Title: Graduate Course on Standard Model & Quantum Field Theory - 3A

Date: Nov 01, 2006 11:00 AM

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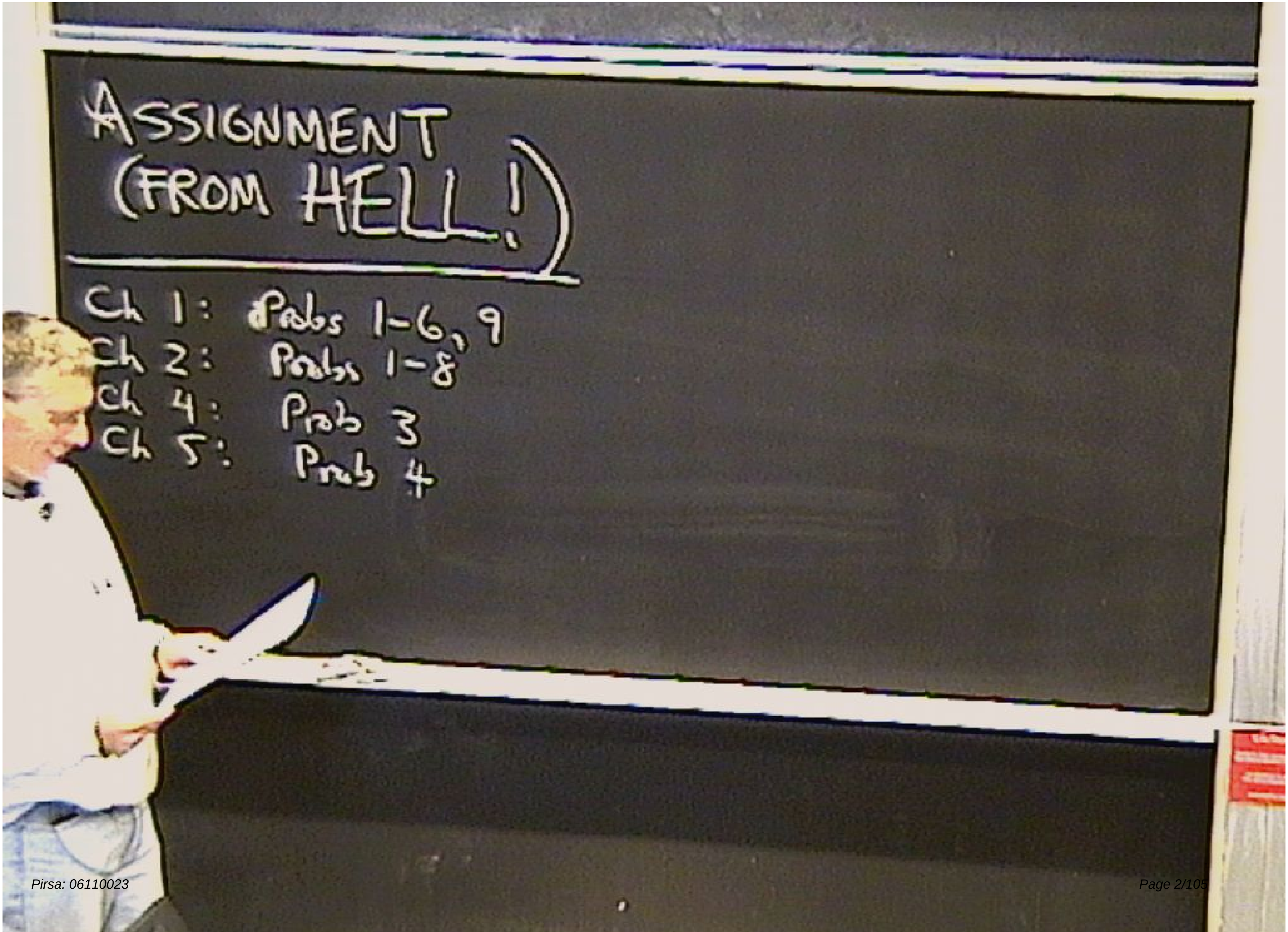
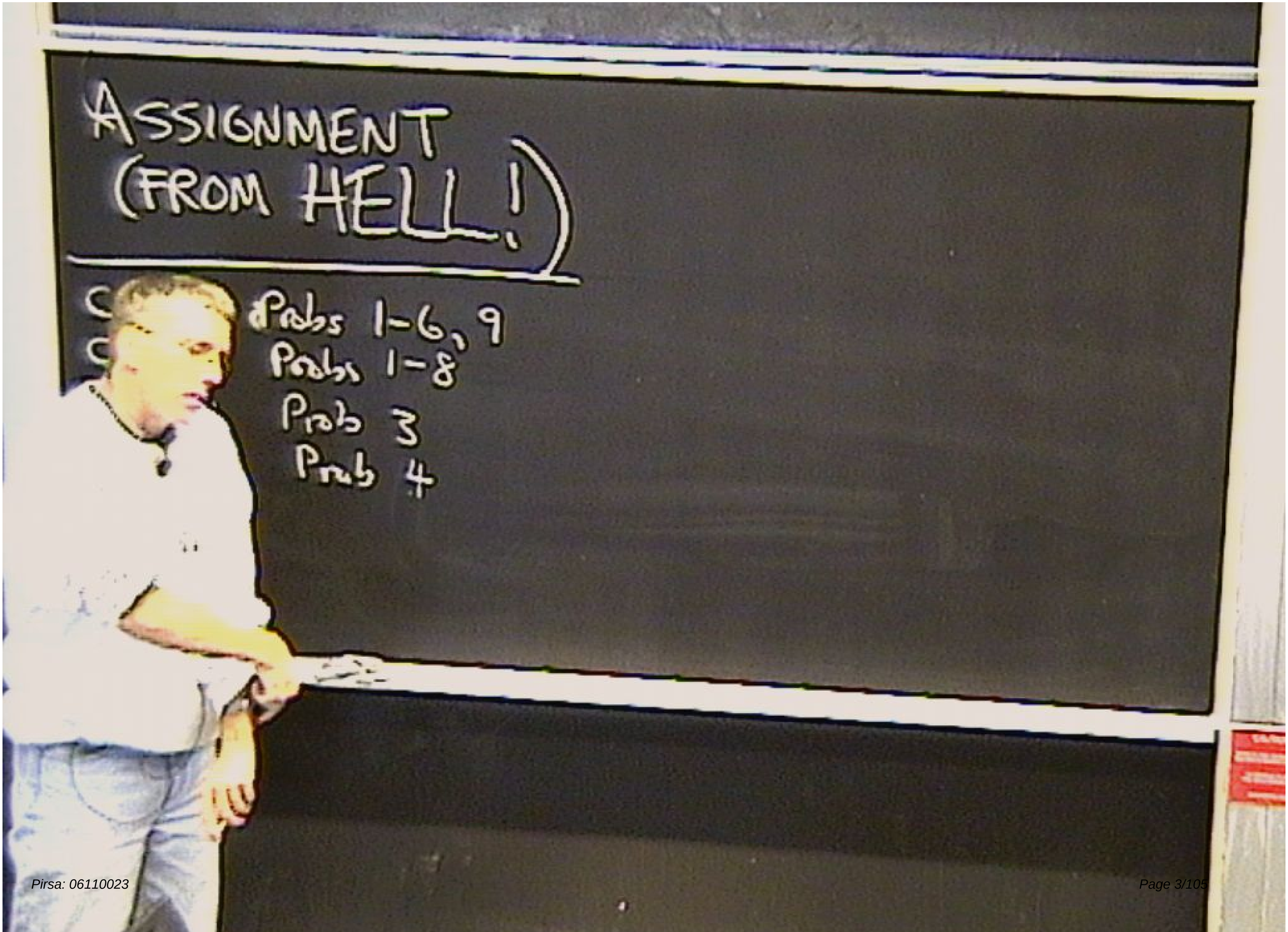
Abstract: Graduate Course on Standard Model & Quantum Field Theory


ASSIGNMENT
(FROM HELL!)
Ch 1: Probs 1-6, 9
Ch 2: Probs 1-8
Ch 4: Prob 3
Ch 5: Prob 4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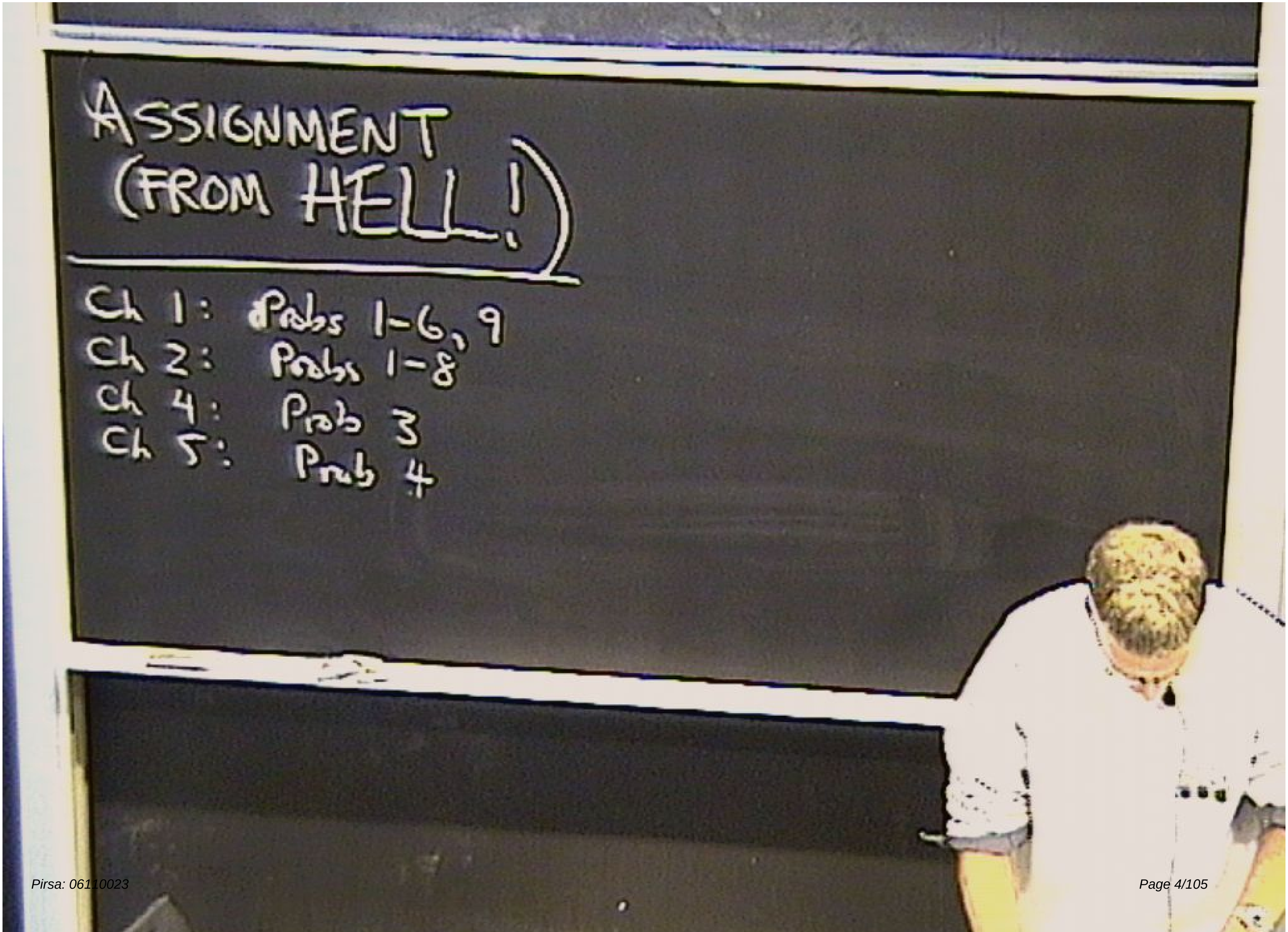
Prob 3
Prob 4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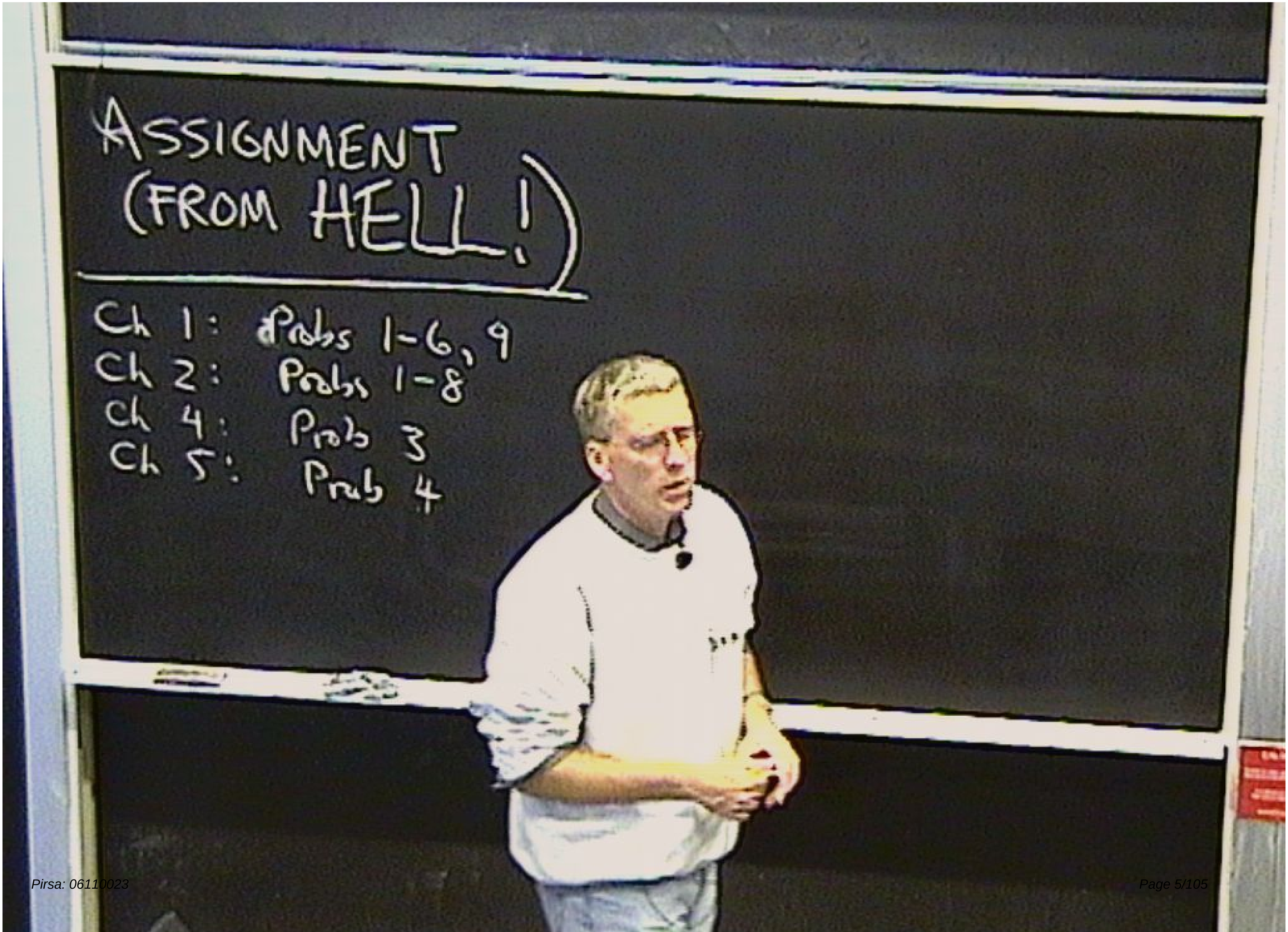
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Ch 1: Probs 1–6, 9
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Due: Weds Dec 6, 2006

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Spin 1/2: Spinor Field

$$\psi = \sum_{\sigma} \int \frac{d^3p}{(2\pi)^{3/2}} \left[u(p,\sigma)\, e^{ipx}\, a_{p\sigma} + v(p,\sigma)\, e^{-ipx}\, a^{*}_{p\sigma} \right]$$

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Spin 1/2 : Spinor Field

$$\psi_m = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} \left[u(p,\sigma)\, e^{ipx}\, a_{p\sigma} + v(p,\sigma)\, e^{-ipx}\, a^*_{p\sigma} \right]$$

where: $$u(p,\sigma) = \frac{1}{\sqrt{2}} \begin{pmatrix} [A_+ - A_- \vec{\sigma}\cdot\hat{p}]\,\chi(\sigma) \\ [A_+ + A_- \vec{\sigma}\cdot\hat{p}]\,\chi(\sigma) \end{pmatrix}$$

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$$\chi(\sigma = +1/2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\chi(\sigma = -1/2) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad A_\pm = \sqrt{E_p \pm m}$$

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$A_\pm = \sqrt{E_p \pm m}$

$\hat{p} = \frac{\vec{p}}{|\vec{p}|}$

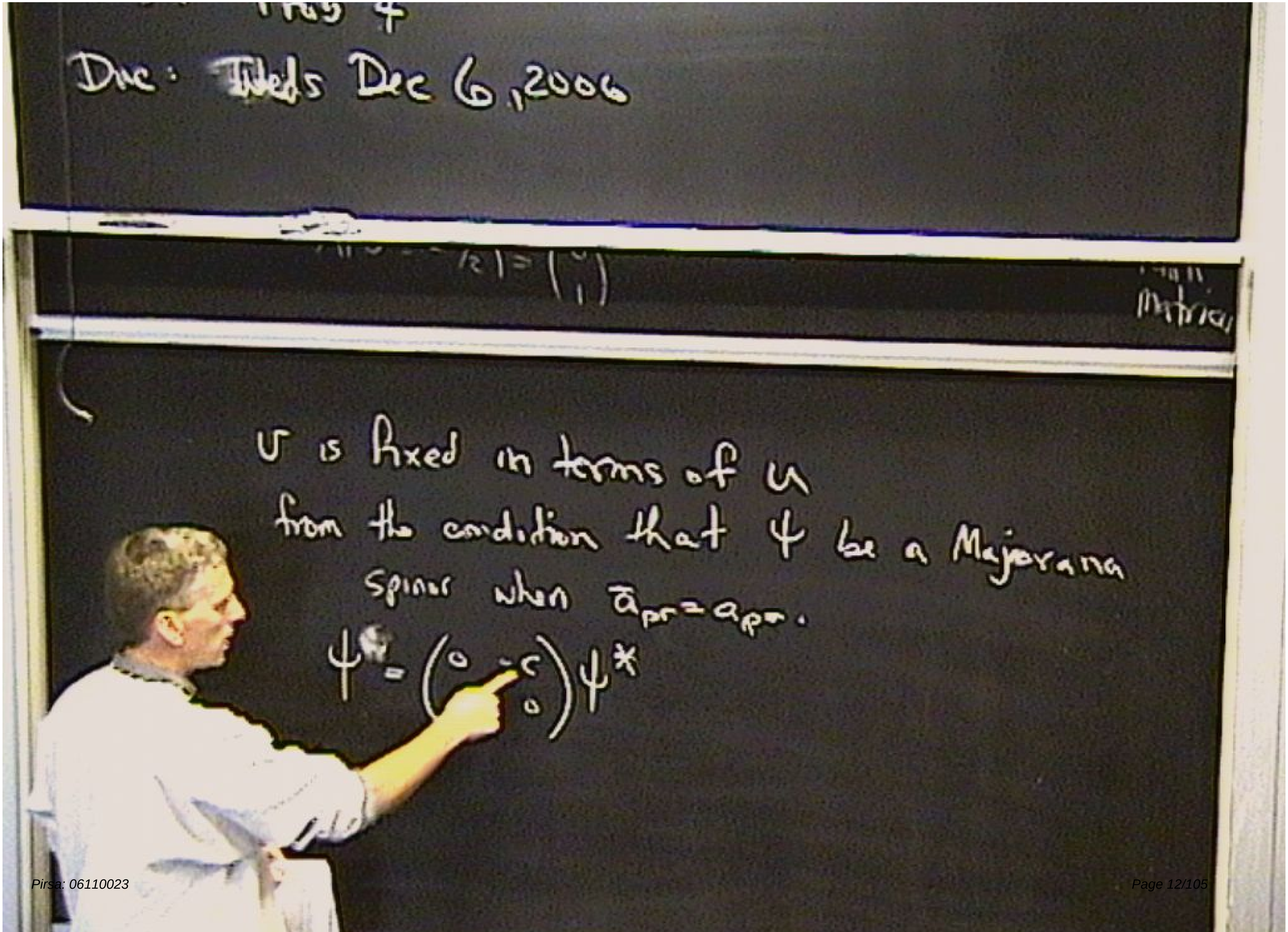
$\vec{\sigma}$ = Pauli matrices

Due: Weds Dec 6, 2006

$(\ldots) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

matrix

v is fixed in terms of u
from the condition that ψ be a Majorana
spinor when $\bar{a}_{p\sigma} = a_{p\sigma}$.


Due: Weds Dec 6, 2006
Matrices
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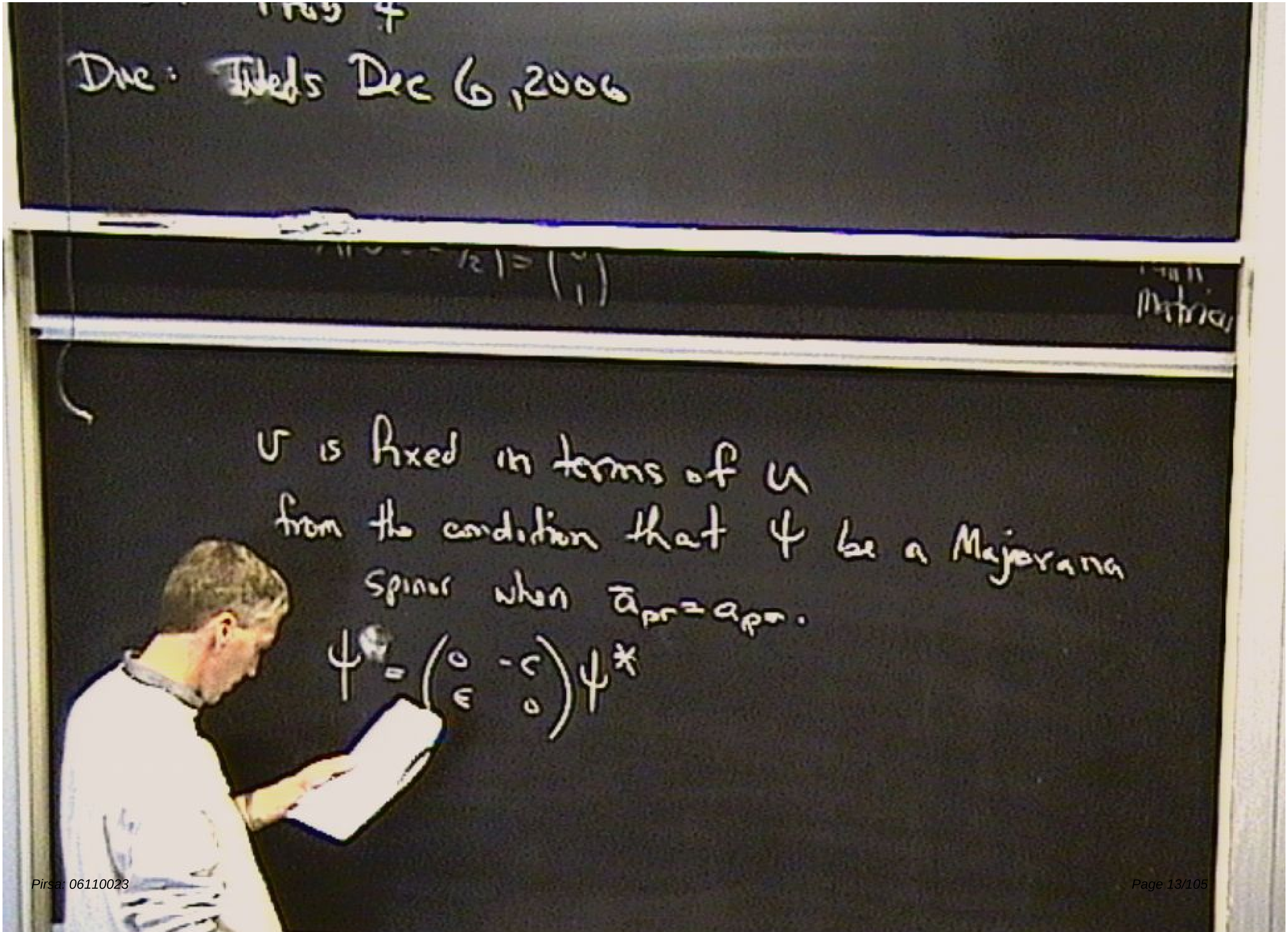
ψ = (0 -c 0) ψ*


Due: Weds Dec 6, 2006
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Due: Weds Dec 6, 2006

v is fixed in terms of u

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$$\psi = \begin{pmatrix} 0 & +\epsilon \\ +\epsilon & 0 \end{pmatrix} \psi^*$$

$$\psi = \begin{pmatrix} \xi \\ \chi \end{pmatrix} \qquad \psi^* = \begin{pmatrix} \xi^* \\ \chi^* \end{pmatrix}$$

