Title: Massless 4D gravitons from 5D Asymptotically AdS spacetime

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Abstract:

Massless 4D Gravitons from Asymptotically AdS_5 Spacetimes

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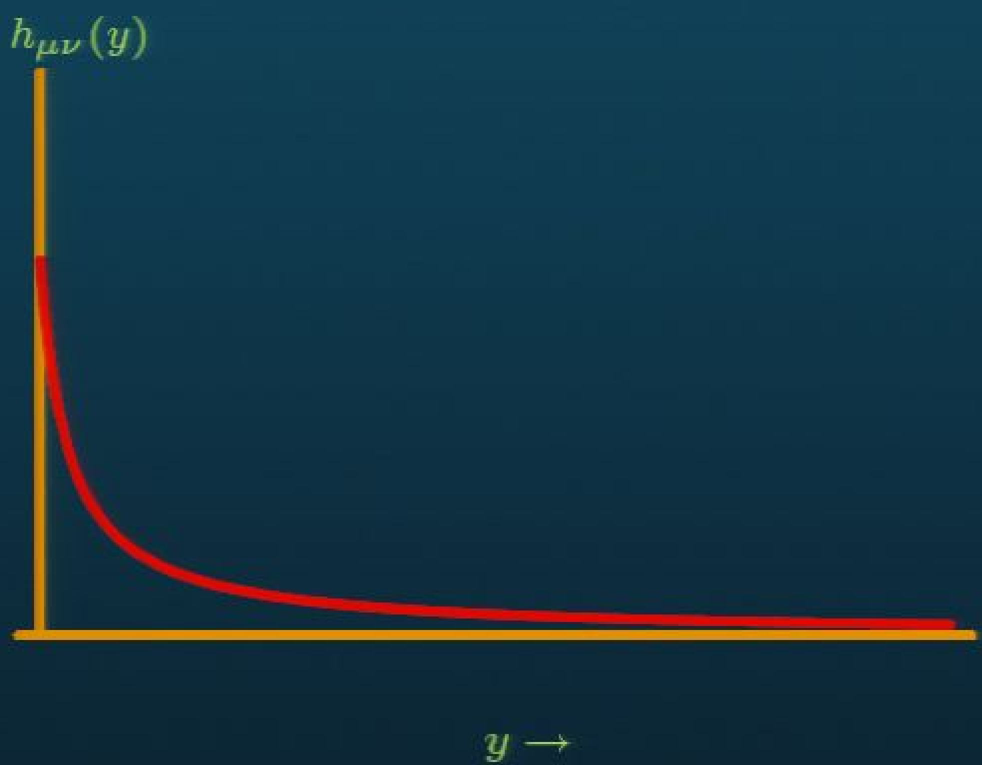
CPhT - Ecole Polytechnique

PI - Novermber 7th 2006

Based on work with E. Kiritsis, to appear.

RS Model:



Prologue

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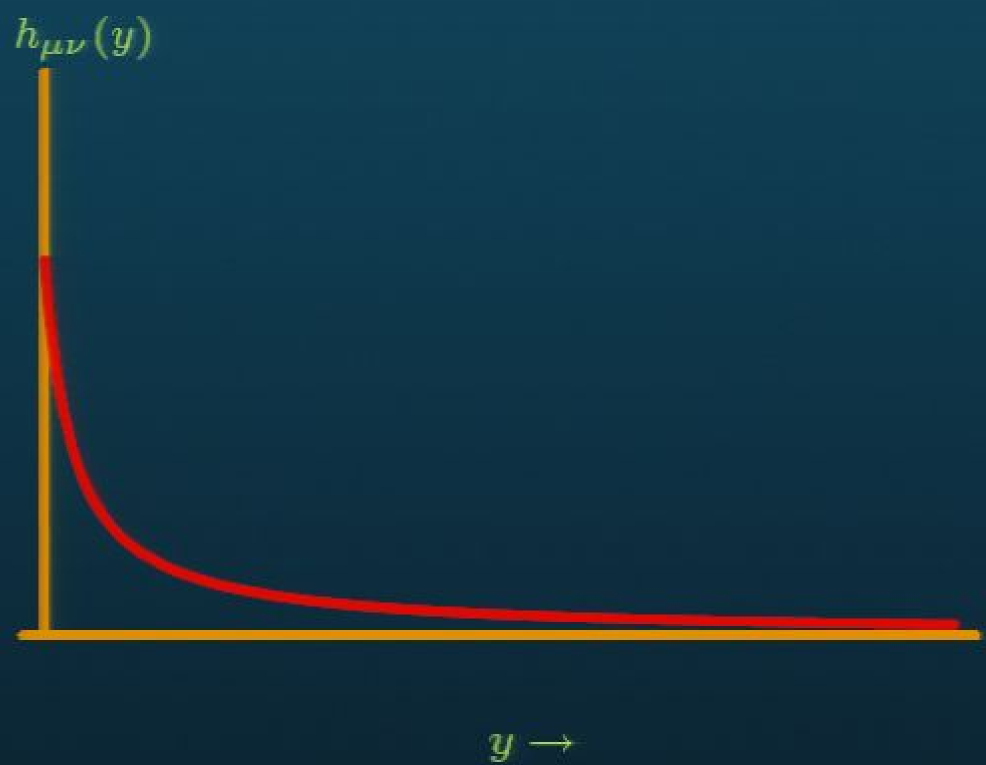

