Title: Generalized Modified Gravity Models: ghosts and dynamical bounds

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Abstract: Modified gravity models seem to have classical instabilities, ghosts degrees of freedom and superluminal modes. Besides these constraints new dynamical bounds have found to be typical of these models. The cosmological nature of all these constraints is discussed.

Introduction

- Problems of modern cosmology

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  - ⋆ Cosmic acceleration – Cosmological constant

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  - ⋆ Dark Matter
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  - ⋆ or not?

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- Chameleon field [Khoury, Weltman PRD69 (2004)]
- Phantom [Caldwell, Phys.Lett.B 545 (2002)]
- Ghost condensation [Arkani-Hamed, Creminelli, Mukohyama, Zaldarriaga JCAP 0404 (2004)]

Intro to MGM

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- DGP model [but ghosts issue] [Dvali, Gabadadze, Porrati Phys.Lett.B485 (2000)] [Koyama PRD 72 123511 (2005)]

- $f(R)$ theories

$$S = \int d^4x \sqrt{-g}\, f(R) + S_m$$

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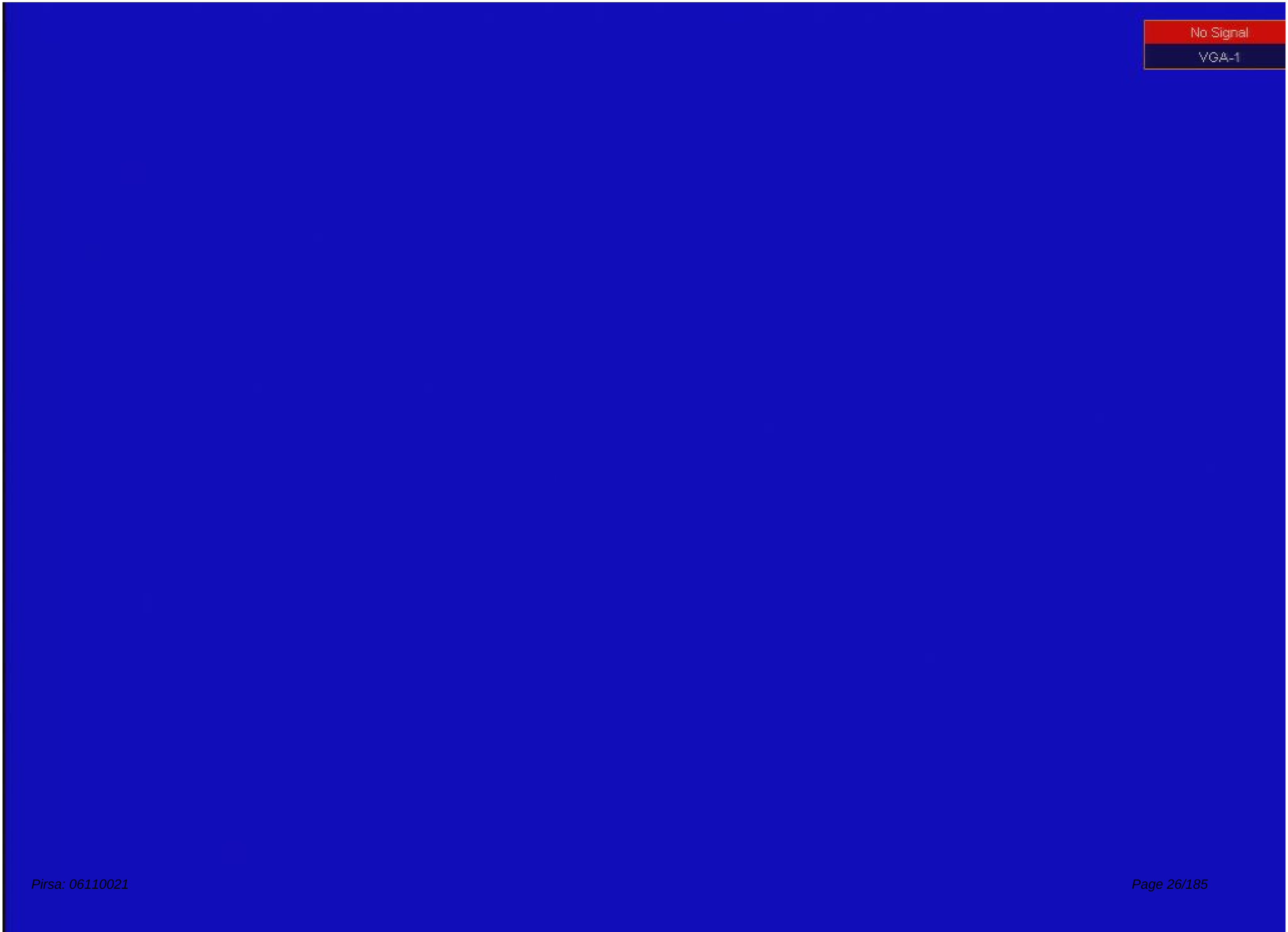
- For CDTT

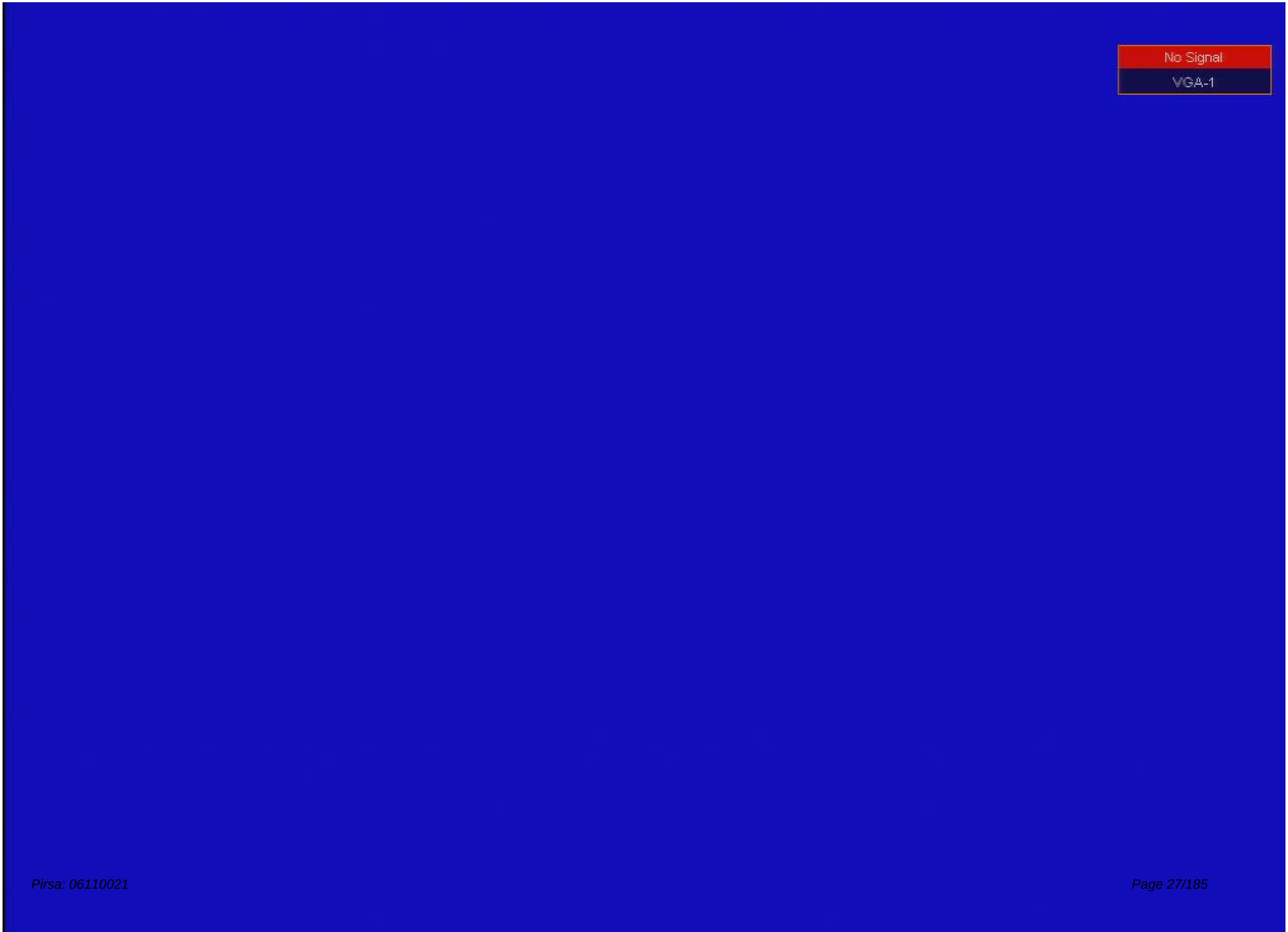
$$f(R) = R - \frac{\mu^4}{R} \qquad \frac{\partial f}{\partial R} = 1 + \frac{\mu^4}{R^2}$$

- CDTT action is equivalent to

$$S = M_P^2 \int d^4x \sqrt{-g}\,[f(\phi) + (R - \phi)\, f'(\phi)] + 16\pi\, S_{\rm m}$$

[Hindawi, Ovrut, Waldram PRD 53 (1996), Chiba Phys.Lett.B 575 (2003)]


No Signal
VGA-1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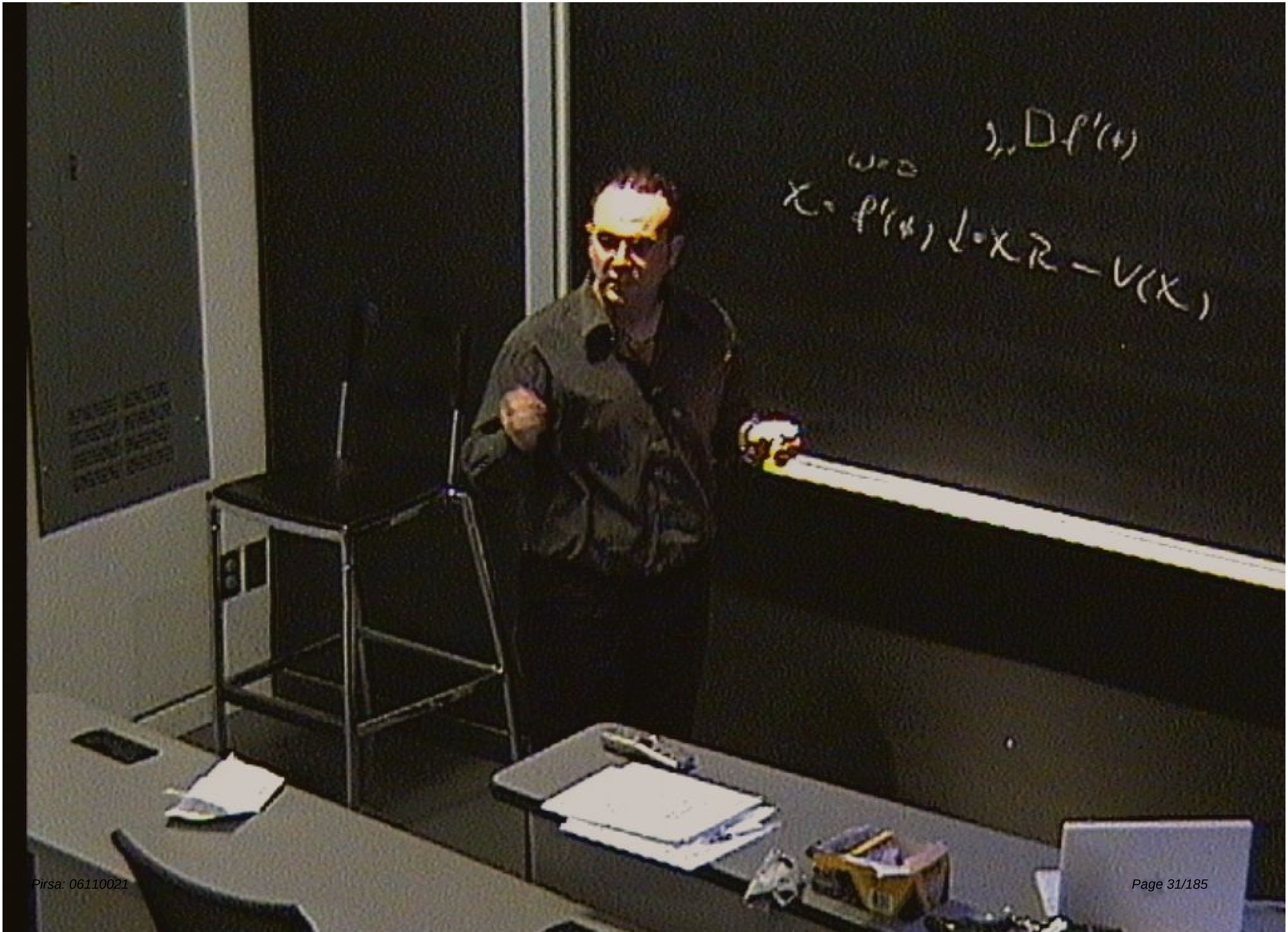

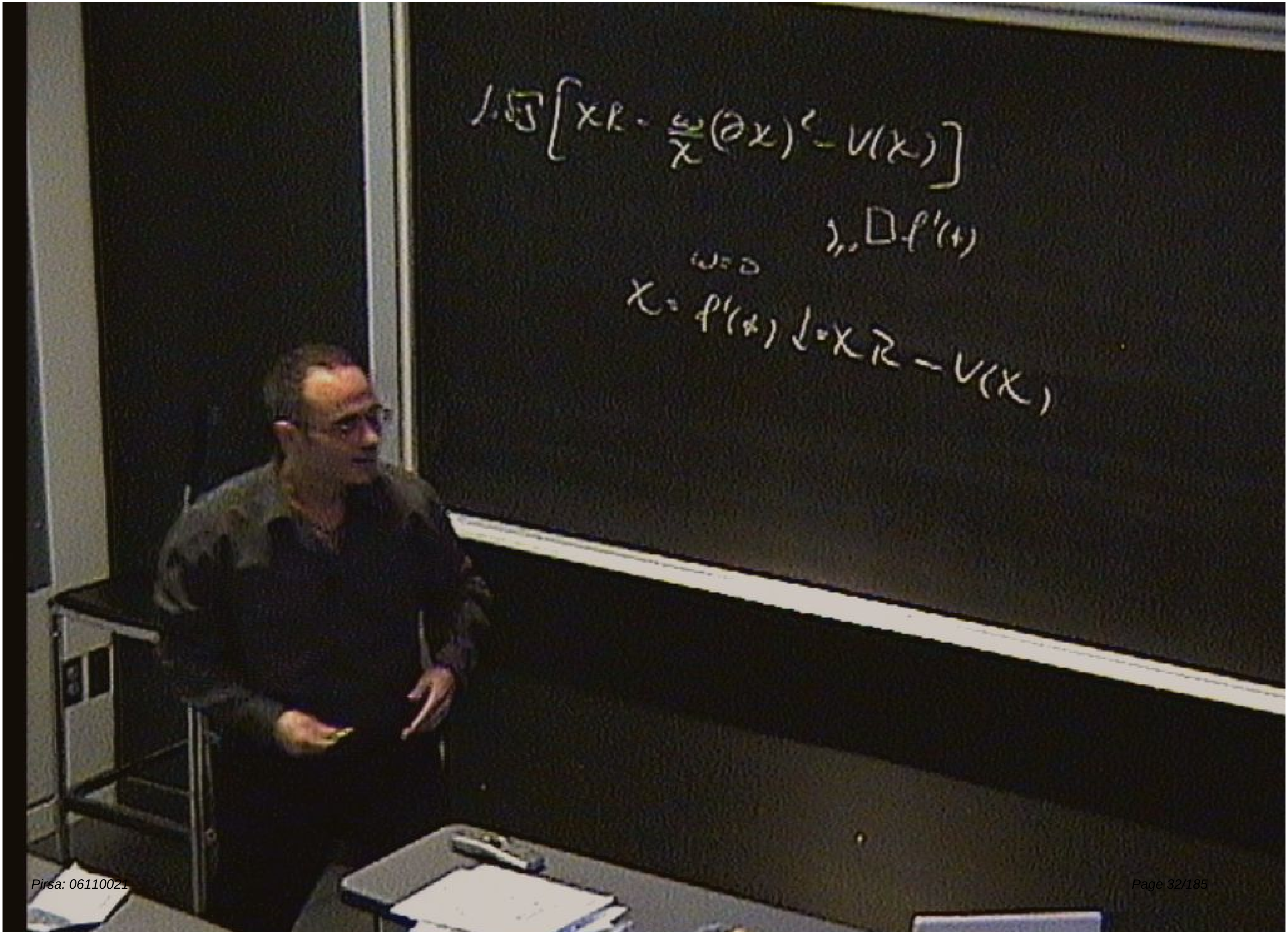

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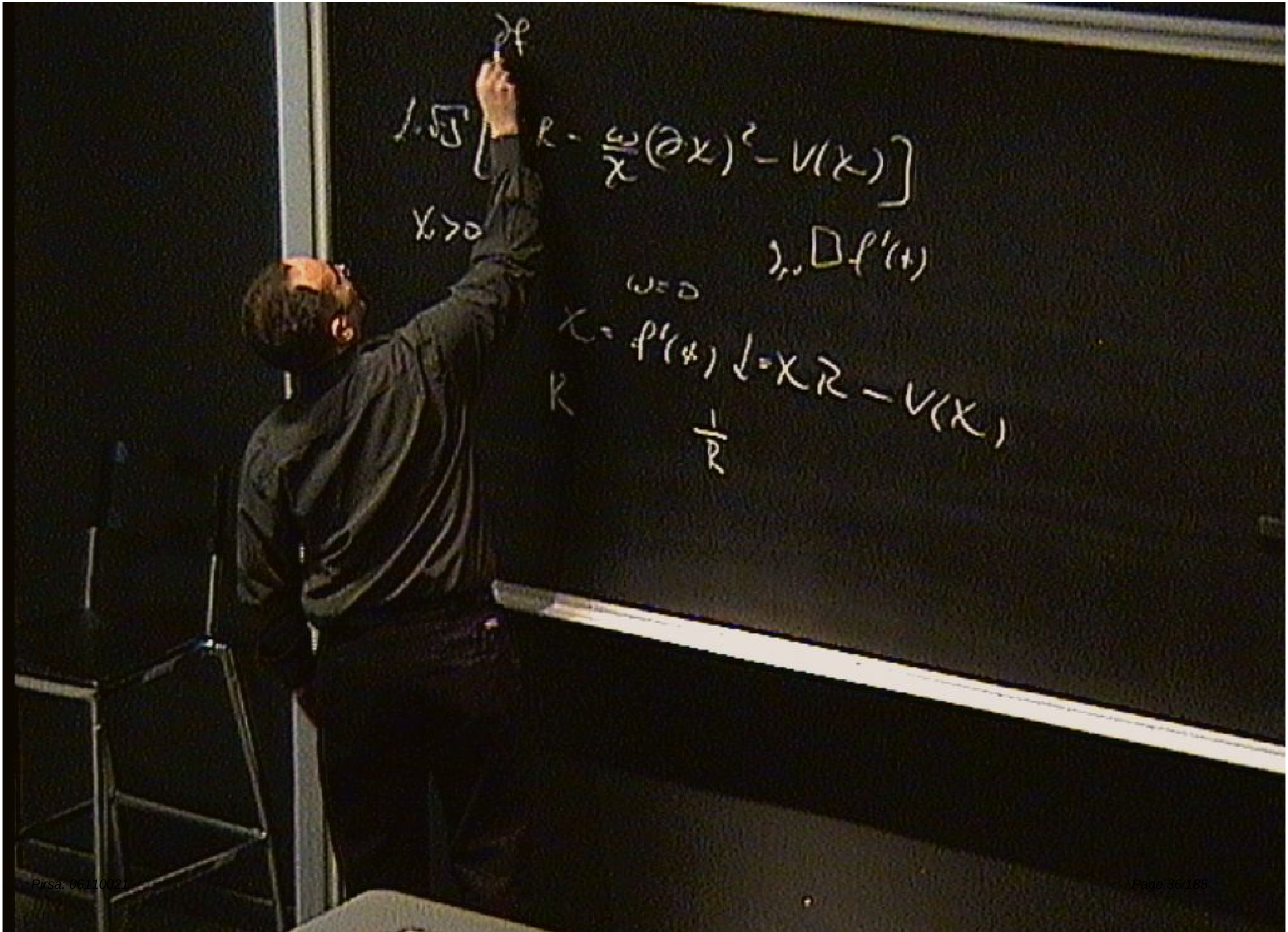
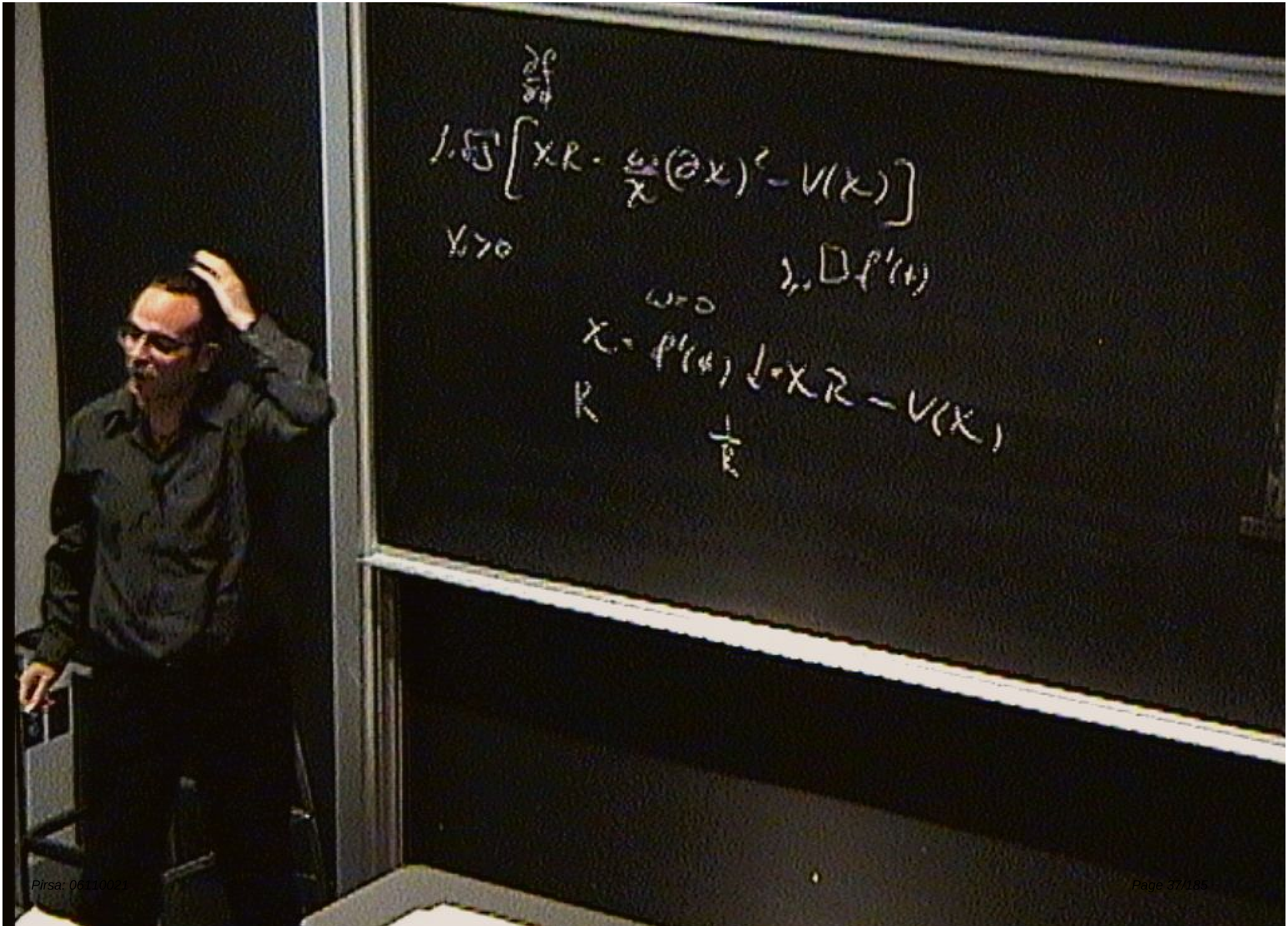
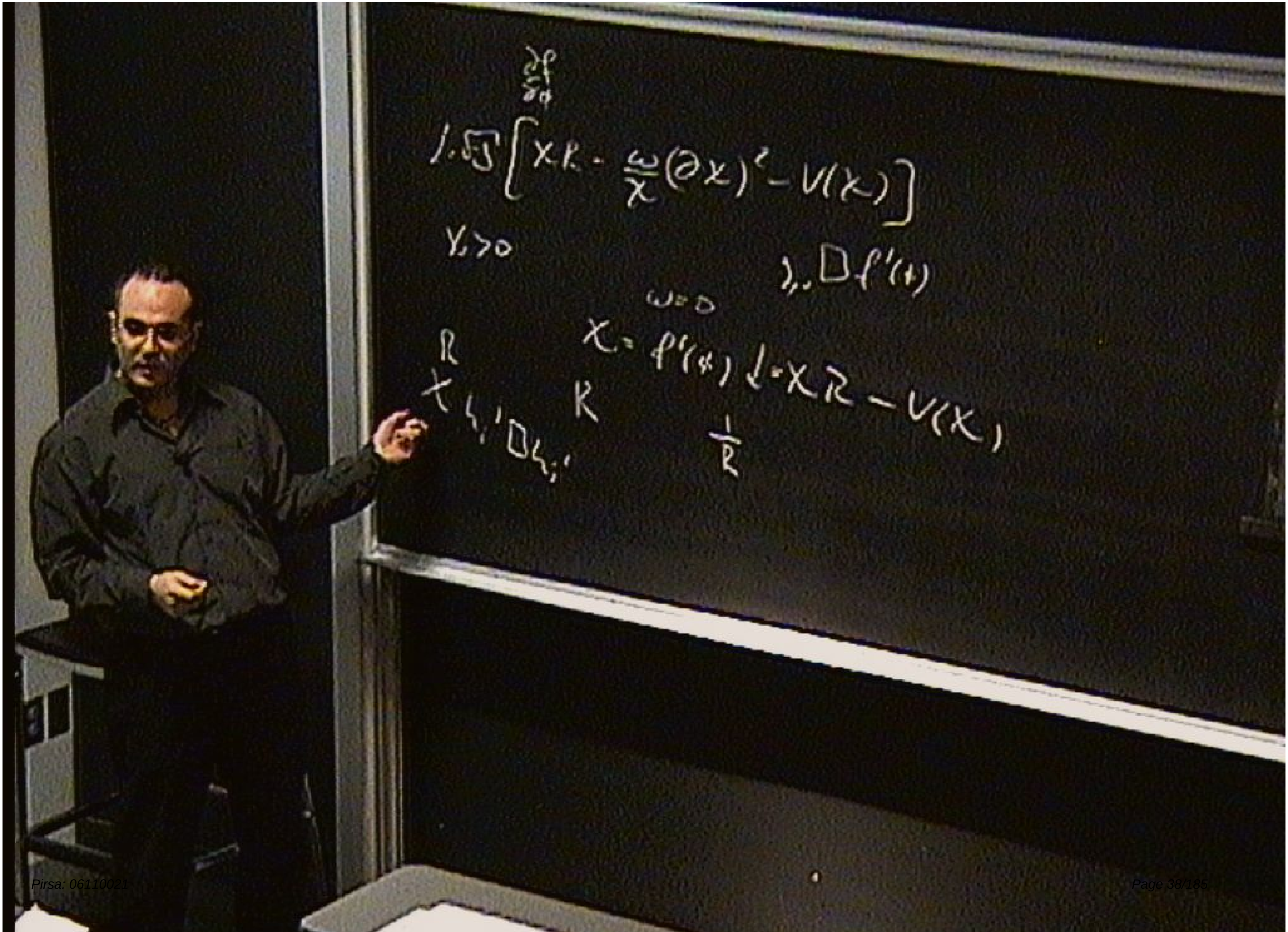
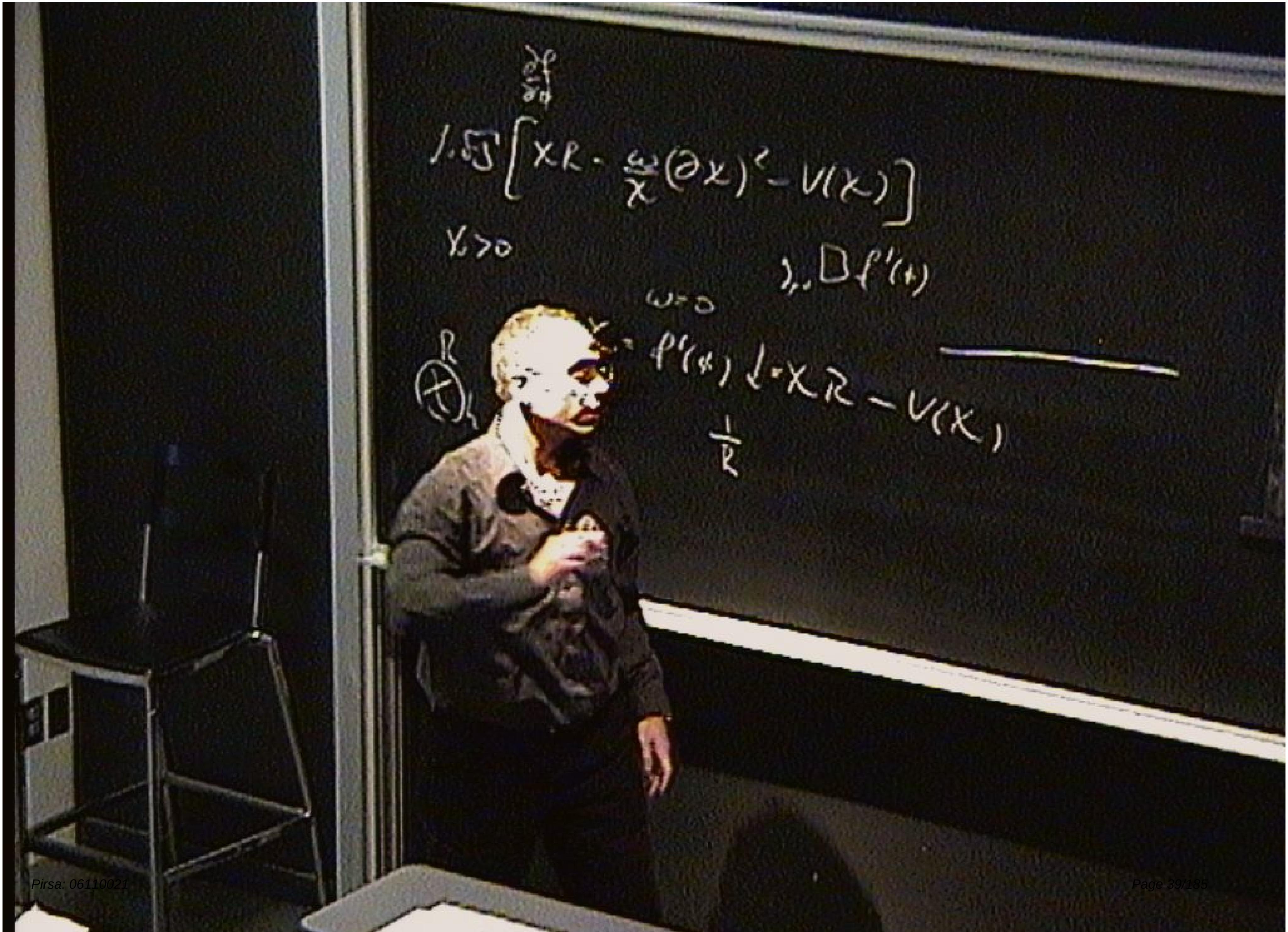
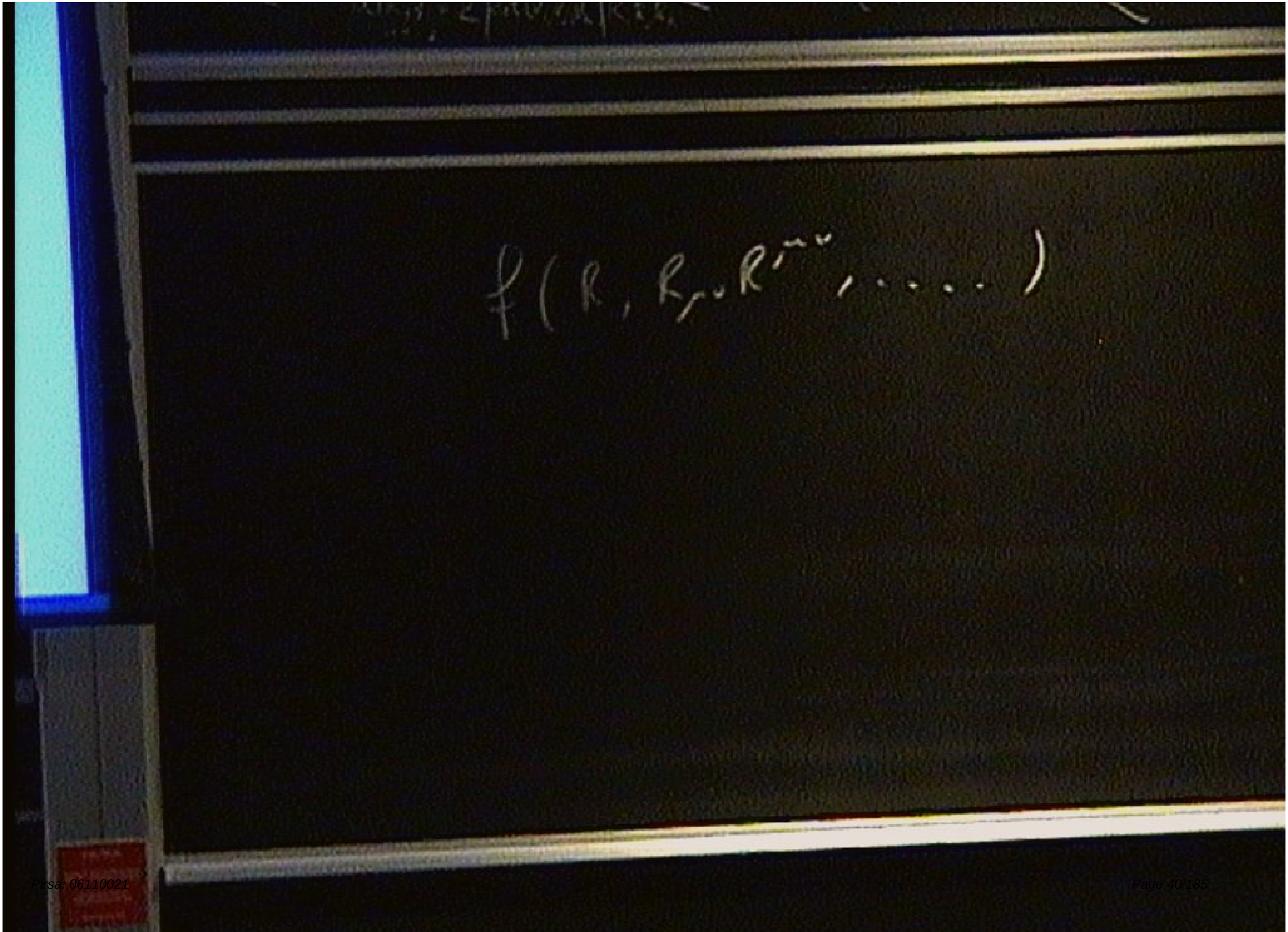
- ϕ is a light field, solar system constraints, $\omega > 40000$, Cassini experiment [Bertotti, Iess, Tortora Nature 425]