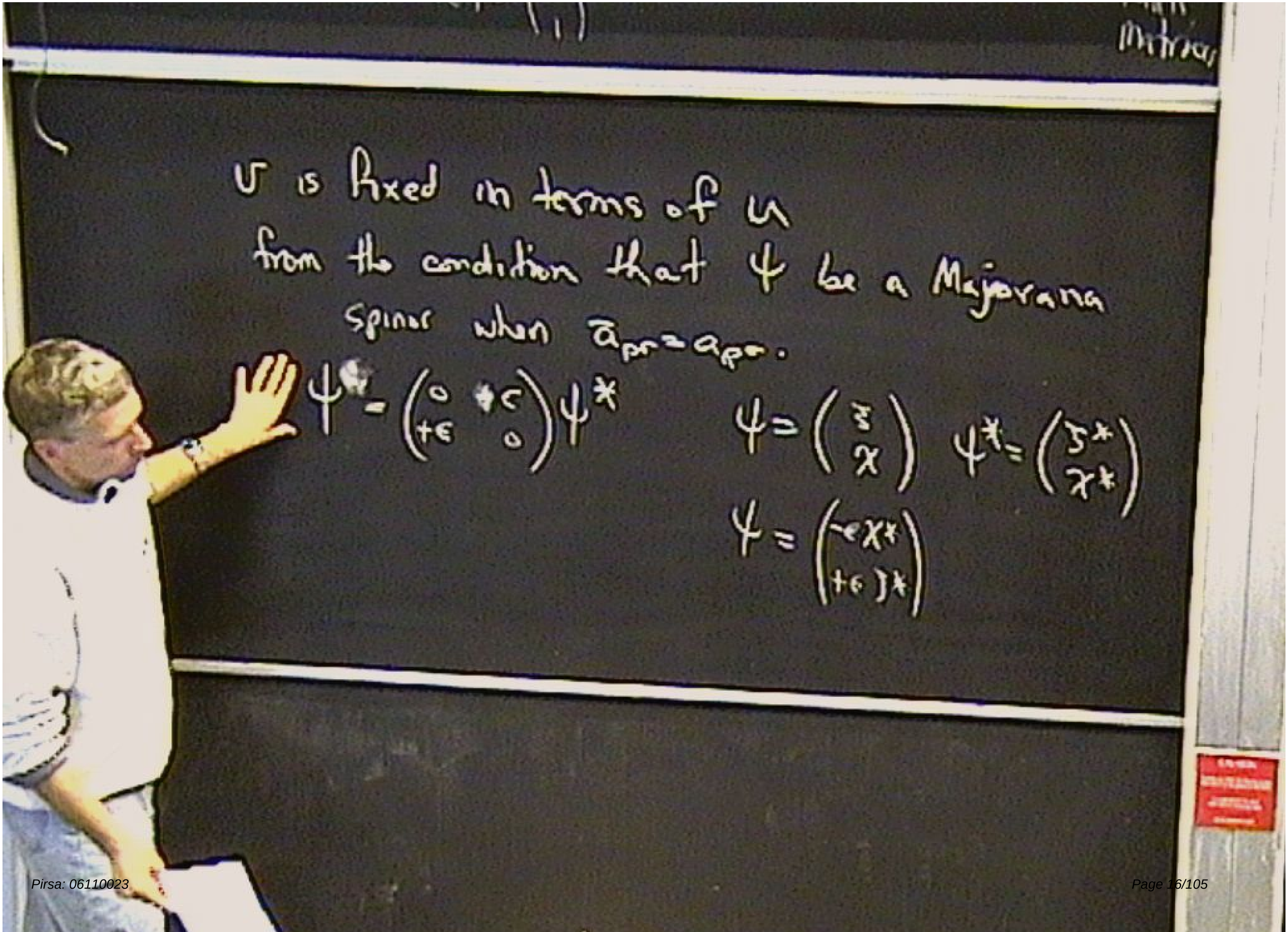
$$\psi = \begin{pmatrix} -\epsilon \chi^* \\ +\epsilon \xi^* \end{pmatrix}$$

the u's satisfy:

$$(i\not{p} + m)\,u = 0 \qquad \not{p} = p^\mu \gamma_\mu$$

$$\gamma^\mu = \{\gamma^0, \gamma^1, \gamma^2, \gamma^3\}$$

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{pmatrix}$$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ψ = (0 ε; +ε 0) ψ*
ψ = (ξ; χ)   ψ* = (ξ*; χ*)
ψ = (−εχ*; +εξ*)

the u's satisfy:

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$$\gamma^\mu=\{\gamma^0,\gamma^1,\gamma^2,\gamma^3\}$$

$$\gamma^0=\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix}=-i\beta \quad \vec{\gamma}=\begin{pmatrix}0 & -i\vec{\sigma}\\ i\vec{\sigma} & 0\end{pmatrix}$$

if $\psi\to D\psi$ under a Lorentz transformation

then $\bar{\psi}=\psi^\dagger\beta=i\psi^\dagger\gamma^0=-i\psi^\dagger\gamma_0 \to \bar{\psi}D^{-1}$ under " " " ".

if $\psi \to D\psi$ under a Lorentz transformation

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the 4 Dirac matrices, γ^μ, ~~transform~~ transform nicely under Lorentz transformations:

$$D^{-1}\gamma^\mu D = \Lambda^\mu{}_\nu \gamma^\nu$$

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$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

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Claim: any 4x4 complex matrix can be expanded as a linear combination of the following basis matrices:

I, γ_5

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from the condition that ψ be a Majorana spinor when $\bar{a}_{p r} = a_{p r}$.

$$\psi = \begin{pmatrix} 0 & +\epsilon \\ +\epsilon & 0 \end{pmatrix} \psi^*$$

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then $\bar{\psi} = \psi^\dagger \beta = i\psi^\dagger \gamma^0 = -i\psi^\dagger \gamma_4 \mapsto \bar{\psi} D^{-1}$ under " " " ".

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$$I \,, \gamma_5 \,, \gamma^\mu \,, \gamma_5 \gamma^\mu \,, \gamma^{\mu\nu} = \tfrac{1}{2}[\gamma^\mu, \gamma^\nu]$$

$$M = A I + B \gamma_5 + C_\mu \gamma^\mu + D_\mu \gamma_5 \gamma^\mu + E_{\mu\nu} \gamma^{\mu\nu}$$

then $\bar{\psi} = \psi^\dagger \beta = i\psi^\dagger \gamma^0 = -i\psi^\dagger \gamma_0 \to \bar{\psi} D^{-1}$ under " " " ".

the 4 Dirac matrices, γ, transform nicely under Lorentz transformations:

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$D^{-1}\gamma_5 D = \gamma_5 (\det \Lambda)$

So the most bilinear of 2 spinors is:

$$\bar{\psi} M \psi = A\,\underset{\sim}{\bar{\psi}\psi} + B\bar{\psi}\gamma_5\psi$$
$$+ C_\mu \bar{\psi}\gamma^\mu\psi + D_\mu \bar{\psi}\gamma_5\gamma^\mu\psi$$
$$+ E_{\mu\nu}\bar{\psi}\gamma^{\mu\nu}\psi$$

$D^{\prime}\gamma_5 D = \gamma_5 (\det \Lambda)$

So the most bilinear of 2 spinors is:

$$\bar{\psi}_1 M \psi_2 = A \underbrace{\bar{\psi}_1 \psi_2} + B \bar{\psi}_1 \gamma_5 \psi_2$$
$$+ C_\mu \bar{\psi}_1 \gamma^\mu \psi_2 + D_\mu \bar{\psi}_1 \gamma_5 \gamma^\mu \psi_2$$
$$+ E_{\mu\nu} \bar{\psi}_1 \gamma^{\mu\nu} \psi_2$$

For N ~~x~~ spin-½ particles

v is fixed in terms of u

from the condition that ψ be a Majorana spinor when $\bar{a}_{pr} = a_{pr}$.

$\gamma_L \psi$

$$\psi = \begin{pmatrix} 0 & -\epsilon \\ +\epsilon & 0 \end{pmatrix} \psi^*$$

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$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

Matrices

v is fixed in terms of u from the condition that ψ be a Majorana spinor when $\bar{a}_{p\sigma} = a_{p\sigma}$.

$\gamma_L \psi_L = \psi_L$

$\gamma_R \psi_R = -\psi_R$

Weyl spinors

$$\psi^{*} = \begin{pmatrix} 0 & -\epsilon \\ +\epsilon & 0 \end{pmatrix} \psi^*$$

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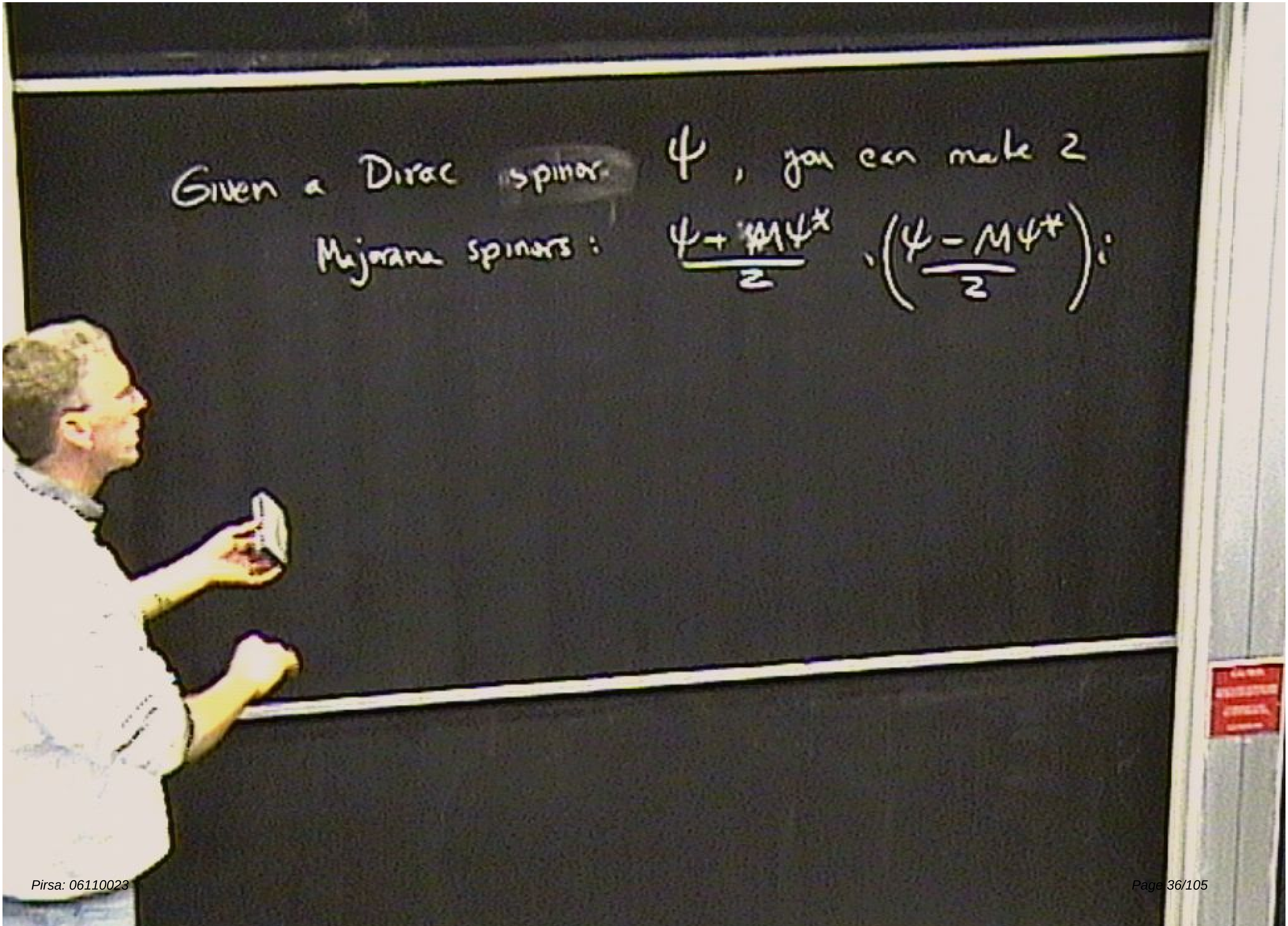
Given a Dirac spinor ψ, you can make 2

Majorana spinors: $\psi +$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and given any 2 majorana spinors, ψ_1, ψ_2,

a general Dirac spinor is $\psi = \psi_1 + i\psi_2$

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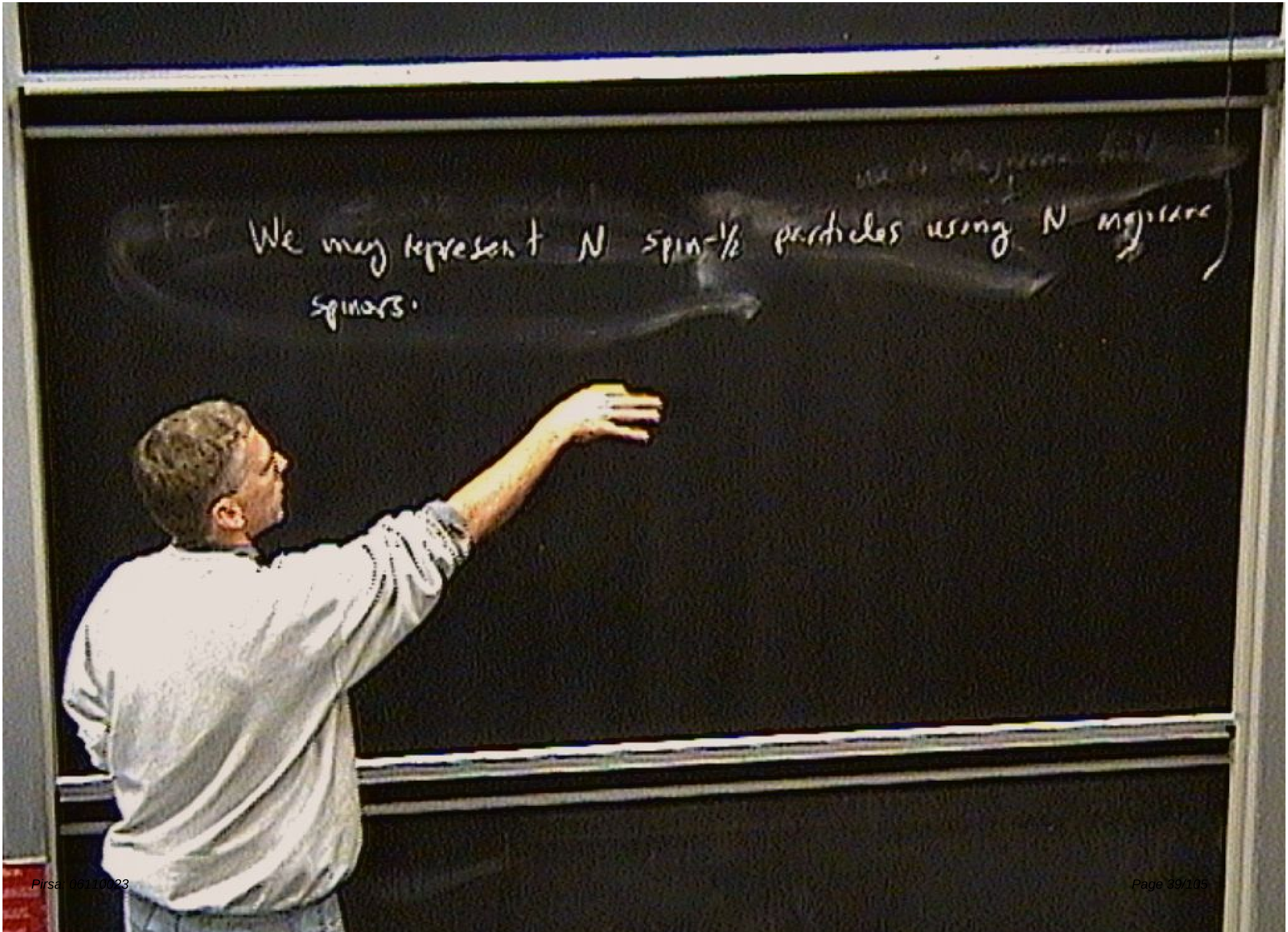
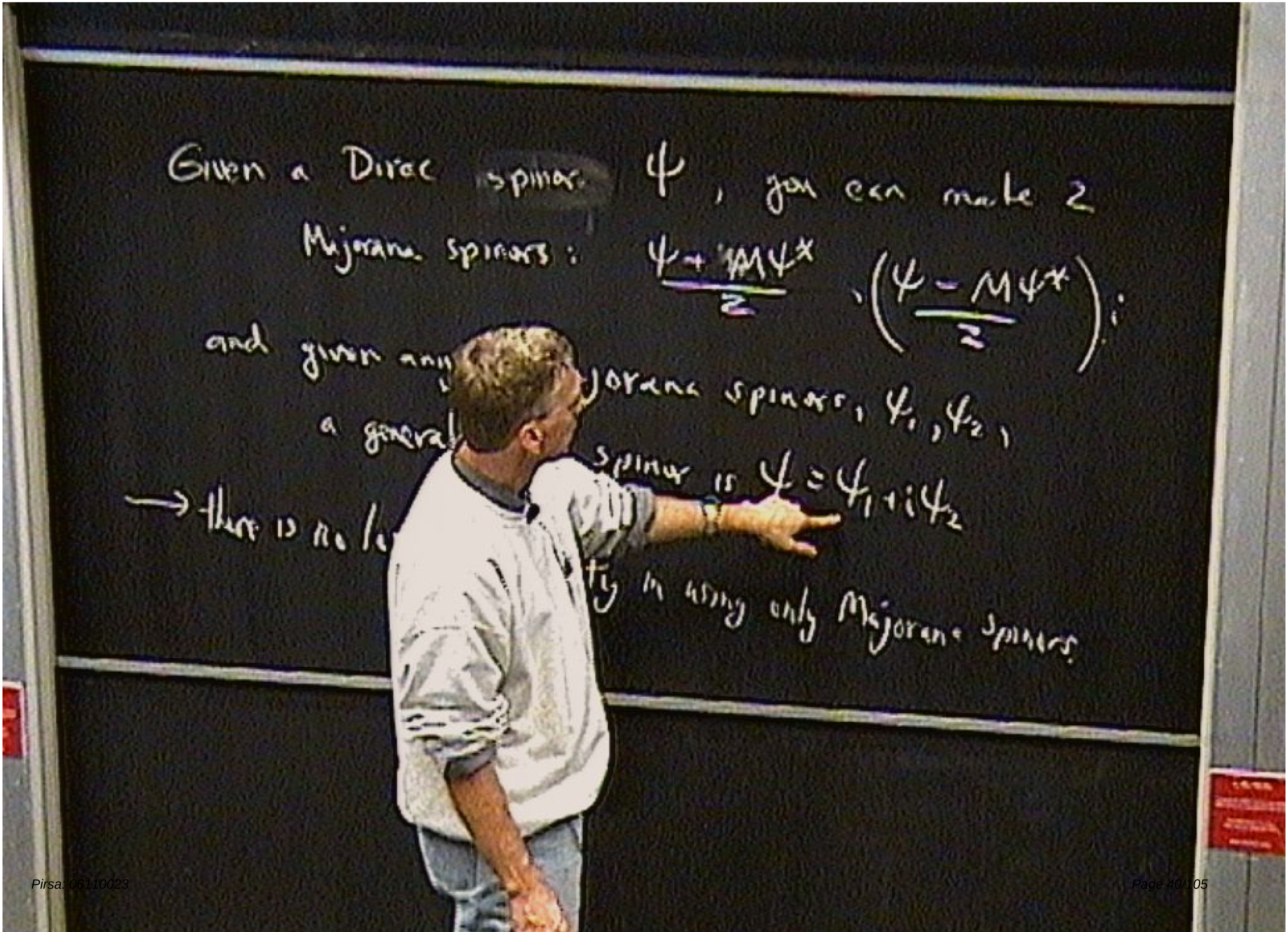
$\longrightarrow$ there is no loss in generality in using only Majorana spinors


We may represent N spin-½ particles using N majorana
spinors.


Given a Dirac spinor ψ, you can make 2
Majorana spinors: (ψ + Mψ*)/2 , ((ψ − Mψ*)/2) i
and given any
jorana spinors, ψ1, ψ2,
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→ there is no l
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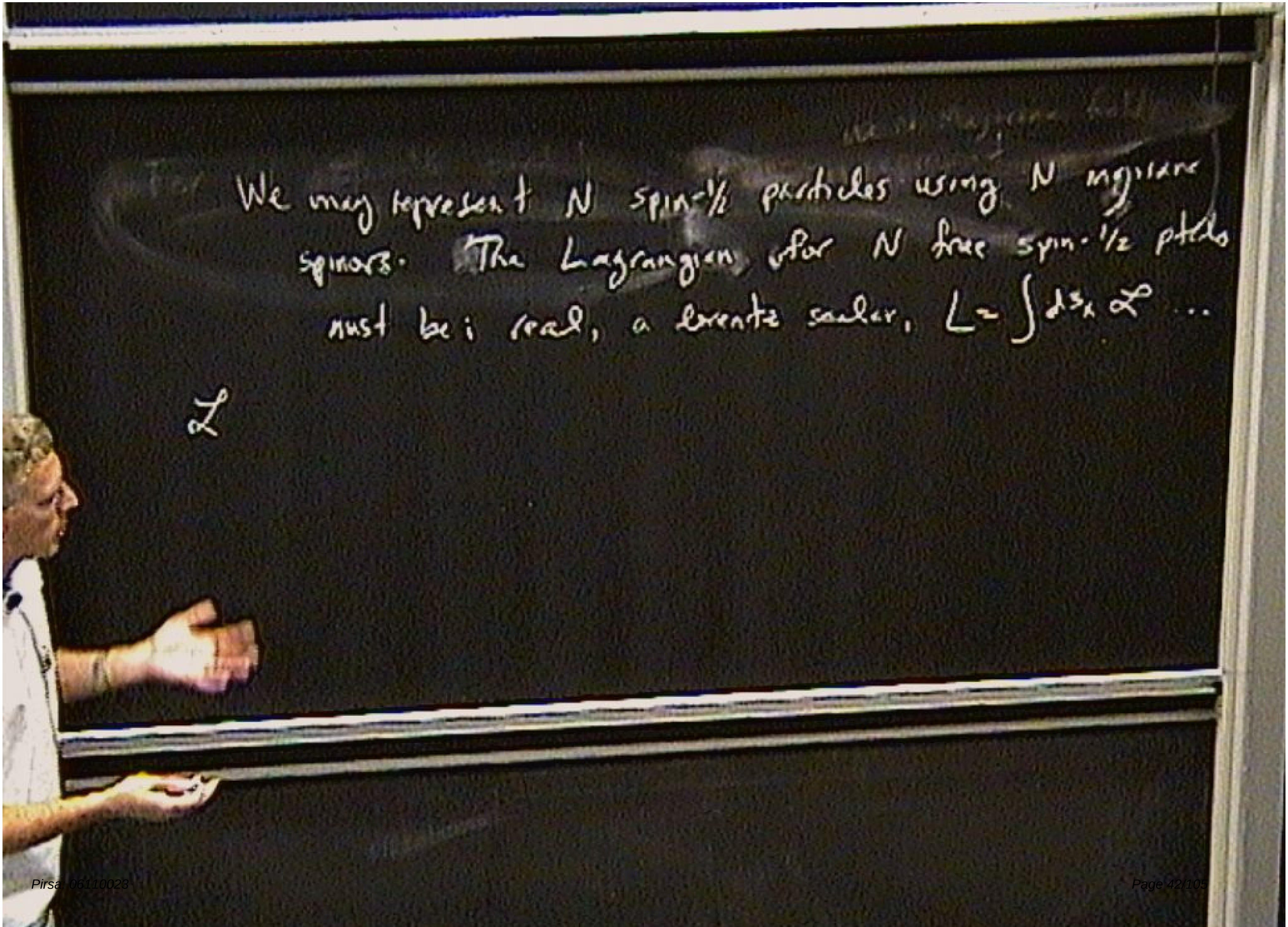
Given a Dirac spinor ψ, you can make 2 Majorana spinors: $\frac{\psi + M\psi^*}{2}$, $\left(\frac{\psi - M\psi^*}{2}\right)i$;

and given any 2 Majorana spinors, ψ_1, ψ_2, a general Dirac spinor is $\psi = \psi_1 + i\psi_2$

$M\psi_1^* = \psi_1$
$M(i\psi_2)^* = -i\psi_2$

$\rightarrow$ there is no loss in generality in using only Majorana spinors.

