Title: Leading Log Solution for Yukawa During Inflation

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Abstract: TBA

Leading Log Solution for Yukawa during Inflation

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Bill Clinton Professor of String Theory

Spacetime Exp. Strengthens QFT

- Why?
  - Loops → classical physics of virtuals
  - Expansion → holds virtuals apart longer
- Maximum Effect for
  - Inflation
  - M=0
  - No conformal invariance (classically)
- Two Particles
  - MMC scalars
  - gravitons

Explicit Results

- MMC φ^4
  - with Onemli
    - gr/qc/0204065
    - gr-qc/0406098
  - with Brunier+Onemli
    - gr-qc/0408080
- SQED
  - Prokopec + Tornkvist
    - astro-ph/0205331
    - gr-qc/0205130
  - with Prokopec
    - astro-ph/0303358
    - gr-qc/0310056
  - with Kahya
    - gr-qc/0508015
- Yukawa
  - with Prokopec
    - astro-ph/0309593
  - with Duffy
    - hep-ph/0505156
- Quantum Gravity
  - with Tsamis
    - hep-ph/9602315-7
- QG + Dirac
  - with Miao
    - gr-qc/0511140
    - gr-qc/0603135

Infrared Logarithms

(1) What: factors of $\ln[a(t)]$ in loop corrections

- $ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x}$
- $a(t) = e^{Ht}$ for de Sitter

(2) Example: $\mathcal{L} = -\partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu} \sqrt{-g} - \frac{\lambda}{4!} \varphi^4 \sqrt{-g} +$ counterterms

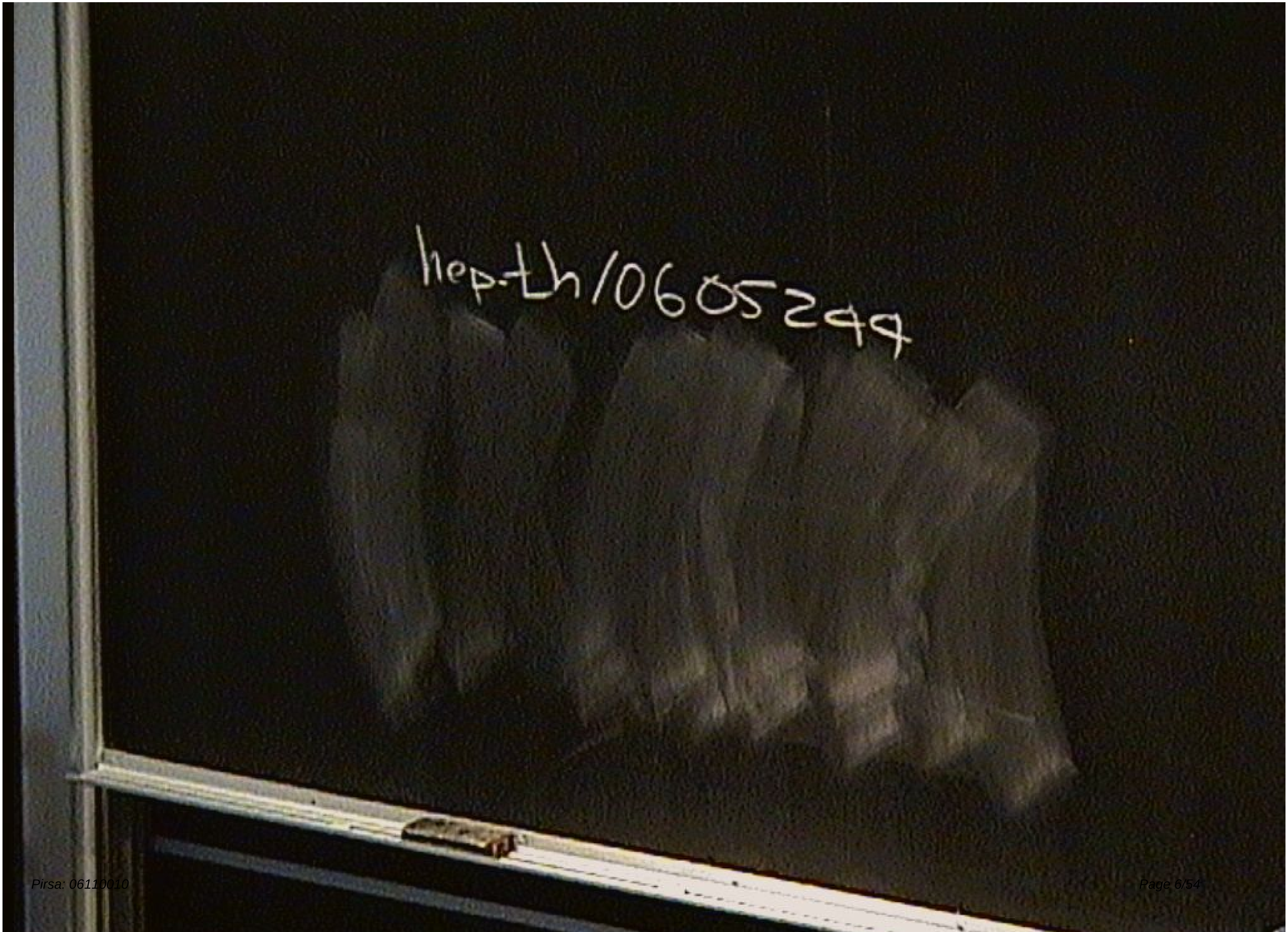
- $\langle \Omega | T_{\mu\nu} | \Omega \rangle = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$
- $\rho = \frac{\lambda H^4}{(2\pi)^4} \left\{ \frac{1}{8} \ln^2(a) \right\} + O(\lambda^2)$
- $p = \frac{\lambda H^4}{(2\pi)^4} \left\{ -\frac{1}{8} \ln^2(a) - \frac{1}{12} \ln(a) \right\} + O(\lambda^2)$

(3) Why: virtuals ripped from vacuum at $k \simeq Ha(t)$

- $\varphi\,,\ h_{\mu\nu} \simeq \sum_{k \lesssim Ha} \big(\text{virtuals}\big)$
- $\langle \Omega | \varphi^2(t, \vec{x}) | \Omega \rangle = \frac{H^2}{4\pi^2} \left\{ \text{UV} + \ln(a) \right\}$
  - ∗ Vilenkin and Ford, PRD26 (1982) 1231.
  - ∗ Linde, PLB116 (1982) 335.
  - ∗ Starobinskii, PLB117 (1982) 175.

(4) Note: may contaminate the **Power Spectrum**

- S. Weinberg, hep – th/0506236.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The Perturbative Conundrum

(1) General form of ρ/H^4

- φ^4: $\lambda \ln^2(a) + \lambda^2\Big[\ln^4(a) + \ln^3(a) + \ln^2(a)\Big] + \ldots + \lambda^{2\ell-2}\Big[\ln^{2\ell-2}(a) + \ldots + \ln^2(a)\Big] + \ldots$
- SQED: $e^2 \ln(a) + e^4\Big[\ln^2(a) + \ln(a)\Big] + \ldots + e^{2\ell-2}\Big[\ln^{\ell-1}(a) + \ldots + \ln(a)\Big] + \ldots$
- Yukawa: $f^2 \ln(a) + f^4\Big[\ln^2(a) + \ln(a)\Big] + \ldots + f^{2\ell-2}\Big[\ln^{\ell-1}(a) + \ldots + \ln(a)\Big] + \ldots$

(2) More IR logs per loop in φ^4

(3) Perturbation theory breaks down at

- $\ln(a) \simeq \dfrac{1}{\sqrt{\lambda}}$
- $\ln(a) \simeq \dfrac{1}{e^2}$
- $\ln(a) \simeq \dfrac{1}{f^2}$

