Title: Linear optics quantum information and Quantum simulation of many-body

Date: Nov 15, 2006 04:00 PM

URL: http://pirsa.org/06110012

Abstract: In this talk, I will show how to efficiently generate graph states
based on realistic linear optics (with imperfect photon detectors and source), how to do scalable quantum computation with probabilistic atom photon
interactions, and how to simulate strongly correlated many-body physics with ultracold atomic gas.

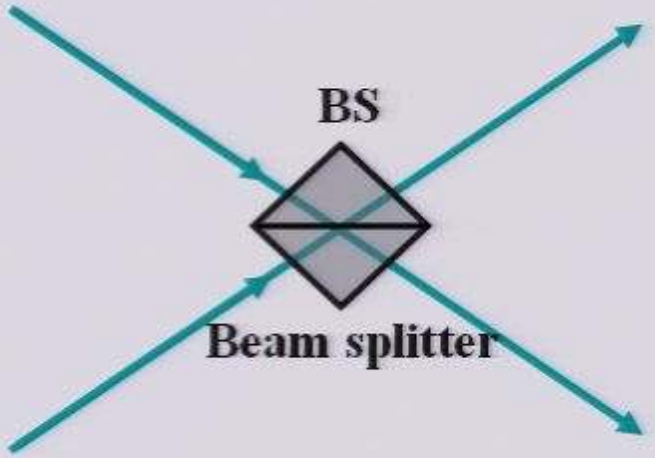
# Scalable quantum information with linear optics

## Quantum simulation of many-body physics with ultracold atoms

**Luming Duan**

**FOCUS center and MCTP, University of Michigan**

# Linear optics device



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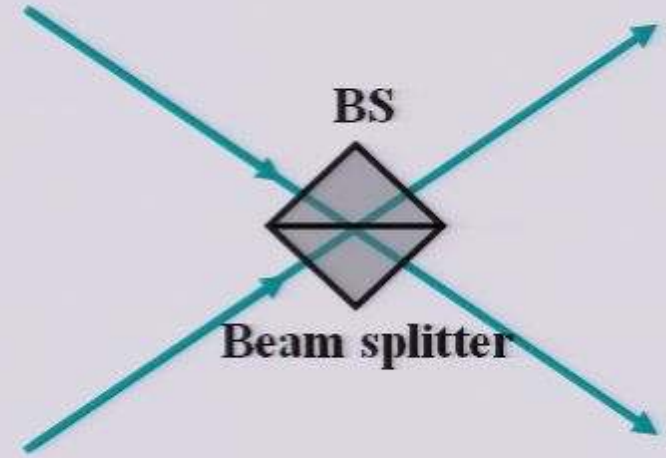


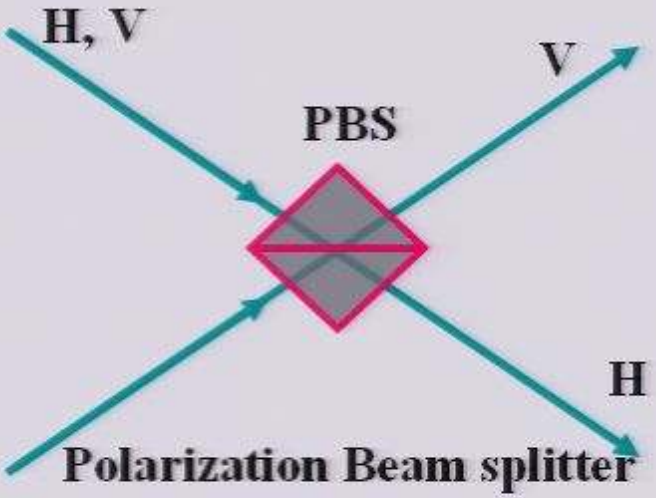

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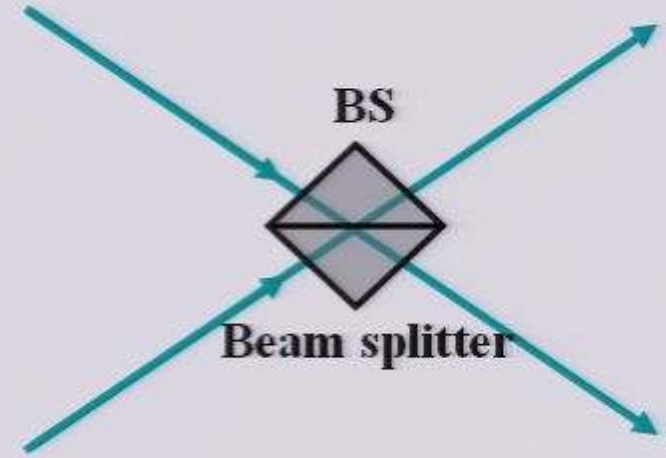
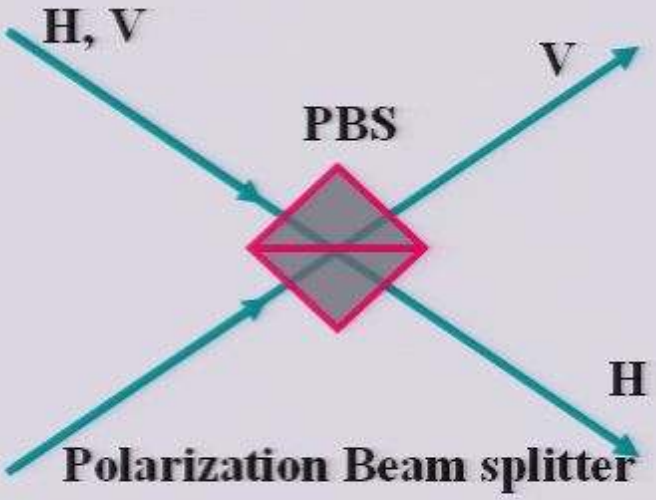
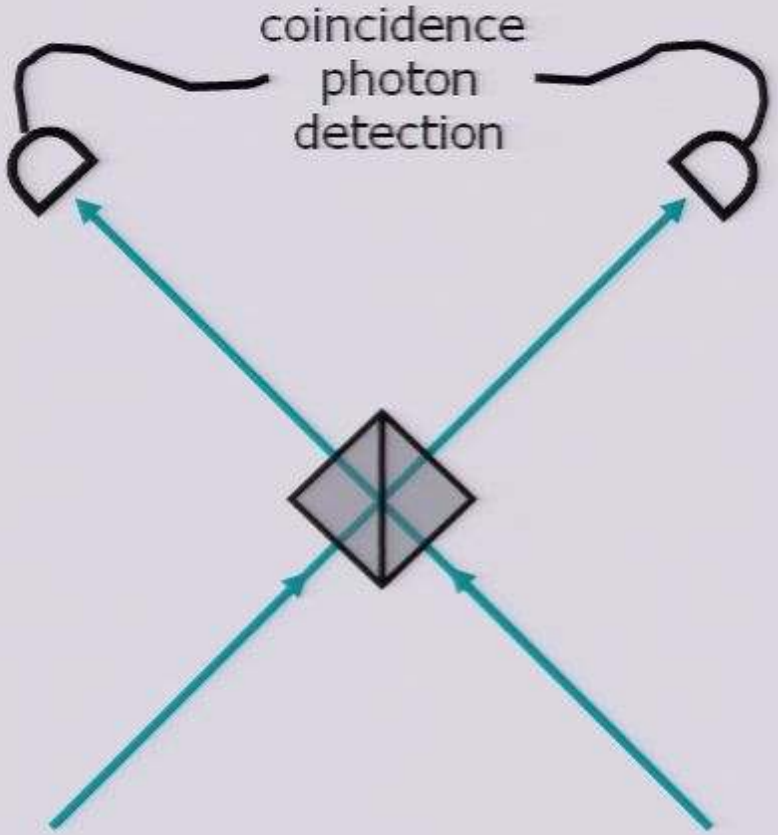


# Post-selection by single-photon detectors




**State evolution in Post-selected subspace**

**Nontrivial!**

**But generally inefficient (non-scalable)!**

# Make things scalable!

- **Linear optics computation scheme (Knill, Laflamme, Milburn, Nature, 2001)**
  - **Through quantum error correction**
  - **Require minimum detector efficiency (threshold)**
  - **Improvements** (Nielsen, Yoran, Reznik, Browne, Rudolph etc.)
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- **Scalable quantum communication (Duan, Lukin, Cirac, Zoller, Nature, 2001)**
  - **Divide and conquer (quantum repeater)**
  - **No requirements on minimum detector efficiency**
  - **More specific purpose**

# Combine the advantages

- No stringent requirement on detector efficiency
- More general applications

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**Results:**

- Scalable quantum state engineering
  - ✓ GHZ state (Duan, PRL, 2002)
  - ✓ Any graph states of tree shape (Bodiya, Duan, PRL, 2006)

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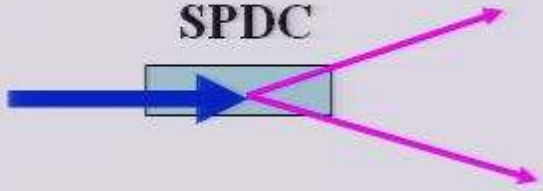
# Input state

$$\rho_s = (1 - \eta_s)\,\rho_{\mathrm{vac}} + \eta_s\,|\Psi\rangle_{12}\langle\Psi|\,,$$

Source efficiency

$$|\Psi\rangle_{12} = (|HH\rangle_{12} + |VV\rangle_{12})/\sqrt{2}$$



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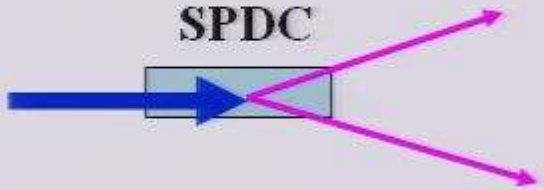


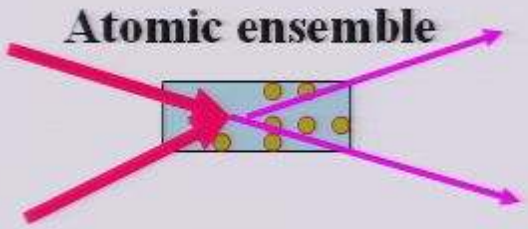

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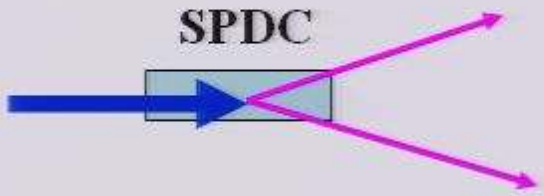
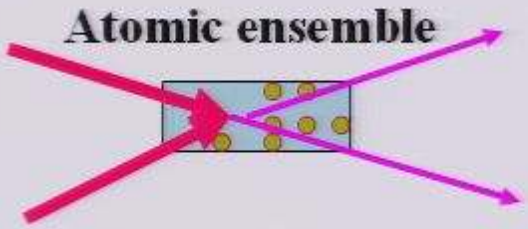
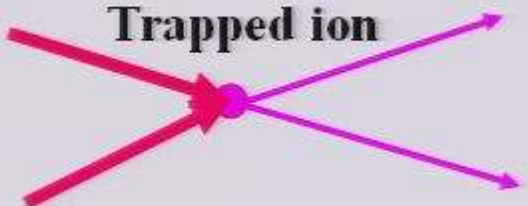
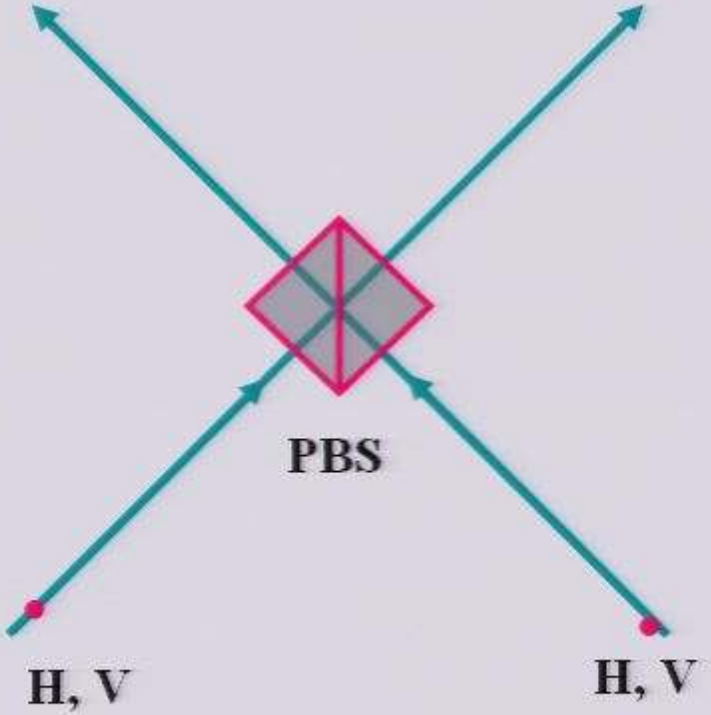


# A PBS gate



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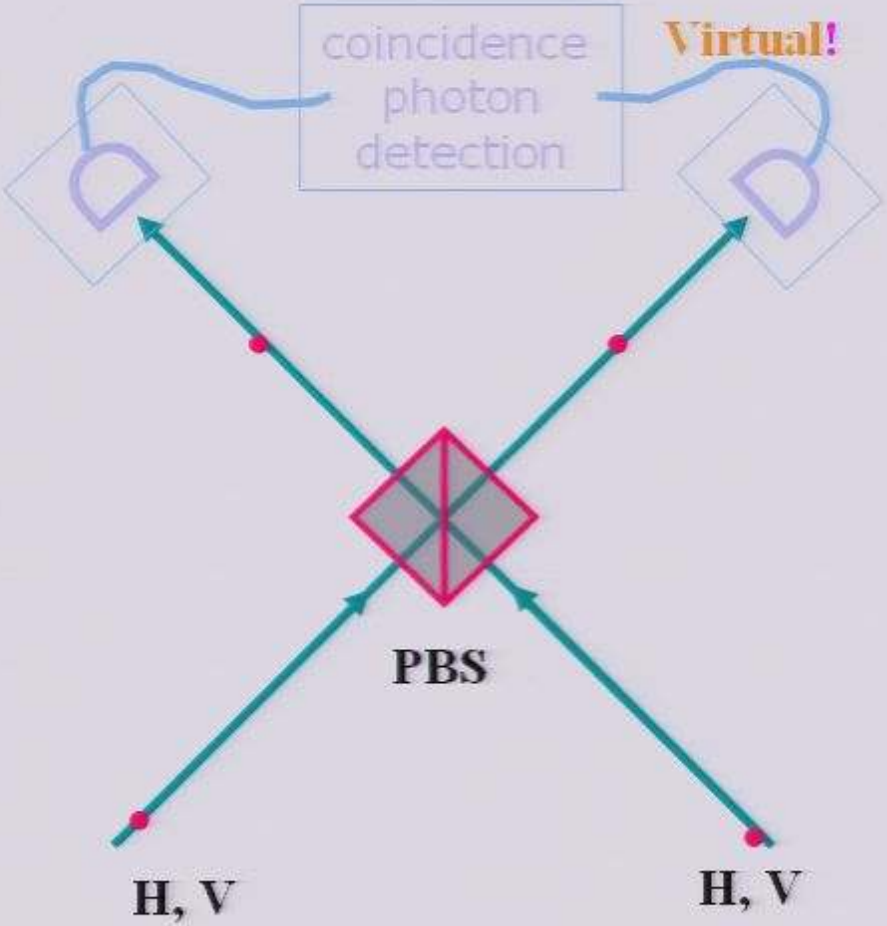

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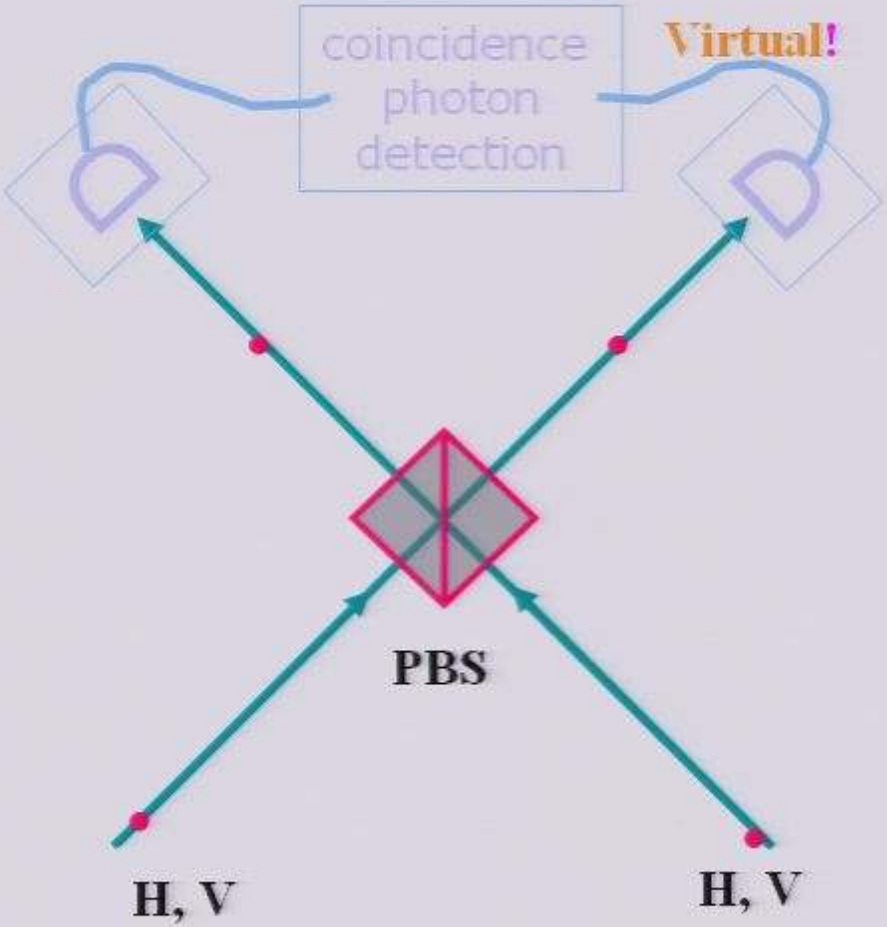


