Title: Linear optics quantum information and Quantum simulation of many-body

Date: Nov 15, 2006 04:00 PM

URL: http://pirsa.org/06110012

Abstract: In this talk, I will show how to efficiently generate graph states
based on realistic linear optics (with imperfect photon detectors and source), how to do scalable quantum computation with probabilistic atom photon
interactions, and how to simulate strongly correlated many-body physics with ultracold atomic gas.

Scalable quantum information with linear optics

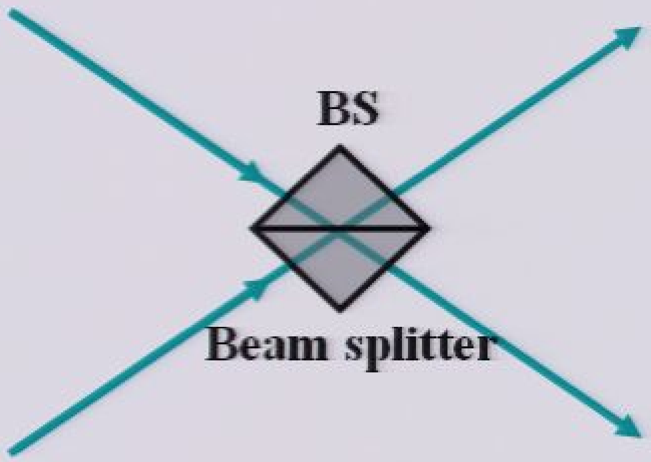
Quantum simulation of many-body physics with ultracold atoms

Luming Duan

FOCUS center and MCTP, University of Michigan

FOCUS MCTP Michigan Center for Theoretical Physics ARDA Sloan NSF

Linear optics device



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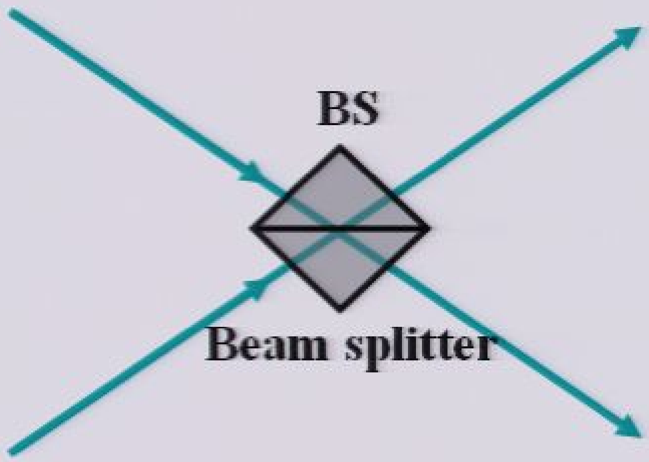

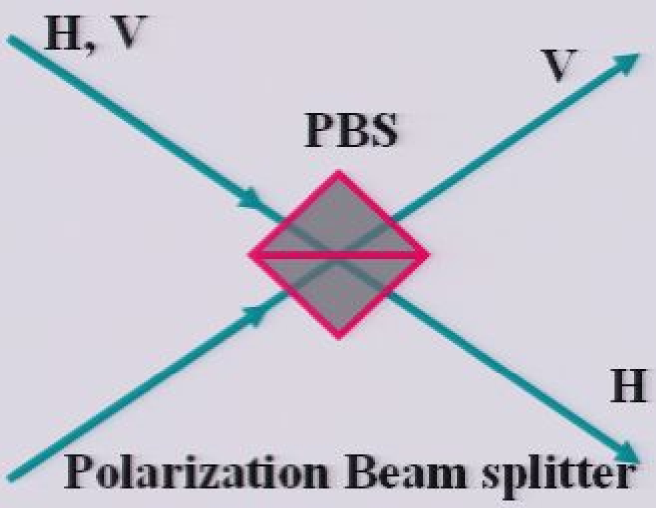

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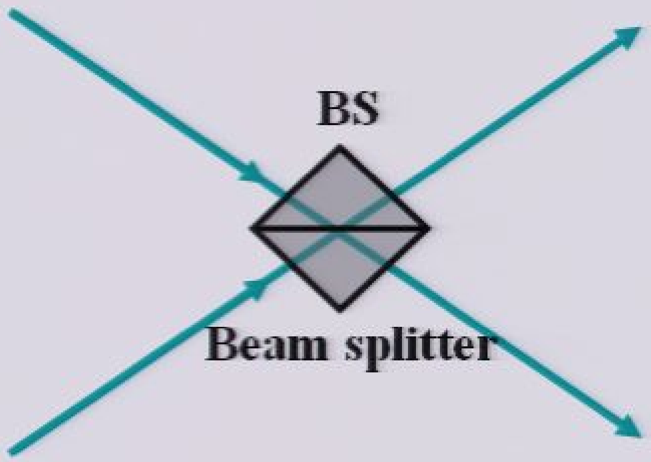
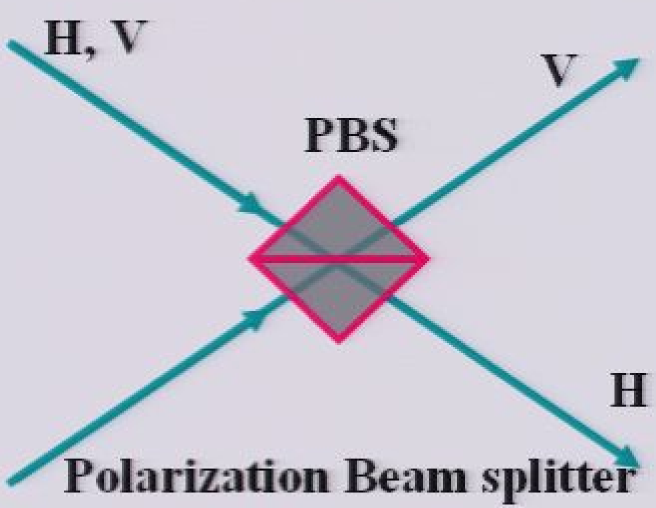
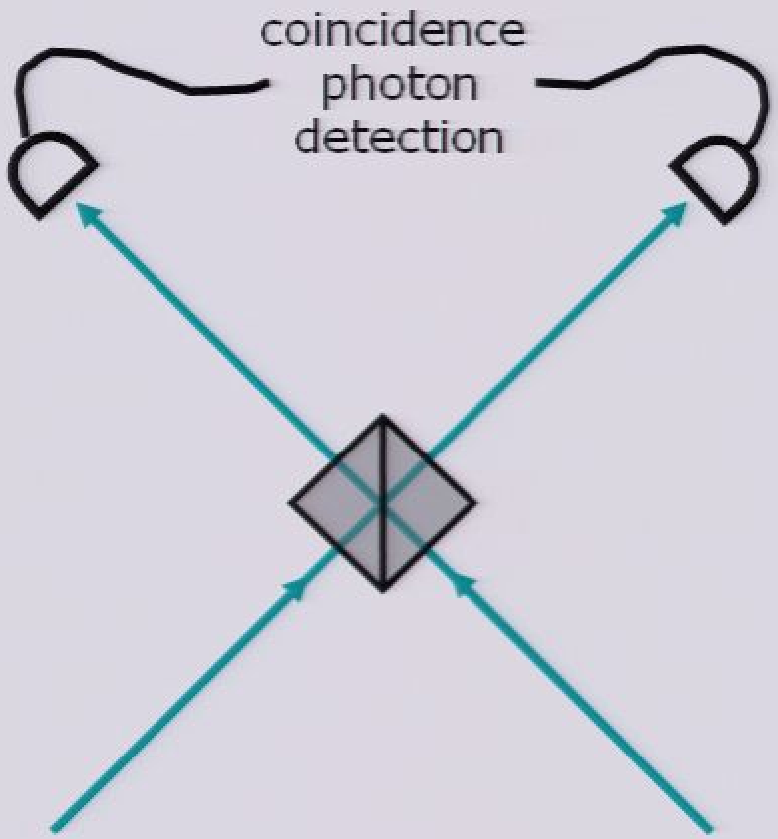




Post-selection by single-photon detectors



State evolution in
Post-selected subspace

Nontrivial!

But generally inefficient (non-scalable)!

Make things scalable!

- Linear optics computation scheme (Knill, Laflamme, Milburn, Nature, 2001)
  - Through quantum error correction
  - Require minimum detector efficiency (threshold)
  - Improvements (Nielsen, Yoran, Reznik, Browne, Rudolph etc.)
  - Latest threshold (efficiency > 99.7%, Nielsen group, PRL 2006)

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- Scalable quantum communication (Duan, Lukin, Cirac, Zoller, Nature, 2001)
  - Divide and conquer (quantum repeater)
  - No requirements on minimum detector efficiency
  - More specific purpose

Combine the advantages

- No stringent requirement on detector efficiency
- More general applications

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Results:

- Scalable quantum state engineering
  - ✓ GHZ state (Duan, PRL, 2002)
  - ✓ Any graph states of tree shape (Bodiya, Duan, PRL, 2006)
- Scalable quantum computation
  - ✓ Need atoms
  - ✓ Probabilistic atom-photon interaction
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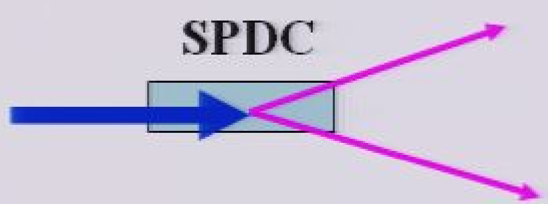
Input state

$$\rho_s = (1-\eta_s)\,\rho_{\text{vac}} + \eta_s\,|\Psi\rangle_{12}\langle\Psi|\,,$$

Source efficiency

$$|\Psi\rangle_{12} = (|HH\rangle_{12} + |VV\rangle_{12})\,/\sqrt{2}$$



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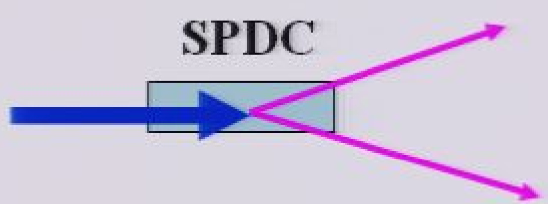

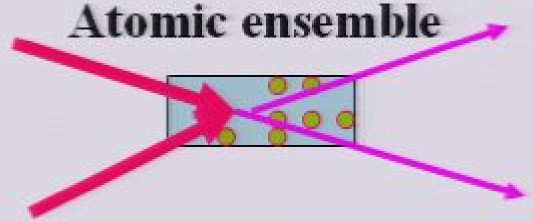

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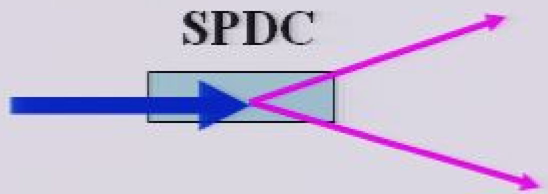
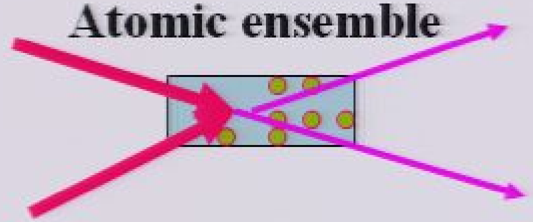
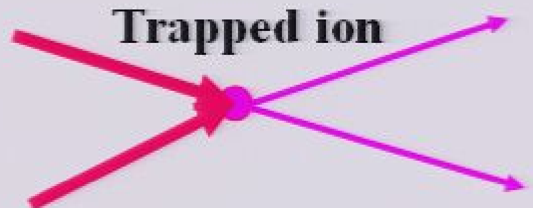
SPDC



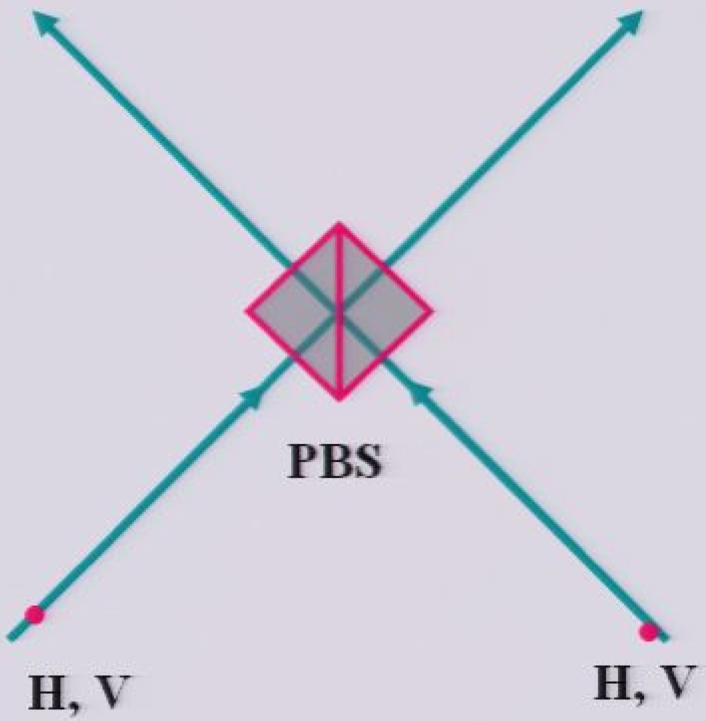
Atomic ensemble



Trapped ion



A PBS gate



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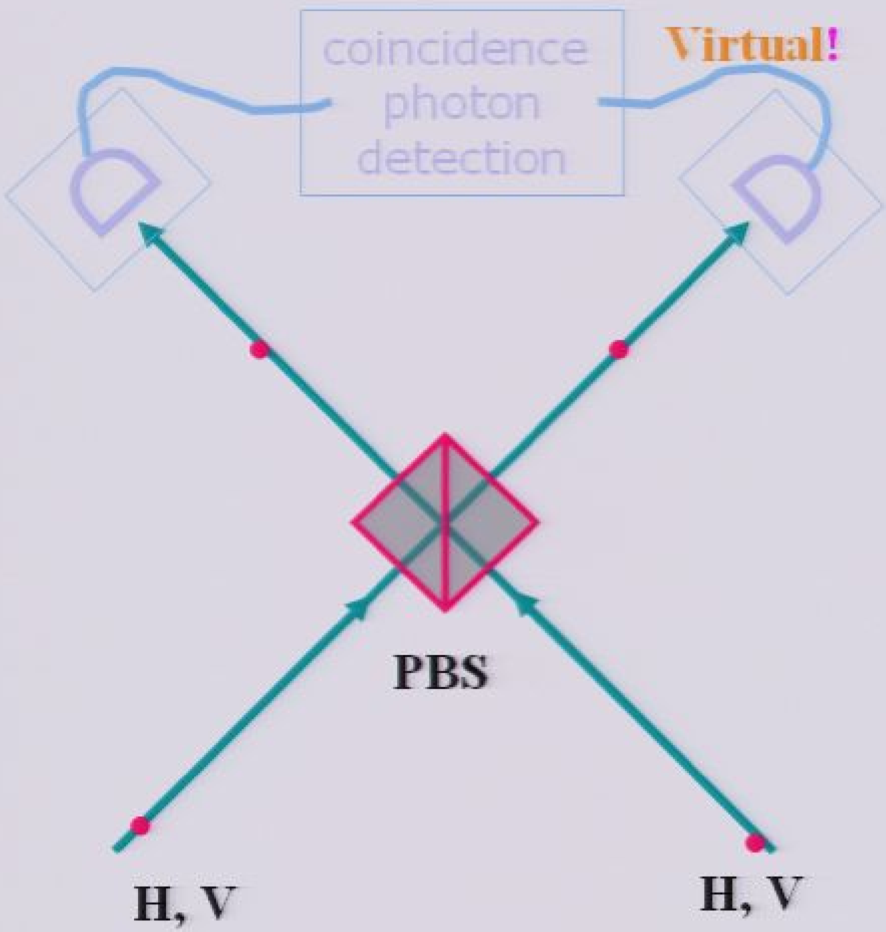

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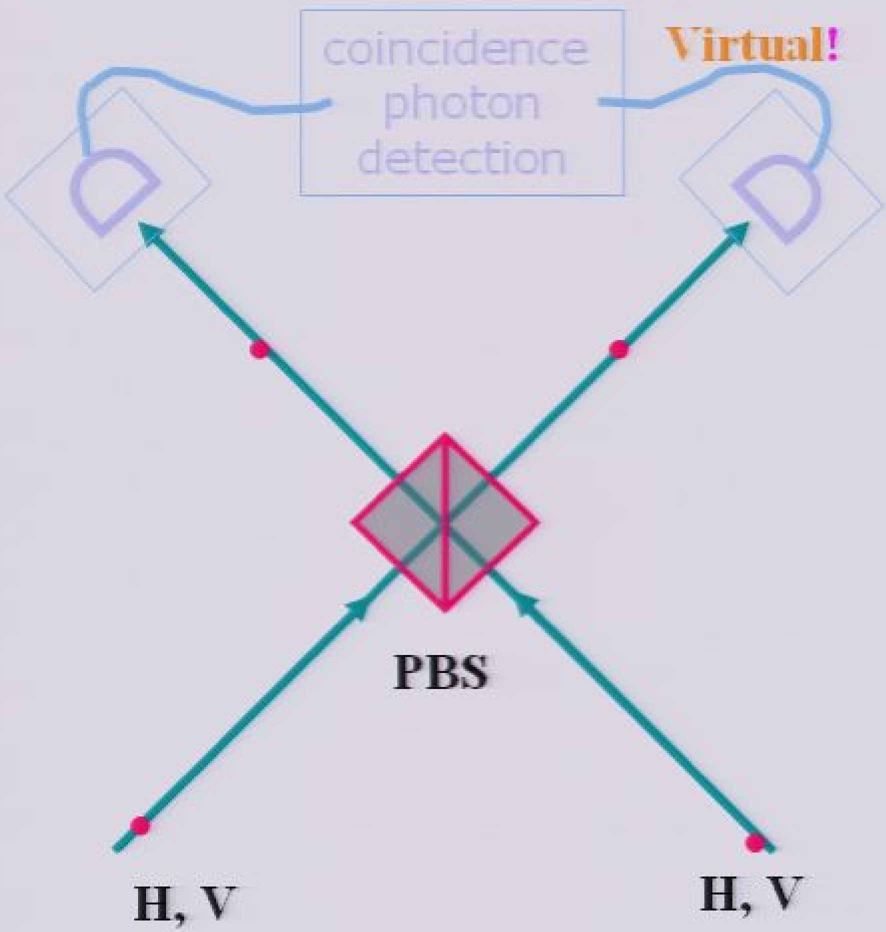