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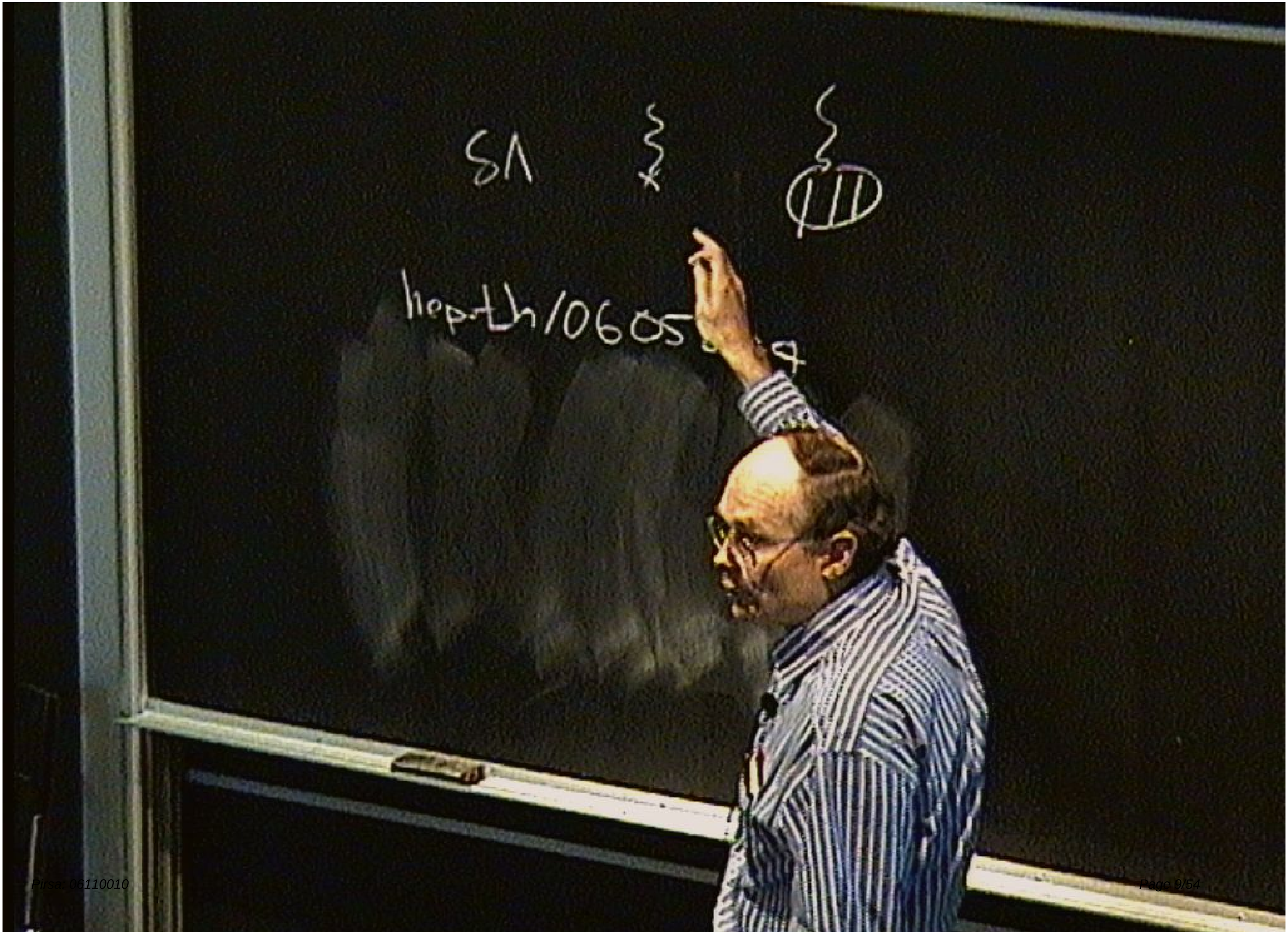

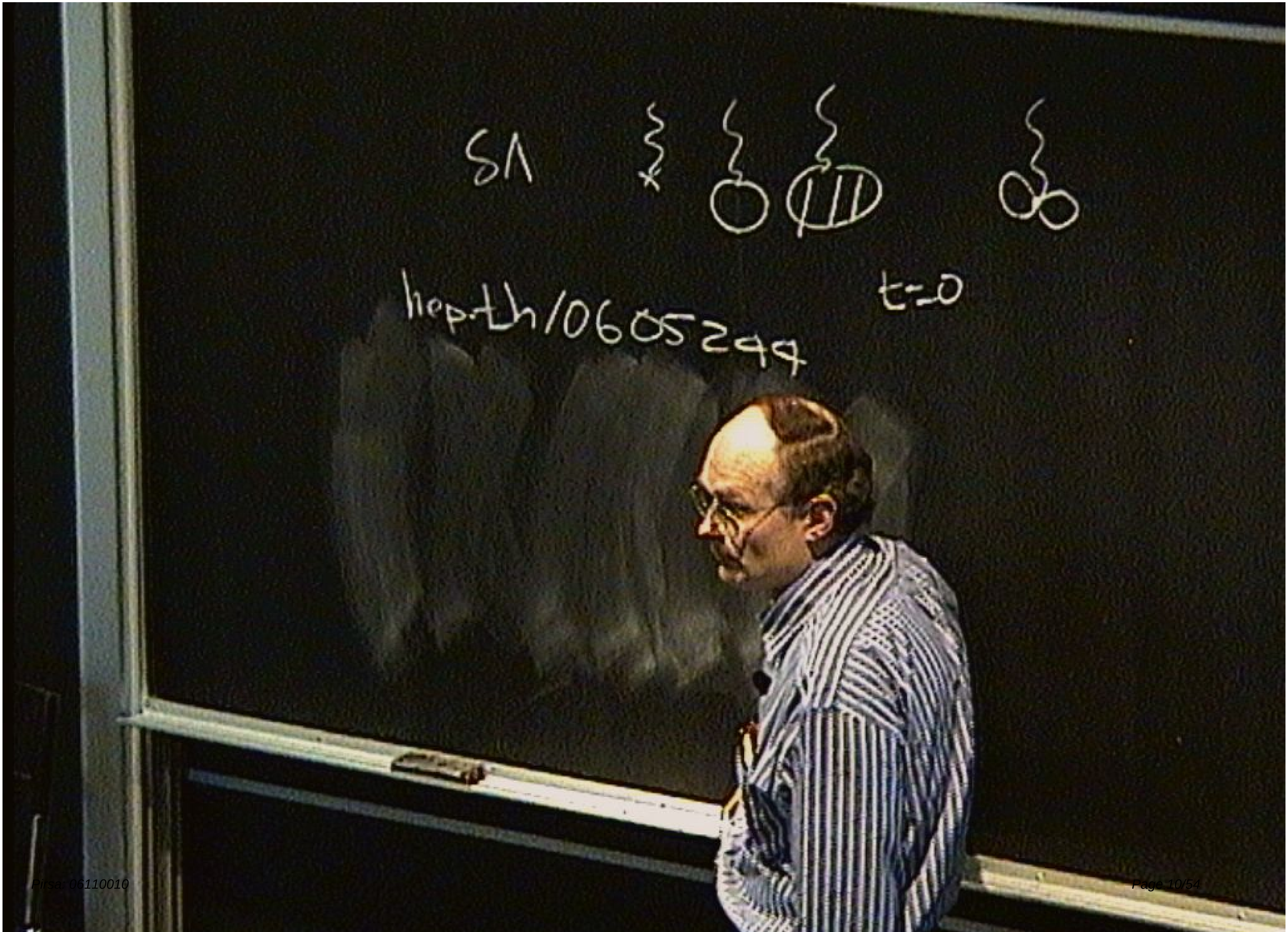
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Leading Log Approximation

(1) $\frac{\rho}{H^4} \sim \lambda \ln^2(a) + \ldots + \lambda^{2\ell-2}\Big[\ln^{2\ell-2}(a) + \ldots + \ln^2(a)\Big] + \ldots$

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- $\Big(\frac{\rho}{H^4}\Big)_{\rm leading} = \sum_{\ell=2}^{\infty} a_\ell \Big(\lambda \ln^2(a)\Big)^{\ell-1}$

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Starobinskii's Rules (astro-ph/9407016)

(1) Simplify: $\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2}{a^2}\varphi + \frac{\lambda}{6}\varphi^3 = 0$