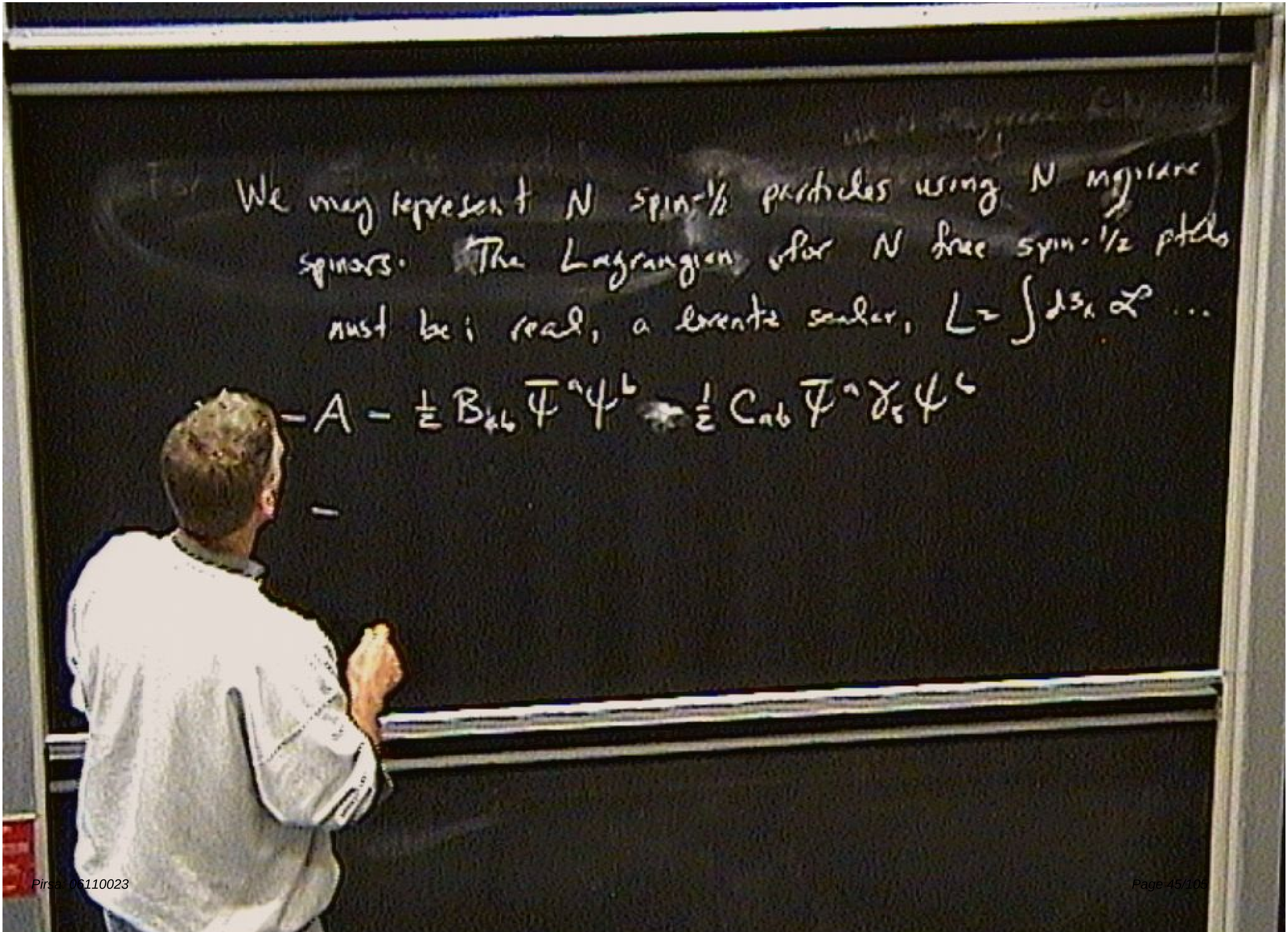

We may represent N spin-½ particles using N majorana
spinors. The Lagrangian for N free spin-½ ptcls
must be: real, a Lorentz scalar, L = ∫d³x ℒ ...
ℒ

We may represent N spin-½ particles using N angular spinors. The Lagrangian for N free spin-½ ptcls must be: real, a Lorentz scalar, $L = \int d^3x\, \mathcal{L}$ ...

$$\mathcal{L} = -A$$

We may represent N spin-1/2 particles using N majorana spinors. The Lagrangian for N free spin-1/2 ptcls must be: real, a Lorentz scalar, $L = \int d^3x\, \mathcal{L}$ ...

$$\mathcal{L} = -A - \frac{1}{2} B_{ab} \bar{\Psi}^a \psi^b$$


We may represent N spin-1/2 particles using N majorana
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$-A - \frac{1}{2} B_{ab} \bar{\psi}^a \psi^b - \frac{1}{2} C_{ab} \bar{\psi}^a \gamma_5 \psi^b$
$-$

We may represent N spin-½ particles using N majorana spinors. The Lagrangian for N free spin-½ ptcls must be: real, a Lorentz scalar, $L = \int d^3x\, \mathcal{L}$ ...

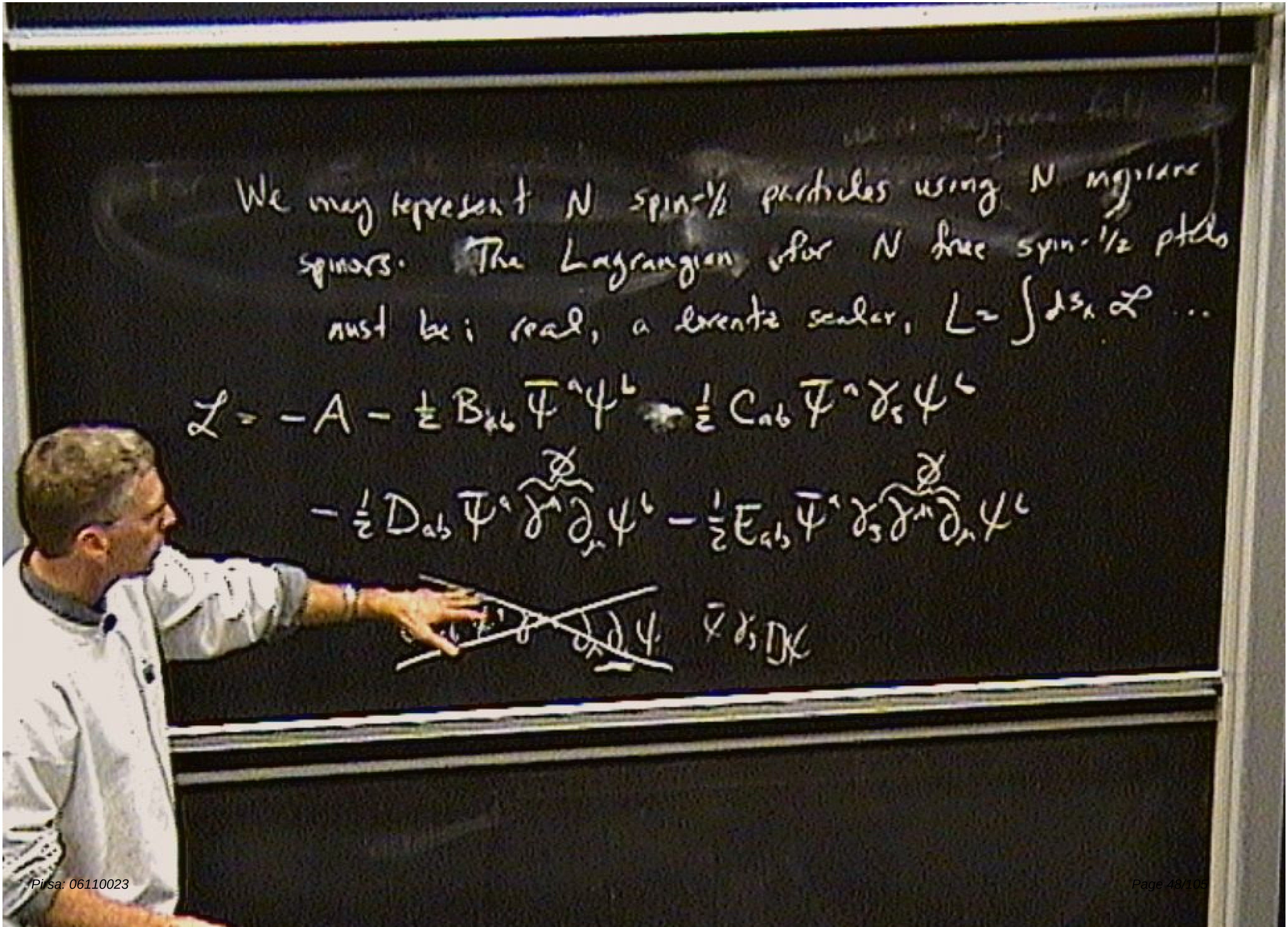
$$\mathcal{L} = -A - \frac{1}{2} B_{ab} \bar{\Psi}^a \psi^b - \frac{1}{2} C_{ab} \bar{\Psi}^a \gamma_5 \psi^b$$

$$- \frac{1}{2} D_{ab} \bar{\Psi}^a \gamma^\mu \partial_\mu \psi^b$$

We may represent N spin-$\frac{1}{2}$ particles using N majorana spinors. The Lagrangian for N free spin-$\frac{1}{2}$ ptcls must be: real, a Lorentz scalar, $L = \int d^3x\, \mathcal{L}$ ...

$$\mathcal{L} = -A - \tfrac{1}{2} B_{ab} \bar{\Psi}^a \psi^b - \tfrac{1}{2} C_{ab} \bar{\Psi}^a \gamma_5 \psi^b$$

$$- \tfrac{1}{2} D_{ab} \bar{\Psi}^a \gamma^\mu \partial_\mu \psi^b - \tfrac{1}{2} E_{ab} \bar{\Psi}^a \gamma_5 \gamma^\mu \partial_\mu \psi^b$$


We may represent N spin-½ particles using N majorana spinors. The Lagrangian for N free spin-½ ptcls must be: real, a Lorentz scalar, $L = \int d^3x \, \mathcal{L}$ ..
$\mathcal{L} = -A - \frac{1}{2} B_{ab} \bar{\Psi}^a \psi^b - \frac{1}{2} C_{ab} \bar{\Psi}^a \gamma_5 \psi^b$
$- \frac{1}{2} D_{ab} \bar{\Psi}^a \overbrace{\gamma^\mu \partial_\mu}^{\not\partial} \psi^b - \frac{1}{2} E_{ab} \bar{\Psi}^a \gamma_5 \overbrace{\gamma^\mu \partial_\mu}^{\not\partial} \psi^b$
$\bar{\psi} \gamma_5 \not D \psi$

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$$\mathcal{L} = -A - \tfrac{1}{2} B_{ab}\bar{\Psi}^a\psi^b - \tfrac{1}{2} C_{ab}\bar{\Psi}^a\gamma_5\psi^b$$

$$- \tfrac{1}{2} D_{ab}\bar{\Psi}^a \overbrace{\gamma^\mu\partial_\mu}^{\not\partial}\psi^b - \tfrac{1}{2} E_{ab}\bar{\Psi}^a\gamma_5\overbrace{\gamma^\mu\partial_\mu}^{\not\partial}\psi^b$$

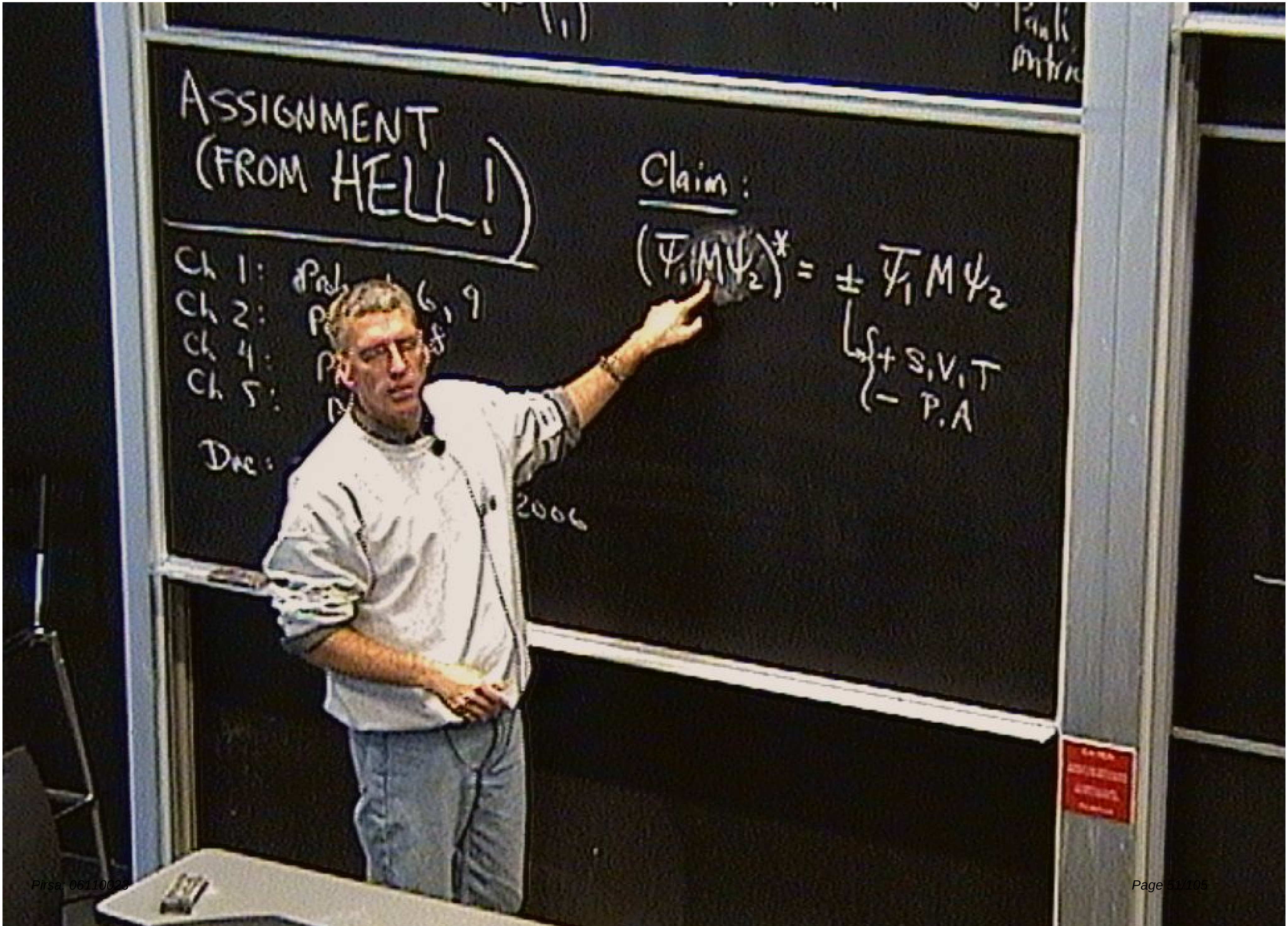
$$\cancel{- \tfrac{1}{2} F_{ab}\bar{\Psi}^a\gamma^\mu\gamma_5\partial_\mu\psi} \qquad \bar{\Psi}\gamma_5\not{D}K$$

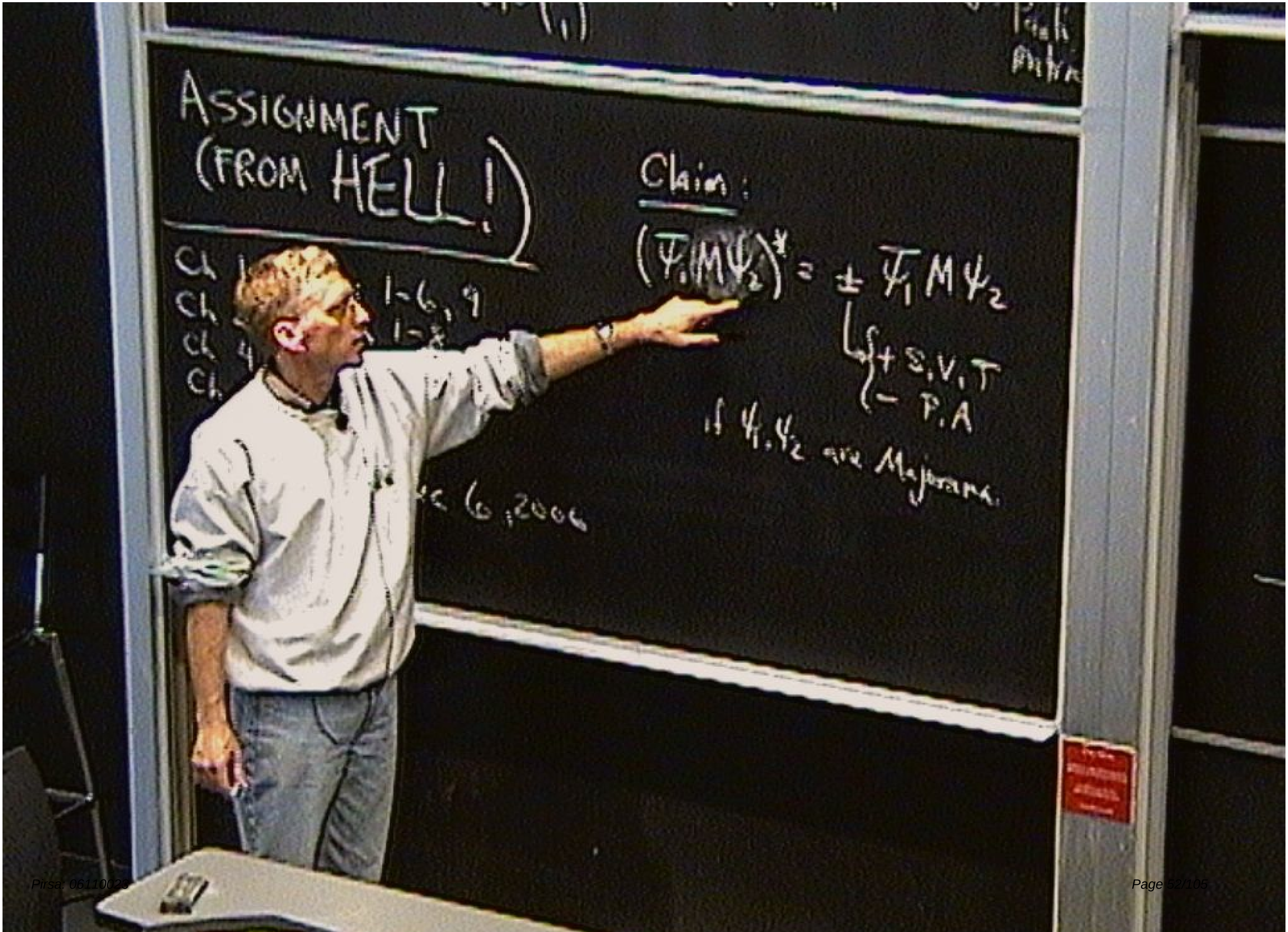
We may represent N spin-½ particles using N majorana spinors. The Lagrangian for N free spin-½ ptcls must be: real, a Lorentz scalar, $L = \int d^3x\, \mathcal{L}$ ...

$$\mathcal{L} = -A - \tfrac{1}{2} B_{ab} \bar{\Psi}^a \psi^b - \tfrac{1}{2} C_{ab} \bar{\Psi}^a \gamma_5 \psi^b$$

$$- \tfrac{1}{2} D_{ab} \bar{\Psi}^a \overbrace{\gamma^\mu \partial_\mu}^{\not\partial} \psi^b - \tfrac{1}{2} E_{ab} \bar{\Psi}^a \gamma_5 \overbrace{\gamma^\mu \partial_\mu}^{\not\partial} \psi^b$$

$$\cancel{- \tfrac{1}{2} F_{ab} \bar{\Psi}^a \gamma\, \partial_\mu \partial \psi} \quad \cancel{\bar{\Psi} \ldots K}$$


ASSIGNMENT
(FROM HELL!)
Ch 1:
Ch 2:
Ch 4:
Ch 5:
Claim:
$(\bar{\psi}_1 M \psi_2)^* = \pm \bar{\psi}_1 M \psi_2$
+ S, V, T
− P, A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Claim:
$(\bar{\psi}_1 M \psi_2)^* = \pm \bar{\psi}_1 M \psi_2$
$+$ S, V, T
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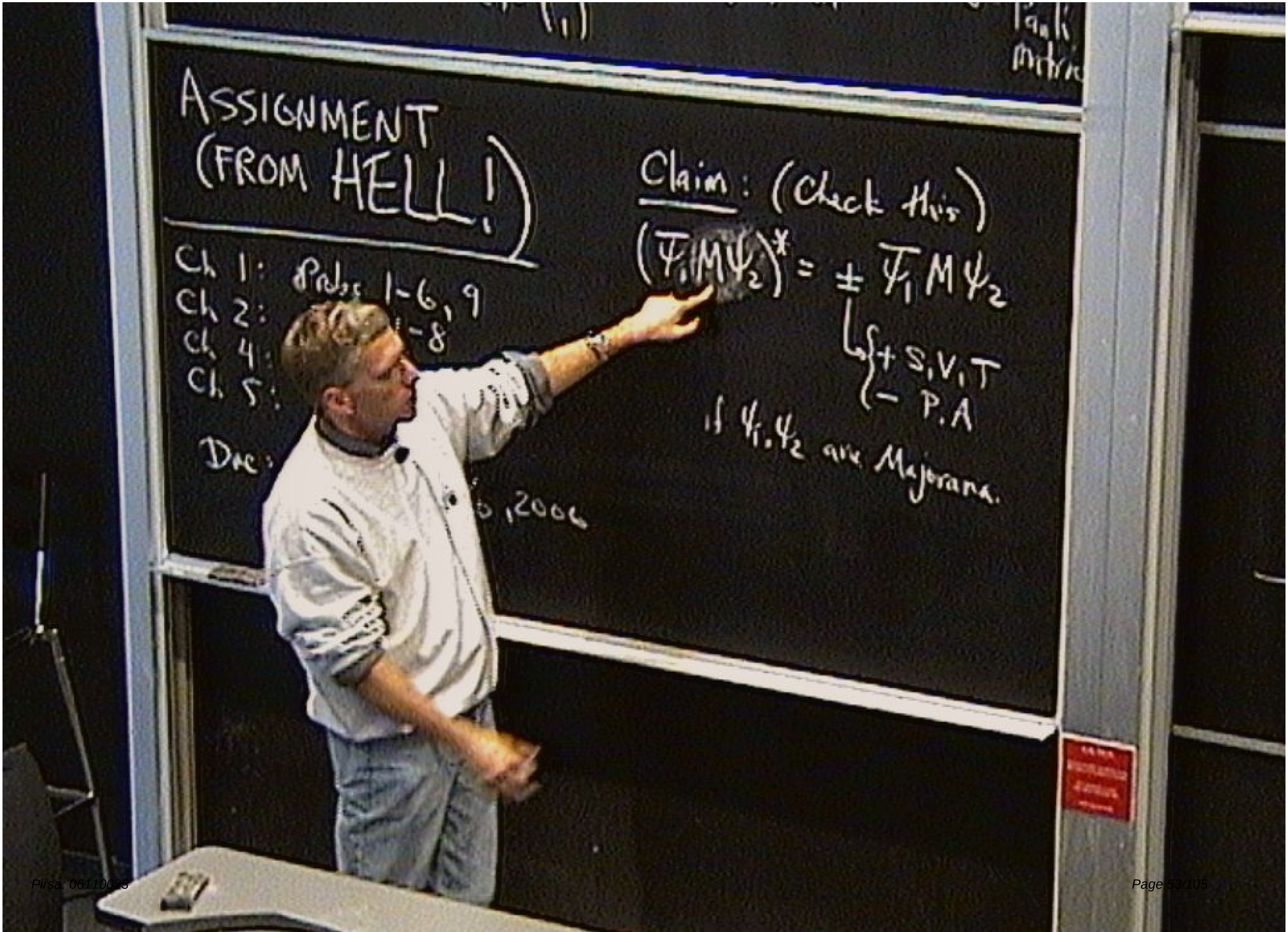
if ψ_1, ψ_2 are Majorana.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Due:
, 2006
Claim: (Check this)
$(\bar{\psi}_1 M \psi_2)^* = \pm \bar{\psi}_1 M \psi_2$
+ S, V, T
− P, A
if ψ_1, ψ_2 are Majorana.

ASSIGNMENT (FROM HELL!)

Ch 1: Probs 1-6, 9
Ch 2: Probs 1-8
Ch 4: Prob 3
Ch 5: Prob 4

Due: Weds Dec 6, 2006

Claim: (Check this)

$$(\bar{\psi}_1 M \psi_2)^* = \pm \bar{\psi}_1 M \psi_2$$

$$\pm = \begin{cases} + & S, V, T \\ - & P, A \end{cases}$$

if ψ_1, ψ_2 are Majorana.

We may represent N spin-1/2 particles using N majorana spinors. The Lagrangian for N free spin-1/2 ptcls must be: real, a Lorentz scalar, $L = \int d^3x\, \mathcal{L}$ ...

$$\mathcal{L} = -A - \tfrac{1}{2} B_{ab} \bar{\Psi}^a \psi^b - \tfrac{i}{2} C_{ab} \bar{\Psi}^a \gamma_5 \psi^b$$

$$- \tfrac{1}{2} D_{ab} \bar{\Psi}^a \gamma^\mu \partial_\mu \psi^b - \tfrac{i}{2} E_{ab} \bar{\Psi}^a \gamma_5 \gamma^\mu \partial_\mu \psi^b$$

$$\cancel{- \tfrac{1}{2} F_{ab} \psi^a \gamma \ldots \partial_\mu \partial \psi} \quad \cancel{\bar{\Psi} \ldots}$$

We may represent N spin-½ particles using N majorana spinors. The Lagrangian for N free spin-½ ptcls must be: real, a Lorentz scalar. $L = \int d^3x\, \mathcal{L}$ ...

$$\mathcal{L} = -A - \tfrac{1}{2} B_{ab} \bar{\Psi}^a \psi^b - \tfrac{i}{2} C_{ab} \bar{\Psi}^a \gamma_5 \psi^b$$

$$- \tfrac{1}{2} D_{ab} \bar{\Psi}^a \gamma^\mu \partial_\mu \psi^b - \tfrac{i}{2} E_{ab} \bar{\Psi}^a \gamma_5 \gamma^\mu \partial_\mu \psi^b$$

$$\cancel{- \tfrac{1}{2} F_{ab} \psi^a \partial_\mu \partial_\nu \psi} \quad \cancel{\bar{\psi} \ldots} \qquad \mathcal{L} = \mathcal{L}^* \Leftrightarrow A, B_{ab}, \ldots E_{ab} \text{ are real.}$$

where: $u(\vec{p},\sigma) = \frac{1}{\sqrt{2}}\left(\ldots\ [A_+ + A_- \vec{\sigma}\cdot\hat{p}]\,\chi(\sigma) \right)$ $\quad \hat{p} = \frac{\vec{p}}{|\vec{p}|}$

$\chi(\sigma = +1/2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

ASSIGNMENT (FROM HELL!)

