

Title: Dark matter: from the early Universe to the Milky Way

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Abstract: The initial conditions for structure formation, and hence the dark matter distribution on sub-galactic scales, depend on the microphysics of the dark matter in the early Universe. I will focus on WIMPs and explain how collisional damping and free-streaming erase perturbations on comoving scales $k > \sim 1/\text{pc}$. Consequently the first structures to form in the Universe are mini-halos with mass of order the Earth. I will then describe the status of calculations of the subsequent dynamical evolution of these mini-halos. Finally, if time permits, I'll briefly overview the microphysics of axions.

Dark matter: from the early Universe to the Milky Way

Anne Green

University of Nottingham

- ~~Evidence for dark matter~~
 - Standard picture of structure formation
 - WIMPS
 - axions
 - PBHs
- } early Universe microphysics
structure formation

work with Stefan Hofmann and Dominik Schwarz

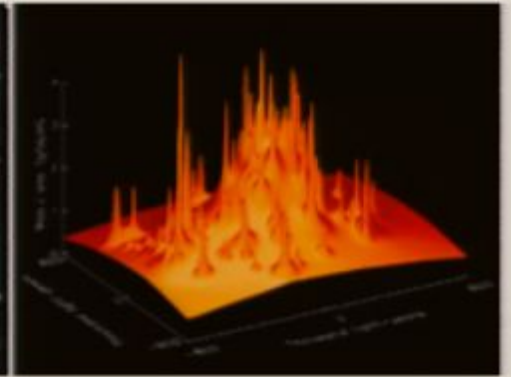
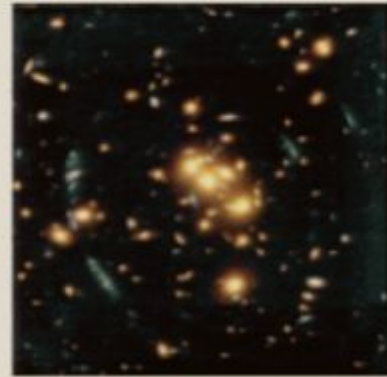
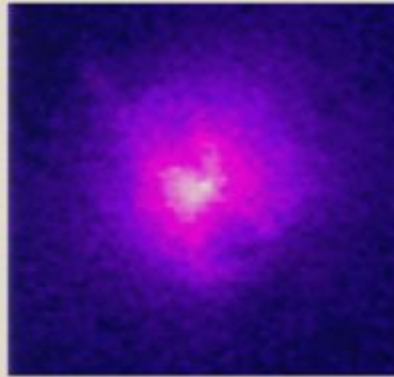
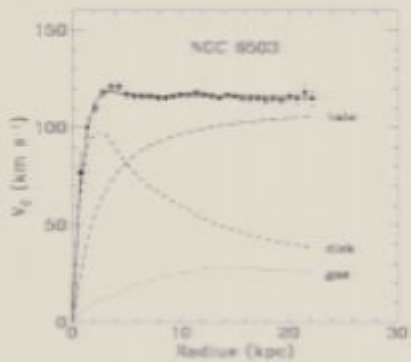
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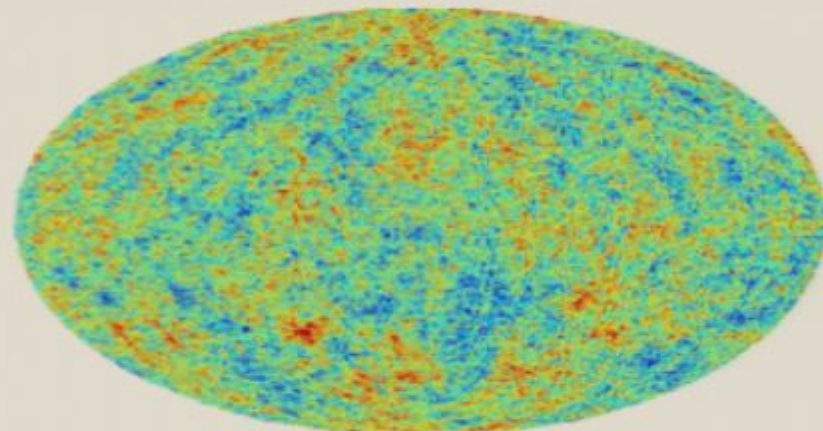
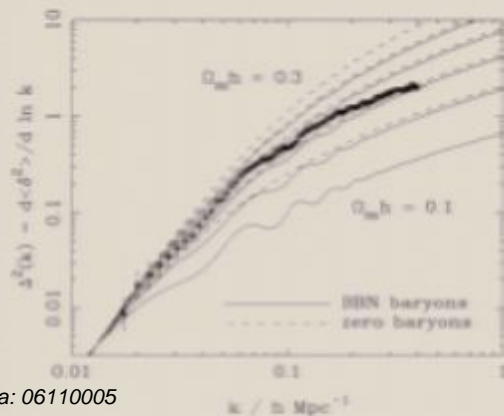
work with Stefan Hofmann and Dominik Schwarz



Lots of evidence for (non-baryonic cold) dark matter from diverse astronomical and cosmological observations

[galaxy rotation curves, galaxy clusters (galaxy velocities, X-ray gas, lensing), galaxy red-shift surveys, Cosmic Microwave Background]

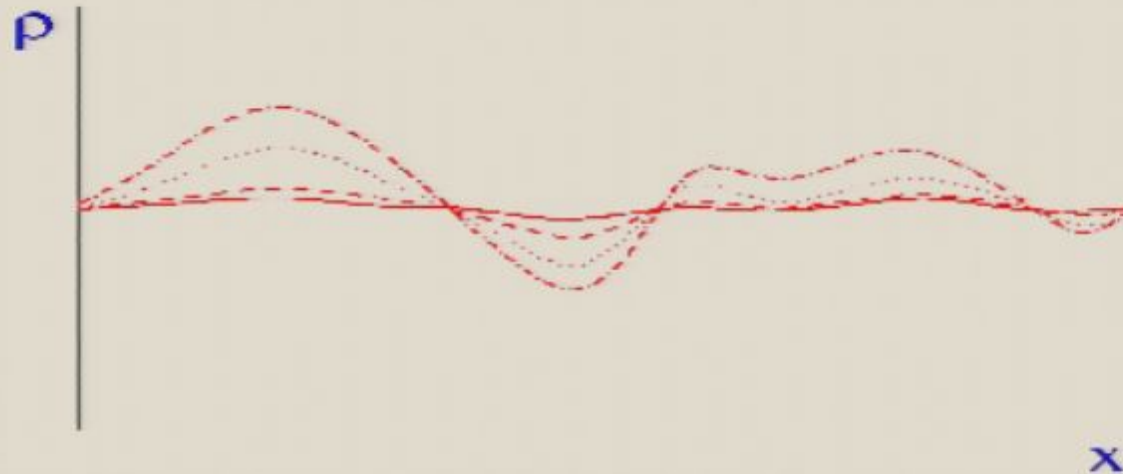
assuming Newtonian gravity/GR is correct.



Structure formation: the standard picture

Structure forms via the growth of small fluctuations (which were created in the early Universe).

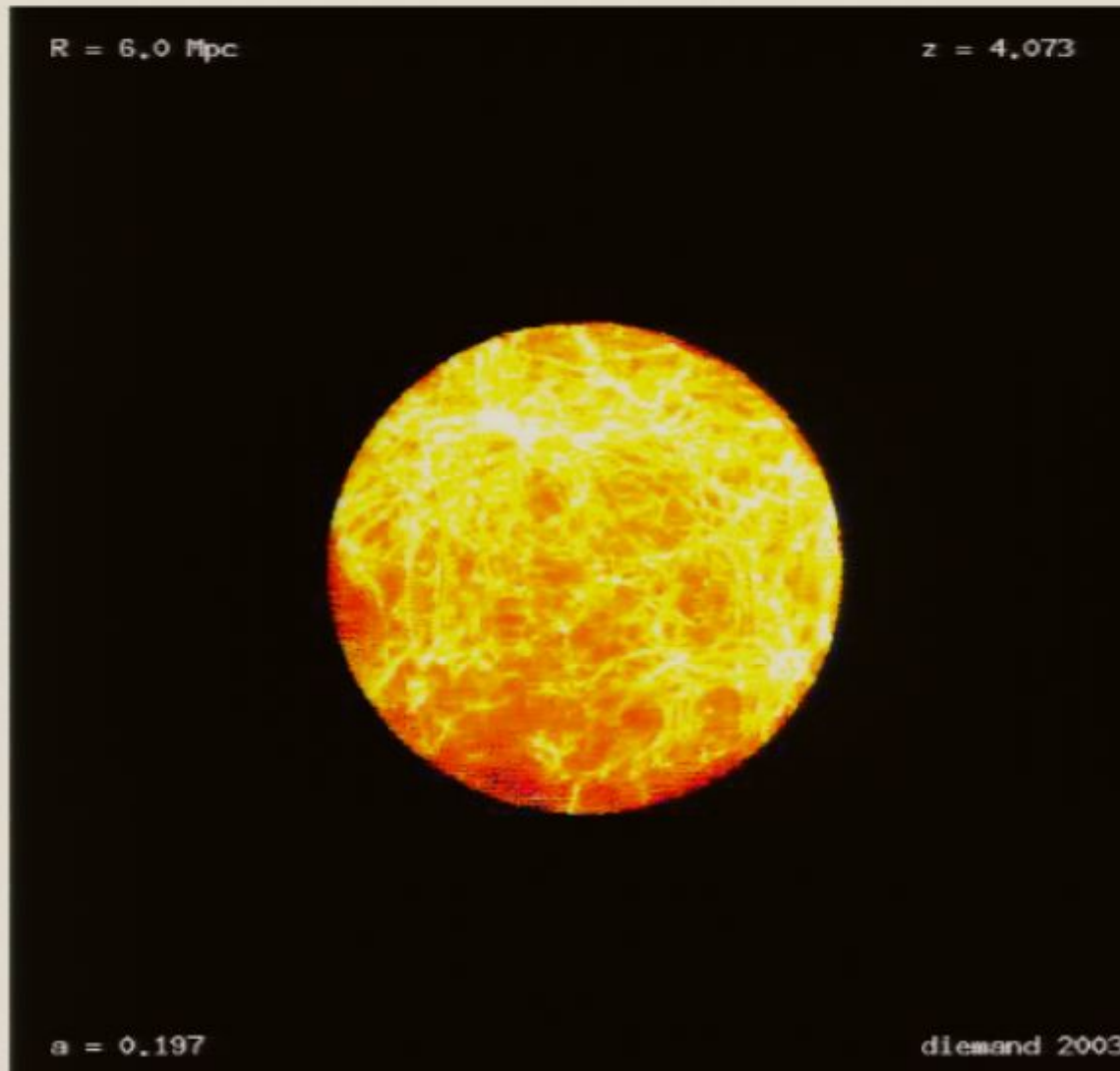
Dense (under-dense) regions become more (less) dense with time.



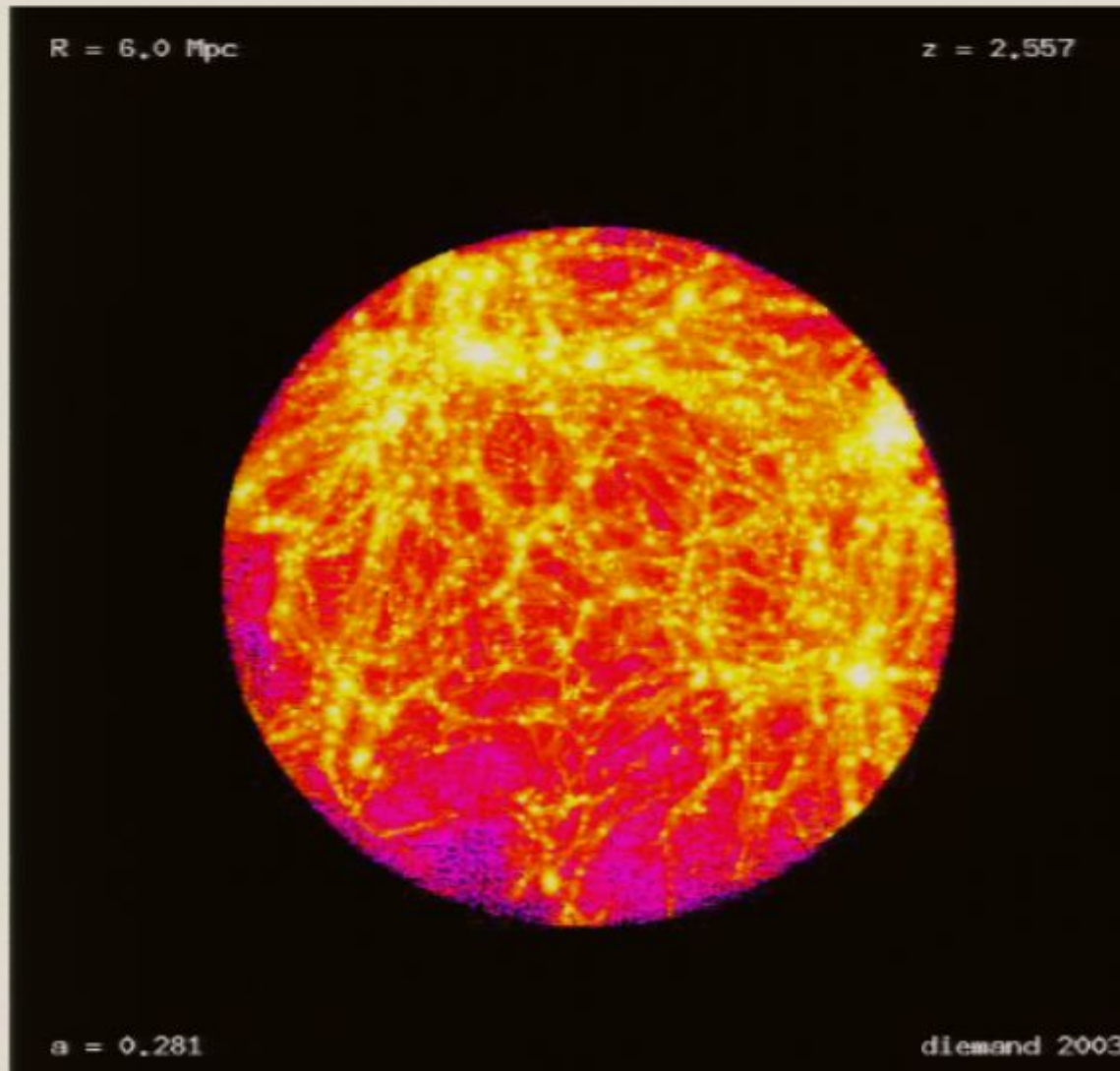
In cold dark matter cosmologies structure forms hierarchically; small halos typically form first, with larger objects forming via mergers and accretion.



