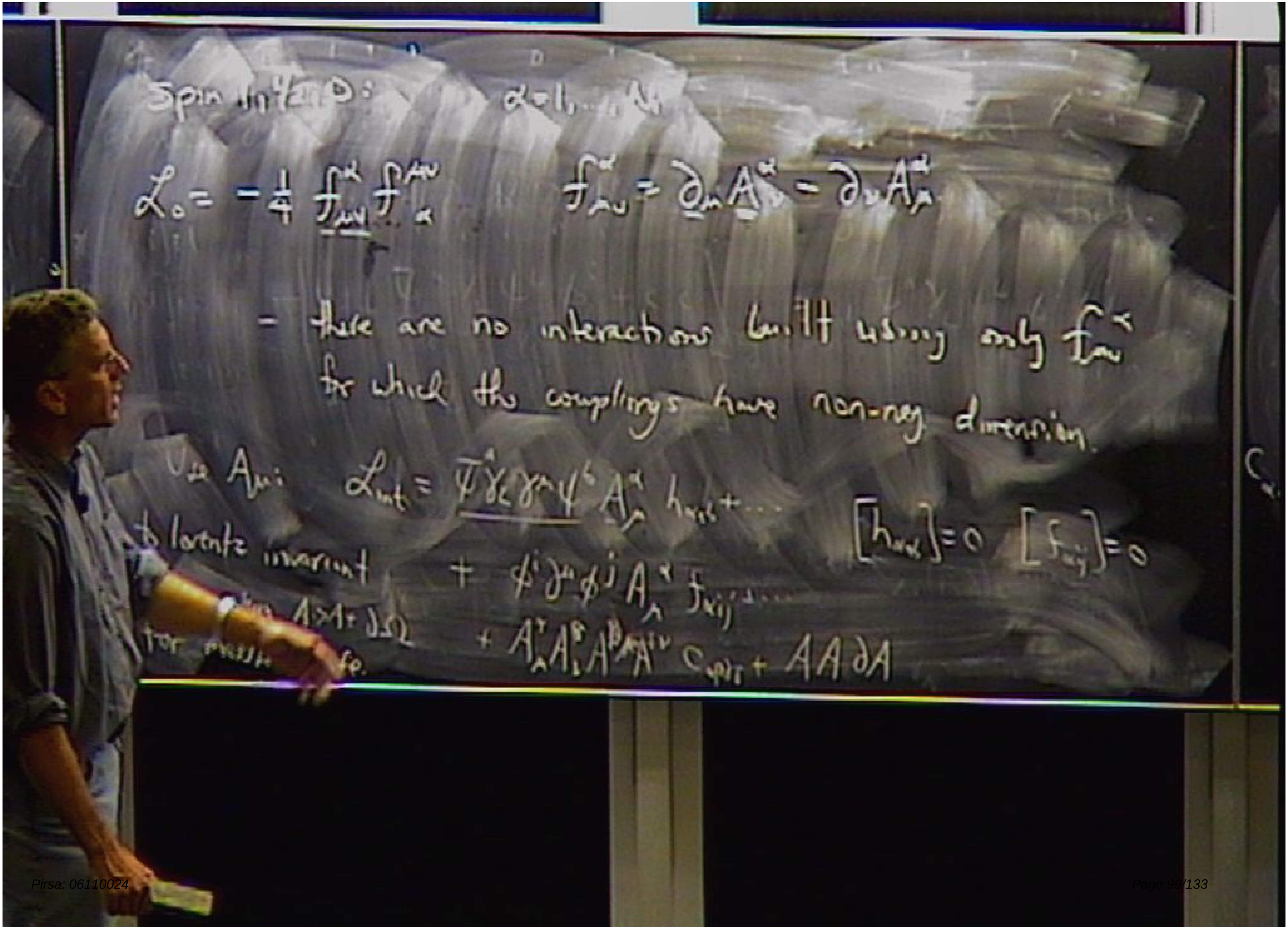
$$\mathcal{L}_0 = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha} \qquad f^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu}$$

- there are no interactions built using only $f^{\alpha}_{\mu\nu}$ for which the couplings have non-neg dimension

Use A_μ: $\mathcal{L}_{int} = \bar{\Psi}^a \gamma_L \gamma^\mu \psi^b A^{\alpha}_{\mu} h_{\alpha ab} + \dots$ $\qquad [h_{\alpha ab}] = 0 \quad [f_{\alpha ij}] = 0$

to Lorentz invariant $\quad + \phi^i \partial^\mu \phi^j A^{\alpha}_{\mu} f_{\alpha ij} + \dots$

require gauge $A \to A + \partial\Omega$ $\quad + A^{\alpha}_{\mu} A^{\beta}_{\nu} A^{\beta\mu} A^{\gamma\nu} c_{\alpha\beta\gamma} + AA\partial A$


there are no interactions built using only
for which the couplings have non-neg dimension
Lorentz invariant

Spin 1 in 4D: $\alpha = 1, \ldots, N$

$$\mathcal{L}_0 = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\mu\nu}_{\alpha} \qquad f^{\alpha}_{\mu\nu} = \partial_\mu A^{\alpha}_{\nu} - \partial_\nu A^{\alpha}_{\mu}$$

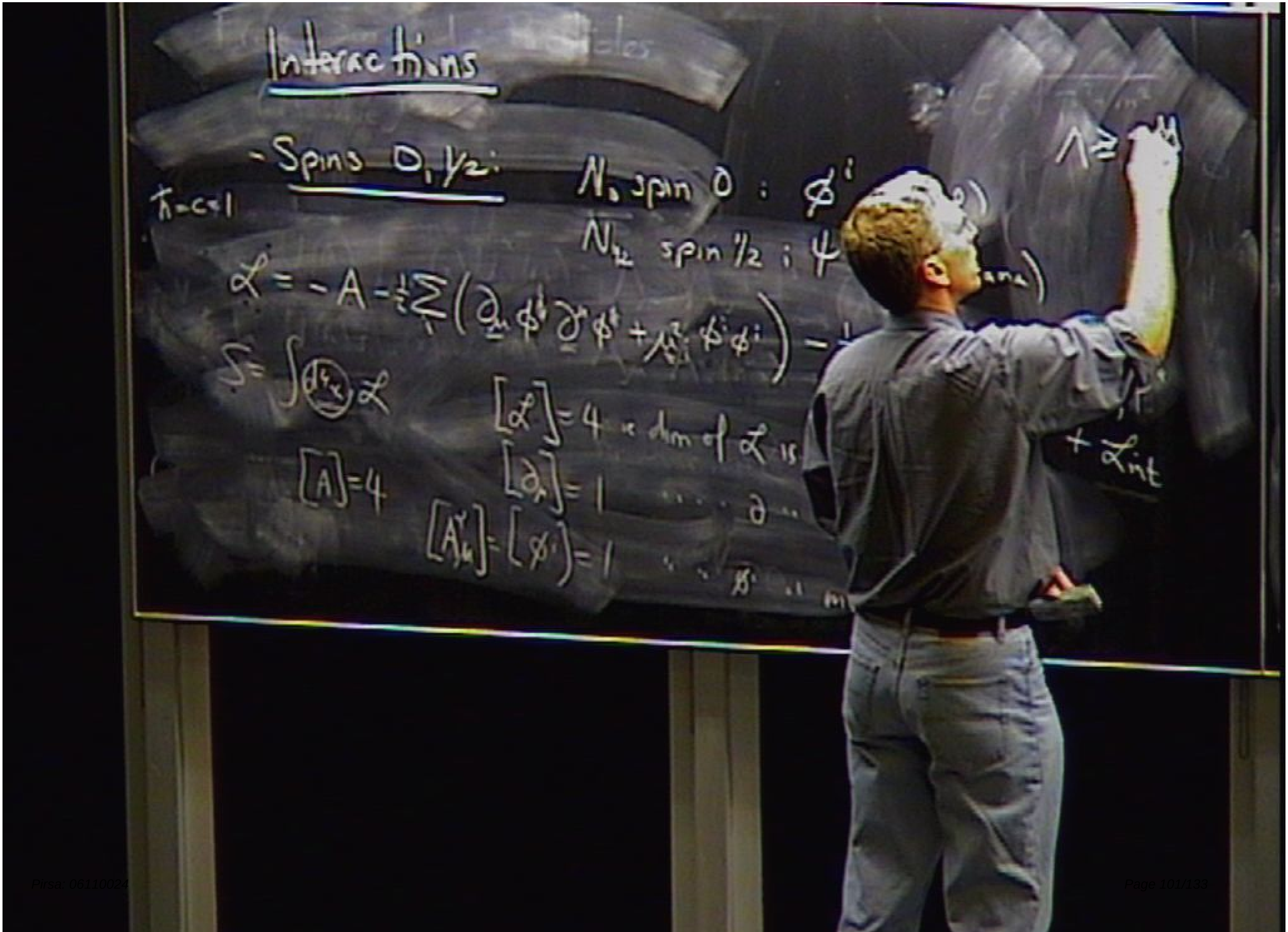
- there are no interactions built using only $f^{\alpha}_{\mu\nu}$ for which the couplings have non-neg dimension.

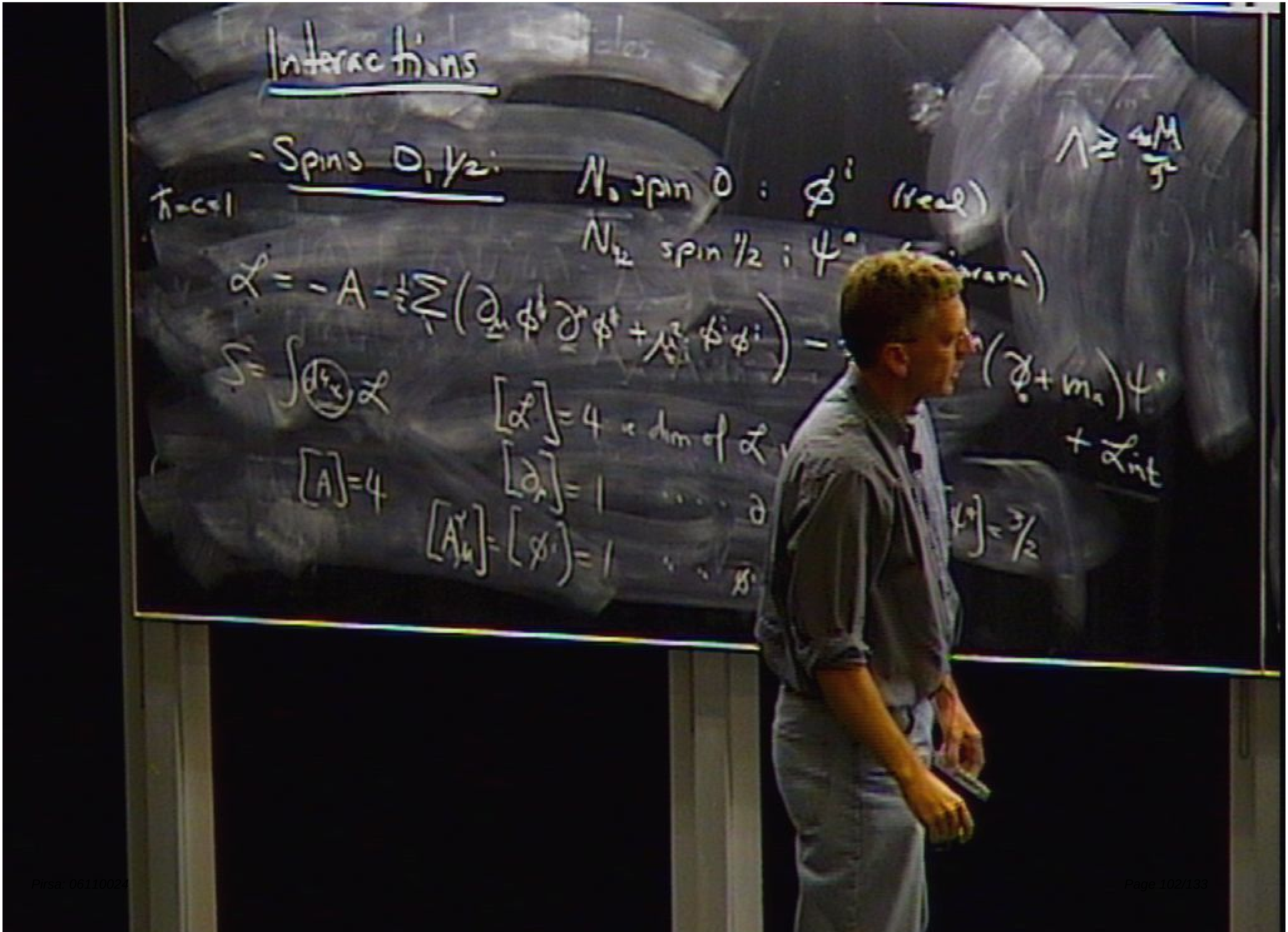
Use A_{μ}: $\mathcal{L}_{int} = \bar{\Psi}^a \gamma_L \gamma^\mu \psi^b A^{\alpha}_{\mu} h_{\alpha ab} + \ldots$ $\quad [h_{\alpha ab}] = 0 \quad [f_{\alpha ij}] = 0$