Ch 1: Probs 1-6, 9
Ch 2: Probs 1-8
Ch 4: Prob 3
Ch 5: Prob 4

Due: Weds Dec 6, 2006

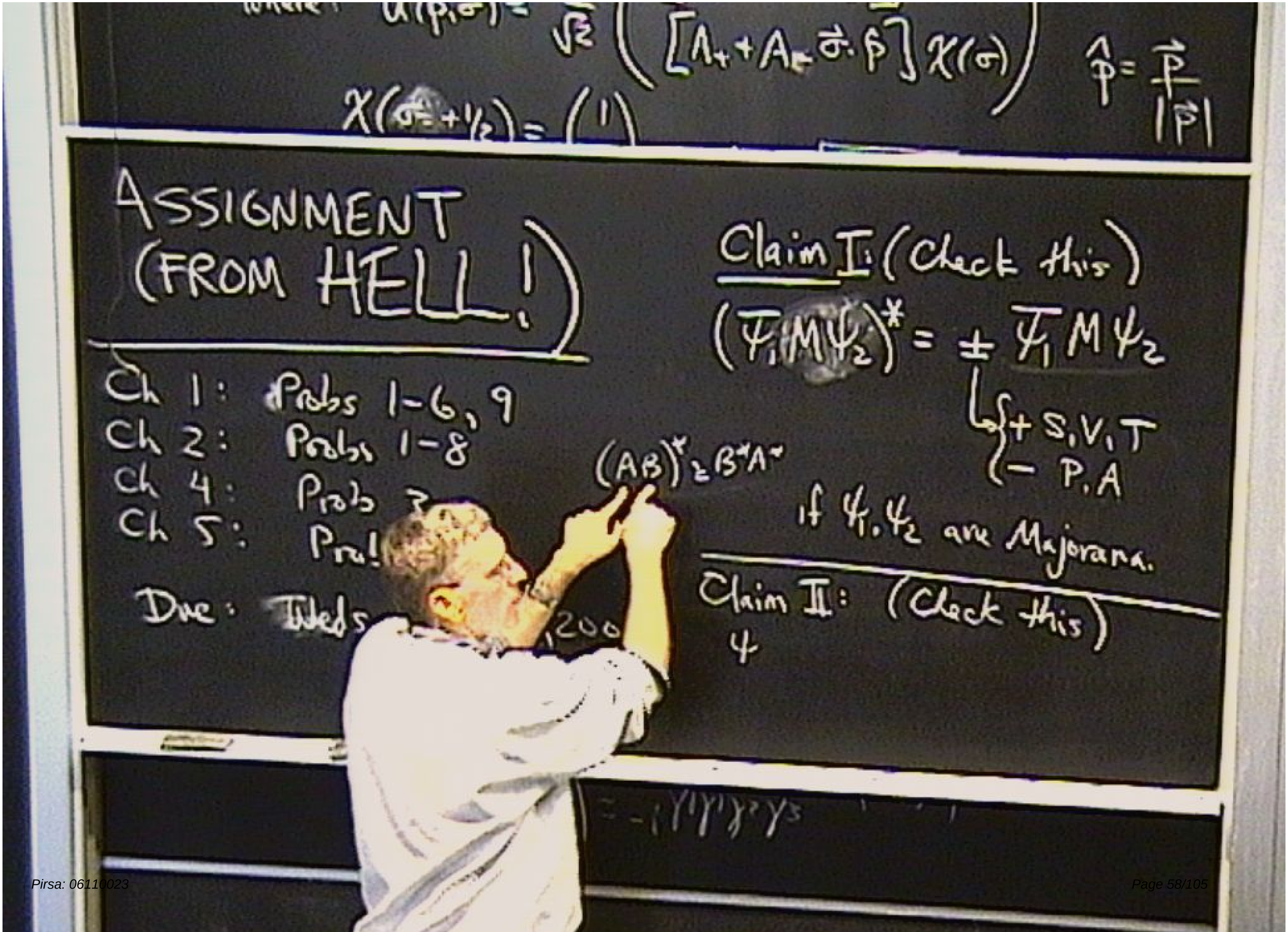
Claim I: (Check this)

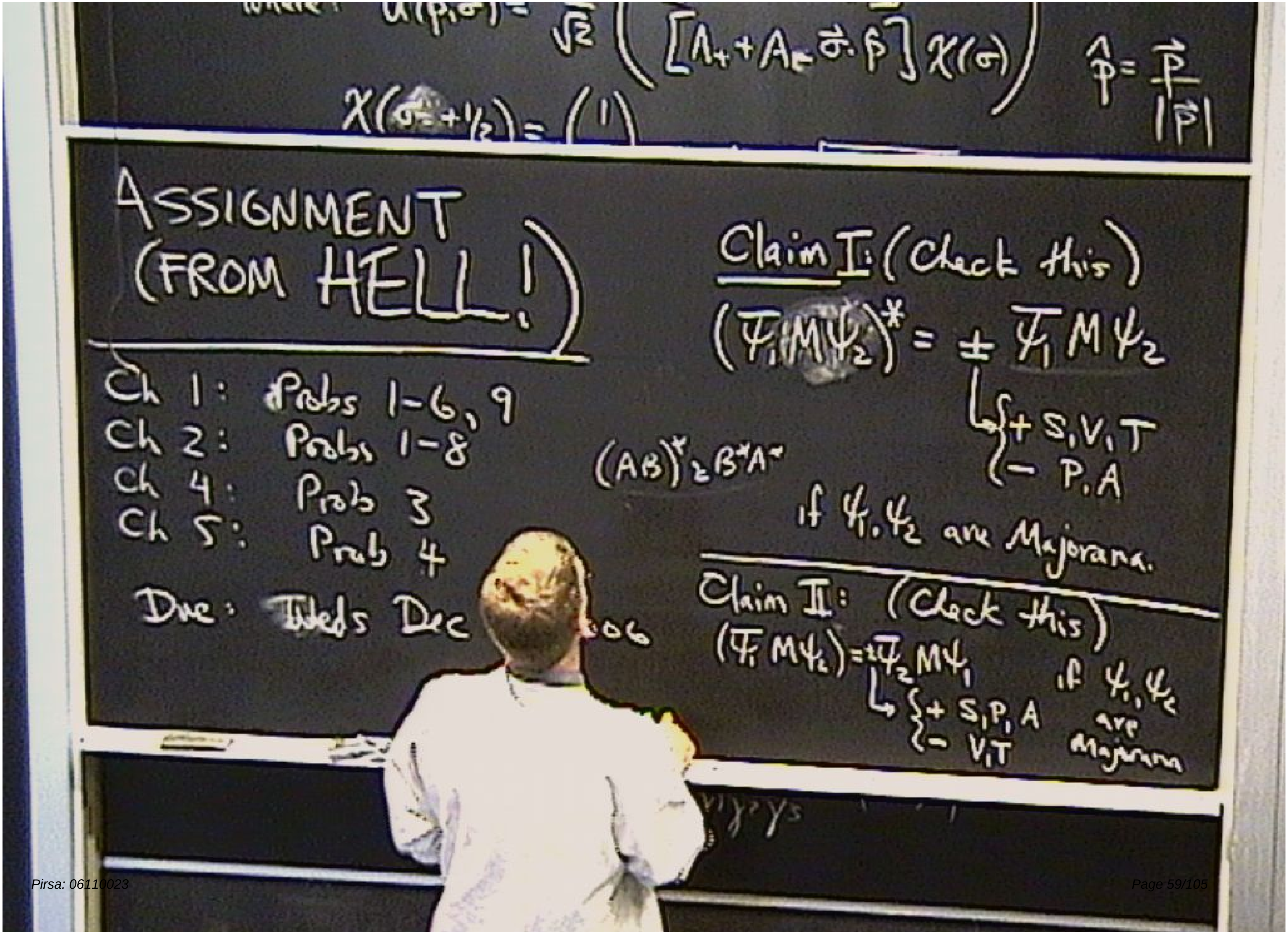
$(\bar{\psi}_1 M \psi_2)^* = \pm\, \bar{\psi}_1 M \psi_2$

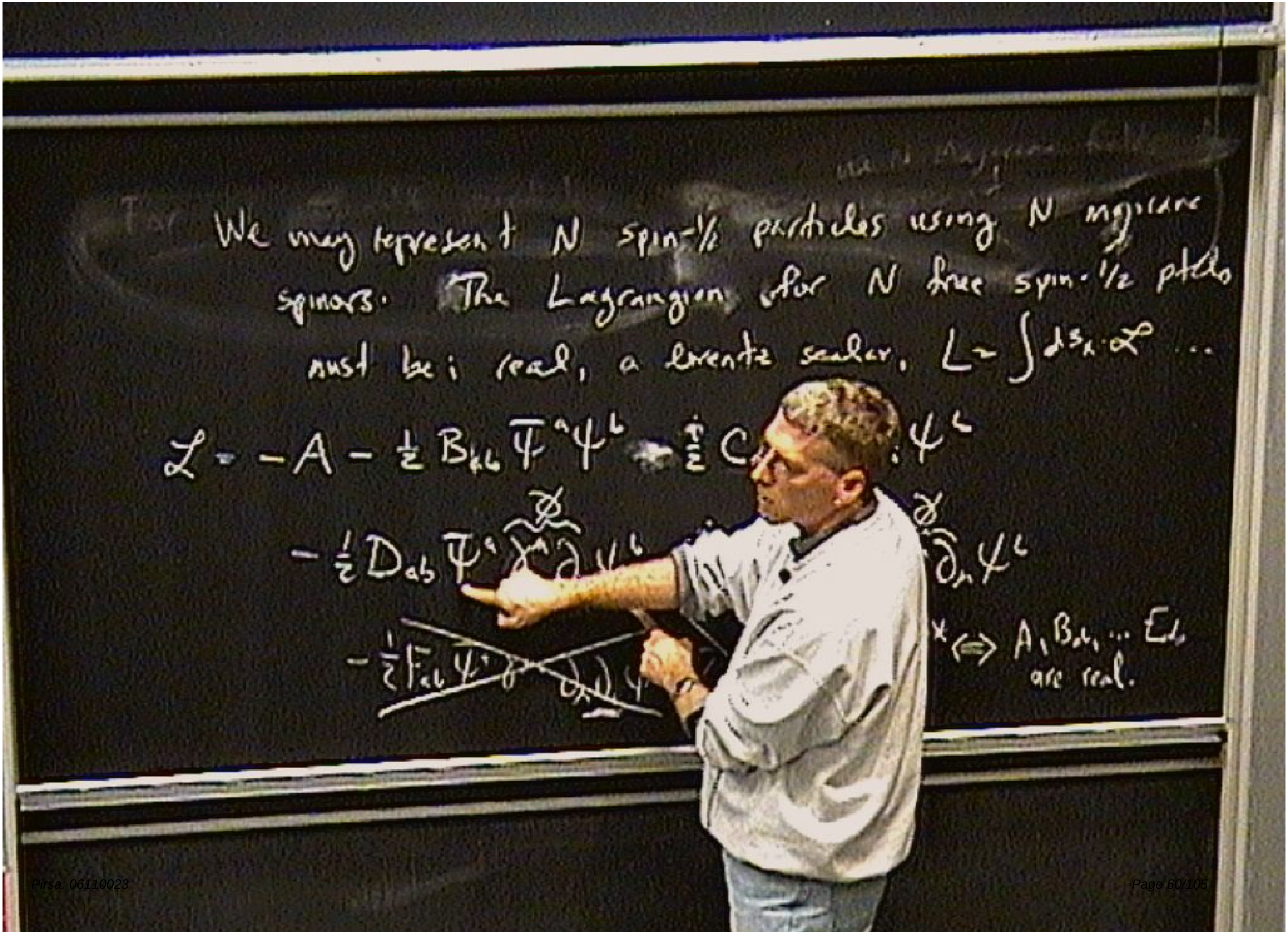
$\begin{cases} + & S, V, T \\ - & P, A \end{cases}$

if ψ_1, ψ_2 are Majorana.

Claim II:


ASSIGNMENT
(FROM HELL!)
Ch 1: Probs 1-6, 9
Ch 2: Probs 1-8
Ch 4: Prob
Ch 5:
Due:
Claim I: (Check this)
if ψ_1, ψ_2 are Majorana.
Claim II: (Check this)


ASSIGNMENT
(FROM HELL!)
Ch 1: Probs 1-6, 9
Ch 2: Probs 1-8
Ch 4: Prob 3
Ch 5: Prob 4
Due:
Claim I: (Check this)
if ψ1, ψ2 are Majorana.
Claim II: (Check this)
if ψ1, ψ2 are Majorana


We may represent N spin-1/2 particles using N majorana
spinors. The Lagrangian for N free spin-1/2 ptcls
must be: real, a Lorentz scalar. L = ∫d³x·ℒ ...
ℒ = −A − ½ B_ab Ψ̄^a ψ^b
− ½ D_ab
A, B_ab, ... E_ab
are real.

We may represent N spin-1/2 particles using N majorana spinors. The Lagrangian for N free spin-1/2 ptcls must be: real, a Lorentz scalar, $L = \int d^3x \cdot \mathcal{L}$ ...

$$\mathcal{L} = -A - \frac{1}{2} B_{ab} \bar{\Psi}^a \psi^b - \frac{i}{2} C_{ab} \bar{\Psi}^a \gamma_5 \psi^b$$

$$- \frac{1}{2} D_{ab} \bar{\Psi}^a \gamma^\mu \partial_\mu \psi^b - \frac{i}{2} E_{ab} \bar{\Psi}^a \gamma_5 \gamma^\mu \partial_\mu \psi^b$$

$$\cancel{- \frac{1}{2} F_{ab} \bar{\Psi}^a \partial_\mu \partial \psi} \quad \cancel{\bar{\Psi} K} \quad \mathcal{L}^* \Leftrightarrow$$

We may represent N spin-1/2 particles using N majorana spinors. The Lagrangian for N free spin-1/2 ptcls must be: real, a Lorentz scalar. $L = \int d^3x \cdot \mathcal{L}$ ...

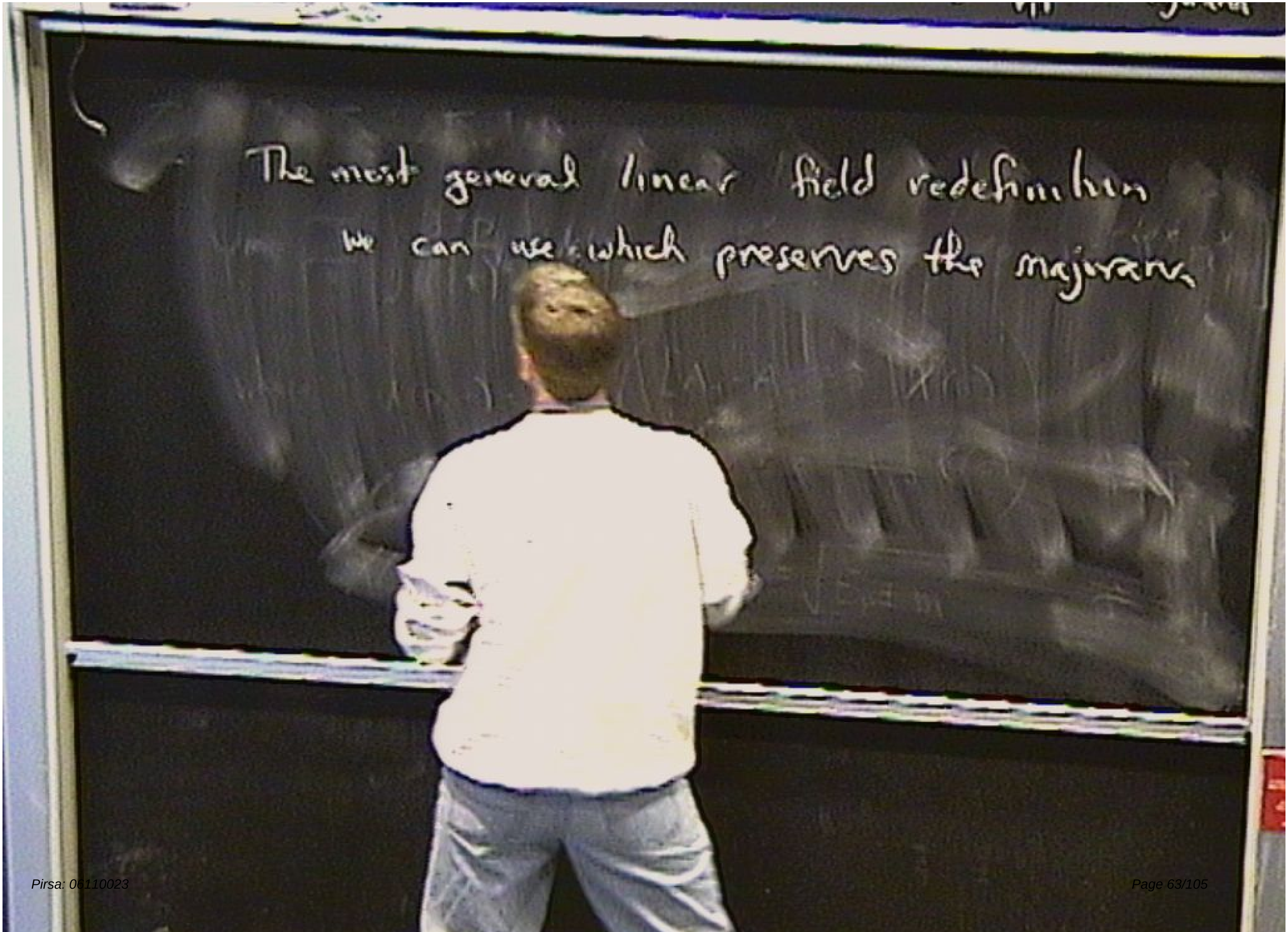
$$\mathcal{L} = -A - \tfrac{1}{2} B_{ab} \bar{\Psi}^a \psi^b - \tfrac{i}{2} C_{ab} \bar{\Psi}^a \gamma_5 \psi^b$$

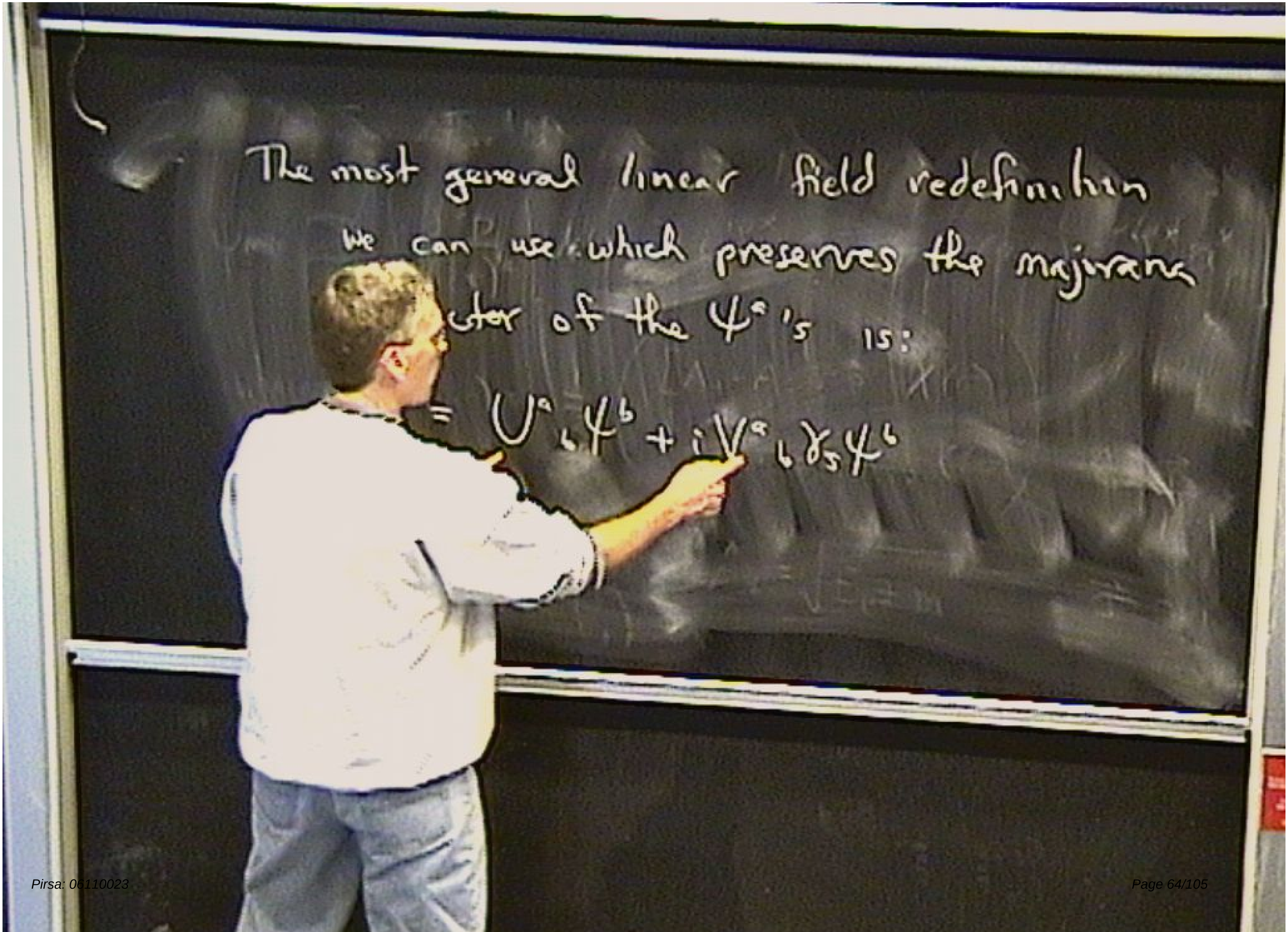
$$- \tfrac{1}{2} D_{ab} \bar{\Psi}^a \underbrace{\gamma^\mu}_{\not\partial} \partial_\mu \psi^b - \tfrac{i}{2} E_{ab} \bar{\Psi}^a \gamma_5 \underbrace{\gamma^\mu}_{\not\partial} \partial_\mu \psi^b$$

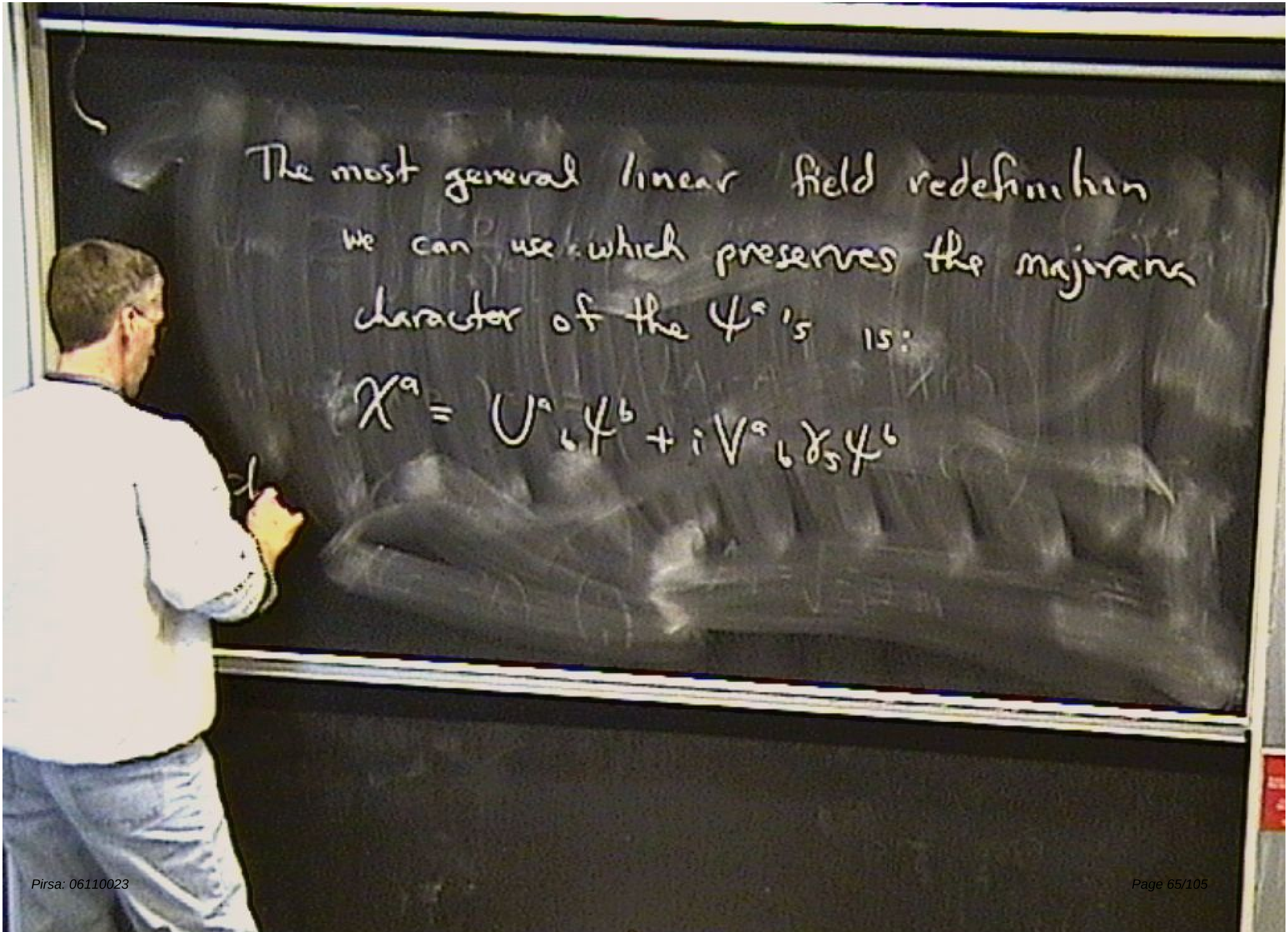
$$\cancel{- \tfrac{1}{2} F_{ab} \bar{\Psi}^a \gamma \partial_\mu \partial \psi} \quad \cancel{\bar{\Psi} \ldots}$$