State evolution in Post-selected subspace:

$$P = |HH\rangle_{12}\langle HH| + |VV\rangle_{12}\langle VV|$$

Cluster-state (Graph-state) Quantum computing

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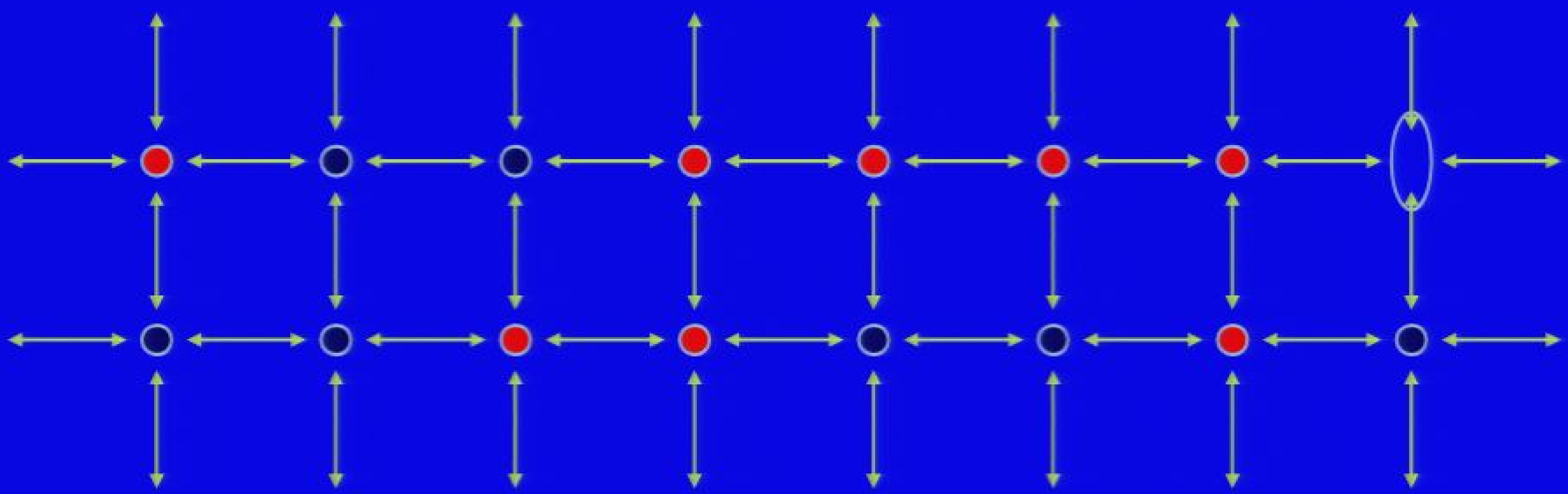
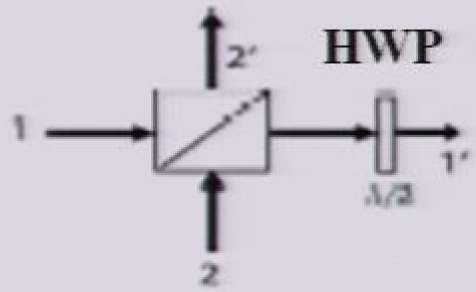
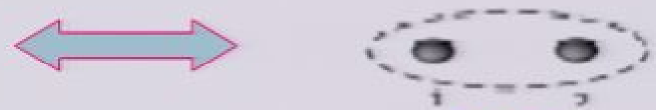


Raussendorf and Briegel, PRL **86**, 910 (2001); PRA (2003).

Construction of graph states with PBS gates





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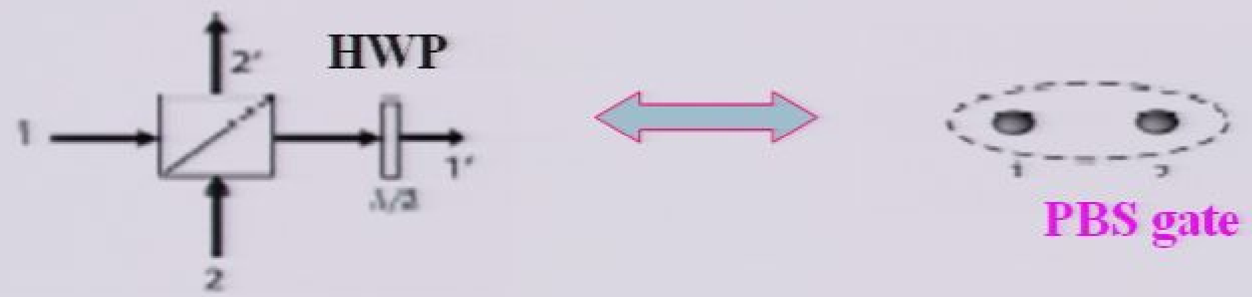

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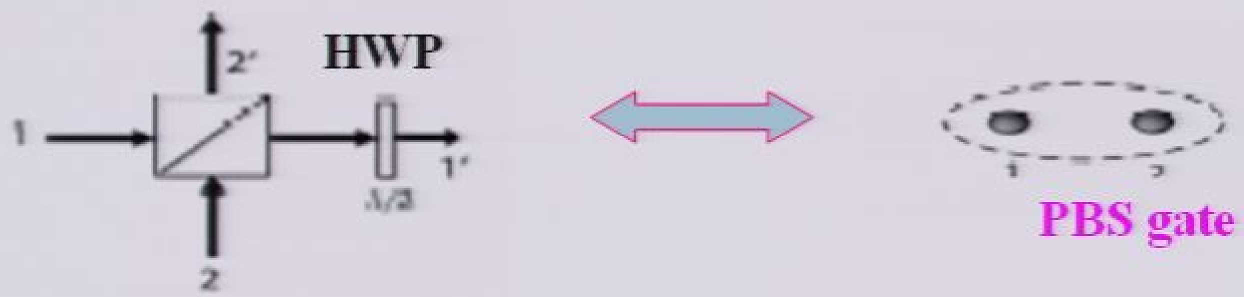
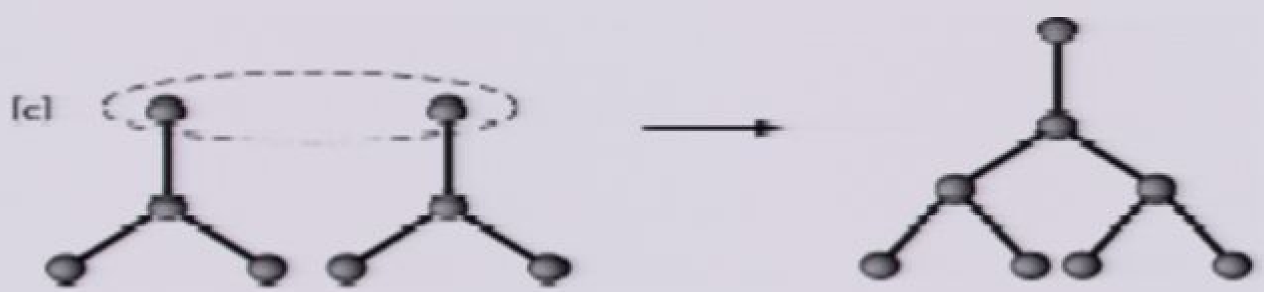
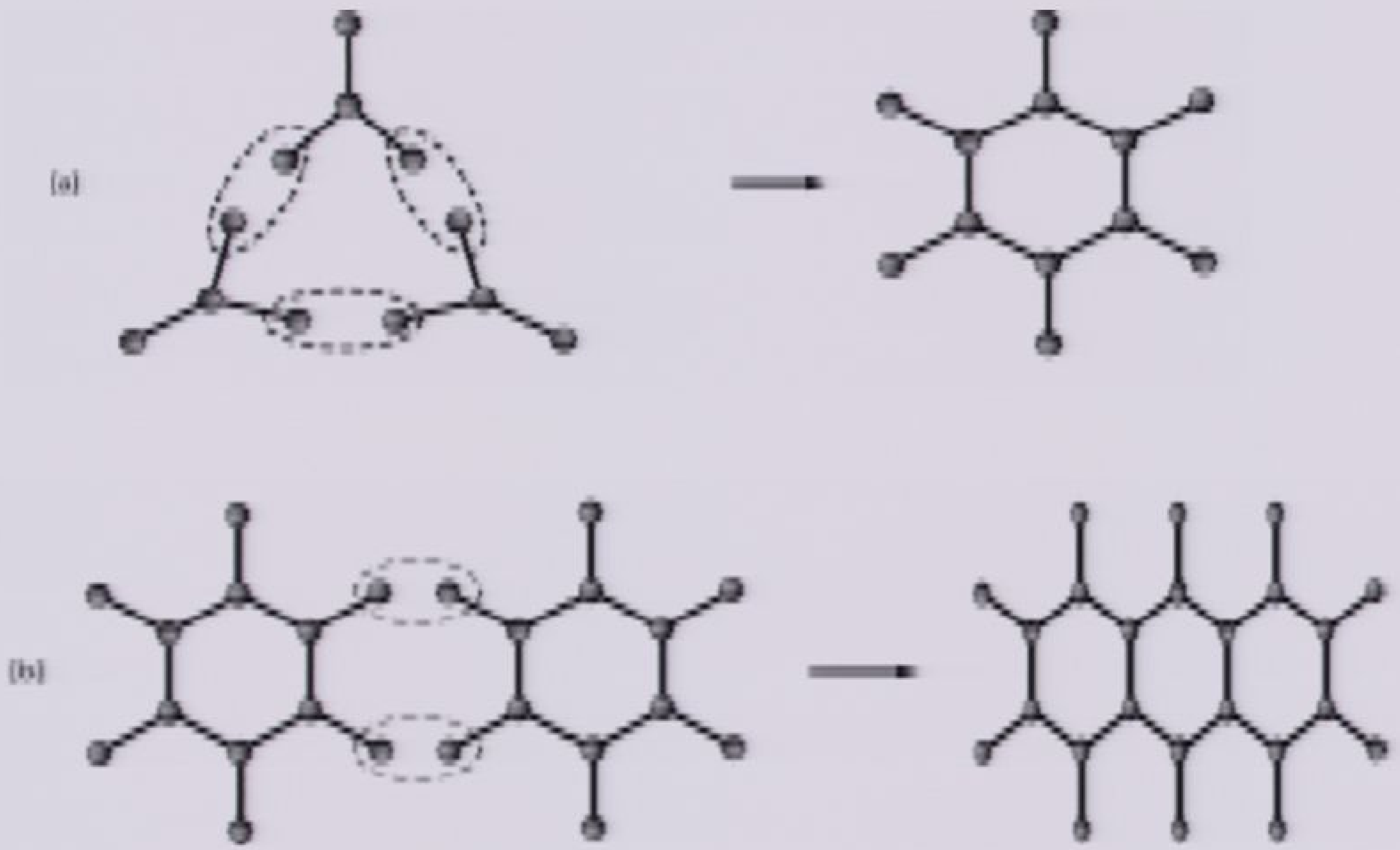




Construction of graph states with PBS gates

(a)

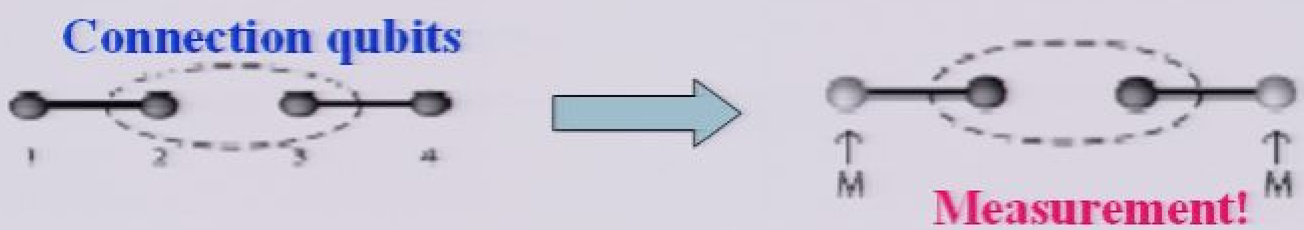
Construction of graph states with PBS gates



More efficient Construction of tree graph states

Connection qubits

More efficient Construction of tree graph states



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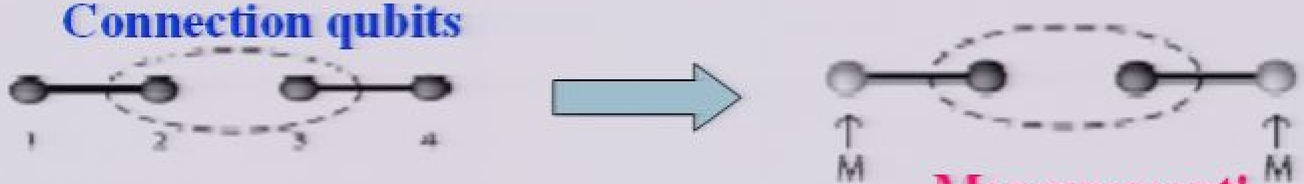
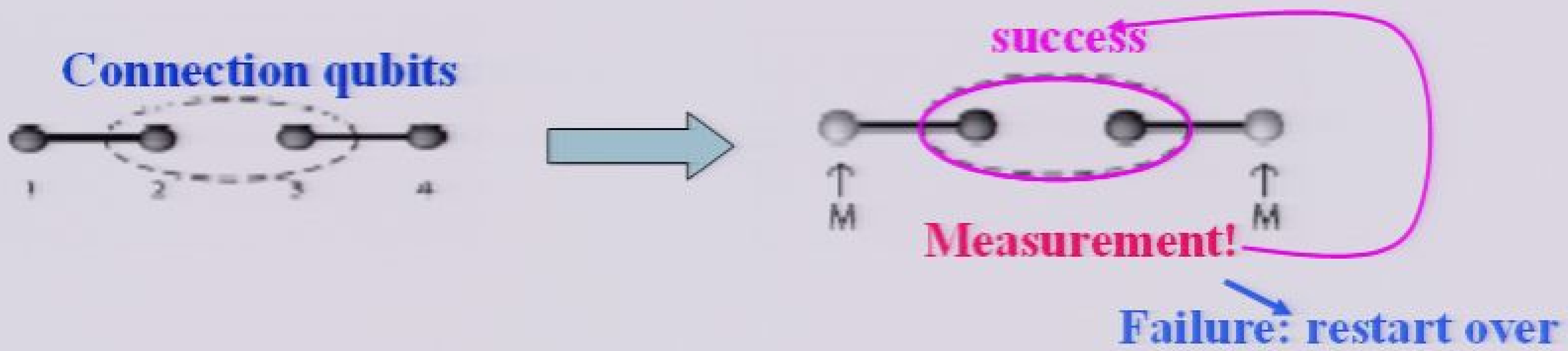


Measurement!

Failure: restart over

More efficient Construction of tree graph states



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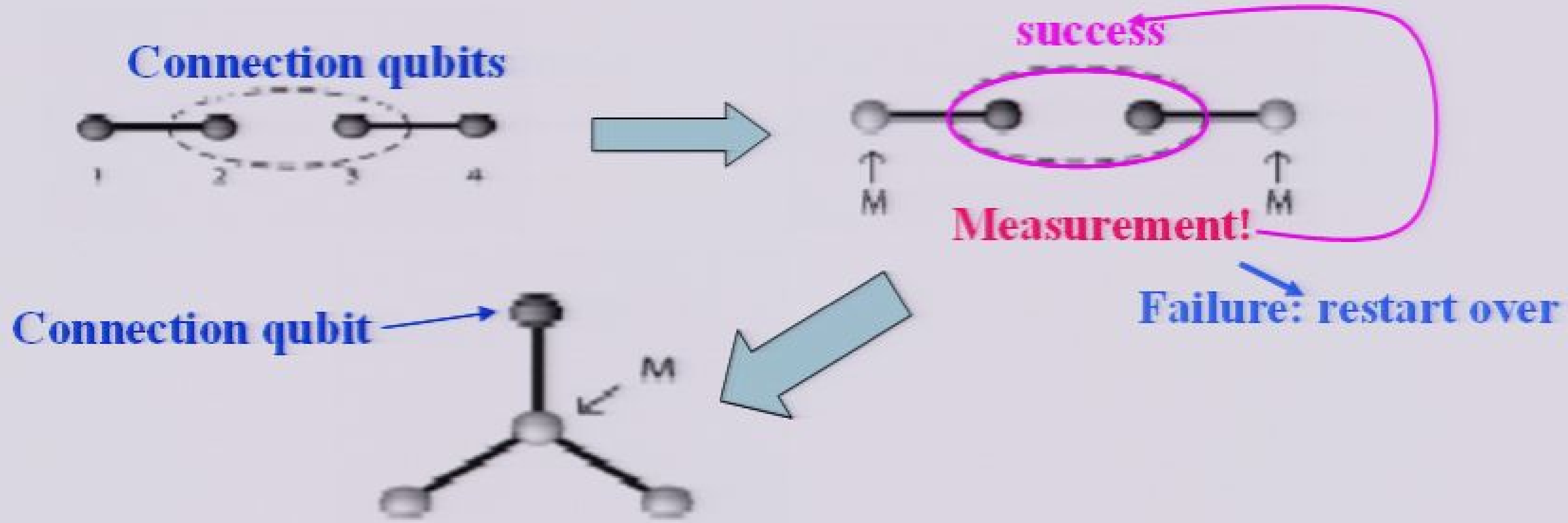