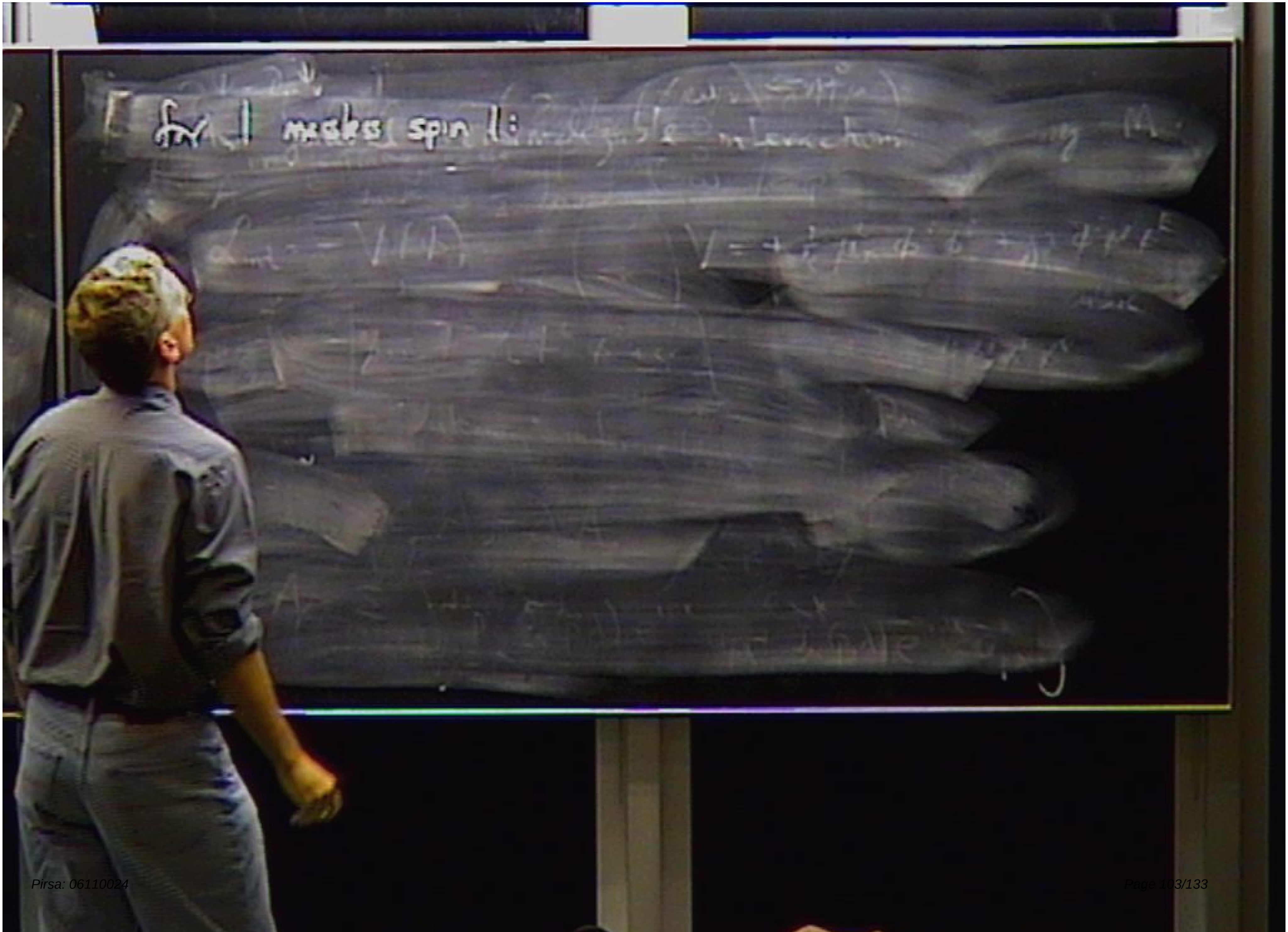
to Lorentz invariant $\quad + \phi^i \partial^\mu \phi^j A^{\alpha}_{\mu} f_{\alpha ij} + \ldots$

require sym $A \to A + \partial\Omega$

for massless case. $\quad + A^{\alpha}_{\mu} A^{\beta}_{\nu} A^{\mu}_{\beta} A^{\gamma\nu} c_{\alpha\beta\gamma} + AA\partial A$

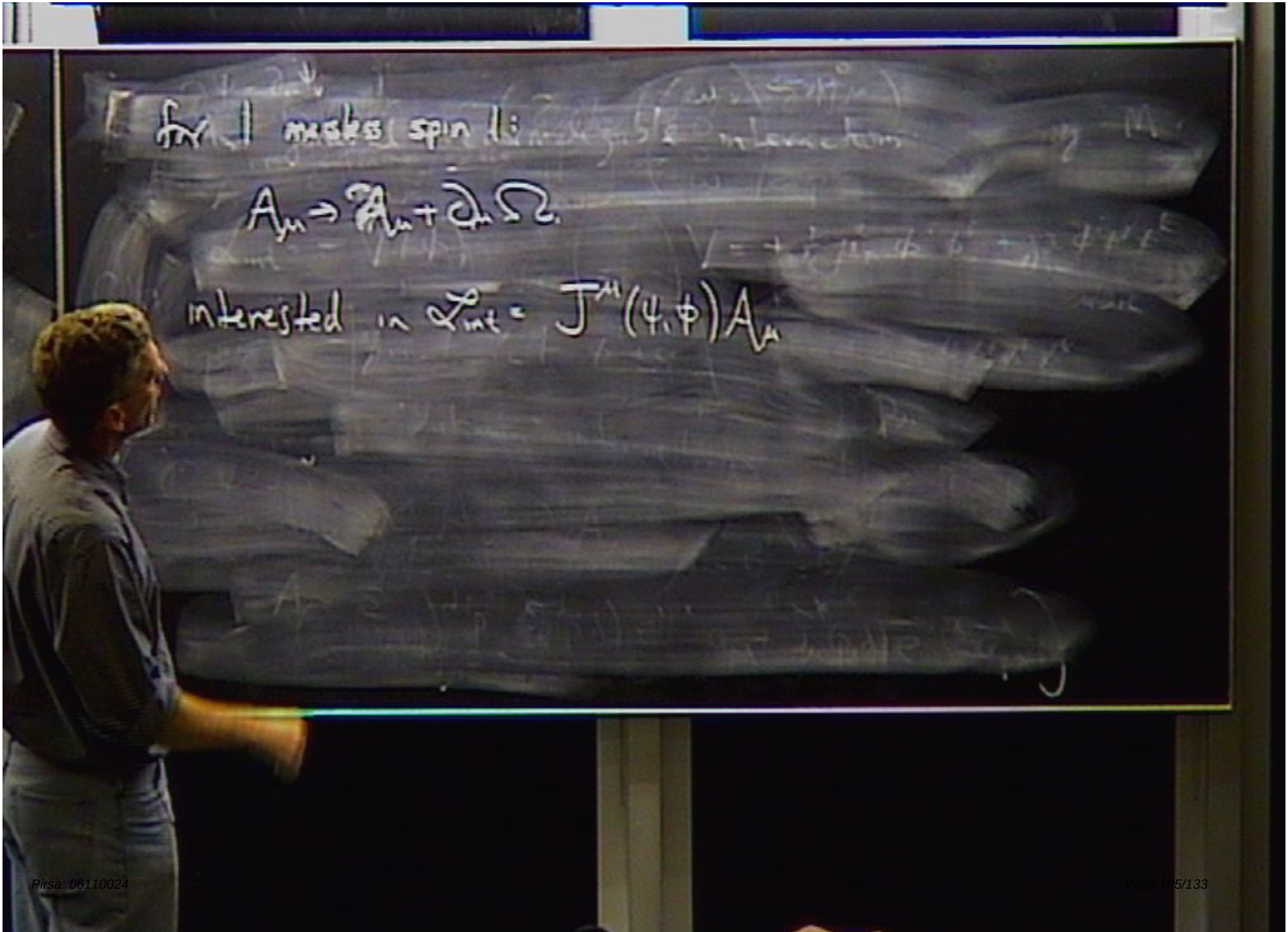

Interactions
- Spins 0, 1/2:
ħ = c = 1
N₀ spin 0 : φⁱ
N½ spin ½ : ψ
ℒ = − A − ½ Σ (∂μ φⁱ ∂μ φⁱ + Λᵢ² φⁱ φⁱ) −
S = ∫ d⁴x ℒ
[ℒ] = 4
[A] = 4
[∂μ] = 1
[Λ] = [φⁱ] = 1
+ ℒint


