

Title: How can AdS/CFT be useful for heavy-ion physics?

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Abstract: TBA

Can AdS/CFT be useful for heavy-ion physics?

Pavel Kovtun

KITP, University of California, Santa Barbara

A review of many people's work

seminar at Perimeter Institute

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What is heavy-ion physics?

Heavy-ion collisions — study QCD at high **energy density**



Experiment: **RHIC** (Brookhaven, NY)

- Started in year 2000
- Collides *Au* nuclei
- CM energy $\sqrt{s}=200$ GeV per nucleon

Quest: find and study QGP [deconfined state of QCD] \rightarrow field theory at finite temperature and density. **Not obvious** *a priori* that a thermal state will be produced.

Evidence for thermalization [lots of data and non-trivial calculations] :

Particle abundances and ratios — reproduced by statistical models

Elliptic flow — reproduced by hydrodynamic models

Jet quenching — indication of short mean free path

Optimistically, QGP is hidden in the collision

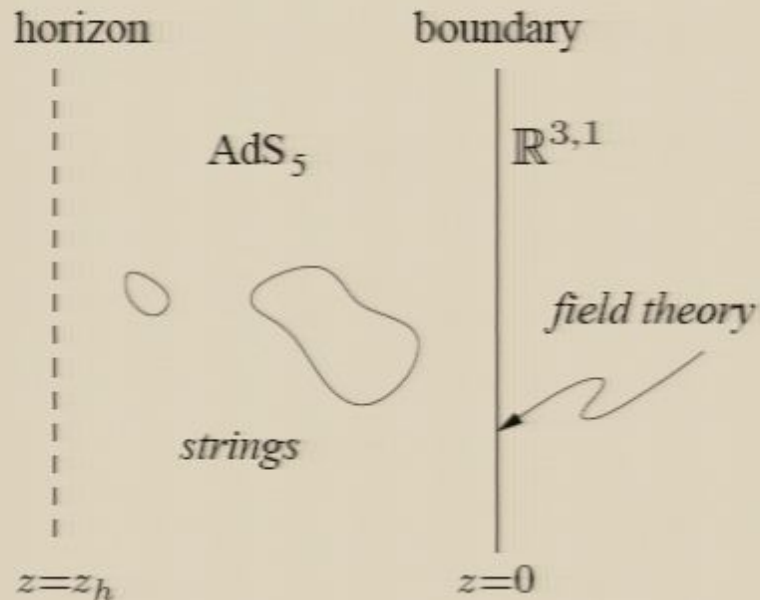
What is AdS/CFT

(J.Maldacena [hep-th/9711200](https://arxiv.org/abs/hep-th/9711200), review: [hep-th/9905111](https://arxiv.org/abs/hep-th/9905111))

large N_c , $d=4$, $\mathcal{N}=4$ SYM = IIB strings on $AdS_5 \times S^5$

$\lambda \leftrightarrow \left(\frac{R^2}{\alpha'}\right)^2$ string corrections to SUGRA

$\frac{\lambda}{4\pi N_c} \leftrightarrow g_s$ string loops



$$\langle e^{\int h(x)T(x)} \rangle_{\text{field}} = Z_{\text{string}}[g(x, z \rightarrow 0) = h(x)]$$

$$T_{\mu\nu}(x) \leftrightarrow h_{\mu\nu}(x, z \rightarrow 0)$$

$$J_\mu(x) \leftrightarrow A_\mu(x, z \rightarrow 0)$$

$$\text{tr}F^2(x) \leftrightarrow \varphi(x, z \rightarrow 0)$$

\vdots

$$\therefore \langle T_{\mu\nu} T_{\alpha\beta} \rangle \sim \frac{\delta^2 \ln Z_{\text{string}}[h]}{\delta h_{\mu\nu} \delta h_{\alpha\beta}} \sim \frac{\delta^2}{\delta h_{\mu\nu} \delta h_{\alpha\beta}} S_{\text{cl}}[h]$$

AdS/CFT is a **tool** to define/perform calculations in field theory

Applies to field theories beyond $\mathcal{N}=4$ SYM

RHIC + AdS/CFT = ♡ ?

Modest: Can AdS/CFT be useful to understand finite-temperature QCD?

Bold: Can AdS/CFT be useful to understand the dynamics of the collision?

$$\text{RHIC} + \text{AdS/CFT} = \heartsuit ?$$

Modest: Can AdS/CFT be useful to understand finite-temperature QCD?

Bold: Can AdS/CFT be useful to understand the dynamics of the collision?

Comments

- $\mathcal{N}=4$ SYM is the simplest example. Theories which are **more similar to QCD** can be treated by AdS/CFT methods.
- Application of AdS/CFT to thermal QCD is **not exhausted**. How far can we push this program?
- It is **not a waste of time** to do these calculations. Results are relatively easy to derive compared to the conventional methods.
- If there were an effective tool to do real-time computations in strongly coupled QCD at finite T and μ — no need to invoke AdS/CFT. In the absence of such a tool, AdS/CFT is **the best we have** (for some questions).
- Understanding finite-temperature field theory is an interesting question by itself. AdS/CFT can be useful in searching for **universal properties** (shear viscosity example is encouraging).

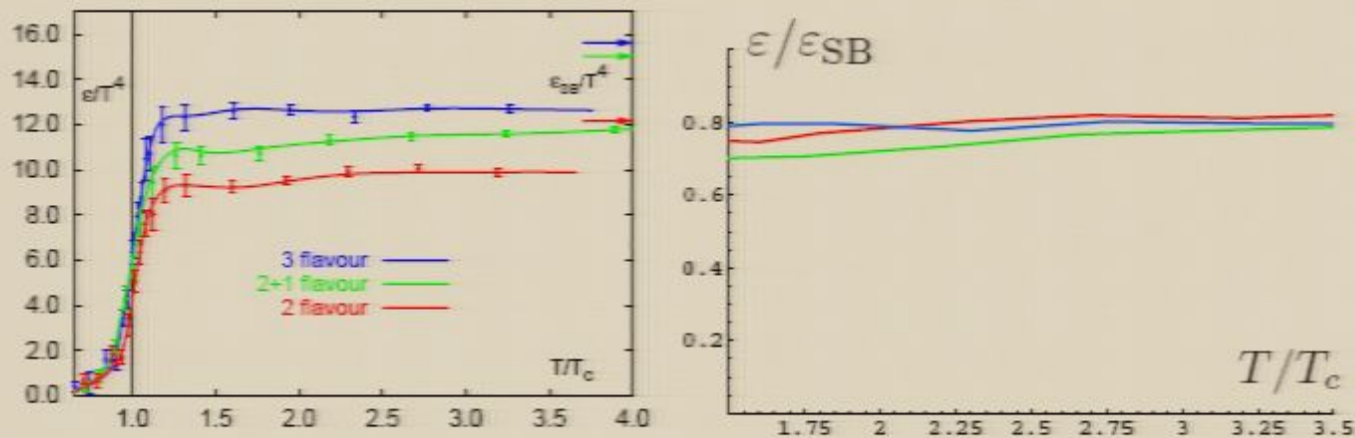
Why $\mathcal{N}=4$ SYM may have something to do with thermal QCD

Q: $\mathcal{N}=4$ SYM is supersymmetric, while QCD is not

A: At finite temperature, supersymmetry is broken anyway

Q: $\mathcal{N}=4$ SYM is conformal, while QCD is asymptotically free

A: Let's look at the thermodynamics of QCD (e.g. F.Karsch, [hep-lat/0106019](https://arxiv.org/abs/hep-lat/0106019))



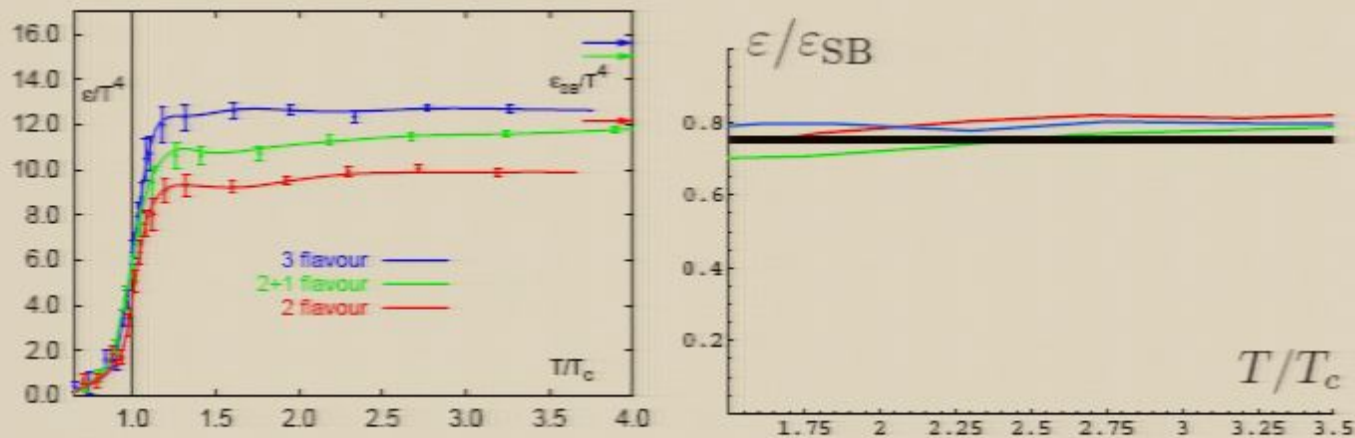
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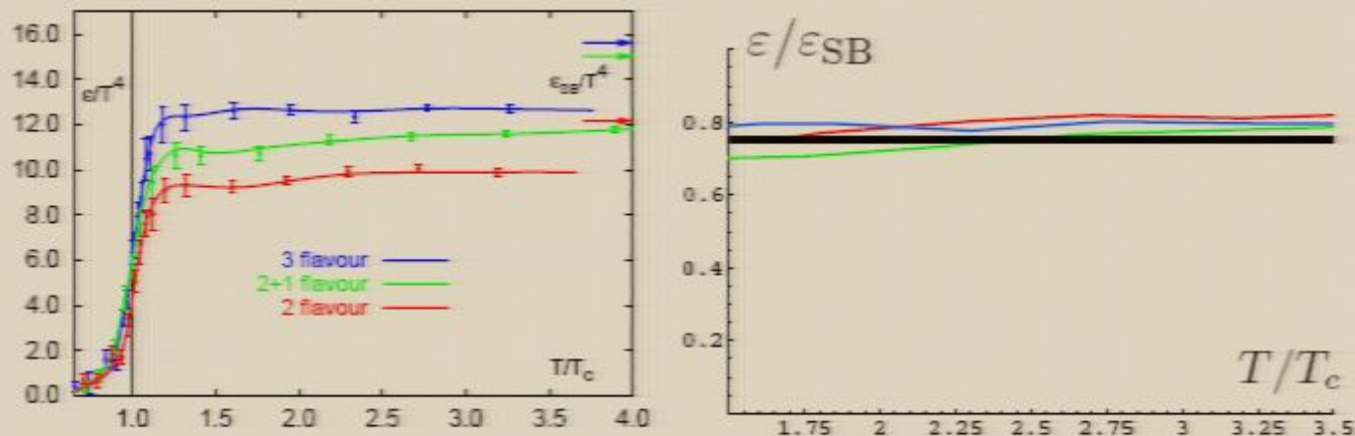
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Q: At high temperature, QCD is weakly coupled, why not use perturbation theory?

A: True only at asymptotically high temperature. At RHIC temperatures,

$T \sim 2T_c$, and QCD is strongly coupled, $\alpha_s(T) = O(1)$.

Q: QCD is a sensible theory, $\mathcal{N}=4$ SYM is just a bunch of non-abelian fields

A: So is QCD, above deconfinement

Lesson:

Use strongly coupled $\mathcal{N}=4$ SYM as a model for QCD at $T \gtrsim T_c$

Will discuss application of AdS/CFT to:

- Momentum transport
 - Electromagnetic response
 - Energy loss by a heavy probe
 - Thermalization
-
- AdS/CFT has more to say!

