Title: How can AdS/CFT be useful for heavy-ion physics?

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Abstract: TBA

Can AdS/CFT be useful for heavy-ion physics?

Pavel Kovtun

KITP, University of California, Santa Barbara

A review of many people's work

seminar at Perimeter Institute

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What is heavy-ion physics?

Heavy-ion collisions — study QCD at high energy density

Experiment: **RHIC** (Brookhaven, NY)

- Started in year 2000
- Collides Au nuclei
- CM energy $\sqrt{s}$=200 GeV per nucleon

Quest: find and study QGP [deconfined state of QCD] → field theory at finite temperature and density. Not obvious *a priori* that a thermal state will be produced.

Evidence for thermalization [lots of data and non-trivial calculations] :

Particle abundances and ratios — reproduced by statistical models

Elliptic flow — reproduced by hydrodynamic models

Jet quenching — indication of short mean free path

Optimistically, QGP is hidden in the collision

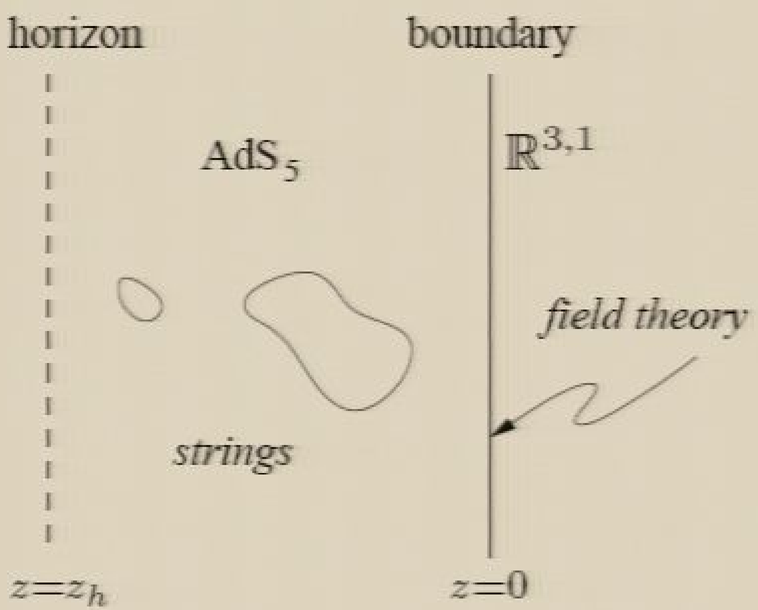
What is AdS/CFT

(J.Maldacena hep-th/9711200, review: hep-th/9905111)

$$\boxed{\text{large } N_c,\ d{=}4,\ \mathcal{N}{=}4 \text{ SYM} \ = \ \text{IIB strings on } AdS_5 \times S^5}$$

$\lambda \leftrightarrow \left(\frac{R^2}{\alpha'}\right)^2$ string corrections to SUGRA

$\frac{\lambda}{4\pi N_c} \leftrightarrow g_s$ string loops



$$\langle e^{\int h(x)T(x)}\rangle_{\text{field}} = Z_{\text{string}}[g(x, z{\to}0) = h(x)]$$

$$T_{\mu\nu}(x) \quad \leftrightarrow \quad h_{\mu\nu}(x, z{\to}0)$$

$$J_\mu(x) \quad \leftrightarrow \quad A_\mu(x, z{\to}0)$$

$$\text{tr}F^2(x) \quad \leftrightarrow \quad \varphi(x, z{\to}0)$$

$$\vdots$$

$$\therefore\ \langle T_{\mu\nu}T_{\alpha\beta}\rangle \sim \frac{\delta^2 \ln Z_{\text{string}}[h]}{\delta h_{\mu\nu}\delta h_{\alpha\beta}} \sim \frac{\delta^2}{\delta h_{\mu\nu}\delta h_{\alpha\beta}} S_{\text{cl}}[h]$$

AdS/CFT is a **tool** to define/perform calculations in field theory

Applies to field theories beyond $\mathcal{N}{=}4$ SYM

RHIC + AdS/CFT = ♡ ?

Modest: Can AdS/CFT be useful to understand finite-tempearture QCD?

Bold: Can AdS/CFT be useful to understand the dynamics of the collision?

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Modest: Can AdS/CFT be useful to understand finite-tempearture QCD?

Bold: Can AdS/CFT be useful to understand the dynamics of the collision?

Comments

- $\mathcal{N}$=4 SYM is the simplest example. Theories which are more similar to QCD can be treated by AdS/CFT methods.
- Application of AdS/CFT to thermal QCD is not exhausted. How far can we push this program?
- It is not a waste of time to do these calculations. Results are relatively easy to derive compared to the conventional methods.
- If there were an effective tool to do real-time computations in strongly coupled QCD at finite T and μ — no need to invoke AdS/CFT. In the absense of such a tool, AdS/CFT is the best we have (for some questions).
- Understanding finite-temperature field theory is an interesting question by itself. AdS/CFT can be useful in searching for universal properties (shear viscosity example is encouraging).

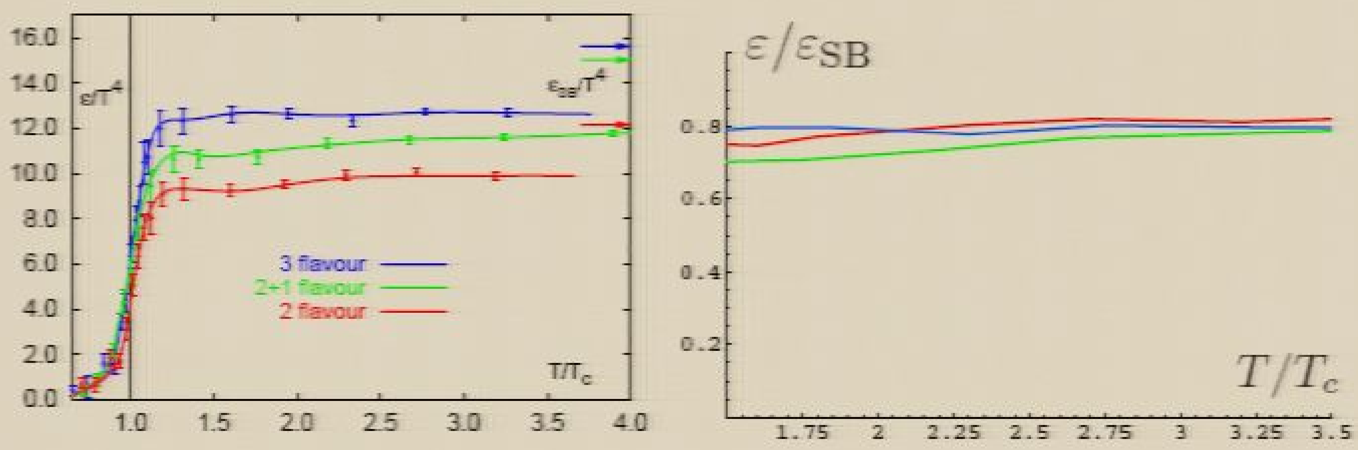
Why $\mathcal{N}$=4 SYM may have something to do with thermal QCD

Q: $\mathcal{N}$=4 SYM is supersymmetric, while QCD is not

A: At finite temperature, supersymmetry is broken anyway

Q: $\mathcal{N}$=4 SYM is conformal, while QCD is asymptotically free

A: Let's look at the thermodynamics of QCD (e.g. F.Karsch, `hep-lat/0106019`)



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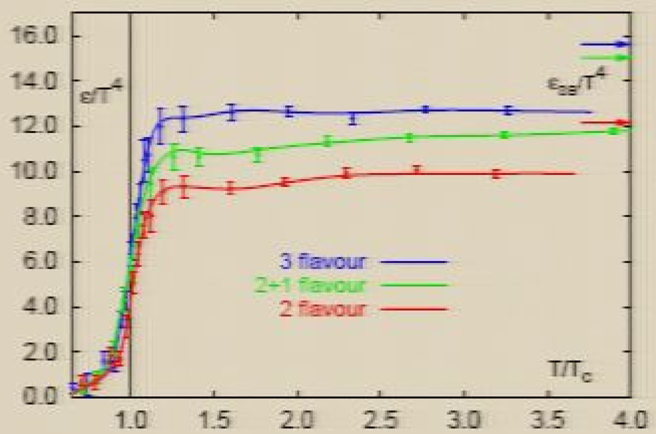

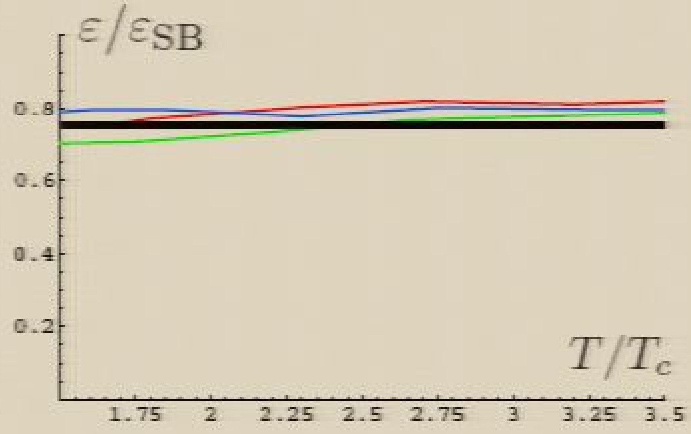

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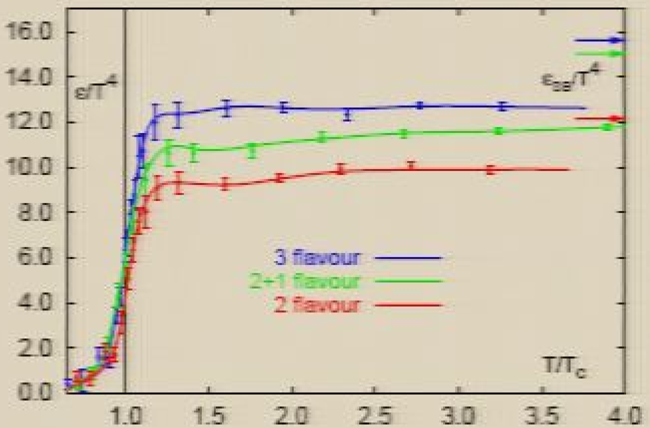

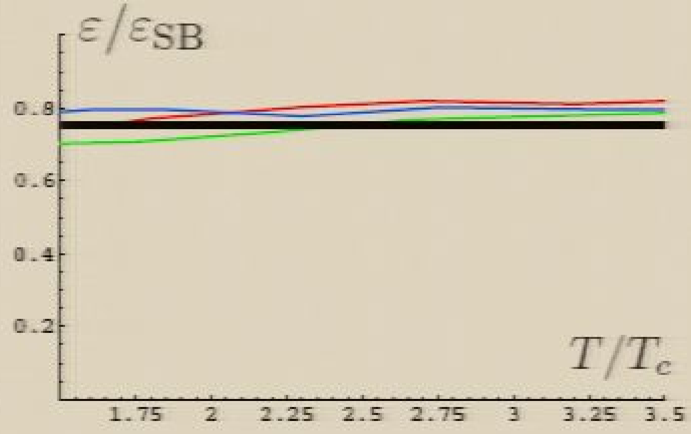

Q: At high temperature, QCD is weakly coupled, why not use perturbation theory?

