Title: Black Hole - String Transition and Rolling D-brane

Date: Nov 24, 2006 02:00 PM

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Abstract: The exact boundary states for the rolling D-brane solution in two-dimensional black hole systems will be presented.I will study the physical significance of the solution in relation to the ``tachyon-radion correspondence\" and the ``black hole - string transition\". When the alpha\' corrections become larger, when at the same time the Hawking temperature coincide with the Hagedorn temperature, the phase transition occurs and the physics changes drastically. It also suggests the universal feature of the decaying D-brane and its failure in the strong quantum regime. The talk is based on my series of works hep-th/0605013, hep-th/0507040 in collaboration with Soo-jong Rey (SNU) and Yuji Sugawara (Tokyo).

Black Hole – String Transition and Rolling D-brane

Yu Nakayama (Tokyo univ.)

Based on :hep-th/0605013 with S.J. Rey, Y. Sugawara,
hep-th/0507040 with S.J. R, Y. S,

Outline

1. Introduction
   - What's 2D black hole?
2. Boundary states for falling D-brane
   - Classical D-brane
   - Wick rotation
   - Contour choice
3. Radiation from falling D-brane
   - Tachyon-Radion correspondence
   - Black hole/String transition
4. Summary

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Purpose of the talk

- Large charge (+BPS) VS Small Charge (+non-BPS)
  - Black hole / String phase transition
  - Hawking temperature VS Hagedorn temperature
  - Is 2D (pure) Black hole really black?
- Analyticity VS Non-analyticity
  - Universality of Tachyon-Radion correspondence
  - Wick rotation in curved space
- Unitarity VS Open/Closed duality
  - Optical theorem
  - Lorentzian world-sheet V.S. Euclidean world-sheet

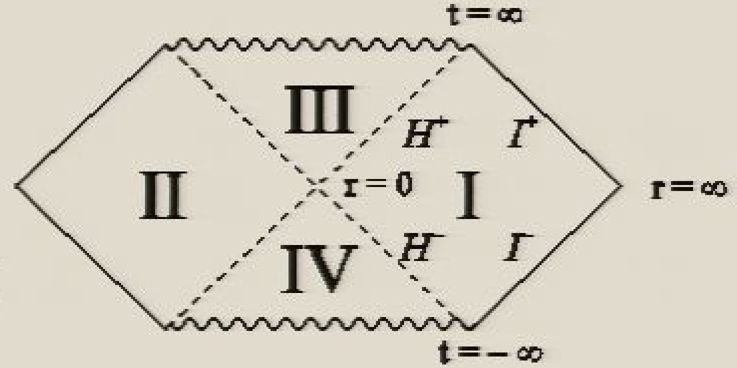
2D (fermionic) Black Hole

- 2D black hole is the simplest black hole geometry as an exact string background (Witten; Mandal, Senguputa, Wadia) $SL(2,R)_k/U(1)$ $\quad c = 3 + \frac{6}{k}$

$$ds^2 = 2k(d\rho^2 - \tanh^2\rho\, dt^2) \quad e^{\Phi} = \frac{e^{\Phi_0}}{\cosh\rho}$$

- Global metric looks Schwarzshild-like

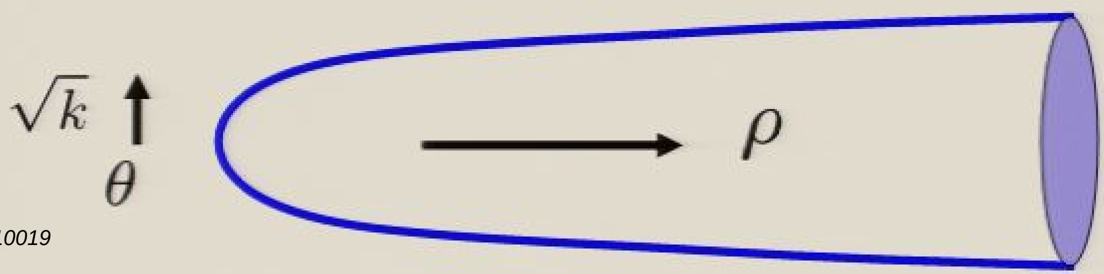
$$u = \sinh\rho\, e^{t} \quad v = -\sinh\rho\, e^{-t} \quad ds^2 = -2k\frac{du\,dv}{1-uv}$$



- In Euclidean geometry, 2D black hole is cigar geometry:

$$ds^2 = 2k(d\rho^2 + \tanh^2\rho\, d\theta^2) \quad e^{\Phi} = \frac{e^{\Phi_0}}{\cosh\rho}$$

$$T_{Hw} = \frac{1}{\beta_{Hw}} = \frac{1}{2\pi\sqrt{2k}}$$



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Applications

- Near horizon limit of nonextremal NS5 brane

$$ds^2 = -\left(1-\frac{r_0^2}{r^2}\right)dt^2 + \left(1+\frac{k\alpha'}{r^2}\right)\left(\frac{dr^2}{1-\frac{r_0^2}{r^2}} + r^2 d\Omega_3^2\right) + d\mathbf{y}_{\mathbf{R}^5}^2$$

- Taking the limit with keeping $r = r_0 \cosh\rho$

$$ds^2 = -\tanh^2\rho\, dt^2 + k\alpha' d\rho^2 + k\alpha' d\Omega_3^2 + d\mathbf{y}_{\mathbf{R}^5}^2\ ,$$

- Level k corresponds to number of NS5 branes.
- k → ∞ is the semiclassical (supergravity) limit. $\frac{1}{k} \sim \alpha'$

2D black hole is important for holographic dual of NS5 branes (Little String Theory)

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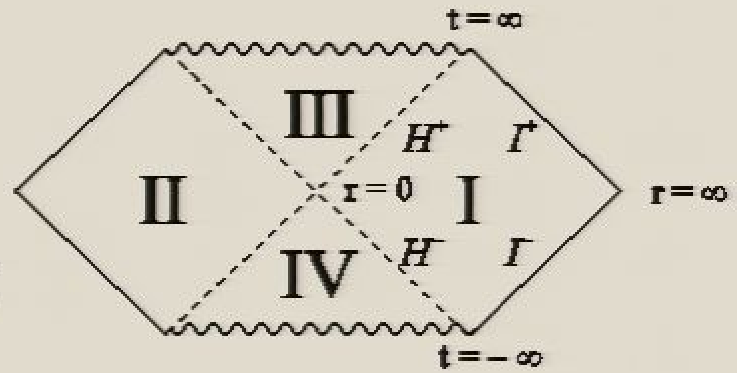

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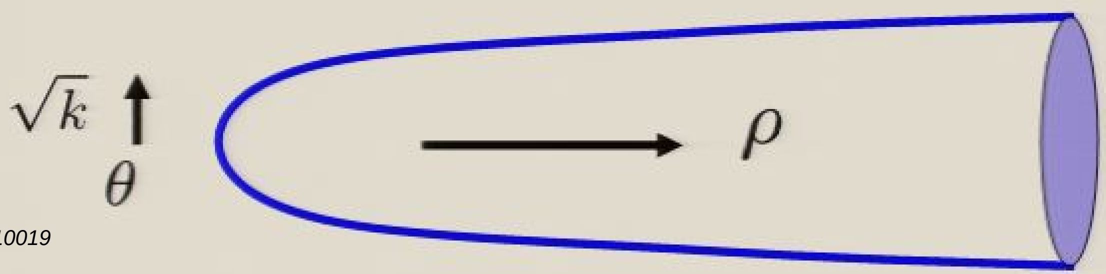

