Title: Self-Testing of Quantum Circuits

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URL: http://pirsa.org/06110011

Abstract: I will explain how a quantum circuit together with measurement apparatuses and EPR sources can be fully verified without any reference to some other trusted set of quantum devices. Our main assumption is that the physical system we are working with consists of several identifiable sub-systems, on which we can apply some given gates locally.
To achieve our goal we define the notions of simulation and equivalence. The concept of simulation refers to producing the correct probabilities when measuring physical systems. The notion of equivalence is used to enable the efficient testing of the composition of quantum operations. Unlike simulation, which refers to measured quantities (i.e., probabilities of outcomes), equivalence relates mathematical objects like states, subspaces or gates.
Using these two concepts, we prove that if a system satisfies some simulation conditions, then it is equivalent to the one it is supposed to implement. In addition, with our formalism, we can show that these statements are robust, and the degree of robustness can be made explicit. Finally, we design a test for any quantum circuit whose complexity is linear in the number of gates and qubits, and polynomial in the required precision.
Joint work with Frederic Magniez, Dominic Mayers and Harold Ollivier.

University of Waterloo

IQC Institute for Quantum Computing

Self-Testing of Quantum Circuits

Michele Mosca

Canada Research Chair in Quantum Computation

Joint with F. Magniez, D. Mayers, H. Ollivier

Perimeter Institute Colloquium

15 November 2006

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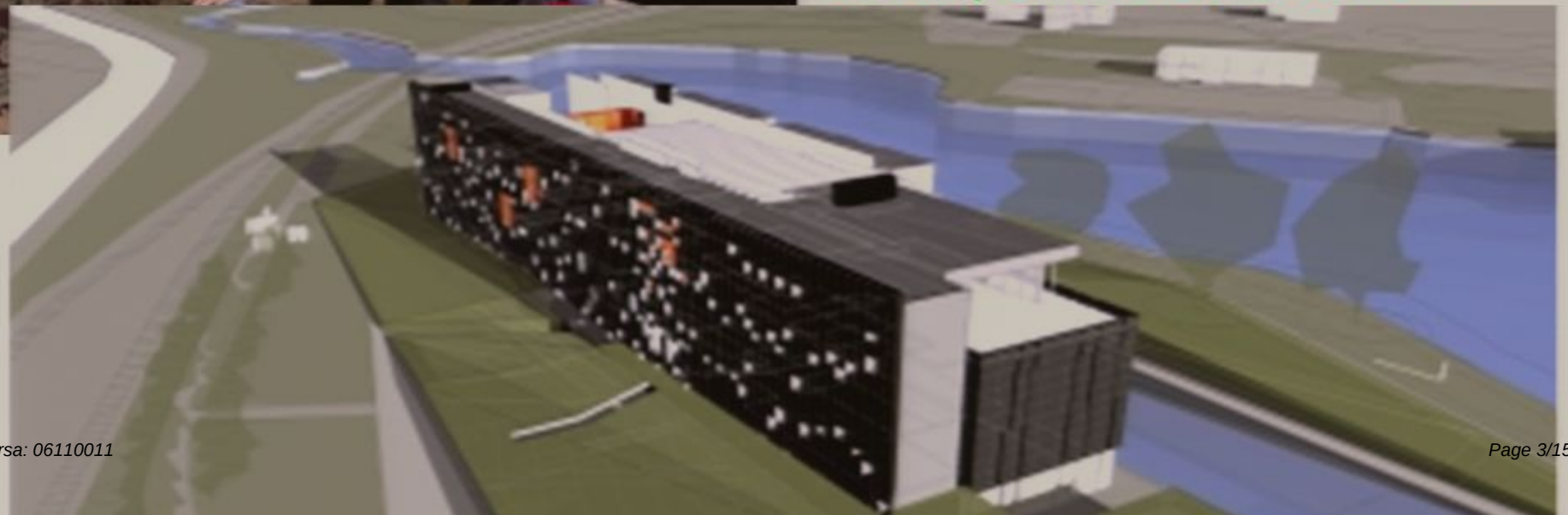

IQC Institute for Quantum Computing

Centre for Applied Cryptographic Research

The University of Waterloo

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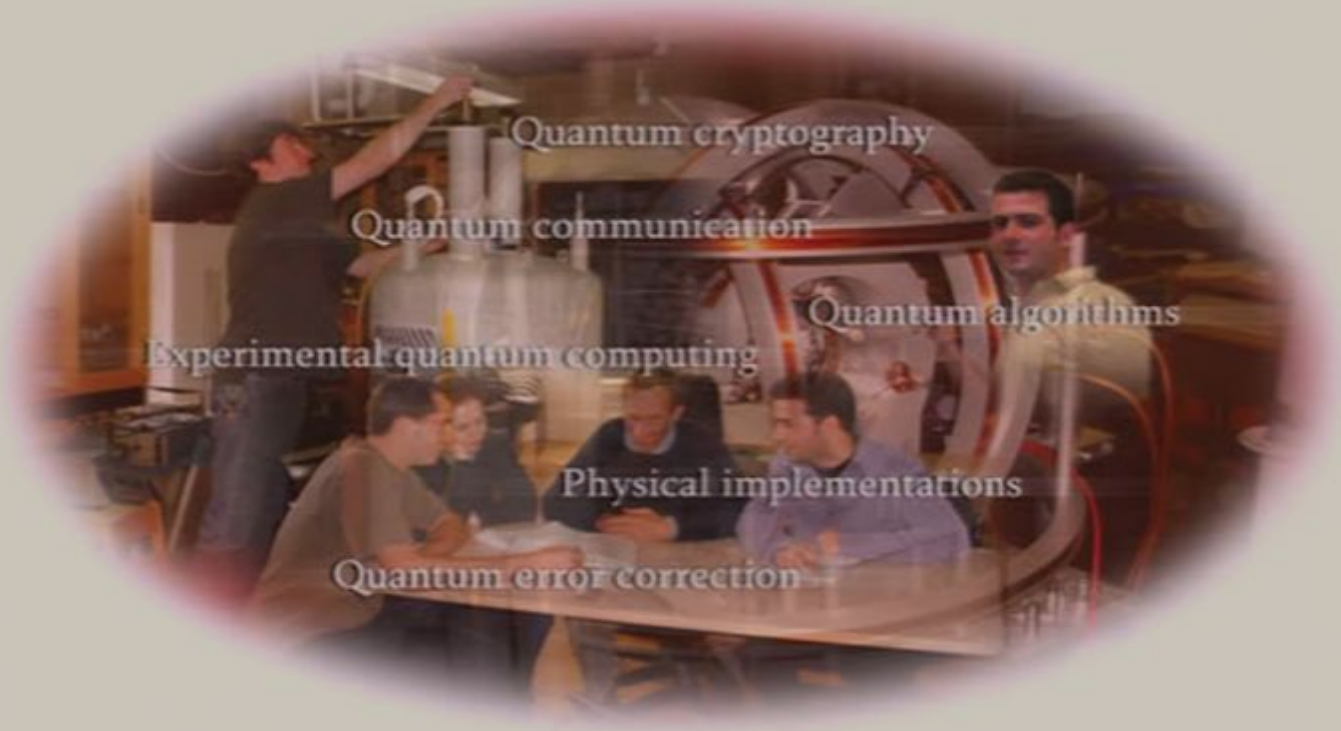

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Outline

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What did we already know?

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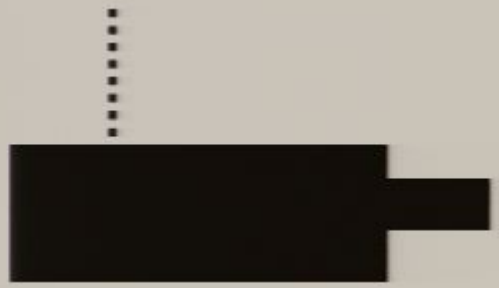

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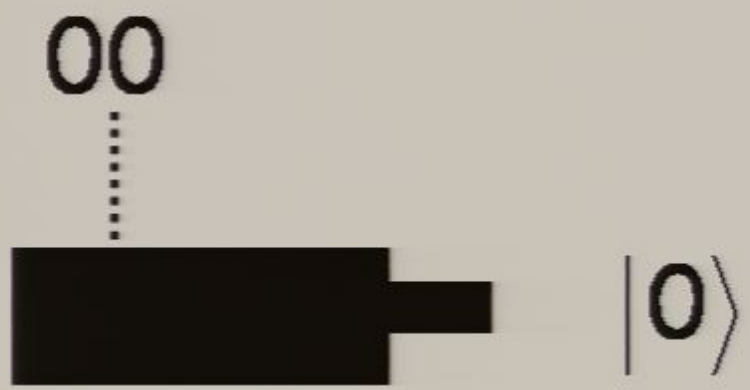

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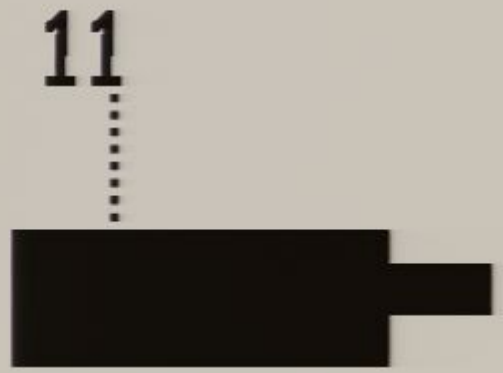

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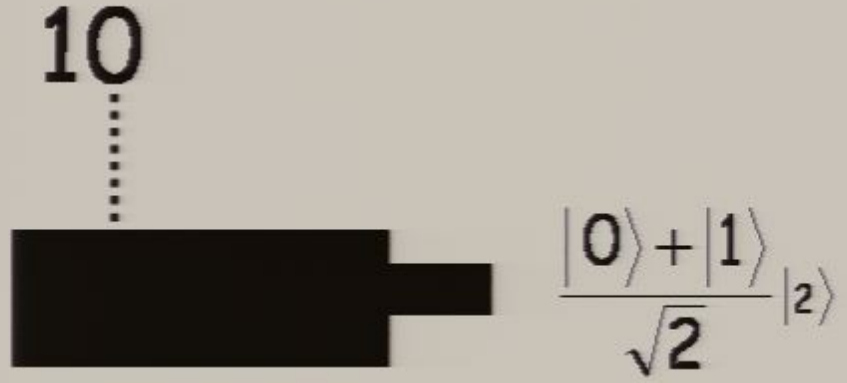

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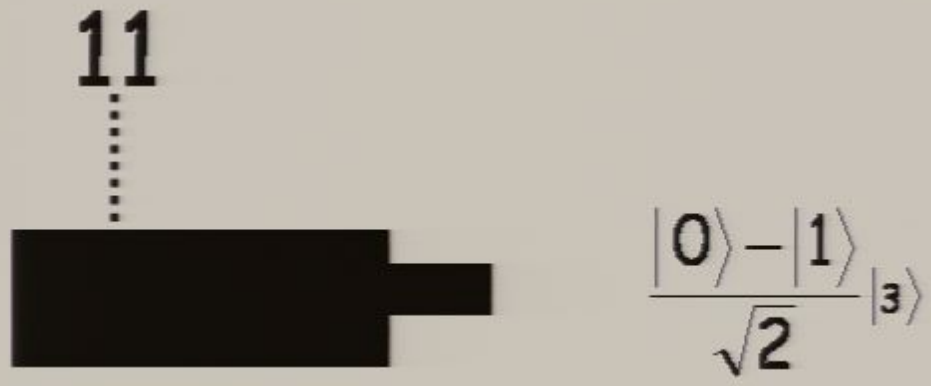

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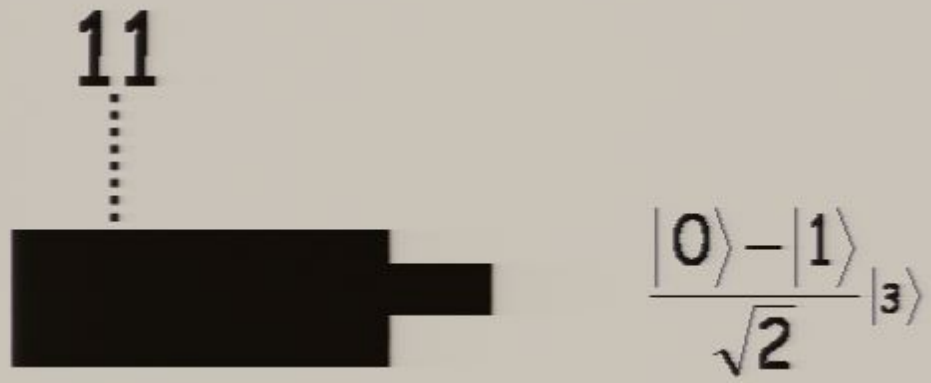

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http://www.research.ibm.com/journal/rd/481/smolin.html

C. H. Bennett, F. Bessett, G. Brassard, L. Salvail, and J. Smolin, " Experimental Quantum Cryptography," J. Cryptol. 5, No. 1, 3–28 (1992).

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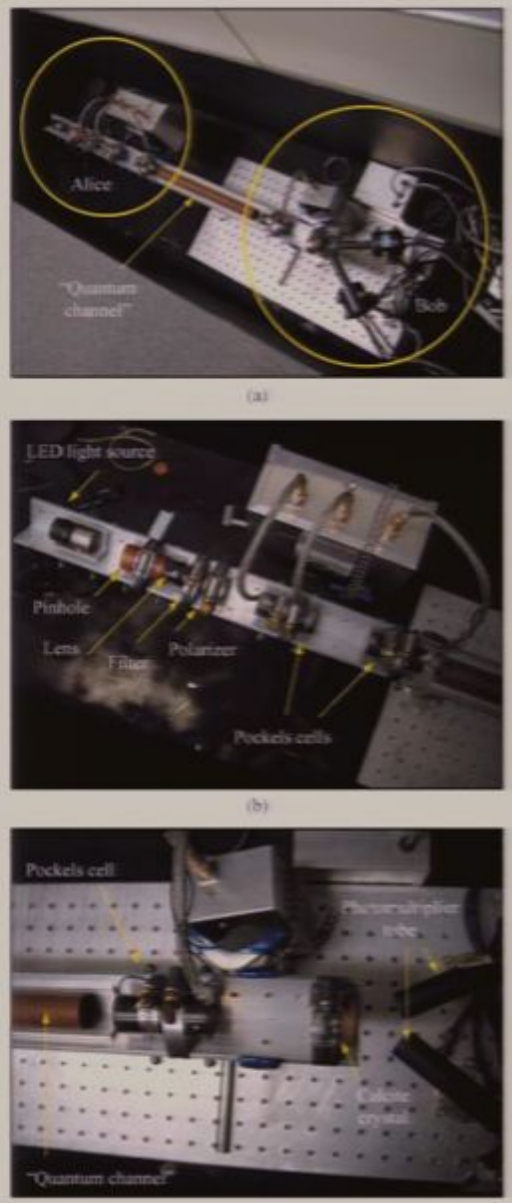

Figure 1

The apparatus used to perform the [illegible] cryptography experiment: (a) The entire apparatus; (b) detailed view of Alice; (c) detailed view of Bob.

We need to make our assumptions and testing procedures explicit.

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We also don't want to rely on some other untrusted apparatus (e.g. in order to "just" do tomography).

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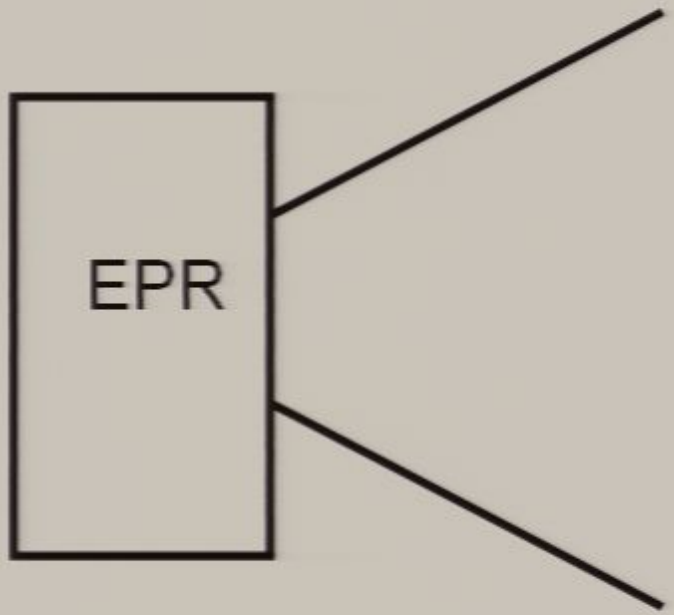

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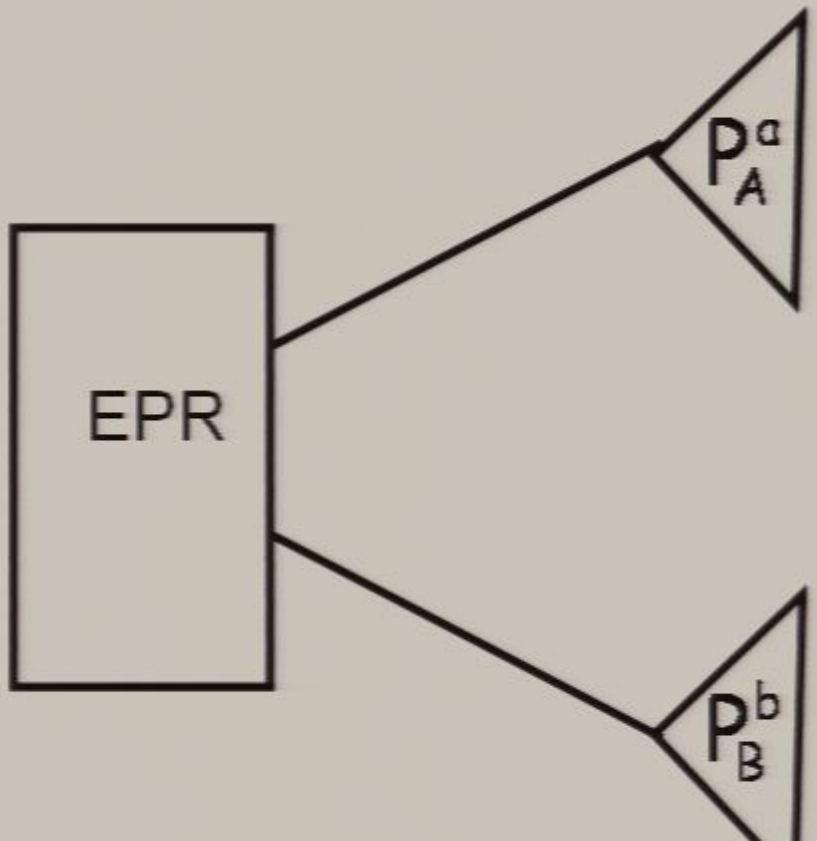

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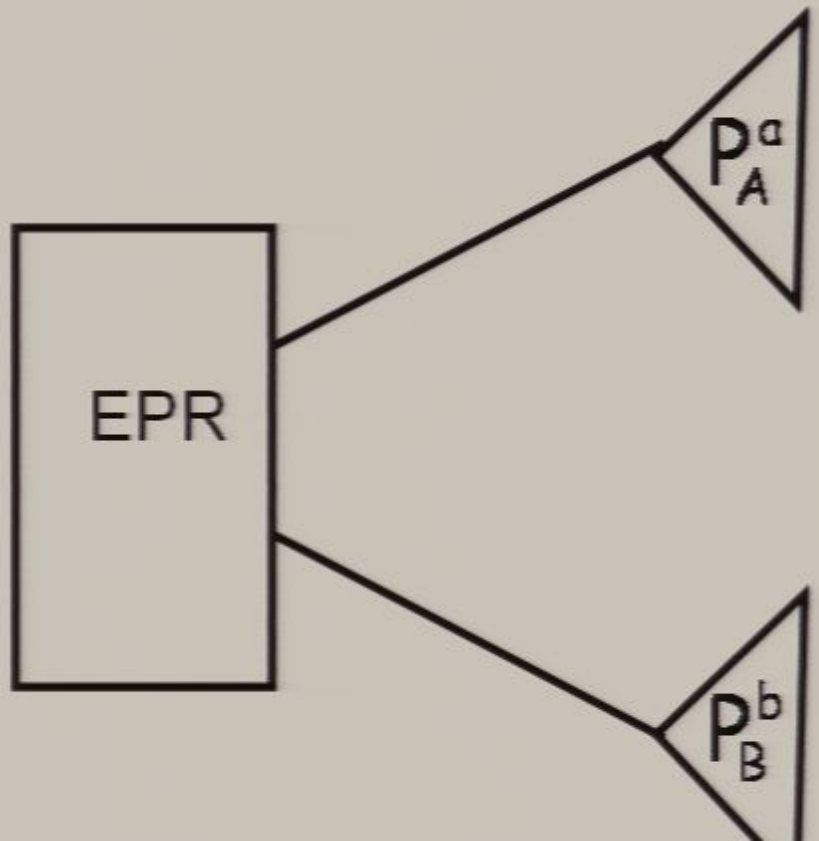

$$P^0 + P^{\pi/2} = I$$

$$P^{\pi/8} + P^{5\pi/8} = I$$

$$P^{\pi/4} + P^{3\pi/4} = I$$

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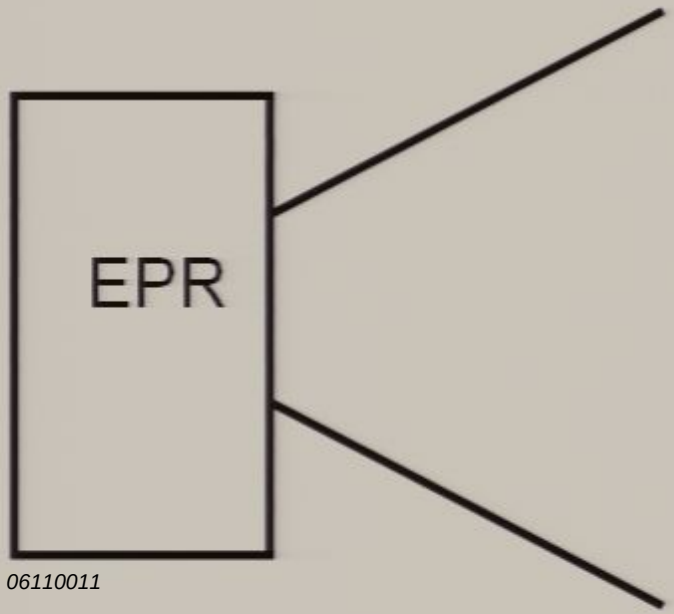

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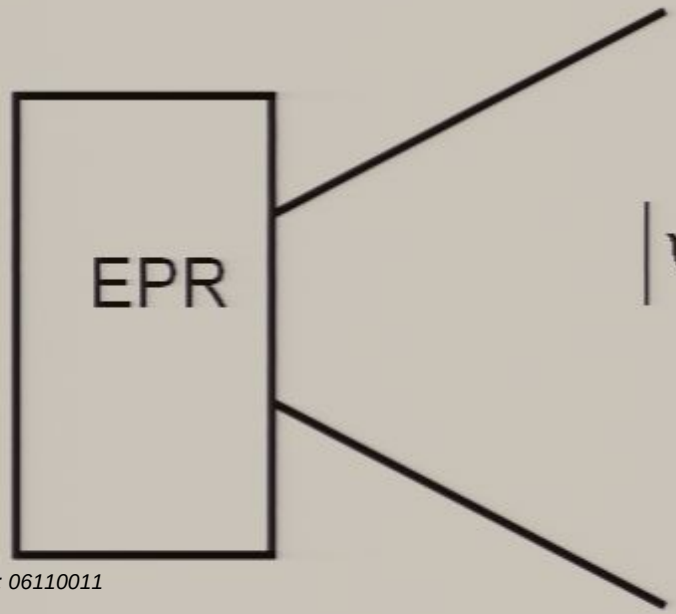

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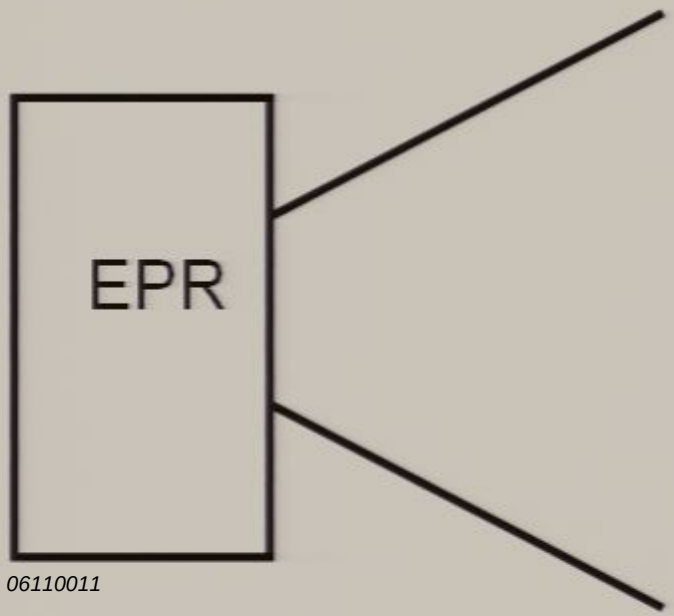

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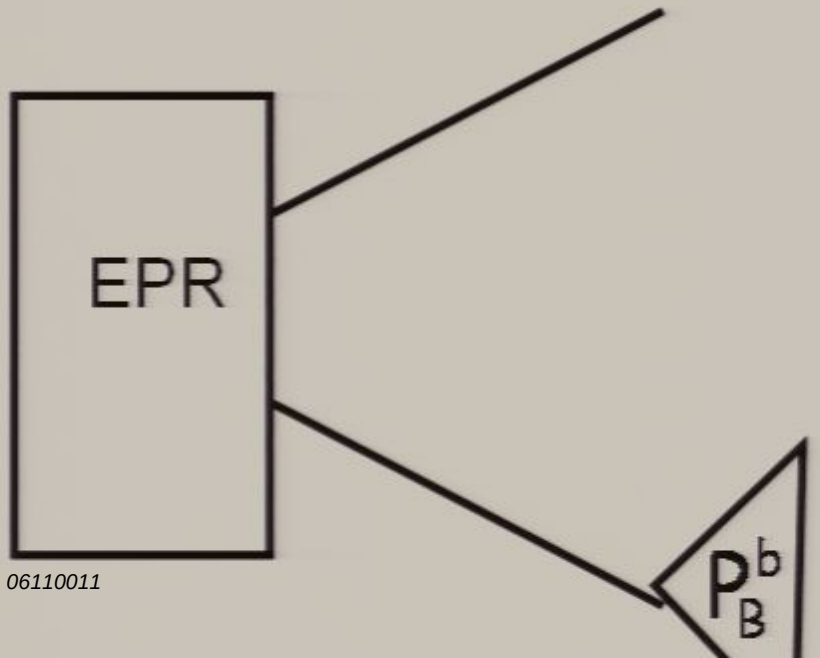