A: True only at asymptotically high temperature. At RHIC temperatures, $T \sim 2T_c$, and QCD is strongly coupled, $\alpha_s(T) = O(1)$.

Q: QCD is a sensible theory, $\mathcal{N}=4$ SYM is just a bunch of non-abelian fields

A: So is QCD, above deconfinement

Lesson:

$\boxed{\text{Use strongly coupled } \mathcal{N}{=}4 \text{ SYM as a model for QCD at } T \gtrsim T_c}$

Will discuss application of AdS/CFT to:

- Momentum transport
- Electromagnetic response
- Energy loss by a heavy probe
- Thermalization

- AdS/CFT has more to say!

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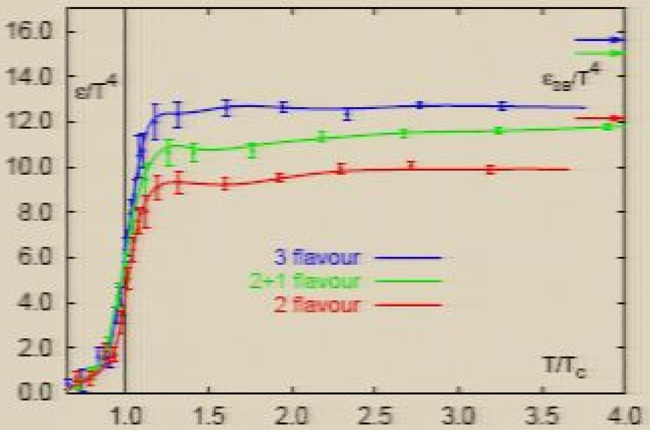

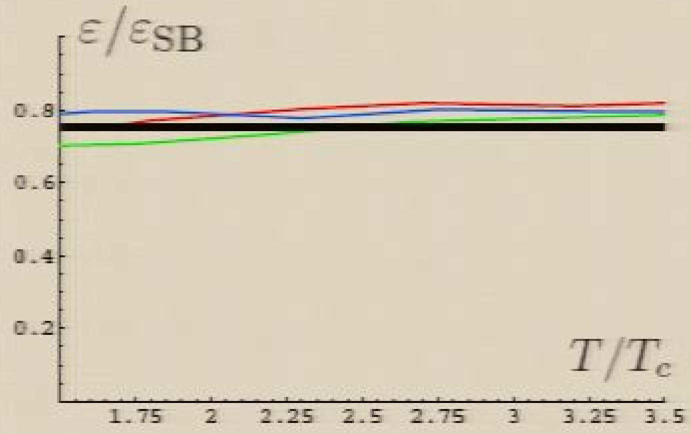

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Momentum transport

- Conservation laws: $\partial_\mu T^{\mu\nu}=0 \quad \Rightarrow \quad \begin{cases} \partial_t \epsilon = -\boldsymbol{\nabla}\cdot\boldsymbol{\pi} \\ \partial_t \pi^i = -\boldsymbol{\nabla}_j T^{ij} \end{cases}$
- Constitutive relations:

$$\begin{cases} T^{ij} = \delta^{ij}\left[\langle P\rangle + v_s^2\delta\epsilon - \gamma_\zeta \boldsymbol{\nabla\pi}\right] - \gamma_\eta\left(\boldsymbol{\nabla}^i\pi^j + \boldsymbol{\nabla}^j\pi^i - \frac{2}{3}\delta^{ij}\boldsymbol{\nabla\pi}\right) + \ldots \\ \gamma_\eta \equiv \dfrac{\eta}{\langle\epsilon+P\rangle},\ \gamma_\zeta \equiv \dfrac{\zeta}{\langle\epsilon+P\rangle},\ v_s^2 = \partial P/\partial\epsilon \end{cases}$$

- Viscosities η, ζ — input from microscopic physics

Two eigenmodes:

Shear mode: $\pi_\perp(t,\boldsymbol{k}) = e^{-\gamma_\eta \boldsymbol{k}^2 t}\,\pi_\perp(0,\boldsymbol{k})$

Sound mode: $\boldsymbol{\pi}_\parallel(t,\boldsymbol{k}) = e^{-\frac{1}{2}(\gamma_\zeta + \frac{4}{3}\gamma_\eta)\boldsymbol{k}^2 t}\times$

$$\times\left[\boldsymbol{\pi}_\parallel(0,\boldsymbol{k})\cos(kv_s t) - i\hat{k}v_s\sin(kv_s t)\delta\epsilon(0,\boldsymbol{k})\right]$$

Long-wavelength response is controlled by a small number of kinetic coefficients

Correlation functions in the hydrodynamic limit

Hydrodynamic modes $\Rightarrow$ hydrodynamic singularities at small ω, k.

Example: $S_{tx,tx}(\omega,k) = \dfrac{2\gamma_\eta k^2}{\omega^2 + (\gamma_\eta k^2)^2}(\epsilon+P)T$ relaxation of transverse momentum

Kubo formulas $\boxed{\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dt\, e^{i\omega t}\int d^3x\, \langle[T_{xy}(t,\boldsymbol{x}), T_{xy}(0)]\rangle}$

$$\boxed{\frac{4}{3}\eta + \zeta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dt\, e^{i\omega t}\int d^3x\, \langle[T_{xx}(t,\boldsymbol{x}), T_{xx}(0)]\rangle}$$

Connection to microscopic physics: Viscosities can be extracted from (small-frequency limits of) real-time correlation functions

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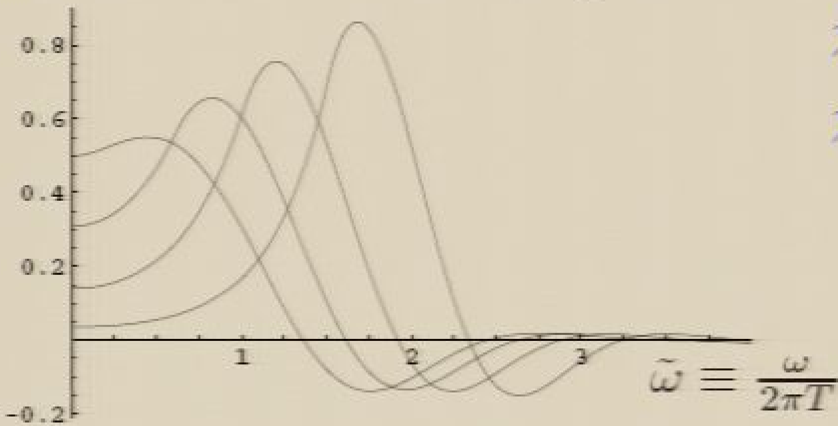
Connection to microscopic physics: Viscosities can be extracted from (small-frequency limits of) real-time correlation functions

Spectral function for stress

(P.K., A.Starinets hep-th/0602059, D.Teaney hep-ph/0602044)

$\frac{1}{\tilde{\omega}}\left(\chi(\tilde{\omega}) - \chi^{T=0}(\tilde{\omega})\right)\ \left[\frac{1}{\pi^2 N_c^2 T^4}\right]$



$$\chi(\omega, k) = -2\,\mathrm{Im}\, G^{\mathrm{ret}}_{xy,xy}(\omega, k)$$

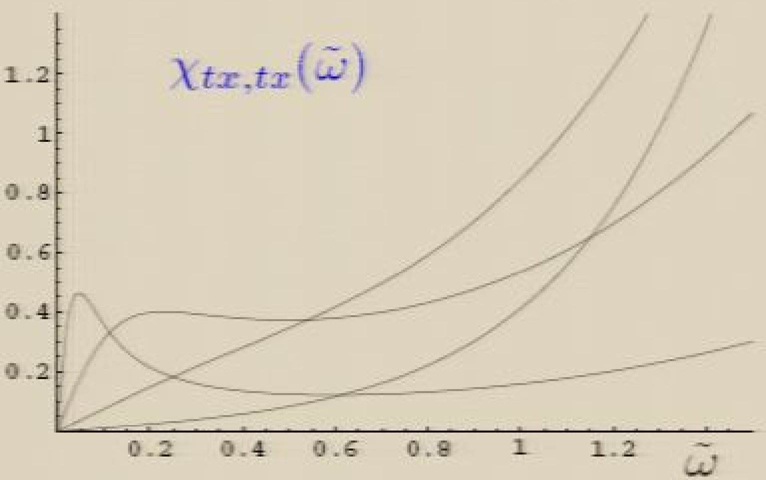
$$\chi(\omega) \sim \omega, \quad \omega \ll 2\pi T$$

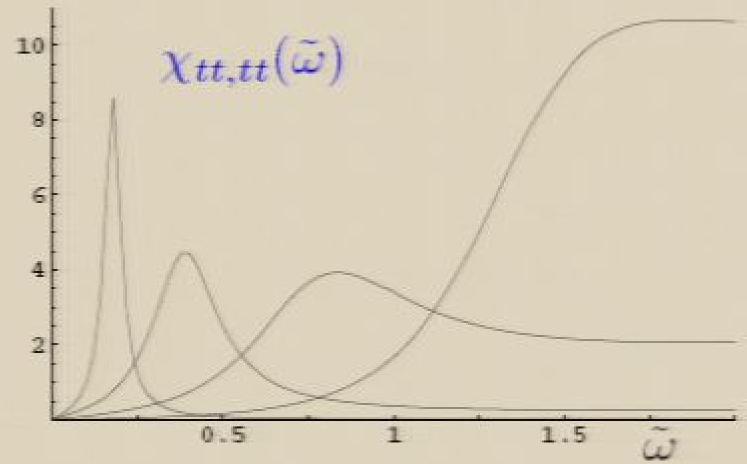
$$\chi(\omega) - \chi^{T=0}(\omega) \sim e^{-\gamma\omega}, \quad \omega \gg 2\pi T$$

$$\boxed{\eta = \frac{\pi}{8} N_c^2 T^3}$$

T^3 by conformal invariance, N_c^2 counts d.o.f.