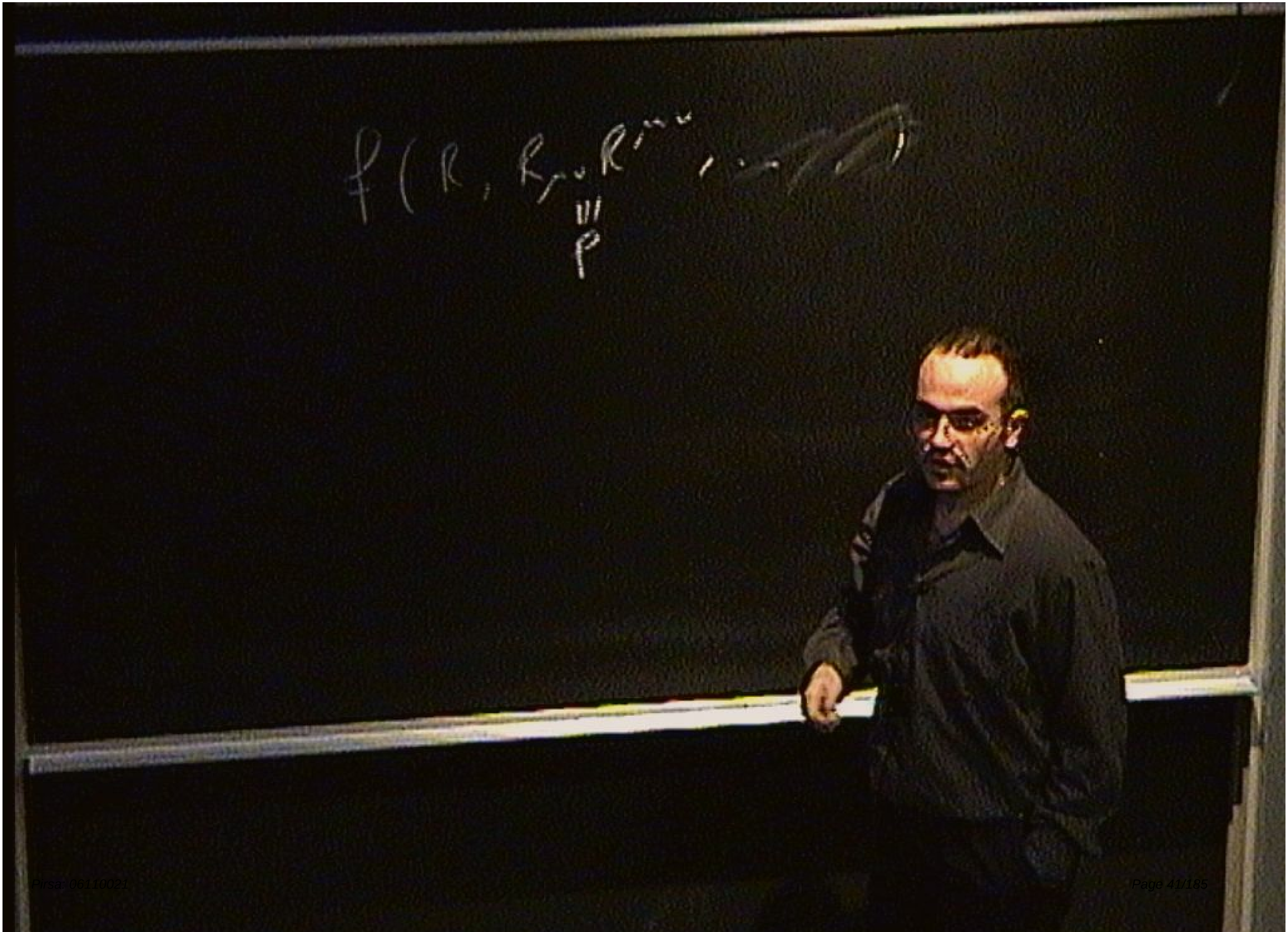


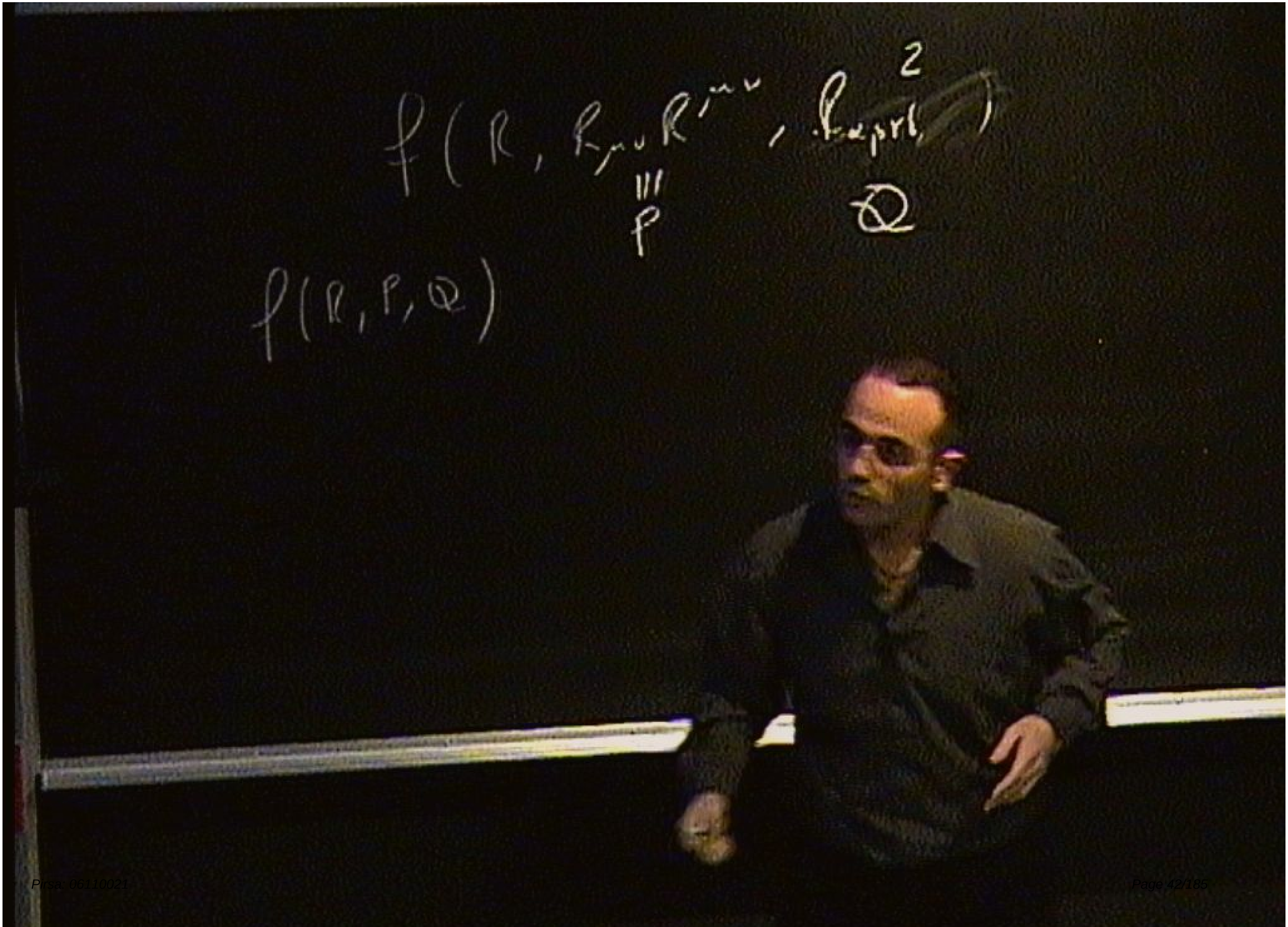







$f(R, R_{\mu\nu}R^{\mu\nu}, \ldots\ldots)$



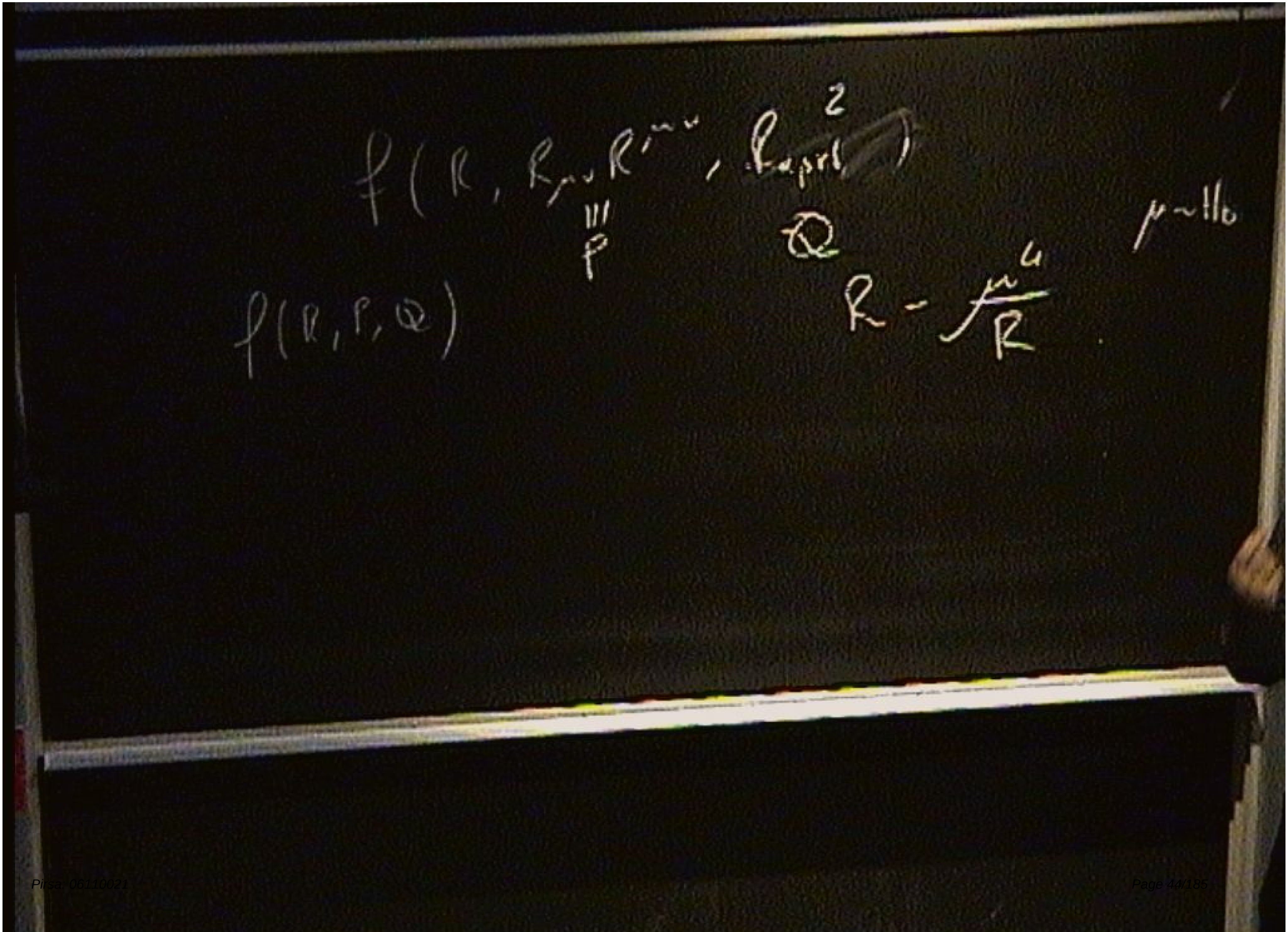

f(R, R_{μν}R^{μν}, R_{αβγδ}^2)
P
Q
f(R, P, Q)

$$f(R,\ R_{\mu\nu}R^{\mu\nu},\ R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})$$

$$R_{\mu\nu}R^{\mu\nu} \equiv P \qquad Q$$

$$f(R, P, Q)$$

$$R - \frac{\mu^4}{R}$$



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P
Q
$f(R, P, Q)$
$R - \frac{\mu^4}{R}$
$\mu \sim H_0$

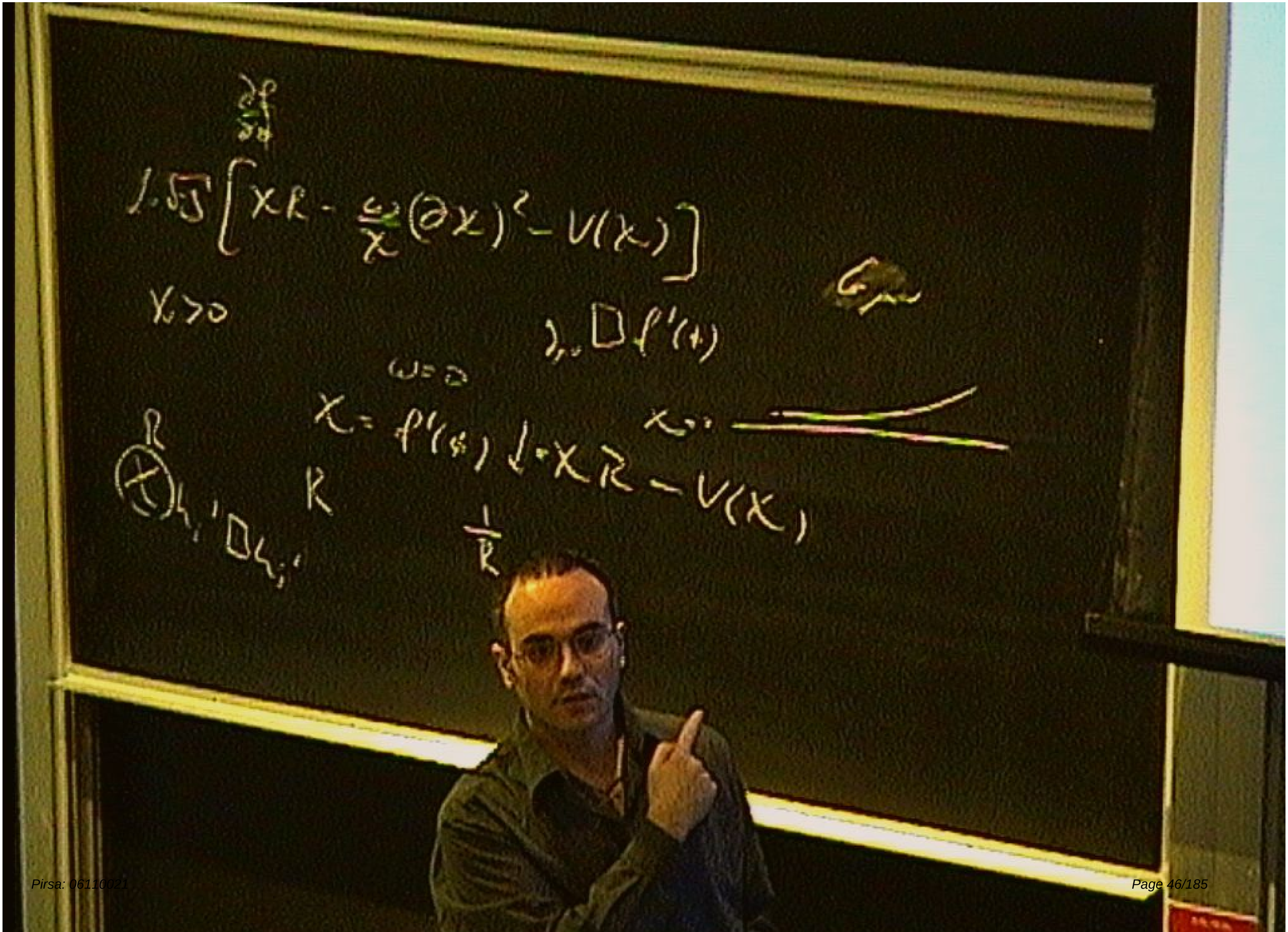
CDDETT Model

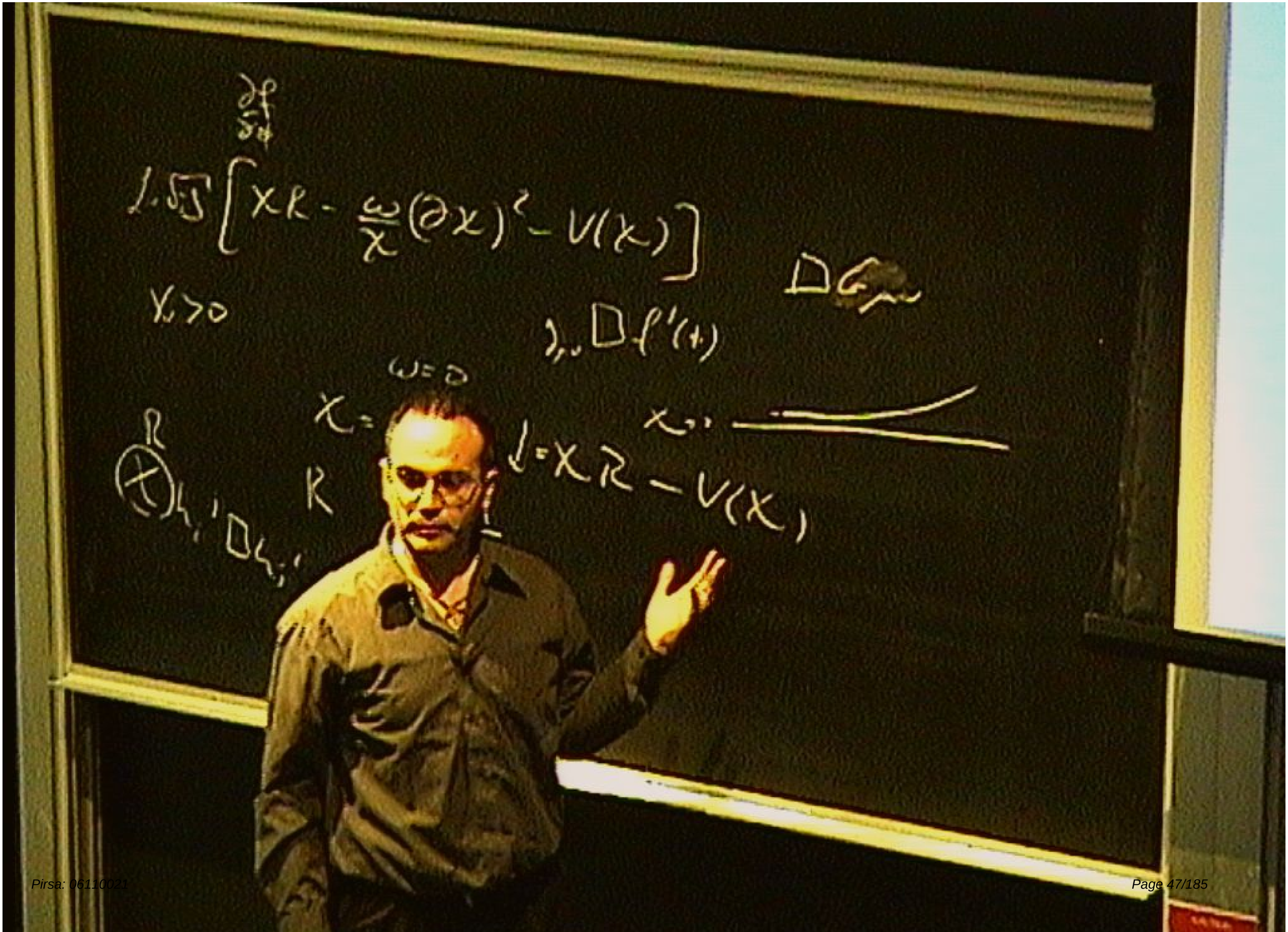
- Other curvature invariants?
- Define $P = R_{\alpha\beta} R^{\alpha\beta}$ and $Q = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$

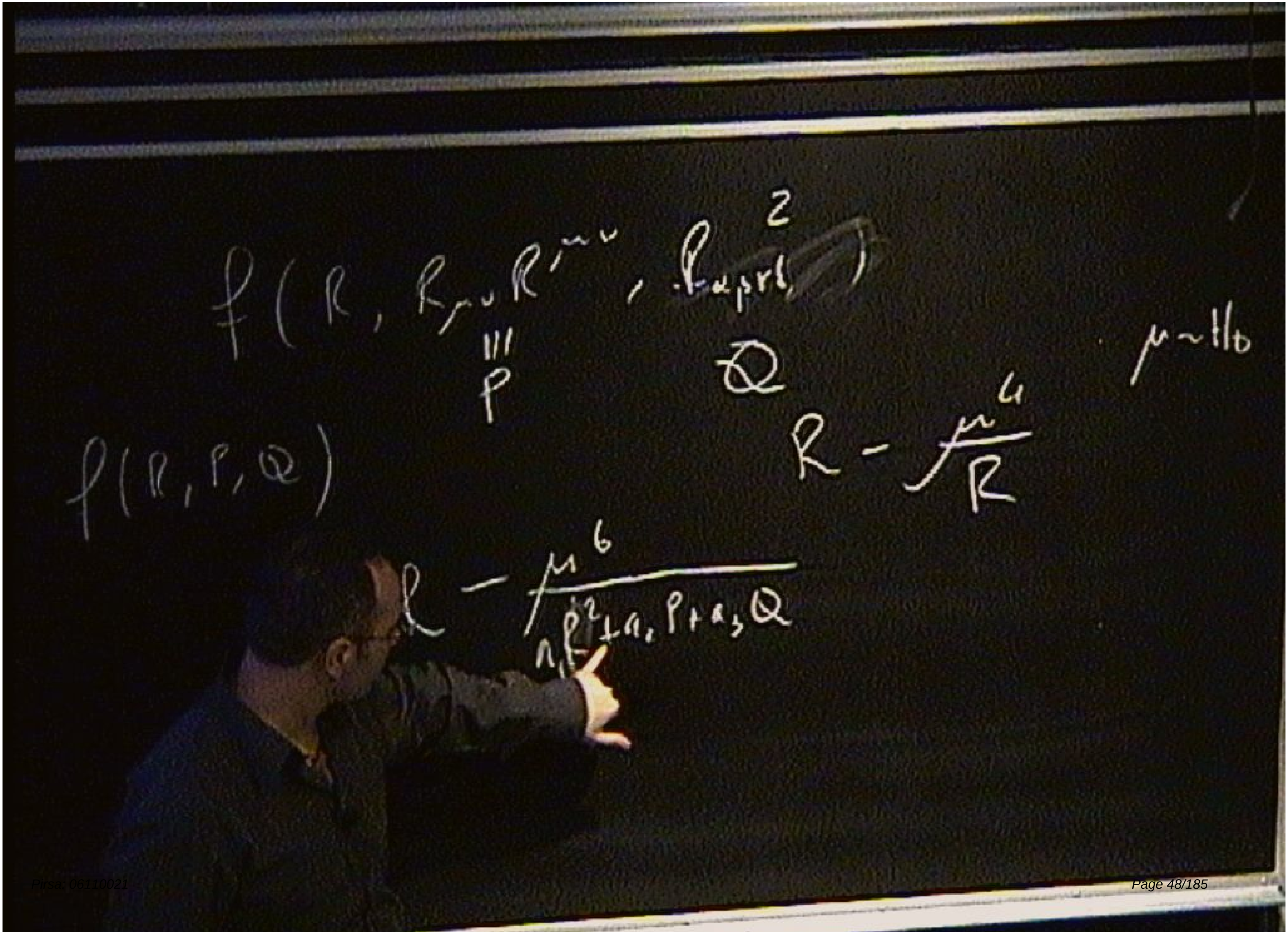
$$S = M_P^2 \int d^4x \sqrt{-g} \left[\gamma R - \theta \frac{\mu^{4n+2}}{(a_1 R^2 + a_2 P + a_3 Q)^n} \right] + 8\pi S_{\rm m}$$

[Carroll, ADF, Duvvuri, Easson, Trodden, Turner PRD 71 (2005)]

- It leads, in general, to a system of non-linear ODEs of fourth order in the scale factor [Mena, Santiago, Weller PRL 96 (2006)]




ω=0
L = X R − V(X)


f(R, R_μν R^μν, R_αβγδ^2)
P
Q
μ ~ H_0
R − μ^4/R
f(R, P, Q)
R − μ^6/(a_1 R^2 + a_2 P + a_3 Q)

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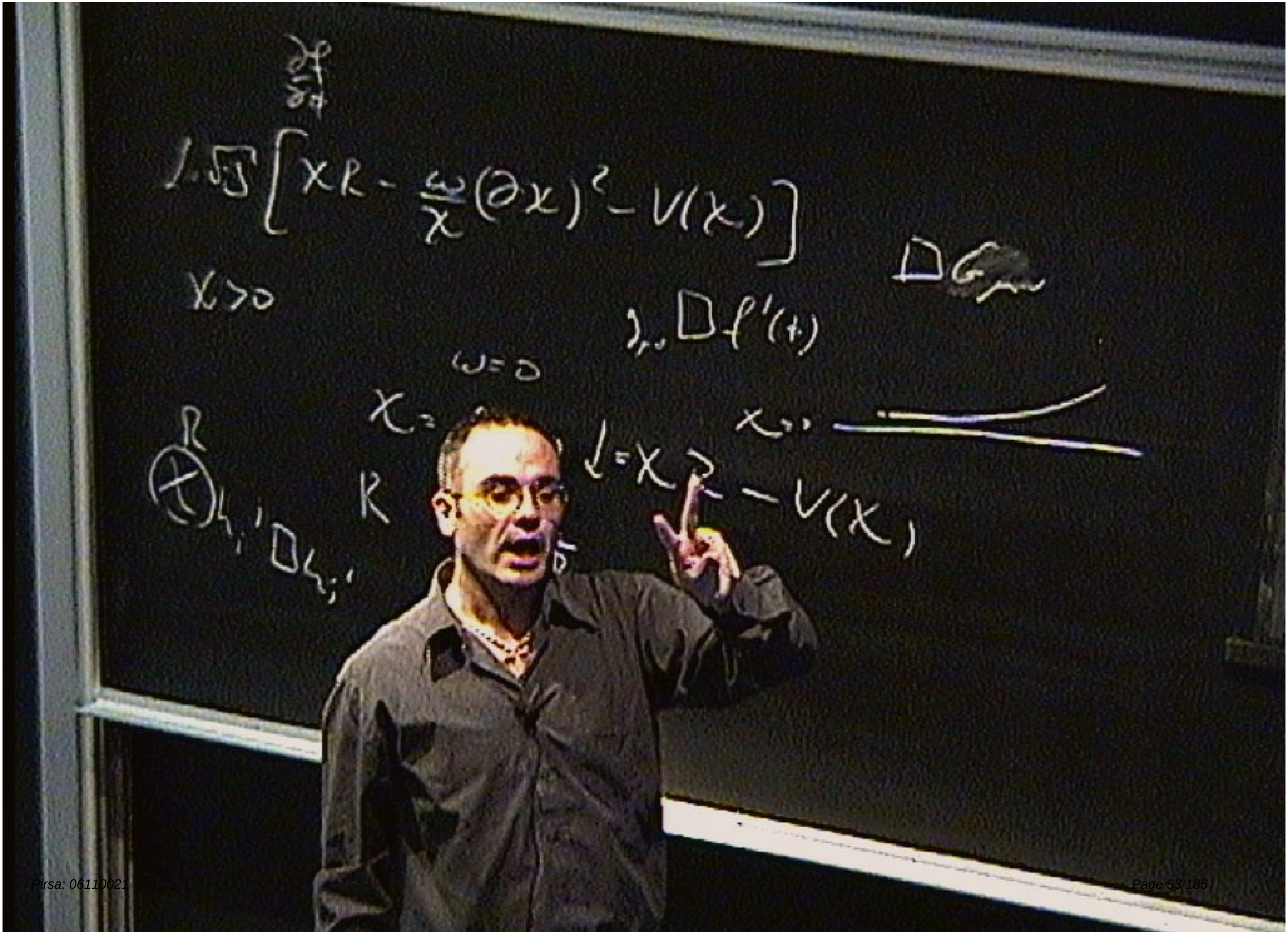