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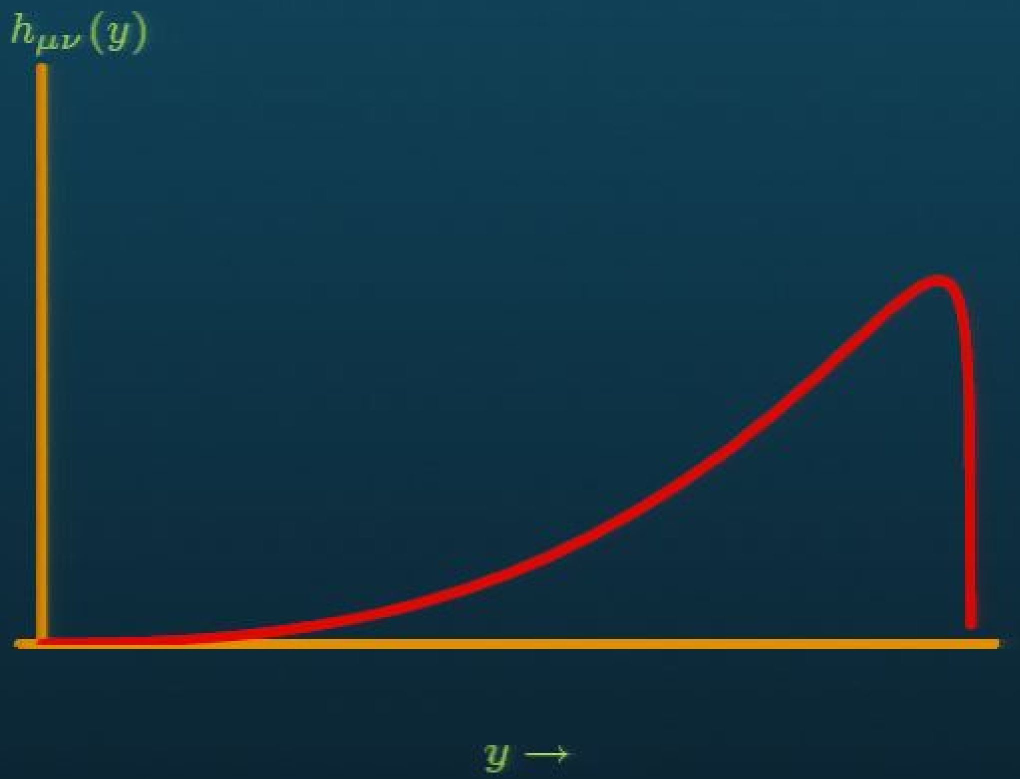
Are there models that exhibit:



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Spectrum of 4D field theory particles = spectrum of normalizable states in the 5D geometry. Witten, '97

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Can we have 4D graviton localized far from the boundary? (this would be emergent, rather than fundamental, in the dual FT).

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- RS-like Models
- Einstein-Dilaton Models

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  - Linear Fluctuation Analysis
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- Conclusion and Perspectives

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Modes of these 5D fields such that:

- they have a fixed 4D mass: $\Box_4\Phi(x,y) = m^2\Phi(x,y)$
- are normalizable w.r.t. to the radial direction y, i.e. they have a finite 4D kinetic term.

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We are interested in 4D-massless, y-normalizable fluctuations of the (tensor part of) the 5D metric component, $h_{\mu\nu}(x,y)$

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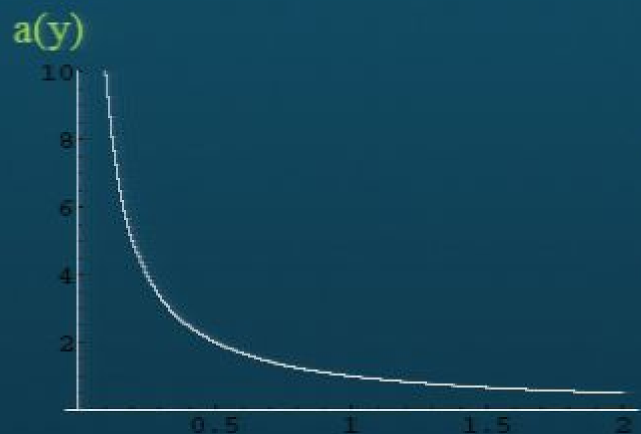
For tensor spin-2,

$$h_{\mu\nu}(x,y) \sim h^{(0)}_{\mu\nu}(x) + y^4 h^{(4)}_{\mu\nu}(x) + \dots$$

AdS_5 vs. RSII



AdS_5:

$$ds^2 = \frac{1}{(ky)^2} \left(dy^2 + dx_\mu^2\right) \quad 0 < y < \infty$$

$$S_{kin}[h^{(0)}] = \int_0 dy \, \frac{1}{(ky)^3} \int d^4x \left(\partial h^{(0)}\right)^2 = \infty$$

- In AdS_5 $h_{\mu\nu}^{(0)}$ is not normalizable $\Rightarrow$ not a state in the 4D FT, rather an external source added to the UV theory.

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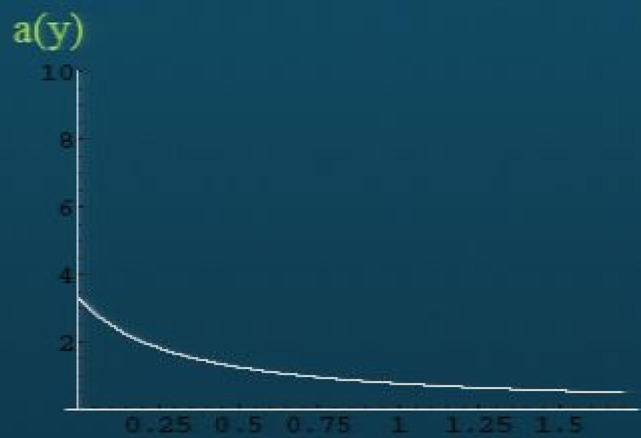

Slice of AdS_5:

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- In RSII it becomes normalizable $\Rightarrow$ the source gets a kinetic term and becomes dynamical. it is promoted to a fundamental d.o.f of the UV theory.