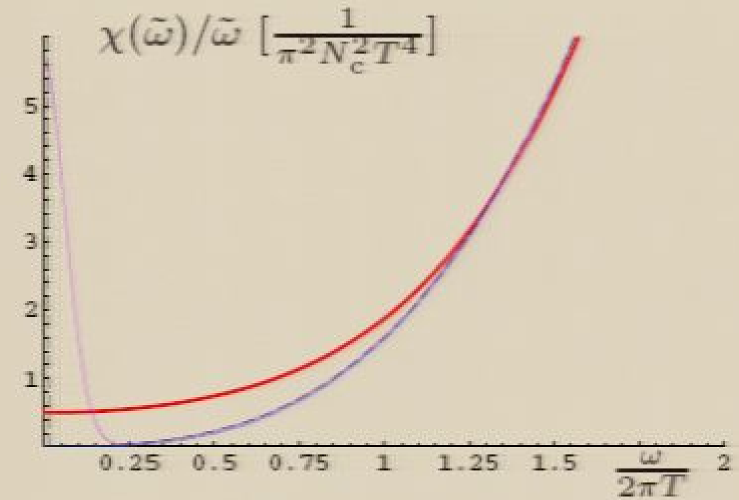
Spectral function for conserved energy-momentum





Hydrodynamic peaks clearly visible in dual classical gravity

Real-time correlators are very different at strong and weak coupling

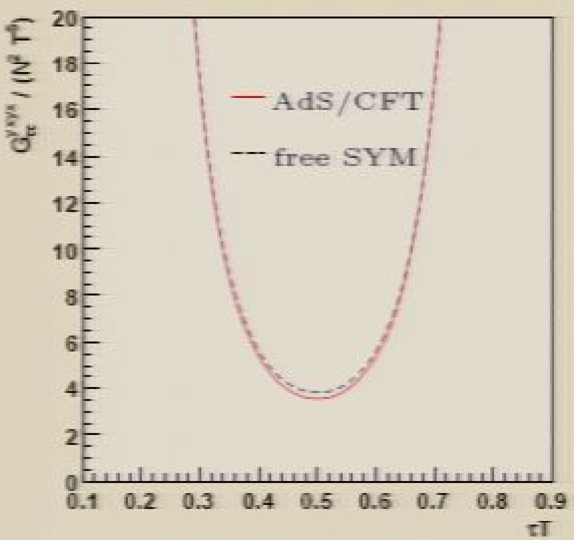


Red — strongly coupled SYM, $\eta = O(1)$

Dashed — free SYM, $\eta = \infty$

Purple — weakly coupled SYM, $\eta \sim \frac{1}{\lambda^2}$

Blue — SYM, $T = 0$



Euclidean correlators are almost the same!

(picture from D.Teaney, hep-ph/0602044)

$$G_{\rm E}(\tau) = \int_0^\infty d\omega\ \chi(\omega) \frac{\cosh[\omega(\tau-\beta/2)]}{\sinh(\beta\omega/2)}$$

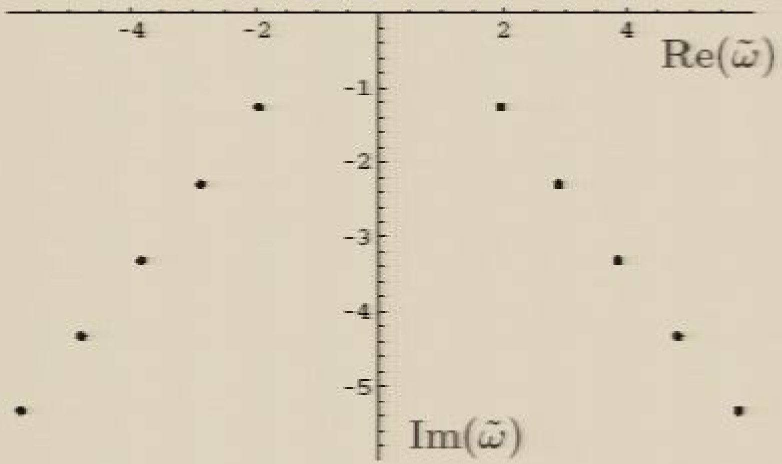
Lattice $G_{\rm E}(\tau)$ has errorbars ~500% (hep-lat/0406009)

$\therefore$ Lattice determination of shear viscosity has no chance

Singularities of $G^{\text{ret}}(\omega, k)$

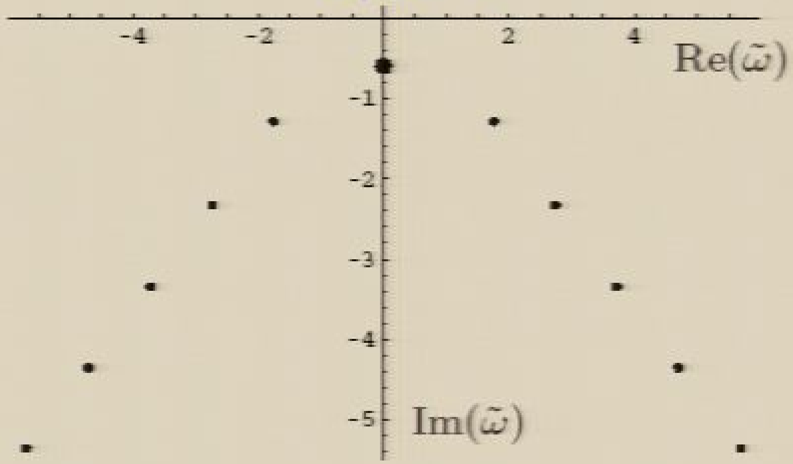
(A.Nunez, A.Starinets hep-th/0302026; P.K., A.Starinets hep-th/0506184)

$G^{\text{ret}}_{xy,xy}(\omega)$

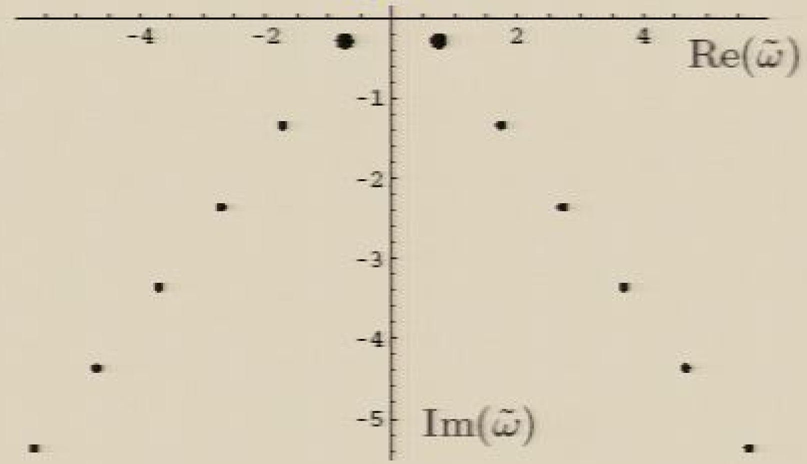


- Infinite series of poles
- $\omega_n = 2\pi n T(\pm 1 - i)$ as $n \to \infty$
- For conserved densities, $\omega_0 \to 0$ as $k \to 0$
- Hydro poles agree with Kubo formula

$G^{\text{ret}}_{tx,tx}(\omega)$

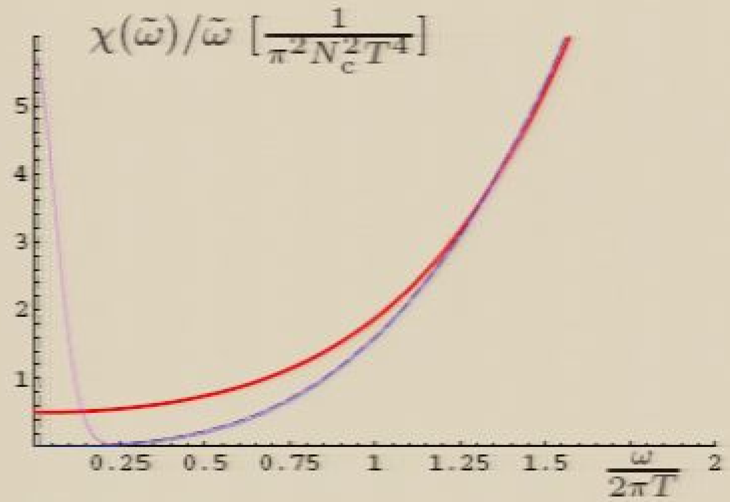


$G^{\text{ret}}_{tt,tt}(\omega)$



Singularities of $G^{\text{ret}}(\omega, k)$ are (quasi)normal modes of the dual gravity background

Real-time correlators are very different at strong and weak coupling



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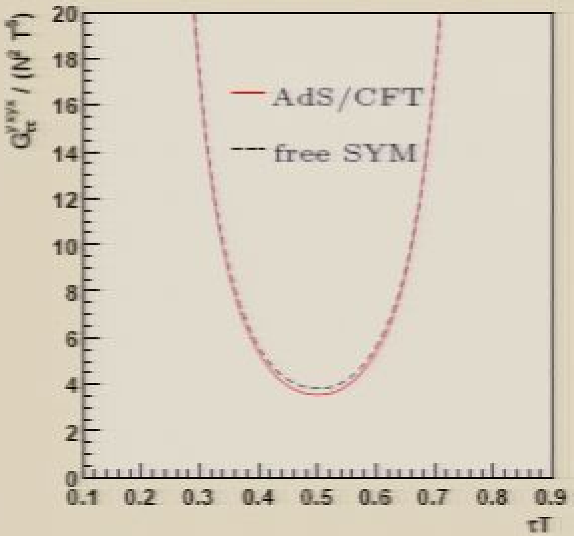

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Universality: Lower bound on shear viscosity?

(P.K., D.Son, A.Starinets hep-th/0405231)

Life at low Reynolds number

E. M. Purcell

he can predict is the order of magnitude of the exponent. But it's more mysterious than that, Viki, because if you look at the Chemical Rubber Handbook table you will find that there is almost no liquid with viscosity much lower than that of water. The viscosities have a big range *but they stop at the same place.* I don't understand that. That's what I'm leaving for him.[1]

!!

- $\eta/s \gg 1$ at small coupling in SYM

  (S.Huot, S.Jeon, G.Moore, hep-ph/0608062)
- $\eta/s = \frac{1}{4\pi}$ is finite at large coupling
- Natural to assume $\eta/s \geqslant \frac{1}{4\pi}$ in SYM

$$\boxed{\text{Is } \frac{\eta}{s} \geqslant \frac{1}{4\pi} \text{ universal?}}$$

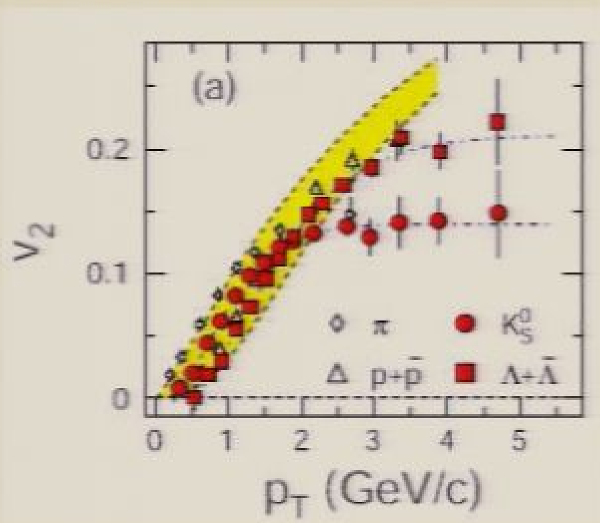
We know $\frac{\eta}{s} = \frac{1}{4\pi}$ is universal — proven for a large class of field theories with gravity duals.

Prove from first principles?

What is the viscosity measured at RHIC?

Of course, RHIC does not "measure" viscosity. Rather, measured angular distributions of particles are confronted with hydrodynamic models of the "fireball" evolution. Quantitatively: lentil-shaped reaction region $\Rightarrow$ azimuthal anisotropy of particle distribution

$$\frac{d^2N}{dp_T d\phi} = N_0 \left[1 + 2\, \underbrace{v_2}_{\text{"elliptic flow"}}(p_T) \cos(2\phi) + \ldots\right]$$



Elliptic flow from PHENIX and STAR (figure from nucl-ex/0501009). Yellow band — hydro calculations. To reproduce elliptic flow and spectra from hydro, η/s must be small, $\eta/s \lesssim 0.3$ [Teaney; Baier, Romastschke].