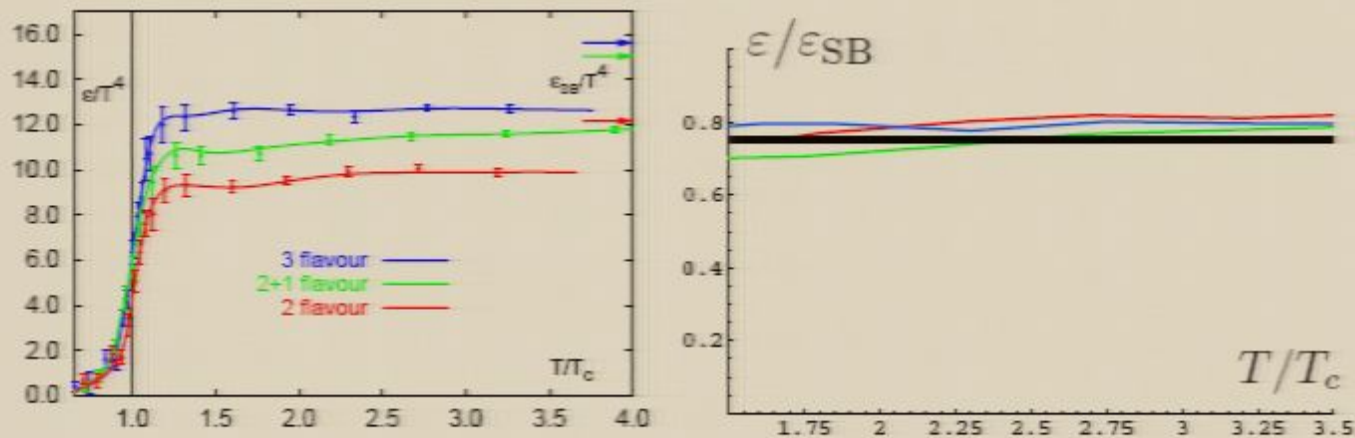
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Momentum transport

- Conservation laws: $\partial_\mu T^{\mu\nu} = 0 \Rightarrow \begin{cases} \partial_t \epsilon = -\nabla \cdot \boldsymbol{\pi} \\ \partial_t \pi^i = -\nabla_j T^{ij} \end{cases}$

- Constitutive relations:

$$\begin{cases} T^{ij} = \delta^{ij} [\langle P \rangle + v_s^2 \delta \epsilon - \gamma_\zeta \nabla \cdot \boldsymbol{\pi}] - \gamma_\eta (\nabla^i \pi^j + \nabla^j \pi^i - \frac{2}{3} \delta^{ij} \nabla \cdot \boldsymbol{\pi}) + \dots \\ \gamma_\eta \equiv \frac{\eta}{\langle \epsilon + P \rangle}, \quad \gamma_\zeta \equiv \frac{\zeta}{\langle \epsilon + P \rangle}, \quad v_s^2 = \partial P / \partial \epsilon \end{cases}$$

- Viscosities η, ζ — input from microscopic physics

Two eigenmodes:

Shear mode: $\pi_\perp(t, \mathbf{k}) = e^{-\gamma_\eta \mathbf{k}^2 t} \pi_\perp(0, \mathbf{k})$

Sound mode: $\pi_\parallel(t, \mathbf{k}) = e^{-\frac{1}{2}(\gamma_\zeta + \frac{4}{3}\gamma_\eta)\mathbf{k}^2 t} \times$

$$\times \left[\pi_\parallel(0, \mathbf{k}) \cos(kv_s t) - i \hat{k} v_s \sin(kv_s t) \delta \epsilon(0, \mathbf{k}) \right]$$

Long-wavelength response is controlled by a small number of kinetic coefficients

Correlation functions in the hydrodynamic limit

Hydrodynamic modes \Rightarrow hydrodynamic singularities at small ω , k .

Example: $S_{tx,tx}(\omega, k) = \frac{2\gamma_\eta k^2}{\omega^2 + (\gamma_\eta k^2)^2} (\epsilon + P) T$ relaxation of transverse momentum

Kubo formulas

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt e^{i\omega t} \int d^3x \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0)] \rangle$$

$$\frac{4}{3}\eta + \zeta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt e^{i\omega t} \int d^3x \langle [T_{xx}(t, \mathbf{x}), T_{xx}(0)] \rangle$$

Connection to microscopic physics: Viscosities can be extracted from (small-frequency limits of) real-time correlation functions

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Connection to microscopic physics: Viscosities can be extracted from (small-frequency limits of) real-time correlation functions

Spectral function for stress

(P.K., A.Starinets [hep-th/0602059](#), D.Teaney [hep-ph/0602044](#))

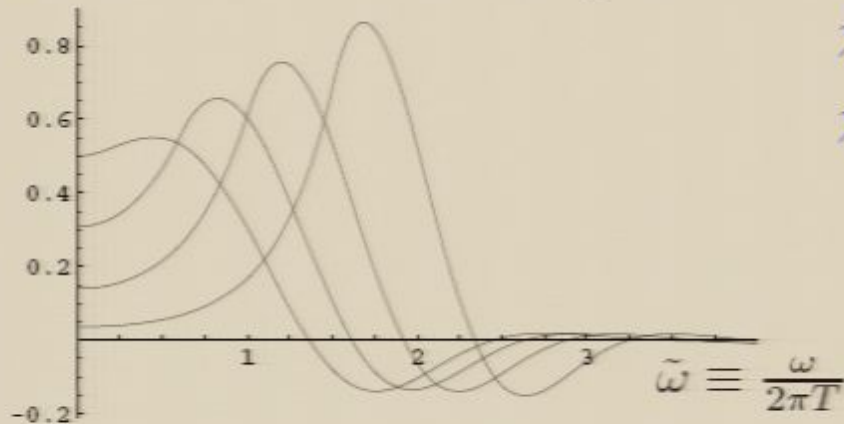
$$\frac{1}{\tilde{\omega}} (\chi(\tilde{\omega}) - \chi^{T=0}(\tilde{\omega})) \left[\frac{1}{\pi^2 N_c^2 T^4} \right]$$

$$\chi(\omega, k) = -2 \text{Im} G_{xy,xy}^{\text{ret}}(\omega, k)$$

$$\chi(\omega) \sim \omega, \quad \omega \ll 2\pi T$$

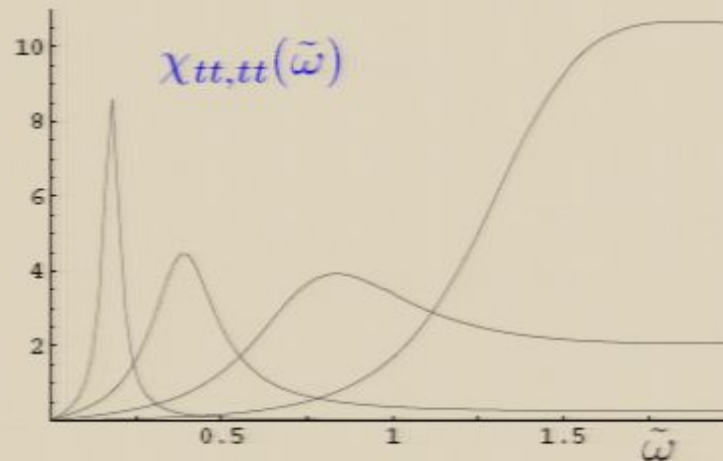
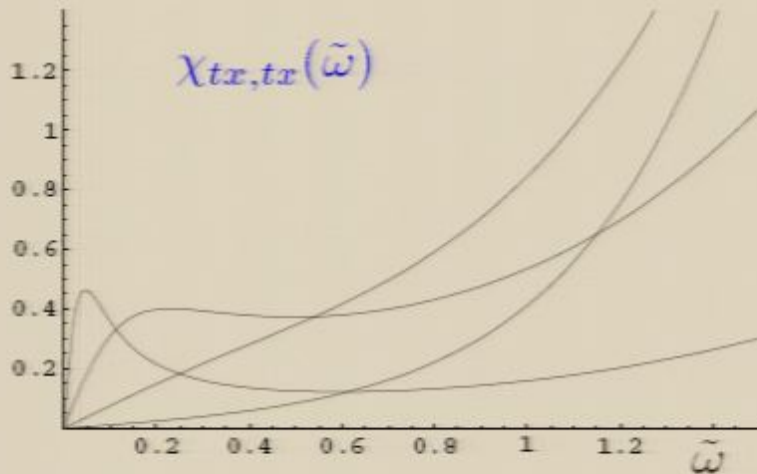
$$\chi(\omega) - \chi^{T=0}(\omega) \sim e^{-\gamma\omega}, \quad \omega \gg 2\pi T$$

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

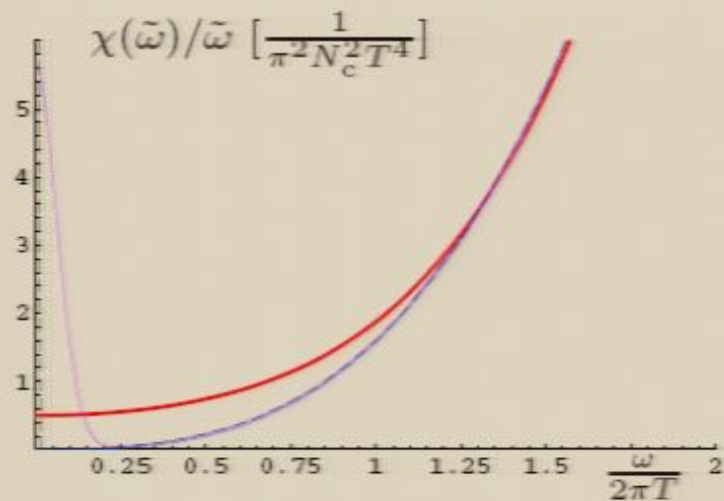


T^3 by conformal invariance, N_c^2 counts d.o.f.

Spectral function for conserved energy-momentum



Real-time correlators are very different at strong and weak coupling

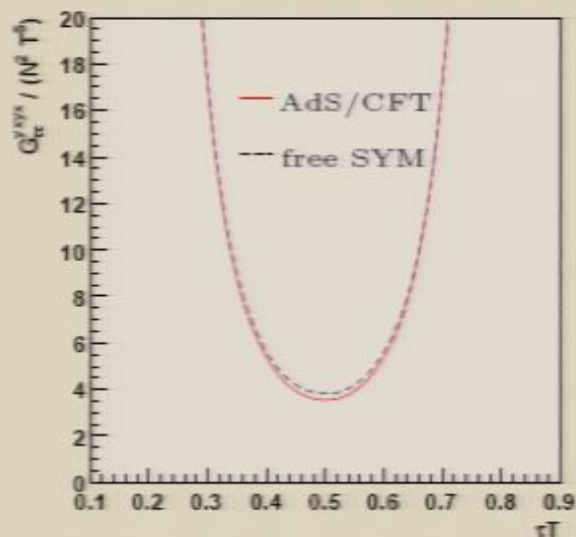


Red — strongly coupled SYM, $\eta = O(1)$

Dashed — free SYM, $\eta = \infty$

Purple — weakly coupled SYM, $\eta \sim \frac{1}{\lambda^2}$

Blue — SYM, $T = 0$



Euclidean correlators are almost the same!