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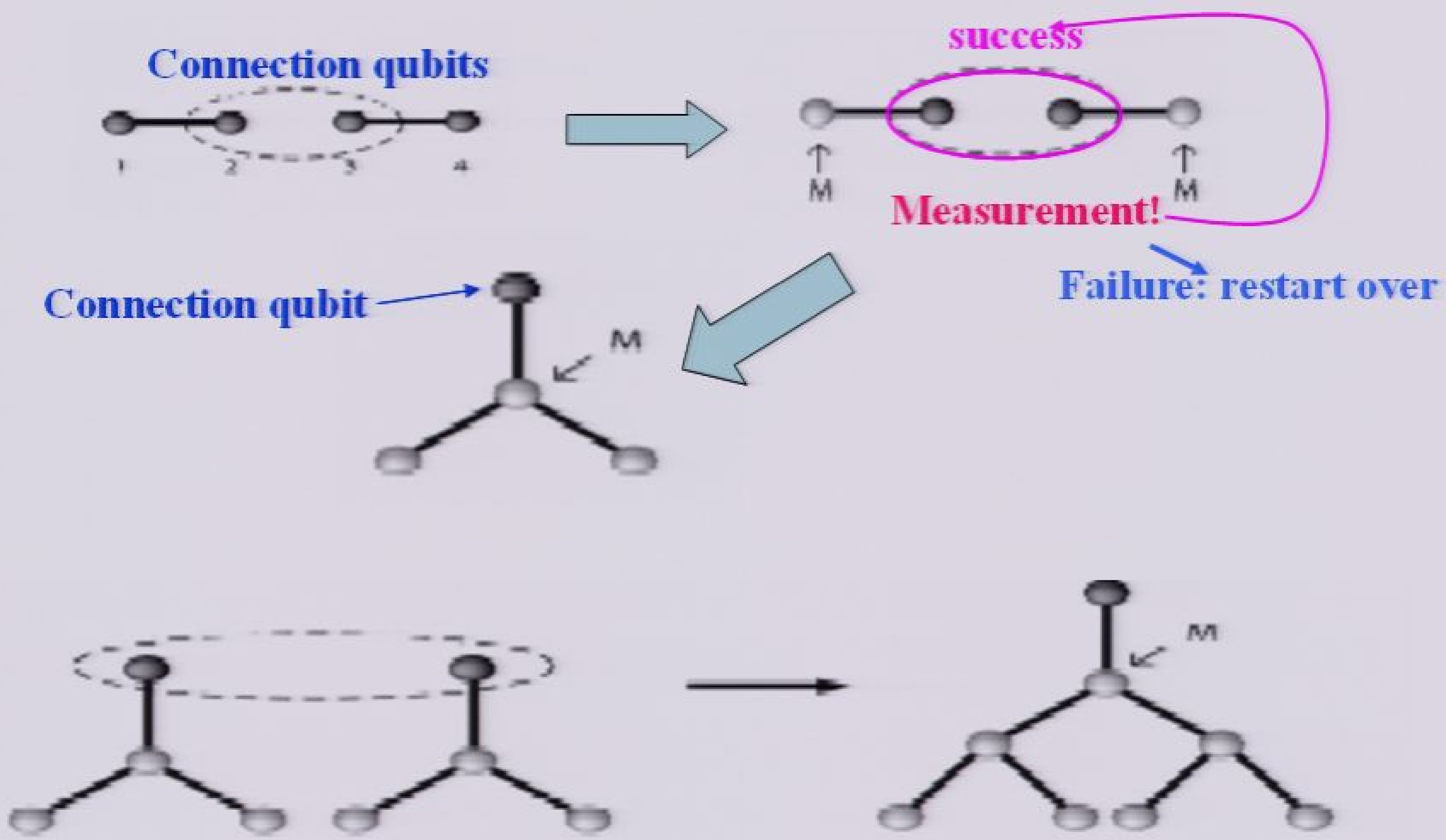

Scaling

Total preparation time:

$$T \approx t_0 \, (\eta_s \eta_d)^{-1} \, n^{[(\log_2 n - 1)/2 + \log_2(1/\eta_d - 1/2)]}$$

number of qubits

detector efficiency

source efficiency

Pulse duration

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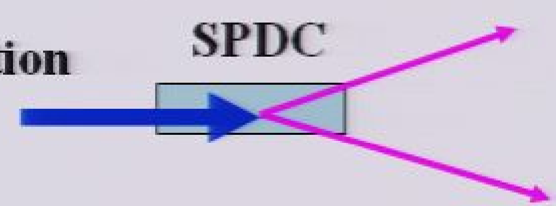

Improvement in efficiency

Example: preparation of 32-qubit graph state

The above method:

$$T/t_0 \sim 3.0 \times 10^7$$

with $\eta_s \sim 1\%$

$$\eta_d \sim 33\%$$

Improvement in efficiency

Example: preparation of 32-qubit graph state

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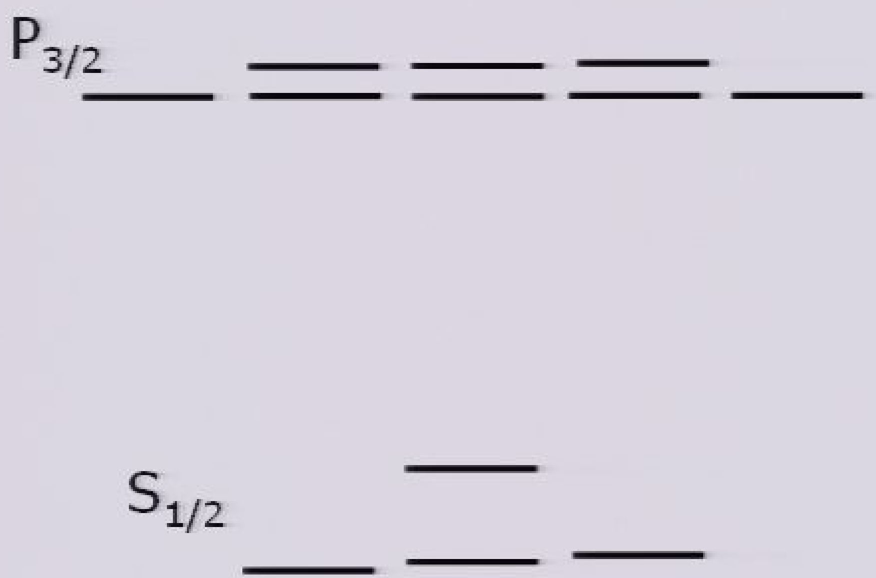
$$\eta_d \sim 33\%$$

Direct preparation:

$$T/t_0 = \eta_s^{-32/2} \eta_d^{-32} 2^{32/2-1} \sim 10^{52}$$

Probabilistic entanglement between 1 ion and 1 photon



Blinov, Moehing, Duan, Monroe Nature 428, 153 (2004).

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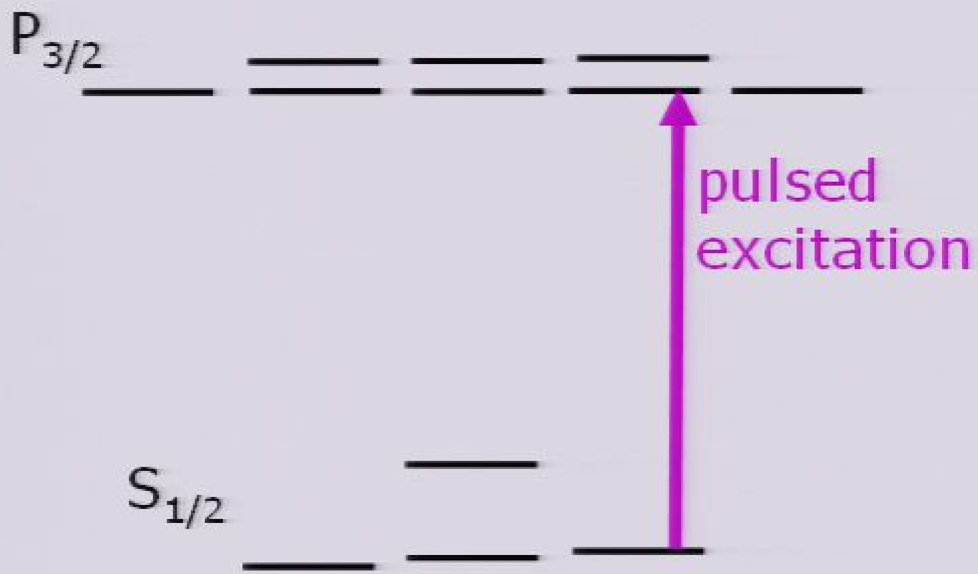

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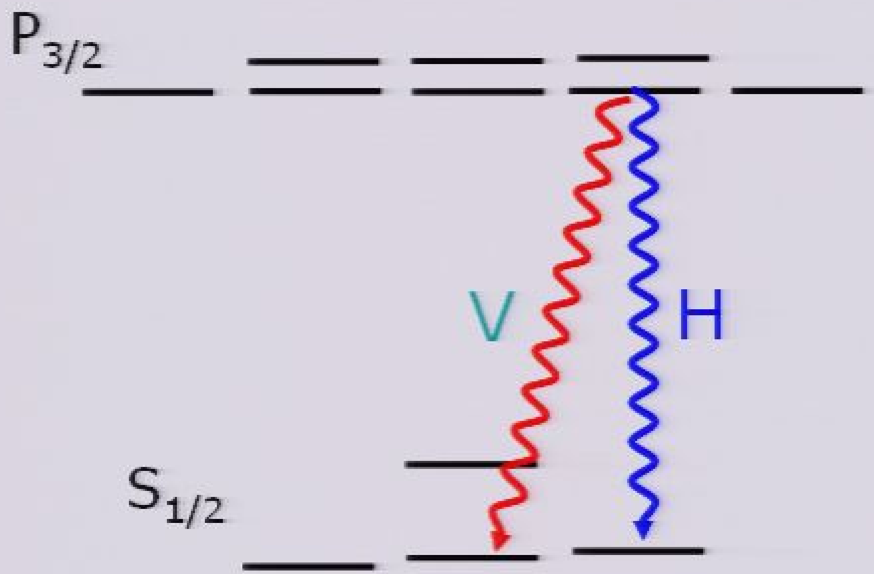

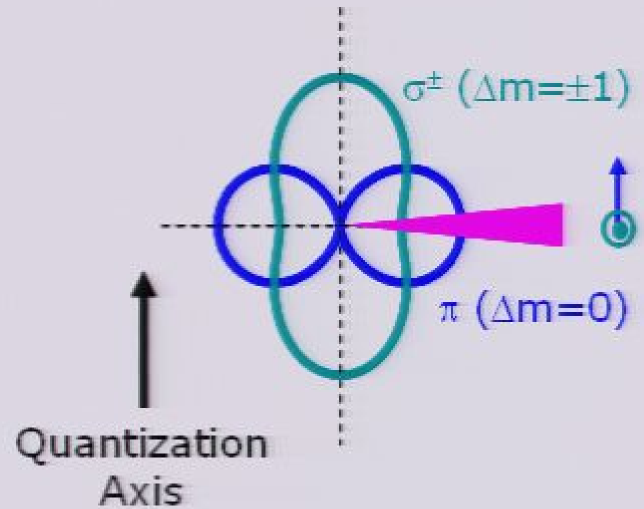

$$|\psi\rangle = |\downarrow\rangle|V\rangle + |\uparrow\rangle|H\rangle$$

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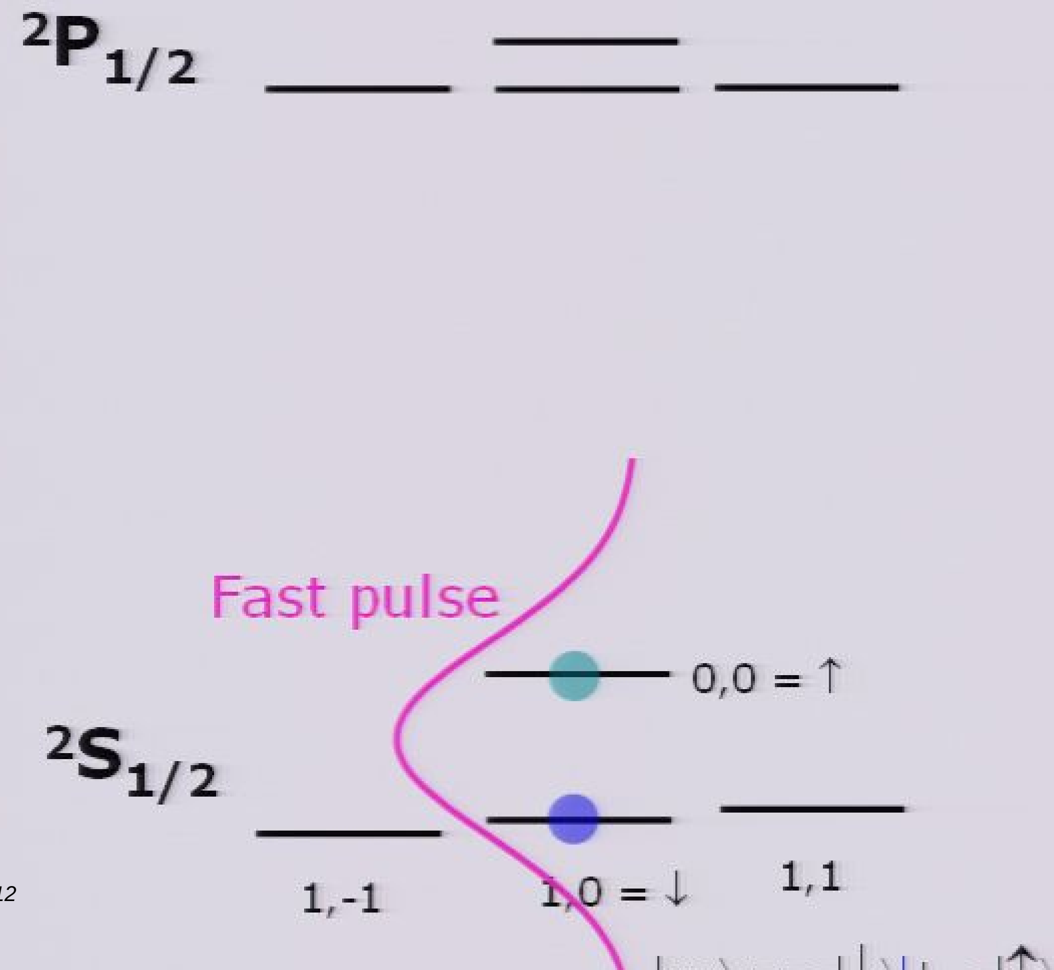
Probabilistic Gate (Madsen et al., PRL, 2006; Duan et al, PRA, 2006) between a single atom and single photon

Frequency Qubit



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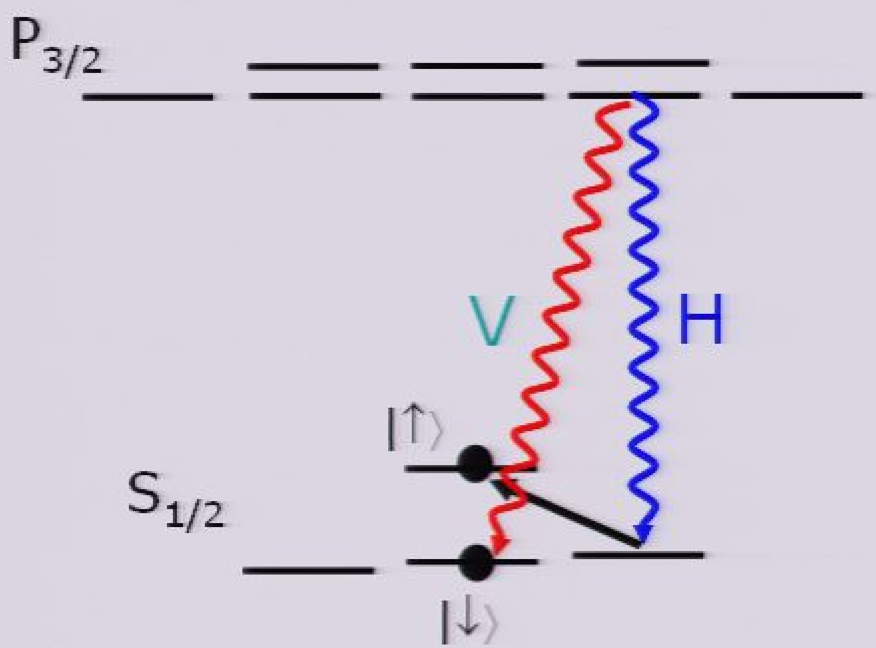

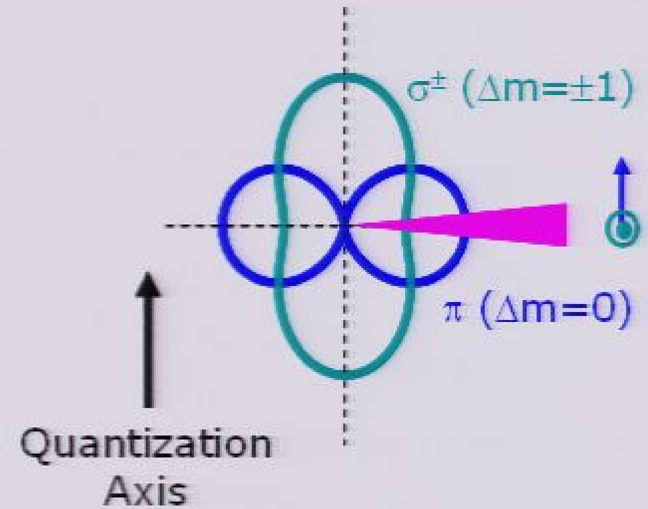

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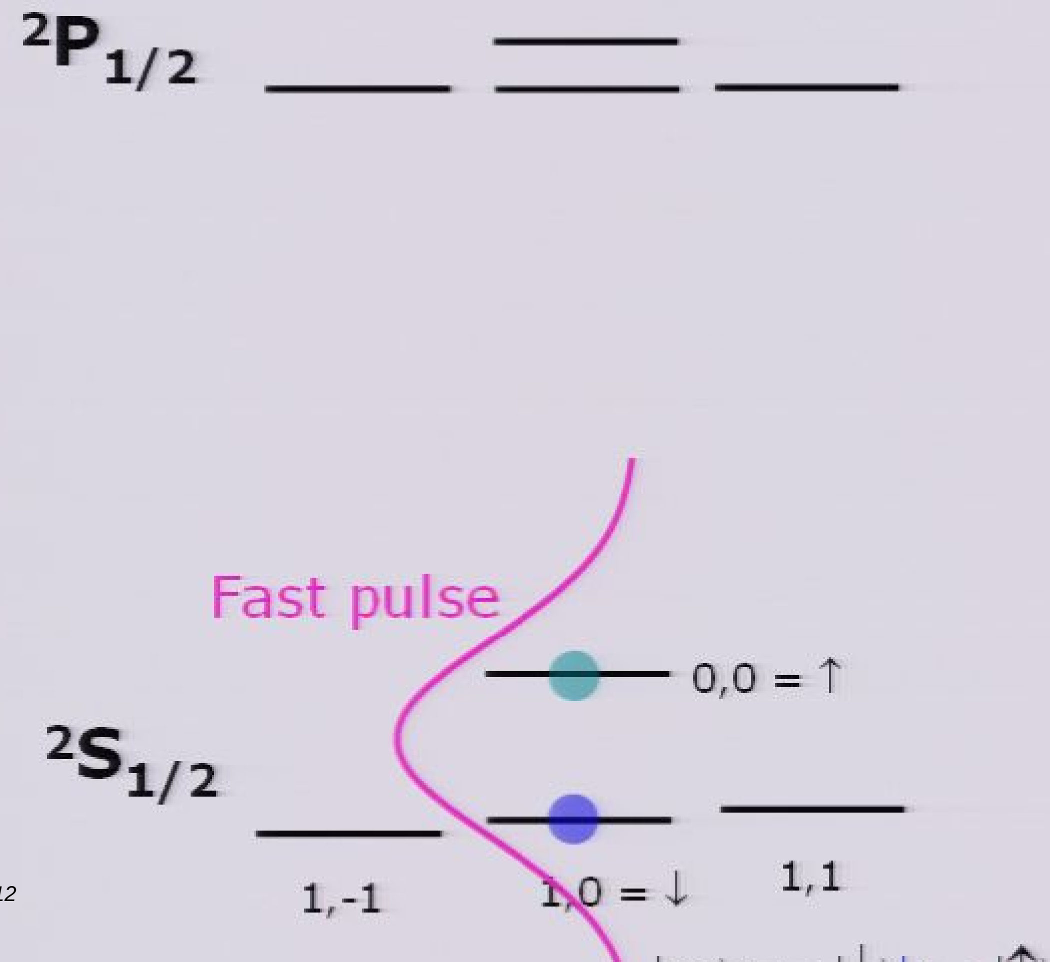