$\Longrightarrow 3H(\dot{\varphi} - f) + \frac{\lambda}{6}\varphi^3 = 0$

- $f(t,\vec{x}) \equiv \int \frac{d^3k}{(2\pi)^3}\delta(k - Ha)\frac{H^2}{\sqrt{2k}}\Big\{e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\Big\}$

(2) Langevin Eqn: $\dot{\varphi} = f - \frac{\lambda}{18H}\varphi^3$

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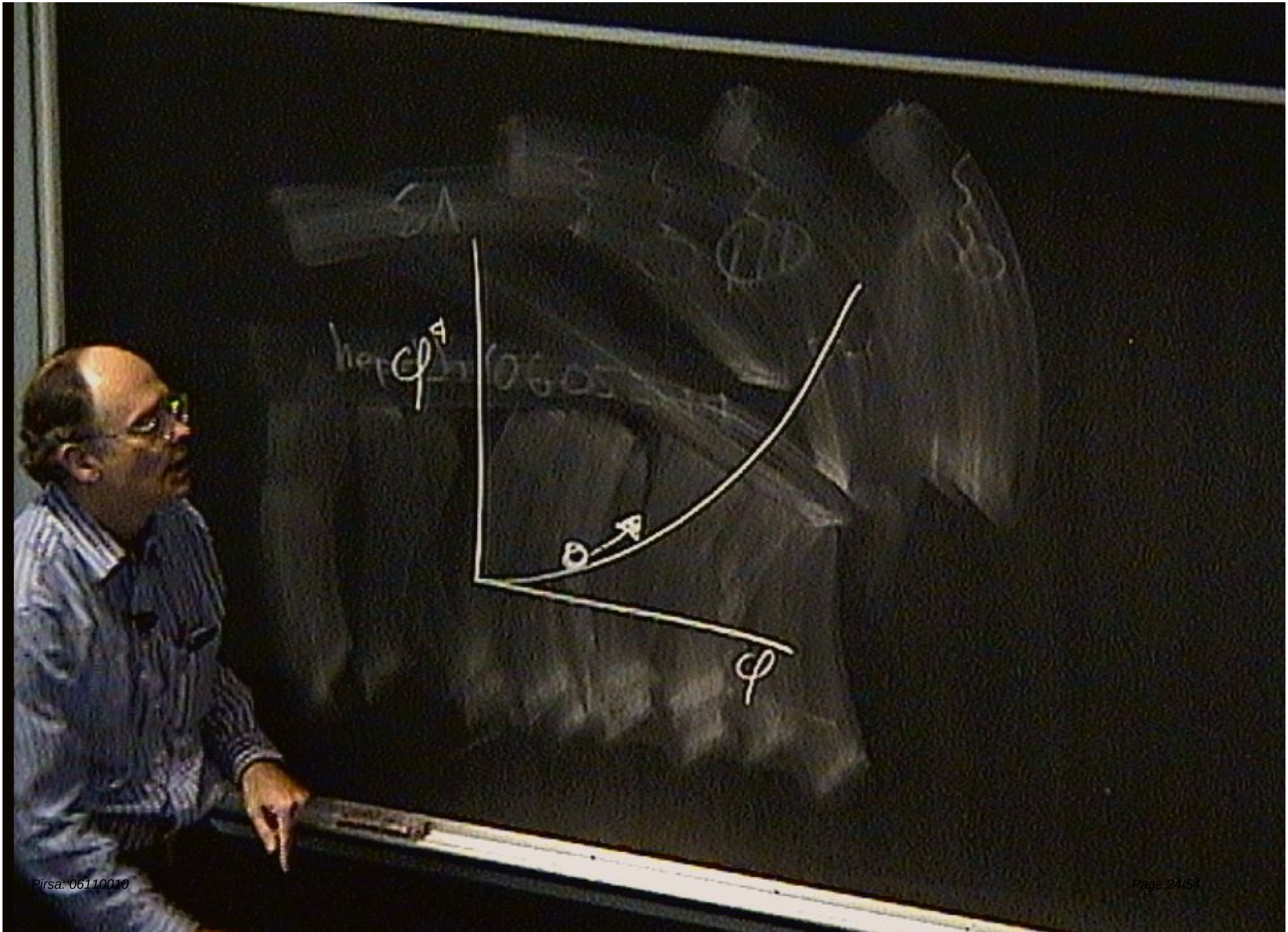
Non-Perturbative Solution

- Physics of competition
  - Inflationary particle production pushes φ up
  - Classical force pushes it back down
- Expect a static limit for ρ(t,φ)

$$\dot{\rho} \rightarrow 0 = \frac{\partial^2}{\partial \varphi^2}\Big[\frac{H^2}{8\pi^2}\rho_\infty\Big] + \frac{\partial}{\partial \varphi}\Big[\frac{\lambda}{18H}\varphi^3 \rho_\infty\Big]$$

- $\frac{\partial}{\partial \varphi}\rho_\infty(\varphi) = -\frac{4\pi^2\lambda}{9H}\Big(\frac{\varphi}{H}\Big)^3 \rho_\infty(\varphi)$
- $\rho_\infty(\varphi) = \frac{2}{\Gamma(\frac{1}{4})}\Big(\frac{\pi^2\lambda}{9H^4}\Big)^{\frac{1}{4}} \exp\Big[-\frac{\pi^2\lambda}{9}\Big(\frac{\varphi}{H}\Big)^4\Big]$
- $\lim_{t\rightarrow\infty}\Big\langle\Omega\Big|\varphi^{2n}(t,\vec{x})\Big|\Omega\Big\rangle = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{1}{4})}\Big(\frac{9H^4}{\pi^2\lambda}\Big)^{\frac{n}{2}}$



Starobinskii's Rules (astro-ph/9407016)

(1) Simplify: $\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2}{a^2}\varphi + \frac{\lambda}{6}\varphi^3 = 0$

$\Longrightarrow 3H(\dot{\varphi} - f) + \frac{\lambda}{6}\varphi^3 = 0$

- $f(t,\vec{x}) \equiv \int \frac{d^3k}{(2\pi)^3}\delta(k - Ha)\frac{H^2}{\sqrt{2k}}\left\{e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\right\}$

(2) Langevin Eqn: $\dot{\varphi} = f - \frac{\lambda}{18H}\varphi^3$

- $\varphi(t,\vec{x})$ a stochastic field driven by noise $f(t,\vec{x})$
- $\left\langle\Omega\middle|f(t,\vec{x})f(t',\vec{x})\middle|\Omega\right\rangle = \frac{H^3}{4\pi^2}\delta(t-t')$

(3) Fokker–Planck Eqn:

- $\left\langle\Omega\middle|F\big(\varphi(t,\vec{x})\big)\middle|\Omega\right\rangle = \int d\varphi\,\rho(t,\varphi)F(\varphi)$
- $\dot{\rho} = \frac{1}{2}\frac{\partial^2}{\partial\varphi^2}\left[\frac{H^3}{4\pi^2}\rho\right] - \frac{\partial}{\partial\varphi}\left[-\frac{\lambda}{18H}\varphi^3\rho\right]$

(4) Reproduces leading IR logs

- but why?
- what about the uncertainty principle?
- where did $f(t,\vec{x})$ come from?

