

Title: Conformal field theory in the laboratory: quantum dots, Kondo effect, non-Fermi liquids and all that

Date: Nov 01, 2006 02:00 PM

URL: <http://pirsa.org/06110000>

Abstract: Boundary conformal field theory finds applications not only to high energy physics but also to condensed matter systems containing quantum impurities, whose world lines can sometimes be modelled as boundaries of 2-dimensional space-time. This technique leads to exact predictions for the low temperature behaviour of gated semi-conductor quantum dot devices which have been recently confirmed experimentally. I will give a non-technical overview of both the theory and the experiments.

Conformal Field Theory in the Laboratory: Quantum Dots, Kondo Effect, Non-Fermi Liquids and all that

Ian Affleck
Perimeter Institute, November 1, 2006



High Energy

Condensed Matter

1970

Wilson

Nozières & Blandin

Wess, Zumino

Andrei & Destri

Witten

Tsvelik

Knizhnik

Cardy

& Zamolodchikov

Affleck & Ludwig

Oregan

& Goldhaber-Gordon

2003

Pustilnik, Borda

Glazman & Van Delft

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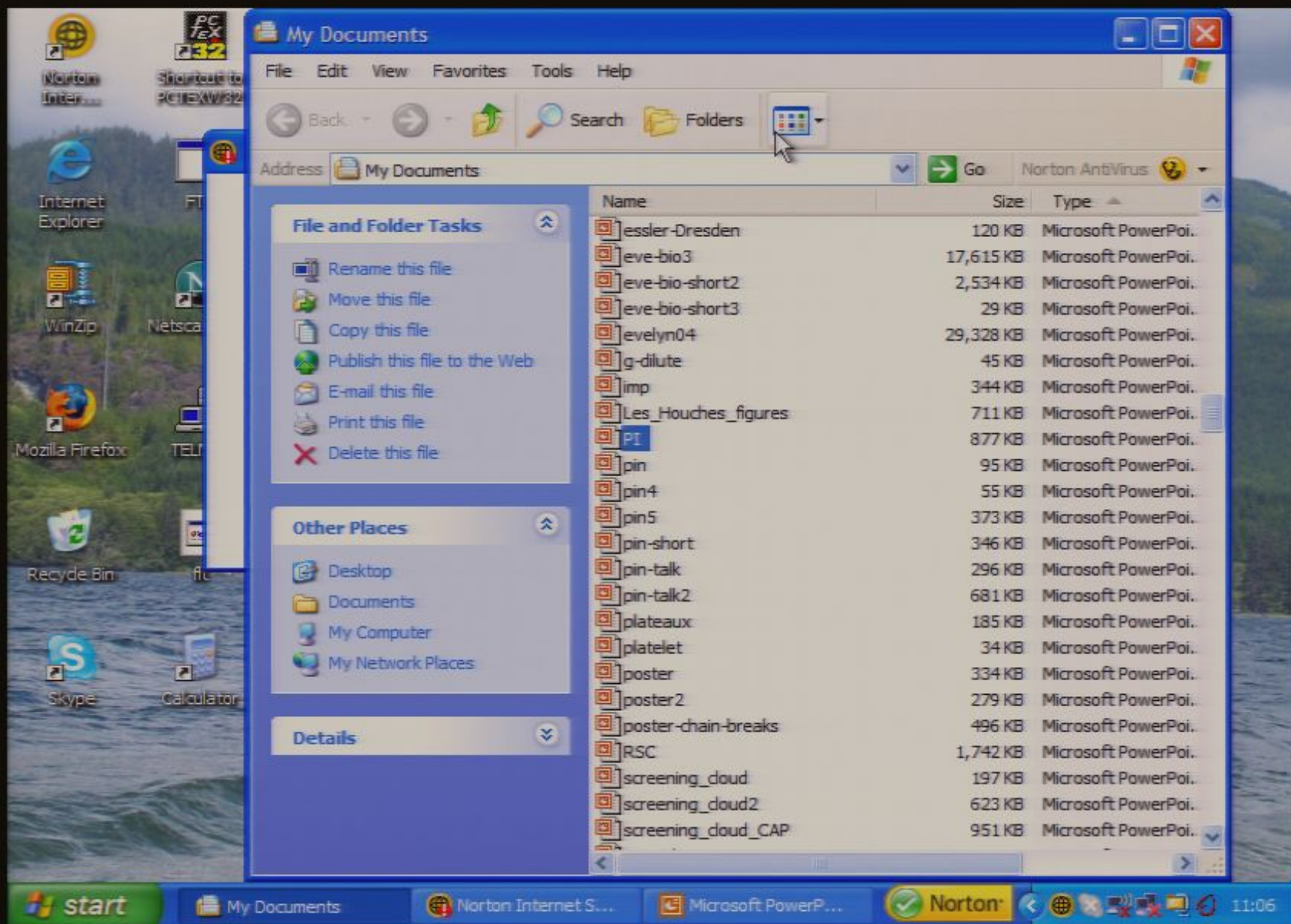
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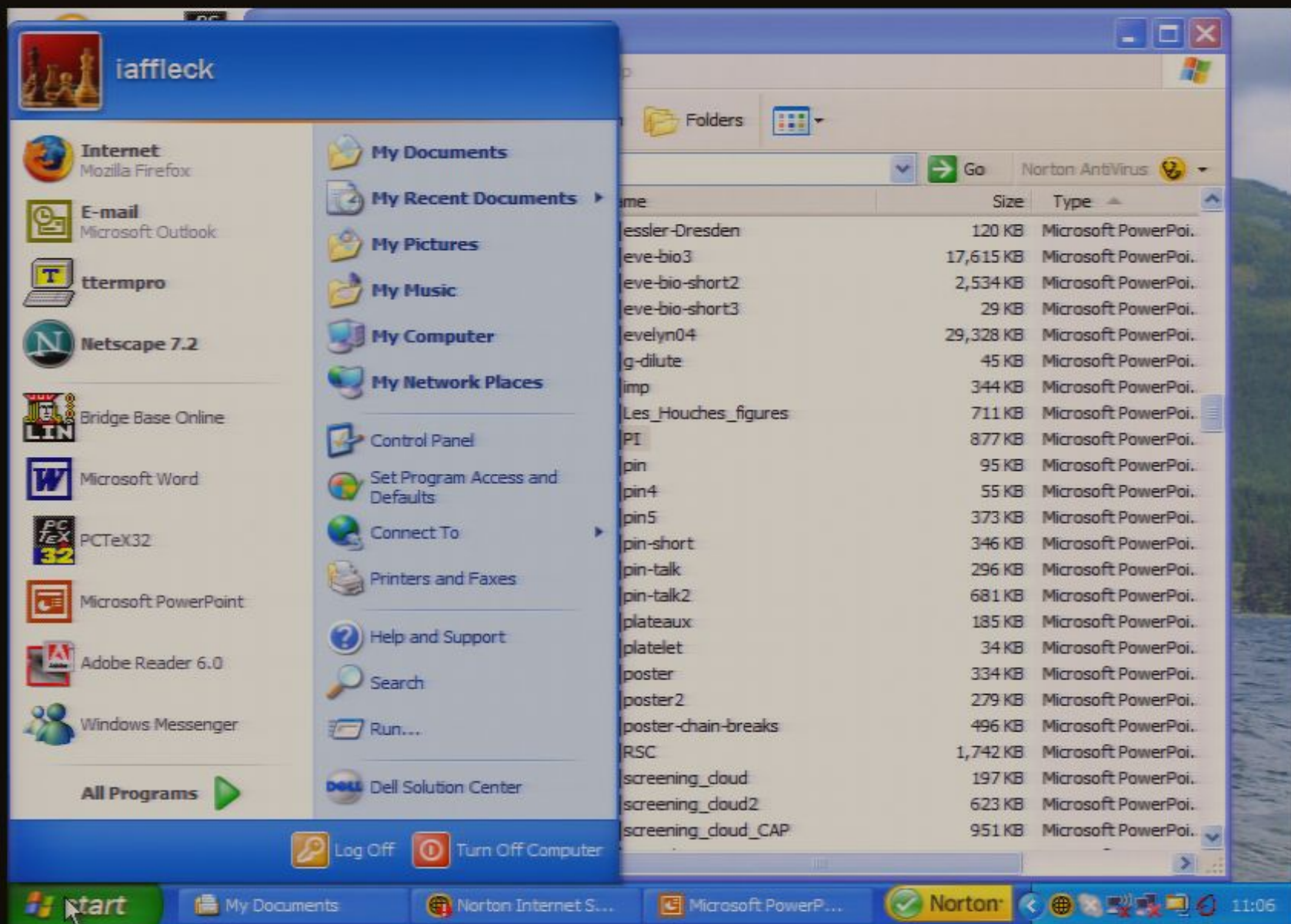
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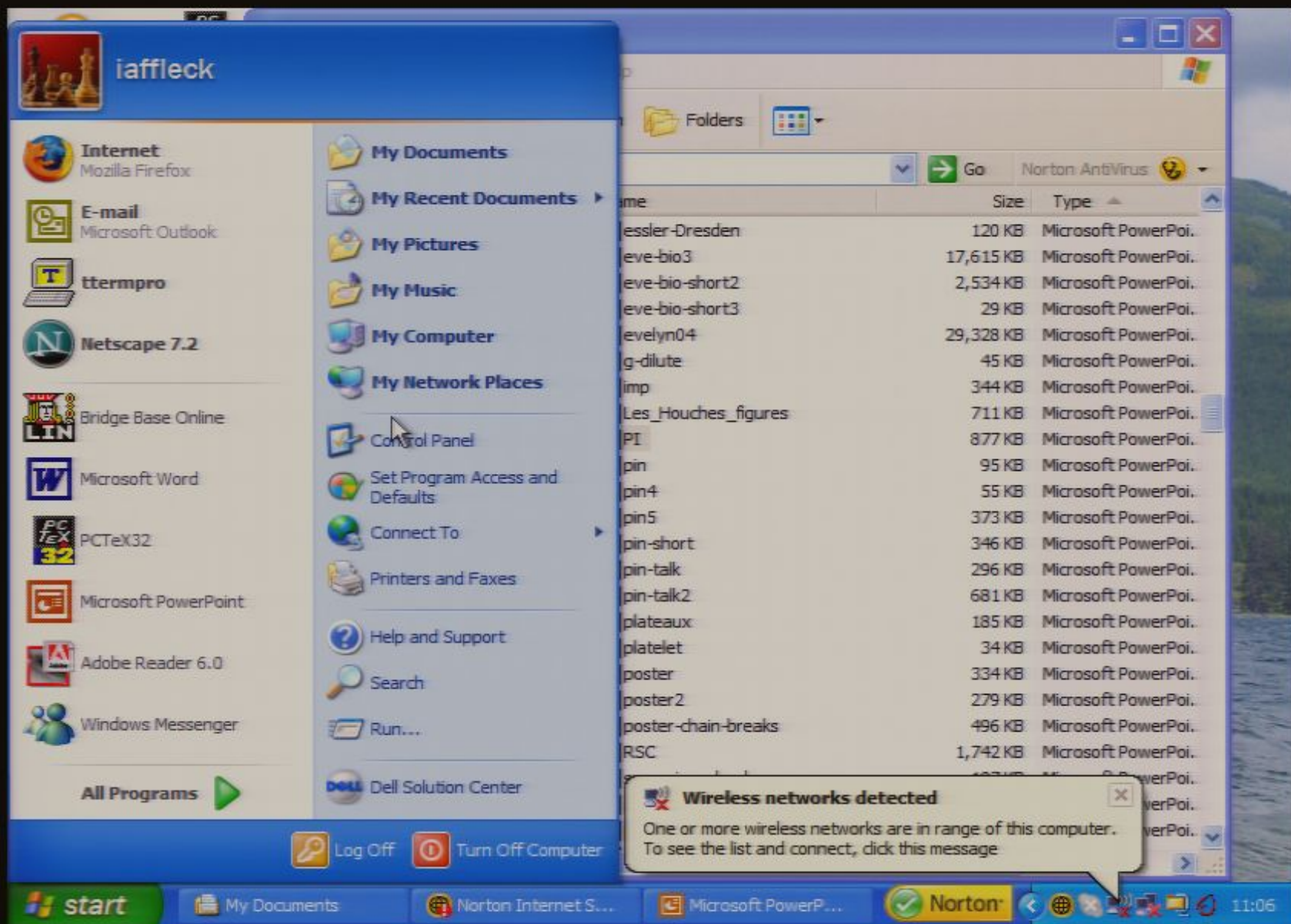
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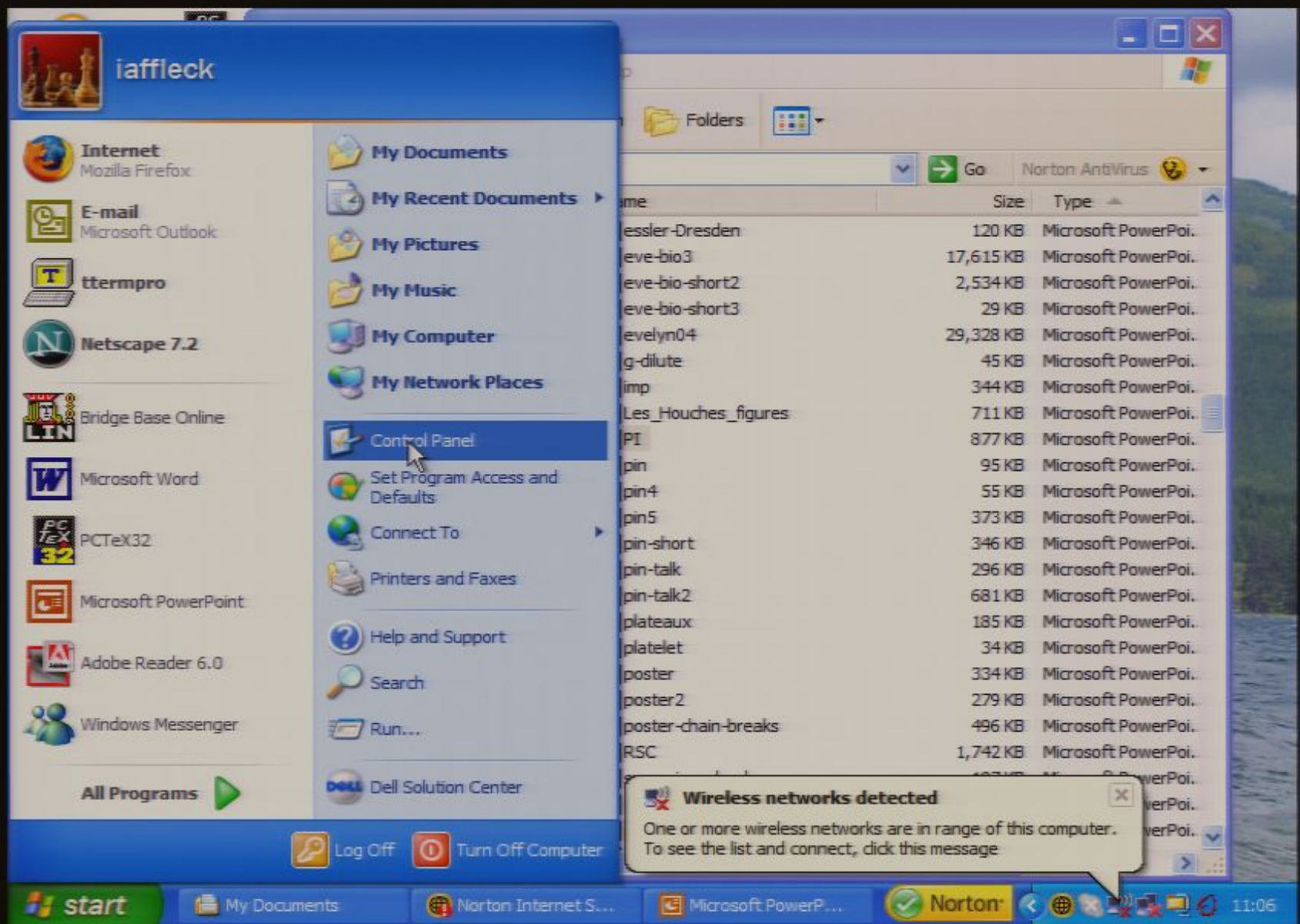
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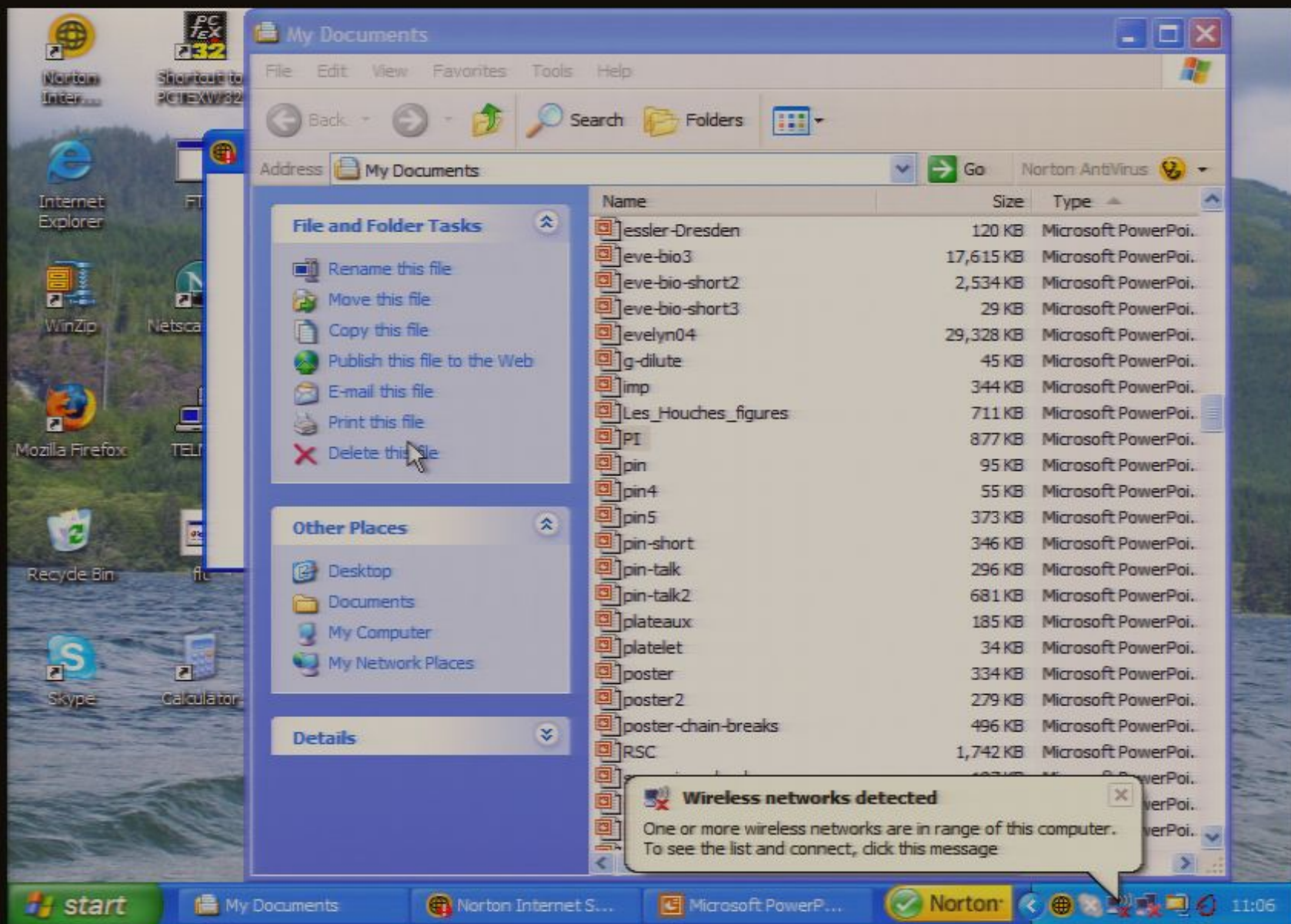
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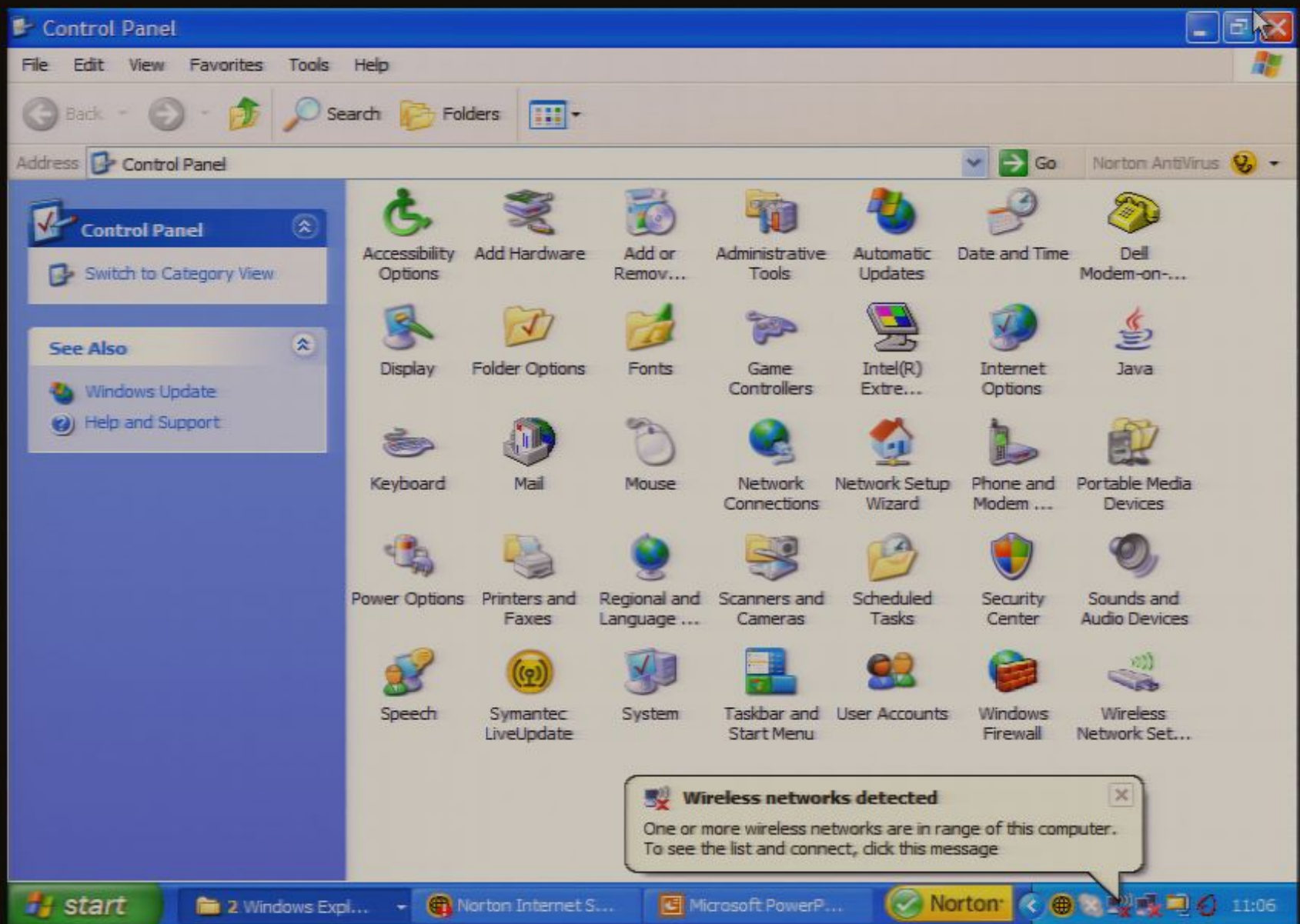