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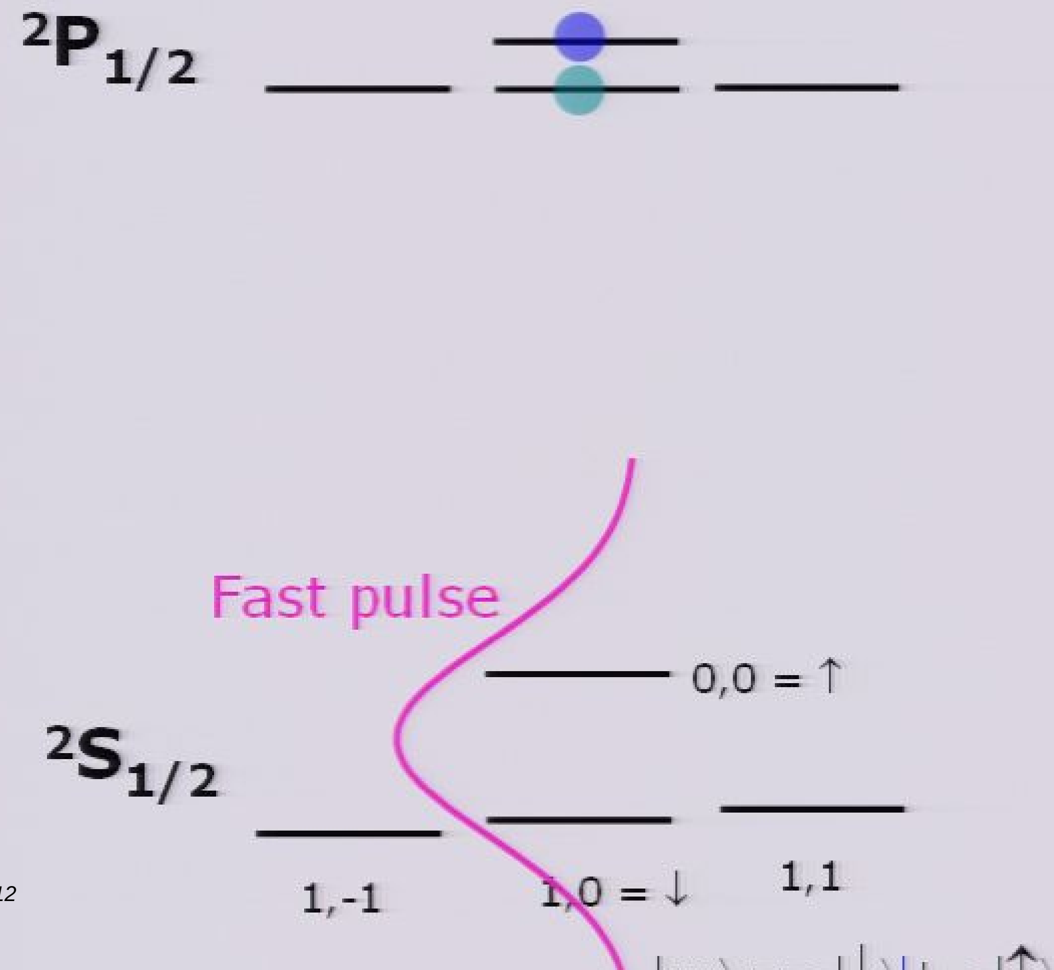

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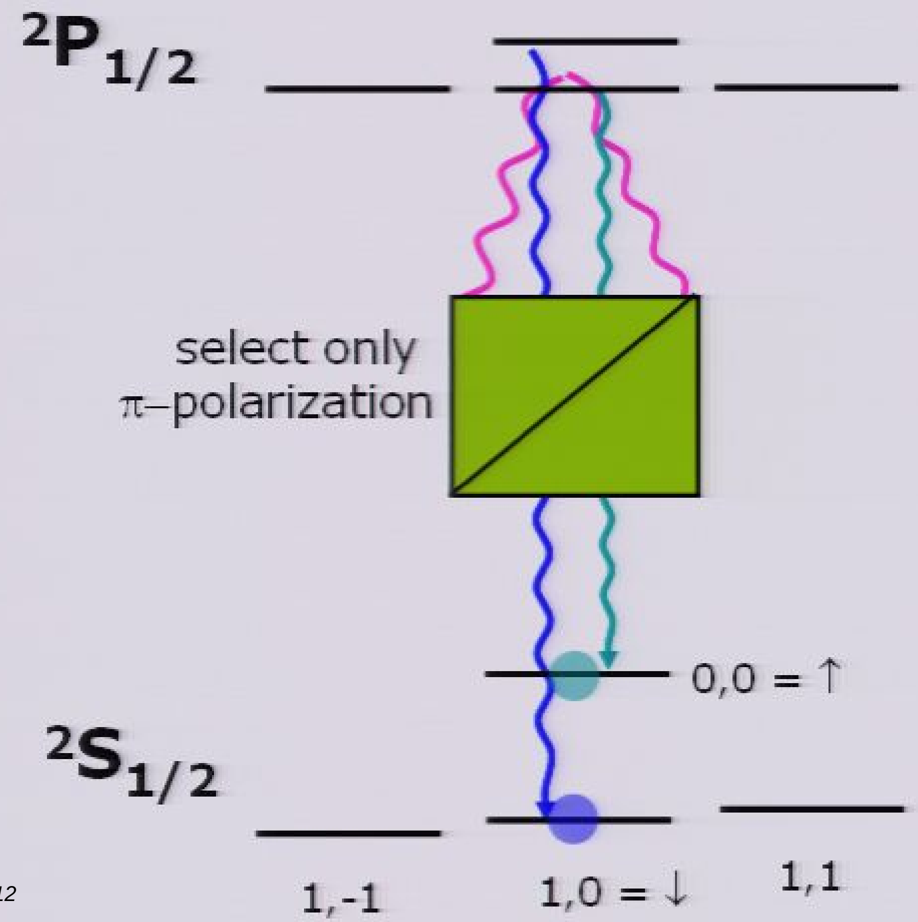

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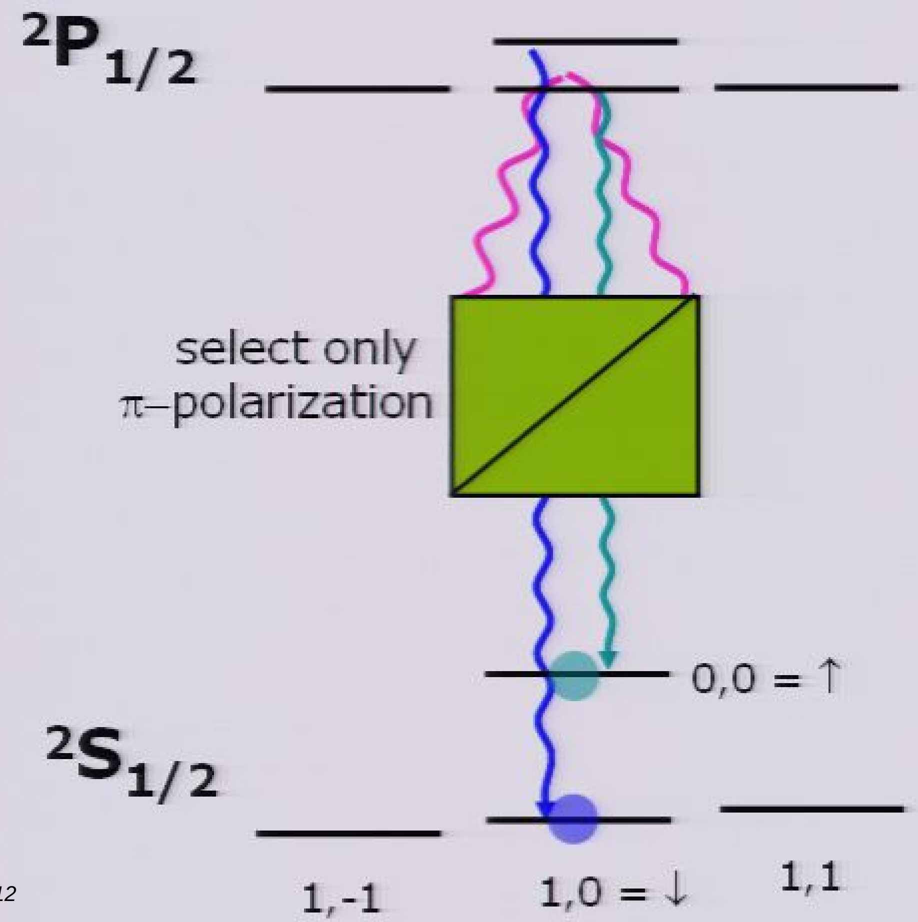


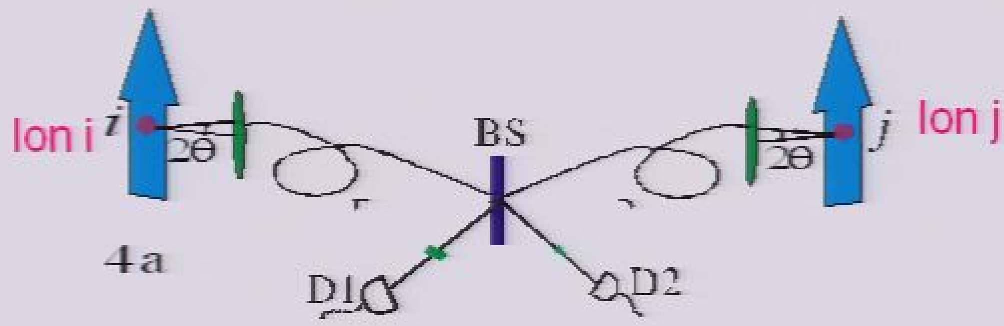
Given photon is emitted into polarizer

$$|\psi\rangle = c_0|\downarrow\rangle|\text{blue}\rangle + c_1|\uparrow\rangle|\text{red}\rangle$$

Keep track of information of the initial state !!

Probabilistic gates on remote ion qubits

Duan et al., PRA, 2006



Joint measurements of emitted photons

- ✓ ions in arbitrary initial states
- ✓ A detection shows one ion in |0> state, and one ion in |1> state with coherence between two possibilities (probabilistic ZZ gate)
- ✓ Probabilistic ZZ is a gate that is universal and efficient for quant. Computation

Efficient computation with probabilistic gates

Problems for computation with probabilistic gates

Exponentially small success probability: p^n

Efficient computation with probabilistic gates

Problems for computation with probabilistic gates

Exponentially small success probability: p^n

Our main result: (Duan, Raussendorf, PRL, 2005)

- **Efficient quantum computation is possible no matter how small the success probability p is.**

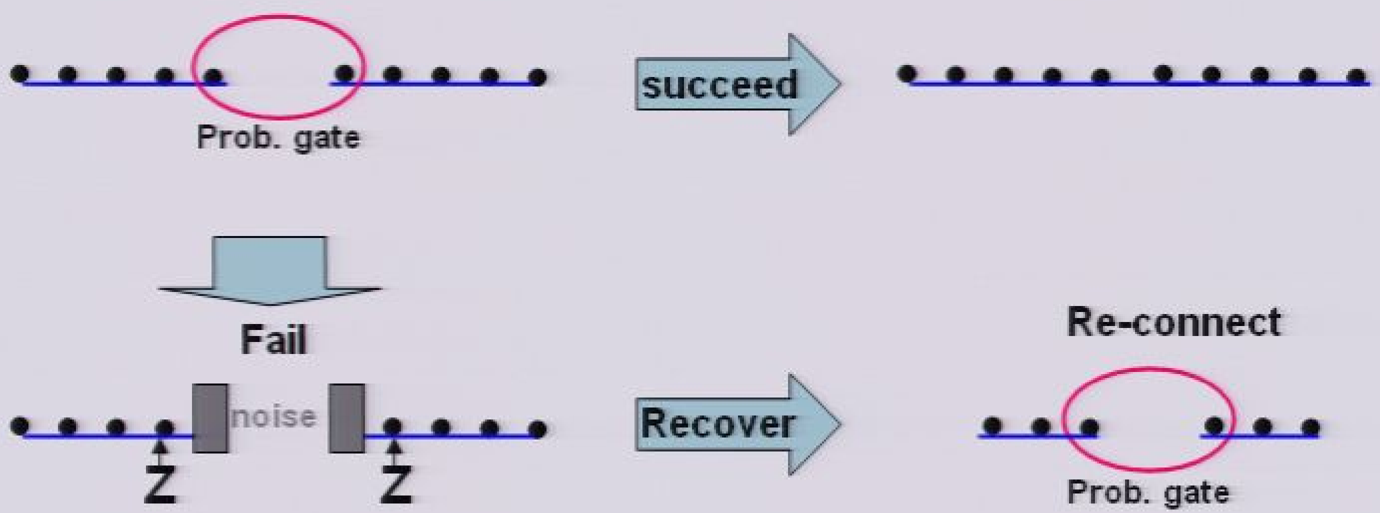
$$p > 0, \text{ or } p_{err} = 1 - p < 100\% = p_{th},$$

- **Computational overheads scale polynomially both with 1/p and n (n: the number of qubits)**

Efficient construction of 1D cluster states

- **Step 1: Start from scratches**

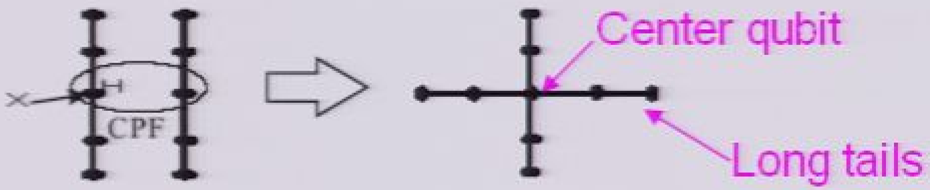
- **Step 2: The cluster chain grows**



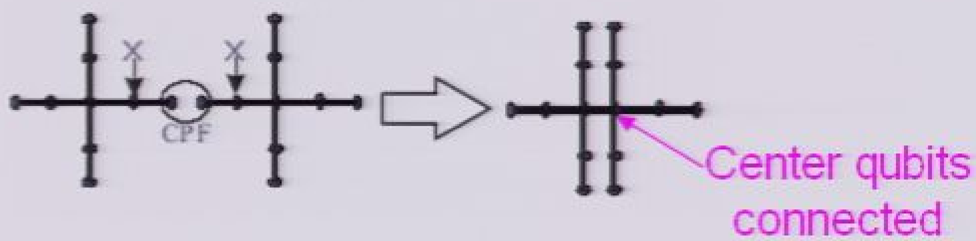
Efficient construction of 2D cluster states

Key idea: transfer the 2D problem to effective 1D connection

- **Step 1: prepare the building blocks ("cross" state)**

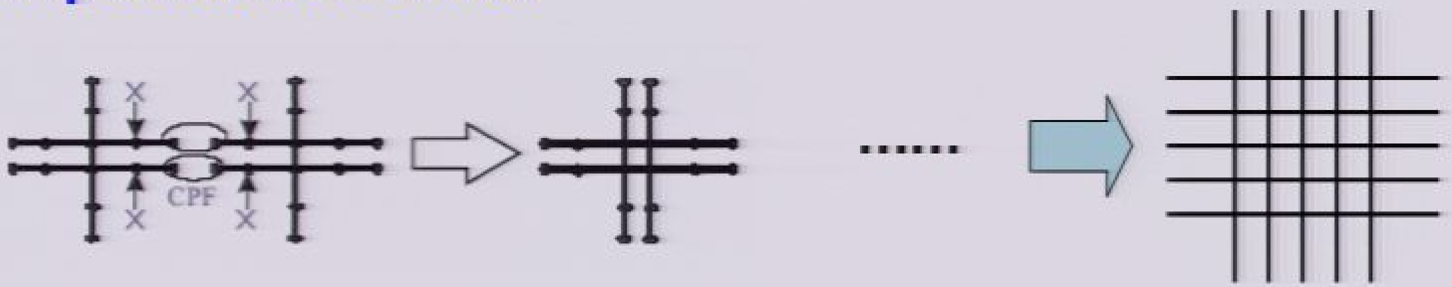


- **Step 2: connect the cross states**



Efficient construction of 2D cluster states

- **Step 3: Form 2D lattice**



Continue connection of center qubits

- **Efficiency**

$$M \propto n \ln(n) \left(\frac{2}{p} \right)^{\log_2(4/p)+2}$$

Summary

- **Quantum state engineering**

  - ✓ A single PBS is a powerful gate in post-selected subspace.
  - ✓ Any graph states can be constructed with PBS gates
  - ✓ Tree graph states can be constructed efficiently with any low detector efficiency

- **Quantum computation**

  - ✓ Probabilistic gates on atoms with linear optics
  - ✓ Scalable computation is possible with probabilistic gates