Perturbative QCD:

See Arnold, Moore, Yaffe, hep-ph/0010177

$\eta/s \approx 1.6 - 1.8$ at relevant T (L.Csernai, J.Kapusta, L.McLerran, nucl-th/0604032)

Optimistically: $\mathcal{N}=4$ SYM does very well with η/s

Will discuss application of AdS/CFT to:

Momentum transport

→ Electromagnetic response

Energy loss by a heavy probe

Thermalization

AdS/CFT has more to say!

In RHIC context see e.g.

C.Gale, hep-ph/0512109

P.Stankus, Ann.Rev.Nucl.Part.Sci.55:517,2005

Production of real and virtual photons

(e.g. L.D.McLerran, T.Toimela, PRD **31**, 545 (1985), H.A.Weldon, PRD **42**, 2384 (1990))

Γ — number of photons per unit time per unit volume

Photon interaction: $e\, J^{\mathrm{EM}}_{\mu} A^{\mu}$; el. charge e small $\Rightarrow$ photons do not thermalize

$$d\Gamma = \frac{d^3k}{(2\pi)^3}\frac{e^2}{2|\boldsymbol{k}|}\,\eta^{\mu\nu}C^{<}_{\mu\nu}(k)\Big|_{\text{lightlike } k} \quad \text{where } C^{<}_{\mu\nu}(x) = \langle J^{\mathrm{EM}}_{\mu}(0)J^{\mathrm{EM}}_{\nu}(x)\rangle$$

Virtual photon can decay into a lepton pair: $d\Gamma = \dfrac{d^4k}{(2\pi)^4}\dfrac{e^2e_\ell^2}{6\pi k^2}\eta^{\mu\nu}C^{<}_{\mu\nu}(k)\Big|_{\text{timelike } k}$

$\boxed{\text{Emission spectra are determined by EM current-current spectral function}}$

true to leading order in e, but to all orders in g

In SYM, can evaluate the whole "blob" at large $\lambda \equiv g^2 N_c$ (using AdS/CFT)

Production of real and virtual photons from $\mathcal{N}=4$ plasma

(S.Caron-Huot, P.K., G.Moore, A.Starinets, L.Yaffe, hep-th/0607237)

But wait... $\mathcal{N}=4$ SYM does not have a photon [$U(1)$ gauge field coupled to a conserved current]

Let's introduce one! To do so:

Gauge a $U(1)$ subgroup of $SU(4)$ R-symmetry with small coupling e

The symmetry is not anomalous

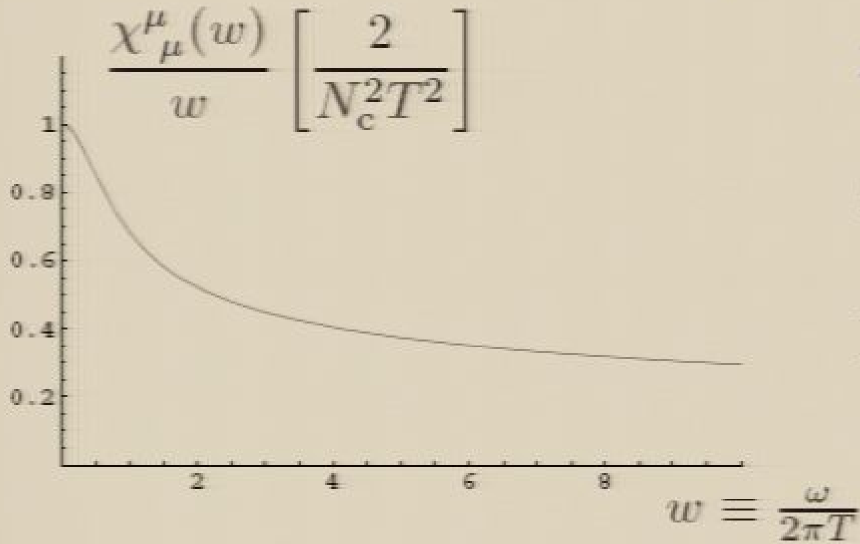
EM current $J_\mu^{\mathrm{EM}}(x) \equiv J_\mu^{\mathrm{R}}(x)$ — conserved, non-anomalous, couples to a $U(1)$ gauge field

> Photon and dilepton emission rate is given by R-current spectral function

In SYM can compute $\chi^\mu{}_\mu(\omega) \equiv -2\,\mathrm{Im}\,\eta^{\mu\nu} C^{\mathrm{RET}}_{\mu\nu}(\omega, q)\Big|_{\omega=q}$ at strong coupling, then emission rate is

$$\boxed{\frac{d\Gamma}{d^3k} \propto n_B(\omega)\frac{\chi^\mu{}_\mu(\omega)}{\omega}}$$

On-shell photons at strong coupling



$$\chi^{\mu}{}_{\mu}(\omega) = -2\,\mathrm{Im}\,\eta^{\mu\nu}C^{\mathrm{RET}}_{\mu\nu}(\omega, q)\Big|_{\omega=q}$$

Small frequency:

$\chi^{\mu}{}_{\mu}(w)\sim\frac{1}{2}N_c^2T^2w$, in accord with hydro

High frequency:

$\chi^{\mu}{}_{\mu}(w) \sim \frac{N_c^2T^2}{4}w^{2/3}\left[3^{5/6}\frac{\Gamma(2/3)}{\Gamma(1/3)}\right]$

Spectral function for on-shell photons can be computed in closed form!

$$\boxed{\chi^{\mu}{}_{\mu}(w) = \frac{N_c^2T^2w}{8}\left|{}_2F_1\left(1-\frac{1}{2}(1+i)w, 1+\frac{1}{2}(1-i)w; 1-iw; -1\right)\right|^{-2}}$$

Emission rate is finite and λ-independent in the limit of large λ

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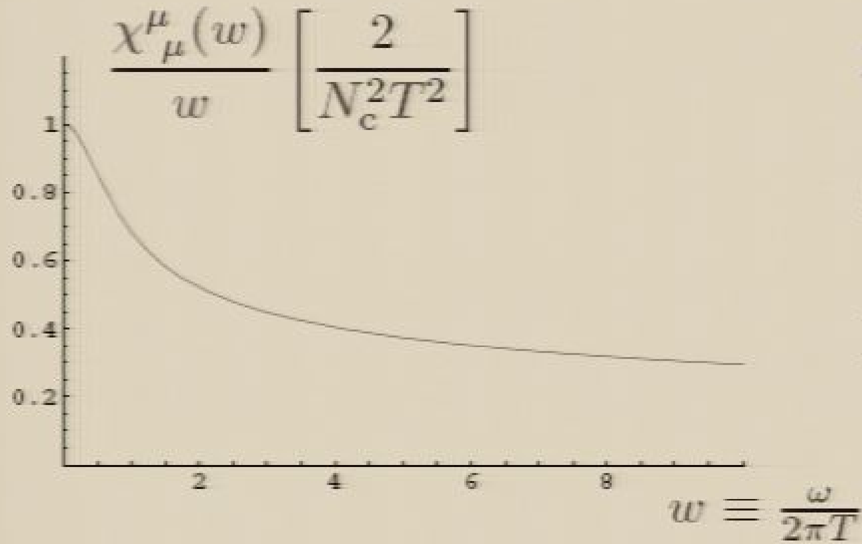

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High frequency:

$\chi^\mu{}_\mu(w) \sim \frac{N_c^2T^2}{4}w^{2/3}\left[3^{5/6}\frac{\Gamma(2/3)}{\Gamma(1/3)}\right]$

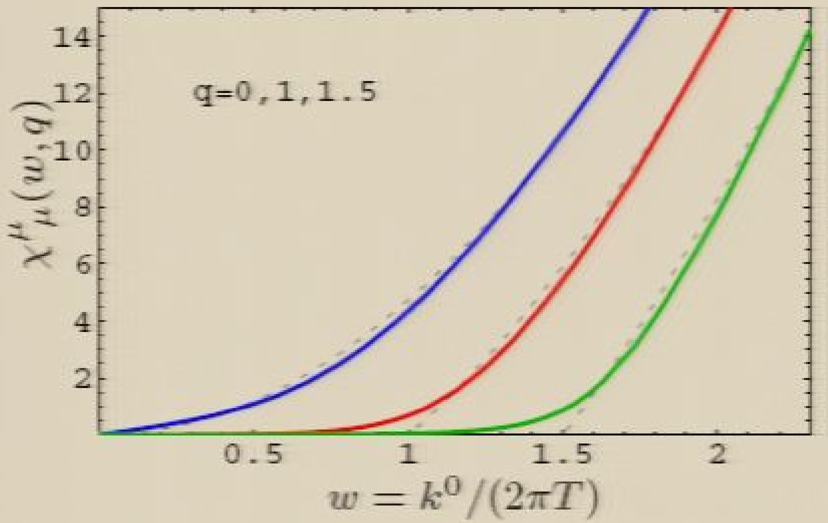
Spectral function for on-shell photons can be computed in closed form!

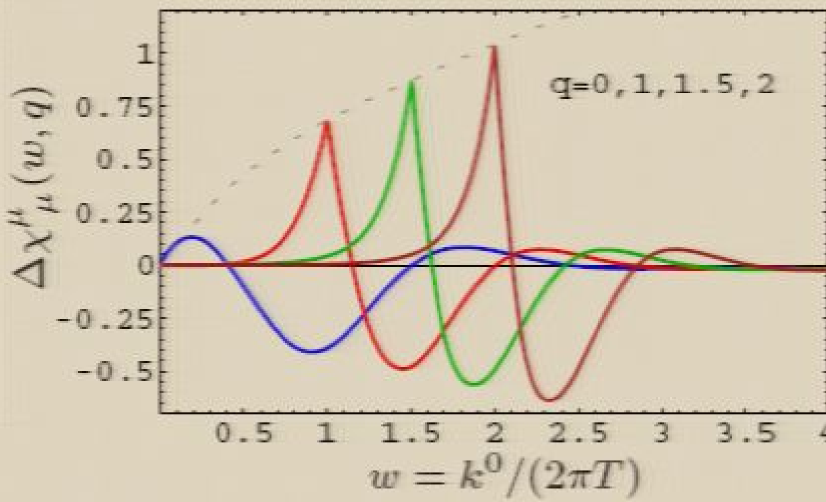
$$\boxed{\chi^\mu{}_\mu(w) = \frac{N_c^2T^2w}{8}\left|{}_2F_1\left(1-\frac{1}{2}(1+i)w, 1+\frac{1}{2}(1-i)w; 1-iw; -1\right)\right|^{-2}}$$

Emission rate is finite and λ-independent in the limit of large λ

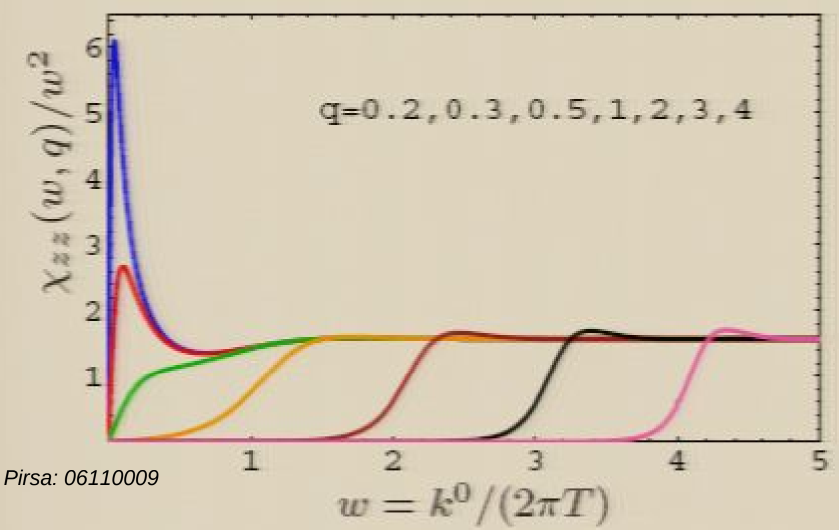
Off-shell photons at strong coupling

Plot spectral function for several values of $q \equiv k/(2\pi T)$





Spectral function oscillates around the zero-temperature value... where is hydro?