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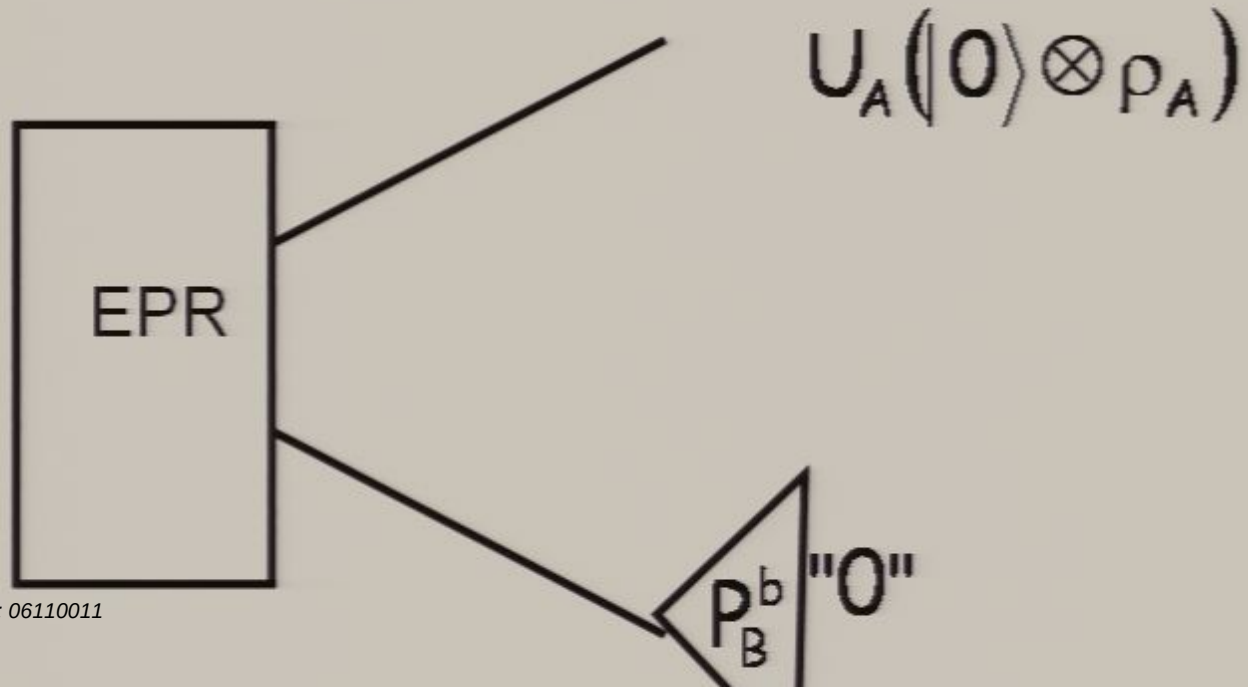

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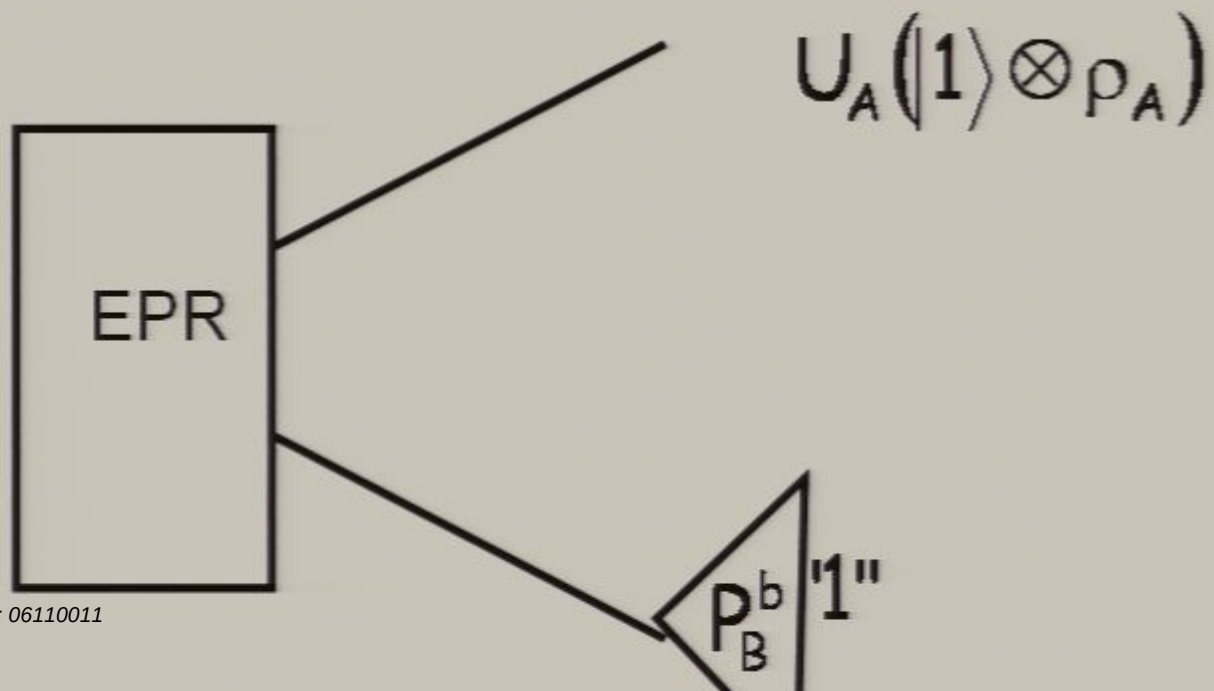

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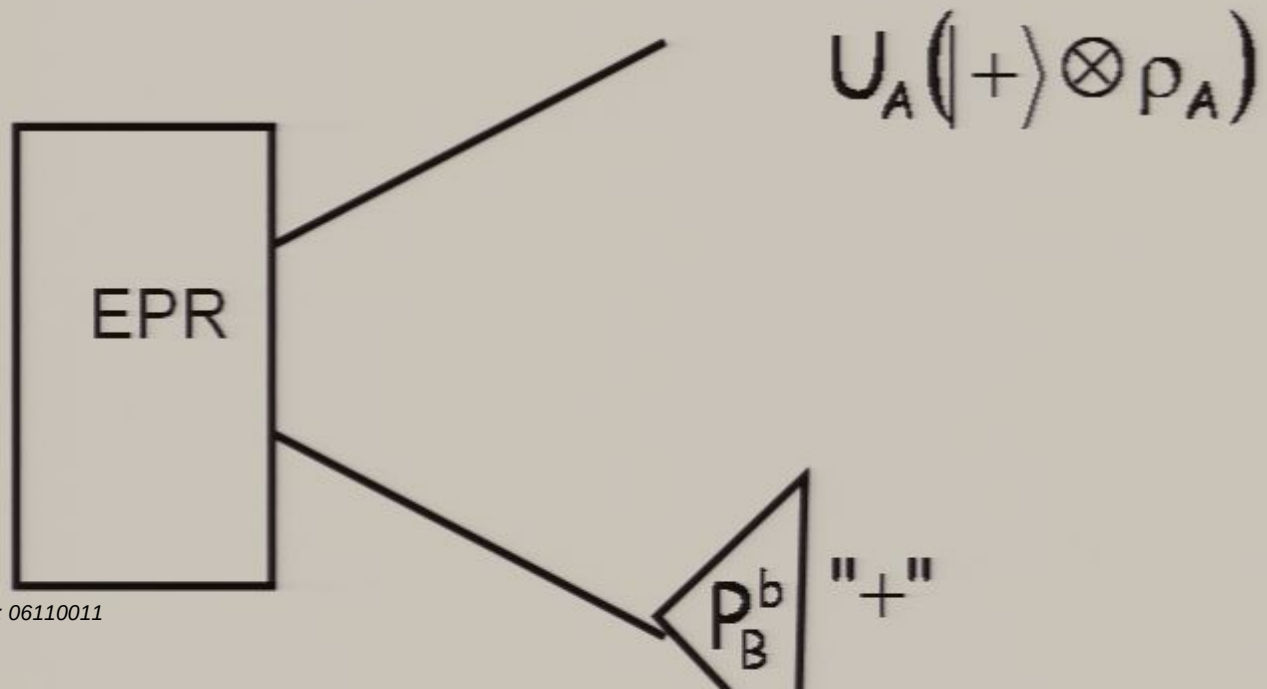

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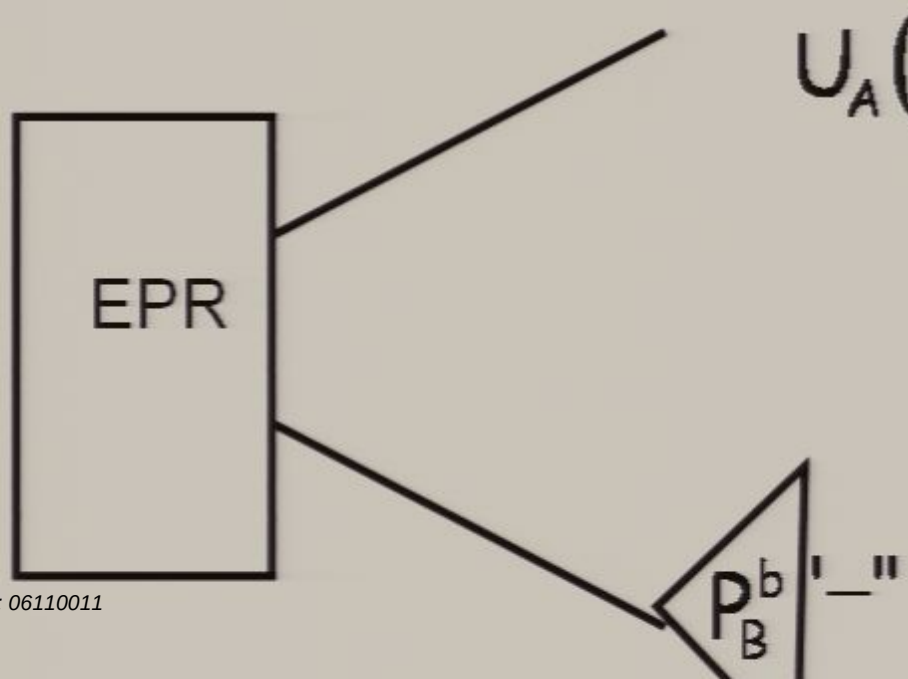

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(Assuming robustness) This might be the only way, using only these assumptions, to verifiably securely

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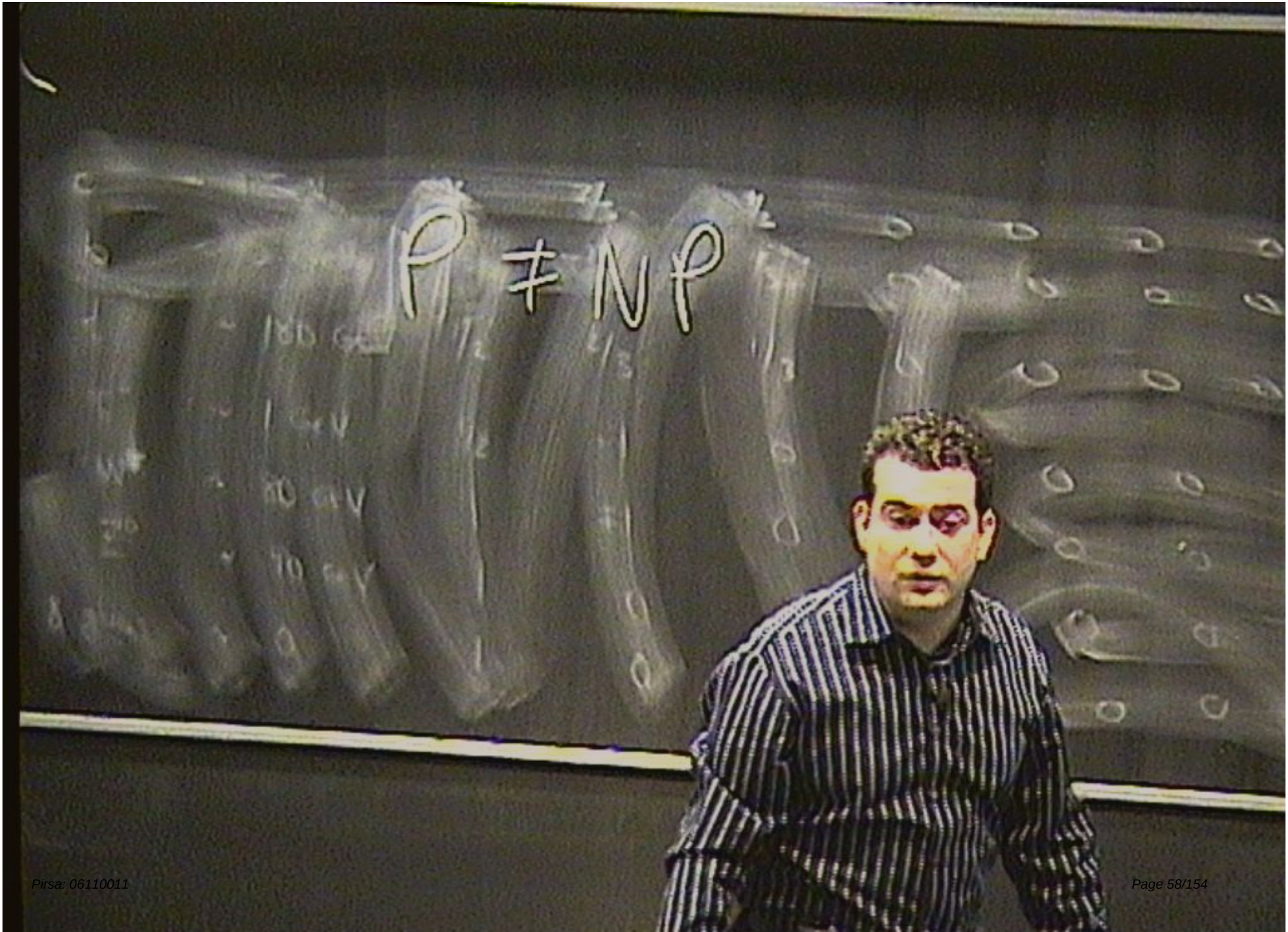
Or, suppose we have proved one of the Clay Institute $1M Millennium problem by a proof that needs to be run on a quantum computer.


P ≠ NP

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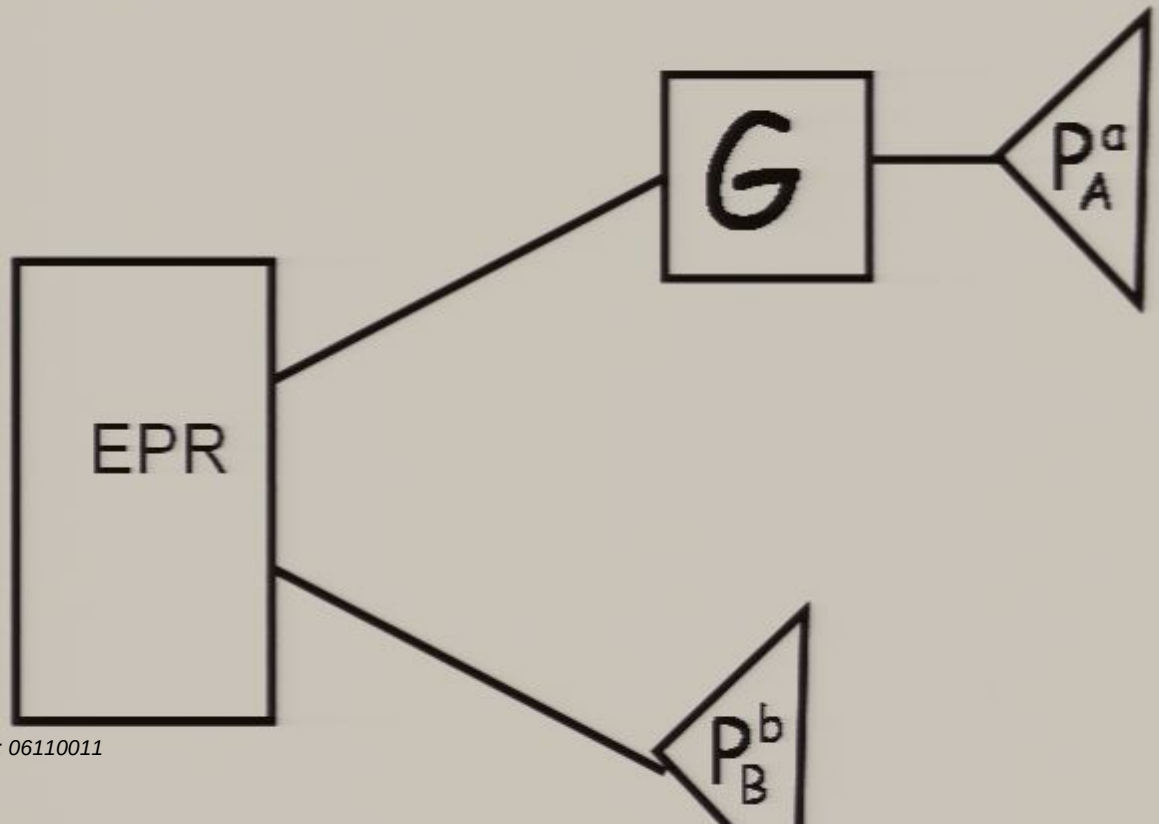

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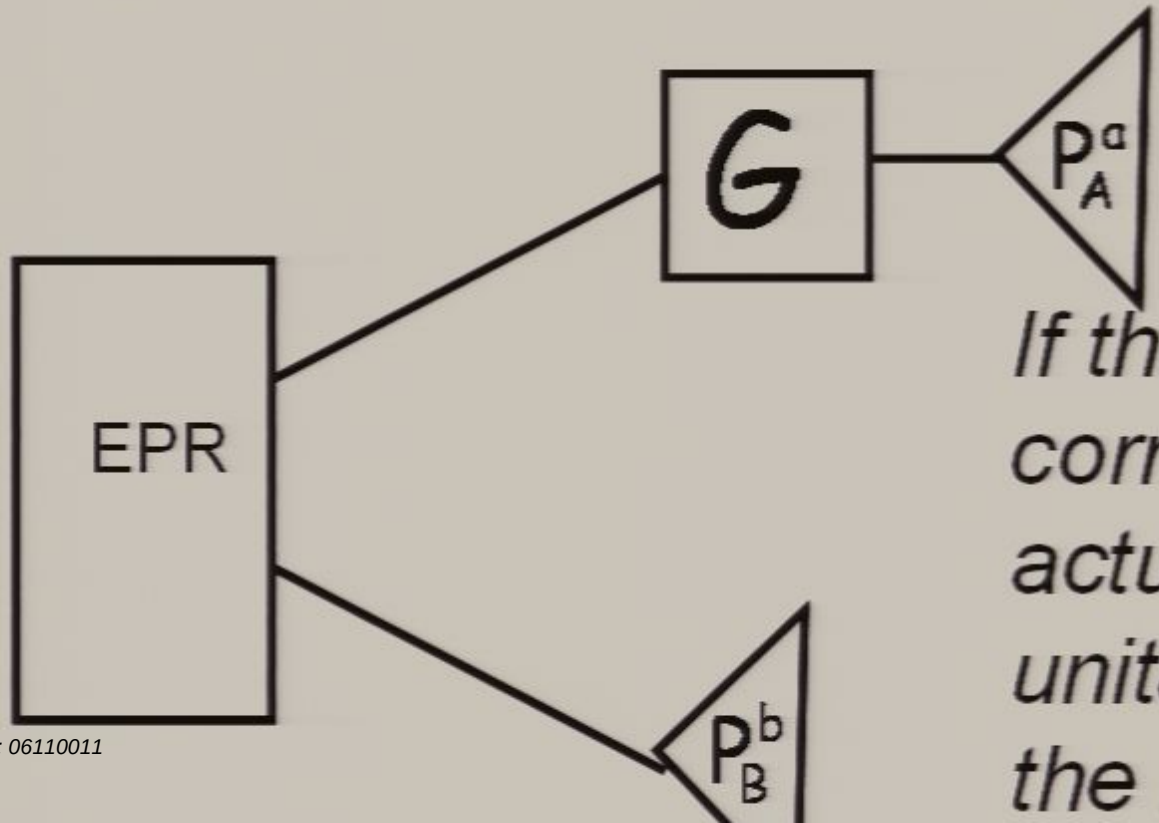

If the statistics are correct, then the actual gate G is locally unitarily equivalent to the ideal gate, T.

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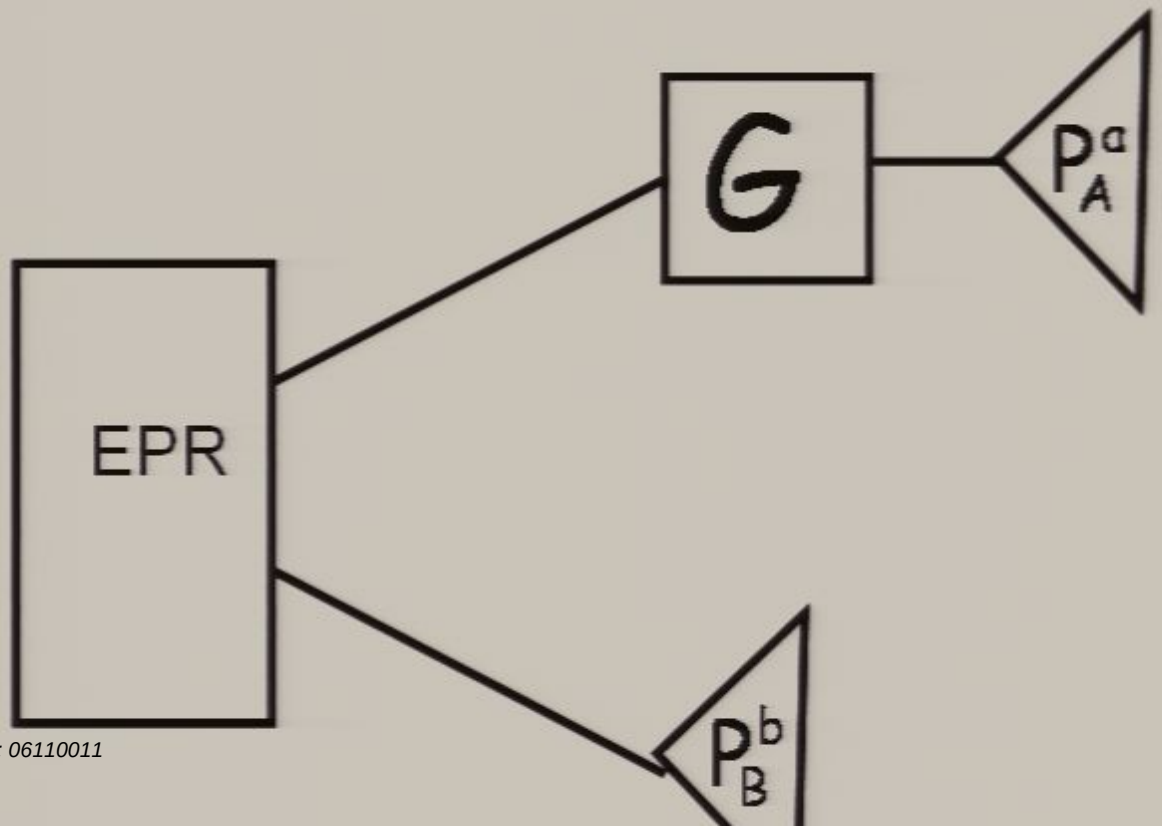

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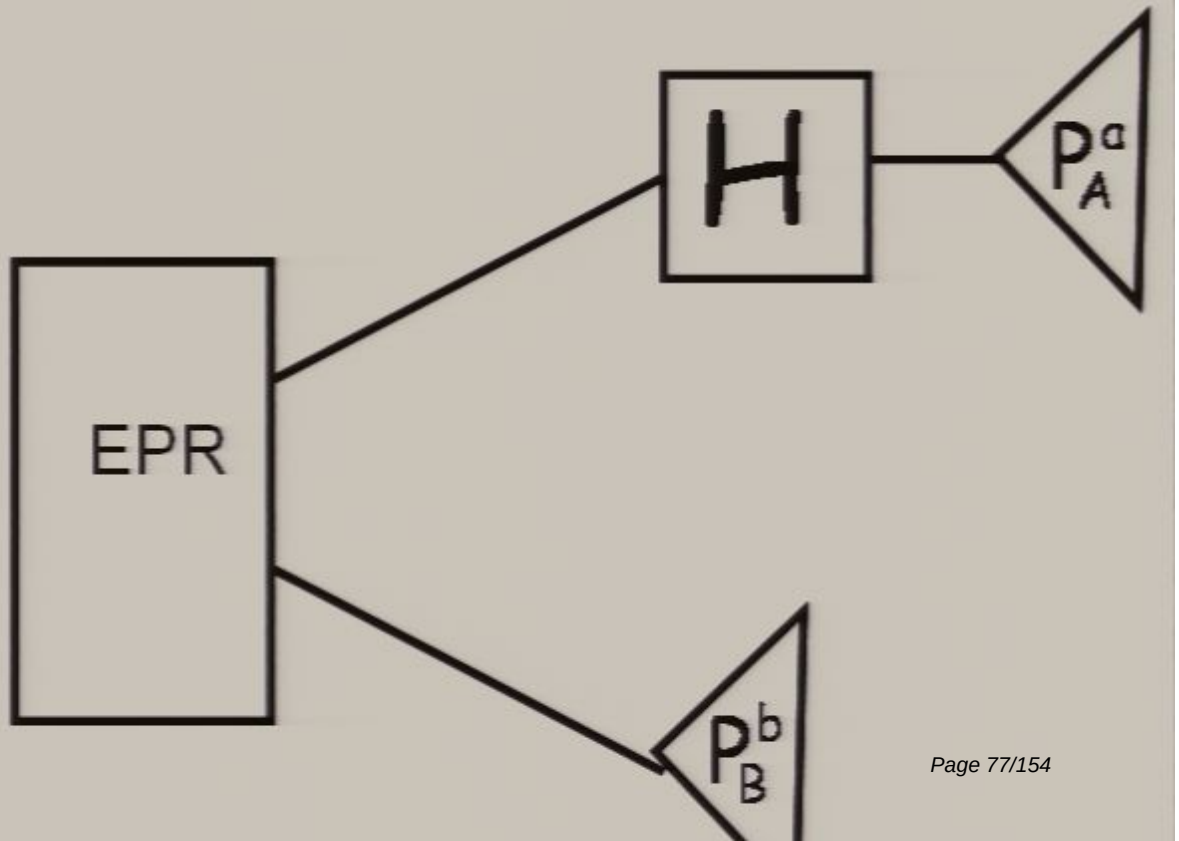