Non-Perturbative Solution

- Physics of competition
  - Inflationary particle production pushes φ up
  - Classical force pushes it back down
- Expect a static limit for ρ(t,φ)

$$\dot{\rho} \to 0 = \frac{\partial^2}{\partial \varphi^2}\Big[\frac{H^2}{8\pi^2}\rho_\infty\Big] + \frac{\partial}{\partial \varphi}\Big[\frac{\lambda}{18H}\varphi^3 \rho_\infty\Big]$$

- $\frac{\partial}{\partial \varphi}\rho_\infty(\varphi) = -\frac{4\pi^2\lambda}{9H}\Big(\frac{\varphi}{H}\Big)^3 \rho_\infty(\varphi)$
- $\rho_\infty(\varphi) = \frac{2}{\Gamma(\frac{1}{4})}\Big(\frac{\pi^2\lambda}{9H^4}\Big)^{\frac{1}{4}} \exp\Big[-\frac{\pi^2\lambda}{9}\Big(\frac{\varphi}{H}\Big)^4\Big]$
- $\lim_{t\to\infty}\Big\langle \Omega\Big|\varphi^{2n}(t,\vec{x})\Big|\Omega\Big\rangle = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{1}{4})}\Big(\frac{9H^4}{\pi^2\lambda}\Big)^{\frac{n}{2}}$

More General Theories

(1) Starobinskii's technique works for

- $\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$

(2) Other models with IR logs

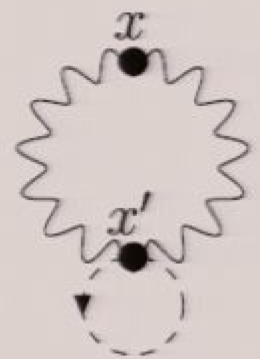
- $\mathcal{L}_{\rm SQED} = -\frac{1}{4}F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g} - \Big(\partial_\mu - ieA_\mu\Big)\varphi^*\Big(\partial_\nu + ieA_\nu\Big)\varphi g^{\mu\nu}\sqrt{-g}$
  $-\delta\xi\varphi^*\varphi\sqrt{-g} - \frac{1}{4}\delta\lambda(\varphi^*\varphi)^2\sqrt{-g}$
- $\mathcal{L}_{\rm Yukawa} = -\frac{1}{2}\partial_\alpha\varphi\partial_\beta\varphi g^{\alpha\beta}\sqrt{-g} - \frac{1}{2}\delta\xi\varphi^2 R\sqrt{-g} - \frac{1}{4!}\delta\lambda\varphi^4\sqrt{-g}$
  $+i\overline{\psi}e^\beta{}_b\gamma^b\mathcal{D}_\beta\psi\sqrt{-g} - f\varphi\overline{\psi}\psi\sqrt{-g}$
- $\mathcal{L}_{\rm QG} = \frac{1}{16\pi G}\Big(R - 2\Lambda\Big)\sqrt{-g}$ + BPHZ counterterms
- $\mathcal{L}_{\rm QG+Dirac} = \frac{1}{16\pi G}\Big(R - 2\Lambda\Big)\sqrt{-g} + i\overline{\psi}e^\beta{}_b\gamma^b\mathcal{D}_\beta\psi\sqrt{-g}$ + BPHZ counterterms

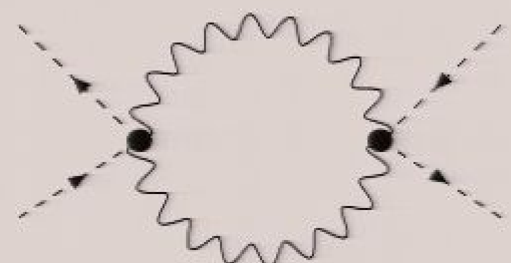
(3) Two kinds of fields

- Active ($\varphi \quad h^{tt}_{ij}$) $\Longrightarrow$ cause IR logs
- Passive ($\psi \quad A_\mu$) $\Longrightarrow$ don't cause IRlogs

Cannot Ignore Passives

- Passives can carry IR logs
  - Eg. < F(x) F(x) >
- Actives interact through passives
  - Eg. Quartic coupling





Cannot Ignore the UV

- IR (H<k<Ha) vs. UV (Ha<k)
  - IR logs only from IR of actives
  - Effective ints from IR+UV of passives
- What to do?
  - Integrate out passive
    - Complicated Effective Action!
  - IR truncate and simplify
    - Reduces to Effective Potential

Yukawa (with Miao, gr-qc/0602110)

(1) $\mathcal{L} = -\frac{1}{2}\partial_\alpha\varphi\partial_\beta\varphi g^{\alpha\beta}\sqrt{-g} - \frac{1}{2}\delta\xi\varphi^2 R\sqrt{-g} - \frac{1}{4!}\delta\lambda\varphi^4\sqrt{-g}$

$$+i\overline{\psi}e^{\beta}{}_{b}\gamma^{b}\mathcal{D}_\beta\psi\sqrt{-g} - f\varphi\overline{\psi}\psi\sqrt{-g}$$

- Active: φ
- Passive: ψ

(2) $e^{i\Gamma[\varphi]} \equiv \int [d\overline{\psi}][d\psi]\, e^{iS[\varphi,\overline{\psi},\psi]} = e^{iS_0[\varphi]}\,\det\Big[\sqrt{-g}\big(i\not{\mathcal{D}} - f\varphi\big)\Big]$

(3) $\frac{\delta\Gamma[\varphi]}{\delta\varphi(x)} = \partial_\mu\Big(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi(x)\Big) - \delta\xi\varphi(x)R\sqrt{-g} - \frac{\delta\lambda}{6}\varphi^3(x)\sqrt{-g}$

$$-\mathrm{Tr}\left[\frac{i}{\sqrt{-g}(i\not{\mathcal{D}} - f\varphi)}\frac{\delta}{\delta\varphi(x)}\sqrt{-g}\big(i\not{\mathcal{D}} - f\varphi\big)\right]$$

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Evaluate for φ(x) = const

- At L.L.O. each φ MUST contr. to a ln(a)
  - So forget about space dependence
- Because passives are passive . . .
  - Green's functions give pos. powers of a'/a

Eg. $G_{\rm cf}(x;x') = \frac{H^2}{4\pi^2}\theta(\Delta t)\frac{\delta(H\Delta x + \frac{1}{a} - \frac{1}{a'})}{aa'H\Delta x}$

$$\bullet \int_0^t dt' a'^3 \int d^3x' G_{\rm cf}(x;x') \times \ln^N(a') = \frac{1}{H}\int_0^t dt' \Big[\frac{a'}{a} - \Big(\frac{a'}{a}\Big)^2\Big] \ln^N(a')$$

$$= \frac{1}{2H^2}\Big\{\ln^N(a) + \ldots\Big\}$$

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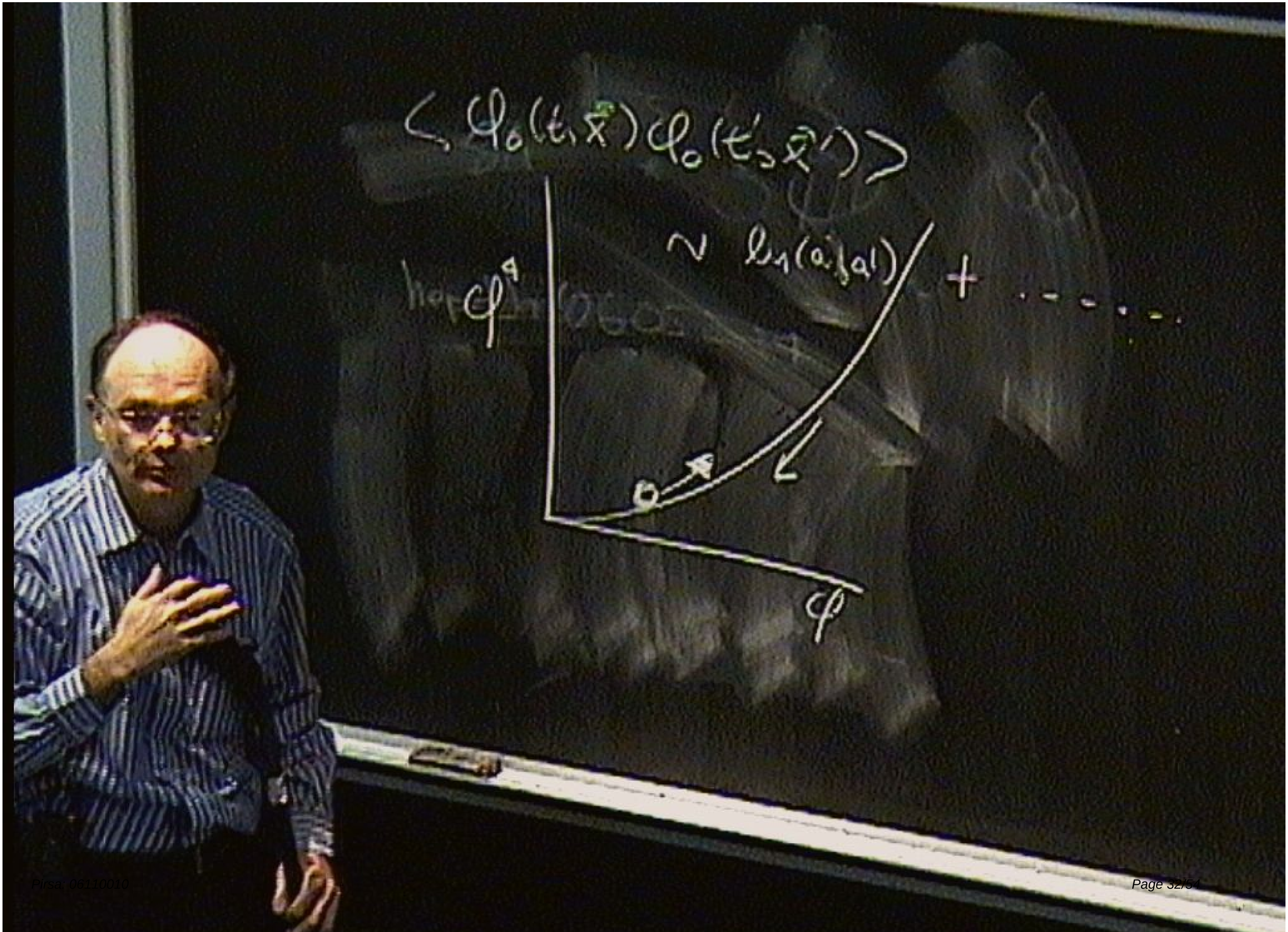