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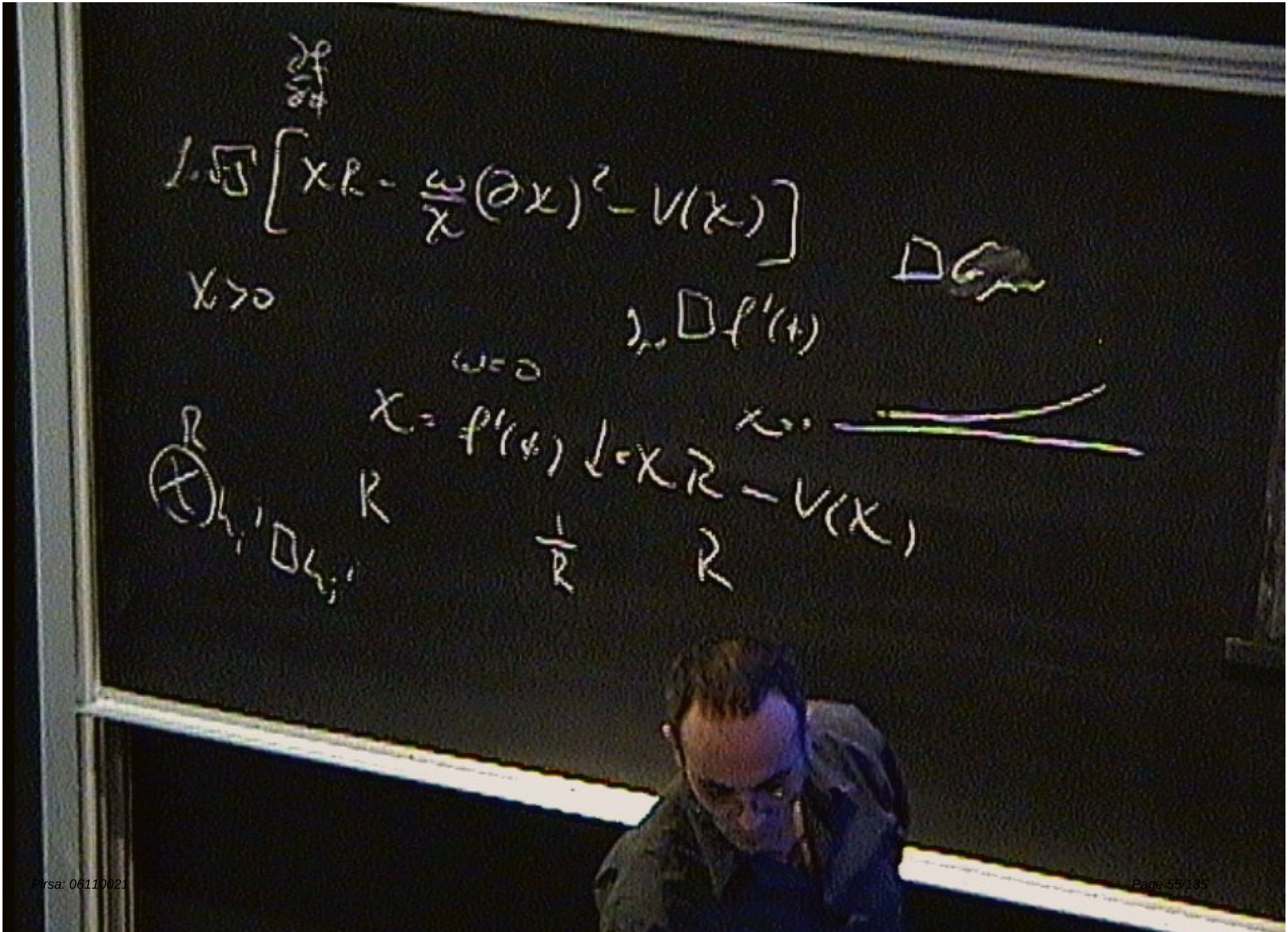
  - ⋆ 1 function, $a(t)$, new Friedmann eqn


ω=0
R
V(X)

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- In vacuum (flat FRW) eqns solved!
  - ⋆ 1 function, $a(t)$, new Friedmann eqn
  - ⋆ Friedmann eqn is a 3$^{\rm rd}$ order non-linear ODE in a
- Recipe to study the equation $F(\dddot{a}, \ddot{a}, \dot{a}, a) = 0$
- Instead of the scale factor use H, $F(\ddot{H}, \dot{H}, H) = 0$
- Define (for a phase-space analysis)

$$x \equiv -H \qquad y \equiv \dot{H}$$

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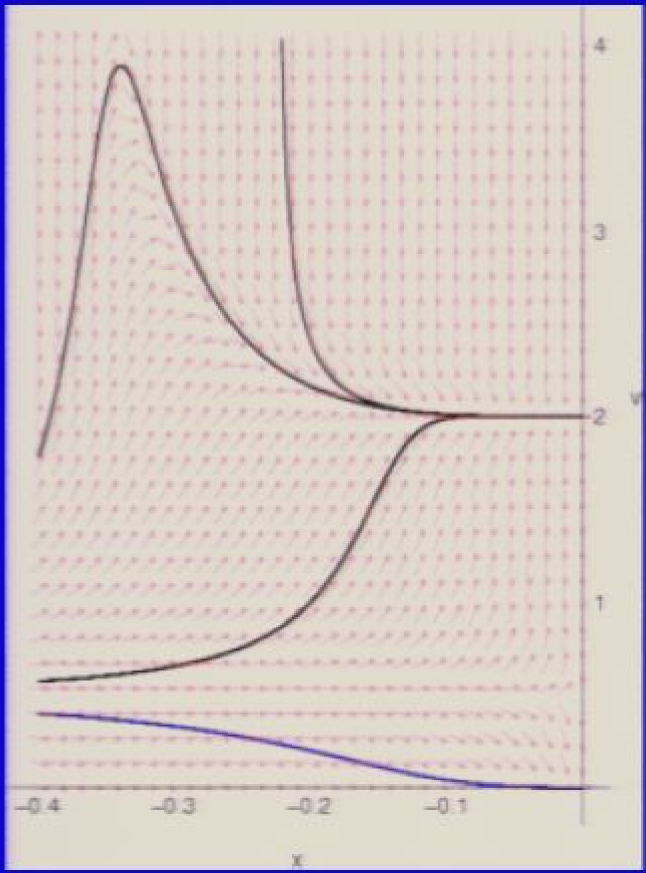
- Looking for possible power-law solutions, define

$$v(x) \equiv -\frac{x^2}{y(x)}$$

- Plot the field v' for any point of the plane x–v.
  Then we have solved, graphically, the ODE!



The solutions that approach $v = \text{constant}$ are power law attractors. Interesting if $p > 1$. Example for CDTT. The solution $v = 2$ is the accelerating attractor.

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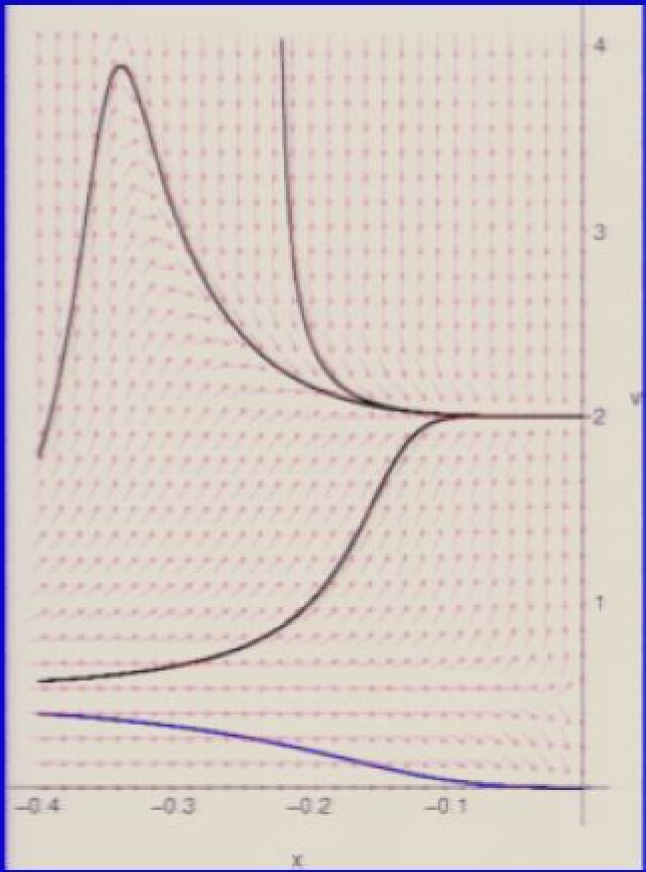
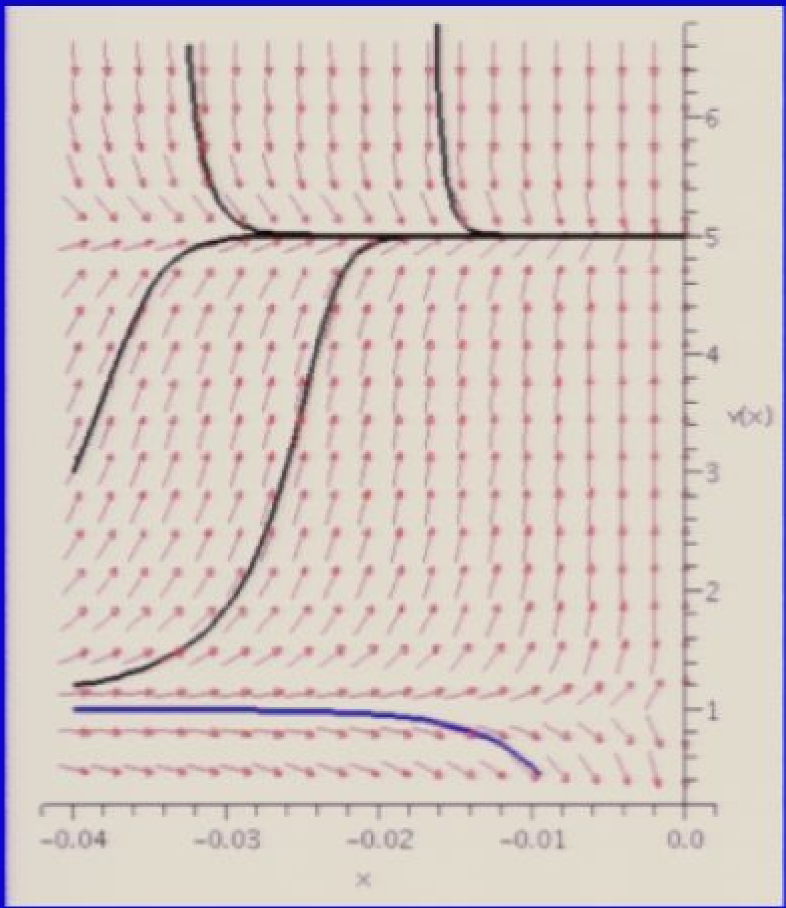


The solutions that approach $v = \mathrm{constant}$ are power law attractors. Interesting if $p > 1$. Example for CDTT. The solution $v = 2$ is the accelerating attractor.

- For Gauss-Bonnet combination, $p = 4n + 1$.



Example for Gauss-Bonnet combination. The solution $v = 5$ is the accelerating attractor. ($\rightarrow$)

Attractors!

- Focusing on the case $n = 1$, we found

$$x\,\frac{dv}{dx} = \frac{72\,x^6\,d(v)^3 + \mu^6\,v^2\,g(v)^2}{2\,\mu^6\,v\,h(v)}$$

$$f(R,\ R_{\mu\nu}R^{\mu\nu},\ R_{\alpha\beta\gamma\delta}^{2})$$

$$R_{\mu\nu}R^{\mu\nu} \equiv P \qquad R_{\alpha\beta\gamma\delta}^{2} \equiv Q$$

$$f(R, P, Q)$$

$$R - \frac{\mu^4}{R} \qquad \mu \sim H_0$$

$$R - \frac{\mu^6}{(a_1 R^2 + a_2 P + a_3 Q)^n}$$

$$R_{GB}^2 = R^2 - 4P + Q$$

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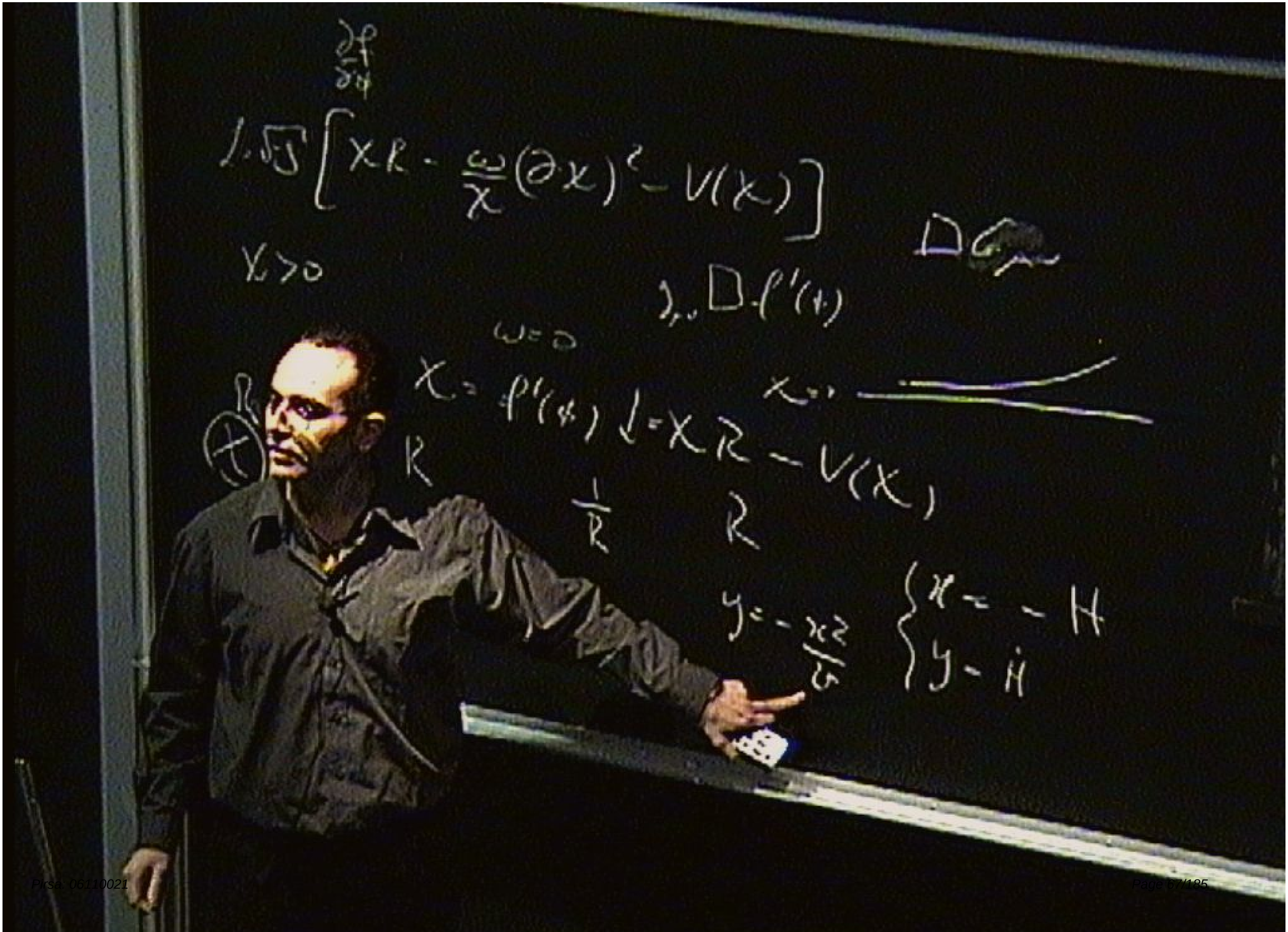

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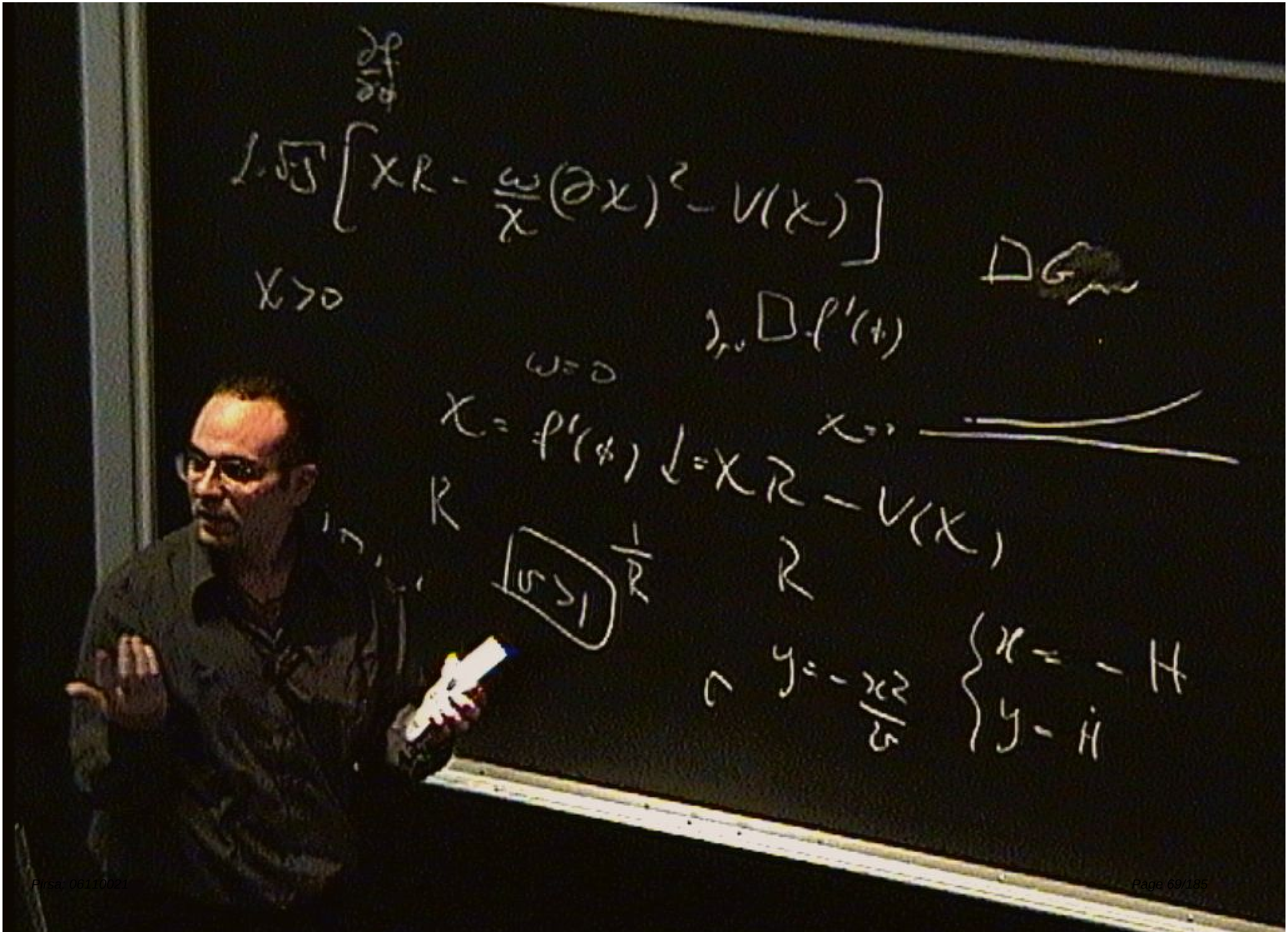
- They all depend on only one parameter $\alpha = \frac{12a_1+4a_2+4a_3}{12a_1+3a_2+2a_3}$


X>0
ω=0
DG

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  - ⋆ The zeros of g are the power-law late-time attractors
  - ⋆ The zeros of d are the singular points of the Frd eqn

Conclusions – Part I

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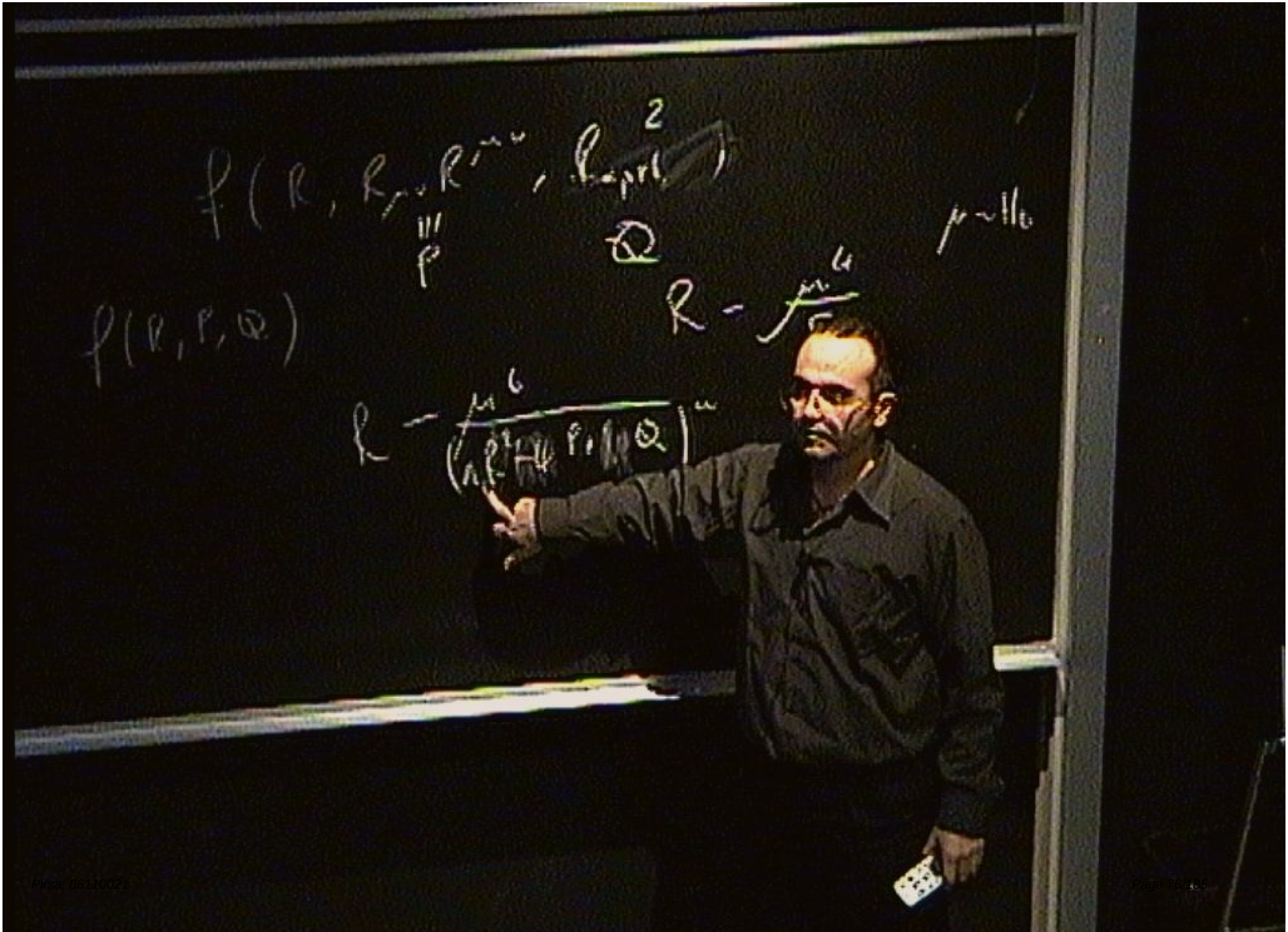
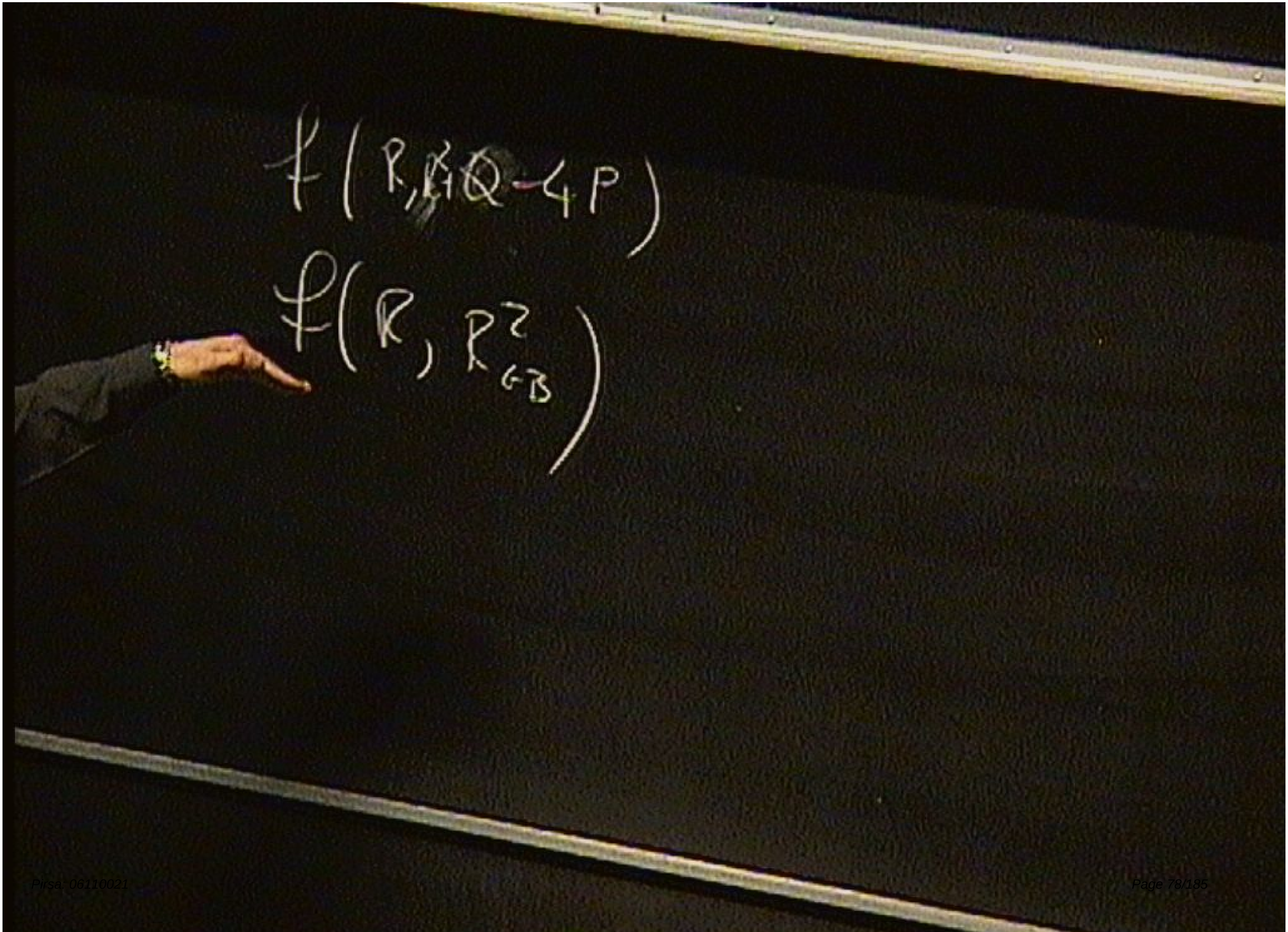
- It can lead to an accelerating universe
- de Sitter is unstable
- No spin-2 ghosts for GB combination, $a_2 = -4a_3$


f(R, P, Q)

$$f(P, Q-4P)$$


$f(R, R^2_{GB})$

$$f(R, Q-4P)$$

$$f(R, R^2_{GB})$$

$$S = \int d^4x \sqrt{-g}\left[f(\lambda, \phi) + (R-\lambda)\frac{\partial f}{\partial \lambda} + (R^2_{GB} - \phi)\frac{\partial f}{\partial \phi} \right]$$