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# Cluster-state (Graph-state) Quantum computing

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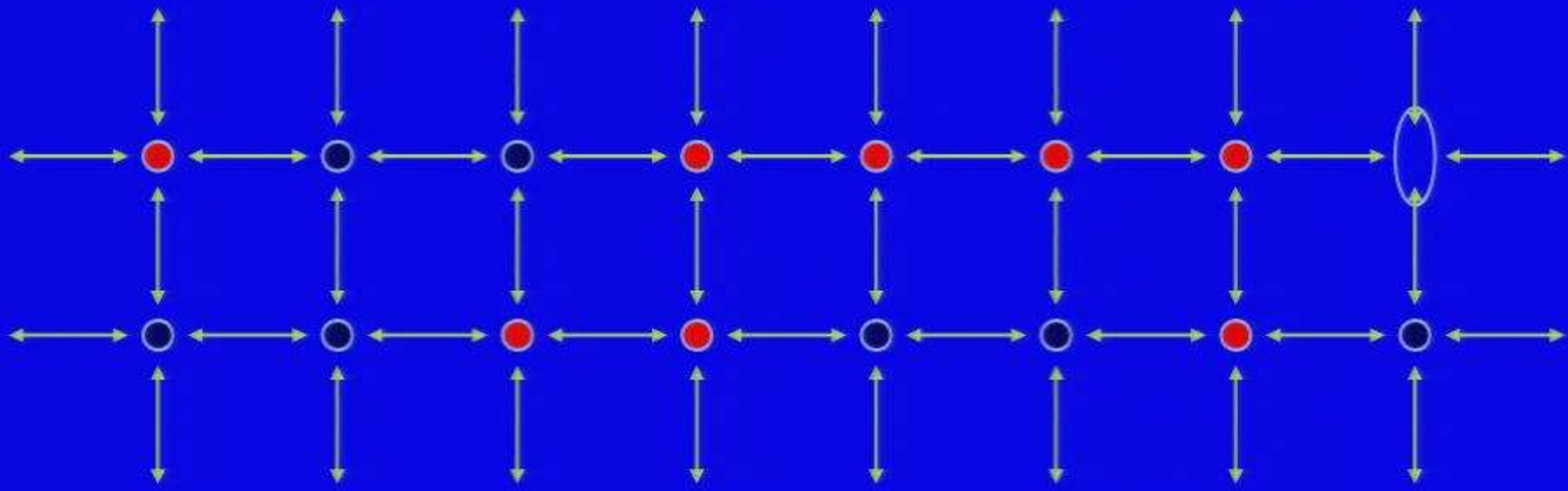
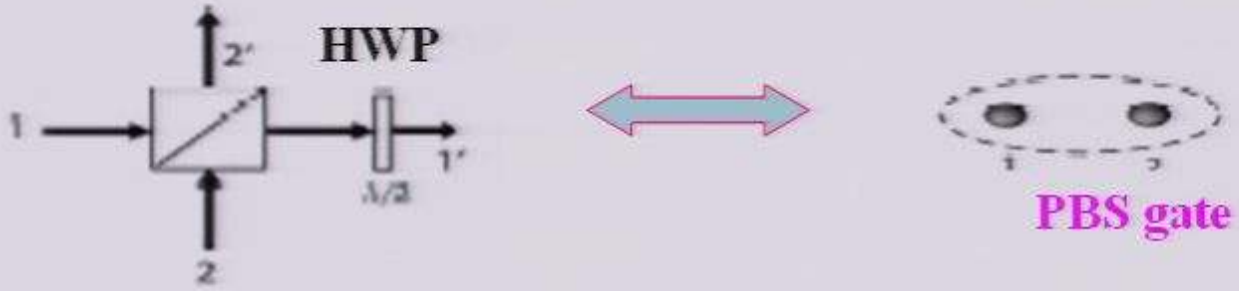


Raussendorf and Briegel, PRL **86**, 910 (2001); PRA (2003).

# Construction of graph states with PBS gates



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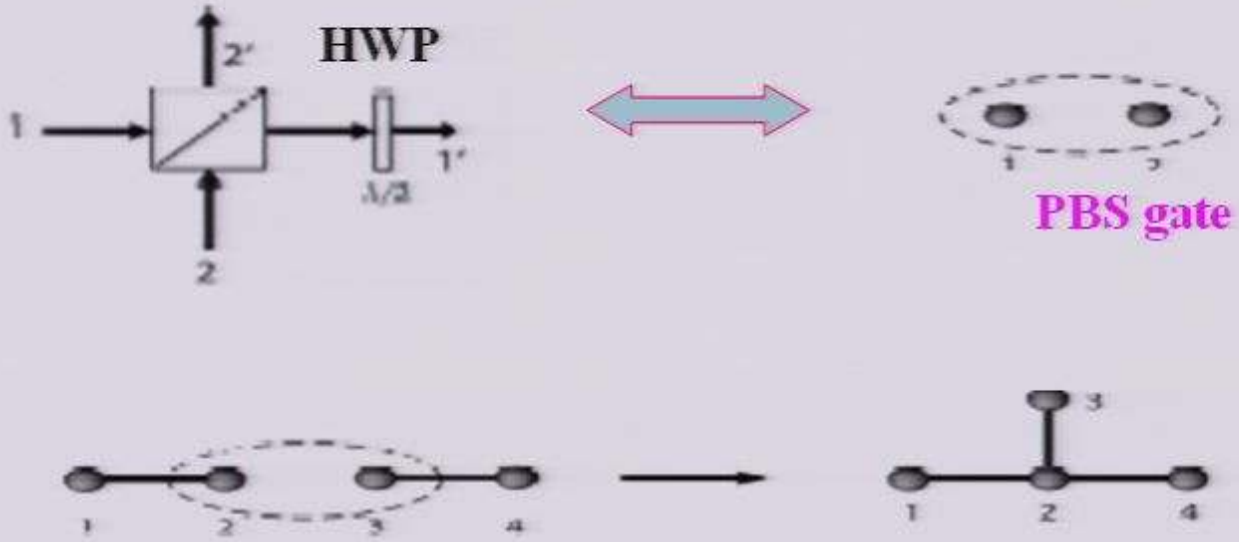

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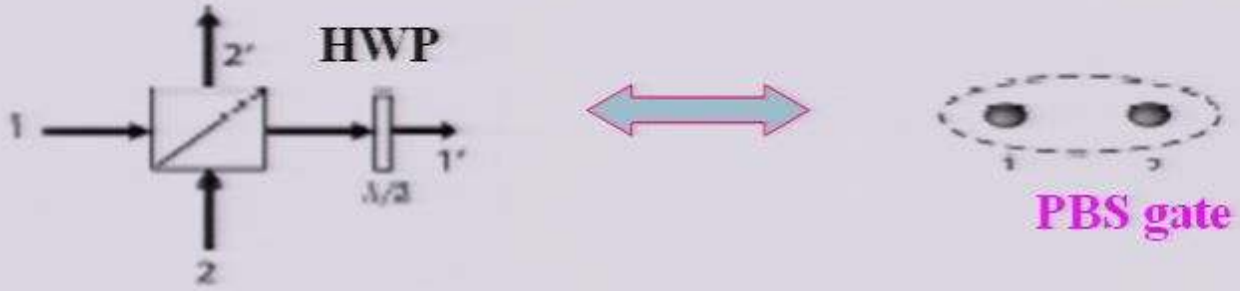


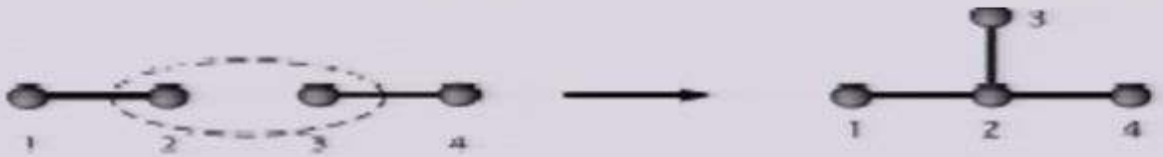


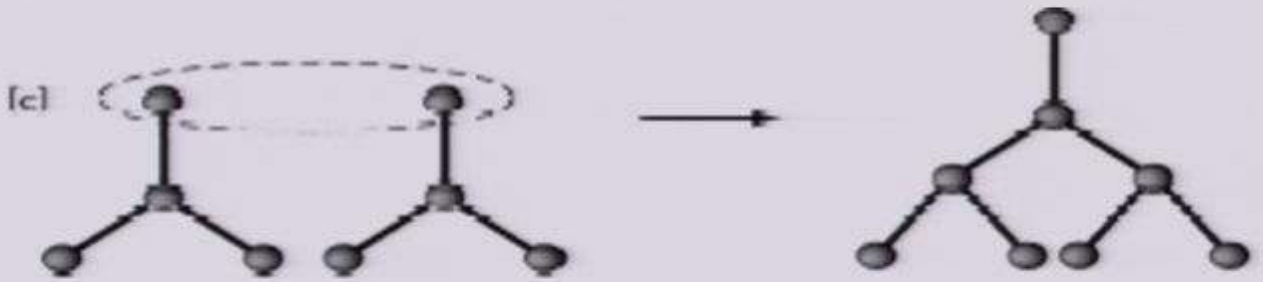

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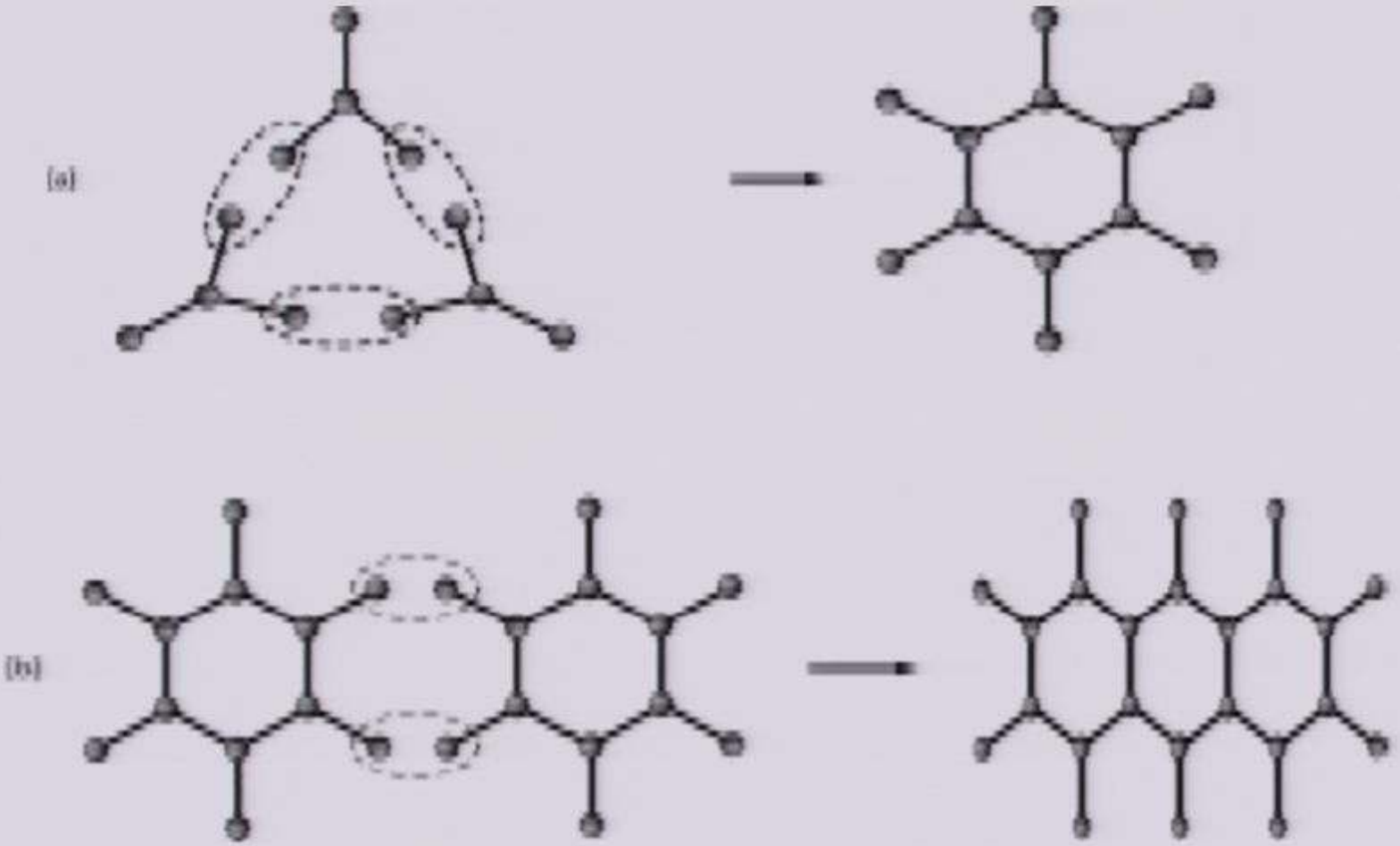

# More efficient Construction of tree graph states

**Connection qubits**

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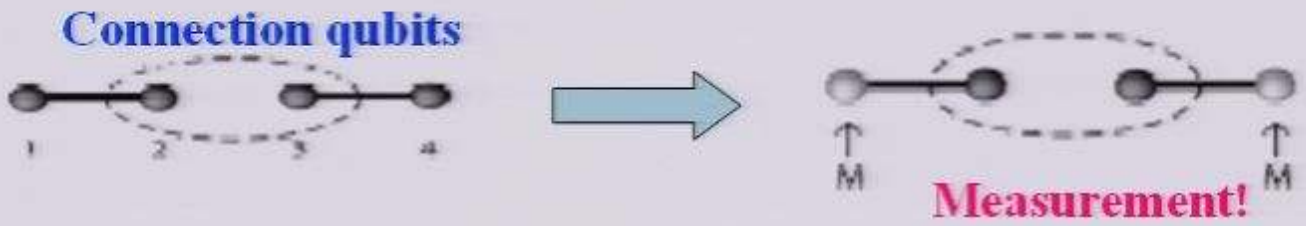

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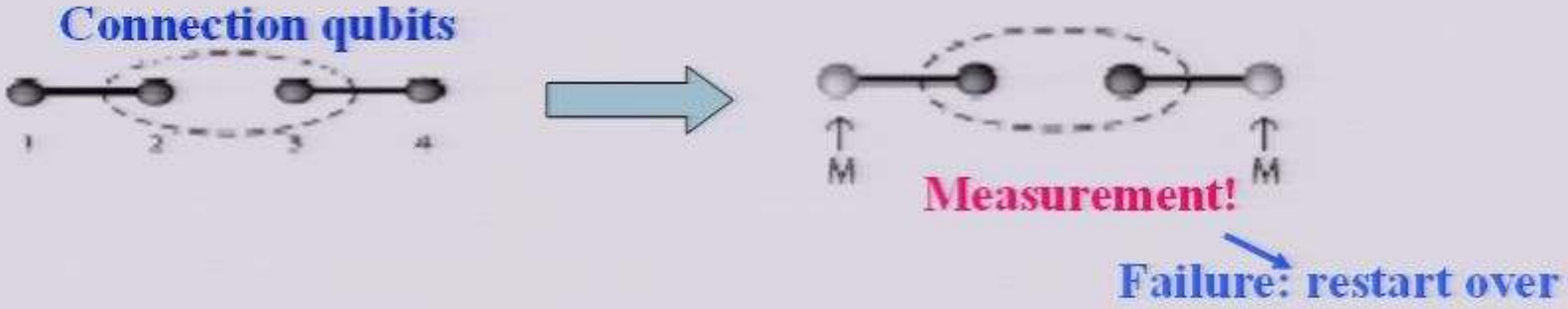

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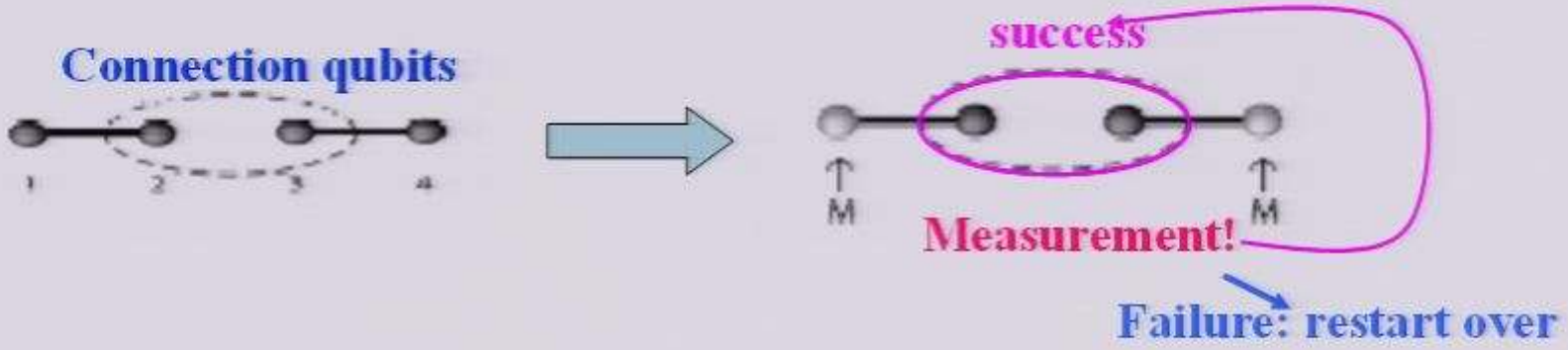