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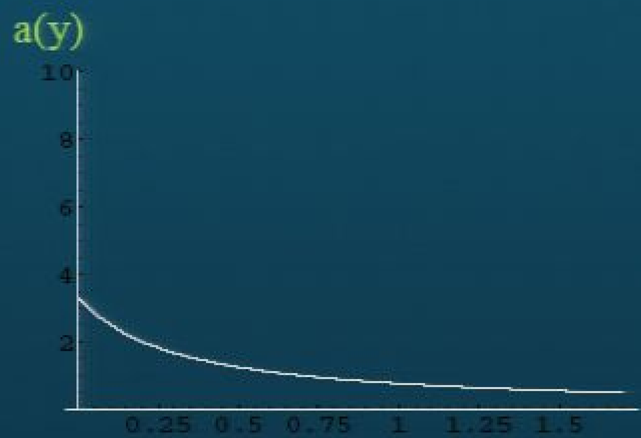

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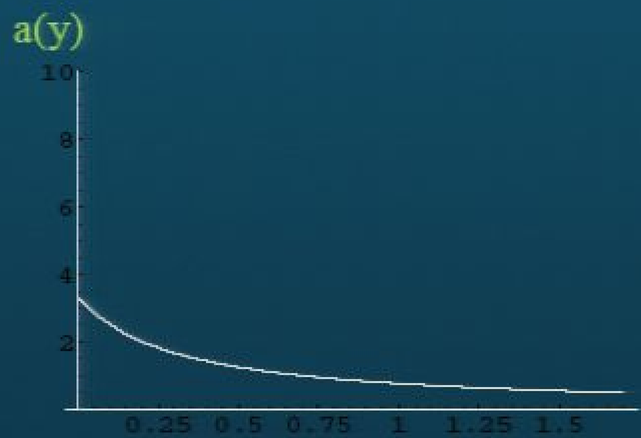

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- A normalizable $h_{\mu\nu}^{(4)}(x)$ would correspond to a low-energy excitation ("glueball") rather than a fundamental one. Can we realize a set-up in which the only normalizable massless tensor mode is $h_{\mu\nu}^{(4)}(x)$?

Massive RSI

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$$g_{AB} = \frac{1}{(1+ky)^2}(\eta_{AB} + h_{AB}) \qquad 0 < y < y_1$$

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- add boundary "massess" to change boundary conditions

$$g_{AB} = \frac{1}{(1+ky)^2}(\eta_{AB} + h_{AB}) \qquad 0 < y < y_1$$

$$\begin{aligned} S[h_{AB}] &= M_5^3 \int d^4x dy \sqrt{-g}\,(R - 2\Lambda) + \lambda\, k^2 M_5^3 S_{FP}[h_{AB}] \\ &+ \alpha_0\, k M_3 S^b_{FP}[h_{\mu\nu}]\Big|_{y=0} + \alpha_1\, k M_3 S^b_{FP}[h_{\mu\nu}]\Big|_{y=y_1} \end{aligned}$$

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$h_{\mu\nu}(x)$ is the massless 4D graviton. In the dual picture it is not a fundamental d.o.f but a low energy effect.

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- general covariance explicitly broken by mass terms
- unclear how to include nonlinearities (+ usual problems of massive gravity)
- scalar sector: one mode has vanishing kinetic term at quadratic level
  - in generally covariant theory it is a gauge mode $\Rightarrow$ expected to decouple to all orders
  - here it could become strongly interacting at nonlinear level

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Take solution asymptotically AdS_5 in the UV $(y \to 0)$:

$$a(y) \sim \frac{1}{ky}; \quad \Phi_0(y) \sim const; \quad V(\Phi_0(y)) \sim 2\Lambda$$

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as long as $\ddot{B} > 0$, i.e. $B' \exp(2B(y)/3)$ is monotonically $\uparrow$.

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under linearized diffeomorphisms $(\delta y = \xi^5, \delta x^\mu = \xi^\mu)$:

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-5 (gauge) - 5 (G_{yA} constraints)

Massive vs. Massless Modes

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This is important for the scalar sector.

Massive Fluctuations

If $m^2 \neq 0$:

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- Constraints: $\phi = -2\psi, \quad V^T_\mu = 0.$
- Field equations:

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$$\zeta'' + \left(3\frac{a'}{a} + 2\frac{z'}{z}\right)\zeta' + m^2\zeta = 0 \qquad \zeta \equiv \psi - \frac{\chi}{z}, \;\; z \equiv \frac{\Phi_0}{a'/a}$$

Massive Fluctuations

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$$\chi, \phi \quad A_\mu = 0, \quad h_{\mu\nu} = 2\psi\, \eta_{\mu\nu} + 2\partial_{(\mu} V^T_{\nu)} + h^{TT}_{\mu\nu}$$

- Constraints: $\phi = -2\psi, \quad V^T_\mu = 0.$
- Field equations:

$$(h^{TT}_{\mu\nu})'' + 3\frac{a'}{a}(h^{TT}_{\mu\nu})' + m^2 h^{TT}_{\mu\nu} = 0$$

$$\zeta'' + \left(3\frac{a'}{a} + 2\frac{z'}{z}\right)\zeta' + m^2\zeta = 0 \qquad \zeta \equiv \psi - \frac{\chi}{z}, \;\; z \equiv \frac{\Phi_0}{a'/a}$$

$(h^{TT}_{\mu\nu}, \zeta)$: 5 (spin 2) + 1 (spin 0) = 6 physical d.o.f. per m^2

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normalizability:

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ψ^{IR} is normalizable, $\psi^{UV}(y)$ is not.