In the hydro limit:

$$\frac{\chi_{zz}(\omega,k)}{\omega^2} = \frac{\omega\, 2D\Xi}{\omega^2 + (Dk^2)^2}$$

Diffusion bump clearly visible at small ω and k

Electrical conductivity

Kubo formula: $\sigma = e^2 \lim_{\omega\to 0} \frac{\chi_{ii}(\omega, q{=}0)}{6\omega}$

In strongly coupled SYM: $\sigma = e^2 \frac{N_c^2 T}{16\pi}$, does not depend on λ

In weakly coupled SYM: $\sigma = e^2 \frac{N_c^2 T}{\lambda^2 \ln(1/\lambda)} C$, where $C{=}O(1)$

Normalize to the number of degrees of freedom: $\frac{\sigma}{e^2 \Xi}$, where Ξ is charge susceptibility. In strongly coupled SYM,

$$\boxed{\frac{\sigma}{e^2\Xi} = \frac{1}{2\pi T}}$$

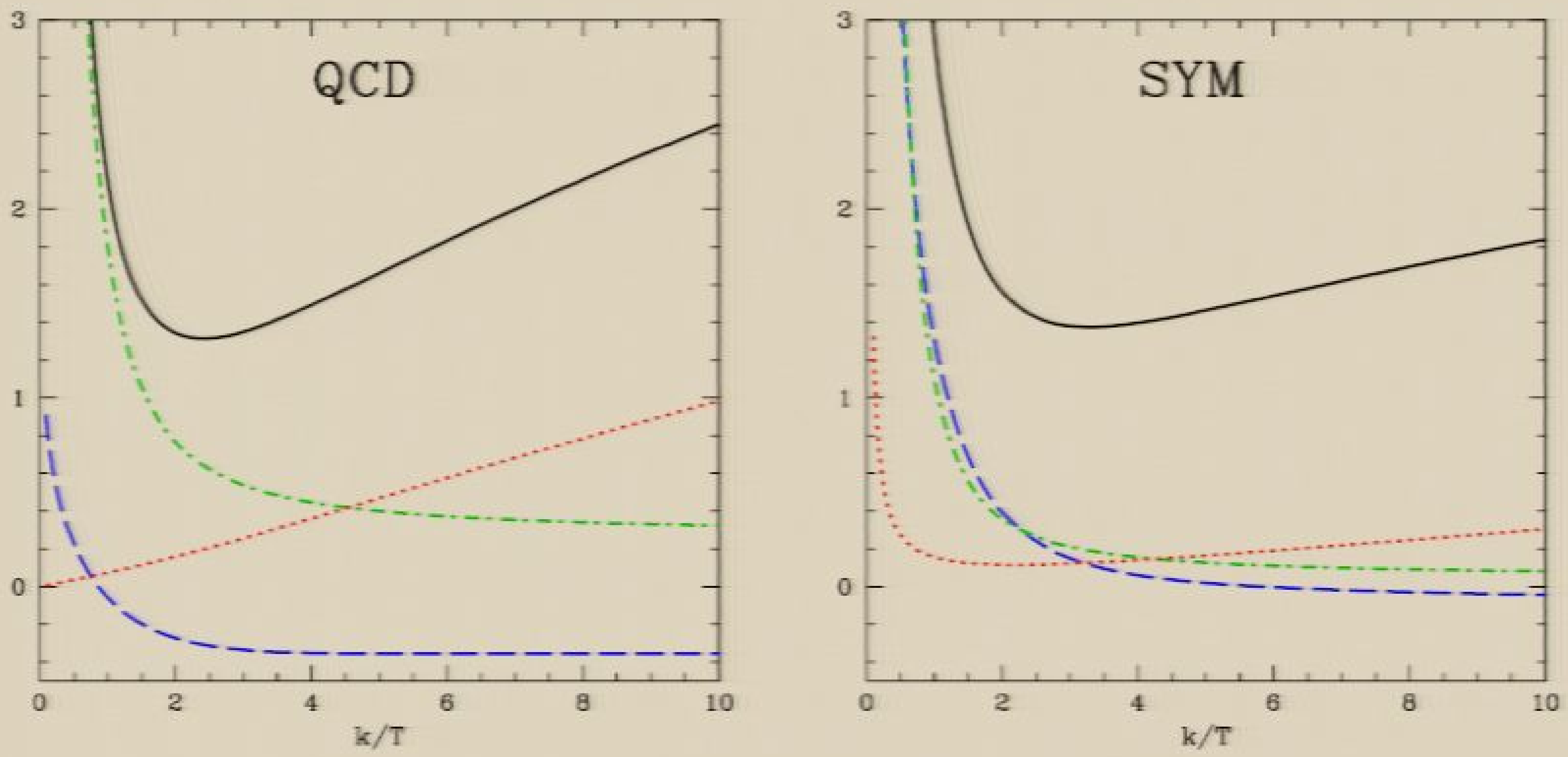
Einstein relation $\frac{\sigma}{e^2\Xi} = D$ is satisfied, as it should.

On-shell photons at weak coupling

$\mathcal{N}=4$ SYM is simply a non-abelian plasma with adjoint representation matter.

Follow approach of Arnold-Moore-Yaffe (hep-ph/0111107) for QCD...

$$\chi^{\mu}{}_{\mu}(k) = \frac{\lambda(N_c^2-1)T^2\, n_F(k)}{4\pi\, n_B(k)} \left[\ln\lambda^{-1/2} + C_{\rm tot}(k/T) + O(\sqrt{\lambda})\right], \quad k \gg \lambda^2 T$$



Momentum dependent $C_{\rm tot}(k/T)$ is not too different between QCD and SYM

Off-shell photons at weak coupling

For time-like momenta, $\chi_{\mu\nu}(k)$ is non-zero already at $O(\lambda^0)$. One-loop diagram:

$$\chi^{\mu}{}_{\mu}(K) = \frac{(N_c^2-1)}{16\pi}(-K^2)\left[3 - \frac{2T}{k}\ln\frac{1+e^{-(k^0-k)/2T}}{1+e^{-(k^0+k)/2T}} + \frac{T}{k}\ln\frac{1-e^{-(k^0+k)/2T}}{1-e^{-(k^0-k)/2T}}\right]$$

LPM resummation for dileptons in $\mathcal{N}=4$ SYM: needs to be done...

Electrical conductivity

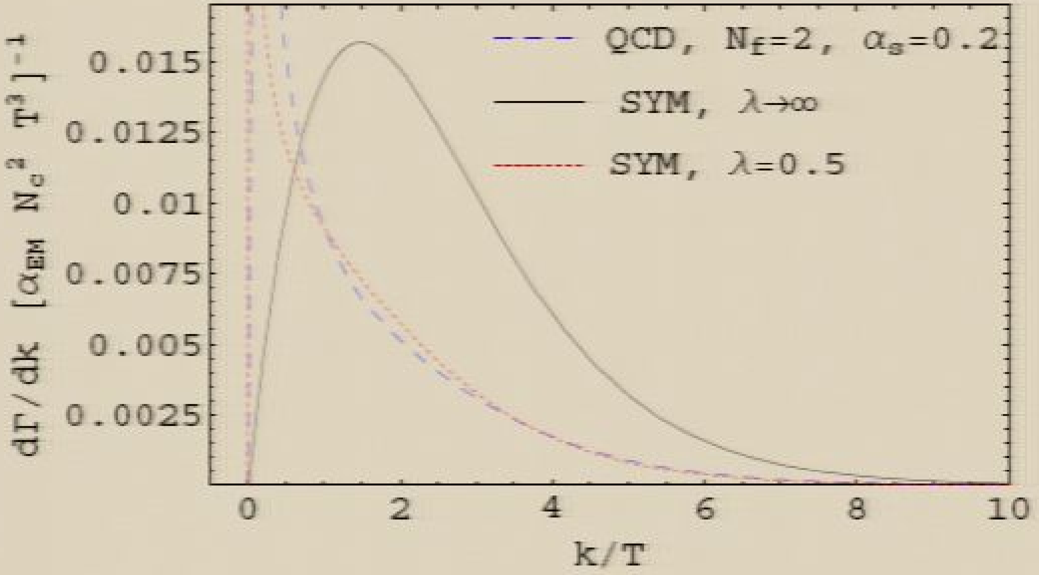
Evaluated at leading log order, following Arnold-Moore-Yaffe, hep-ph/0010177

$$\sigma = 1.28349\,\frac{e^2(N_c^2-1)T}{\lambda^2\left[\ln(\lambda^{-1/2}) + O(1)\right]}\,.$$

The $O(1)$ constant: needs to be done...

Now can compare weakly coupled SYM and strongly coupled SYM

Photon emission



At weak coupling:

$k \gg \lambda^2 T$, follow AMY

$k \ll \lambda^2 T$, hydro regime

$k \sim \lambda^2 T$: technically hard. Height of the peak is unknown at weak coupling in either SYM or QCD.