Interactions
- Spins 0, 1/2:
ħ = c = 1
N_0 spin 0 : ϕ^i (real)
$N_{1/2}$ spin 1/2 : ψ^a
$\Lambda \gtrsim \frac{4\pi M}{g}$
$\mathcal{L} = -A - \frac{1}{2}\sum_i (\partial_\mu \phi^i \partial^\mu \phi^i + \lambda_i^2 \phi^i \phi^i)$
$(\not\partial + m_a)\psi^a$
$+ \mathcal{L}_{int}$
$S = \int d^4x \, \mathcal{L}$
$[\mathcal{L}] = 4$ = dim of $\mathcal{L}$
$[A] = 4$
$[\partial_\mu] = 1$
$[A_\mu^\nu] = [\phi^i] = 1$
$[\psi^a] = 3/2$


makes spin 1:

for all massless spin 1:

$$A_\mu \to A_\mu + \partial_\mu \Omega$$

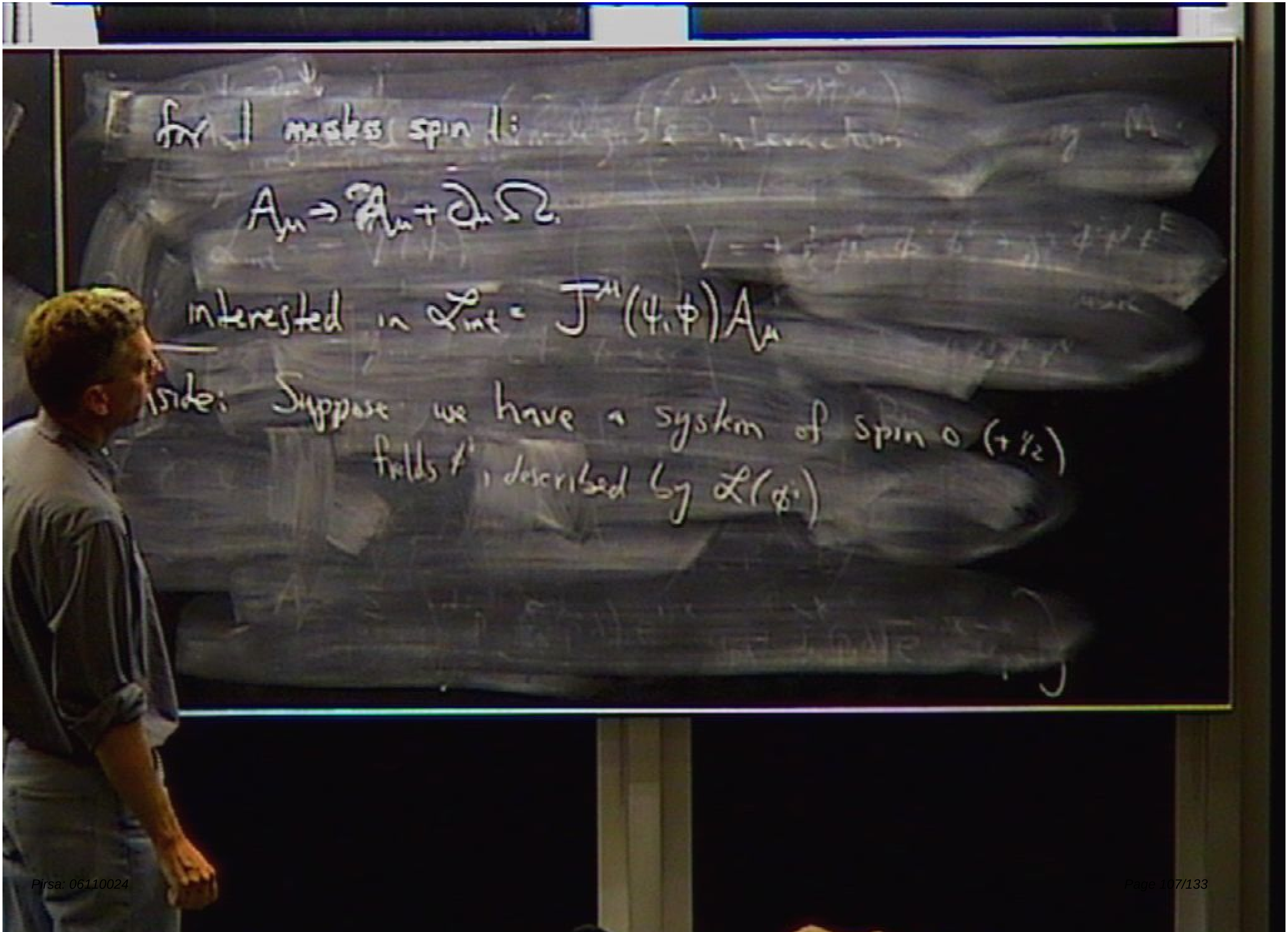

for a massless spin 1:
A_μ → A_μ + ∂_μ Ω
interested in L_int = J^μ(ψ,ψ) A_μ

for massless spin 1:

$$A_\mu \to A_\mu + \partial_\mu \Omega$$

interested in $\mathcal{L}_{int} = J^\mu(\psi, \bar\psi) A_\mu$

Aside:


for 1 massless spin 1:
$A_\mu \to A_\mu + \partial_\mu \Omega$
interested in $\mathcal{L}_{int} = J^\mu(\psi, \phi) A_\mu$
Aside: Suppose we have a system of spin 0 (+1/2)
fields ϕ^i, described by $\mathcal{L}(\phi^i)$

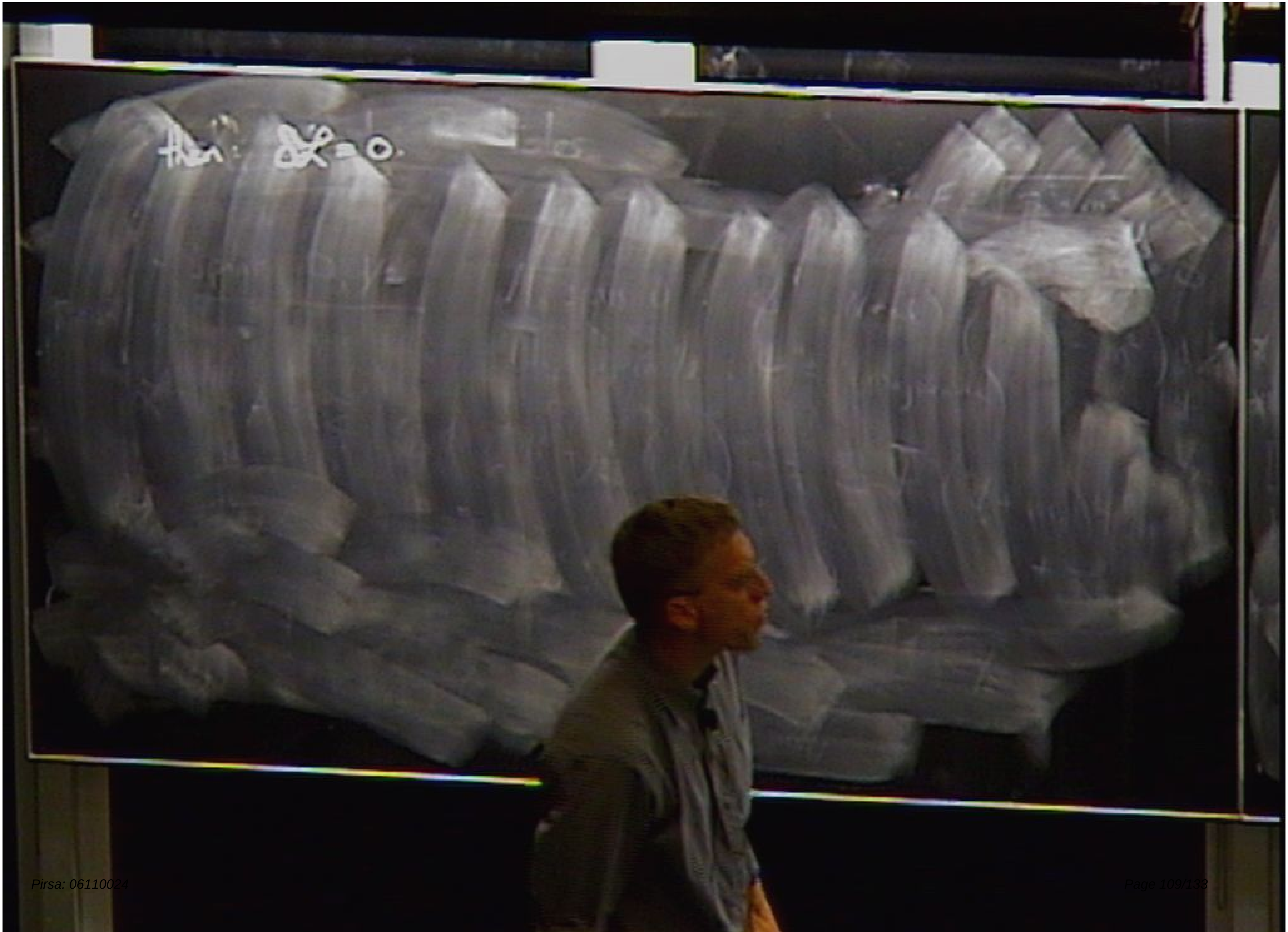
for massless spin 1:

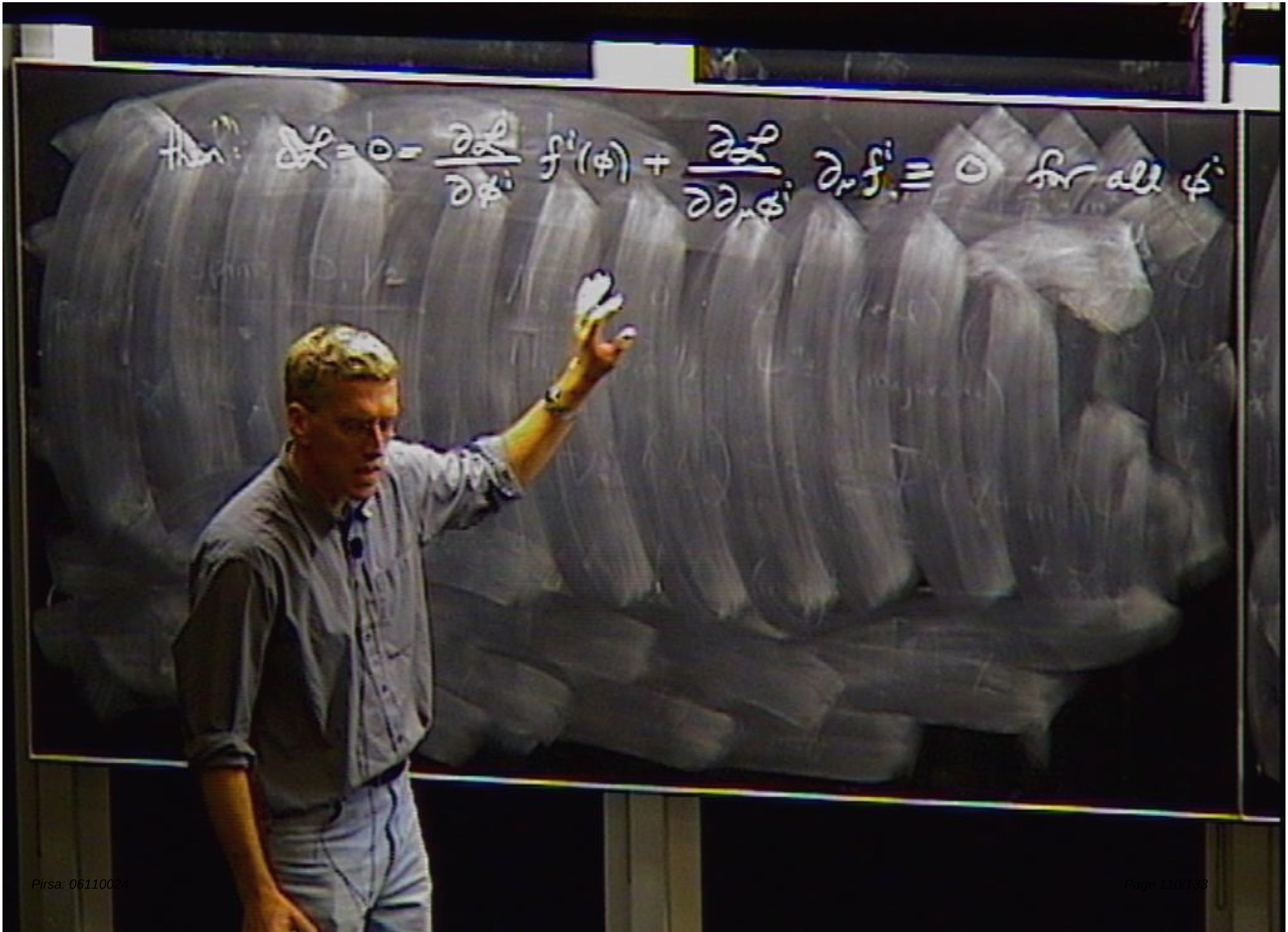
$$A_\mu \to A_\mu + \partial_\mu \Omega$$