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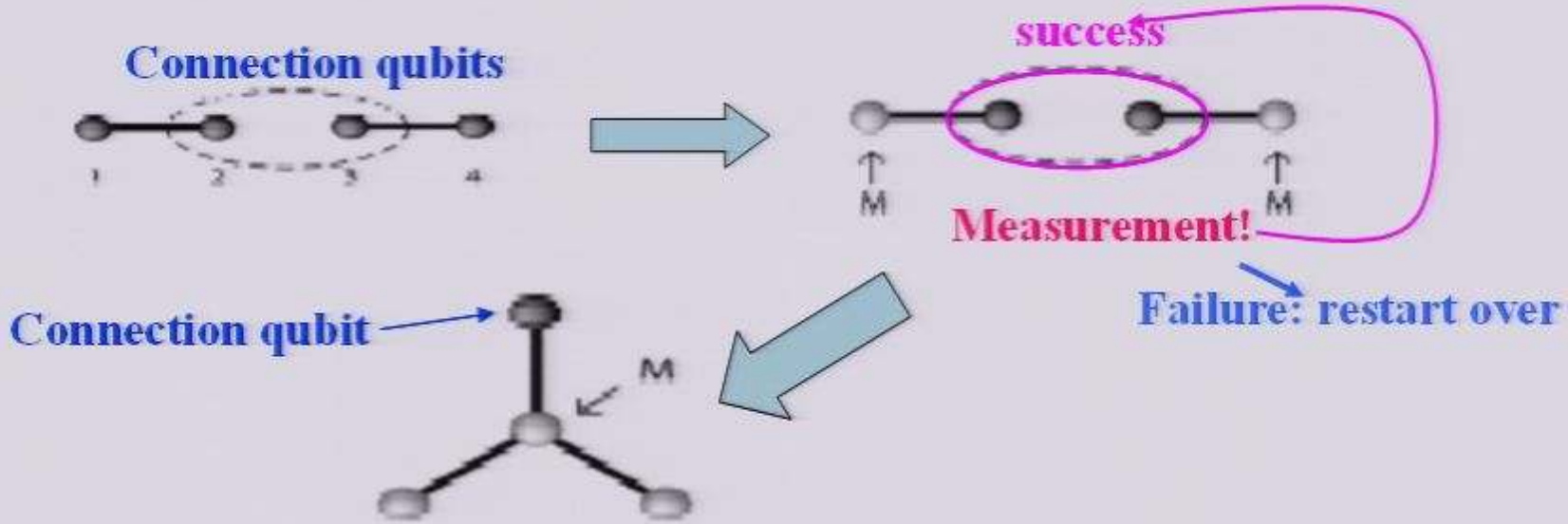

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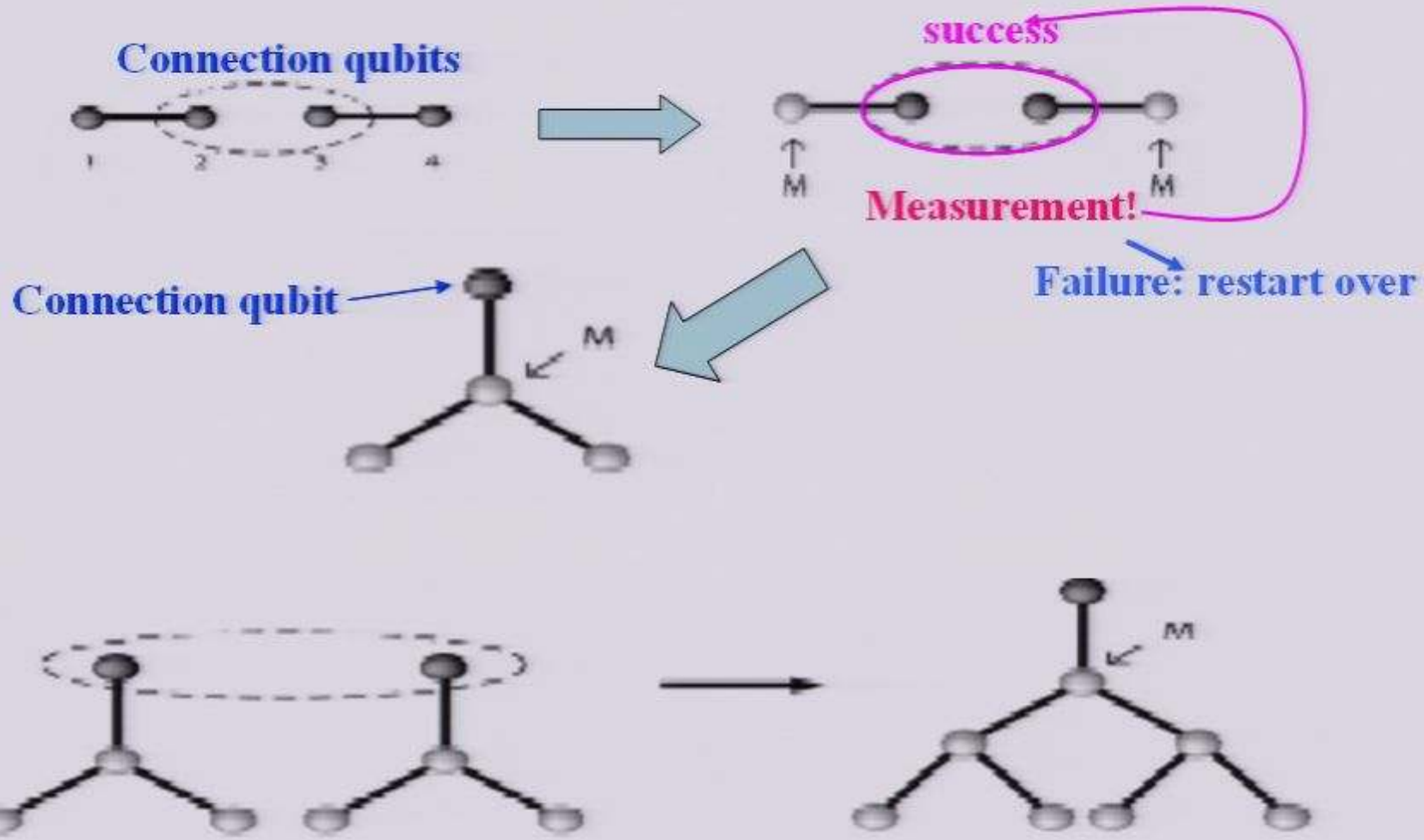

# Scaling

**Total preparation time:**

$$T \approx t_0 (\eta_s \eta_d)^{-1} n^{[(\log_2 n - 1)/2 + \log_2(1/\eta_d - 1/2)]}$$

number of qubits

detector efficiency

source efficiency

Pulse duration

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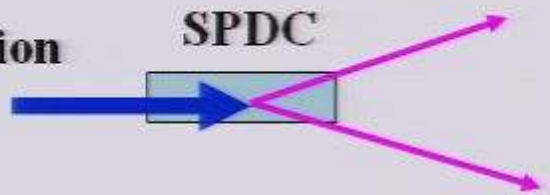

# Improvement in efficiency

**Example: preparation of 32-qubit graph state**

**The above method:**

$$T/t_0 \sim 3.0 \times 10^7$$

**with** $\eta_s \sim 1\%$

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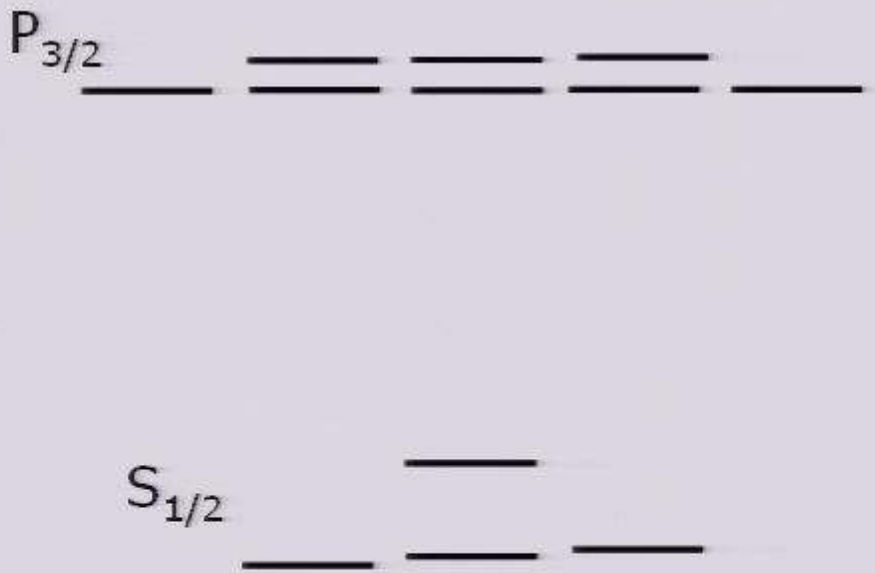
**Direct preparation:**

$$T/t_0 = \eta_s^{-32/2} \eta_d^{-32} 2^{32/2-1} \sim 10^{52}$$

# Probabilistic entanglement between 1 ion and 1 photon

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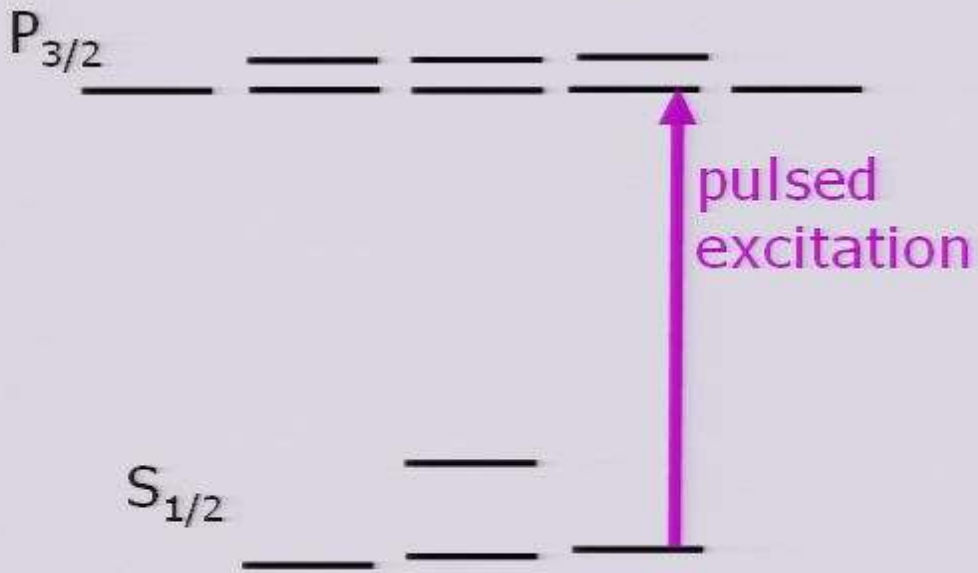


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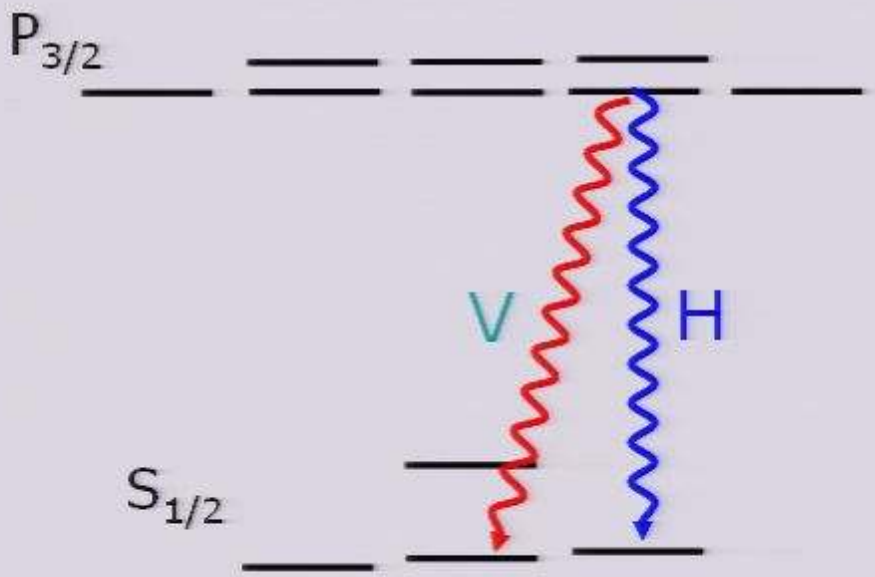


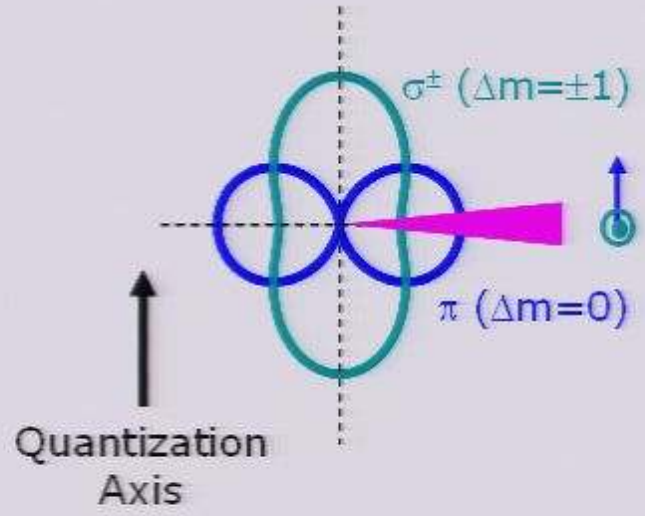


$$|\psi\rangle = |\downarrow\rangle|V\rangle + |\uparrow\rangle|H\rangle$$

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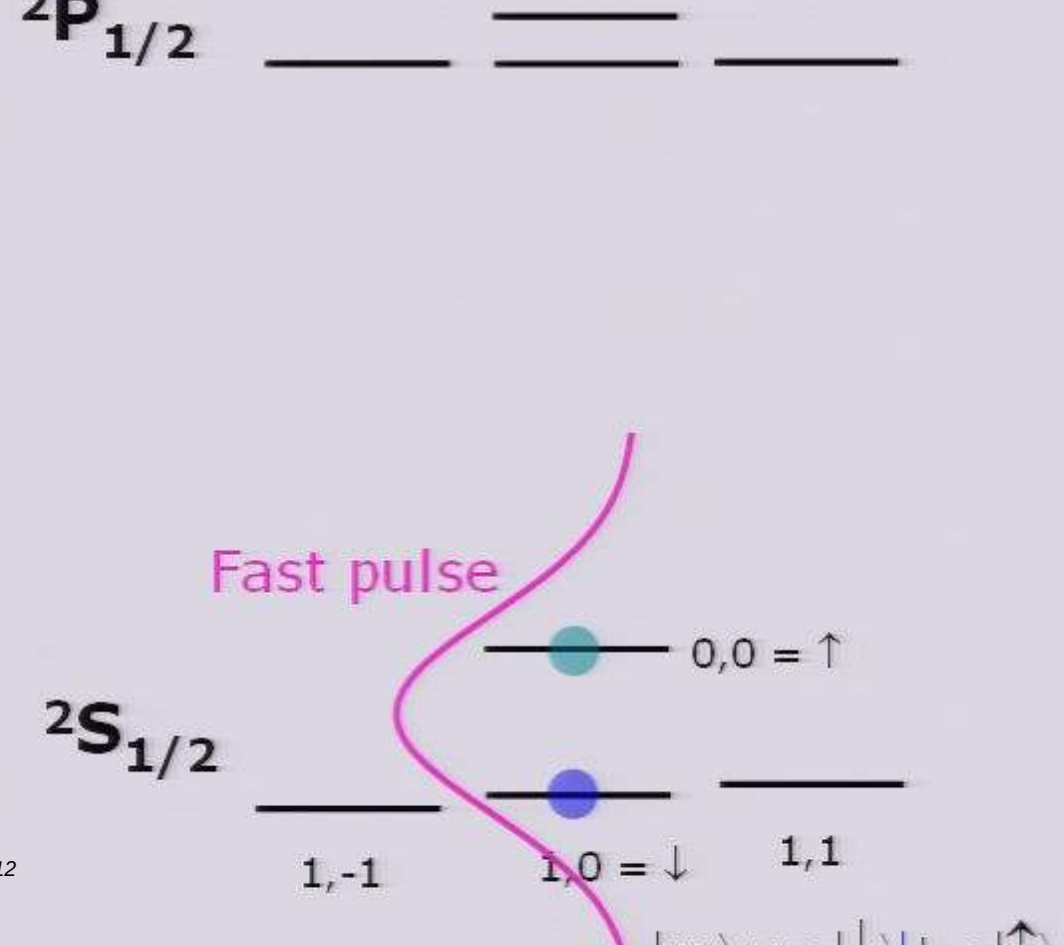
*Probabilistic* Gate (Madsen et al., PRL, 2006; Duan et al, PRA, 2006) between a single atom and single photon

**Frequency Qubit**



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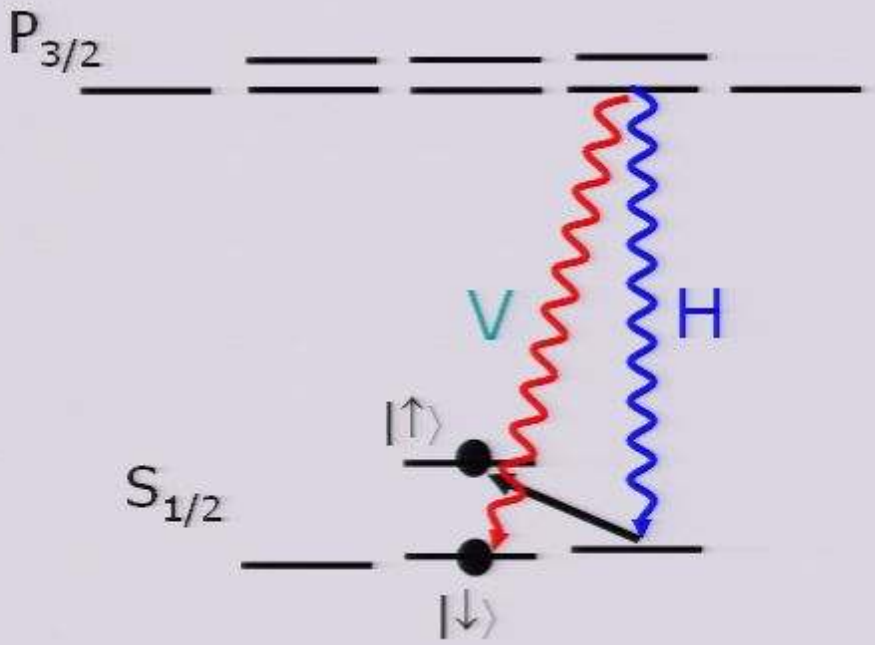