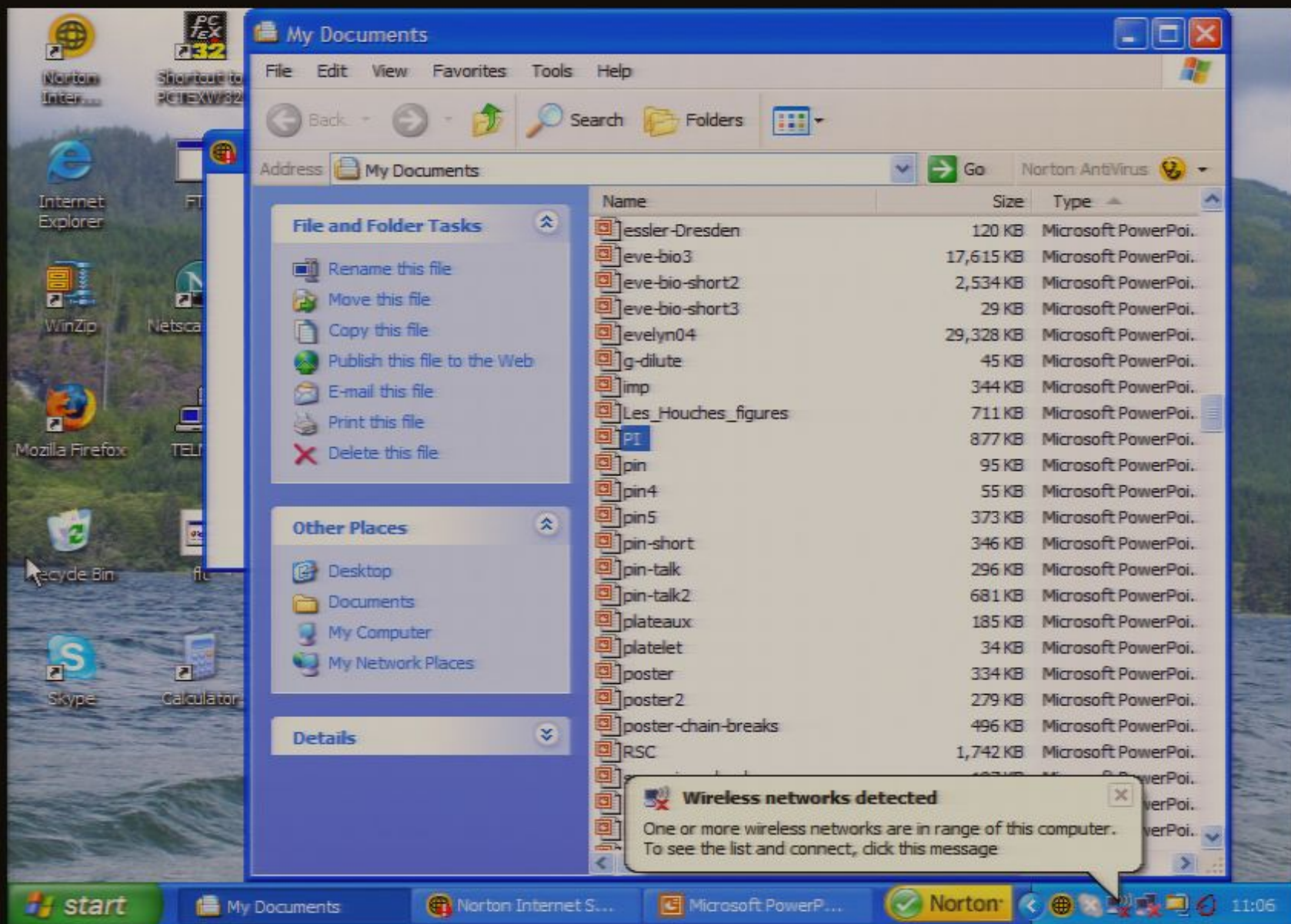


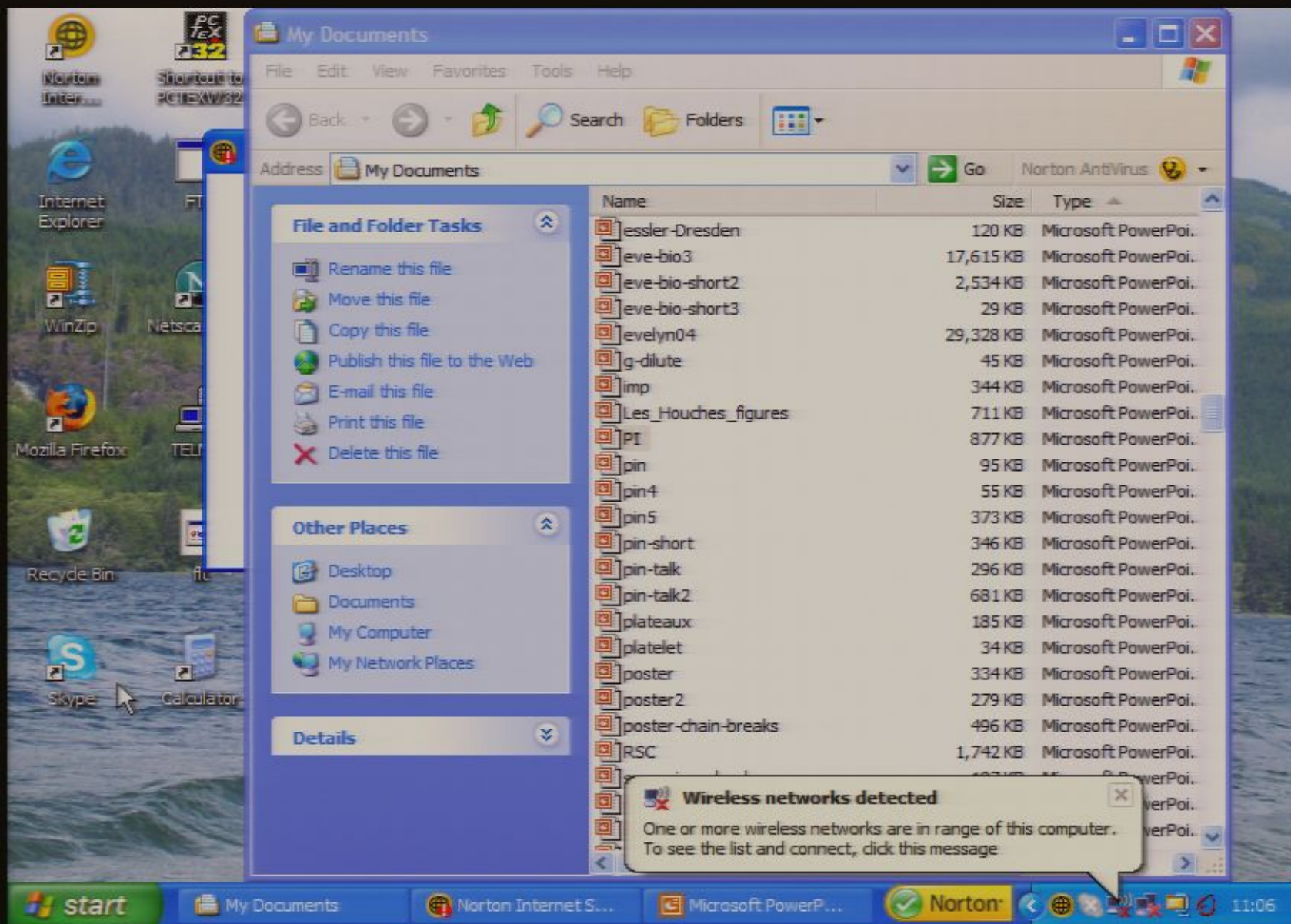


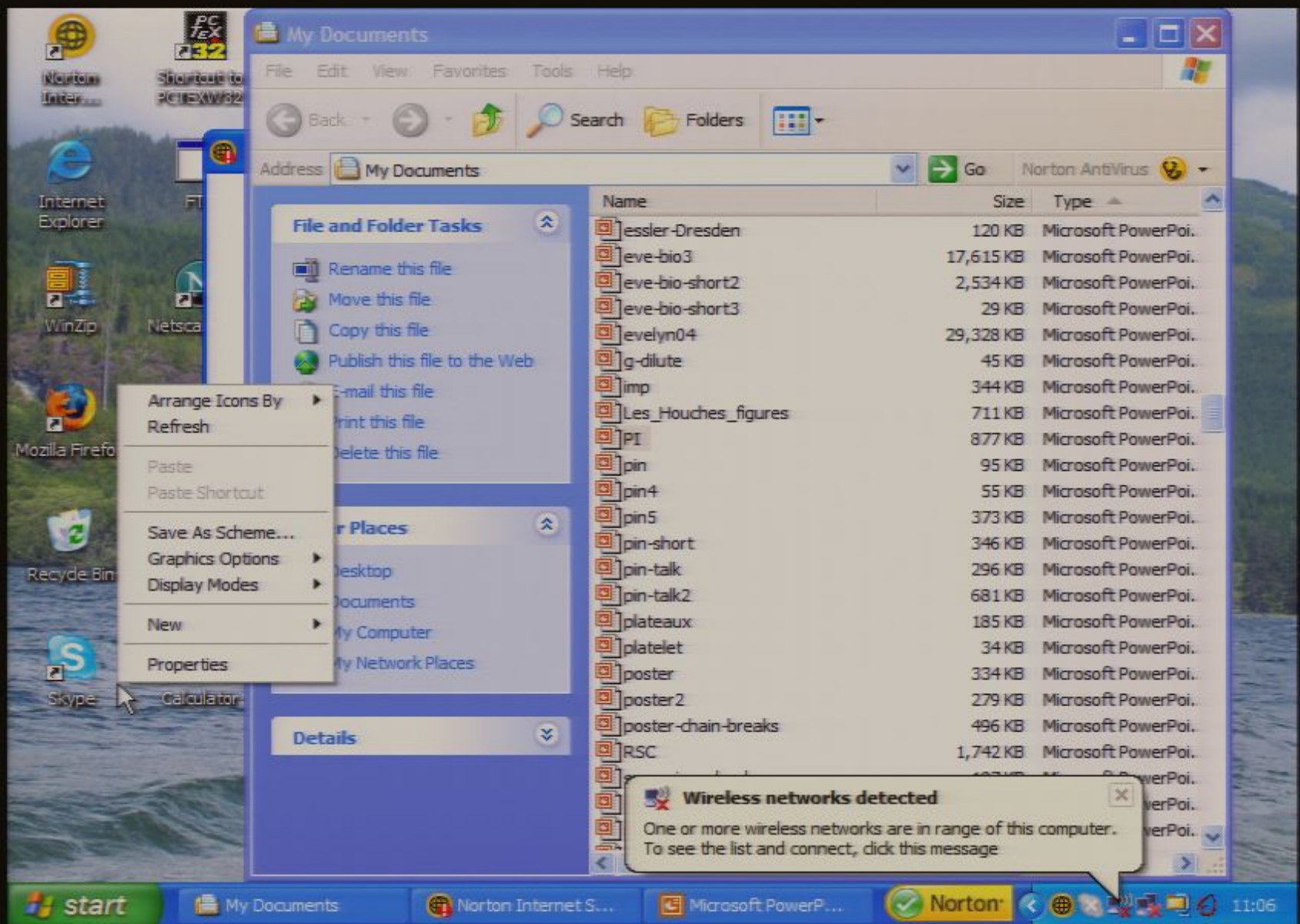


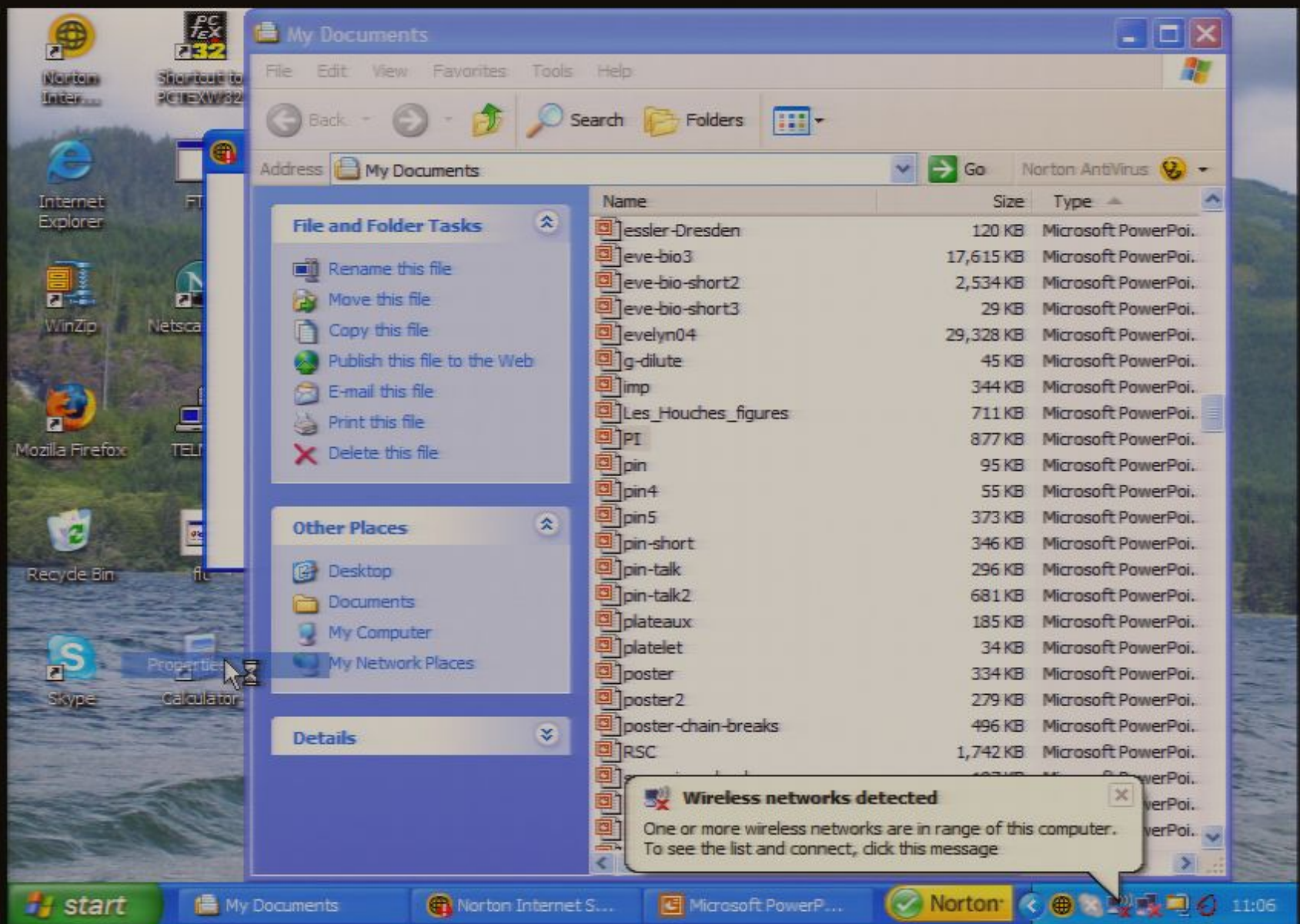


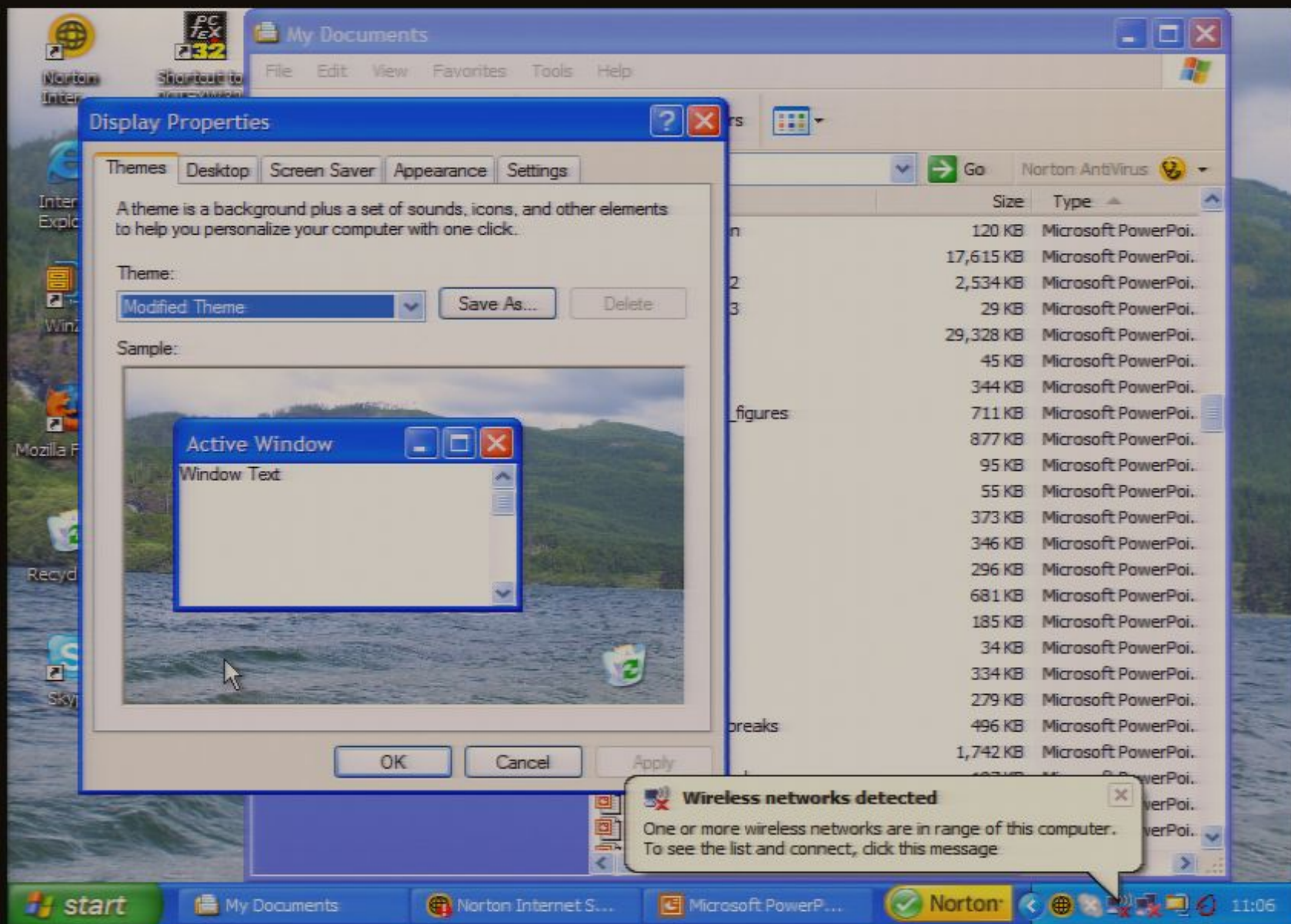


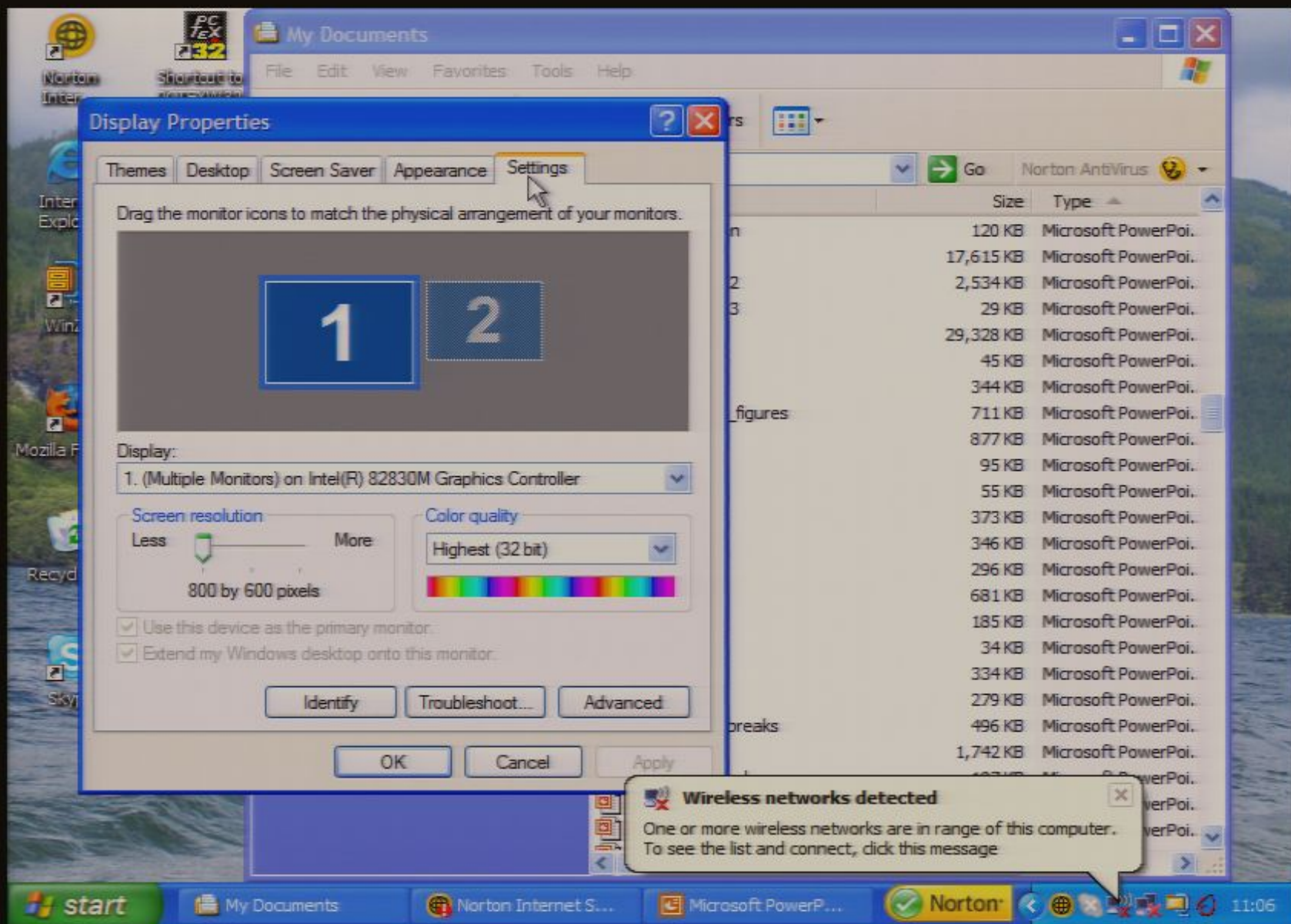


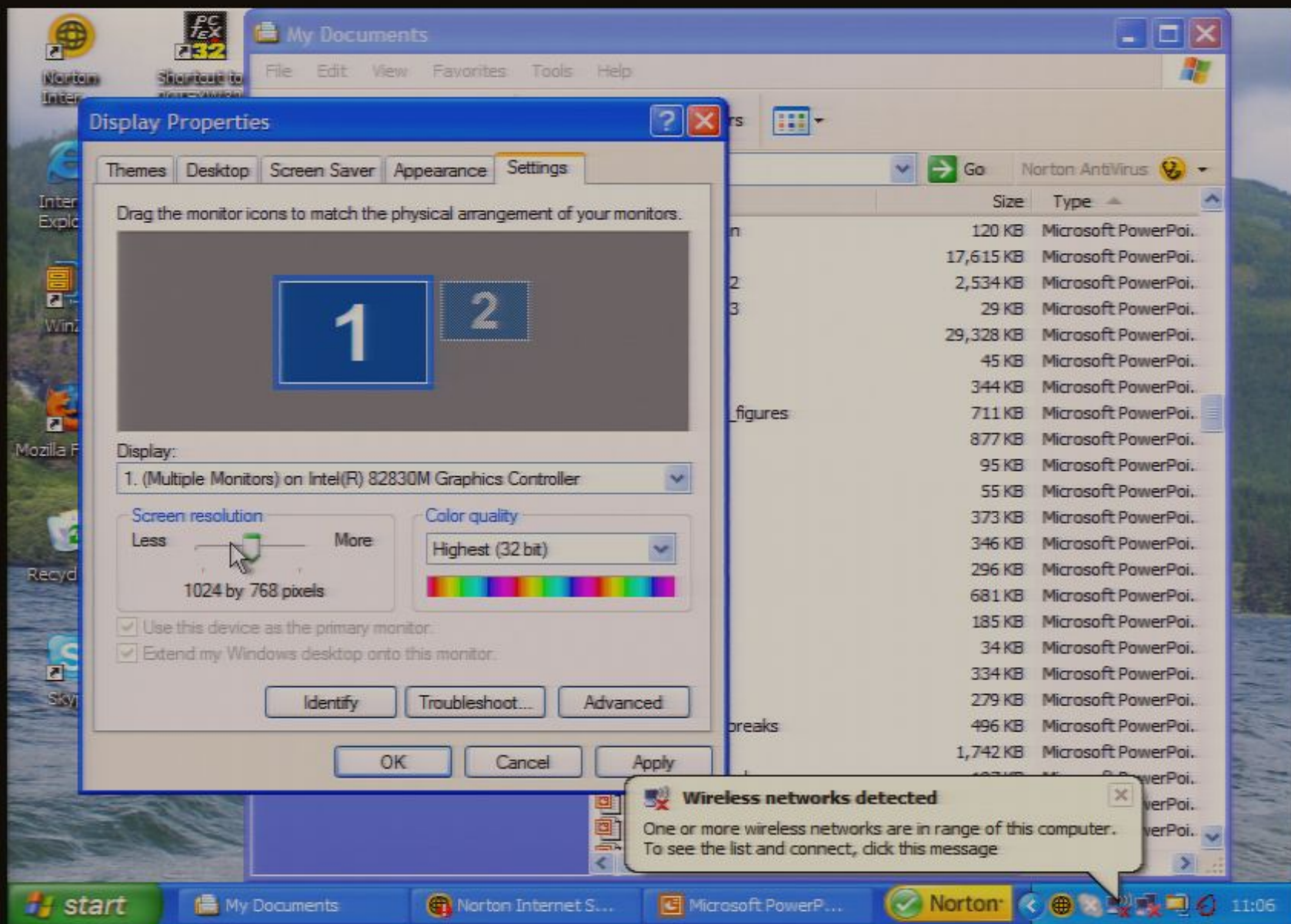


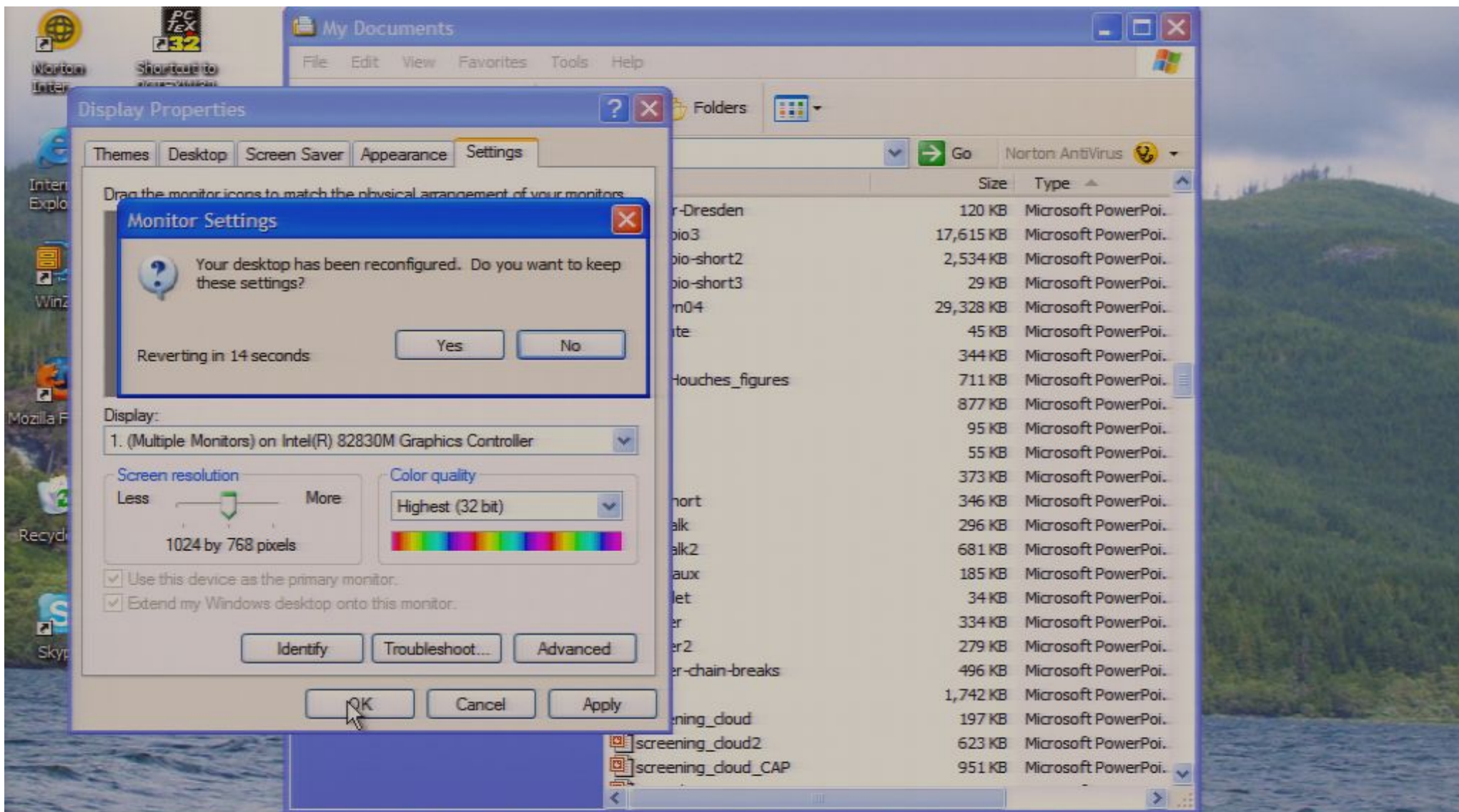


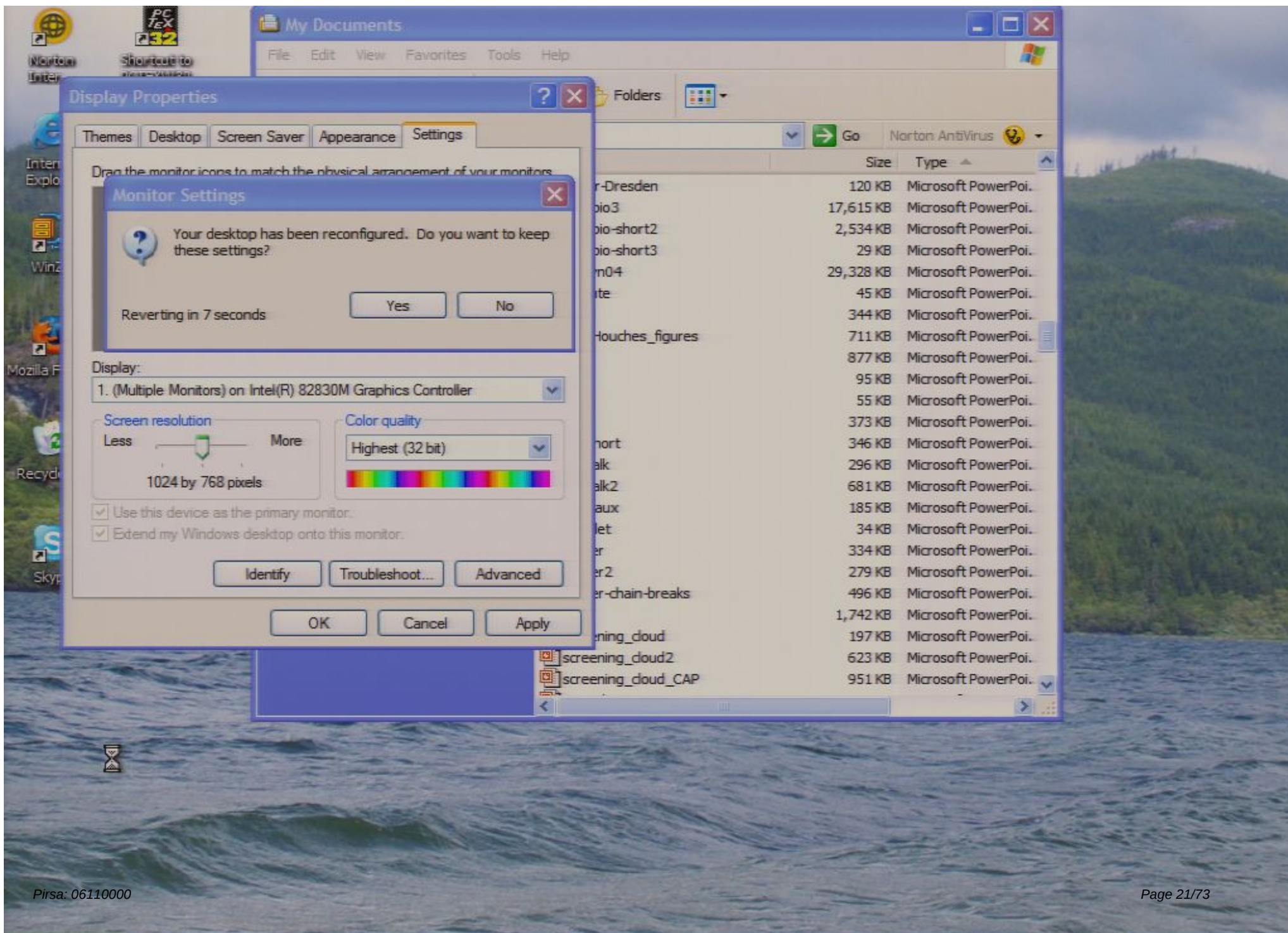


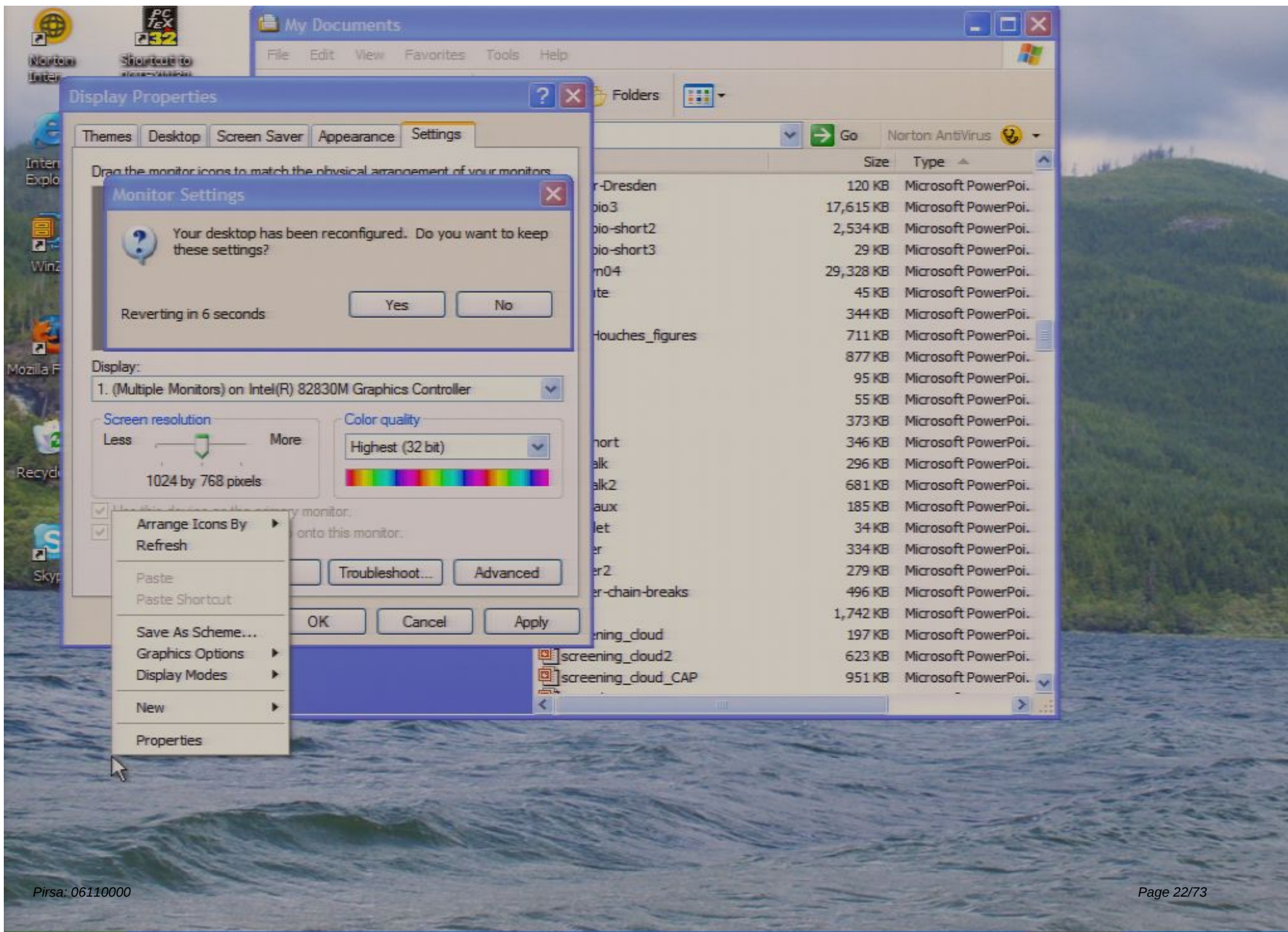


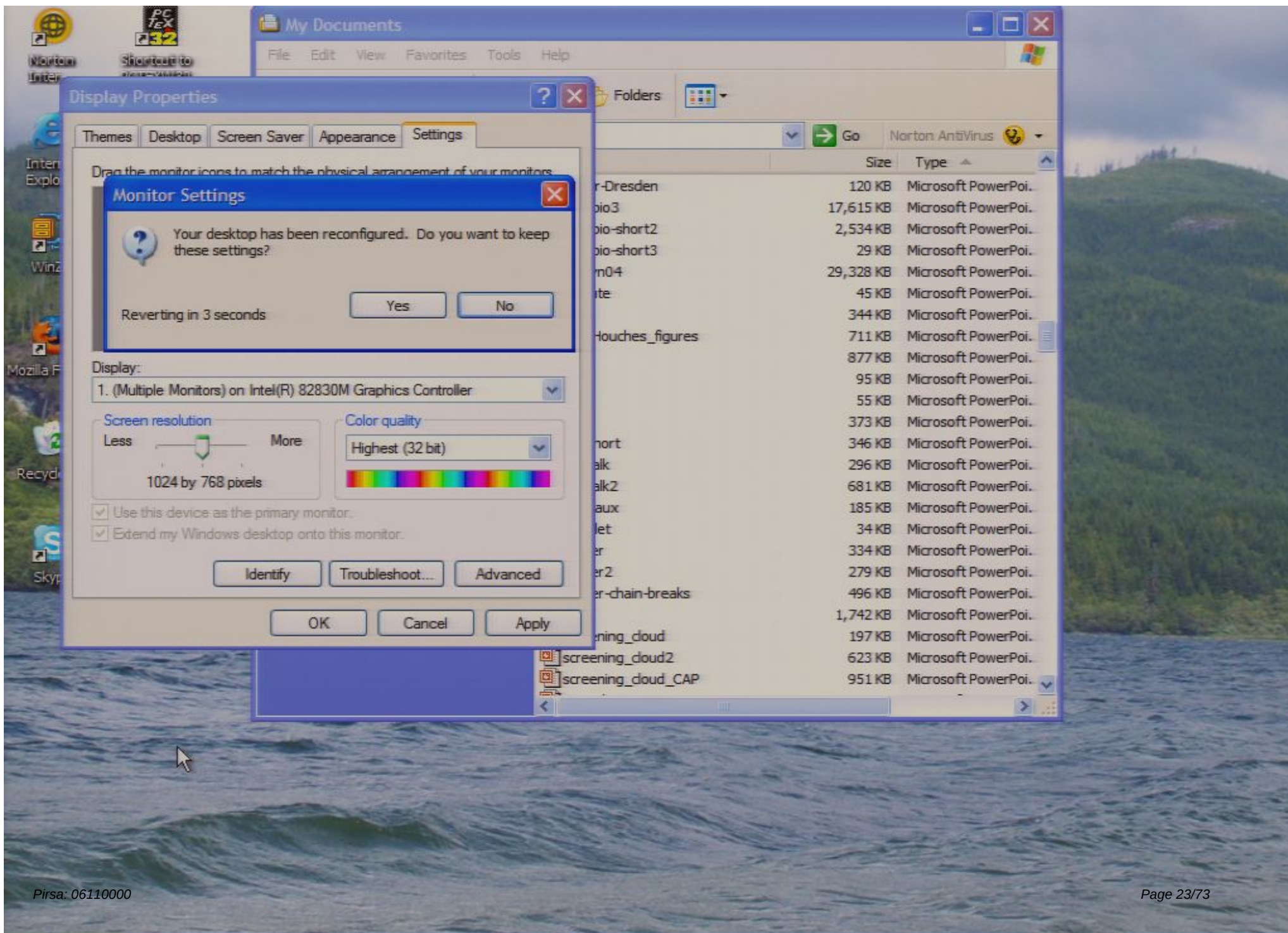


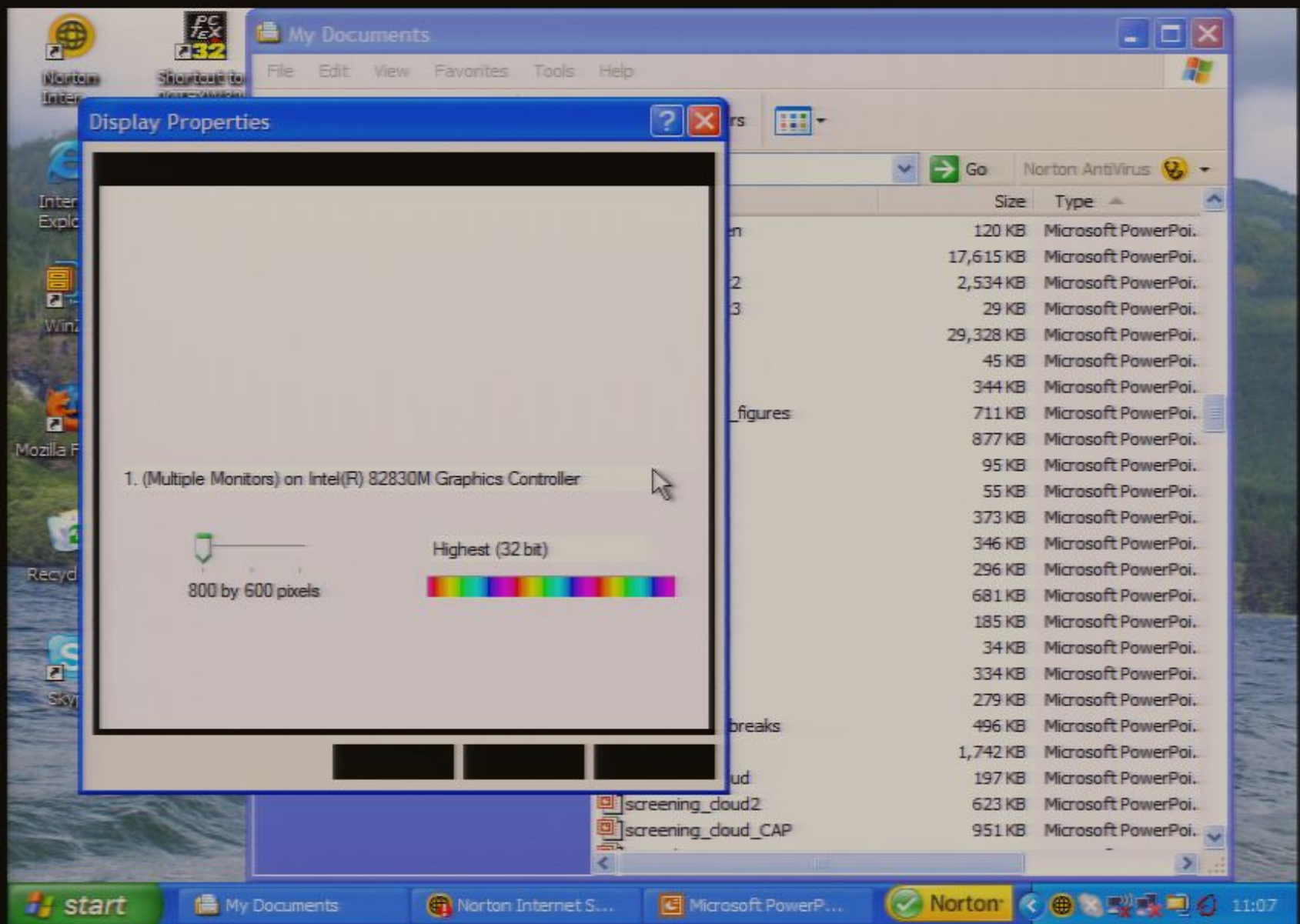


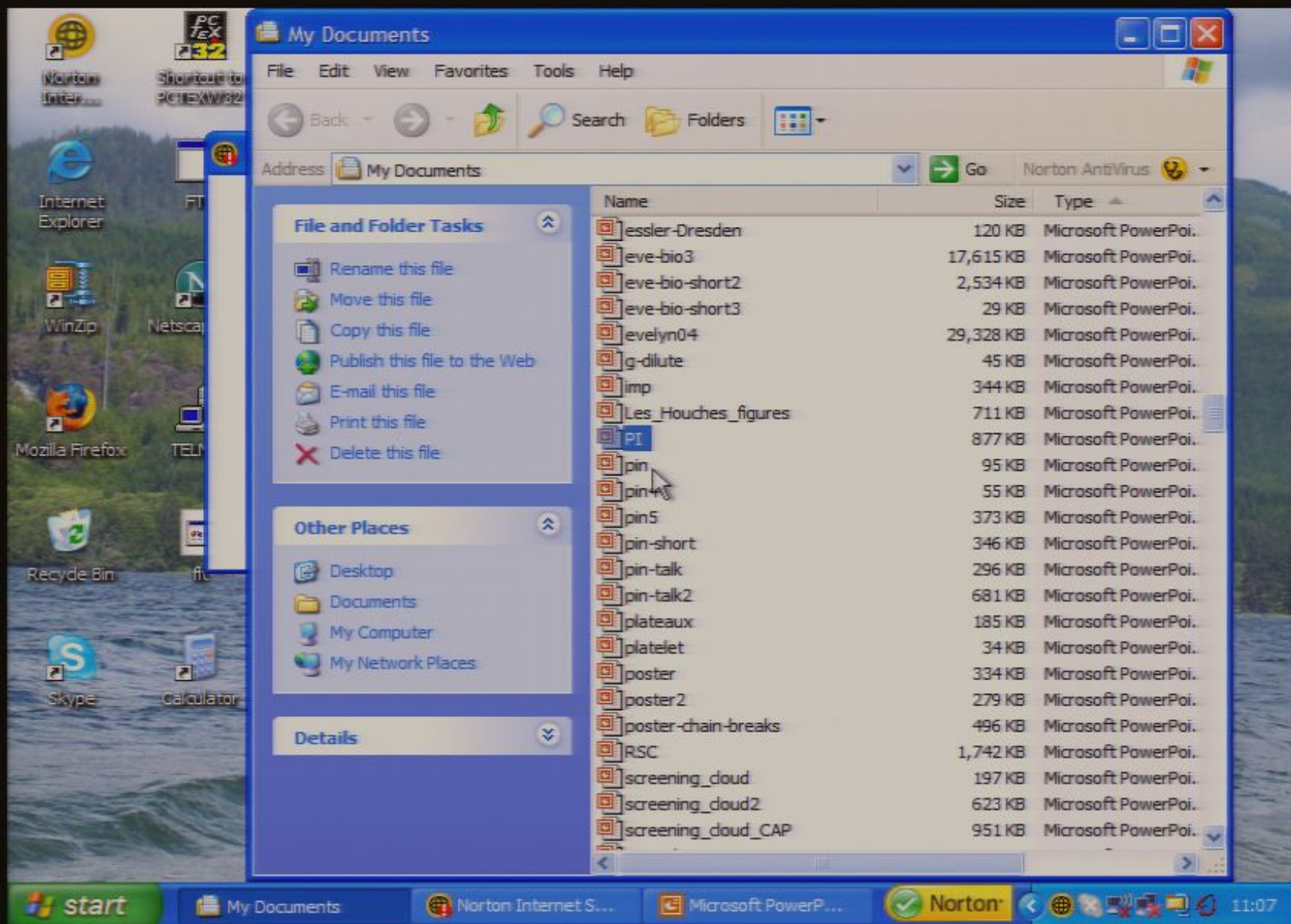












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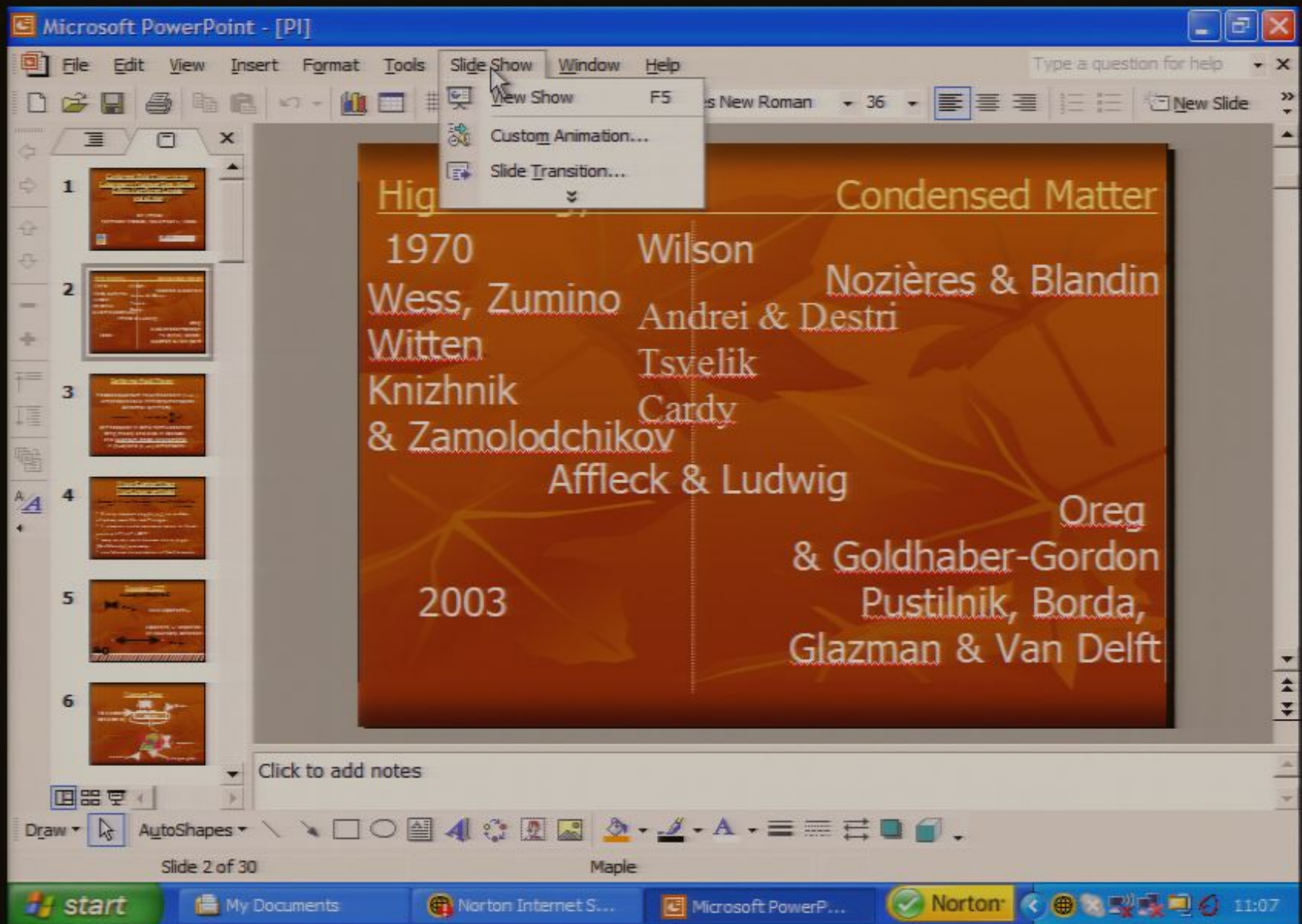
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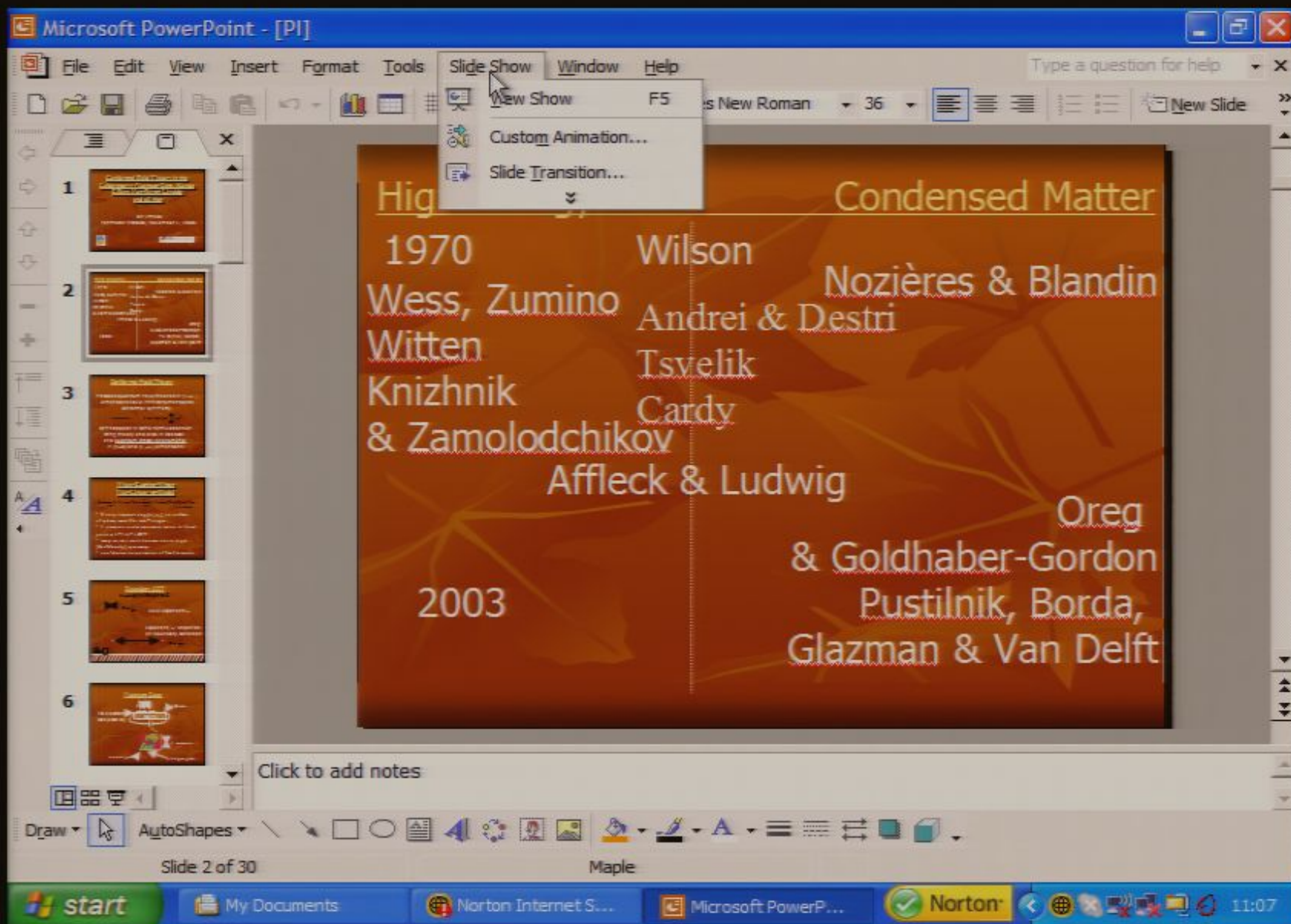
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Conformal Field Theory

massless quantum field theories in (1+1) dimensions have infinite-dimensional conformal symmetry

$$z \equiv \tau + ix \qquad z \rightarrow w(z) = \sum_{n=0}^{\infty} a_n z^n$$

CFT's appear in some formulations of string theory and also in classical and quantum critical phenomena in (2+0) and (1+1) dimensions