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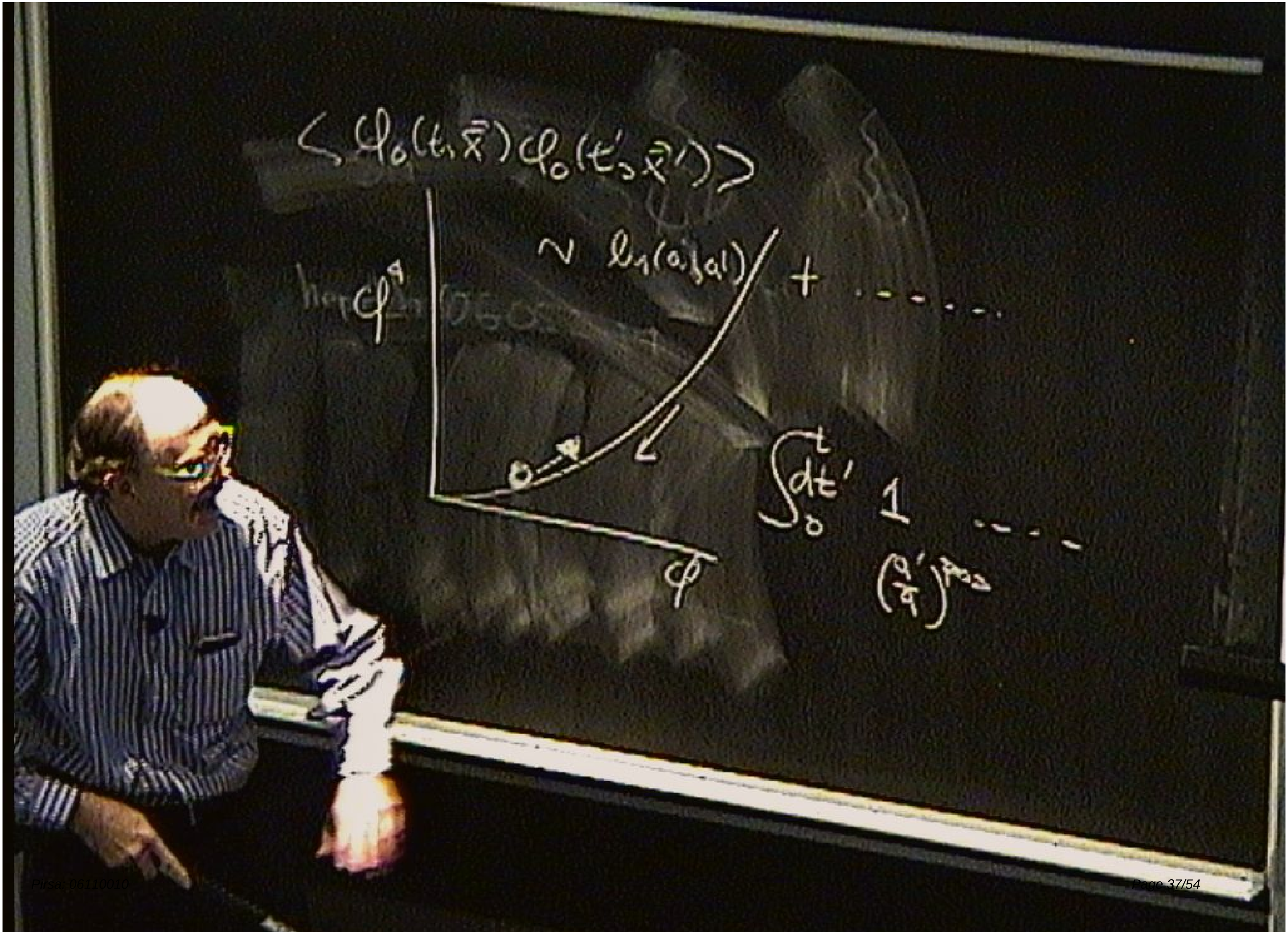
⟨φ₀(t, x̄) φ₀(t', x̄')⟩
~ ln(a/|a'|)
+ ......
∫₀ᵗ dt' 1
φ

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Candelas-Raine, PRD12 (1975) 965

(1) We just need coincidence limits

(2) $K \equiv \frac{mH^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\| \frac{\Gamma(\frac{D}{2} + \frac{im}{H})}{\Gamma(1 + \frac{im}{H})} \right\|^2 \Gamma\Big(1 - \frac{D}{2}\Big)$

- $\Big\langle x \Big| \frac{i}{\sqrt{-g}(i\not{\mathcal{D}} - f\varphi)} \Big| x' \Big\rangle \quad \longrightarrow K \times I$
- $\mathcal{D}_\mu \Big\langle x \Big| \frac{i}{\sqrt{-g}(i\not{\mathcal{D}} - f\varphi)} \Big| x' \Big\rangle \longrightarrow K \times \frac{i}{D} ma\gamma_\mu$
- $\Big\langle x \Big| \frac{i}{\sqrt{-g}(i\not{\mathcal{D}} - f\varphi)} \Big| x' \Big\rangle \overleftarrow{\mathcal{D}}'_\mu \longrightarrow K \times -\frac{i}{D} ma\gamma_\mu$

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Leading Log Potential

(1) After renormalization

- $V(\varphi) = -\frac{H^4}{8\pi^2}\Big\{2\gamma\Big(\frac{f\varphi}{H}\Big)^2 - \big[\zeta(3)-\gamma\big]\Big(\frac{f\varphi}{H}\Big)^4 + 2\int_0^{\frac{f\varphi}{H}} dx (x+x^3)\big[\psi(1+ix)+\psi(1-ix)\big]\Big\}$