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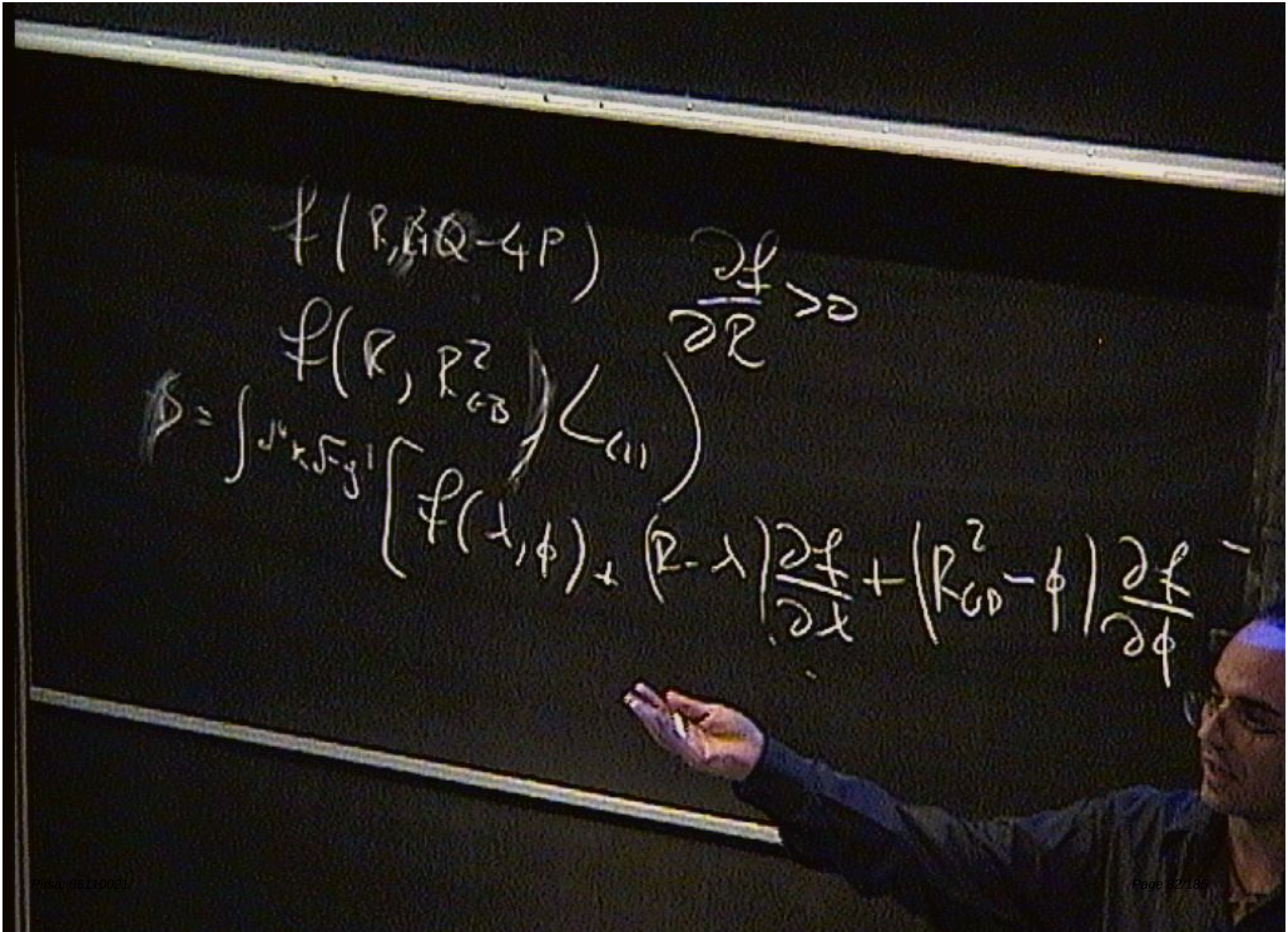
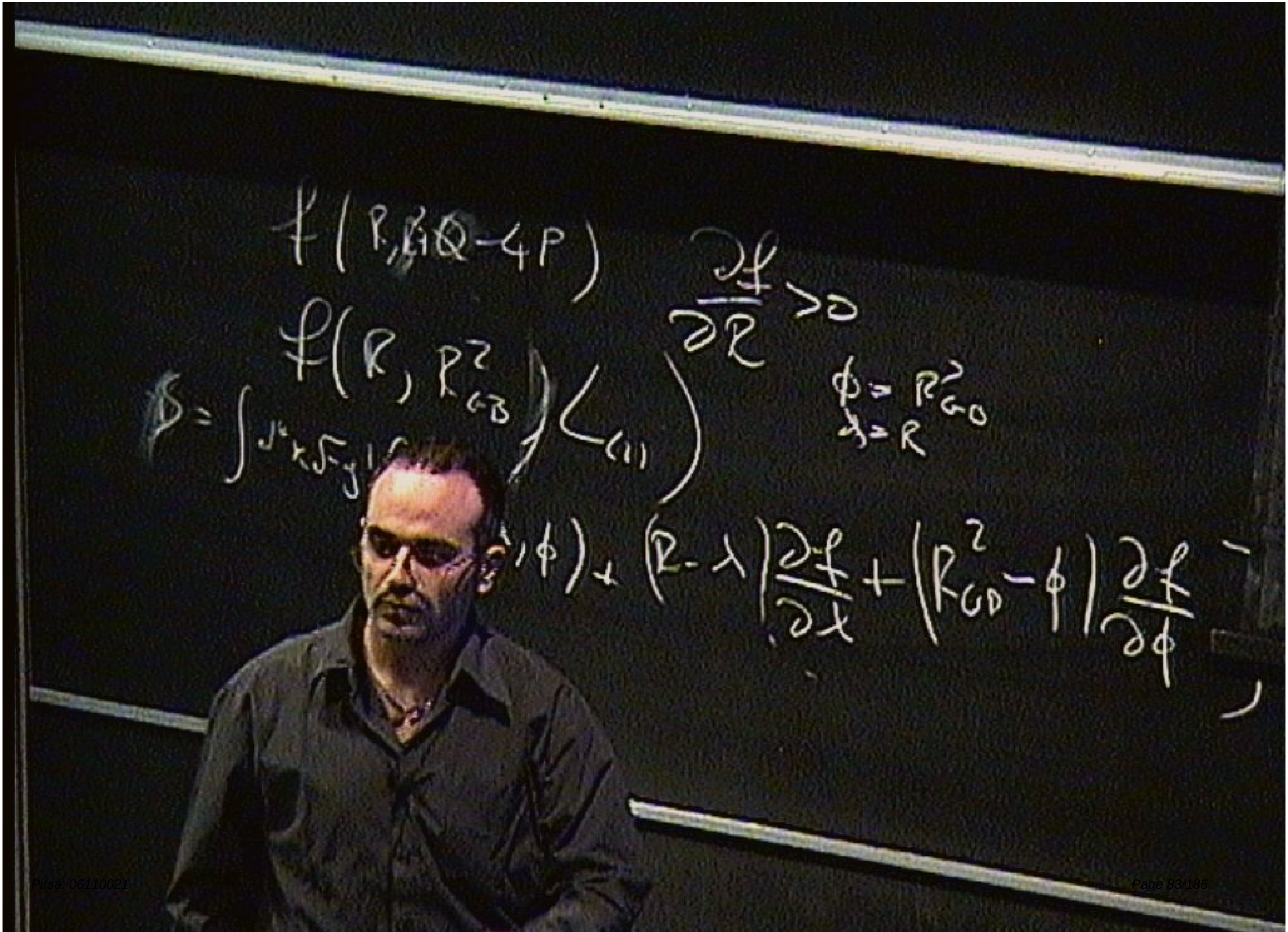
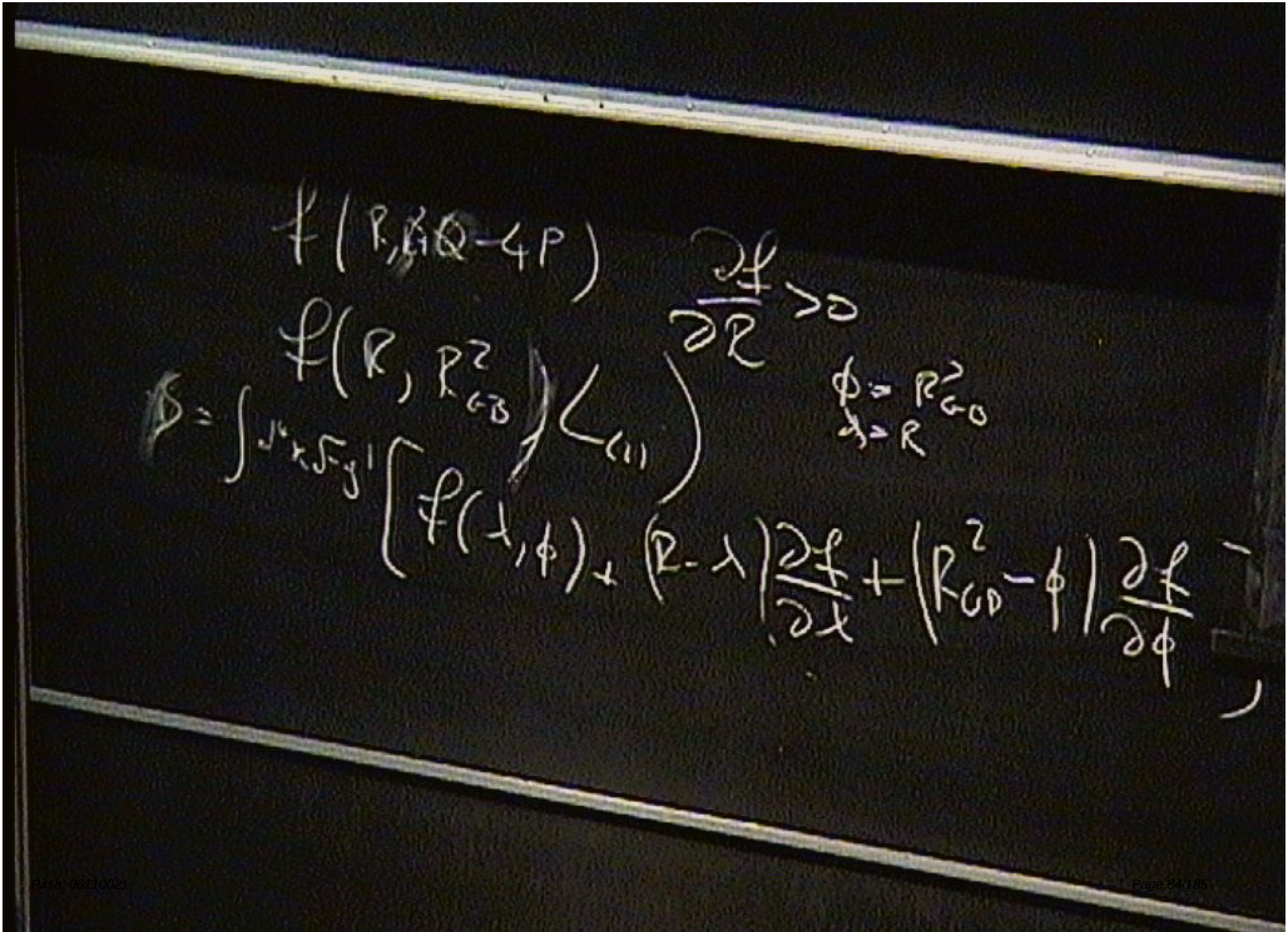
$$f\left(R, R^2 + Q - 4P\right) \qquad \frac{\partial f}{\partial R} > 0$$

$$f\left(R, R^2_{GB}\right)$$

$$S = \int d^4x \sqrt{-g}\left[f(\lambda, \phi) + (R - \lambda)\frac{\partial f}{\partial \lambda} + \left(R^2_{GB} - \phi\right)\frac{\partial f}{\partial \phi} \right]$$


f(R, G, Q-4P)
∂f/∂R > 0
f(R, R²_GB) < (1)
S = ∫ d⁴x √-g [f(λ, φ) + (R-λ) ∂f/∂λ + (R²_GB - φ) ∂f/∂φ


∂f/∂R > 0
φ = R²_GB


f(R, GQ−4P)
∂f/∂R > 0
f(R, R²_GB)
S = ∫d⁴x √−g [f(λ, φ) + (R − λ) ∂f/∂λ + (R²_GB − φ) ∂f/∂φ]
φ = R²_GB
λ = R

Degrees of freedom

- CDDETT action is equivalent to

$$S = M_P^2 \int d^4x \sqrt{-g}\,[(\gamma + \chi)\,R - U + \xi\, R_{GB}^2]$$

if $b = \frac{a_1}{a_3} - 1$

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if $b = \frac{a_1}{a_3} - 1$, $a_2 = -4a_3$, $\zeta = \theta\mu^{4n+2}/a_3^n$ then

$$\chi = \frac{2bn\zeta\lambda}{\phi^{n+1}} \quad U = \frac{\zeta}{\phi^n}\left[bn\frac{\lambda^2}{\phi} + n + 1\right] \quad \xi = \frac{\zeta\,n}{\phi^{n+1}}$$

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- The eqns are 2^{nd} order: no spin-2 ghost! [Nunez, Solganik PLB 608 (2005)][Chiba JCAP 0503 (2005)][Navarro, van Acoleyen gr-qc/0511045]

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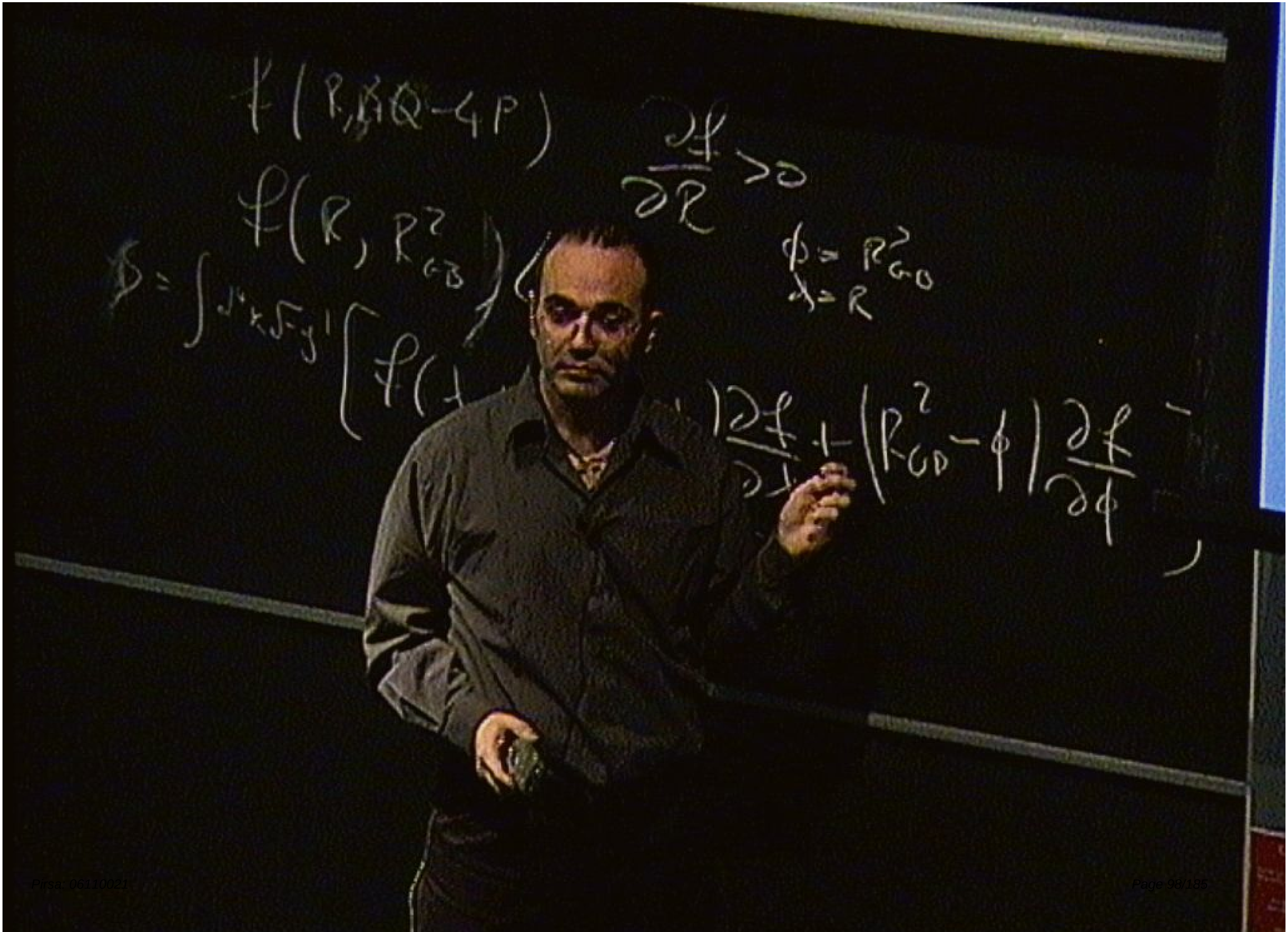
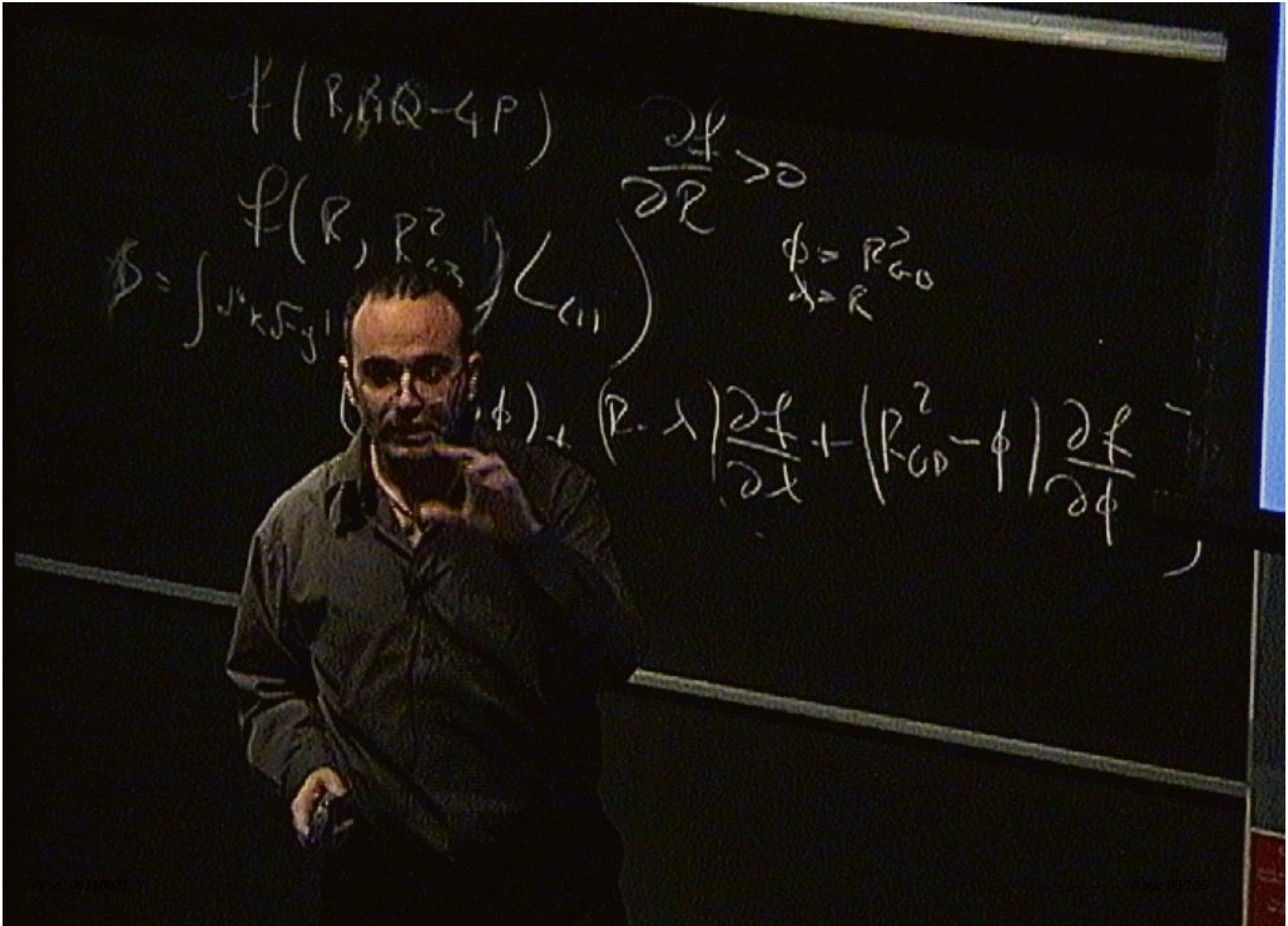
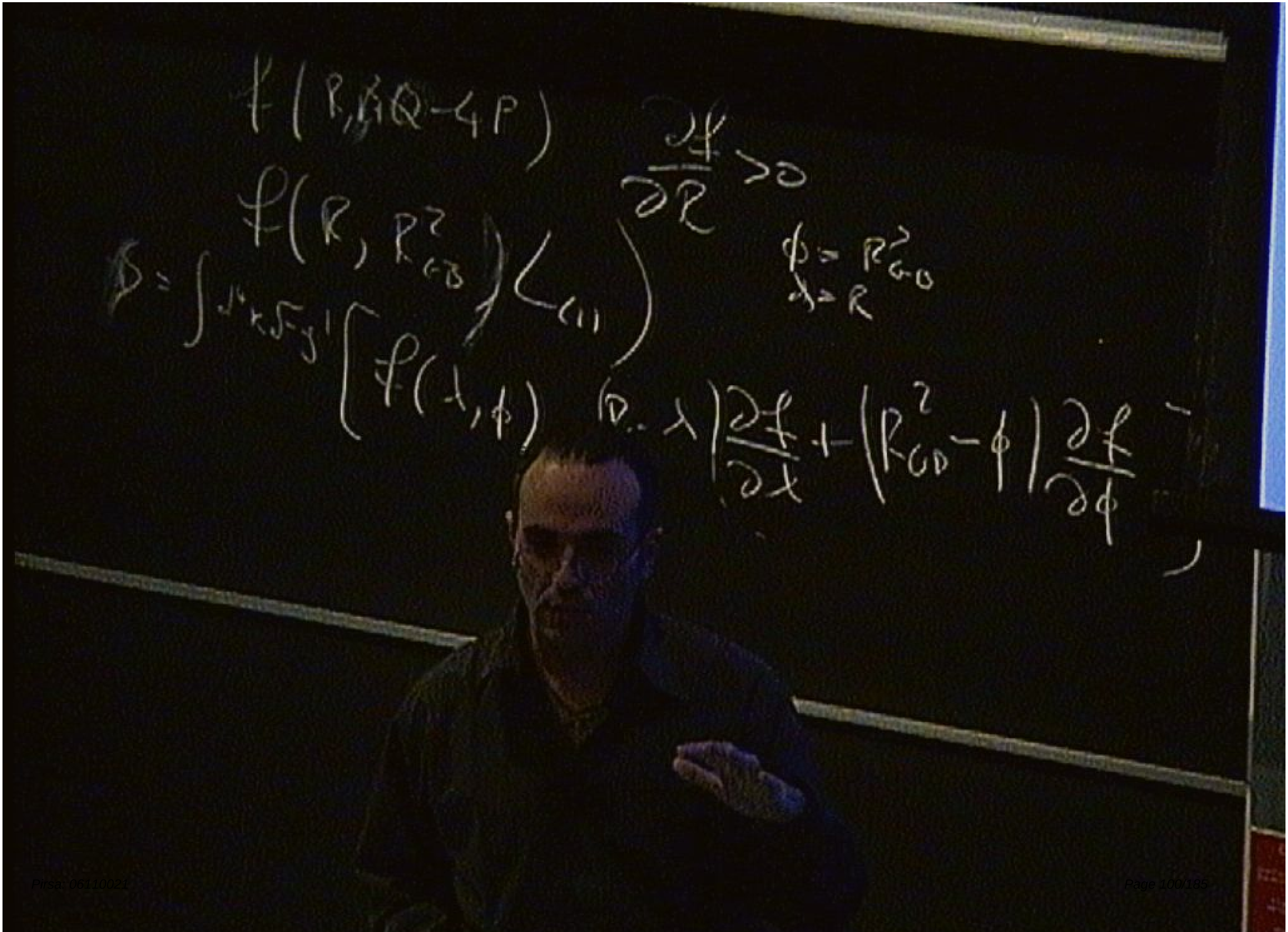
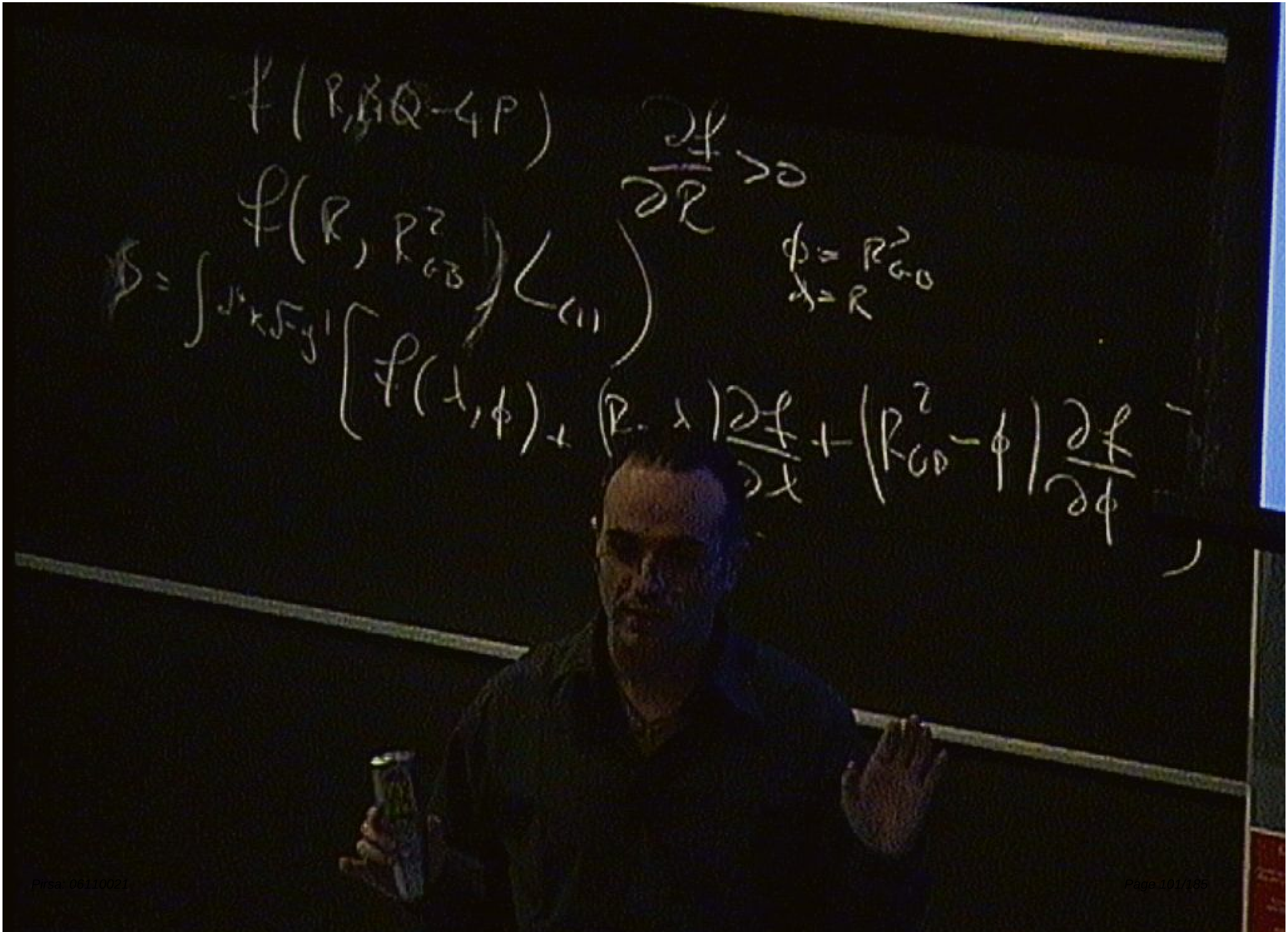
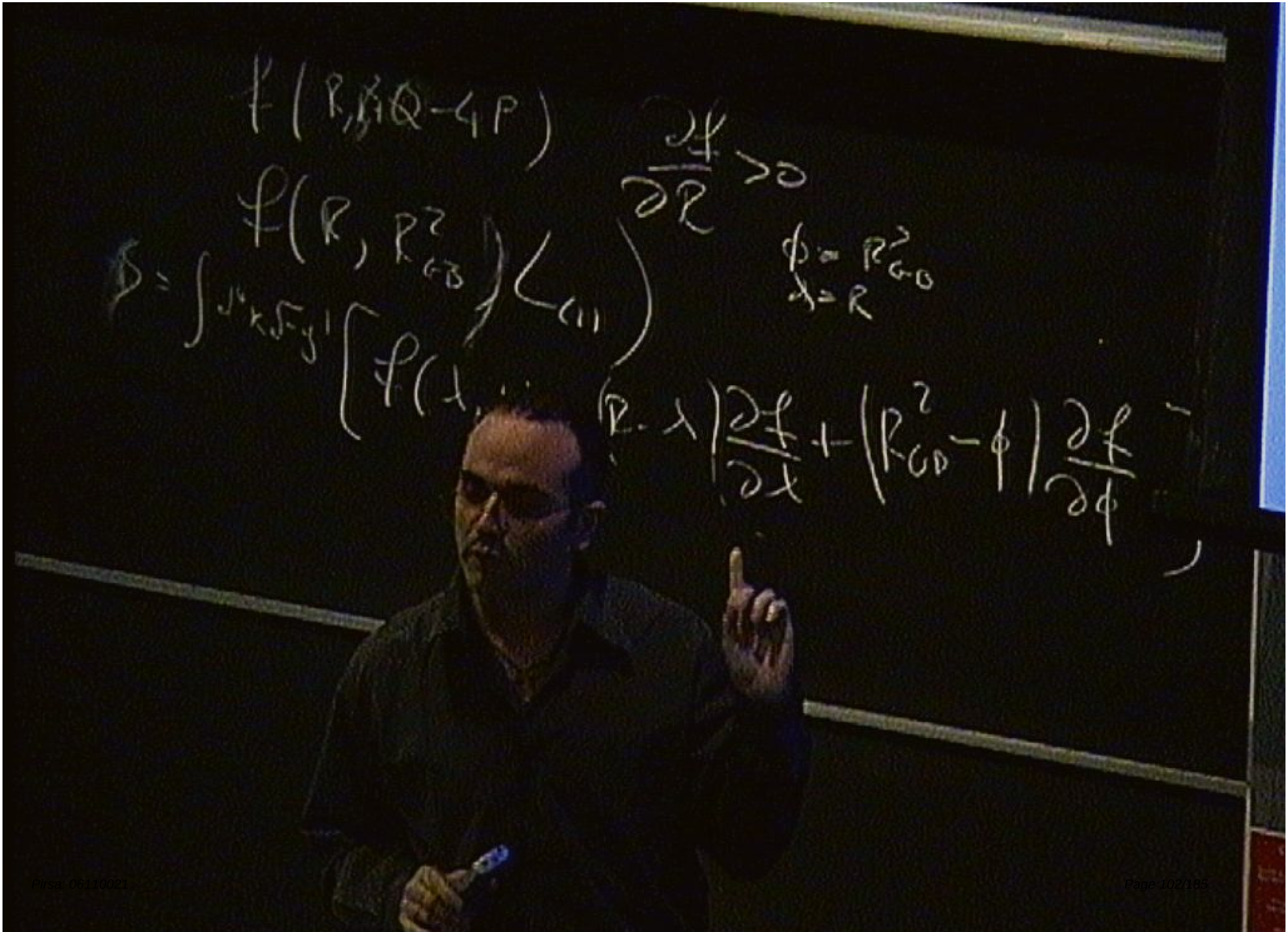


f(R, R²_GB)
∂f/∂R > 0
φ = R²_GB
χ = R




f(R, R^2_{GB})
∂f/∂R > 0
φ = R^2_{GB}
λ = R


$f(R, G, Q-4P)$
$\frac{\partial f}{\partial R} > 0$
$f(R, R^2_{GB})$
$\phi = R^2_{GB}$
$\lambda = R$
$S = \int d^4x \sqrt{-g} \left[f(\lambda, \phi) + (R - \lambda) \frac{\partial f}{\partial \lambda} + (R^2_{GB} - \phi) \frac{\partial f}{\partial \phi} \right]$