(Notice: both are normalizable in RS, where $y > 1/\Lambda$)

IR Asymptotics

We have one candidate Zero-Mode, normalizable around $y = 0$:

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- y range extends to $+\infty \Rightarrow \Psi_{IR} \to \infty$ as $y \to \infty$, not normalizable
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- close to y_0:

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$\Rightarrow$ IR behavior is already encoded in fundamental UV theory.

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zero mode exists in the spectrum. skip

Summary

	$y \in (0, y_0)$		
$B(y_0)$	$-\alpha\, log(y_0 - y)$		
	$0 < \alpha < 1/2$	$1/2 < \alpha < 1$	$1 < \alpha < 3/2$
Spin 2	○	○	○
Spin 1	○	–	–
Spin 0	–	–	–
$V(y_0)$	$-1/(y_0 - y)^2$	$-1/(y_0 - y)^2$	$+1/(y_0 - y)^2$
$\Psi(y)$	$(y_0 - y)^\alpha$	$(y_0 - y)^{1-\alpha}$	$1/(y_0 - y)^{\alpha-1}$
$R(y_0)$	$+\infty$	$+\infty$	$-\infty$

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$\Rightarrow$ this is the boundary conditions we need to impose on the fluctuations to keep zero-mode is in the spectrum

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setting PF mass term = 0 in 4D is not considered tuning.

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$$S = \frac{1}{2k_5^2} \int dy \frac{a^3(y)}{a^2(y_b)} \left(\partial_\rho h_{\mu\nu}(y)\right)^2 + \int_{y=y_b} h_{\mu\nu}(y_b) T^{\mu\nu}$$

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$$S = \frac{1}{2k_5^2} \int dy \frac{a^3(y)}{a^2(y_b)} \left(\partial_\rho h_{\mu\nu}(y)\right)^2 + \int_{y=y_b} h_{\mu\nu}(y_b) T^{\mu\nu}$$

$$\left[h_{\mu\nu}(y,x) = a^{-3/2}(y)\Psi(y)h_{\mu\nu}(x)\right]$$

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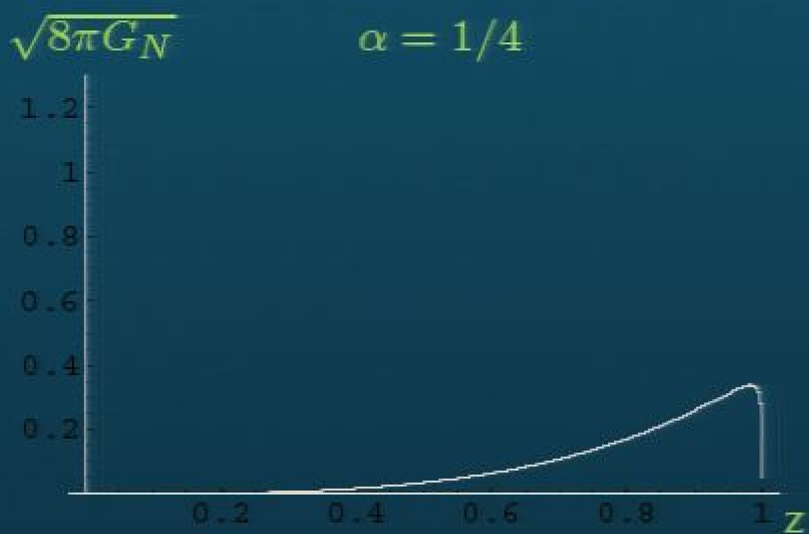
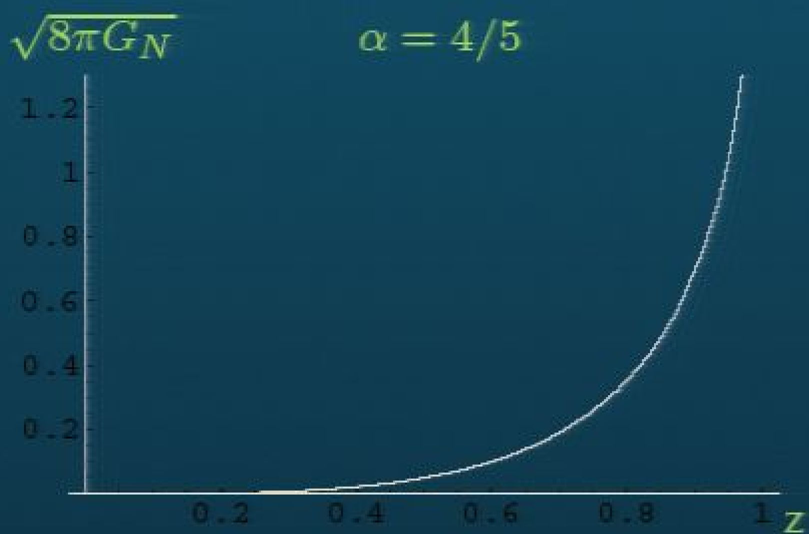
$$z_b \equiv y_b/y_0$$