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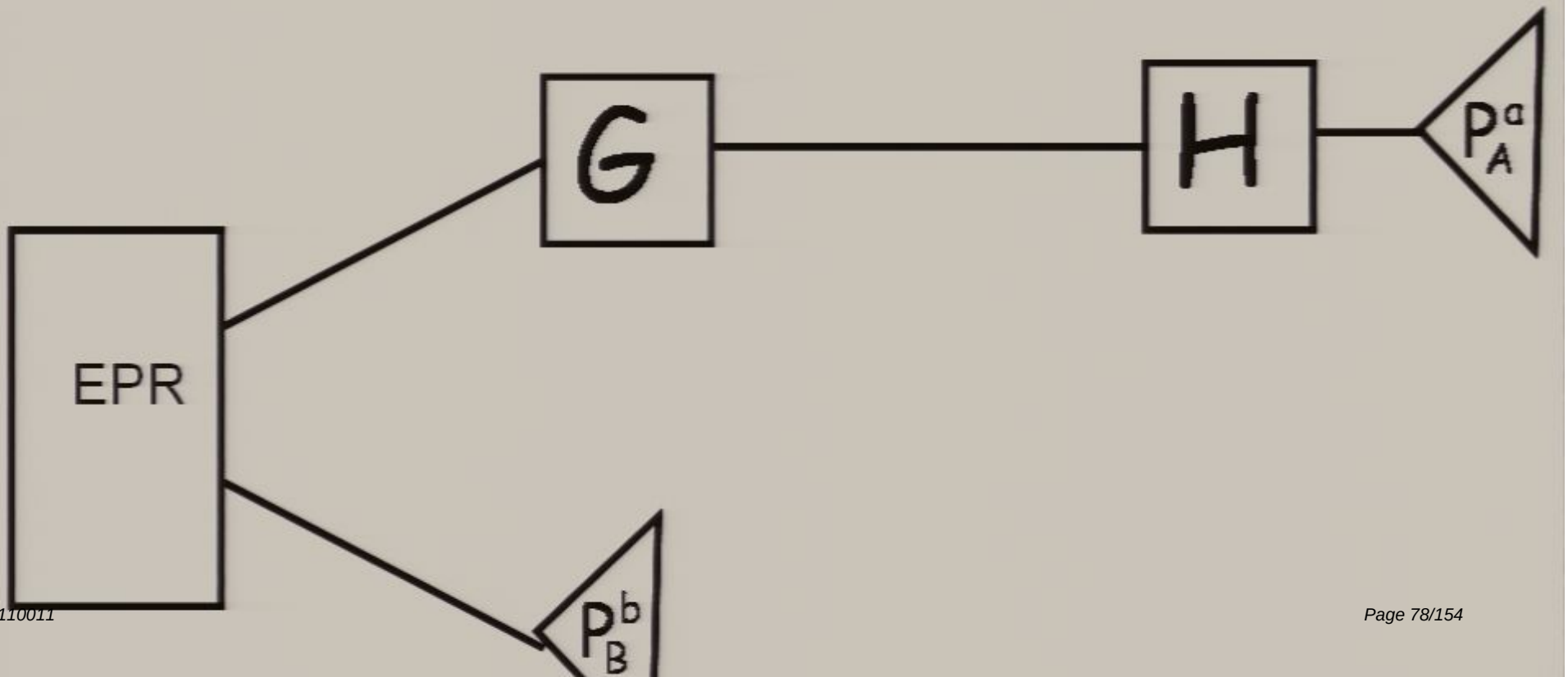

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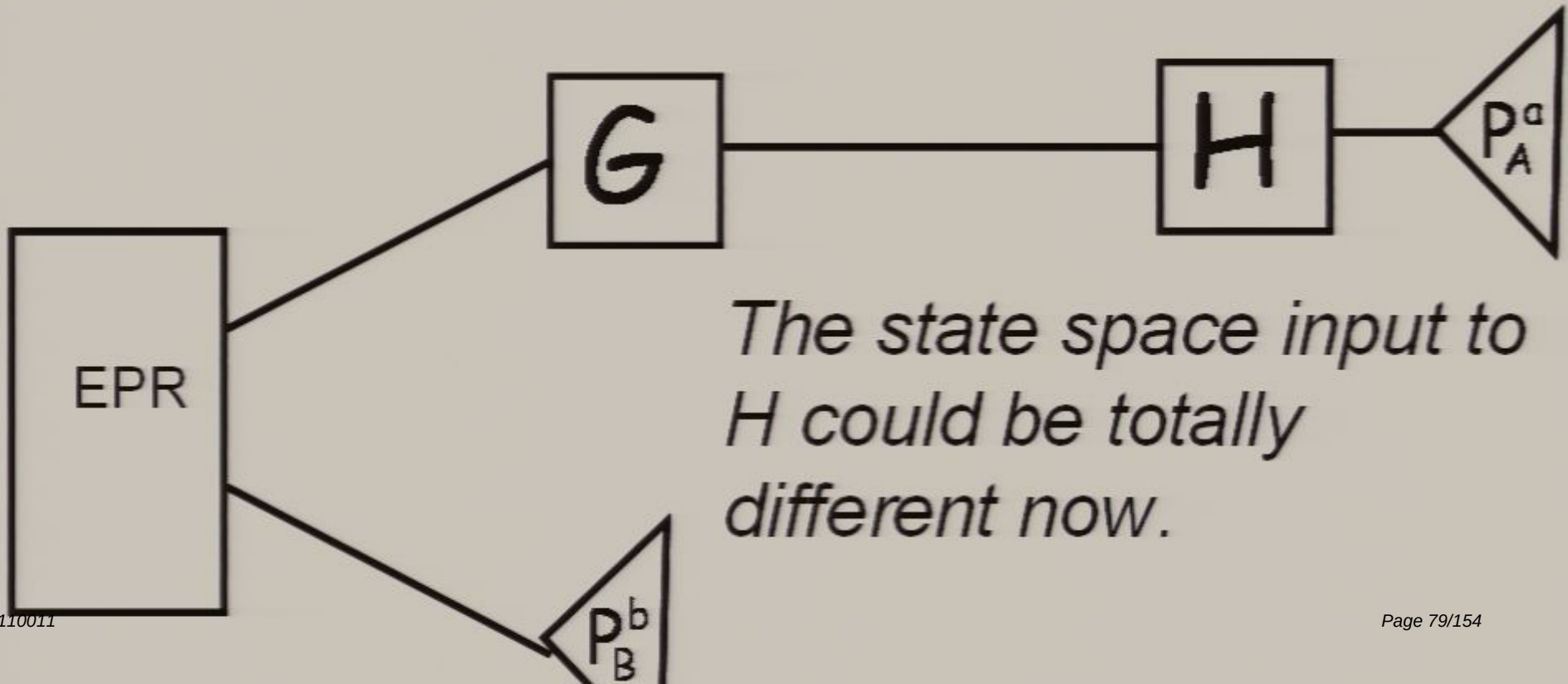

The state space input to H could be totally different now.

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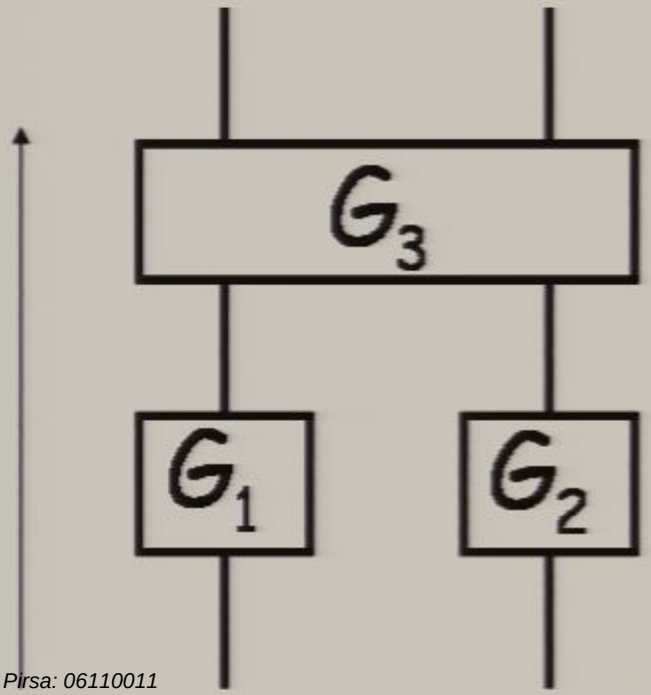

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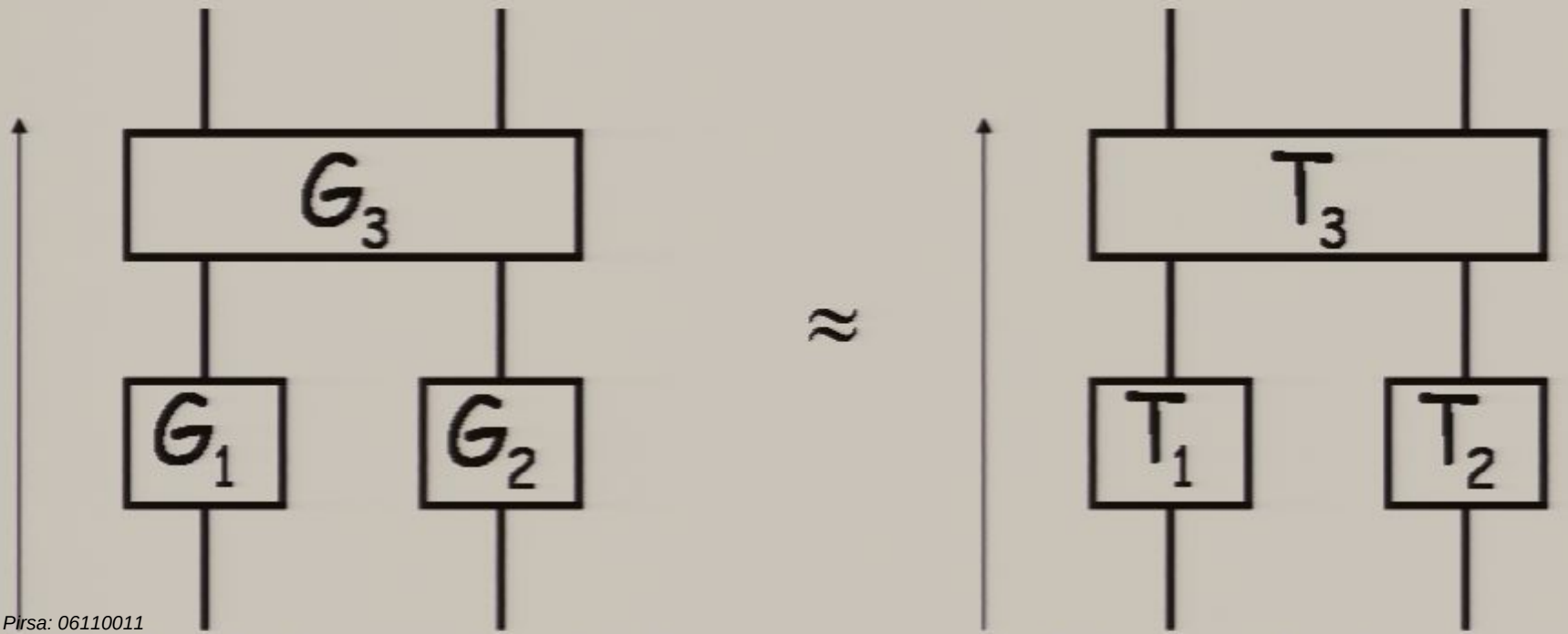

"actual physical gates" *"ideal gates"*

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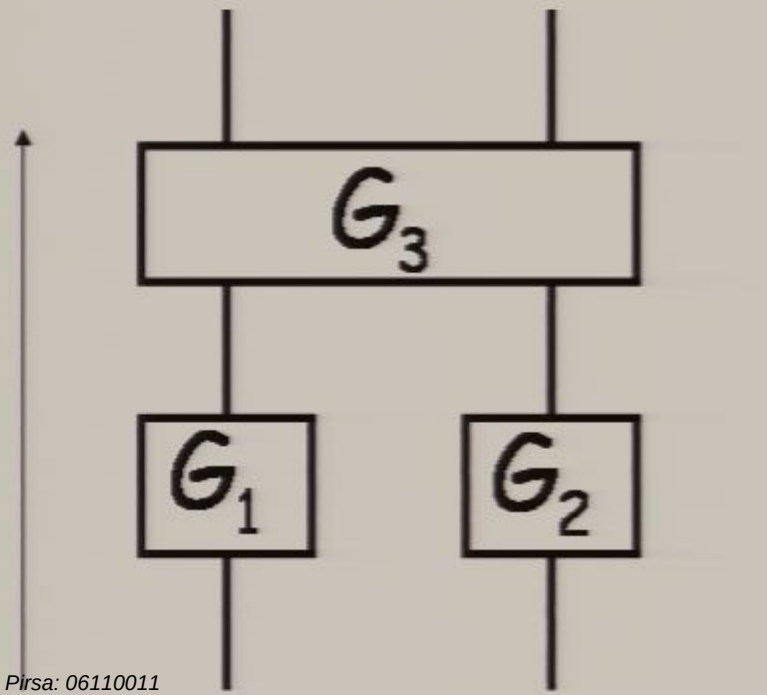

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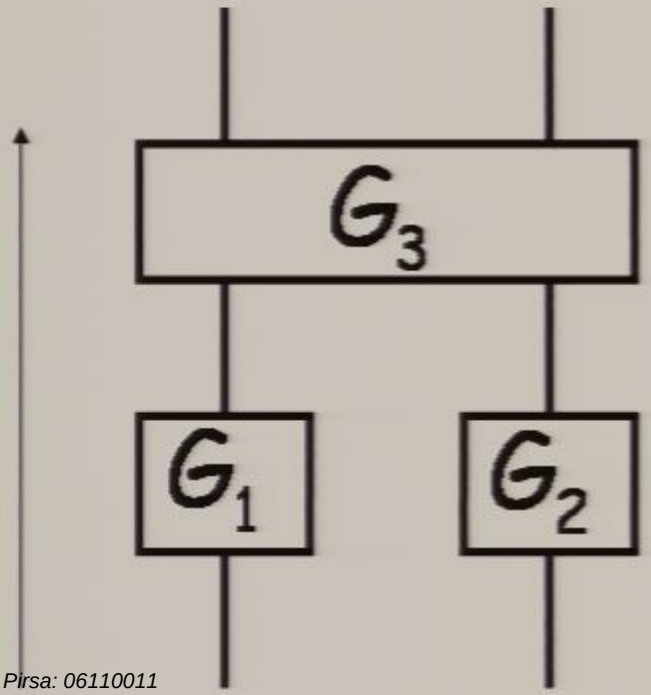

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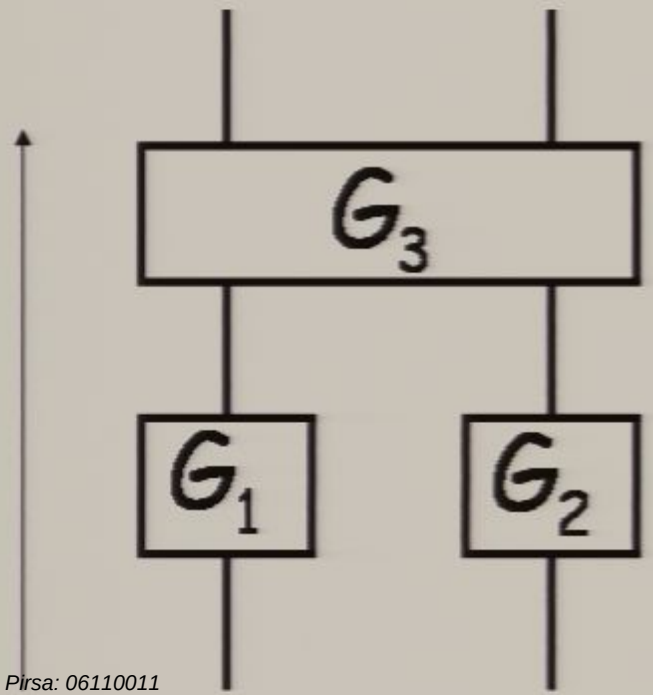

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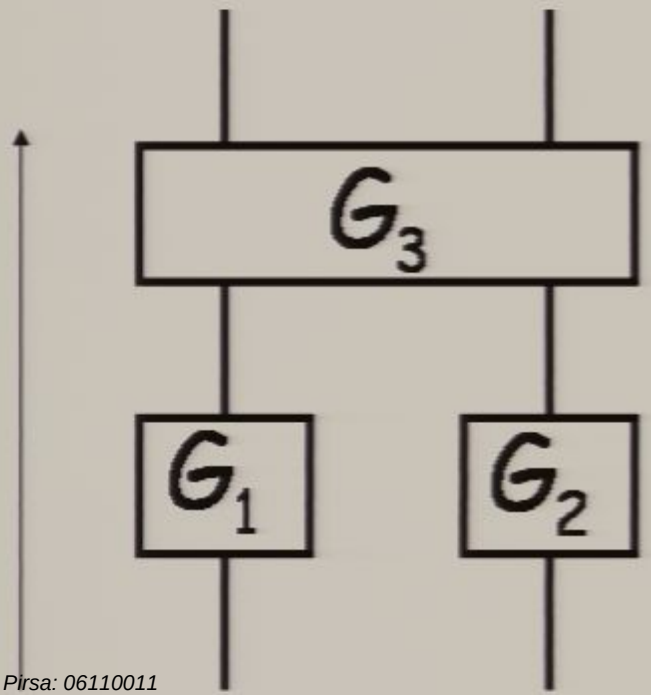

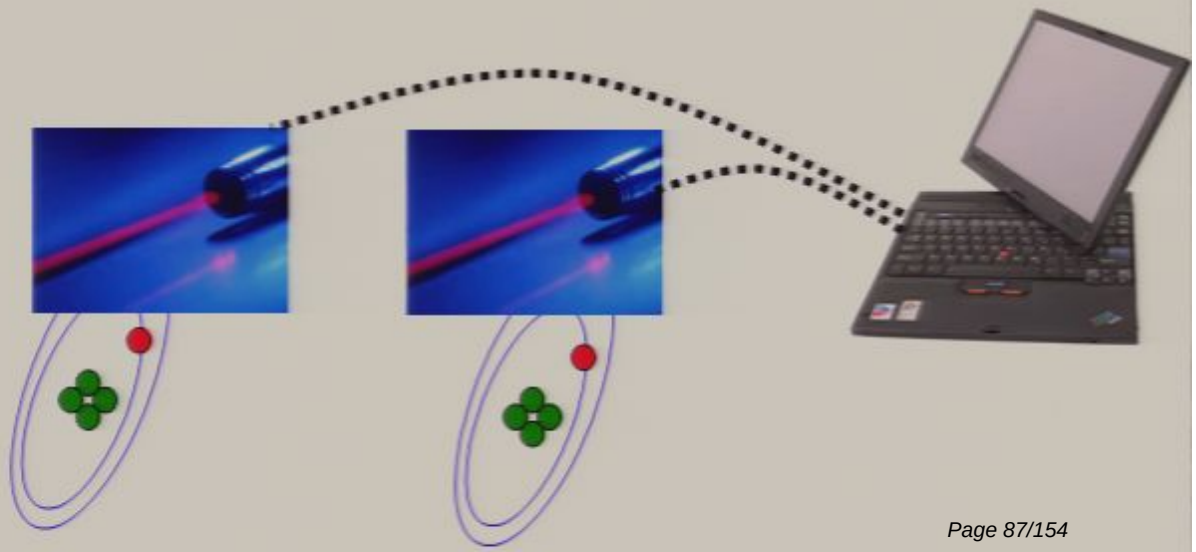

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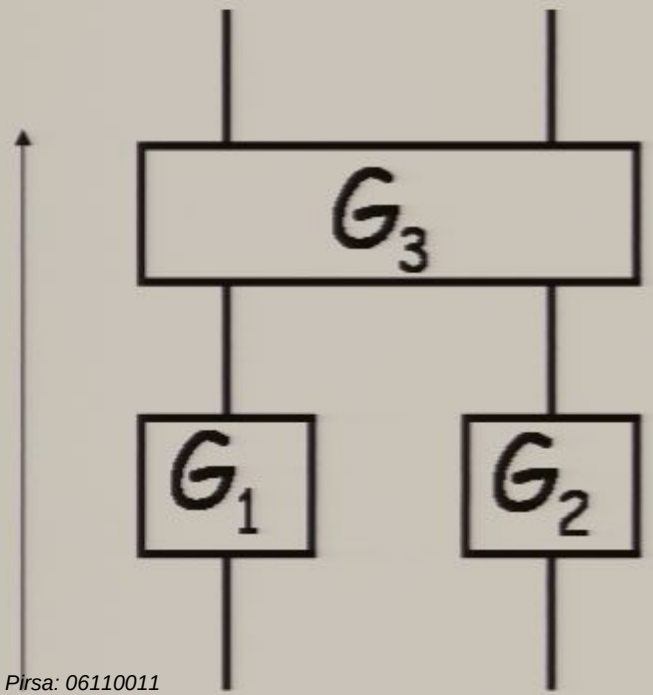

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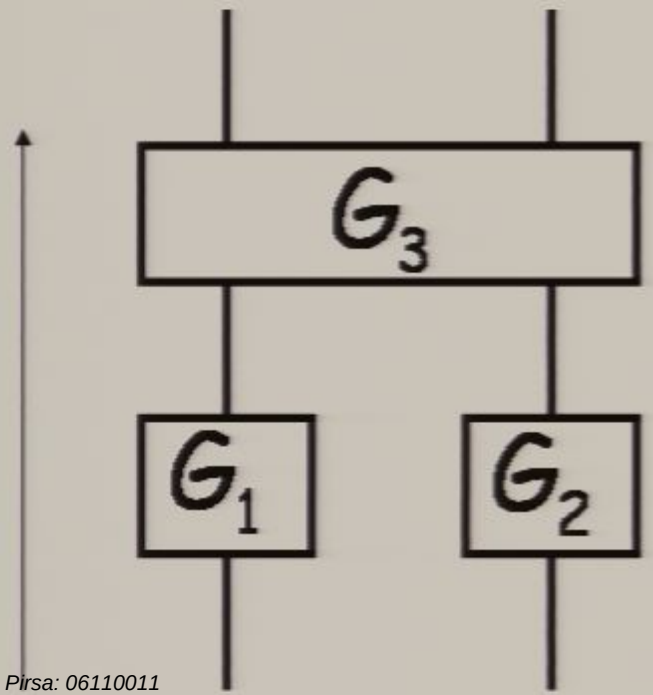