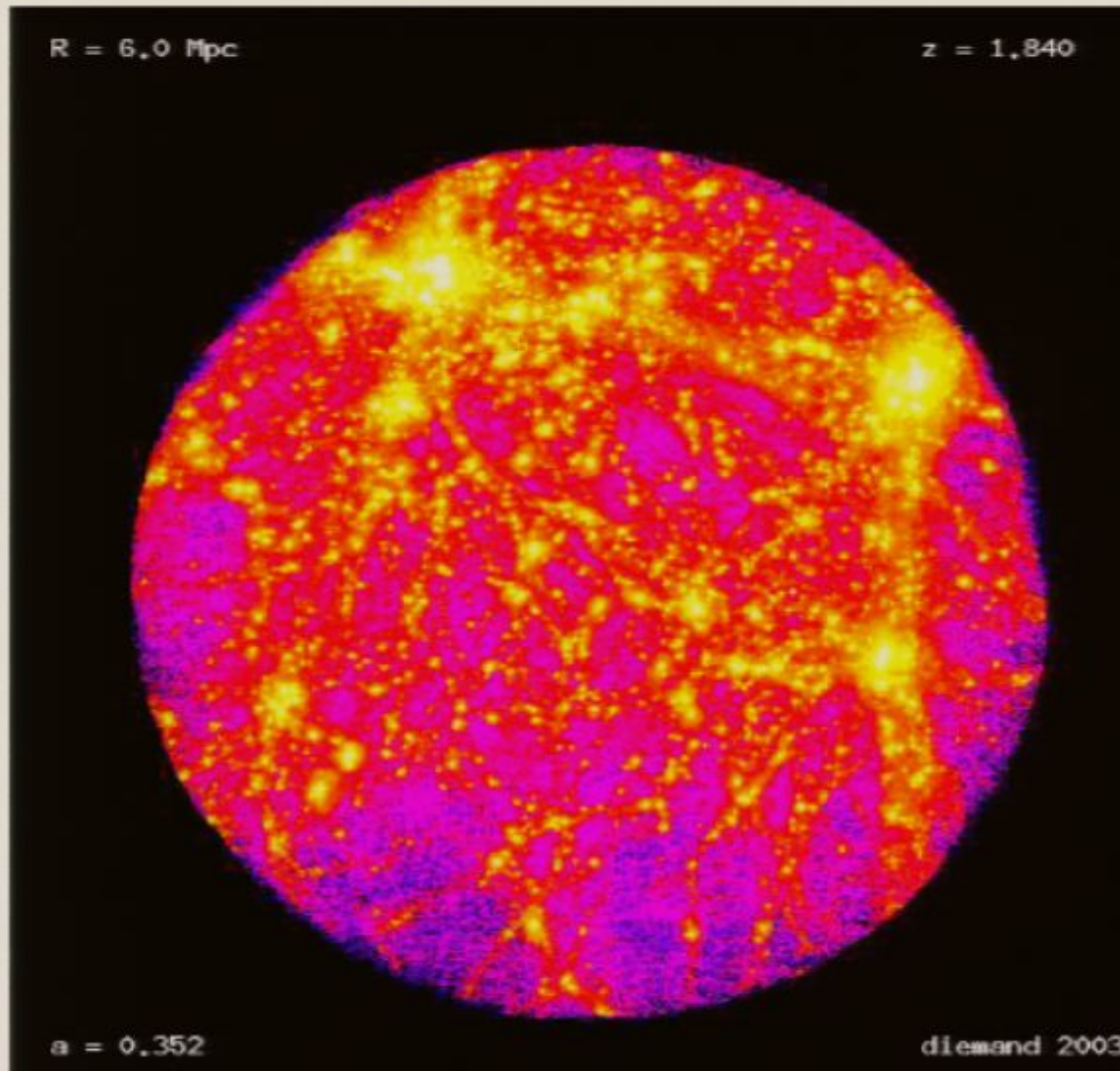
Simulation of the formation of a Galaxy Cluster by Juerg Diemand, Joakim Stadel, Ben Moore (University of Zurich) on the zBox Supercomputer at the University of Zurich.