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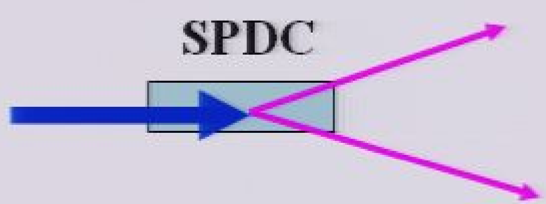
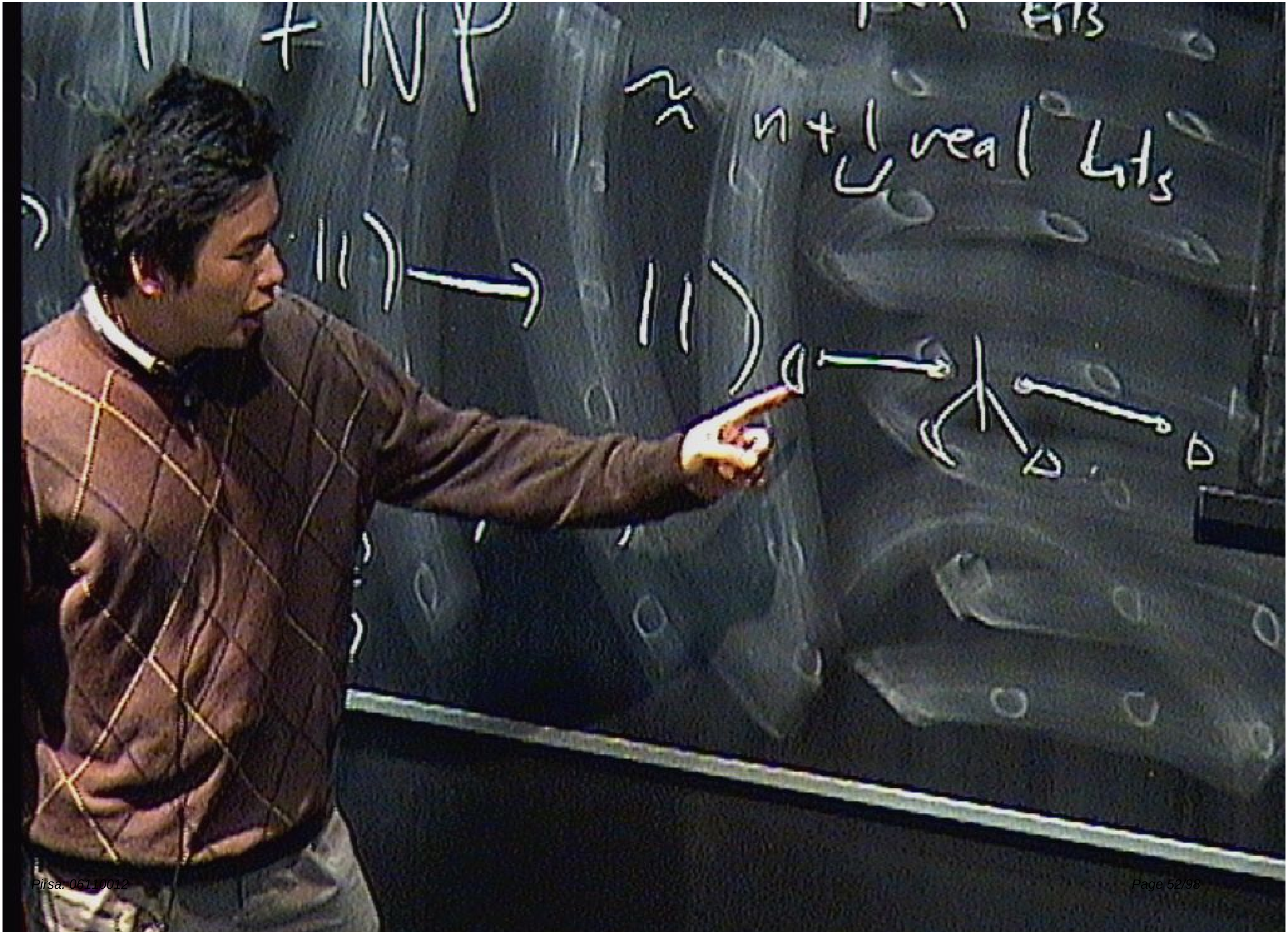
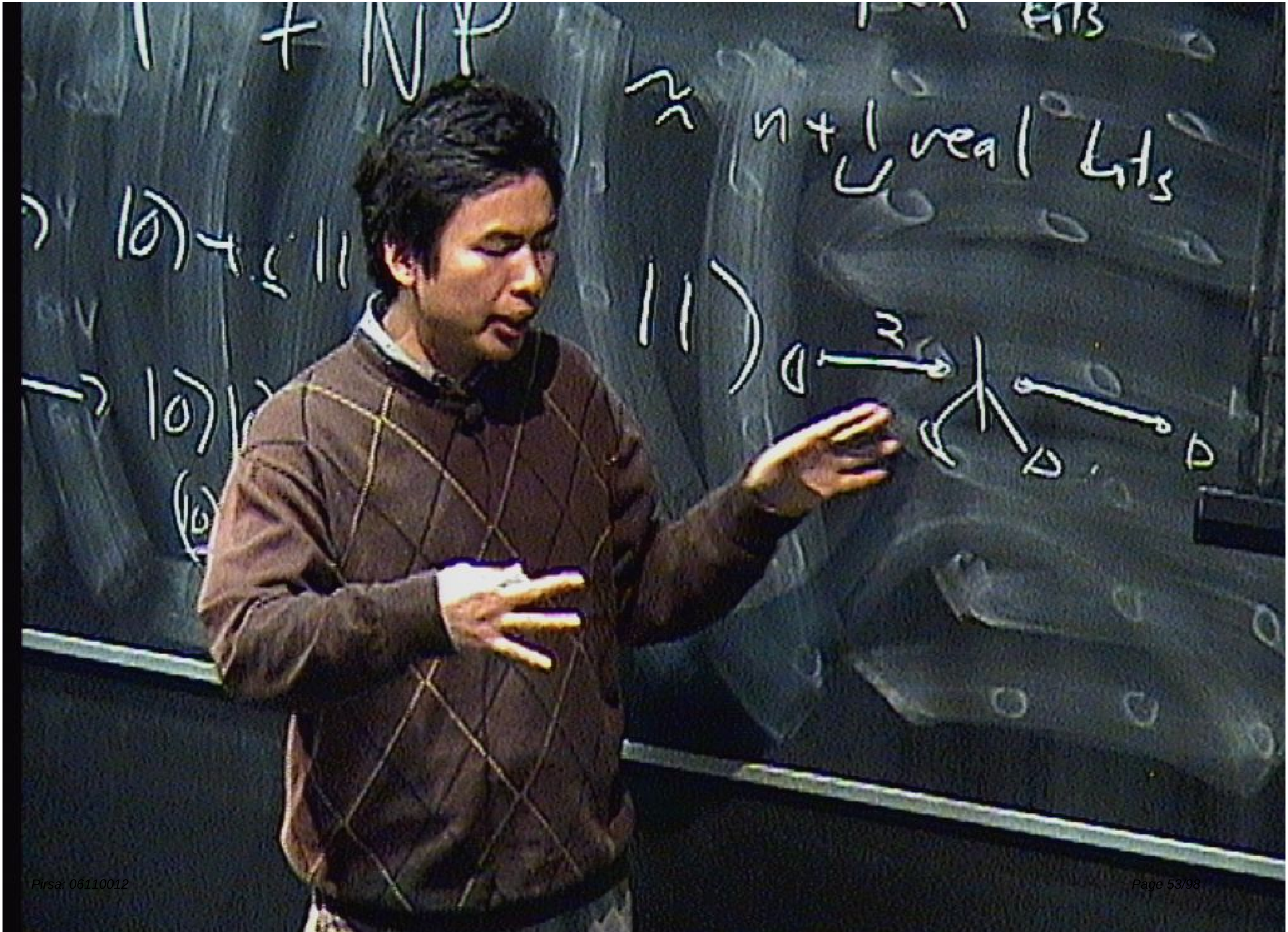



n+1 real bits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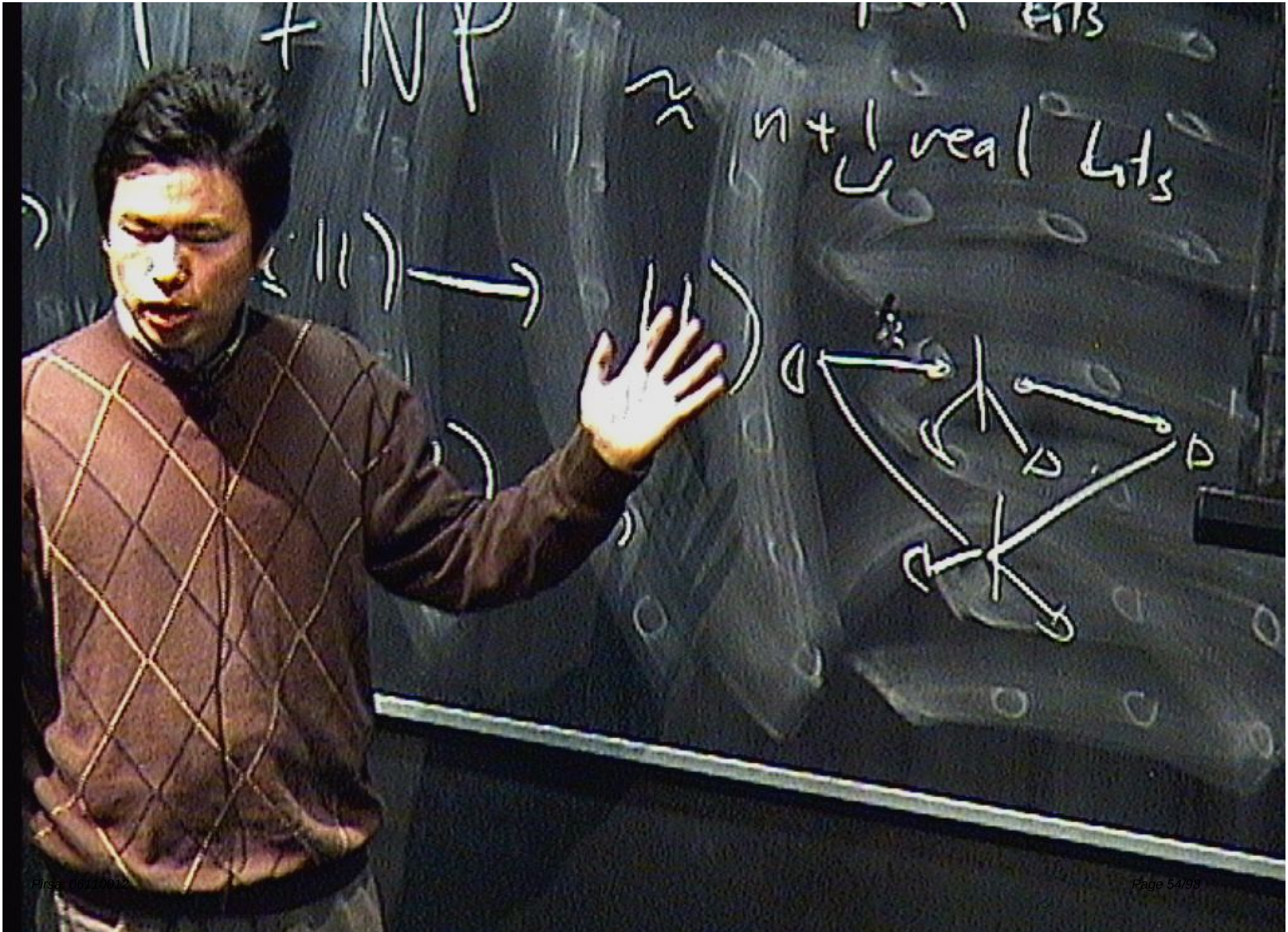
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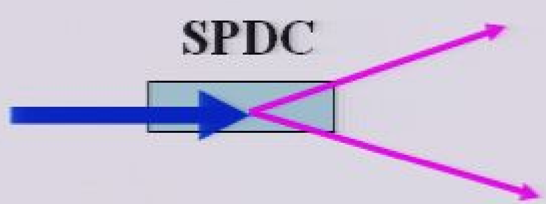

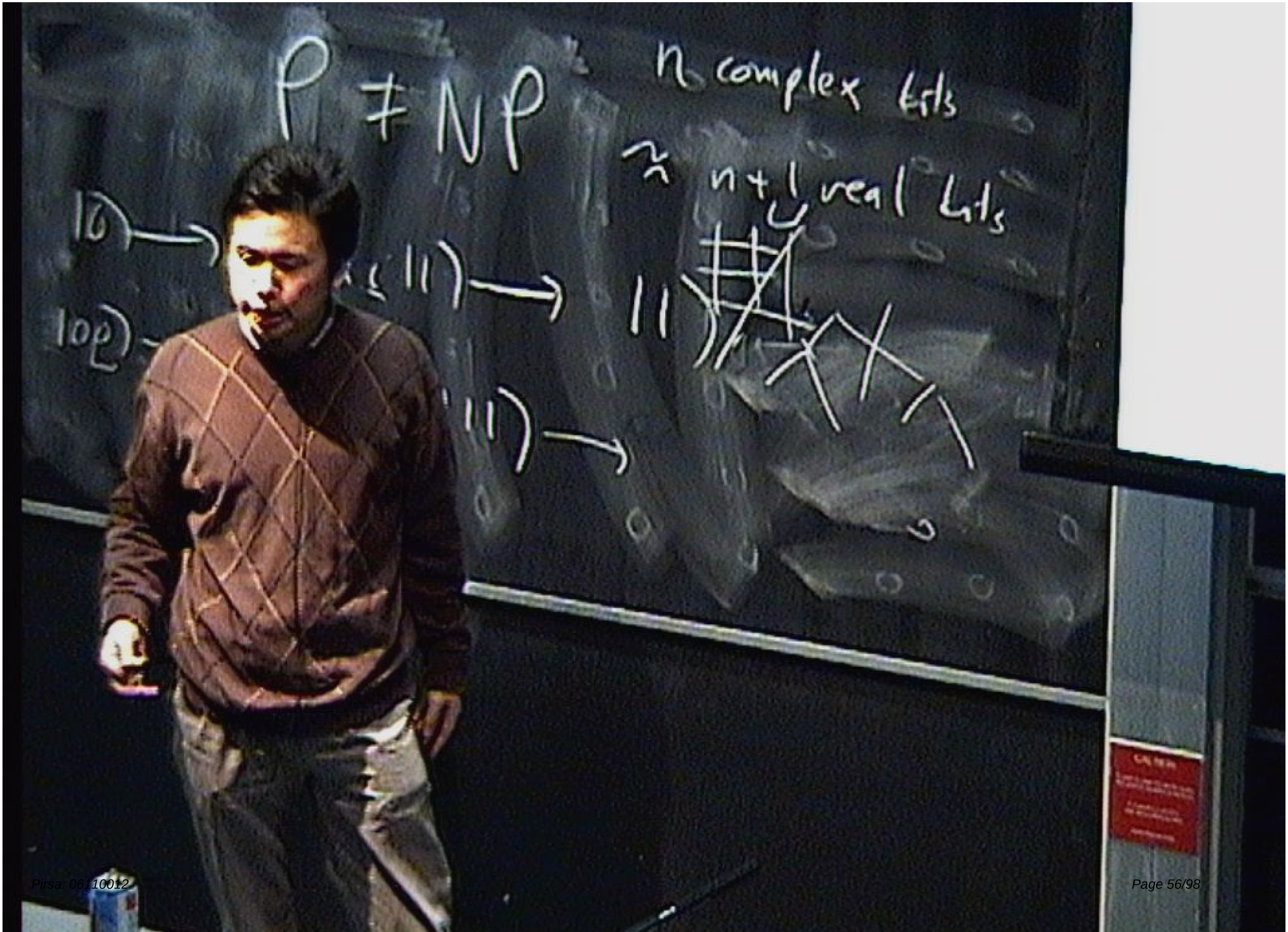
P ≠ NP
n complex bits
n+1 real bits

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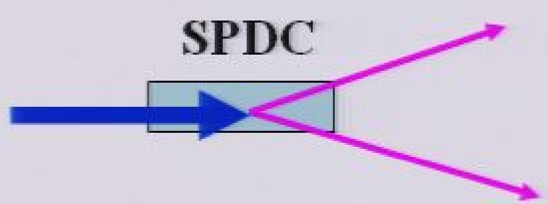
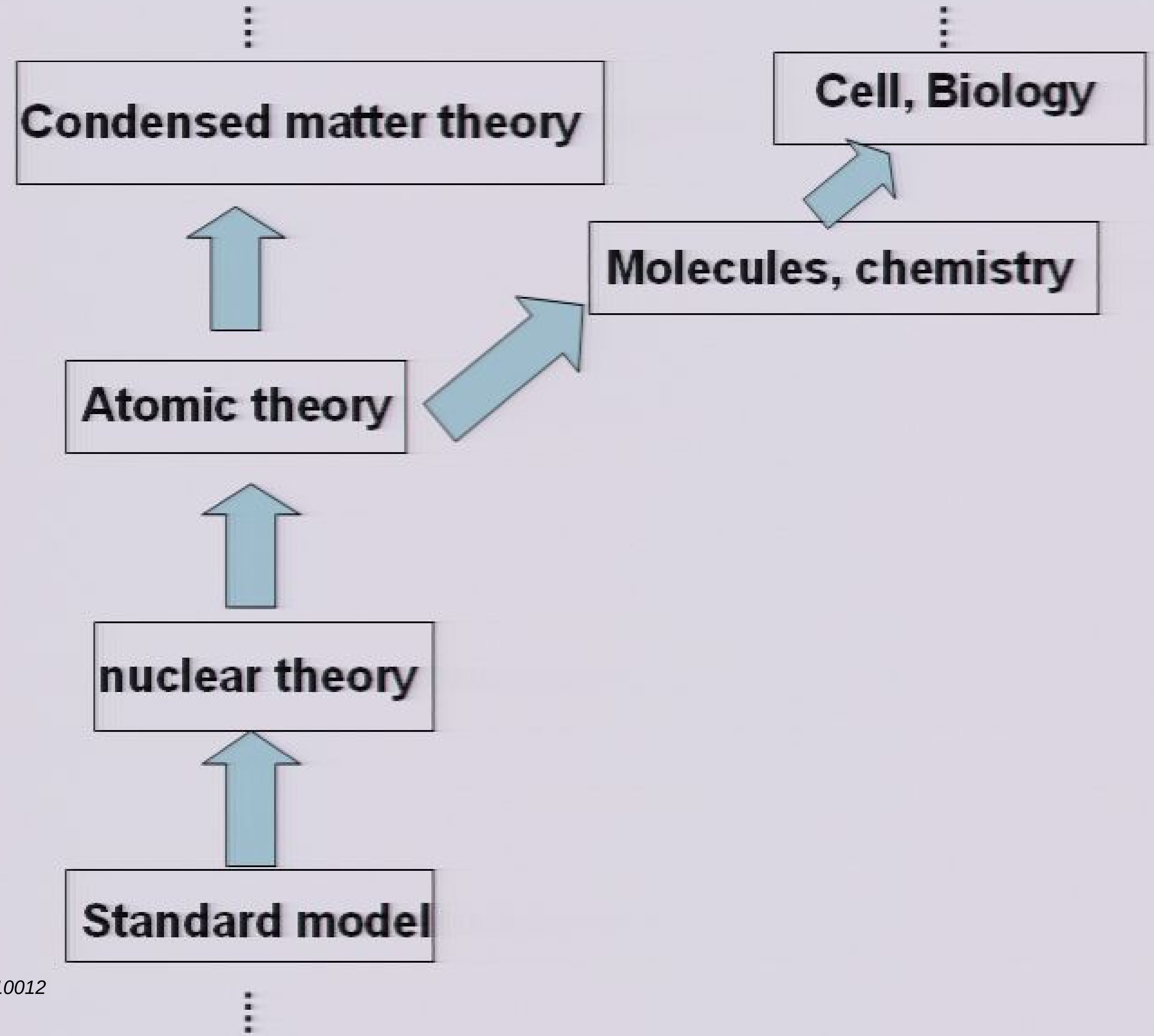