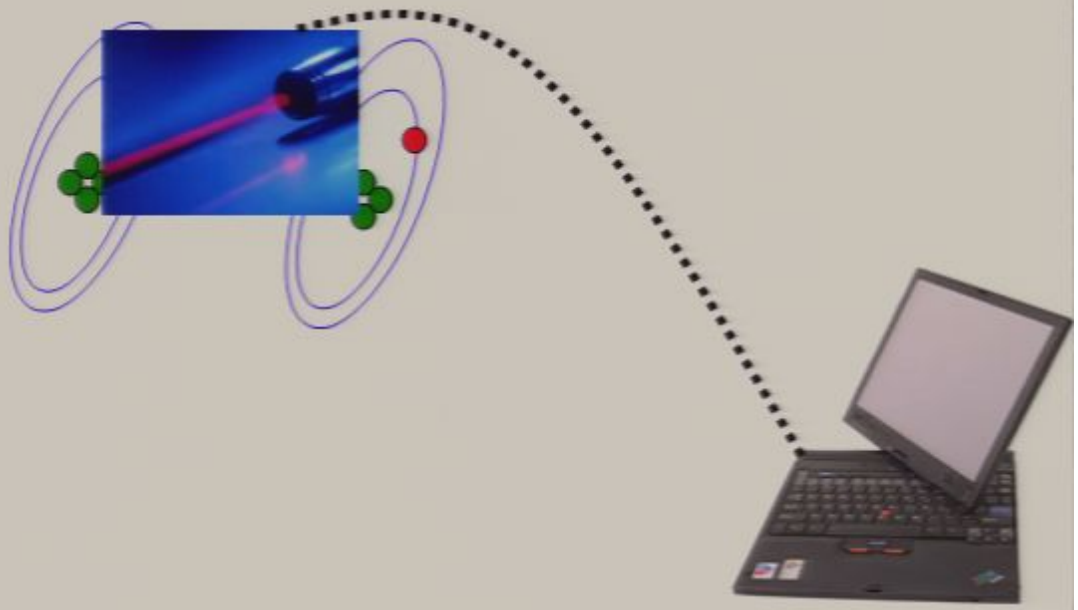

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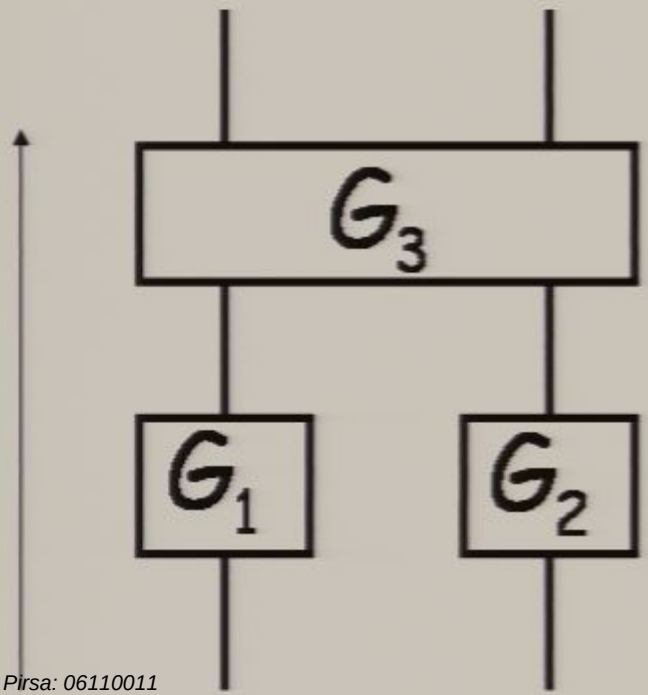
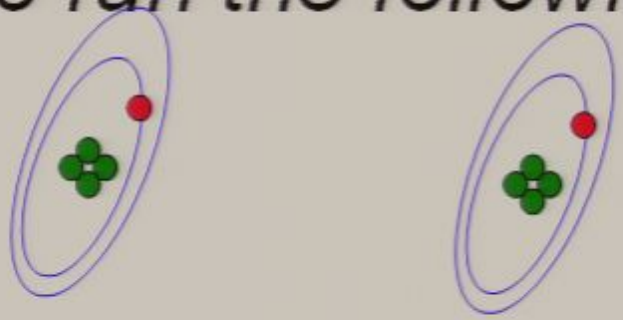
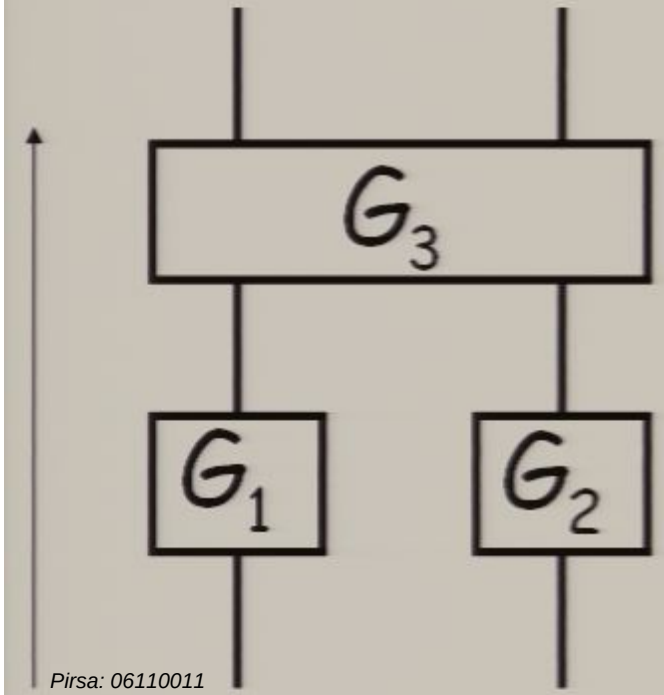
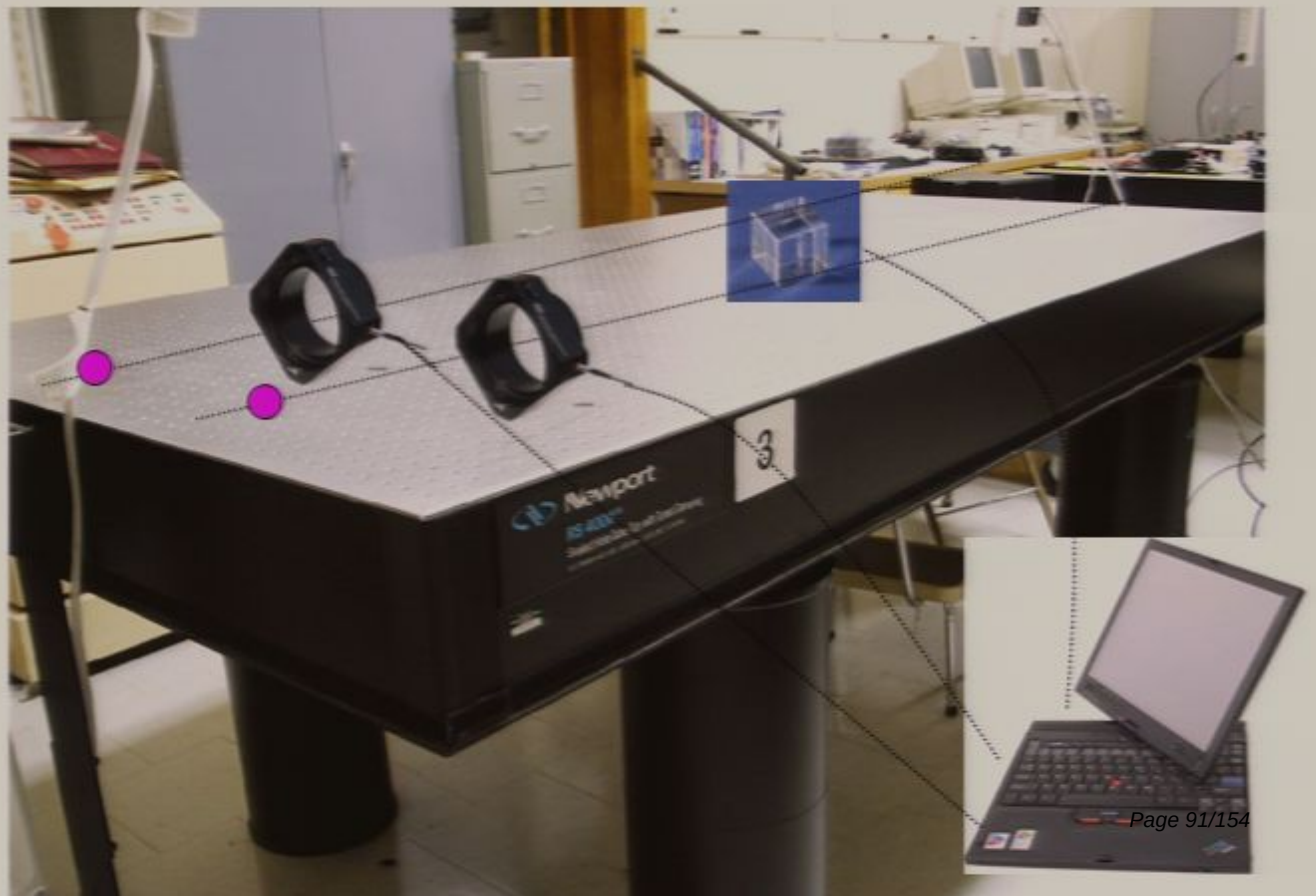




Example 2

Suppose we wish to run the following circuit





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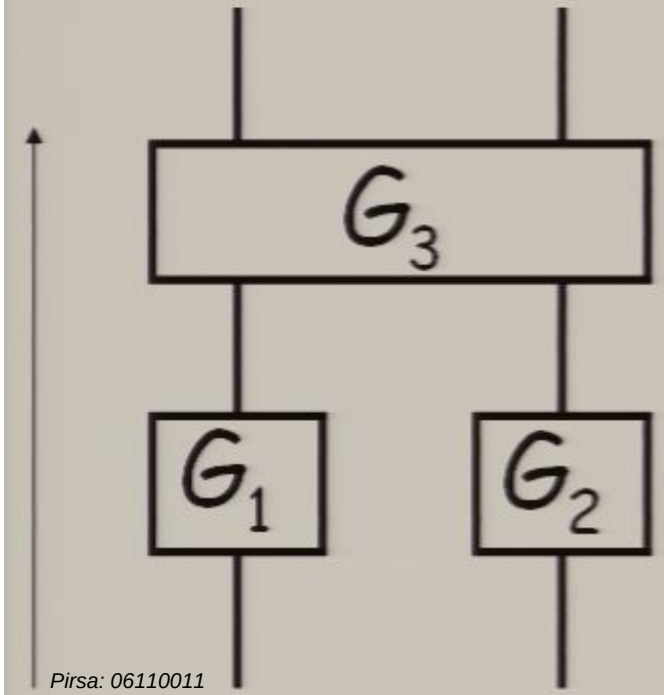

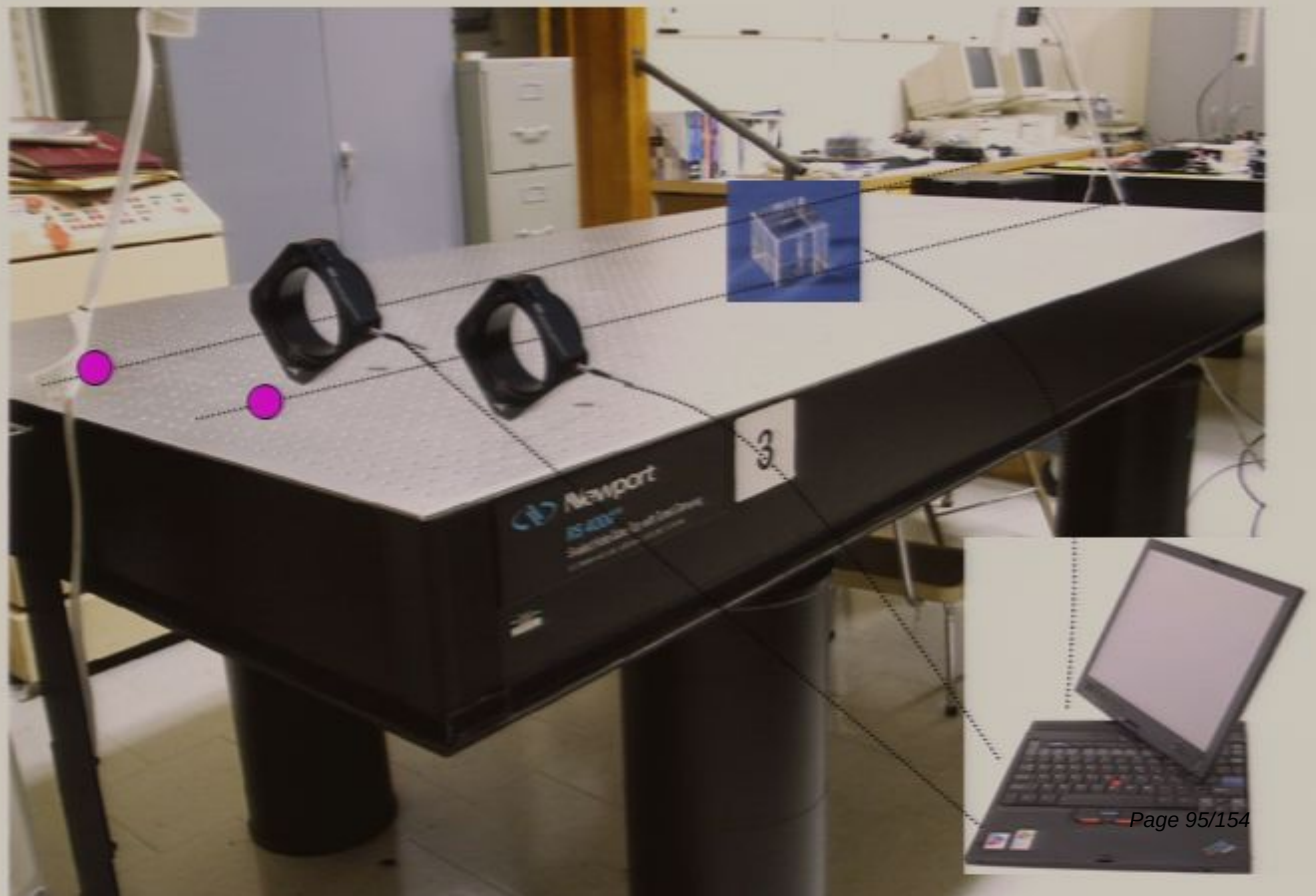

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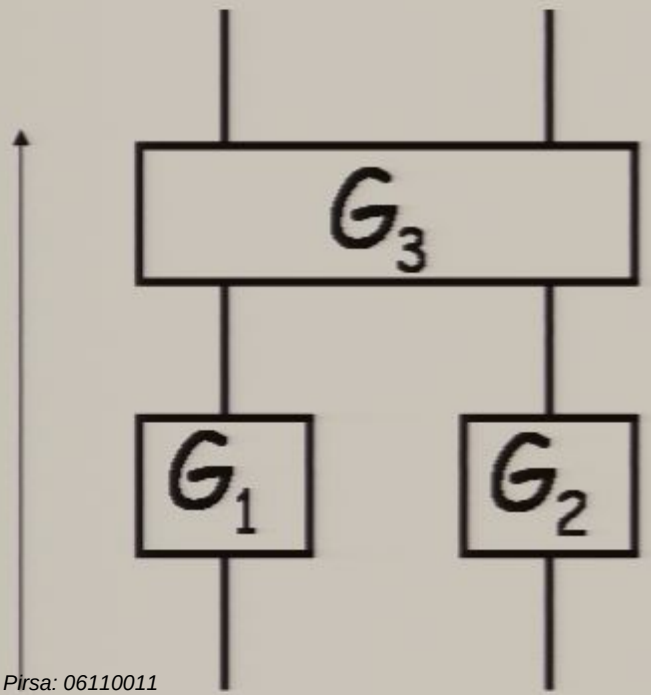

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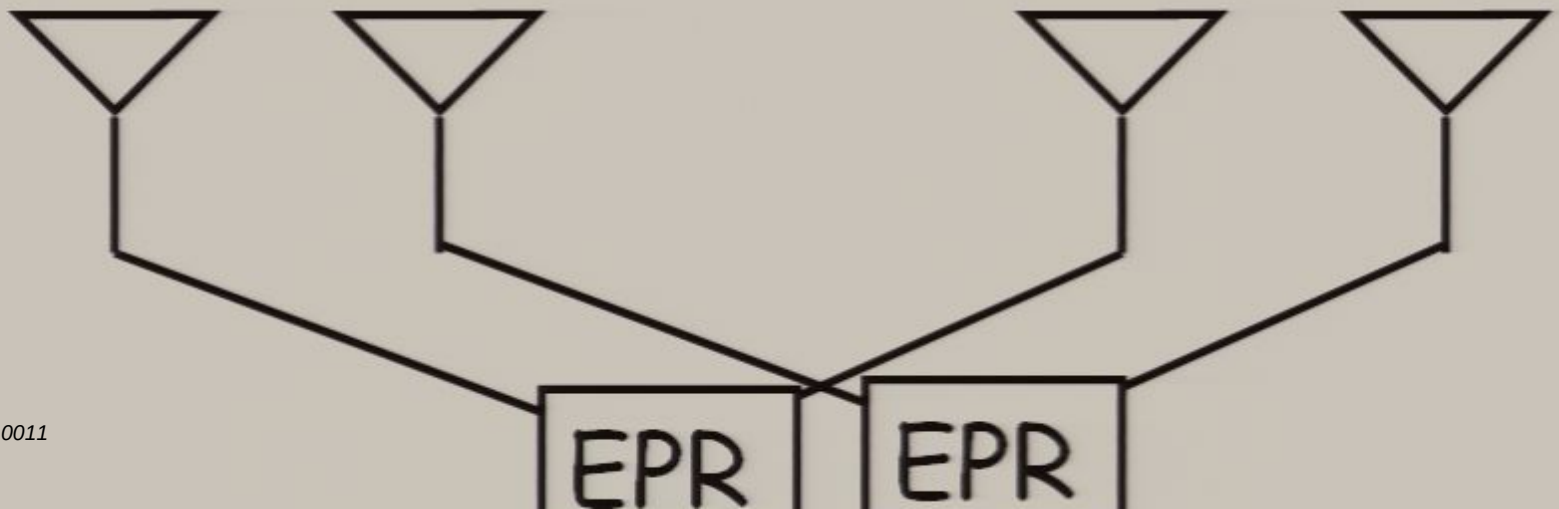
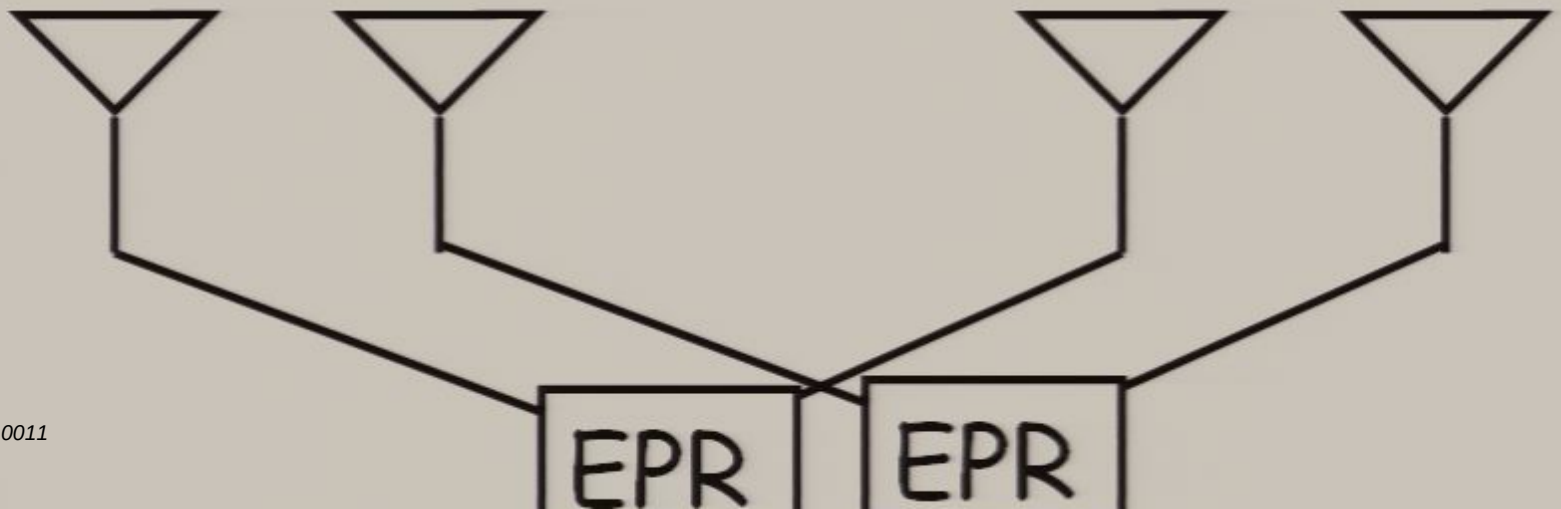
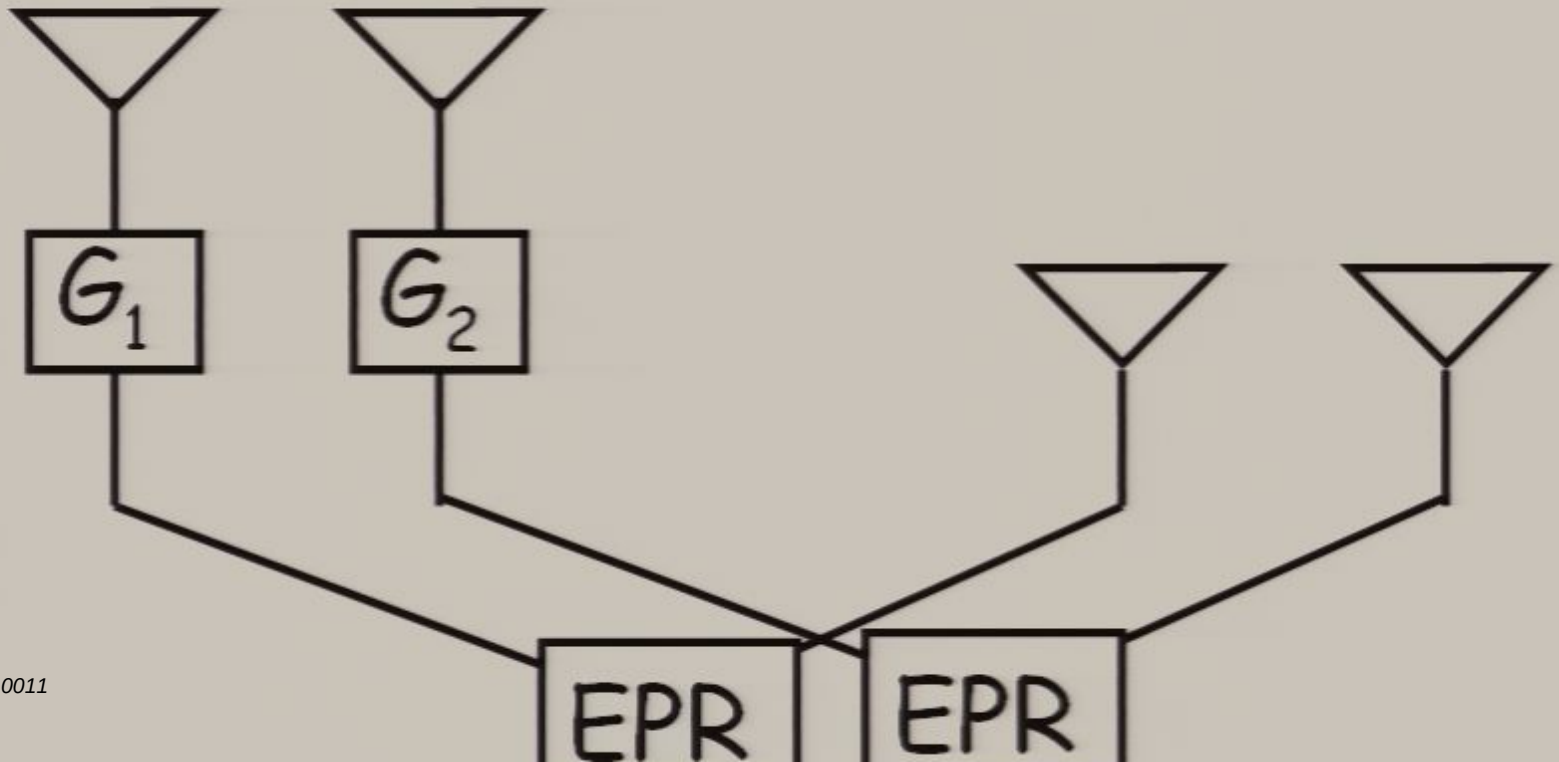


example

Verify the initial qubit sources.



“tomography test”



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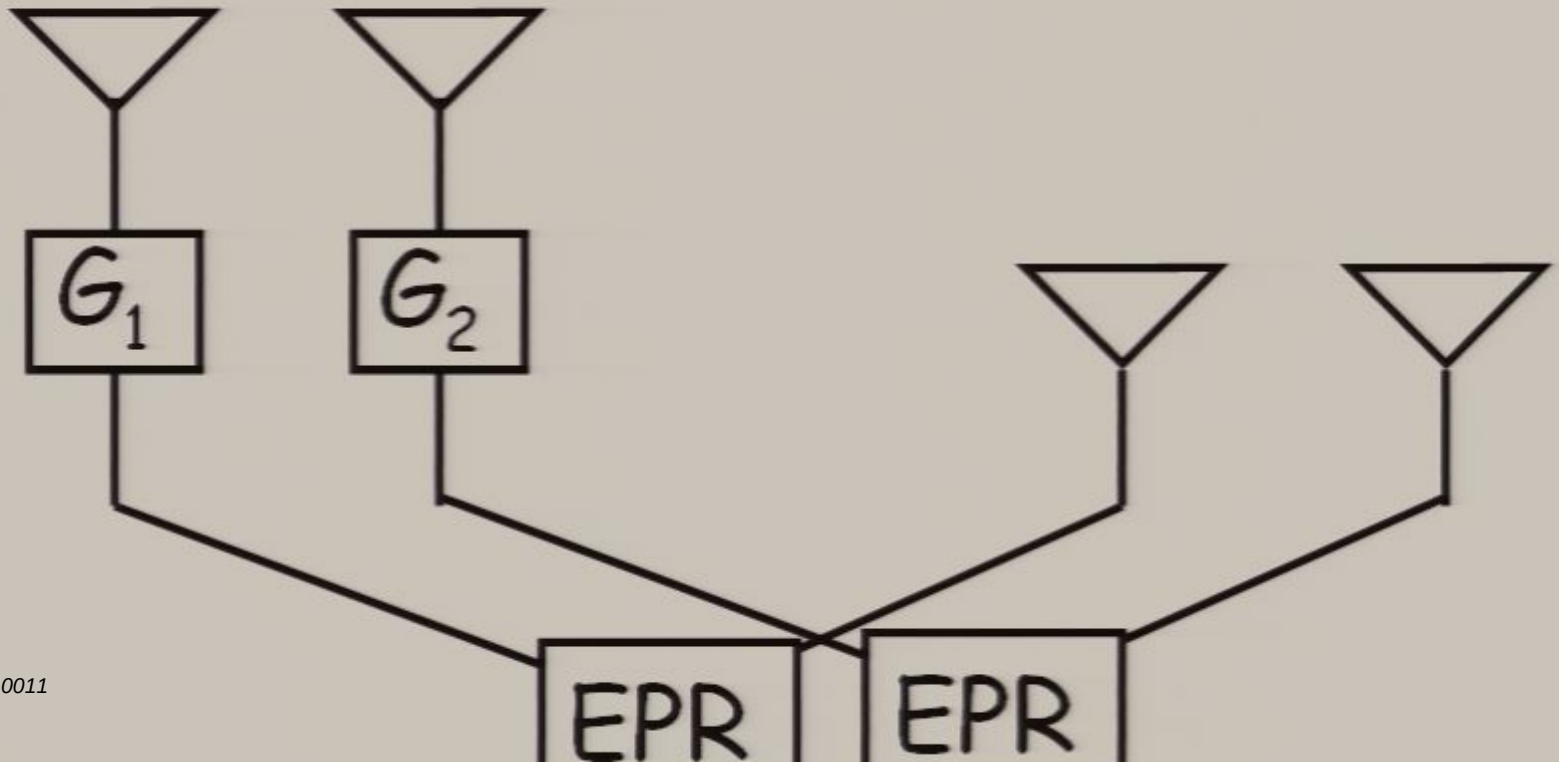
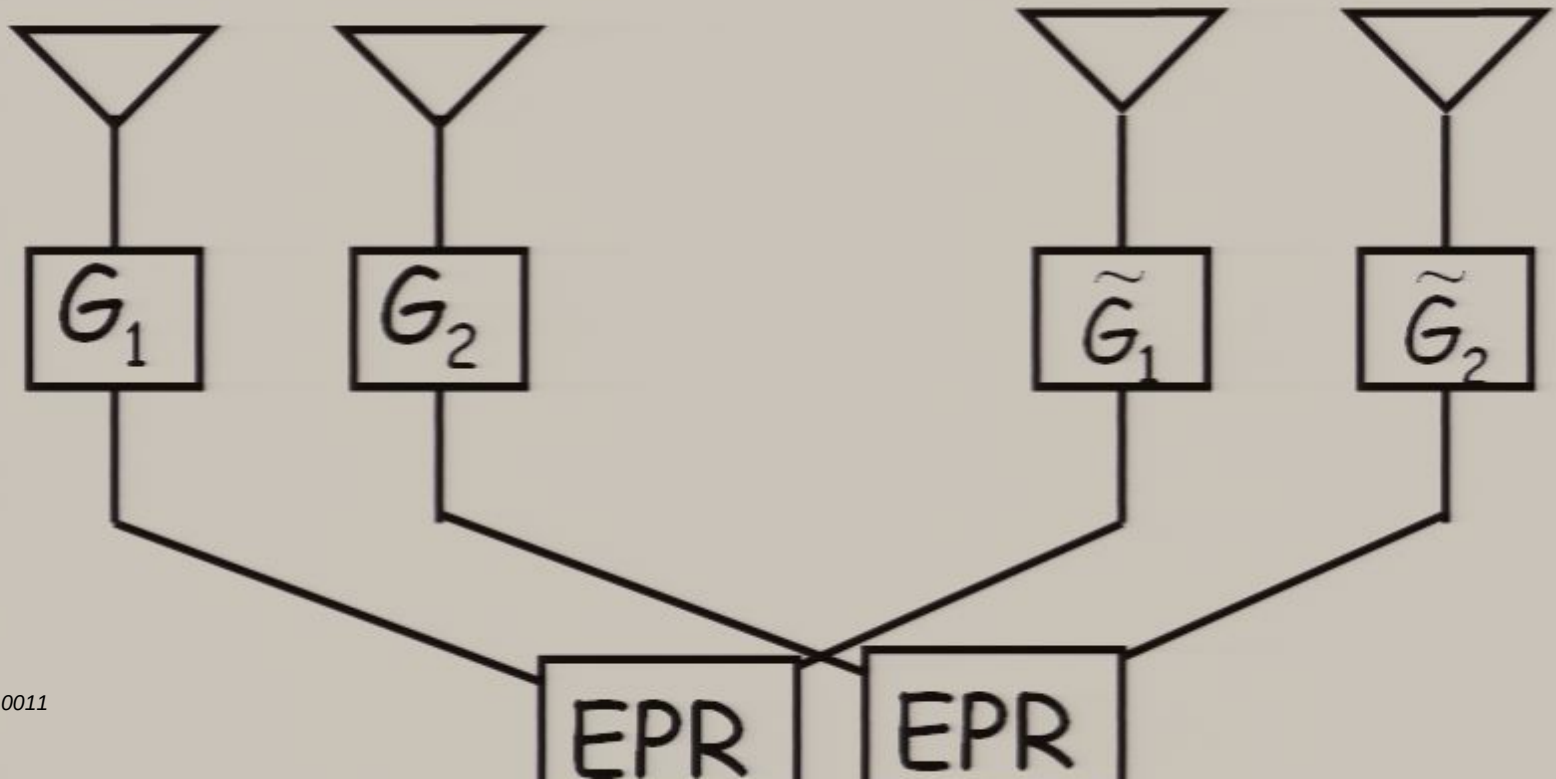
We verify the behaviour of the gates G_1 and G_2