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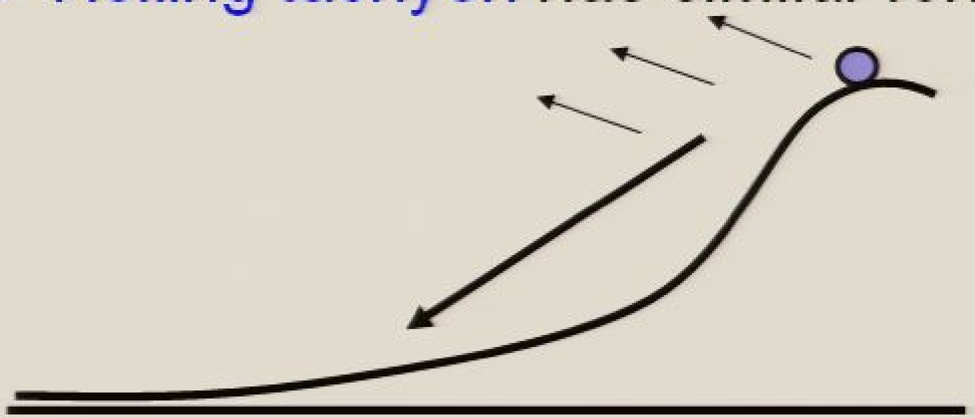

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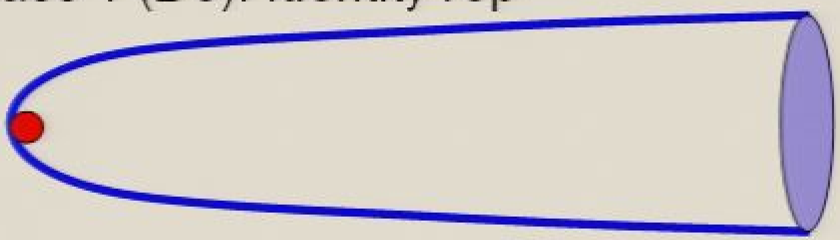
Is tachyon-radion correspondence universal?
Artifact at the level of effective action?

Boundary states for falling D-brane

Branes in 2D Black Hole geometries

Classical D-branes are classified by solutions of DBI action.

Class 1 (D0): identity rep



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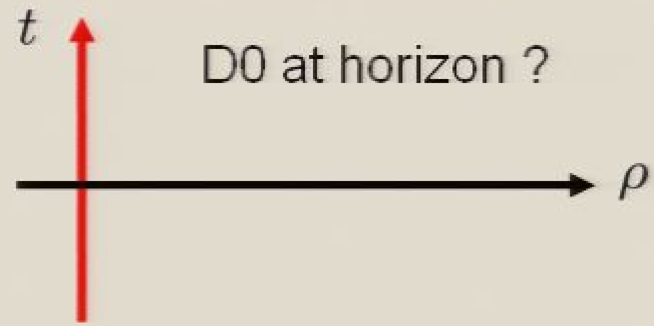
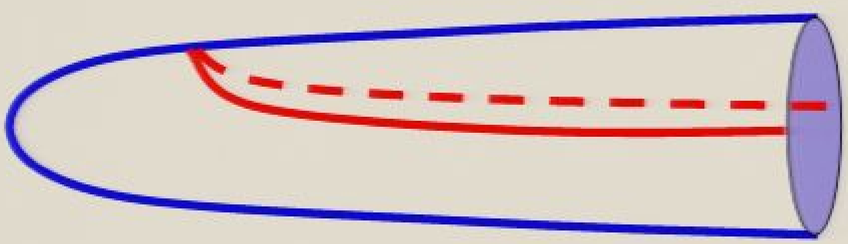
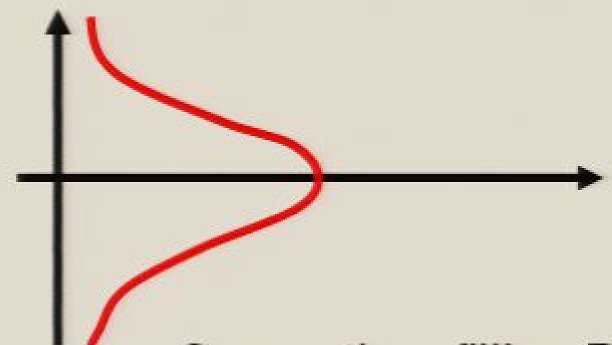


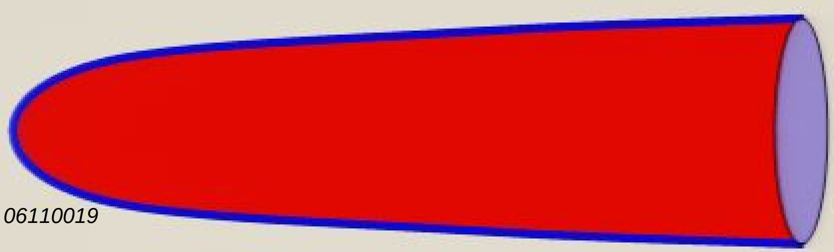
Class 2' (D1): continuous rep



Infalling brane



Class 3 (D2): discrete rep



Space-time filling D1?

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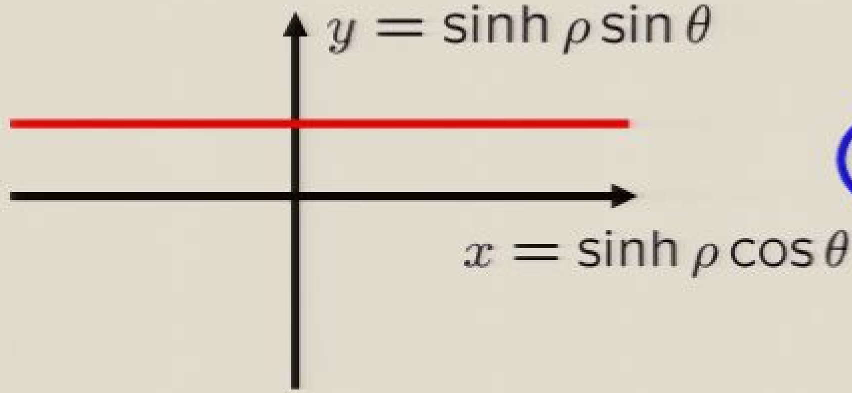