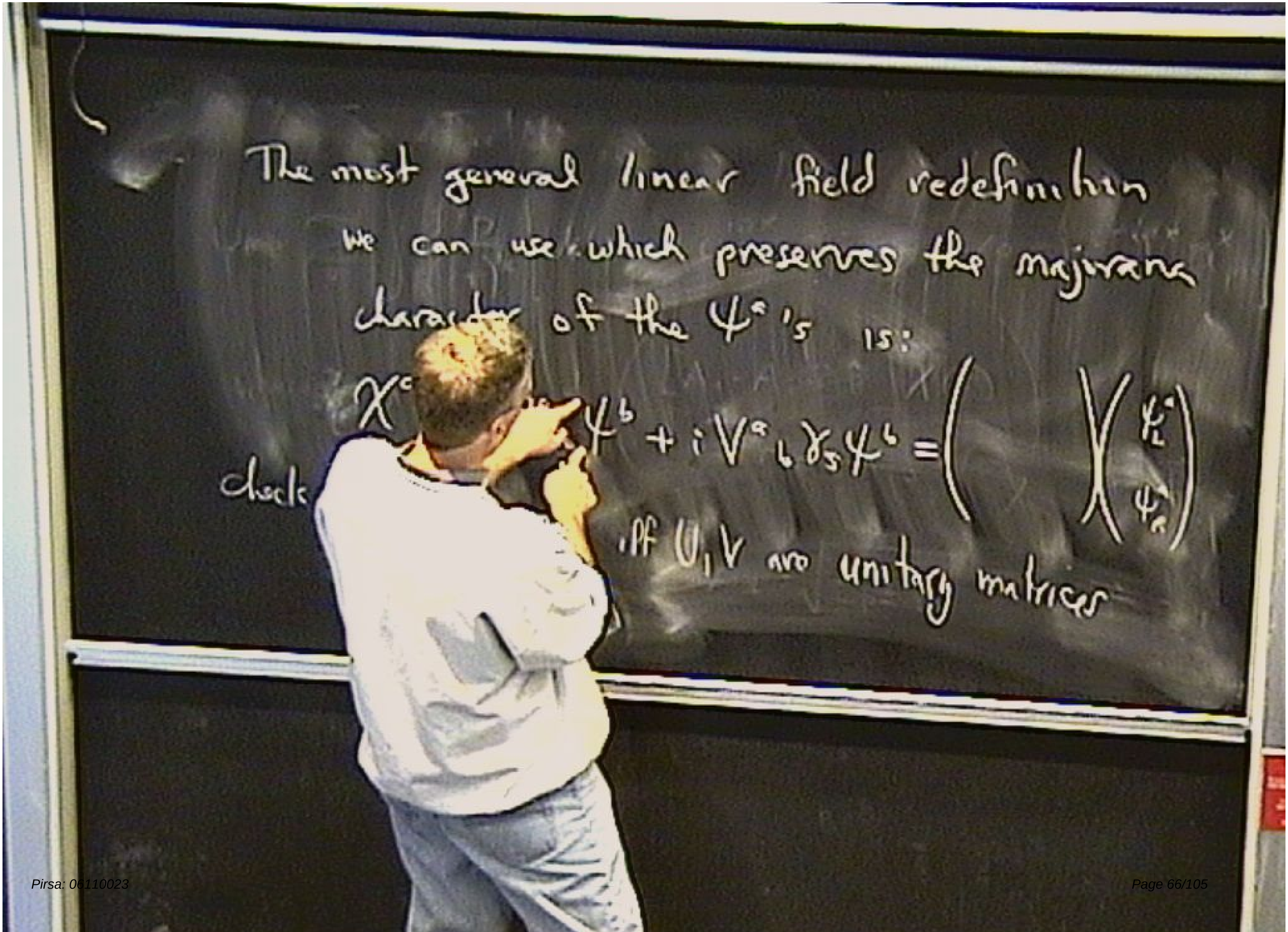
$\mathcal{L} = \mathcal{L}^* \Leftrightarrow A, B_{ab}, \ldots E_{ab}$ are real.

$$\boxed{\text{No loss to assume: } B, C, D \text{ are symmetric}, \; E = -E^T}$$


The most general linear field redefinition
we can use which preserves the majorana


The most general linear field redefinition
we can use which preserves the majorana
cter of the ψ^a's is:
$= U^a{}_b \psi^b + i V^a{}_b \gamma_5 \psi^b$

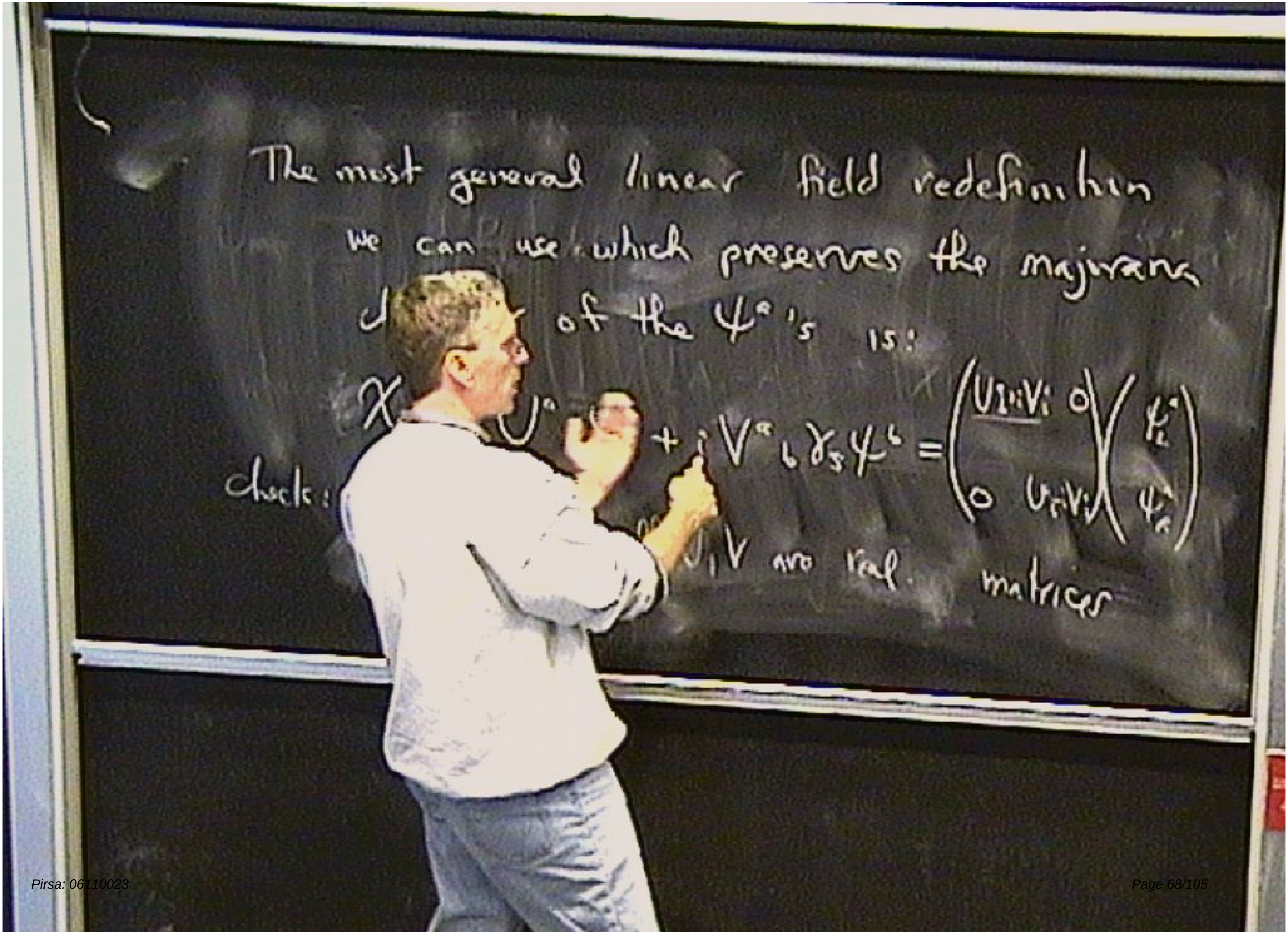

The most general linear field redefinition
we can use which preserves the majorana
character of the ψ^a's is:
$\chi^a = U^a{}_b \psi^b + i V^a{}_b \gamma_5 \psi^b$


The most general linear field redefinition
we can use which preserves the majorana
character of the ψ^a 's is:
χ^a ψ^b + i V^a_b γ_5 ψ^b = ( )(ψ_L ψ_R)
check
iff U, V are unitary matrices

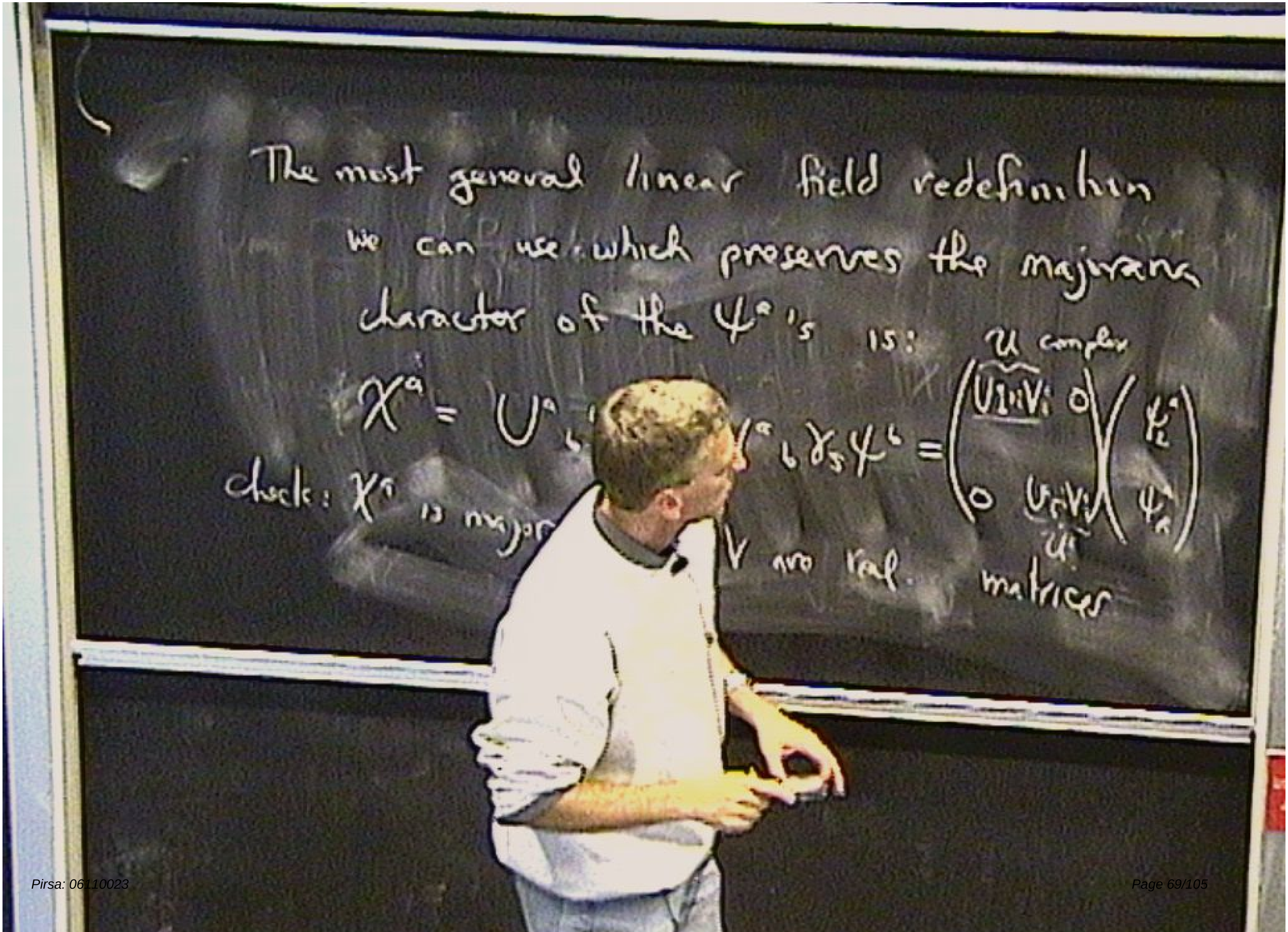
The most general linear field redefinition we can use which preserves the majorana character of the ψ^a's is:

$$\chi^a = U^a{}_b \psi^b + i V^a{}_b \gamma_5 \psi^b = \begin{pmatrix} U+iV & 0 \\ 0 & U-iV \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

check: χ^a is majorana iff U, V are unitary matrices



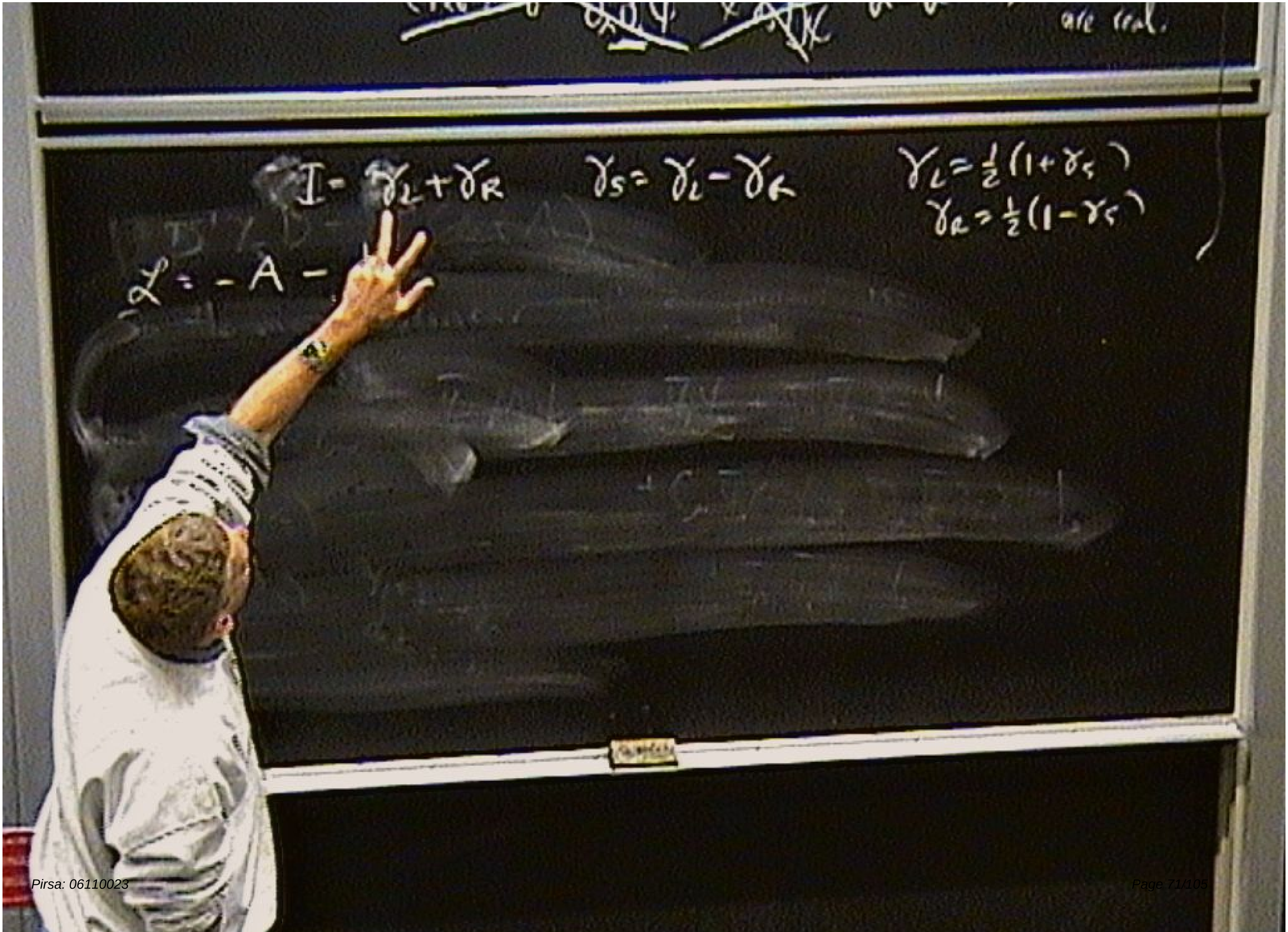
The most general linear field redefinition
we can use which preserves the majorana
of the ψ^a's is:
+ i V^a_b γ_5 ψ^b =
check:
U, V are real. matrices

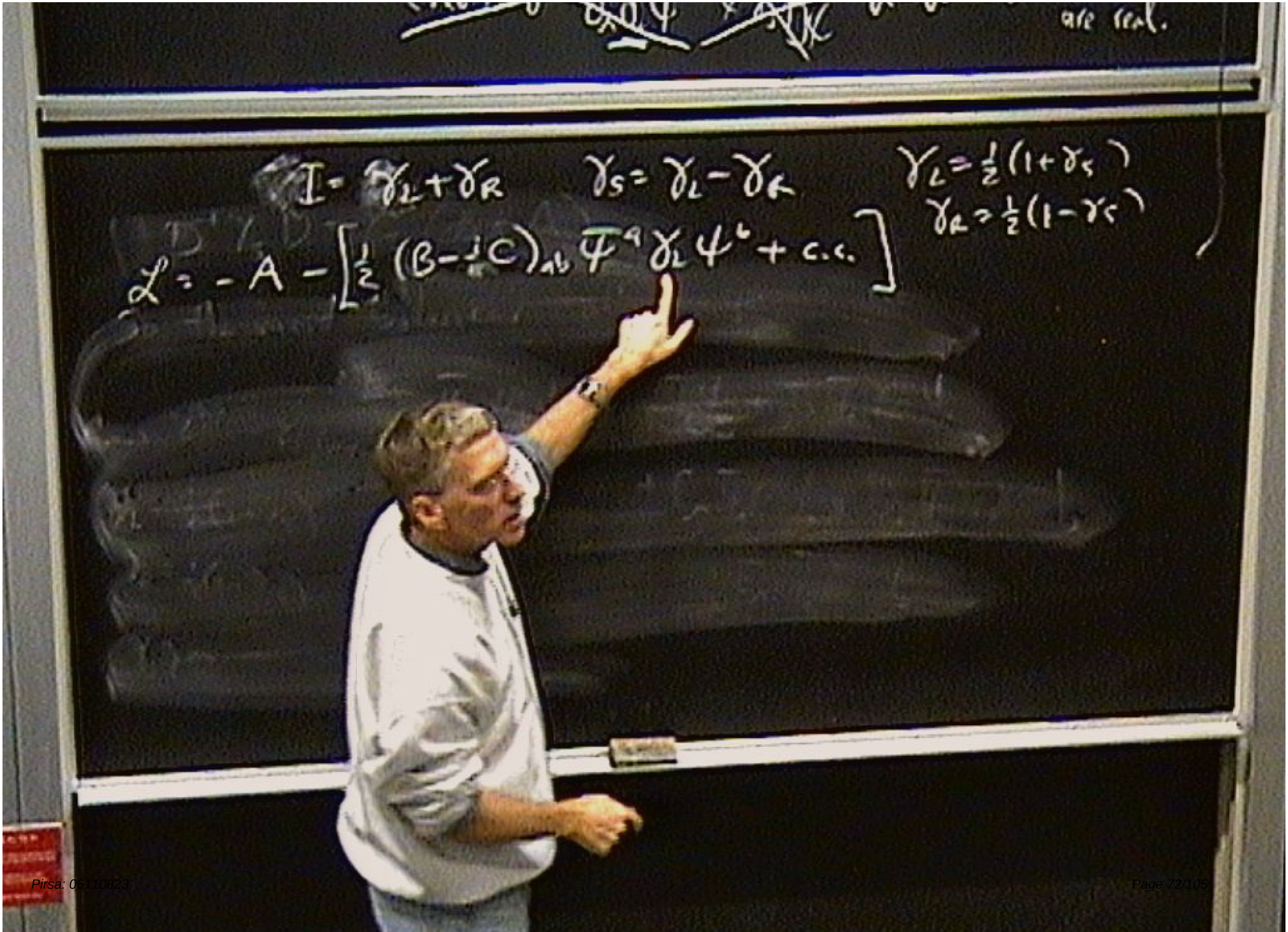

The most general linear field redefinition
we can use, which preserves the majorana
character of the ψ^a's is:
U complex
check: χ^a is major
V are real matrices
matrices

The most general linear field redefinition we can use which preserves the majorana character of the ψ^a's is:

$$\chi^a = U^a{}_b \psi^b + i V^a{}_b \gamma_5 \psi^b = \begin{pmatrix} \overbrace{U+iV}^{\mathcal{U}\ \text{complex}} & 0 \\ 0 & \underbrace{U-iV}_{\mathcal{U}^*} \end{pmatrix} \begin{pmatrix} \psi_L^a \\ \psi_R^a \end{pmatrix}$$

check: χ^a is majorana iff U, V are real. matrices


are real.
I = γL + γR
γ5 = γL − γR
γL = ½(1 + γ5)
γR = ½(1 − γ5)
L = − A −


are real.
I = γL + γR
γ5 = γL − γR
γL = ½(1 + γ5)
γR = ½(1 − γ5)
L = − A − [½ (B − i C)ab Ψ̄a γL ψb + c.c.]

are real.

$$I = \gamma_L + \gamma_R \qquad \gamma_5 = \gamma_L - \gamma_R \qquad \gamma_L = \tfrac{1}{2}(1+\gamma_5)$$
$$\gamma_R = \tfrac{1}{2}(1-\gamma_5)$$

$$\mathcal{L} = -A - \left[\tfrac{1}{2}\left(B - \tfrac{i}{4}C\right)_{ab} \bar{\Psi}^a \gamma_L \psi^b + \text{c.c.}\right]$$

are real.

$$I = \gamma_L + \gamma_R \qquad \gamma_5 = \gamma_L - \gamma_R \qquad \gamma_L = \tfrac{1}{2}(1+\gamma_5) \qquad \gamma_R = \tfrac{1}{2}(1-\gamma_5)$$

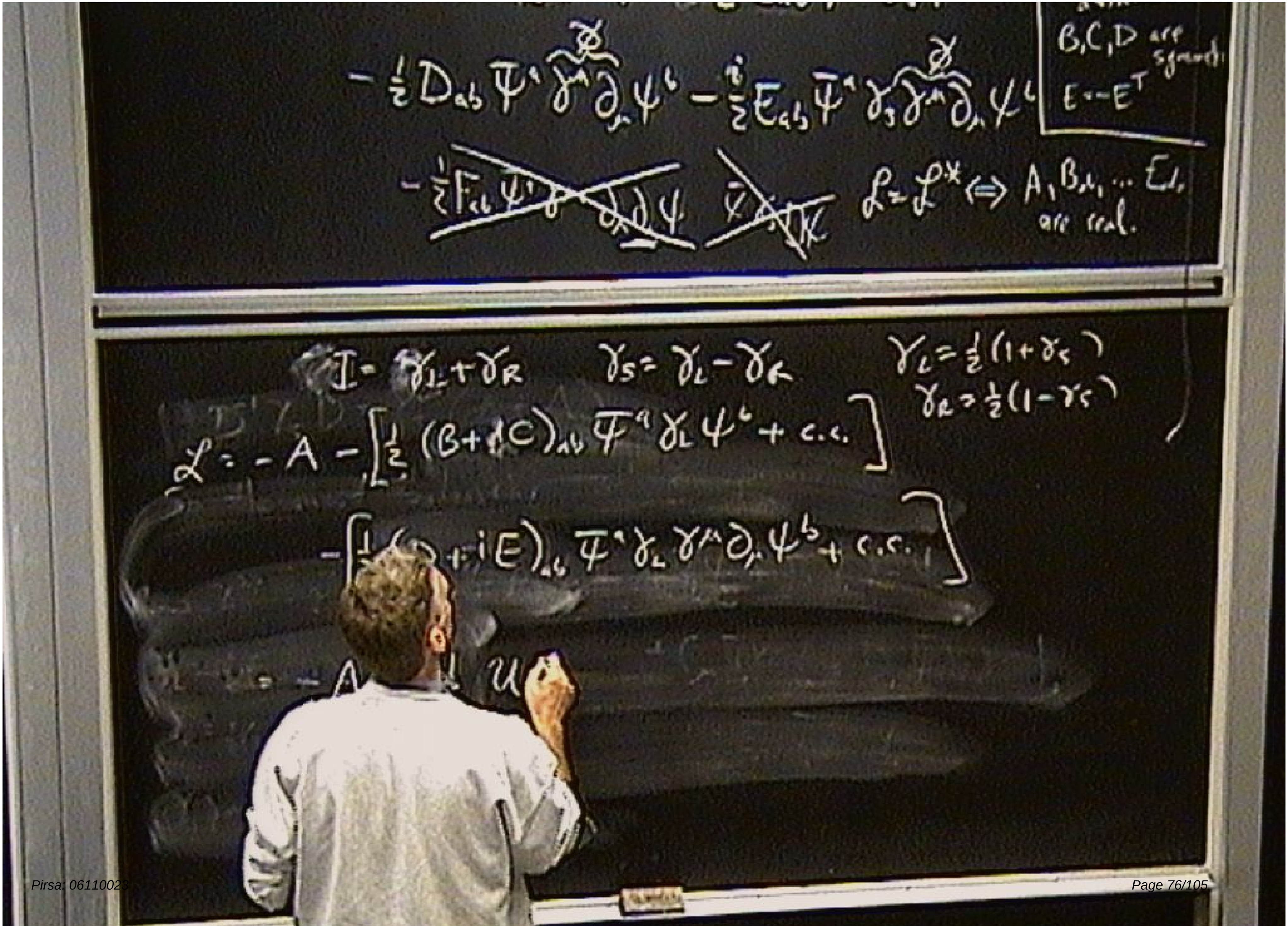
$$\mathcal{L} = -A - \left[\tfrac{1}{2}(B+iC)_{ab}\,\bar{\Psi}^a \gamma_L \psi^b + c.c.\right]$$