“conspiracy test”



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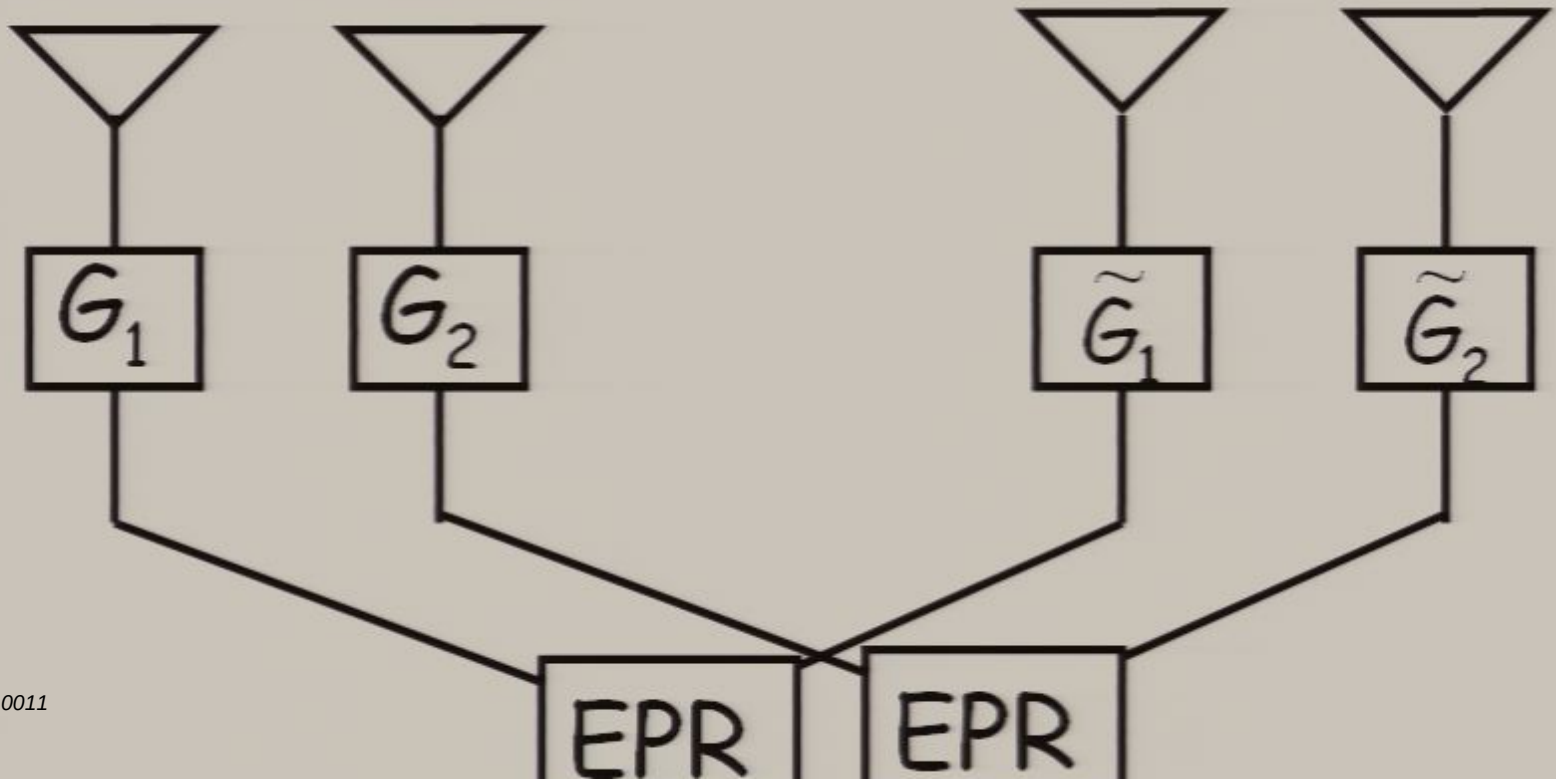
If $\tilde{G}_1 = G_1^{\dagger}$, *then this should recreate two EPR pairs.*



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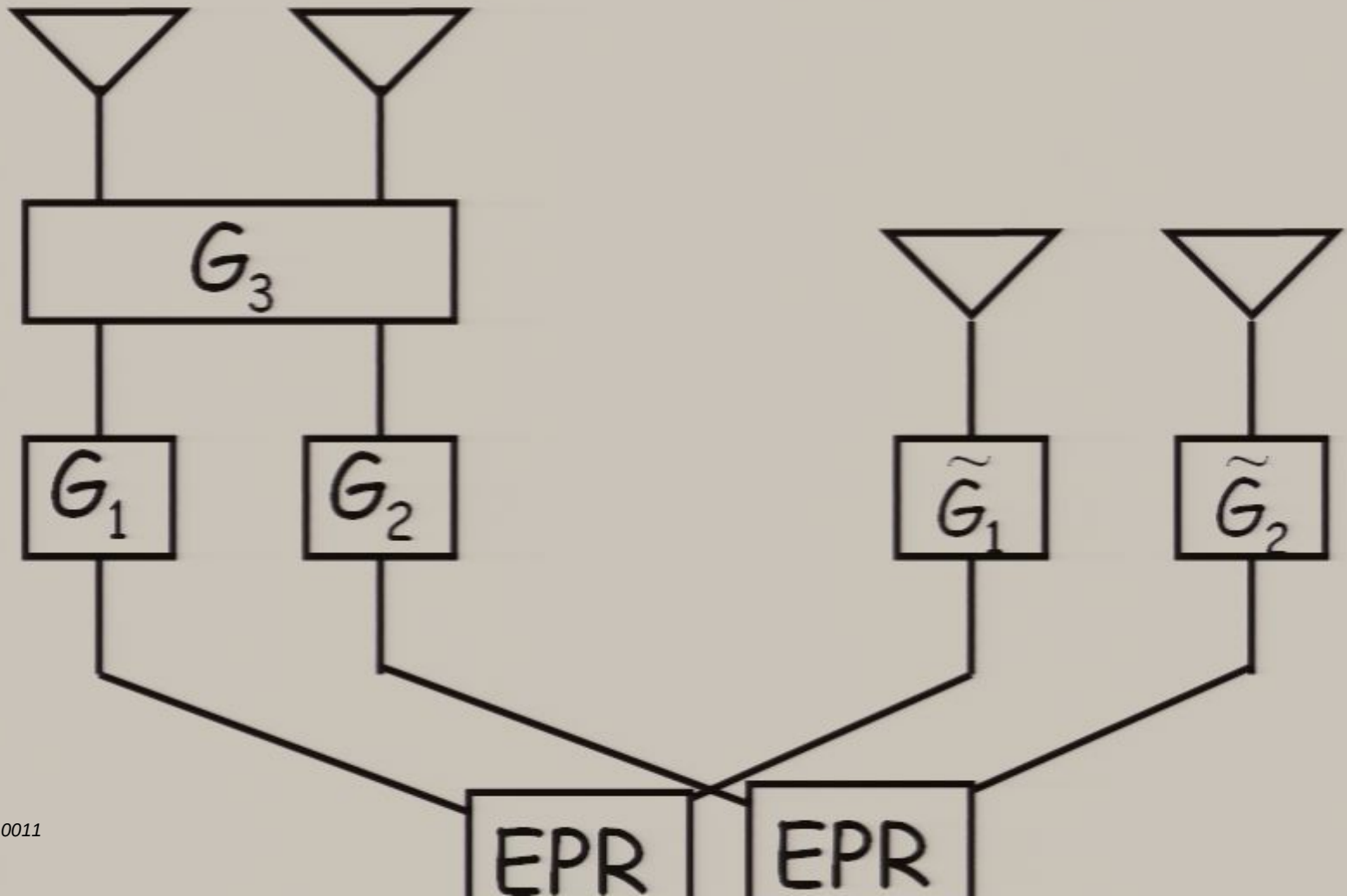

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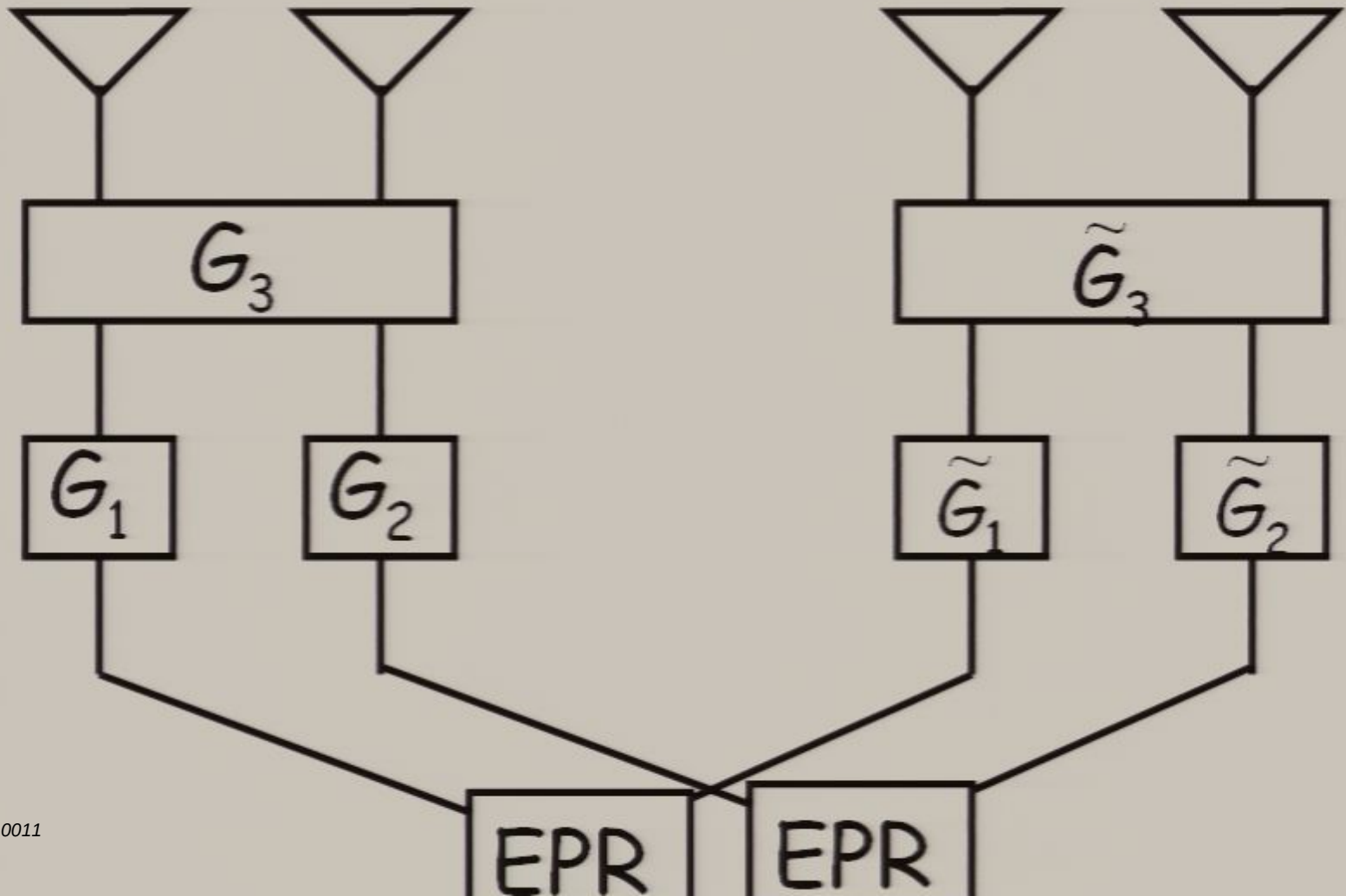
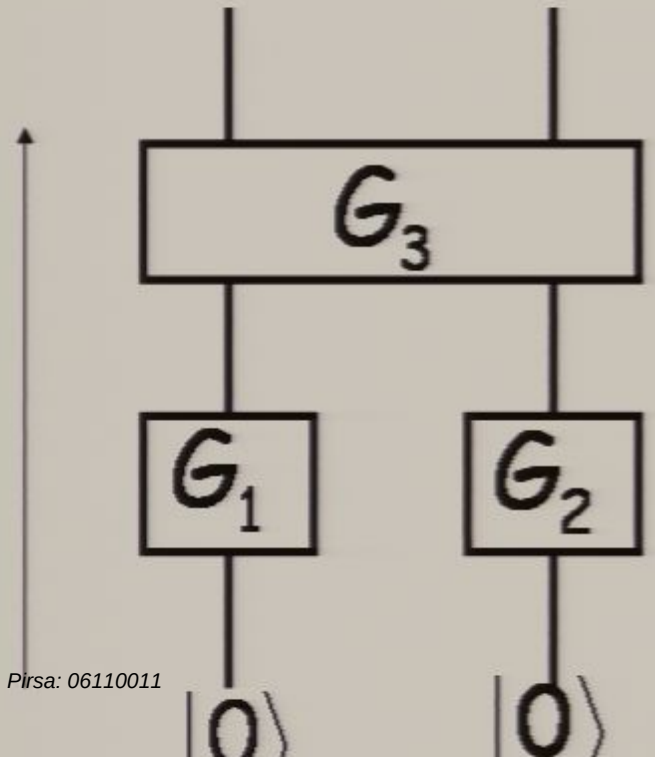


One last technical glitch



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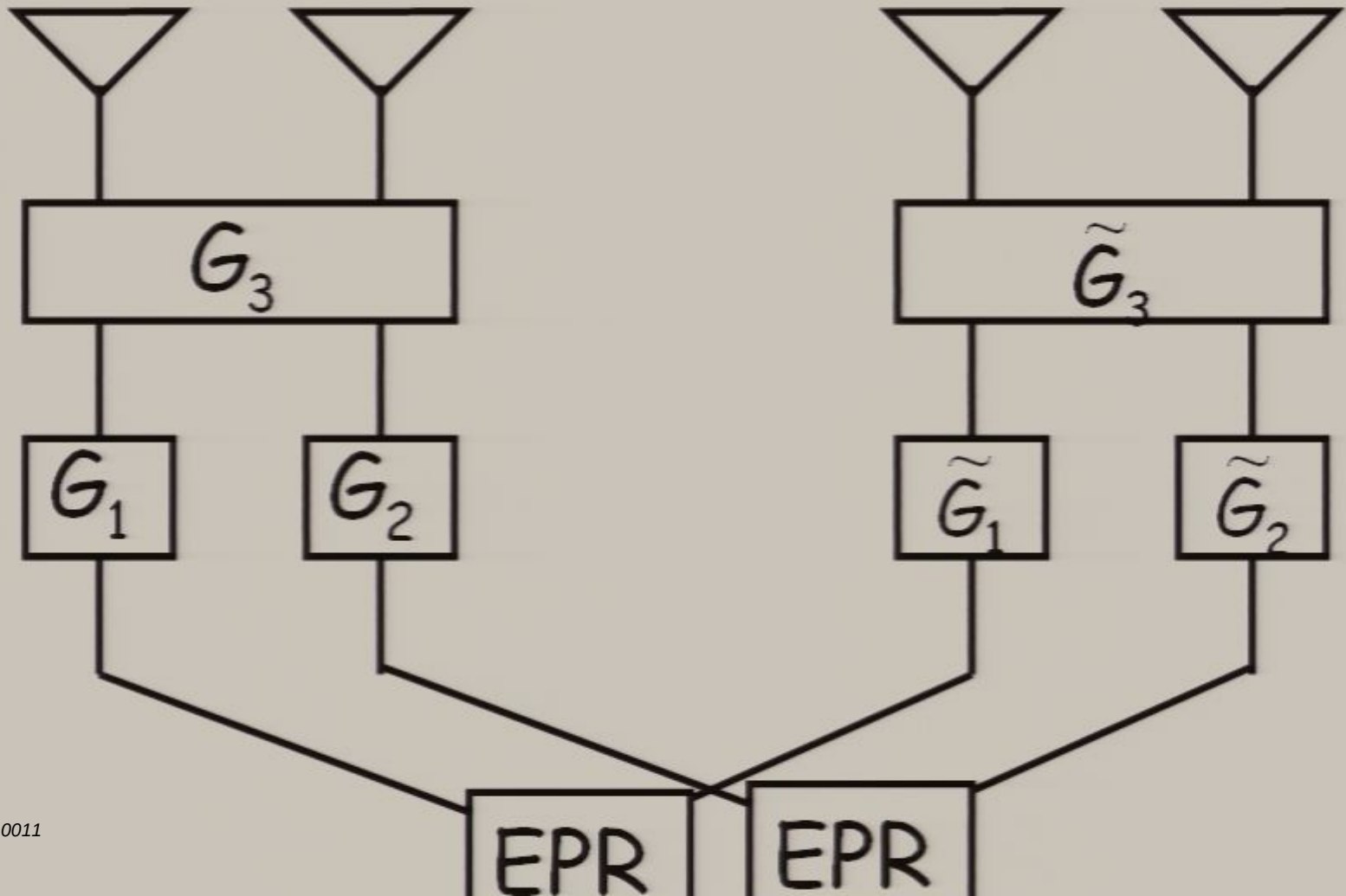

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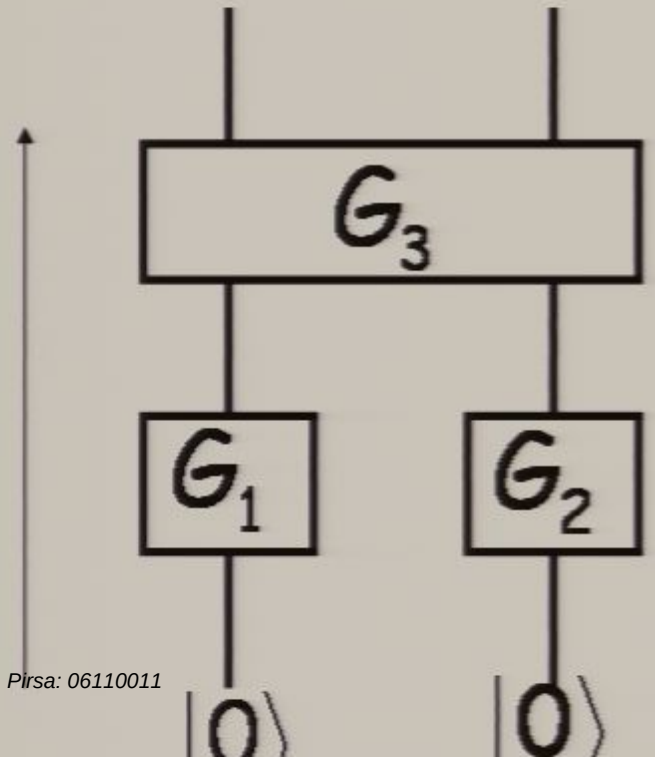

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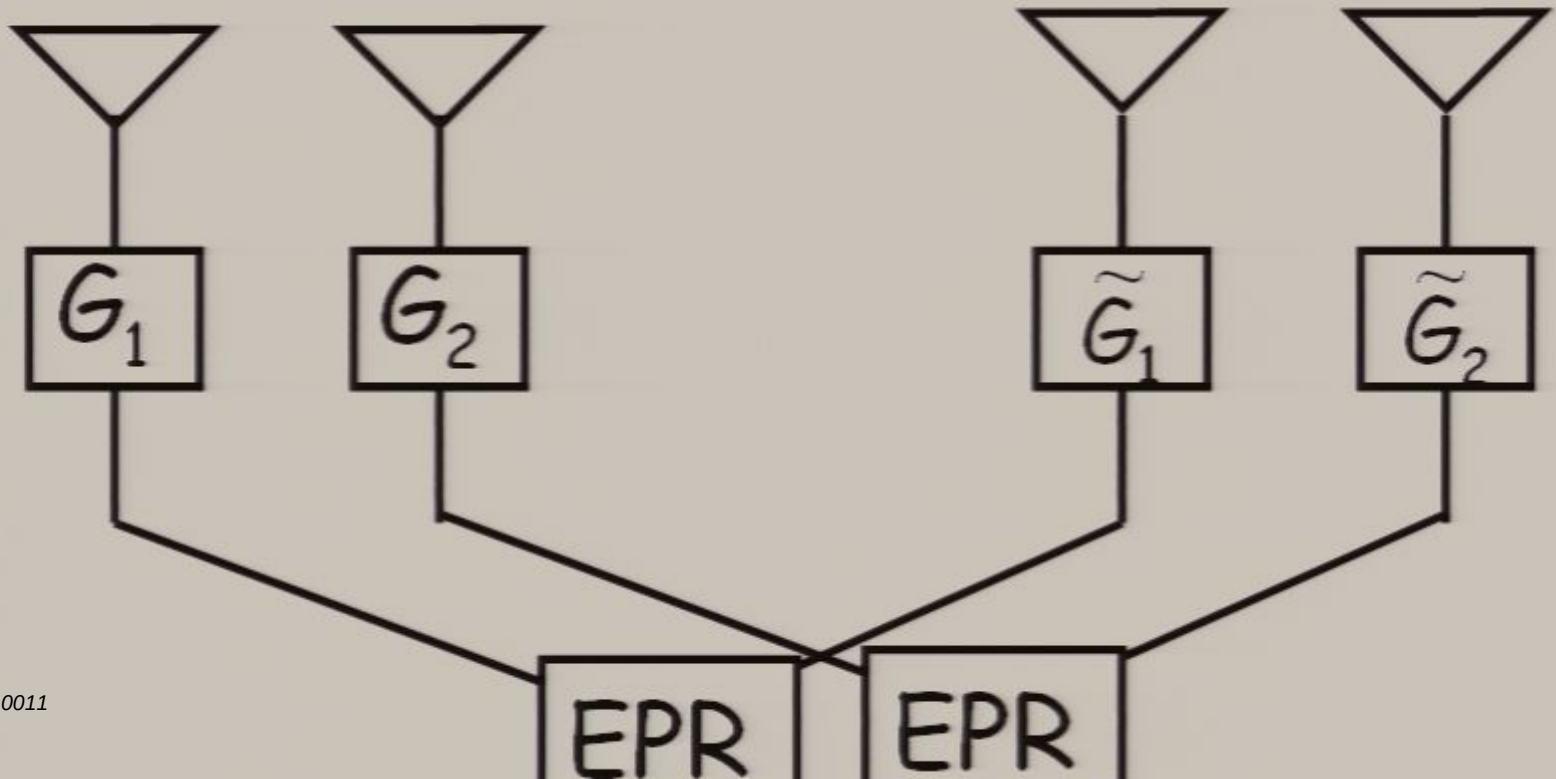

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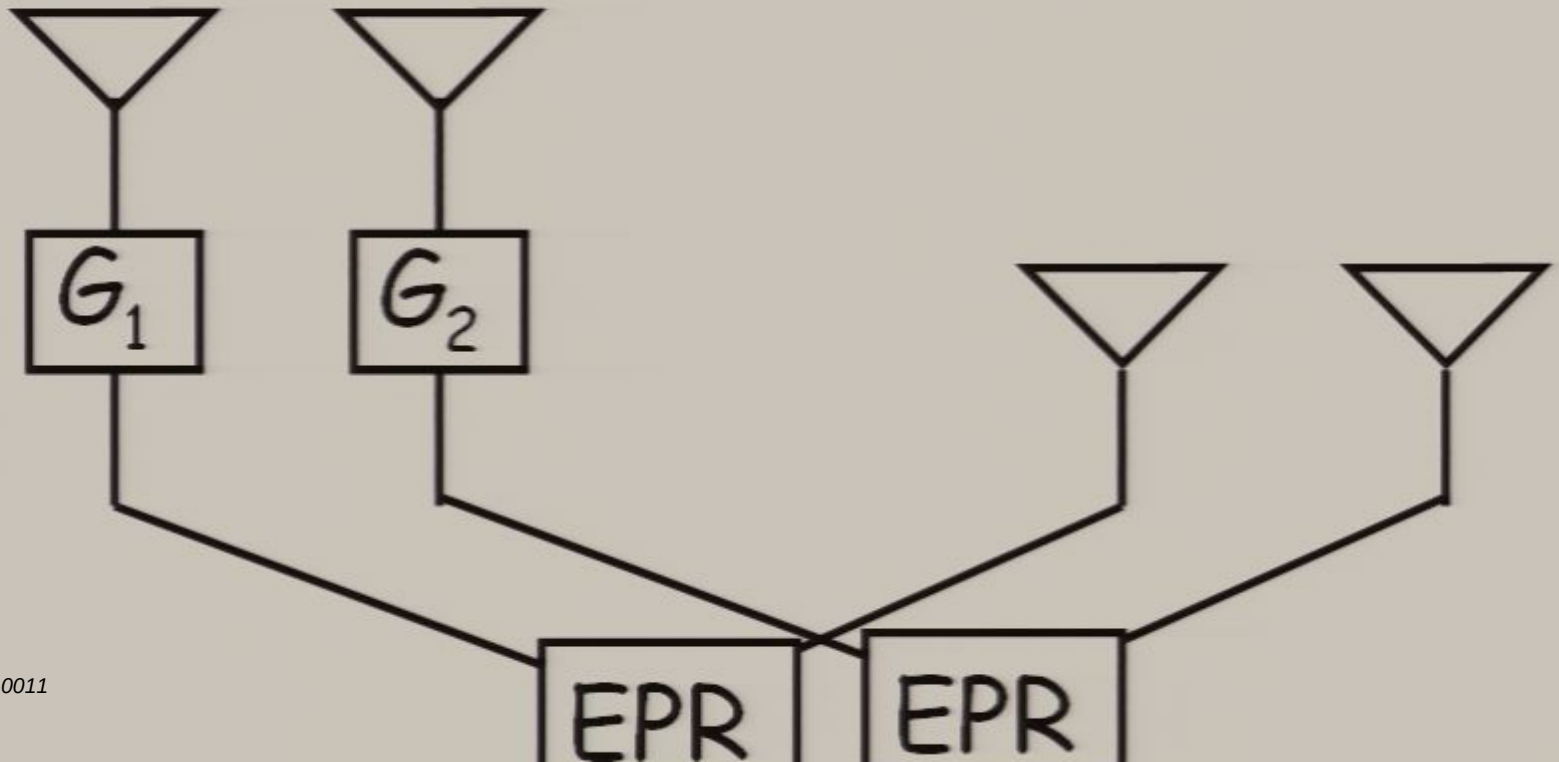