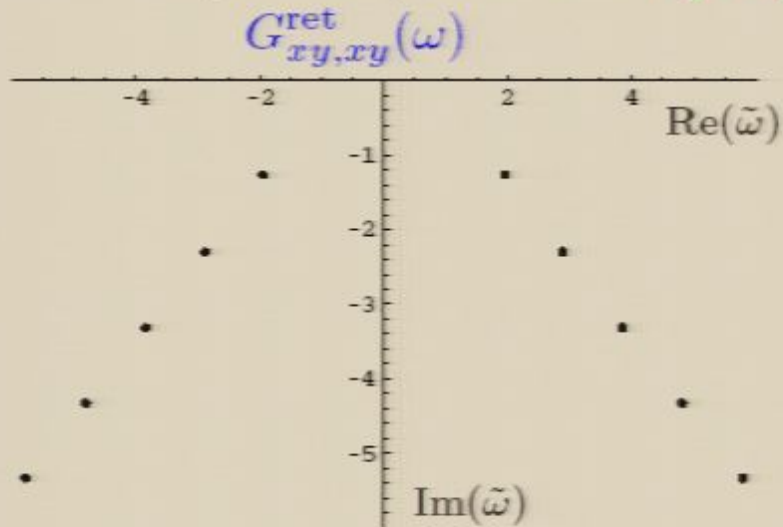
(picture from D. Teaney, [hep-ph/0602044](https://arxiv.org/abs/hep-ph/0602044))

$$G_E(\tau) = \int_0^\infty d\omega \chi(\omega) \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh(\beta\omega/2)}$$

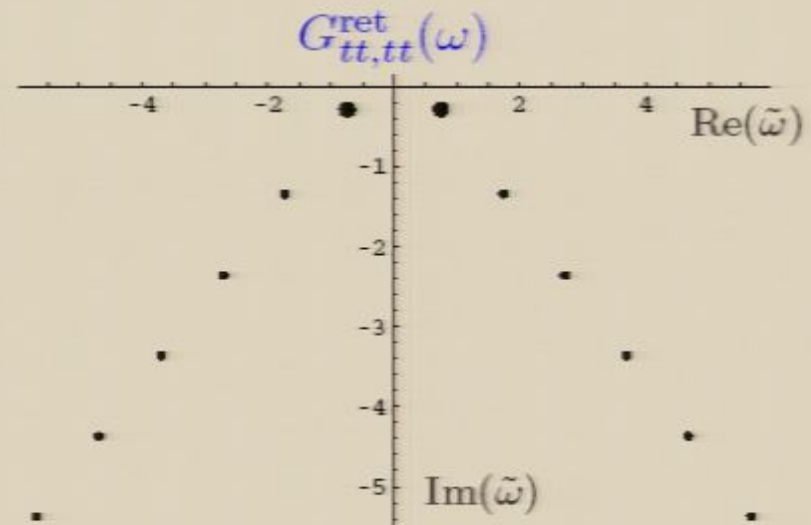
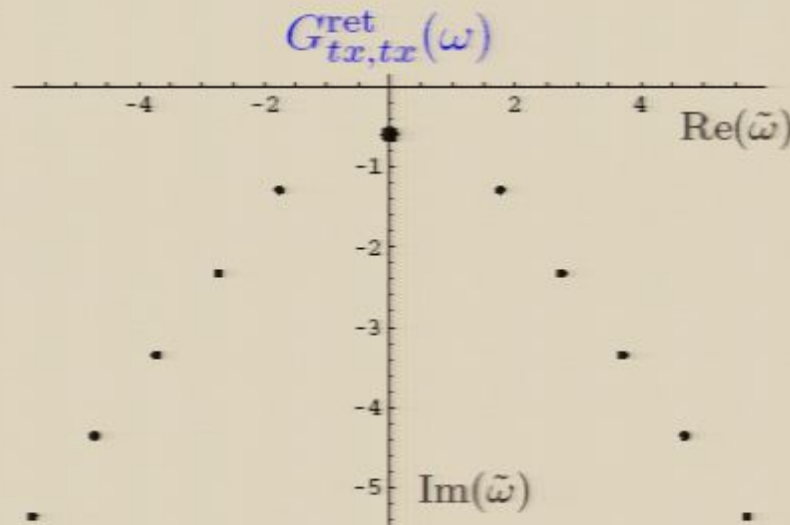
Lattice $G_E(\tau)$ has errorbars $\sim 500\%$ ([hep-lat/0406009](https://arxiv.org/abs/hep-lat/0406009))

Singularities of $G^{\text{ret}}(\omega, k)$

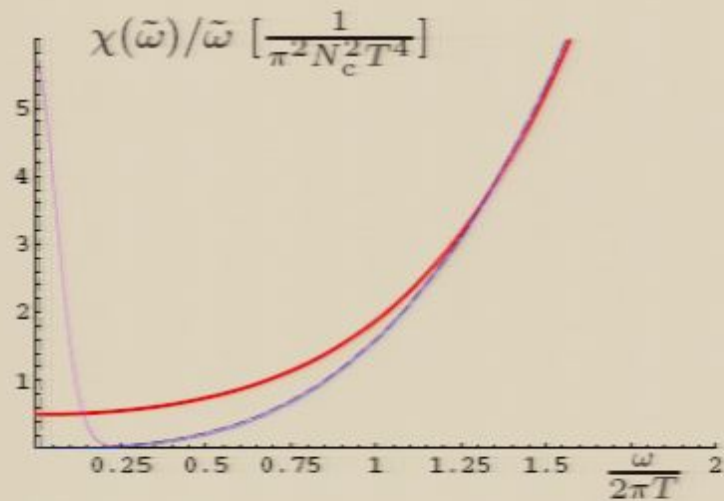
(A.Nunez, A.Starinets [hep-th/0302026](#); P.K., A.Starinets [hep-th/0506184](#))



- Infinite series of poles
- $\omega_n = 2\pi nT(\pm 1 - i)$ as $n \rightarrow \infty$
- For conserved densities, $\omega_0 \rightarrow 0$ as $k \rightarrow 0$
- Hydro poles agree with Kubo formula



Real-time correlators are very different at strong and weak coupling

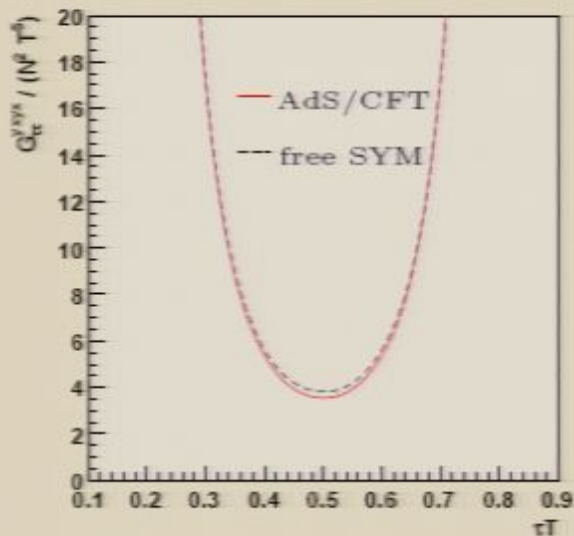


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Universality: Lower bound on shear viscosity?

(P.K., D.Son, A.Starinets [hep-th/0405231](#))

Life at low Reynolds number

E. M. Purcell

Lowell Laboratory, Harvard University, Cambridge, Massachusetts 02138
(Received 12 June 1978)

Editor's note: This is a reprint (slightly edited) of a paper of the same title that appeared in the book *Physic and Our World: A Symposium in Honor of Victor F. Weisskopf*, published by the American Institute of Physics (1978). The personal tone of the original talk has been preserved in the paper, which was itself a slightly edited transcript of a tape. The figures reproduce transparencies used in the talk. The demonstration involved a tall rectangular transparent vessel of corn syrup, projected by an overhead projector viewed on its side. Some essential hand waving could not be reproduced.

This is a talk that I would not, I'm afraid, have the nerve to give under any other circumstances. It's a story I've been saving up to tell Viki. Like so many of you here, I've enjoyed from time to time the wonderful experience of exploring with Viki some part of physics, or anything to which we can apply physics. We wander around strictly as amateurs equipped only with some elementary physics, and in the end, it turns out, we improve our understanding of the elementary physics even if we don't throw much light on the other subjects. Now this is that kind of a subject, but I have still another reason for wanting to, as it were, needle Viki with it, because I'm going to talk for a while about viscosity. Viscosity in a liquid will be the dominant theme here and you know Viki's program of explaining everything, including the heights of mountains, with the elementary constants. The viscosity of a liquid is a very tough nut to crack, as he well knows, because when the stuff is cooled by merely 40 degrees, its viscosity can change by a factor of a million. I was really amazed by fluid viscosity in the early days of NMR, when it turned out that glycerine was just what we needed to explore the behavior of spin relaxation. And yet if you were a little bug inside the glycerine, looking around, you wouldn't see much change in your surroundings as the glycerine cooled. Viki will say that he can at least predict the logarithm of the viscosity. And that, of course, is correct because the reason viscosity changes is that it's got one of these activation energy things and what he can predict is the order of magnitude of the exponent. But it's more mysterious than that, Viki, because if you look at the Chemical Rubber Handbook table you will find that there is almost no liquid with viscosity much lower than that of water. The viscosities have a big range but they stop at the same place. I don't understand that. That's what I'm leaving for him.¹

Now, I'm going to talk about a world which, as physicists, we almost never think about. The physicist hears about viscosity in high school when he's repeating Millikan's oil drop experiment and he never hears about it again, at least not in what I teach. And Reynolds's number, of course, is something for the engineers. And the low Reynolds number regime most engineers aren't even interested in—except possibly chemical engineers, in connection with fluidized beds, a fascinating topic I heard about from a chemical engineering friend at MIT. But I want to take you into the world of very low Reynolds numbers—a world which is inhabited by the overwhelming majority of the organisms in this room. This world is quite different from the one that we have developed our imaginations in.

I might say what got me into this. To introduce something

that will come later, I'm going to talk partly about how microorganisms swim. That will not, however, turn out to be the only important question about them. I got into this through the work of a former colleague of mine at Harvard, Howard Berg. Berg got his Ph.D. with Norman Ramsey, working on a hydrogen maser, and then he went back into biology which had been his early love, and into cellular physiology. He is now at the University of Colorado at Boulder, and has recently participated in what seems to me one of the most astonishing discoveries about the questions we're going to talk about. So it was partly Howard's work, tracking *E. coli* and finding out this strange thing about them, that got me thinking about this elementary physics stuff.

Well, here we go. In Fig. 1, you see an object which is moving through a fluid with velocity v . It has dimension a . In Stokes's law, the object is a sphere, but here it's anything; η and ρ are the viscosity and density of the fluid. The ratio of the inertial forces to the viscous forces, as Osborne Reynolds pointed out slightly less than a hundred years ago, is given by $\rho v a / \eta$ or $\rho v^2 a / \eta$, where ρ is called the kinematic viscosity. It's easier to remember its dimensions: for water, $\rho = 10^{-2}$ cm²/sec. The ratio is called the Reynolds number and when that number is small the viscous forces dominate. Now there is an easy way, which I didn't realize at first, to see who should be interested in small Reynolds numbers. If you take the viscosity η and square it and divide by the density, you get a force (Fig. 2). No other dimensions come in at all; η^2/ρ is a force. For water, since $\eta = 10^{-2}$ and $\rho = 1$, $\eta^2/\rho = 10^{-4}$ dyn. That is a force that will tow anything, large or small, with a Reynolds number of order of magnitude 1. In other words, if you want to tow a submarine with Reynolds number 1 (or strictly speaking, 1/6r if it's a spherical submarine) tow it with 10^{-4} dyn. So it's clear in this case that you're interested in small Reynolds number if you're interested in small forces in an absolute sense. The only other people who are interested in low Reynolds number, although they usually don't have to locate it, are the geophysicists. The Earth's mantle is supposed to have a viscosity of 10^{21} P. If you now work out η^2/ρ , the force is 10^{41} dyn. That is more than 10^6 times the gravitational force that hold the Earth's crust on the other half! So the conclusion is, of course, that in the flow of the mantle of the Earth the Reynolds number is very small indeed.