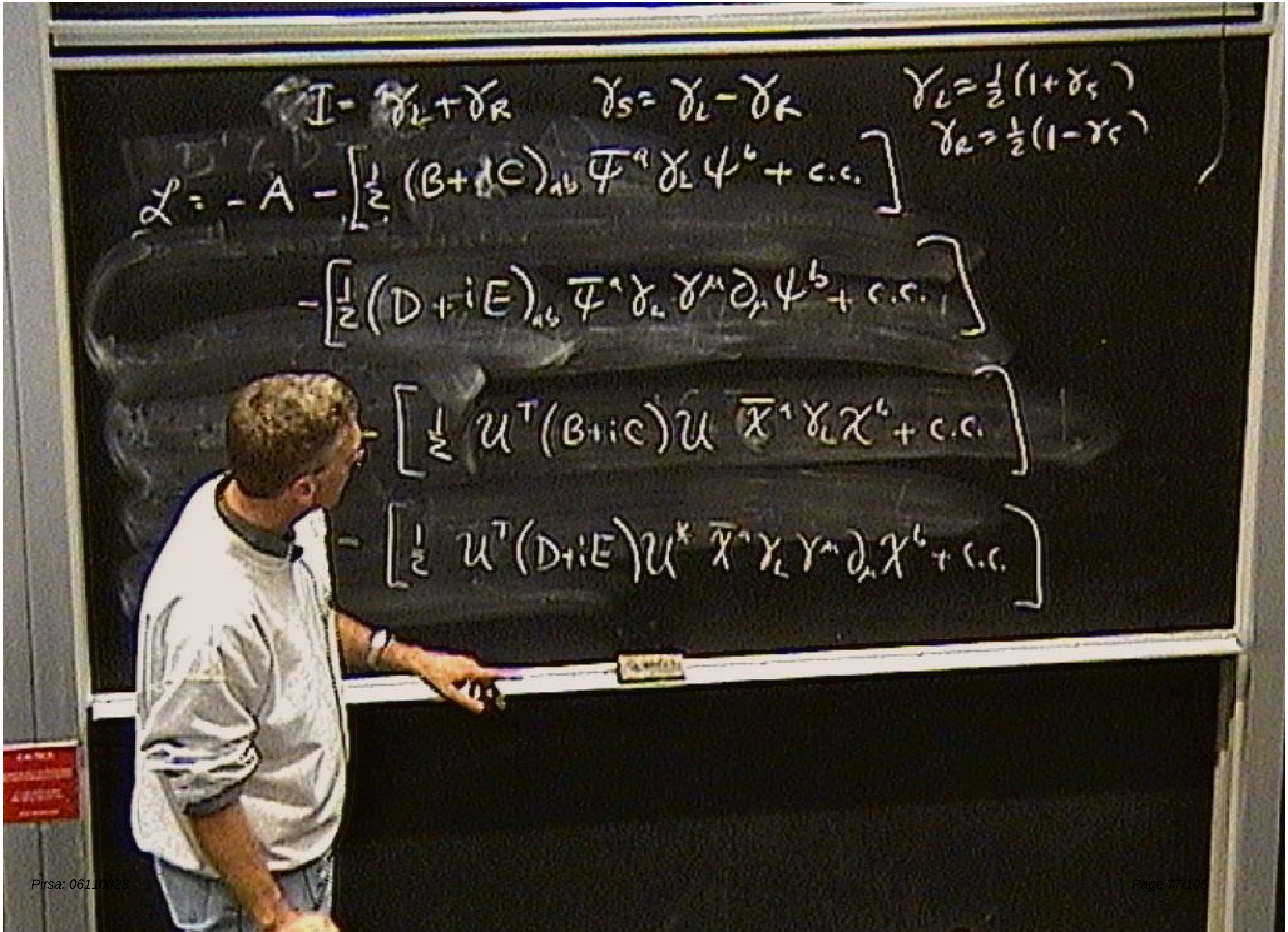
$$-\left[\tfrac{1}{2}(D+iE)_{ab}\,\bar{\Psi}^a \gamma_L \gamma^\mu \partial_\mu \psi^b + c.c.\right]$$

The most general linear field redefinition we can use which preserves the majorana character of the ψ^a's is:

$$\psi^a = U^a{}_b\,\chi^b + i V^a{}_b\,\gamma_5\,\chi^b = \begin{pmatrix} \overbrace{U - iV}^{\mathcal{U}\ \text{complex}} & 0 \\ 0 & \underbrace{U + iV}_{\mathcal{U}^*} \end{pmatrix} \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$$

check: χ^a is majorana iff U, V are real matrices


$-\frac{1}{2} D_{ab} \bar{\Psi}^a \gamma^\mu \partial_\mu \psi^b - \frac{i}{2} E_{ab} \bar{\Psi}^a \gamma_5 \gamma^\mu \partial_\mu \psi^b$
B, C, D are
$E = -E^T$
$\mathcal{L} = \mathcal{L}^* \Leftrightarrow A, B_{ab}, \ldots E_{ab}$ are real.
$I = \gamma_L + \gamma_R$
$\gamma_5 = \gamma_L - \gamma_R$
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\gamma_R = \frac{1}{2}(1-\gamma_5)$
$\mathcal{L} = -A - \left[\frac{1}{2}(B+iC)_{ab} \bar{\Psi}^a \gamma_L \psi^b + c.c.\right]$



$I = \gamma_L + \gamma_R \qquad \gamma_5 = \gamma_L - \gamma_R \qquad \gamma_L = \frac{1}{2}(1+\gamma_5)$
$\gamma_R = \frac{1}{2}(1-\gamma_5)$
$\mathcal{L} = -A - \left[\frac{1}{2}(B+iC)_{ab}\,\bar{\Psi}^a \gamma_L \psi^b + c.c.\right]$
$- \left[\frac{1}{2}(D+iE)_{ab}\,\bar{\Psi}^a \gamma_L \gamma^\mu \partial_\mu \psi^b + c.c.\right]$
$- \left[\frac{1}{2}\,\mathcal{U}^T(B+iC)\mathcal{U}\;\bar{\chi}^a \gamma_L \chi^b + c.c.\right]$
$- \left[\frac{1}{2}\,\mathcal{U}^T(D+iE)\mathcal{U}^*\;\bar{\chi}^a \gamma_L \gamma^\mu \partial_\mu \chi^b + c.c.\right]$

$$I = \gamma_L + \gamma_R \qquad \gamma_5 = \gamma_L - \gamma_R \qquad \gamma_L = \tfrac{1}{2}(1+\gamma_5)$$

$$\gamma_R = \tfrac{1}{2}(1-\gamma_5)$$

$$\mathcal{L} = -A - \left[\tfrac{1}{2}(B+iC)_{ab}\,\overline{\Psi}^a \gamma_L \Psi^b + c.c.\right]$$

$$-\left[\tfrac{1}{2}(D+iE)_{ab}\,\overline{\Psi}^a \gamma_L \gamma^\mu \partial_\mu \Psi^b + c.c.\right]$$

$$\mathcal{L} = -A - \left[\tfrac{1}{2}\,\mathcal{U}^T(B+iC)\,\mathcal{U}\;\overline{\chi}^a \gamma_L \chi^b + c.c.\right]$$

$$-\left[\tfrac{1}{2}\,\mathcal{U}^T(D+iE)\,\mathcal{U}^*\;\overline{\chi}^a \gamma_L \gamma^\mu \partial_\mu \chi^b + c.c.\right]$$

$$I = \gamma_L + \gamma_R \qquad \gamma_5 = \gamma_L - \gamma_R \qquad \gamma_L = \tfrac{1}{2}(1+\gamma_5)$$
$$\gamma_R = \tfrac{1}{2}(1-\gamma_5)$$

$$\mathcal{L} = -A - \left[\tfrac{1}{2}(B+iC)_{ab}\,\bar{\Psi}^a \gamma_L \psi^b + c.c.\right]$$
$$-\left[\tfrac{1}{2}(D+iE)_{ab}\,\bar{\Psi}^a \gamma_L \gamma^\mu \partial_\mu \psi^b + c.c.\right]$$

$$\bar{\Psi} = \psi^T N$$

$$= -A - \left[\tfrac{1}{2}\,\mathcal{U}^T(B+iC)\mathcal{U}\;\bar{\chi}^a \gamma_L \chi^b + c.c.\right]$$
$$-\left[\tfrac{1}{2}\,\mathcal{U}^T(D+iE)\mathcal{U}^*\;\bar{\chi}^a \gamma_L \gamma^\mu \partial_\mu \chi^b + c.c.\right]$$

$\mathbb{I} = \gamma_L + \gamma_R \qquad \gamma_5 = \gamma_L - \gamma_R \qquad \gamma_L = \frac{1}{2}(1+\gamma_5)$

$\gamma_R = \frac{1}{2}(1-\gamma_5)$

$$\mathcal{L} = -A - \left[\frac{1}{2}(B+iC)_{ab}\,\bar{\Psi}^a \gamma_L \psi^b + c.c.\right]$$

$$- \left[\frac{1}{2}(D+iE)_{ab}\,\bar{\Psi}^a \gamma_L \gamma^\mu \partial_\mu \psi^b + c.c.\right]$$

$\bar{\Psi} = \psi^T N$

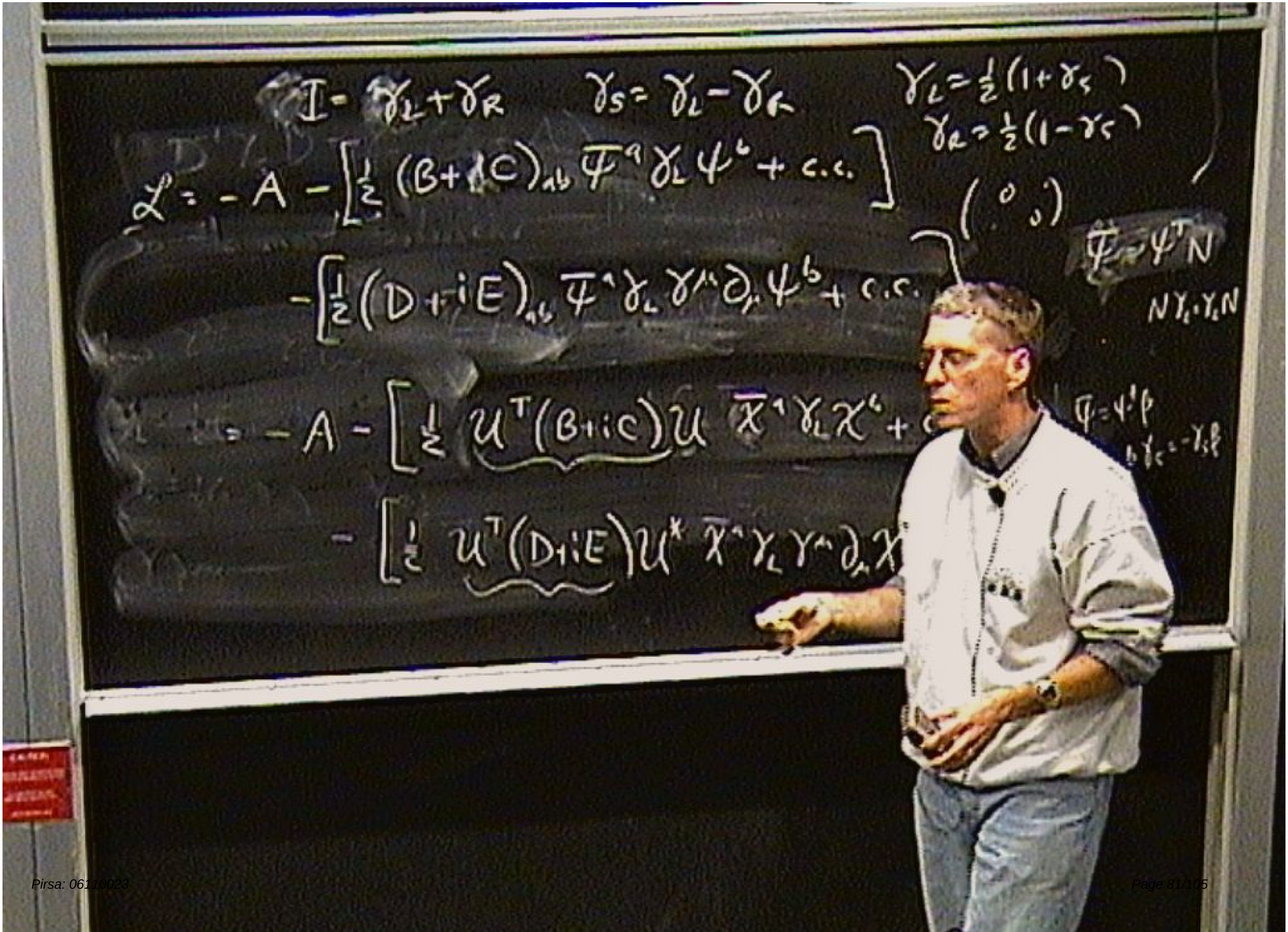
$N\gamma_L = \gamma_L N$

$$\mathcal{L} = -A - \left[\frac{1}{2}\,\mathcal{U}^T (B+iC)\,\mathcal{U}\,\bar{\chi}^a \gamma_L \chi^b + c.c.\right]$$

$$- \left[\frac{1}{2}\,\mathcal{U}^T (D+iE)\,\mathcal{U}^*\,\bar{\chi}^a \gamma_L \gamma^\mu \partial_\mu \chi^b + c.c.\right]$$

$\bar{\Psi} = \psi^\dagger \beta$

$\beta\gamma_5 = -\gamma_5\beta$



$I = \gamma_L + \gamma_R$
$\gamma_5 = \gamma_L - \gamma_R$
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\gamma_R = \frac{1}{2}(1-\gamma_5)$
$\mathcal{L} = -A - \left[\frac{1}{2}(B+iC)_{ab}\bar{\Psi}^a\gamma_L\psi^b + c.c.\right]$
$-\left[\frac{1}{2}(D+iE)_{ab}\bar{\Psi}^a\gamma_L\gamma^\mu\partial_\mu\psi^b + c.c.\right]$
$\bar{\Psi} = \psi^T N$
$N\gamma_L = \gamma_L N$
$= -A - \left[\frac{1}{2}\,\mathcal{U}^T(B+iC)\mathcal{U}\,\bar{\chi}^a\gamma_L\chi^b + c\right.$
$-\left[\frac{1}{2}\,\mathcal{U}^T(D+iE)\mathcal{U}^*\,\bar{\chi}^a\gamma_L\gamma^\mu\partial_\mu\chi\right.$

$I = \gamma_L + \gamma_R \qquad \gamma_5 = \gamma_L - \gamma_R \qquad \gamma_L = \frac{1}{2}(1+\gamma_5)$

$\gamma_R = \frac{1}{2}(1-\gamma_5)$

$$\mathcal{L} = -A - \left[\frac{1}{2}(B+iC)_{ab}\,\bar{\Psi}^a \gamma_L \psi^b + c.c.\right]$$

$$-\left[\frac{1}{2}(D+iE)_{ab}\,\bar{\Psi}^a \gamma_L \gamma^\mu \partial_\mu \psi^b + c.c.\right]$$

$\begin{pmatrix} 0 & \\ & 0 \end{pmatrix}$

$\bar{\Psi} = \psi^T N$

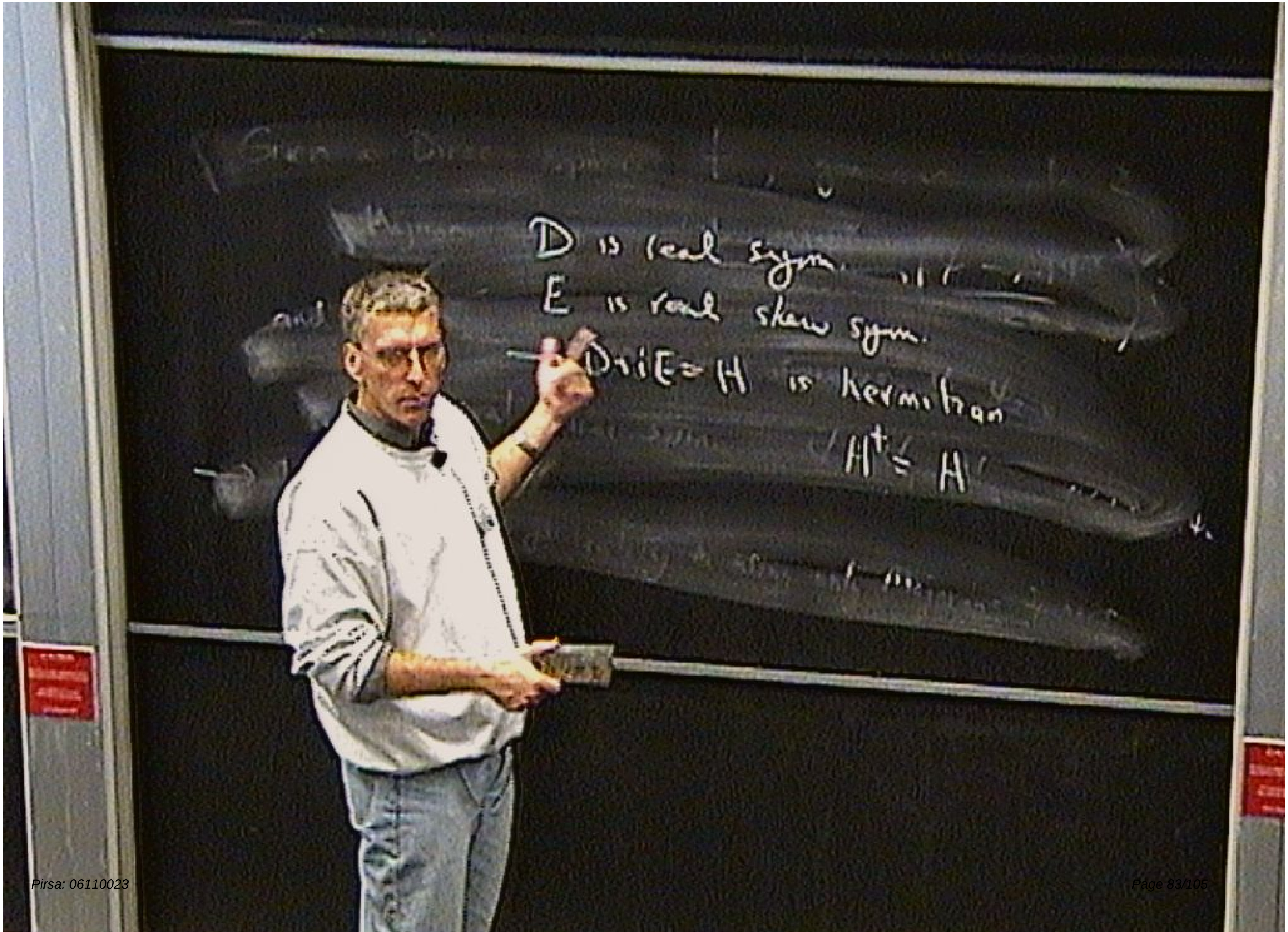
$N\gamma_L = \gamma_L N$

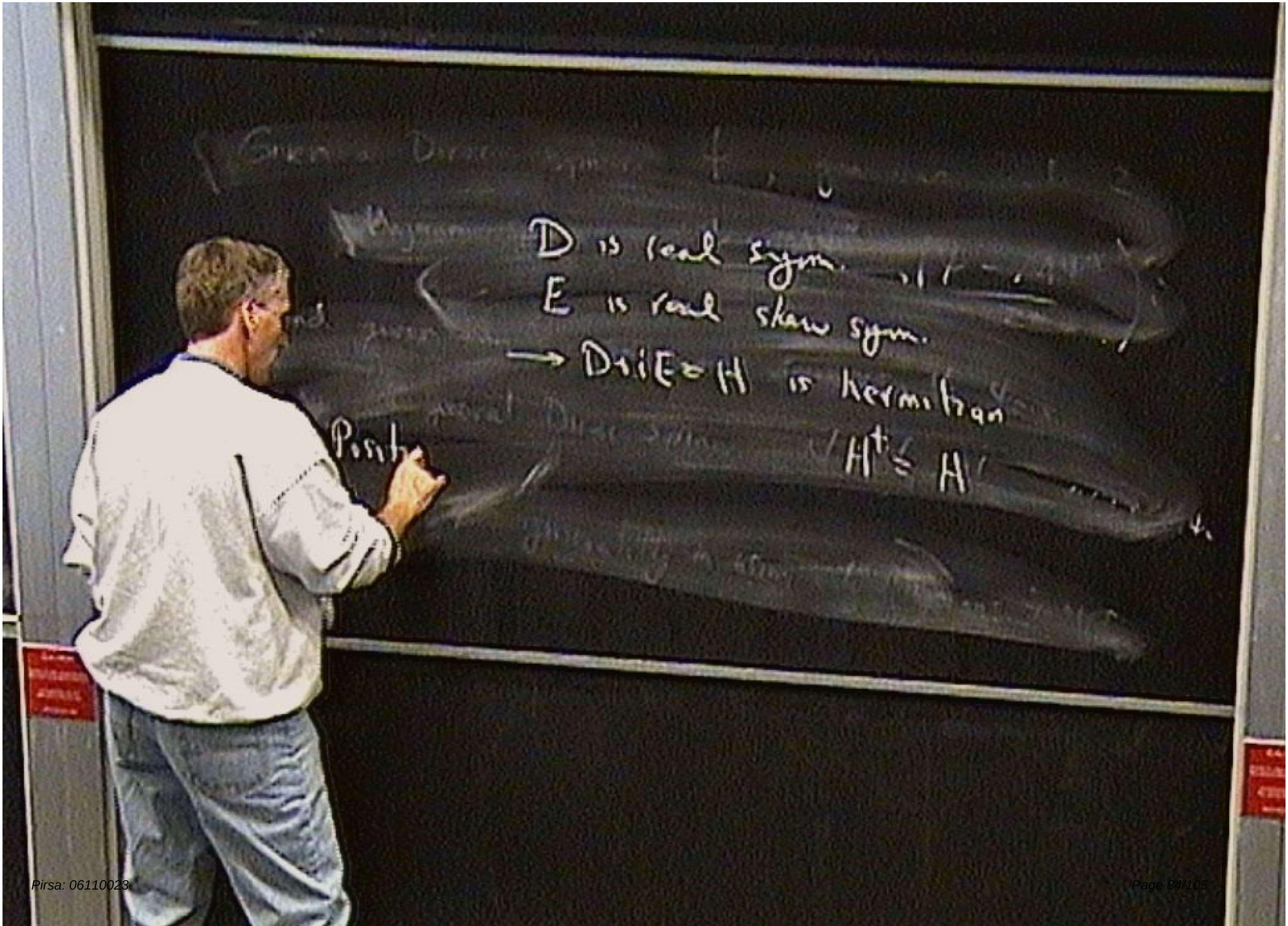
$$= -A - \left[\frac{1}{2}\underbrace{\mathcal{U}^T(B+iC)\mathcal{U}}\,\bar{\chi}^a \gamma_L \chi^b + c.c.\right]$$

$$-\left[\frac{1}{2}\underbrace{\mathcal{U}^T(D+iE)\mathcal{U}^*}\,\bar{\chi}^a \gamma_L \gamma^\mu \partial_\mu \chi^b + c.c.\right]$$

$\bar{\Psi} = \psi^\dagger \beta$

$\beta\gamma_L = -\gamma_L\beta$


D is real sym.
E is real skew sym.
D+iE = H is hermitian
H† = H


D is real sym.
E is real skew sym.
→ D+iE = H is hermitian
H† = H

D is real sym.

and E is real skew sym.

$\longrightarrow D + iE = H$ is hermitian

Stability: $H^\dagger = H$

D is real sym.

E is real skew sym.

$\longrightarrow D + iE = H$ is hermitian

$A^\dagger = A$

Stability: eigenvalues of H are all positive.

D is real sym.

E is real skew sym.

$\rightarrow D + iE = H$ is hermitian

$H^\dagger = H$

Stability: eigenvalues of H are all positive.