(2) Small φ

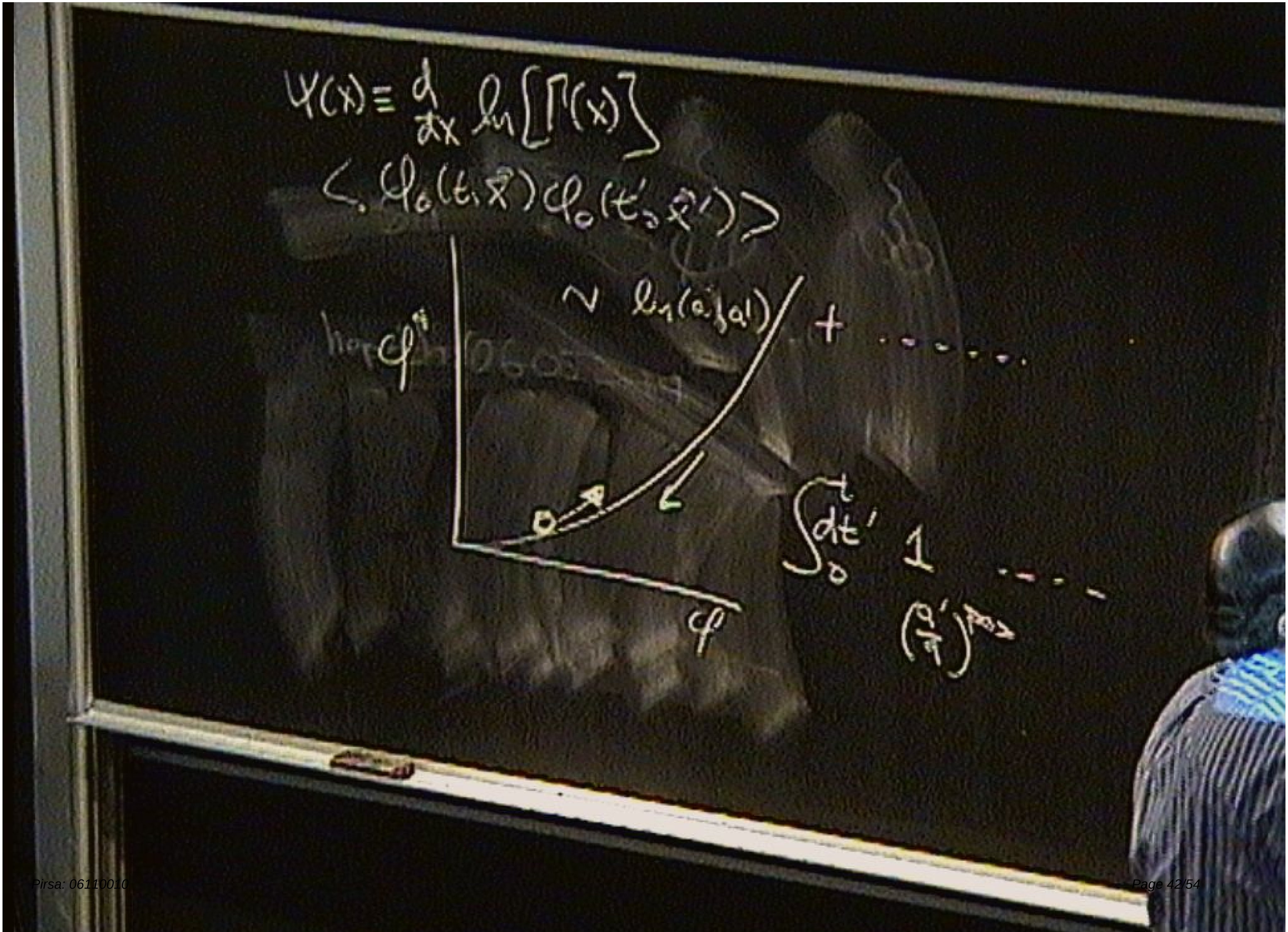
- $V(\varphi) = -\frac{H^4}{8\pi^2}\Big\{\frac{2}{3}\big[\zeta(3)-\zeta(5)\big]\Big(\frac{f\varphi}{H}\Big)^6 + \dots\Big\}$

(3) Large φ

- $V(\varphi) = -\frac{H^4}{8\pi^2}\Big\{\Big(\frac{f\varphi}{H}\Big)^4 \ln\Big(\frac{f|\varphi|}{H}\Big) + \dots\Big\}$
- Cf. Eqn (6.10) of Coleman – Weinberg

(4) Physics

- Inflationary particle production increases φ
- φ increases fermion mass
- Increasing fermion mass makes vacuum energy more negative


ψ(x) ≡ d/dx ln [Γ(x)]
<0| φ₀(t, x⃗) φ₀(t', x⃗') >
~ ln(a/a') + ...
φ
∫₀ᵗ dt' 1

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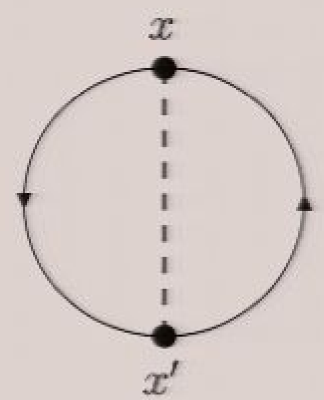
Reality Check

$$\left\langle \Omega \middle| T\left[\varphi(x)\overline{\psi}(x)\psi(x)\right] \middle| \Omega \right\rangle$$
$$= \text{Constant} + \frac{fH^4}{8\pi^2}\frac{\ln(a)}{4-D} + O(f^3)$$



$$\int [d\overline{\psi}][d\psi] e^{iS[\varphi,\overline{\psi},\psi]}\varphi(x)\overline{\psi}(x)\psi(x) = \varphi(x) \times e^{i\Gamma[\varphi]}\,\mathrm{Tr}\left[-\left\langle x \middle| \frac{i}{\sqrt{-g}(i\not{D} - f\varphi)} \middle| x \right\rangle\right]$$
$$\longrightarrow -4f\varphi^2 \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\| \frac{\Gamma(\frac{D}{2} + \frac{im}{H})}{\Gamma(1 + \frac{im}{H})} \right\|^2 \Gamma\Big(1 - \frac{D}{2}\Big) e^{i\Gamma[\varphi]}$$
$$= \left\{ -\frac{4fH^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\Big(1 - \frac{D}{2}\Big)\Gamma^2\Big(\frac{D}{2}\Big)\varphi^2 + O(f^3) \right\} e^{i\Gamma[\varphi]}$$

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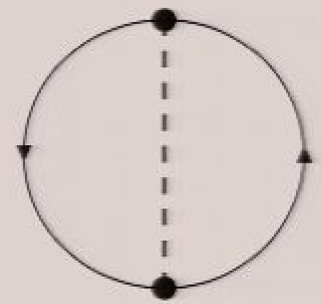

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Gravitational Response

$$T_{\mu\nu} = -\frac{i}{2}\Big[\overline{\psi}e_{(\mu b}\gamma^b\mathcal{D}_{\nu)}\psi - \overline{\psi}\,\overleftarrow{\mathcal{D}}_{(\mu}e_{\nu)b}\gamma^b\psi\Big] + \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi$$

$$+\delta\xi\Big[(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)\varphi^2 + g_{\mu\nu}(\varphi^2)^{;\rho}{}_{\rho} - (\varphi^2)_{;\mu\nu}\Big] - \frac{\delta\lambda}{4!}\varphi^4 g_{\mu\nu}$$

$$\int[d\overline{\psi}][d\psi]e^{iS[\varphi,\overline{\psi},\psi]}T_{\mu\nu} \longrightarrow -g_{\mu\nu}V_s(\varphi)e^{i\Gamma[\varphi]}$$

- $V_s(\varphi) = \frac{H^4}{8\pi^2}\Big\{\Big[\frac{1}{2} - \gamma\Big]\Big(\frac{f\varphi}{H}\Big)^2 + \Big[\frac{1}{4} - \gamma + \zeta(3)\Big]\Big(\frac{f\varphi}{H}\Big)^4 - \frac{1}{2}\Big[\Big(\frac{f\varphi}{H}\Big)^2 + \Big(\frac{f\varphi}{H}\Big)^4\Big]\Big[\psi\Big(1 + \frac{if\varphi}{H}\Big) + \psi\Big(1 - \frac{if\varphi}{H}\Big)\Big]\Big\}$

V_s(φ) ≠ V(φ)

(1) Small φ

$$\bullet V_s(\varphi) = +\frac{H^4}{16\pi^2}\Big(\frac{f\varphi}{H}\Big)^2 + \ldots$$

$$\bullet V(\varphi) = -\frac{H^4}{12\pi^2}\big[\zeta(3) - \zeta(5)\big]\Big(\frac{f\varphi}{H}\Big)^6 + \ldots$$