Wess-Zumino-Witten Non-Linear σ -Model

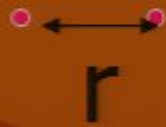
$$S_{WZW}^k = \frac{1}{8\pi\lambda} \int d^2x \operatorname{tr} \partial_\mu g^+ \partial_\mu g + \frac{k}{12\pi} \int d^3x \varepsilon_{\mu\nu\rho} \operatorname{tr} g^+ \partial_\mu g g^+ \partial_\nu g g^+ \partial_\rho g$$

$g \in SU(2)$

- S only depends on $g(x_1, x_2)$, on surface of sphere, $\text{mod } 2\pi i \Rightarrow k = \text{integer}$
- λ increases under renormalization to fixed point at $\lambda = 1/k$ – a CFT
- many exact results known due to high (Kac-Moody) symmetry
- non-abelian bosonization of free fermions

Boundary CFT

$z \rightarrow w(z)$ with $w(\tau) \in R$



$$G \propto \frac{1}{r^\eta}$$

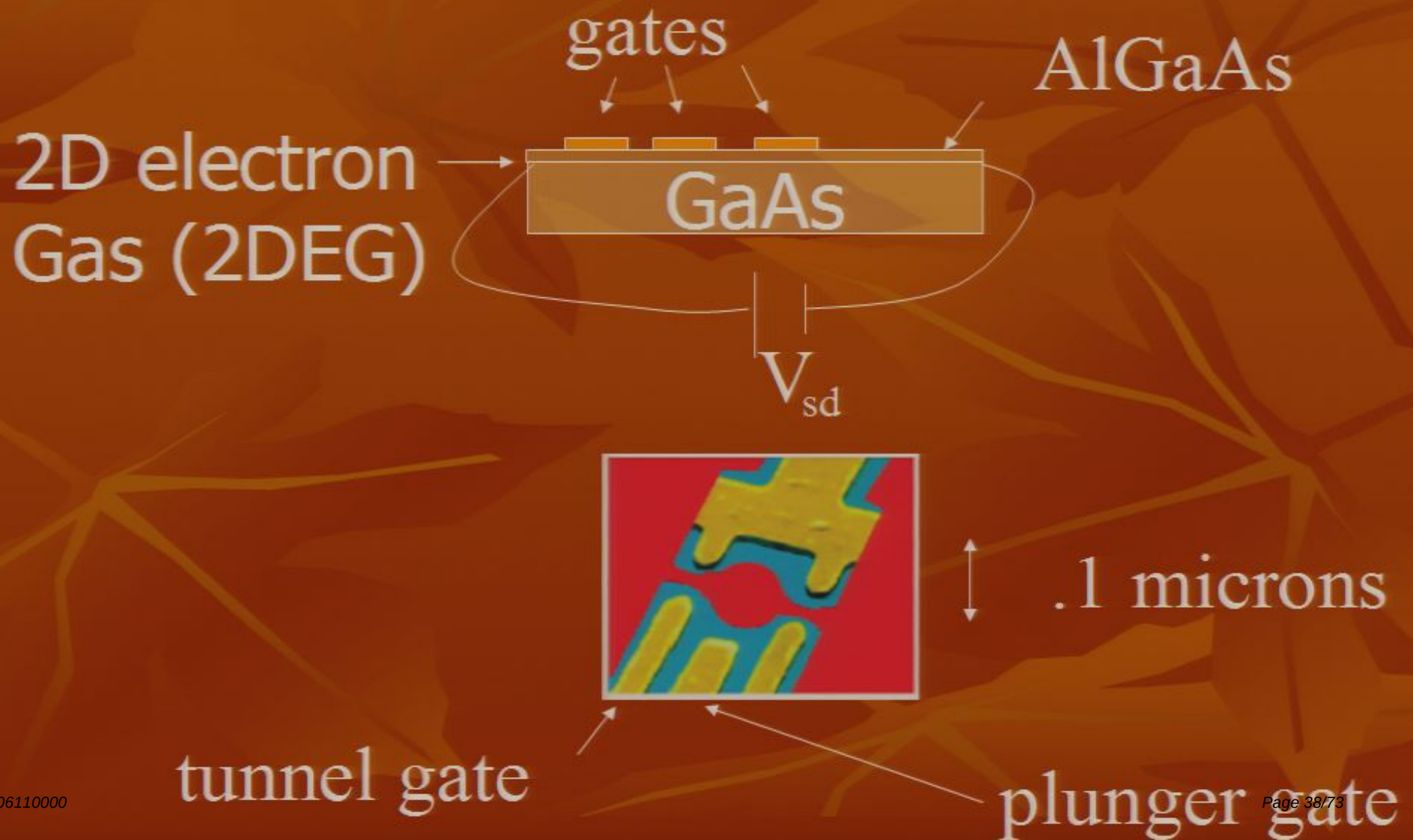
bulk exponent η

exponent, η' depends
on boundary condition



$$G \propto \frac{1}{r^{\eta'}}$$

Quantum Dots



- when dot is small and tunneling on/off dot is weak, dot contains a well-defined number of electrons, due to charging energy
- this number is controlled in single steps by plunger gate
- when number is odd, dot has spin-1/2
- at low energies, spin of quantum dot is its only active degree of freedom