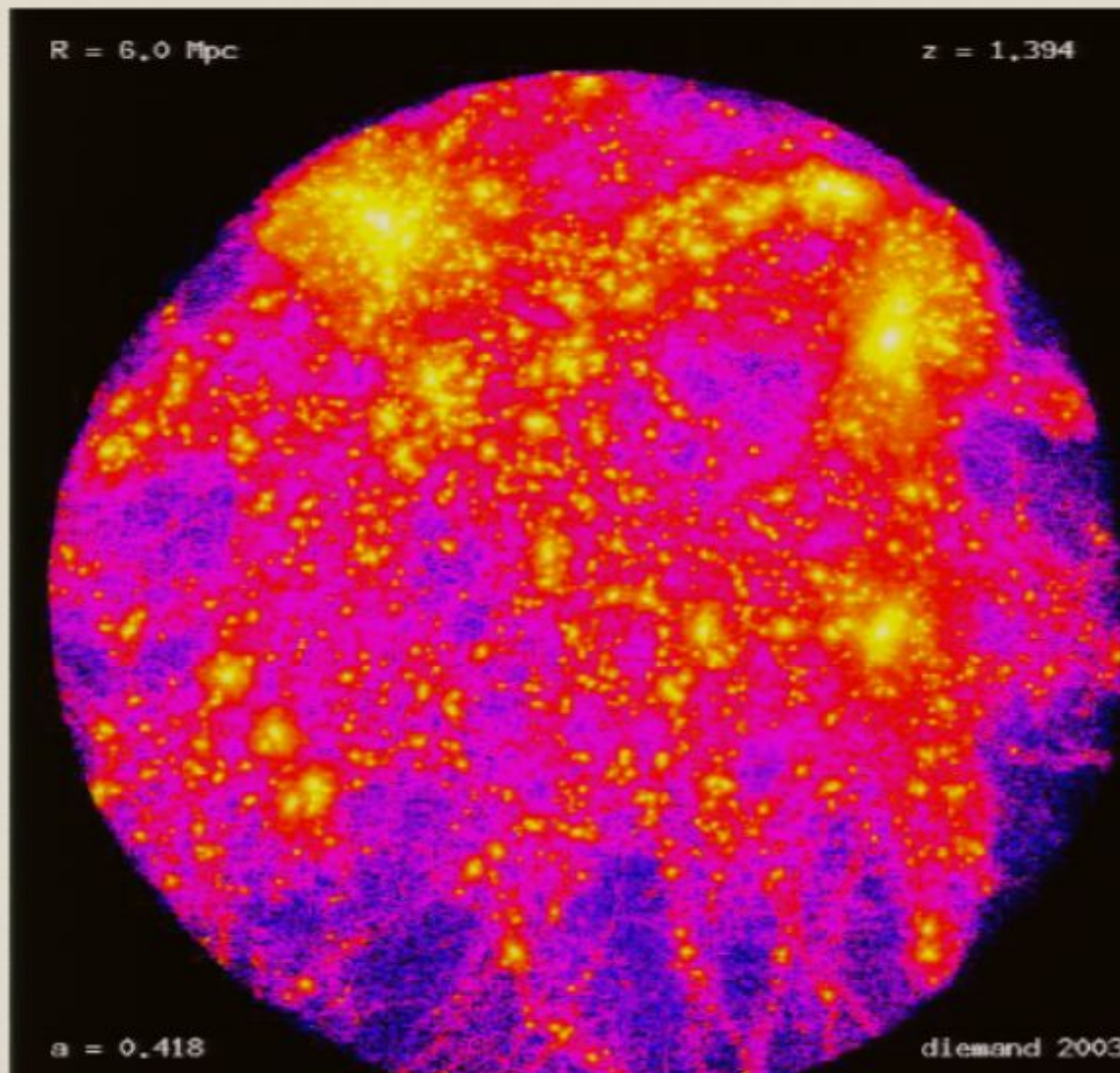
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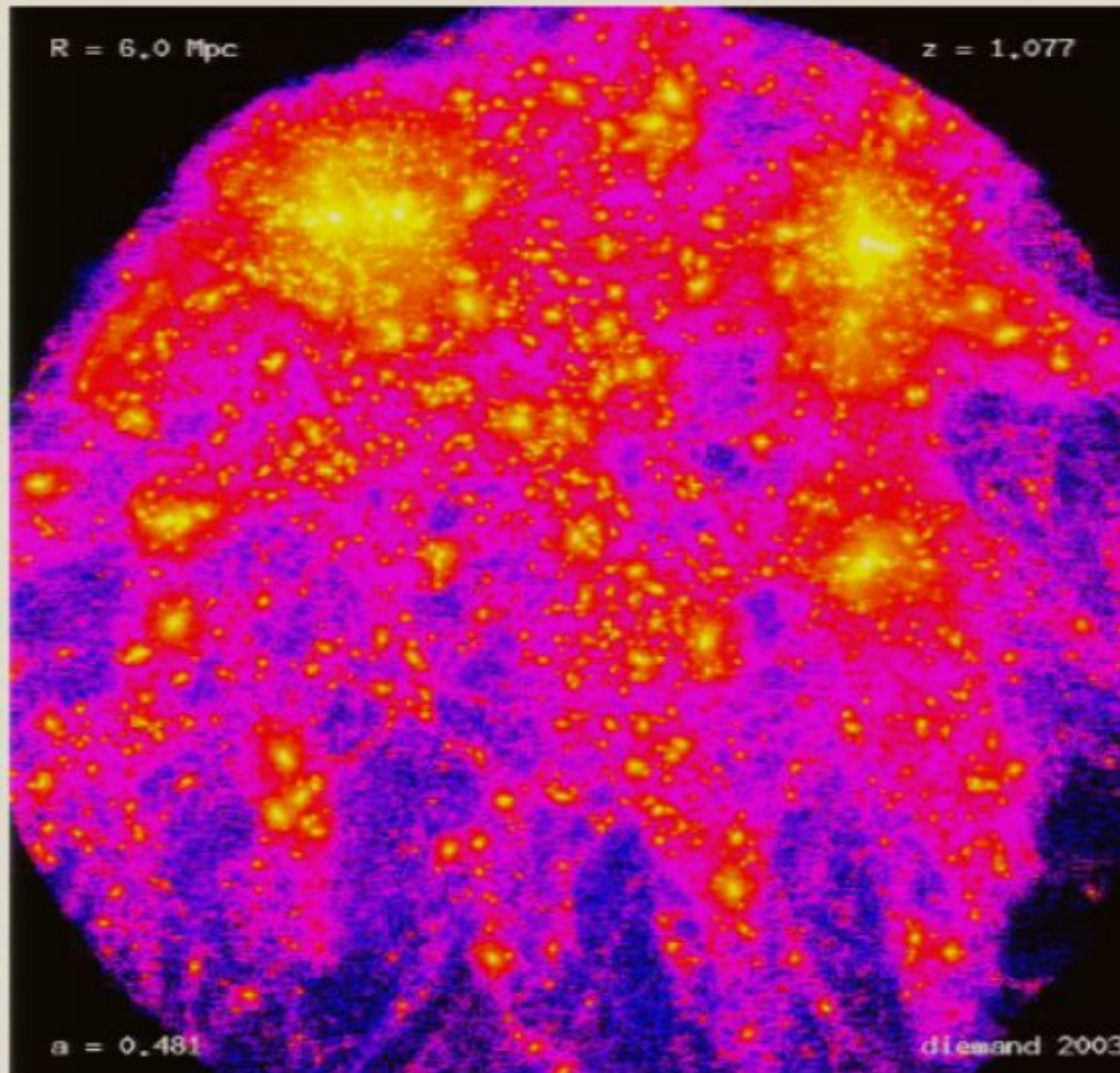
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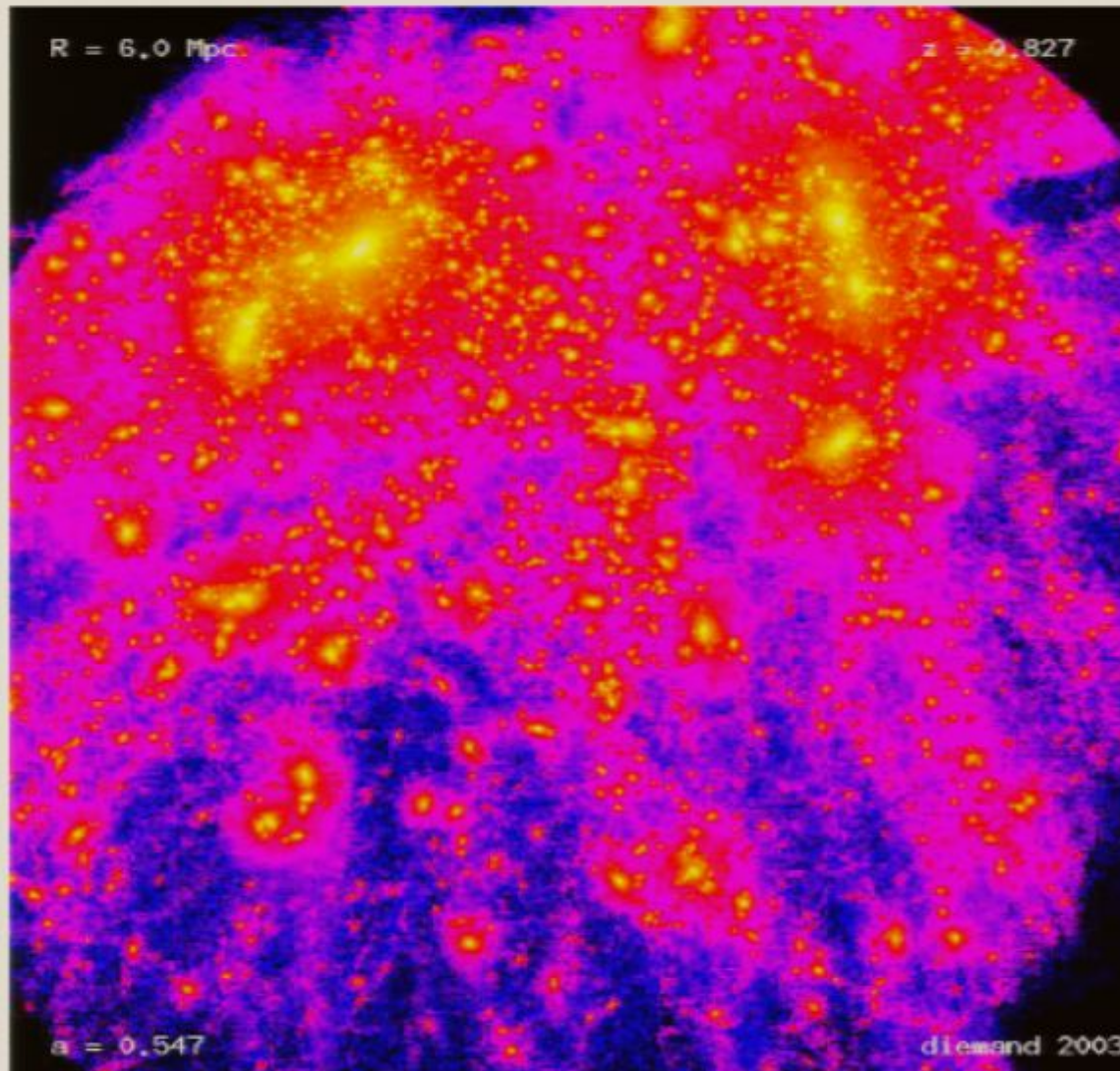
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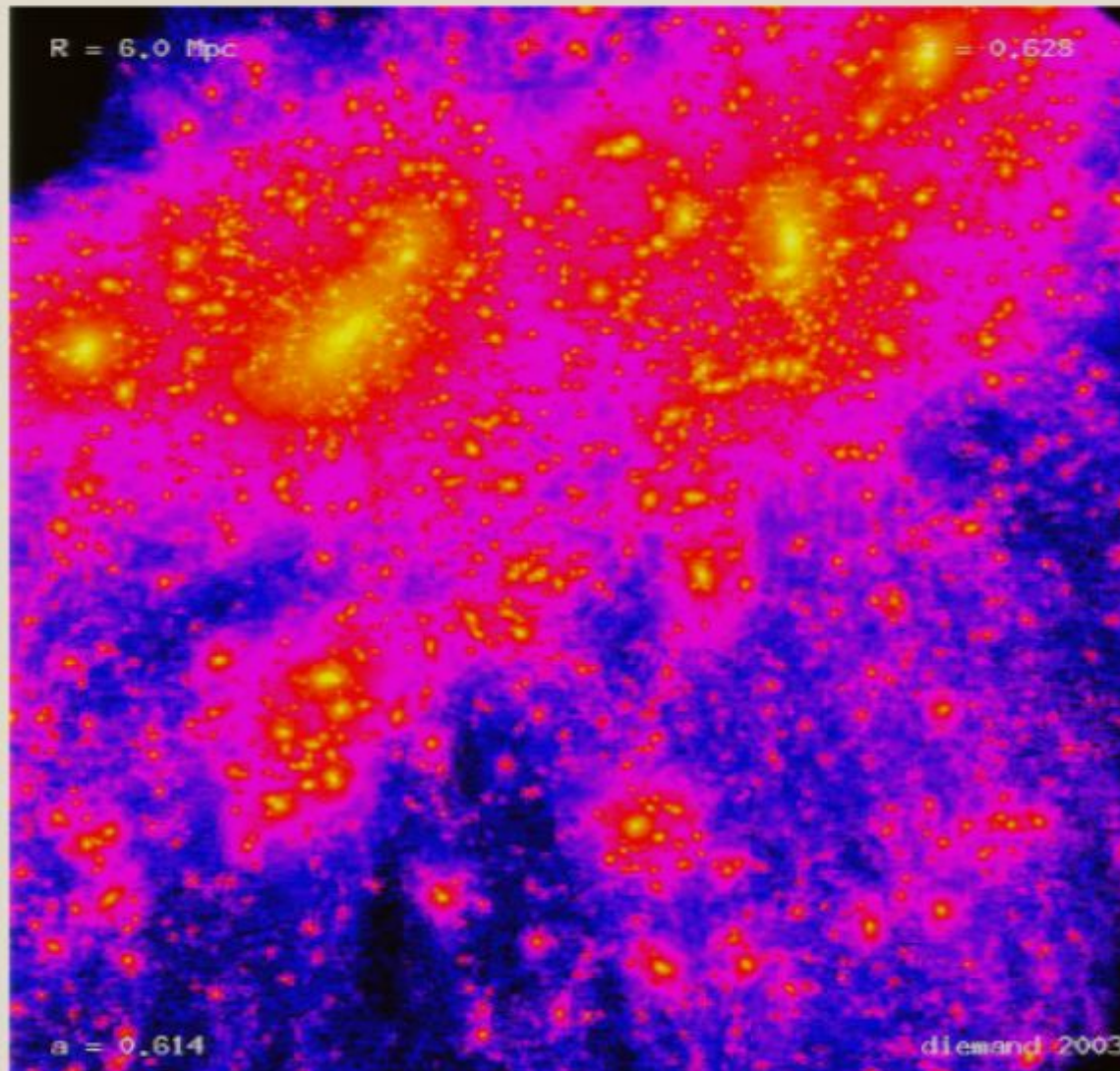
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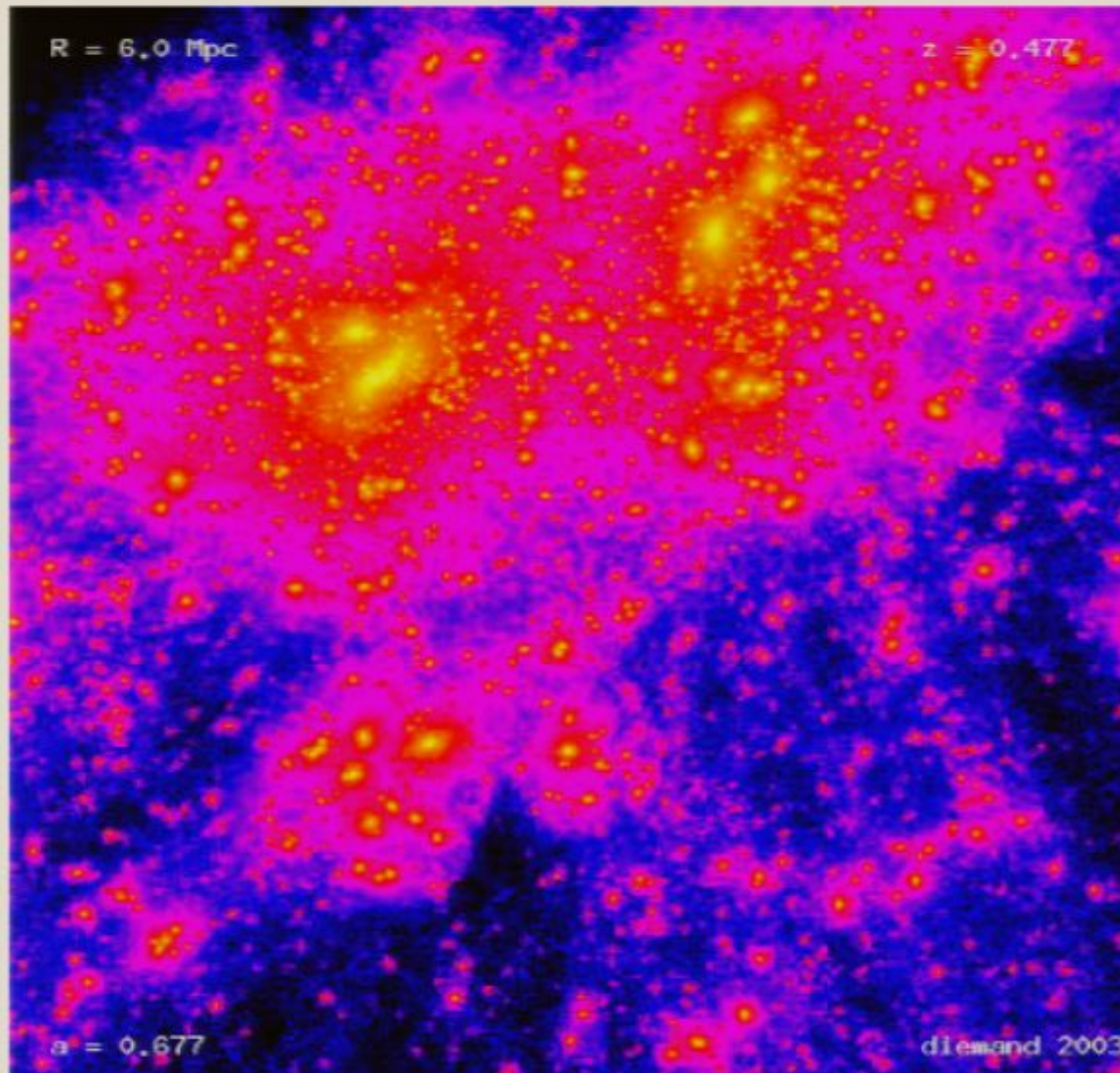
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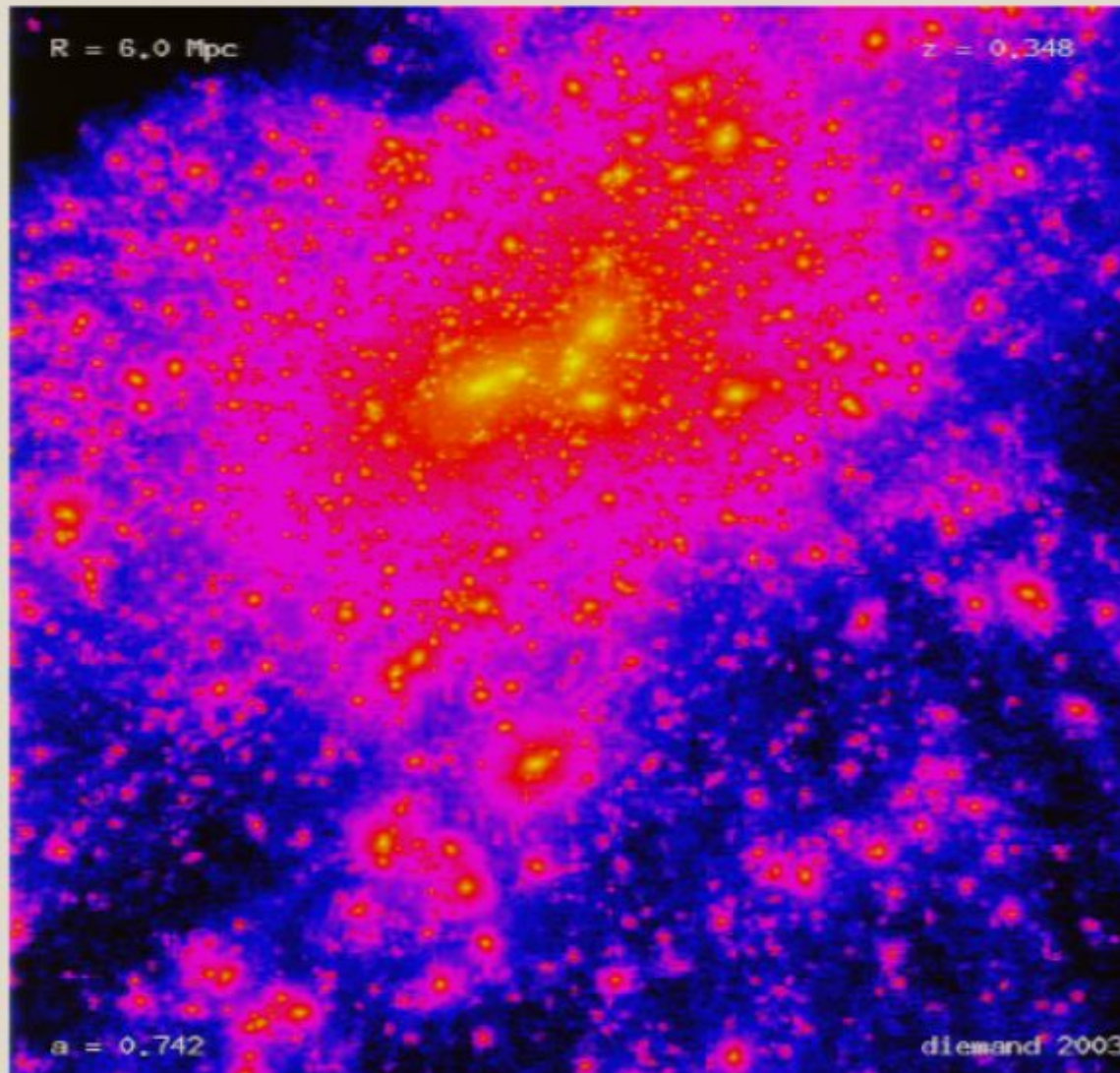
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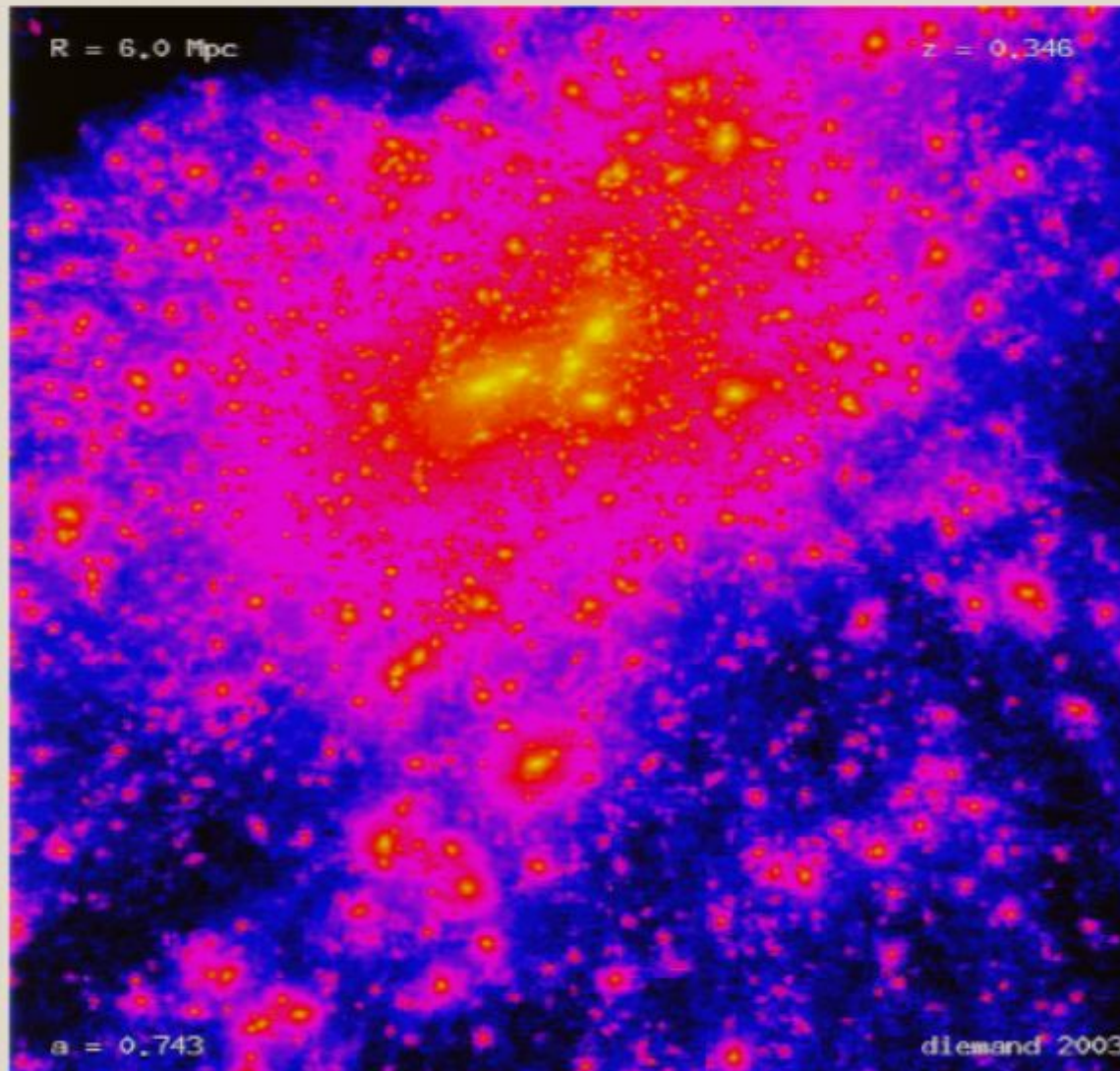
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A closer look:

Density perturbations “=”

initial perturbations \times gravitational growth \times microphysics

$$\delta_{\mathbf{k}} = \frac{1}{(2\pi)^{3/2}} \int \delta(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} d^3x$$

$$\mathcal{P}_\delta(k) = \frac{k^3}{2\pi^2} \langle |\delta_{\mathbf{k}}|^2 \rangle$$

Initial distribution

Initial distribution of fluctuations is close to scale invariant (i.e. the initial amplitude is roughly constant, independent of scale).

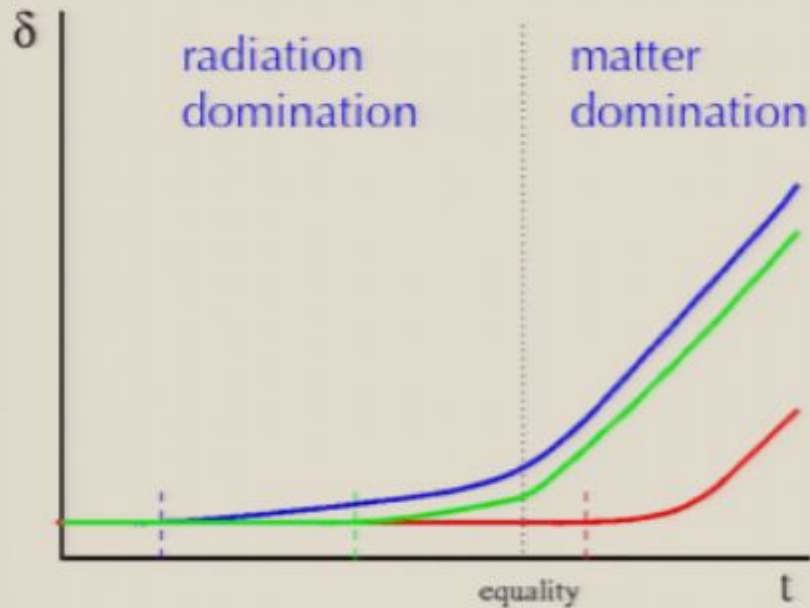
Observational evidence: CMB and large scale structure ($n \sim 1$),

Theoretical expectations: from inflation

(inflation \Leftarrow ‘slow roll’ \Rightarrow weak scale dependence)

Gravitational growth

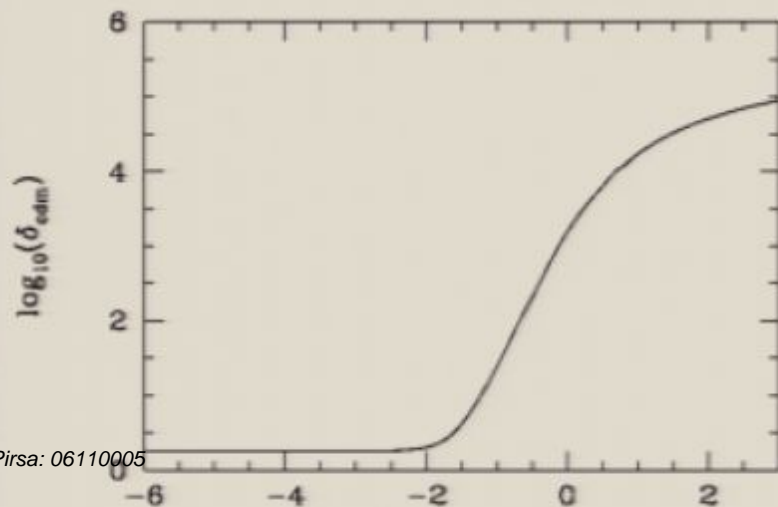
Depends on size of fluctuation (relative to horizon scale) and whether Universe is matter or radiation (or dark energy) dominated.



Small scale enters horizon long before matter-radiation equality.

Larger scale, enters horizon before equality.

Large scale enters horizon after equality.



Size of fluctuations as a function of co-moving wavenumber at $t=0$.

Why is structure formation on small (sub-Galactic) scales interesting?

General interest:

When do the first structures in the Universe form?
And how big are they?

Technical issues:

Does perturbations theory make sense without a small scale cut-off?

Practical reasons:

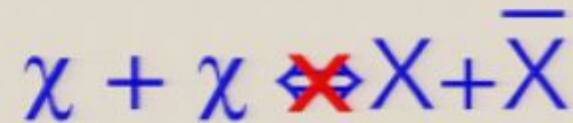
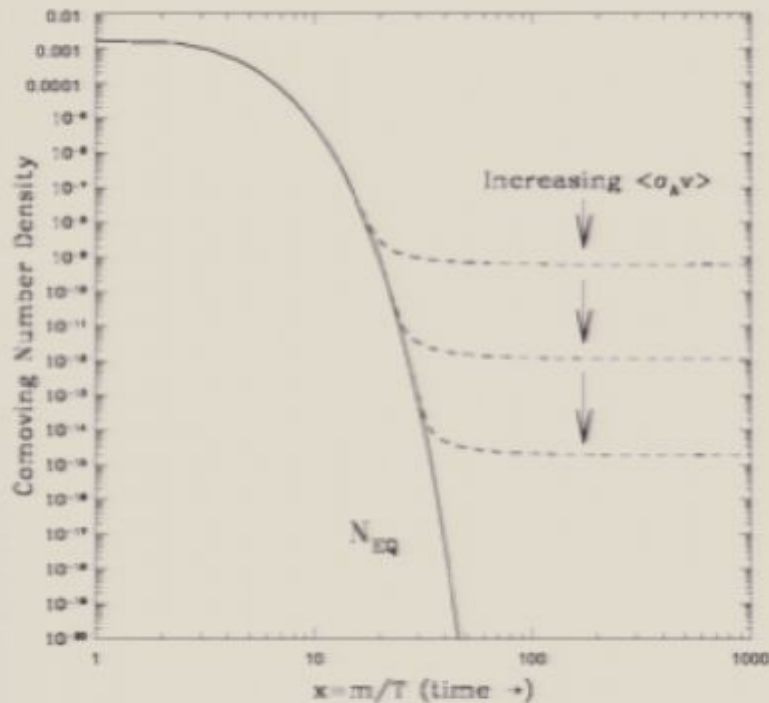
Lab based dark matter detection experiments probe the dark matter distribution on sub-milli-pc scales.