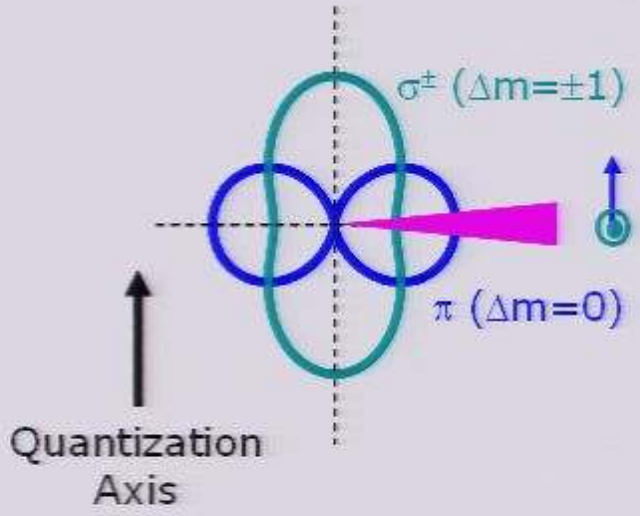


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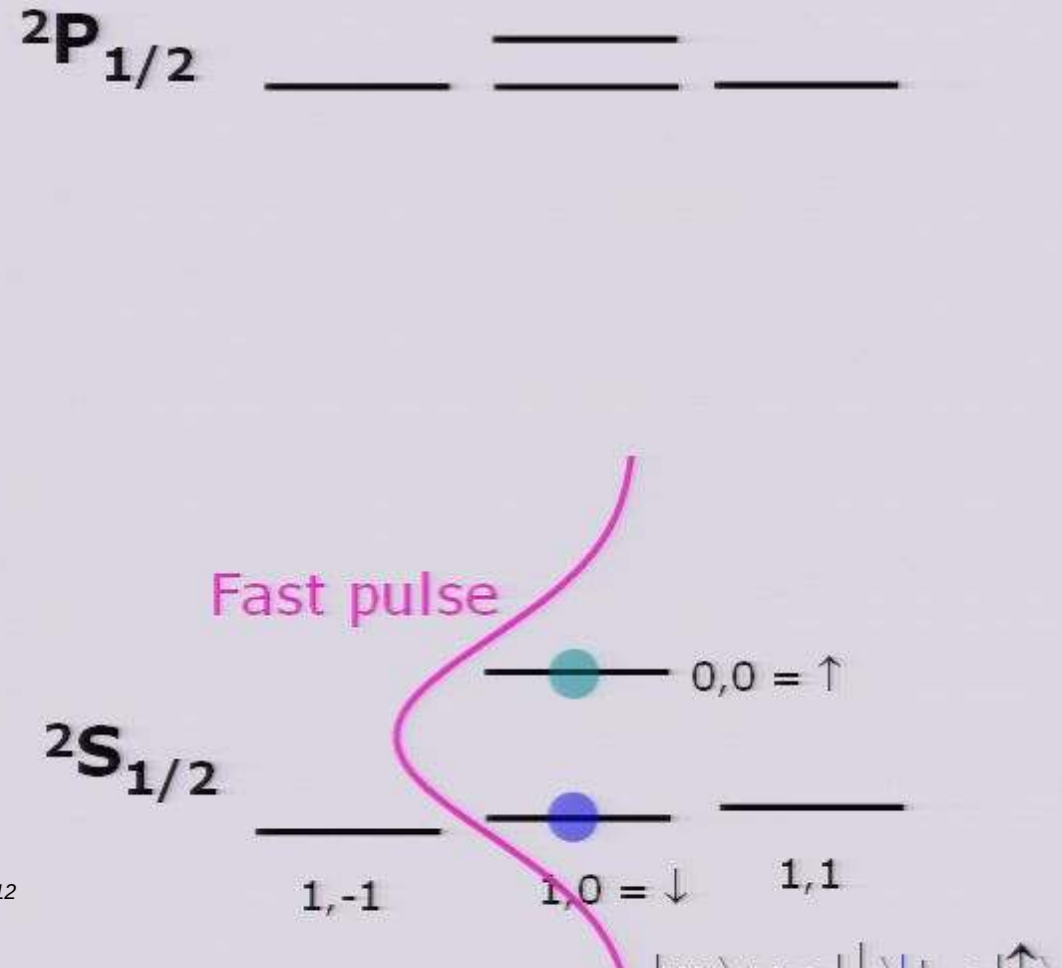

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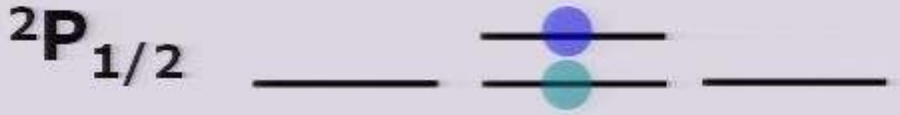


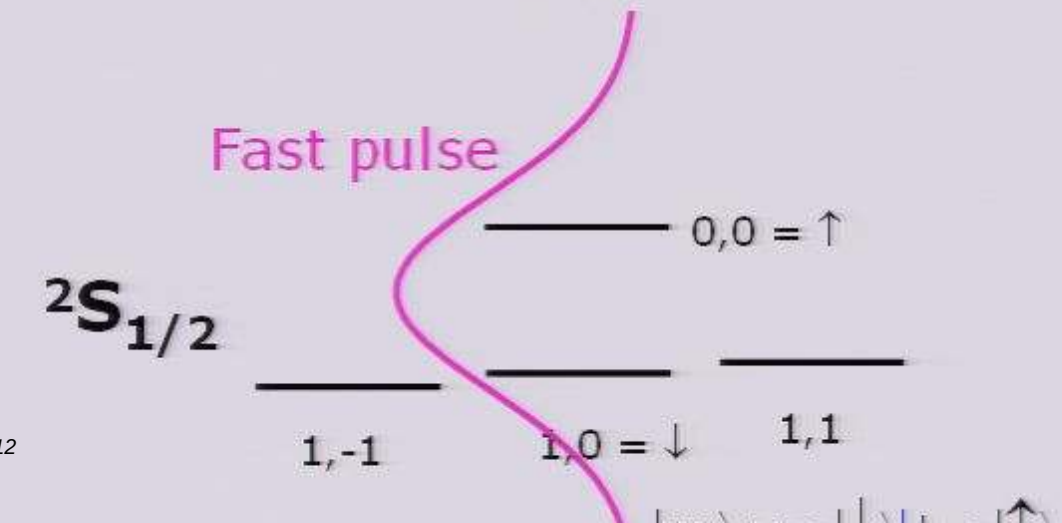

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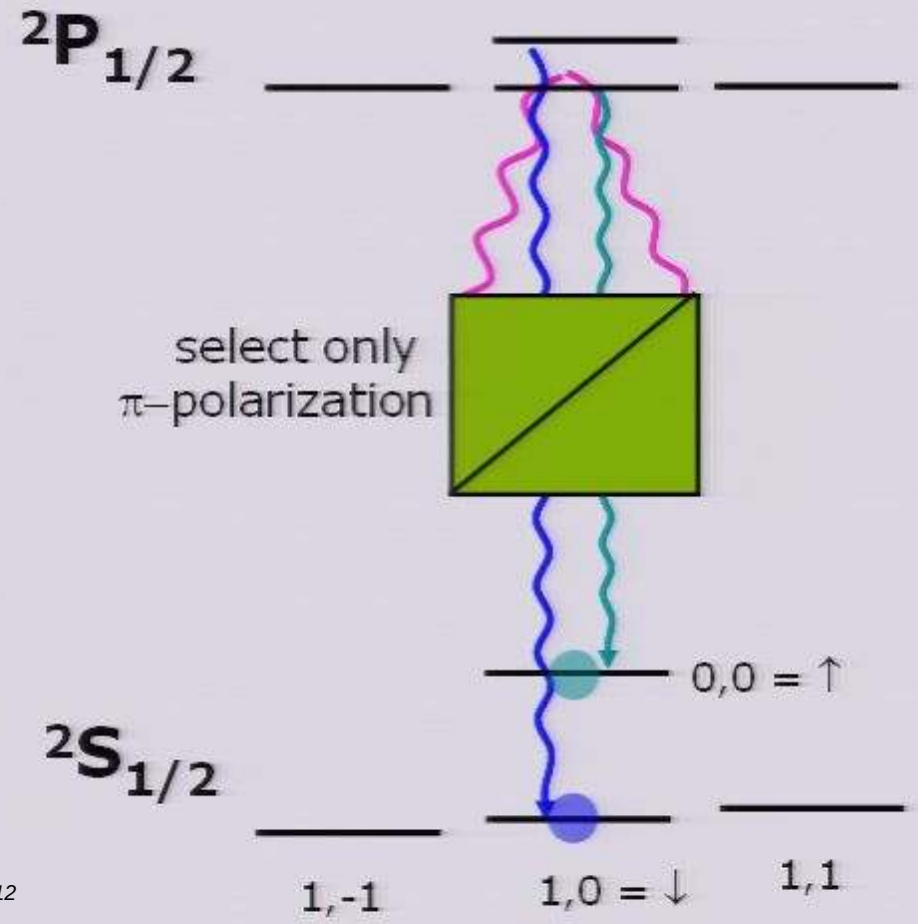

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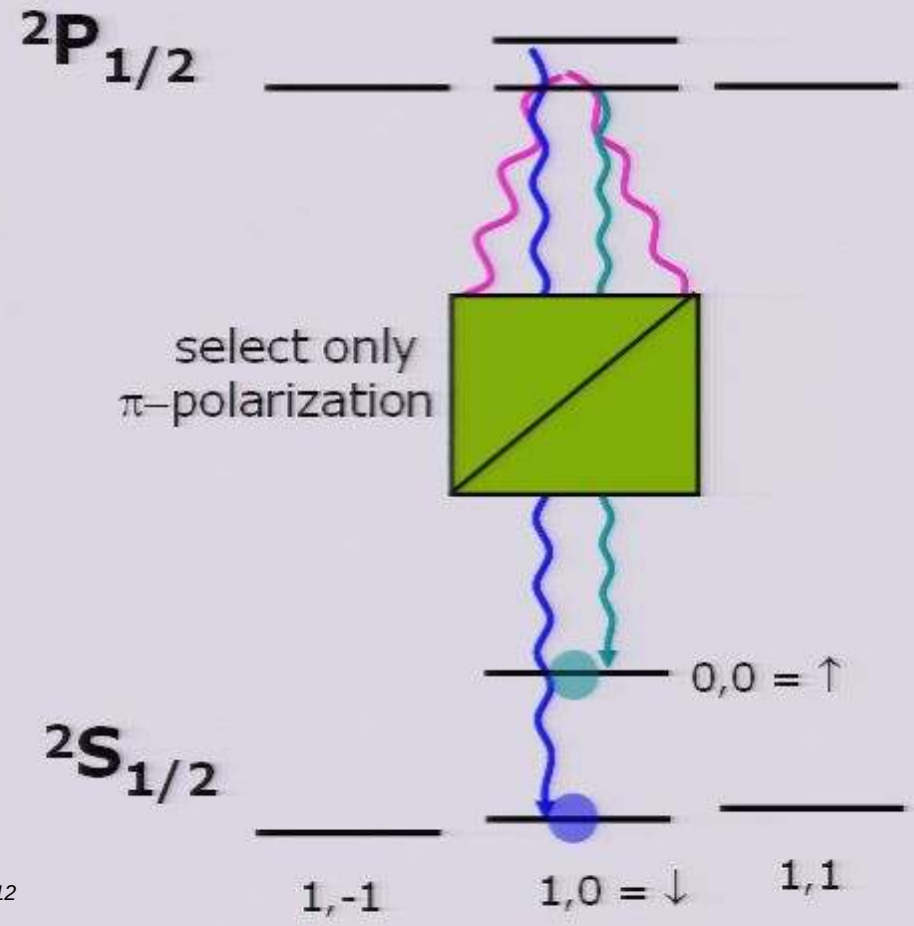


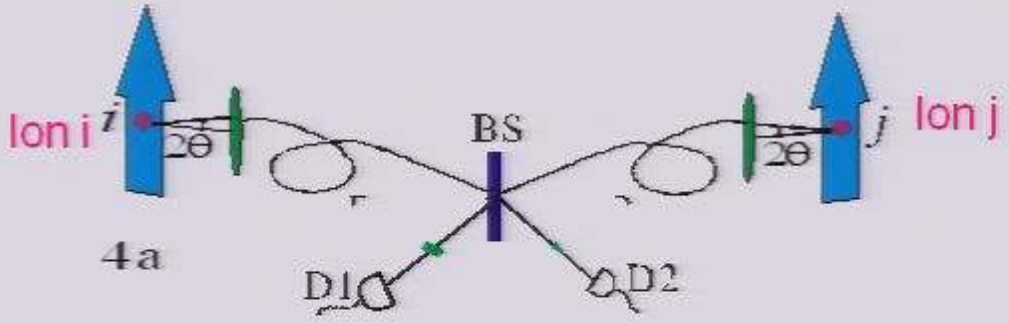

Given photon is emitted into polarizer

$|\psi\rangle = c_0|\downarrow\rangle|\text{blue}\rangle + c_1|\uparrow\rangle|\text{red}\rangle$

Keep track of information of the initial state !!

# Probabilistic gates on remote ion qubits

**Duan et al., PRA, 2006**




**Joint measurements of emitted photons**

- ✓ ions in arbitrary initial states
- ✓ A detection shows one ion in |0> state, and one ion in |1> state with coherence between two possibilities (probabilistic ZZ gate)
- ✓ Probabilistic ZZ is a gate that is universal and efficient for quant. Computation

# Efficient computation with probabilistic gates

**Problems for computation with probabilistic gates**

Exponentially small success probability: $p^n$

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**Our main result:** (Duan, Raussendorf, PRL, 2005)

- **Efficient quantum computation is possible no matter how small the success probability p is.**

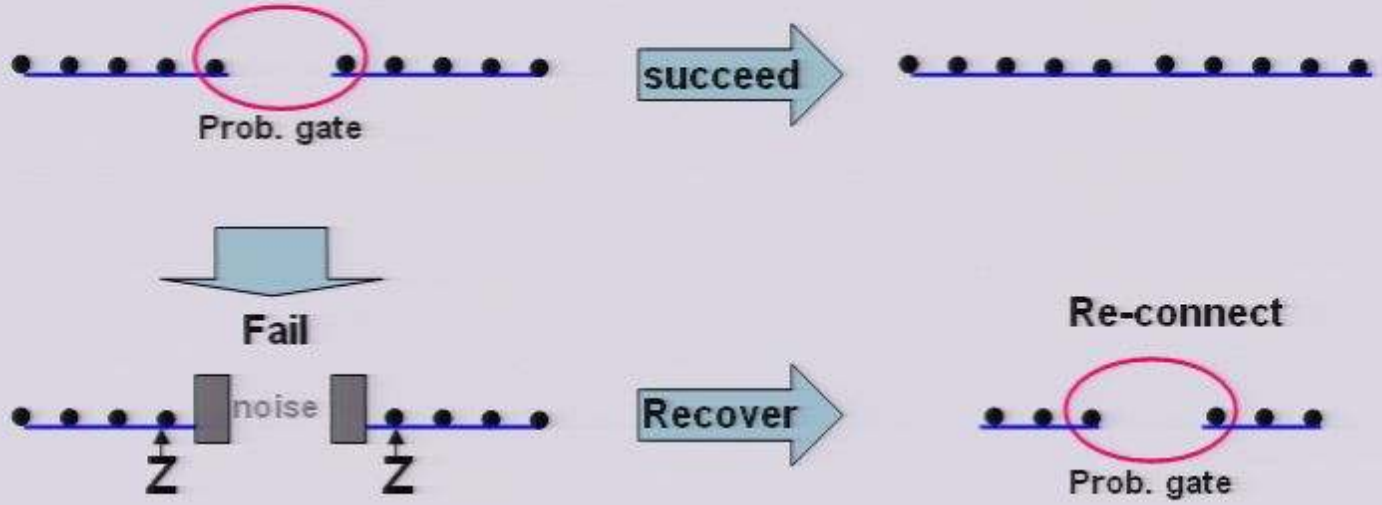
$$p > 0, \text{ or } p_{err} = 1 - p < 100\% = p_{th},$$

- **Computational overheads scale polynomially both with 1/p and n (n: the number of qubits)**

# Efficient construction of 1D cluster states

- **Step 1: Start from scratches**

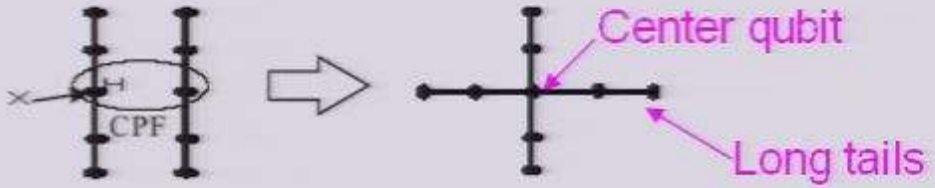
- **Step 2: The cluster chain grows**



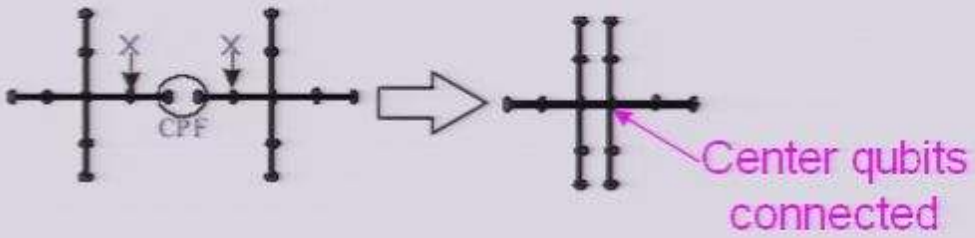
# Efficient construction of 2D cluster states