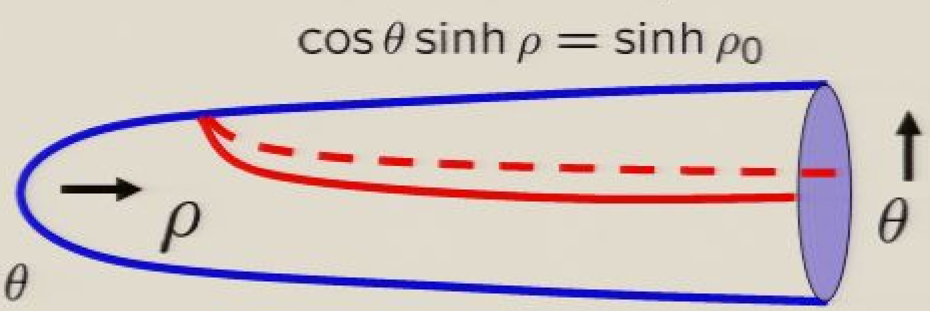

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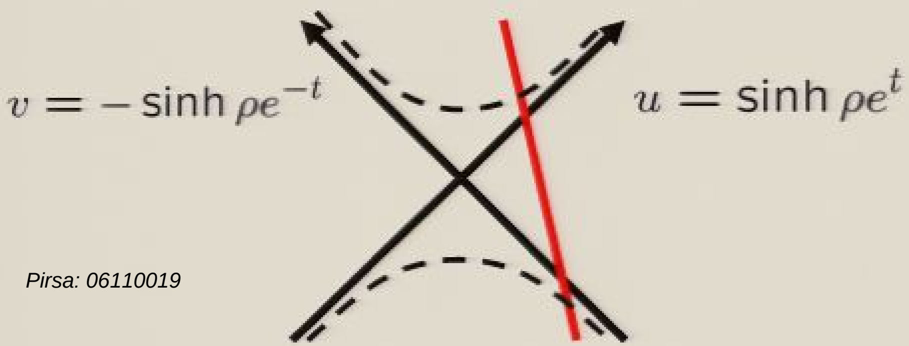
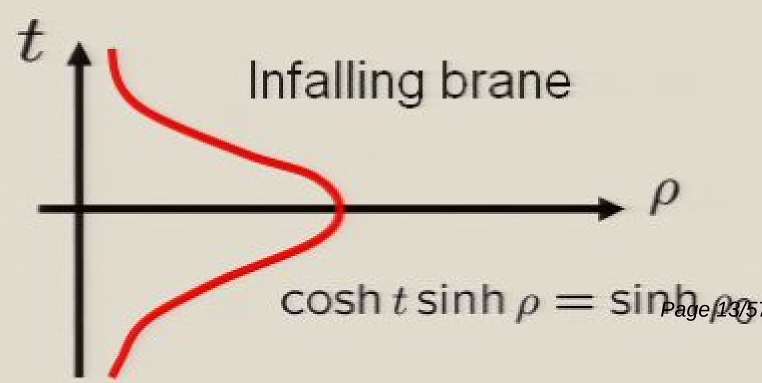
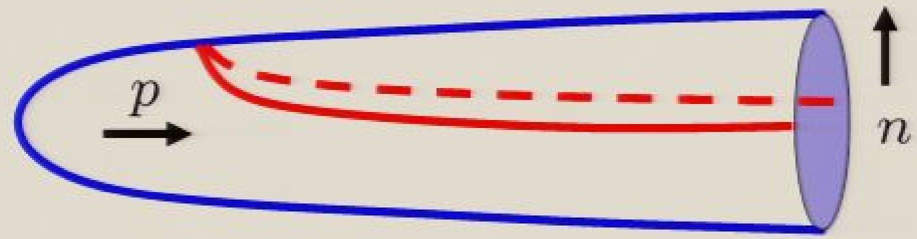




Euclidean boundary states

- Class 2' boundary states in Euclidean BH

$$\cos\theta \sinh\rho = \sinh\rho_0$$



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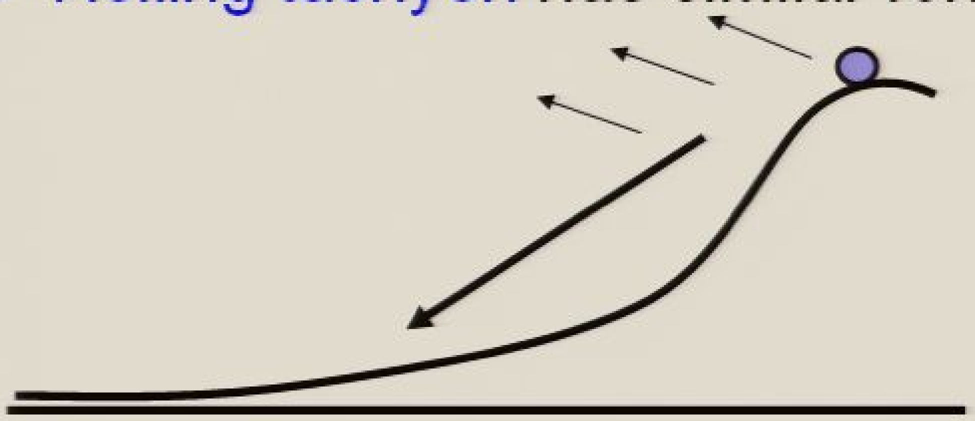

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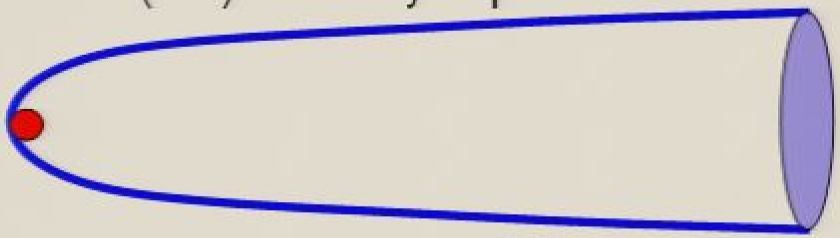

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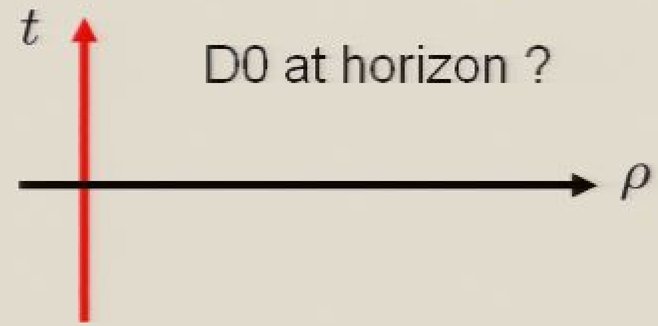

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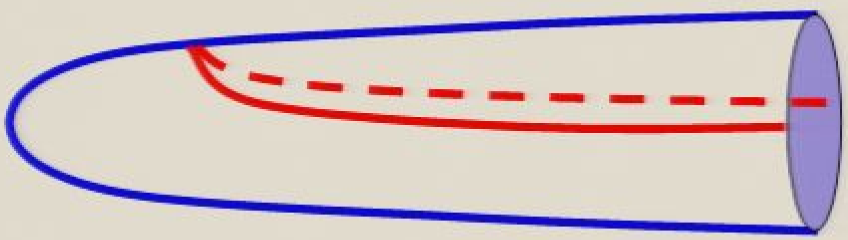

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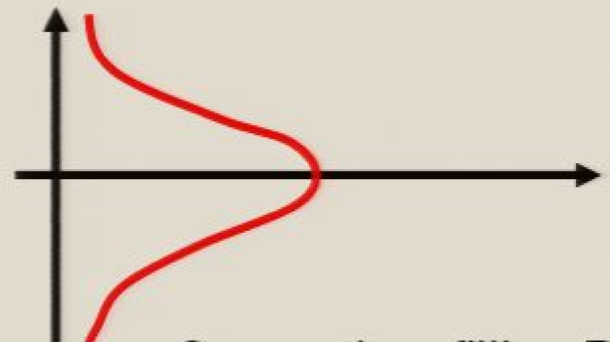

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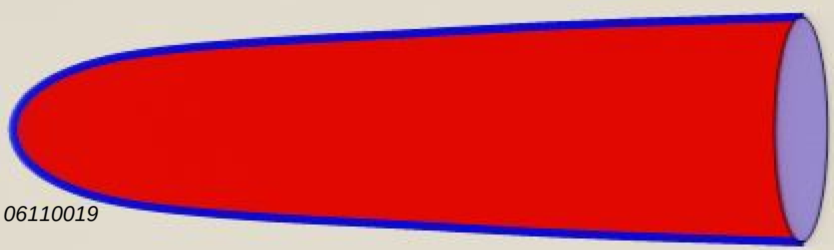