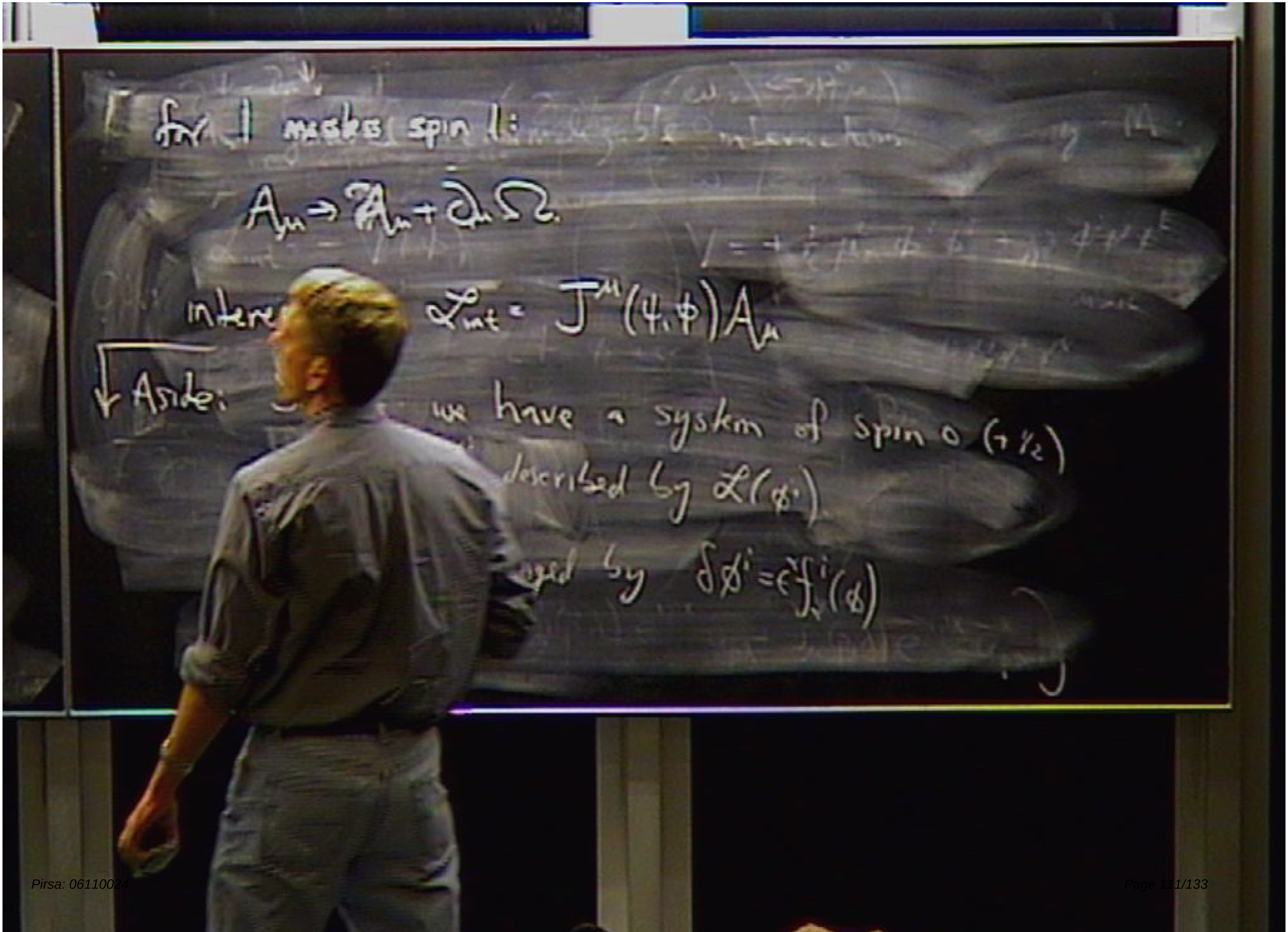
interested in $\mathcal{L}_{int} = J^\mu(\psi,\phi) A_\mu$

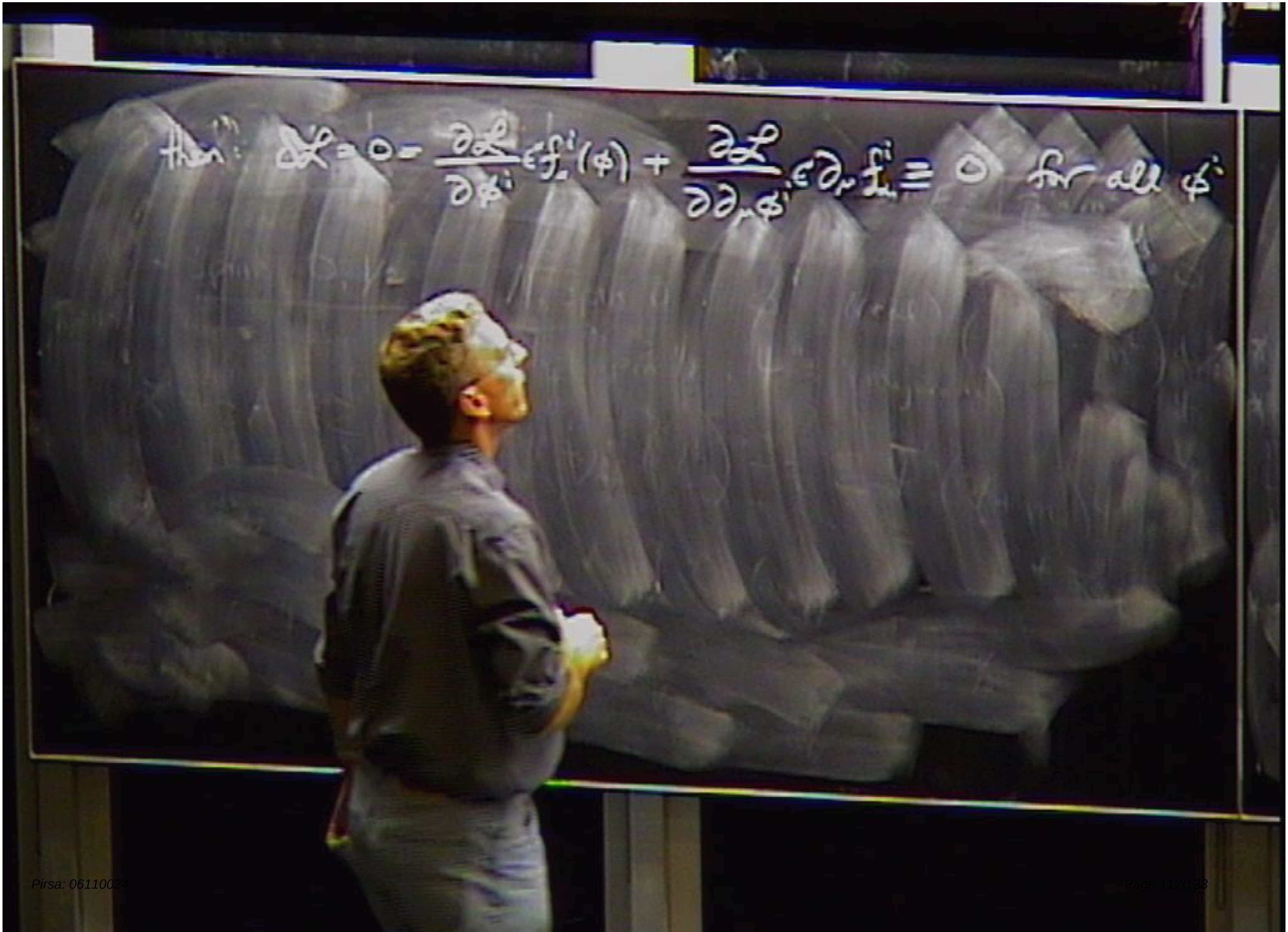
Aside: Suppose we have a system of spin 0 (+ 1/2) fields ϕ^i, described by $\mathcal{L}(\phi^i)$

Suppose $\mathcal{L}$ is unchanged by $\delta\phi^i = f^i(\phi)$


then: δℒ = 0.


then (*): $\delta\mathcal{L} = 0 = \frac{\partial \mathcal{L}}{\partial \phi^i} f^i(\phi) + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^i} \partial_\mu f^i \equiv 0$ for all ϕ^i


for A makes spin 1:
Aμ → Aμ + ∂μ Ω
intere 𝓛int = J^μ(ψ,φ)Aμ
Aside: we have a system of spin 0 (+ 1/2)
described by 𝓛(φ^i)
ged by δφ^i = ε^a f_a^i(φ)


then: $\delta \mathcal{L} = 0 = \frac{\partial \mathcal{L}}{\partial \phi^i} \epsilon f^i_a(\phi) + \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^i} \epsilon \partial_\mu f^i_a = 0$ for all ϕ^i

for 1 massless spin 1:

$$A_\mu \to A_\mu + \partial_\mu \Omega$$

interested in $\mathcal{L}_{int} = J^\mu(\psi,\phi) A_\mu$

Aside: Suppose we have a system of spin 0 (+ 1/2) fields ϕ^i, described by $\mathcal{L}(\phi^i)$.