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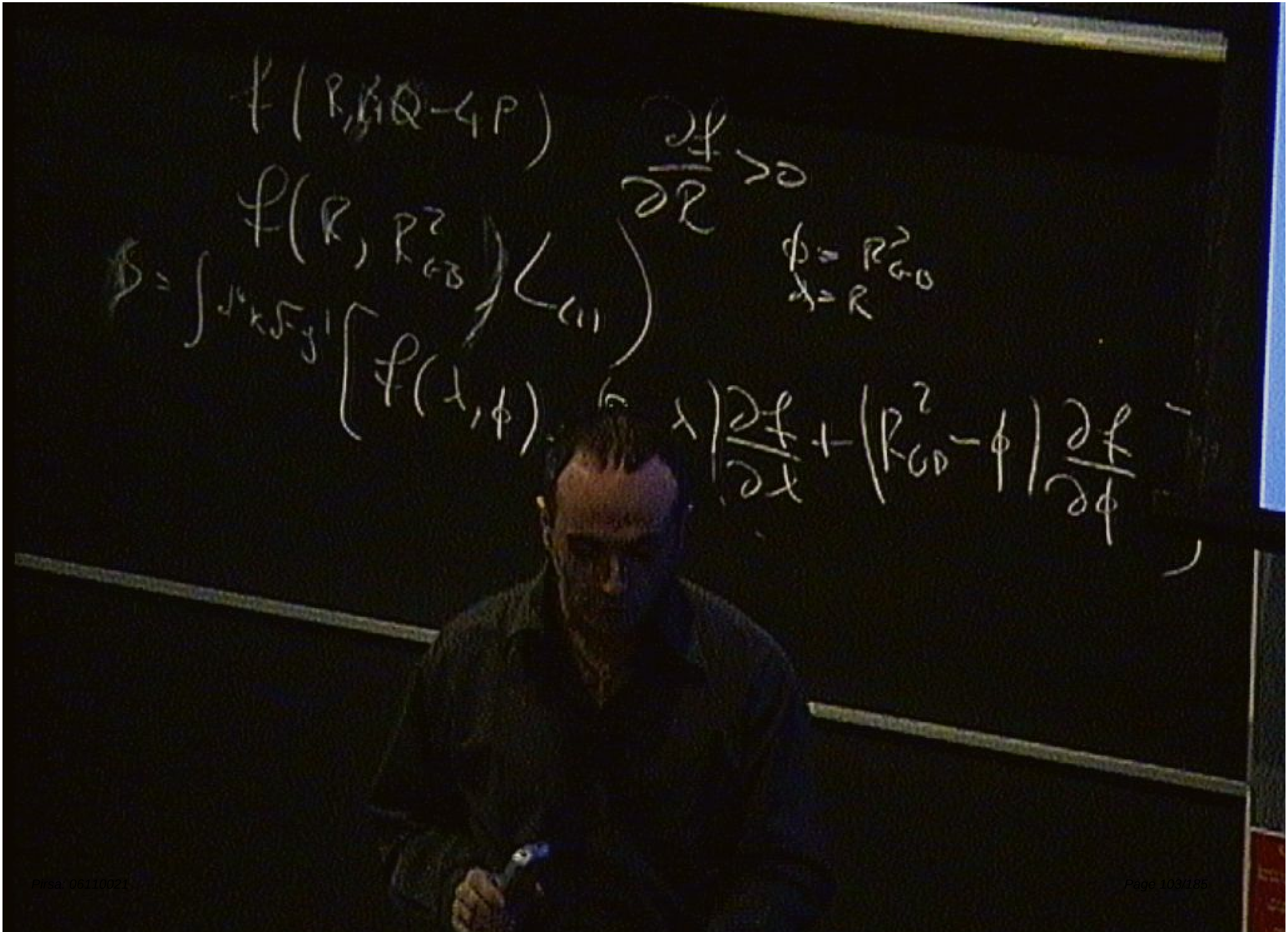
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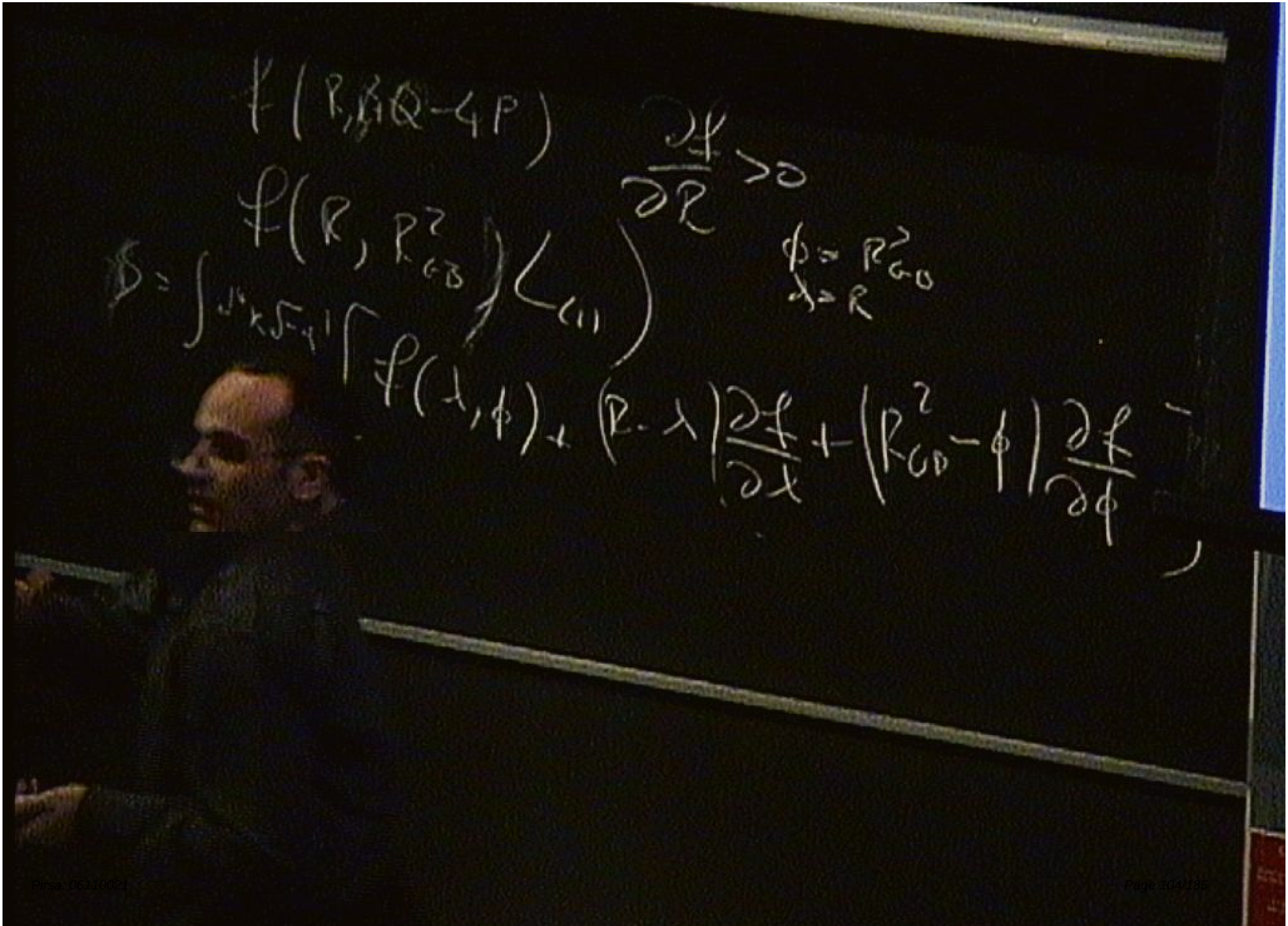
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$f(R, R_{\mu\nu}Q-4P)$ $\quad \frac{\partial f}{\partial R} > 0$

$f(R, R^2_{GB})$ $\quad \phi = R^2_{GB}$, $\lambda = R$

$S = \int d^4x \sqrt{-g} \left[f(\lambda, \phi) + (R-\lambda)\frac{\partial f}{\partial \lambda} + (R^2_{GB} - \phi)\frac{\partial f}{\partial \phi} \right] \mathcal{L}_{(1)}$

$$f(R, R_{ab}, Q - 4P)$$

$$\frac{\partial f}{\partial R} > 0$$

$$f(R, R^2_{GB}) \quad \mathcal{L}_{(1)}$$

$$\phi = R^2_{GB}$$
$$\lambda = R$$

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$$f(R, R^2_{GB}) \quad \mathcal{L}_{(1)}$$

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$$f(R, R_{\mu\nu}Q - 4P) \qquad \frac{\partial f}{\partial R} > 0$$

$$f(R, R^2_{GB}) \qquad \phi = R^2_{GD}, \quad \lambda = R$$

$$S = \int d^4x \sqrt{-g} \left[f(\lambda, \phi) + (R - \lambda) \frac{\partial f}{\partial \lambda} + (R^2_{GD} - \phi) \frac{\partial f}{\partial \phi} \right]$$

$f(R, G, Q-4P)$

$\frac{\partial f}{\partial R} > 0$

$f(R, R^2_{GB}) \quad \mathcal{L}_{(11)}$

$\phi = R^2_{GB}$

$\lambda = R$

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$$f(R, R_{\mu\nu}Q - 4P) \qquad \frac{\partial f}{\partial R} > 0$$

$$f(R, R^2_{GB}) \qquad \phi = R^2_{GB}, \quad \lambda = R$$

$$S = \int d^4x \sqrt{-g}\left[f(R, R^2_{GB}) \mathcal{L}_{(m)} \right]$$

$$\left[f(\lambda, \phi) + (R - \lambda)\frac{\partial f}{\partial \lambda} + (R^2_{GD} - \phi)\frac{\partial f}{\partial \phi} \right]$$

$$f(R, R_{ab}Q - GP)$$

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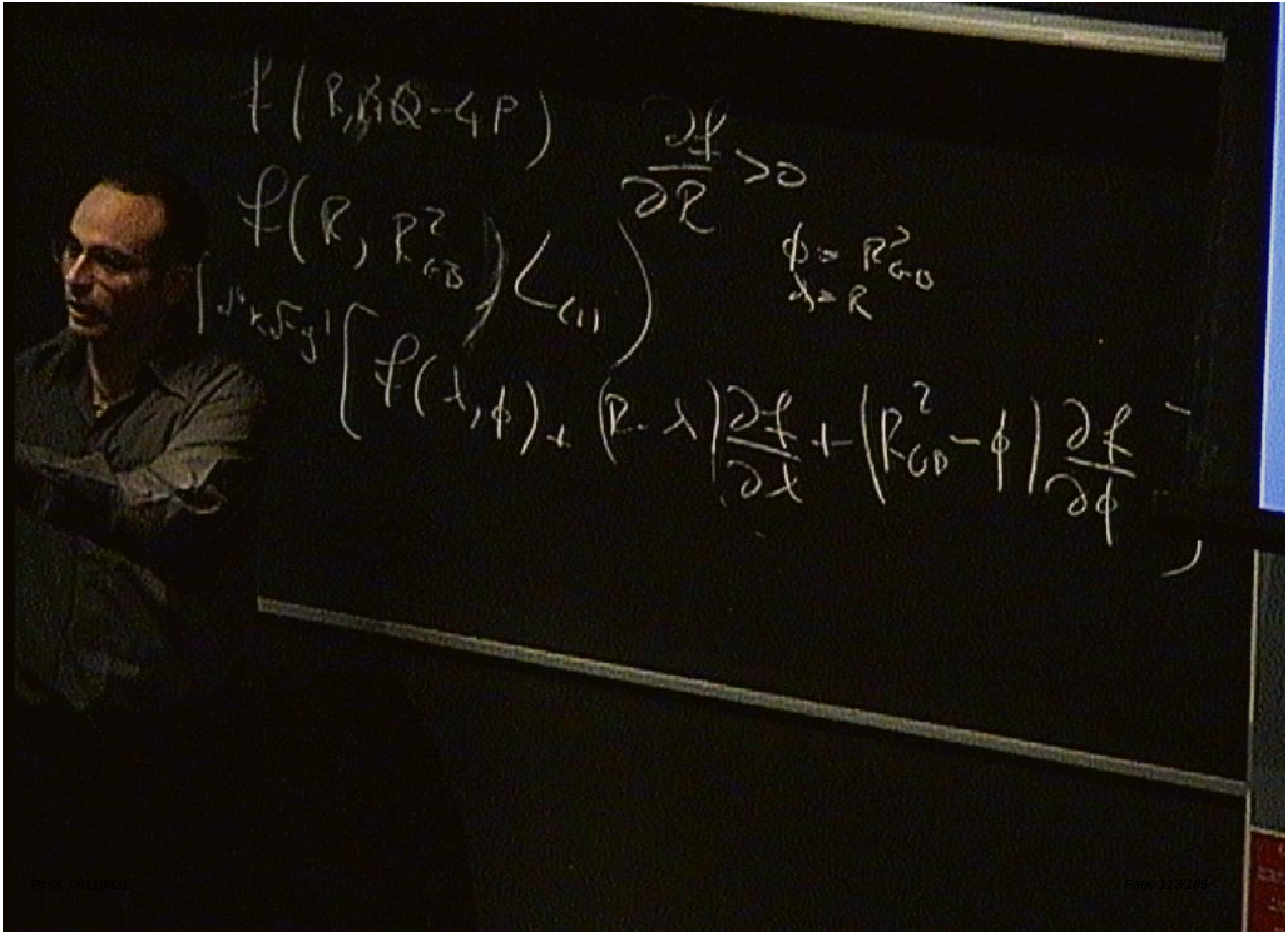
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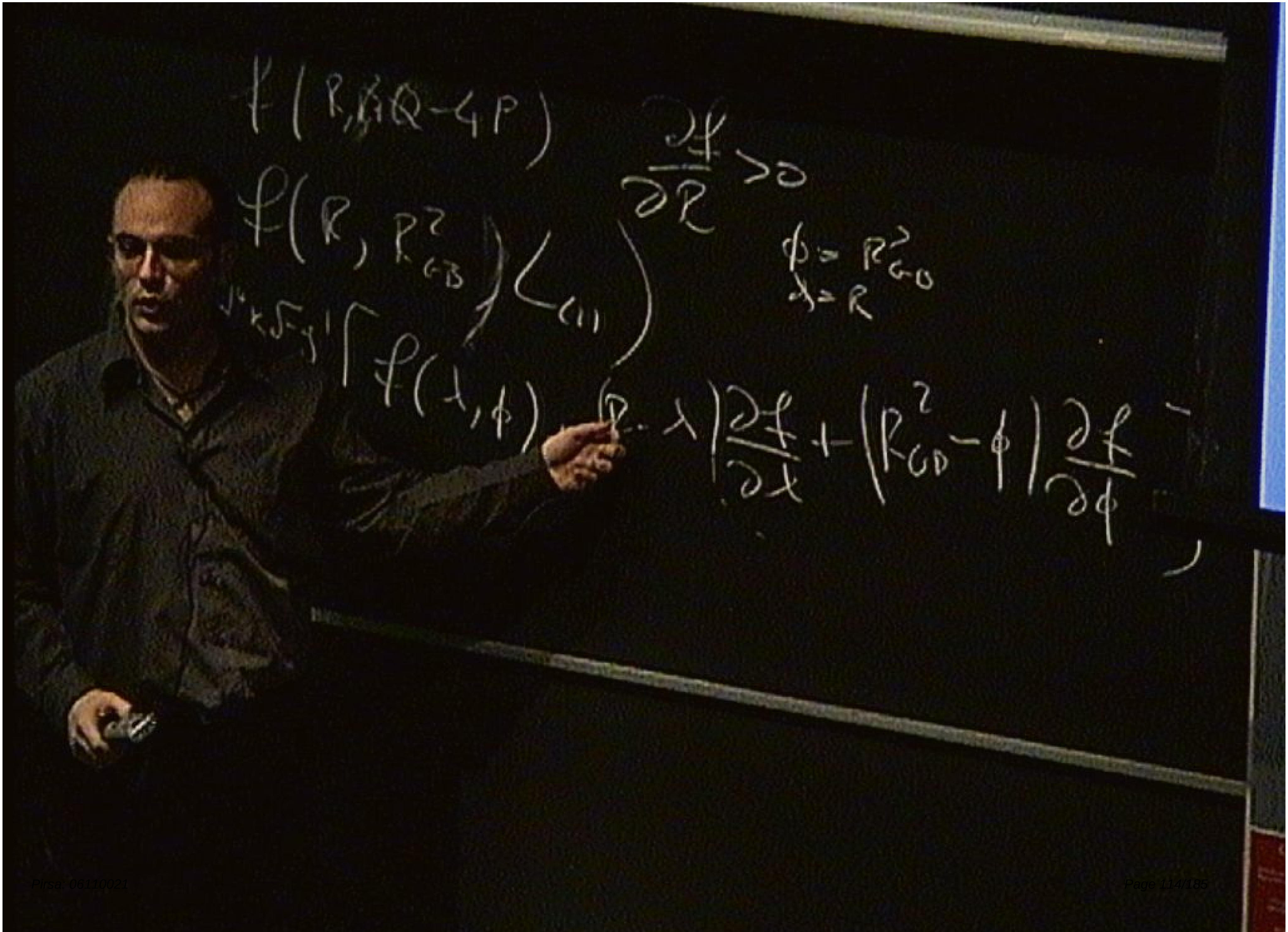

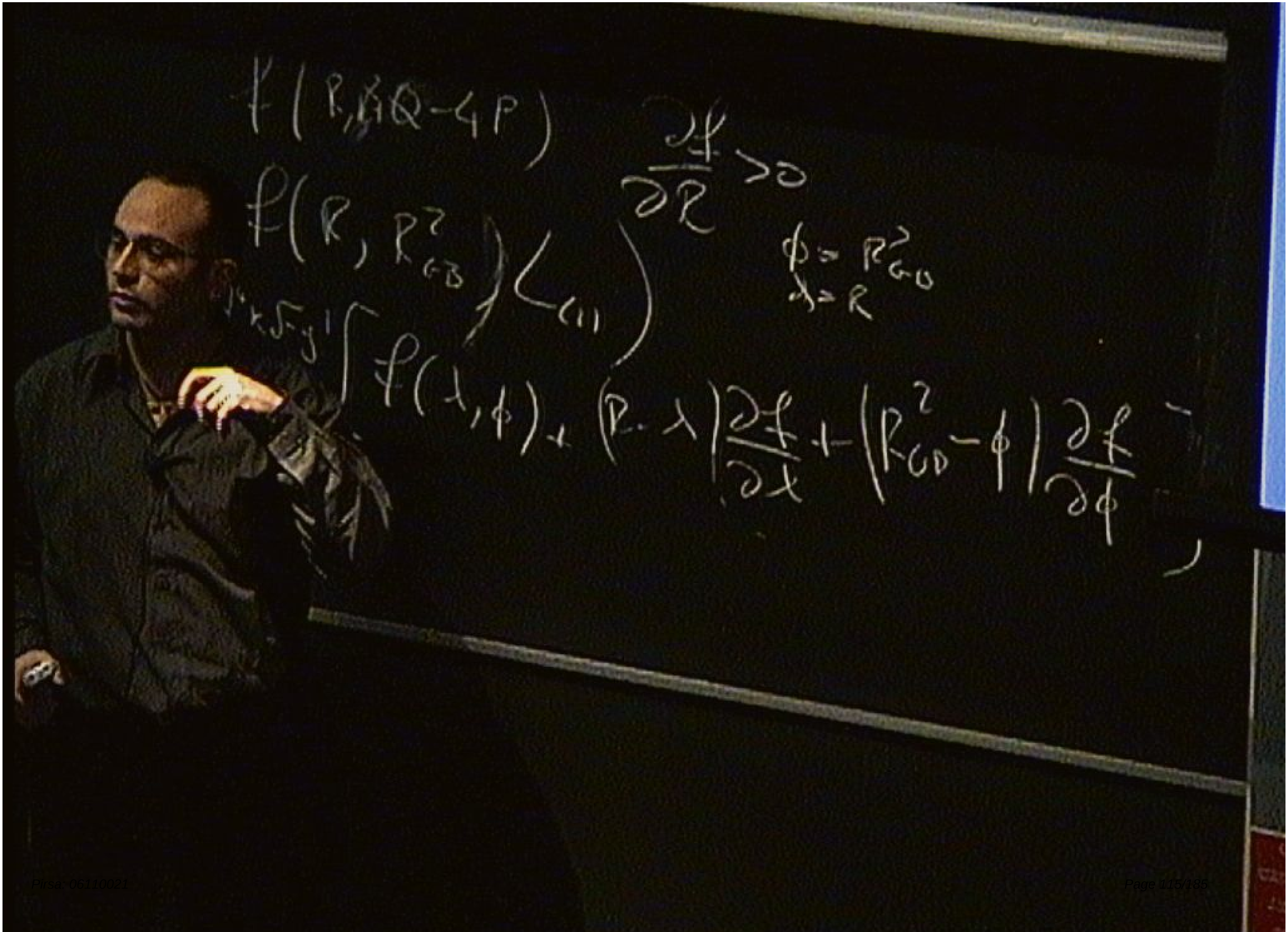

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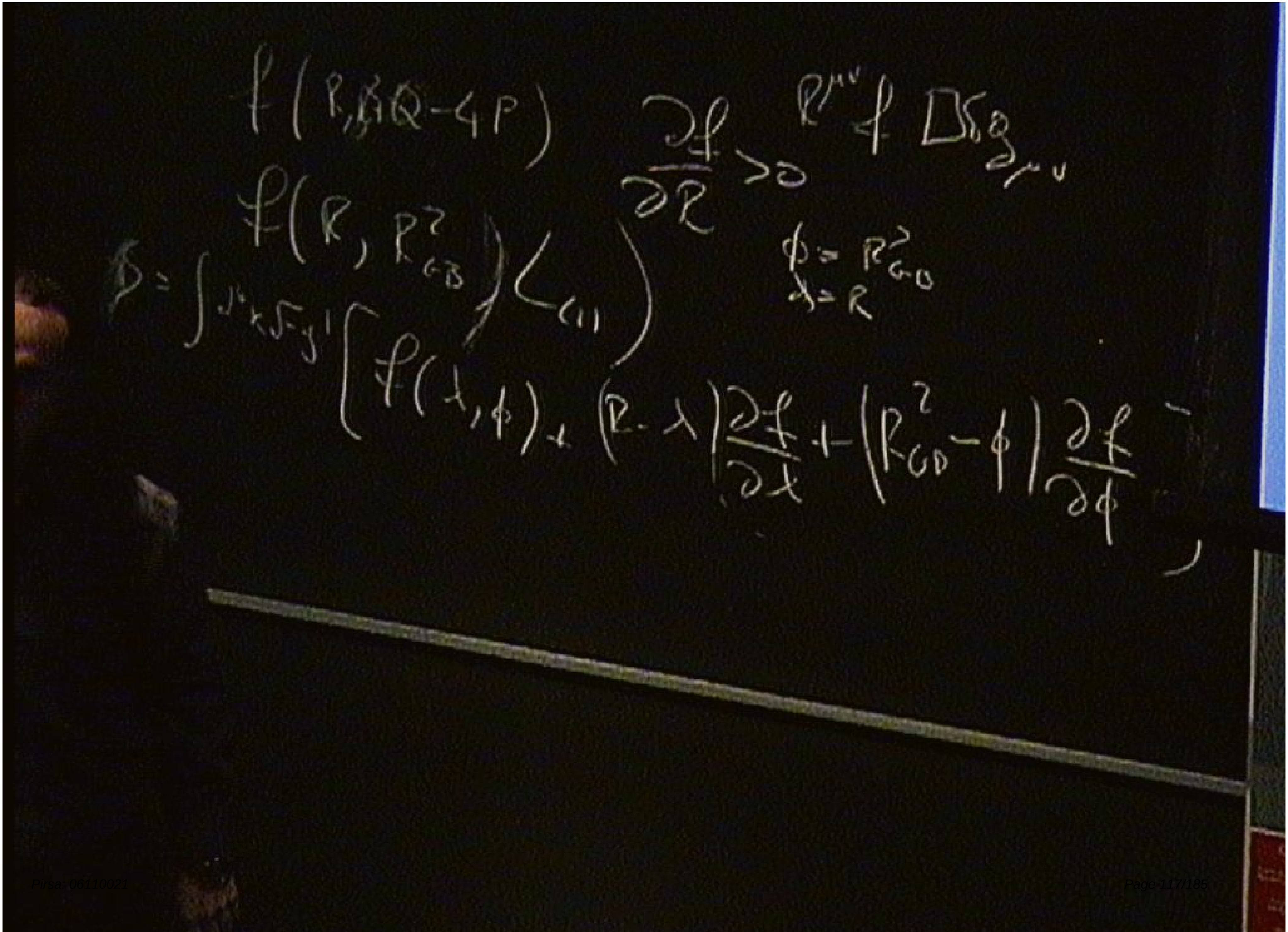

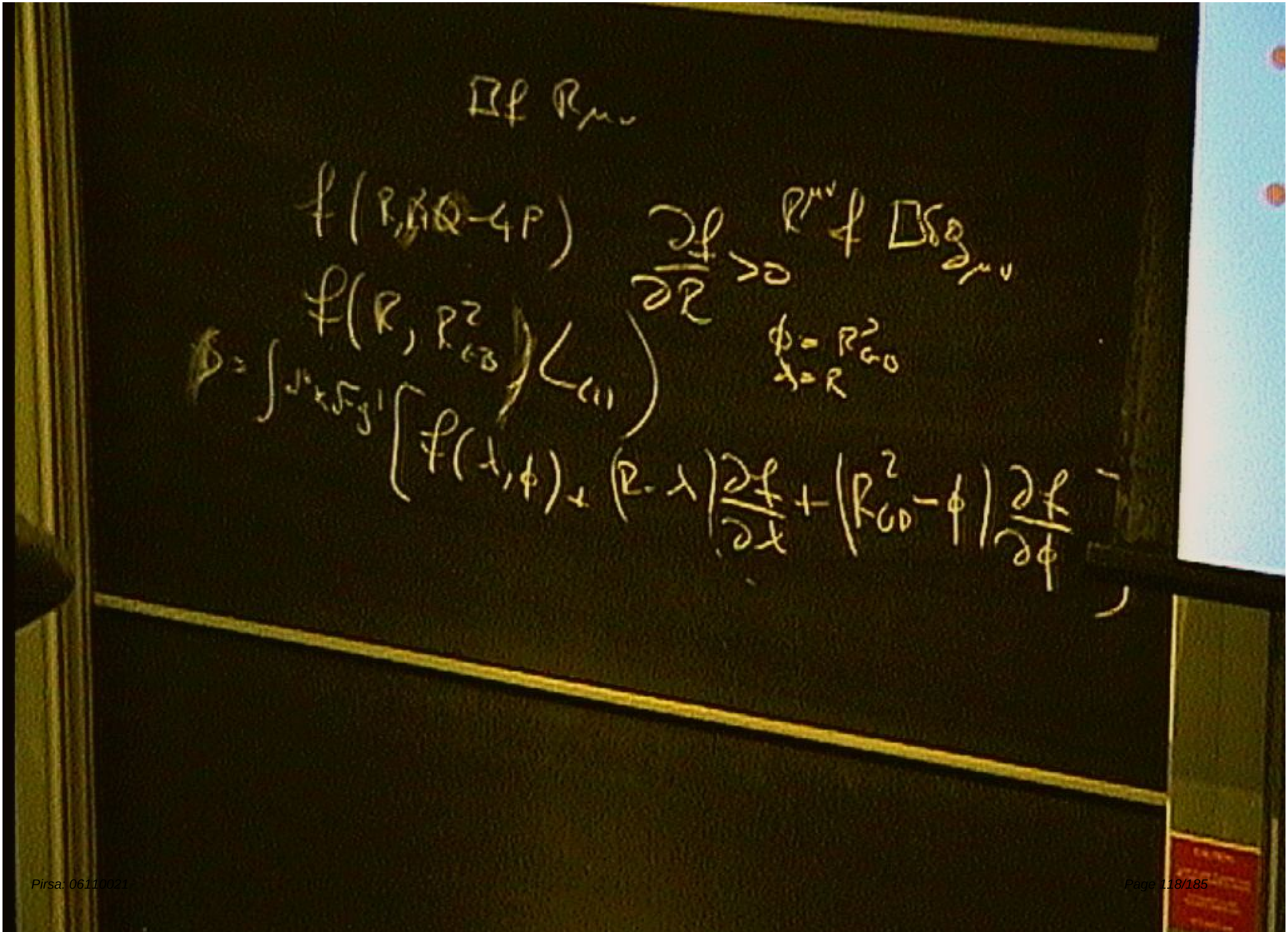
□f R_{μν}
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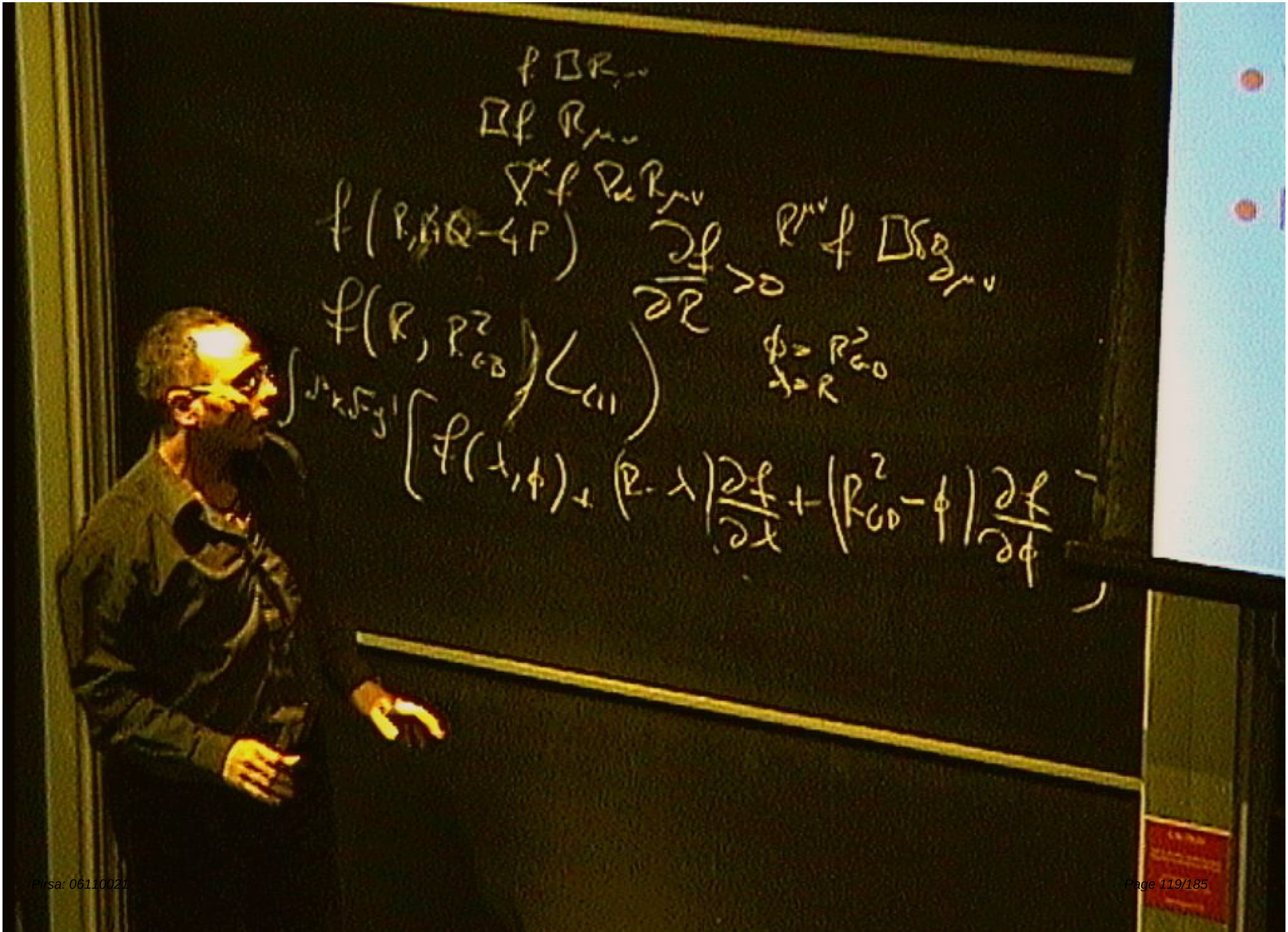
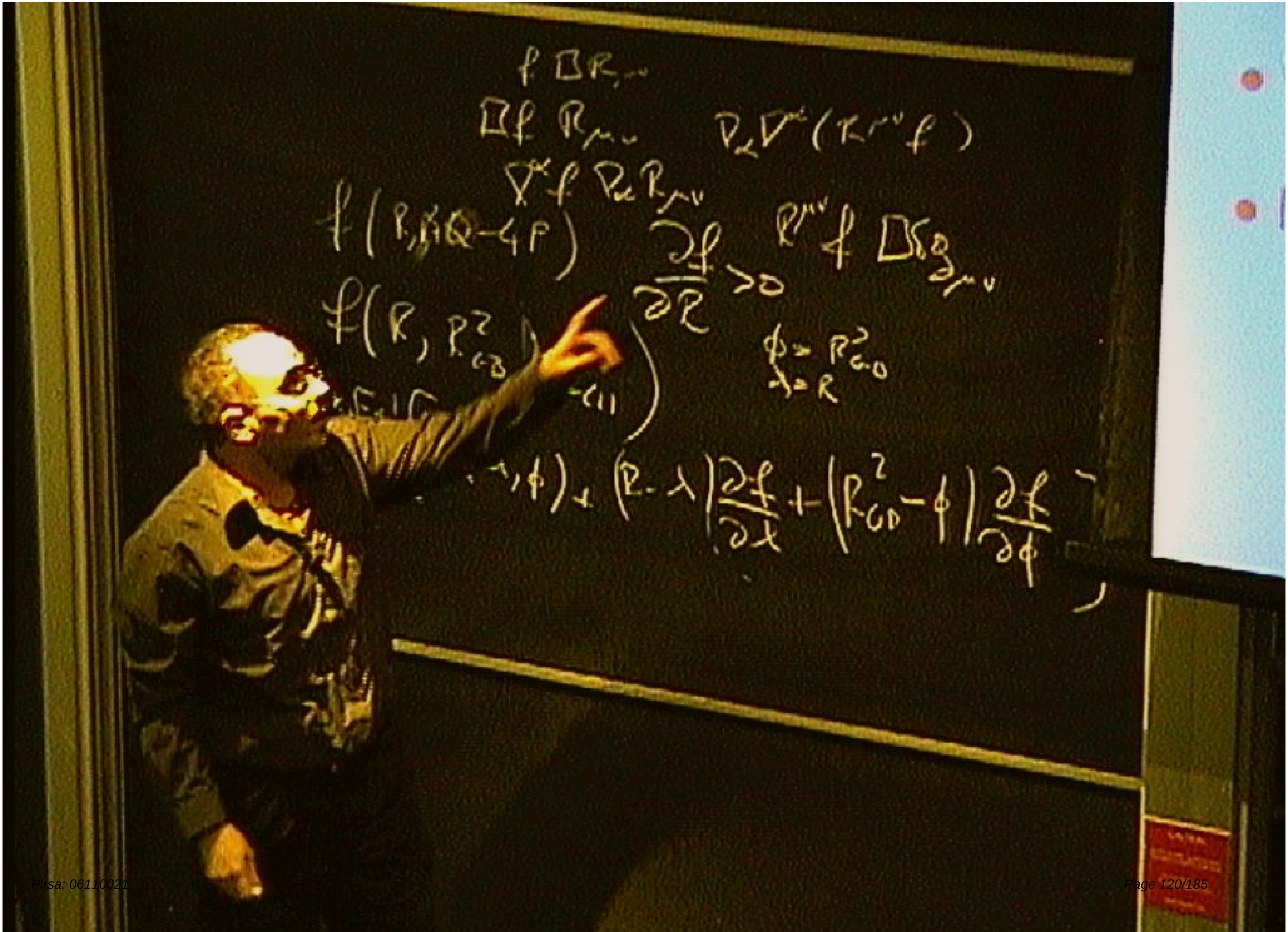
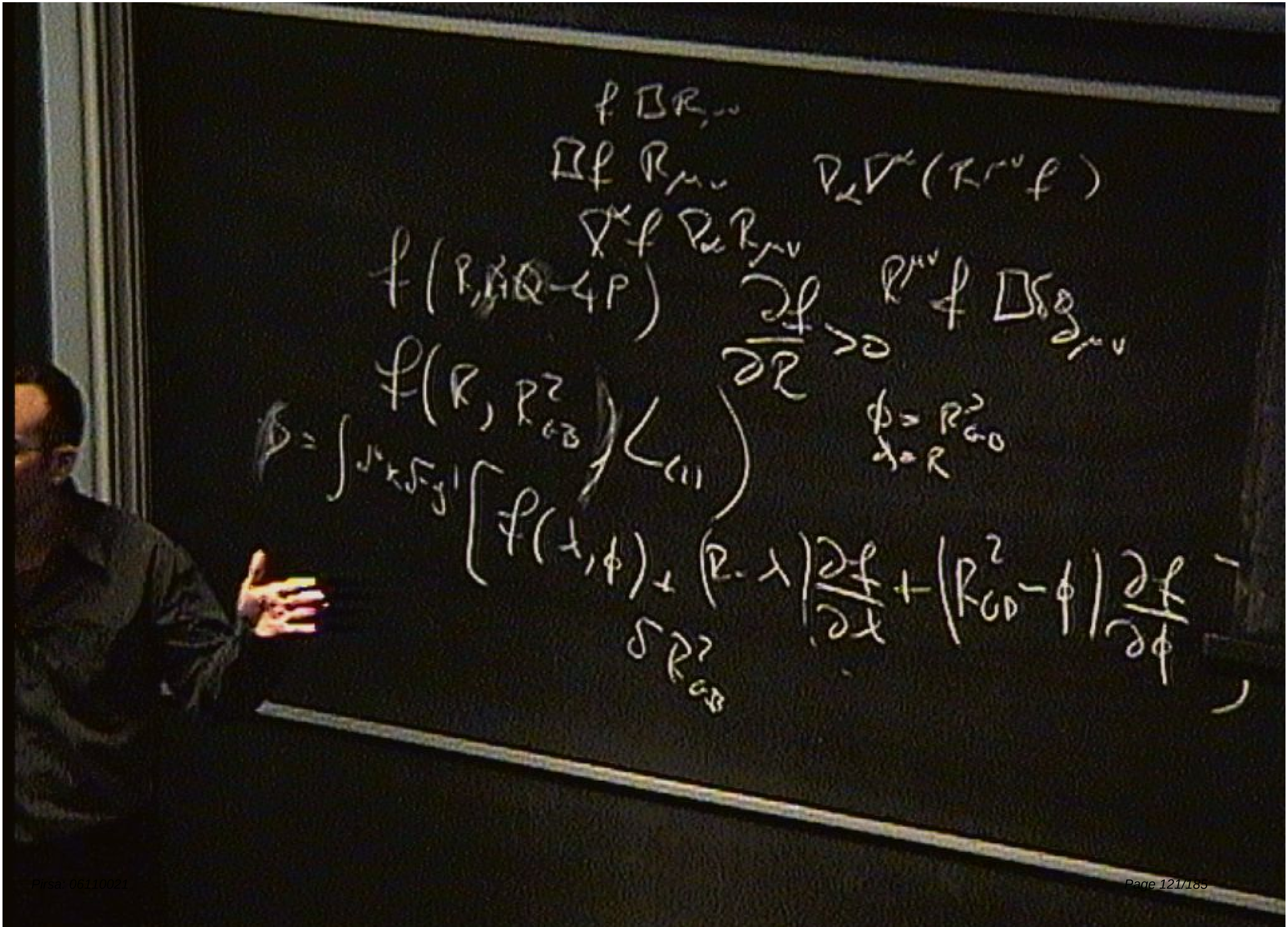
φ = R²_{GB}
λ = R
f(λ, φ) + (R − λ) ∂f/∂λ + (R²_{GB} − φ) ∂f/∂φ