Depending on the nature of the dark matter there may be other observable consequences. *(how sensitive is structure formation to the nature of the dark matter?)*

WIMPs

Introduction

Any Weakly Interacting Massive Particle in thermal equilibrium in the early Universe will have an interesting density today.



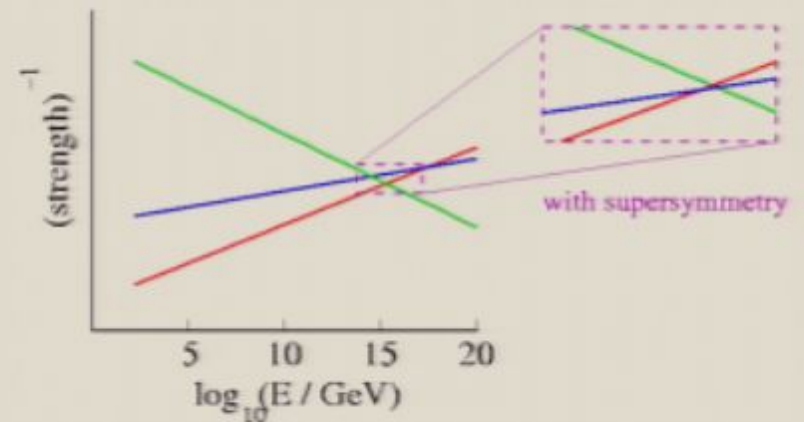
$$\Omega_{\text{wimp}} \sim 0.2 \frac{(m/T_{\text{fo}})/25}{\langle \sigma_{\text{ann}} v \rangle / 1\text{pb}}$$

Supersymmetry (the particle physicists' favoured extension of the standard model) provides us with a concrete WIMP candidate.

Every standard model particle has a supersymmetric partner. (Bosons have a fermion spartner and vice versa)

Motivations:

- ◆ Gauge hierarchy problem ($M_W \sim 100 \text{ GeV} \ll M_{Pl} \sim 10^{19} \text{ GeV}$)
- ◆ Unification of coupling constants
- ◆ String theory



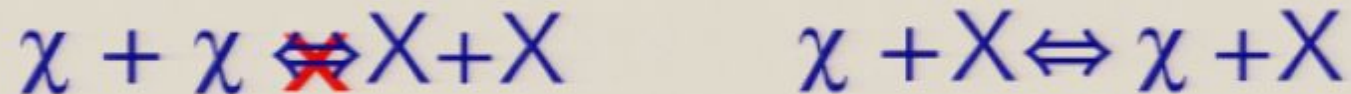
In most models the **Lightest Supersymmetric Particle** (which is usually the lightest neutralino, a mixture of the susy partners of the photon, the Z and the Higgs) is stable (R parity is conserved) and is a good CDM candidate.

Early Universe Microphysics

Kinetic decoupling

[Schmid, Schwarz & Widerin; Boehm, Fayet & Schaeffer; Chen, Kamionkowski & Zhang; **Hofmann, Schwarz & Stöcker**; Schwarz, Hofmann & Stöcker; Berezhinsky, Dokuchaev & Eroshenko; Green, Hofmann & Schwarz x2; Loeb & Zaldarriaga]

After freeze-out (chemical decoupling) WIMPS carry on interacting kinetically with radiation:



The WIMPs kinetically decouple when

$$\tau_{\text{relax}} = H^{-1}$$

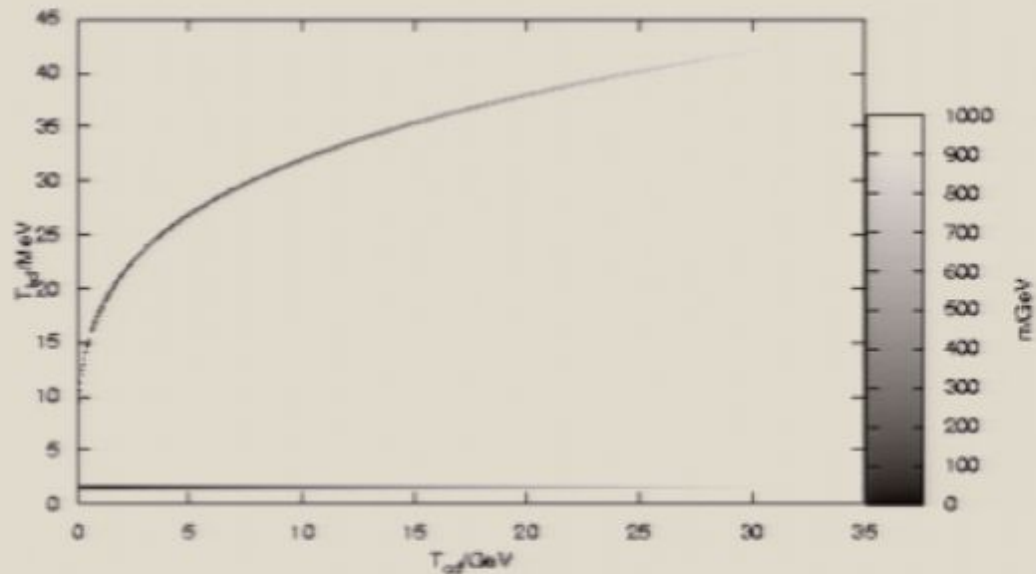
n.b. the momentum transfer per scattering ($\sim T$) is small compared with the WIMP momentum ($\sim M$), therefore a very large number of collisions are required to keep or establish thermal equilibrium.

$$\tau_{\text{relax}} \gg \tau_{\text{col}}$$

Dependence of decoupling temperatures on WIMP mass, for WIMPs with present day density compatible with WMAP measurements, for $l=1$ for Majorana particles (i.e. neutralinos interacting via sfermion exchange) and $l=0$ Dirac particles (i.e. standard model-like particles interacting via Z^0 exchange).

$$\langle \sigma_{\text{el}} \rangle = \sigma_0^{\text{el}} \left(\frac{T}{m} \right)^{l+1} \quad \sigma_0^{\text{el}} \approx \frac{(G_F m_W^2)^2 m^2}{m_Z^2}$$

Kinetic
decoupling
temperature
in MeV



mass in GeV

Chemical decoupling temperature in GeV

Collisional damping

Energy transfer between radiation and WIMP fluids (due to bulk and shear viscosity) leads to collisional damping of density perturbations. [Hofmann, Schwarz & Stöcker]

$$\Delta'' + \frac{\zeta_{\text{vis}} + 4\eta_{\text{vis}}/3}{\rho_{\text{wimp}}} \frac{k^2}{a} \Delta' + c_{\text{wimp}}^2 k^2 \Delta = 0$$

$$\zeta_{\text{vis}} \approx \frac{5}{3} nT \tau_{\text{relax}}$$

$$\eta_{\text{vis}} \approx nT \tau_{\text{relax}}$$

$$\tau_{\text{relax}} \sim N \tau_{\text{coll}} \sim \frac{m}{T} \tau_{\text{coll}}$$

Free-streaming

After kinetic decoupling WIMPs free-stream, leading to further (collision-less) damping.

$$\frac{1}{k_{\text{fs}}} \sim l_{\text{fs}}(\eta) = \bar{v}_{\text{kd}} a_{\text{kd}} \int_{\eta_{\text{kd}}}^{\eta} \frac{d\eta'}{a(\eta')}$$

Calculate free-streaming length by solving the collisionless Boltzmann equation, taking into account perturbations present at kinetic decoupling.

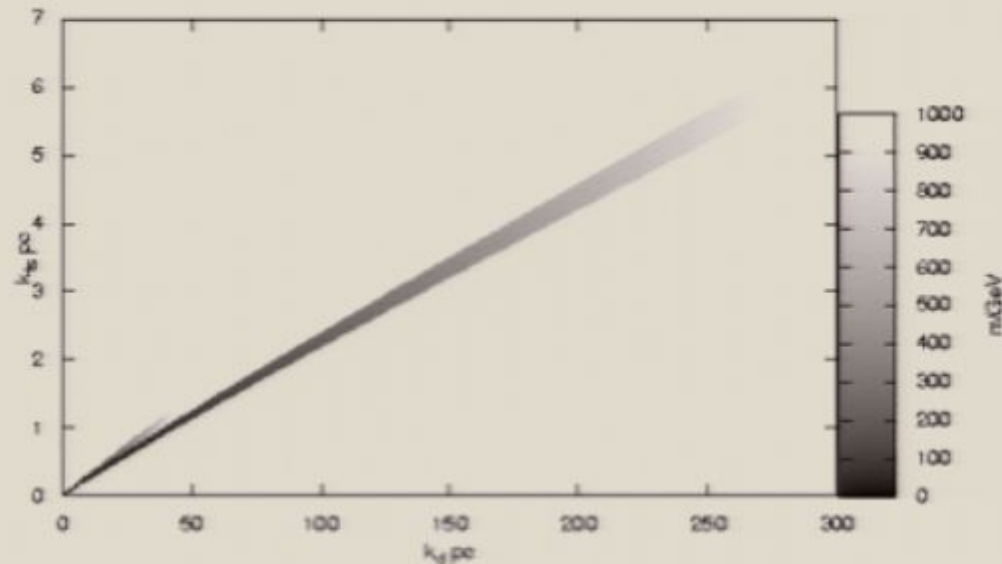
Net damping factor:

$$D(k) \equiv \frac{\delta_{\text{WIMP}}(k, \eta)}{\delta_{\text{WIMP}}(k, \eta_i)} = D_{\text{cd}}(k) D_{\text{fs}}(k) = \left[1 - \frac{2}{3} \left(\frac{k}{k_{\text{fs}}} \right)^2 \right] \exp \left[- \left(\frac{k}{k_{\text{fs}}} \right)^2 - \left(\frac{k}{k_{\text{d}}} \right)^2 \right]$$

Dependence of damping scales on WIMP mass, for WIMPs with present day density compatible with WMAP measurements, and $l = 0/1$ (top and bottom).

Free-streaming
comoving
wavenumber
(pc)

mass in GeV



Collisional damping comoving wavenumber (pc)

Loeb & Zaldarriaga numerical treatment:

Memory of coupling to radiation fluid leads to acoustic oscillations of CDM fluid and additional damping

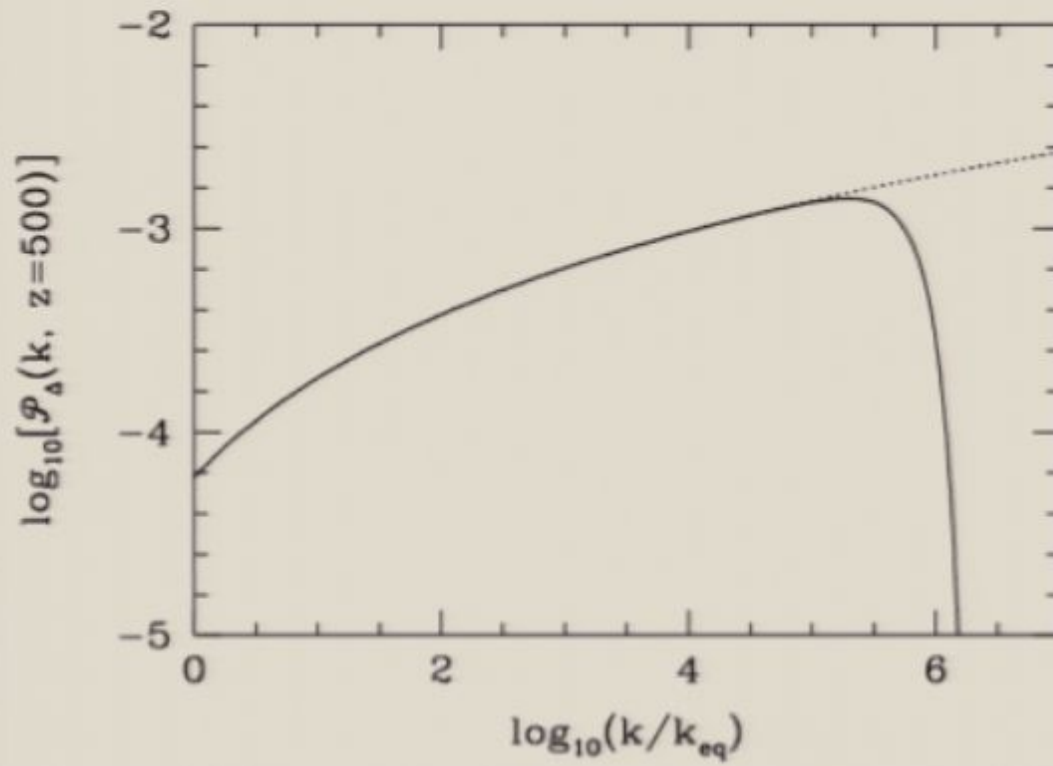
Bertschinger:

Numerically solved full Boltzmann equation describing WIMP-lepton scattering (Fokker-Planck equation describing diffuse in velocity space due to elastic scattering, advection and gravitational forces).