- interaction of this spin with conduction electrons in 2DEG lead to interesting physics

Kondo Effect

$$S = \int d^2x d\tau \left[\psi^\dagger \left(\partial_\tau - \frac{1}{2m} \nabla^2 \right) \psi + J \delta^2(\vec{x}) \psi^\dagger \frac{\vec{\sigma}}{2} \psi \cdot \vec{S} \right]$$

- fermions (with spin) & $S=1/2$ impurity spin
- although interactions only occur at origin, this model exhibits non-trivial many body effects!
- Kondo coupling, J , grows large under renormalization at low energies (for $J>0$)
- first (1965) example of asymptotic freedom

effective coupling becomes large at energy scale T_K : $T_K = De^{-(1/J\rho)}$

where ρ is density of states, D is band width (ultra-violet cut-off)

- T_K is like Λ_{QCD} in high energy physics
- strong coupling physics is very simple:
 - one electron forms a singlet with impurity spin (remember it has $S=1/2$)
 - other electrons must “stay away” or go into orthogonal wave-functions due to Pauli principle

- screening electron is from s-wave channel
- strong coupling fixed point described by a $\pi/2$ s-wave phase shift, corresponding to an infinite short-range repulsion at origin

• this is clear if we introduce a lattice: electrons live on lattice sites only and hop between sites with amplitude t



- if $J \gg t$ we solve single site problem first
- 1 electron sits at the origin and forms singlet
- other electrons avoid origin, otherwise free

for small T_K , we may linearize the dispersion relation: $\vec{k}^2 / 2m - \varepsilon_F \approx v_F(k - k_F)$ and use a quantum field theory which is Lorentz invariant when $J=0$:

$$S = \int d\tau \int_0^\infty dx \left[\psi_L^+ (\partial_\tau - i\partial_x) \psi_L + \psi_R^+ (\partial_\tau + i\partial_x) \psi_R \right] \\ + J \int d\tau \psi_L^+(\tau, 0) \frac{\vec{\sigma}}{2} \psi_L(\tau, 0) \cdot \vec{S}, \quad \psi_L(\tau, 0) = -\psi_R(\tau, 0)$$

strong coupling fixed point corresponds to:

$$\psi_L(\tau, 0) = +\psi_R(\tau, 0)$$

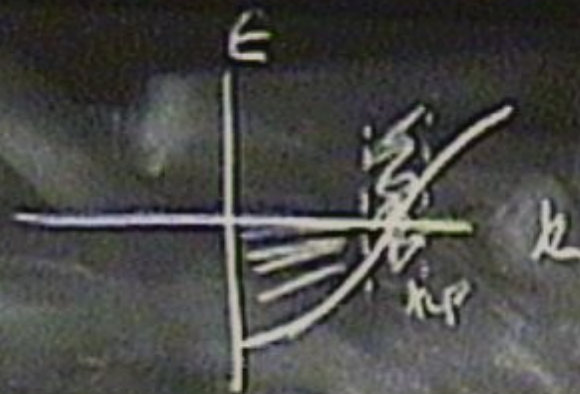
- this is a simple example of how interactions with a quantum impurity renormalize to a fixed point corresponding to a conformally invariant boundary condition
- we can use non-abelian bosonization to replace free fermions (with spin) by a
- ($k=1$) WZW model for spin degrees of freedom and a free boson model for charge degrees of freedom