Logarithmic $B(y)$ - 4D Planck Scale II



$$\sqrt{8\pi G_N} \propto \frac{z_b^{1/2} F(z_b)}{(1 - z_b)^{\alpha/3}}$$

Logarithmic $B(y)$ - 4D Planck Scale II



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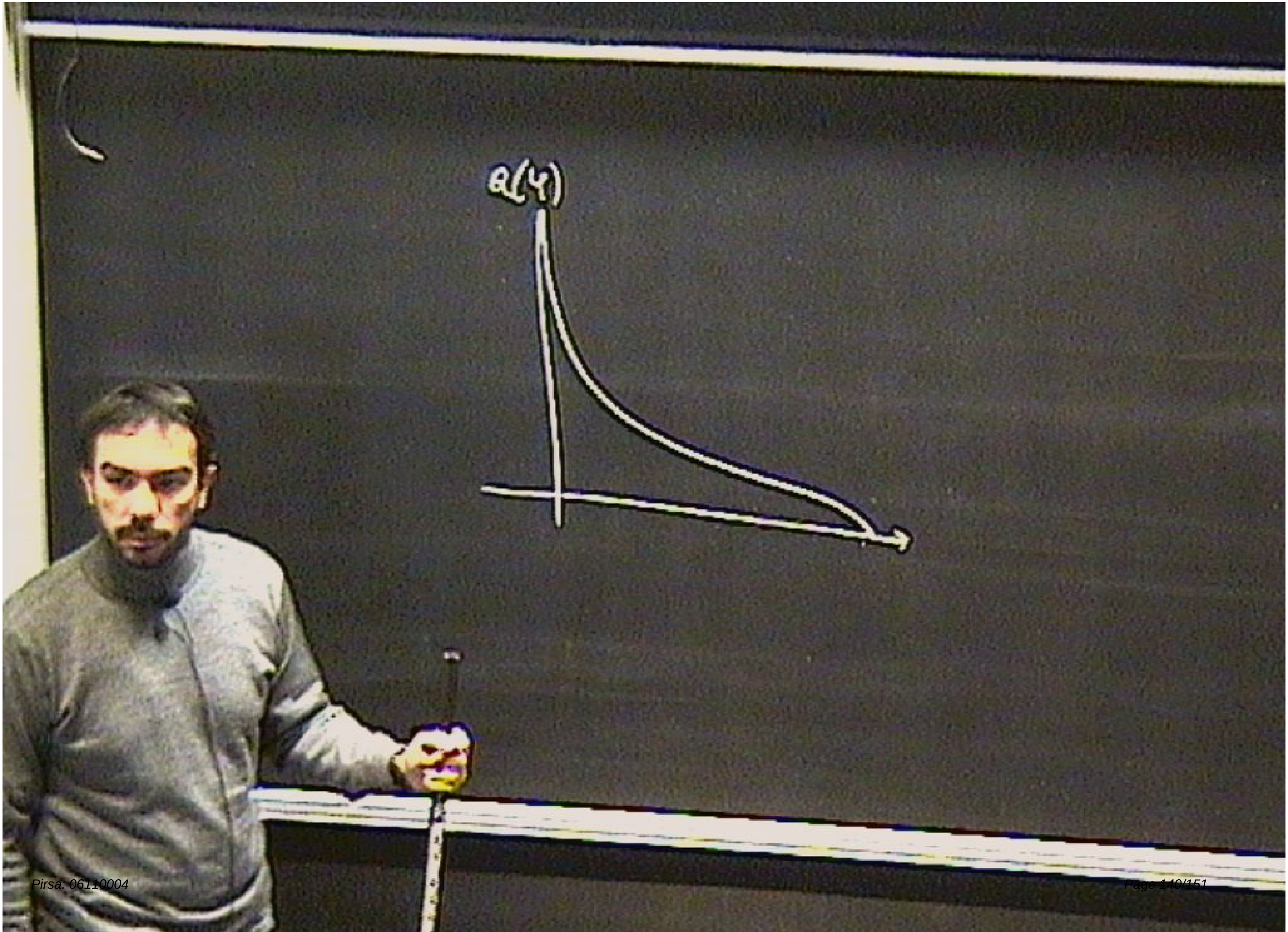
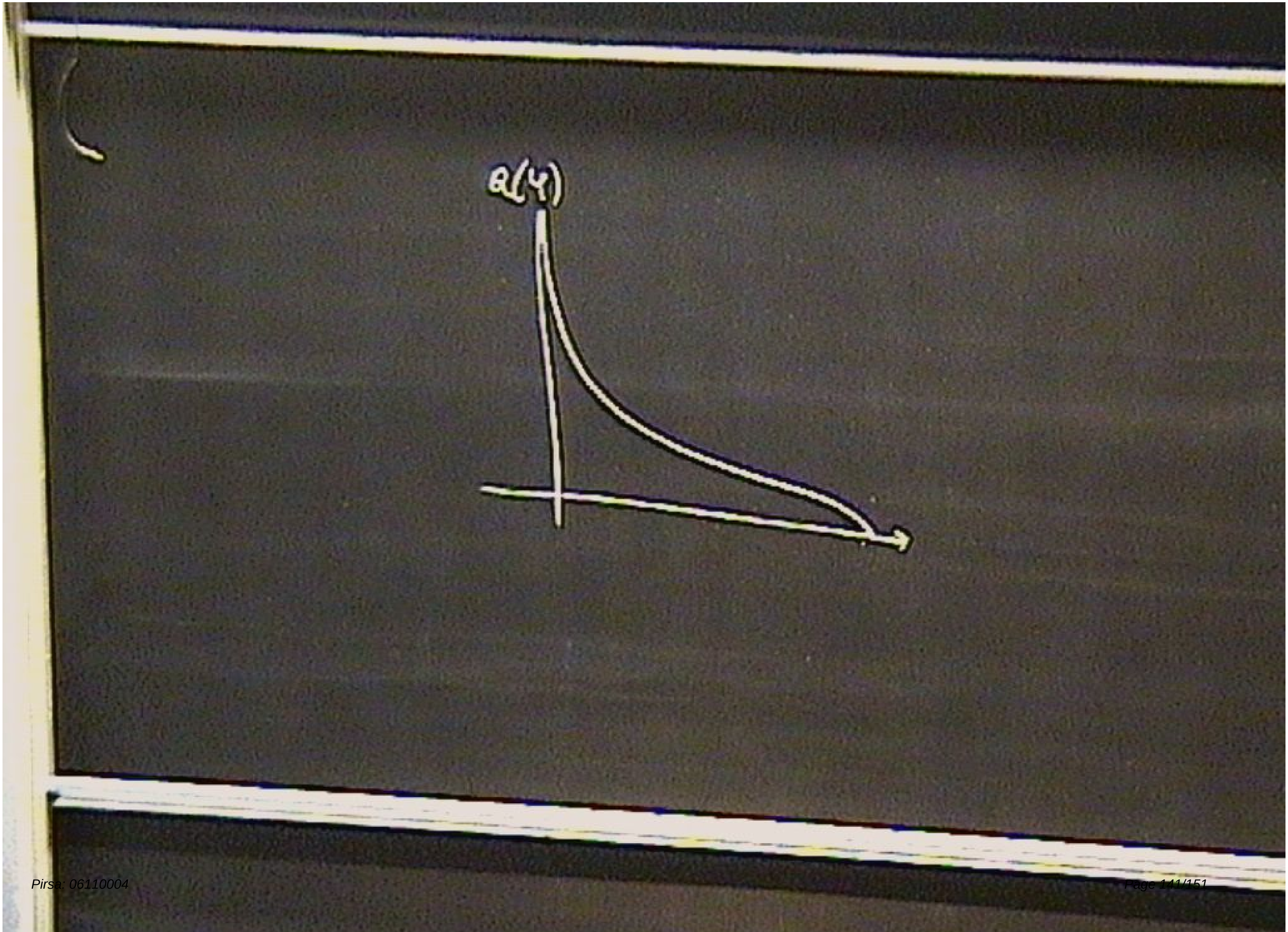
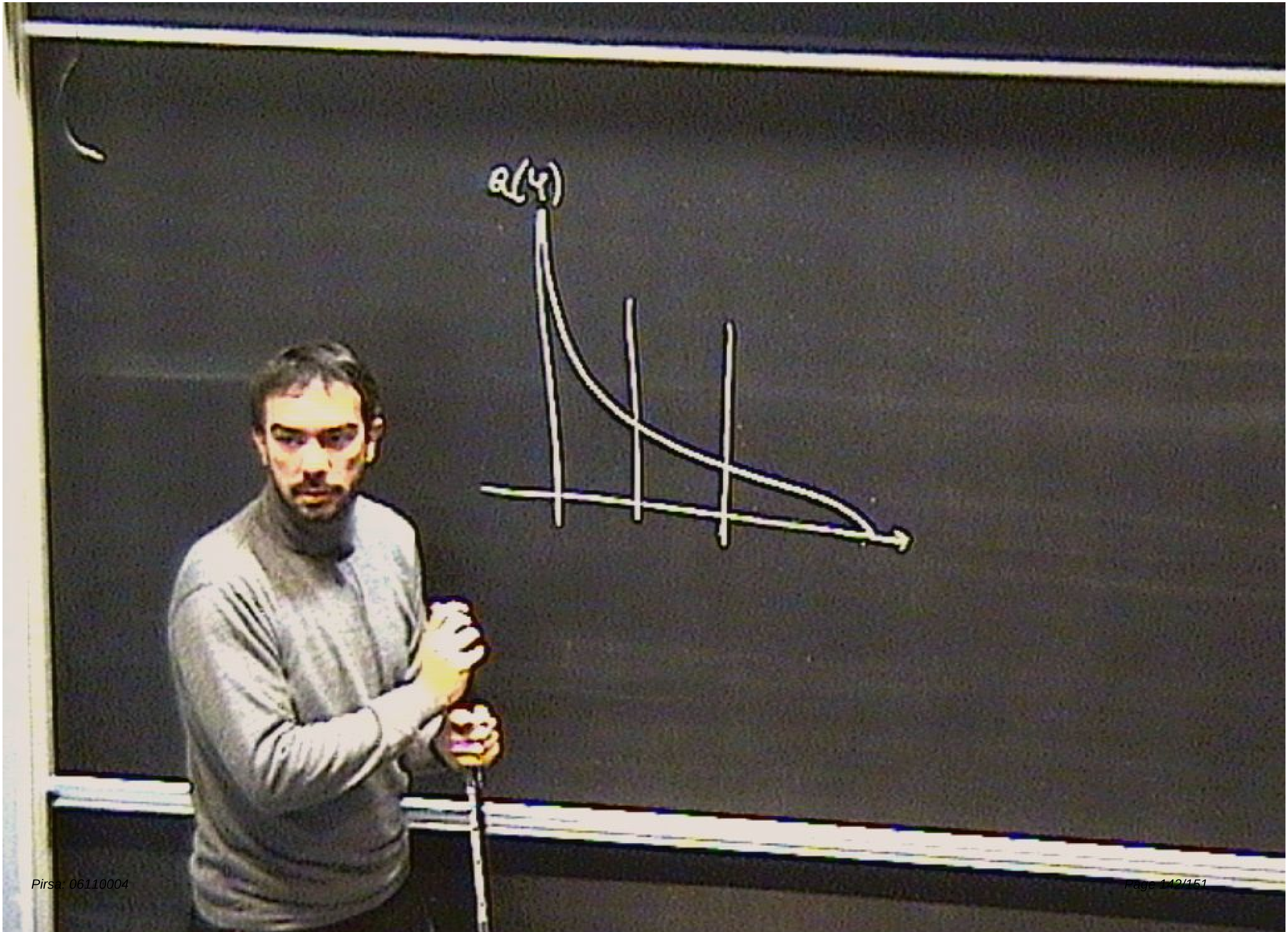
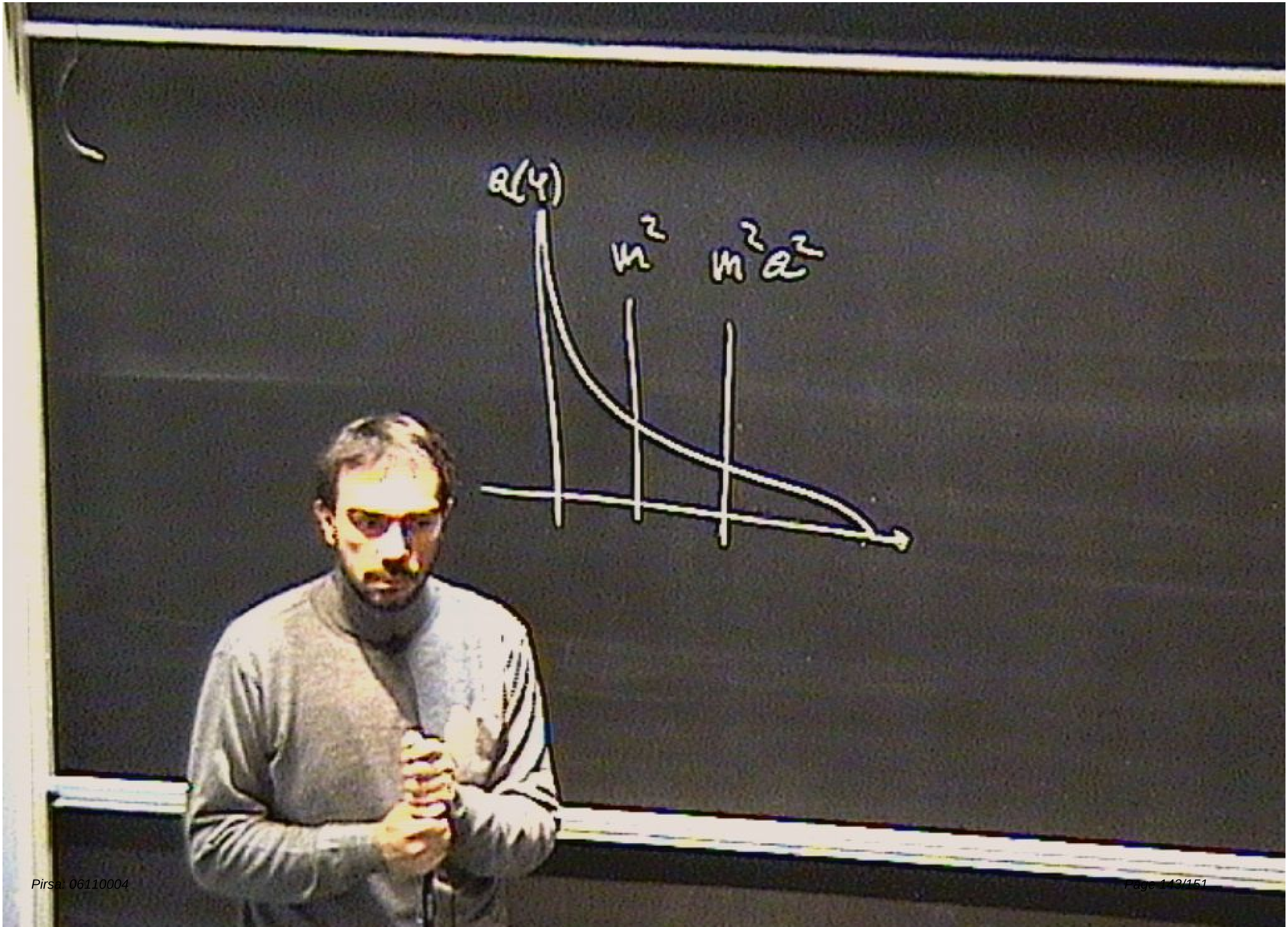