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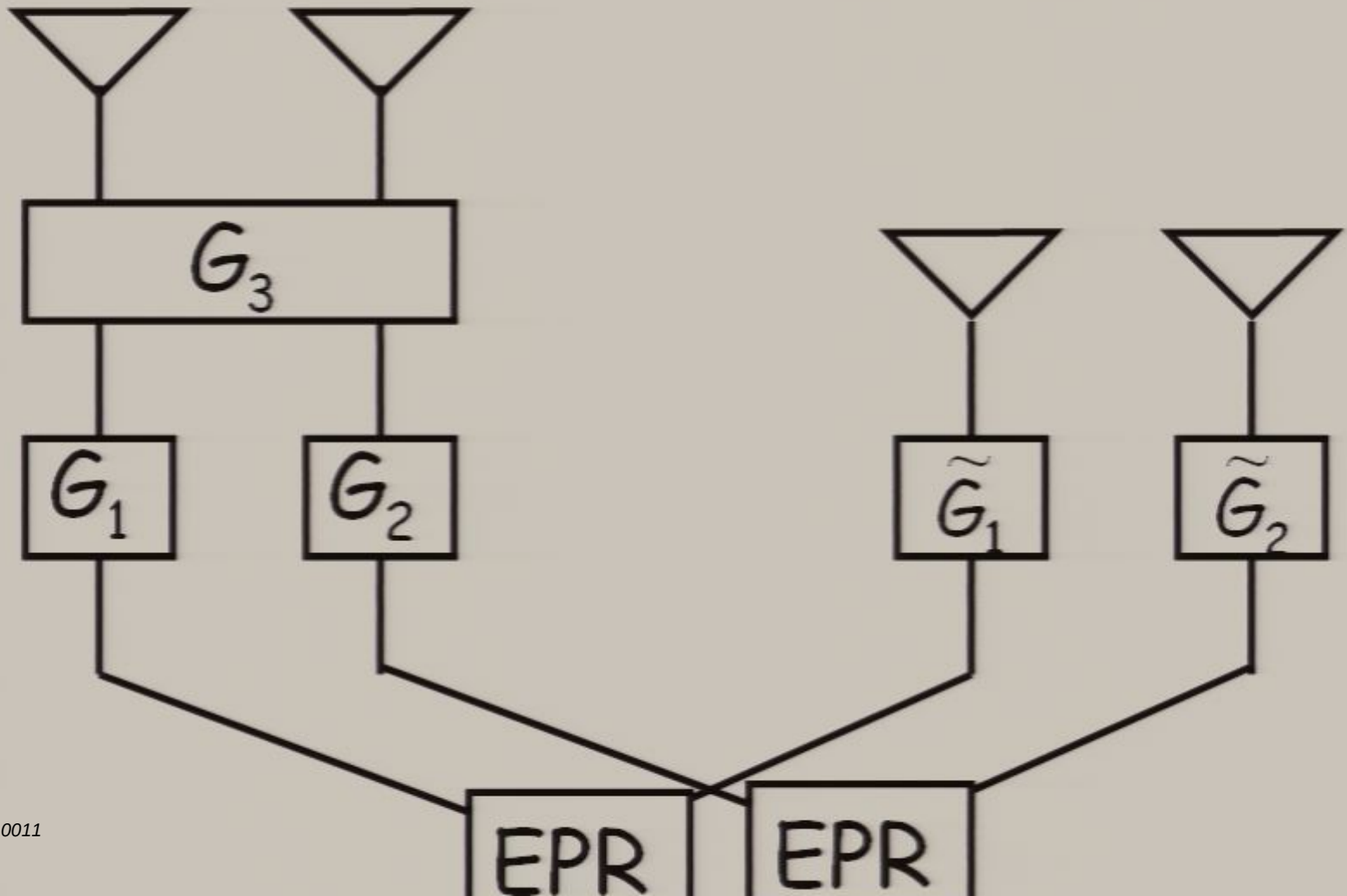

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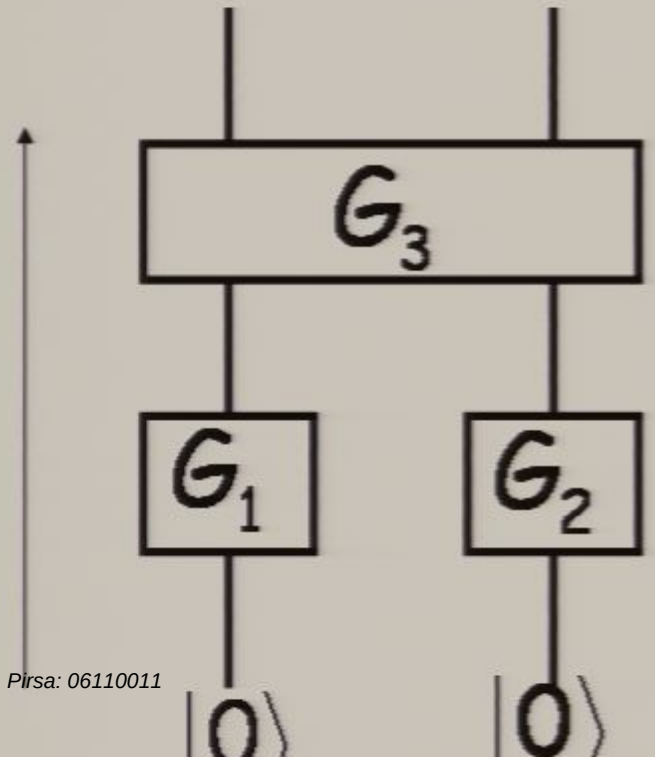

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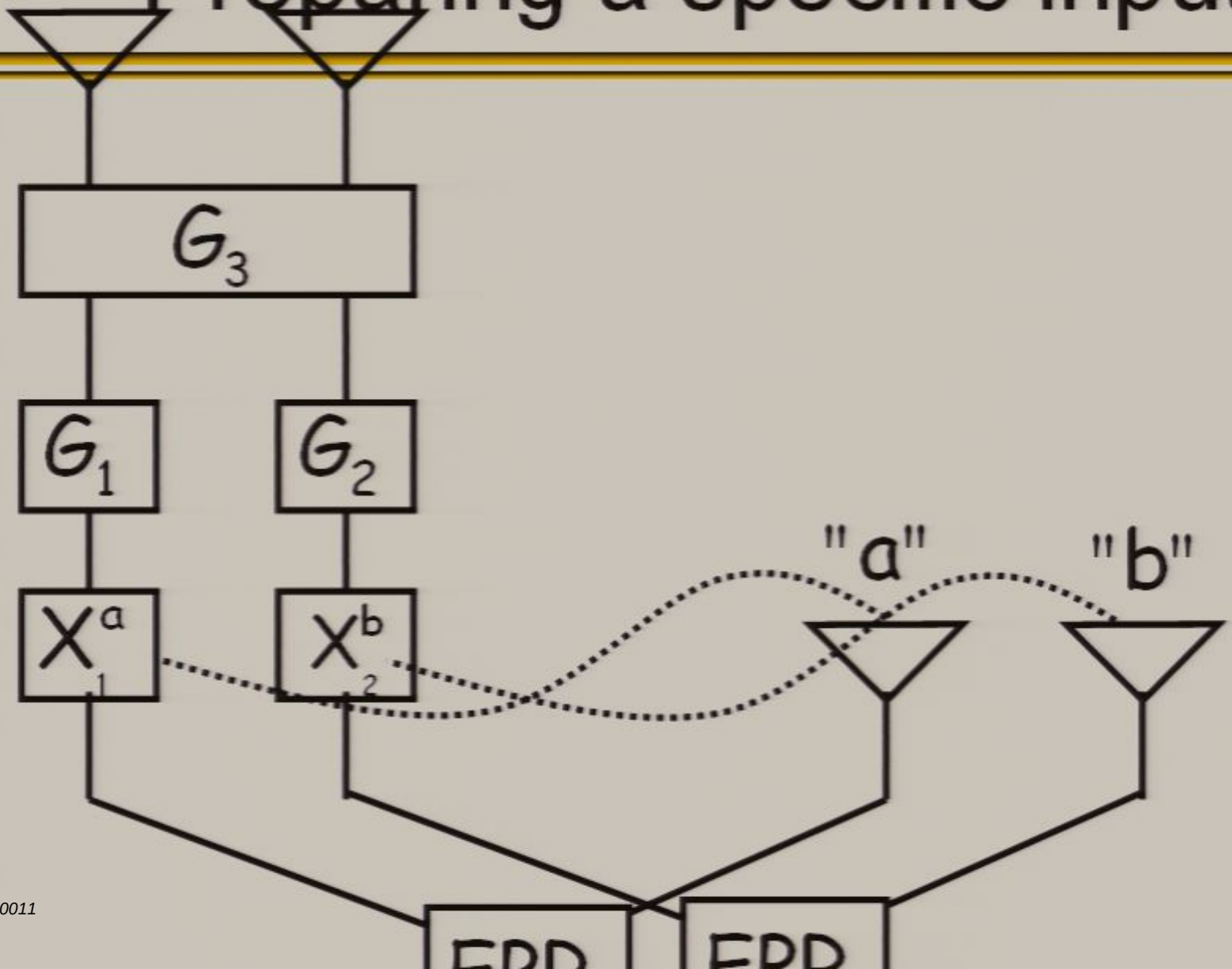

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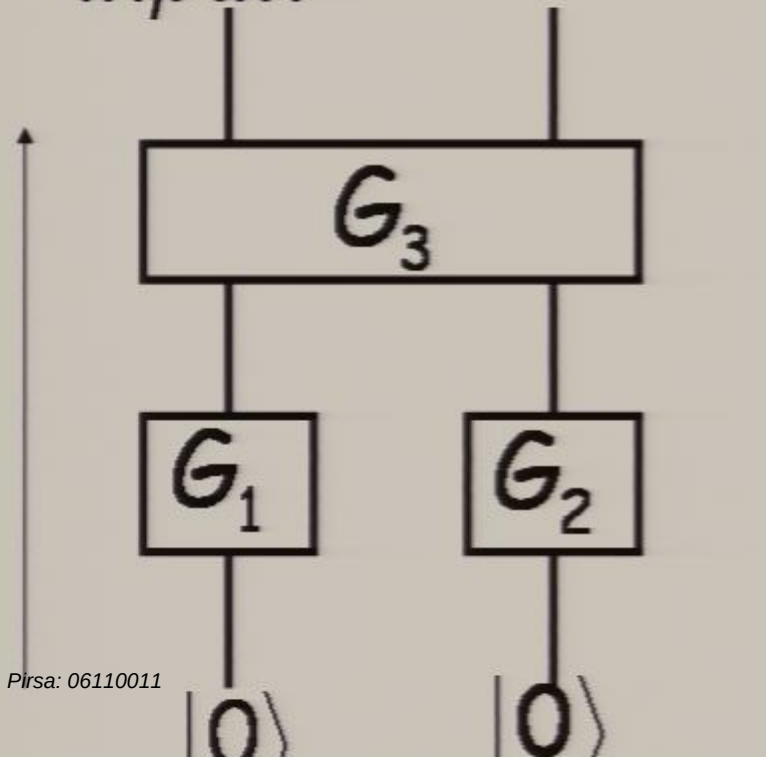

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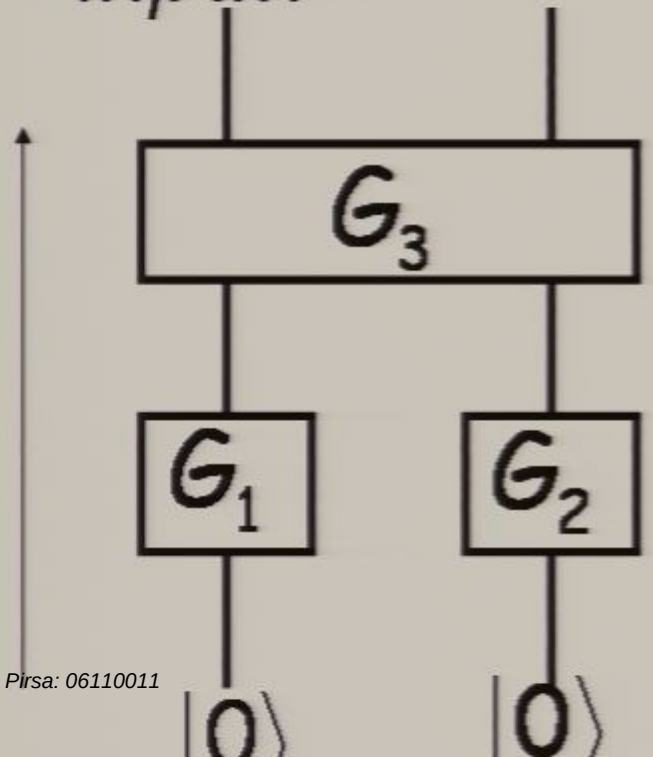

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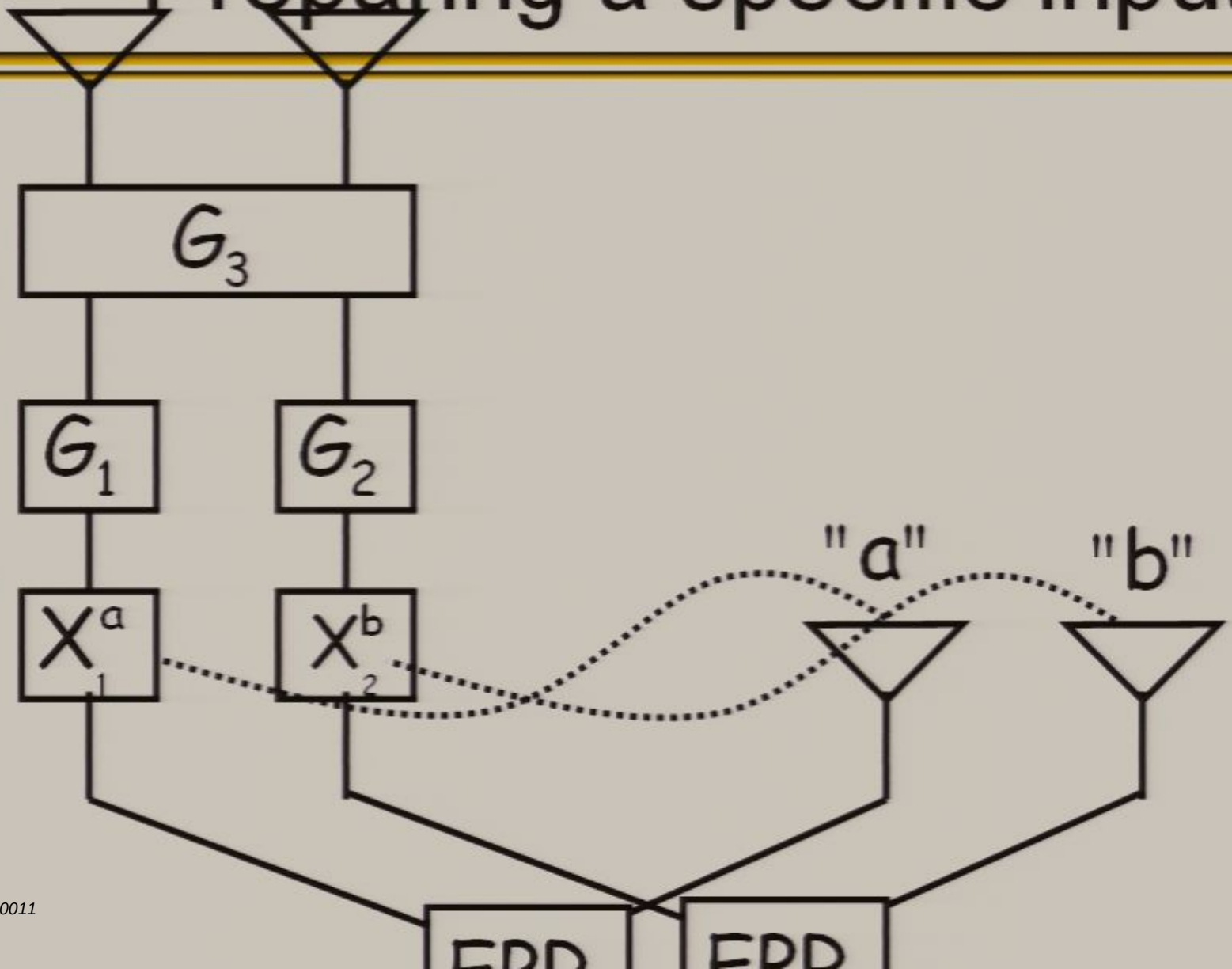

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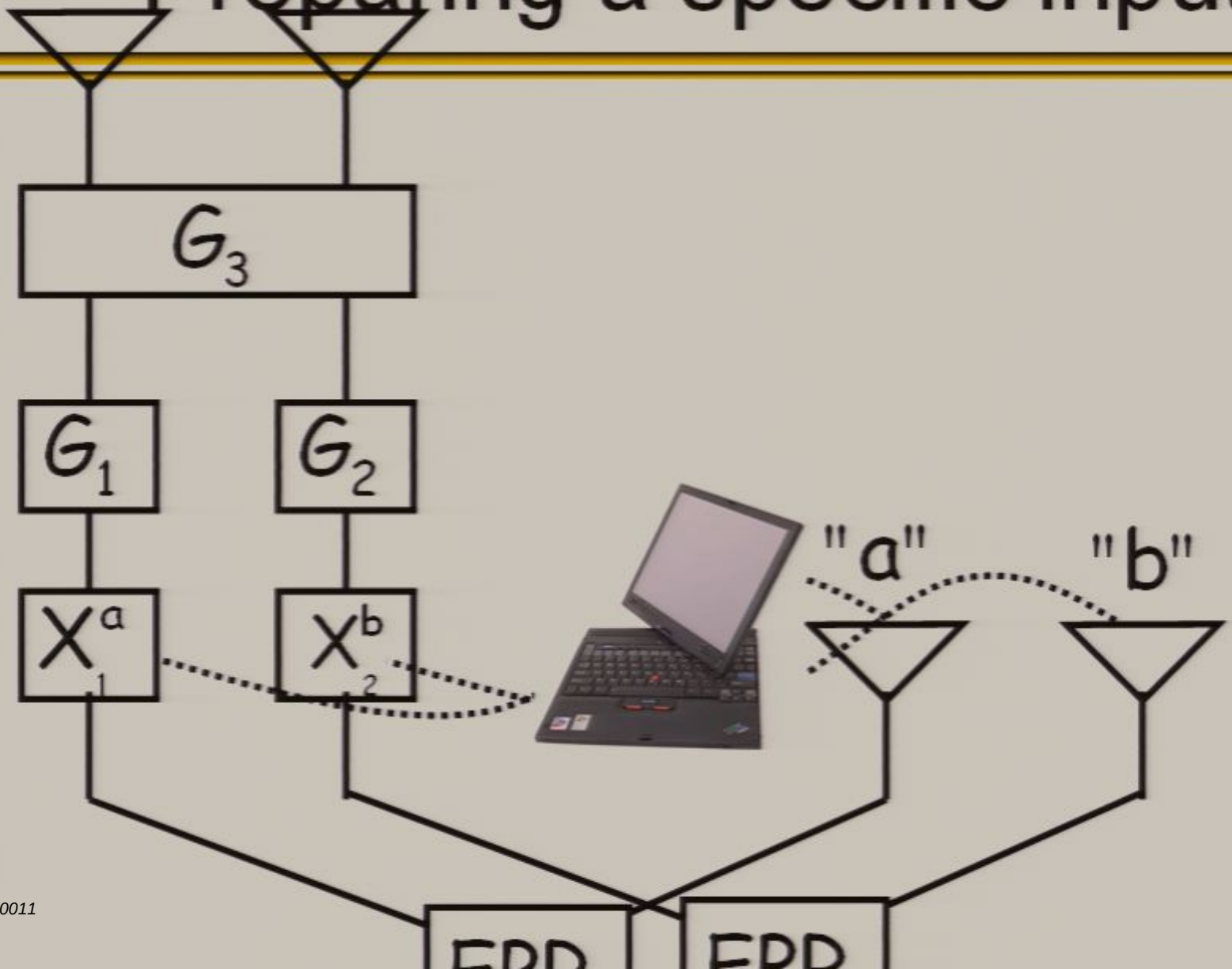

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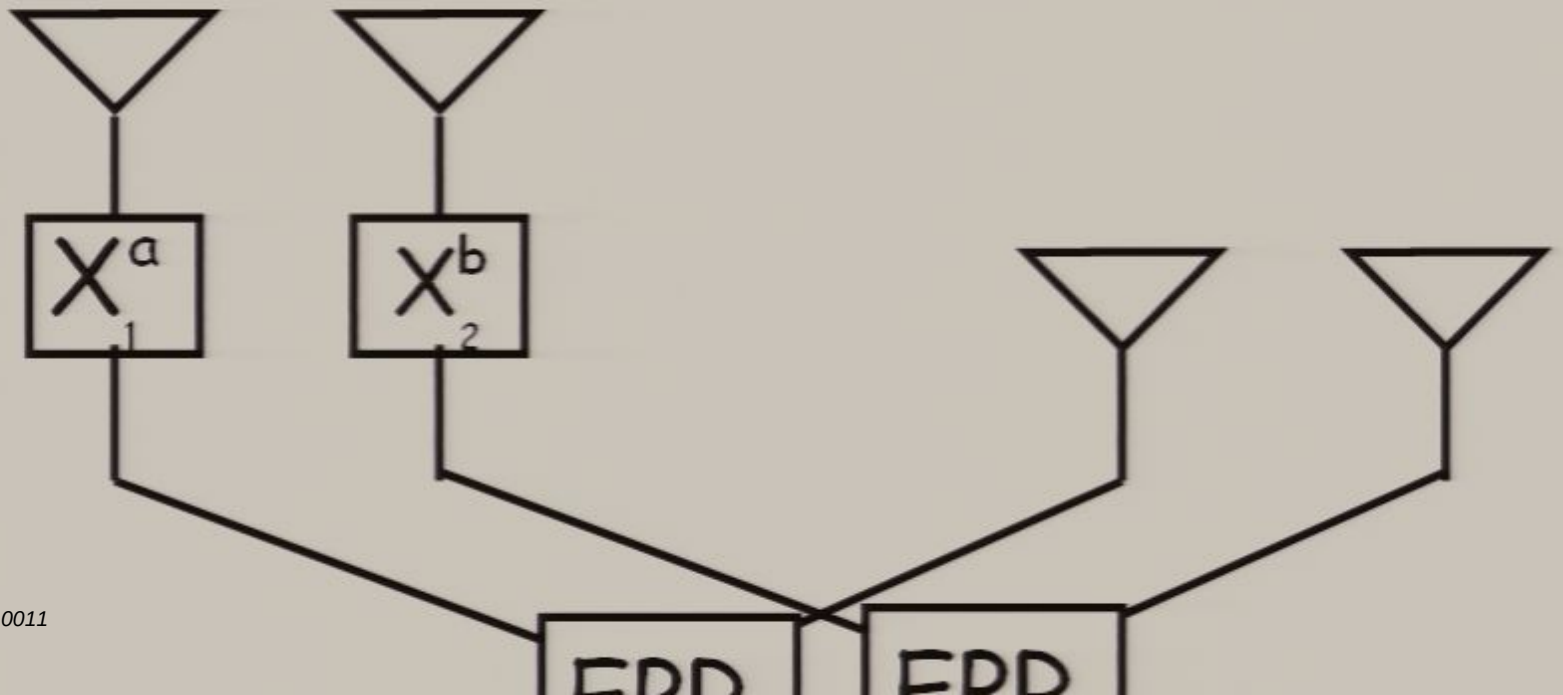

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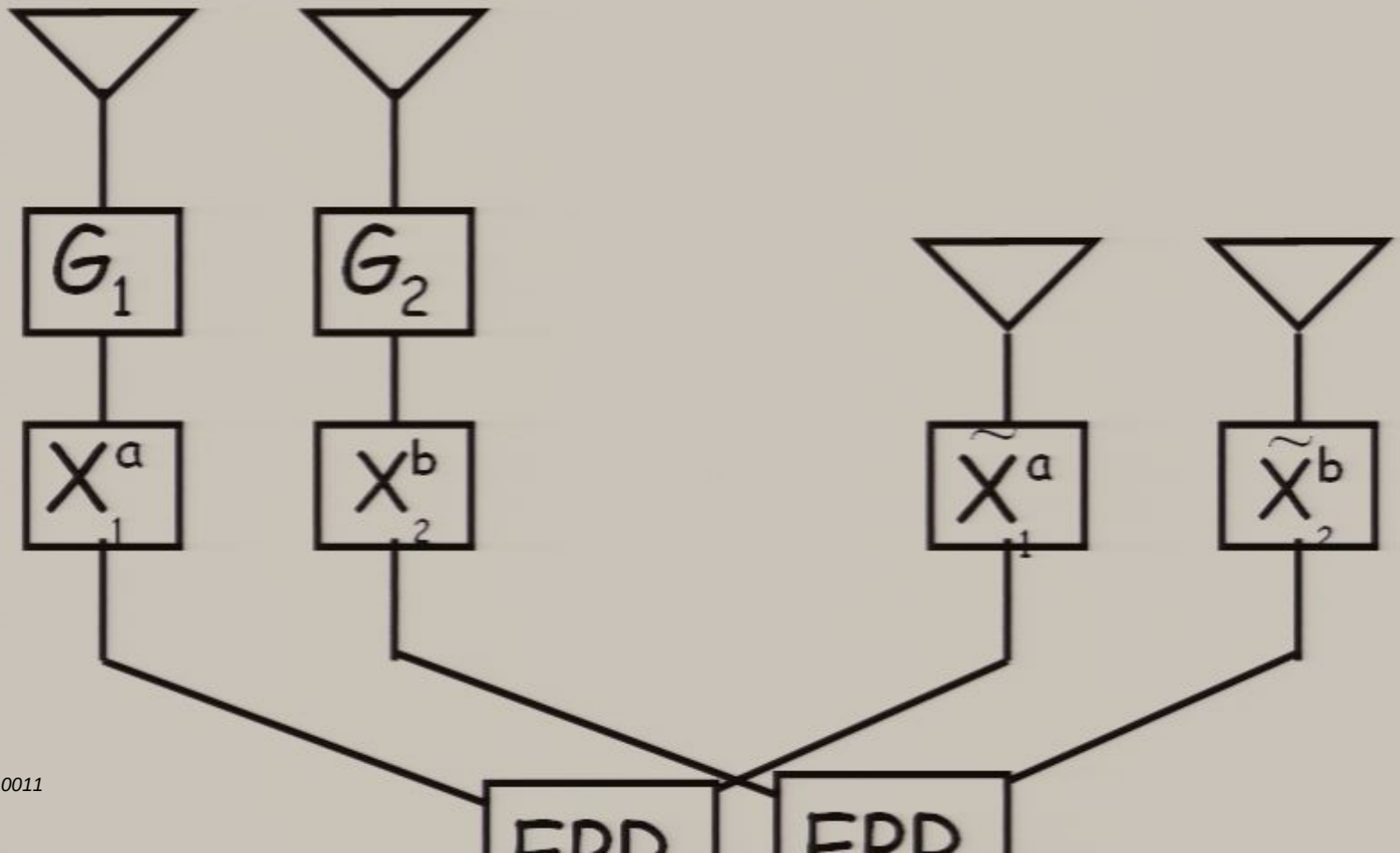