Suppose $\mathcal{L}$ is unchanged by $\delta\phi^i = \epsilon^a f_a^i(\phi)$ $\quad \partial_\mu \epsilon^a = 0$

then: $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon f_a^i(\phi) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\epsilon\partial_\mu f_a^i \equiv 0$ for all ϕ^i

for massless spin 1:

$$A_\mu \to A_\mu + \partial_\mu \Omega$$

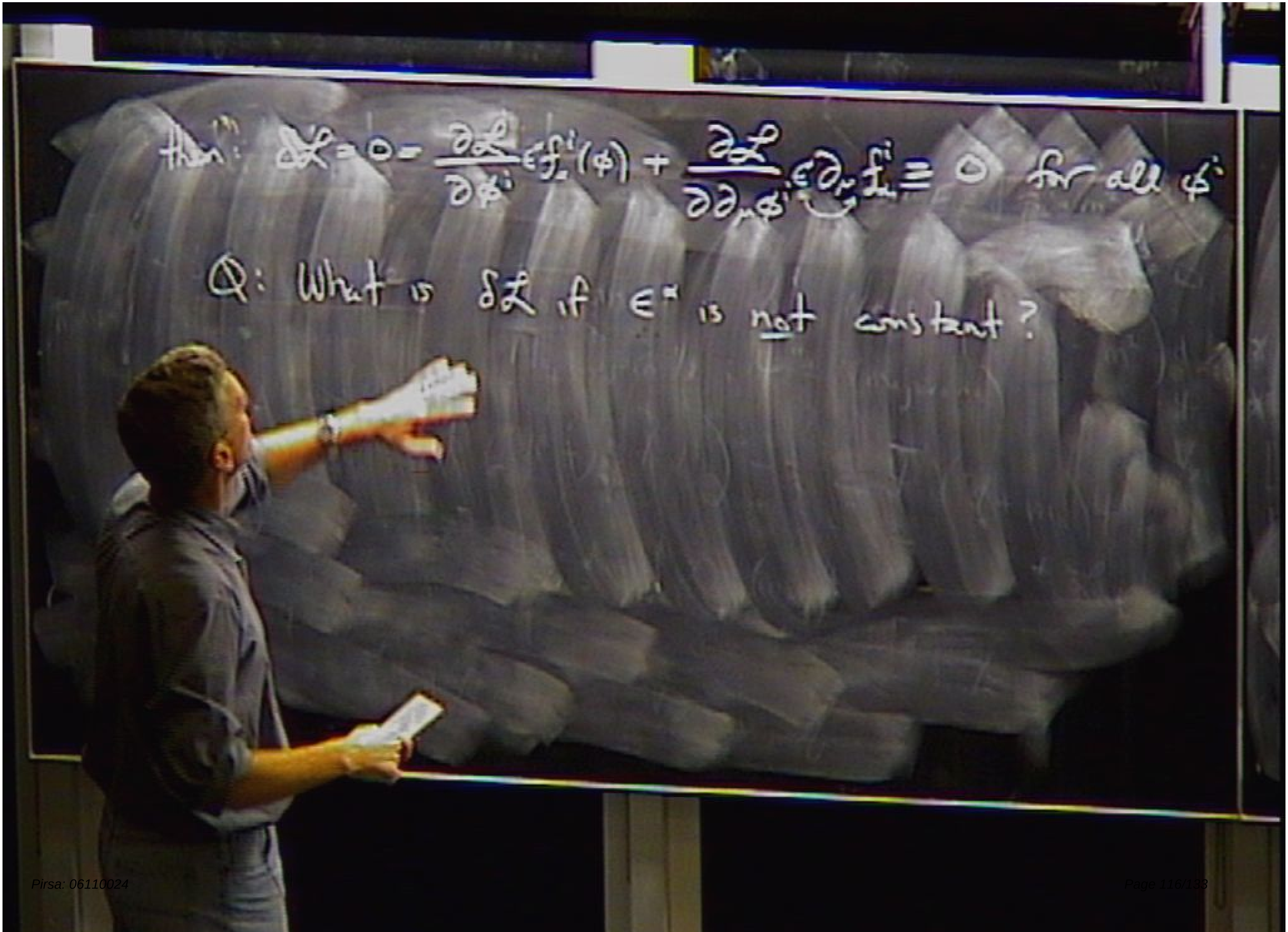
interested in $\mathcal{L}_{int} = J^\mu(\psi,\phi)A_\mu$

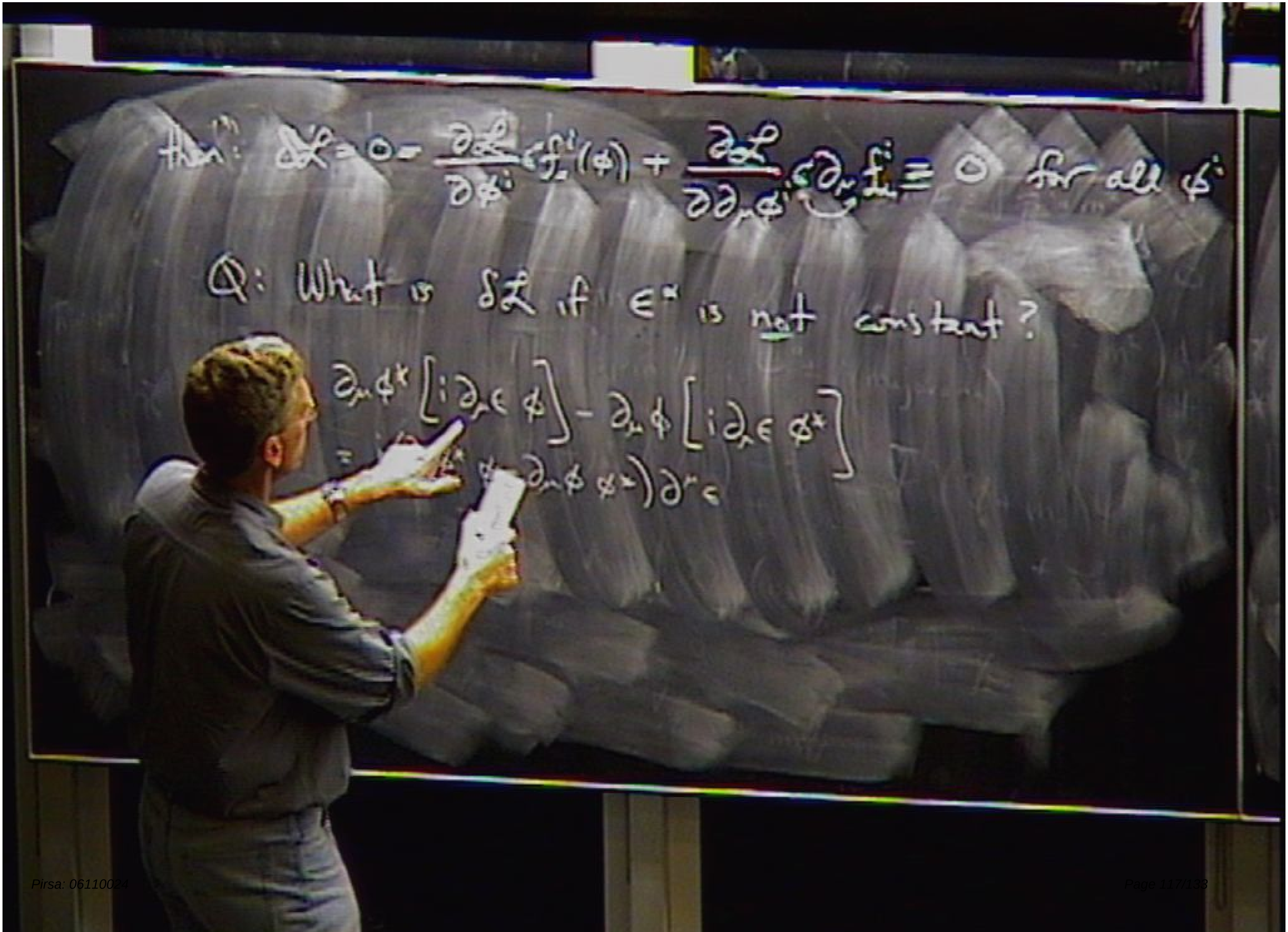
Aside: Suppose we have a system of spin 0 (+ 1/2) fields ϕ^i, described by $\mathcal{L}(\phi^i)$

Suppose $\mathcal{L}$ is unchanged by $\delta\phi^i = \epsilon^a f_a^i(\phi)$ $\quad \partial_\mu \epsilon^a = 0$

eg

$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi^*\phi)$ $\quad \delta\phi = i\epsilon\phi$

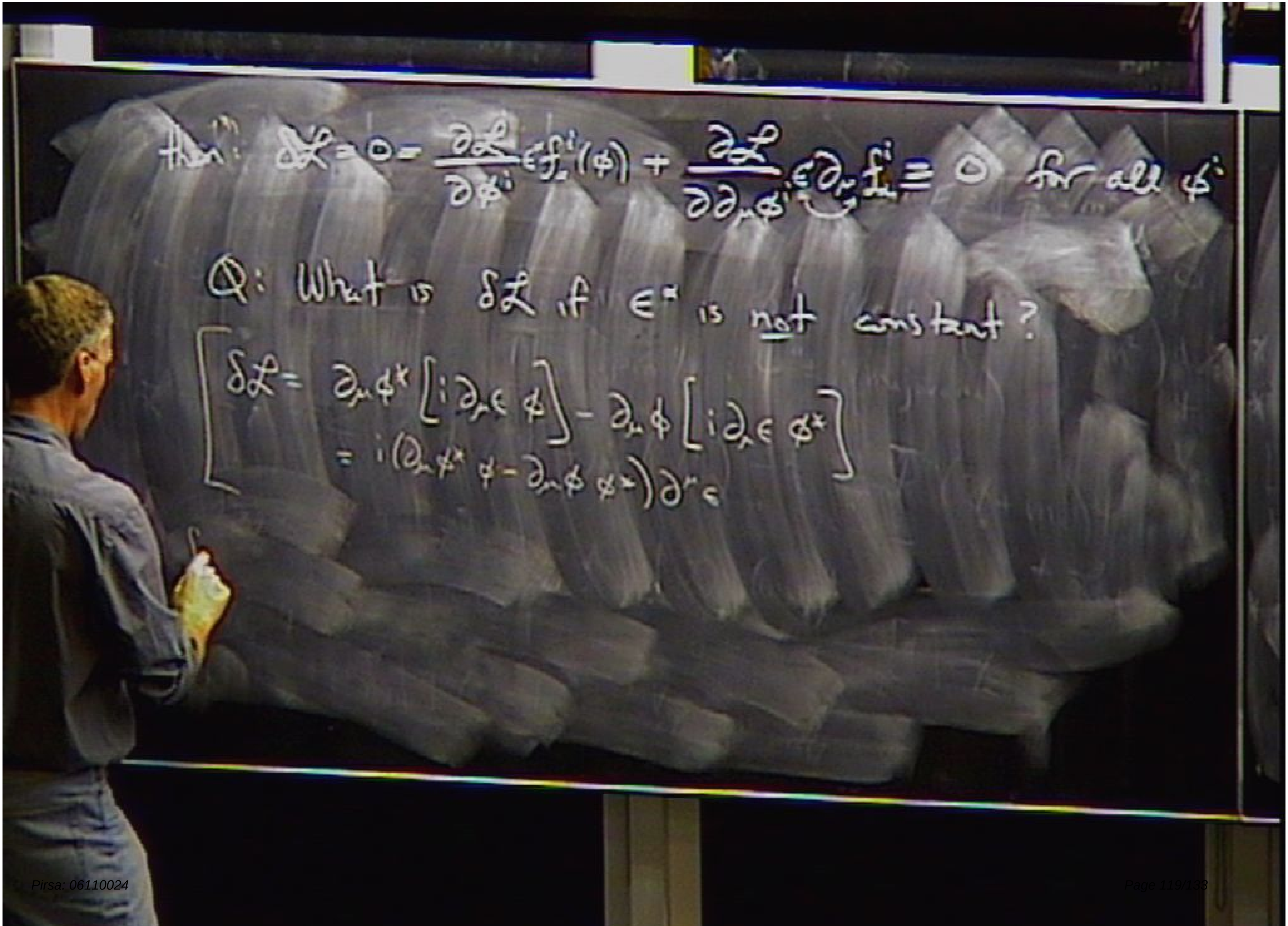

then: $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon f^i_a(\phi) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\epsilon\partial_\mu f^i_a \equiv 0$ for all ϕ^i
Q: What is $\delta\mathcal{L}$ if ϵ^a is not constant?