Now consider things that move through a liquid (Fig. 3). The Reynolds number for a man swimming in water might be 10^6 , if we put in reasonable dimensions. For a goldfish or a tiny guppy it might get down to 10^2 . For the animals that we're going to be talking about, as we'll see in a mo-

he can predict is the order of magnitude of the exponent. But it's more mysterious than that, Viki, because if you look at the Chemical Rubber Handbook table you will find that there is almost no liquid with viscosity much lower than that of water. The viscosities have a big range *but they stop at the same place*. I don't understand that. That's what I'm leaving for him.¹ !!

- $\eta/s \gg 1$ at small coupling in SYM
(S.Huot, S.Jeon, G.Moore, [hep-ph/0608062](#))
- $\eta/s = \frac{1}{4\pi}$ is finite at large coupling
- Natural to assume $\eta/s \geq \frac{1}{4\pi}$ in SYM

$$\text{Is } \frac{\eta}{s} \geq \frac{1}{4\pi} \text{ universal?}$$

We know $\frac{\eta}{s} = \frac{1}{4\pi}$ is universal — proven for a large class of field theories with gravity duals.

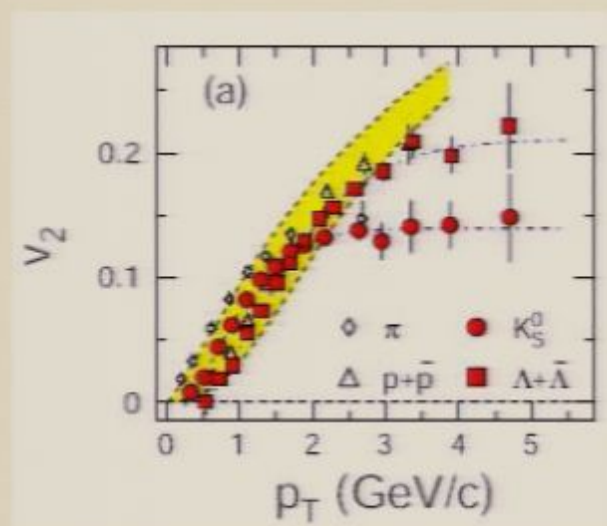
Prove from first principles?

What is the viscosity measured at RHIC?

Of course, RHIC does not “measure” viscosity. Rather, measured angular distributions of particles are confronted with hydrodynamic models of the “fireball” evolution. Quantitatively: lentil-shaped reaction region \Rightarrow **azimuthal anisotropy** of particle distribution

$$\frac{d^2N}{dp_T d\phi} = N_0 [1 + 2 v_2(p_T) \cos(2\phi) + \dots]$$

\swarrow “elliptic flow”



Elliptic flow from PHENIX and STAR (figure from [nucl-ex/0501009](https://arxiv.org/abs/nuc1-ex/0501009)). **Yellow band** — hydro calculations. To reproduce elliptic flow and spectra from hydro, η/s **must be small**, $\eta/s \lesssim 0.3$ [Teaney; Baier, Romastschke].

Perturbative QCD:

See Arnold, Moore, Yaffe, [hep-ph/0010177](https://arxiv.org/abs/hep-ph/0010177)

$\eta/s \approx 1.6 - 1.8$ at relevant T (L.Csernai, J.Kapusta, L.McLerran, [nucl-th/0604032](https://arxiv.org/abs/nuc1-th/0604032))

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→ Electromagnetic response

Energy loss by a heavy probe

Thermalization

AdS/CFT has more to say!

In RHIC context see e.g.

C.Gale, [hep-ph/0512109](#)

Pirsa: 06110009
P.Stankus, [Ann.Rev.Nucl.Part.Sci.55:517,2005](#)

Production of real and virtual photons

(e.g. L.D.McLerran, T.Toimela, *PRD* **31**, 545 (1985), H.A.Weldon, *PRD* **42**, 2384 (1990))

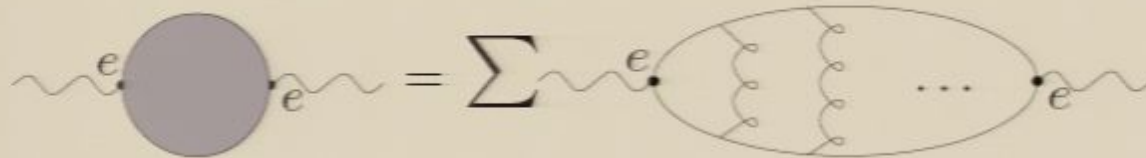
Γ — number of photons per unit time per unit volume

Photon interaction: $e J_\mu^{\text{EM}} A^\mu$; el. charge e small \Rightarrow photons **do not** thermalize

$$d\Gamma = \frac{d^3k}{(2\pi)^3} \frac{e^2}{2|\mathbf{k}|} \eta^{\mu\nu} C_{\mu\nu}^<(k) \Big|_{\text{lightlike } k} \quad \text{where } C_{\mu\nu}^<(x) = \langle J_\mu^{\text{EM}}(0) J_\nu^{\text{EM}}(x) \rangle$$

Virtual photon can decay into a lepton pair: $d\Gamma = \frac{d^4k}{(2\pi)^4} \frac{e^2 e_\ell^2}{6\pi k^2} \eta^{\mu\nu} C_{\mu\nu}^<(k) \Big|_{\text{timelike } k}$

Emission spectra are determined by EM current-current spectral function



true to leading order in e ,
but to **all orders** in g

In SYM, can evaluate the whole “blob” at large $\lambda \equiv g^2 N_c$ (using AdS/CFT)

Production of real and virtual photons from $\mathcal{N}=4$ plasma

(S.Caron-Huot, P.K., G.Moore, A.Starinets, L.Yaffe, [hep-th/0607237](#))

But wait... $\mathcal{N}=4$ SYM does not have a photon [$U(1)$ gauge field coupled to a conserved current]

Let's introduce one! To do so:

Gauge a $U(1)$ subgroup of $SU(4)$ R-symmetry with small coupling e

The symmetry is not anomalous

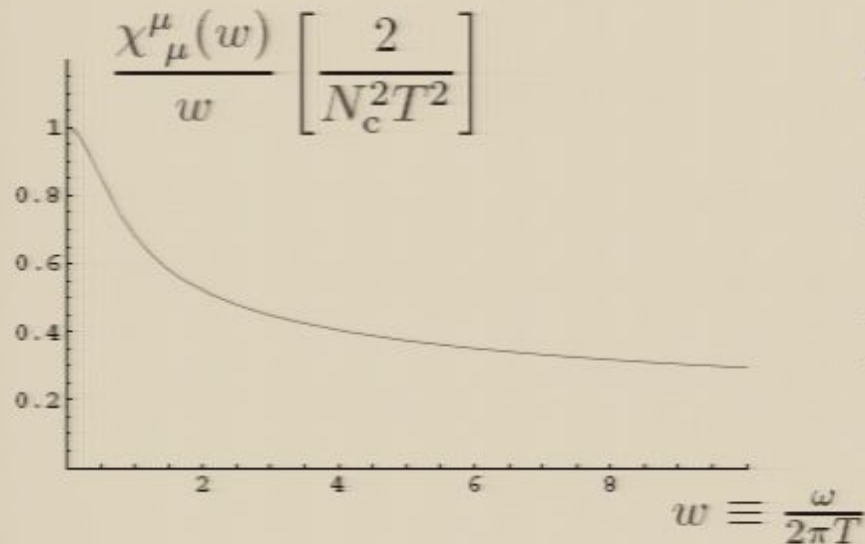
EM current $J_\mu^{\text{EM}}(x) \equiv J_\mu^{\text{R}}(x)$ conserved
non-anomalous
couples to a $U(1)$ gauge field

Photon and dilepton emission rate is given by R-current spectral function

In SYM can compute $\chi_\mu^\mu(\omega) \equiv -2 \text{Im} \eta^{\mu\nu} C_{\mu\nu}^{\text{RET}}(\omega, q) \Big|_{\omega=q}$ at strong coupling, then emission rate is

$$\frac{d\Gamma}{d^3k} \propto n_B(\omega) \frac{\chi_\mu^\mu(\omega)}{\omega}$$

On-shell photons at strong coupling



$$\chi^\mu_\mu(\omega) = -2 \operatorname{Im} \eta^{\mu\nu} C_{\mu\nu}^{\text{RET}}(\omega, q) \Big|_{\omega=q}$$

Small frequency:

$$\chi^\mu_\mu(w) \sim \frac{1}{2} N_c^2 T^2 w, \text{ in accord with hydro}$$

High frequency:

$$\chi^\mu_\mu(w) \sim \frac{N_c^2 T^2}{4} w^{2/3} \left[3^{5/6} \frac{\Gamma(2/3)}{\Gamma(1/3)} \right]$$

Spectral function for on-shell photons can be computed in closed form!

$$\chi^\mu_\mu(w) = \frac{N_c^2 T^2 w}{8} \left| {}_2F_1 \left(1 - \frac{1}{2}(1+i)w, 1 + \frac{1}{2}(1-i)w; 1-iw; -1 \right) \right|^{-2}$$

Emission rate is finite and λ -independent in the limit of large λ

Production of real and virtual photons from $\mathcal{N}=4$ plasma

(S.Caron-Huot, P.K., G.Moore, A.Starinets, L.Yaffe, [hep-th/0607237](#))

But wait... $\mathcal{N}=4$ SYM does not have a photon [$U(1)$ gauge field coupled to a conserved current]

Let's introduce one! To do so:

Gauge a $U(1)$ subgroup of $SU(4)$ R-symmetry with small coupling e

The symmetry is not anomalous

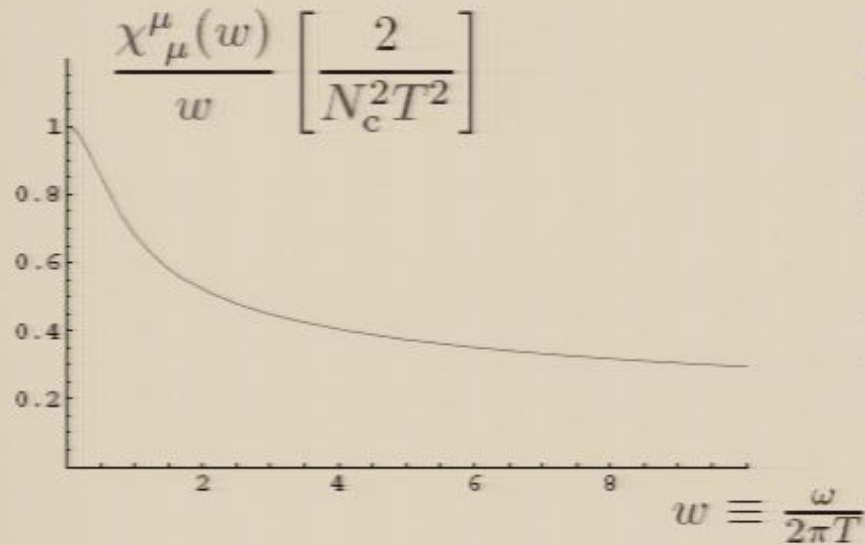
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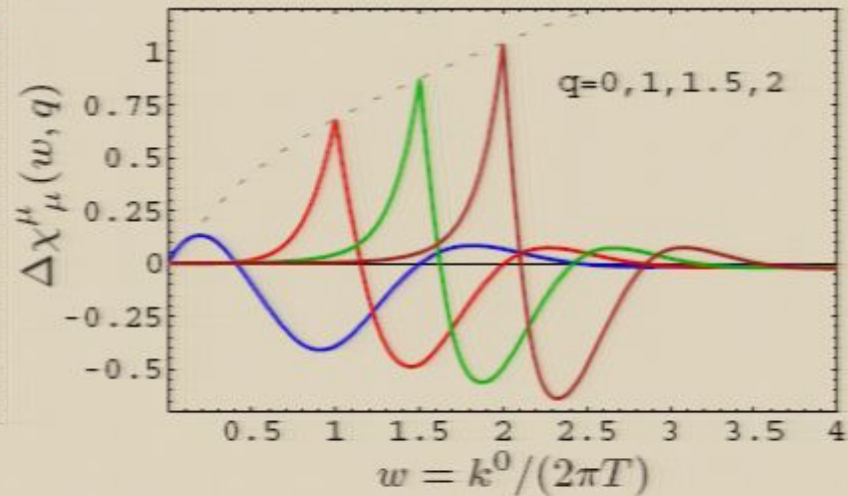
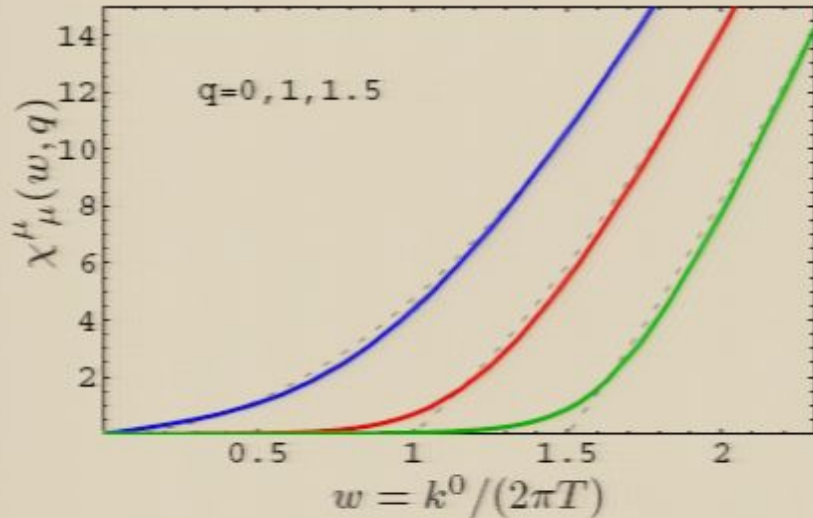
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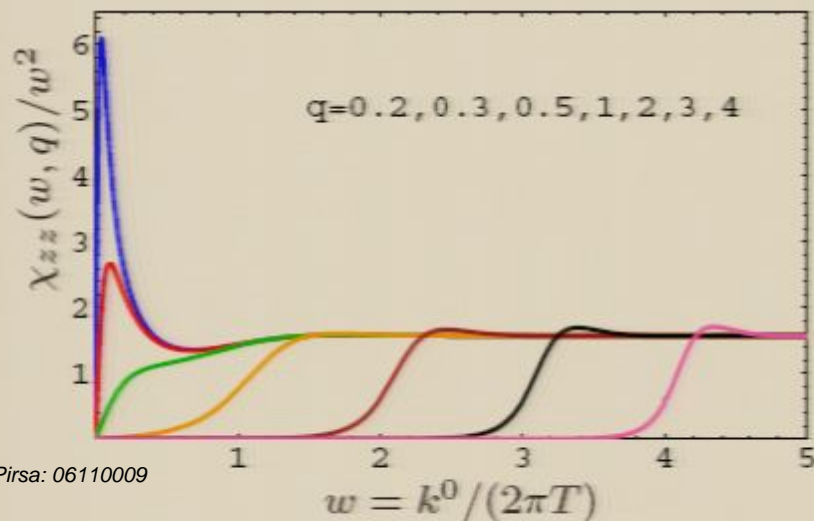
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Off-shell photons at strong coupling

Plot spectral function for several values of $q \equiv k/(2\pi T)$



Spectral function oscillates around the zero-temperature value... where is hydro?



In the hydro limit:

$$\frac{\chi_{zz}(\omega, k)}{\omega^2} = \frac{\omega 2D\Xi}{\omega^2 + (Dk^2)^2}$$

Diffusion bump clearly visible at small ω and k

Electrical conductivity

Kubo formula: $\sigma = e^2 \lim_{\omega \rightarrow 0} \frac{\chi_{ii}(\omega, q=0)}{6\omega}$

In **strongly** coupled SYM: $\sigma = e^2 \frac{N_c^2 T}{16\pi}$, does not depend on λ

In **weakly** coupled SYM: $\sigma = e^2 \frac{N_c^2 T}{\lambda^2 \ln(1/\lambda)} C$, where $C=O(1)$

Normalize to the number of degrees of freedom: $\frac{\sigma}{e^2 \Xi}$, where Ξ is charge susceptibility. In strongly coupled SYM,

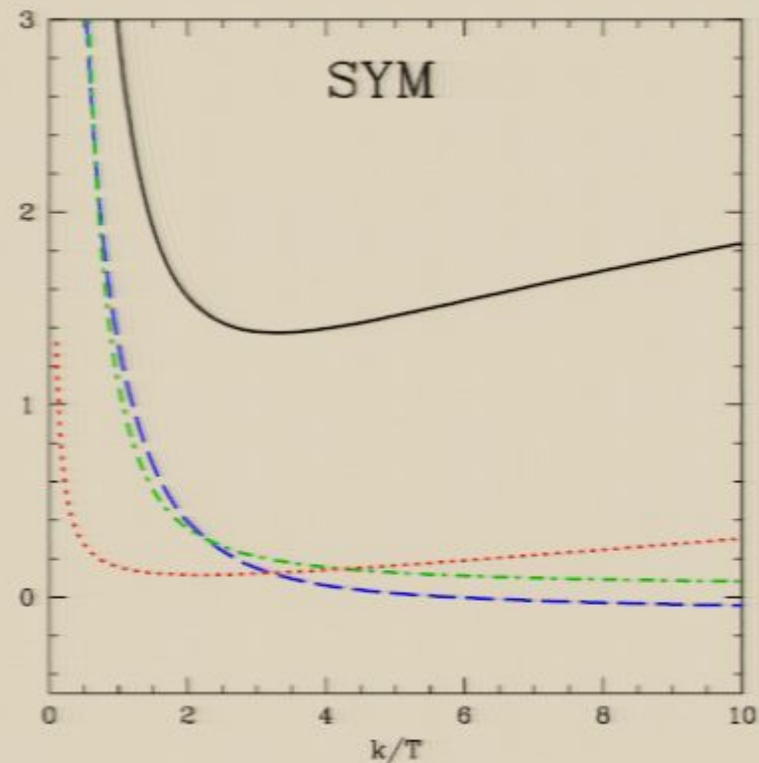
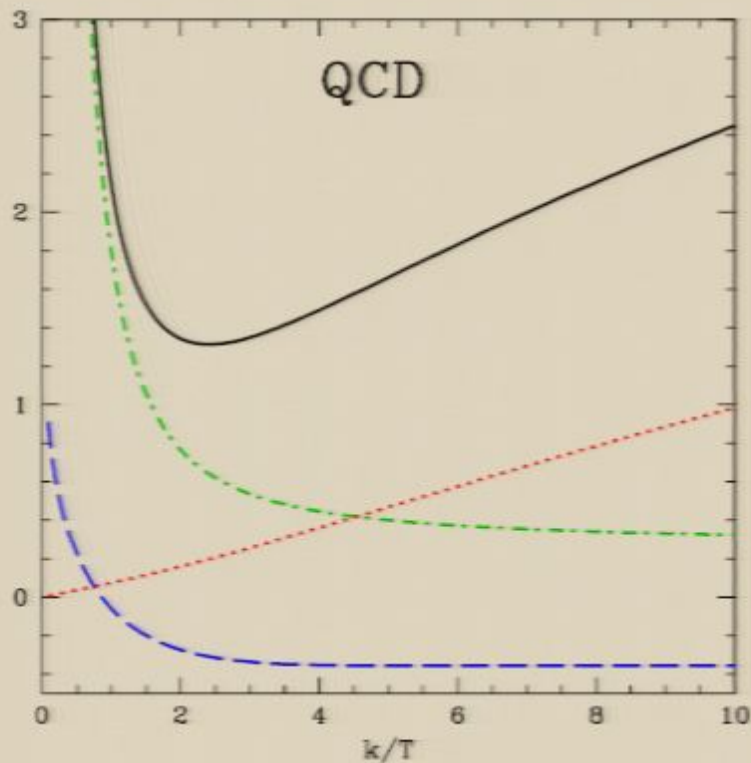
$$\frac{\sigma}{e^2 \Xi} = \frac{1}{2\pi T}$$

Einstein relation $\frac{\sigma}{e^2 \Xi} = D$ is satisfied, as it should.

On-shell photons at weak coupling

$\mathcal{N} = 4$ SYM is simply a non-abelian plasma with adjoint representation matter.
 Follow approach of Arnold-Moore-Yaffe ([hep-ph/0111107](https://arxiv.org/abs/hep-ph/0111107)) for QCD...

$$\chi_{\mu}^{\mu}(k) = \frac{\lambda(N_c^2 - 1)T^2 n_F(k)}{4\pi n_B(k)} \left[\ln \lambda^{-1/2} + C_{\text{tot}}(k/T) + O(\sqrt{\lambda}) \right], \quad k \gg \lambda^2 T$$



Off-shell photons at weak coupling

For time-like momenta, $\chi_{\mu\nu}(k)$ is non-zero already at $O(\lambda^0)$. One-loop diagram:

$$\chi_{\mu}^{\mu}(K) = \frac{(N_c^2 - 1)}{16\pi} (-K^2) \left[3 - \frac{2T}{k} \ln \frac{1 + e^{-(k^0 - k)/2T}}{1 + e^{-(k^0 + k)/2T}} + \frac{T}{k} \ln \frac{1 - e^{-(k^0 + k)/2T}}{1 - e^{-(k^0 - k)/2T}} \right]$$

LPM resummation for dileptons in $\mathcal{N} = 4$ SYM: needs to be done...

Electrical conductivity

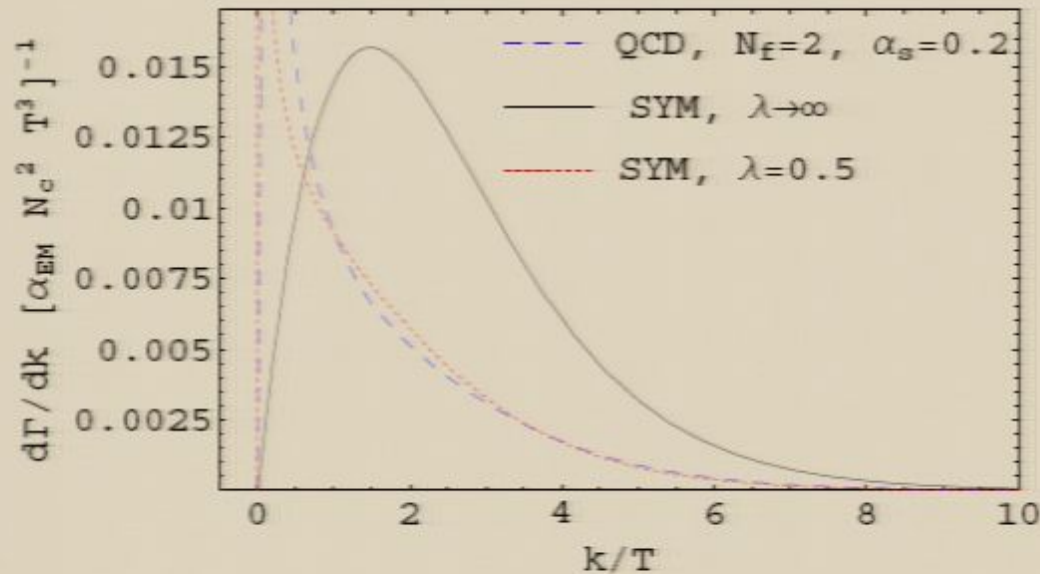
Evaluated at leading log order, following Arnold-Moore-Yaffe, [hep-ph/0010177](https://arxiv.org/abs/hep-ph/0010177)

$$\sigma = 1.28349 \frac{e^2 (N_c^2 - 1) T}{\lambda^2 [\ln(\lambda^{-1/2}) + O(1)]}$$

The $O(1)$ constant: needs to be done...