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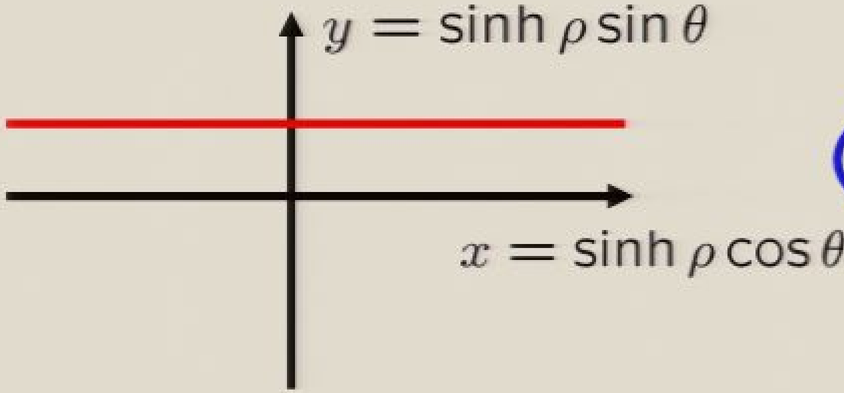

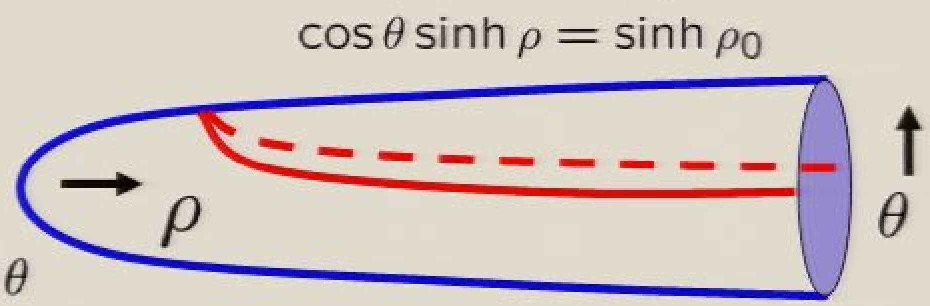

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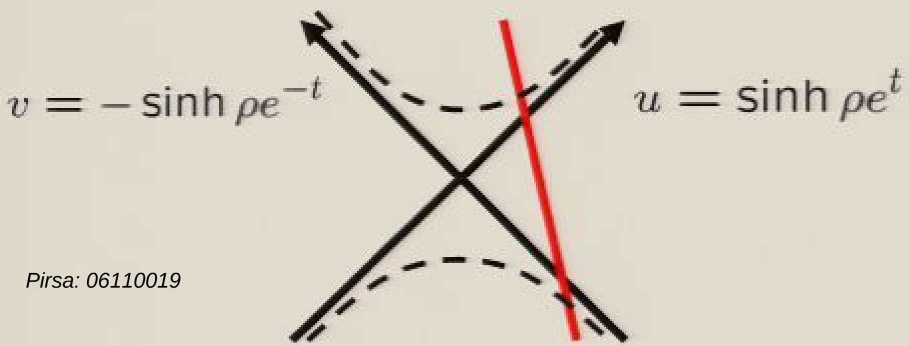

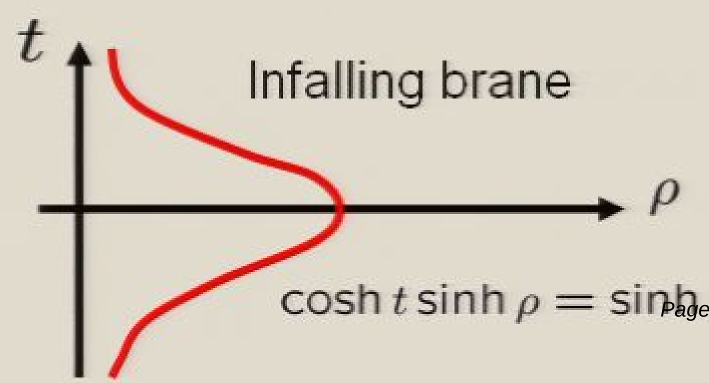

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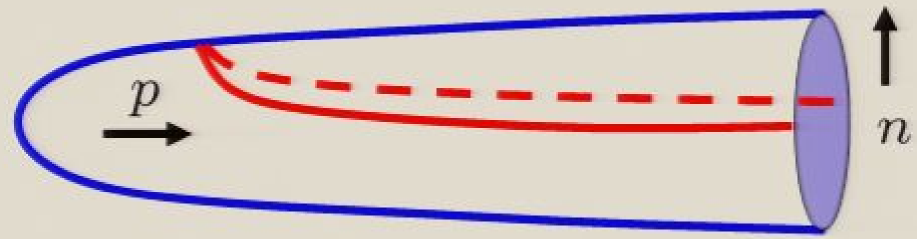

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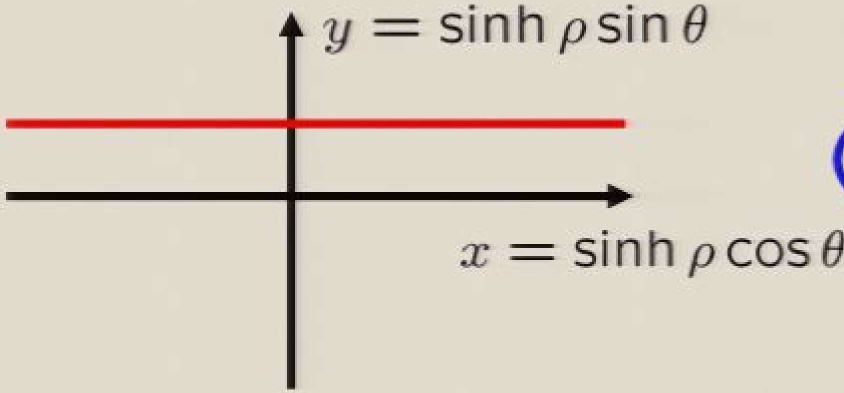

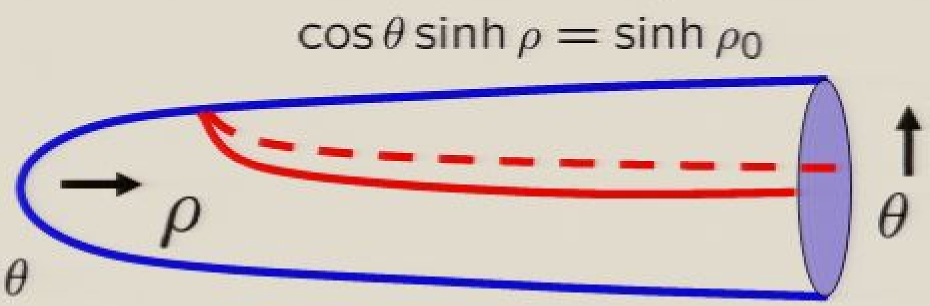

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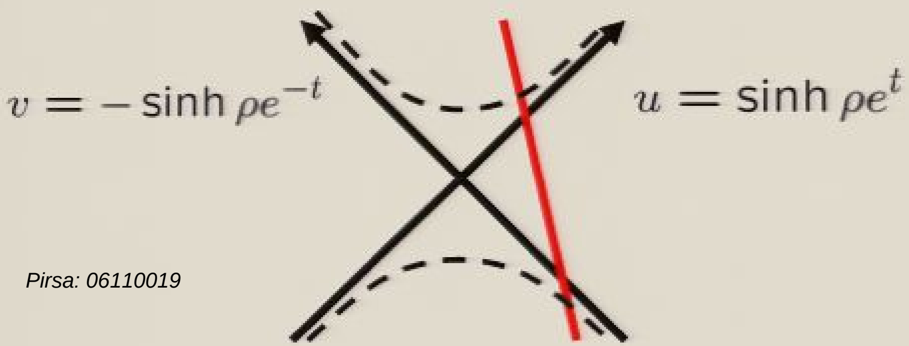