**Key idea: transfer the 2D problem to effective 1D connection**

- **Step 1: prepare the building blocks ("cross" state)**

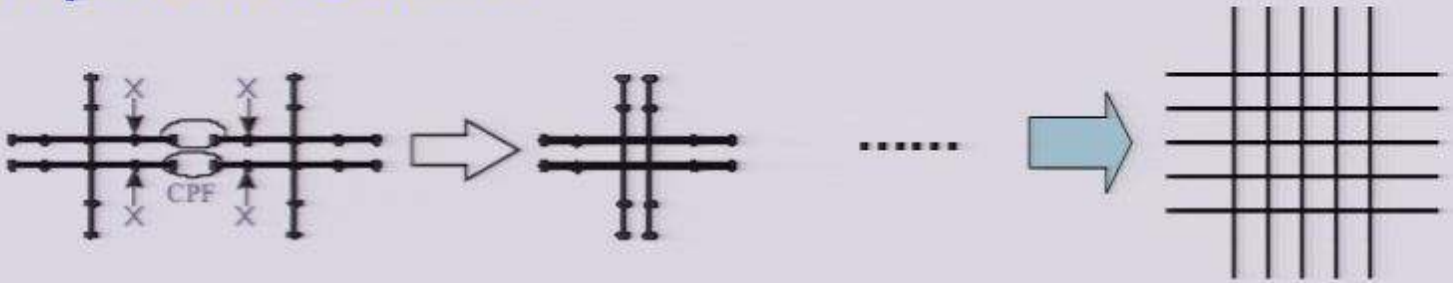



- **Step 2: connect the cross states**



# Efficient construction of 2D cluster states

- **Step 3: Form 2D lattice**




Continue connection of center qubits

- **Efficiency**

$$M \propto n \ln(n) \left(\frac{2}{p}\right)^{\log_2(4/p)+2}$$

# Summary

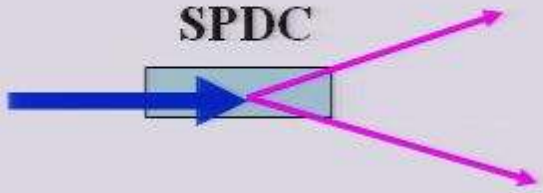
- **Quantum state engineering**
  - ✓ A single PBS is a powerful gate in post-selected subspace.
  - ✓ Any graph states can be constructed with PBS gates
  - ✓ Tree graph states can be constructed efficiently with any low detector efficiency
- **Quantum computation**
  - ✓ Probabilistic gates on atoms with linear optics
  - ✓ Scalable computation is possible with probabilistic gates

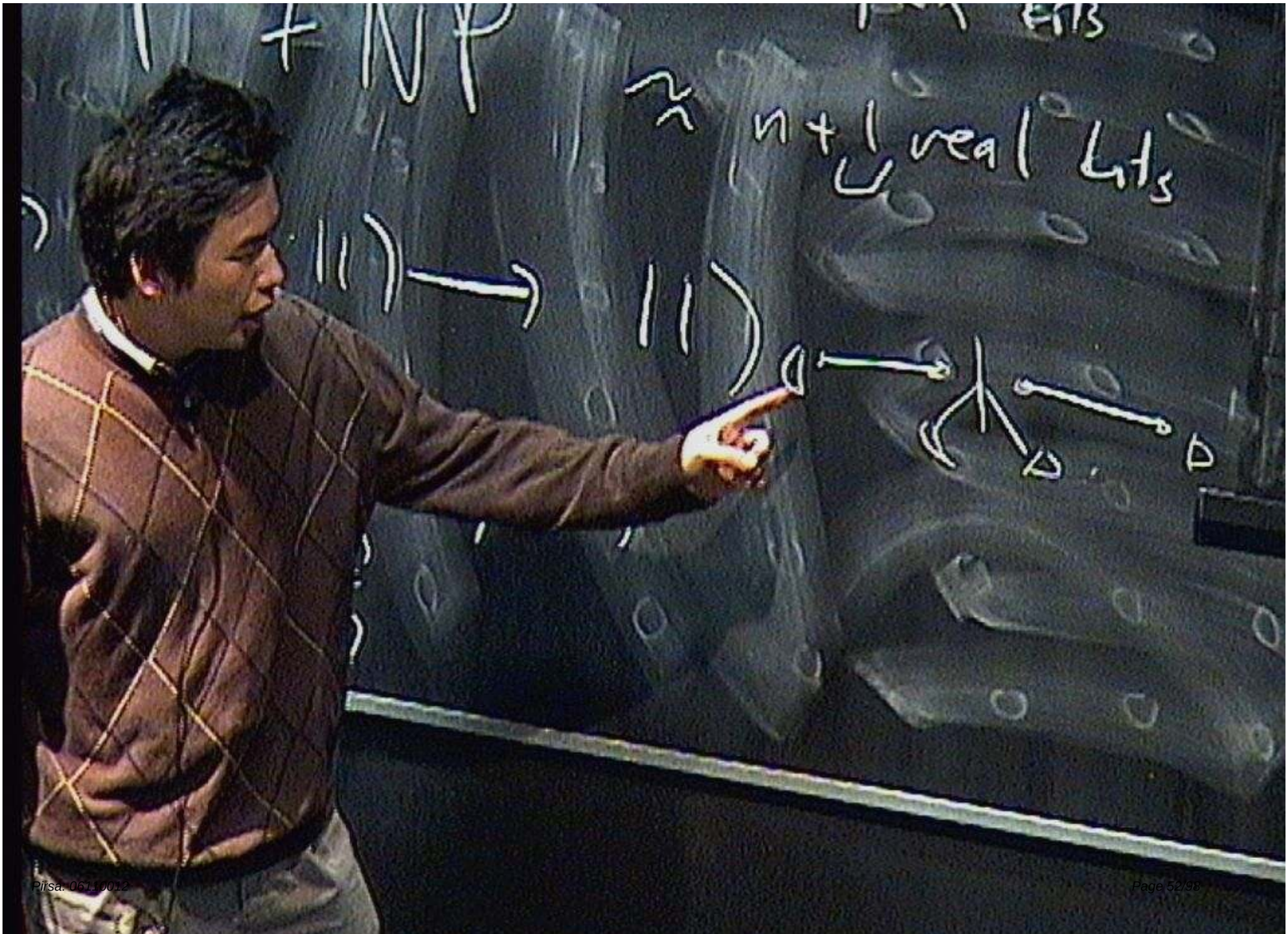
# Input state

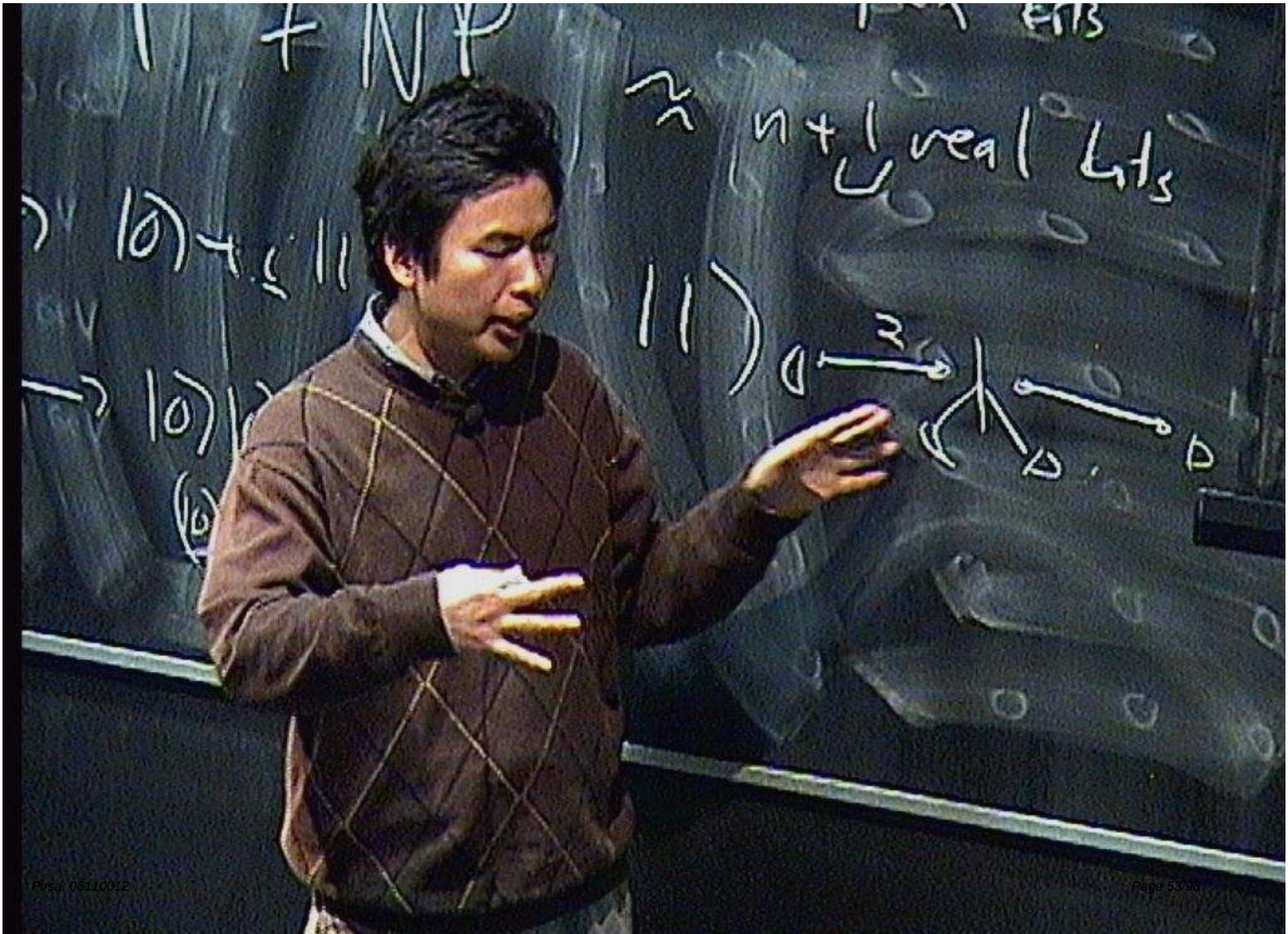
$$\rho_s = (1 - \eta_s)\,\rho_{\mathrm{vac}} + \eta_s\,|\Psi\rangle_{12}\,\langle\Psi|\,,$$

Source efficiency

$$|\Psi\rangle_{12} = (|HH\rangle_{12} + |VV\rangle_{12})/\sqrt{2}$$




NP
n+1 real Lt's


+ NP
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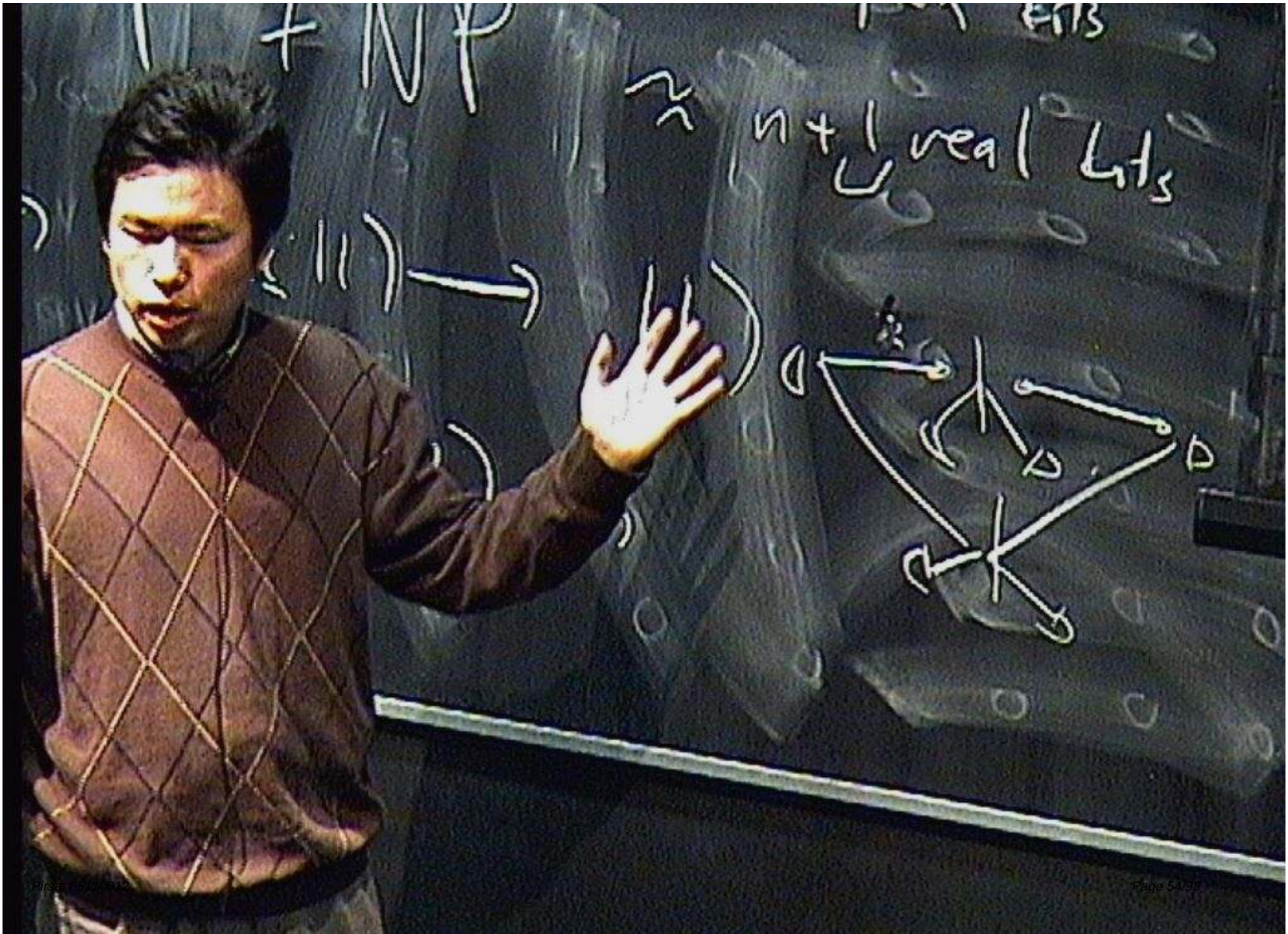
Page 53/98


+NP
≈ n+1 real Lt's

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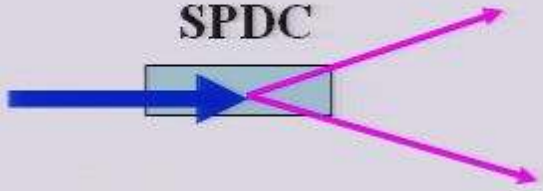