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$$|k - k_F| < 1.$$

$$\frac{E}{\hbar} = A + \chi$$

$$\frac{E}{\hbar} = A - 2\chi$$

$$- \left[\frac{1}{2} \sum_i \bar{\chi}_i \chi_i \phi \chi_i + \text{c.c.} \right]$$

$$= \underbrace{-A}_{\text{Canonical form}} - \left[\frac{1}{2} \sum_i \left[\bar{\chi}_i (\phi + \mu_i) \chi_i \right] \right]$$

$$(\phi + \mu) \chi$$

$$(i\phi + \mu)$$

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$$S = S_0(\varphi) + S_{WZW}^{k=1}(g) + \lambda \int d\tau \vec{J}_L(\tau, 0) \cdot \vec{S}(\tau), \quad \vec{J}_L = \text{tr } g^+ \partial_- g \vec{\sigma}, \quad \lambda \prec J$$

apart from modified boundary condition,
strong coupling fixed point is also
characterized by a leading irrelevant

interaction: $S_{\text{int}} = S_{WZW}^{k=1}(g) - \frac{1}{T_K} \int d\tau [\vec{J}_L(\tau, 0)]^2$

- N.B. screened impurity spin doesn't appear
- single power of $1/T_K$ follows from fact that operator has dimension 2
- at low temperature, $T \ll T_K$, we can do perturbation theory in T/T_K

Kondo interaction between spin of quantum dot (when number of electrons is odd) and conduction electrons in 2DEG:

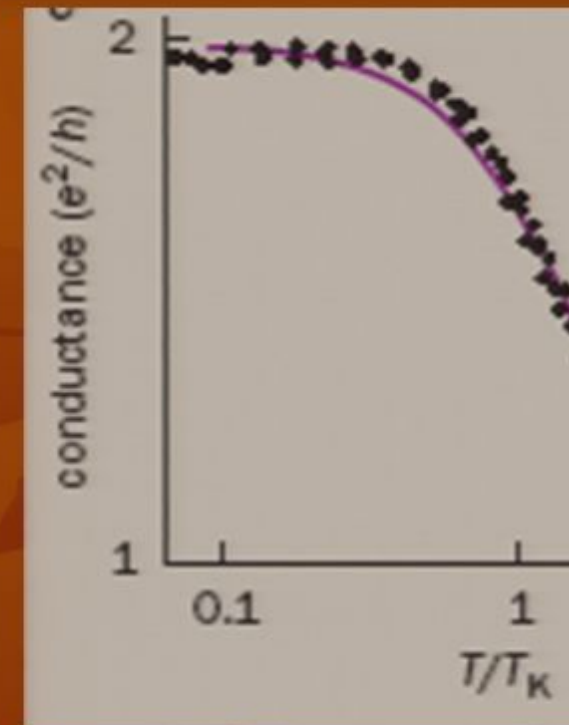
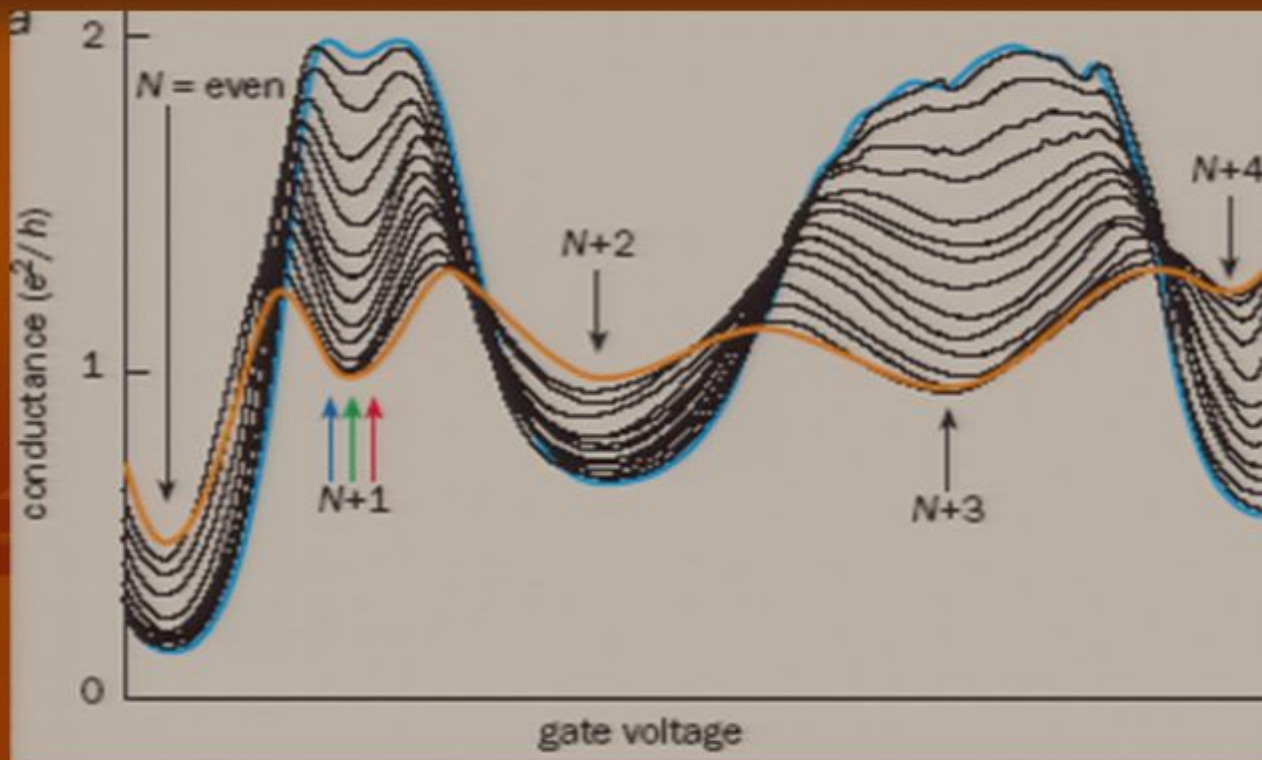
$$S = S_{0,1} + S_{0,2} + J[\psi^{\dagger 1}(0_-) + \psi^{\dagger 2}(0_+)]\vec{\sigma}[\psi^1(0_-) + \psi^2(0_+)] \cdot \vec{S}$$



Kondo interaction transmits electrons across
Quantum dot

- conductance gets *enhanced* at low temperature by growth of effective Kondo coupling
- electrons are transported thru quantum dot in p-wave channel with no resistance at $T \rightarrow 0$
- $I = G(T) \Delta V$
- $G = 2e^2/h$ for a dot which conducts perfectly in 1 channel (only) – Landauer formula
- for a small quantum dot, conductance is very small except at $T \ll T_K$ where:

$$G \rightarrow 2e^2/h [1 - c(T/T_K)^2]$$
(2nd order in perturbation theory in $1/T_K$)



Coulomb blockade suppresses conductance except for transition gate voltages where 2 electron numbers on dot have same energy and on plateaus where N is odd, at $T \ll T_K$

- this is a simple example of a much more general, and interesting, phenomena (IA, AL)
- very general interactions with various types of quantum impurities lead to low energy behaviour which is characterized by generalized conformally invariant “boundary conditions”
- in general, these boundary conditions are *not* free particle like – they encode interactions at the boundary

- non-interacting boundary conditions correspond to “Fermi liquid” behaviour in usual condensed matter terminology
- interacting boundary conditions give “non-Fermi liquid” behaviour
- models can be solved by Cardy’s “boundary state” formalism



$$Z_{AB} = \text{tr} \exp[-\beta H_{AB}^L] = \langle A | \exp[-LH_P^\beta] | B \rangle$$

finite temperature, $T=1/\beta$, finite size, L ,
 partition function with boundary conditions
 A, B is equivalent to propagation for time L
 between initial and final *boundary states* $|A\rangle$
 and $|B\rangle$ with Hamiltonian with periodic b.c.
 on circle of circumference β

one of simplest physical examples is provided by 2-channel Kondo model

$$S = \int d^2x d\tau \left[\psi^{a+} \left(i\partial_\tau - \frac{1}{2m} \nabla^2 \right) \psi_a + J \delta^2(\vec{x}) \psi^{a+} \frac{\vec{\sigma}}{2} \psi_a \cdot \vec{S} \right]$$

- here channel index, a , is summed over 2 values as is spin index (not written)
- J renormalizes towards strong coupling at low energies, as before
- but now strong coupling fixed point is unstable!

easily seen from lattice model:

- if we assume $J \gg t$ and solve single site model, impurity is overscreened
- i.e. 1 electron from each channel sits at origin in a symmetric $S=1$ configuration
- ground state of electron-spin system has $S=1/2$



- now low energy effective Hamiltonian still contains an effective $S=1/2$ impurity, and therefore a Kondo interaction: $J' \sim 1/J^2$
- J' gets stronger under renormalization implying that a very large J gets weaker



- intermediate coupling stable fixed point is described by a non-trivial conformally invariant boundary condition

- we can again map low energy effective action into a Lorentz invariant model
- interacting part is same as before except now we get WZW model with topological coupling constant $k=2$

$$S = S_{WZW}^{k=2}(g) + \lambda \int \vec{J}_L(\tau, 0) \cdot \vec{S}(\tau)$$

we identified stable fixed point with a particular non-trivial boundary condition by a phenomenological argument and checked our conjecture against numerical and Bethe ansatz results

- important consequence of our solution is that leading irrelevant operator has dimension $3/2$ (1^{st} descendent of $s=1$ primary)

$$S_{\text{int}} = S_{WZW}^{k=2}(g) + \frac{1}{\sqrt{T_K}} \int d\tau \vec{J}_{-1} \cdot \vec{\phi}_{s=1}(\tau, 0)$$

- now perturbation theory gives a series in $(T/T_K)^{1/2}$, rather than T/T_K
- again we can calculate 1^{st} order terms although calculations are rather different and more complicated

Experimental Realization

- a variety of different experiments have been interpreted in terms of 2 channel Kondo model
- have all remained controversial until recent (unpublished) quantum dot experiments by D. Goldhaber-Gordon's group at Stanford

- non-Fermi liquid fixed point is unstable against channel anisotropy:

$$S = \int d^2x d\tau \left[\psi^{a+} \left(i\partial_\tau - \frac{1}{2m} \nabla^2 \right) \psi_a + \sum_a J_a \delta^2(\vec{x}) \psi^{a+} \frac{\vec{\sigma}}{2} \psi_a \cdot \vec{S} \right]$$

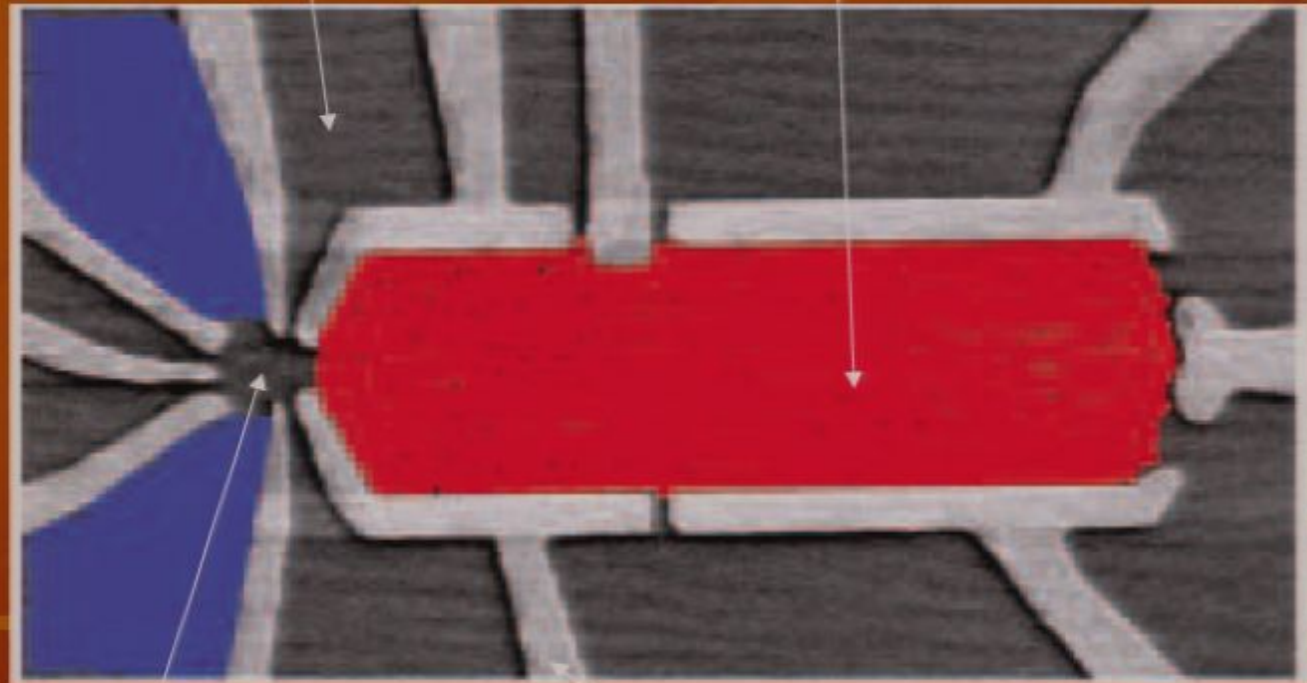
if $J_1 > J_2$ then channel 1 screens spin and channel 2 decouples at low energies yielding a Fermi liquid fixed point:

$$\psi_{L1}(\tau, 0) = +\psi_{R1}(\tau, 0), \quad \psi_{L2}(\tau, 0) = -\psi_{R2}(\tau, 0)$$

- very difficult to make $J_1 = J_2$, while avoiding magnetic fields and other destructive effects