(2) Large φ

$$\bullet V_s(\varphi) = -\frac{H^4}{8\pi^2}\Big\{\Big(\frac{f\varphi}{H}\Big)^4 \ln\Big(\frac{f|\varphi|}{H}\Big) + \ldots\Big\} \longrightarrow V(\varphi)$$

- Why don't they agree?
  - V(φ) + H^4 F(fφ/H) and H^2 → R/12 generally
  - Dependence upon R has gravitational consequences
- Which is right?
  - V(φ) controls evolution of φ(x)
  - V_s(φ) controls gravitational response

SQED (with Prokopec and Tsamis)

(1) $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} - \left(\partial_\mu - ieA_\mu\right)\varphi^* \left(\partial_\nu + ieA_\nu\right)\varphi g^{\mu\nu} \sqrt{-g}$
$-\delta\xi \varphi^* \varphi \sqrt{-g} - \frac{1}{4}\delta\lambda (\varphi^*\varphi)^2 \sqrt{-g}$

- Active: φ
- Passive: A_μ

(2) $\mathcal{L}_{\rm stoch} = -\partial_\mu \varphi^* \partial_\nu \varphi g^{\mu\nu} \sqrt{-g} - V(\varphi^*\varphi)\sqrt{-g}$

- Large φ $\quad V = \frac{3H^4}{16\pi^2} \left(\frac{e^2 \varphi^* \varphi}{H^2}\right)^2 \ln\left(\frac{e^2 \varphi^* \varphi}{H^2}\right) + \dots$
- Cf. eqn (4.5) of Coleman – Weinberg

(3) Explicit Leading Log Results for

- $\left\langle \Omega \middle| F_{\mu\nu}(x) F_{\rho\sigma}(x) \middle| \Omega \right\rangle$
- $\left\langle \Omega \middle| (D_\mu \varphi)^* D_\nu \varphi \middle| \Omega \right\rangle$
- $\left\langle \Omega \middle| \varphi^*(x) \varphi(x) \middle| \Omega \right\rangle$

Conclusions

- Infl. + m=0 + no conf. → enhanced QFT
  - MMC scalars and gravitons
  - Effects manifest as factors of ln[a(t)]
- Starobinskii's stochastic formalism
  - Gives leading IR logs for $V(\varphi)$ models
  - Can be summed for $V(\varphi)$ bounded below
  - UV doesn't matter
- General models have passive fields
  - Eg ψ in Yukawa
  - Don't cause IR logs but can carry them
  - Also mediate interactions between actives

Conclusions II

- Integrate out passives, then IR truncate
  - Same as computing effective potential!
- Done for Yukawa and SQED
  - $V(\varphi)$ unbounded below for Yukawa!
    - IR logs DO NOT always sum to a constant
  - SQED φ reaches ~ H/e!
    - Non-pert. confirmation of Davis, Dimopoulos, Prokopec and Tornkvist (PLB501 (2001) 165)
- Next up: Quantum Gravity

$V_s(\varphi) \neq V(\varphi)$

(1) Small φ

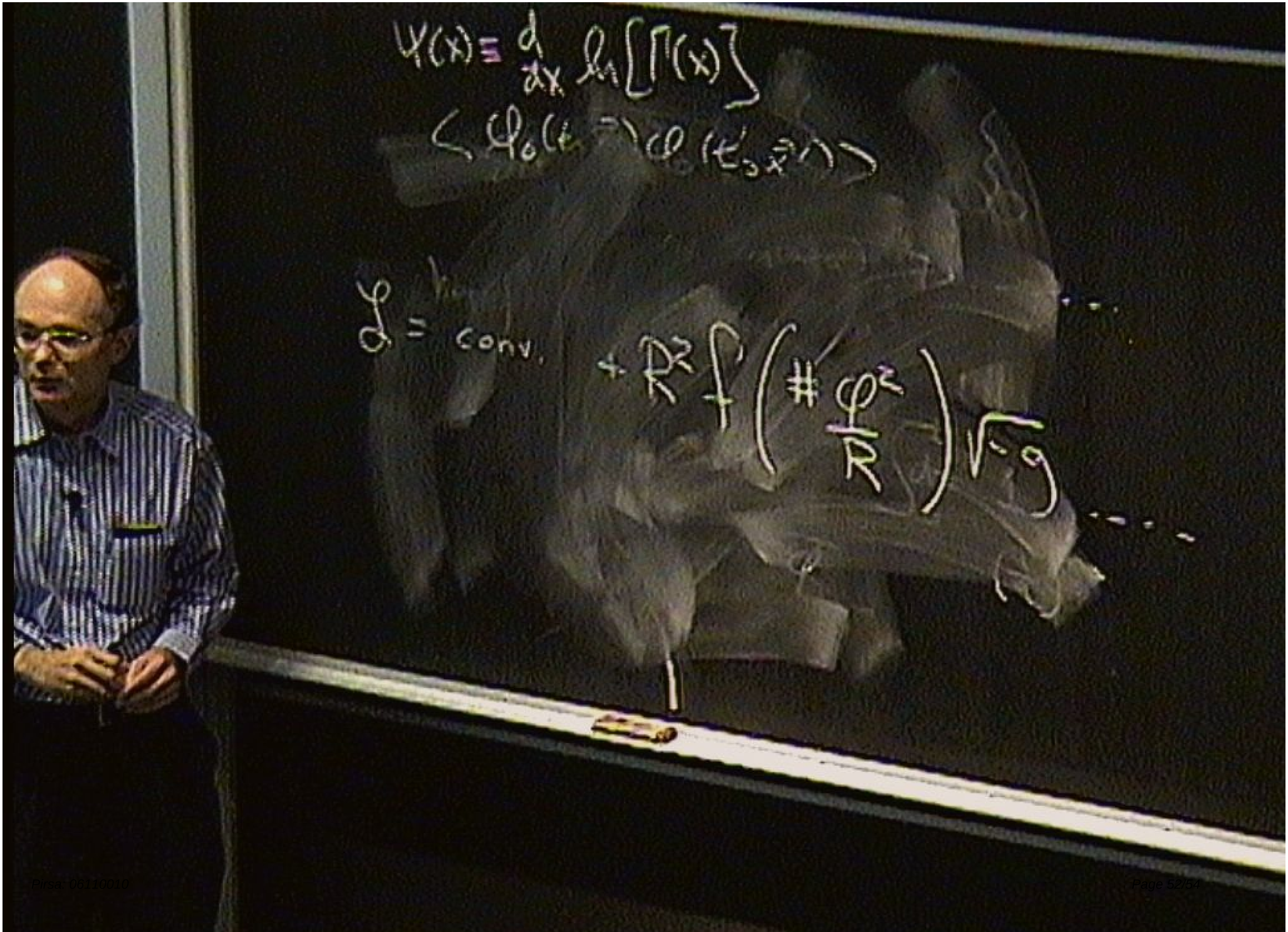
$$\bullet V_s(\varphi) = +\frac{H^4}{16\pi^2}\Big(\frac{f\varphi}{H}\Big)^2 + \dots$$