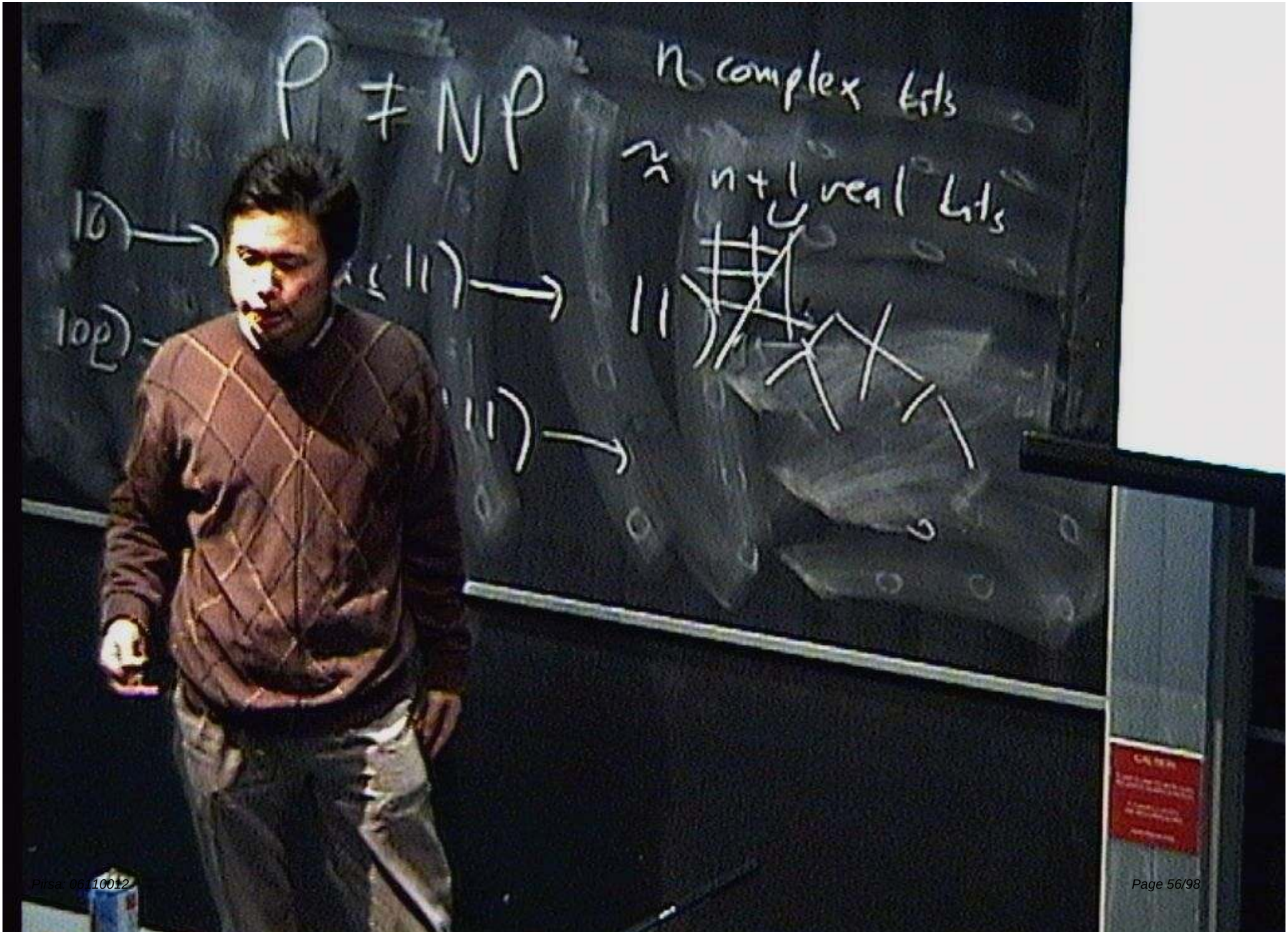
P ≠ NP
n complex bits
≅ n+1 real bits

# Input state

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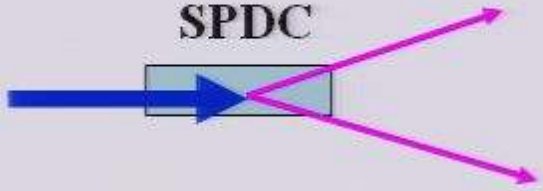
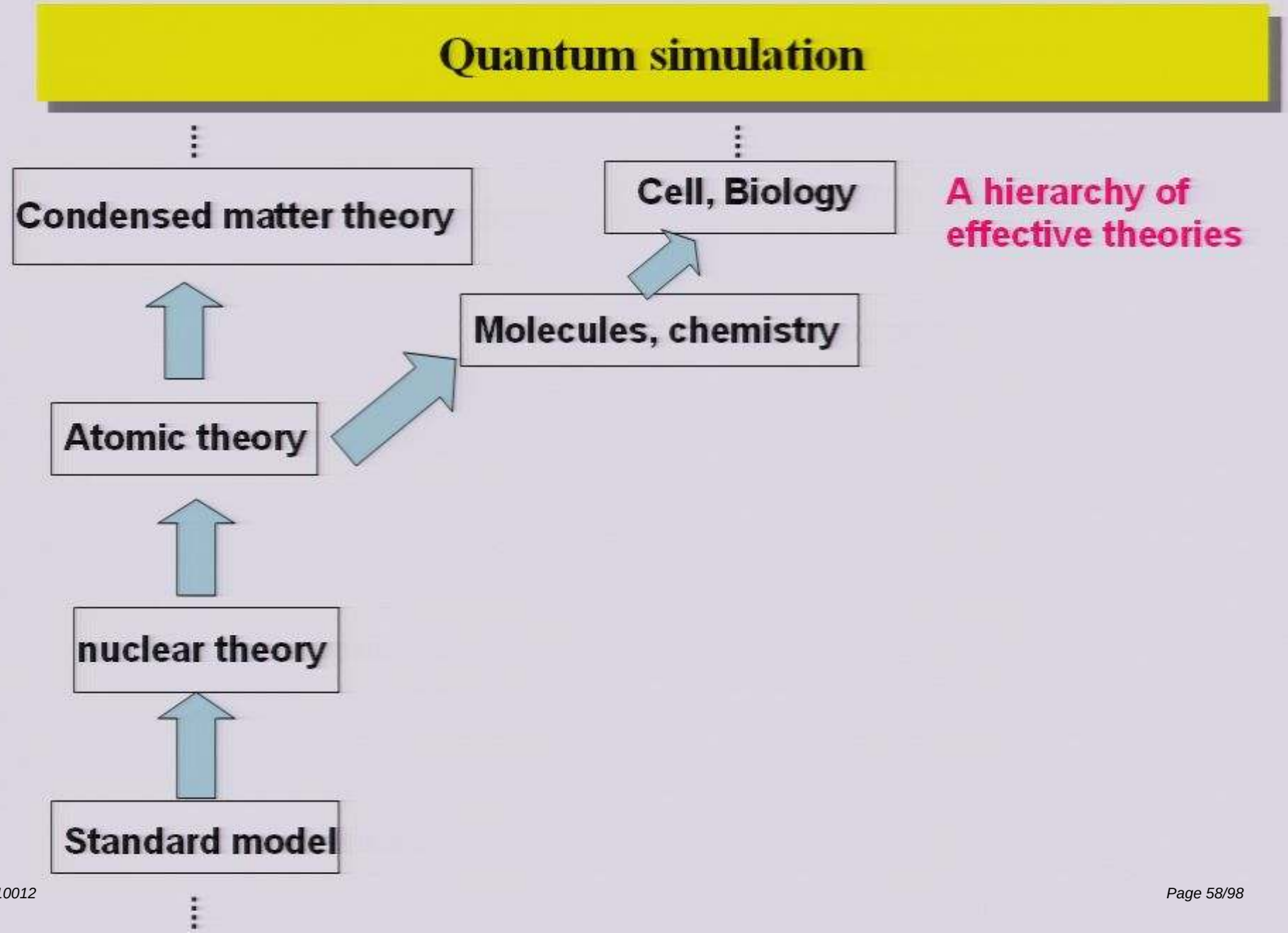
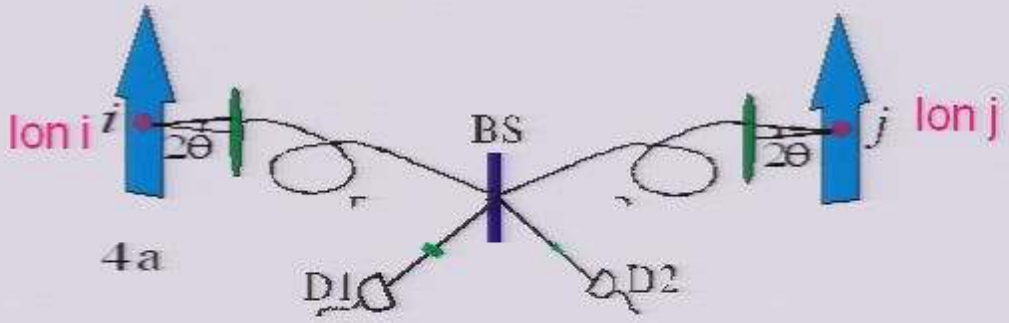



Quantum simulation
Condensed matter theory
Cell, Biology
A hierarchy of effective theories
Molecules, chemistry
Atomic theory
nuclear theory
Standard model

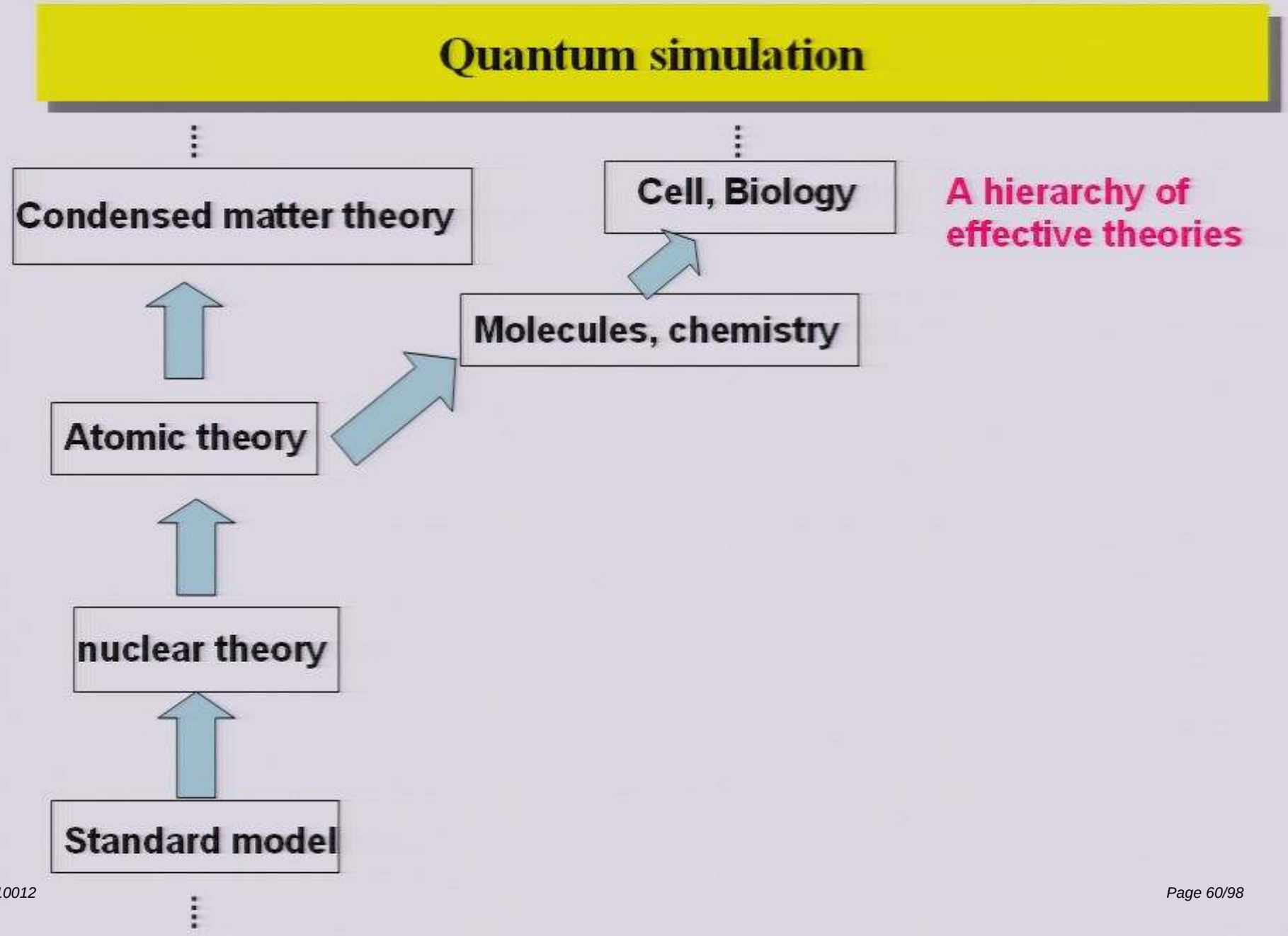
# Probabilistic gates on remote ion qubits

**Duan et al., PRA, 2006**




**Joint measurements of emitted photons**

- ✓ ions in arbitrary initial states
- ✓ A detection shows one ion in |0> state, and one ion in |1> state with coherence between two possibilities (probabilistic ZZ gate)
- ✓ Probabilistic ZZ is a gate that is universal and efficient for quant. Computation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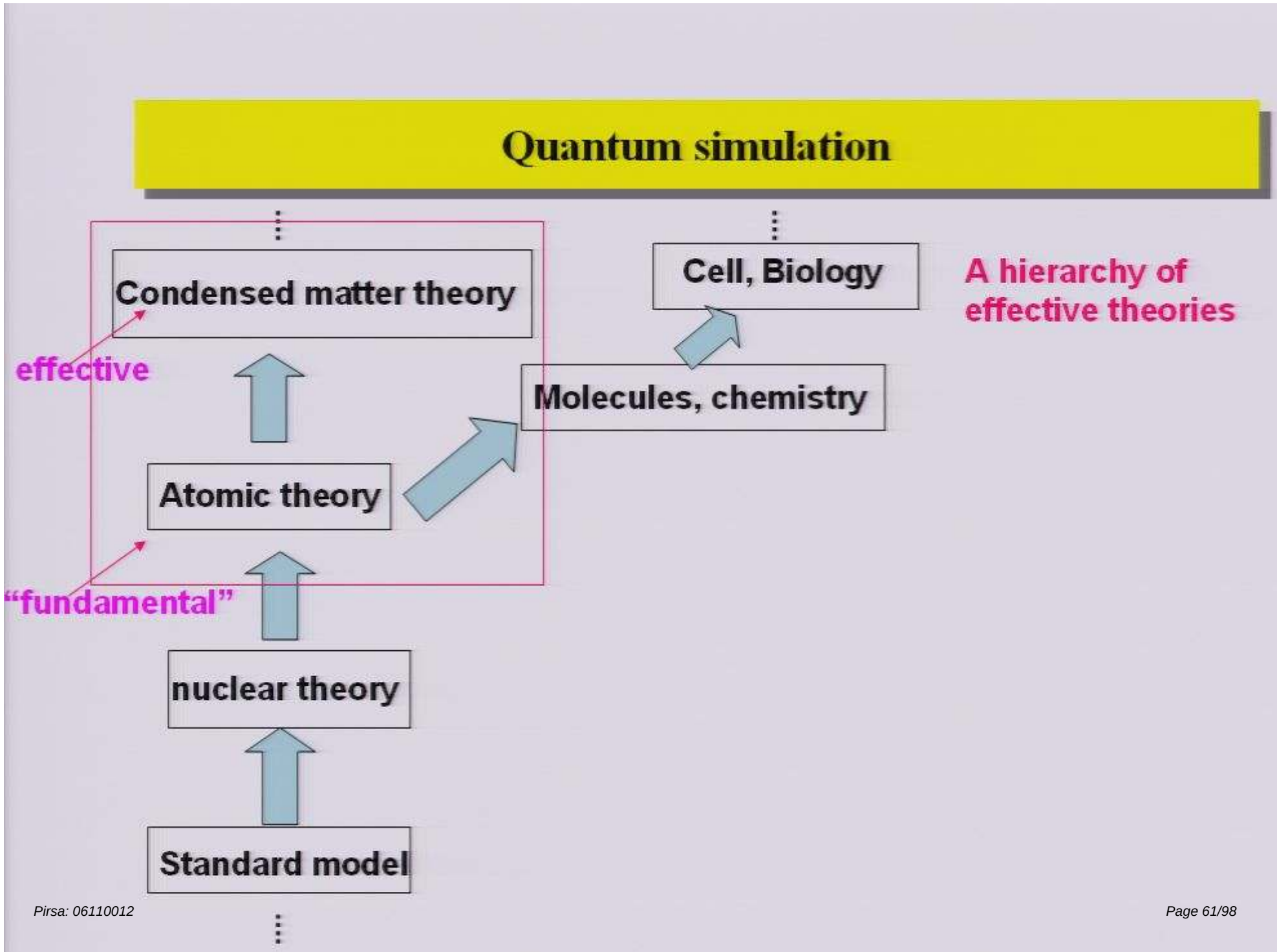
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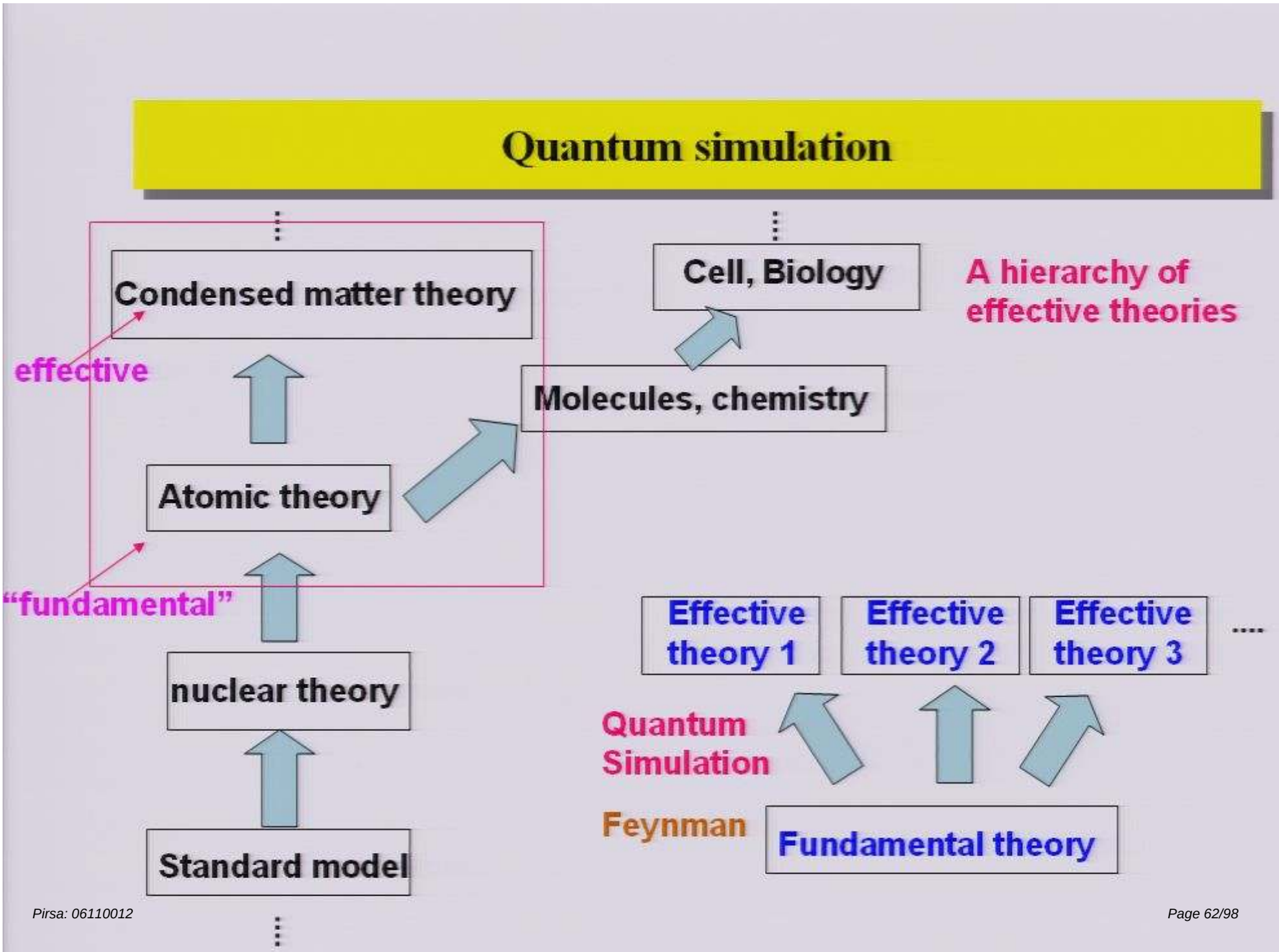
Quantum simulation
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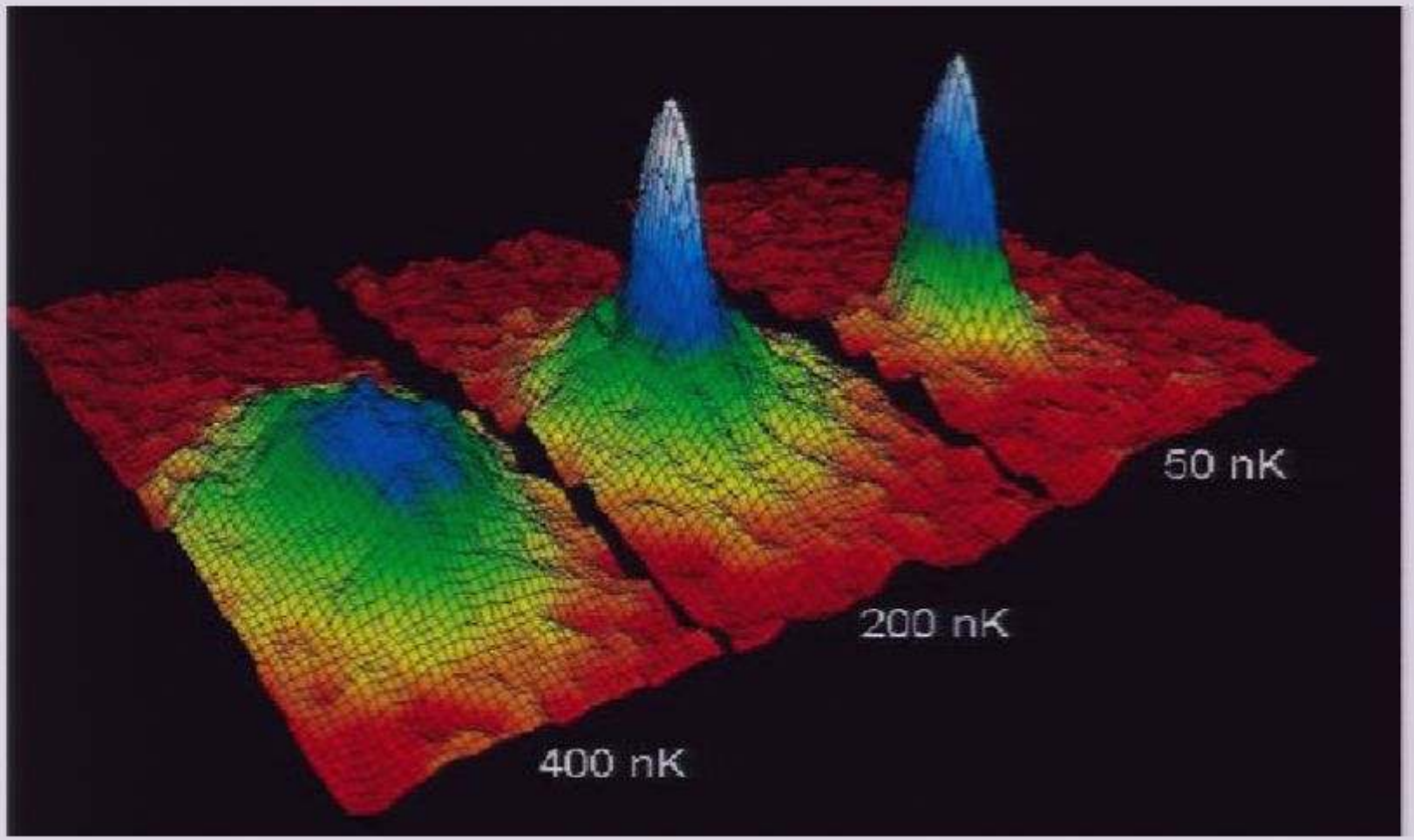
"fundamental"
Effective theory 1
Effective theory 2
Effective theory 3
....
nuclear theory
Quantum Simulation
Feynman
Fundamental theory
Standard model