f □R_μν
□f R_μν   ∇_α∇^α(R^μν f)
∇^α f ∇_α R_μν
f(R, RQ−4P)   ∂f/∂R > 0   R^μν f □δg_μν
f(R, R²_GB)
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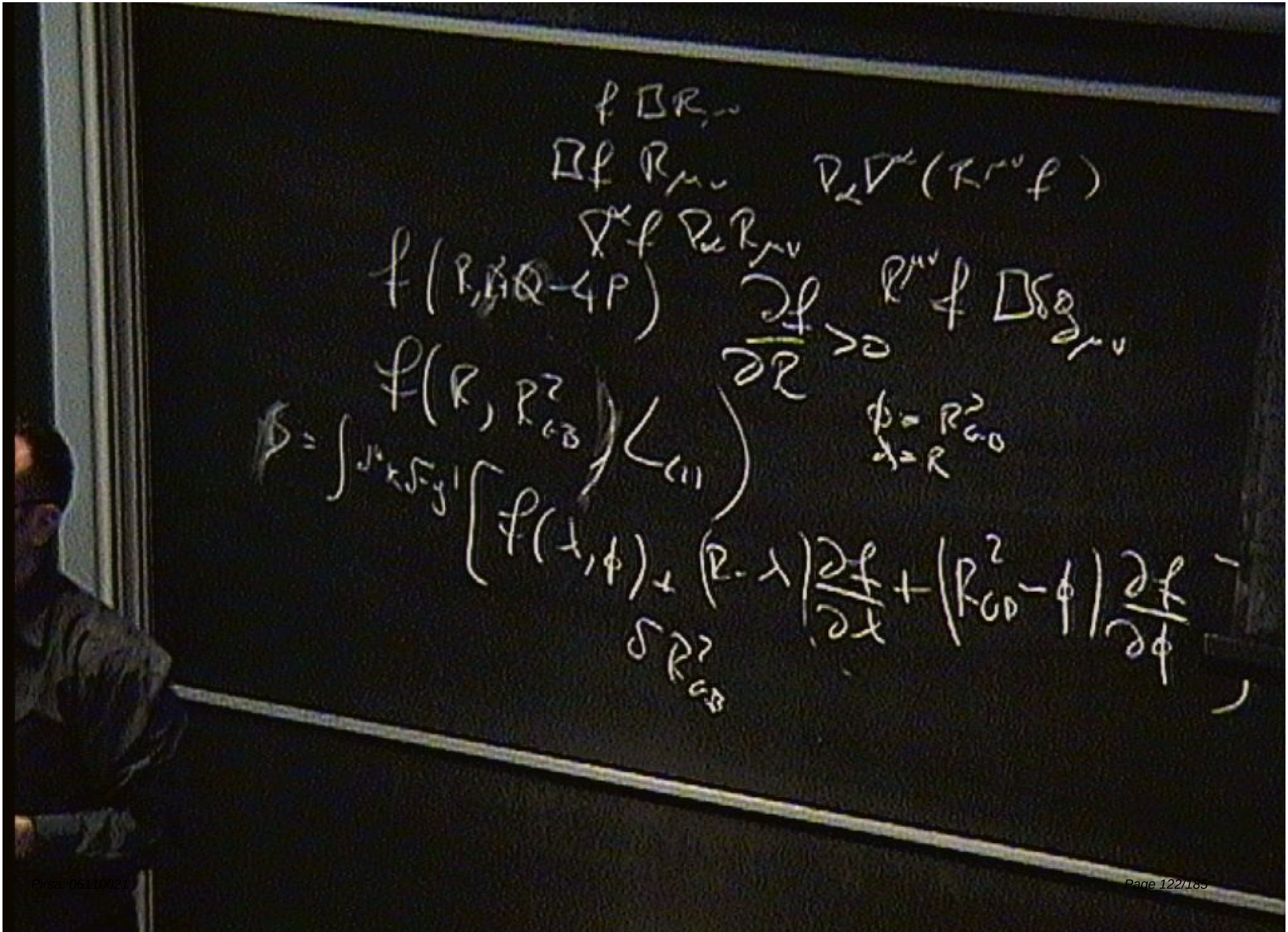
S = ∫d⁴x√−g [f(λ, φ) + (R − λ) ∂f/∂λ + (R²_GB − φ) ∂f/∂φ]
δR²_GB


f □R_μν
□f R_μν
∇_α∇^α(R^μν f)
∇^α f ∇_α R_μν
f(R, RQ-4P)
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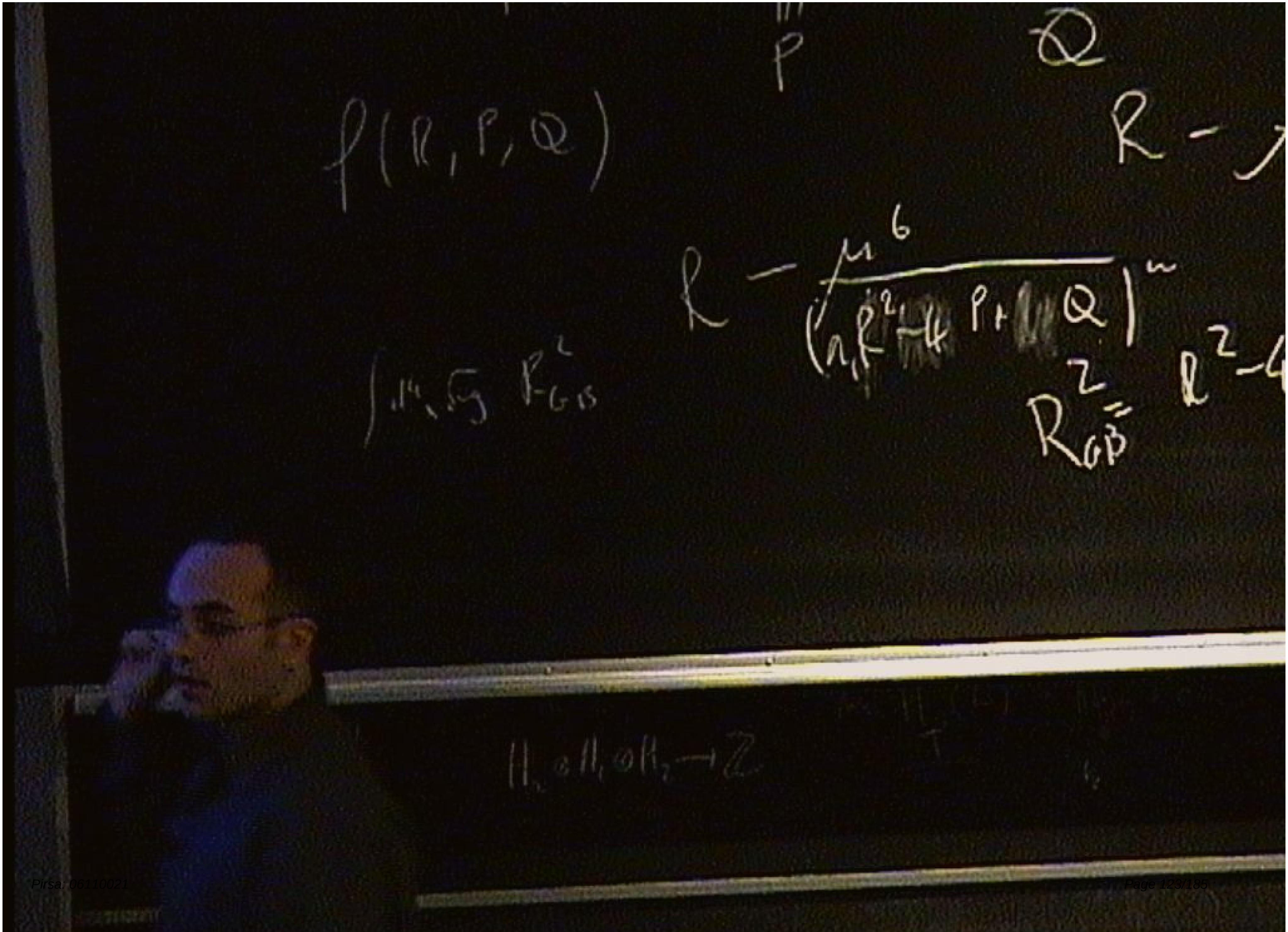
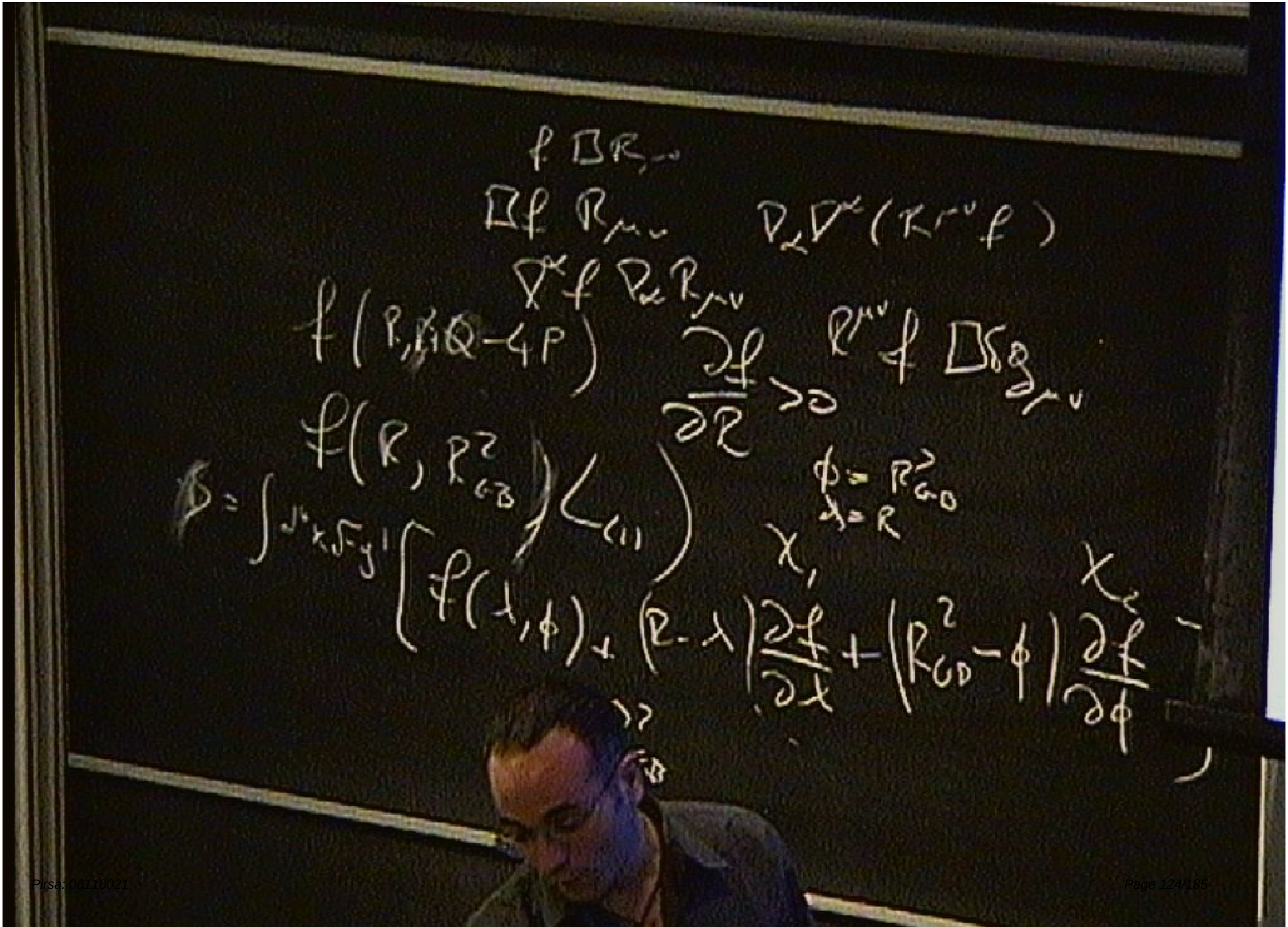

f(R, P, Q)


f □R_μν
□f R_μν
∇_α∇^α(R^μν f)
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Cosmological perturbations

- Background is FRW

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- Cosmology needs FRW, inflation

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- Cosmology needs FRW, inflation
- Expand the action about general flat FRW!

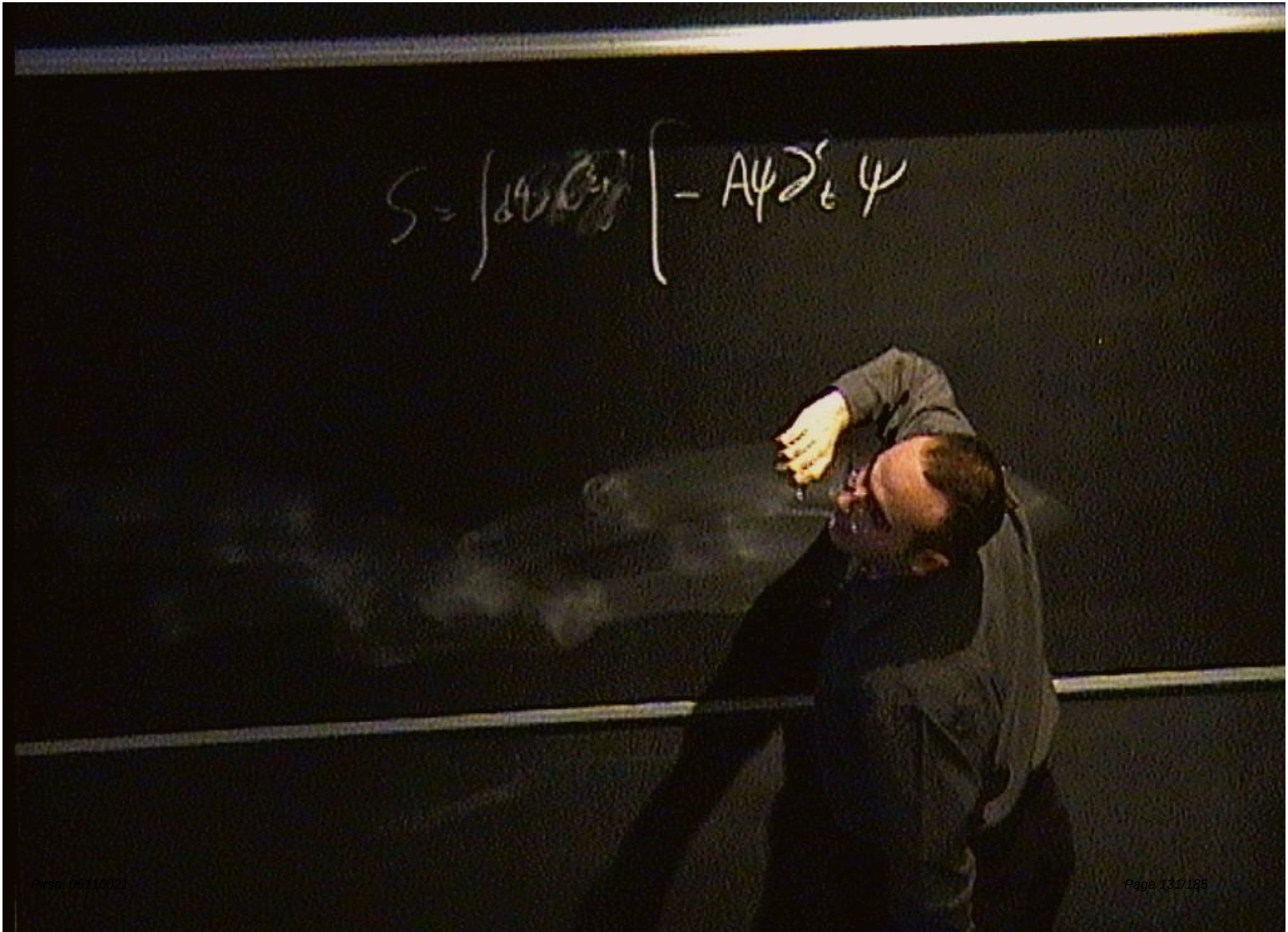
Propagating modes

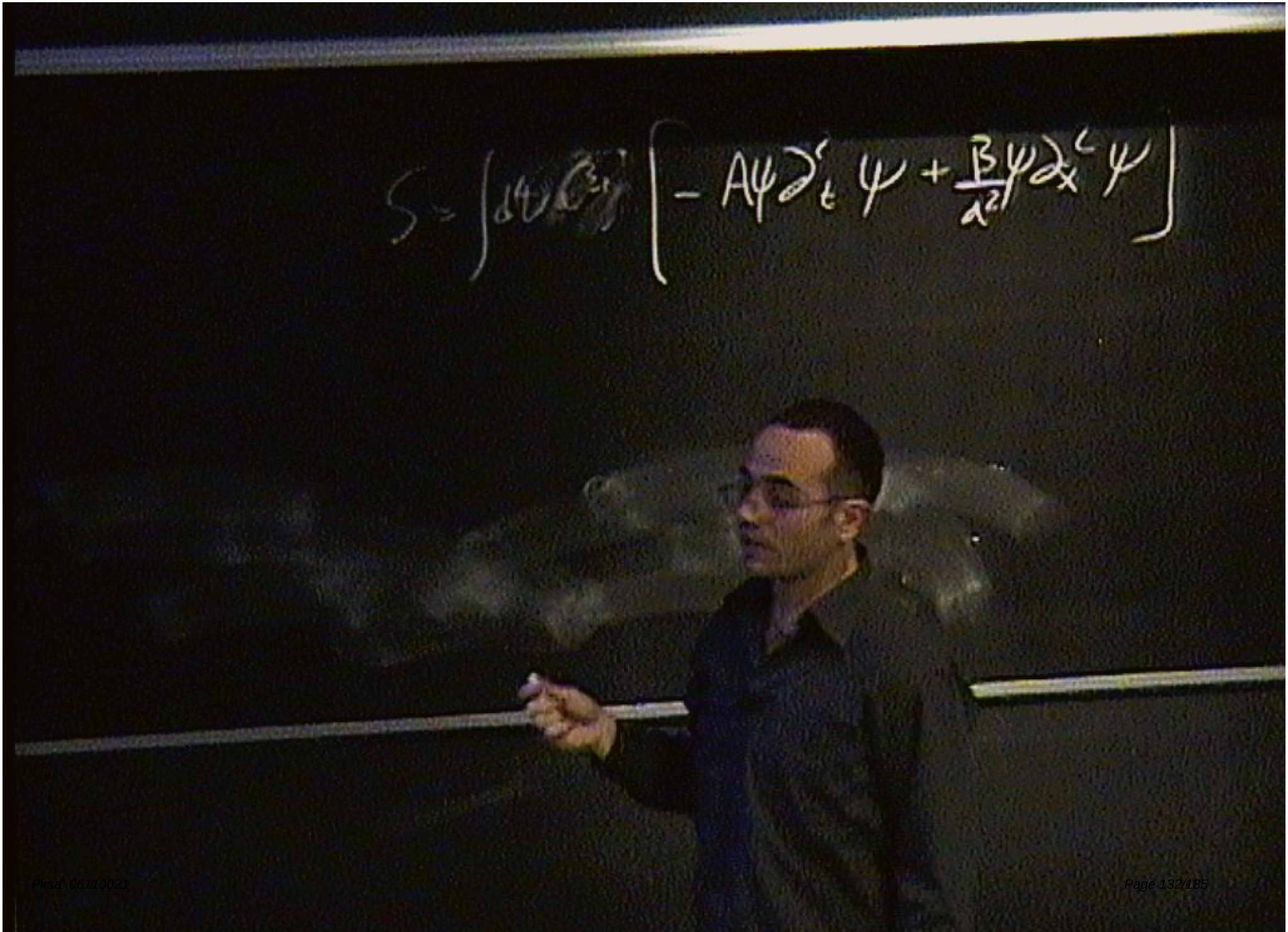
- Study of the scalar, vector and tensor perturbations

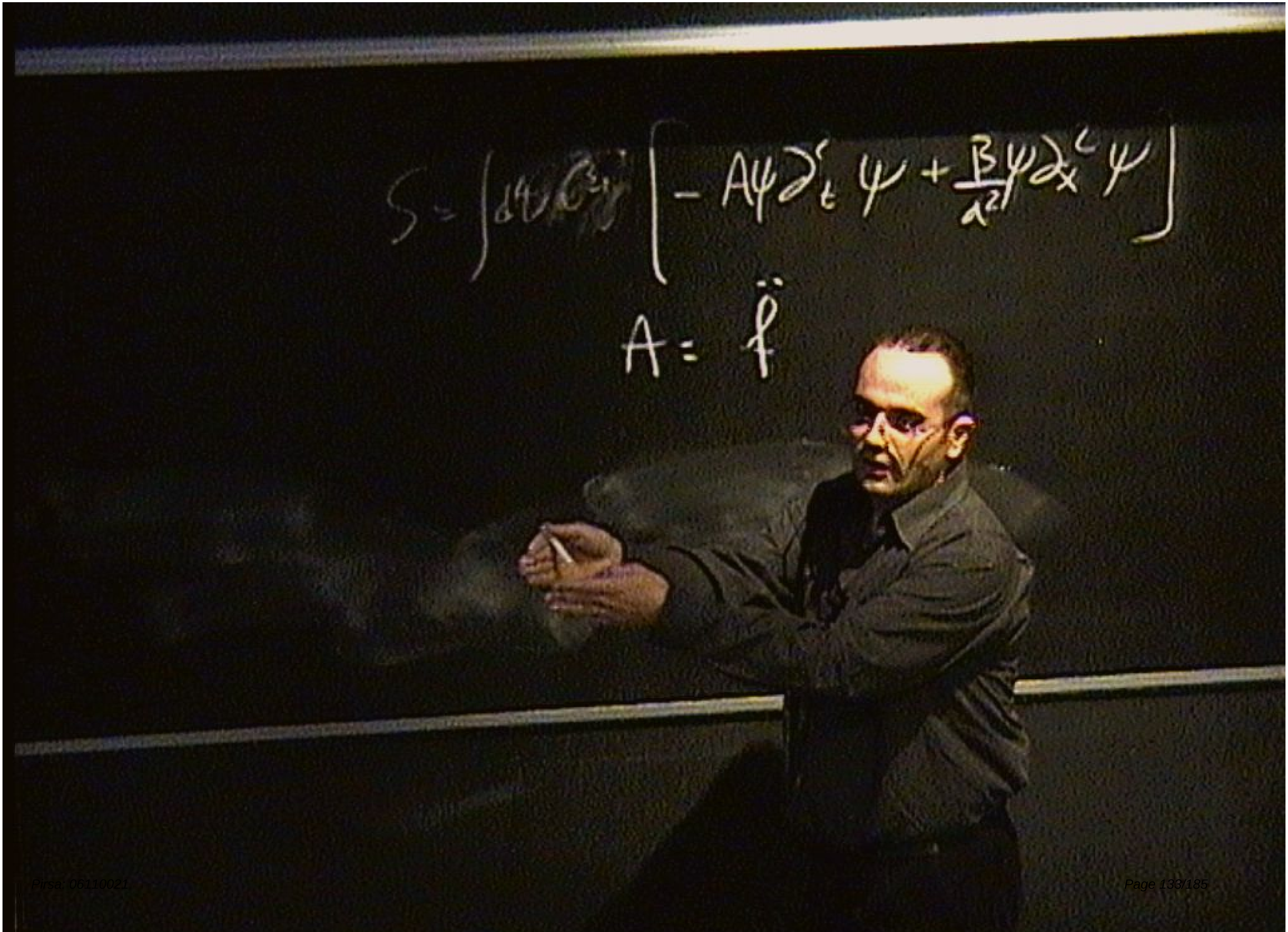
Propagating modes

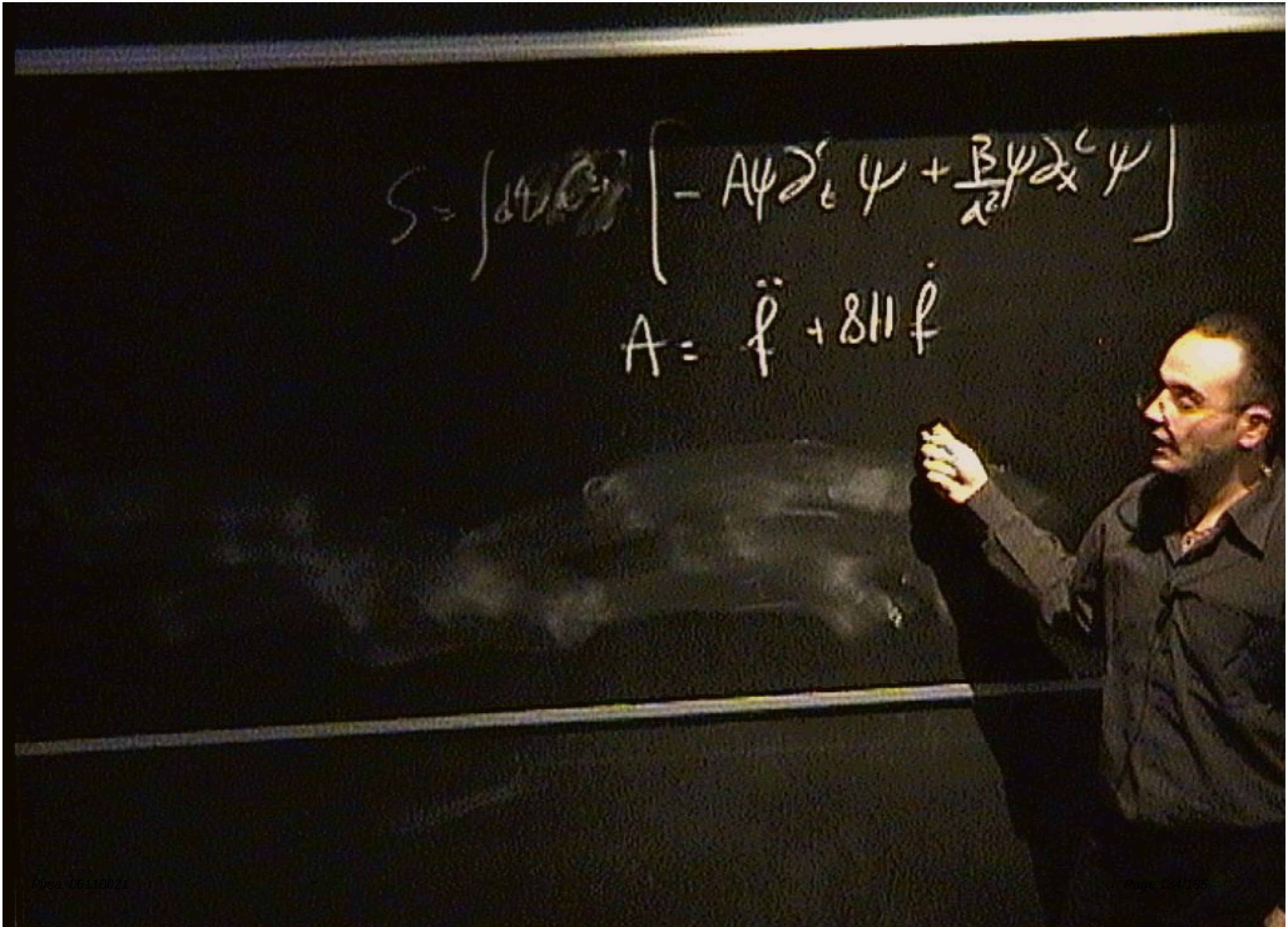
- Study of the scalar, vector and tensor perturbations
- Expansion of the action at the second order about FRW
- General expression

$$S = \int dt\, a^3 \left[-A(t)\, \Psi \ddot{\Psi} + \frac{B(t)}{a^2} \Psi \nabla^2 \Psi \right]$$