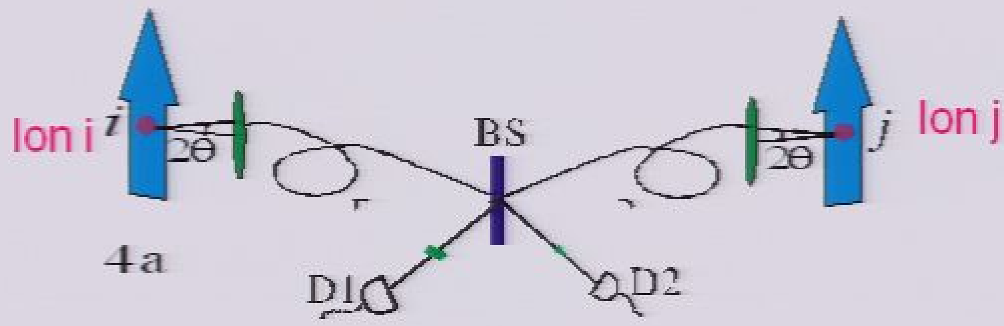
Quantum simulation



A hierarchy of effective theories

Probabilistic gates on remote ion qubits

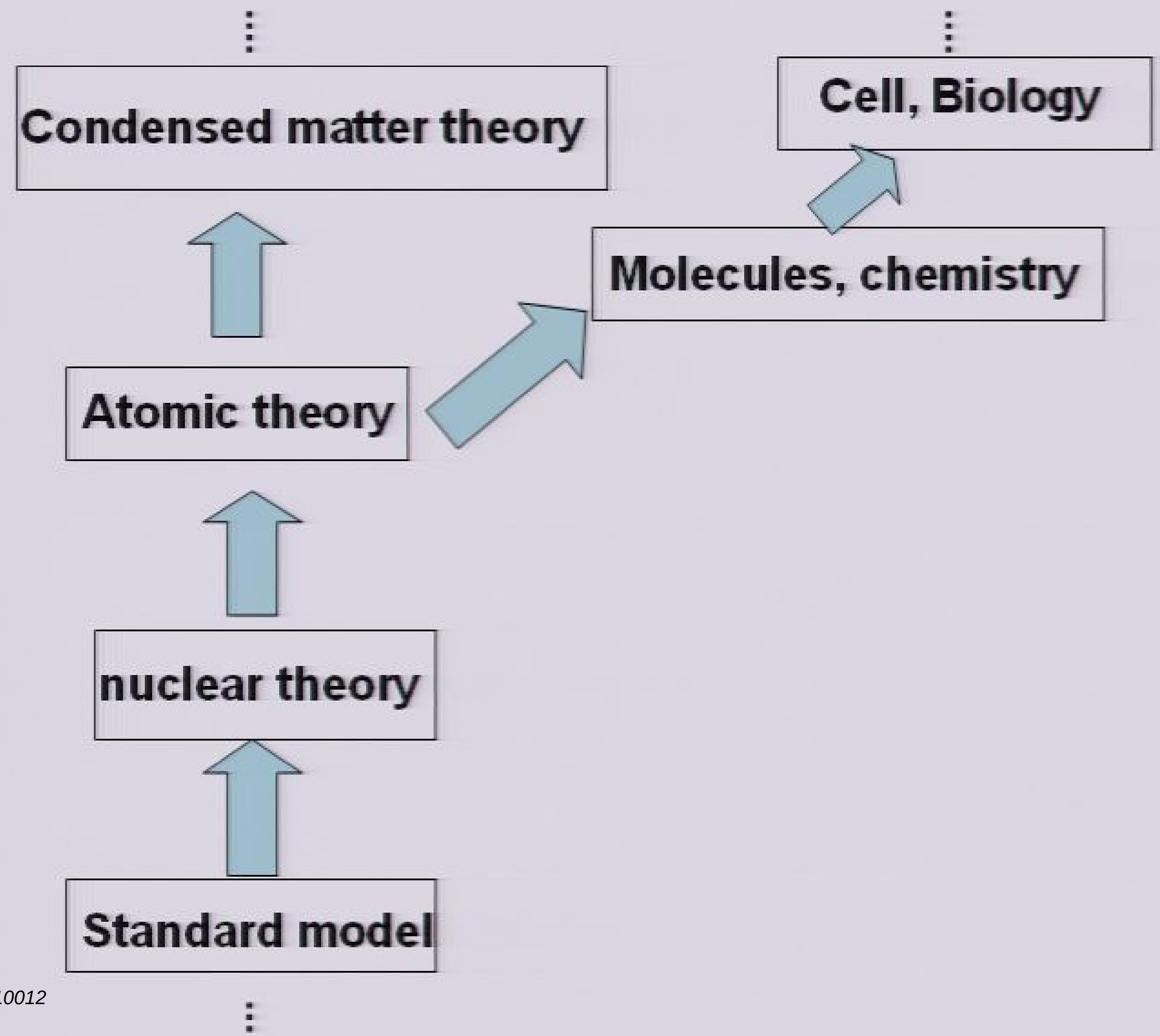
Duan et al., PRA, 2006



Joint measurements of emitted photons

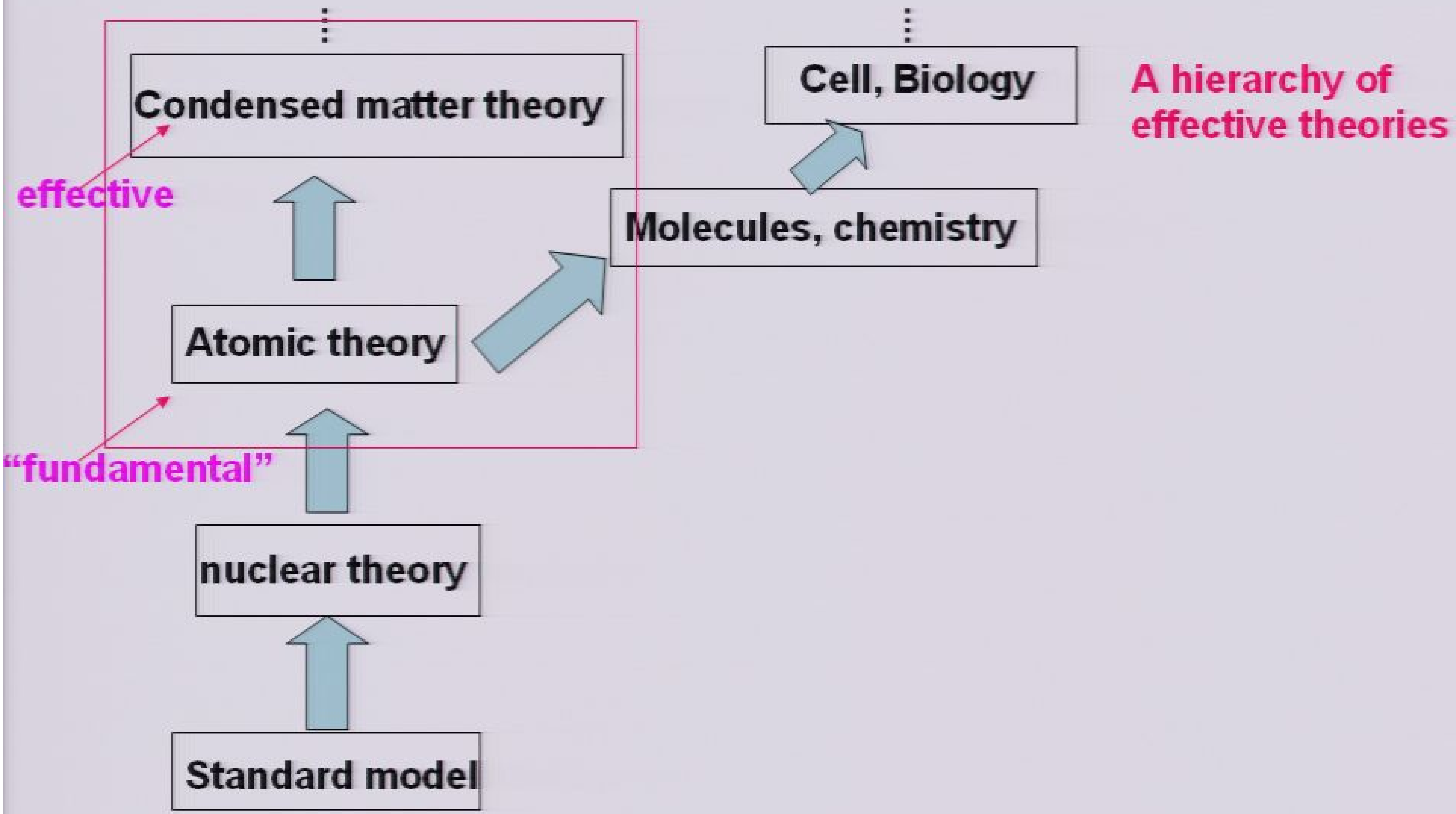
- ✓ ions in arbitrary initial states
- ✓ A detection shows one ion in |0> state, and one ion in |1> state with coherence between two possibilities (probabilistic ZZ gate)
- ✓ Probabilistic ZZ is a gate that is universal and efficient for quant. Computation

Quantum simulation

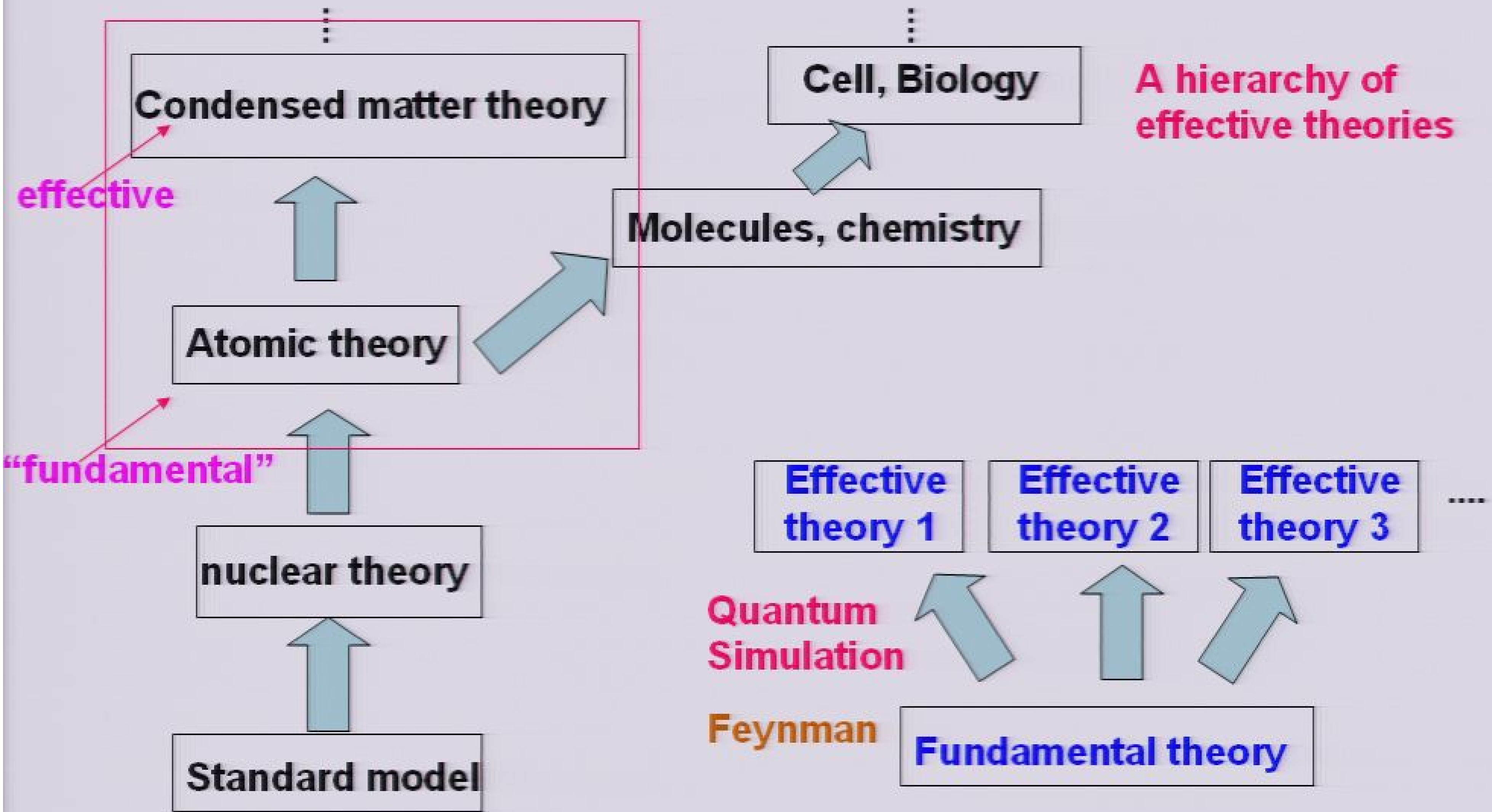


A hierarchy of effective theories

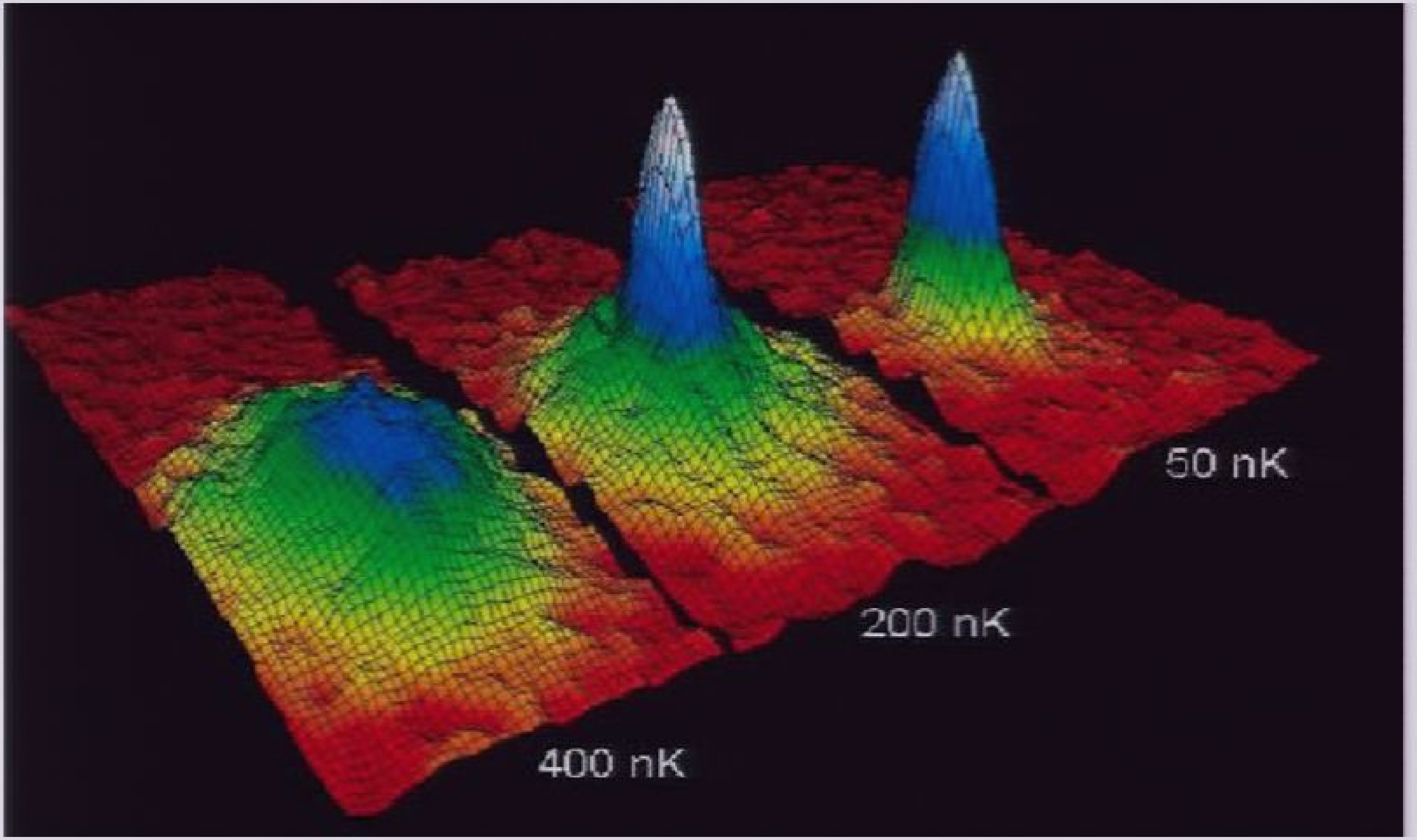
Quantum simulation



Quantum simulation



Ultracold atoms



JILA

Control for ultracold atoms

- Features of Ultracold atoms: quantum material

--- remarkable controllability and diversity

  - Various geometry of optical lattice and superlattice
  - Both Bosons and fermions, different internal (spin) states
  - Interaction strength well controllable
  - Dynamical properties (kinetics) as well as equilibrium properties

A few illustrative examples

- **Simulation of gauge fields and spin Hall effects**
- **Simulation of quantum magnetism and anyon model**
- **Simulating Kondo Physics**
- **Simulation of t-J model and RVB** (resonating valence bonds) **physics**

A few illustrative examples

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  Optical lattice
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- Simulation of t-J model and RVB (resonating valence bonds) physics

  Optical lattice+ Feshbach resonance

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Optical lattice

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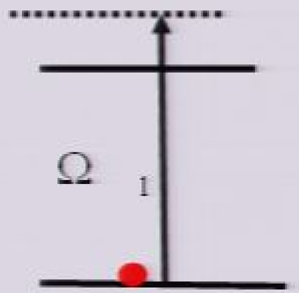
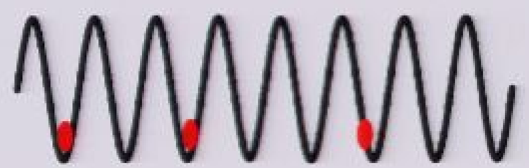
Optical lattice+ Feshbach resonance

Optical lattice





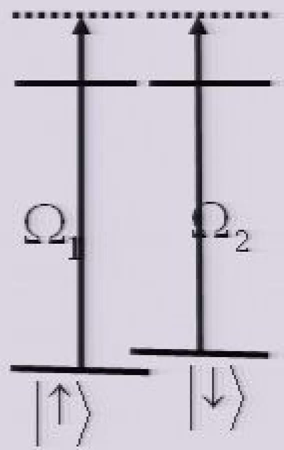
Hansch-Bloch's group

Experimental superfluid-Mott insulator transition etc.