At strong coupling:

$$\frac{d\Gamma_\gamma}{dk} = \frac{\alpha_{\mathrm{EM}} N_c^2 T^3}{16\pi^2} \frac{(k/T)^2}{e^{k/T}-1} \left| {}_2F_1\!\left(1 - \frac{(1+i)k}{4\pi T}, 1 + \frac{(1-i)k}{4\pi T}; 1 - \frac{ik}{2\pi T}; -1\right)\right|^{-2}$$

– emission rate for *all* k is given by a simple *analytic* expression

– hydro limit is naturally reproduced when $k \ll T$

– as coupling grows, the rate becomes finite and *coupling-independent*

At small k/T, photon emission rate is greater in *weakly-coupled* theory

At large k/T, photon emission rate is greater in *strongly-coupled* theory

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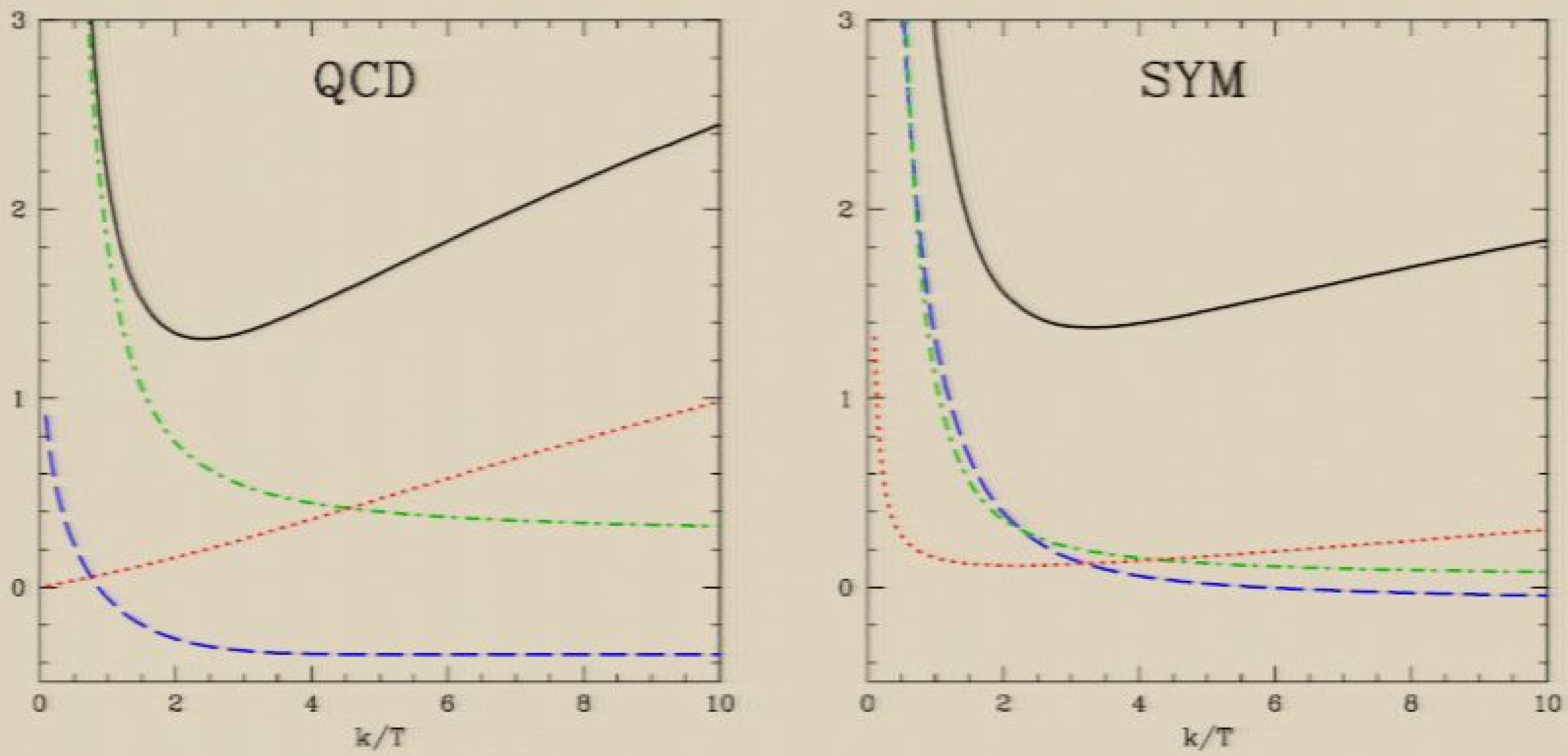

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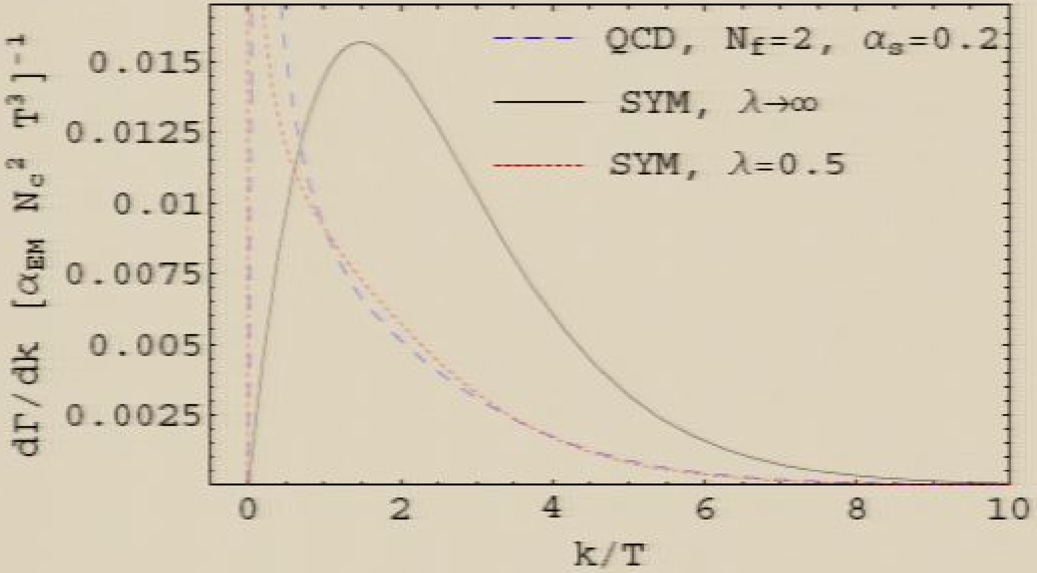

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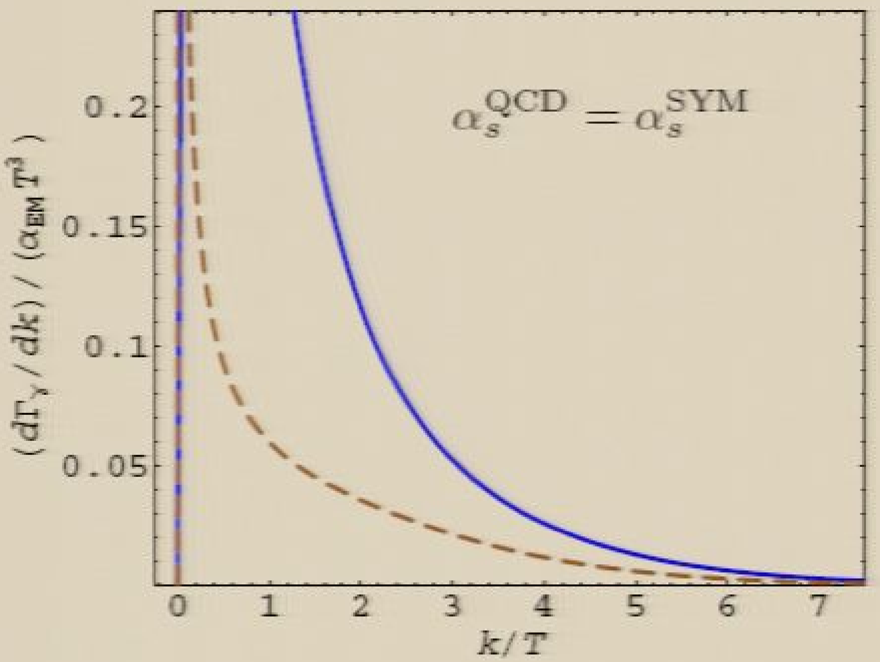
Comapre SYM and QCD at weak coupling

To mimic QCD, choose:

– the leading short-distance behavior of $C_{\mu\nu}(x)$ to coincide in QCD and SYM

– a value of the coupling constant in SYM

When compare at the same value of the coupling constant:



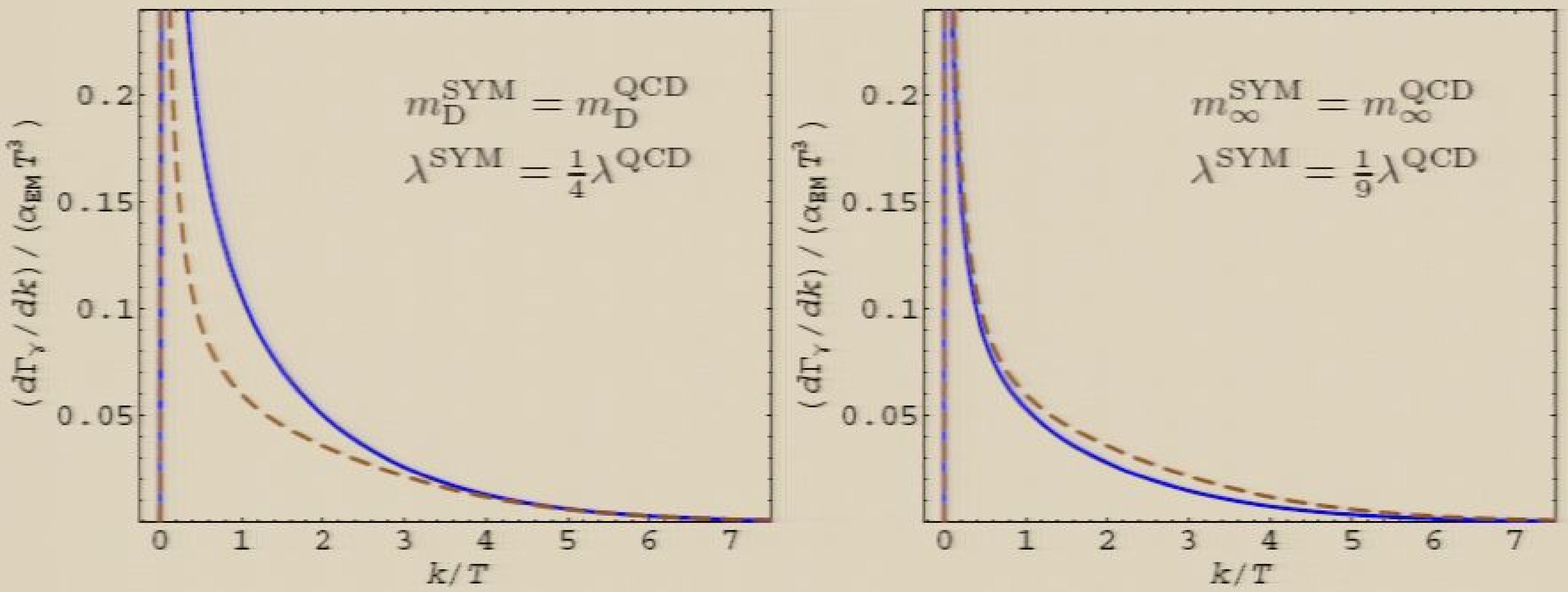
Photon emission rate in SYM is much greater than in QCD

Comapre SYM and QCD at weak coupling

To mimic QCD, choose:

– the leading short-distance behavior of $C_{\mu\nu}(x)$ to coincide in QCD and SYM
– a value of the coupling constant in SYM

When compare at the same value of a physical quantity:



Photon emission rate in SYM is very similar to QCD

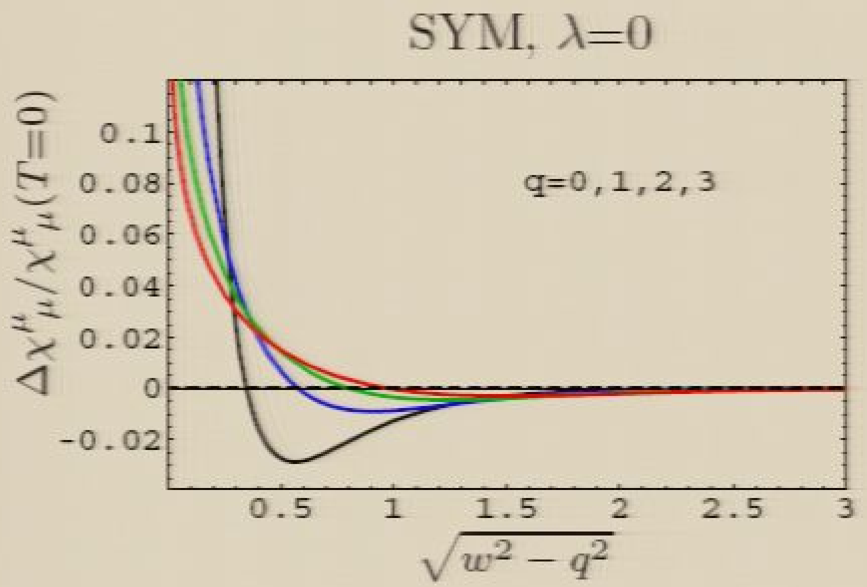
Lesson: Do not compare QCD and SYM at the same 't Hooft coupling.