then: $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon f^i_\alpha(\phi) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\epsilon\partial_\mu f^i_\alpha = 0$ for all ϕ^i
Q: What is $\delta\mathcal{L}$ if ϵ^α is not constant?
$\partial_\mu\phi^*[i\partial^\mu\epsilon\,\phi] - \partial_\mu\phi[i\partial^\mu\epsilon\,\phi^*]$
$\partial_\mu\phi\,\phi^*)\partial^\mu\epsilon$

then: $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon f^i_a(\phi) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\epsilon\,\partial_\mu f^i_a \equiv 0$ for all ϕ^i

Q: What is $\delta\mathcal{L}$ if ϵ^a is <u>not</u> constant?

$$\delta\mathcal{L} = \partial_\mu\phi^* \left[i\partial_\mu\epsilon\,\phi \right] - \partial_\mu\phi \left[i\partial_\mu\epsilon\,\phi^* \right]$$
$$= i(\partial_\mu\phi^*\,\phi - \partial_\mu\phi\,\phi^*)\partial^\mu\epsilon$$


then: $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon f_a^i(\phi) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\epsilon\partial_\mu f_a^i = 0$ for all ϕ^i
Q: What is $\delta\mathcal{L}$ if ϵ^a is not constant?
$\delta\mathcal{L} = \partial_\mu\phi^* [i\partial^\mu\epsilon\,\phi] - \partial_\mu\phi [i\partial^\mu\epsilon\,\phi^*]$
$= i(\partial_\mu\phi^*\phi - \partial_\mu\phi\,\phi^*)\partial^\mu\epsilon$

Then: $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon f^i_\alpha(\phi) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\epsilon\,\partial_\mu f^i_\alpha = 0$ for all ϕ^i

Q: What is $\delta\mathcal{L}$ if ϵ^α is <u>not</u> constant?

$$\left[\begin{aligned}\delta\mathcal{L} &= \partial_\mu\phi^*\left[i\partial_\mu\epsilon\,\phi\right] - \partial_\mu\phi\left[i\partial_\mu\epsilon\,\phi^*\right]\\ &= i(\partial_\mu\phi^*\,\phi - \partial_\mu\phi\,\phi^*)\partial^\mu\epsilon\end{aligned}\right.$$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon^\alpha f^i_\alpha + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\partial_\mu(\epsilon^\alpha f^i_\alpha) = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^i)}\partial_\mu\epsilon^\alpha f^i_\alpha$$

thm: $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon f^i_\alpha(\phi) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\epsilon\partial_\mu f^i_\alpha \equiv 0$ for all ϕ^i

Q: What is $\delta\mathcal{L}$ if ϵ^α is not constant?

$$\left[\begin{aligned}\delta\mathcal{L} &= \partial_\mu\phi^*\left[i\partial_\mu\epsilon\,\phi\right] - \partial_\mu\phi\left[i\partial_\mu\epsilon\,\phi^*\right]\\ &= i(\partial_\mu\phi^*\,\phi - \partial_\mu\phi\,\phi^*)\partial^\mu\epsilon\end{aligned}\right.$$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon^\alpha f^i_\alpha + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\partial_\mu(\epsilon^\alpha f^i_\alpha) = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^i)}\partial_\mu\epsilon^\alpha f^i_\alpha$$

$$= J^\mu_\alpha(x)\,\partial_\mu\epsilon^\alpha \leftarrow \text{Noether current}$$

eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right] = 0.$

Use A_μ: $\mathcal{L}_{int} = \underline{\bar{\Psi}^a \gamma_L \gamma^\mu \Psi^b} A_\mu^\alpha h_{\alpha a b} + \ldots$ $\quad [h_{\alpha a b}] = 0 \quad [f_{\alpha ij}] = 0$

to Lorentz invariant $\quad + \phi^i \partial^\mu \phi^j A_\mu^\alpha f_{\alpha ij} + \ldots$

require gau $A \to A + \partial \Omega$

for massless case. $\quad + A_\mu^\alpha A_\nu^\beta A^{\beta\mu} A^{\gamma\nu} c_{\alpha\beta\gamma\delta} + AA\partial A$

$\frac{\partial \mathcal{L}}{\partial \phi^i}\epsilon\, \partial_\mu f^i_\alpha = 0$ for all ϕ^i

not constant?

$\partial_\mu \epsilon\, \phi^\alpha]$

$= \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \partial_\mu \epsilon^\alpha f^i_\alpha$

$J^\mu_\alpha(x)\, \partial_\mu \epsilon^\alpha$ Noether current

eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)}\right] = 0$

$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha\right] = 0$

Use A_μ: $\mathcal{L}_{int} = \underline{\bar{\Psi}^a \gamma_L \gamma^\mu \psi^b} A^\alpha_\mu h_{\alpha ab} + \ldots$

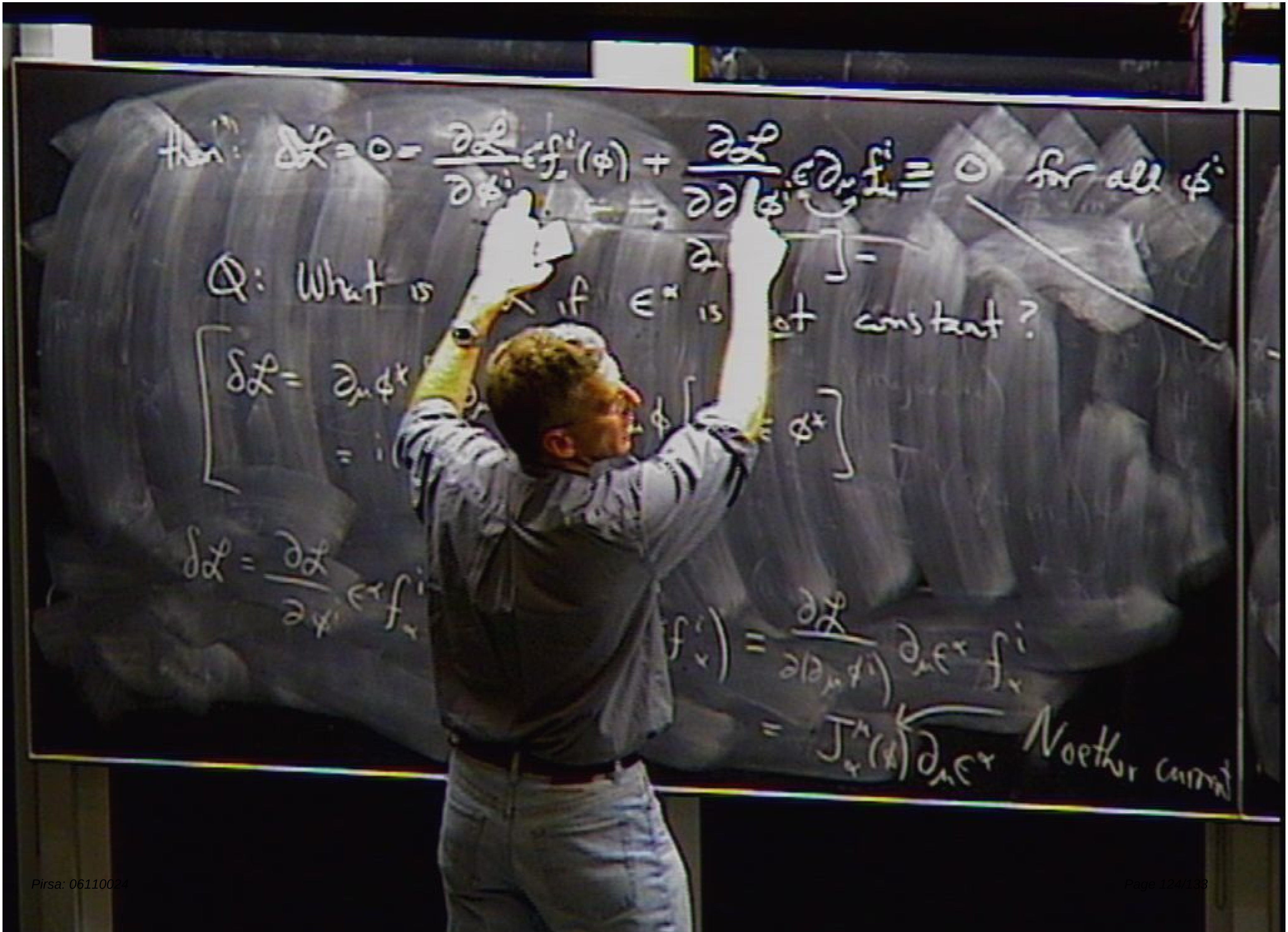
to lorentz invariant

$+ \phi^i \partial^\mu \phi^j A^\alpha_\mu f_{\alpha ij} + \ldots$

require sym $A \to A + \partial \Omega$

for massless case.