a(t)



Q(Y)


Q(Y)


a(y)
m²
m²a²

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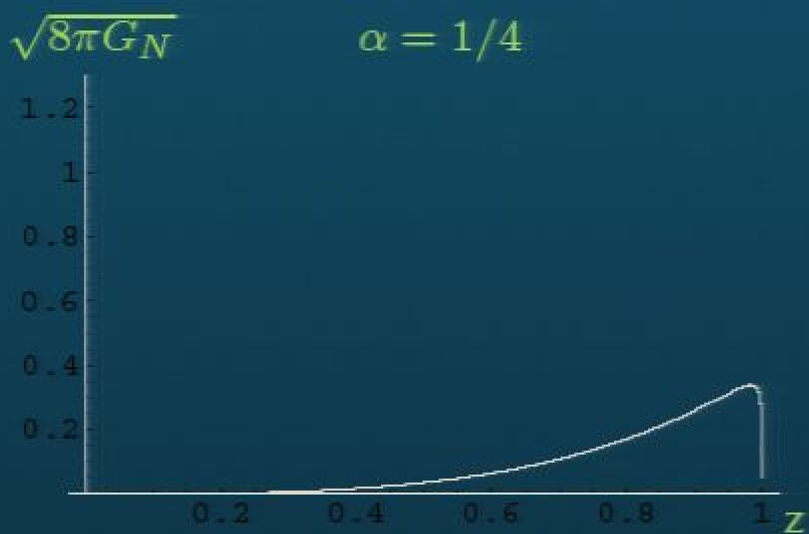

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Perspectives: Avoiding the need of BC - 1

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- singular IR behavior
- need extra input in the IR (boundary conditions): spectrum is not purely specified by UV data.
- Singularity might be resolved (e.g. in string theory), but need to do it in a very specific way to give correct b.c.