$\rightarrow H^{1/2}$ exists

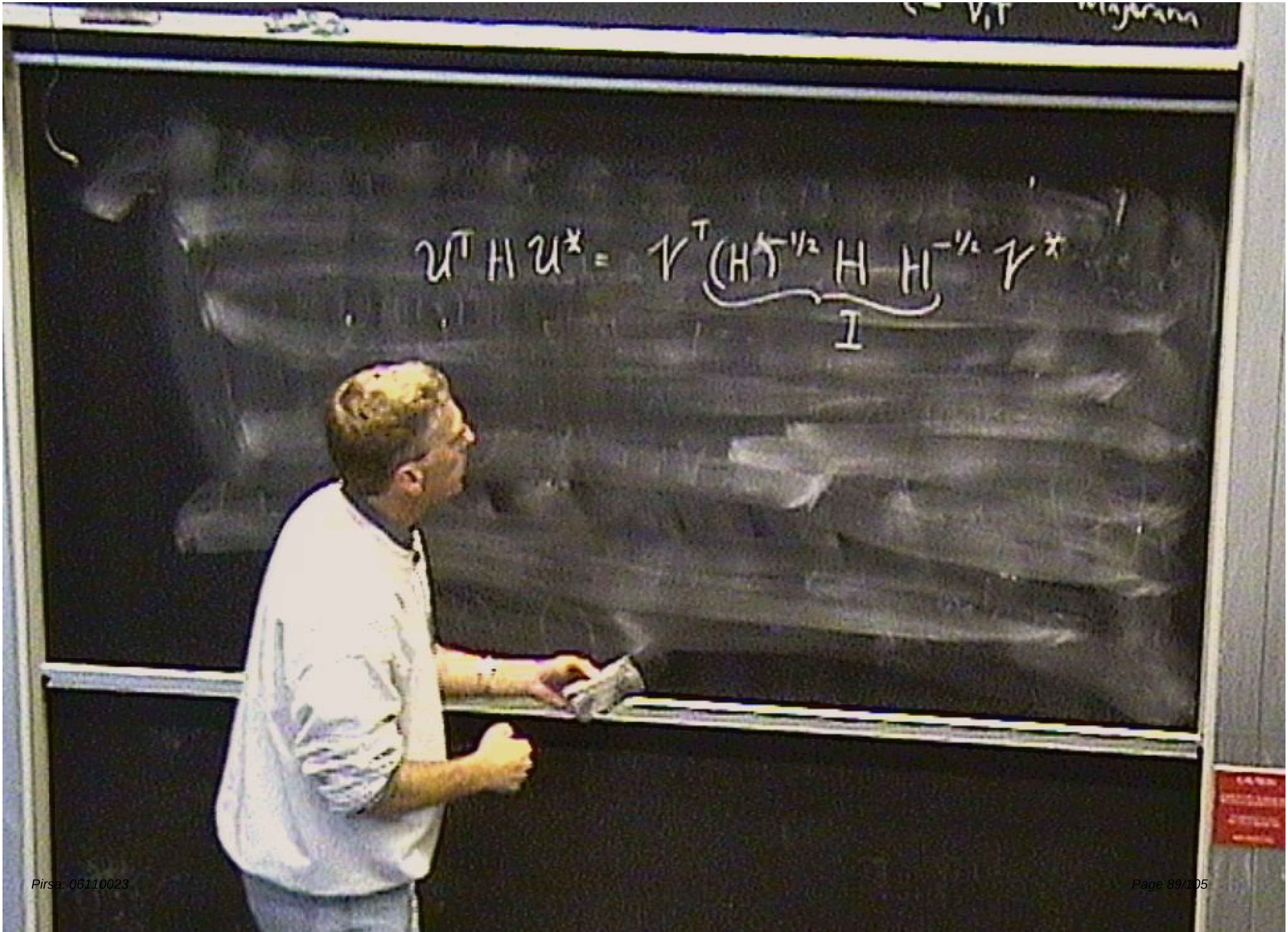
D is real sym.

E is real skew sym.

$\longrightarrow D + iE = H$ is hermitian $\quad H^\dagger = H$

Stability: eigenvalues of H are all positive.

$\rightarrow H^{1/2}$ exists ; Choose: $U = (H^\dagger)^{1/2} V$

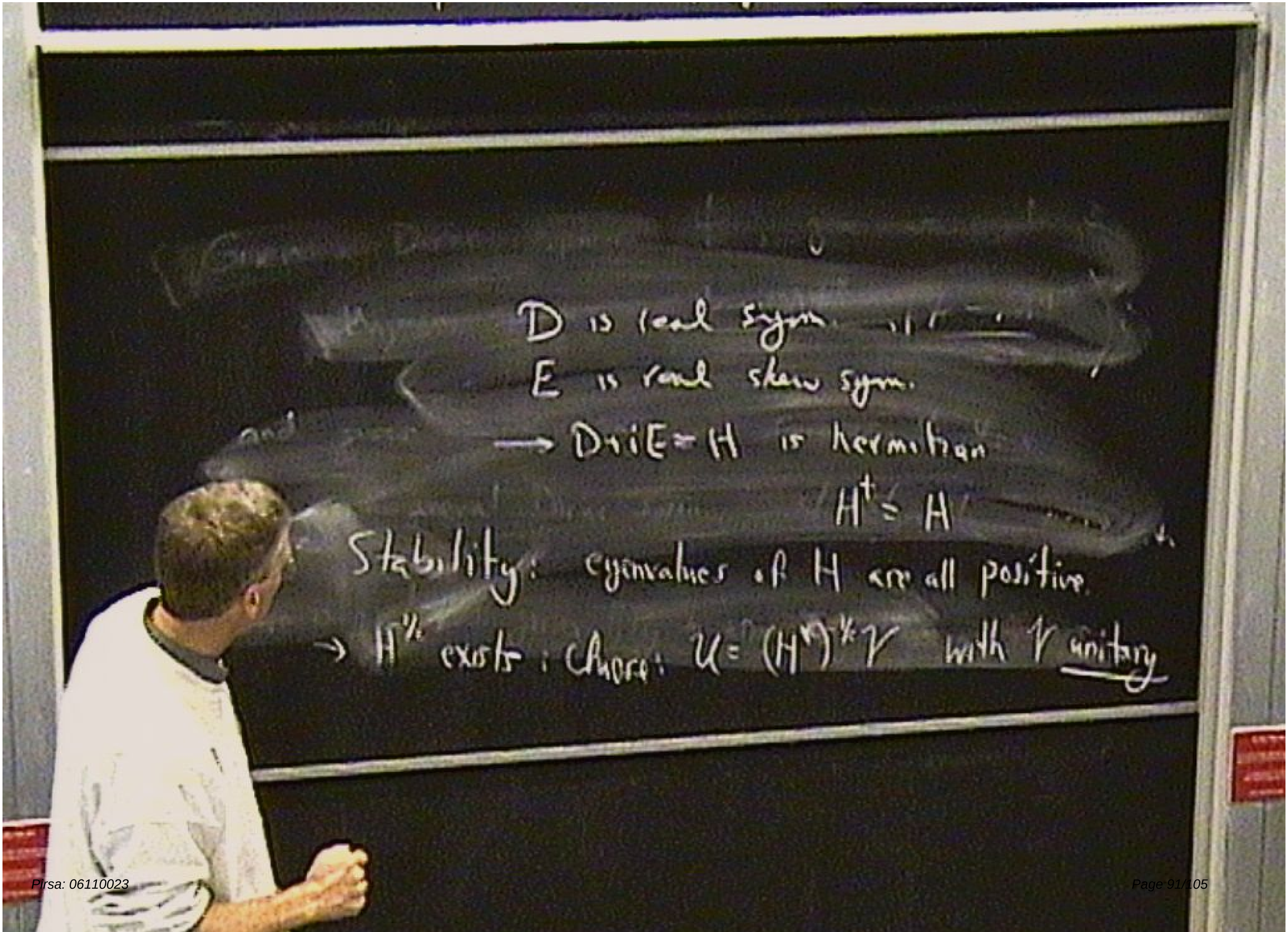

$\mathcal{U}^T H\, \mathcal{U}^* = \mathcal{V}^T \underbrace{(H^*)^{-1/2}\, H\, H^{-1/2}}_{I} \mathcal{V}^*$
Majorana

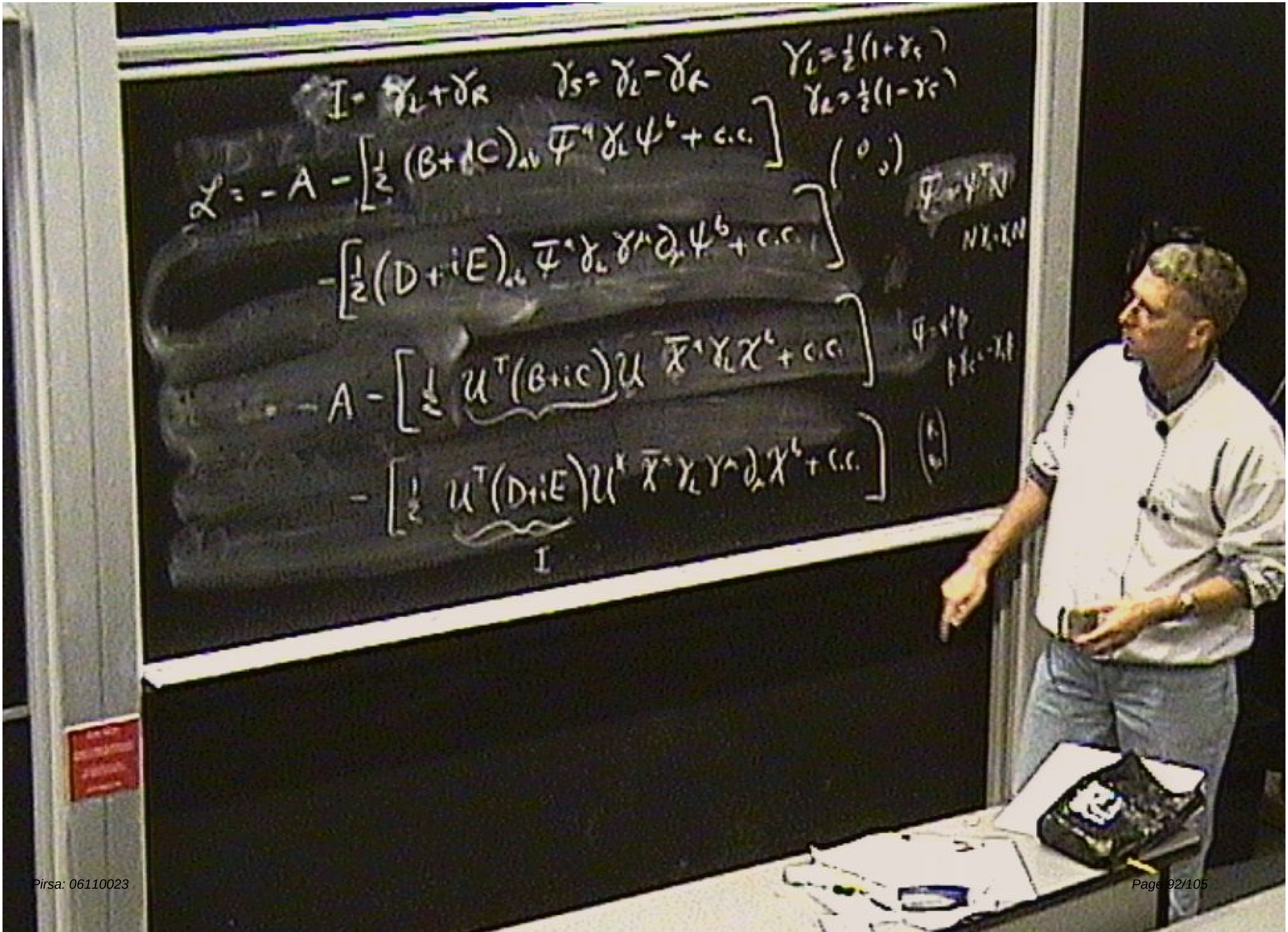
V, T Majorana

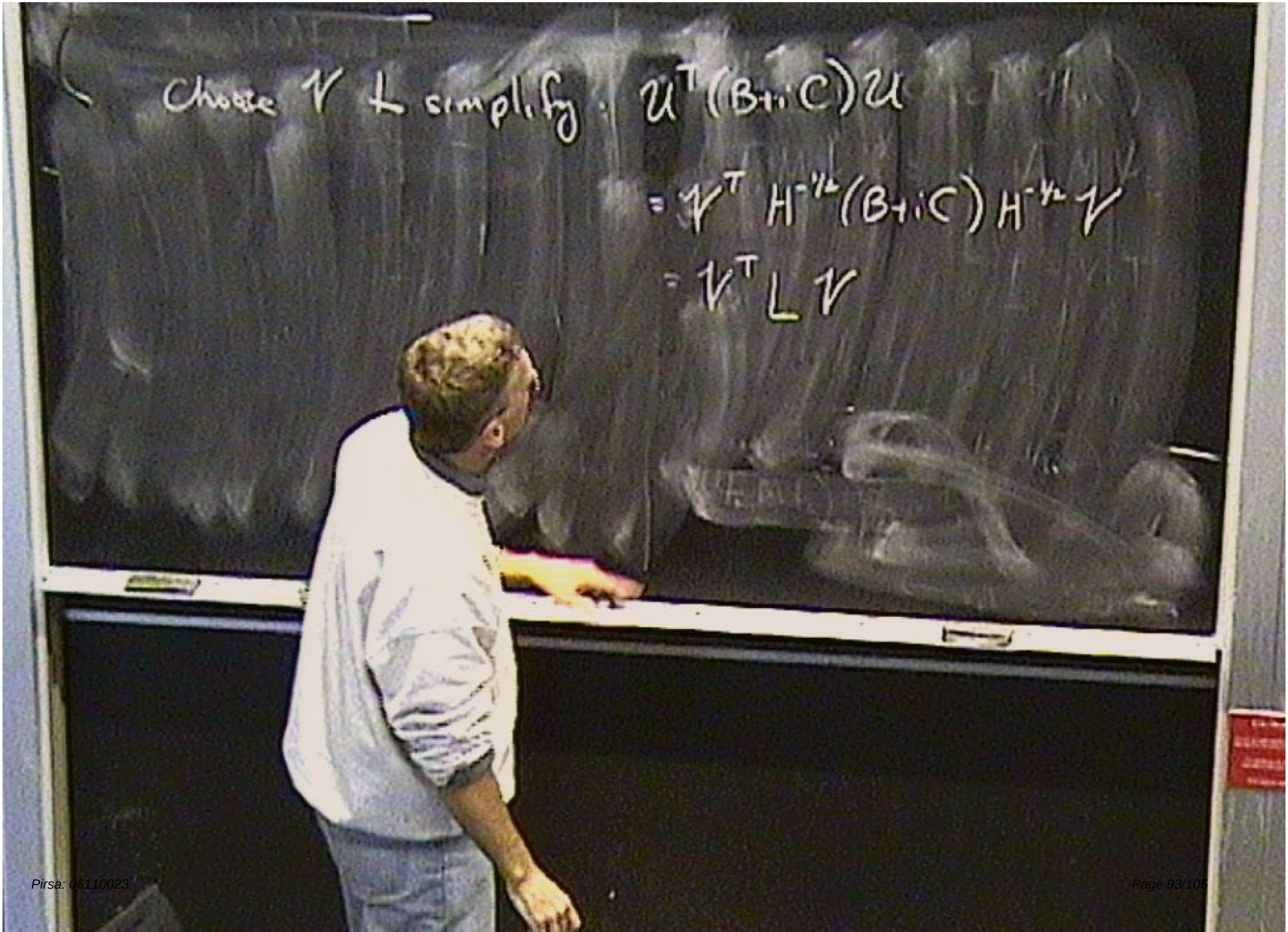
$$\mathcal{U}^T H\, \mathcal{U}^* = \mathcal{V}^T \underbrace{(H^*)^{-1/2}\, H\, H^{-1/2}}_{I}\, \mathcal{V}^*$$

$$= \mathcal{V}^T \mathcal{V}^*$$

$$= (\mathcal{V}^\dagger \mathcal{V})^*$$


D is real symm.
E is real skew symm.
→ D+iE = H is hermitian
H† = H
Stability: eigenvalues of H are all positive.
→ H^½ exists: Choose: U = (H^T)^½ V with V unitary


I = γL + γR
γ5 = γL − γR
γL = ½(1+γ5)
γR = ½(1−γ5)
ℒ = − A − [½ (B+iC)ab Ψ̄a γL ψb + c.c.]
− [½ (D+iE)ab Ψ̄a γL γμ ∂μ ψb + c.c.]
A − [½ UT(B+iC)U χ̄a γL χb + c.c.]
− [½ UT(D+iE)U* χ̄a γL γμ ∂μ χb + c.c.]
I


Choose $\mathcal{V}$ L simplify: $\mathcal{U}^T(B+iC)\mathcal{U}$
$= \mathcal{V}^T H^{-1/2}(B+iC)H^{-1/2}\mathcal{V}$
$= \mathcal{V}^T L \mathcal{V}$

Choose $\mathcal{V}$ to simplify: $\mathcal{U}^T (B+iC)\,\mathcal{U}$

$$= \mathcal{V}^T H^{-1/2}(B+iC)H^{-1/2}\mathcal{V}$$

$$= \mathcal{V}^T L\, \mathcal{V}$$

B, C real symmetric, H is hermitian

$$\gamma^T \underbrace{(H)^{-1/2}\, H\, H^{-1/2}}_{I}\, \gamma^*$$

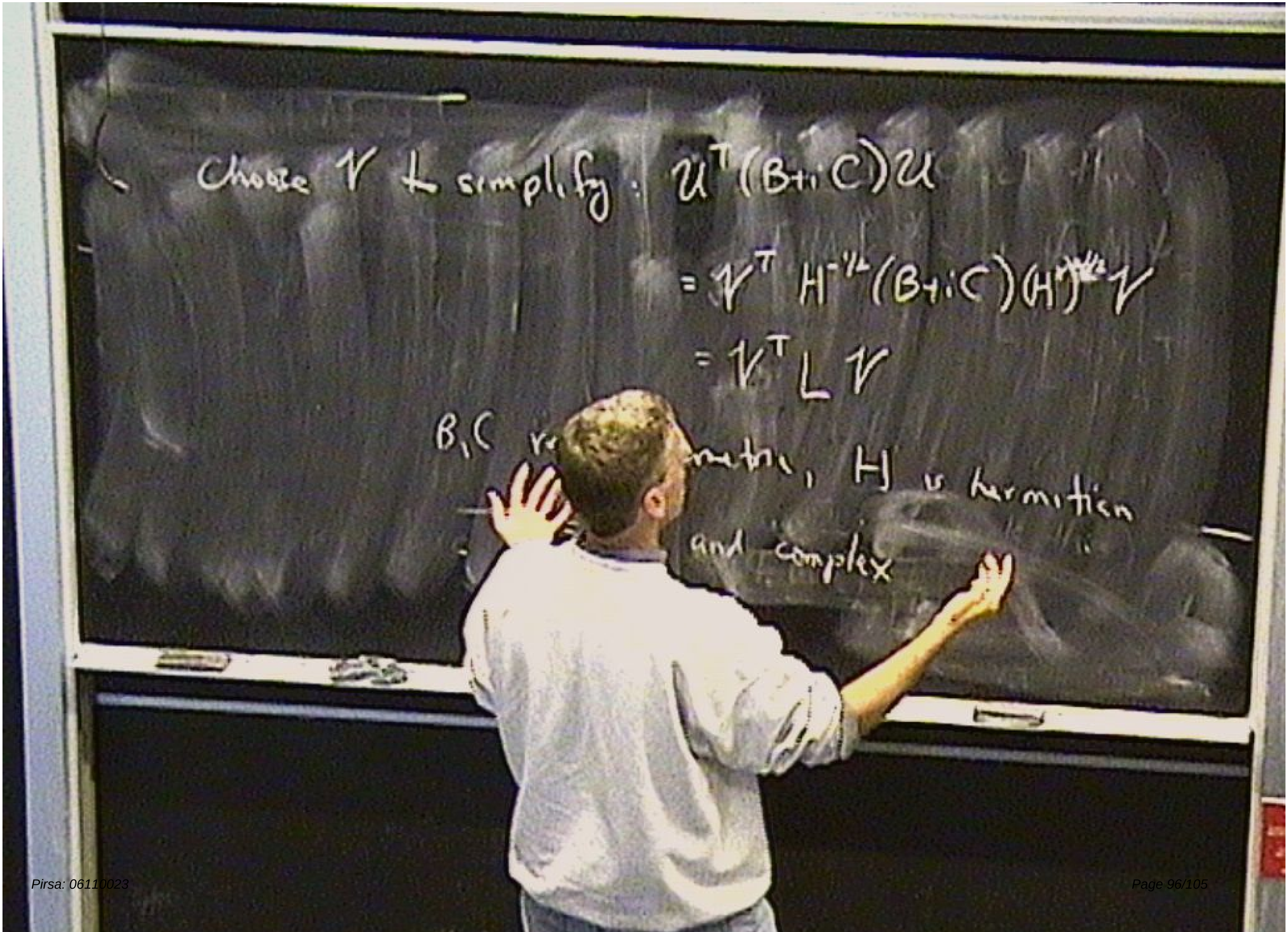
$$= \gamma^T \gamma^*$$

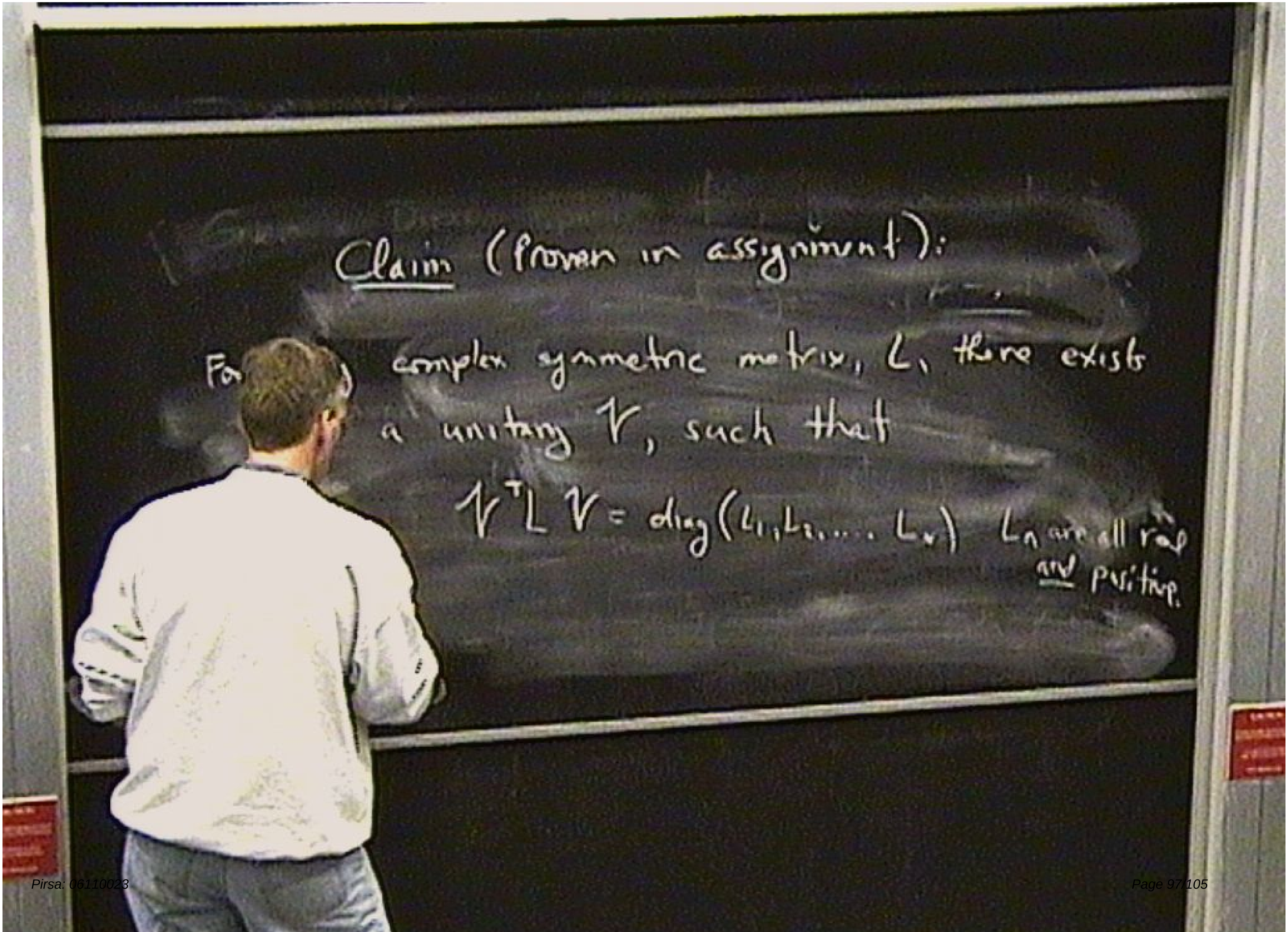
$$= (\gamma^\dagger \gamma)^*$$

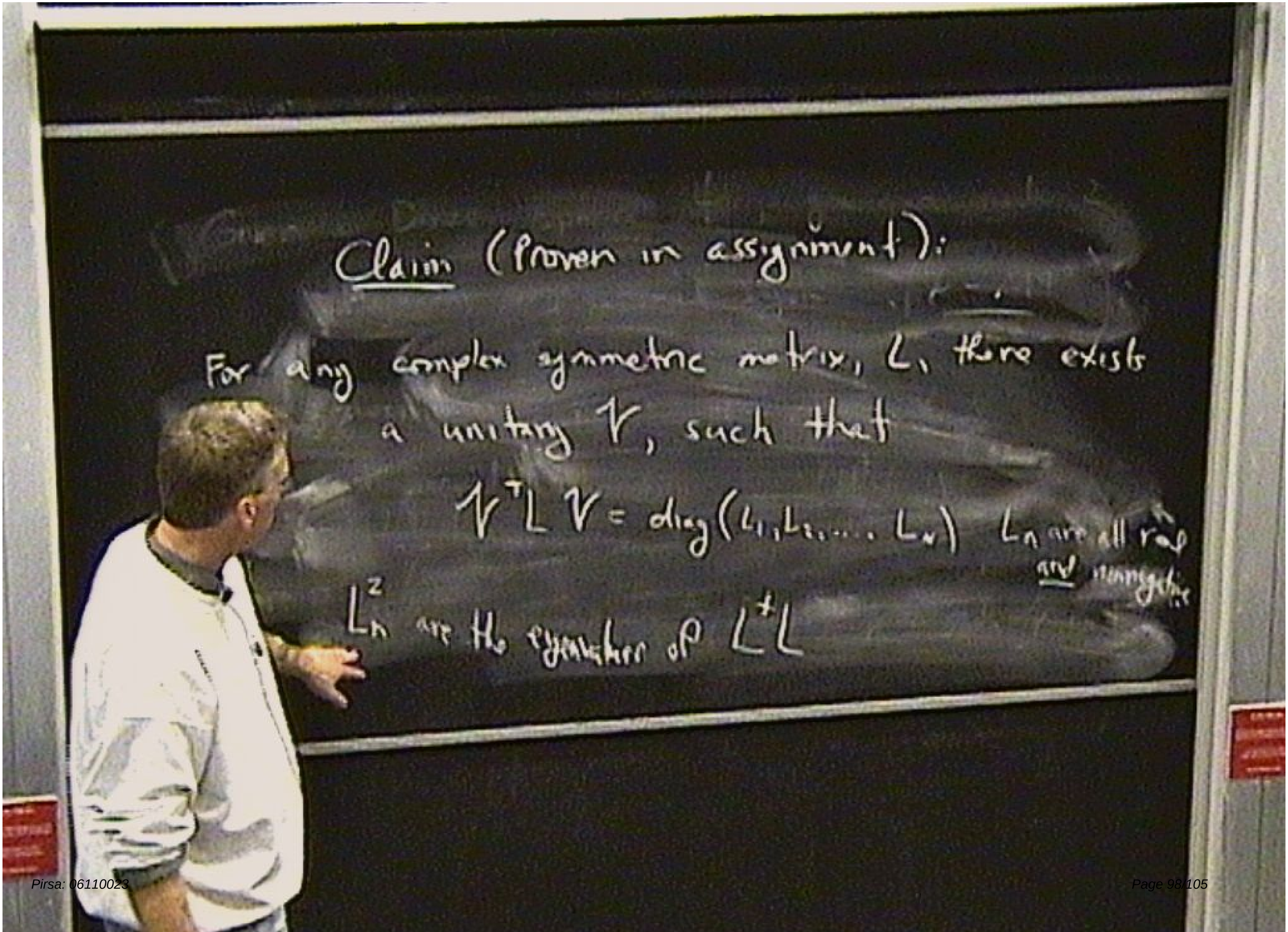
Choose γ & simplify: $\mathcal{U}^T(B+iC)\mathcal{U}$

$$= \gamma^T H^{-1/2}(B+iC)H^{-1/2}\gamma$$

$$= \gamma^T L\, \gamma$$


Choose V & simplify: U^T (B+iC) U
= V^T H^-1/2 (B+iC)(H^-1/2)* V
= V^T L V
B, C
H is hermitian
and complex


Claim (Proven in assignment):
Fo complex symmetric matrix, L, there exists
a unitary V, such that
$V^T L V = diag(L_1, L_2, \dots L_N)$ L_n are all real and positive.