Spin-dependent tunneling

L.-M Duan, E. Demler, M. Lukin, PRL 91, 090402 (2003).

- ✓ Spin-1/2 (two-component) atoms in optical lattice
- ✓ Spin-dependent tunneling!



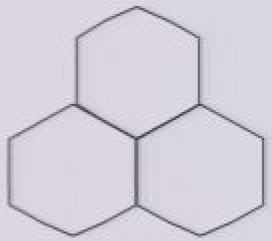
The effective Hamiltonian

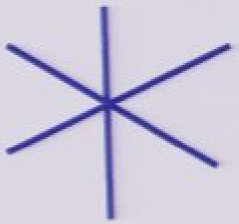
- The effective Hamiltonian in the Mott insulator region

$$H=\sum_{\langle i,j\rangle}\left[\lambda_{\mu z}\sigma_i^z\sigma_j^z+\lambda_{\mu\perp}\left(\sigma_i^x\sigma_j^x+\sigma_i^y\sigma_j^y\right)\right],$$

XXZ model

- **The parameters $\lambda_{\mu z},\lambda_{\mu\perp}$ fully controllable in each direction**
- **Different magnetic coupling along different directions in lattice**

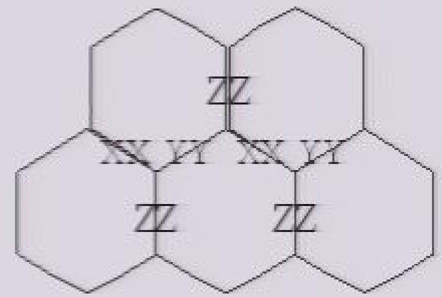




Realization of more exotic magnetic model

- An exact anyon model (Kitaev)

  - Anisotropic spin interaction in a hexagonal lattice

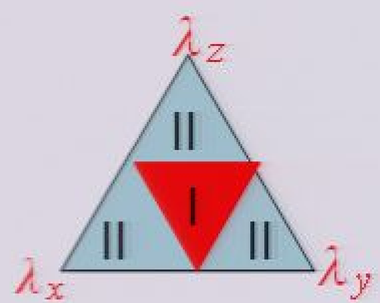


$$H = \lambda_x \sum_{\langle i,j \rangle \in D_x} \sigma_i^x \sigma_j^x + \lambda_y \sum_{\langle i,j \rangle \in D_y} \sigma_i^y \sigma_j^y + \lambda_z \sum_{\langle i,j \rangle \in D_z} \sigma_i^z \sigma_j^z,$$

  - Properties:

    I: non-abelian anyon excitations

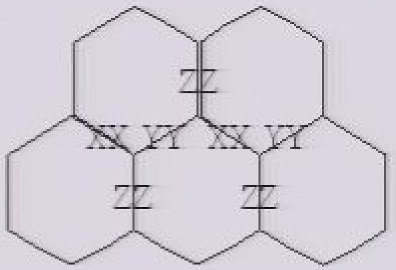
    II: abelian anyon excitations



Implementation of the anyon model

L.-M Duan, E. Demler, M. Lukin, PRL (2003).

Three steps:



- **Forming a 2D hexagonal lattice**
- **Introduce spin-dependent tunneling**
- **Different tunneling interactions along different spatial directions in the lattice**

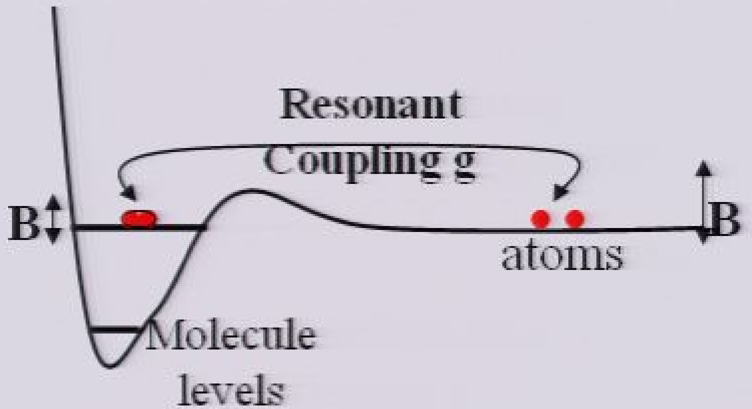
Feshbach resonance + optical lattice

- **Optical lattice** -- diverse interaction configurations

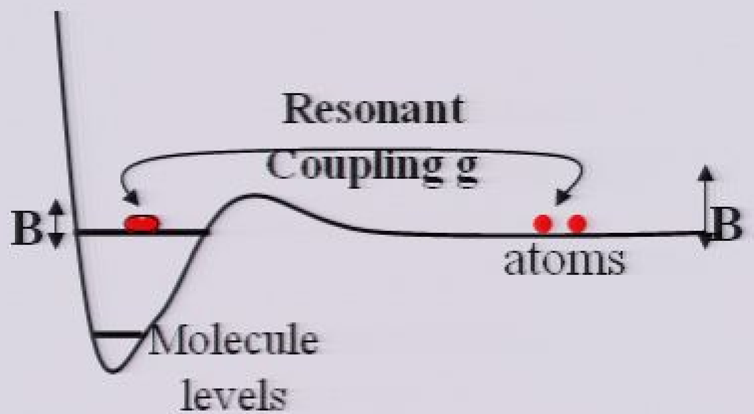
  -- interaction weak (ultra-low temperature)

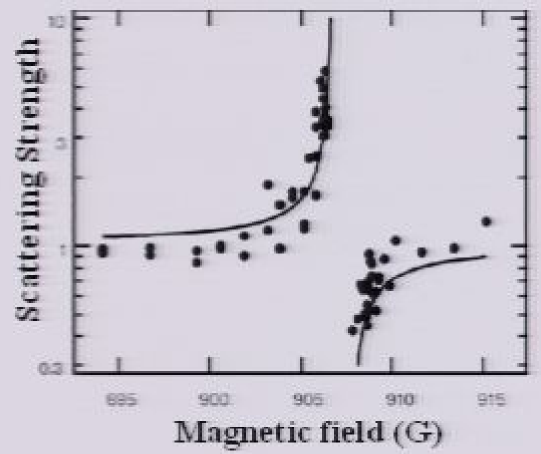
- **Feshbach resonance** -- dramatically increase interaction magnitude

Feshbach Resonance



Feshbach Resonance





Experimental Feshbach resonance

S. Inouye, et al., Nature (98).

--- Control of interaction strength with magnetic field

Feshbach resonance + optical lattice

Feshbach resonance -- magnitude
Optical lattice -- diversity

Experiments: Esslinger, Ketterle etc.

(L.-M Duan, Phys. Rev. Lett. 95, 243202 (2005))

- Derivation of effective Hamiltonian for strongly interacting fermions in an optical lattice across Feshbach resonance
- Connect t-J model and XXZ models, simulation of RVB physics

Strong interaction effects

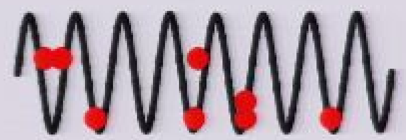
What is the complication?

- **Multi-band populations** (Diener, T.-L. Ho, PRL 2006)

$$g_{on} > E_{bg}$$

On-site coupling rate

Band gap



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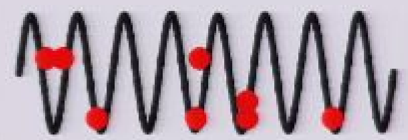

$$\sum_{i;pqr} g_{pqr} b_{ip}^{+} a_{iq\uparrow} a_{ir\downarrow}$$

Different bands

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On-site coupling rate

Band gap

$$\sum_{i;pqr} g_{pqr} b^{+}_{ip} a_{iq\uparrow} a_{ir\downarrow}$$

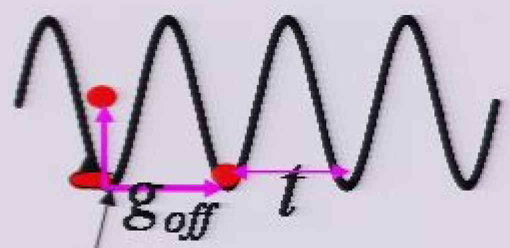
Different bands

- **Off-site collision couplings** (Duan, PRL 2005)

$$g_{off} > t$$

Off-site coupling rate

Tunneling rate



The lattice Hamiltonian

- **The multi-band Hamiltonian**

$$
\begin{aligned}
H_I = & \sum_{i\mathrm{pqs}} \left(c^{(am)}_{0;\mathrm{pss}'} b^\dagger_{i\mathrm{p}} a_{i\mathrm{s}\uparrow} a_{i\mathrm{s}'\downarrow} + h.c. \right) + \sum_{i\mathrm{pqss}'} c^{(aa)}_{0;\mathrm{pqss}'} a^\dagger_{i\mathrm{p}\downarrow} a^\dagger_{i\mathrm{q}\uparrow} a_{i\mathrm{s}\uparrow} a_{i\mathrm{s}'\downarrow} \\
& + \sum_i \sum_{j \in N(i)} \left[\left(\sum_{\mathrm{pss}'} c^{(am)}_{1;\mathrm{pss}'} b^\dagger_{i\mathrm{p}} + \sum_{\mathrm{pqss}'} c^{(aa)}_{1;\mathrm{pqss}'} a^\dagger_{i\mathrm{p}\downarrow} a^\dagger_{i\mathrm{q}\uparrow} \right) \left(a_{i\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} - a_{i\mathrm{s}\downarrow} a_{j\mathrm{s}'\uparrow} \right) + \sum_{\mathrm{pss}'} c^{(am)}_{2;\mathrm{pss}'} b^\dagger_{i\mathrm{p}} a_{j\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} + h.c. \right] \\
& + \sum_{i\mathrm{pqss}'} \sum_{j \in N(i)} \left[c^{(aa)}_{2;\mathrm{pqss}'} a^\dagger_{i\mathrm{p}\downarrow} a^\dagger_{i\mathrm{q}\uparrow} a_{j\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} + c^{(aa)}_{3;\mathrm{pqss}'} a^\dagger_{i\mathrm{p}\downarrow} a^\dagger_{j\mathrm{q}\uparrow} \left(a_{i\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} - a_{i\mathrm{s}\downarrow} a_{j\mathrm{s}'\uparrow} \right) \right], \qquad (1)
\end{aligned}
$$

The lattice Hamiltonian

- The multi-band Hamiltonian

$$
\begin{aligned}
H_I = & \sum_{i\mathrm{pqs}} \left(c^{(am)}_{0;\mathrm{pss}'} b^\dagger_{i\mathrm{p}} a_{i\mathrm{s}\uparrow} a_{i\mathrm{s}'\downarrow} + h.c. \right) + \sum_{i\mathrm{pqss}'} c^{(aa)}_{0;\mathrm{pqss}'} a^\dagger_{i\mathrm{p}\downarrow} a^\dagger_{i\mathrm{q}\uparrow} a_{i\mathrm{s}\uparrow} a_{i\mathrm{s}'\downarrow} \\
& + \sum_i \sum_{j\in N(i)} \left[\left(\sum_{\mathrm{pss}'} c^{(am)}_{1;\mathrm{pss}'} b^\dagger_{i\mathrm{p}} + \sum_{\mathrm{pqss}'} c^{(aa)}_{1;\mathrm{pqss}'} a^\dagger_{i\mathrm{p}\downarrow} a^\dagger_{i\mathrm{q}\uparrow} \right) \left(a_{i\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} - a_{i\mathrm{s}\downarrow} a_{j\mathrm{s}'\uparrow} \right) + \sum_{\mathrm{pss}'} c^{(am)}_{2;\mathrm{pss}'} b^\dagger_{i\mathrm{p}} a_{j\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} + h.c. \right] \\
& + \sum_{i\mathrm{pqss}'} \sum_{j\in N(i)} \left[c^{(aa)}_{2;\mathrm{pqss}'} a^\dagger_{i\mathrm{p}\downarrow} a^\dagger_{i\mathrm{q}\uparrow} a_{j\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} + c^{(aa)}_{3;\mathrm{pqss}'} a^\dagger_{i\mathrm{p}\downarrow} a^\dagger_{j\mathrm{q}\uparrow} \left(a_{i\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} - a_{i\mathrm{s}\downarrow} a_{j\mathrm{s}'\uparrow} \right) \right], \qquad (1)
\end{aligned}
$$

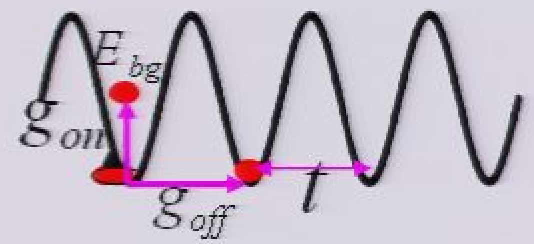
Hopeless to solve!

How to simplify ???

- **Separation of different energy scales** (Duan, PRL 2005)

Large energy scale: $g_{on} \gtrsim E_{bg}$

Small energy scale: $g_{off} \gtrsim t$



Energy scale separation: $g_{on}, E_{bg} >> g_{off}, t$

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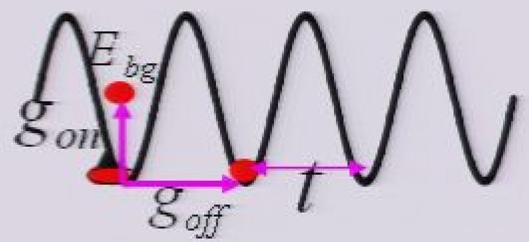

Energy scale separation: $g_{on}, E_{bg} >> g_{off}, t$

$$E_{on} \sim 23E_r,\ E_{bg} \sim 6.3E_r,$$

$$E_{off} \sim 0.09E_r,\ E_t \sim 0.02E_r$$

for $V_0 = 10E_r$ and ^{40}K

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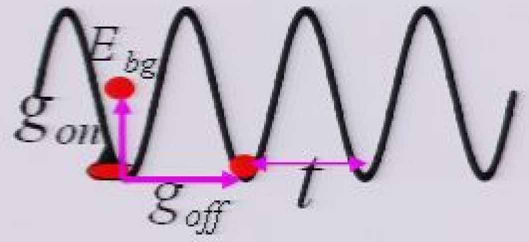

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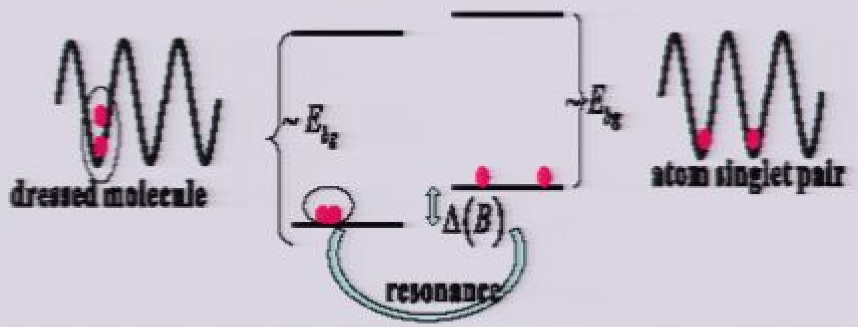
for $V_0 = 10E_r$ and ^{40}K

First solve the single-site problem!