Now can compare weakly coupled SYM and
strongly coupled SYM

Photon emission



At weak coupling:

$k \gg \lambda^2 T$, follow AMY

$k \ll \lambda^2 T$, hydro regime

$k \sim \lambda^2 T$: technically hard.

Height of the peak is unknown at weak coupling in either SYM or QCD.

At strong coupling:

$$\frac{d\Gamma_\gamma}{dk} = \frac{\alpha_{\text{EM}} N_c^2 T^3}{16\pi^2} \frac{(k/T)^2}{e^{k/T} - 1} \left| {}_2F_1\left(1 - \frac{(1+i)k}{4\pi T}, 1 + \frac{(1-i)k}{4\pi T}; 1 - \frac{ik}{2\pi T}; -1\right) \right|^{-2}$$

- emission rate for *all* k is given by a simple *analytic* expression
- hydro limit is naturally reproduced when $k \ll T$
- as coupling grows, the rate becomes finite and *coupling-independent*

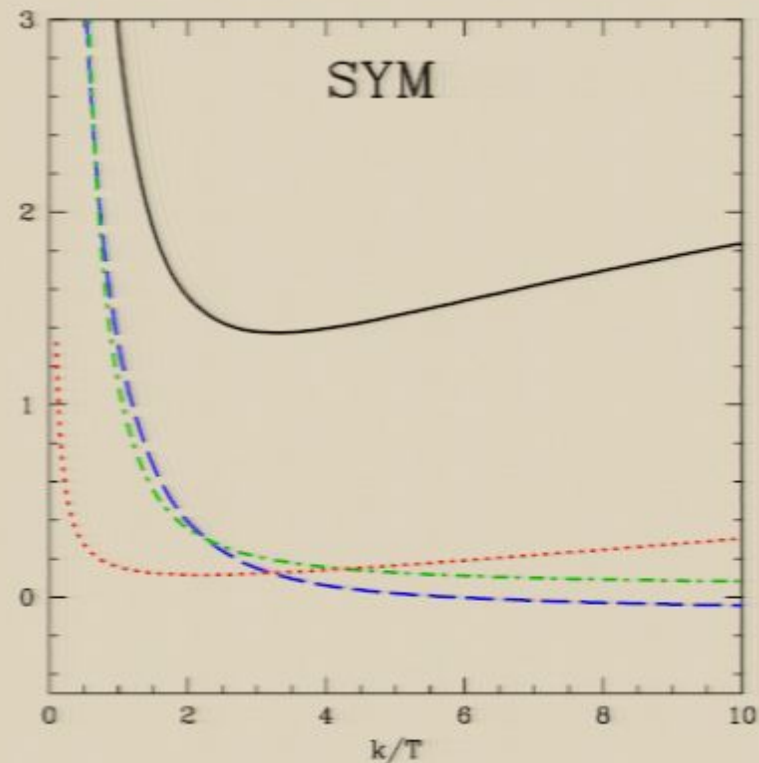
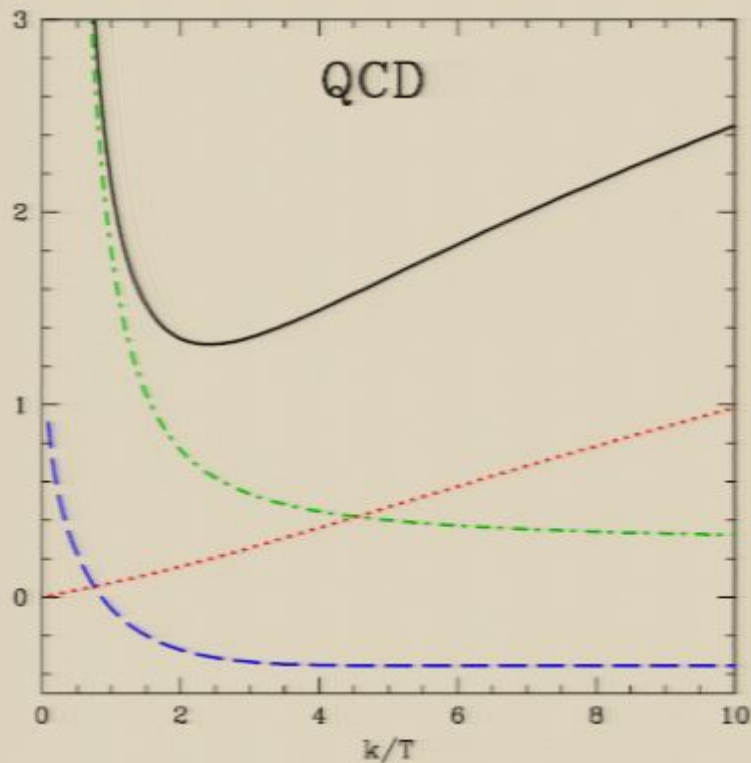
At small k/T , photon emission rate is greater in *weakly-coupled* theory

At large k/T , photon emission rate is greater in *strongly-coupled* theory

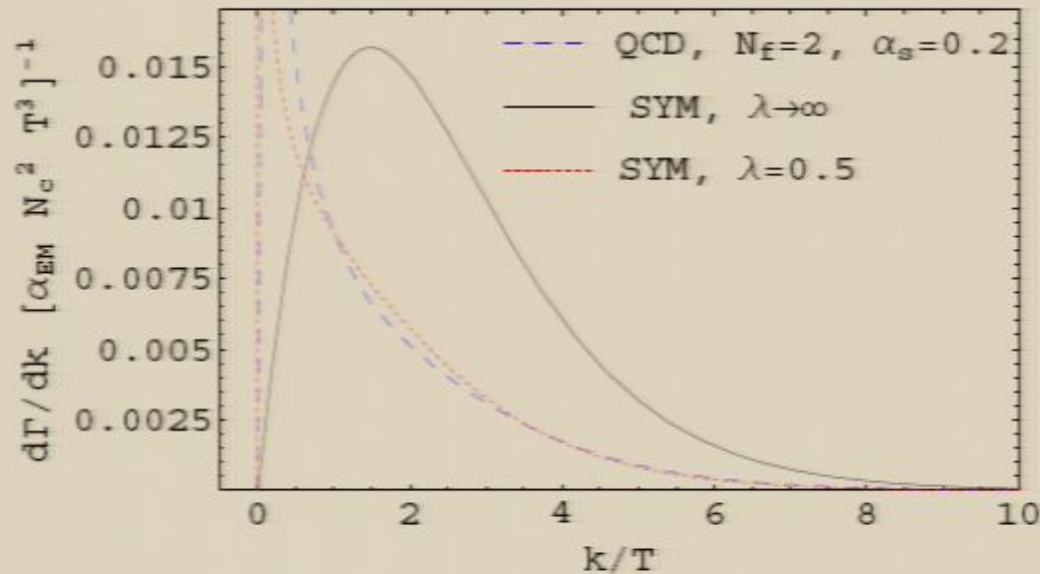
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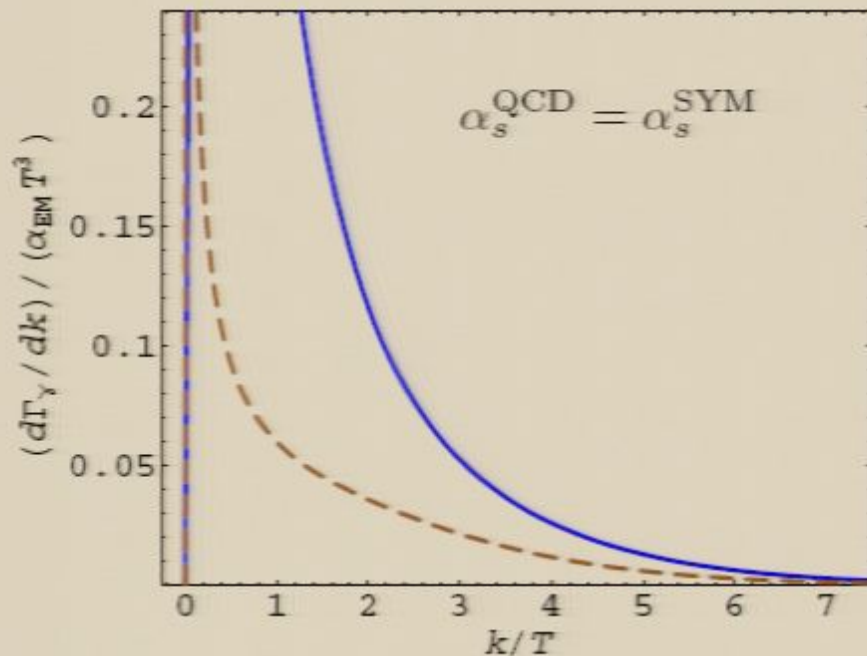
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Compare SYM and QCD at weak coupling

To mimic QCD, choose:

- the leading short-distance behavior of $C_{\mu\nu}(x)$ to coincide in QCD and SYM
- a value of the coupling constant in SYM

When compare at the same value of the coupling constant:



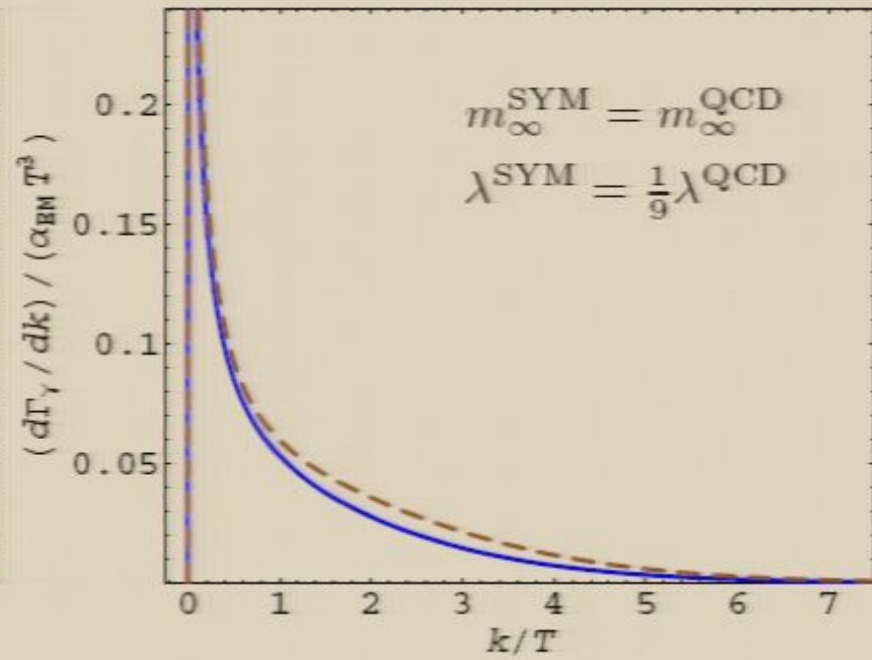
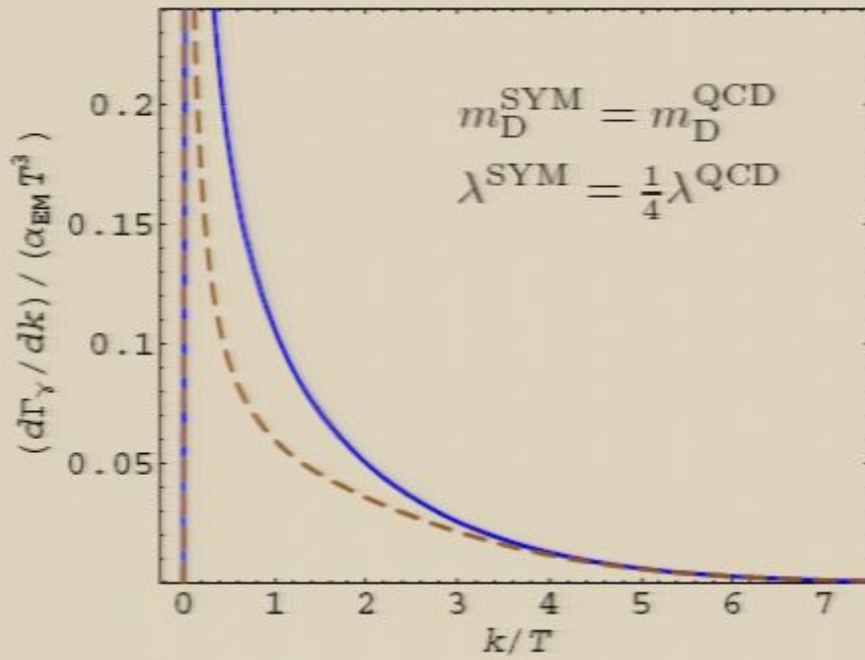
Photon emission rate in SYM is much greater than in QCD

Compare SYM and QCD at weak coupling

To mimic QCD, choose:

- the leading short-distance behavior of $C_{\mu\nu}(x)$ to coincide in QCD and SYM
- a value of the coupling constant in SYM

When compare at the same value of a physical quantity:



Photon emission rate in SYM is very similar to QCD

Lesson: Do not compare QCD and SYM at the same 't Hooft coupling.