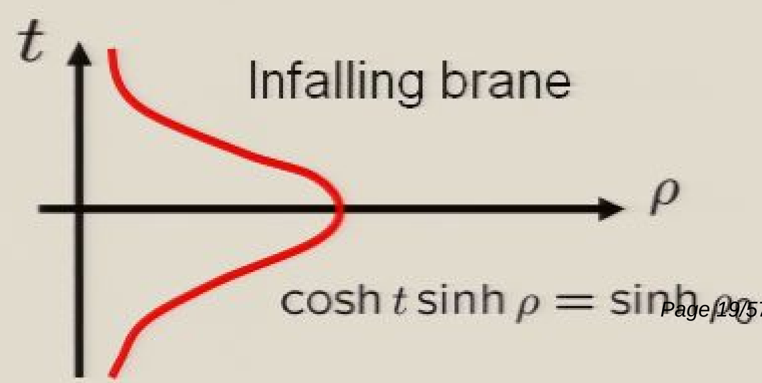

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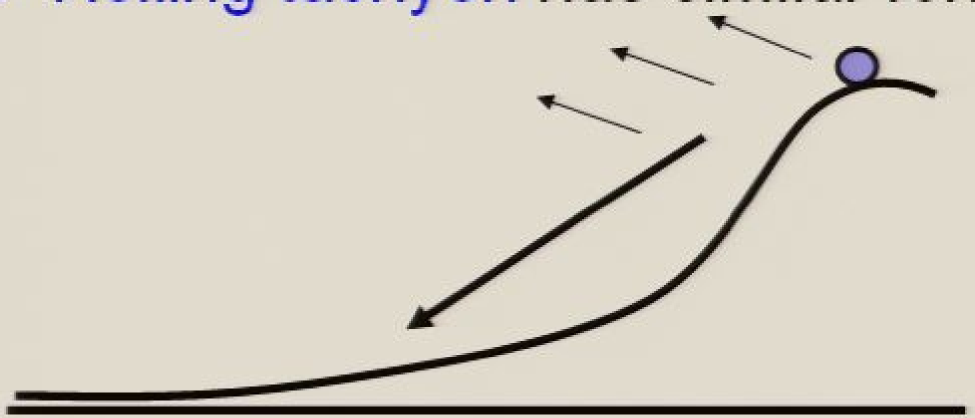

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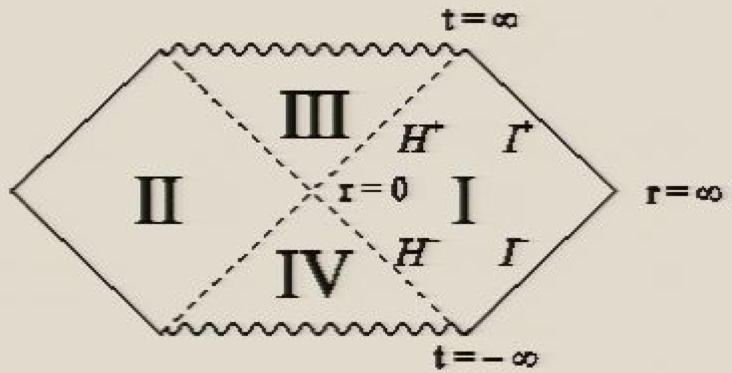

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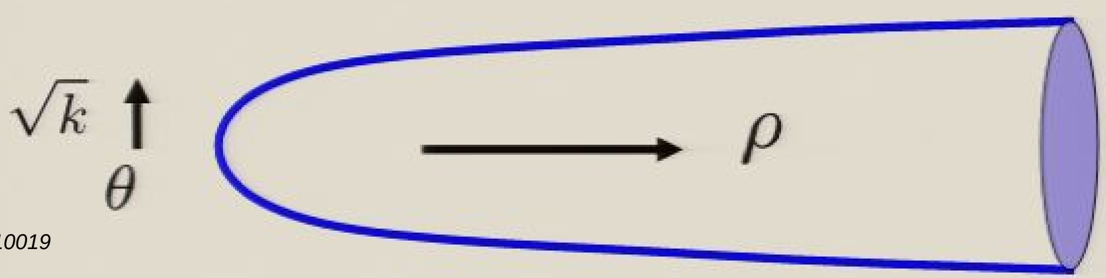

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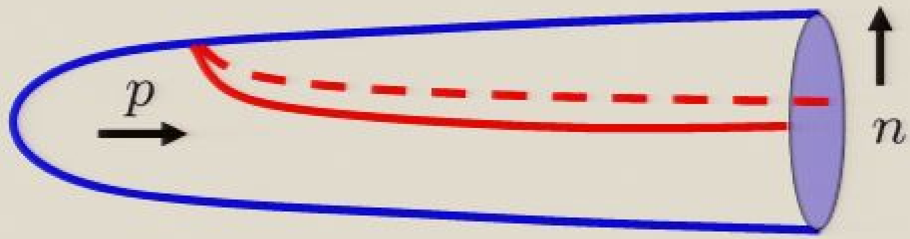

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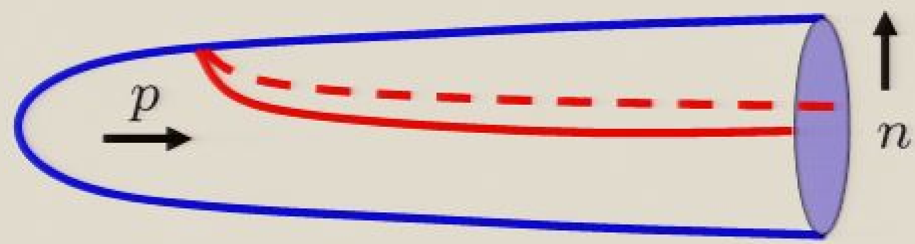