# Ultracold atoms




**JILA**

# Control for ultracold atoms

- **Features of Ultracold atoms: quantum material**

  **--- remarkable controllability and diversity**

  - **Various geometry of optical lattice and superlattice**
  - **Both Bosons and fermions, different internal (spin) states**
  - **Interaction strength well controllable**
  - **Dynamical properties (kinetics) as well as equilibrium properties**

# A few illustrative examples

- **Simulation of gauge fields and spin Hall effects**
- **Simulation of quantum magnetism and anyon model**
- **Simulating Kondo Physics**
- **Simulation of t-J model and RVB** (resonating valence bonds) **physics**

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  Optical lattice+ Feshbach resonance

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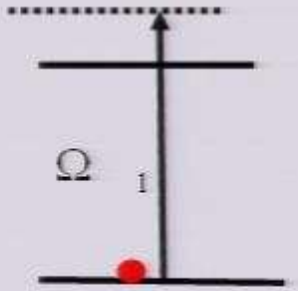
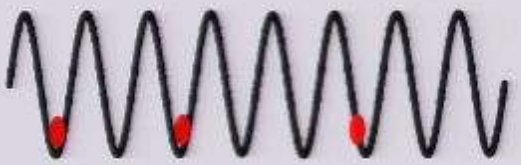
# Optical lattice



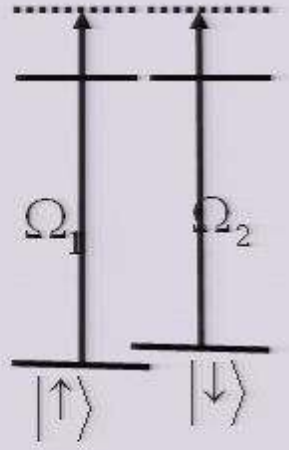
**Hansch-Bloch's group**

**Experimental superfluid-Mott insulator transition etc.**

# Spin-dependent tunneling

L.-M Duan, E. Demler, M. Lukin, PRL 91, 090402 (2003).

- ✓ Spin-1/2 (two-component) atoms in optical lattice
- ✓ Spin-dependent tunneling!



# The effective Hamiltonian

- The effective Hamiltonian in the Mott insulator region

$$H = \sum_{\langle ij \rangle} \left[ \lambda_{\mu z} \sigma_i^z \sigma_j^z + \lambda_{\mu \perp} \left( \sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \right],$$

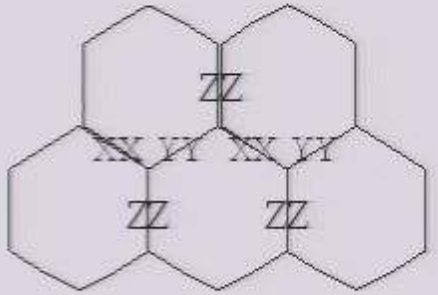
**XXZ model**

- **The parameters $\lambda_{\mu z}, \lambda_{\mu \perp}$ fully controllable in each direction**
- **Different magnetic coupling along different directions in lattice**

# Realization of more exotic magnetic model

- An exact anyon model (Kitaev)

  - Anisotropic spin interaction in a hexagonal lattice

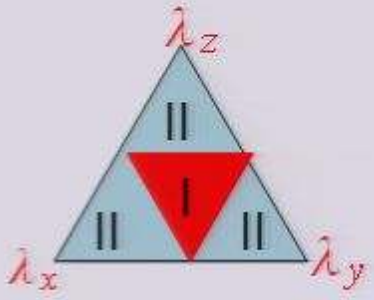



$$H = \lambda_x \sum_{\langle i,j\rangle \in D_x} \sigma_i^x \sigma_j^x + \lambda_y \sum_{\langle i,j\rangle \in D_y} \sigma_i^y \sigma_j^y + \lambda_z \sum_{\langle i,j\rangle \in D_z} \sigma_i^z \sigma_j^z,$$

  - Properties:

    I: non-abelian anyon excitations

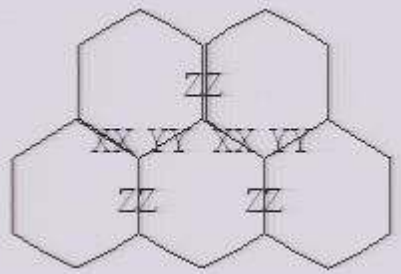
    II: abelian anyon excitations



# Implementation of the anyon model

L.-M Duan, E. Demler, M. Lukin, PRL (2003).

**Three steps:**




- **Forming a 2D hexagonal lattice**
- **Introduce spin-dependent tunneling**
- **Different tunneling interactions along different spatial directions in the lattice**

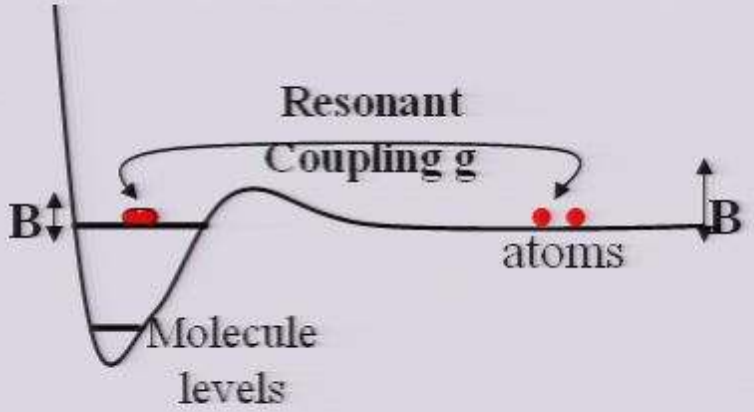
# Feshbach resonance + optical lattice

- **Optical lattice** -- diverse interaction configurations

  -- interaction weak (ultra-low temperature)

- **Feshbach resonance** -- dramatically increase interaction magnitude

# Feshbach Resonance



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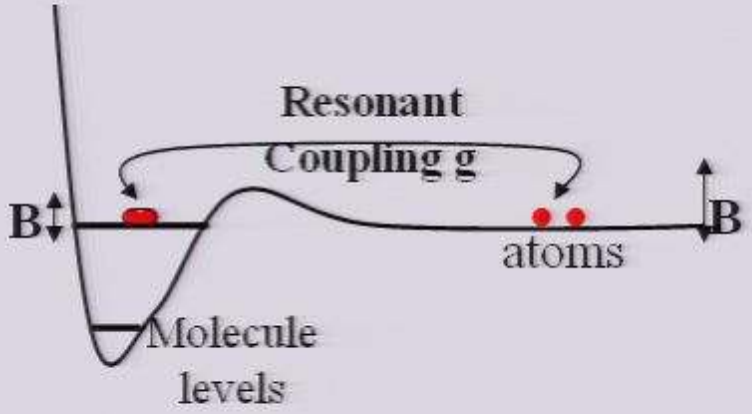


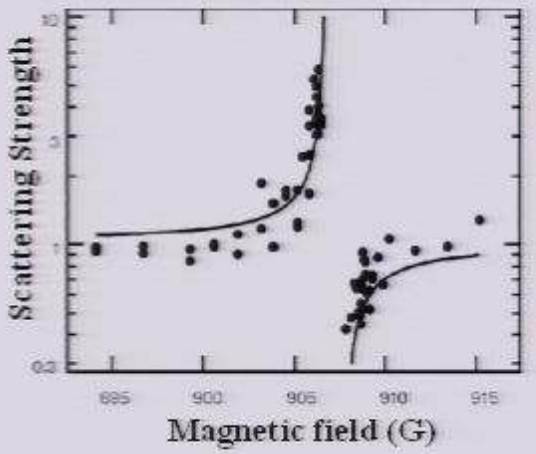


**Experimental Feshbach resonance**

*S. Inouye, et al., Nature (98).*

**--- Control of interaction strength with magnetic field**

# Feshbach resonance + optical lattice

Feshbach resonance -- magnitude
Optical lattice -- diversity

Experiments: Esslinger, Ketterle etc.

(L.-M Duan, Phys. Rev. Lett. 95, 243202 (2005)

- **Derivation of effective Hamiltonian for strongly interacting fermions in an optical lattice across Feshbach resonance**
- **Connect t-J model and XXZ models, simulation of RVB physics**

# Strong interaction effects

## What is the complication?

- **Multi-band populations** (Diener, T.-L. Ho, PRL 2006)

$$g_{on} > E_{bg}$$

On-site coupling rate

Band gap

# Strong interaction effects

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Band gap

$$\sum_{i;pqr} g_{pqr} b_{ip}^{+} a_{iq\uparrow} a_{ir\downarrow}$$

Different bands

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$$g_{on} > E_{bg}$$

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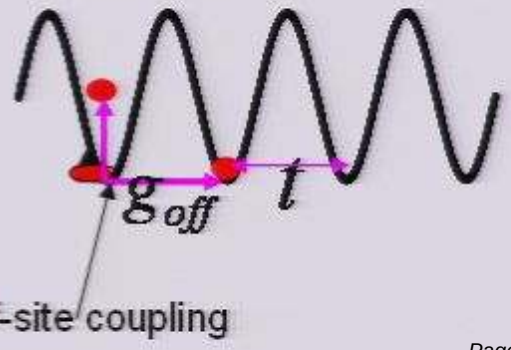
Different bands

- **Off-site collision couplings** (Duan, PRL 2005)

$$g_{off} > t$$

Off-site coupling rate

Tunneling rate



# The lattice Hamiltonian

- **The multi-band Hamiltonian**

$$
\begin{aligned}
H_I = & \sum_{i\mathrm{pqs}} \left( c^{(am)}_{0;\mathrm{pss}'} b^{\dagger}_{i\mathrm{p}} a_{i\mathrm{s}\uparrow} a_{i\mathrm{s}'\downarrow} + h.c. \right) + \sum_{i\mathrm{pqss}'} c^{(aa)}_{0;\mathrm{pqss}'} a^{\dagger}_{i\mathrm{p}\downarrow} a^{\dagger}_{i\mathrm{q}\uparrow} a_{i\mathrm{s}\uparrow} a_{i\mathrm{s}'\downarrow} \\
& + \sum_{i} \sum_{j \in N(i)} \left[ \left( \sum_{\mathrm{pss}'} c^{(am)}_{1;\mathrm{pss}'} b^{\dagger}_{i\mathrm{p}} + \sum_{\mathrm{pqss}'} c^{(aa)}_{1;\mathrm{pqss}'} a^{\dagger}_{i\mathrm{p}\downarrow} a^{\dagger}_{i\mathrm{q}\uparrow} \right) \left( a_{i\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} - a_{i\mathrm{s}\downarrow} a_{j\mathrm{s}'\uparrow} \right) + \sum_{\mathrm{pss}'} c^{(am)}_{2;\mathrm{pss}'} b^{\dagger}_{i\mathrm{p}} a_{j\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} + h.c. \right] \\
& + \sum_{i\mathrm{pqss}'} \sum_{j \in N(i)} \left[ c^{(aa)}_{2;\mathrm{pqss}'} a^{\dagger}_{i\mathrm{p}\downarrow} a^{\dagger}_{i\mathrm{q}\uparrow} a_{j\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} + c^{(aa)}_{3;\mathrm{pqss}'} a^{\dagger}_{i\mathrm{p}\downarrow} a^{\dagger}_{j\mathrm{q}\uparrow} \left( a_{i\mathrm{s}\uparrow} a_{j\mathrm{s}'\downarrow} - a_{i\mathrm{s}\downarrow} a_{j\mathrm{s}'\uparrow} \right) \right], \qquad (1)
\end{aligned}
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\end{aligned}
$$

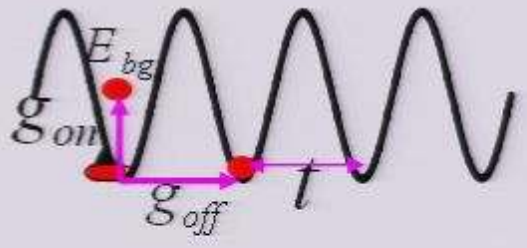
**Hopeless to solve!**

# How to simplify ???

- **Separation of different energy scales** (Duan, PRL 2005)

**Large energy scale:** $g_{on} \gtrsim E_{bg}$

**Small energy scale:** $g_{off} \gtrsim t$

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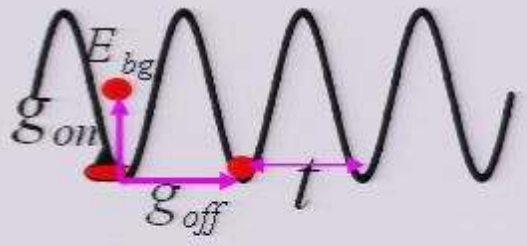


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$$E_{on} \sim 23E_r,\ E_{bg} \sim 6.3E_r,$$
$$E_{off} \sim 0.09E_r,\ E_t \sim 0.02E_r$$

for $V_0 = 10E_r$ and $^{40}K$

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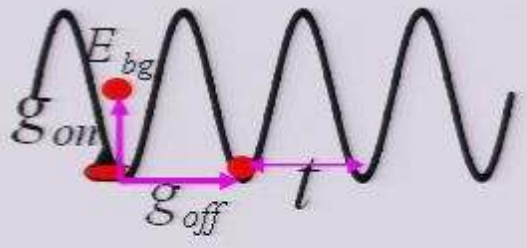