1st channel

2nd channel



quantum dot

V_c controls J_2

- complicated device has many knobs

- can tune to non-Fermi liquid fixed point

- lowest order perturbation theory in irrelevant interaction now gives a conductance:

$$G=(1/2)G_0 [1-(T/T_K)^{1/2}]$$

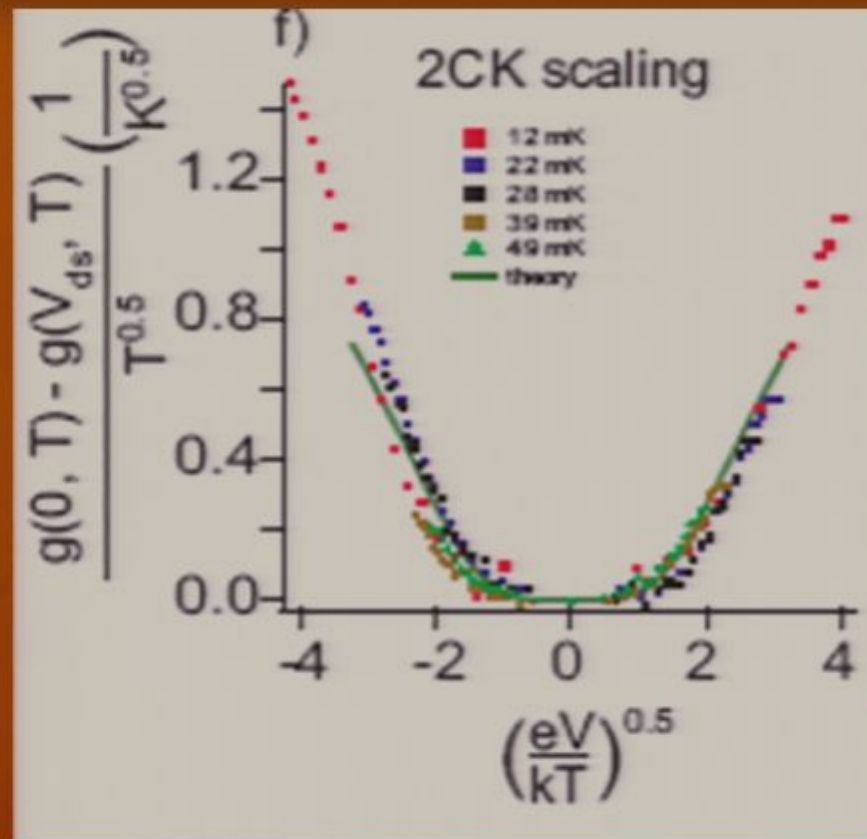
- $T=0$ value is $1/2$ as big as for 1-channel case and T -dependence has exponent $1/2$ instead of 2

for finite bias voltage:

$$dI/dV = G_0[1-(T/T_K)^2 f(eV/T)] \text{ (for FL)}$$

$$dI/dV = (1/2)G_0[1-(T/T_K)^{1/2} h(eV/T)] \text{ (NFL)}$$

- scaling functions $f(x)$ and $h(x)$ are known in certain limits



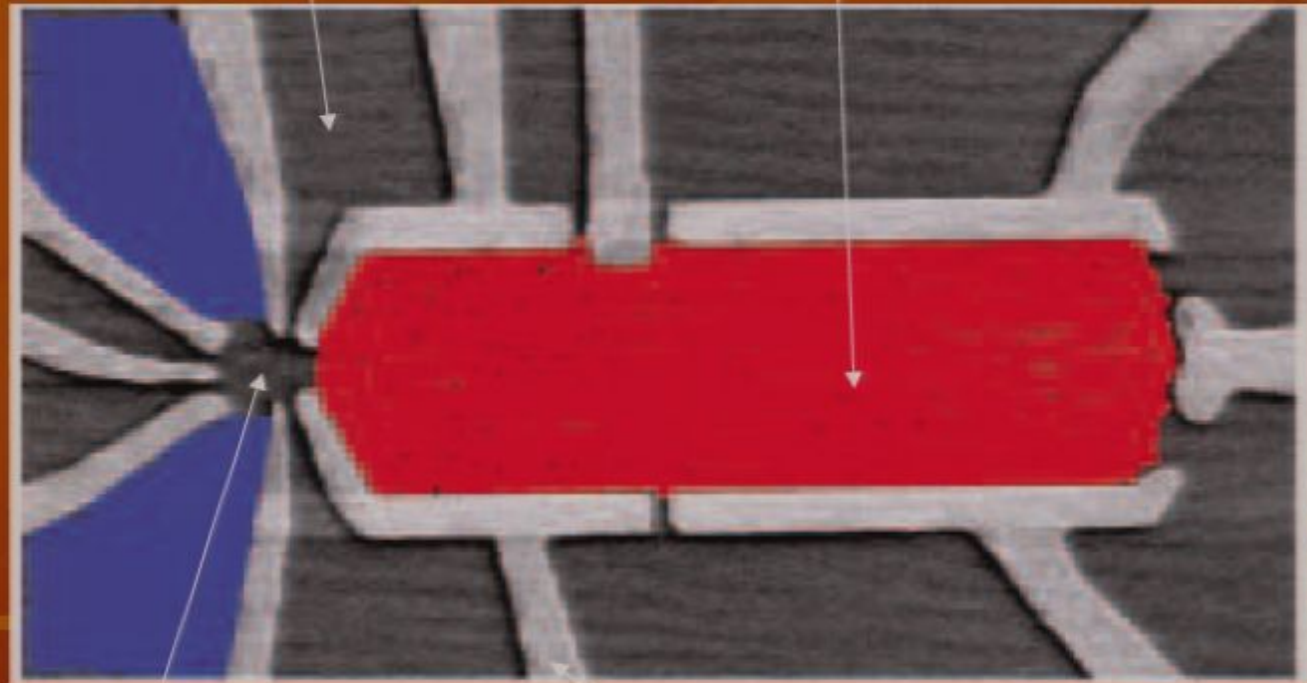
- conductance measurement, versus T and bias voltage, V , with all parameters tuned to non-Fermi liquid fixed point exhibits expected scaling (note $T^{0.5}$)

Conclusions

- boundary CFT describes quantum impurity models of condensed matter physics
- 2-channel Kondo, $k=2$ WZW, model has a non Fermi liquid boundary condition
- solution of this model by CFT methods leads to predictions verified by quantum dot experiments
- CFT may or may not be part of a “theory of everything” but we now know it is definitely a theory of something 😊

1st channel

2nd channel



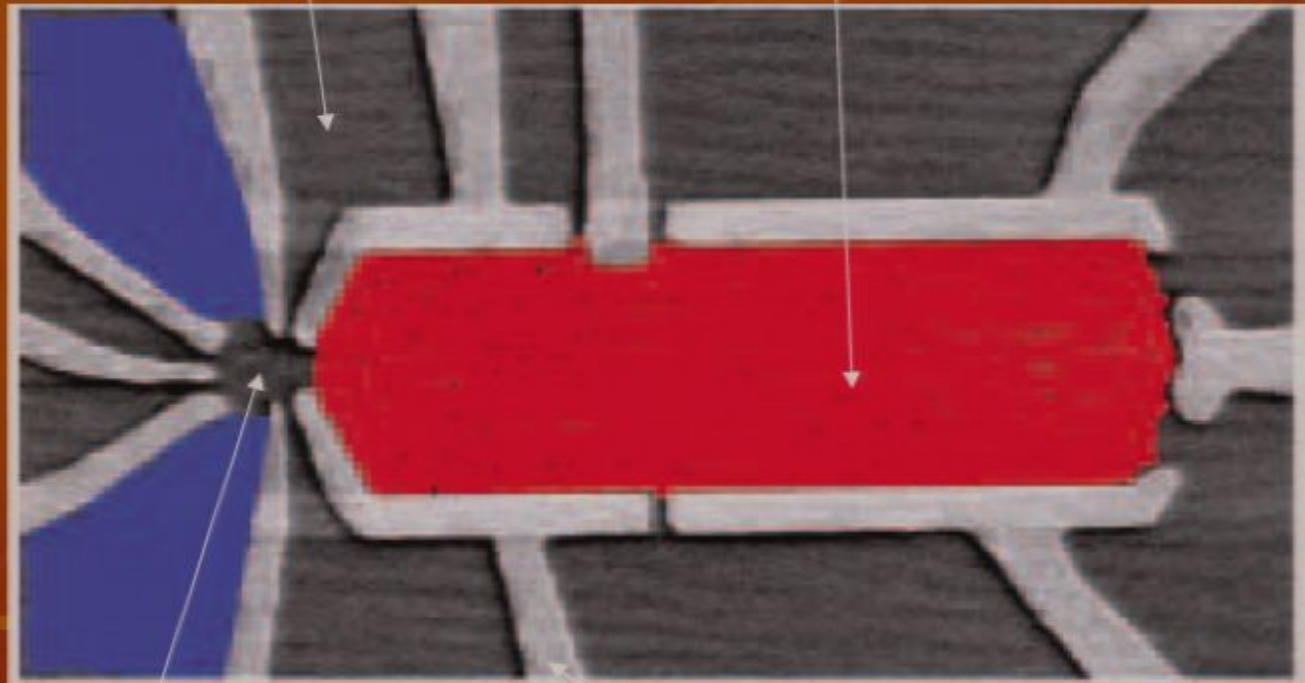
quantum dot

V_c controls J_2

- complicated device has many knobs
- can tune to non-Fermi liquid fixed point

1st channel

2nd channel



quantum dot

V_c controls J_2

- complicated device has many knobs

- can tune to non-Fermi liquid fixed point

$$S = \int d^4x \left(\bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 \right)$$

$$\mathcal{L} = A - 2X$$

$$- \left[\frac{1}{2} \sum_i \bar{\chi}_i \gamma^\mu \partial_\mu \chi_i + \text{c.c.} \right]$$

$$- A - \left[\frac{1}{2} \sum_i \left[\bar{\chi}_i (\not{\partial} + m_i) \chi_i \right] \right] \quad \begin{aligned} (\not{\partial} + m) \chi &= 0 \\ (i\not{\partial} + m) u &= 0 \end{aligned}$$

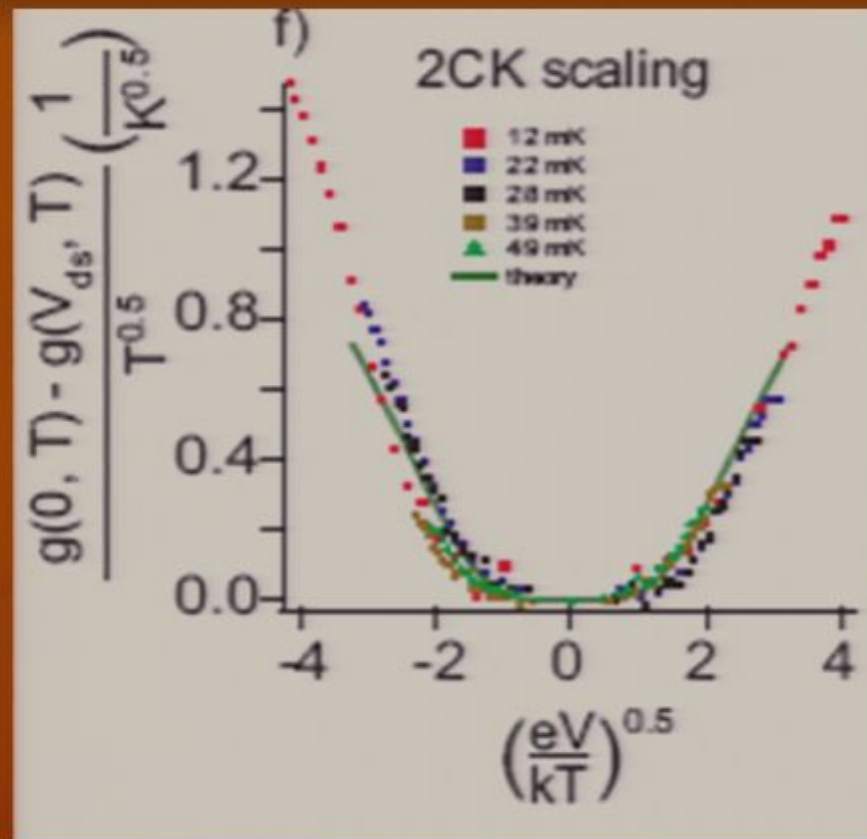
Canonical form

$$S \rightarrow \underbrace{0}_{\text{base}} \cdot \left(D \underbrace{(4_{-1} + 4_n)}_{\text{1012}} + \frac{1}{5} (4_{-1} + 4_n) \cdot \bar{S} \right)$$

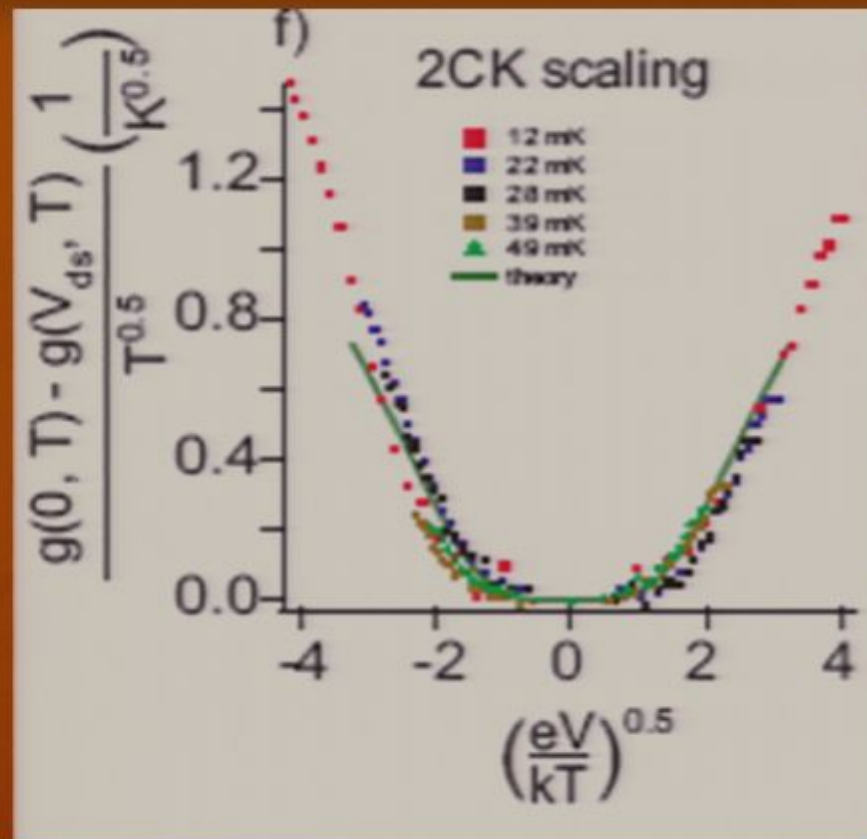
$\bar{\chi} = A - 2\chi$

$$- \left[\frac{1}{2} \sum_i \bar{\chi}^i \gamma_i \not{\partial} \chi^i + \text{c.c.} \right] \quad (\not{\partial} + \mu) \chi = 0$$

$$- A - \left[\frac{1}{2} \sum_i [\bar{\chi}^i (\not{\partial} + \mu_i) \chi^i] \right] \quad (\not{\partial} + \mu_i) u = 0$$
 Canonical form



- conductance measurement, versus T and bias voltage, V , with all parameters tuned to non-Fermi liquid fixed point exhibits expected scaling (note $T^{0.5}$)



- conductance measurement, versus T and bias voltage, V , with all parameters tuned to non-Fermi liquid fixed point exhibits expected scaling (note $T^{0.5}$)