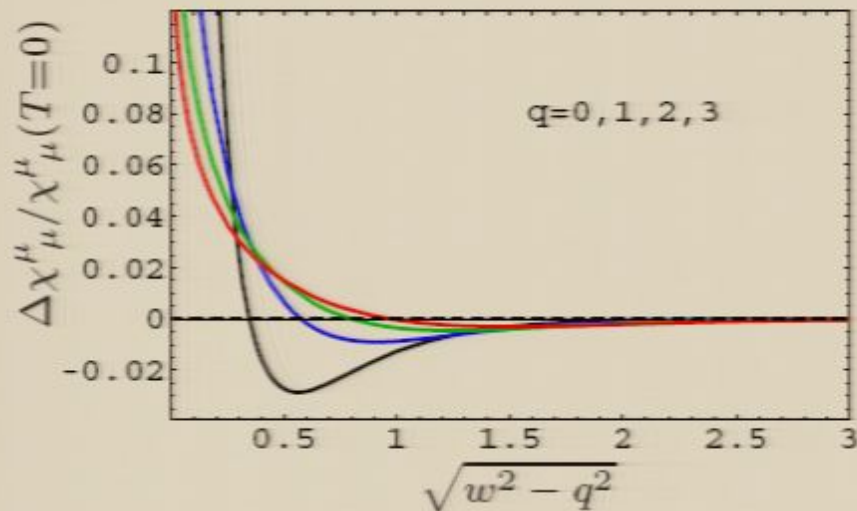
For example, $\alpha_s = 0.5$ in QCD does *not* correspond to $\lambda^{\text{SYM}} = g^2 N_c = (4\pi\alpha_s)N_c \approx 20$, but to $\lambda^{\text{SYM}} \approx 2-5$.

$1/\lambda$ corrections are important for better comparison with QCD at RHIC

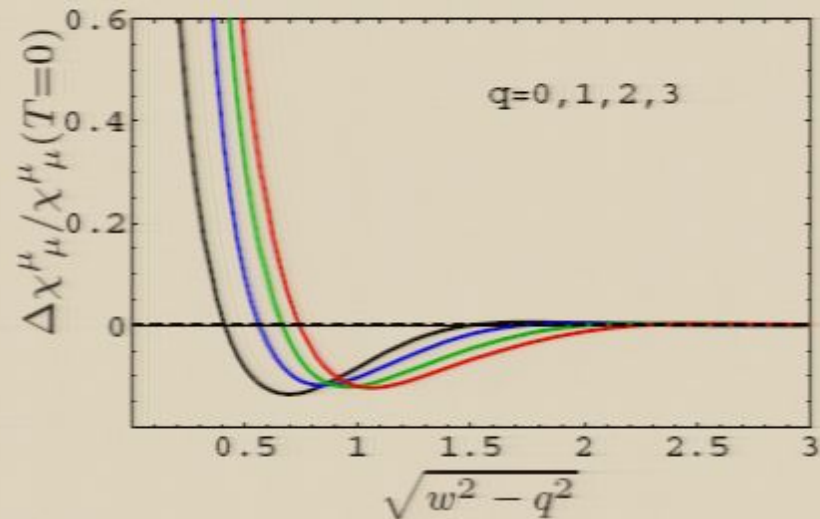
Dilepton emission

Plot relative thermal correction to the trace of the spectral function, $q \equiv k/(2\pi T)$, $w \equiv k^0/(2\pi T)$

SYM, $\lambda=0$



SYM, $\lambda=\infty$



For $\sqrt{-K^2} \gtrsim 2\pi T$, thermal correction to the spectral function are $< 2\%$ at weak coupling, and $< 15\%$ at strong coupling.

Dilepton spectrum is nearly identical at weak and strong coupling, for large invariant mass of the pair

Will discuss application of AdS/CFT to:

Momentum transport

Electromagnetic response

→ Energy loss by a heavy probe

Thermalization

AdS/CFT has more to say!

Related:

J.Casalderrey-Solana, D.Teaney, [hep-ph/0605199](#)

H.Liu, K.Rajagopal, U.Wiedemann, [hep-ph/0605178](#)

J.Piess, S.Gubser, G.Michalogiorgakis, S.Pufu, [hep-th/0607022](#)

Heavy probe energy loss

(C.Herzog, A.Karch, P.K., C.Kozcaz, L.Yaffe, [hep-th/0605158](#))

Setup: Particle of mass M (“probe”), moving through a *strongly* interacting thermal medium ($\lambda \rightarrow \infty$) with temperature $T \ll M$

(charm quark $M \approx 1.3 \text{ GeV}$, moving in a plasma at $T = 200 \text{ MeV}$)

Questions:

What is meant by “energy” of a probe?

How localized is the probe?

How to separate energy of the probe from the energy of the medium?

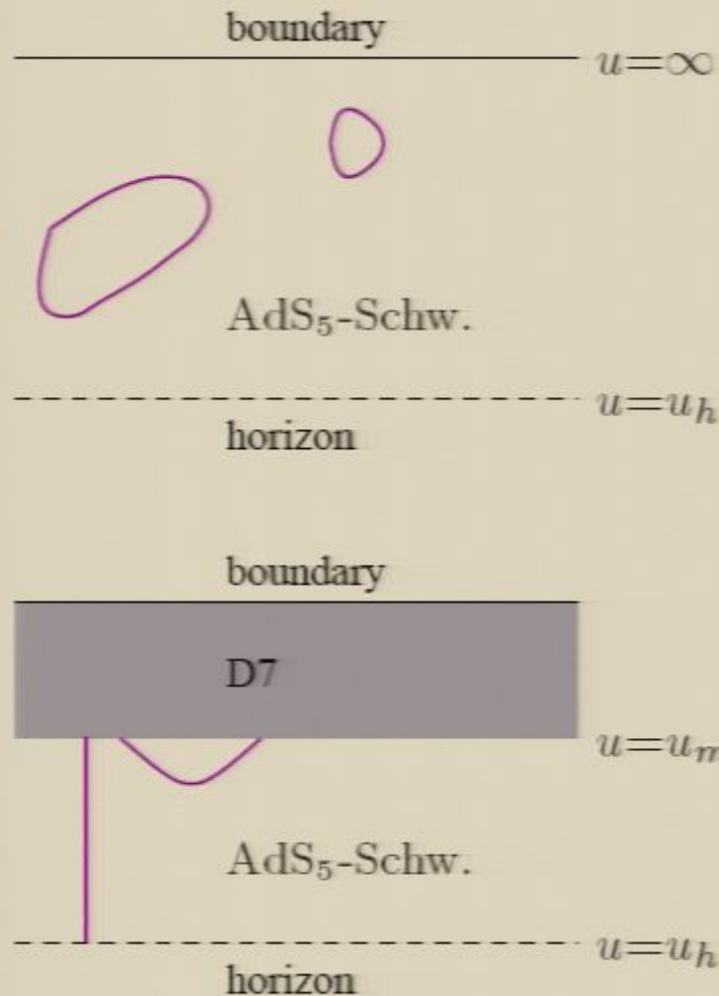
How to define energy loss?

What is the physical mechanism of the energy loss?

How does energy loss depend on mass, coupling, temperature?

These are **non-trivial** questions (keep in mind that perturbative language such as “bremsstrahlung”, “quark-gluon scattering” etc is useless when coupling is strong)

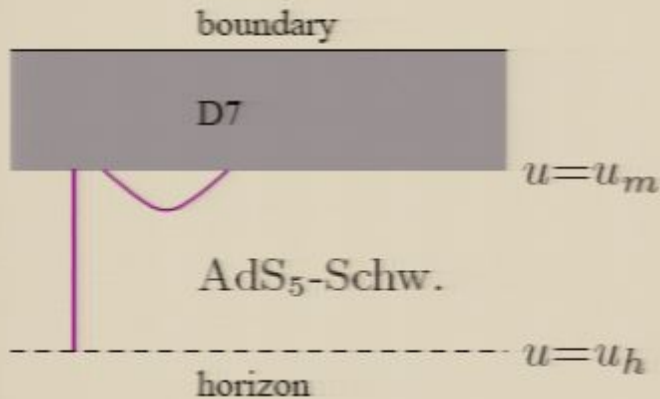
Model: $\mathcal{N}=4$ SYM + fundamental matter $N_f \ll N_c$



- Adjoint d.o.f. \Leftrightarrow closed strings (gravity) in the bulk
- Quantum fluctuations of SYM fields $(T_{\mu\nu}, J_\mu) \Leftrightarrow$ classical fluctuations of fields $(g_{\mu\nu}, A_\mu)$ in the bulk
- Finite temperature \Leftrightarrow black hole (brane) in the bulk
- When add fundamental d.o.f. \Leftrightarrow open strings ending on D7 brane in the bulk (A.Karch, A.Katz, [hep-th/0205236](https://arxiv.org/abs/hep-th/0205236))
- “Quark” mass set by u_m
- Quark configuration \Leftrightarrow classical string

Energy/momentum of a quark = energy/momentum of a classical string

How heavy a quark can one treat this way?