Our calculation correctly gives the envelope of the damping factor. A high accuracy calculation of the cut-off scale and the detailed shape of the processed power spectrum both require numerical calculations.

Power spectrum

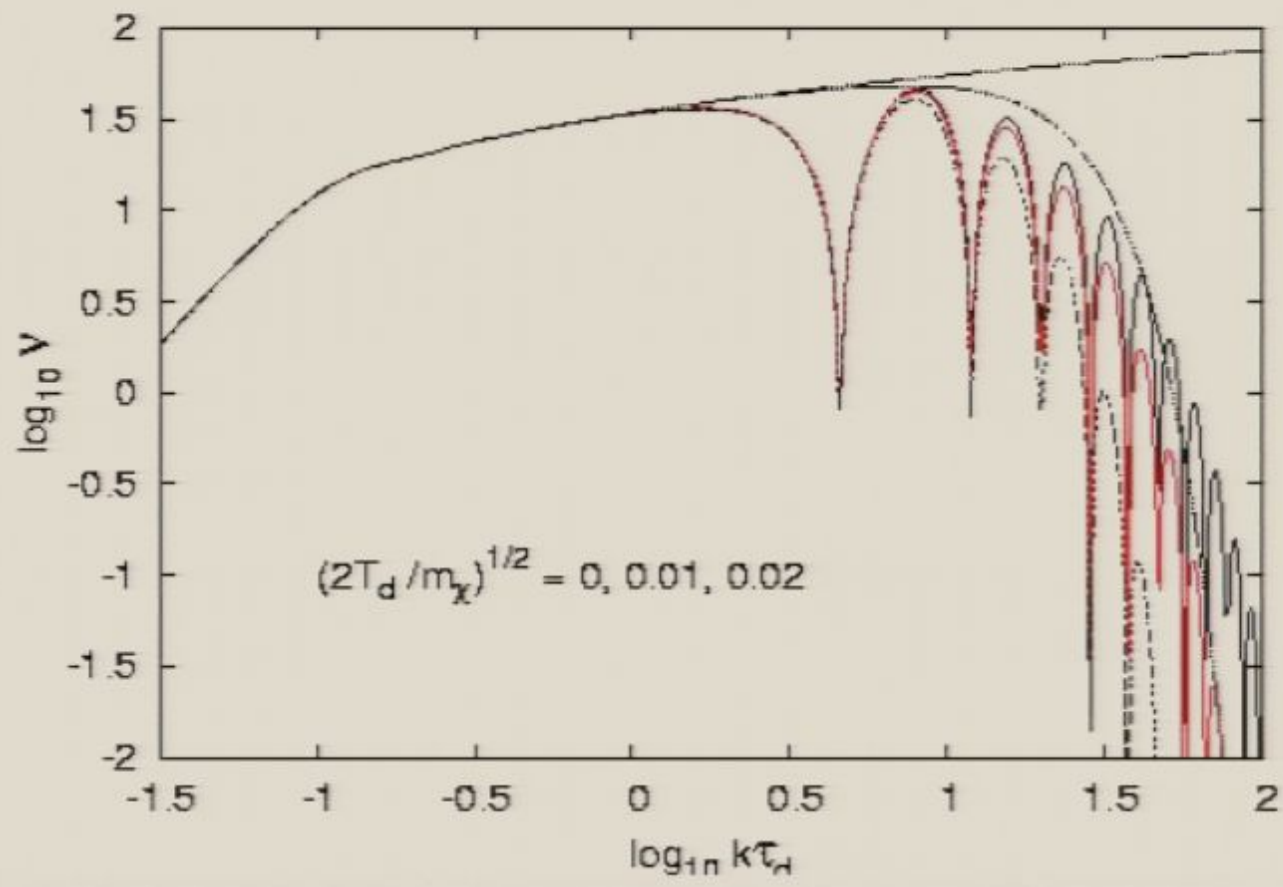
For a 100 GeV bino-like WIMP and a scale invariant, WMAP normalised, primordial power spectrum at $z=500$:



$$\mathcal{P}_\delta(k) = \frac{k^3}{2\pi^2} \langle |\delta^2| \rangle$$

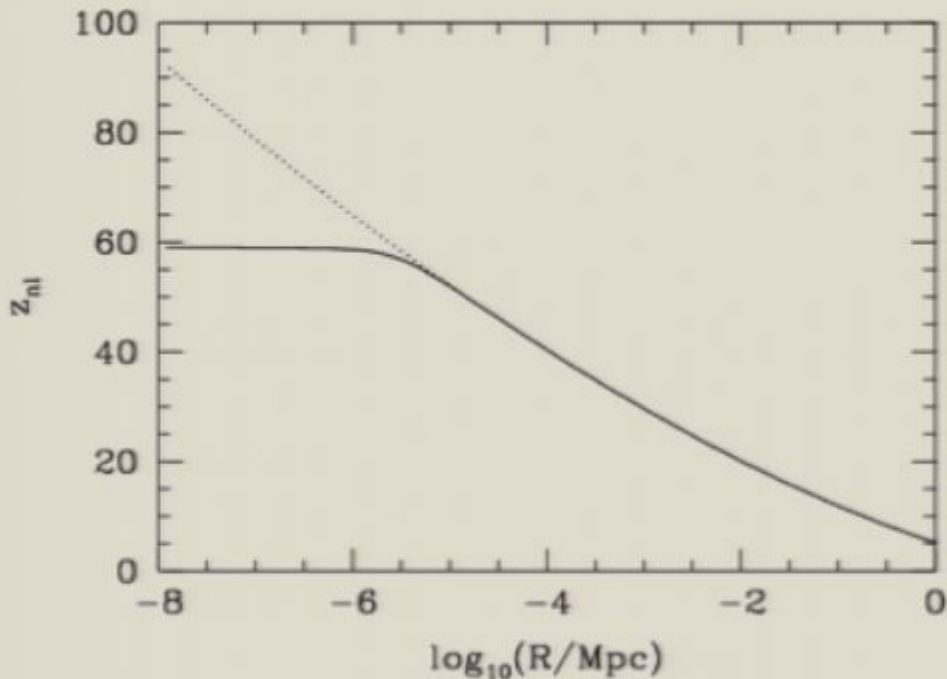
Sharp cut-off at $k = k_{fs} \sim 1/\text{pc}$

Bertschinger's refined calculation:



z_{nl}

The red-shift at which typical fluctuations on co-moving physical scale R go non-linear can be estimated via the mass variance:



$$\sigma(R, z_{nl}) = 1$$

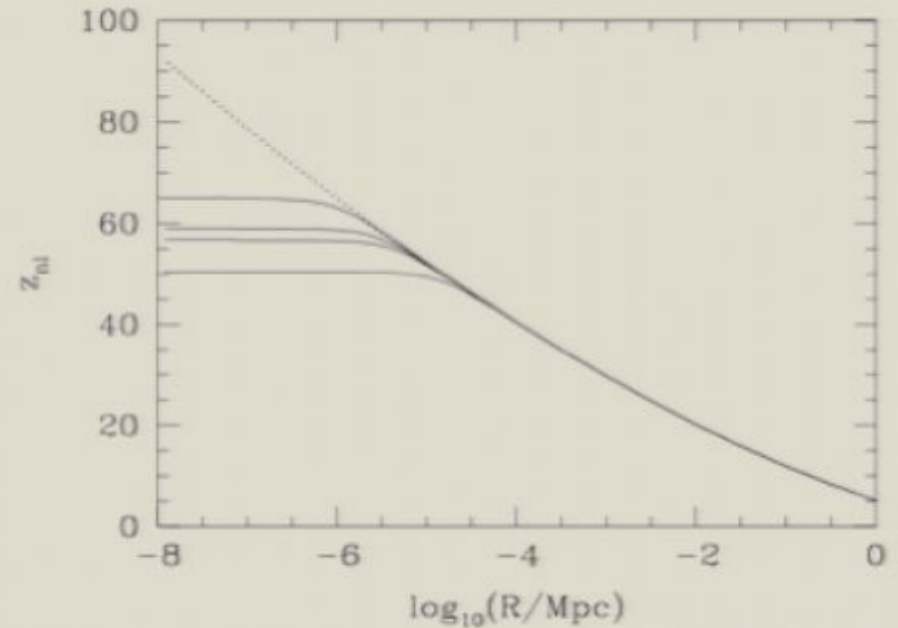
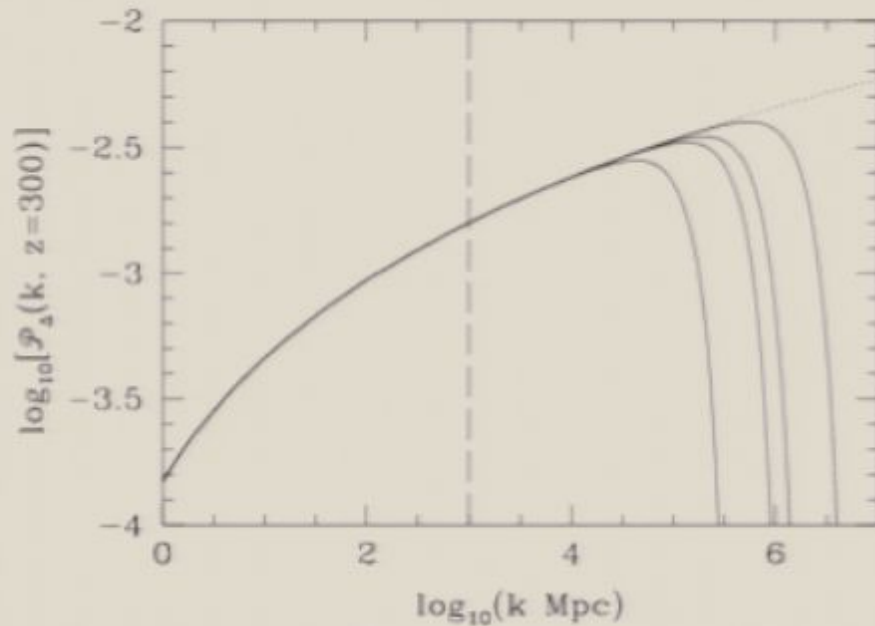
$$\sigma^2(R, z) = \int_0^\infty W^2(kR) \mathcal{P}_\delta(k, z) \frac{dk}{k}$$

Typical one-sigma fluctuations collapse at $z_{nl} \sim 60$.

(N -sigma fluctuations collapse at $z_{nl} \sim 60N$)

Effect of varying:

i) WIMP properties



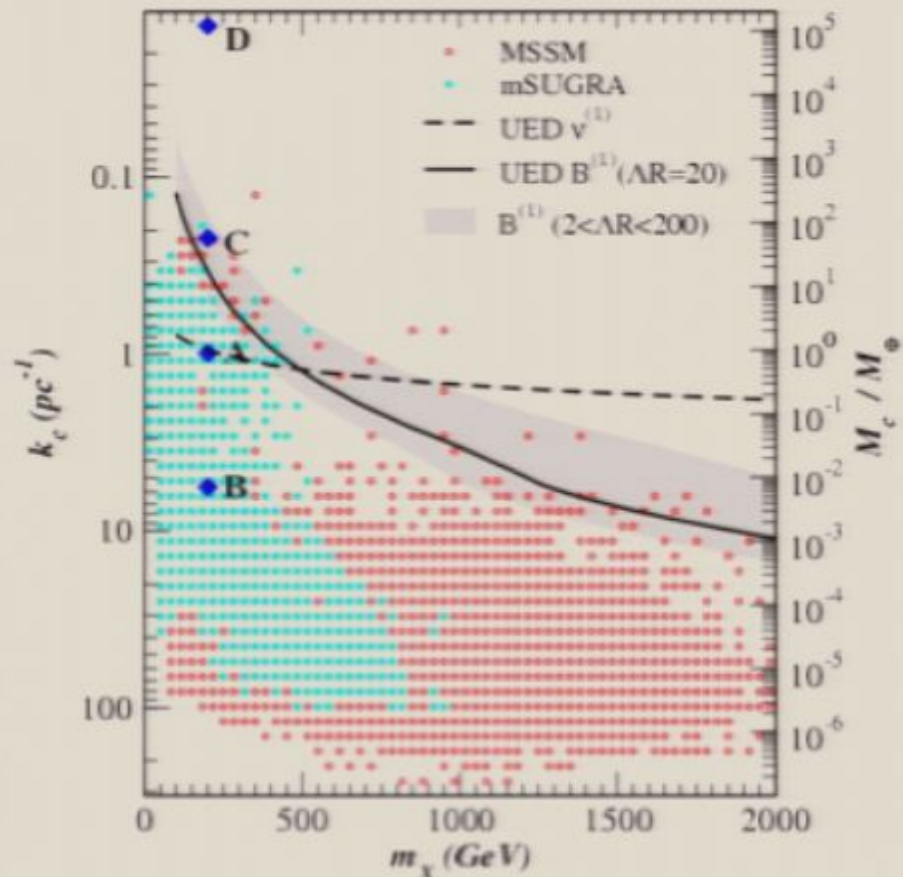
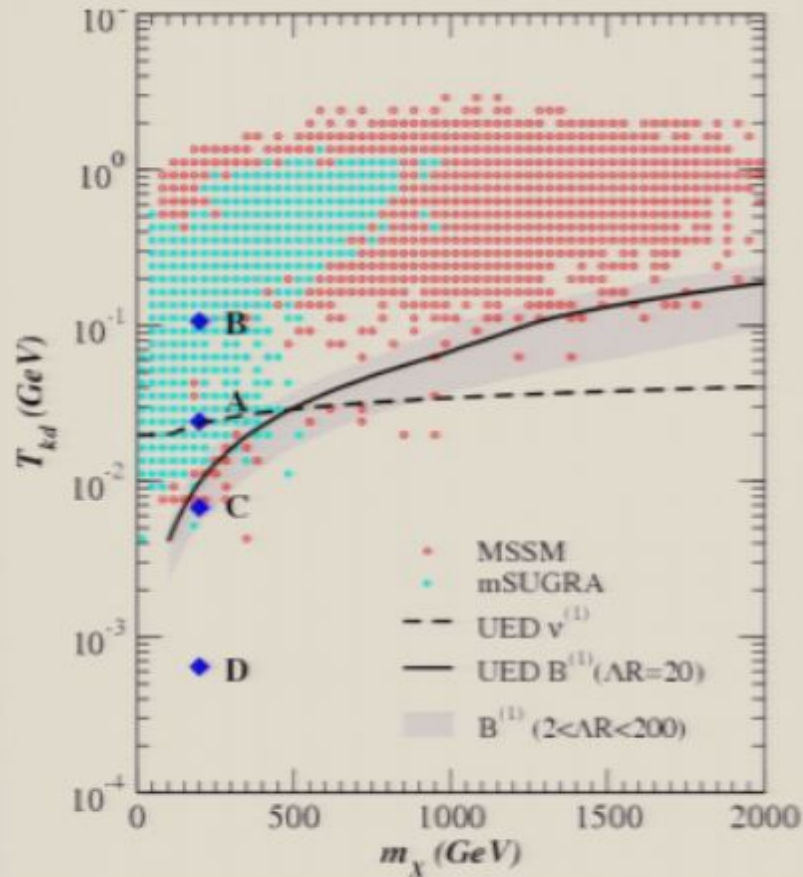
left to right/bottom to top:

Dirac (elastic scattering mediated by Z_0 exchange) $m = 100 \text{ GeV}$

Majorana (Z_0 exchange suppressed) $m = 50, 100, 500 \text{ GeV}$

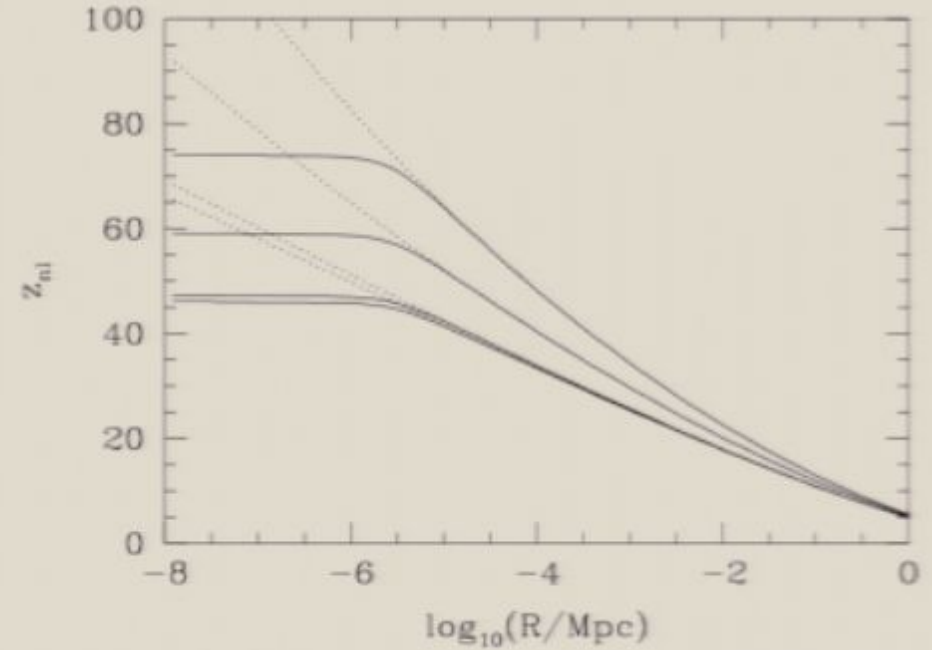
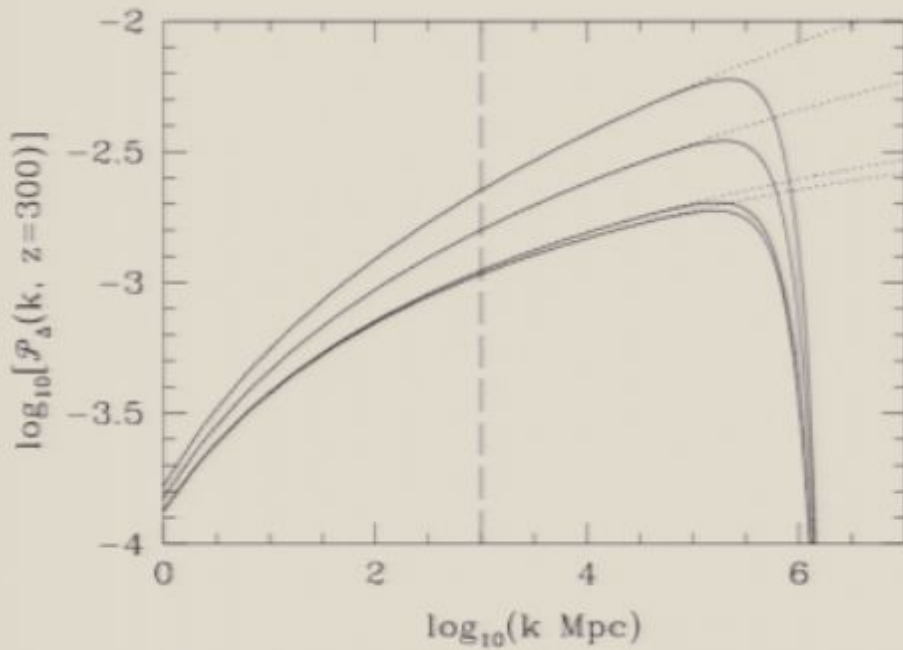
Profumo, Sigurdson & Kamionkowski:

Scan MSSM and also consider Universal Extra Dimensions and heavy neutrino like dark matter.



- A: coannihilation region, light scalar sparticles, (quasi-degenerate) NLSP is stau
 - B: focus point region, heavy scalars, scattering from light fermions is via Z0 exchange
 - C: $\Delta m_{\tilde{\nu}_{e,\mu}} \equiv m_{\tilde{\nu}_{e,\mu}} - m_\chi = 1 \text{ GeV}$
 - D: $\Delta m_{\tilde{\nu}_{e,\mu}} \equiv m_{\tilde{\nu}_{e,\mu}} - m_\chi = 0.01 \text{ GeV}$
- } Sfermion resonances. At high T scattering from light fermions energy independent.

ii) primordial power spectrum



top to bottom:

false vacuum dominated hybrid inflation

$$n=1.036, \quad \alpha=0$$

scale invariant

$$n=1.000, \quad \alpha=0$$

power law inflation

$$n=0.964, \quad \alpha=0$$

$m^2 \Phi^2$ chaotic inflation

$$n=0.964, \quad \alpha=-0.0006$$

$$\alpha = \frac{d\eta}{dk}$$

Structure Formation

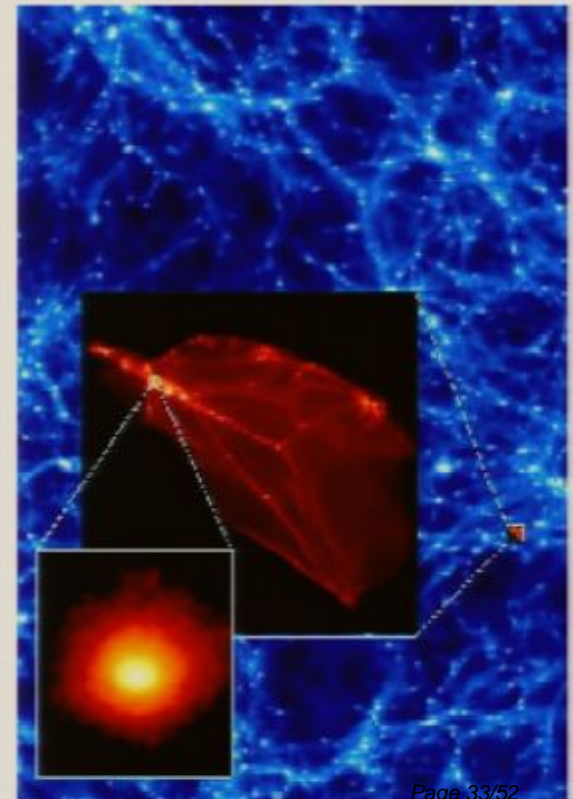
Our estimate of properties of first typical WIMP halos
(using spherical collapse model):

Form at $z \sim 50$

$M \sim 10^{-6} M_{\odot}$

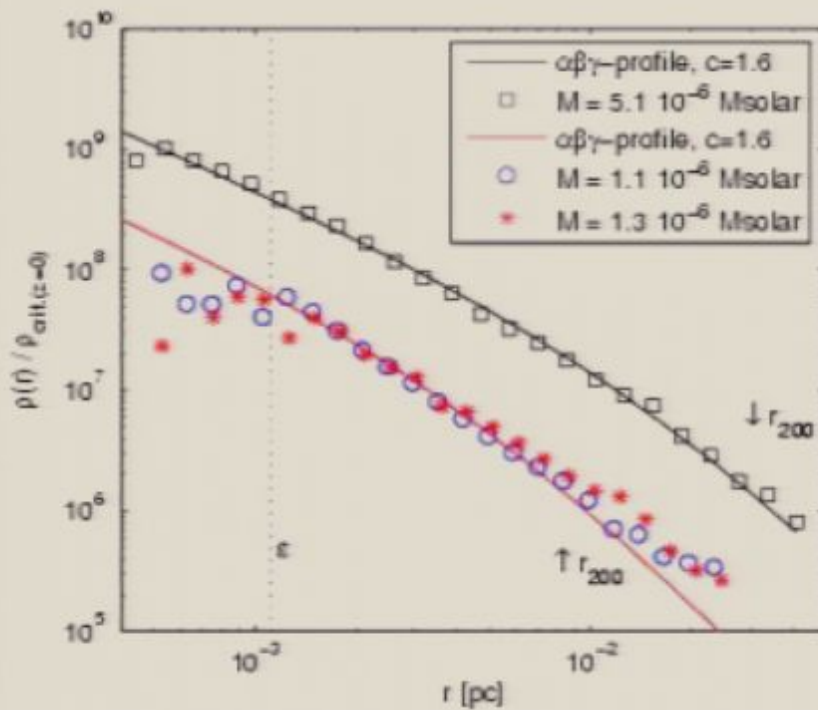
$r \sim 0.02$ pc

Diemand, Moore & Stadel, (re-) simulate a small, ~ 0.3 pc, region starting at $z=350$ (when the fluctuations are still linear) up until $z=26$ (when the high resolution region begins to merge with surrounding low resolution regions).