Claim (Proven in assignment):
For any complex symmetric matrix, L, there exists
a unitary V, such that
V^T L V = diag(L_1, L_2, ..., L_N)
L_n are all real and nonnegative
L_n^2 are the eigenvalues of L^† L

Claim (Proven in assignment):

For any complex symmetric matrix, L, there exists a unitary V, such that

$$V^T L V = \mathrm{diag}(L_1, L_2, \ldots, L_N)$$

L_n are all real and nonnegative

L_n^2 are the eigenvalues of $L^\dagger L$

then $\bar{\psi} = \psi^\dagger \beta = i\psi^\dagger \gamma^0 = -i\psi^\dagger \gamma_0 \to \bar{\psi} D^{-1}$ under " " " ".

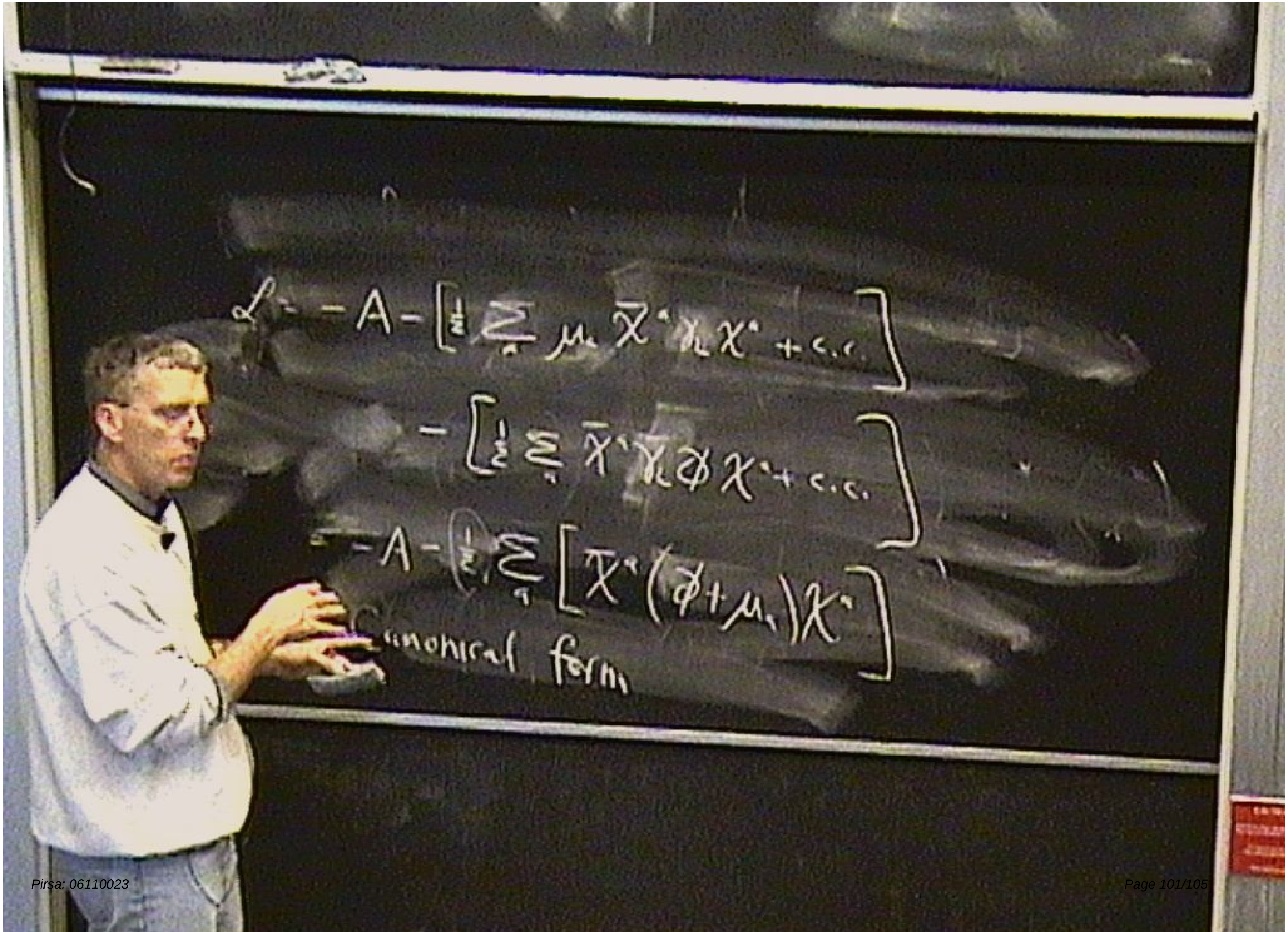
Claim (Proven in assignment):

For any complex symmetric matrix, L, there exists a unitary V, such that

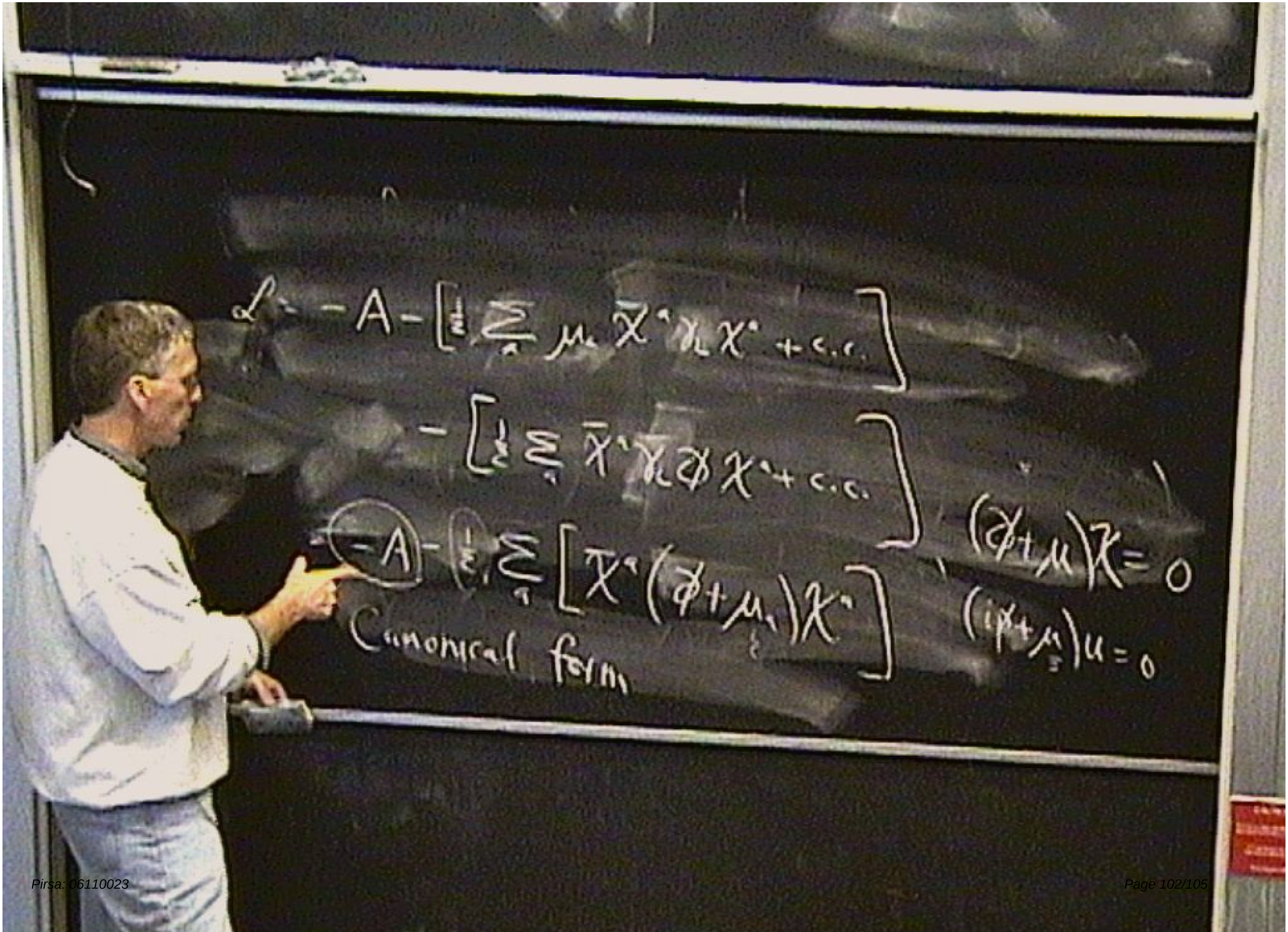
$$V^T L V = \mathrm{diag}(\mu_1, \ldots, \mu_N)$$

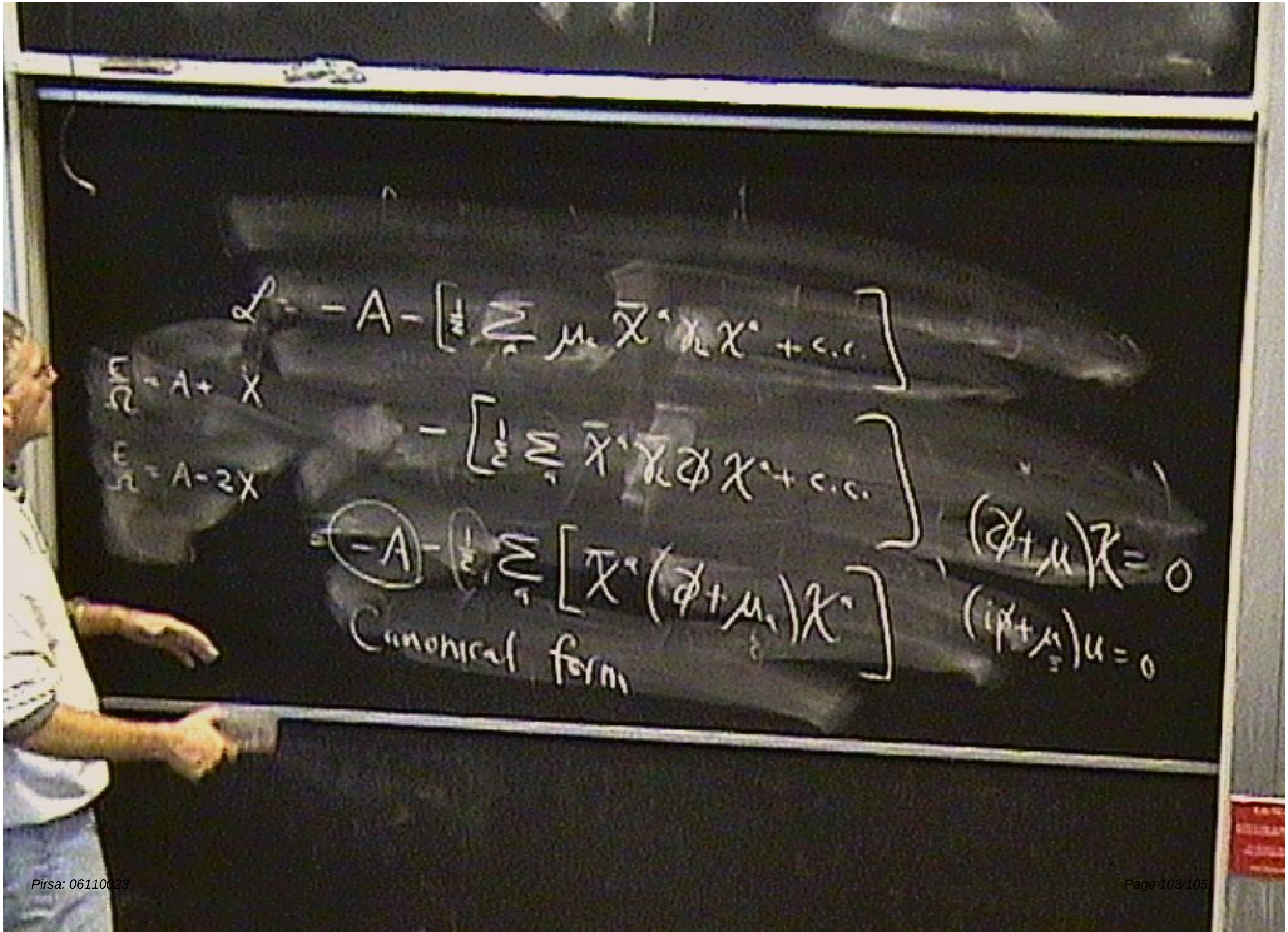
μ_n are all real and nonnegative

μ_n^2 are the eigenvalues of $L^\dagger L$

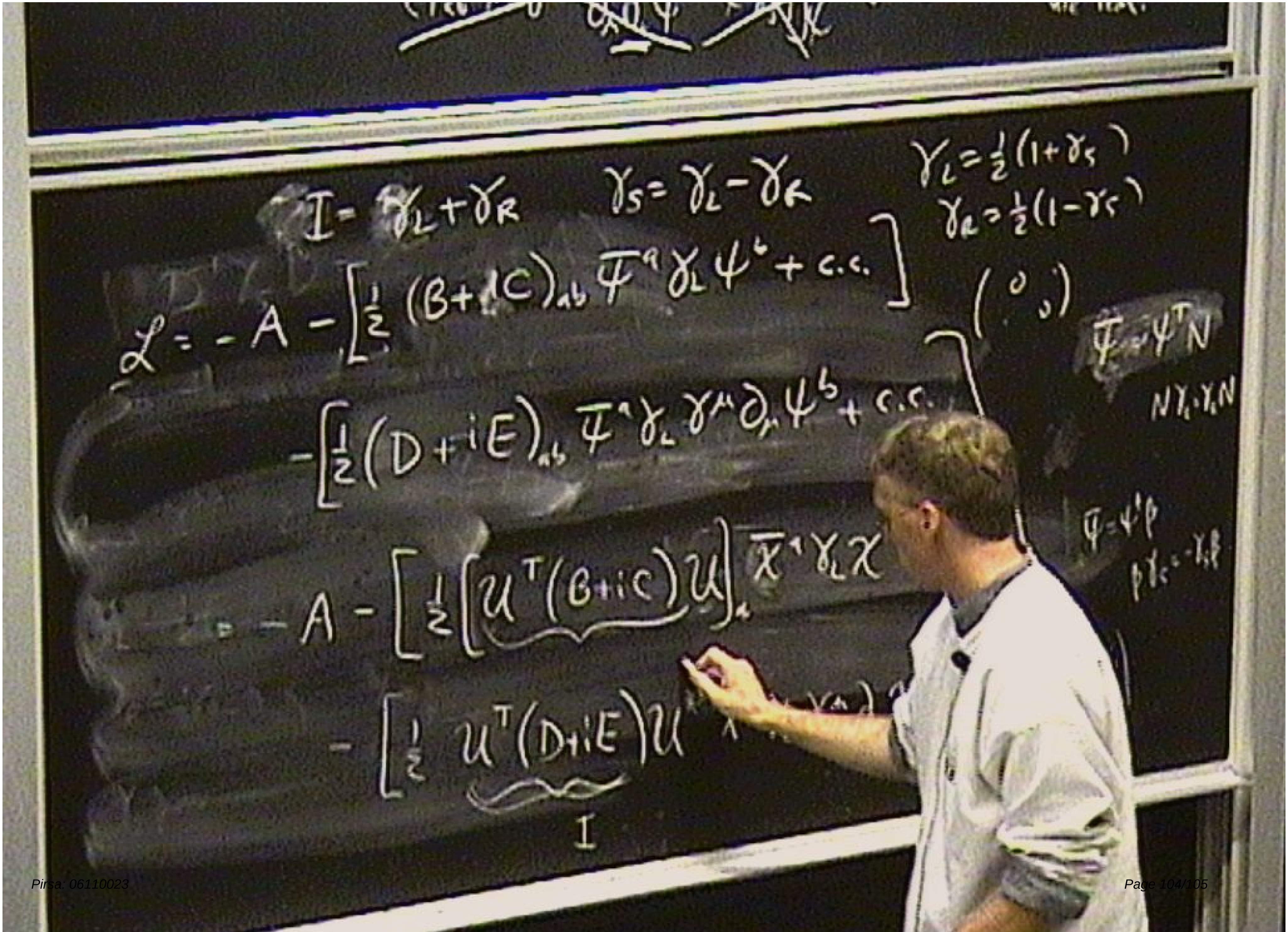


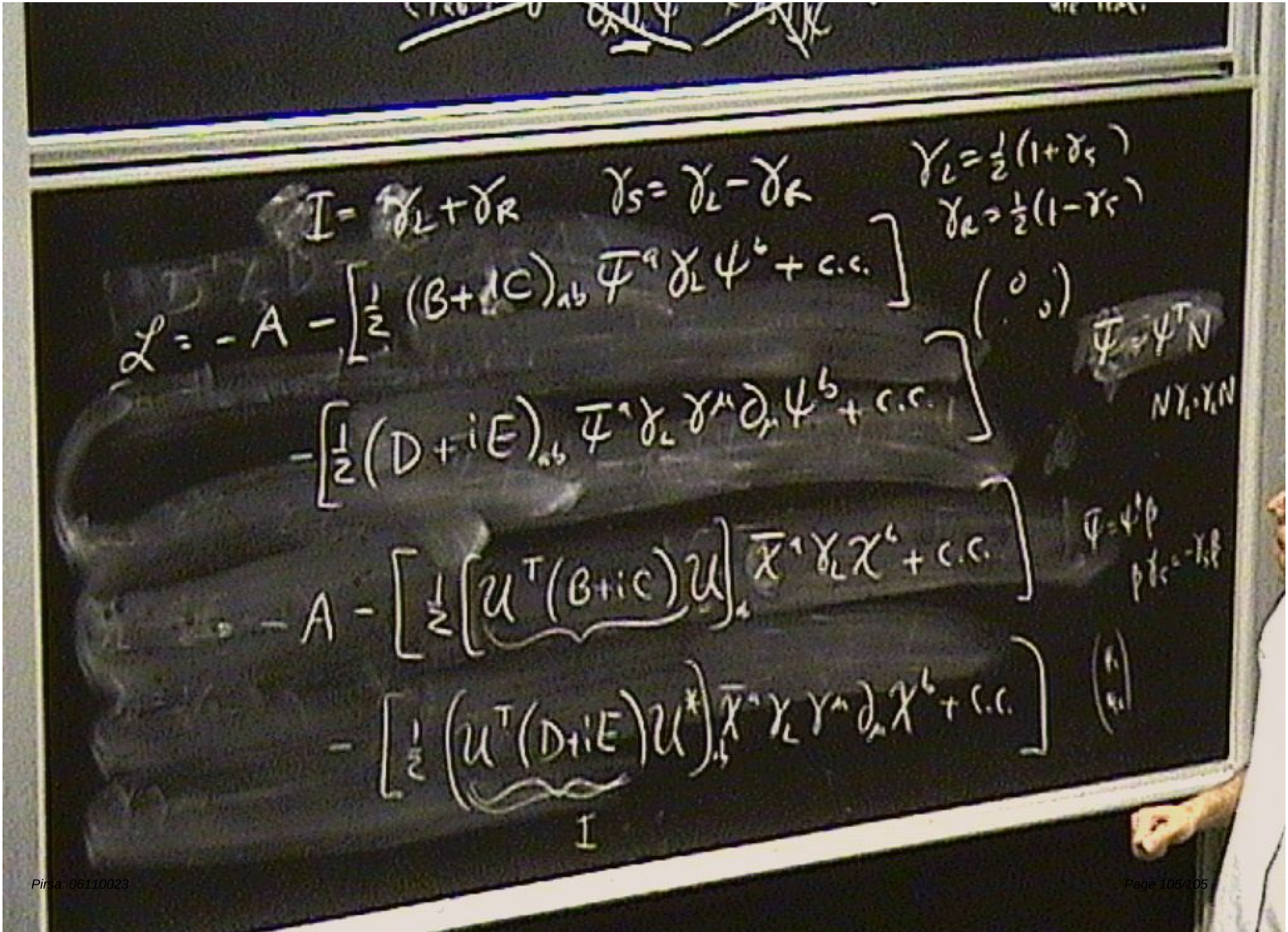
$\mathcal{L} = -A - \left[\frac{1}{2}\sum_a \mu_a \bar{\chi}^a \gamma_L \chi^a + c.c.\right]$
$- \left[\frac{1}{2}\sum_a \bar{\chi}^a \gamma_L \not{\partial} \chi^a + c.c.\right]$
$= -A - \frac{1}{2}\sum_a \left[\bar{\chi}^a (\not{\partial} + \mu_a) \chi^a\right]$
Canonical form


$\mathcal{L} = -A - \left[\frac{1}{2} \sum_a \mu_a \bar{\chi}^a \gamma_L \chi^a + c.c.\right]$
$- \left[\frac{1}{2} \sum_a \bar{\chi}^a \gamma_L \partial\!\!\!/ \chi^a + c.c.\right]$
$= -A - \frac{1}{2} \sum_a \left[\bar{\chi}^a (\partial\!\!\!/ + \mu_a) \chi^a\right]$
Canonical form
$(\partial\!\!\!/ + \mu) \chi = 0$
$(i\partial\!\!\!/ + \mu) u = 0$



$\mathcal{L} = -A - \left[\frac{1}{2}\sum_a \mu_a \bar{\chi}^a \gamma_5 \chi^a + c.c.\right]$
$\frac{E}{\Omega} = A + X$
$- \left[\frac{1}{2}\sum_a \bar{\chi}^a \gamma_5 \not{\partial} \chi^a + c.c.\right]$
$\frac{E}{\Omega} = A - 2X$
$= -A - \frac{1}{2}\sum_a \left[\bar{\chi}^a (\not{\partial} + \mu_a)\chi^a\right]$
$(\not{\partial} + \mu)\chi = 0$
$(i\not{p} + \mu_5)u = 0$
Canonical form


$I = \gamma_L + \gamma_R$
$\gamma_5 = \gamma_L - \gamma_R$
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\gamma_R = \frac{1}{2}(1-\gamma_5)$
$\mathcal{L} = -A - \left[\frac{1}{2}(B+iC)_{ab}\,\bar{\Psi}^a \gamma_L \psi^b + c.c.\right]$
$-\left[\frac{1}{2}(D+iE)_{ab}\,\bar{\Psi}^a \gamma_L \gamma^\mu \partial_\mu \psi^b + c.c.\right.$
$\bar{\Psi} = \psi^T N$
$N\gamma_L = \gamma_L N$
$\psi = \psi' \rho$
$\rho\gamma_5 = -\gamma_5\rho$
$= -A - \left[\frac{1}{2}\left[\mathcal{U}^T(B+iC)\mathcal{U}\right]_{a}\bar{\chi}^a \gamma_L \chi\right.$
$-\left[\frac{1}{2}\,\underbrace{\mathcal{U}^T(D+iE)\mathcal{U}}_{I}\right.$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$\psi = \psi' \rho$
$\rho\gamma_5 = -\gamma_5\rho$
$= -A - \left[\frac{1}{2}\left[\mathcal{U}^T(B+iC)\mathcal{U}\right]_{ab}\bar{\chi}^a\gamma_L\chi^b + c.c.\right]$
$-\left[\frac{1}{2}\left(\underbrace{\mathcal{U}^T(D+iE)\mathcal{U}^*}_{I}\right)_{ab}\bar{\chi}^a\gamma_L\gamma^\mu\partial_\mu\chi^b + c.c.\right]$