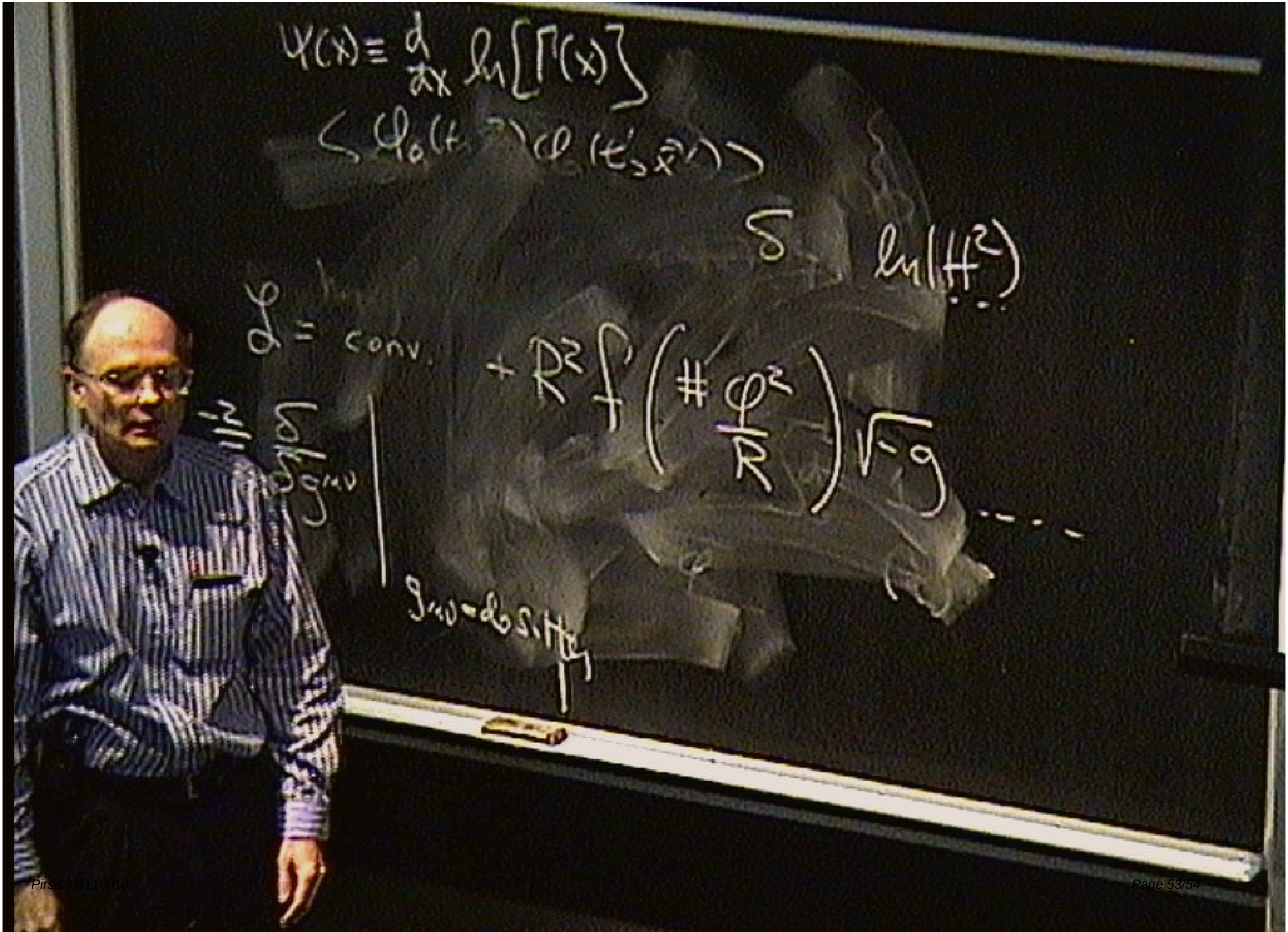
$$\bullet V(\varphi) = -\frac{H^4}{12\pi^2}\big[\zeta(3) - \zeta(5)\big]\Big(\frac{f\varphi}{H}\Big)^6 + \dots$$

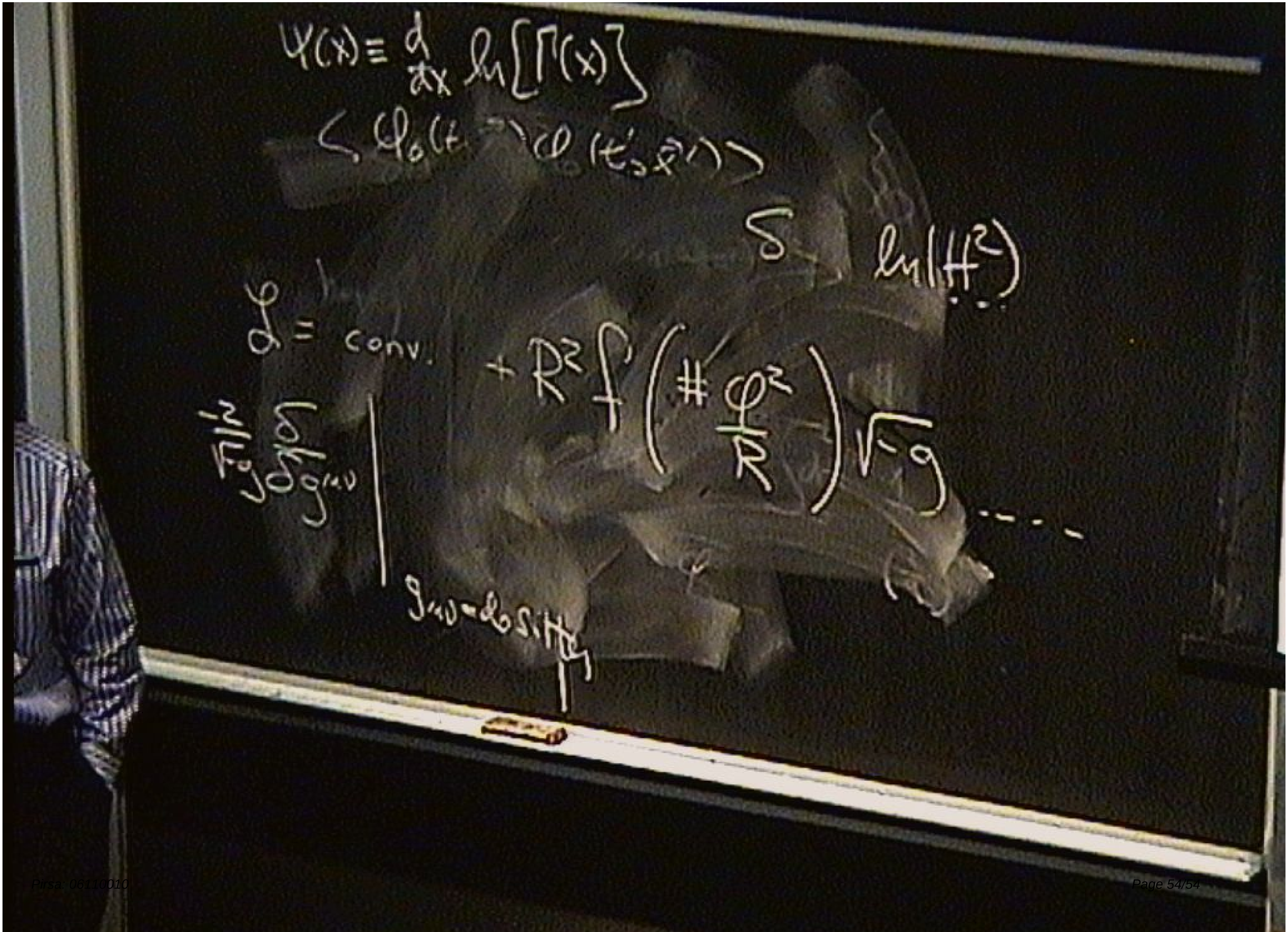
(2) Large φ

$$\bullet V_s(\varphi) = -\frac{H^4}{8\pi^2}\Big\{\Big(\frac{f\varphi}{H}\Big)^4 \ln\Big(\frac{f|\varphi|}{H}\Big) + \dots\Big\} \longrightarrow V(\varphi)$$

- Why don't they agree?
  - V(φ) + H^4 F(fφ/H) and H^2 → R/12 generally
  - Dependence upon R has gravitational consequences
- Which is right?
  - V(φ) controls evolution of φ(x)
  - V_s(φ) controls gravitational response


Ψ(x) = d/dx ln[Γ(x)]
ℒ = conv.
+ R² f(# φ²/R) √-g


ψ(x) ≡ d/dx ln[Γ(x)]
ln(H²)
ℒ = conv. + R² f(# φ²/R) √-g


ψ(x) ≡ d/dx ln[Γ(x)]
ln(H²)
L = conv. + R² f(# φ²/R) √-g ----
g_μν = de Sitter