Static string: $E = \frac{\sqrt{\lambda}}{2\pi} u_m, \quad T = 0$

$E = \frac{\sqrt{\lambda}}{2\pi} (u_m - u_h), \quad T \neq 0$

thermal mass

$E_{\text{rest}} = m - \frac{1}{2} \sqrt{\lambda} T + \dots$

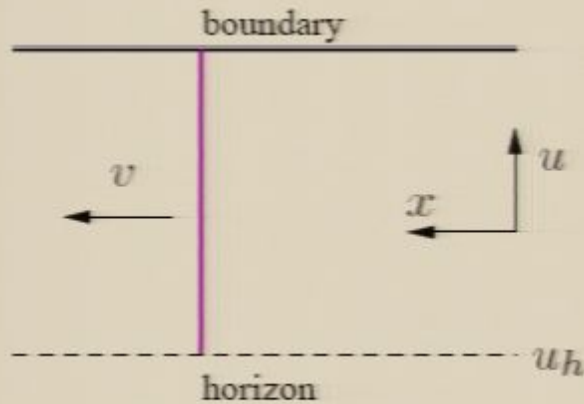
$\Delta m(T)$

When $m > 0.92 \Delta m(T)$ — D7 above the horizon, string is classical

To analyze energy loss of a heavy quark ($m > \frac{1}{2} \sqrt{\lambda} T$), solve classical equations of motion for a moving string

Friction coefficient – dissipation

Take infinitely heavy “quark” ($u_m \rightarrow \infty$), move it at constant speed

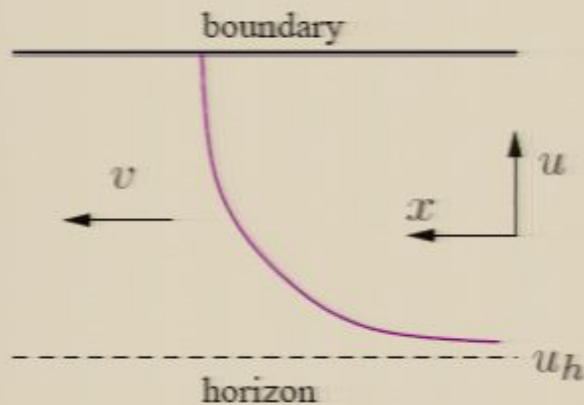


String profile: $x(u, t) = vt$ solves e.o.m.

however:

$-g$ flips sign at $u = \frac{u_h}{(1-v^2)^{1/4}} > u_h$

E, P become complex — solution unphysical



$x(u, t) = X(u) + vt$

Can find $X(u)$ s.t. $-g$ is positive everywhere

Source moves at constant speed

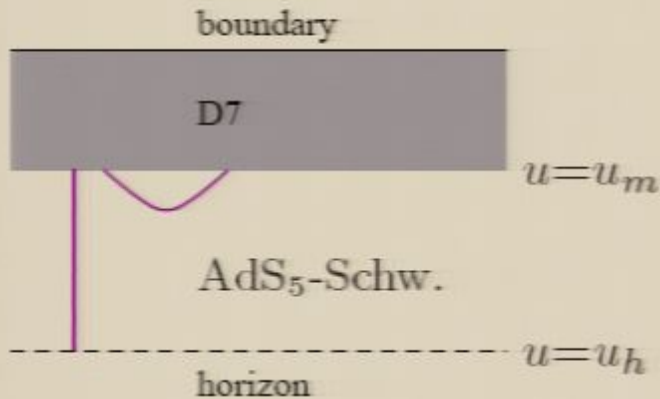
Momentum pumped in at the boundary

Momentum leaks off at the horizon

$$\frac{dP}{dt} = -\pi_x^1 \Big|_{u=u_h} = -\frac{\sqrt{\lambda}}{2\pi} \frac{v}{\sqrt{1-v^2}} (\pi T)^2 = -\left(\frac{\sqrt{\lambda} T^2 \pi}{2m} \right) \left(\frac{mv}{\sqrt{1-v^2}} \right) = -\mu P$$

Note that friction coefficient μ has a finite $\lambda \rightarrow \infty$ limit

Quasinormal modes – fluctuations

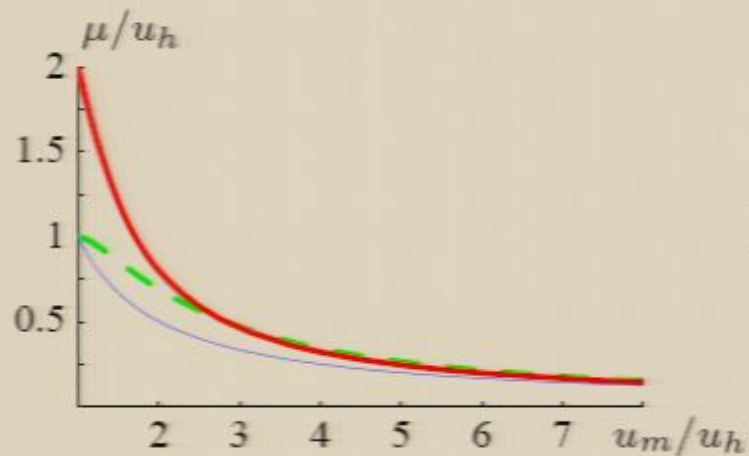


Linearize around static string

Neumann b.c. at the brane $u = u_m$

Outgoing b.c. at the horizon $u = u_h$

Lowest quasinormal mode on the worldsheet is purely imaginary, $x \sim e^{-\mu t}$



Numerical QNM:

Agrees with analytic result for $m \gg \Delta m(T)$

Gives μ for arbitrary $O(1)$ $m/\Delta m(T)$

Langevin dynamics – fluctuation-dissipation theorem

From field theory point of view:

effective equation of motion for the quark is stochastic,

$$\dot{\mathbf{p}} = -\mu\mathbf{p} + \boldsymbol{\xi}(t), \text{ with } \langle \xi_i(t)\xi_j(t') \rangle = 2T\mu m\delta_{ij}\delta(t-t')$$

Long-time behavior = diffusion, $|\Delta x| \sim \sqrt{Dt}$, with $D = \frac{T}{\mu m}$

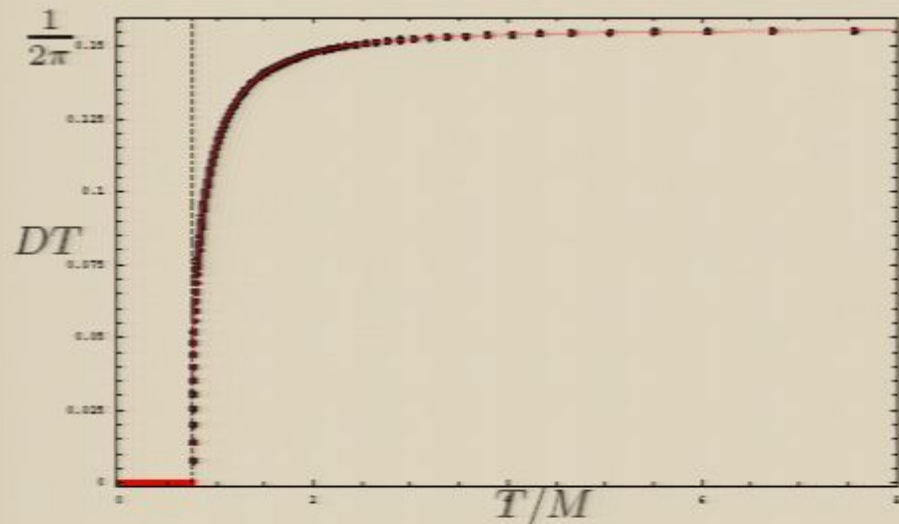
$$D = \frac{2}{\pi T \sqrt{\lambda}}$$

(Computed independently by J.Casalderrey-Solana, D.Teaney, [hep-ph/0605199](#))

Neglecting noise requires $\frac{1}{2}mv^2 \gg T$, or $v \gg \frac{1}{\lambda^{1/4}}$

Classical string does not see Brownian motion

Diffusion constant from current-current correlator



Picture from R.Myers, A.Starinets,
R.Thomson, **to appear**
(see also R.Myers, Strings-2006 talk)

Note that supergravity predicts $D = 0$ in the low-temperature phase.

However, this zero is really $D = \frac{2}{\pi T \sqrt{\lambda}}$, in the limit $\lambda \rightarrow \infty$

Evaluating D from current-current correlator remains to be done

Let's plug in the numbers: $\alpha_s = 0.5$

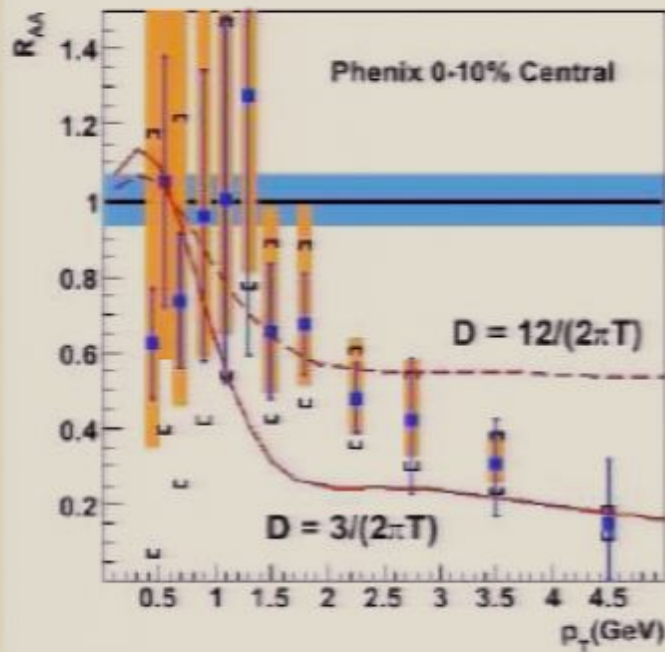
- Naively translates to $\lambda=19$, $\Rightarrow D \approx \frac{0.9}{2\pi T}$
- However, need to take λ smaller, e.g. $\lambda=4 \Rightarrow D \approx \frac{2}{2\pi T}$
- In perturbative QCD: $D \approx \frac{6}{2\pi T}$ (G.Moore, D.Teaney, [hep-ph/0412346](#))

Can these numbers be compared with RHIC data?

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Can these numbers be compared with RHIC data?



Nuclear modification factor $R_{AA}(p_T)$,
PHENIX data ([nucl-ex/0510047](https://arxiv.org/abs/nuc1-ex/0510047)).

Curves by D.Teaney in Langevin model

$D = \frac{3}{2\pi T}$ seems to work best

1/ λ corrections needed!

Lessons for heavy quark

The coupling of the probe to the surrounding medium is strong:

- energy loss is **not** due to collisions (there are particles to collide with)
- energy loss is **not** due to radiation of gluons (there are no gluons to radiate)
- energy loss is **not** due to emission of sound waves (string solution does not change at all as the supersonic barrier $v_s=1/\sqrt{3}$ is crossed)

What is the physics of energy loss?

Will discuss application of AdS/CFT to:

Momentum transport

Electromagnetic response

Energy loss by a heavy probe

→ Thermalization

AdS/CFT has more to say!

Thermalization

Thermal equilibrium in a collision established? If so:

On the partonic level at early stage, or on the hadronic level at a later stage?

At what temperature? How does the state evolve?

From AdS/CFT perspective [both solid results and speculations] :

H.Nastase, [hep-th/0501068](#), [hep-th/0603176](#), O.Aharony, S.Minwalla, T.Wiseman, [hep-th/0507219](#), E.Shuryak, S.Sin, I.Zahed, [hep-th/0511199](#), R.Janik, R.Peschanski, [hep-th/0512162](#), R.Janik, [hep-th/0610144](#)

however:

Any attempt to understand thermalization in heavy-ion collisions from AdS/CFT must:

- be able to distinguish between hadron-hadron and Au+Au collisions
- be able to see transition from weak to strong coupling
- quantitatively understand the role of finite N_c

Will discuss application of AdS/CFT to:

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→ AdS/CFT has more to say!

What's next? Formalism

- How to compute transport coefficients in the confining phase? Not a single AdS/CFT calculation exists! (Euclidean correlation functions are in practice useless to extract viscosity)
- How is hydrodynamics encoded in the bulk? Linear hydro is only understood from correlation functions; non-linear hydro is not understood at all.
- How to describe Brownian motion from the bulk point of view? Need α' corrections.
- Isotropization may be easier than thermalization. How does an anisotropic state relax to equilibrium? (Initial-value problem for gravity in the bulk)
- Hadronization as the fireball cools. Keep in mind however that in AdS/QCD deconfinement transitions are typically strongly first order, while in QCD it is a crossover.

What's next? Practice

- Photons: try SYM spectral functions instead of QCD spectral functions in hydro simulations of photon production at RHIC.
- Finite coupling corrections to photon production. Remember, one needs $\lambda \sim 2-5$, not $\lambda = \infty$.
- Photons from fundamental representation matter fields ($\mathcal{N}=4$ SYM has adjoints only)?
- Now that one has the method to compute spectral functions: what does AdS/CFT say about heavy-quark resonances? Lattice studies for $c\bar{c}$ are available...
- Energy loss by light quarks.
- All of the above, now with non-zero chemical potential.
- Prove universality of η/s , or the bound $\eta/s \geq 1/4\pi$?
- Hydrodynamic long-time tails from AdS/CFT: there is a non-trivial and universal field theory prediction! String calculation needed.

THE END