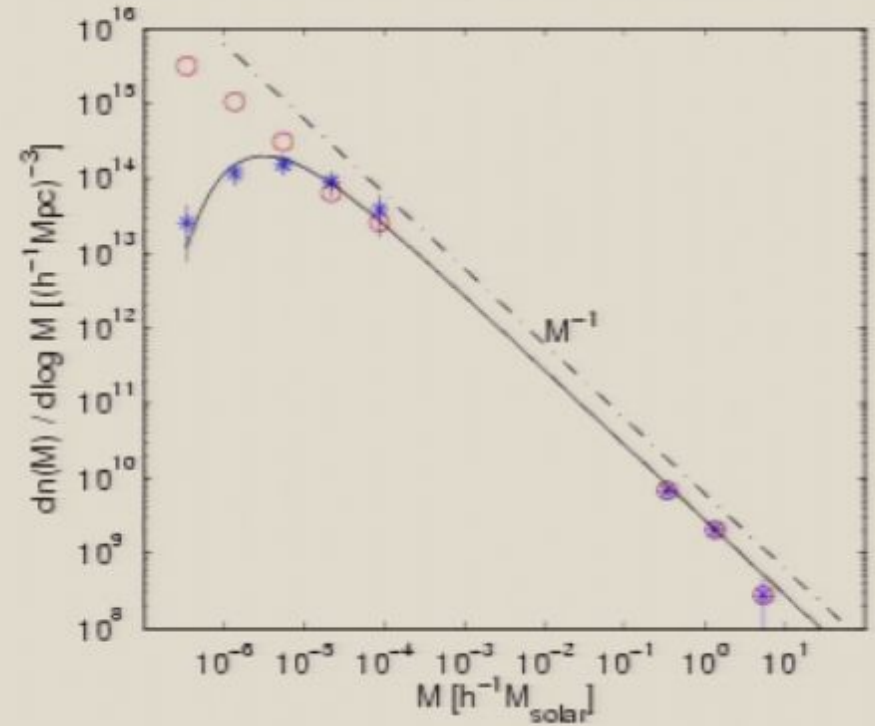


Properties of halos at $z=26$:

density profiles of
3 sample halos



mass function



$$\rho \propto r^{-\gamma}$$

$$\gamma \approx 1.5 - 2$$

$$\frac{dn}{d\log M} \propto M^{-1}$$

What happens to these mini-halos after $z=26$?

disrupted by: mergers with similar mass halos
tidal stripping
interactions with stars

Lots of recent work, in particular on interactions with stars

[Zhao et al.; Moore et al.; Berenzinsky et al.; Green & Goodwin; Goerdet et al.; Angus & Zhao]

Rough summary:

Tidal stripping only significant for mini-halos in the very inner regions of the Milky Way.

Mini-halos which pass through the MW disc will experience significant energy input due to interactions with stars.

Working out the (spatial and mass) distribution of mini-halos in the solar neighbourhood is an extremely non-trivial problem.

Survival probability (and distribution of remains of disrupted mini-halos) is very much an open question

BUT even if WIMP mini-halos in the solar-neighbourhood loose almost all of their mass the densest inner regions may survive (and be detectable by GLAST as gamma-ray sources with measurable proper motion?? [Koushiappas])

AND the distribution of the material which is removed is important for WIMP direct detection experiments.

Health Warning!

Everything I've said up until now (apart from perhaps the discussion of mini-halo survival) is reasonably well established and understood.

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Most of the rest of the talk is about issues which are not well understood (by me at least....) and should be taken in the spirit of `thinking aloud`.

Axions

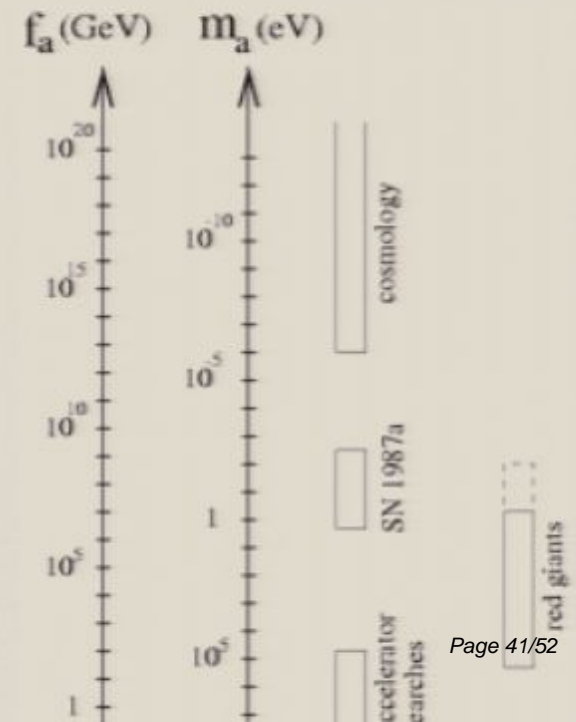
Introduction

✧ consequence of Pecci-Quinn symmetry proposed to solve strong CP problem

("why is the electric dipole moment of the neutron so small?")

✧ very light and very weakly interacting (never in thermal equilibrium in the early Universe)

✧ constraints on mass from cosmology, lab searches and from cooling of stars and supernovae



early Universe microphysics

Axions interact so weakly they are never in thermal equilibrium. Are produced by other, non-thermal (and messier...) mechanisms.

mis-alignment angle:

axion field generated at Peccei-Quinn phase transition at $T \sim f_a$

axion mass 'switches on' much later, at quark-gluon phase transition



Coherent oscillations of field \equiv cold dark matter

Axion density depends on initial value of field:

$$n_a(T_1) = \frac{1}{2} m_a(T_1) \theta_1^2 f_a^2$$

$$m_a(T_1) \approx 3H(T_1) \quad T_1 \sim 1 \text{ GeV}$$

$$V(\theta) = m_a^2(T) f_a^2 (1 - \cos \theta) \quad \theta \equiv a/f_a$$

$$\delta\rho_a = -\delta\rho_{\text{rad}} \rightarrow \text{isocurvature perturbations}$$

If either i) inflation does not occur

or ii) inflation occurs but re-heat temperature is high enough that Peccei-Quinn symmetry is restored ($T_{RH} > f_a$).

Coherence length of axion field \sim Horizon scale at PQ PT
 \ll Horizon scale at QCD PT

Large spatial variations in axion field:

i) Large spatial variations in density of axions produced by mis-alignment mechanism.

Mean density depends only on axion mass/ f_a :

$$\Omega_a h^2 \approx \left(\frac{\Lambda_{\text{QCD}}}{200 \text{ MeV}} \right)^{-0.7} \left(\frac{m_a}{10^{-5} \text{ eV}} \right)^{-1.18}$$

ii) Network of (global) axion strings produced after PQ phase transition.

Strings radiate axions-controversy over spectrum (and density) of axions produced (impossible to do simulations of physically relevant regime, therefore calculations rely on extrapolations.....). [e.g. Sikivie, Battye & Shellard, Yamaguchi et al.]

Density roughly comparable to density of mis-alignment axions.

After QCD PT unstable network of domain walls forms, decays producing further axions.

If iii) inflation occurs and re-heat temperature is lower than Peccei-Quinn scale ($T_{RH} < f_a$)

Entire observable Universe originates from region smaller than horizon scale at PQ PT.

Axion field uniform (+quantum fluctuations during inflation):

i) Density of axions produced by mis-alignment mechanism uniform(-ish?), and depends on axion mass and (unknown) value of field at PQ-PT

$$\Omega_a h^2 \approx 0.1 \left(\frac{\Lambda_{\text{QCD}}}{200 \text{ MeV}} \right)^{-0.7} \left(\frac{m_a}{10^{-5} \text{ eV}} \right)^{-1.18} f_{\text{an}}(\theta_1) \theta_1^2$$

ii) Axionic strings are inflated away.

structure formation

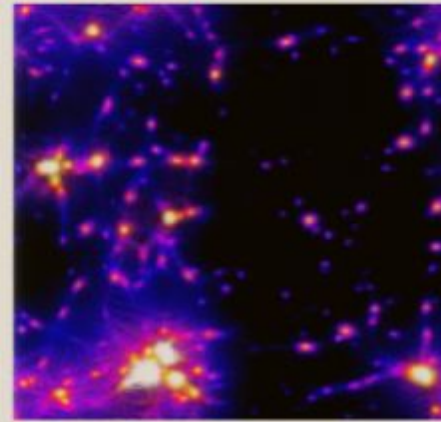
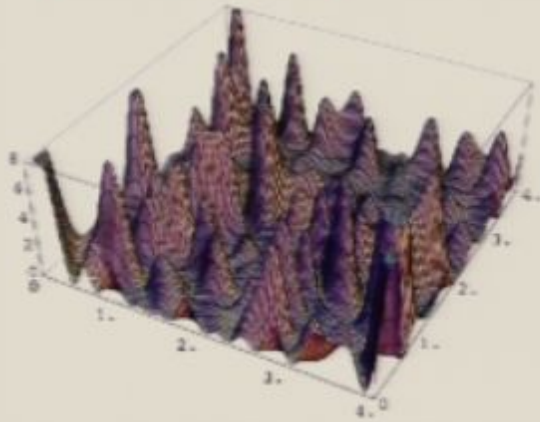
i) no inflation or $T_{\text{RH}} \geq f_a$

Large spatial fluctuations in value of axion field (and hence axion density) on horizon scale at QCD phase transition.
First axion halos form around matter-radiation equality.

[Hogan & Rees,
Kolb & Tkachev]

Zurek, Hogan & Quinn, astro-ph/0607341:

2-d slice of
initial (white
noise) axion
field.



simulated
density dist.
at $z \sim 3000$

Properties of first axionic halos:

$$M \sim 10^{-12} M_{\odot}$$

$$r \sim 10^{-5} \text{ pc}$$

Mass function and subsequent evolution haven't yet been calculated.

potential observational/experimental consequences?

Pico- and femto-lensing of gamma-ray bursts

First axion halos dense enough to cause lensing of cosmological point sources.

Image splitting tiny (pico or femto arc-seconds depending on mass).

Detectable either as fringes in energy spectrum or a GRB observed by two detectors (separated by ~ 1 AU) with very different intensities.

$$\text{ii) } T_{\text{RH}} \ll f_a$$

Axion is mass-less scalar field during inflation, acquires scale-invariant spectrum of fluctuations:

$$\mathcal{P}_a = \frac{k^3}{2\pi^2} \langle |a_k|^2 \rangle = \text{const} \quad \rightarrow \quad \langle |a_k|^2 \rangle \propto k^{-3/2}$$

CMB and LSS constraints on amplitude of isocurvature perturbations
-> constraint on energy scale of inflation [Lyth, Linde]

Axion can not be dominant component of CDM?? [Beltran et al.]

Primordial Black Holes

Introduction/early Universe

Primordial Black Holes (PBHs) can form via various mechanisms (collapse of large density perturbations, cosmic string loops, bubble wall collisions) in the early (radiation dominated) Universe.

Collapse of density perturbations:

If density fluctuation in a given region is sufficiently large gravity overcomes pressure and fluctuation collapses to form a black hole soon after horizon entry with mass roughly equal to the horizon mass.

[Carr; Musco et al.] $\delta > \delta_c \sim 0.3 - 0.5$

[Nishibata & Sasaki] $\zeta_0 > \zeta_c \sim 1$

Initial PBH density:

$$\Omega_{\text{PBH}}^i(M) = \text{erfc} \left(\frac{\delta_c}{\sigma(M)} \right)$$



$\frac{\rho_{\text{PBH}}}{\rho_{\text{rad}}} \propto \frac{a^{-3}}{a^{-4}} \sim a$ and $M_{\text{H}} = \frac{4\pi}{3}\rho(H^{-1})^3 \propto T^{-2}$ therefore, for PBHs with $M > 5 \times 10^{14}$ g which have lifetime greater than the age of the Universe:

$$\Omega_{\text{PBH}}^0(M) \sim \frac{T_i}{T_{\text{eq}}} \text{erfc}\left(\frac{\delta_c}{\sigma(M)}\right) \sim \left(\frac{M_{\text{H,eq}}}{M}\right)^{1/2} \text{erfc}\left(\frac{\delta_c}{\sigma(M)}\right)$$

and

$$\Omega_{\text{PBH}}^0 \sim \mathcal{O}(1) \quad \rightarrow \quad \sigma(M) \sim 0.05$$

This requires the density perturbations on small scales to be significantly larger on small scales than they are on cosmological scales ($\sigma(M) \approx 10^{-5}$).

This could happen if the primordial power spectrum has a feature or positive running, $dn/d \ln k > 0$.

But need fine tuning in order not to produce far too many, or too few, PBHs.

Structure formation

PBHs form from very rare high peaks in the density field.

High peaks in density field are clustered. [Bond, Bardeen, Kaiser & Szalay].

This could have consequences for structure formation with dark matter in the form of PBHs. [Afshordi, MacDonald & Spergel; Chisholm]

e.g. formation (and possible evaporation) of PBH clusters.....
constraints from isocurvature nature of fluctuations

Conclusions

- ★ There is lots of evidence for dark matter from diverse astronomical and cosmological observations (assuming that Newtonian gravity/GR is correct....).
- ★ There are several well motivated dark matter candidates, with different early Universe microphysics and (probably) different present day distributions on small (sub-galactic) scales.
- ★ WIMPs: microphysics well established, still some (tricky..) open issues for small scale distribution today.
- ★ axions: microphysics and cosmology more complex, work still to be done?
- ★ PBHs: microphysics straight forward (but fine-tuning needed to produce the right number density?), clustering not yet well understood.

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