This may be related with Witten-Weinberg theorem: "you cannot get a composite massless spin-2 state from a Lorentz-covariant 4D field theory." This might indicate that the singularity should be resolved in a Lorentz-non-invariant way.

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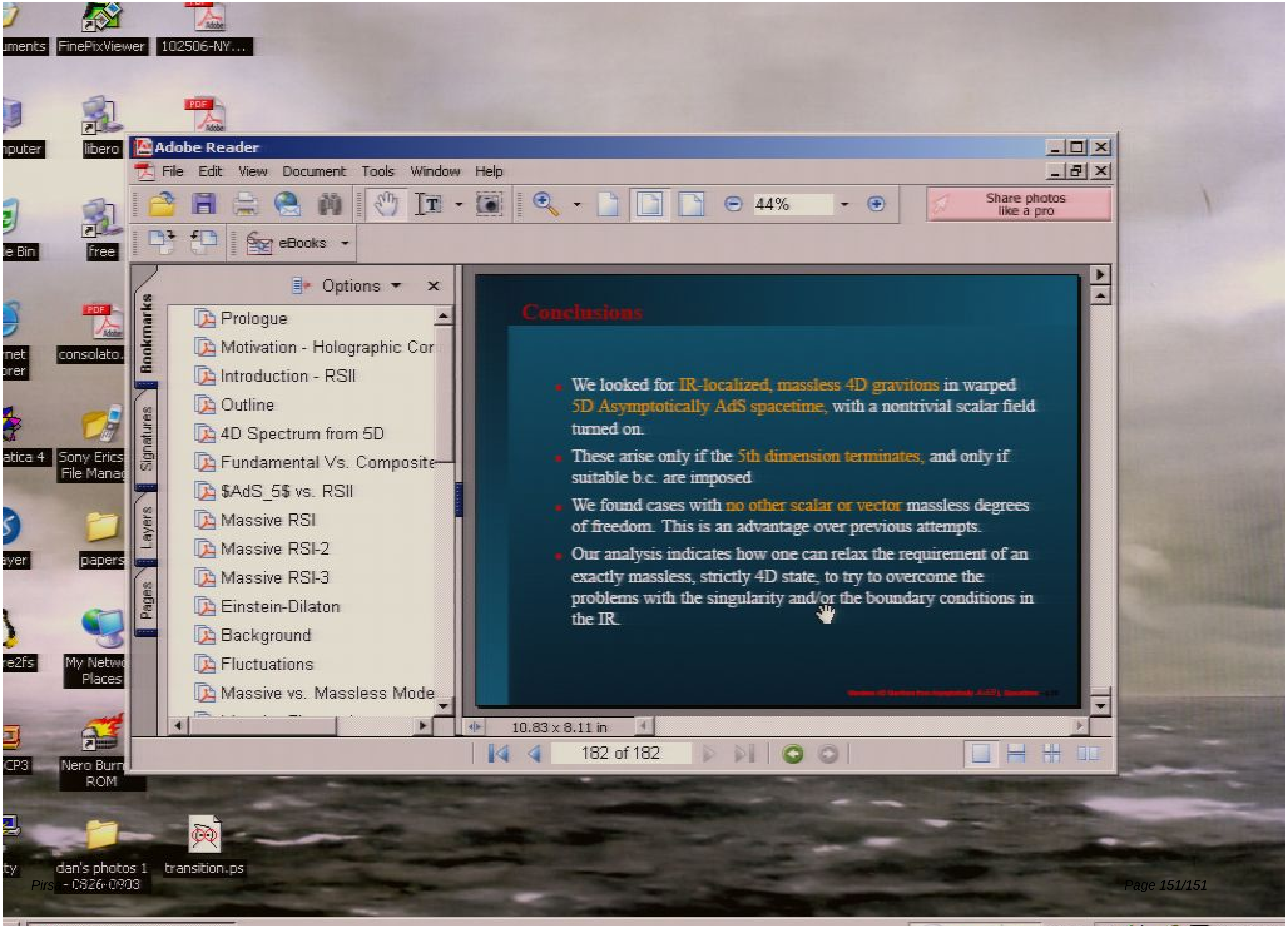
- look for light, long lived, spin-2 resonance.

end

Conclusions

- We looked for IR-localized, massless 4D gravitons in warped 5D Asymptotically AdS spacetime, with a nontrivial scalar field turned on.
- These arise only if the 5th dimension terminates, and only if suitable b.c. are imposed
- We found cases with no other scalar or vector massless degrees of freedom. This is an advantage over previous attempts.
- Our analysis indicates how one can relax the requirement of an exactly massless, strictly 4D state, to try to overcome the problems with the singularity and/or the boundary conditions in the IR.


Adobe Reader
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Prologue
Motivation - Holographic Cor
Introduction - RSII
Outline
4D Spectrum from 5D
Fundamental Vs. Composite
AdS_5 vs. RSII
Massive RSI
Massive RSI-2
Massive RSI-3
Einstein-Dilaton
Background
Fluctuations
Massive vs. Massless Mode
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10.83 x 8.11 in