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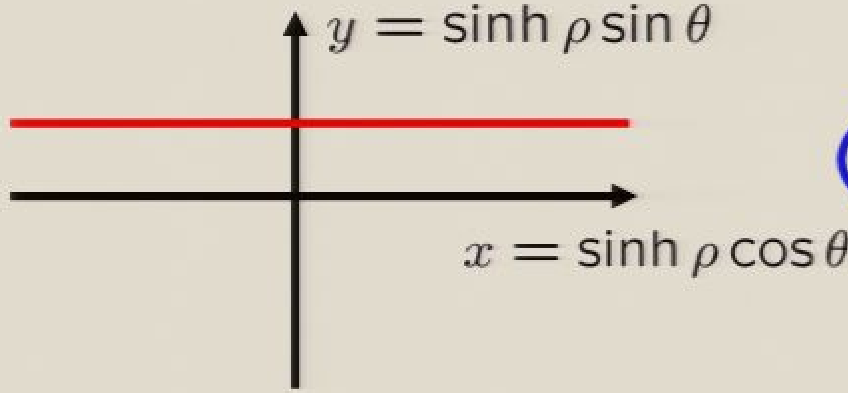
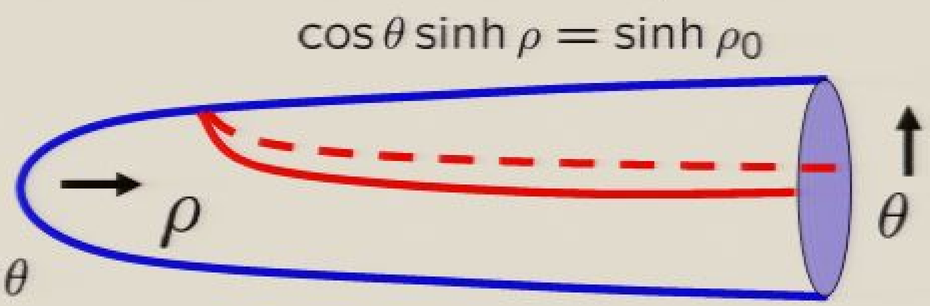
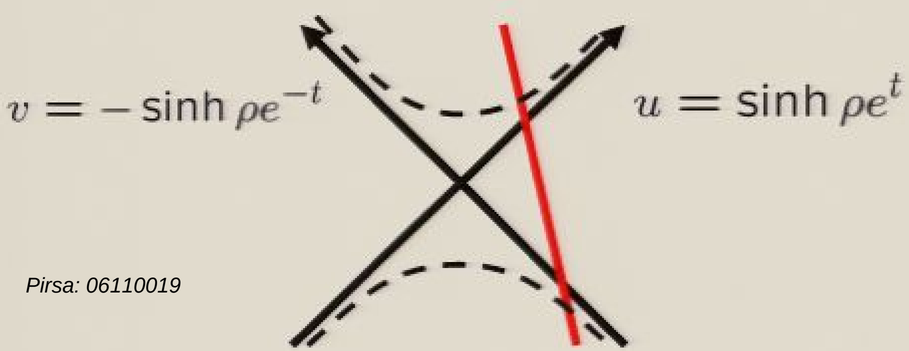
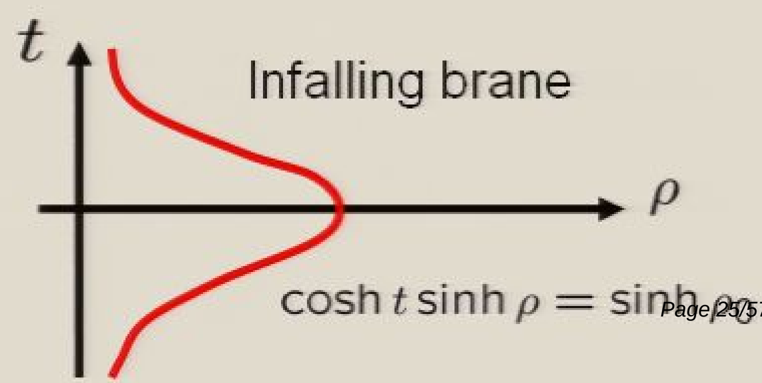




- Lorentzian case ("Rolling D-brane": Kutasov, NST, Yogendran, NRS)

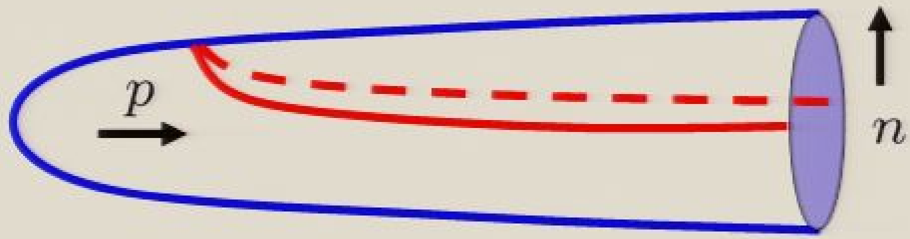




Euclidean boundary states

- Class 2' boundary states in Euclidean BH

$$\cos\theta \sinh\rho = \sinh\rho_0$$



$$|B;\rho_0,\theta_0\rangle = \int_0^\infty \frac{dp}{2\pi} \sum_{n\in Z} \Psi_{D1}(\rho_0;p,n)\,|p,n\rangle,$$

$$\Psi_{D1}(\rho_0;p,n) = \mathcal{N}(k)\frac{\Gamma(ip)\Gamma\left(1+\frac{ip}{k}\right)}{\Gamma\left(\frac{1}{2}+\frac{ip+n}{2}\right)\Gamma\left(\frac{1}{2}+\frac{ip-n}{2}\right)} [e^{-ip\rho_0}+(-1)^n e^{ip\rho_0}] \,.$$

- Effect of 1/k correction
  - Delta function localized trajectory → smeared wavefunction
    Poisson distribution: $\delta(\phi) \sim \sqrt{\frac{1}{k-1}}$
  - The steeper the hairpin, the wider the trajectory (NRPT).

Wick rotation: rolling D-brane boundary states

Naïve momentum space Wick rotation $n \to i\omega$ does not work.

- Performing Wick rotation in coordinate space, or choosing the contour integral properly,

$$\Psi(\rho_0;p,\omega)=\frac{\Gamma(\frac{1}{2}-i\frac{p+\omega}{2})\Gamma(\frac{1}{2}-i\frac{p-\omega}{2})\Gamma(1+\frac{ip}{k})}{\Gamma(1-ip)}\left[e^{-ip\rho_0}-\frac{\cosh\left(\pi\frac{p-\omega}{2}\right)}{\cosh\left(\pi\frac{p+\omega}{2}\right)}e^{ip\rho_0}\right]$$

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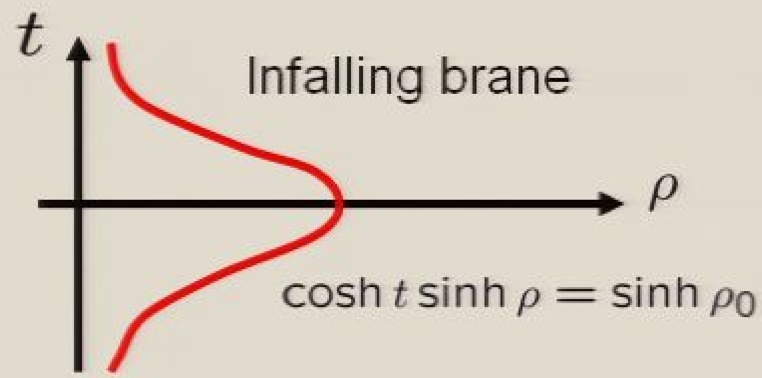

- Finite k correction:
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  - Rolling D-brane gathers moss (Kutasov)

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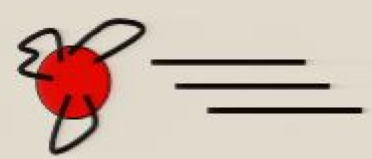


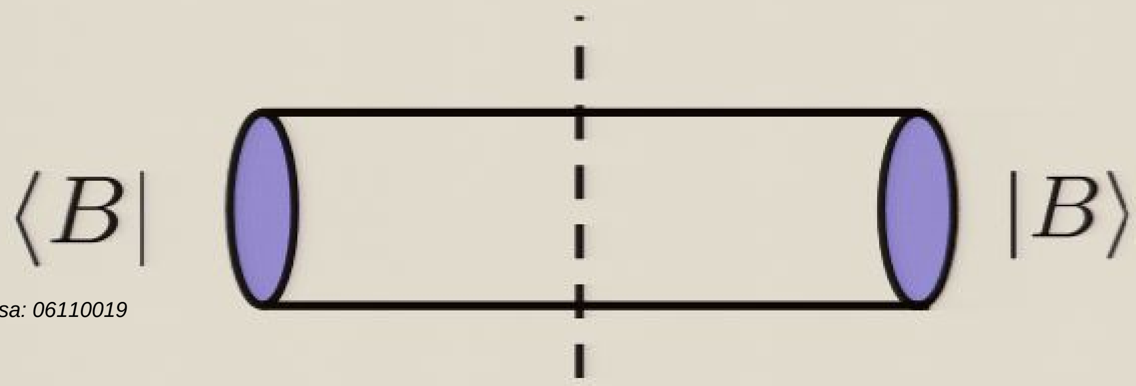
3. Radiation from falling D-brane

Radiation from falling brane

From boundary states, we can compute closed string emission from falling D-branes.