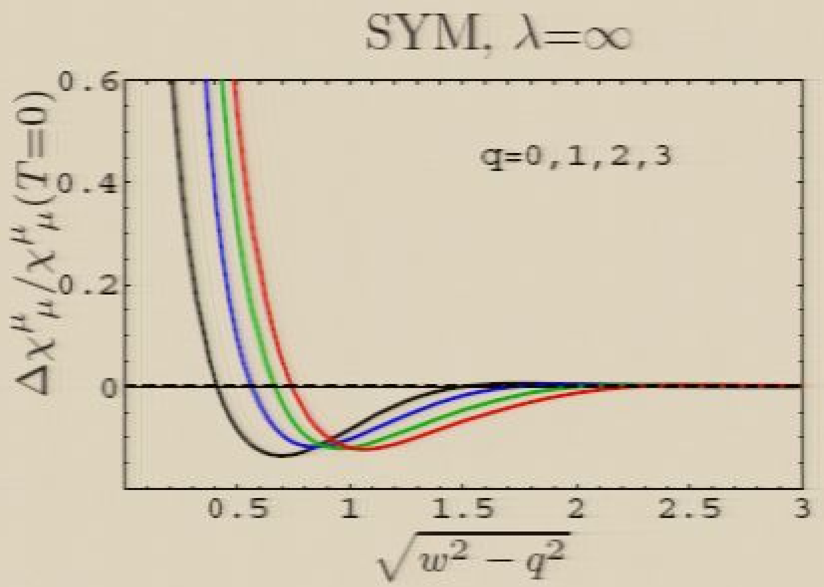
For example, $\alpha_s = 0.5$ in QCD does *not* correspond to $\lambda^{\rm SYM} = g^2 N_c = (4\pi\alpha_s) N_c \approx 20$, but to $\lambda^{\rm SYM} \approx 2$–$5$.

$1/\lambda$ corrections are important for better comparision with QCD at RHIC

Dilepton emission

Plot relative thermal correction to the trace of the spectral function, $q \equiv k/(2\pi T)$, $w \equiv k^0/(2\pi T)$





For $\sqrt{-K^2} \gtrsim 2\pi T$, thermal correction to the spectral function are $< 2\%$ at weak coupling, and $< 15\%$ at strong coupling.

Dilepton spectrum is nearly identical at weak and strong coupling, for large invariant mass of the pair

Will discuss application of AdS/CFT to:

Momentum transport

Electromagnetic response

→ Energy loss by a heavy probe

Thermalization

AdS/CFT has more to say!

Related:

J.Casalderrey-Solana, D.Teaney, hep-ph/0605199

H.Liu, K.Rajagopal, U.Wiedemann, hep-ph/0605178

J.Friess, S.Gubser, G.Michalogiorgakis, S.Pufu, hep-th/0607022

Heavy probe energy loss

(C.Herzog, A.Karch, P.K., C.Kozcaz, L.Yaffe, `hep-th/0605158`)

Setup: Particle of mass M ("probe"), moving through a *strongly* interacting thermal medium ($\lambda \to \infty$) with temperature $T \ll M$

(charm quark $M \approx 1.3\,\mathrm{GeV}$, moving in a plasma at $T = 200\,\mathrm{MeV}$)

Questions:

What is meant by "energy" of a probe?

How localized is the probe?

How to separate energy of the probe from the energy of the medium?

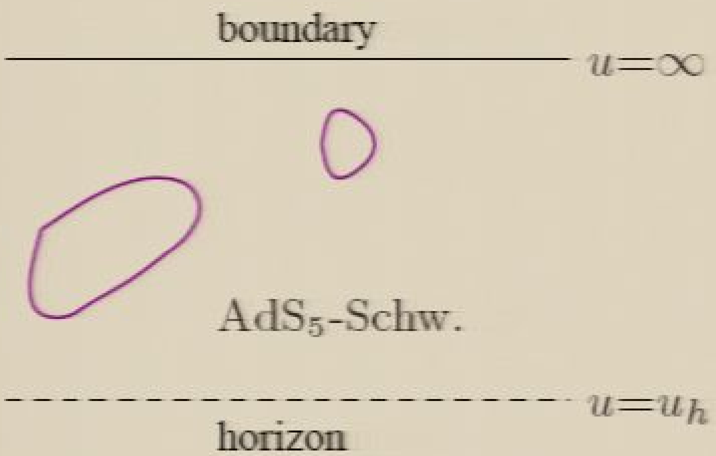
How to define energy loss?

What is the physical mechanism of the energy loss?

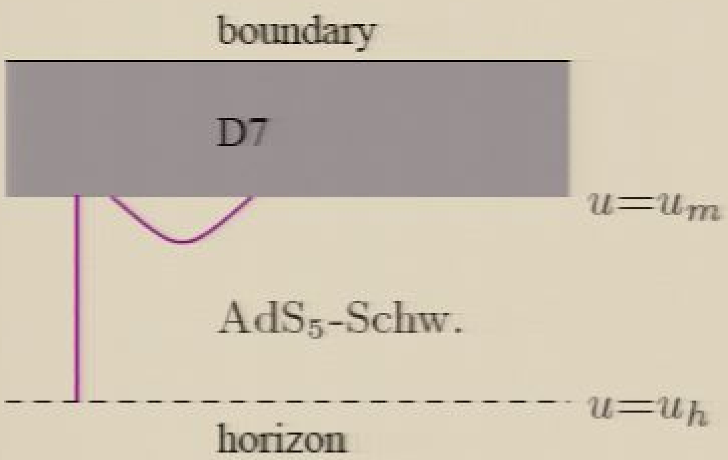
How does energy loss depend on mass, coupling, temperature?

These are non-trivial questions (keep in mind that perturbative language such as "bremsstrahlung", "quark-gluon scattering" etc is useless when coupling is strong)

Model: $\mathcal{N}=4$ SYM + fundamental matter $N_f \ll N_c$



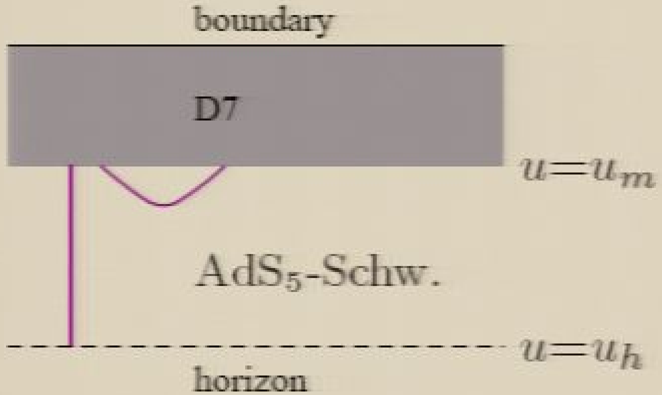
- Adjoint d.o.f. $\Leftrightarrow$ closed strings (gravity) in the bulk
- Quantum fluctuations of SYM fields $(T_{\mu\nu}, J_\mu)$ $\Leftrightarrow$ classical fluctuations of fields $(g_{\mu\nu}, A_\mu)$ in the bulk
- Finite temperature $\Leftrightarrow$ black hole (brane) in the bulk



- When add fundamental d.o.f. $\Leftrightarrow$ open strings ending on D7 brane in the bulk (A.Karch, A.Katz, hep-th/0205236)
- "Quark" mass set by u_m
- Quark configuration $\Leftrightarrow$ classical string

Energy/momentum of a quark = energy/momentum of a classical string

How heavy a quark can one treat this way?



Static string: $E = \frac{\sqrt{\lambda}}{2\pi} u_m, \quad T = 0$

$$E = \frac{\sqrt{\lambda}}{2\pi}(u_m - u_h), \quad T \neq 0$$

thermal mass

$$E_{\text{rest}} = m - \tfrac{1}{2}\sqrt{\lambda}T + \ldots$$

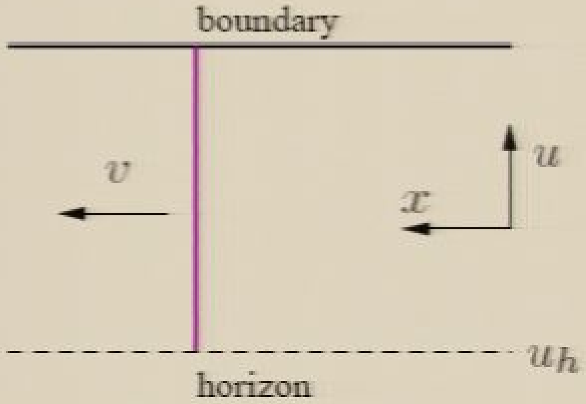
$\Delta m(T)$

When $m > 0.92\,\Delta m(T)$ — D7 above the horizon, string is classical

To analyze energy loss of a heavy quark ($m > \frac{1}{2}\sqrt{\lambda}T$), solve classical equations of motion for a moving string

Friction coefficient – dissipation

Take infinitely heavy "quark" ($u_m \to \infty$), move it at constant speed

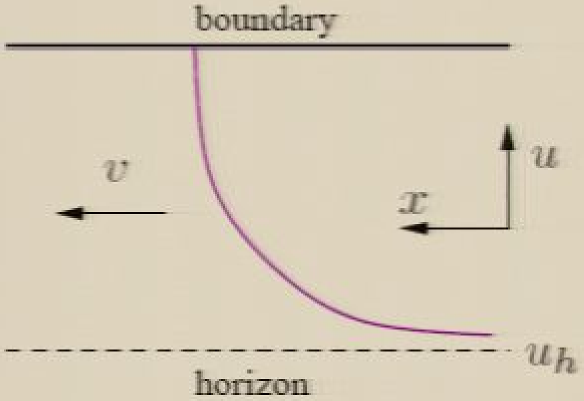


String profile: $x(u,t) = vt$ solves e.o.m.

however:

$-g$ flips sign at $u = \dfrac{u_h}{(1-v^2)^{1/4}} > u_h$

E, P become complex — solution unphysical



$x(u,t) = X(u) + vt$

Can find $X(u)$ s.t. $-g$ is positive everywhere

Source moves at constant speed

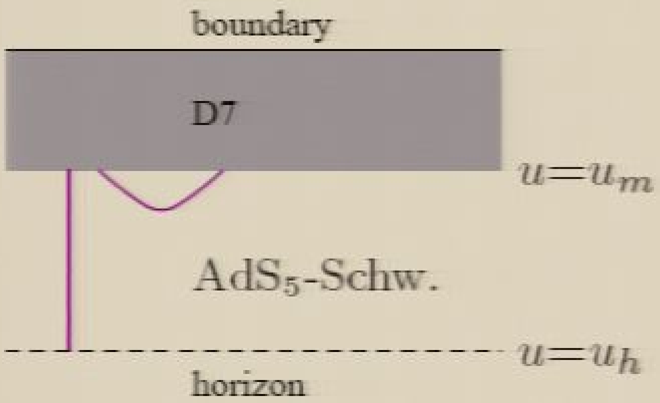
Momentum pumped in at the boundary

Momentum leaks off at the horizon

$$\frac{dP}{dt} = -\pi_x^1\Big|_{u=u_h} = -\frac{\sqrt{\lambda}}{2\pi}\frac{v}{\sqrt{1-v^2}}(\pi T)^2 = -\left(\frac{\sqrt{\lambda}T^2\pi}{2m}\right)\left(\frac{mv}{\sqrt{1-v^2}}\right) = -\mu P$$

Note that friction coefficient μ has a finite $\lambda \to \infty$ limit

Quasinormal modes – fluctuations

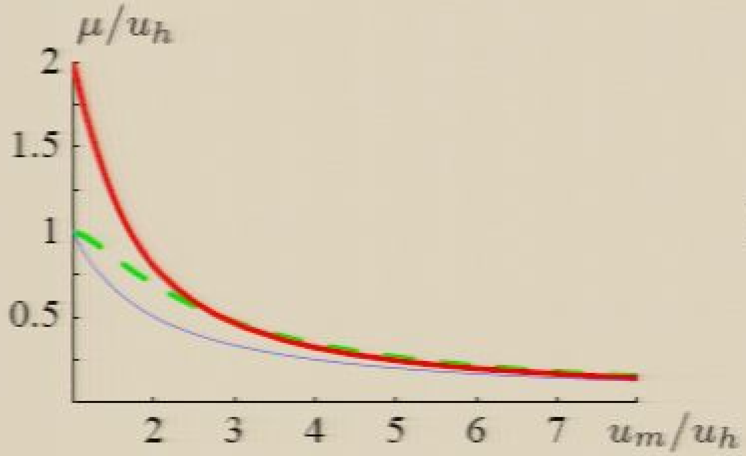


Linearize around static string

Neumann b.c. at the brane $u = u_m$

Outgoing b.c. at the horizon $u = u_h$

Lowest quasinormal mode on the worldsheet is purely imaginary, $x \sim e^{-\mu t}$



Numerical QNM:

Agrees with analytic result for $m \gg \Delta m(T)$

Gives μ for arbitrary $O(1)$ $m/\Delta m(T)$

Langevin dynamics – fluctuation-dissipation theorem

From field theory point of view:

effective equation of motion for the quark is stochastic,

$\dot{\boldsymbol{p}} = -\mu \boldsymbol{p} + \boldsymbol{\xi}(t)$, with $\langle \xi_i(t)\xi_j(t')\rangle = 2T\mu m \delta_{ij}\delta(t-t')$

Long-time behavior = diffusion, $|\Delta x| \sim \sqrt{Dt}$, with $D = \dfrac{T}{\mu m}$

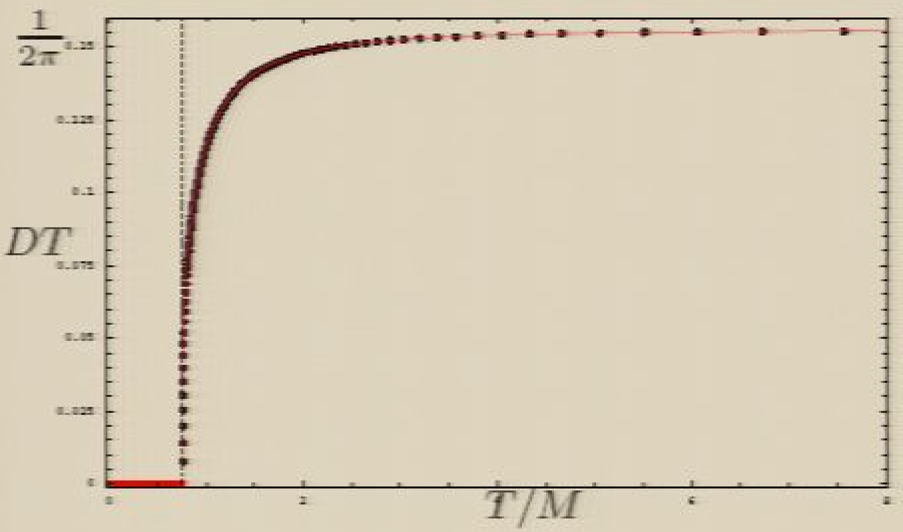
$$\boxed{D = \frac{2}{\pi T\sqrt{\lambda}}}$$

(Computed independently by J.Casalderrey-Solana, D.Teaney, hep-ph/0605199)

Neglecting noise requires $\frac{1}{2}mv^2 \gg T$, or $v \gg \dfrac{1}{\lambda^{1/4}}$

Classical string does not see Brownian motion

Diffusion constant from current-current correlator



Picture from R.Myers, A.Starinets, R.Thomson, `to appear`
(see also R.Myers, Strings-2006 talk)

Note that supergravity predicts $D = 0$ in the low-temperature phase.

However, this zero is really $D = \dfrac{2}{\pi T \sqrt{\lambda}}$, in the limit $\lambda \to \infty$

Evaluating D from current-current correlator remains to be done

Let's plug in the numbers: $\alpha_s = 0.5$

– Naively translates to λ=19, $\Rightarrow D \approx \dfrac{0.9}{2\pi T}$

– However, need to take λ smaller, e.g. λ=4 $\Rightarrow D \approx \dfrac{2}{2\pi T}$

– In perturbative QCD: $D \approx \dfrac{6}{2\pi T}$ (G.Moore, D.Teaney, `hep-ph/0412346`)

Can these numbers be compared with RHIC data?

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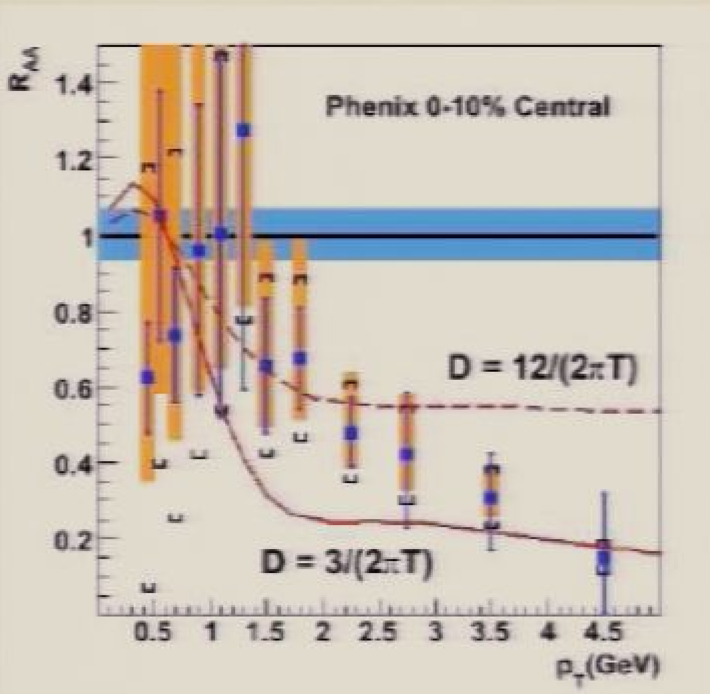

Nuclear modification factor $R_{AA}(p_T)$, PHENIX data (`nucl-ex/0510047`). Curves by D.Teaney in Langevin model $D = \dfrac{3}{2\pi T}$ seems to work best

$1/\lambda$ corrections needed!

Lessons for heavy quark

The coupling of the probe to the surrounding medium is strong:

- energy loss is **not** due to collisions (there are particles to collide with)
- energy loss is **not** due to radiation of gluons (there are no gluons to radiate)
- energy loss is **not** due to emission of sound waves (string solution does not change at all as the supersonic barrier $v_s = 1/\sqrt{3}$ is crossed)

What is the physics of energy loss?

Will discuss application of AdS/CFT to:

Momentum transport

Electromagnetic response

Energy loss by a heavy probe

→ Thermalization

AdS/CFT has more to say!

Thermalization

Thermal equilibrium in a collision established? If so:
On the partonic level at early stage, or on the hadronic level at a later stage?
At what temperature? How does the state evolve?

From AdS/CFT perspective [both solid results and speculations] :

H.Nastase, hep-th/0501068, hep-th/0603176, O.Aharony, S.Minwalla, T.Wiseman, hep-th/0507219, E.Shuryak, S.Sin, I.Zahed, hep-th/0511199, R.Janik, R.Peschanski, hep-th/0512162, R.Janik, hep-th/0610144

however:

Any attempt to understand thermalization in heavy-ion collisions from AdS/CFT must:

- be able to distinguish between hadron-hadron and Au+Au collisions
- be able to see transition from weak to strong coupling
- quantitatively understand the role of finite N_c

AdS/CFT is not (currently) useful for understanding thermalization

Will discuss application of AdS/CFT to:

- Momentum transport
- Electromagnetic response
- Energy loss by a heavy probe
- Thermalization

→ AdS/CFT has more to say!

What's next? Formalism

- How to compute transport coefficients in the confining phase? Not a single AdS/CFT calculation exists! (Euclidean correlation functions are in practice useless to extract viscosity)
- How is hydrodynamics encoded in the bulk? Linear hydro is only understood from correlation functions; non-linear hydro is not understood at all.
- How to describe Brownian motion from the bulk point of view? Need α' corrections.
- Isotropization may be easier than thermalization. How does an anisotropic state relax to equilibrium? (Initial-value problem for gravity in the bulk)
- Hadronization as the fireball cools. Keep in mind however that in AdS/QCD deconfinement transitions are typically strongly first order, while in QCD it is a crossover.

What's next? Practice

- Photons: try SYM spectral functions instead of QCD spectral functions in hydro simulations of photon production at RHIC.
- Finite coupling corrections to photon production. Remember, one needs $\lambda \sim 2\text{–}5$, not $\lambda = \infty$.
- Photons from fundamental representation matter fields ($\mathcal{N}$=4 SYM has adjoints only)?
- Now that one has the method to compute spectral functions: what does AdS/CFT say about heavy-quark resonances? Lattice studies for $c\bar{c}$ are available...
- Energy loss by light quarks.
- All of the above, now with non-zero chemical potential.
- Prove universality of η/s, or the bound $\eta/s \geqslant 1/4\pi$?
- Hydrodynamic long-time tails from AdS/CFT: there is a non-trivial and universal field theory prediction! String calculation needed.

AdS/CFT has already been useful for heavy-ion physics!

THE END