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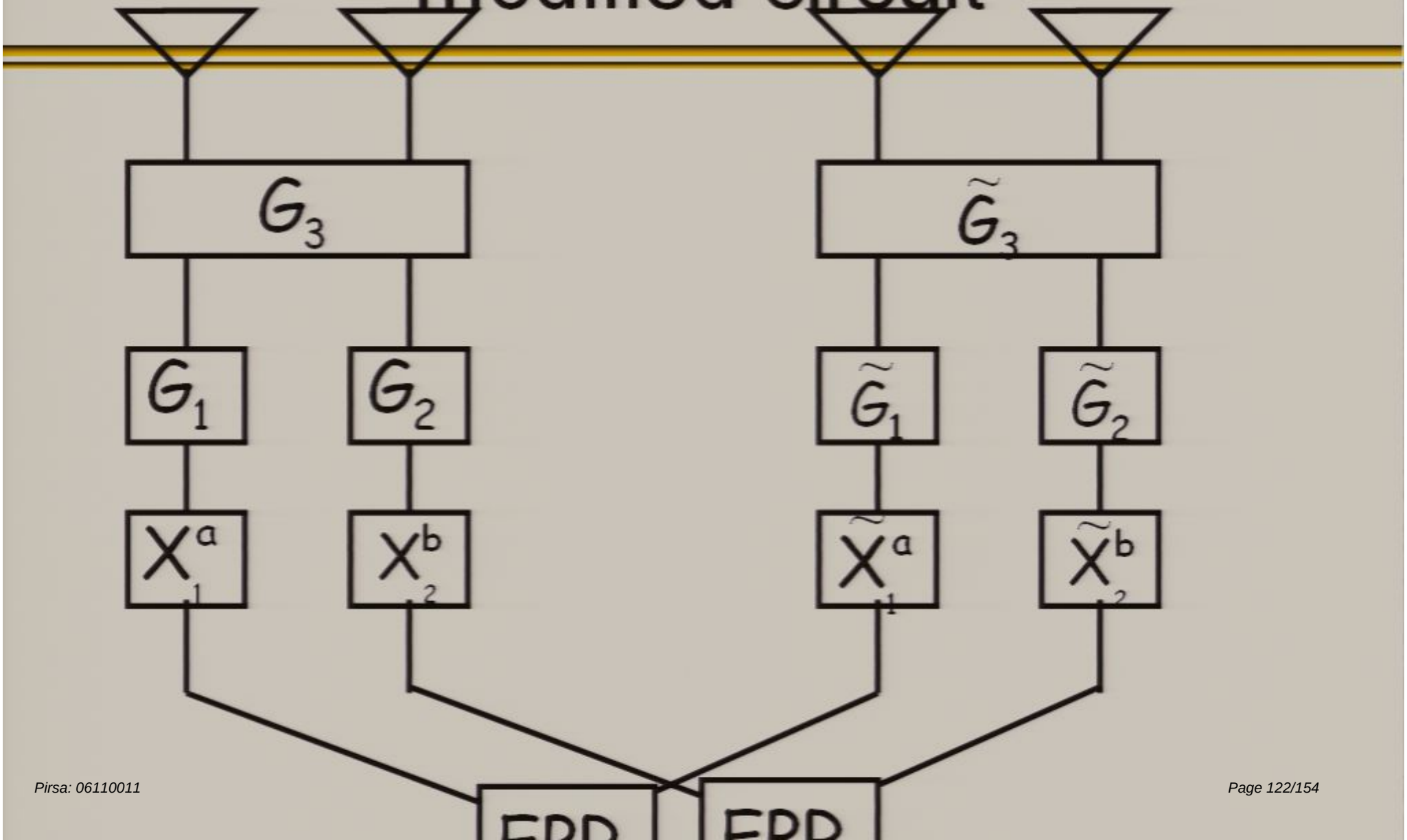

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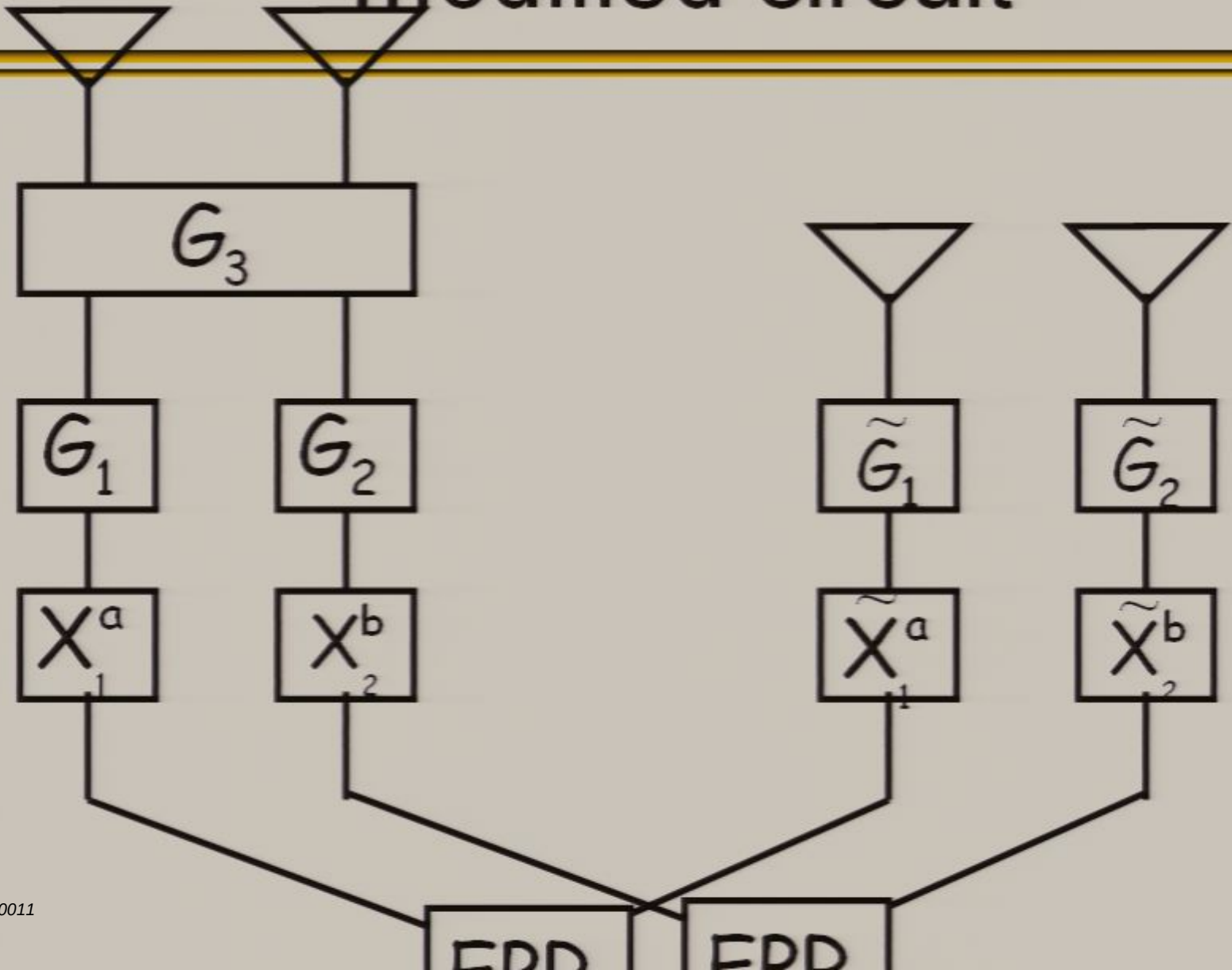

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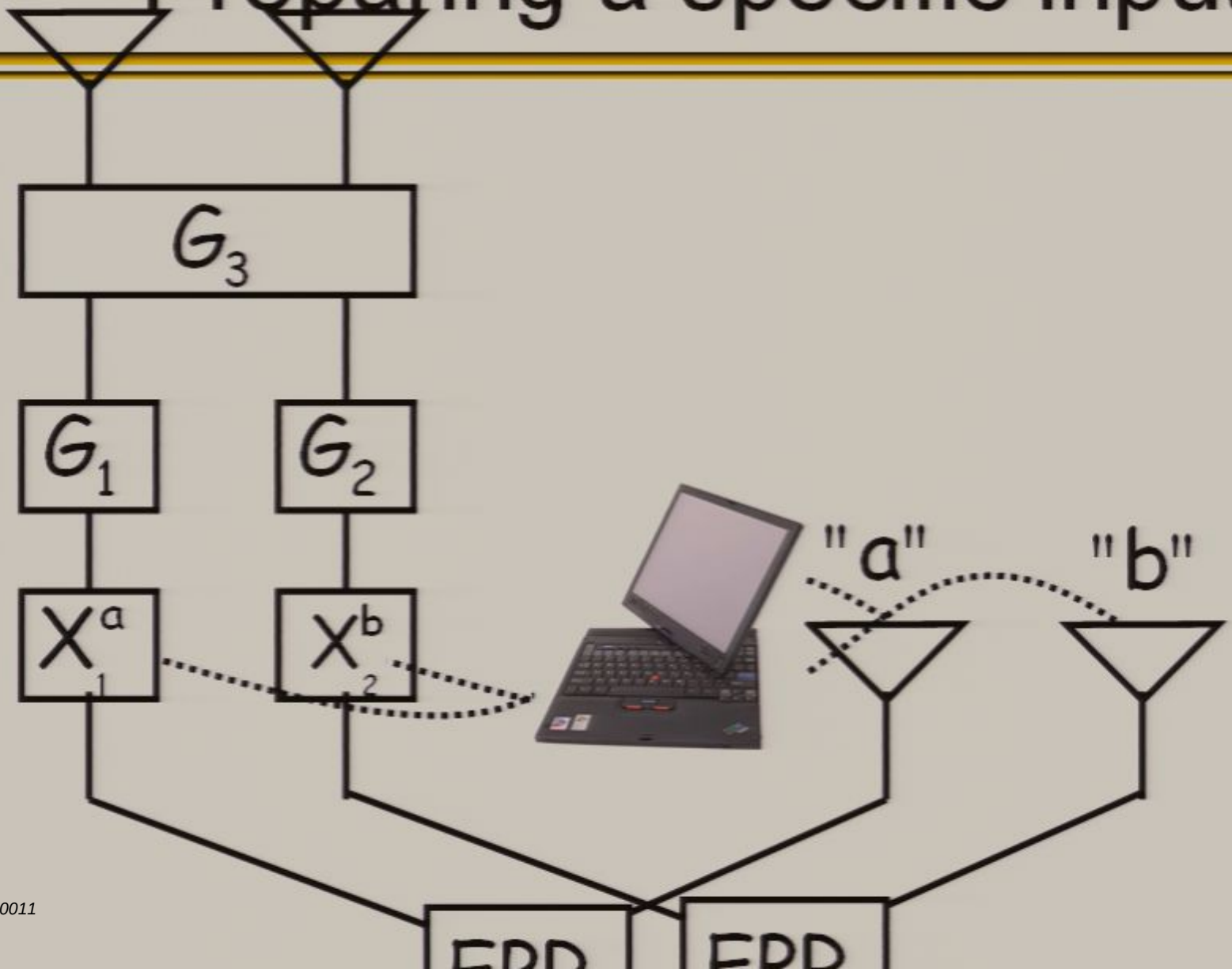

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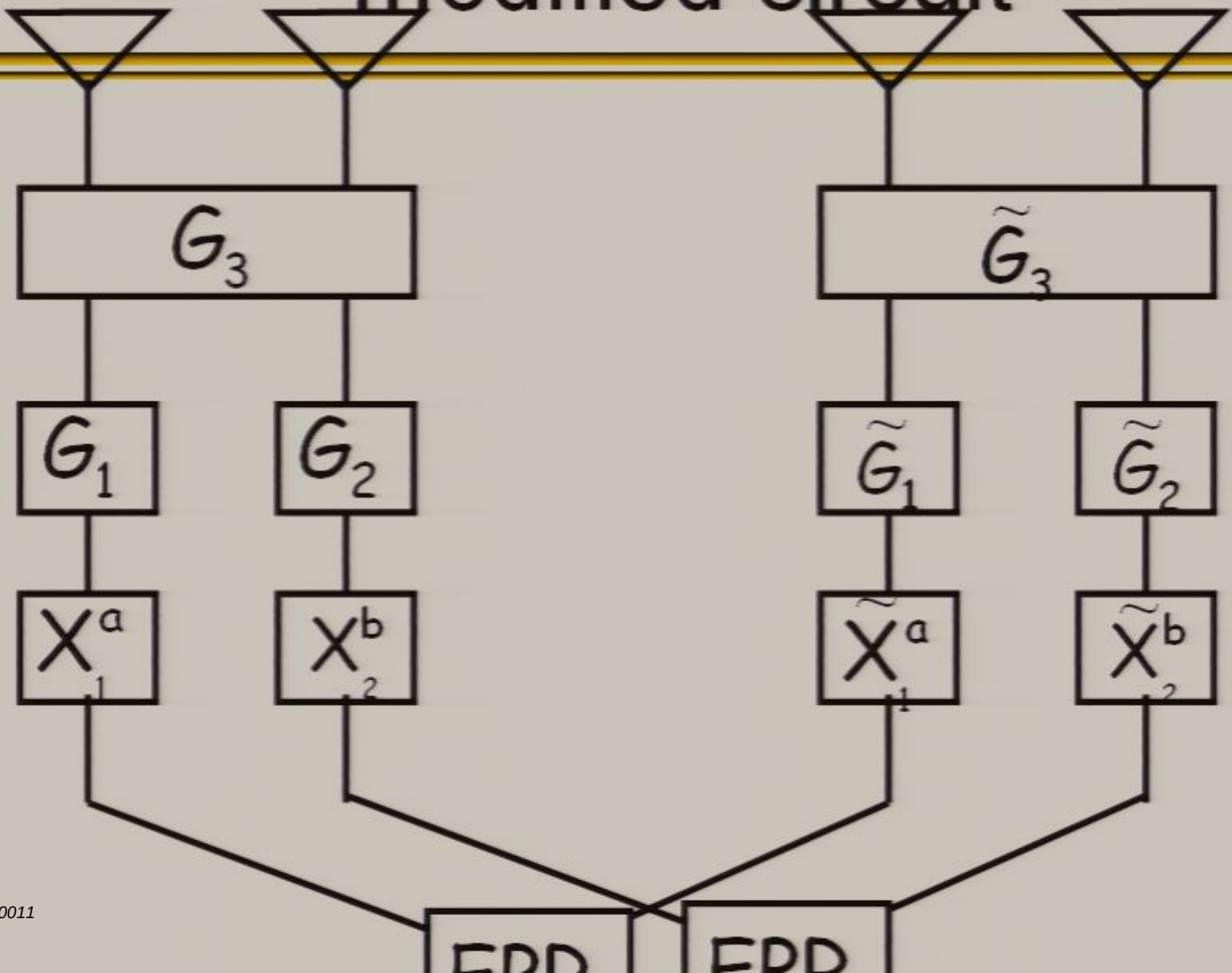

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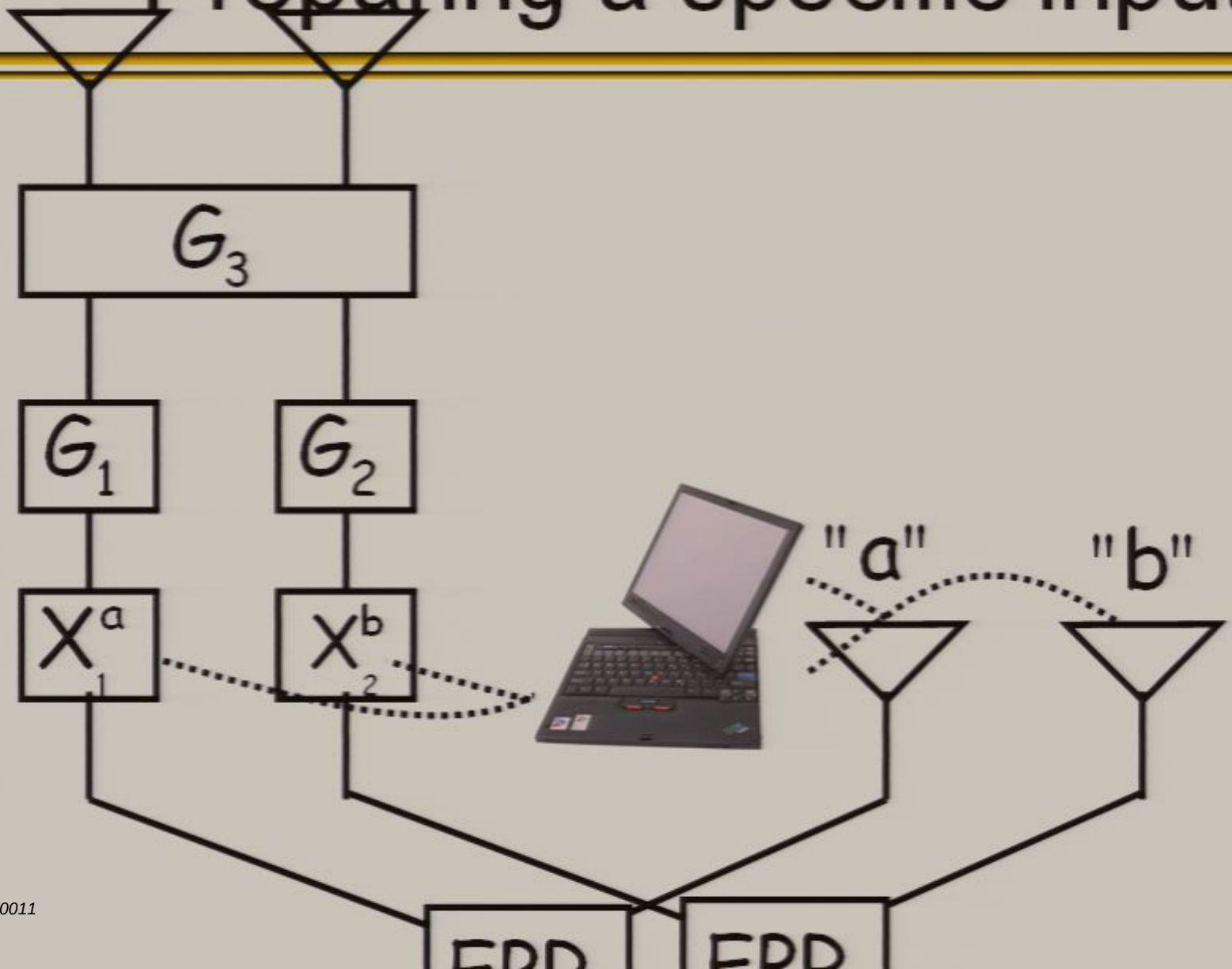

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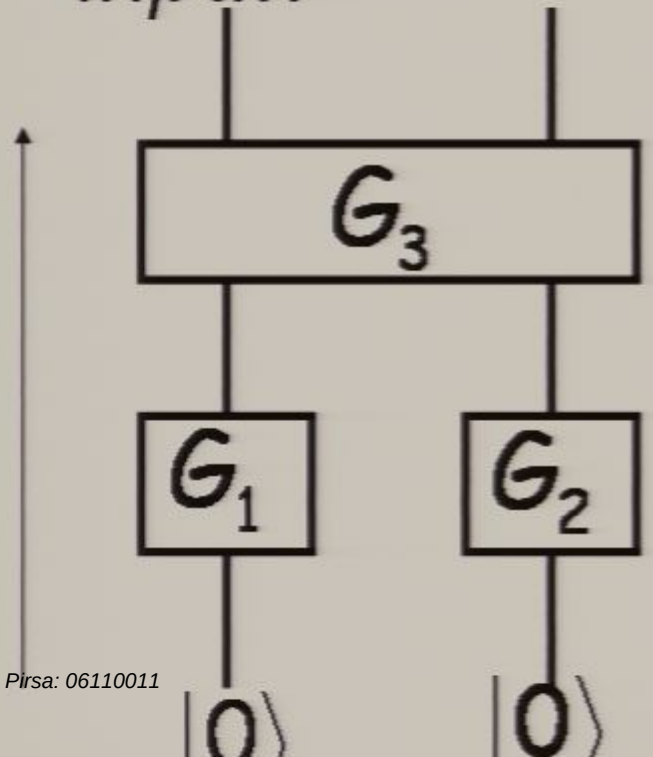

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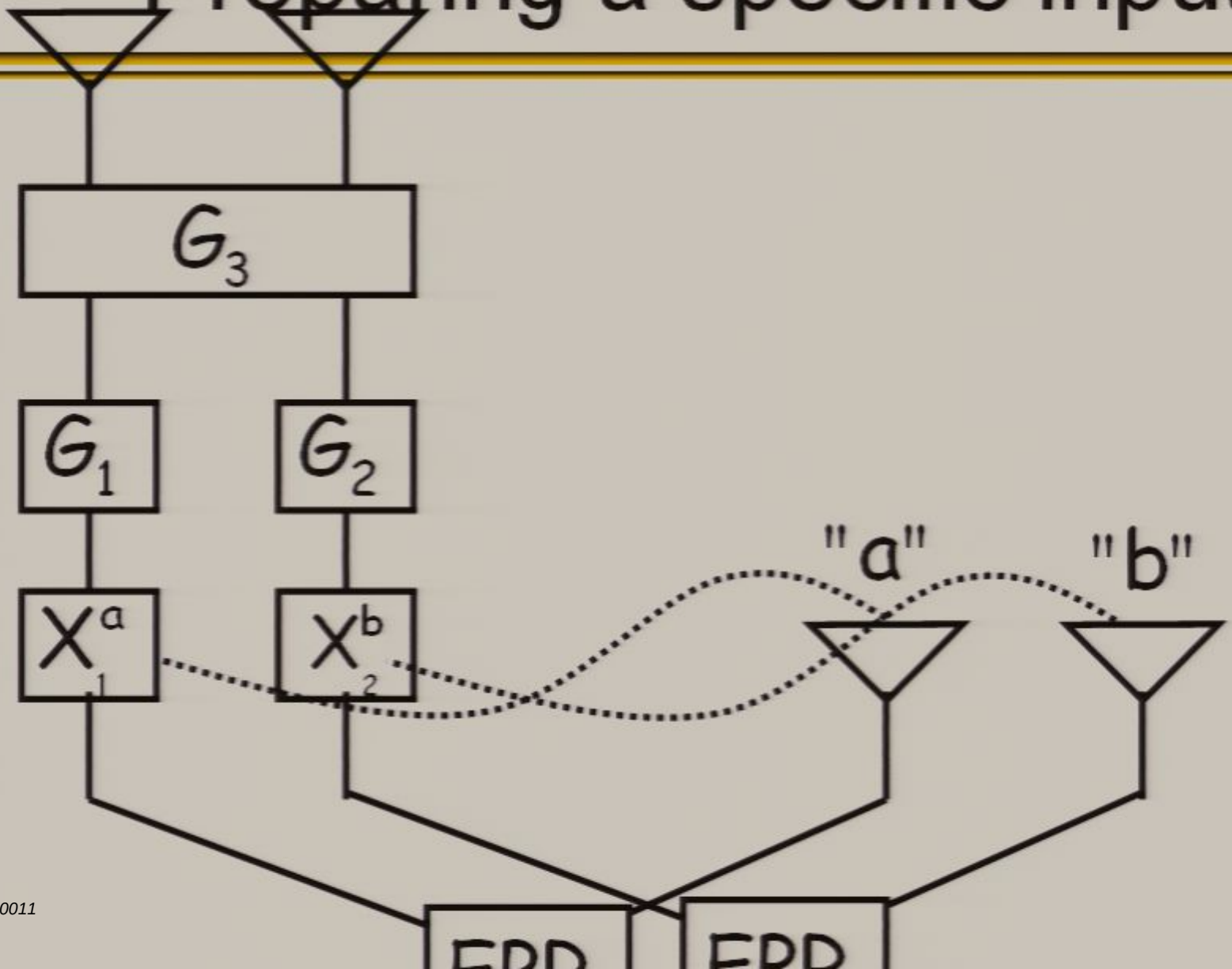