Lattice Resonance

- Resonance between atoms and dressed molecules



Projection:

$$P \equiv \bigotimes_i P_i,$$

$$P_i \equiv |0\rangle_i \langle 0| + |\uparrow\rangle_i \langle\uparrow| + |\downarrow\rangle_i \langle\downarrow| + |d\rangle_i \langle d|.$$

Effective Hamiltonian

- **Effective Hamiltonian** (Duan, PRL, 2005)

$$
\begin{aligned}
H_{eff} &= \sum_i \Delta(B) d_i^\dagger d_i + \sum_{i;j\in N(i)} t_d P d_i^\dagger d_j P \\
&+ \sum_{i;j\in N(i)} \sum_\sigma \left(t_a P a_{i\sigma}^\dagger a_{j\sigma} P + t_{da} d_i^\dagger d_j a_{j\sigma}^\dagger a_{i\sigma} \right) \\
&+ \sum_{i;j\in N(i)} \left(g d_i^\dagger \left(a_{i\uparrow} a_{j\downarrow} - a_{i\downarrow} a_{j\uparrow} \right) + h.c. \right), \qquad (2)
\end{aligned}
$$

Effective Hamiltonian

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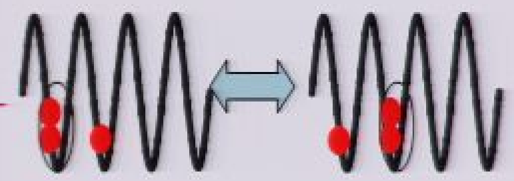

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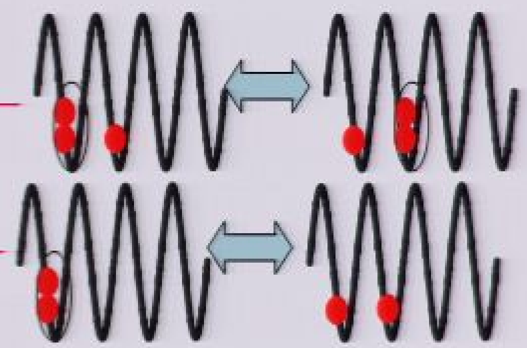
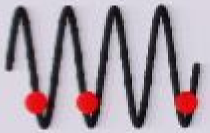


t-J model for atoms

- **Limiting case 1: atom limit**



Atom effective Hamiltonian:

$$H_{tJ} = \sum_{i;j\in N(i)} \left[t_a \sum_{\sigma} P_a a^{\dagger}_{i\sigma} a_{j\sigma} P_a + J\left(\mathbf{s}_i \cdot \mathbf{s}_j - n_i n_j/4\right)\right]$$

Effective Hamiltonian

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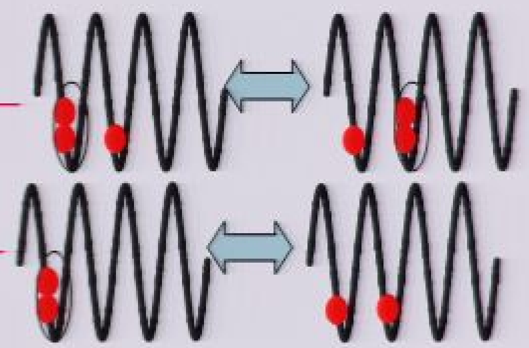

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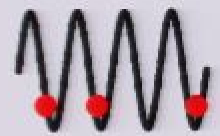

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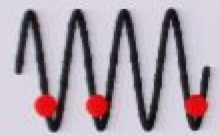

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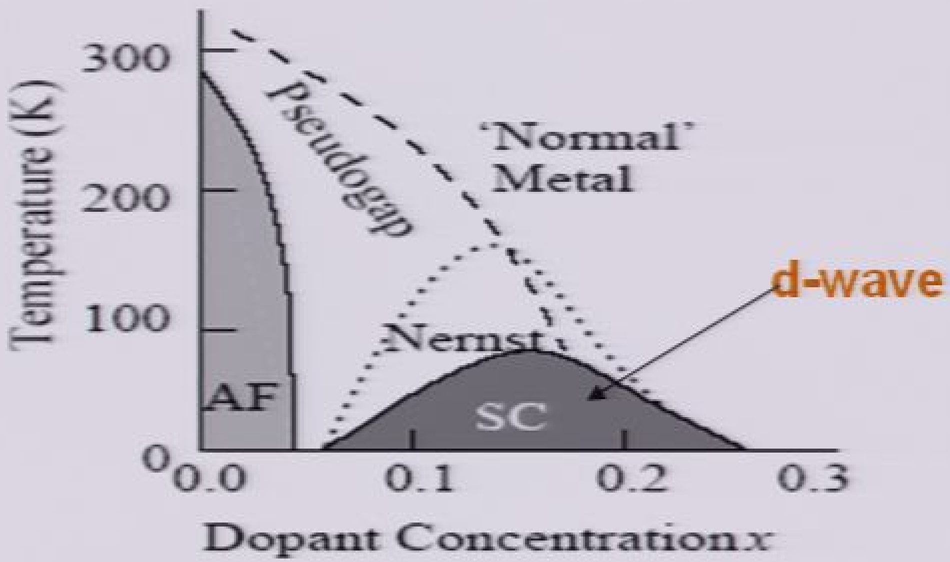
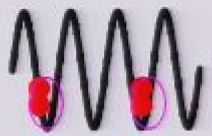
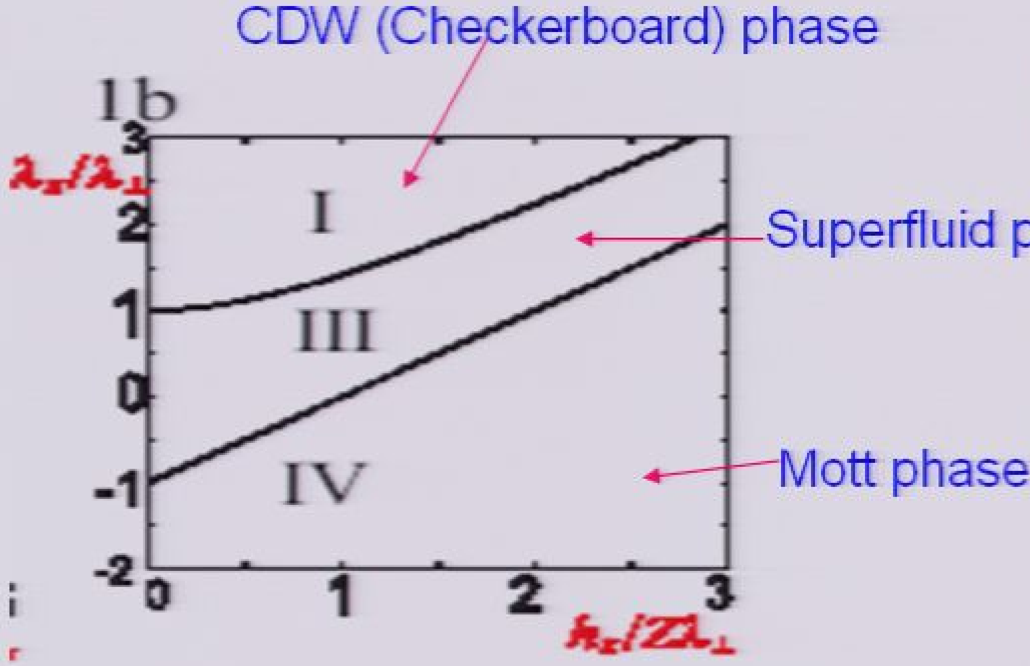


XXZ model for dressed molecules

- **Limiting case2: molecule limit**

$$H_{XXZ} = (t'_d/4)\sum_i \left[B_{eff} Z_i + \sum_{j\in N(i)} (X_i X_j + Y_i Y_j + \delta_z Z_i Z_j) \right]$$





Duan, Demler, Lukin
PRL 2003

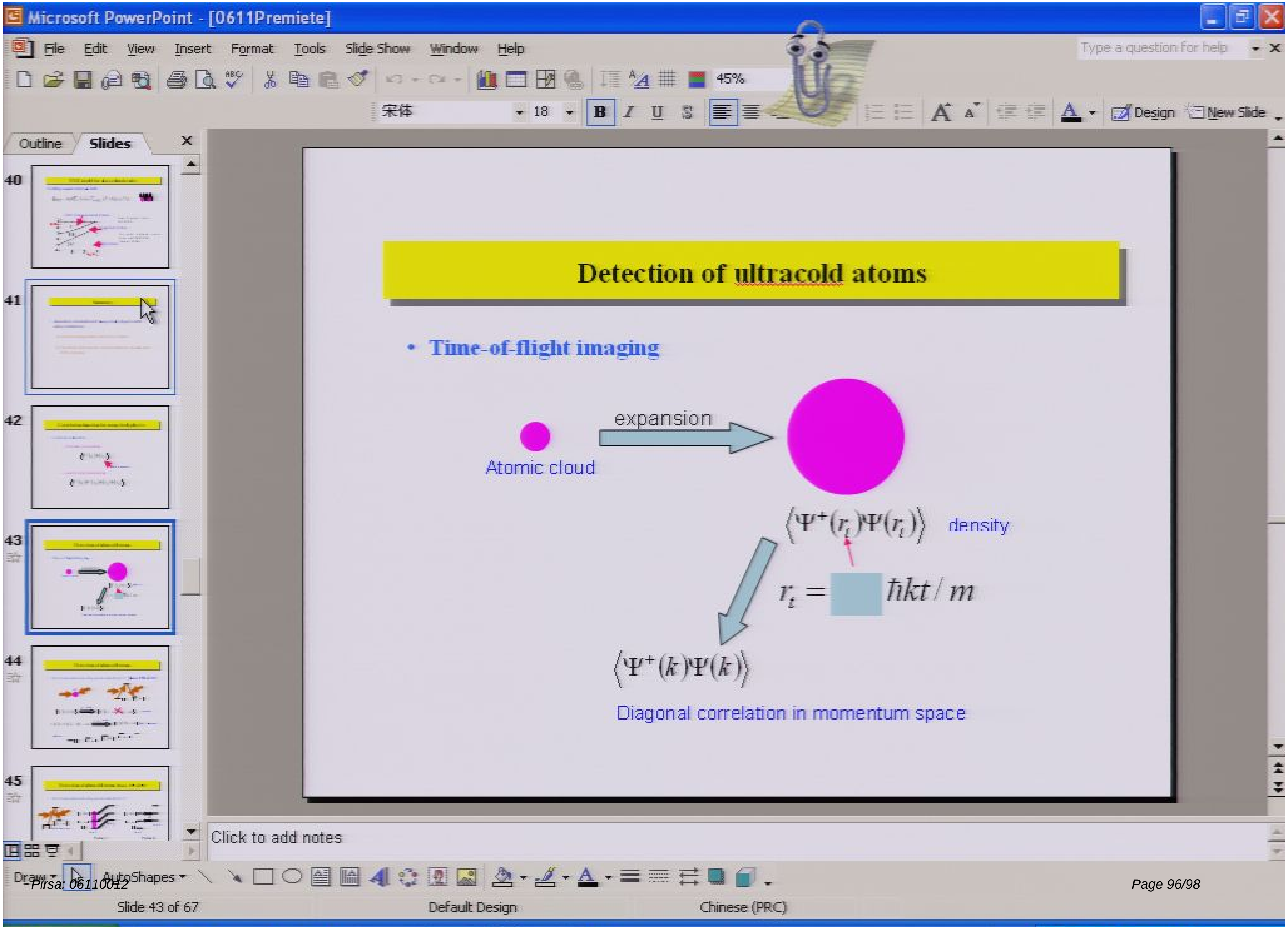
Chin et al., Ketterle' group
Cond-mat/0607004,
Nature (2006)

Summary

- **Quantum simulation of many-body physics with ultracold atoms**

    - Quantum magnetism and anyon model

    - Feshbach resonance + optical lattice (t-J model and RVB physics)


Microsoft PowerPoint - [0611Premiete]
Detection of ultracold atoms
Time-of-flight imaging
expansion
Atomic cloud
$\langle \Psi^+(r_t)\Psi(r_t)\rangle$
density
$r_t = \hbar k t / m$
$\langle \Psi^+(k)\Psi(k)\rangle$
Diagonal correlation in momentum space
Click to add notes
Slide 43 of 67
Default Design
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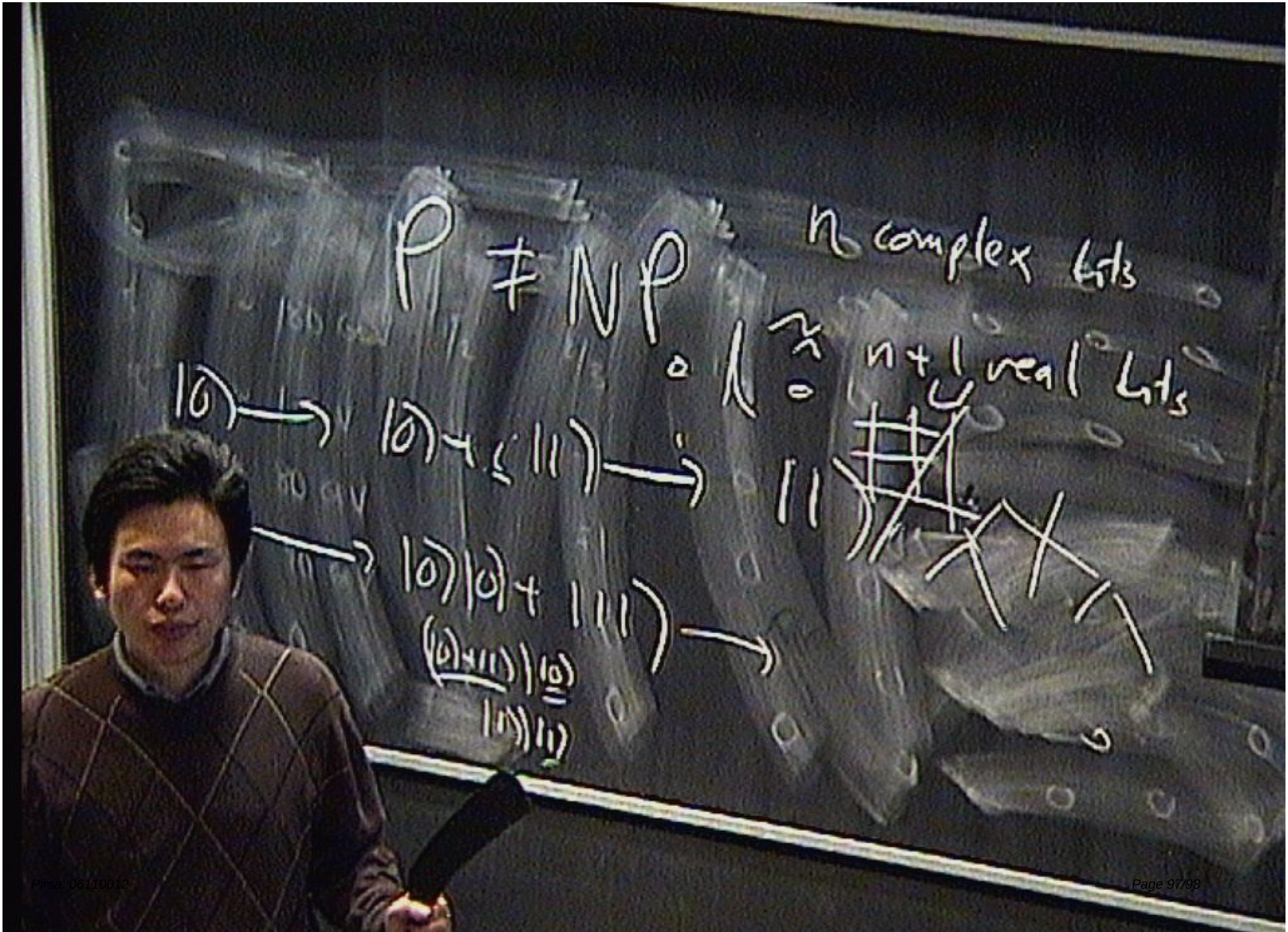
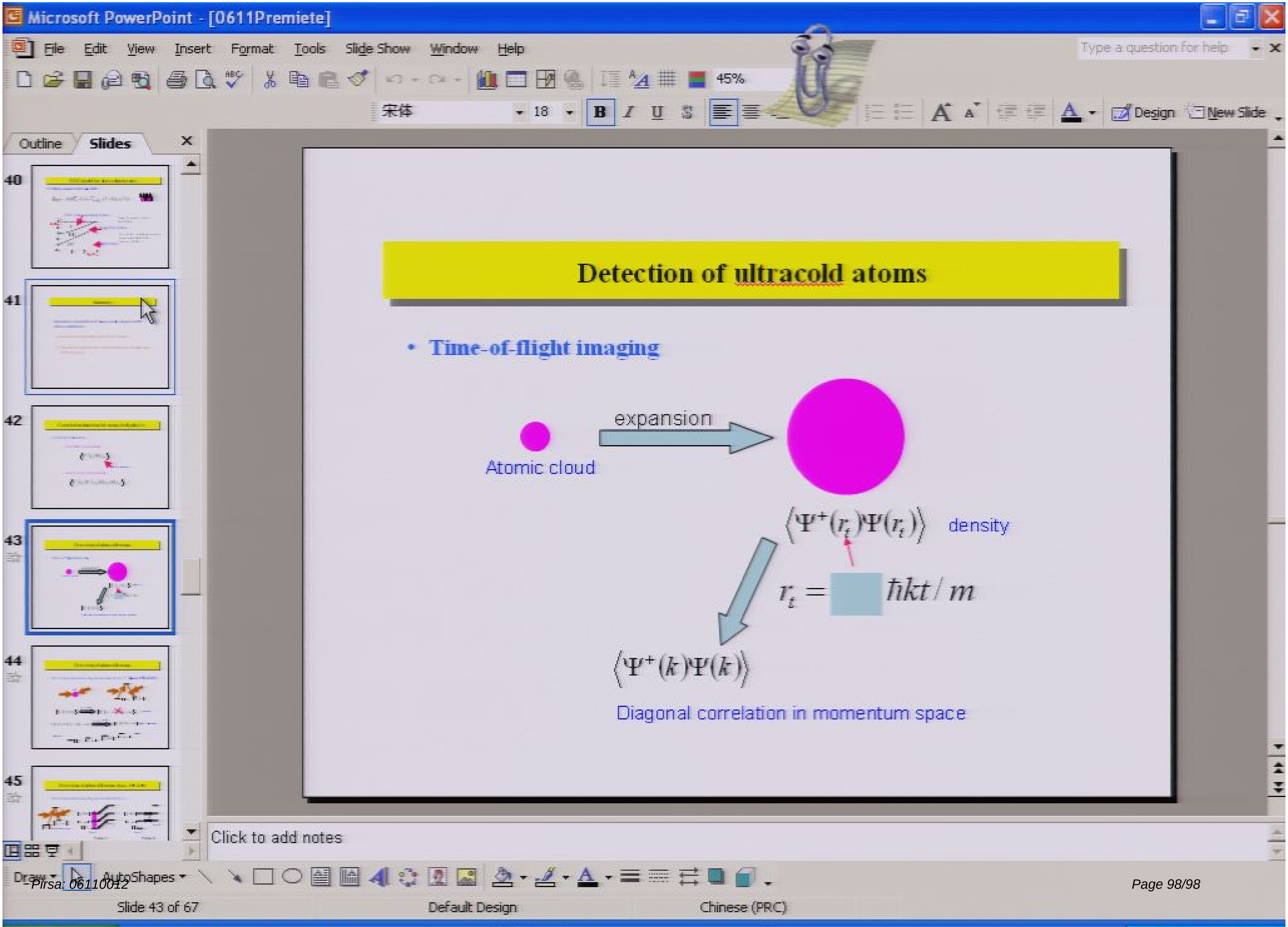

P ≠ NP
n complex bits
n+1 real bits
|0⟩ → |0⟩+i|1⟩ → |1⟩
|0⟩|0⟩+|1⟩|1⟩


Microsoft PowerPoint - [0611Premiete]
Detection of ultracold atoms
• Time-of-flight imaging
expansion
Atomic cloud
$\langle \Psi^{+}(r_t)\Psi(r_t) \rangle$ density
$r_t = \hbar k t / m$
$\langle \Psi^{+}(k)\Psi(k) \rangle$
Diagonal correlation in momentum space
Click to add notes
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