- From the optical theorem, imaginary part of one-loop amplitude gives total emission rate.

$$\bar{N} = \text{Im}\int dt\langle B|e^{-tH}|B\rangle$$
$$= \sum_M \int dk^{5-p}\int \frac{dp}{2E_p}|\Psi(p,E_p)|^2, \quad E_p = \sqrt{p^2+k^2+M^2}$$



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- Let us assume k>1. For fixed mass level, $\omega^2(p,M) = p^2 + 2kM^2$

$$N(M) \sim \int \frac{d^{5-p}\mathbf{k}_\perp}{(2\pi)^{5-p}} \int_0^\infty \frac{dp}{2\pi} \frac{1}{2\omega(p,M)} |\Psi(\rho_0; p, \omega(p,M))|^2$$
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$$\sim M^{2-\frac{p}{2}} e^{-2\pi M\sqrt{1-\frac{1}{2k}}} = M^{2-\frac{p}{2}} e^{-2\pi M \frac{\beta_{Hg}}{2}}$$

- Saddle point approximation is used as M → ∞
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Tachyon-radion correspondence

- We can sum over all the final states

$$N = \sum_M N(M) \sim \int dM \rho(M) N(M) \sim \int \frac{dM}{M} M^{-p/2}$$

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- Density of states exactly cancels with the radiation density → shows the same behavior in rolling tachyon (Lambert-Liu-Maldacena)

> Tachyon-radion correspondence is true at the stringy level.

- Remarkable cancellation of stringy corrections → universal property of rolling (falling) D-brane?

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Evaluation changes drastically at k=1 (BH/String transition)

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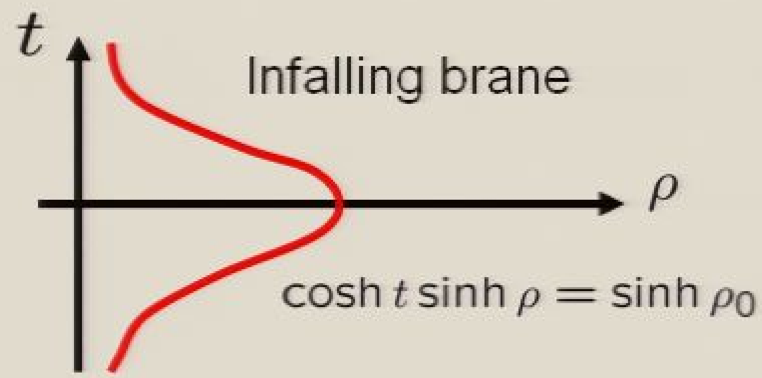

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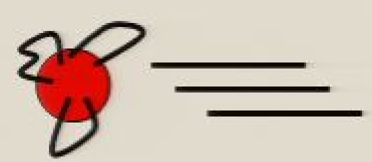

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Unitarity and Open/Closed duality (NRS: hep-th/0605013)

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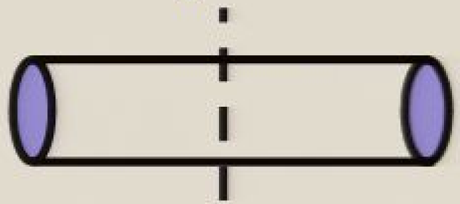
$$\mathrm{Im} Z_{cylinder} = \quad \langle B|$$  $$|B\rangle$$

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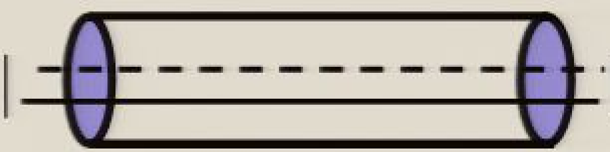
$$\mathrm{Im} Z_{cylinder} \stackrel{?}{=} \langle B|$$  $$|B\rangle$$

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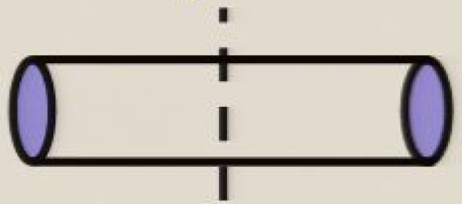
$\mathrm{Im} Z_{cylinder} = \quad \langle B|$  $|B\rangle$

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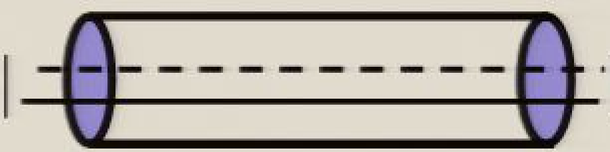
$\mathrm{Im} Z_{cylinder} \stackrel{?}{=} \quad \langle B|$  $|B\rangle$

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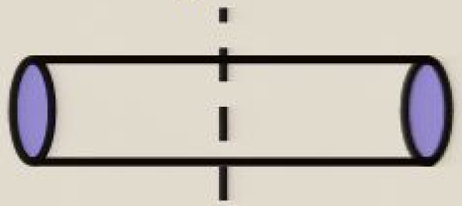

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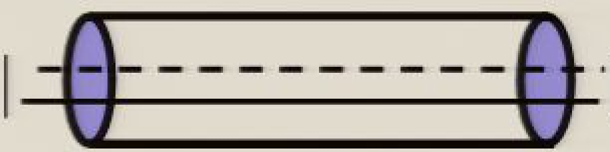

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Unitarity meets open/closed duality

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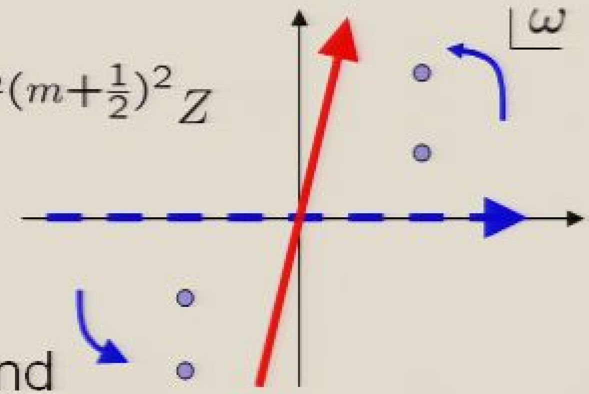

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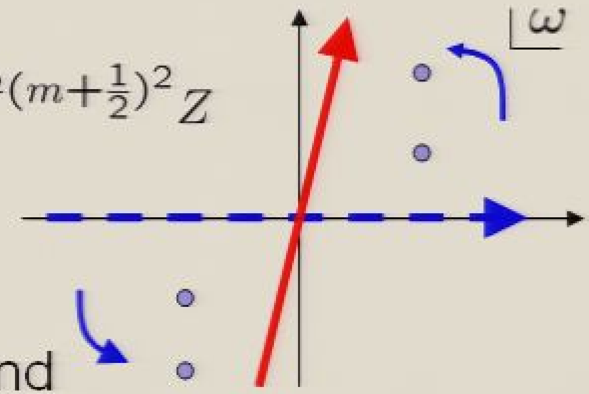