$S = \int dt \ldots \left[- A\psi \partial_t^2 \psi + \frac{B}{a^2} \psi \partial_x^4 \psi \right]$

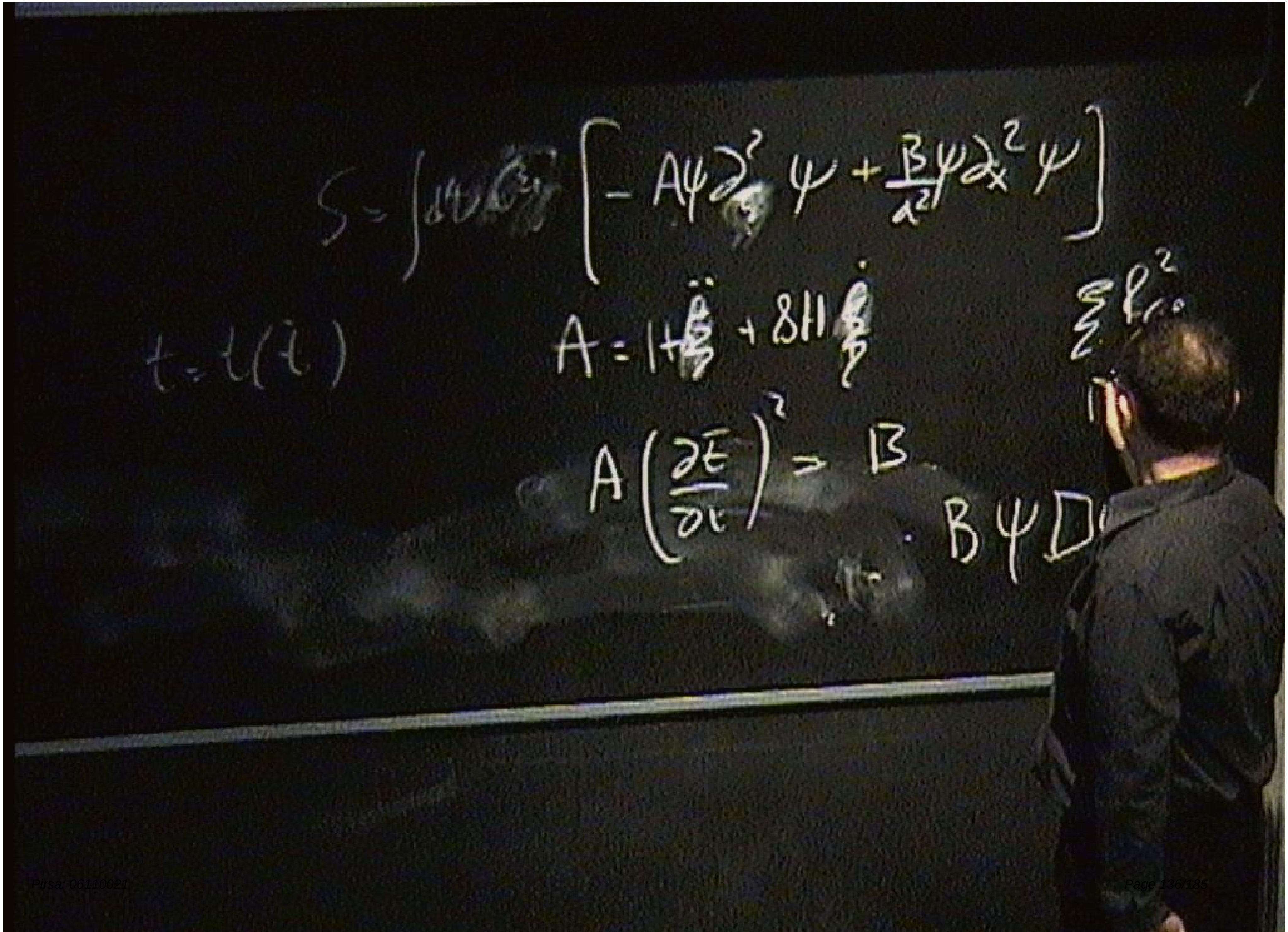


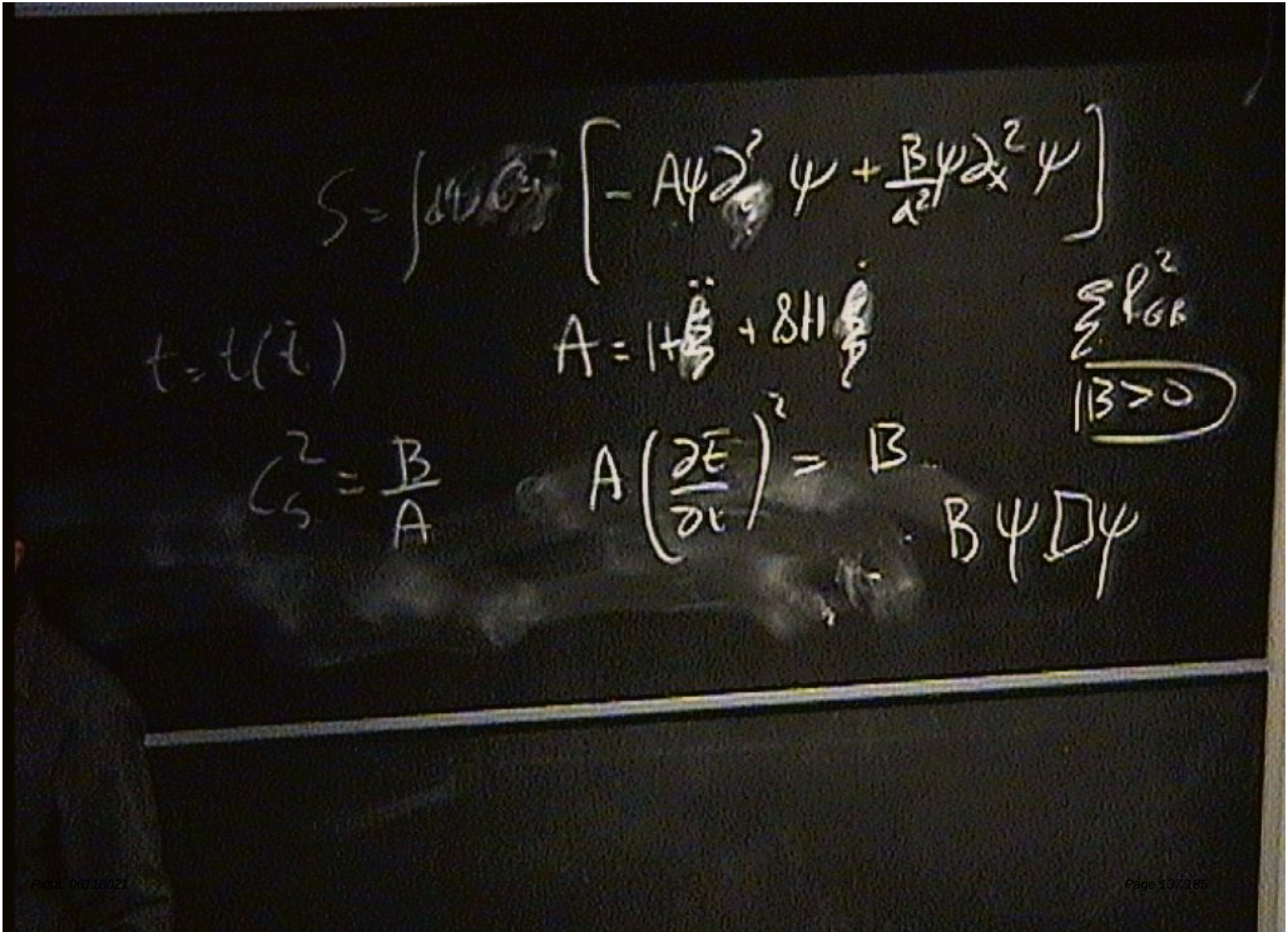

S = ∫ d⁴x [− Aψ∂_t² ψ + B/a² ψ∂_x⁴ ψ]
A = f̈ + 8H ḟ

$$S = \int d^4x \, [\text{illegible}] \left[-A\psi \partial_t^2 \psi + \frac{B}{a^2} \psi \partial_x^2 \psi \right]$$

$$A = 1 + \ddot{\xi} + 8H\dot{\xi}$$

$$\xi \rho_{GB}^2$$


t = t(t̃)
A = 1 + 8H
A(∂t/∂t̃)² = B
BψD


S = ∫ d^4x [− Aψ∂_t^2 ψ + B/λ^2 ψ∂_x^2 ψ]
t = t(t̃)
A = 1 + ξ̈ + 8H ξ̇
ξ ℓ_GR^2
B > 0
c_s^2 = B/A
A (∂E/∂t)^2 = B
B ψ □ ψ

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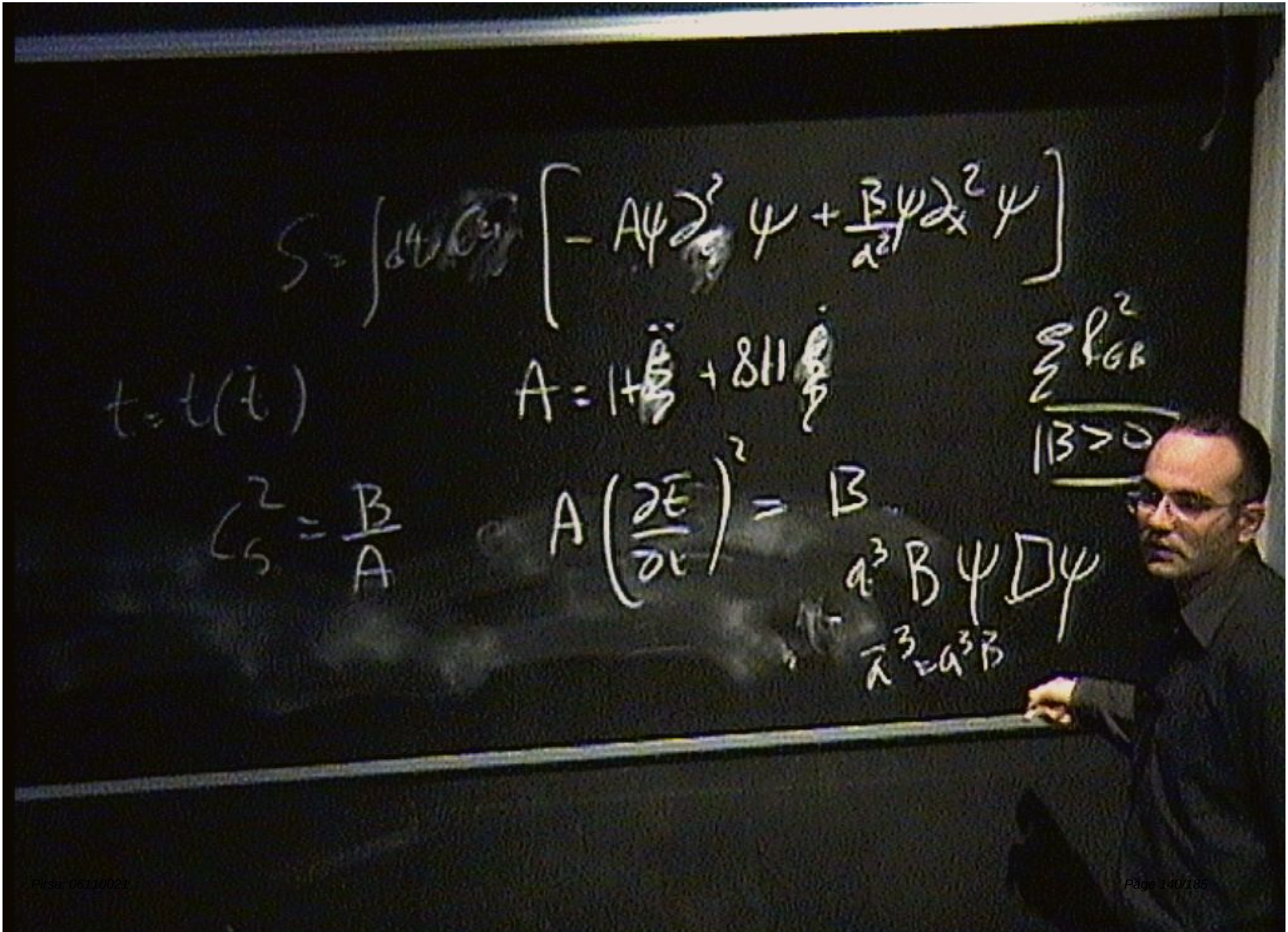
Ghosts and instabilities

- No-Ghosts and instabilities: $A(t), B(t) > 0$ so that $s > 0$


$S = \int d^4x \left[-A\psi \partial_t^2 \psi + \frac{B}{a^2}\psi \partial_x^2 \psi \right]$
$A = 1 + \ddot{\xi} + 8H\dot{\xi}$
$\xi \ell_{GB}^2$
$B > 0$
$c_s^2 = \frac{B}{A}$
$A\left(\frac{\partial E}{\partial t}\right)^2 = B$
$a^3 B \psi \Box \psi$
$\tilde{a}^3 = a^3 B$

Problems

- Ghosts: violation of unitarity, failure of energy conservation, wild particle production $A, B < 0$

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- Ghosts: violation of unitarity, failure of energy conservation, wild particle production $A, B < 0$
- Instability: No wave-propagation, Euclidean box, exponential behaviour, $AB < 0$
- Superluminal: No unicity of the future cone, ill-defined Cauchy problem, $B/A > 1$

Ghosts

- Negative kinetic term

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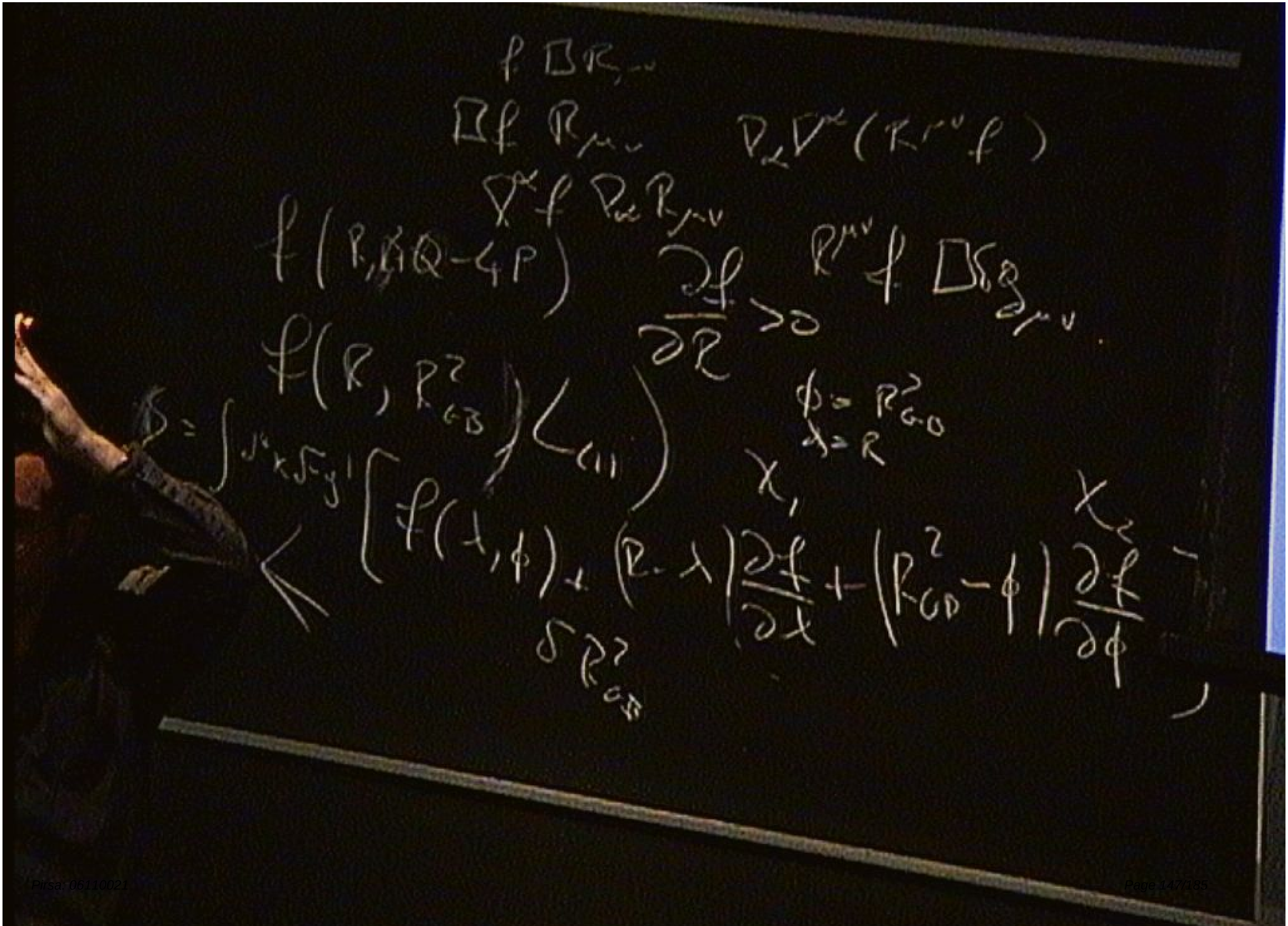
- SR: $p^\mu = (-\sqrt{\mathbf{p}^2 + m^2}, \mathbf{p})$


f □ R_μν
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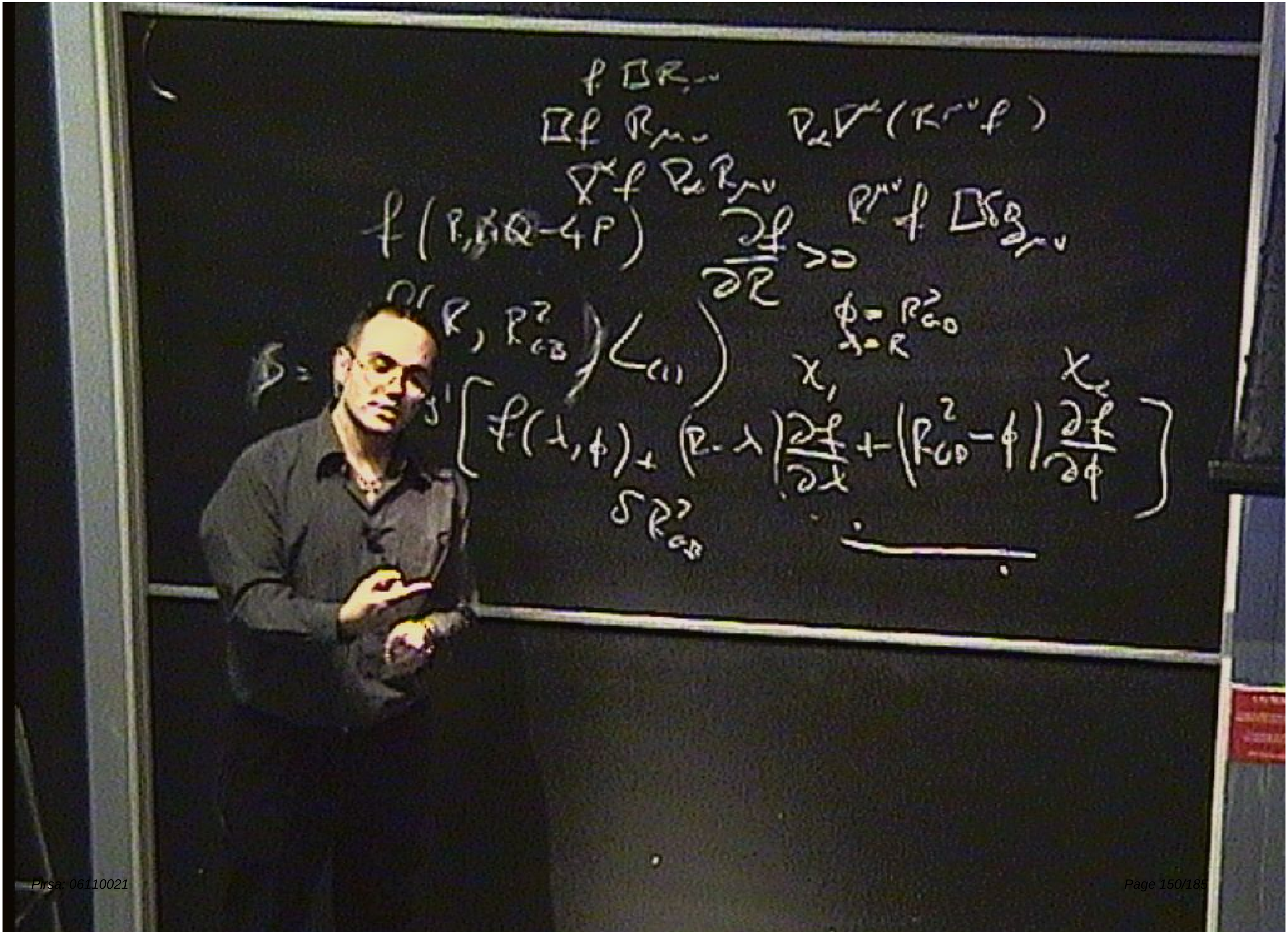
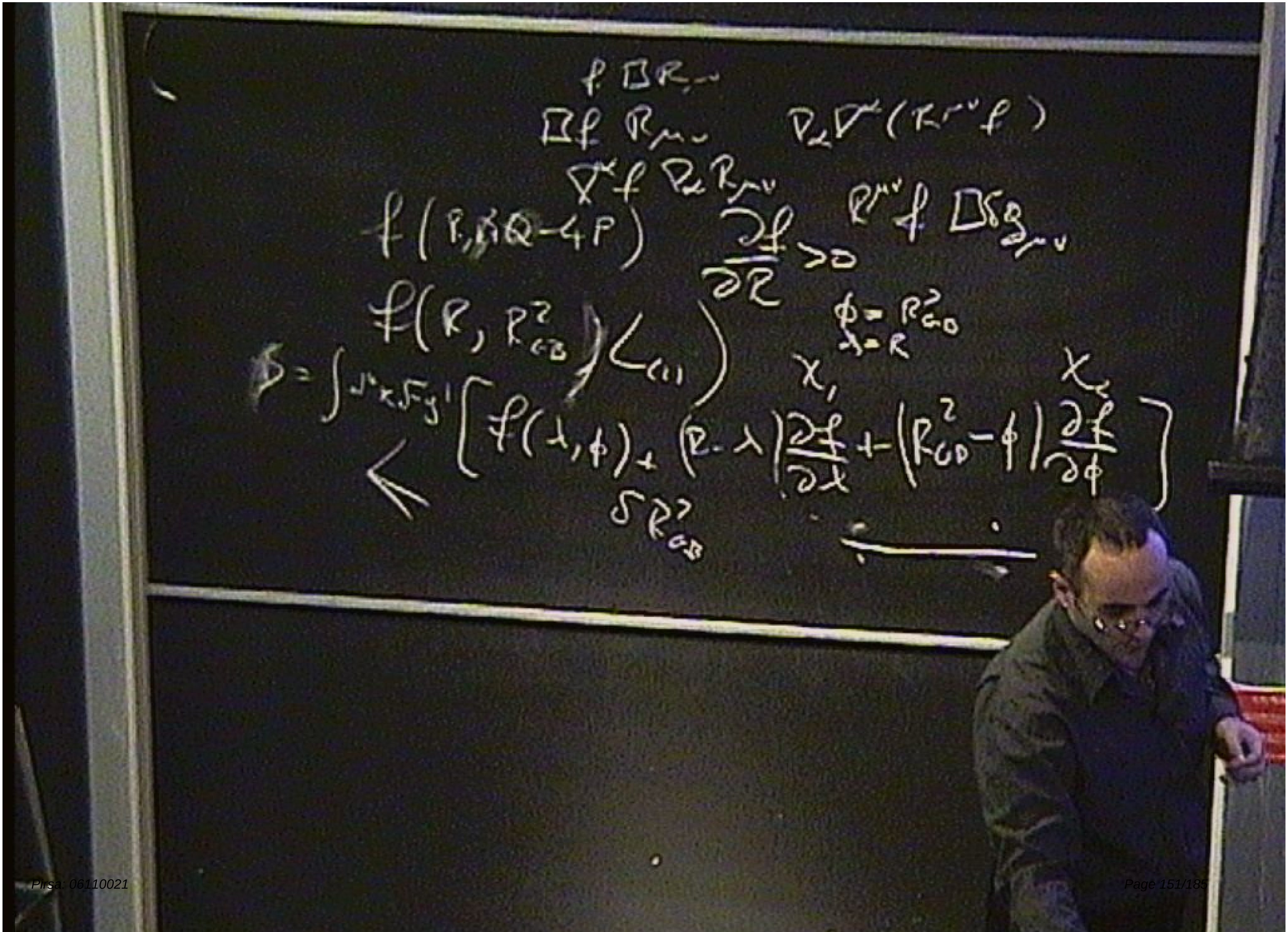
- In QFT, either you violate unitarity or negative energy states




f □R_{μν}
□f R_{μν}
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∇^α f ∇_α R_{μν}
R^{μν} f □δg_{μν}
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δR²_{GB}

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  - ⋆ High energy: higher powers contributions

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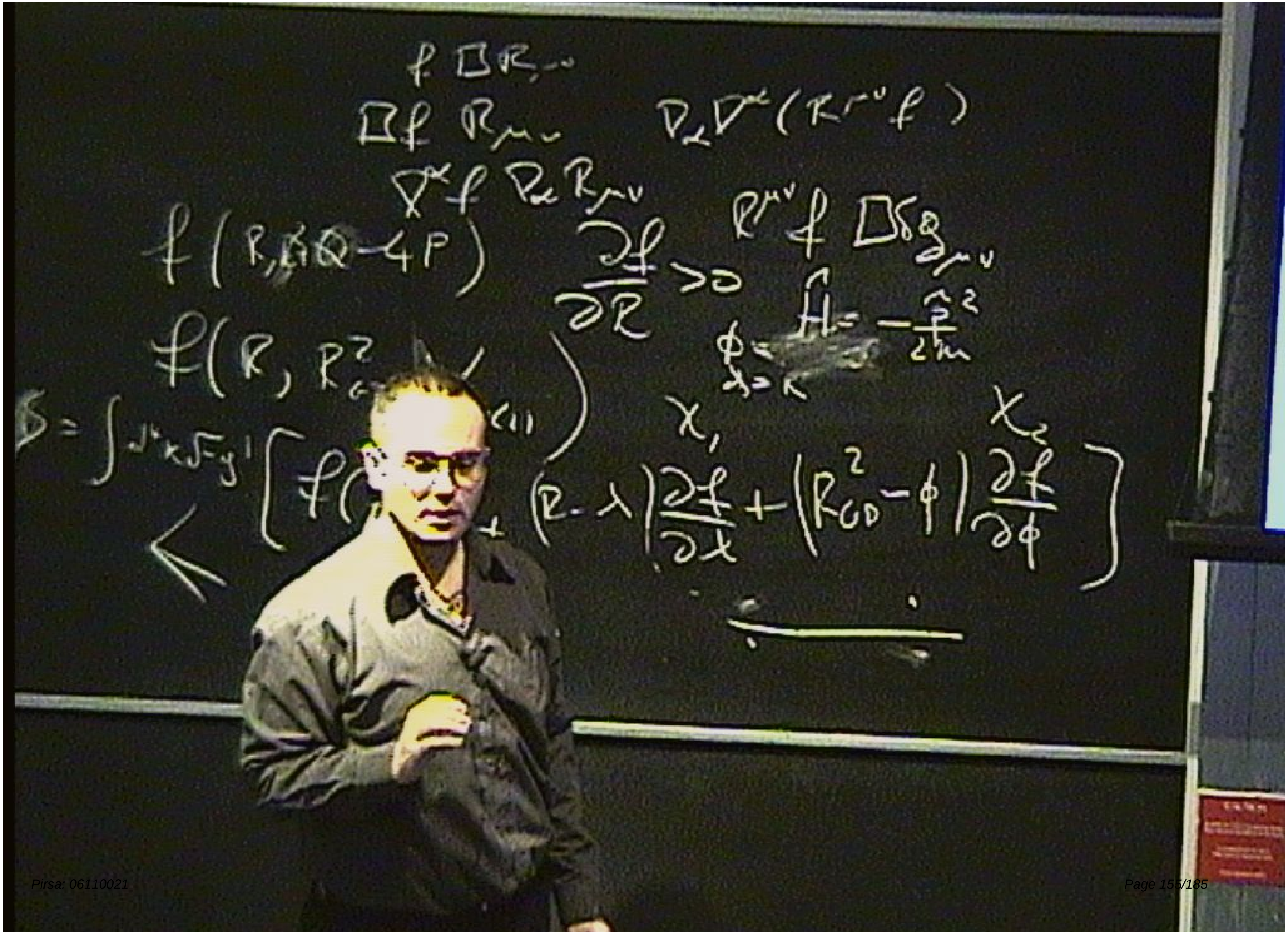
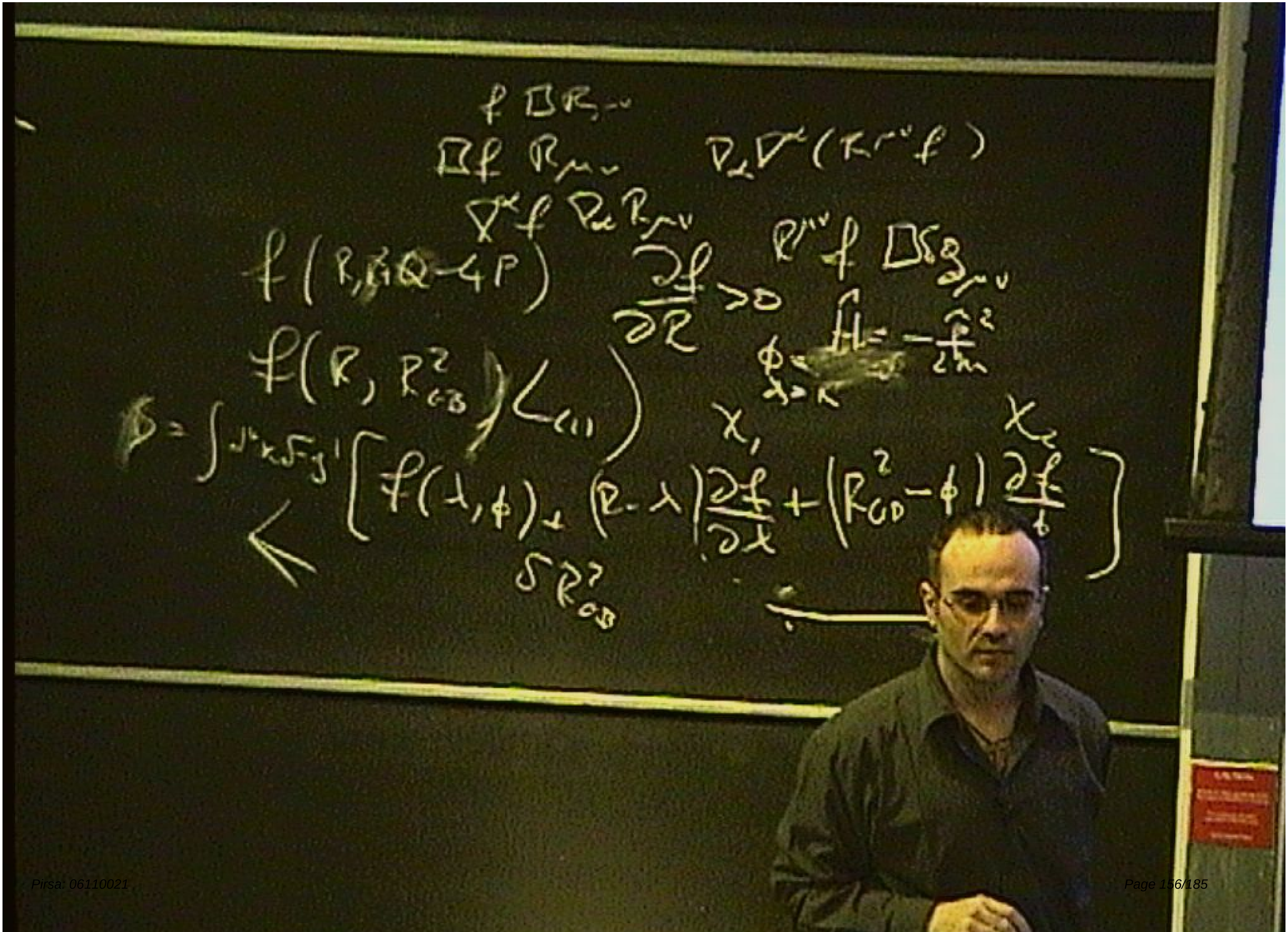
  - ⋆ High energy: higher powers contributions
  - ⋆ Low energy
- SL? No problem
  - ⋆ Not unique definition of future