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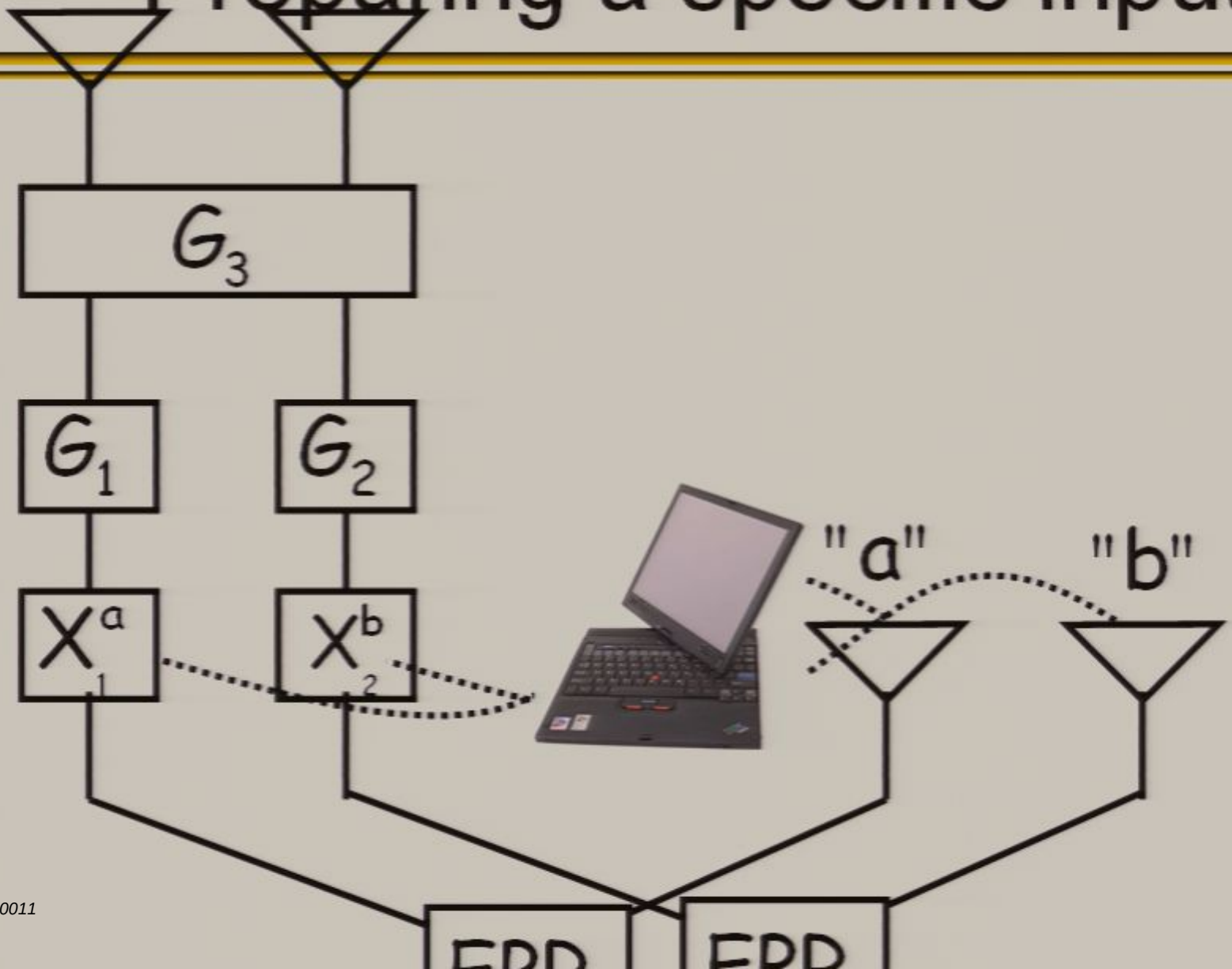

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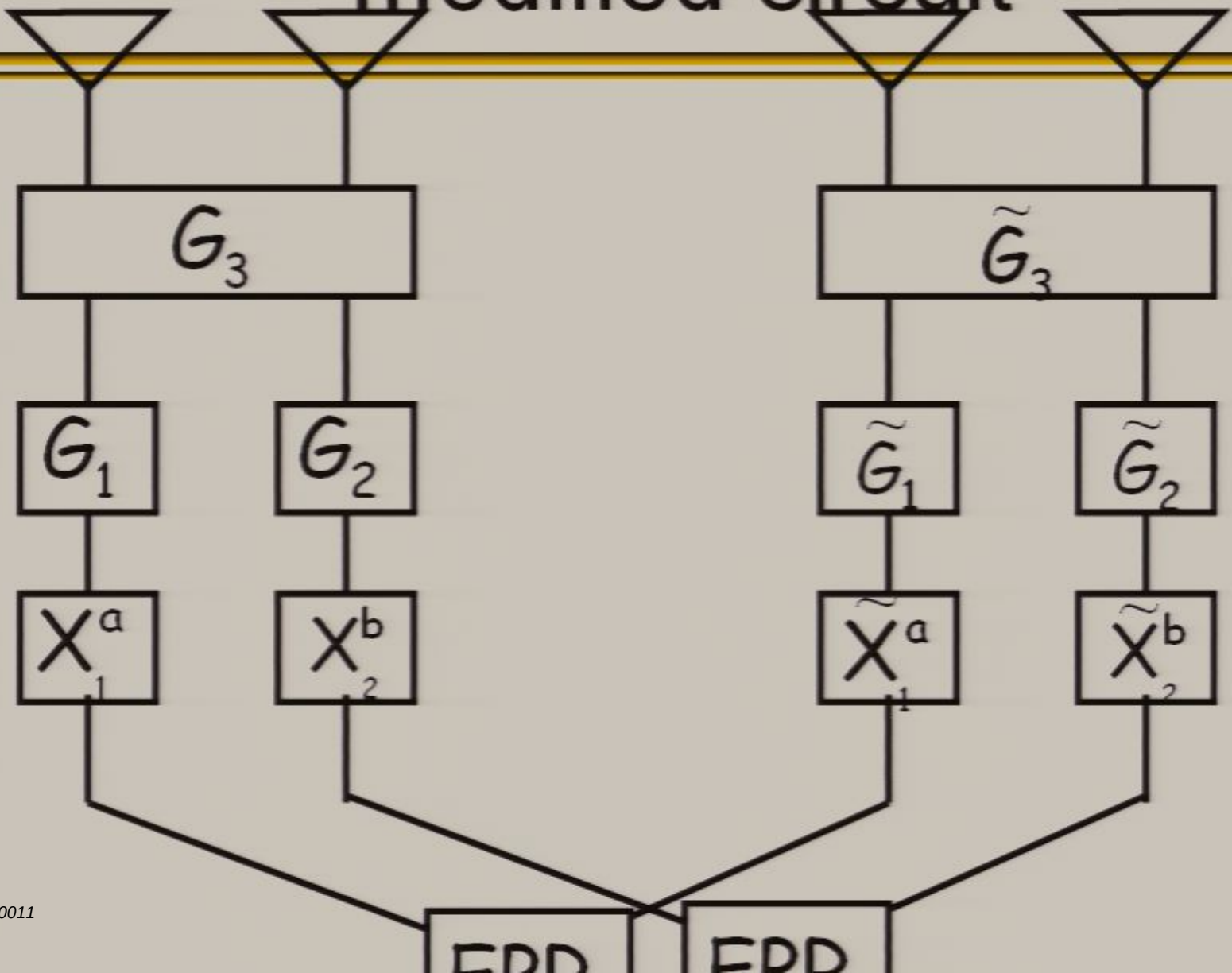

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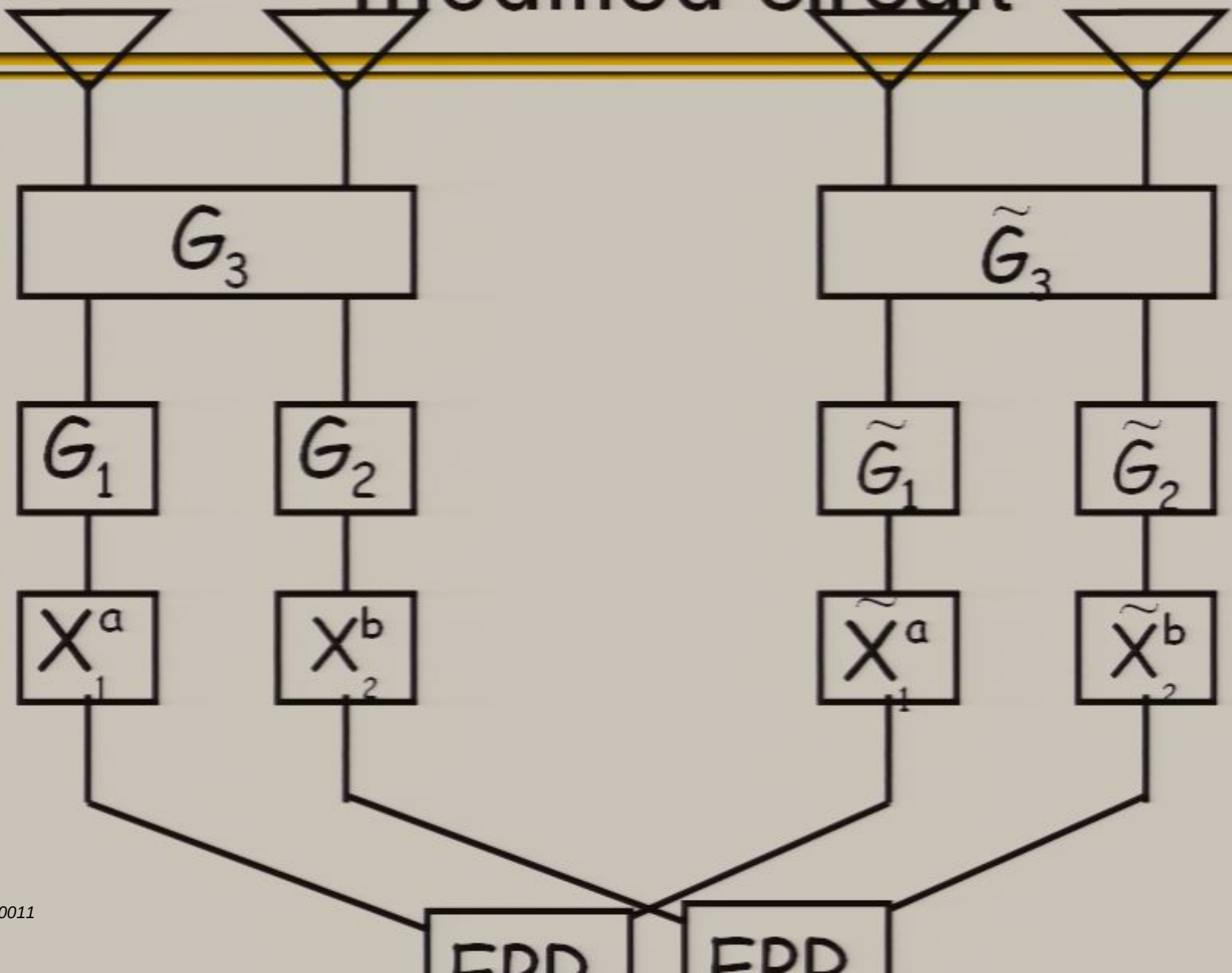

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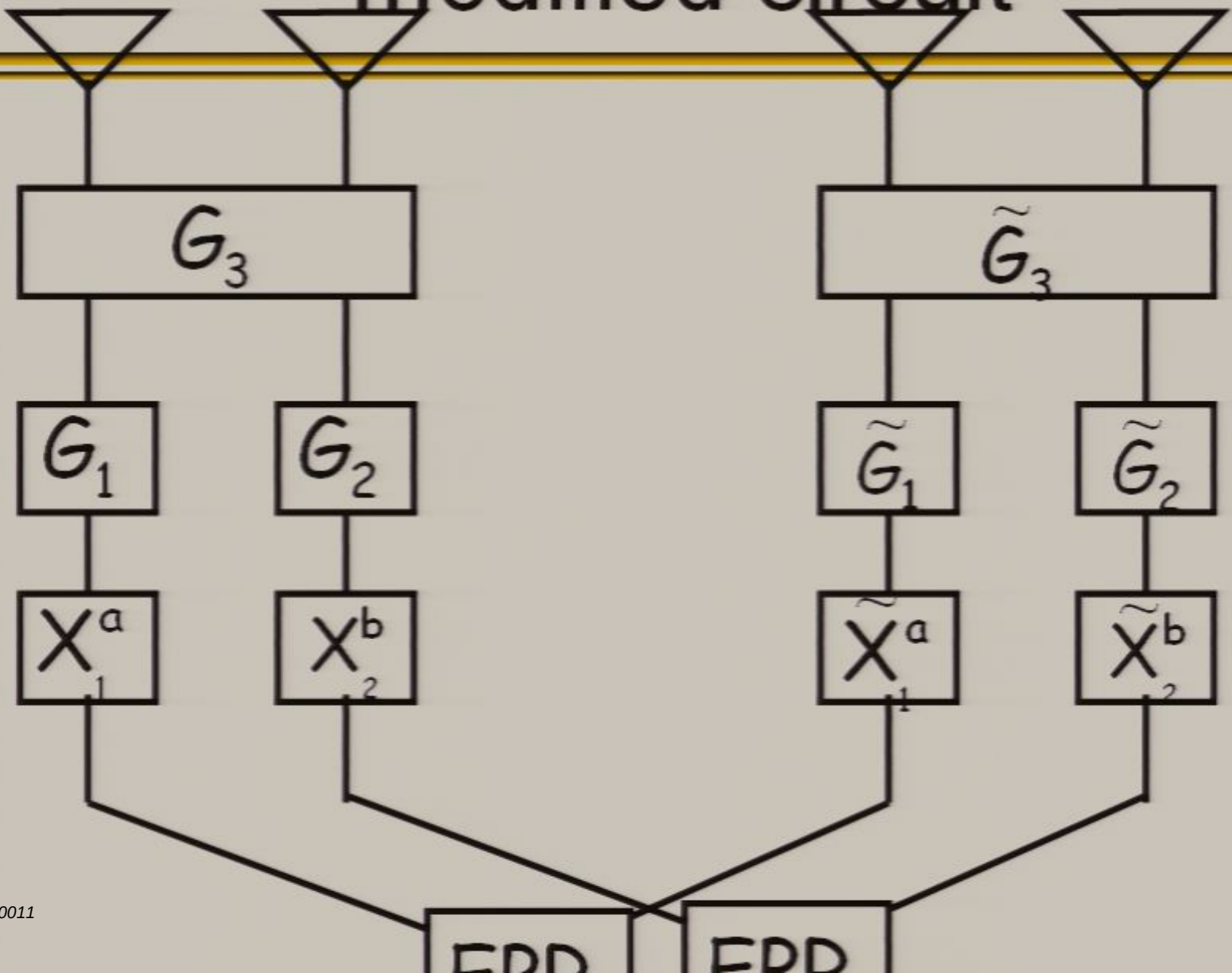

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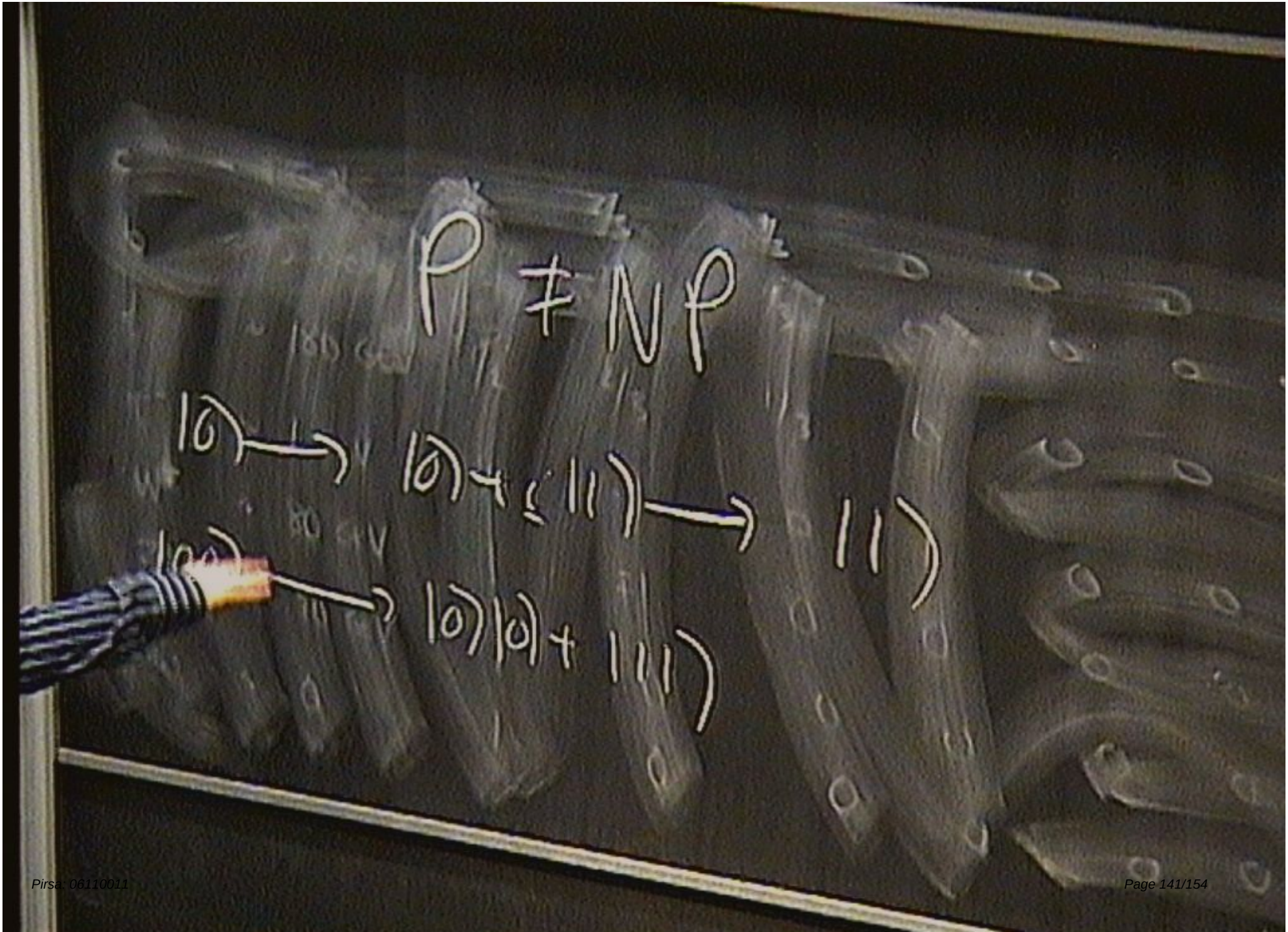
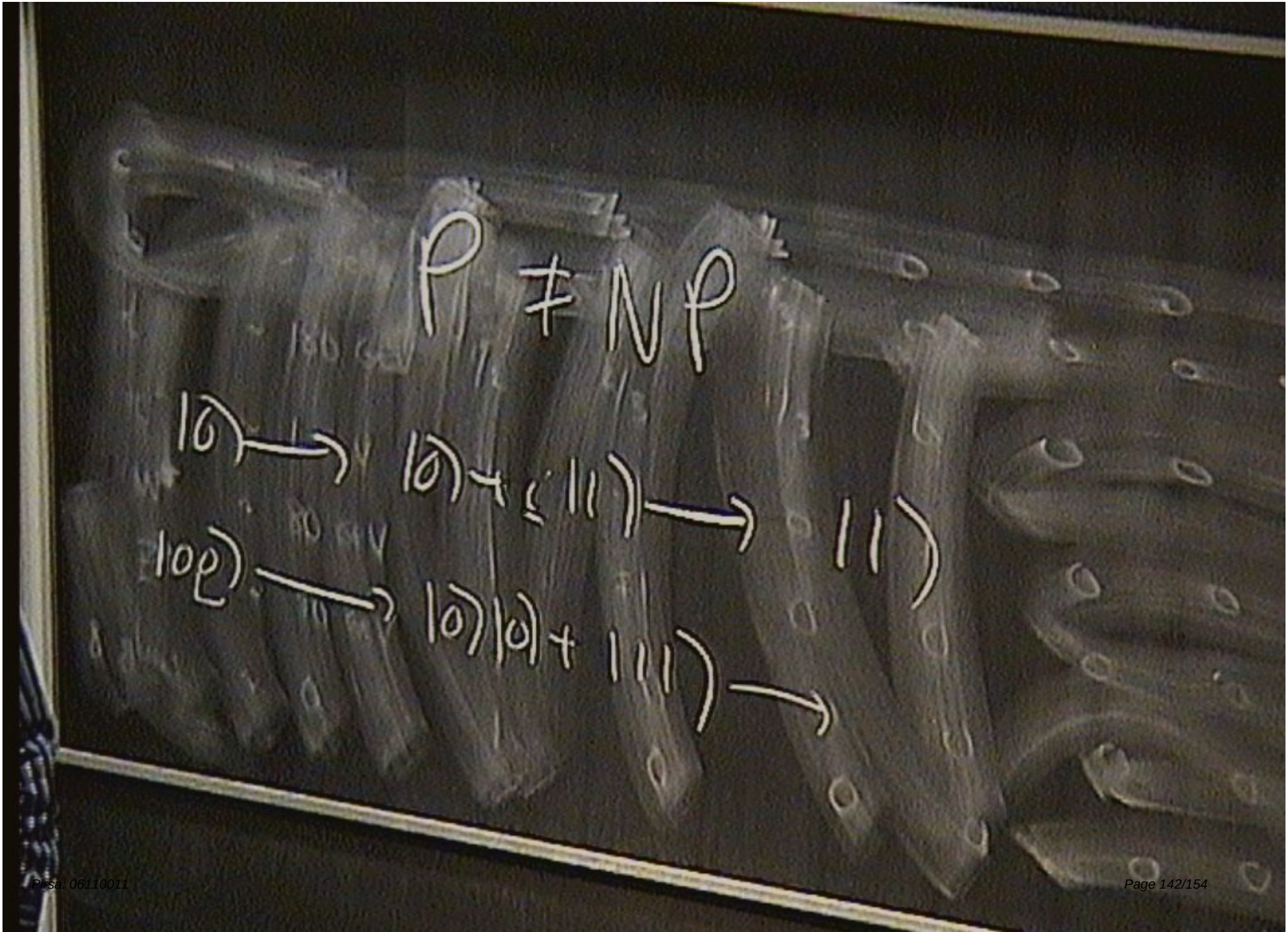
1 complex bit can be simulated by 2 real bits (see e.g. Rudolph and Grover quant-ph/0210187). But the two systems are not "equivalent" according to our notion of equivalence. E.g. inner products are not


P ≠ NP
|0⟩ → |0⟩ + |1⟩ → |1⟩
|0⟩ → |0⟩|0⟩ + |1⟩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|0⟩ → |0⟩+ε|1⟩ → |1⟩
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Precise statement of our main result

Let $T^1, T^2, \ldots, T^k \in U(2^n)$ *(acting on a constant number of qubits each)*

$x \in \{0,1\}^n, \varepsilon > 0, \gamma > 0$

If $\mathrm{CircuitTest}(T^1, T^2, \ldots, T^k, x, \varepsilon, \gamma)$ *accepts, then with probability* $1 - O(\gamma)$ *the outcome probability distribution of the circuit is at total variation distance* $O((k+n)\varepsilon^{1/8})$ *from the distribution that comes from the measurement of* $T^k T^{k-1} \ldots T^2 T^1 |x\rangle$ *in the computational basis.*

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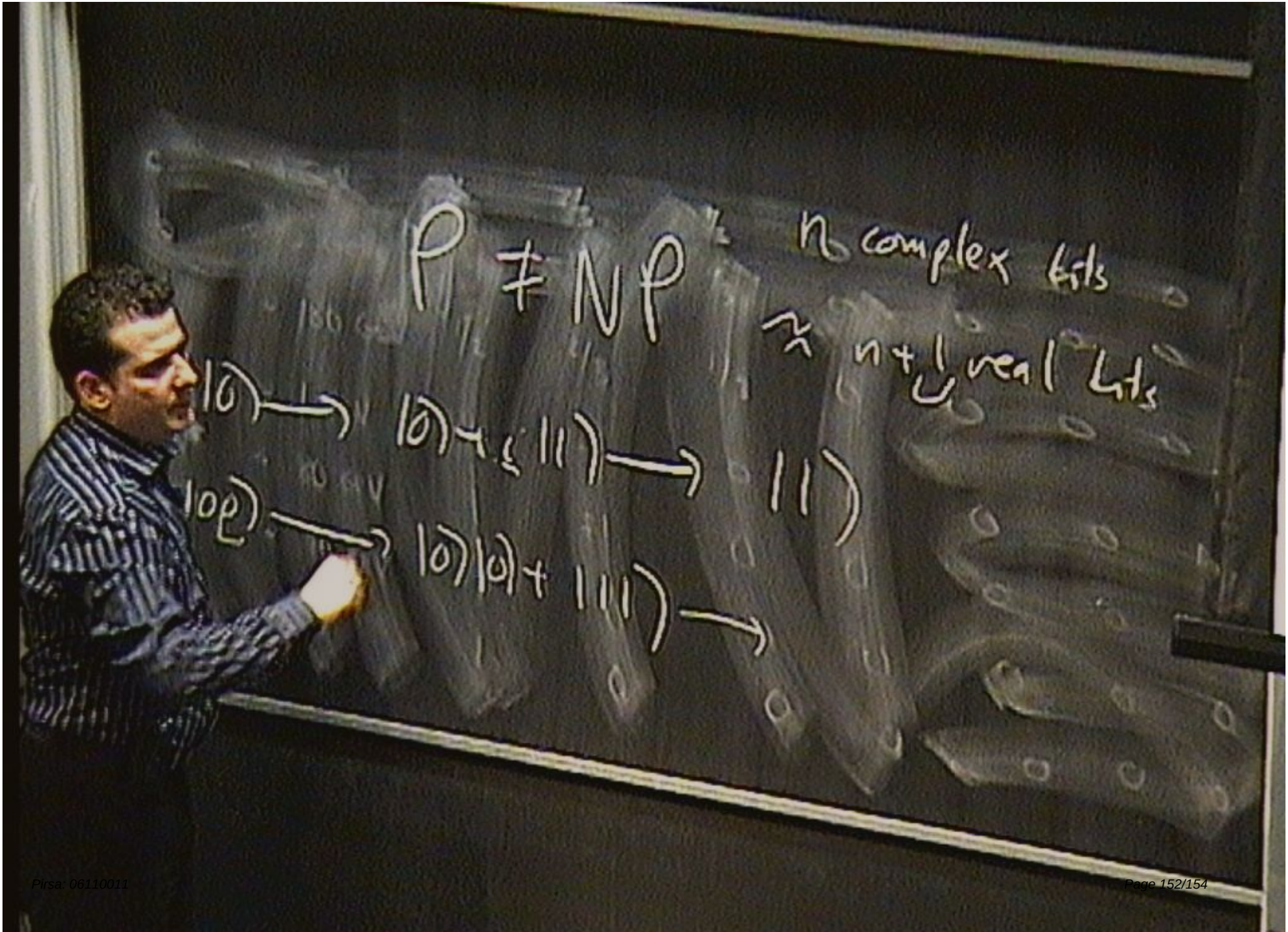
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- *Can we improve the asymptotics?*
- *Can we add some other reasonable and testable assumptions that allow prepare-and-send QKD?*
- *Can something be done about the complex case?*

The main assumptions of Mayers and Yao are

1) *Locality (i.e. measurements at A commute with those at B)*
2) *Repeatability of experiments*
3) *Trusted classical apparatus*

However, the results are not "robust". The results hold exactly if the statistics are satisfied exactly.

Any realistic application will need to be robust.

(Assuming robustness) This might be the only way, using only these assumptions, to verifiably securely