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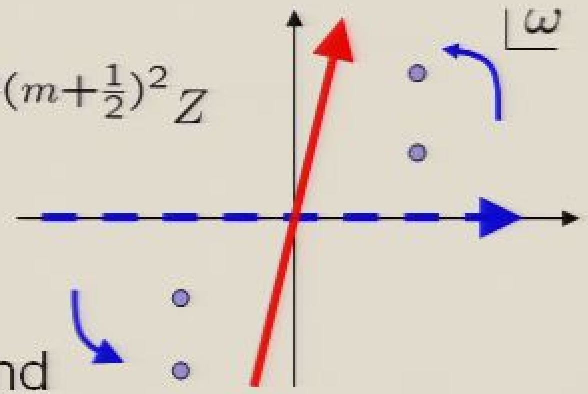
- Pole part comes from poles in Euclidean (Wick) rotation

$$\mathrm{Im} Z_{pole} = 2\int_0^\infty \frac{dt_o}{t_o} \sum_{n=1}^{\infty} \sum_{m=0}^{[kn-\frac{1}{2}]} (-1)^{n+1} e^{-\pi t_o n(2m+1) + \frac{2\pi t_o}{k}(m+\frac{1}{2})^2} Z$$



- Both contributions are imperative to understand
  - Unitarity
  - Tachyon-Radion correspondence
- Summary
  - In Euclidean approach, no apparent reason to include/exclude pole contributions.
  - Unitarity demands its existence, and Lorentzian theory automatically knows it.
  - Fortuitously no pole contribution in rolling tachyon (in 2D ZZ-brane decay as well).

4. Discussion/Summary

Summary and Outlook

- Exact boundary states for rolling D-brane is constructed.
- Tachyon-Radion correspondence is proved in α' exact way.

  → Full proof in string field theory?
- BH/String transition is observed at k=1.

  → Is 2D pure BH really black? Matrix model?
- Consistency between unitarity and open/closed duality requires careful analytic continuation (Wick rotation).

The shortest path between two truths in the real domain passes through the complex domain. ---- J. Hadamard

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Unitarity meets open/closed duality

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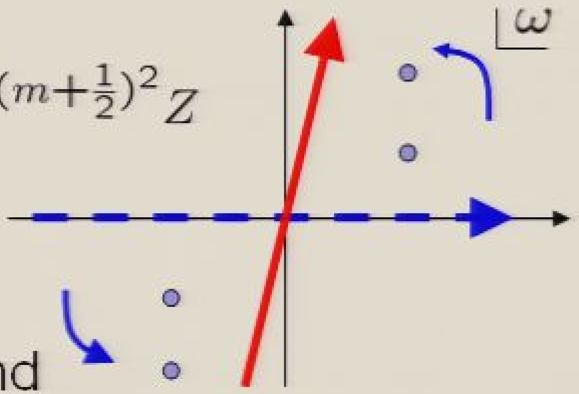

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Open string computation

- Modular transform is (only) well-defined in Lorentzian signature world sheet.

$$\int ds_c \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sinh(\pi p)}{(\cosh(\pi\omega)+\cosh(\pi p))\sinh(\pi p/k)} q^{\frac{1}{2k}(p^2-\omega^2)} Z_{osc} \quad \text{closed}$$

$$= \int \frac{ds_o}{s_o} \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sinh(\pi\omega)}{(\cosh(\pi p)+\cosh(\pi\omega))\sinh(\pi\omega/k)} \tilde{q}^{\frac{1}{2k}(p^2-\omega^2)} Z_{osc} \quad \text{open}$$

- Imaginary part consists of two parts

$$\mathrm{Im} Z_{cyl} = \mathrm{Im} Z_{naive} + \mathrm{Im} Z_{pole}$$

- Naïve part corresponds to contribution easily guessed in the Euclidean approach (but not enough)

$$Z_{naive} = \int_0^\infty \frac{dt_o}{t_o} \int_0^\infty dp \int_{-\infty}^\infty d\omega \frac{\sin(\pi\omega)\tilde{q}^{\frac{1}{2k}(\omega^2+p^2)}}{(\cos(\pi\omega)+\cosh(\pi p))\sin(\pi\omega/k)} Z_{osc}$$

$$\mathrm{Im} Z_{naive} = \sum_{n=1}^\infty \int \frac{dt_o}{t_o} \int_{-\infty}^\infty dp \frac{(-1)^{n+1}\sin(\pi nk) e^{-2\pi t_o(\frac{p^2}{2k}+\frac{kn^2}{2})}}{\cos(\pi nk)+\cosh(\pi p)} Z_{osc}$$

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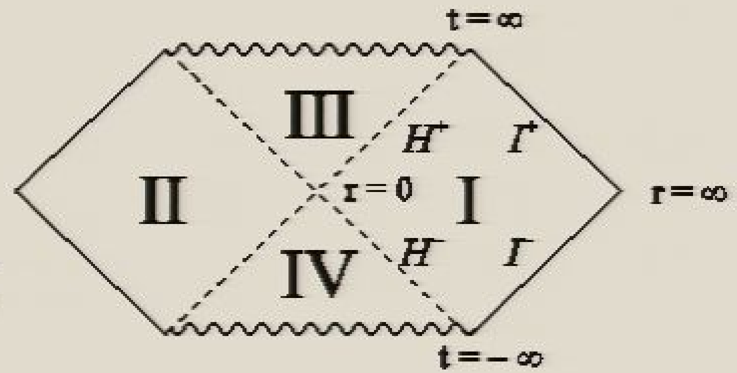
2D (fermionic) Black Hole

- 2D black hole is the simplest black hole geometry as an exact string background (Witten; Mandal, Senguputa, Wadia) $SL(2,R)_k/U(1)$ $\quad c = 3 + \frac{6}{k}$

$$ds^2 = 2k(d\rho^2 - \tanh^2 \rho dt^2) \quad e^{\Phi} = \frac{e^{\Phi_0}}{\cosh \rho}$$

- Global metric looks Schwarzshild-like

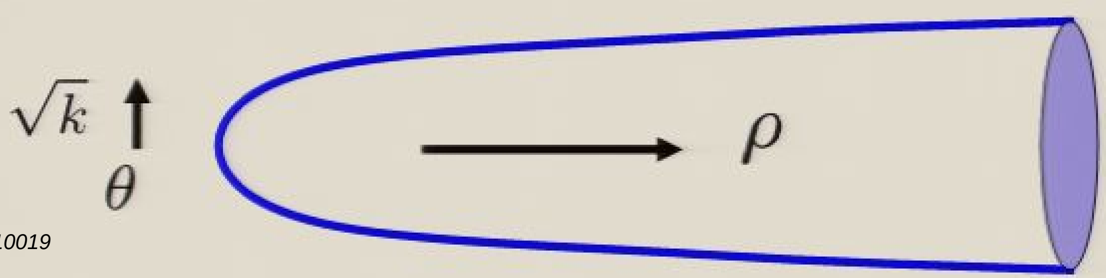
$$u = \sinh \rho e^t \quad v = -\sinh \rho e^{-t} \quad ds^2 = -2k \frac{dudv}{1-uv}$$



- In Euclidean geometry, 2D black hole is cigar geometry:

$$ds^2 = 2k(d\rho^2 + \tanh^2 \rho d\theta^2) \quad e^{\Phi} = \frac{e^{\Phi_0}}{\cosh \rho}$$

$$T_{Hw} = \frac{1}{\beta_{Hw}} = \frac{1}{2\pi\sqrt{2k}}$$



$$T_{Hg} = \frac{1}{\beta_{Hg}} = \frac{1}{4\pi\sqrt{1 - \frac{1}{2k}}}$$