$+ A^\alpha_\mu A^\beta_\nu A^{\gamma\mu} A^{\delta\nu} c_{\alpha\beta\gamma\delta} + AA\partial$


Q: What is
if
is
constant ?
for all
Noether current

eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)}\right] = 0.$

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha \right] = 0$$

$$J^\mu_\alpha$$

Use $A_{\mu i}$ $\quad \mathcal{L}_{int} = \bar{\Psi}^a \gamma_L \gamma^\mu \Psi^b A^\alpha_\mu h_{\alpha ab} + \dots \qquad [h_{\alpha ab}] = 0 \quad [f_{\alpha ij}] = 0$

to Lorentz invariant $\quad + \phi^i \partial^\mu \phi^j A^\alpha_\mu f_{\alpha ij} + \dots$

"gauge sym $A \to A + \partial\Omega$"

for massless case. $\quad + A^\alpha_\mu A^\beta_\nu A^{\mu} A^{\nu} c_{\alpha\beta\gamma} + AA\partial A$

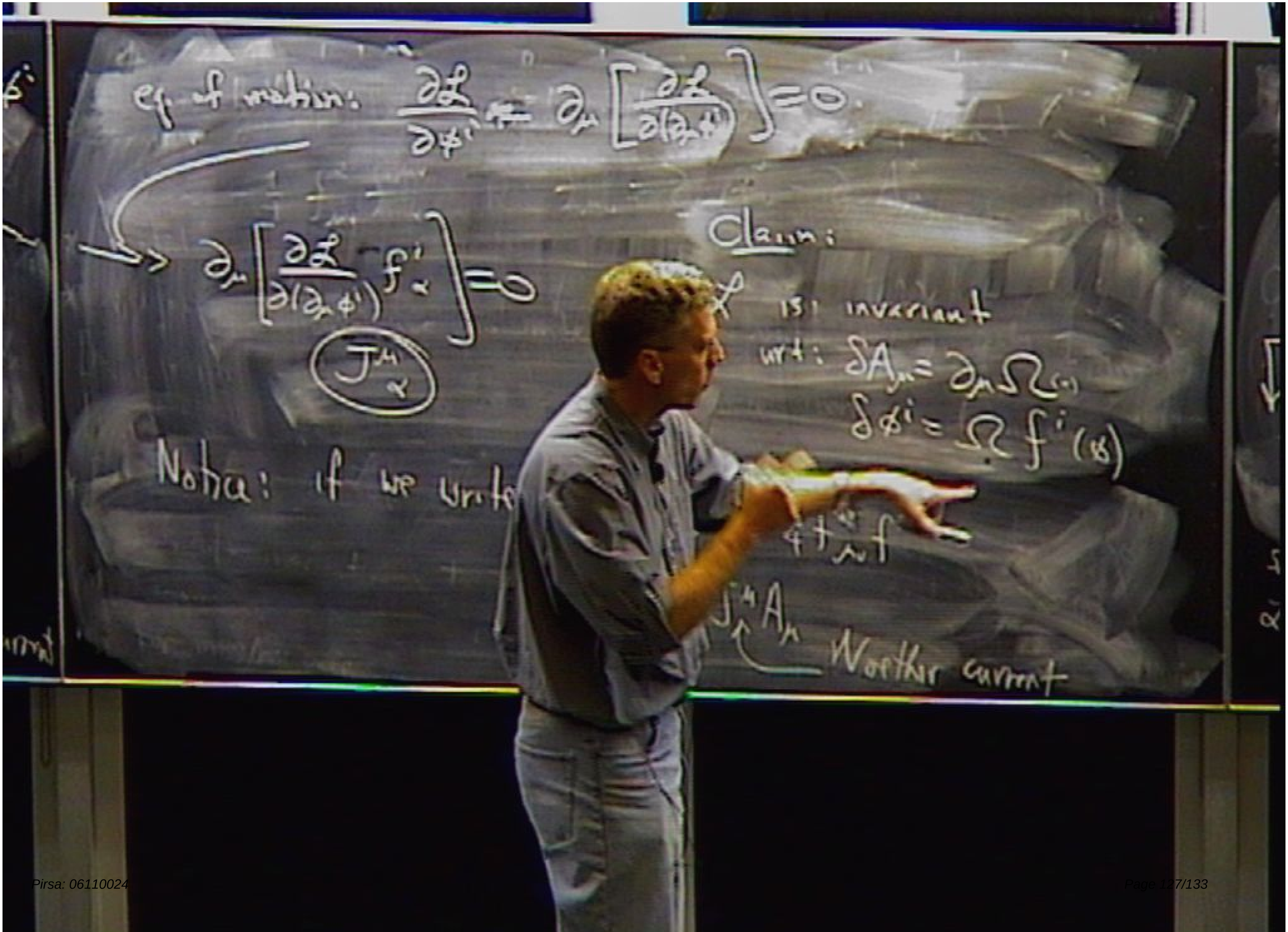
eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \right] = 0.$

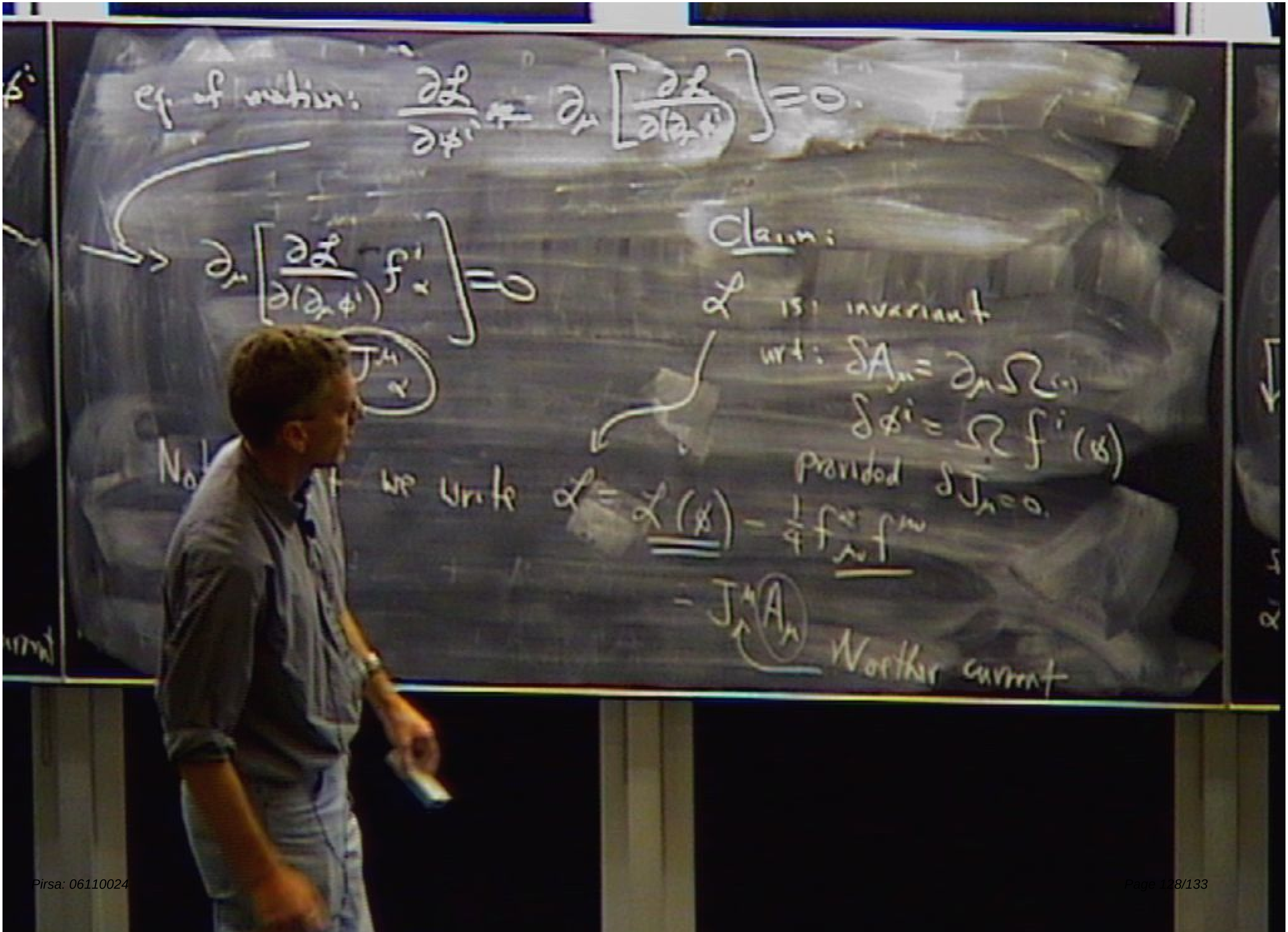
$$\partial_\mu \left[\underbrace{\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha}_{J^\mu_\alpha} \right] = 0$$

Notice: if we write $\mathcal{L} = \mathcal{L}(\phi) - \frac{1}{4} f^\alpha_{\mu\nu} f^{\mu\nu}$

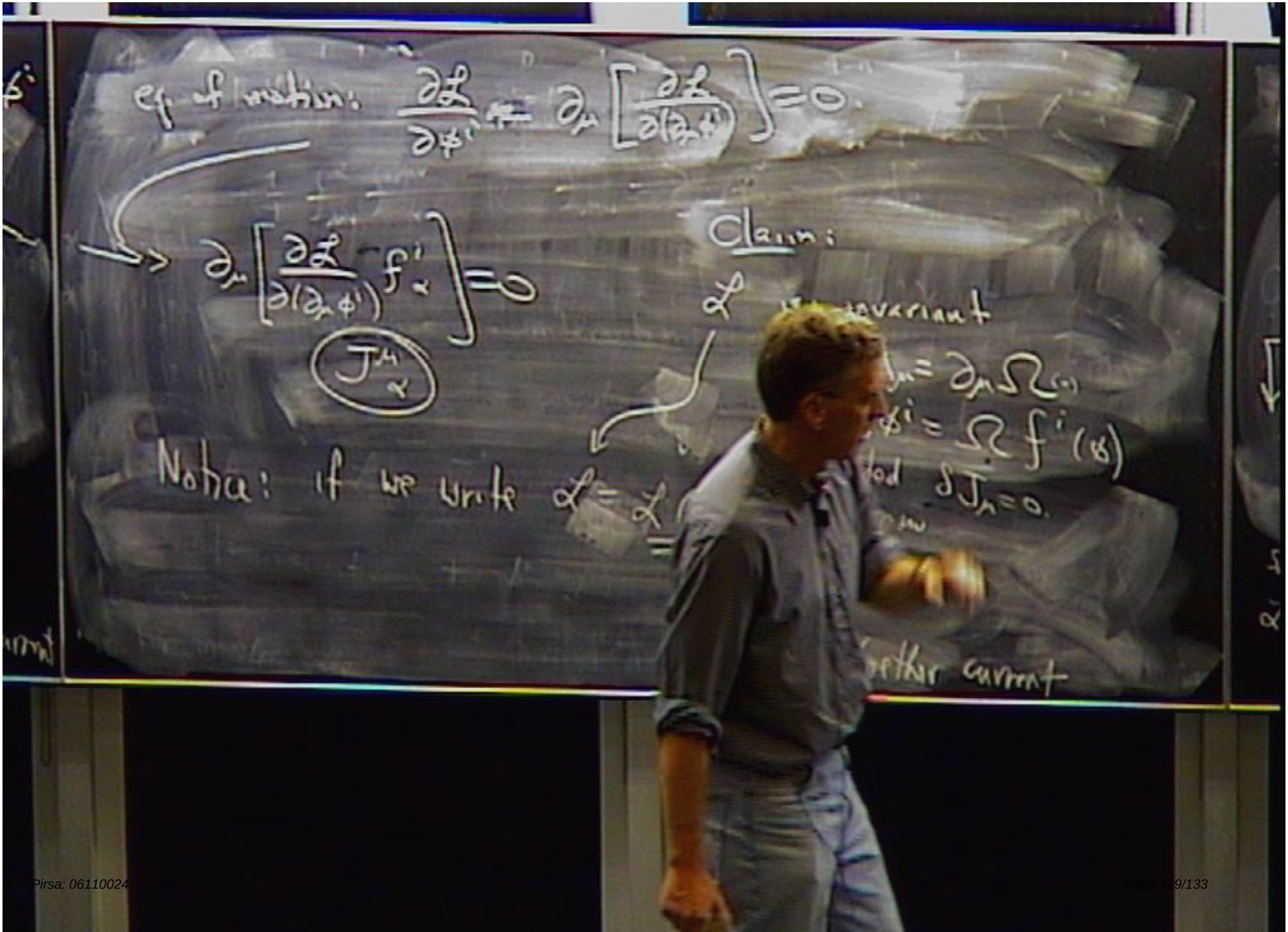
$- J^\mu A_\mu$

Noether current


eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \right] = 0.$
$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha \right] = 0$
J^μ_α
Claim:
$\mathcal{L}$ is invariant
wrt: $\delta A_\mu = \partial_\mu \Omega(x)$
$\delta \phi^i = \Omega f^i(x)$
Notia: if we write
Noether current



eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \right] = 0.$
$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha \right] = 0$
J^μ_α
Claim:
$\mathcal{L}$ is invariant
wrt: $\delta A_\mu = \partial_\mu \Omega(x)$
$\delta \phi^i = \Omega f^i(\phi)$
provided $\delta J_\mu = 0$.
we write $\mathcal{L} = \mathcal{L}(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$- J^\mu A_\mu$
Noether current


eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu\left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)}\right] = 0.$
$\partial_\mu\left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha\right] = 0$
J^μ_α
Claim:
$\mathcal{L}$
invariant
$= \partial_\mu \Omega(x)$
$= \Omega f^i(\phi)$
$\delta J_\mu = 0.$
Note: if we write $\mathcal{L} = \mathcal{L}$
current

eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \right] = 0.$

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha \right] = 0$$

J^μ_α

Claim:

$\mathcal{L}$ is invariant

wrt: $\delta A_\mu = \partial_\mu \Omega(x)$

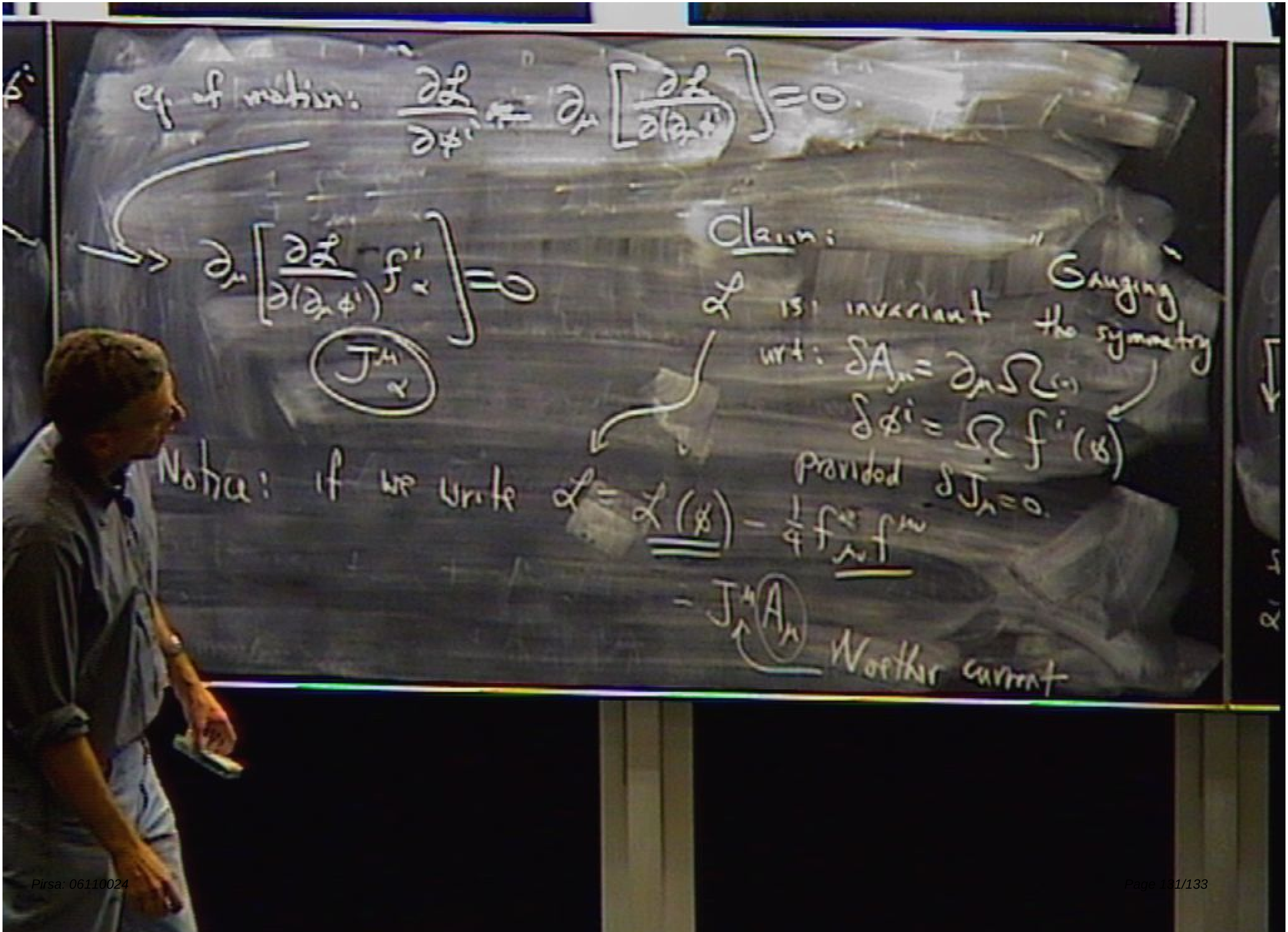
$\delta \phi^i = \Omega f^i(\phi)$

provided $\delta J_\mu = 0$.

Notice: if we write $\mathcal{L} = \mathcal{L}(\phi) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}$

$- J^\mu A_\mu$

Noether current



eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \right] = 0.$
$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha \right] = 0$
J^μ_α
Claim:
$\mathcal{L}$ is invariant
wrt: $\delta A_\mu = \partial_\mu \Omega(x)$
$\delta \phi^i = \Omega f^i(\phi)$
Gauging the symmetry
provided $\delta J_\mu = 0$.
Notice: if we write $\mathcal{L} = \mathcal{L}(\phi) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}$
$- J^\mu A_\mu$
Noether current

eq. of motion: $\frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \right] = 0.$

$$\rightarrow \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} f^i_\alpha \right] = 0$$

J^μ_α

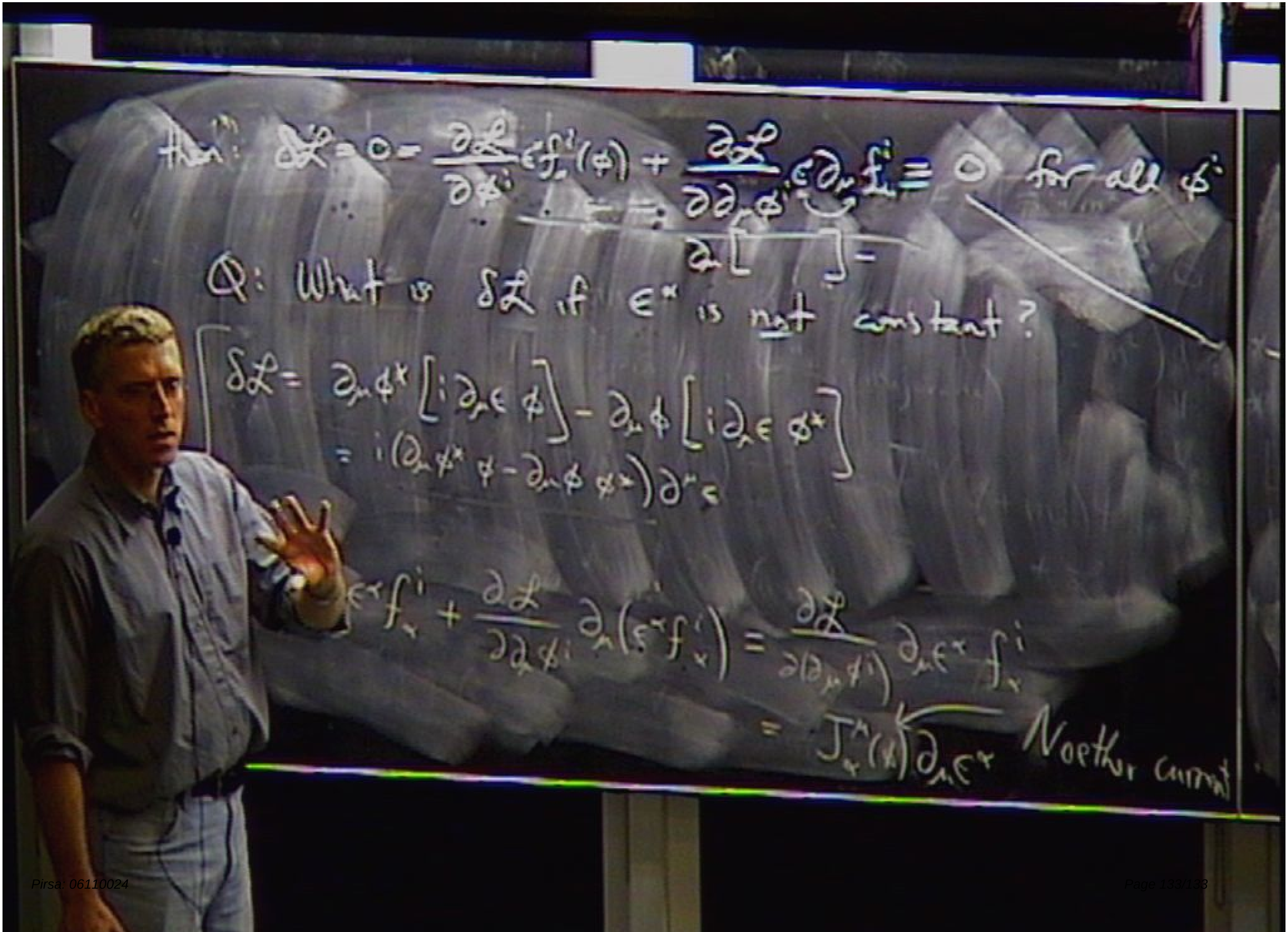
Claim:

$\mathcal{L}$ is invariant wrt: $\delta A_\mu = \partial_\mu \Omega(x)$, $\delta \phi^i = \Omega f^i(\phi)$ provided $\delta J_\mu = 0$.

Gauging the symmetry

Notice: if we write $\mathcal{L} = \mathcal{L}(\phi) - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - J^\mu A_\mu$

Noether current


then: $\delta\mathcal{L} = 0 = \frac{\partial\mathcal{L}}{\partial\phi^i}\epsilon f^i_\alpha(\phi) + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\epsilon\partial_\mu f^i_\alpha = 0$ for all ϕ^i
$\partial_\mu[\quad] =$
Q: What is $\delta\mathcal{L}$ if ϵ^α is not constant?
$\delta\mathcal{L} = \partial_\mu\phi^*[i\partial_\mu\epsilon\,\phi] - \partial_\mu\phi[i\partial_\mu\epsilon\,\phi^*]$
$= i(\partial_\mu\phi^*\phi - \partial_\mu\phi\,\phi^*)\partial^\mu\epsilon$
$\epsilon^\alpha f^i_\alpha + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^i}\partial_\mu(\epsilon^\alpha f^i_\alpha) = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^i)}\partial_\mu\epsilon^\alpha f^i_\alpha$
$= J^\mu_\alpha(x)\partial_\mu\epsilon^\alpha$
Noether current