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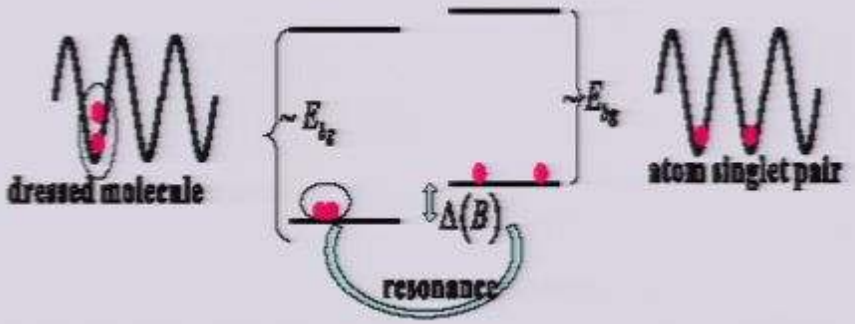
**First solve the single-site problem!**

# Lattice Resonance

- **Resonance between atoms and dressed molecules**




**Projection:**

$$\hat{P} \equiv \bigotimes_i \hat{P}_i,$$

$$\hat{P}_i \equiv |0\rangle_i \langle 0| + |\uparrow\rangle_i \langle\uparrow| + |\downarrow\rangle_i \langle\downarrow| + |\tilde{d}\rangle_i \langle d|.$$

# Effective Hamiltonian

- **Effective Hamiltonian** (Duan, PRL, 2005)

$$H_{eff} = \sum_i \Delta(B) d_i^\dagger d_i + \sum_{i;j\in N(i)} t_d P d_i^\dagger d_j P$$
$$+ \sum_{i;j\in N(i)} \sum_\sigma \left( t_a P a_{i\sigma}^\dagger a_{j\sigma} P + t_{da} d_i^\dagger d_j a_{j\sigma}^\dagger a_{i\sigma} \right)$$
$$+ \sum_{i;j\in N(i)} \left( g d_i^\dagger (a_{i\uparrow} a_{j\downarrow} - a_{i\downarrow} a_{j\uparrow}) + h.c. \right), \quad (2)$$

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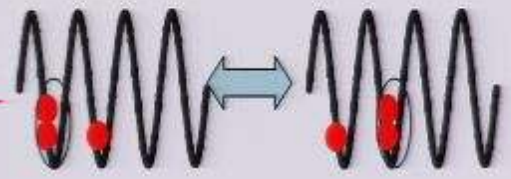

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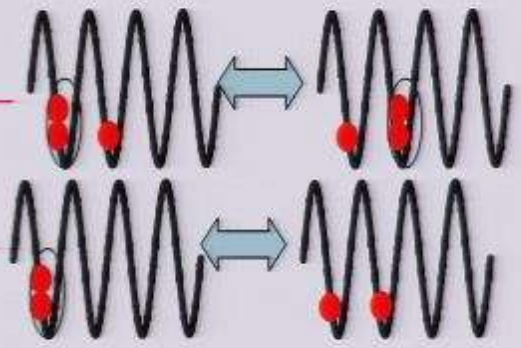
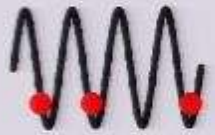


# t-J model for atoms

- **Limiting case 1: atom limit**

  **Atom effective Hamiltonian:**



$$H_{tJ} = \sum_{i;j\in N(i)} \left[ t_a \sum_{\sigma} P_a a^{\dagger}_{i\sigma} a_{j\sigma} P_a + J\left(\mathbf{s}_i \cdot \mathbf{s}_j - n_i n_j/4\right) \right]$$

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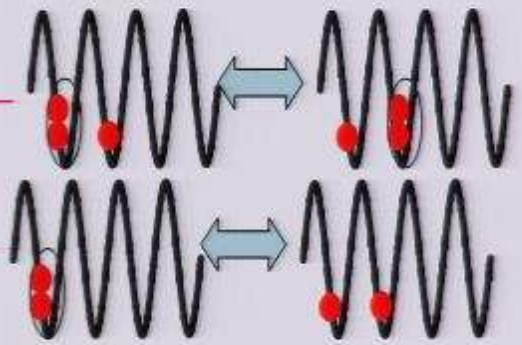

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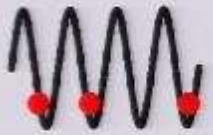

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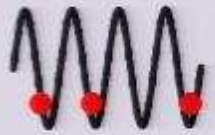

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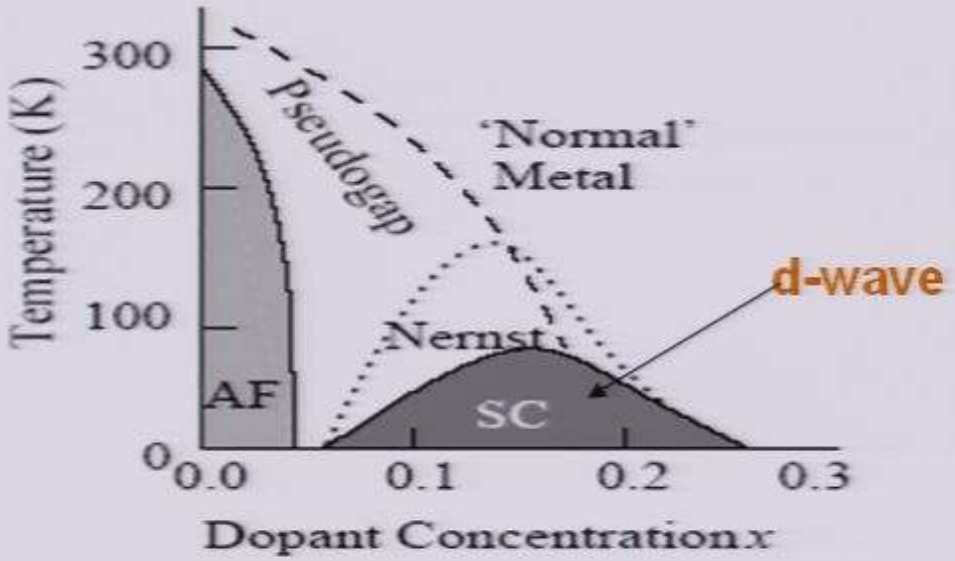
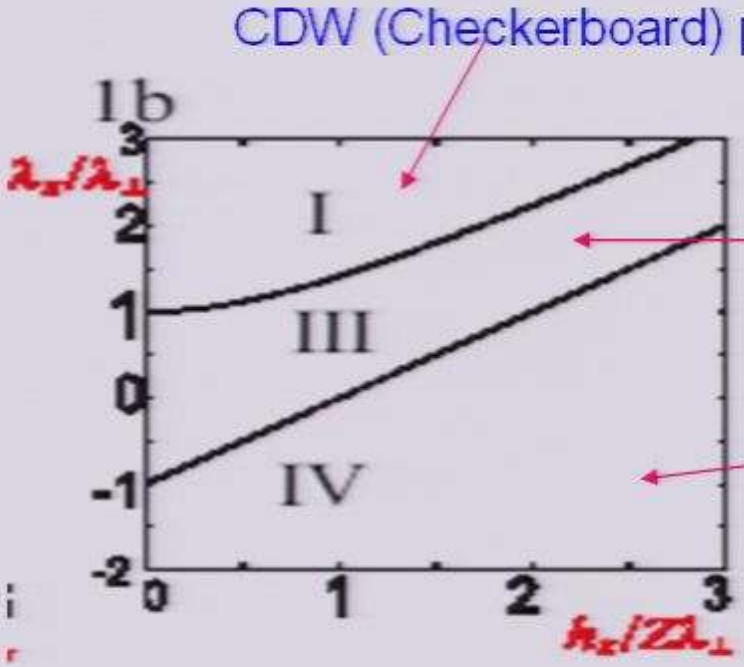


# XXZ model for dressed molecules

- **Limiting case2: molecule limit**

$$H_{XXZ} = (t_d'/4)\sum_i \left[ B_{eff} Z_i + \sum_{j\in N(i)} (X_i X_j + Y_i Y_j + \delta_z Z_i Z_j) \right]$$

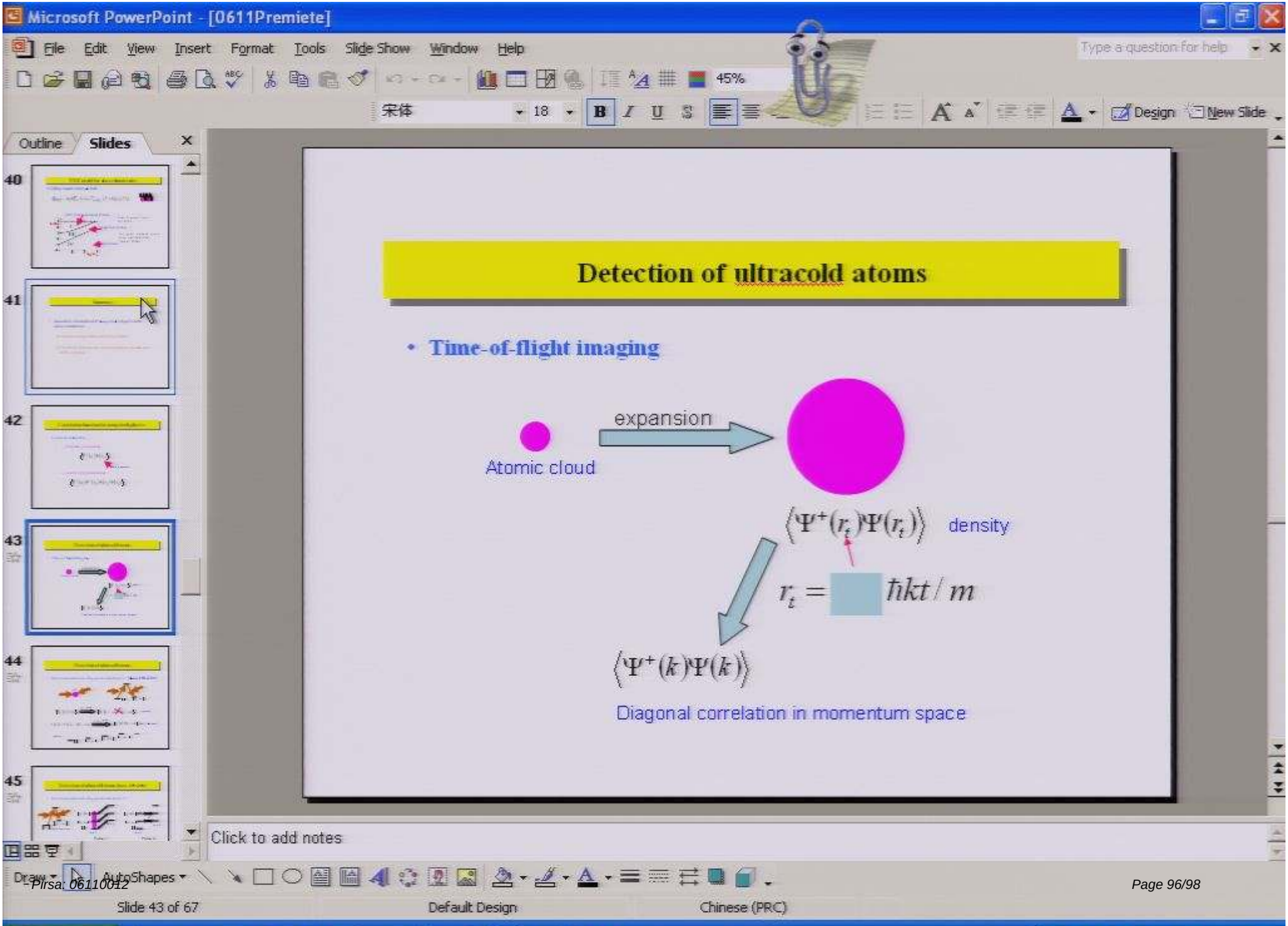



Duan, Demler, Lukin
PRL 2003

Chin et al., Ketterle' group
Cond-mat/0607004,
Nature (2006)

# Summary

- **Quantum simulation of many-body physics with ultracold atoms**
  - Quantum magnetism and anyon model
  - Feshbach resonance + optical lattice (t-J model and RVB physics)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40
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43
44
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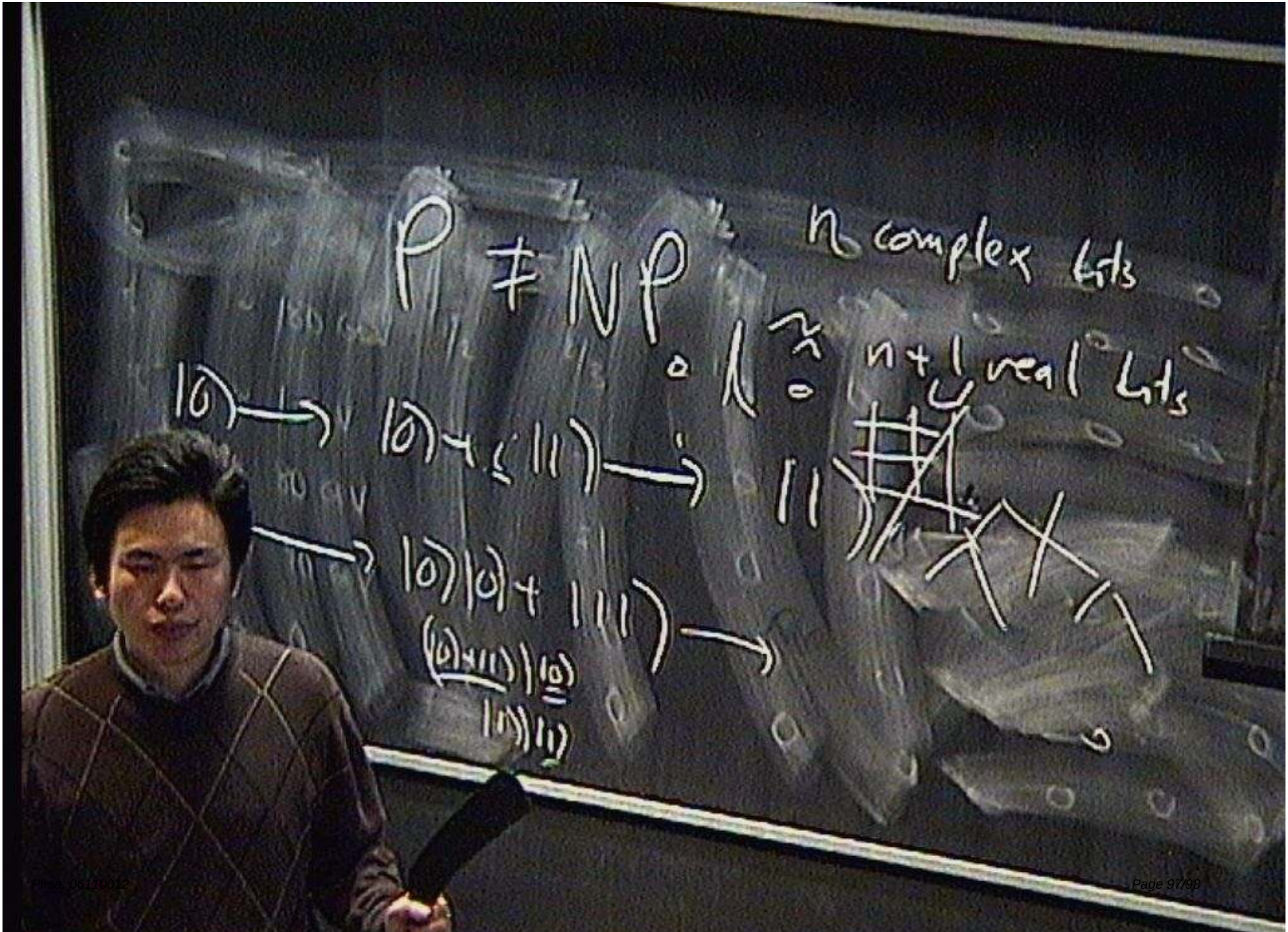
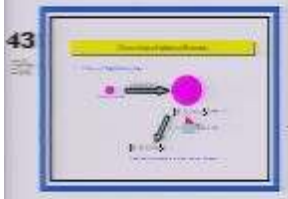
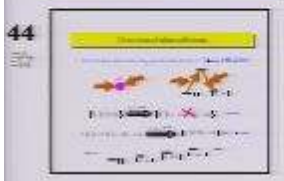
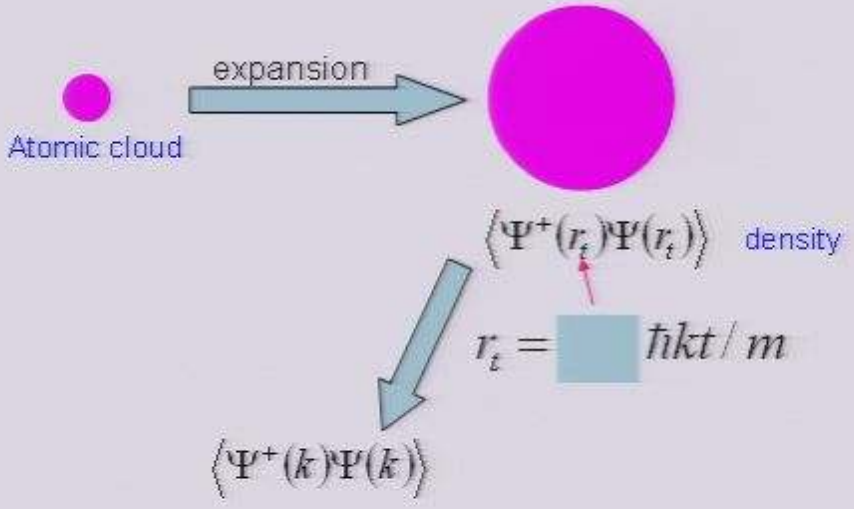
Detection of ultracold atoms
• Time-of-flight imaging
expansion
Atomic cloud
$\langle \Psi^+(r_t)\Psi(r_t)\rangle$ density
$r_t = \hbar kt/m$
$\langle \Psi^+(k)\Psi(k)\rangle$
Diagonal correlation in momentum space
Click to add notes
Draw AutoShapes
Slide 43 of 67
Default Design
Chinese (PRC)


P ≠ NP
n complex bits
n+1 real bits

## Detection of ultracold atoms

- **Time-of-flight imaging**

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