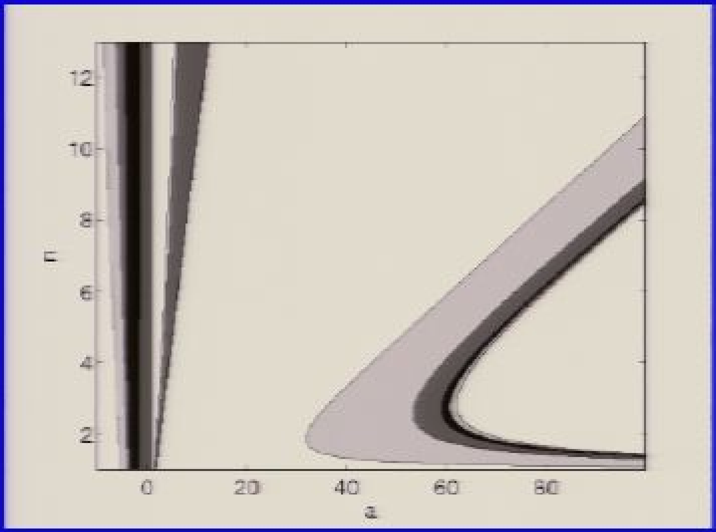


Objections

- Only an effective action
  - ⋆ High energy: higher powers contributions
  - ⋆ Low energy
- SL? No problem
  - ⋆ Not unique definition of future
  - ⋆ Black hole may not be black
  - ⋆ Spatial sections could be causally connected
  - ⋆ Ill-posed Cauchy problem
- Ghosts instabilities, no FRW

Constraints from attractors

- Look for accelerating power-law attractors space



Conclusions – Part II

- MGM cosmology and ghosts

Dynamical variables

- Use u and N

$$ds^2 = -\frac{e^{-2u}}{\beta^2\, H_0^2}\, dN^2 + a_0^2\, e^{2N}\, d\vec{x}^2,$$

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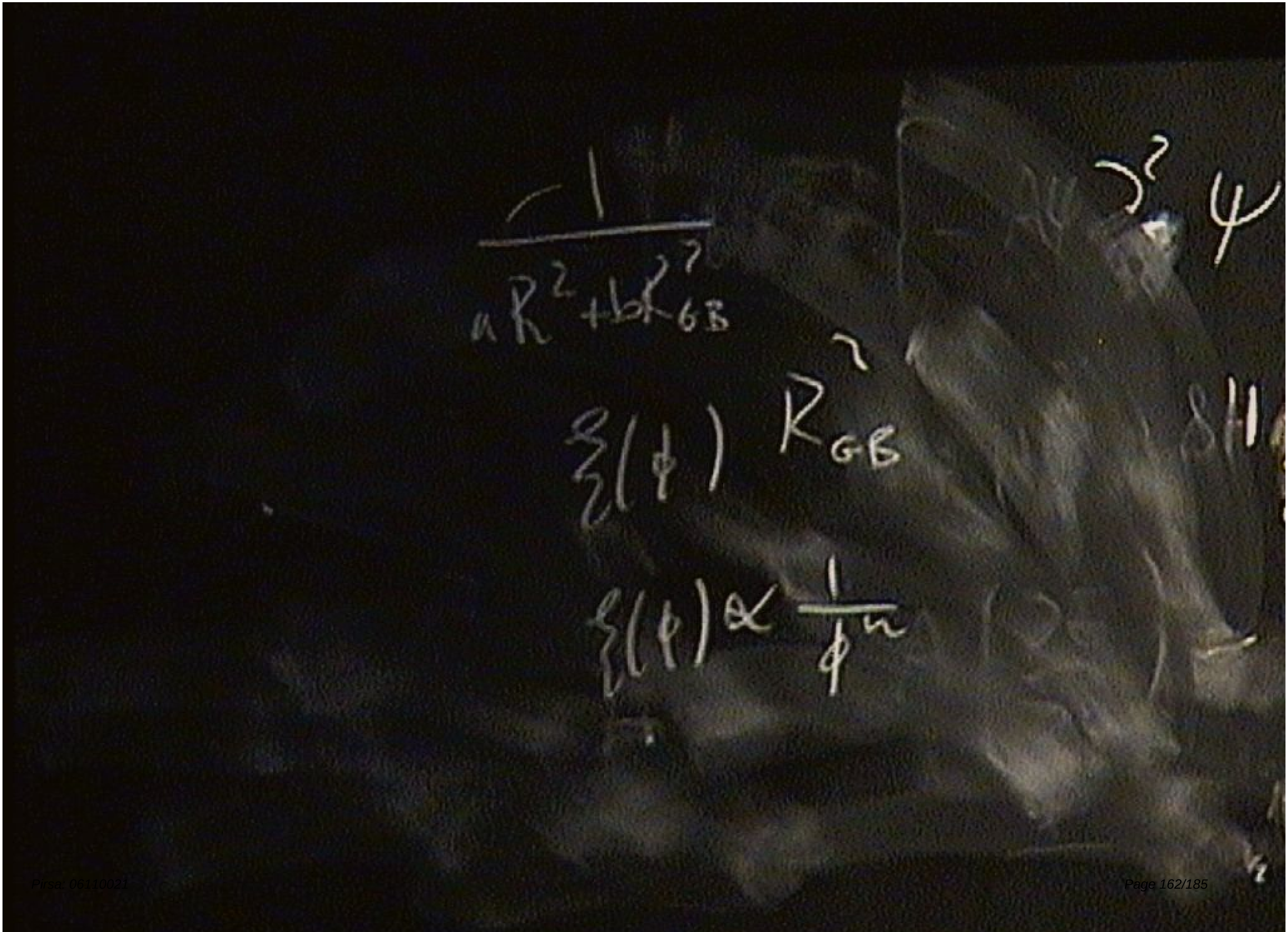
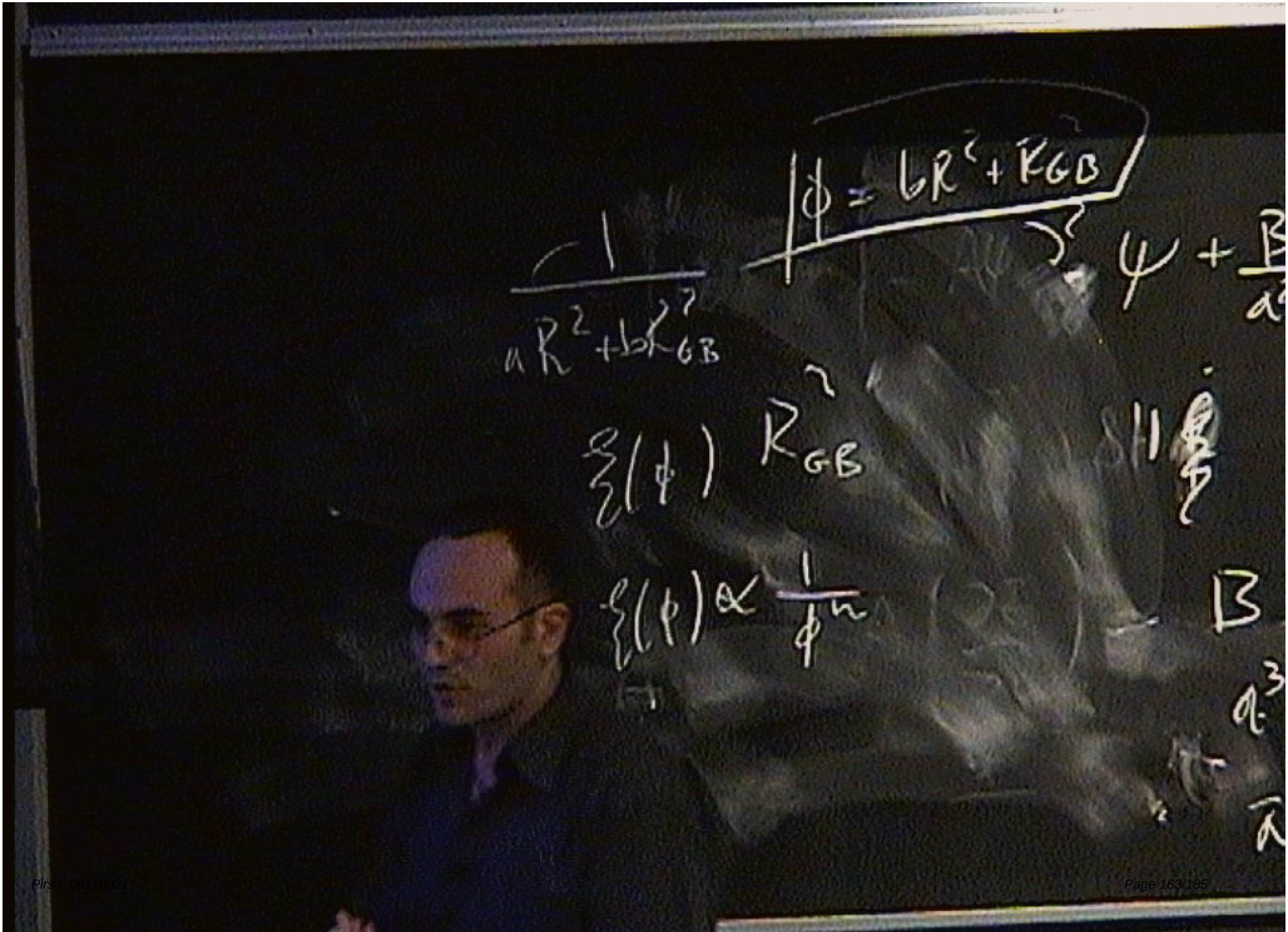
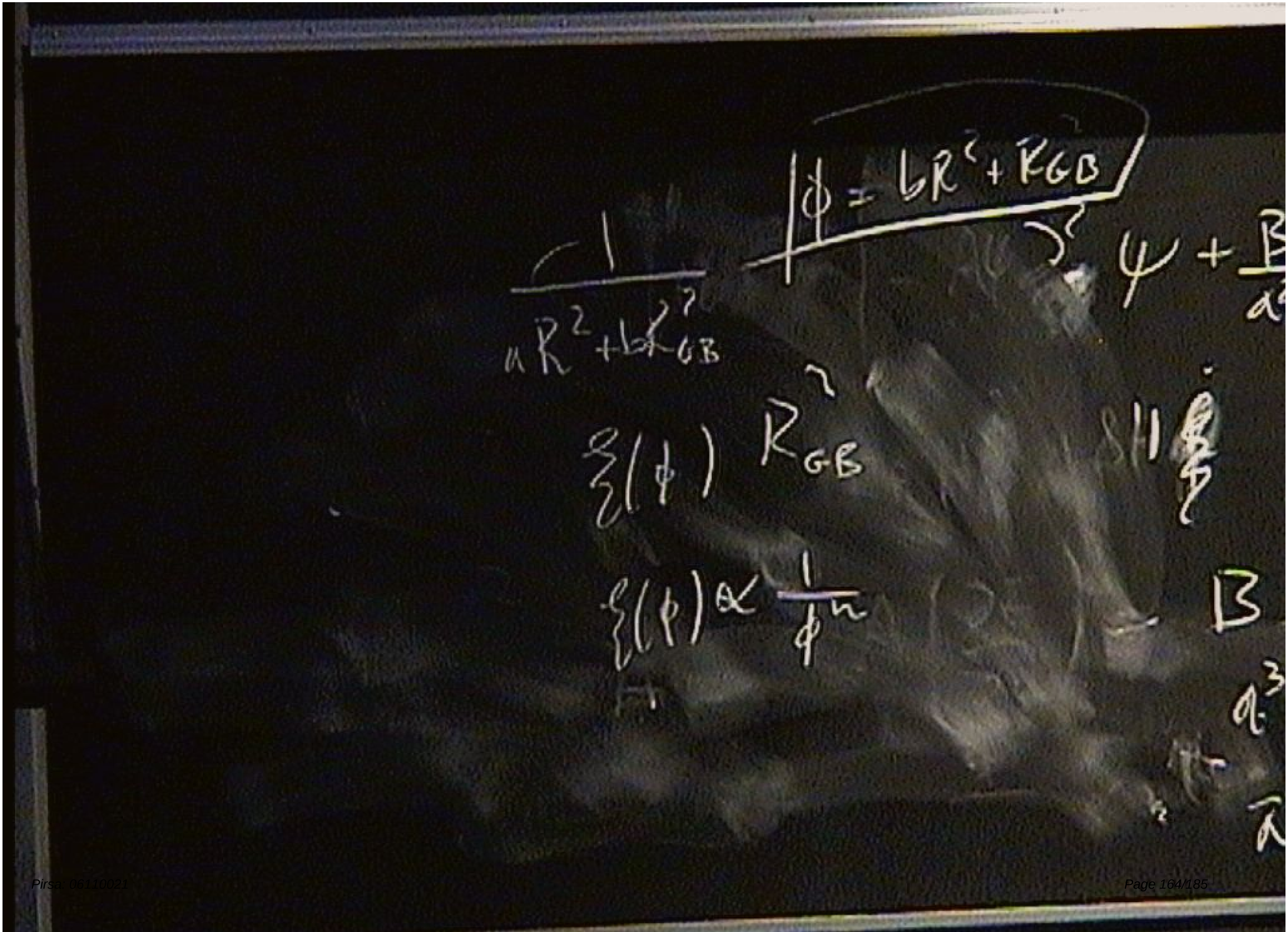
where $H = \beta e^u$, $N = \ln(a/a_0)$


1 / aR² + bR²GB
ξ(φ) R²GB
ξ(φ) ∝ 1/φⁿ
∂² ψ




ξ(φ) R_GB
ξ(φ) ∝ 1/φ^n

- Relation $\phi = b\lambda^2 + 4\beta^2 e^{2u}[\lambda - 6\beta^2 e^{2u}]$
- Choose $u = u(\lambda, \phi)$
- Friedmann $+ \lambda = R$, two 1st order eqns, $(\rightarrow)$
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- Note $\phi = 12\, H^4\, [3bq^2 - 2(1+3b)q + 3b]$

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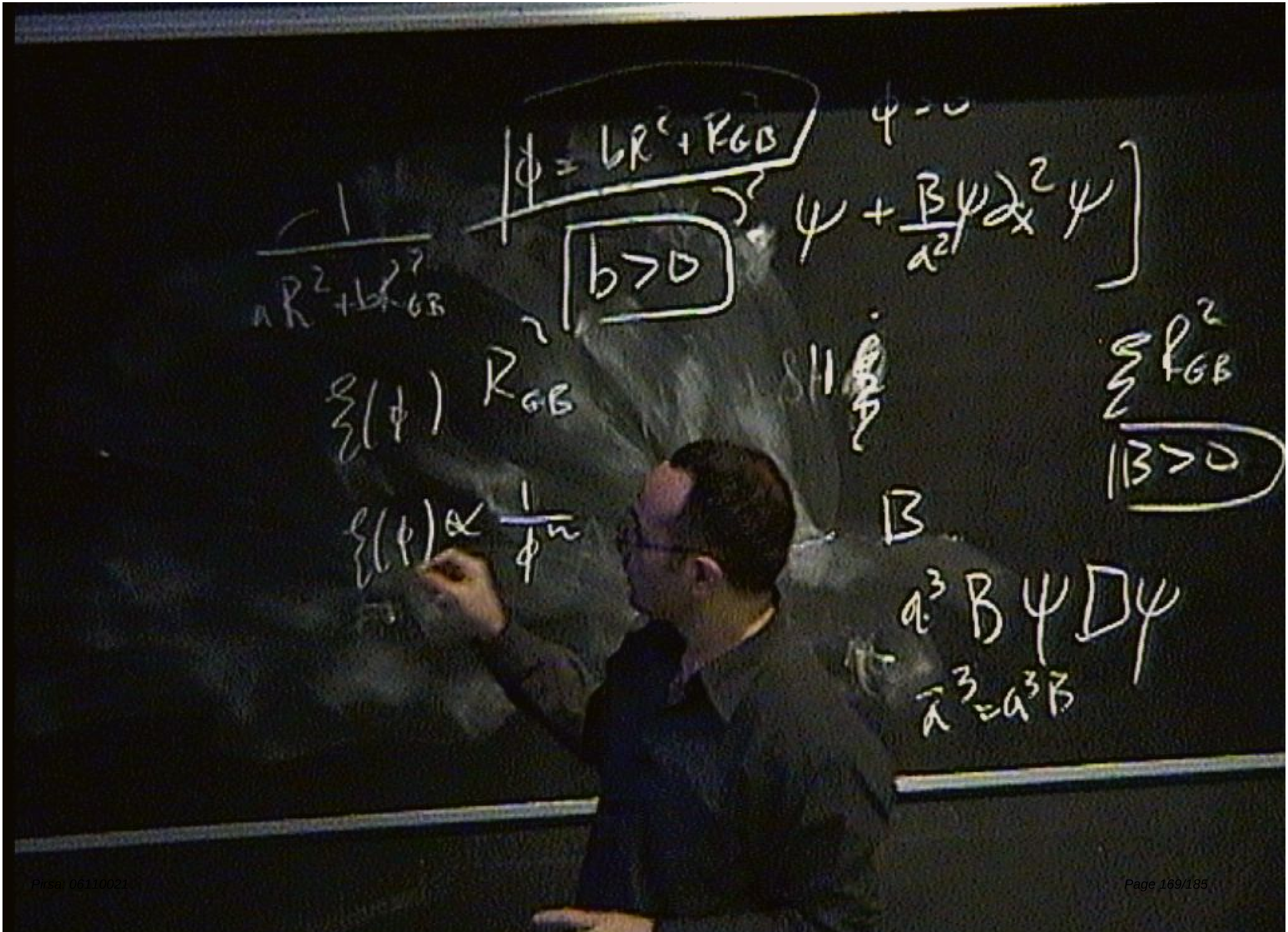
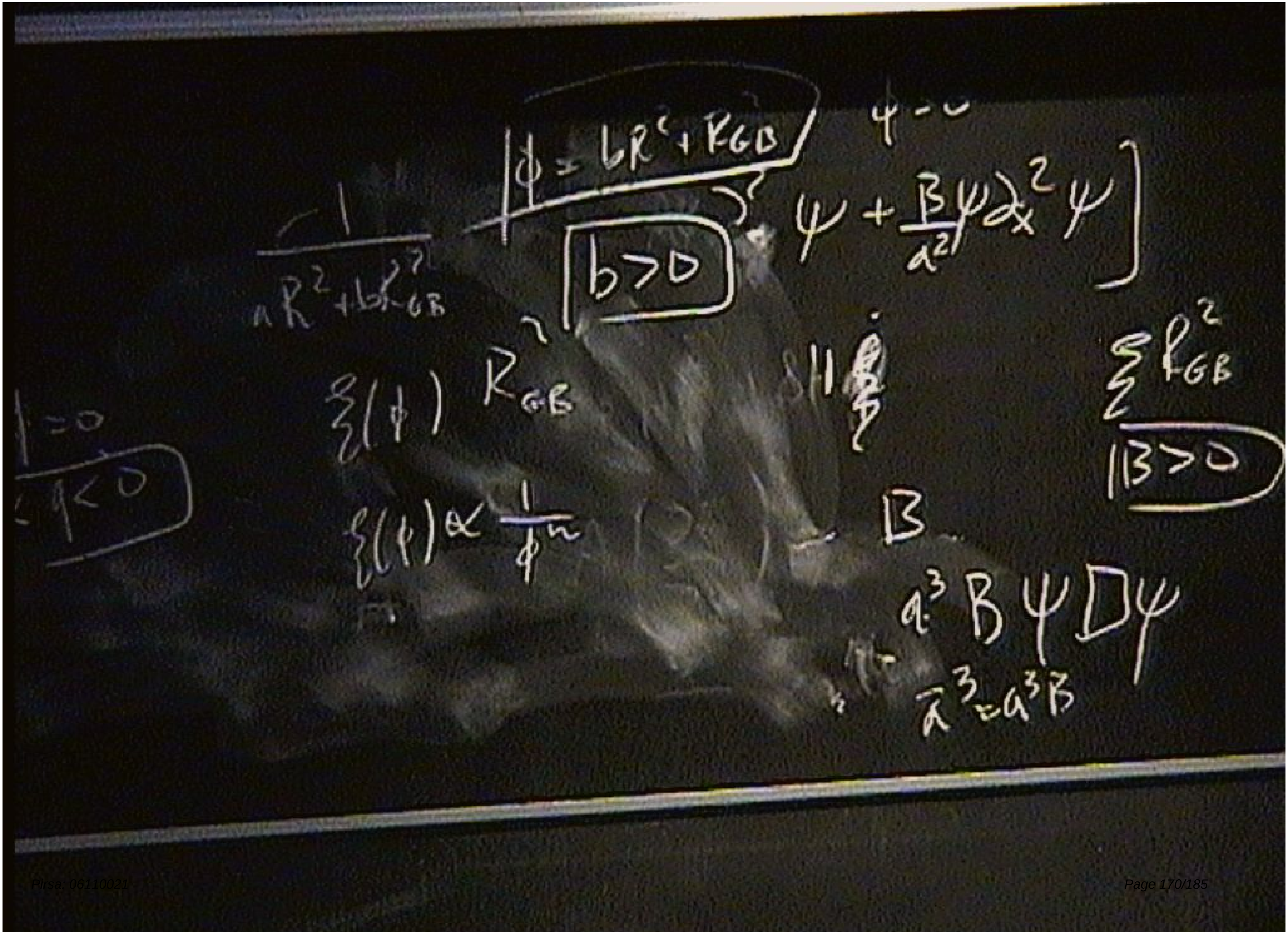

b>0
B>0


b>0
B>0
a<0
a³ B ψ □ψ
ã³ = a³ B

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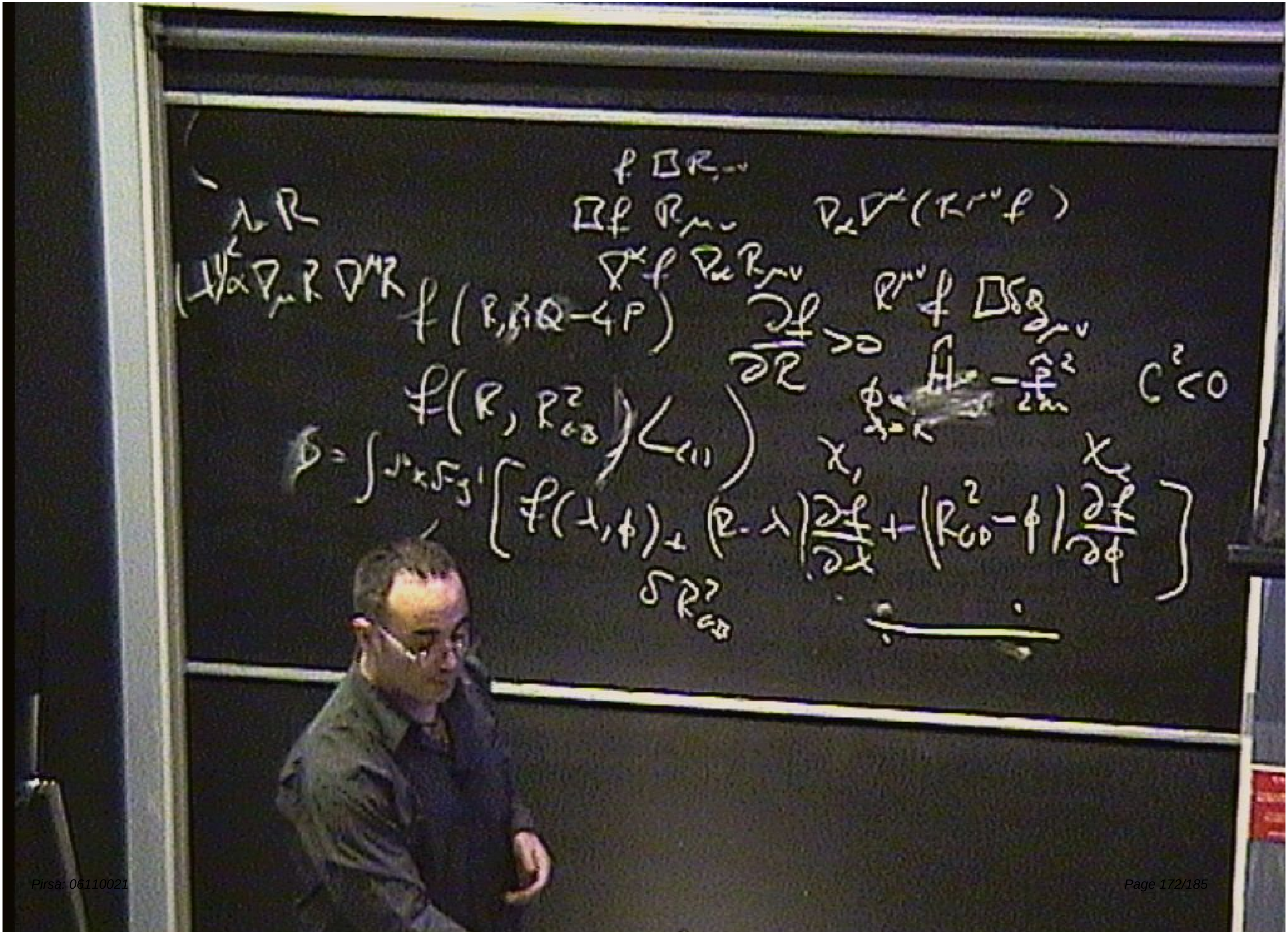

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  where $\gamma_\kappa = \sqrt{1+6b-6\kappa}$
- q-κ relation: $\kappa = \dfrac{3bq^2 - 2(3b+1)q + 3b}{3(q-1)^2}$
- In cosmology $-\infty < \kappa < b$
- $\Delta = 0$ unviable if $b \le \bar{b}(n)$, $-\frac{1}{3} < \bar{b} \le -\frac{2}{9}$

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- At given N, use ghosts conditions to restrict IC
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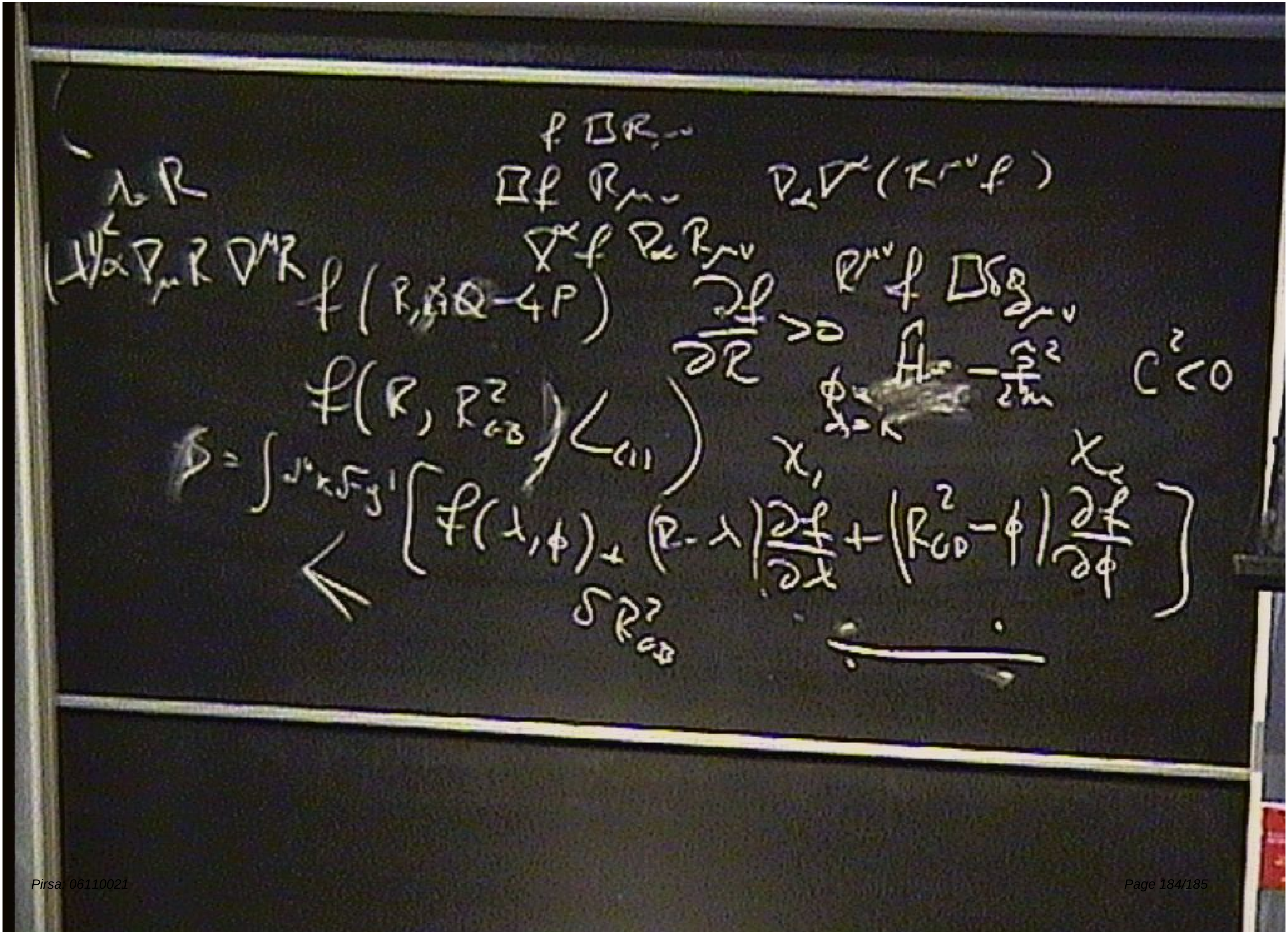
λ R
f □R
□f R
∇f ∇R
f(R, R², Q − 4P)
∂f/∂R > 0
f(R, R²)
S = ∫ d⁴x √-g [f(λ, φ) + (R − λ) ∂f/∂λ + (R² − φ) ∂f/∂φ]
C² < 0

λR

$(-1)^{c}\alpha \nabla_\mu R \nabla^\mu R$

$f \Box R_{\mu\nu}$

$\Box f \, R_{\mu\nu}$ $\quad \nabla_\alpha \nabla^\alpha (R^{\mu\nu} f)$

$\nabla^\alpha f \, \nabla_\alpha R_{\mu\nu}$

$f(R, 6Q-4P)$ $\quad \frac{\partial f}{\partial R} > 0$ $\quad R^{\mu\nu} f \, \Box \delta g_{\mu\nu}$

$f(R, R^2_{GB}) \ll \mathcal{L}_{(1)}$ $\quad A = -\frac{\hat{p}^2}{2m}$ $\quad C^2 < 0$

χ_1 $\quad \chi_2$

$$S = \int d^4x \sqrt{-g} \left[f(\lambda,\phi) + (R-\lambda)\frac{\partial f}{\partial \lambda} + (R^2_{GB}-\phi)\frac{\partial f}{\partial \phi} \right]$$

$G \propto \frac{1}{\chi}$

δR^2_{GB}